On the Optimality of Transparency

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August 24, 2022

Research question

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  - S and R's payoffs depend on state and action
- **Question:** When is full disclosure ex-ante optimal for S?

Literature (optimal info design with a "large" state space)

- Dworczak and Kolotilin (2019), Dworczak and Martini (2019), Gentzkow and Kamenica (2016), Kolotilin (2018), Kolotilin et al. (2021), Arieli et al. (2020)
    - *Indirect utility* of the sender depends only on the expected state (perhaps on other moments of posterior distribution)
- Dworczak and Martini (2019), Kolotilin (2018), Kolotilin et al. (2020)
    - Optimality of full disclosure, conditions in terms of sender's indirect utility
- Mensch (2021)
    - Conditions for full disclosure jointly on receiver's utility function and a transformation of sender's utility function that takes into account the incentive compatibility constraint of the receiver ("virtual utility")
- **Our paper:**
    - Very general assumptions on utilities
    - Conditions directly on "primitives of the model", i.e., shapes of the parties' utility functions, easy to check

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- Disclosure rule is common knowledge
- Assumption: $U(\omega, a)$ is twice continuously differentiable and strictly concave in a.

Receiver's problem

- Message m generates posterior beliefs $\pi(\omega|m)$

$$\max_a E(U(\omega, a)|m)$$

- Assume, for each $\pi(\omega|m)$, FOC has a unique finite solution $a^*(m)$:

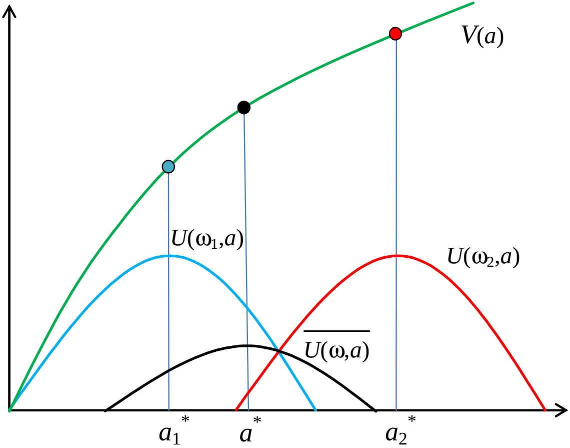
$$\frac{dE(U(\omega, a)|m)}{da} = 0 \Rightarrow a^*(m)$$

What we are after

- Suppose message m generates $\pi(\omega|m)$ with a non-singleton support
- We want to obtain a condition under which, for any such m, revealing the states in the support instead of sending m improves S' ex-ante welfare
- Kolotilin et al. (2022): there exists an optimal disclosure rule in which all messages have an at most binary support

$\Rightarrow$ We can restrict ourselves to messages with support $\{\omega_1, \omega_2\}$

Main idea



Compare probability-weighted marginal changes in S' utility pairwise as action moves from a^* to a_1^* and to a_2^* in states ω_1 and ω_2 respectively

Deriving the condition

- Let $\pi_1 := \Pr(\omega_1 | m)$, $\pi_2 := \Pr(\omega_2 | m)$
- Disclosing states is better iff

$$\pi_1 V(\omega_1, a_1^*) + \pi_2 V(\omega_2, a_2^*) > \pi_1 V(\omega_1, a^*) + \pi_2 V(\omega_2, a^*)$$

$$\Leftrightarrow \pi_2 \left[V(\omega_2, a_2^*) - V(\omega_2, a^*) \right] > \pi_1 \left[V(\omega_1, a^*) - V(\omega_1, a_1^*) \right]$$

$$\Leftrightarrow \int_{a^*}^{a_2^*} \pi_2 V_a(\omega_2, a) da > \int_{a_1^*}^{a^*} \pi_1 V_a(\omega_1, a) da$$

Deriving the condition

$$\int_{a^*}^{a_2^*} \pi_2 V_a(\omega_2, a)da > \int_{a_1^*}^{a^*} \pi_1 V_a(\omega_1, a)da$$

$$x_2 := \frac{U_a(\omega_2, a)}{U_a(\omega_2, a^*)}, \;\; x_1 := 1 - \frac{U_a(\omega_1, a)}{U_a(\omega_1, a^*)}$$

$$\pi_1 U_a(\omega_1, a^*) + \pi_2 U_a(\omega_2, a^*) = 0 \text{ (FOC under ``pooling'')}$$

$$\Rightarrow \int_0^1 \frac{V_a(\omega_2, a_2(x))}{-U_{aa}(\omega_2, a_2(x))}dx > \int_0^1 \frac{V_a(\omega_1, a_1(x))}{-U_{aa}(\omega_1, a_1(x))}dx$$

As x runs from 0 to 1, $a_2(x)$ runs from a_2^* to a^*, $a_1(x)$ — from a^* to a_1^*

Sufficient condition:

$$\frac{V_a(\omega_2, a_2(x))}{-U_{aa}(\omega_2, a_2(x))} > \frac{V_a(\omega_1, a_1(x))}{-U_{aa}(\omega_1, a_1(x))}, \text{ for each } x \in [0, 1]$$

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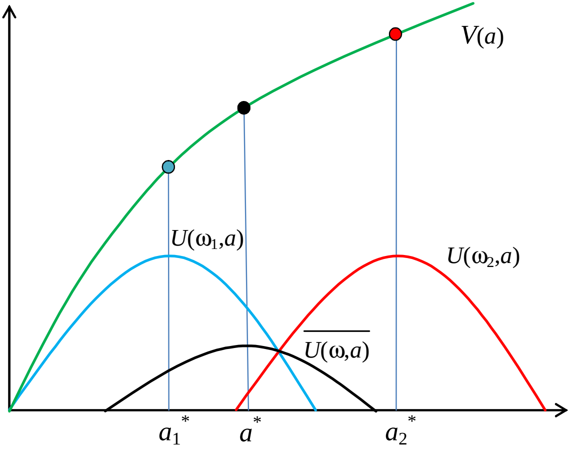


Sufficient condition for full disclosure

$$\text{For all } a_1,\ a_2,\ \omega_1,\ \omega_2,$$

$$\begin{cases} a_2 > a_1 \\ U_a(\omega_2, a_2) > U_a(\omega_1, a_1) \end{cases} \Rightarrow \frac{V_a(\omega_2, a_2)}{-U_{aa}(\omega_2, a_2)} > \frac{V_a(\omega_1, a_1)}{-U_{aa}(\omega_1, a_1)}. \quad (1)$$

Theorem

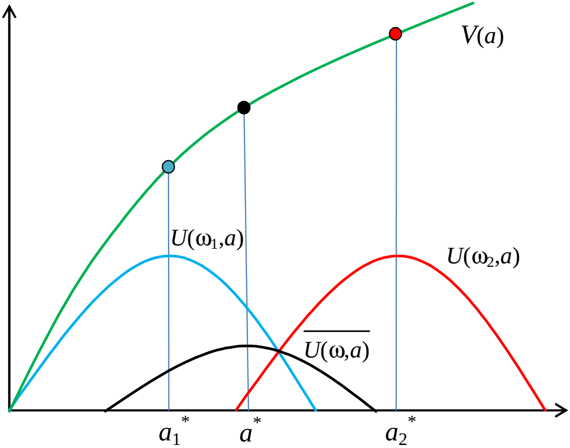
If (1) holds, full disclosure maximizes Sender's expected utility.

Alternative formulation:

$$-\frac{\partial V(\omega_2, a_2)}{\partial U_a(\omega_2, a_2)} > -\frac{\partial V(\omega_1, a_1)}{\partial U_a(\omega_1, a_1)}$$

Sufficient condition for full disclosure

$$-\frac{\partial V(\omega_2, a_2)}{\partial U_a(\omega_2, a_2)} > -\frac{\partial V(\omega_1, a_1)}{\partial U_a(\omega_1, a_1)}$$



Sufficient condition for suboptimality of full disclosure

Fix $\omega_1,\ \omega_2$. Consider condition

$$\text{For all } a_1,\ a_2,$$
$$\begin{cases} a_2 > a_1 \\ U_a(\omega_2, a_2) > U_a(\omega_1, a_1) \end{cases} \Rightarrow \frac{V_a(\omega_2, a_2)}{-U_{aa}(\omega_2, a_2)} < \frac{V_a(\omega_1, a_1)}{-U_{aa}(\omega_1, a_1)}\ . \quad (2)$$

Theorem

If there exist ω_1, ω_2 such that (2) holds, full disclosure is suboptimal.

Some well known cases. "Simple case"

$$U(\omega, a) = -\frac{1}{2}(a-\omega)^2, \ V(\omega, a) = V(a)$$
$$U_a = \omega - a, \ U_{aa} = 0$$

- The condition becomes

$$\begin{cases} a_2 > a_1 \\ \omega_2 + (a_1 - a_2) > \omega_1 \end{cases} \Rightarrow V'(a_2) > V'(a_1)$$

or

$$a_2 > a_1 \Rightarrow V'(a_2) > V'(a_1)$$

- Coincides with the necessary and sufficient condition in the “simple case” (Kolotilin et al. (2022))

Some well known cases. "Simple receiver case"

$$U_a(\omega, a) = -\frac{1}{2}(a-\omega)^2, \;\; V(\omega, a)$$

- The condition becomes

$$\begin{cases} a_2 > a_1 \\ \omega_2 + (a_1 - a_2) > \omega_1 \end{cases} \Rightarrow V_a(\omega_2, a_2) > V_a(\omega_1, a_1)$$

- Kolotilin et al. (2022)'s sufficient condition in the "simple receiver case":

$$\begin{cases} \text{For any } \omega \text{ and } a_2 > a_1, \; V_a(\omega, a_2) > V_a(\omega, a_1) \\ \text{For any } a \text{ and } \omega_2 > \omega_1, \; V_a(\omega_2, a) > V_a(\omega_1, a) \end{cases}$$

- Our condition is *weaker*, because it requires that V_a increases along a *subset* of all directions in which both a and ω weakly increase (and one of them strictly increases).

Application. Principal-Agent model

- Principal – Sender, Agent – Receiver
- A's effort $a \Rightarrow$ output $y(\omega, a)$
- A's wage $w(y) = \delta y$, P receives $(1-\delta)y$
- $U(\omega, a) = \delta y(\omega, a) - a, \; V(\omega, a) = (1-\delta)y(\omega, a)$

Example 1. Role of risk-aversion

- Output $y(\omega, a) = \omega\sqrt{a}$
- $U(\omega, a) = \delta y(\omega, a) - a, \; V(\omega, a) = (1-\delta) y(\omega, a)$
- A's and P's utilites of money

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}, \; v(x) = \frac{x^{1-\rho}}{1-\rho}$$

Example 1. Role of risk-aversion

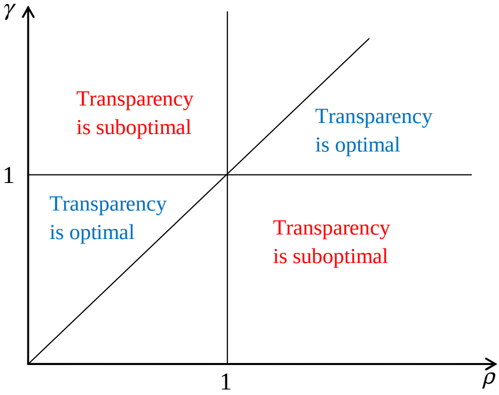
$$y(\omega, a) = \omega\sqrt{a},\ \ u(x) = \frac{x^{1-\gamma}}{1-\gamma},\ \ v(x) = \frac{x^{1-\rho}}{1-\rho}$$

$$\begin{aligned}
U(\omega, a) &= \frac{1}{1-\gamma}(\delta\omega)^{1-\gamma} a^{\frac{1}{2}(1-\gamma)} - a \\
V(\omega, a) &= \frac{1}{1-\rho}((1-\delta)\omega)^{1-\rho} a^{\frac{1}{2}(1-\rho)} \\
U_a(\omega, a) &= \frac{1}{2}(\delta\omega)^{1-\gamma} a^{-\frac{1}{2}-\frac{1}{2}\gamma} - 1 \\
V_a(\omega, a) &= \frac{1}{2}((1-\delta)\omega)^{1-\rho} a^{-\frac{1}{2}-\frac{1}{2}\rho} \\
U_{aa}(\omega, a) &= -(1+\gamma)\frac{1}{4}(\delta\omega)^{1-\gamma} a^{-\frac{3}{2}-\frac{1}{2}\gamma}
\end{aligned}$$

Example 1. Role of risk-aversion

$$\frac{V_a(\omega, a)}{-U_{aa}(\omega, a)} = const \cdot (U_a(\omega, a) + 1)^{\frac{\gamma-\rho}{1-\gamma}} \cdot a^{\frac{1-\rho}{1-\gamma}},$$

There exists a direction in which both a and U_a go up (if $\gamma < 1$ ($\gamma > 1$) just increase (decrease) ω sufficiently when $a \uparrow$)



Role of risk-aversion. Intuition

$$U(\omega, a) = \frac{1}{1-\gamma}(\delta\omega)^{1-\gamma} a^{\frac{1}{2}(1-\gamma)} - a$$
$$V(\omega, a) = \frac{1}{1-\rho}((1-\delta)\omega)^{1-\rho} a^{\frac{1}{2}(1-\rho)}$$

- $\gamma < 1 < \rho \Rightarrow$ State and effort are complements for A but substitutes for P $\Rightarrow$ disclosing states boosts/reduces effort in states where P benefits less/more from effort $\Rightarrow$ transparency is suboptimal

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- $\gamma < 1 < \rho \Rightarrow$ State and effort are complements for A but substitutes for P $\Rightarrow$ disclosing states boosts/reduces effort in states where P benefits less/more from effort $\Rightarrow$ transparency is suboptimal
- $\gamma < 1,\ \rho < 1 \Rightarrow$ State and effort are complements for both P and A $\Rightarrow$ state-contingent preferences are more aligned $\Rightarrow$ transparency gets a chance

Complementarity/substitutability. Relation to the general sufficient condition

$$\begin{cases} a_2 > a_1 \\ U_a(\omega_2, a_2) > U_a(\omega_1, a_1) \end{cases} \Rightarrow \frac{V_a(\omega_2, a_2)}{V_a(\omega_1, a_1)} > \frac{-U_{aa}(\omega_2, a_2)}{-U_{aa}(\omega_1, a_1)}$$

- Suppose a and ω are complements for R $\Rightarrow$ $a_2 > a_1$ and $U_a(\omega_2, a_2) > U_a(\omega_1, a_1)$ imply $\omega_2 > \omega_1$ $\Rightarrow$
- For given $V_a(\omega_1, a_1)$, $V_a(\omega_2, a_2)/V_a(\omega_1, a_1)$ is higher if a and ω are complements for S rather than substitutes
- Hence, given complementarity for R, complementarity (substitutability) for S makes the condition easier (harder) to satisfy

Example 2. Risk neutrality, more general production function

$$U(\omega, a) = \delta y(\omega, a) - a, \;\; V(\omega, a) = (1-\delta) y(\omega, a)$$

$$\begin{cases} a_2 > a_1 \\ y_a(\omega_2, a_2) > y_a(\omega_1, a_1) \end{cases} \Rightarrow \frac{y_a(\omega_2, a_2)}{-y_{aa}(\omega_2, a_2)} > \frac{y_a(\omega_1, a_1)}{-y_{aa}(\omega_1, a_1)} .$$

If ω and a are complements in the production function, the condition can be expressed as

$$\begin{cases} y_{aa\omega} y_a \geq y_{a\omega} y_{aa} \\ y_{aaa} y_{a\omega} \geq y_{aa\omega} y_{aa} \end{cases} ,$$

with at least one inequality being strict

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- Suppose $y(\omega, a) = \alpha(\omega) + \beta(\omega)\varphi(a) + \xi(a)$

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- Suppose $y(\omega, a) = \alpha(\omega) + \beta(\omega)\varphi(a) + \xi(a)$
- $\varphi(a) = ka^s,\ \xi(a) = la^t$, with $k > 0,\ l > 0,\ s \in (0,1),\ t \in (0,1) \Rightarrow$ sufficient condition becomes $s \geq t$

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- $\varphi(a) = ka^s, \ \xi(a) = la^t$, with $k > 0, \ l > 0, \ s \in (0,1), \ t \in (0,1) \Rightarrow$ sufficient condition becomes $s \geq t$
- $\varphi(e) = s \cdot \ln a$ and $\xi(e) = t \cdot \ln a \Rightarrow$ sufficient condition is always satisfied

Conclusion

- Interpretable and easily verifiable sufficient condition for the optimality of transparency in a general setting
- Sufficient condition for suboptimality of transparency
- Further research:
  - When is complete non-transparency optimal?
  - Interaction between explicit compensation schemes for the agent and disclosure policy?
    - Joint determination of the optimal compensation and disclosure