Testing For Time-Varying Exogeneity: A Bootstrap Approach

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Motivation & Questions

- New-Keynesian Phillips Curve(NKPC) Gali and Gertler(1999) GG Model

$$\pi_t = c + \beta_f E_t \pi_{t+1} + \beta_b \pi_{t-1} + \beta_x x_t + \epsilon_t \quad (1)$$

where π_t is the inflation rate, $E_t\pi_{t+1}$ denotes expected inflation for $t+1$ given information set at t and x_t is some forcing variable e.g unemployment.

- Endogeneity issue as Expected inflation and forcing variable are endogenous variables. Let $X_t = (E_t\pi_{t+1}, x_t)$ then

$$X_t = \gamma Z_t + \nu_t \quad (2)$$

where Z_t is a vector of appropriate instruments e.g: lags of expected inflation rate and forcing variable.

Questions

- Is the NKPC stable over time?
- Is the forcing variable indeed endogenous?
- Does the endogeneity status change over time?

¿Solution and More Questions?

- Giraitis, Kapetanios and Marcelino(2020), proposed a **non-parametric**, **kernel**-based estimation and inferential theory for time varying IV with either deterministic or stochastic coefficients and derived a time-varying version of the **Hausman** exogeneity test.
- However,
  - test appears to experience **size distortions** and have **low power**,
  - its size and power is **sensitive** to the bandwidth parameter and the sample size.

¿Solution and More Questions?

- Giraitis, Kapetanios and Marcelino(2020), proposed a **non-parametric**, **kernel**-based estimation and inferential theory for time varying IV with either deterministic or stochastic coefficients and derived a time-varying version of the **Hausman** exogeneity test.
- However,
  - test appears to experience **size distortions** and have **low power**,
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Proposal:

Use bootstrap to reduce size distortion of the test statistic.

Model

To fix ideas,

$$y_t = x_t^{'}\beta_t + u_t \tag{3}$$

$$x_t = \Psi_t^{'} z_t + v_t \tag{4}$$

where $x_t = (x_{1,t}, ..., x_{p,t})^{'}$ is a $p \times 1$ vector of random variables, $\beta_t = (\beta_{1,t}, ..., \beta_{p,t})^{'}$ is a $p \times 1$ parameter vector and u_t is a 1×1 noise. In (4) , $z_t = (z_{1,t}, ..., z_{n,t})^{'}$ is a $n \times 1$ vector of random variables, $\Psi_t^{'} = (\psi_{lk,t})$ is a $p \times n$ parameter matrix and $v_t = (v_{1,t}, ..., v_{p,t})^{'}$ is a $p \times 1$ noise vector. Assume endogenous variables x_t are correlated with u_t but there exist some exogenous instruments z_t such that:

$$E[z_t u_t] = 0, \;\; E[z_t v_t^{'}] = 0, \;\; t \geq 1 \tag{5}$$

IV Estimator

GKM introduced a kernel type estimator for β_t

$$\tilde{\beta}_t = (\sum_{j=1}^{T} b_{H,|j-t|} \hat{\Psi}_j' z_j x_j')^{-1} (\sum_{j=1}^{T} b_{H,|j-t|} \hat{\Psi}_j' z_j y_j) \tag{6}$$

where $b_{H,|j-t|} = K(\frac{|j-t|}{H})$ are the kernel weights with bandwidth parameter H where $H \to \infty$, $H = o(T)$, and $\hat{\Psi}_j$ is the kernel OLS estimator

$$\hat{\Psi}_t = (\sum_{j=1}^{T} b_{L,|j-t|} z_j z_j')^{-1} (\sum_{j=1}^{T} b_{L,|j-t|} z_j x_j') \tag{7}$$

which is a consistent estimate of Ψ_j.

OLS Estimator

The OLS estimator:

$$\hat{\beta}_t = (\sum_{j=1}^{T} b_{H,|j-t|} x_j x_j^{'})^{-1} (\sum_{j=1}^{T} b_{H,|j-t|} x_j y_j) \quad (8)$$

Kernel Choices: $K(x) = exp(-cx^{\alpha})$ with $c, \alpha > 0$ and $K(x) = 0.75(1 - x^2)$ etc

Time-Varying Hausman Test

GKM introduced the following test statistic

$$H = \frac{K_t^2}{K_{2,t}} V_{T,t}' \hat{\Sigma}_{\hat{v}\hat{v},t}^{-1} V_{T,t} \hat{\sigma}_{\hat{u},t}^{-2} \sim \chi_p^2 \tag{9}$$

where:

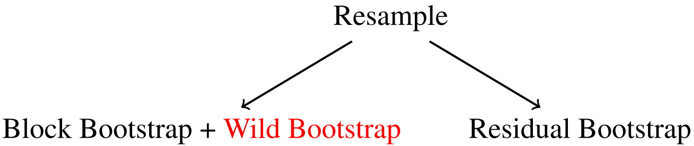
- $V_{T,t} = (S_{\hat{x}\hat{x},t})^{1/2}(S_{xx,t})^{1/2}(\hat{\beta}_t - \tilde{\beta}_t)$ where $S_{xx,t} = K_t^{-1} b_{H,|j-t|} x_j x_j'$, $S_{\hat{x}\hat{x},t} = K_t^{-1} \sum_{j=1}^T b_{H,|j-t|} \hat{x}_j \hat{x}_j'$, $\hat{x}_j = \hat{\Psi}_j z_j$.
- $\hat{\Sigma}_{\hat{v}\hat{v},t} := K_t^{-1} \sum_{j=1}^T b_{H,|j-t|} \hat{v}_j \hat{v}_j'$, $\hat{\sigma}_{\hat{u},t}^2 := K_t^{-1} \sum_{j=1}^T b_{H,|j-t|} \hat{u}_j^2$ based on residuals $\hat{u}_j = y_j - x_j' \tilde{\beta}_{1,j}$ and $\hat{v}_j = x_j - \hat{\Psi}_j' z_j$ respectively.

Null Hypothesis:

$$H_{0,\mathbf{t}} : E[v_t u_t] = 0$$

Bootstrap I

Mode of resampling so that the null is imposed.



For Block Bootstrap(BB):

- Resample blocks of rows of $[x_t, z_t]$ using Block Bootstrap (Kunsch,1989).
- Use Wild bootstrap to impose the null so that $u_t^\star = \eta_t \hat{u}_t$ where $\eta_t \sim N(0,1)$ and hence $E^\star[x_t^\star u_t^\star] = 0$.

Bootstrap II

For Residual Bootstrap(RB):

- Resample y_t using Parametric Bootstrap through (3).
- By virtue of LS, residuals are guaranteed to satisfy orthogonality condition.

Which estimator to use(2SLS or LS)? Power considerations

Monte Carlo Simulations

- 5,000 Monte Carlo Simulations
- Two cases: Exactly Identified and Overidentified
- Three sample sizes considered $T = 100$, 200 and 400
- Bandwidth parameter values for $H = T^{h_1}$ and $L = T^{h_2}$: $h_1, h_2 = 0.4, 0.5, 0.7$
- Gaussian, Epanechnikov and Triangular Kernels
- 999 Bootstrap Replications
- Block sizes $b \propto T^{1/3}$

Monte Carlo: Exactly Identified

As data generating process (DGP) under the exactly identified case, I consider the following model:

$$y_t = \beta_t x_t + u_t \tag{10}$$
$$x_t = \psi_t z_t + v_t \tag{11}$$

for $t = 1, ..., T$. Correlation between u_t and v_t is introduced by specifying them as

$$u_t = se_{1,t} + (1-s)e_{2,t} \;\; v_t = se_{1,t} + (1-s)e_{3,t} \tag{12}$$

where $s = 0, 0.2, 0.5, 0.8, 0.9$ and $\{e_{1,t}\}$,$\{e_{2,t}\}$ and $\{e_{3,t}\}$ are mutually independent $NIID(0,1)$ sequences. The parameters $\beta_t = T^{-1/2}\xi_{1,t}$, $\psi_t = T^{-1/2}\xi_{2,t}$,$t = 1, ..., T$ are generated as two independent rescaled random walks, such that $\xi_{l,t} - \xi_{l,t-1} \sim N(0,1)$ for $l = 1, 2$ that are also independent of $\{\psi_t\}$,$\{u_t\}$ and $\{v_t\}$.

Results

RB Hausman test outperforms both Asymptotic test and BB in terms of power and size irrespective of the bandwidth parameter chosen.

- Exactly Identified case: Size of H_{RB} is close to nominal 5% while the i power of H_{BB} is at least 38.5% higher than the power of the asymptotic test. T=100
- Overidentified case: Size of asymptotic test is close to nominal value although it varies more than the H_{RB}.(e.g for $h_1 = 0.5$ and $h_2 = 0.4$ size for asymptotic test is 0.069 compared to 0.056 for RB.) T=100

For all sample sizes, power refinements of RB increase with the degree of endogeneity.

Empirical Application

Estimate a Forward Looking (NK) **Phillips curve** as in Gali and Gertler(2008):

$$\Delta\pi_t = c_t + \rho_t \Delta\pi^e_{t+1} + \gamma_t \Delta\pi_{t-1} + \alpha_t \Delta u_t + v_t \tag{13}$$

or

$$\Delta\pi_t = c_t + \rho_t \Delta\pi_{t+1} + \gamma_t \Delta\pi_{t-1} + \alpha_t \Delta u_t + \epsilon_t \tag{14}$$

where

- $\epsilon_t = \rho_t(\Delta\pi^e_{t+1} - \Delta\pi_{t+1}) + v_t$.
- US sample for period 1959:6-2021:11
- Kernel: Gaussian Kernel
- Bandwidth parameters: $H = L = T^{0.7}$ and $H = L = T^{0.5}$
- Instruments: $\{\Delta u_{t-j}\}_{j=1}^4$ and $\{\Delta\pi_{t-j}\}_{j=1}^4$

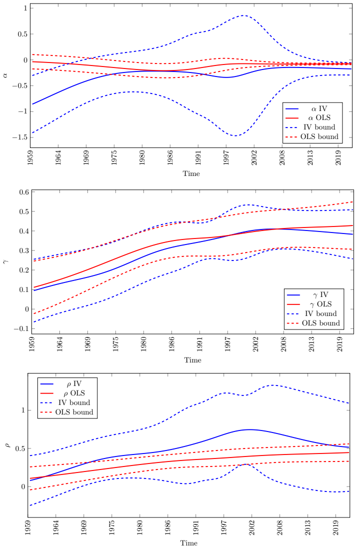


Figure: The three panels graph the OLS and IV coefficient estimates for α_t, γ_t and ρ_t respectively using $H = L = T^{0.7}$.

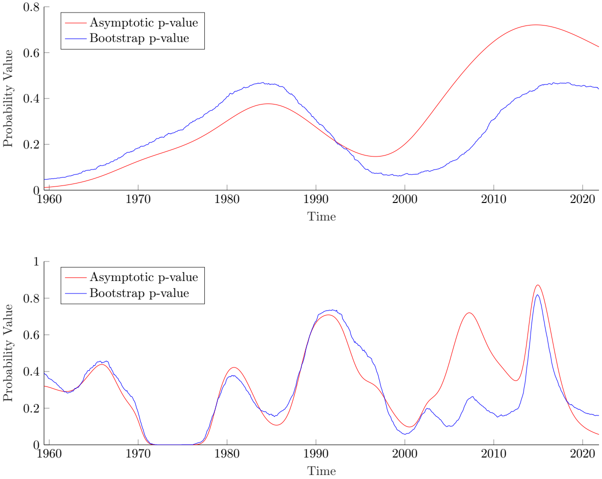


Figure: Empirical p-values of the time-varying Hausman test and its bootstrap version for $H = L = T^{0.7}$ and $H = L = T^{0.5}$ respectively.

Conclusion

GKM proposed

- a non-parametric IV estimation based on kernels, allowing for both deterministic and random coefficients.
- Time-varying Hausman exogeneity test statistic to test for a possible switching endogeneity status at a specific point.
  - However, test appears to experience size distortions and have low power.
  - Its size and power is sensitive to the bandwidth parameter, and the sample size.

I propose:

To use Residual Bootstrap so that the test has proper size and high power and it is insensitive to the bandwidth parameter choice and the sample size.

Thank you!

Table: Rejection frequencies for the local Hausman test at $t = T/2$ and for $\alpha = 0.05$.

			T=100					
s	h_1	h_2	***Asymptotic***	***RB***	$b = 2$	$b = 4$	$b = 6$	$b = 8$
0	0.4	0.4	0.024	0.047	0.060	0.057	0.054	0.054
	0.4	0.5	0.029	0.047	0.052	0.052	0.050	0.049
	0.5	0.4	0.034	0.048	0.056	0.057	0.055	0.053
	0.5	0.5	0.033	0.047	0.056	0.056	0.053	0.055
0.2	0.4	0.4	0.032	0.049	0.060	0.058	0.058	0.059
	0.4	0.5	0.039	0.048	0.056	0.054	0.053	0.053
	0.5	0.4	0.040	0.054	0.064	0.064	0.064	0.063
	0.5	0.5	0.042	0.055	0.060	0.059	0.058	0.059
0.5	0.4	0.4	0.305	0.362	0.376	0.373	0.368	0.369
	0.4	0.5	0.306	0.358	0.353	0.347	0.344	0.338
	0.5	0.4	0.452	0.506	0.528	0.527	0.526	0.524
	0.5	0.5	0.452	0.509	0.524	0.518	0.513	0.516
0.8	0.4	0.4	0.705	0.846	0.805	0.801	0.798	0.797
	0.4	0.5	0.687	0.814	0.771	0.764	0.763	0.761
	0.5	0.4	0.804	0.909	0.881	0.877	0.876	0.875
	0.5	0.5	0.785	0.899	0.875	0.873	0.870	0.870
0.9	0.4	0.4	0.705	0.867	0.810	0.805	0.803	0.803
	0.4	0.5	0.684	0.821	0.772	0.766	0.764	0.763
	0.5	0.4	0.807	0.919	0.883	0.880	0.879	0.877
	0.5	0.5	0.786	0.907	0.877	0.875	0.870	0.869

Table: Rejection frequencies for the local Hausman test at $t = T/2$ and for $\alpha = 0.05$.

T=200								
s	h_1	h_2	***Asymptotic***	***RB***	$b = 4$	$b = 6$	$b = 8$	$b = 16$
0	0.4	0.4	0.025	0.054	0.062	0.061	0.061	0.057
	0.4	0.5	0.029	0.050	0.053	0.050	0.051	0.048
	0.5	0.4	0.033	0.054	0.066	0.064	0.065	0.061
	0.5	0.5	0.031	0.053	0.062	0.063	0.063	0.060
0.2	0.4	0.4	0.032	0.053	0.069	0.066	0.066	0.063
	0.4	0.5	0.037	0.056	0.059	0.058	0.055	0.052
	0.5	0.4	0.043	0.062	0.072	0.070	0.068	0.064
	0.5	0.5	0.042	0.059	0.070	0.066	0.068	0.063
0.5	0.4	0.4	0.369	0.443	0.460	0.457	0.455	0.449
	0.4	0.5	0.368	0.435	0.424	0.419	0.415	0.410
	0.5	0.4	0.551	0.609	0.626	0.625	0.626	0.617
	0.5	0.5	0.542	0.607	0.621	0.619	0.619	0.613
0.8	0.4	0.4	0.721	0.850	0.805	0.804	0.803	0.801
	0.4	0.5	0.701	0.811	0.770	0.765	0.764	0.758
	0.5	0.4	0.823	0.921	0.887	0.883	0.883	0.880
	0.5	0.5	0.804	0.911	0.876	0.873	0.873	0.868
0.9	0.4	0.4	0.713	0.869	0.810	0.808	0.805	0.801
	0.4	0.5	0.683	0.810	0.769	0.763	0.760	0.758
	0.5	0.4	0.822	0.930	0.890	0.887	0.885	0.882
	0.5	0.5	0.792	0.912	0.875	0.872	0.869	0.867

Table: Rejection frequencies for the local Hausman test at $t = T/2$ and for $\alpha = 0.05$.

			T=100					
s	h_1	h_2	***Asymptotic***	***RB***	$b = 2$	$b = 4$	$b = 6$	$b = 8$
0	0.4	0.4	0.055	0.052	0.073	0.074	0.076	0.073
	0.4	0.5	0.051	0.052	0.071	0.070	0.067	0.067
	0.5	0.4	0.069	0.056	0.085	0.085	0.084	0.084
	0.5	0.5	0.056	0.047	0.069	0.069	0.071	0.069
0.2	0.4	0.4	0.065	0.058	0.079	0.081	0.080	0.079
	0.4	0.5	0.061	0.058	0.074	0.072	0.072	0.070
	0.5	0.4	0.084	0.062	0.095	0.094	0.094	0.094
	0.5	0.5	0.069	0.059	0.075	0.077	0.077	0.076
0.5	0.4	0.4	0.379	0.391	0.410	0.410	0.408	0.410
	0.4	0.5	0.386	0.414	0.399	0.398	0.395	0.395
	0.5	0.4	0.535	0.525	0.555	0.555	0.557	0.555
	0.5	0.5	0.542	0.556	0.562	0.559	0.560	0.563
0.8	0.4	0.4	0.889	0.929	0.914	0.914	0.912	0.912
	0.4	0.5	0.896	0.944	0.923	0.920	0.919	0.918
	0.5	0.4	0.944	0.953	0.957	0.957	0.956	0.957
	0.5	0.5	0.950	0.968	0.965	0.964	0.964	0.963
0.9	0.4	0.4	0.907	0.951	0.935	0.934	0.932	0.934
	0.4	0.5	0.906	0.958	0.936	0.935	0.933	0.933
	0.5	0.4	0.956	0.968	0.968	0.968	0.967	0.968
	0.5	0.5	0.958	0.976	0.972	0.972	0.971	0.972

Table: Rejection frequencies for the local Hausman test at $t = T/2$ and for $\alpha = 0.05$.

				T=200				
s	h_1	h_2	***Asymptotic***	***RB***	$b = 2$	$b = 4$	$b = 6$	$b = 8$
0	0.4	0.4	0.049	0.055	0.075	0.075	0.074	0.075
	0.4	0.5	0.047	0.053	0.064	0.064	0.064	0.063
	0.5	0.4	0.067	0.060	0.087	0.088	0.089	0.088
	0.5	0.5	0.054	0.057	0.074	0.074	0.074	0.071
0.2	0.4	0.4	0.053	0.055	0.078	0.077	0.075	0.074
	0.4	0.5	0.052	0.056	0.064	0.061	0.062	0.061
	0.5	0.4	0.078	0.065	0.095	0.095	0.096	0.094
	0.5	0.5	0.064	0.063	0.079	0.079	0.079	0.078
0.5	0.4	0.4	0.474	0.500	0.522	0.521	0.518	0.517
	0.4	0.5	0.476	0.517	0.503	0.499	0.498	0.493
	0.5	0.4	0.666	0.672	0.696	0.694	0.698	0.693
	0.5	0.5	0.675	0.700	0.704	0.700	0.700	0.699
0.8	0.4	0.4	0.927	0.963	0.952	0.951	0.950	0.949
	0.4	0.5	0.920	0.960	0.943	0.942	0.940	0.938
	0.5	0.4	0.970	0.986	0.982	0.982	0.981	0.980
	0.5	0.5	0.967	0.988	0.982	0.981	0.981	0.981
0.9	0.4	0.4	0.930	0.975	0.958	0.959	0.956	0.955
	0.4	0.5	0.920	0.966	0.947	0.946	0.944	0.944
	0.5	0.4	0.975	0.992	0.985	0.984	0.984	0.982
	0.5	0.5	0.970	0.990	0.984	0.983	0.982	0.981

NKPC Theoretical Formulation I

Based on Gali and Gertler(1999): NKPC

- Each firm is able to adjust its price in any given period with probability $1-\theta$ independent of the time the price has been fixed.
- A fraction $1-\omega$ of the firms are forward looking while a fraction ω are backward looking.
- Aggregate price level evolves as

$$p_t = \theta p_{t-1} + (1-\theta)\bar{p}_t^\star \tag{15}$$

where $\bar{p}_t^\star$ is

$$\bar{p}_t^\star = (1-\omega)p_t^f + \omega p_t^b \tag{16}$$

- Likewise, p_t^f is expressed

$$p_t^f = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{mc_{t+k}^n\} \tag{17}$$

- Finally,

$$p_t^b = \bar{p}_{t-1}^\star + \pi_{t-1} \tag{18}$$

NKPC Theoretical Formulation II

Combining (15)-(18), Gali and Gertler(1999) derive the hybrid Philips curve as:

$$\pi_t = \lambda mc_t + \gamma_f E_t\{\pi_{t+1}\} + \gamma_b \pi_{t-1} \tag{19}$$

where

$$\lambda \equiv (1-\omega)(1-\theta)(1-\beta\theta)\phi^{-1} \tag{20}$$

$$\gamma_f \equiv \beta\theta\phi^{-1} \tag{21}$$

$$\gamma_b \equiv \omega\phi^{-1} \tag{22}$$

$$\phi \equiv \theta + \omega[1-\theta(1-\beta)] \tag{23}$$

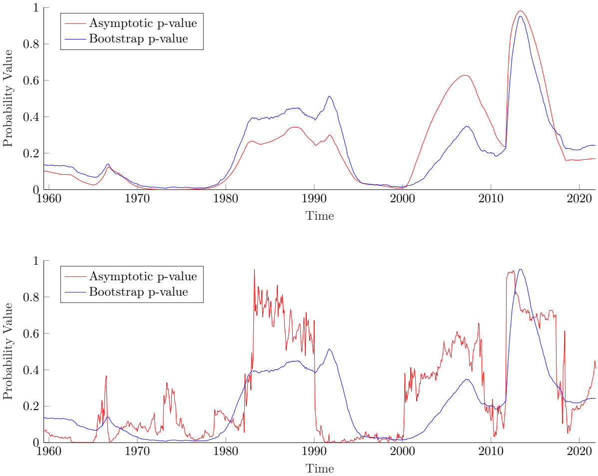


Figure: The two panels graph the empirical p-values of the aymptotic and bootstrap Hausman tests using $H = L = T^{0.7}$. The upper pane uses the Epanechnikov kernel while the lower uses the rectangular.