

A Behavioral Explanation for the Puzzling Persistence of the Aggregate Real Exchange Rate

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The Law-of-One-Price and Purchasing Power Parity

▶ LOP

$$\frac{S_t P_{\text{beer},t}^*}{P_{\text{beer},t}} = 1$$

where S_t is the nominal exchange rate, * denotes "foreign"

▶ PPP

$$\frac{S_t P_t^*}{P_t} = 1$$

where P_t and P_t^* are CPIs of home and foreign countries

The Law-of-One-Price and Purchasing Power Parity

- ▶ LOP deviations (or good-level RER)

$$\frac{S_t P_{\text{beer},t}^*}{P_{\text{beer},t}} \neq 1$$

where S_t is the nominal exchange rate, * denotes "foreign"

- ▶ PPP deviations (aggregate RER)

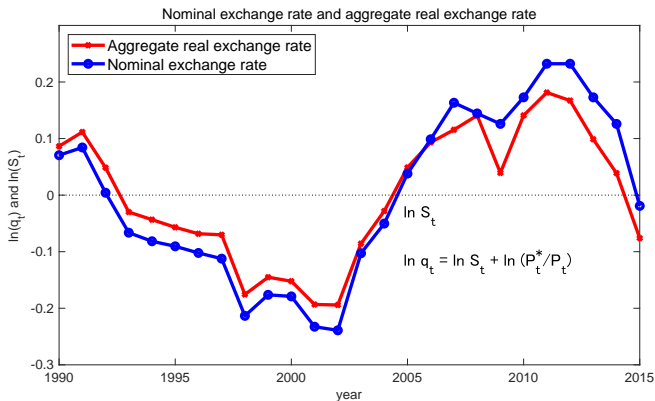
$$\frac{S_t P_t^*}{P_t} \neq 1$$

where P_t and P_t^* are CPIs of home and foreign countries

- ▶ In reality, LOP & PPP do not hold

PPP puzzle 1

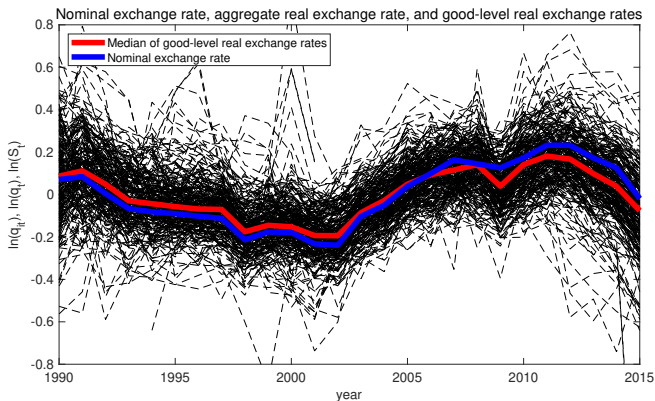
- ▶ PPP deviations are extremely persistent



- ▶ too persistent to be explained by reasonable degree of nominal price rigidities (Rogoff 1996)
- ▶ Half-life = 3-5 years

PPP puzzle 2

- ▶ LOP deviations are much less persistent than PPP deviations



- ▶ Half-life ($\simeq 1.2$ year) is lower than that of q_t (3-5 years)
- ▶ Imbs, Mumtaz, Ravn and Rey (2005), Crucini and Shintani (2008), Carvalho and Nechio (2011)

Research questions

Q1: Does the behavioral inattention model help to solve the PPP puzzle?

- ▶ We consider a behavioral model of rational inattention (“sparse-based model”) by Gabaix (2014, 2020)
- ▶ Gabaix (2014, 2020) discusses models with attention parameter m :

$m = 1$ (if agents are fully attentive)

$m < 1$ (if agents are inattentive)

- ▶ We introduce m into the model of LOP deviations used in Crucini, Shintani and Tsuruga (2010a, 2010b, 2013, 2015)

Q2: Do micro price data support the behavioral inattention?

Theoretical finding

Q1: Does the behavioral inattention model help to solve the PPP puzzle?

A1: Yes

- ▶ We derive the relationship between LOP deviations and PPP deviations
- ▶ If $m = 1$

$$\ln q_{it} = \lambda \ln q_{it-1} + e_t + e_{it}$$

(λ : the degree of price stickiness, e : iid shocks)

- ▶ If $m < 1$

$$\ln q_{it} = \lambda \ln q_{it-1} + (1 - m)(1 - \lambda) \ln q_t + e_t + e_{it}$$

- ▶ **Complementarity** generates persistence of q_t and q_{it}

Theoretical finding

Q1: Does the behavioral inattention model help to solve the PPP puzzle?

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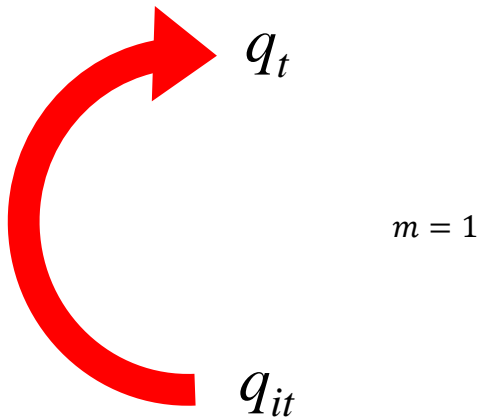
▶ If $m < 1$

$$\ln q_{it} = \lambda \ln q_{it-1} + (1 - m)(1 - \lambda) \ln q_t + e_t + e_{it}$$

▶ **Complementarity** generates persistence of q_t and q_{it}

Pricing complementarity

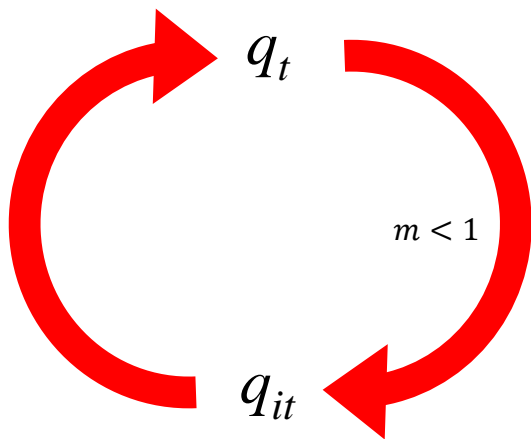
$$\ln q_{it} = \lambda \ln q_{it-1} + e_t + e_{it}$$



- ▶ No complementarities if $m = 1$

Pricing complementarity

$$\ln q_{it} = \lambda \ln q_{it-1} + (1 - m)(1 - \lambda) \ln q_t + e_t + e_{it}$$



- ▶ Complementarity generates slow aggregate real exchange rate if $m < 1$

Empirical findings

Q2: Do micro price data support the behavioral inattention?

A2: Yes

- ▶ We test the model of LOP deviations with behavioral inattention

- ▶ Competing Hypotheses

H_0 : $m = 1$ (fully attentive)

H_1 : $m < 1$ (inattentive)

- ▶ $H_0: m = 1$ is strongly rejected by the data
- ▶ Our estimates of m are $m = 0.1 - 0.2$
- ▶ Under the estimated m , the model explains the PPP puzzle

The persistence of the PPP and LOP deviations

- ▶ With $m = 1$ (**full attention**)

	Model ($m = 1$)	Model ($m < 1$)	Data
Aggregate real exchange rates (PPP deviations)			
Half-life	0.64		2.4-4.9
Good-level real exchange rates (LOP deviations)			
Half-life	0.64		1.2-1.6

- ▶ Note: The unit of a period is year

The persistence of the PPP and LOP deviations

- ▶ With $m = 0.1$ (**behavioral inattention**)

	Model ($m = 1$)	Model ($m < 1$)	Data
Aggregate real exchange rates (PPP deviations)			
Half-life	0.6	2.5-3.7	2.4-4.9
Good-level real exchange rates (LOP deviations)			
Half-life	0.6	1.0-1.2	1.2-1.6

- ▶ Note: The unit of a period is year

Overview of the model

- ▶ Follows Kehoe and Midrigan (2007), Crucini, Shintani and Tsuruga (2010, 2013)
- ▶ Households
 - ▶ $U(c_t, n_t) = \ln c_t - \chi n_t$ ▶ max. problem
- ▶ Firms
 - ▶ Set prices in monopolistically competitive market (Home and Foreign, local currency pricing) ▶ CES
 - ▶ Calvo pricing with its parameter λ
 - ▶ Use technology: $y_{it}(z) = a_{it}n_{it}(z)$
 - ▶ Must pay trade cost to send goods from a country to the other ▶ resources
 - ▶ We introduce Gabaix's behavioral inattention to price setting
- ▶ Governments
 - ▶ Control money supply

Rational firms' pricing: step 1

- ▶ Home firm's pricing under **full attention**

$$\hat{p}_{Hit} = (1 - \lambda\delta)\mathbb{E}_t \sum_{k=0}^{\infty} (\lambda\delta)^k (\widehat{m}c_{Hit+k})$$

$$\hat{p}_{Hit}^* = (1 - \lambda\delta)\mathbb{E}_t \sum_{k=0}^{\infty} (\lambda\delta)^k (\widehat{m}c_{Hit+k}^*)$$

where all variables are the log-deviations, p_{Hit} : relative price of good i , a_{it} : productivity, δ : discount factor,

- ▶ \hat{p}_{Fit}^* and \hat{p}_{Fit} are analogously derived

Behavioral firms' pricing: step 2

- ▶ Home firm's pricing under **behavioral inattention**

$$\hat{p}_{Hit}(m_H) = (1 - \lambda\delta)\mathbb{E}_t \sum_{k=0}^{\infty} (\lambda\delta)^k (m_H \widehat{mc}_{Hit+k})$$

$$\hat{p}_{Hit}^*(m_H^*) = (1 - \lambda\delta)\mathbb{E}_t \sum_{k=0}^{\infty} (\lambda\delta)^k (m_H^* \widehat{mc}_{Hit+k}^*)$$

- ▶ $\hat{p}_{Fit}^*(m_F^*)$ and $\hat{p}_{Fit}(m_F)$ are analogously derived
- ▶ The optimal (relative) prices are insensitive to the aggregate shocks ▶ price index

Proposition 1

Under the preferences given by $U(c, n) = \ln c - \chi n$, the CIA constraints, the stochastic processes of money supply, the stochastic processes of the labor productivity, and the Calvo pricing with the degree of price stickiness $\lambda \in (0, 1)$, the stochastic process of the good-level real exchange rate is given by:

$$\ln q_{it} = \lambda \ln q_{it-1} + (1 - m)(1 - \lambda) \ln q_t + \lambda \varepsilon_t^n + \psi \varepsilon_{it}^r$$

where

- ▶ $m \in (0, 1]$: the degree of attention, $m = \omega m_H + (1 - \omega) m_F$
- ▶ ψ : param. for real frictions (> 0 with trade cost)
- ▶ ε_t^n : nominal shock, $\varepsilon_t^n = \Delta \ln S_t \sim i.i.d.(0, \sigma_n^2)$
- ▶ ε_{it}^r : real shock, $\varepsilon_{it}^r = (\varepsilon_{it} - \varepsilon_{it}^*) \sim i.i.d.(0, \sigma_r^2)$

Takeaway: Direct relationship between $\ln q_{it}$ and $\ln q_t$

Testable implication of behavioral LOP

- ▶ We define modified LOP deviations \tilde{q}_{it} and PPP deviations \tilde{q}_t

$$\underbrace{\ln q_{it} - \lambda \ln q_{it-1} - \lambda \varepsilon_t^n}_{\ln \tilde{q}_{it}} = (1 - m) \underbrace{(1 - \lambda) \ln q_t}_{\ln \tilde{q}_t} + \underbrace{\psi \varepsilon_{it}^r}_{u_{it}}$$

where we replace ε_t^n by $\Delta \ln S_t$

- ▶ We can use this regression as a test for full attention

$$\ln \tilde{q}_{it} = \alpha + \beta \ln \tilde{q}_t + u_{it},$$

- ▶ The null hypothesis of full attention ($H_0 : m = 1$ or $\beta = 0$) is significantly rejected in favor of behavioral inattention

Data

- ▶ We use the annual micro price data of US-Canadian city pairs and UK-Euro city pairs
 - ▶ The *Worldwide Cost of Living Survey* by Economic Intelligence Unit
 - ▶ Our regression has variations in three dimensions

$$\ln q_{ijt}$$

- ▶ 274 goods (i)
- ▶ 17 US cities and 4 Canadian cities = 68 city pairs (j)
- ▶ (or 19 Euro cities and 2 UK cities = 38 city pairs (j))
- ▶ 26 years from 1990 to 2015 (t)

Test for behavioral inattention (US–Canada)

- ▶ Modified RER with common λ

$$\ln \tilde{q}_{ijt} = \alpha_{ij} + \beta \ln \tilde{q}_t + \gamma' X_{ijt} + u_{ijt}$$

	(1)	(2)	(3)	(4)
$\ln \tilde{q}_t$	0.844*** (0.030)	0.802*** (0.028)	0.812*** (0.029)	0.806*** (0.029)
# of Obs.	389,500	389,500	389,500	389,500
city-pairs FE	N	Y	N	Y
Control for productivity	N	N	Y	Y
\hat{m}	0.156	0.198	0.188	0.194

Note: *10% significance level, **5% significance level, ***1% significance level. For λ , the median value of the degrees of price stickiness ($\lambda = 0.34$) is used. All specifications include the fixed effect at the good level. The standard errors are clustered by goods.

Test for behavioral inattention (UK–Euro)

- ▶ Modified RER with common λ

$$\ln \tilde{q}_{ijt} = \alpha_{ij} + \beta \ln \tilde{q}_t + \gamma' X_{ijt} + u_{ijt}$$

	(1)	(2)	(3)	(4)
$\ln \tilde{q}_t$	0.791*** (0.041)	0.749*** (0.042)	0.782*** (0.041)	0.779*** (0.041)
# of Obs.	225,922	225,922	224,871	224,871
city-pairs FE	N	Y	N	Y
Control for productivity	N	N	Y	Y
\hat{m}	0.209	0.251	0.218	0.221

Note: *10% significance level, **5% significance level, ***1% significance level. For λ , the median value of the degrees of price stickiness ($\lambda = 0.34$) is used. All specifications include the fixed effect at the good level. The standard errors are clustered by goods.

Test for behavioral inattention (US–Canada)

- ▶ Modified RER with good-specific λ_i

$$\ln \tilde{q}_{ijt} = \alpha_{ij} + \beta \ln \tilde{q}_{it} + \gamma' X_{ijt} + u_{ijt}$$

	(1)	(2)	(3)	(4)
$\ln \tilde{q}_{it}$	0.894*** (0.029)	0.862*** (0.028)	0.883*** (0.032)	0.880*** (0.033)
# of Obs.	389,500	389,500	389,500	389,500
city-pairs FE	N	Y	N	Y
Control for productivity	N	N	Y	Y
\hat{m}	0.106	0.138	0.117	0.120

Note: *10% significance level, **5% significance level, ***1% significance level. For λ , the good-specific values of the degrees of price stickiness (λ_i) are used. All specifications include the fixed effect at the good level. The standard errors are clustered by goods.

Test for behavioral inattention (UK–Euro)

- ▶ Modified RER with good-specific λ_i

$$\ln \tilde{q}_{ijt} = \alpha_{ij} + \beta \ln \tilde{q}_{it} + \gamma' X_{ijt} + u_{ijt}$$

	(1)	(2)	(3)	(4)
$\ln \tilde{q}_{it}$	0.838*** (0.043)	0.789*** (0.044)	0.827*** (0.042)	0.803*** (0.043)
# of Obs.	208,073	208,073	207,095	207,095
city-pairs FE	N	Y	N	Y
Control for productivity	N	N	Y	Y
\hat{m}	0.162	0.211	0.173	0.197

Note: *10% significance level, **5% significance level, ***1% significance level. For λ , the good-specific values of the degrees of price stickiness (λ_i) are used. All specifications include the fixed effect at the good level. The standard errors are clustered by goods.

Test for behavioral inattention: Summary

- ▶ The null hypothesis of $m = 1$ is significantly rejected and robust to various specifications
- ▶ The estimated degree of inattention ranges between 0.11–0.25
 - ▶ Baseline estimate $m = 0.106$ for US-Canada
 - ▶ Baseline estimate $m = 0.162$ for UK-Euro
- ▶ What are the implications for PPP puzzle?

Propositions 2 & 3

Under the same assumptions in Proposition 1,

$$\rho_q > \lambda \text{ (PPP puzzle 1)}$$

$$\rho_q > \rho_{qi} \text{ (PPP puzzle 2)}$$

provided $m \in (0,1)$, $\lambda \in (0,1)$, $\tau \in (0,\infty)$, and $\sigma_r/\sigma_n \in (0,\infty)$

▶ Eq Prop 2

▶ Eq Prop 3

The persistence of the PPP and LOP deviations

- ▶ For US-Canada with $m = 0.106$

	Model ($m = 1$)	Model ($m < 1$)	95% CI	Data
Aggregate real exchange rates (PPP deviations)				
Half-life	0.6	3.7	[2.5, 7.6]	4.9
Good-level real exchange rates (LOP deviations)				
Half-life	0.6	1.2	[1.0, 2.1]	1.6

- ▶ Note: The unit of a period is year

The persistence of the PPP and LOP deviations

- ▶ For UK-Euro with $m = 0.162$

	Model ($m = 1$)	Model ($m < 1$)	95% CI	Data
Aggregate real exchange rates (PPP deviations)				
Half-life	0.6	2.5	[1.7, 4.9]	2.4
Good-level real exchange rates (LOP deviations)				
Half-life	0.6	1.0	[0.8, 1.5]	1.2

- ▶ Note: The unit of a period is year

Conclusion

- ▶ Two puzzles on PPP and LOP deviations
 1. The persistence of the aggregate RER is too high to be explained by reasonable degree of nominal price rigidities
 2. The good-level RER is less persistent than the aggregate RER
- ▶ The behavioral model by Gabaix (2014, 2020) could explain these puzzles

	Model ($m = 1$)	Model ($m < 1$)	Data
Aggregate real exchange rates (PPP deviations)			
Half-life	0.6	2.5-3.7	2.4-4.9
Good-level real exchange rates (LOP deviations)			
Half-life	0.6	1.0-1.2	1.2-1.6

Thank you!

Households

- ▶ Domestic household solves

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t (\ln c_t - \chi n_t)$$

$$\begin{aligned} \text{s.t. } M_t + B_t &= W_t n_t + R_{t-1} B_{t-1} + (M_{t-1} - P_t c_{t-1}) + T_t + \Pi_t \\ M_t &\geq P_t c_t \end{aligned}$$

M_t : nominal money holding, B_t : nominal bond holding, W_t : nominal wage, R_t : nominal interest, P_t : price level, T_t : transfers, Π_t : profits, δ : discount factor

- ▶ Foreign household's problem is analogously defined except for the budget const.

$$\begin{aligned} \text{s.t. } M_t^* + \frac{B_t^*}{S_t} &= W_t^* n_t^* + \frac{R_{t-1}}{S_t} B_{t-1}^* + (M_{t-1}^* - P_{t-1}^* c_{t-1}^*) + T_t^* + \Pi_t^* \\ M_t^* &\geq P_t^* c_t^* \end{aligned}$$

S_t : nominal exchange rate

Households (2)

► FOC

$$\frac{W_t}{P_t} = \chi c_t, \quad \frac{W_t^*}{P_t^*} = \chi c_t^*$$
$$M_t = P_t c_t, \quad M_t^* = P_t^* c_t^*$$

$$\frac{1}{R_t} = \delta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-1} \frac{P_t}{P_{t+1}} \right] = \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}^*}{c_t^*} \right)^{-1} \frac{S_t}{S_{t+1}} \frac{P_t^*}{P_{t+1}^*} \right]$$
$$q_t \frac{U_{c,t}}{U_{c,t}^*} = q_{t-1} \frac{U_{c,t-1}}{U_{c,t-1}^*} = \dots = q_0 \frac{U_{c,0}}{U_{c,0}^*} = 1$$

► back

CES aggregators

- ▶ Home and Foreign (*)
- ▶ Consumption of good i

$$c_{it} = \left[\int c_{it}(z)^{(\varepsilon-1)/\varepsilon} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad c_{it}^* = \left[\int c_{it}^*(z)^{(\varepsilon-1)/\varepsilon} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- ▶ where z denotes the brand z of good i
- ▶ $z \in [0, 1/2]$ is produced in home and $z \in (1/2, 1]$ is produced in foreign
- ▶ Aggregate consumption

$$c_t = \left[\int c_{it}^{(\varepsilon-1)/\varepsilon} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad c_t^* = \left[\int c_{it}^{*(\varepsilon-1)/\varepsilon} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Resource constraint

- ▶ Production function

$$y_{it}(z) = a_{it}n_{it}(z)$$

- ▶ $z \in [0, 1/2]$ are domestic firms

$$y_{it}^*(z) = a_{it}^*n_{it}^*(z)$$

- ▶ $z \in (1/2, 1]$ are foreign firms

- ▶ Resource constraint

$$c_{it}(z) + (1 + \tau)c_{it}^*(z) = y_{it}(z) \text{ for } z \in [0, 1/2]$$

$$(1 + \tau)c_{it}(z) + c_{it}^*(z) = y_{it}^*(z) \text{ for } z \in (1/2, 1]$$

- ▶ Firms supply their goods to home and foreign cities [▶ back](#)

Price indexes for good i

- ▶ Under Calvo pricing,

$$\hat{p}_{it} = \lambda(\hat{p}_{it-1} - \pi_t) + (1 - \lambda)\hat{p}_{it}^{opt}(m_H, m_F)$$

$$\hat{p}_{it}^* = \lambda(\hat{p}_{it-1}^* - \pi_t^*) + (1 - \lambda)\hat{p}_{it}^{opt*}(m_F^*, m_H^*)$$

- ▶ $\hat{p}_{it}^{opt}(m_H, m_F)$, $\hat{p}_{it}^{opt*}(m_F^*, m_H^*)$ are the weighted average of reset prices:

$$\hat{p}_{it}^{opt}(m_H, m_F) = \omega\hat{p}_{Hi}(m_H) + (1 - \omega)\hat{p}_{Fi}(m_F)$$

$$\hat{p}_{it}^{opt*}(m_F^*, m_H^*) = \omega\hat{p}_{Fi}(m_F^*) + (1 - \omega)\hat{p}_{Hi}(m_H^*)$$

where $1/2 < \omega < 1$ is the degree of home bias as a function of trade costs τ [▶ back](#)

Proposition 2

Under the same assumptions in Proposition 1,

$$\rho_q \geq \lambda$$

provided $m \in (0, 1]$ and $\lambda \in (0, 1)$.

- ▶ Aggregate the LOP deviations

$$\ln q_{it} = \lambda \ln q_{it-1} + (1 - m)(1 - \lambda) \ln q_t + \lambda \varepsilon_t^n + \psi \varepsilon_{it}^r$$

to get

$$\ln q_t = \rho_q \ln q_{t-1} + \rho_q \varepsilon_t^n,$$

where

$$\rho_q = \frac{\lambda}{1 - (1 - m)(1 - \lambda)}$$



▶ back

Proposition 3

Under the same assumptions in Proposition 1,

$$\rho_q \geq \rho_{qi}$$

provided $m \in (0, 1]$, $\lambda \in (0, 1)$, $\tau \in [0, \infty)$, $\varepsilon \in (1, \infty)$, and $\sigma_r/\sigma_n \in [0, \infty)$.

- ▶ The relationship btwn ρ_q and ρ_{qi} is

$$\rho_q = \left[\frac{1}{1 - (1 - m)(1 - \lambda) \frac{A}{1+A}} \right] \rho_{qi}$$

where

$$A = \psi^2 \frac{1 - \rho_q^2}{\rho_q^2 (1 - \lambda^2)} \left(\frac{\sigma_r}{\sigma_n} \right)^2$$