Universität St.Gallen

Choice Under Fundamental Uncertainty: The Case of Aggregate Consumption

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Introduction and Motivation

The Point of Departure

In real world situations, households, managers, policy makers, often struggle to understand the true properties of the data generating process (DGP) of the "economic system".

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> *Economists are struggling to forecast how many people who left the workforce in 2020 will eventually return [...] They are also grappling with doubts over when consumers will shift their spending back to services, easing the upward pressure on goods prices caused by bunged-up supply chains.* ***Economic data have become harder to interpret. If retail sales fall, for example, does it reflect economic weakening, or a welcome return to normal patterns of consumption?***
>
> — The Economist (29th January of 2022)

Fundamental Uncertainty (FU)

FU surrounds many questions relevant for economic decision-makers:

- How will AI affect the productivity of labor and capital in the coming 10 years?
- Are we in a stagflationary period?
- (Asked a year ago:) Is the current (U.S.) inflation transitory or permanent?

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Many potential reasons for FU (agents not "knowing" the DGP): unknown state-space, changing (unstable) economic environment, irreducible disagreement about cause and effect, local vs. global properties, indeterminancy, observational equivalency, different identifying assumptions, bounded rationality, ...

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Decision-makers need to be able to operate under these uncertain conditions.

Fundamental Uncertainty

The Key Questions

How can we model an economy where decision-makers are (partially) ignorant about the data-generating processes?

Is such a model able to replicate real-world patterns in say aggregate consumption?

The Economic Setting

We model a representative agent (RA) maximising lifetime utility

- RA confronted with stochastic stream of labour income at the aggregate level of the economy
- **RA does not know the data-generating process (DGP) of labour income**
- RA's **only** information about the DGP of income is **past experience**
- RA decides consumption (and assets), forms income expectation
- Benchmark: Full information rational expectations (FIRE)
- We simulate c_t, a_t decisions out of the benchmark (**FIRE model**) and out of the model of fundamentally uncertain agents (**FU model**) based on empirical income (real US GDP)
- We compare simulated with empirical patterns (correlations, ...)

Let us begin with: Full Information Rational Expectations (FIRE)

Agents choose consumption as to maximise their lifetime utility:

$$\max_{(c_t, c_{t+1}, c_{t+2}, \ldots) \in \mathbb{B}_t} \mathbb{E}_t \sum_{h=0}^{\infty} \beta^h u(c_{t+h}) \quad \text{s.t. standard constraints} \tag{1}$$

The assets evolve according to:

$$(a_t + y_t - c_t)(1 + r) = a_{t+1} \tag{2}$$

Income evolves according to some true DGP (e.g. an AR(1) process) denoted by $f^*(y)$.

Solution (policy function) can be derived from Bellman equation for c_t:

$$V_t(a_t, y_t; f^*(y)) = \max_{c_t} \left\{ u(c_t) + \beta \cdot \mathbb{E}_{t; f^*(y)} \left[V_{t+1}(a_{t+1}, y_{t+1}; f^*(y)) \right] \right\} \tag{3}$$

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This problem cannot be solved without information about the true DGP. What is $E_t V_{t+1}$?

The Model

A Decision-Making Problem Feasible under FU

Our Proposal:

$$V_t(a_t, y_t; \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \omega \cdot V^{FU} \left(a_{t+1} + \mathbb{E}_{t;\hat{f}(y)} \left[y_{t+1} \right] \right) \right\} \tag{4}$$

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In case of logarithmic functions for agents' utility and valuation of next period resources:

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ \log\left(c_t\right) + \omega \cdot \log\left(a_{t+1} + y^*_{t+1|t} \right) \right\} \tag{5}$$

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Key differences vis-a-vis the FIRE agent's problem:

- The V^{FU} value function is independent of any models about DGP after $t+1$. V^{FU} is a valuation index for estimated next period financial resources.

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- Agents use a "mental model" of the DGP of income, $\widehat{f}(y)$, to make a point forecast for income in $t+1$. Certainty equivalence in V^{FU} results. $\mathbb{E}_{\hat{f}(y),t}\left[y_{t+1}\right] = y^*_{t+1|t}$

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- ω is a weight of importance of the future for the agent.

Interpreting the FU Decision Problem

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ \log\left(c_t\right) + \omega \cdot \log\left(a_{t+1} + y^*_{t+1|t}\right) \right\}$$

This is de-facto a two-period optimisation problem.

Agents face a trade-off between consumption today and their experience-based estimate of the value of resources available in the future.

Alternative way to think about it: Agent anticipates bequests of resources to their own future self. V^{FU} is a valuation index for the estimated bequested resources.

The Expectations under Fundamental Uncertainty: $\mathbb{E}_{t;\hat{f}(y)} = y^*_{t+1|t}$

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The model is able to accommodate a wide range of different forecasts or forecasting rules:

- **Heuristic rule**: simple extrapolation, constant forecasts
- **Empirically measured**: University of Michigan Index of Consumer Expectations, re-scaled
- **Learnt from the experience**: historical analogies (with k−nearest neighbors), regression, ...

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Example of Historical Analogies:

> *It was clear that the Fed needed to do more – but what? In response to current events, people often reach for historical analogies, and this occasion was no exception. The trick is to choose the right analogy. In August 2007, the analogies that came to mind—both inside and outside the Fed—were October 1987, [...], and August 1998, [...]. With help from the Fed, markets had rebounded each time with little evident damage to the economy.*
>
> — Bernanke (2015)

The Solution of FU Agents

$$
\begin{aligned}
V^{FU}(a_t, y_t, \bar{b}_t, \widehat{f}(y)) = \max_{c_t} & \left\{ \log(c_t) + \omega \cdot \log\left(a_{t+1} + y^*_{t+1|t} - \bar{b}^*_{t+1|t}\right)\right\} \quad s.t. \\
& (a_t + y_t - c_t)(1+r) = a_{t+1} \\
& a_t \geq \bar{b}_t
\end{aligned} \tag{6}
$$

The optimal consumption under FU becomes:

$$
c_t = \frac{1}{1+\omega}\left[a_t + y_t + \left(y^*_{t+1|t} - \bar{b}^*_{t+1|t}\right)/(1+r)\right] \quad \textit{if borrowing-constraint not binding} \tag{7}
$$

Expectations? Propagate with a factor of $((1+\omega)(1+r))^{-1}$ into c_t

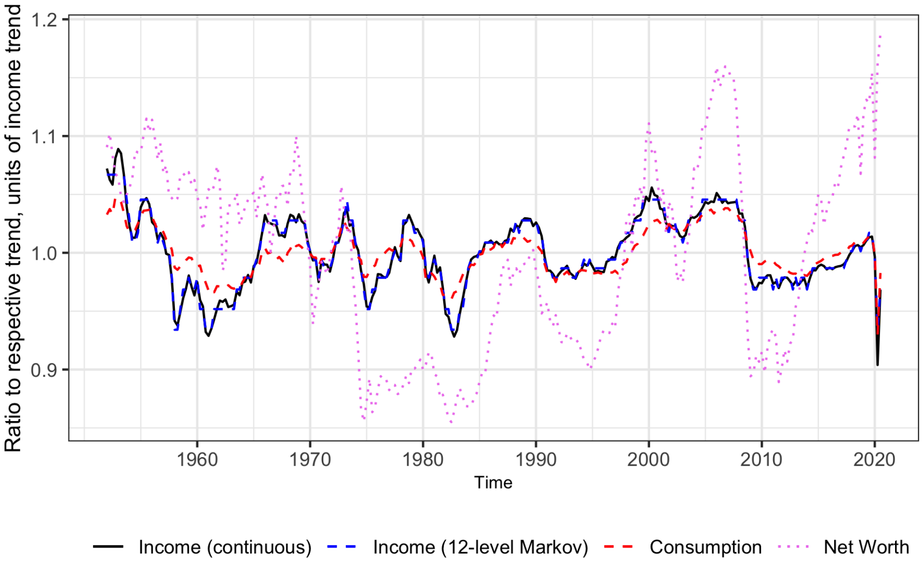
Results

Empirical Series (Income, Consumption, Assets)

- Data: Real U.S. GDP, Real Personal Consumption Expenditures, Net Worth of Households (all quarterly, from FRED)
- Start: Q1 1952, End: Q2 2021
- Trend estimated (in logs) as cubic smoothing spline with three knots
- y_t, c_t and a_t represented as ratios of realisations to their respective estimated trends
- For example: $y_t = 1$ means US Real GDP was equal to its estimated trend
- Another example: $c_t = 0.95$ means that US Real PCE were 5% below its trend that quarter
- For RE optimization: y_t modelled to be Markov with 12 states. Transition probabilities estimated by simulating a long series from an estimated AR(1) process of y_t.

The representative agent observes and decides on the "cyclical component" of income.

Empirical Series (Income, Consumption, Assets)



Correlations Between Simulated and Empirical Series

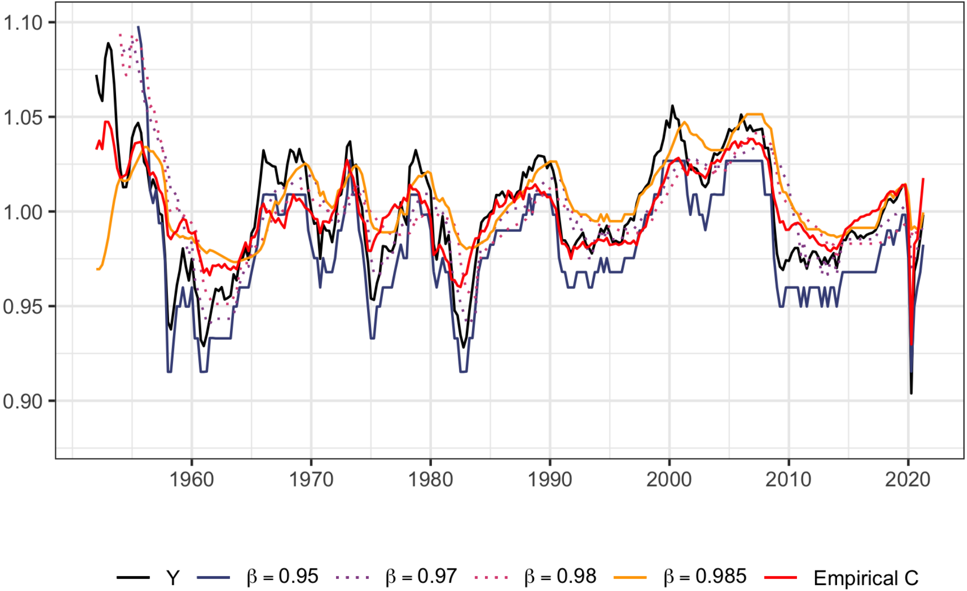
Panel A: FU model

	Historical Analogies		*Constant forcast*		*Simple Extrapolation*		*UoM Survey*	
Omega	Cons	Assets	Cons	Assets	Cons	Assets	Cons	Assets
1.0	0.856	0.047	0.848	0.535	0.872	-0.532	0.792	-0.15
2.0	**0.857**	**0.527**	**0.821**	**0.530**	**0.869**	**0.529**	**0.794**	**0.124**
3.0	0.846	0.525	0.798	0.516	0.857	0.516	0.783	0.247
4.0	0.833	0.496	0.777	0.496	0.843	0.496	0.769	0.288
6.0	0.804	0.446	0.741	0.447	0.814	0.447	0.719	0.261

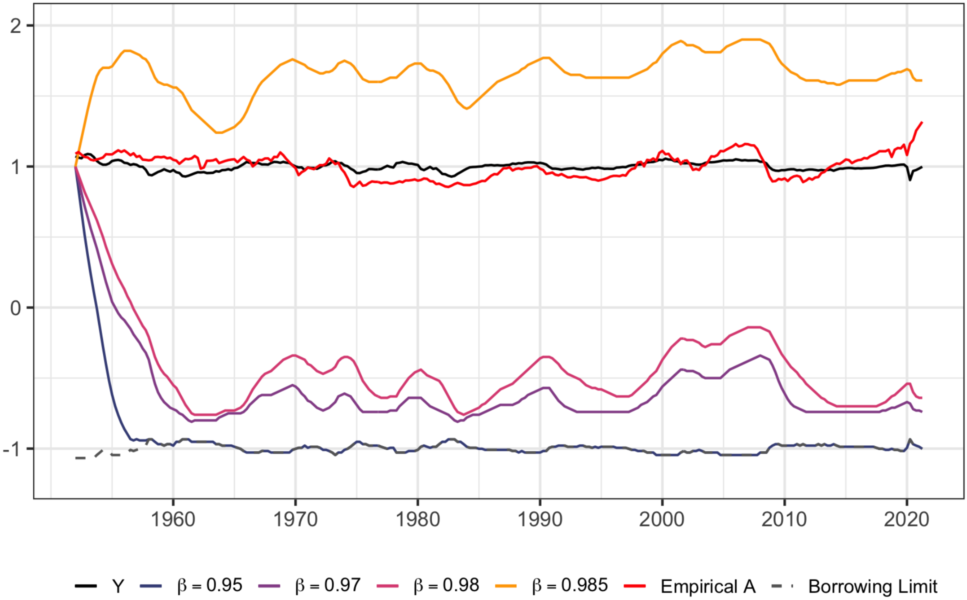
Panel B: FIRE model

Discount factor	Consumption	Assets
0.950	**0.862**	**-0.538**
0.970	0.757	0.396
0.980	**0.694**	**0.251**
0.985	0.785	0.311

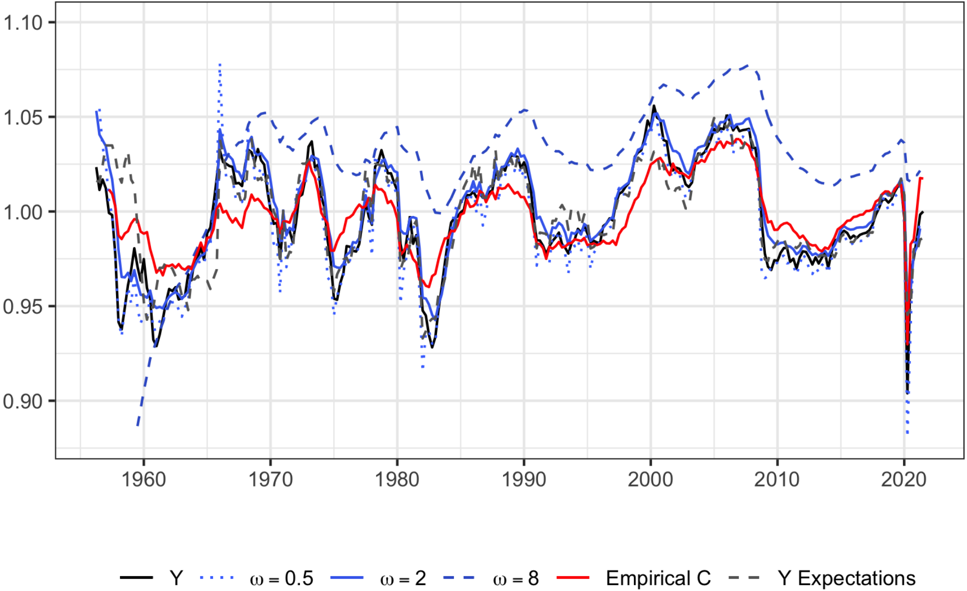
Consumption: Simulated for Benchmark (FIRE) and Empirical



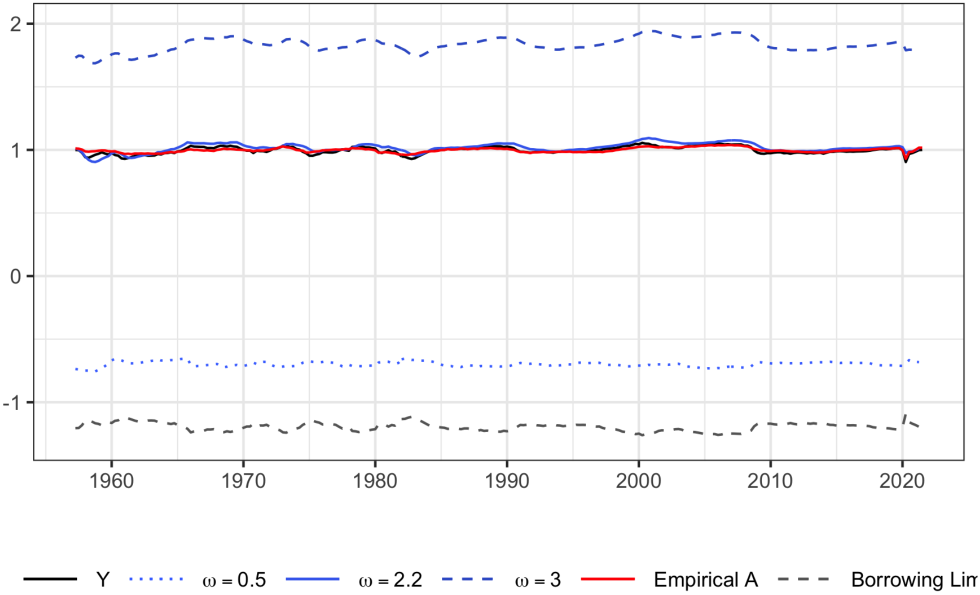
Assets: Simulated for Benchmark (FIRE) and Empirical



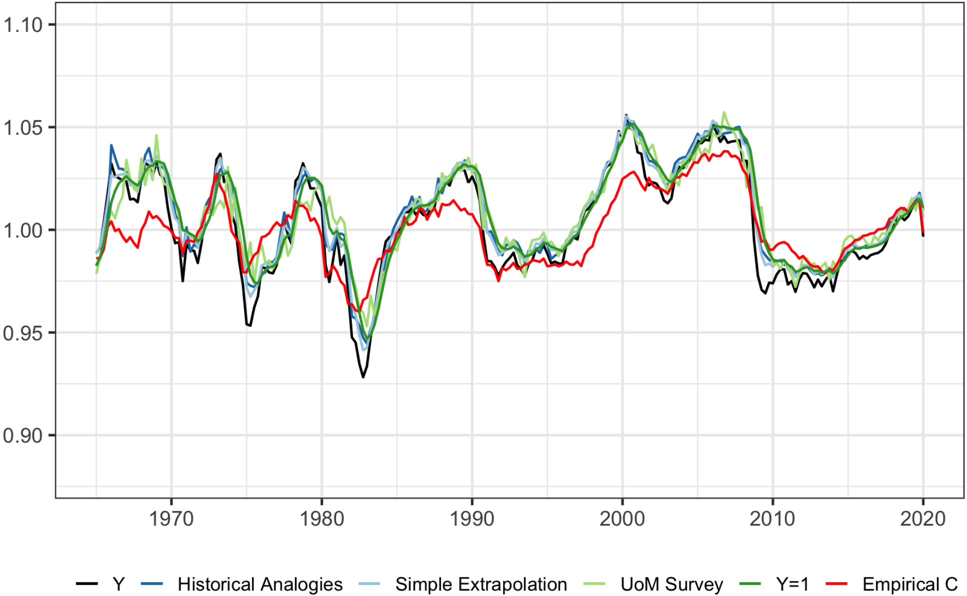
Consumption: Simulated for FU (Expect. from Historical Analogies)



Assets: Simulated for FU (Expectations from Historical Analogies)



Consumption: Simulated for FU; Various Expectations



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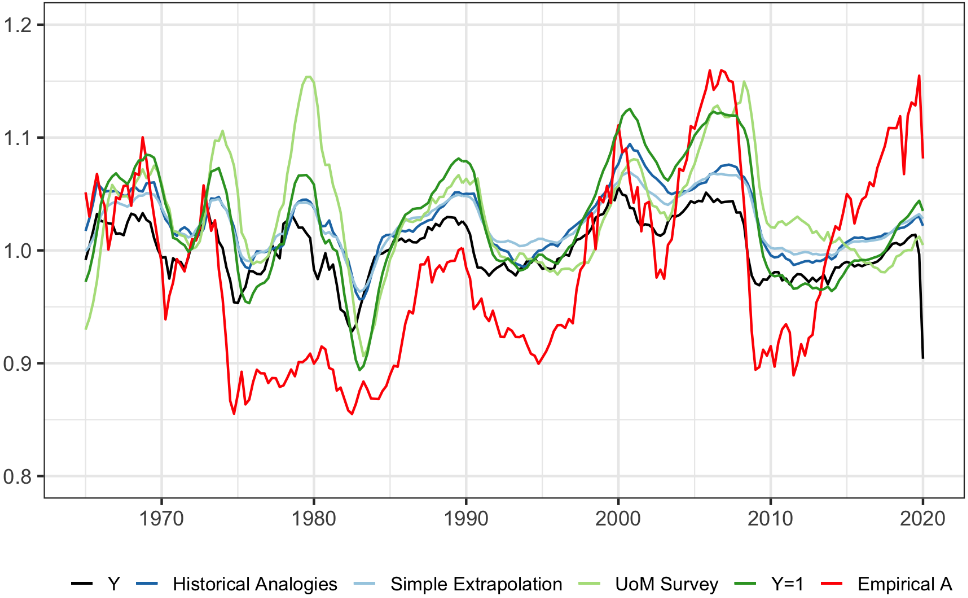


Conclusion

Our Goal Was...

To propose a model of decision-making under FU that

- Stays true to the standard intertemporal optimisation and utility maximisation as much as possible
- Enables the agent to operate with minimal information about the DGP
- Is cognitively and psychologically plausible

We have seen that:

- Forward-looking decisions under FU are possible and can be modelled.
- Even with very limited knowledge about DGPs, agents' decisions can match several first-order facts about consumption and assets quite well and with substantial degree of flexibility.

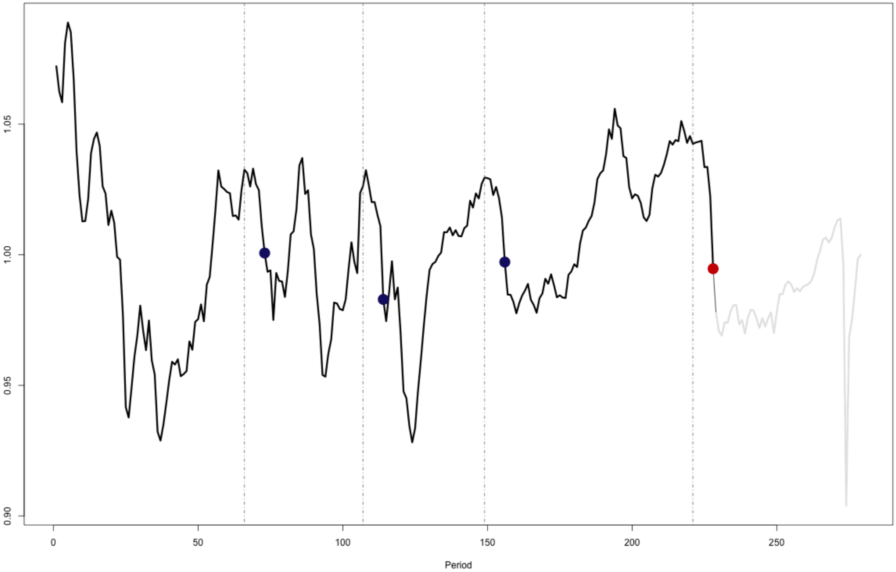
Much Work Remains...

- General equilibrium (decisions of firms, ...)
- Further intuition about the model (meaning of risk aversion, importance of expectations, ...)
- Are and if so in what ways are FU agents more "robust" than RE agents?
- Endogenous business cycles

Thank you for your attention

Appendix

Historical Analogies



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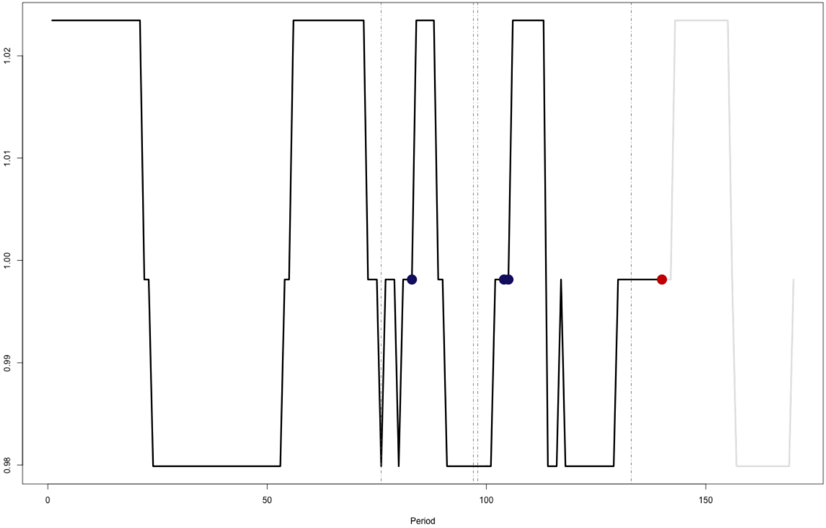

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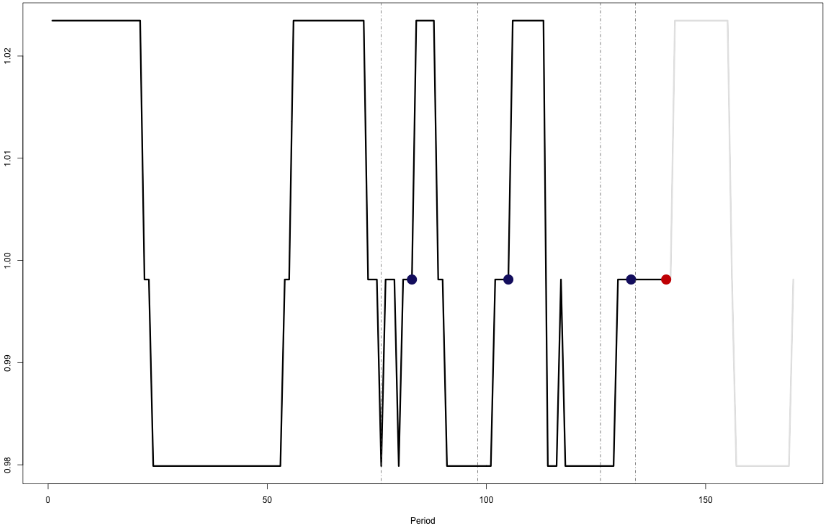

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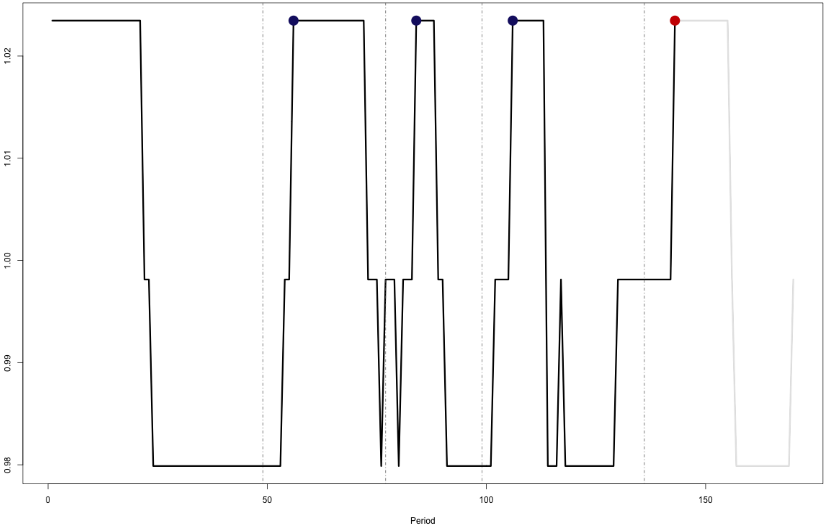

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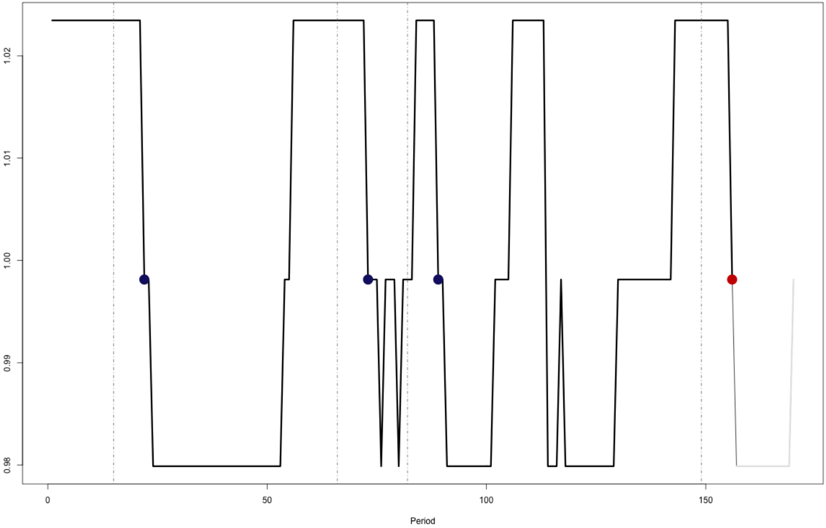

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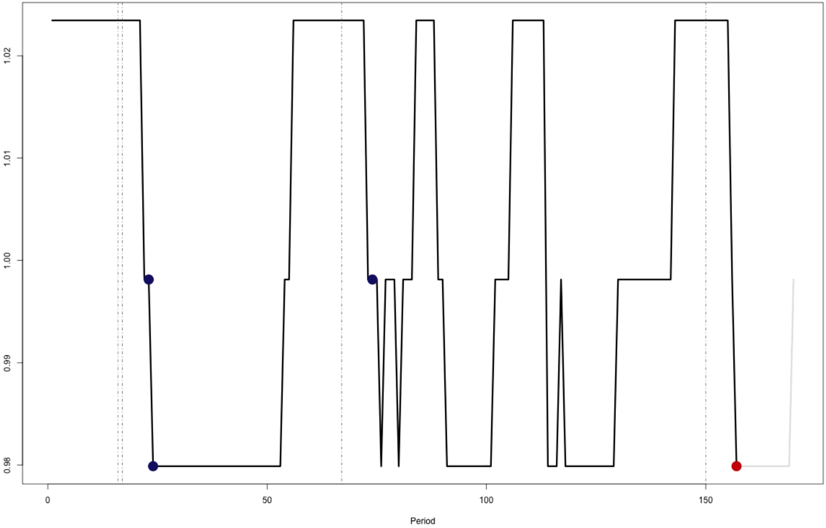

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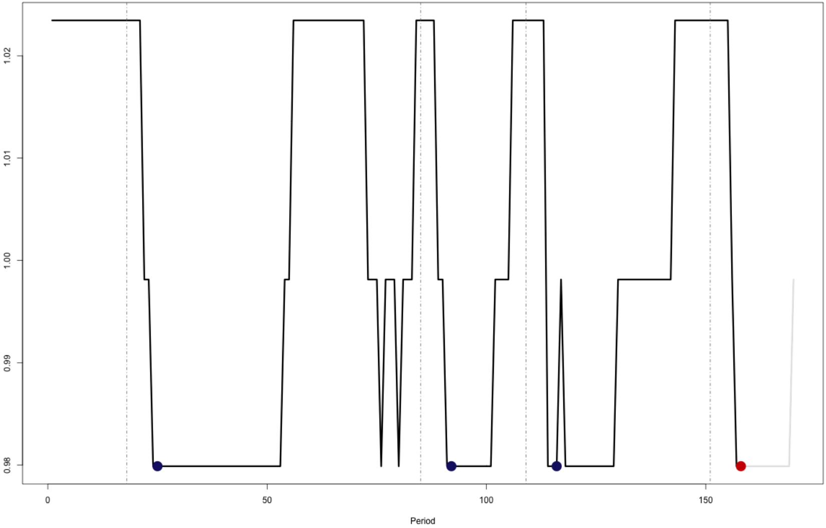

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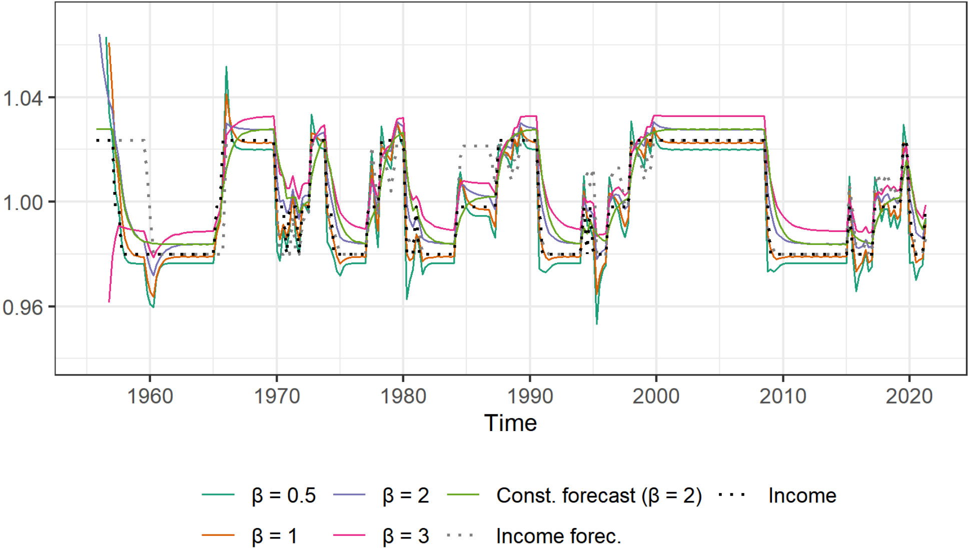

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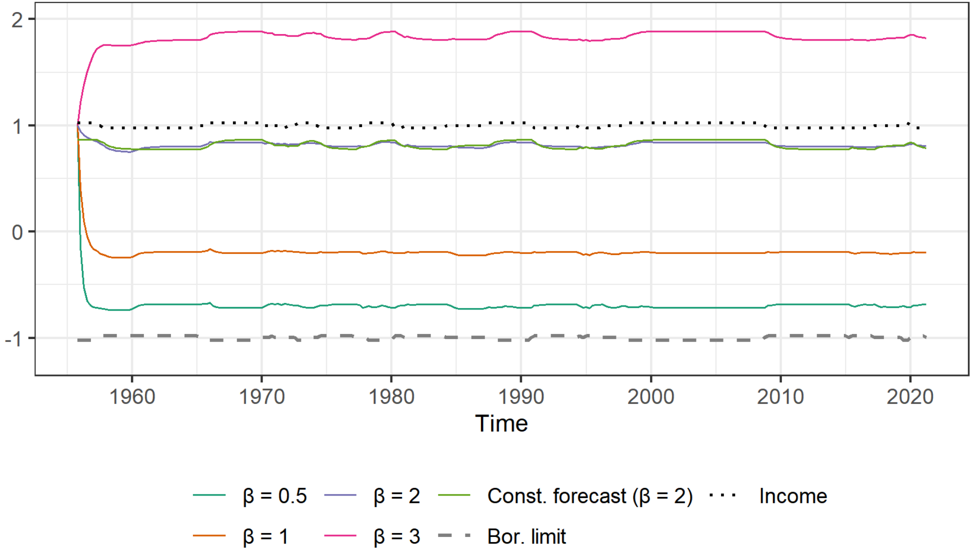

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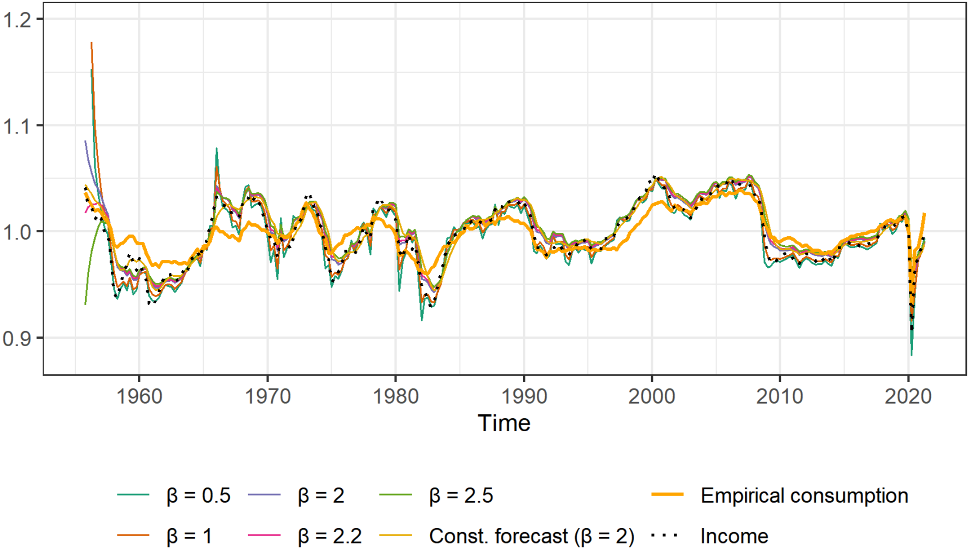

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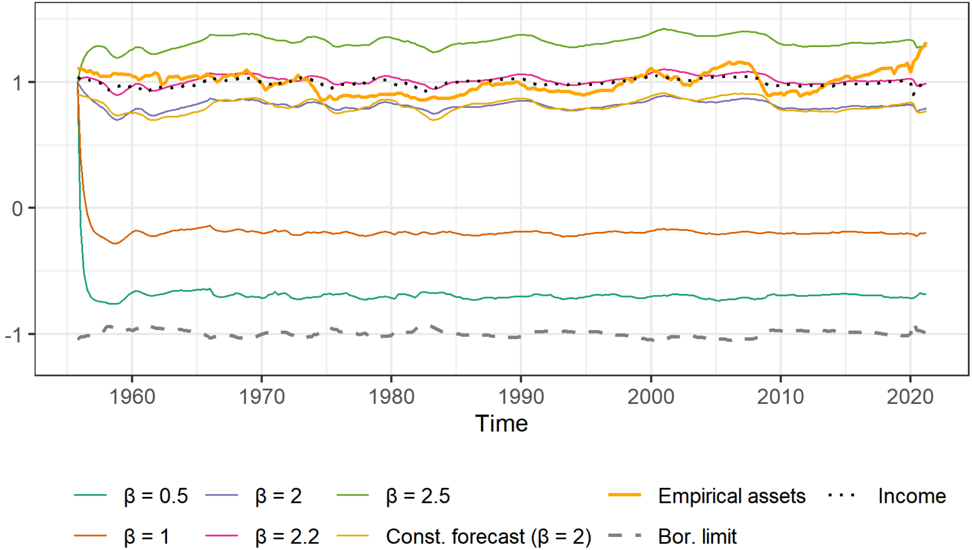

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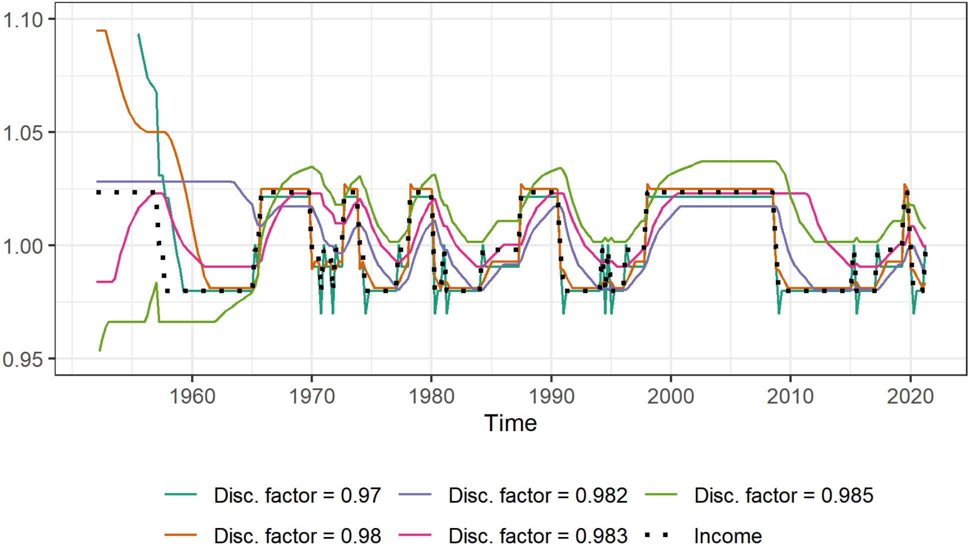

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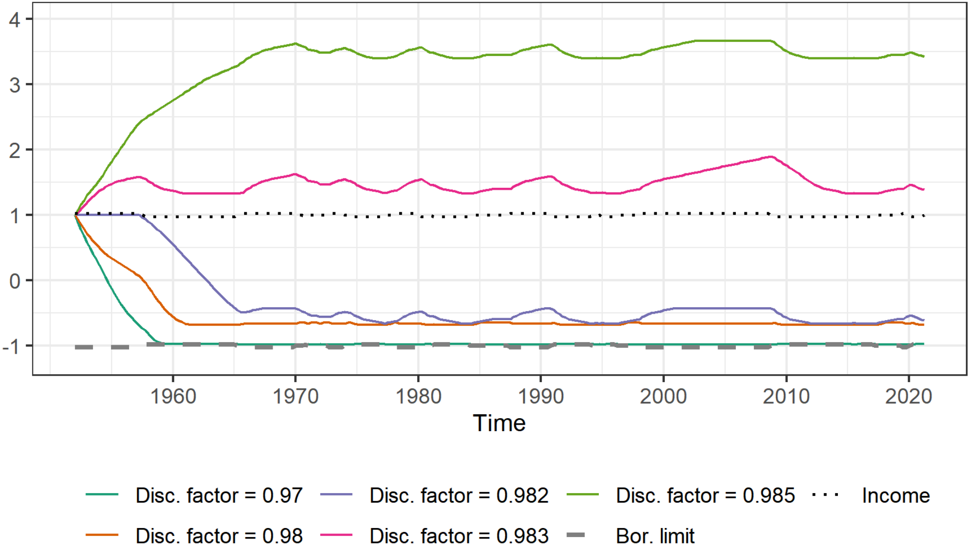

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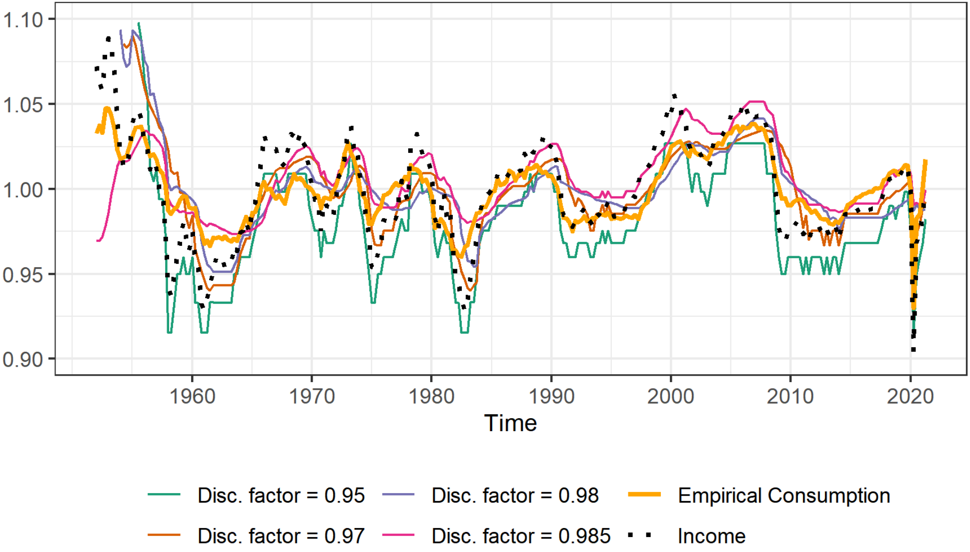

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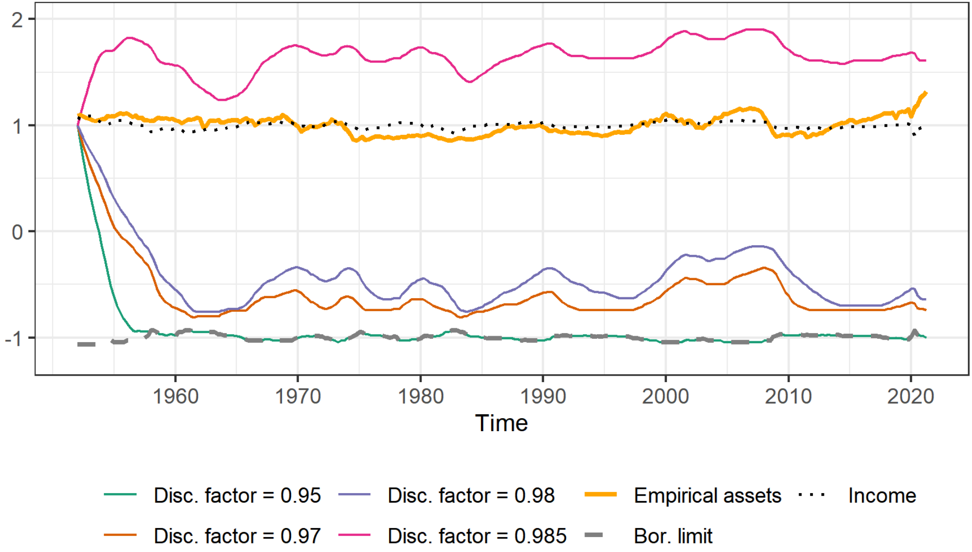

How Rational are the FU Agents?

	Dependent variable:		
	Y, kNN	Y, Simple Extrapolation	Y, RE
	(1)	(2)	(3)
$\mathbb{E}_{t-1}[Y_t]$	0.868***	0.913***	1.016***
	(0.052)	(0.027)	(0.027)
Constant	0.133**	0.087***	−0.016
	(0.052)	(0.027)	(0.027)
Observations	225	225	225
R^2	0.559	0.835	0.861
Note:			*p<0.1; **p<0.05; ***p<0.01

Hall's RE-PIH Test

	Dependent variable:
	C_empirical
lag(C_empirical)	0.931***
	(0.022)
Constant	0.069***
	(0.022)
Observations	271
R^2	0.869
Adjusted R^2	0.869
Residual Std. Error	0.007 (df = 269)
F Statistic	1,786.107*** (df = 1; 269)
Note:	*p<0.1; **p<0.05; ***p<0.01

Correctness of Income Forecasts

Panel A: FU model				
	Historical Analogies	*Constant forcast*	*Simple Extrapolation*	*UoM Survey*
RMSE	0.193	0.392	0.163	0.469
Panel B: RE model				
RMSE		0.155		

Deriving A Problem Solvable Under Fundamental Uncertainty (FU)

$$V_t(a_t, y_t, f^*(y)) = \max_{c_t} \left\{ u(c_t) + \beta \cdot \mathbb{E}_{f^*(y),t} \left[V_{t+1}(a_{t+1}, y_{t+1}, f^*(y)) \right] \right\}$$

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Approximation

$$\Downarrow$$

$$\widehat{V_t(a_t, y_t)} = \max_{c_t} \left\{ u(c_t) + \widehat{\beta} \cdot \widehat{\mathbb{E}_t} \left[\widehat{V_{t+1}(a_{t+1}, y_{t+1})} \right] \right\}$$

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Approximation

$$\Downarrow$$

$$\widehat{V_t(a_t, y_t)} = \max_{c_t} \left\{ u(c_t) + \widehat{\beta} \cdot \widehat{\mathbb{E}_t} \left[\widehat{V_{t+1}(a_{t+1}, y_{t+1})} \right] \right\}$$

$$\Downarrow$$

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \beta^{FU} \cdot \mathbb{E}_{\hat{f}(y),t} \left[V_{t+1}(a_{t+1}, y_{t+1}, \widehat{f}(y)) \right] \right\}$$

Deriving A Problem Solvable Under Fundamental Uncertainty (FU)

$$V_t(a_t, y_t, f^*(y)) = \max_{c_t} \left\{ u(c_t) + \beta \cdot \mathbb{E}_{f^*(y),t} \left[V_{t+1}(a_{t+1}, y_{t+1}, f^*(y)) \right] \right\}$$

Approximation

$$\Downarrow$$

$$\widehat{V_t(a_t, y_t)} = \max_{c_t} \left\{ u(c_t) + \widehat{\beta} \cdot \widehat{\mathbb{E}_t} \left[\widehat{V_{t+1}(a_{t+1}, y_{t+1})} \right] \right\}$$

$$\Downarrow$$

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \beta^{FU} \cdot \mathbb{E}_{\hat{f}(y),t} \left[V_{t+1}(a_{t+1}, y_{t+1}, \widehat{f}(y)) \right] \right\}$$

Simplification

$$\Downarrow$$

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \beta^{FU} \cdot \mathbb{E}_{\hat{f}(y),t} \left[V^{FU}(a_{t+1}, y_{t+1}) \right] \right\}$$

Deriving A Problem Solvable Under Fundamental Uncertainty (FU)

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \beta^{FU} \cdot \mathbb{E}_{\hat{f}(y),t} \left[V^{FU}(a_{t+1}, y_{t+1}) \right] \right\}$$

Deriving A Problem Solvable Under Fundamental Uncertainty (FU)

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \beta^{FU} \cdot \mathbb{E}_{\hat{f}(y),t} \left[V^{FU}(a_{t+1}, y_{t+1}) \right] \right\}$$

$$\Downarrow$$

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \beta^{FU} \cdot V^{FU} \left(a_{t+1}, \mathbb{E}_{\hat{f}(y),t} \left[y_{t+1} \right] \right) \right\}$$

Deriving A Problem Solvable Under Fundamental Uncertainty (FU)

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \beta^{FU} \cdot \mathbb{E}_{\hat{f}(y),t} \left[V^{FU}(a_{t+1}, y_{t+1}) \right] \right\}$$

$$\Downarrow$$

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \beta^{FU} \cdot V^{FU} \left(a_{t+1}, \mathbb{E}_{\hat{f}(y),t} \left[y_{t+1} \right] \right) \right\}$$

$$\Downarrow$$

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ \log\left(c_t\right) + \beta^{FU} \log\left(a_{t+1} + \hat{y}_{t+1|t}\right) \right\}$$

Deriving A Problem Solvable Under Fundamental Uncertainty (FU)

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \beta^{FU} \cdot \mathbb{E}_{\hat{f}(y),t} \left[V^{FU}(a_{t+1}, y_{t+1}) \right] \right\}$$

$$\Downarrow$$

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \beta^{FU} \cdot V^{FU} \left(a_{t+1}, \mathbb{E}_{\hat{f}(y),t} \left[y_{t+1} \right] \right) \right\}$$

$$\Downarrow$$

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ \log(c_t) + \beta^{FU} \log\left(a_{t+1} + \hat{y}_{t+1|t} \right) \right\}$$

This can be solved under FU!