The Dynamics of Firm-level Pay: Theory and Evidence from Portugal.*

Rui Castro[†] Gian Luca Clementi[‡]

June 2022

Submission draft – Preliminary and incomplete – Please do not circulate.

Abstract

Recent empirical work has emphasized the role played by firms in shaping earnings inequality. In this paper, we build a tractable model of firm-pay heterogeneity by introducing labor market's monopsonistic power in a Hopenhayn-style firm dynamics framework. The model naturally generates wage-size premia. We use our theory to study the dynamics of earnings inequality in Portugal. Similarly to other developed economies, earnings inequality in Portugal has increased sharply from the mid-1980s until the mid-1990s. However, it has been steeply decreasing ever since. We find that the decline in the variance of firm pay was the main force behind the decline in inequality. Our model suggests that the joint rise in the real minimum wage and in the fraction of workers with college degree may go a long way towards rationalizing this evidence.

Keywords: Firm Dynamics, Earnings Inequality, Labor Share.

JEL Codes: D33, E24, E25, L11, L25.

*We thank Pedro Bento, Sílvia Gonçalves, Pedro Raposo, and seminar participants at Banco de Portugal and Universidade Católica Portuguesa for helpful comments. All errors are ours. Rui Castro acknowledges financial support from SSHRC and the generous hospitality of Universidade Católica Portuguesa and the Bank of Portugal while working on this project. Both authors thank the Portugal's Instituto Nacional de Estatística for the data access.

†Department of Economics and CIREQ, McGill University. Email: rui.castro@mcgill.ca. Web: rui-castro.research.mcgill.ca.

‡Department of Economics, Stern School of Business, New York University and NBER. Email: clem@nyu.edu. Web: http://people.stern.nyu.edu/gclement/.

1 Introduction

Recent empirical work made possible by the availability of employer-employee matched datasets has shown that firms may have played an important role in shaping the dynamics of earnings inequality in both developed and developing countries.[1] In this paper, we build a tractable theory of firm-level pay heterogeneity, by introducing labor market's monopsonistic power in the familiar firm dynamics model due to Hopenhayn (1992).

Our model generates a monotone-increasing map from productivity into wages, the shape of which has equilibrium implications for earnings inequality. We use our theory to shed light on the evolution of earnings inequality in Portugal over the last 25 years.

Using high-quality administrative data, we document that, after a fast-paced increase between the mid-80s and the mid-90s, earnings inequality has been steeply declining, driven exclusively by a compression in firm pay. In fact, other components such as cross-worker inequality and assortative matching between workers and firms have pushed inequality in the opposite direction.

We did not find any appreciable decline in the heterogeneity of observable firm–level characteristics, such as productivity. The lower firm pay dispersion is instead associated to a decline in the pass-through from firm-level productivity onto wages.

The quantitative exercise informed by our theory suggests that the joint increase in the real minimum wage and in the fraction of workers with college degree may go a long way towards rationalizing this evidence. We show that, in terms of a AKM-type decomposition, both factors lower the variance of firm fixed effects and the variance of worker fixed effects. In turn, this means that for the model's equilibrium to generate a data-conforming increase in the variance of worker fixed effect, a substantial skill-biased technical change must have taken place in the meantime.

The remainder of this paper is structured as follows. In Section 2, we carefully document the evolution of earnings inequality in Portugal and we highlight the role played by firms. In Section 3 we introduce our model and define the equilibrium. In Section 4 we characterize firm pay in stationary equilibrium. In Section 5 we introduce worker heterogeneity and gauge

[1] See for example Abowd et al. (1999); Card et al. (2013); Song et al. (2018); Alvarez et al. (2018).

the role played by the rise in minimum wage and in educational attainment in shaping the dynamics of inequality.

2 Earnings inequality in Portugal: Evidence

2.1 Quadros de Pessoal

The earnings data is from Quadros de Pessoal (QP), an administrative data set covering all private-sector firms employing at least one salaried worker, including those owned by the government. The data is collected annually by the Ministry of Labor to ensure compliance with labor laws. The reporting is for the month of March until 1993 and for October since. The data is unavailable for 1990 and 2001.

QP covers all workers employed in Portugal, except civil servants and independent contractors, as well as temporary rural and domestic workers. It is a matched employer-employee data set, providing comprehensive information on workers and some basic information on firms. We focus on workers aged 18–65, working full-time (at least 140 normal monthly hours), and earning at least the minimum wage as base pay.[2] We exclude firms in farming and fishing, since earnings in these sectors are often not reported on a monthly basis, and focus on for-profit non-financial market-oriented sectors.

Our favored measure of compensation is total monthly earnings, i.e. base salary plus overtime and regular monthly supplements such as productivity and seniority premia, as well as compensating differentials.[3] To avoid outliers, we trim the top 1% earnings observations per year. Further details on the data are in Appendix A.

2.2 The dynamics of earnings inequality

Figure 1 plots the variance of log earnings since 1986, the year Portugal joined the European Union, then known as the European Economic Community.

[2]The data on the (general) minimum wage and on the (private consumption, base year 2016) CPI are from https://www.pordata.pt/en/Home

[3]When running our analysis using hourly wages instead, the magnitudes change slightly, but the message does not.

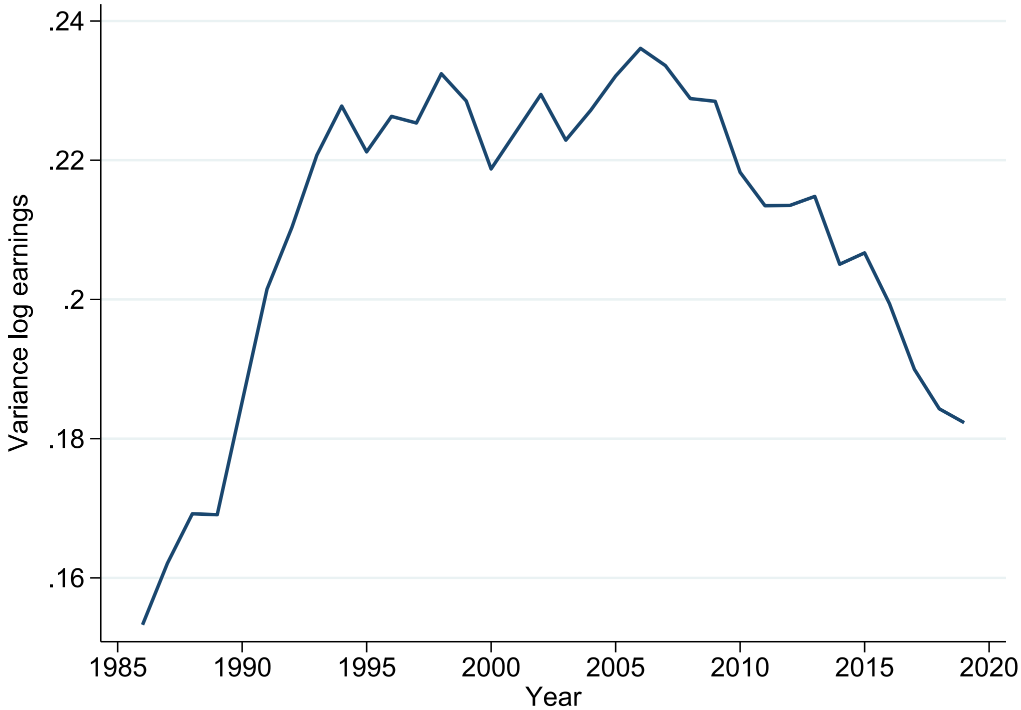


Figure 1: Quadros de Pessoal, 1986-2019

Inequality has increased dramatically from 1986 until the mid-1990s. The variance grew by about 50% over just a decade. From the mid-1990s onwards, however, inequality has first plateaued and then briskly declined since the financial and sovereign debt crisis of 2008-9. The variance of earnings dropped by more than one fifth between the mid-1990s' peak and 2019. An yearly rate of decline of about 1% is economically significant and comparable to the rate of increase in the same measure of inequality reported by Song et al. (2018) for the United States between 1985 and 2009.

As far as we know, this non-monotonic dynamics is unique in the developed world. The sharp increase in inequality from the mid-1980s until the mid-1990s was also documented by Cardoso (1999), using the same data. This paper focuses on inequality past the mid-90s, a period during which, thanks to the reduction in informality, Quadros de Pessoal has significantly increased worker coverage.

Inspired by the recent empirical literature highlighting the key role that firms have played in shaping inequality in Germany (Card et al., 2013), the U.S. (Song et al., 2018) and Brazil (Alvarez et al., 2018), we ask whether employers have played a substantial role in Portugal as well.

2.2.1 The role of firms: A first look

Figure 2 displays a decomposition of the variance of log earnings into the variance of average earnings between firms and the employment-weighted average of the within-firm earnings variances.

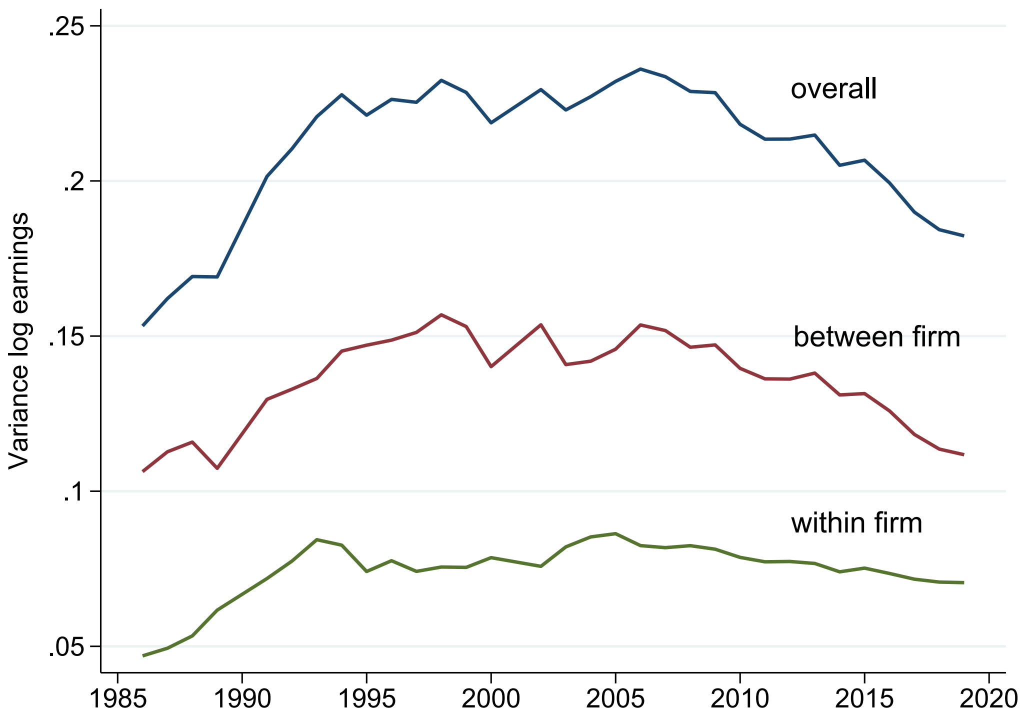


Figure 2: Between-within firm decomposition

The decomposition gives a first indication of the importance of firms in the Portuguese context. The between-firm component accounts for about 60% of the level. Most importantly for our purpose, it is also the main component behind the dynamics, accounting about 3/4 of the increase until the mid-1990s and for about 90% of the decline since the mid-1990s.

A natural question is whether the evolution of the between-firm component is driven by changes over time in the composition of the workforce at the firm level or by variation in intrinsic firm–level characteristics.

2.2.2 AKM regression

To isolate the role played by intrinsic firm–level characteristics, we now resort to the regression framework due to Abowd et al. (1999). The AKM approach extends the prototypical Mincer wage regression specification by adding firm fixed effects to control for observed and unobserved systematic firm-level characteristics, in addition to worker fixed effects and a set of time-varying observables. Our regression equation for the two-way fixed effect (AKM) model is

$$y_{ijt} = \mathrm{WFE}_i + \mathrm{FFE}_j + \mathbf{X}'_{ijt}\beta + \varepsilon_{ijt}. \tag{1}$$

For the time-varying observables we follow Card et al. (2013) and include a full set of year dummies and a worker age cubic polynomial, both fully interacted with schooling attendance dummies.[4] As is standard in the literature, we estimate (1) over six non-overlapping subperiods p of four/five years each. This is a simple way to capture variation over time in the estimated coefficients.[5] We therefore estimate cross-sectional distributions of WFE_i^p and FFE_j^p per subperiod p, plus the coefficients in β^p.

For estimation purposes, we restrict our sample in a number or ways. First, we focus on the largest connected set of all firms (and their workers) for which every worker in the set has ever worked for. Second, we discard singletons (unique firm-worker observations). Appendix C describes further sample restrictions, such as dropping very small firms to mitigate the limited mobility bias when estimating the cross-sectional variances, and restricting to workers sufficiently away from the minimum wage. Our main results are not very sensitive to excluding these firms.

[4] As in Card et al. (2013), age is normalized (by deviations from the age of 40) so that the polynomial implies flat experience profiles at age 40. The linear effect of age is dropped as it is captured by the worker fixed effects together with time. Likewise, the independent effect of schooling is also captured by the worker fixed effects.

[5] Engbom et al. (2021) consider an alternative approach by extending the original AKM's identification requirement. They show that firm-specific time trends are identified within a set of workers which are mobile not just across firms, but across firm-years.

2.2.3 Basic variance decomposition

Ignoring the time-varying observables for now, the AKM regression delivers the following basic variance decomposition by subperiod p:

$$\mathrm{var}\left(y_{ijt}\right) = \underbrace{\underbrace{\mathrm{var}\left(\mathrm{WFE}_i\right)}_{\text{fixed}} + \underbrace{\mathrm{var}\left(\varepsilon_{ijt}\right)}_{\text{random}}}_{\text{workers}} + \underbrace{\underbrace{\mathrm{var}\left(\mathrm{FFE}_j\right)}_{\text{firm pay}} + \underbrace{2\mathrm{cov}\left(\mathrm{WFE}_i, \mathrm{FFE}_j\right)}_{\text{assortative matching}}}_{\text{firms}}. \tag{2}$$

The first two components are attributed to workers. $\mathrm{var}(\mathrm{WFE}_i)$ is the dispersion in the systematic component that captures observable features such as schooling and gender, as well as unobservable characteristics such as motivation and inter-personal traits. $\mathrm{var}(\varepsilon_{ijt})$ is the dispersion in the random component capturing permanent or transitory earnings changes due to shocks to, say, health or training. The two remaining components are attributed to firms. The first is the dispersion in the systematic firm-level component capturing characteristics such as productivity, size, or management quality. This is the firm pay component, enjoyed by any worker employed by a specific firm. The second is an assortative matching component conveying the extent to which high-wage workers are employed by high-wage firms.

The dispersion in worker fixed effects could vary across subperiods due to either persistent changes in the systematic components of continuing workers, or to cohort effects induced by the characteristics of either labor force entrants or exiters. In turn, the latter could stem from the significant increases in college graduation and female labor force participation which have taken place since the mid-1980s. The dispersion in firm pay may have changed due to cross-period changes in incumbents' pay policies, or may be due to cohort effects induced by the specific characteristics of entrants and exiters within each subperiod.

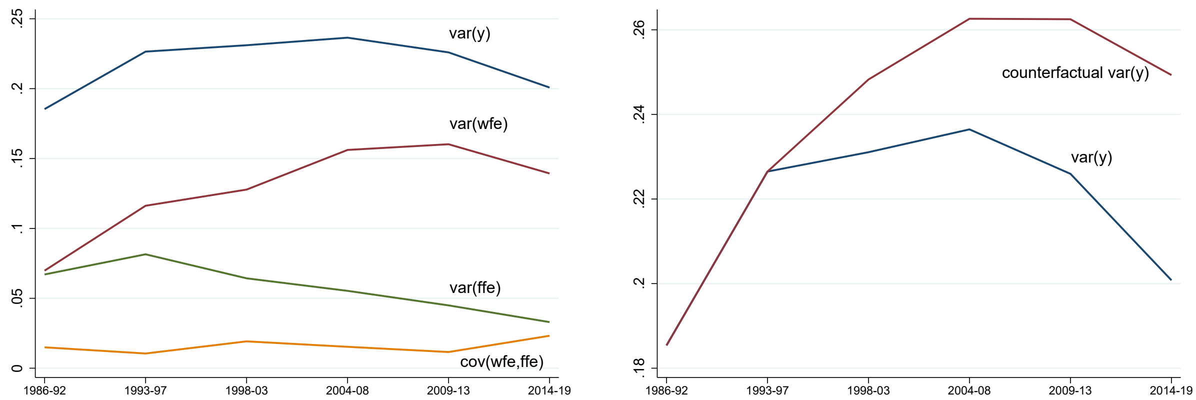
The results of this decomposition exercise are in Table 1. We emphasize two main trends across three subperiods: The increase in inequality from the mid-1980s until the mid-1990s and the decline since the mid-1990s. The table contains also the time-varying observables (Xb) that were omitted in (2).

Figure 3a shows the period-by-period dynamics of the most significant components from Table 1. The variance of firm pay is a significant driver of the increase in overall earnings inequality until the mid-1990s and it's by far the key force behind the decline since then. In

	Interval 1 (1986-1992)		Interval 2 (1993-1997)		Interval 6 (2014-2019)		Change 1–2		Change 2–6	
	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share
Var(y)	.185	–	.226	–	.201	–	.041	–	-.026	–
Var(WFE)	.07	37.6	.116	51.3	.139	69.4	.046	112.9	.023	-89.9
Var(FFE)	.067	36.2	.081	36	.033	16.4	.014	35.1	-.049	188.7
Var(Xb)	.012	6.7	.004	1.8	.008	3.9	-.008	-20.1	.004	-14.6
Var(ϵ)	.015	8	.014	6.3	.011	5.3	0	-1	-.004	14.1
2Cov(WFE, FFE)	.015	8.1	.011	4.6	.023	11.6	-.004	-10.8	.013	-49.3
2Cov(WFE, Xb)	.002	1	-.002	-.7	-.012	-6.2	-.003	-8.1	-.011	42.7
2Cov(FFE, Xb)	.005	2.5	.001	.6	-.001	-.4	-.003	-8	-.002	8.3

Table 1: Basic Decomposition

fact, it is the only major force pushing for earnings compression. It accounts for 189% of the overall decline in inequality and outmatches the impact of greater worker heterogeneity, responsible for a negative contribution of 90%.



(a) Main components

(b) Counterfactual earnings variance

Figure 3: Basic variance decomposition

An alternative way to appreciate the importance of the compression in firm pay for the overall inequality decline is to evaluate (2) while counterfactually holding the variance of firm fixed effects constant at the mid-1990s level. Figure 3b shows that earnings inequality would have increased rather than declined.

Such outsize role of firm pay as a driver of earnings inequality is in common with the experiences of Brazil and Ecuador, where inequality has steadily declined, and with those of U.S. and Germany, where inequality has steadily increased.[6]

2.2.4 Detailed variance decomposition

A more detailed decomposition, along the lines of Song et al. (2018), splits the overall variance into a within-firm and a between-firm component. Once again abstracting from the

[6]See Alvarez et al. (2018) for Brazil, Messina and Silva (2019) for Ecuador, Card et al. (2013) for Germany, and Song et al. (2018) for the United States

time-varying observables:

$$\text{var}(y_{ijt}) = \underbrace{\text{var}\left(\text{WFE}_i - \overline{\text{WFE}}_j\right) + \text{var}(\varepsilon_{ijt})}_{\text{within-firm}} + \underbrace{\underbrace{\text{var}(\text{FFE}_j)}_{\text{firm pay}} + \underbrace{2\text{cov}\left(\overline{\text{WFE}}_j, \text{FFE}_j\right)}_{\text{sorting}} + \underbrace{\text{var}\left(\overline{\text{WFE}}_j\right)}_{\text{segregation}}}_{\text{between-firm}}. \quad (3)$$

The various components add up to the between and within firm components of Figure 2. The payoff consists in gaining further insight into the worker-firm matching patterns. We now have a sorting component reflecting the tendency for high-wage firms to attract high-wage workers and a segregation component reflecting the tendency for firms to employ similar workers.

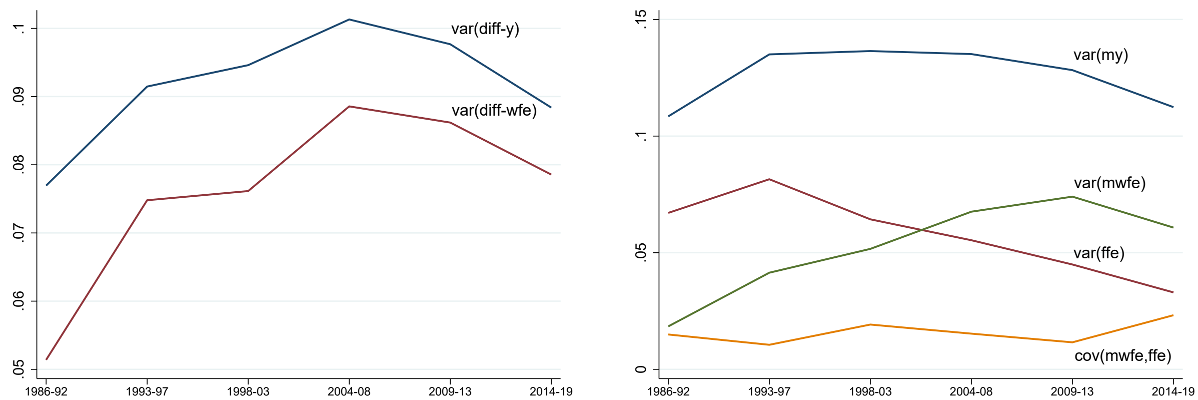
Table 2 provides a comprehensive account of the results, while Figure 4 zeroes in on the main components. Figure 4a suggests that the overall increase in the variance of worker fixed effects is mirrored within Portuguese firms. Within-firm inequality rose simply because the dispersion in relevant worker-level characteristics rose in Portugal. Figure 4b confirms that the compression in firm pay since the 1990s has been the chief driver of earnings inequality. Whereas segregation exerts only a slight force towards an increase in inequality (-12.7% of the total variance since the mid-1990s), sorting between Portuguese workers and firms has increased significantly, a powerful force leading to higher inequality (-49.3%). The bottom-line is that compression in firm pay has counteracted both of these forces and has single-handedly brought earnings inequality down.

2.3 Determinants of compression in firm pay since the mid-1990s

What are the economic forces primarily responsible for the greater compression in firm pay? The literature has suggested that firm-level heterogeneity in pay may be intimately tied to the variation in firm-level productivity in a non-competitive labor market. This motivates us to investigate the association between firm pay and firm-level productivity. We postulate the following log-linear specification for firm j's pay policy in subperiod p:

	Interval 1 (1986-1992)		Interval 2 (1993-1997)		Interval 6 (2014-2019)		Change 1–2		Change 2–6	
	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share
Var(y)	.185	–	.226	–	.201	–	.041	–	-.026	–
Var(m_y)	.108	58.5	.135	59.6	.112	56	.027	64.7	-.023	88
Var(m_WFE)	.018	9.9	.041	18.3	.061	30.3	.023	56	.019	-75.3
Var(FFE)	.067	36.2	.081	36	.033	16.4	.014	35.1	-.049	188.7
Var(m_Xb)	.002	1	.001	.2	.001	.5	-.001	-2.9	0	-1.9
2Cov(m_WFE, FFE)	.015	8.1	.011	4.6	.023	11.6	-.004	-10.8	.013	-49.3
2Cov(m_WFE, m_Xb)	.002	.9	0	-.1	-.005	-2.4	-.002	-4.7	-.004	17.4
2Cov(FFE, m_Xb)	.005	2.5	.001	.6	-.001	-.4	-.003	-8	-.002	8.3
Var(diff_y)	.077	41.5	.091	40.4	.088	44	.015	35.3	-.003	12
Var(diff_WFE)	.051	27.7	.075	33	.079	39.1	.023	56.9	.004	-14.6
Var(diff_Xb)	.011	5.7	.004	1.6	.007	3.4	-.007	-17.2	.003	-12.7
Var(ϵ)	.015	8	.014	6.3	.011	5.3	0	-1	-.004	14.1
2Cov(diff_WFE, diff_Xb)	0	.1	-.001	-.5	-.008	-3.8	-.001	-3.4	-.006	25.2

Table 2: Detailed Decomposition



(a) Within-firm

(b) Between-firm

Figure 4: Detailed Decomposition

$$\text{FFE}_{jp} = \alpha_p + \gamma_p \log z_{jp}, \tag{4}$$

where z_{jp} is the firm's within-period systematic productivity level, and γ_p is the pass-through from productivity to the firm-level component of wages. From the firm pay policy (4) we obtain

$$\text{var}\left(\text{FFE}_{jp}\right) = \gamma_p^2 \text{var}\left(\log z_{jp}\right), \tag{5}$$

which indicates two possible sources for the compression in firm pay: (i) A compression in the distribution of firm–level productivity and (ii) a decline in the pass-through.

We first ask whether the Portuguese data is consistent with a specification such as (4). We do this by regressing the estimated AKM firm fixed effects on measures of firm–level productivity, following Alvarez et al. (2018) very closely.

2.3.1 Sistema de Contas Integradas das Empresas

We link the Quadros de Pessoal with the Sistema de Contas Integradas das Empresas (SCIE), which allows us to compute measures of firm-level productivity. The SCIE provides a broad coverage of private-sector non-financial and for-profit firms operating in Portugal. However, it is only available since 1996.

Between 1996 and 2004, the information is drawn from the Inquérito à Empresa Harmonizado (IEH), a survey compulsory for all large private-sector corporations and a random sample of small corporations (less than 100 employees, or sales less than 5 million euros).

Starting in 2005, it is based upon administrative information (Informação Empresarial Simplificada, IES) on the universe of private-sector non-financial and for-profit firms operating in Portugal, drawn in particular from balance-sheet data. This is significantly higher quality information, used by the Public Administration and the Bank of Portugal. The number of available variables was increased with the IES, and SCIE coverage was also extended to include entrepreneurs and independent contractors, although we restrict our sample to corporations. In 2010 there was a major overhaul of the data reporting, for harmonization purposes with the rest of the E.U., which was applied retroactively to some of the variables starting in 2004. Some of the variables are therefore only available since 2004, and some others since 2010. Further details can be found in Appendix B.

2.3.2 Firm–level productivity and firm pay

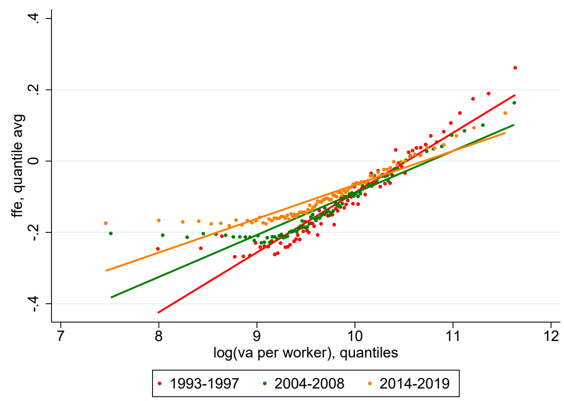
We compute two alternative measures of firm-level productivity. Our favorite is firm–level TFP, computed for firm j in sector s and year t as

$$z_{jt} = \frac{y_{jt}}{n_{jt}^{\alpha_n^s} m_{jt}^{\alpha_m^s} k_{jt}^{1-\alpha_n^s-\alpha_m^s}},$$

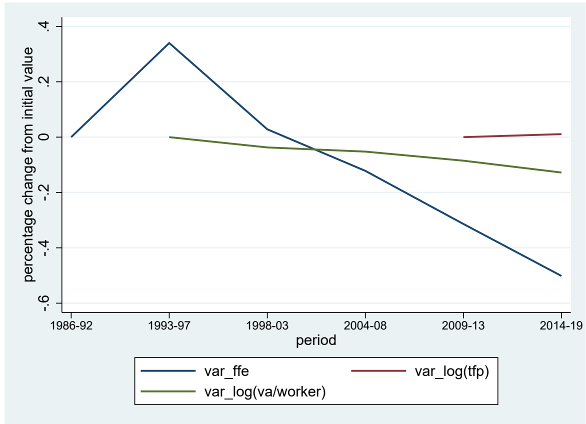
where y is total operating revenue (CPI deflated), n is the number of employees, m is the real value of intermediates, k is the real book value of capital, and the α's are the sector-specific averages of the firm–level revenue shares of total expenditures in labor and materials and services.[7] Unfortunately information on the book value of capital is only available since 2010, and so is firm-level TFP. We therefore rely mostly on real value-added per worker (CPI deflated), which is available since 1996 and hence covers the whole period of firm pay compression. We average each measure of productivity over each subperiod. To avoid outliers, we trim the top and bottom 1% observations by subperiod.

Figure 5 displays the evolution of the two potential determinants of firm pay compression. Figure 5a illustrates the relationship between firm pay and value-added per worker across different subperiods. It plots the average log value-added per worker quantiles against the

[7] We average over observations with strictly positive value added.



(a) Firm pay and value added per worker



(b) Variance of firm pay, value-added per worker, and TFP

Figure 5: Determinants of Firm Pay Compression

corresponding firm fixed effects, averaged over each quantile. Quite clearly, high-productivity firms pay more. However, it is also quite clear that the pass-through has weakened over time, especially from the mid-1990s to the mid-2000s.

Figure 5b displays the evolution of firm–level productivity dispersion against the dispersion in firm fixed effects which we wish to explain. The dispersion in value-added per worker decline only very slightly over time, whereas the dispersion in firm–level TFP increased slightly, although our sample is unfortunately very short as explained above. The evidence therefore suggests that the compression of firm pay is entirely due to the weakening of the pass-through from productivity to wages.

To provide a quantitative assessment of the contribution of the pass-through dynamics, in each subperiod we regress the firm fixed effects on log value-added per worker (regression weighted by worker-years, for comparability with the variance of firm fixed effects computed out of the AKM regression). We also consider sector and firm size controls, since these are likely co-determinants of firm pay. The results, reported in Table 3, show a reduction in the pass-through. The baseline regression without controls displays a decline from 0.173 to 0.117. These figures are somewhat lower but yet comparable to those estimated by Alvarez et al. (2018) for Brazil – a decline from 0.26 to 0.14 between 1996-2000 and 2008-2012.[8]

[8]Our figures are also very much in line with the 0.1-0.15 range reported by Card et al. (2018) for Portugal, although they restrict to male workers in the late 2000s and employ a different data set for firm–level characteristics.

	1993-97			1998-03			2004-08			2009-13			2014-19		
log va per worker	.173	.16	.132	.169	.156	.14	.147	.131	.124	.124	.117	.111	.117	.113	.107
log size	–	.05	.04	–	.041	.03	–	.031	.03	–	.018	.016	–	.017	.016
sector dummies	no	no	yes	no	no	yes	no	no	yes	no	no	yes	no	no	yes
R^2	.235	.364	.469	.281	.395	.468	.2	.269	.284	.169	.208	.219	.198	.249	.261

Notes: All coefficients are significant at 1%. Weighted by worker-years.

Table 3: Regression of firm pay on firm characteristics

3 Model

Our model nests a monopsonistic labor market in a off-the-shelf theory of firm dynamics along the lines of Hopenhayn (1992). The framework has clear similarities with those introduced in Gouin-Bonenfant (2022) and Bilal et al. (2021), respectively. Our environment is more parsimonious and closer in spirit to the firm dynamics literature. We concentrate our attention on the implications for wages.

Time is discrete. In every period, a continuum of firms produce an homogeneous good by means of the production function $y = zl^\gamma$, where l is labor, z is a firm-level (idiosyncratic) productivity index, and $0 < \gamma \leq 1$. Aggregate product demand is infinitely elastic at the unitary price. Production entails a fixed cost $c_f > 0$.

Productivity takes value on a discrete support, i.e. $z \in Z = \{z_1, \ldots, z_N\}$, and its evolution is traced by the Markov chain Π with typical element $\pi_{ij} \equiv \Pr(z_{t+1} = z_j | z_t = z_i)$. Firms whose expected discounted profits are negative exit.

There is a constant mass $N_w > 0$ of hand-to-mouth consumers that sell labor services to firms. The value to a worker of a wage offer w is

$$u(w) = w + \varepsilon,$$

where ε is a match-specific amenity shock distributed according to the Gumbel distribution with location parameter 0 and scale parameter $\sigma \geq 0$. In other words, ε proxies for non-pecuniary aspects of the job offer that the worker may value, such as proximity, colleagues' characteristics, and required effort.

In every period, each worker is matched with two firms, drawn randomly from the population. This means that each worker will get at most two offers. When she indeed gets two wage offers $\{w_i, w_j\}$, her optimization problem is

$$\max\ \{w_i + \varepsilon_i, w_j + \varepsilon_j\}.$$

Workers' optimization, jointly with firms' wage offers, induces a firm-level labor supply curve $m\Phi(w)$, where m is the mass of binding offers extended by the firm and $\Phi(w)$ is the probability that a wage offer w is accepted. The firm's value contingent on productivity realization z_i is

$$v_t(z_i, \Phi_t) = \max \left\{ 0, \max_{m,w}\ z_i[l(w)]^\gamma - wl(w) - c_f + \beta \sum_{j=1}^{N} \pi_{ij} v_{t+1}(z_j, \Phi_{t+1}) \right\}$$

$$\text{s.t. } l(w) = m\Phi_t(w), \tag{6}$$

$$m \leq \frac{2N_w}{N_f}. \tag{7}$$

where $\beta \in (0,1)$ is the discount factor. The constraint $m \leq \frac{2N_w}{N_f}$ reflects the requirement that each firm is matched with $\frac{2N_w}{N_f}$ workers and decides how many binding offers to make. In the simple framework considered here, it will always be the case that $m = \frac{2N_w}{N_f}$. This will not be the case in Section 5 below.

Notice that the optimal wage is the solution to a static optimization problem. Necessary condition for optimality is

$$z\gamma l^{\gamma-1} - w = \frac{\Phi_t(w)}{\Phi_t'(w)}. \tag{8}$$

Since the elasticity of labor supply is $\epsilon_s(w) = \frac{\Phi_t'(w)w}{\Phi(w)}$, equation (8) can be rewritten as the familiar condition relating a monopsonistic firm's wage markdown to the labor supply elasticity:

$$\frac{z\gamma l^{\gamma-1} - w}{w} = \frac{1}{\epsilon_s(w)}.$$

The wage is pushed up to the point where the marginal gain deriving from adding to labor equals the marginal cost from adding to the wage bill of inframarginal workers.

3.1 Entry

Entry is modeled along the lines of Clementi and Palazzo (2016). Every period, there is a mass of potential entrants $M > 0$ that draw a signal $\eta > 0$ about their initial productivity from the continuous distribution H. We assume that $\Pr(z_t \leq z_i|\eta) = G_i(\eta)$. For every η, the collection $\{G_i(\eta)\}_{i=1,N}$ is a cumulative distribution function. Crucially, G_i is strictly decreasing in η for all i. A potential entrant starts operating if and only if

$$\sum_{i=1}^{N} v_t(z_i, \Phi_t) g_i(\eta) \geq c_e,$$

where g_i is the pdf associated to G_i.

3.2 Stationary recursive equilibrium

As long as the firm's value function v is monotone increasing in the productivity index, a stationary firm distribution will be entirely characterized by two thresholds. There will exist a level of productivity z_{i*} such that $v(z_i, \Phi) \geq 0$ if and only if $z_i \geq z_{i*}$. We will refer to it as the exit threshold. Furthermore, there will be a signal η^* such that potential entrants will enter if and only if $\eta \geq \eta^*$. It follows that the mass of entrants will be

$$E = M[1 - H(\eta^*)] \int_{\eta^*} [1 - G_{i*}(\eta)] dH(\eta).$$

The measure of entrants will be

$$\hat{\mu}_e(z_i) = M \int_{\eta^*} g_i(\eta) dH(\eta), \quad \text{for } z_i \geq z_{i*},$$
$$\hat{\mu}_e(z_i) = 0, \quad \text{otherwise.} \tag{9}$$

Let $\widehat{\Pi}$ denote the incumbents' $N \times N$ transition matrix implied by Π together with the exit threshold z_{i*}. We have that $\hat{\pi}_{ij} = \pi_{ij}$ for $j \geq i^*$ and $\hat{\pi}_{ij} = 0$ otherwise. Then, the stationary measure of firms $\hat{\mu}$ will satisfy

$$\hat{\mu} = \hat{\Pi}' \hat{\mu} + \hat{\mu}_e. \tag{10}$$

The mass of operating firms will be $N_f = ||\hat{\mu}||_1$. From now on, let $\mu_i = \mu(z_i) = \hat{\mu}(z_i)/N_f$.

When comparing two wage offers $\{w_i, w_j\}$, the probability that a worker accepts a job offer from a z_i-productivity firm is

$$\Pr(w_i + \varepsilon_i \geq w_j + \varepsilon_j) = \frac{1}{1 + e^{\frac{w_j - w_i}{\sigma}}}.$$

Restricting firms to pure strategies, the share of workers a firm is able to attract by offering a wage w when firms' wage policy is given by $w_j \equiv w(z_j)$, is

$$\Phi(w) = \sum_{j=1}^{N} \frac{1}{1 + e^{\frac{w_j - w}{\sigma}}} \mu_j. \tag{11}$$

In turn, this means that the vector of equilibrium wage policies $w_i = w(z_i)$ is uniquely determined by the system of equations

$$\left[z_i \gamma \left(m \sum_j \frac{1}{1 + e^{\frac{w_j - w_i}{\sigma}}} \mu_j \right)^{\gamma - 1} - w_i \right] \sum_j \frac{1}{\sigma} \frac{e^{\frac{w_j - w_i}{\sigma}}}{\left[1 + e^{\frac{w_j - w_i}{\sigma}} \right]^2} \mu_j - \sum_j \frac{1}{1 + e^{\frac{w_j - w_i}{\sigma}}} \mu_j = 0, \tag{12}$$

for $i = 1, 2, ..., N$.

We close this section by providing our definition of stationary equilibrium.

Definition. A stationary equilibrium consists of a time-invariant firm distribution μ, a mass of firms N_f, a measure of entrants $\hat{\mu}_e$, a value function v, an exit threshold z_{i^*}, an entry signal threshold η^*, an individual wage function w, a wage distribution Φ and a number of offers m such that

- v solves equation (7), given Φ,
- $\hat{\mu}_e$ solves (9),
- $\mu = \hat{\mu}/N_f$, where $\hat{\mu}$ solves (10) and $N_f = ||\hat{\mu}||_1$,
- $m = 2N_w/N_f$,
- $z_i[l(w)]^\gamma - wl(w) - c_f + \beta \sum_{j=1}^{N} \pi_{ij} v(z_j, \Phi) \geq 0$ if and only if $z_i > z_{i*}$, for $l(w) = m\Phi(w)$,

- the vector w solves the system of equations (12),
- Φ solves (11).

4 Equilibrium firm-level pay

Consistent with the empirical evidence, our theory predicts that more productive firms are larger and pay higher wages. The reason is straightforward: Higher productivity means higher marginal product of labor. Hiring more labor requires paying higher wages.

The slope of the productivity-wage map depends on the parameter σ. This is the case because σ shapes firms' labor market power by disciplining the importance of job amenities relative to wages in workers' choice. For $\sigma = 0$, workers will simply choose the highest salary. This is the scenario where firms enjoy the least labor market power. For σ arbitrary large, wages essentially don't matter. In the limit, the probability of the worker accepting a job is 1/2 no matter the wage offers. This is the least competitive scenario.

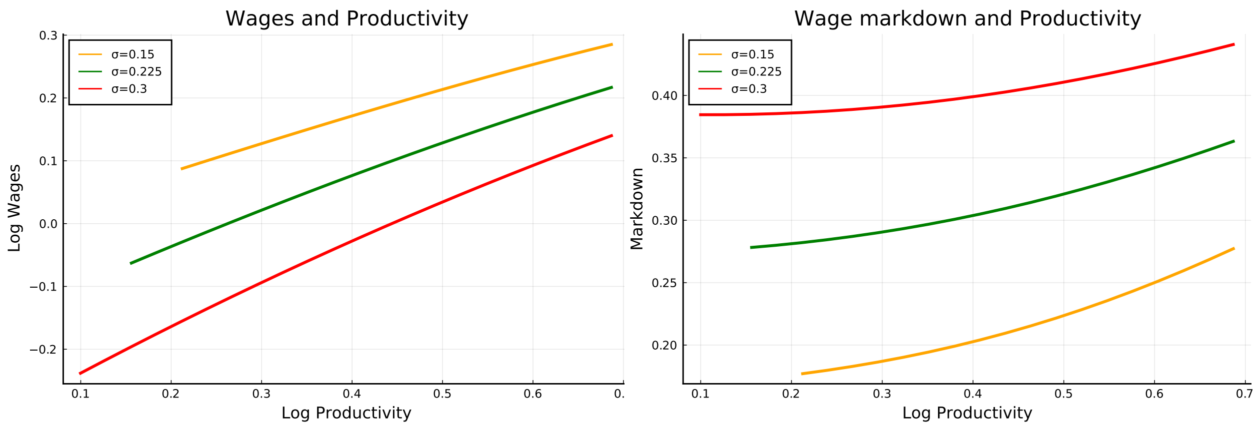
We solve for the equilibrium wage choices in a parameterized version of the model. We then perform a comparative statics exercise by considering the steady-state implications of changes in the value of σ.

Figure 6a illustrates the equilibrium pay policy for three different levels of σ. As competition increases, i.e. as σ declines, wages increase too. This is the case because, for low σ, a marginal increase in wages has a greater yield in terms of hires. In other words, firms face a more elastic labor supply function.

Notice however that this effect is proportionally larger for low-productivity firms, leading to a flatter wage-productivity profile. This is confirmed by the observation that the wage markdown declines with σ – at a faster rate for low-productivity firms. See Figure 6b.

For given wage offers, the probability that a worker will accept the lowest of the two is decreasing in σ. This implies that, for given wage policies, a decline in σ redistributes employment from low-productivity firms, who offer lower wages, to high-productivity firms. This is why, in equilibrium, low-productivity firms will respond to a lower σ by raising wages more aggressively than their high-productivity counterparts.

Figure 7 displays the equilibrium firm size distribution. A decline in σ generates more



(a) Wages

(b) Markdown

Figure 6: Wages, Markdown, and Productivity

selection. In face of the fixed operating cost $c_f > 0$, higher wages induced by more competitive labor markets lead low-productivity firms to exit. The distribution over productivity shifts to the right.

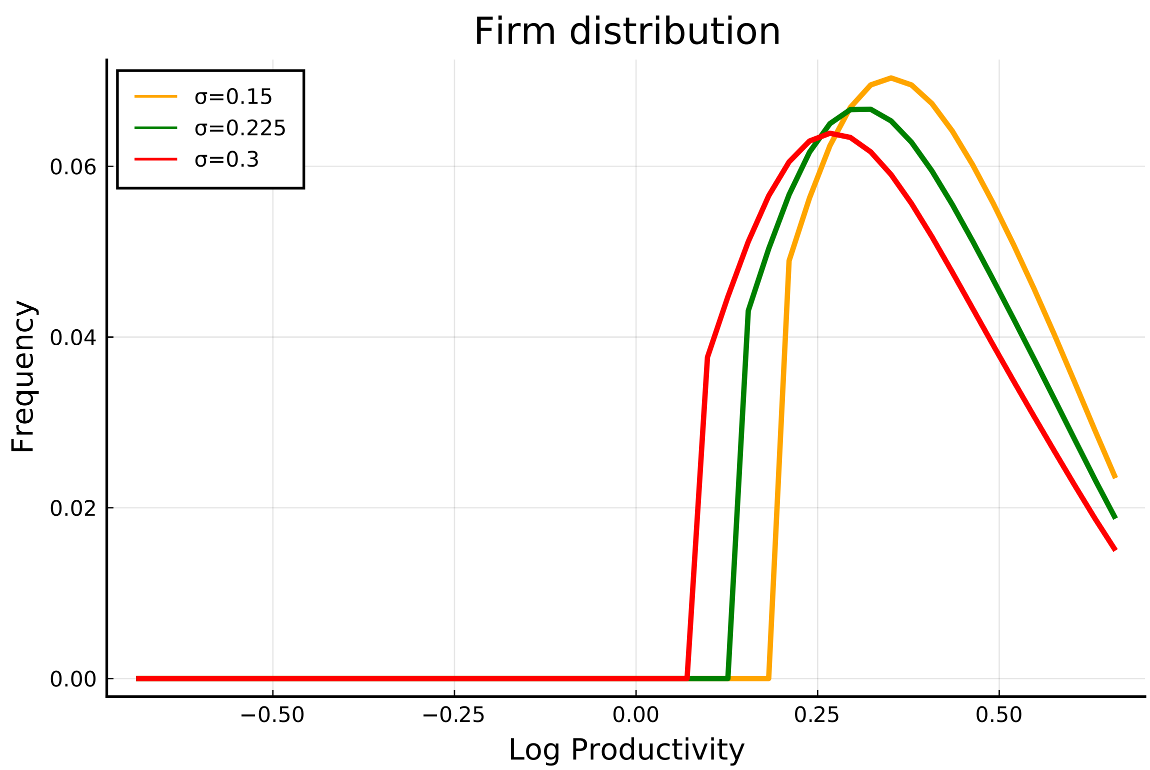


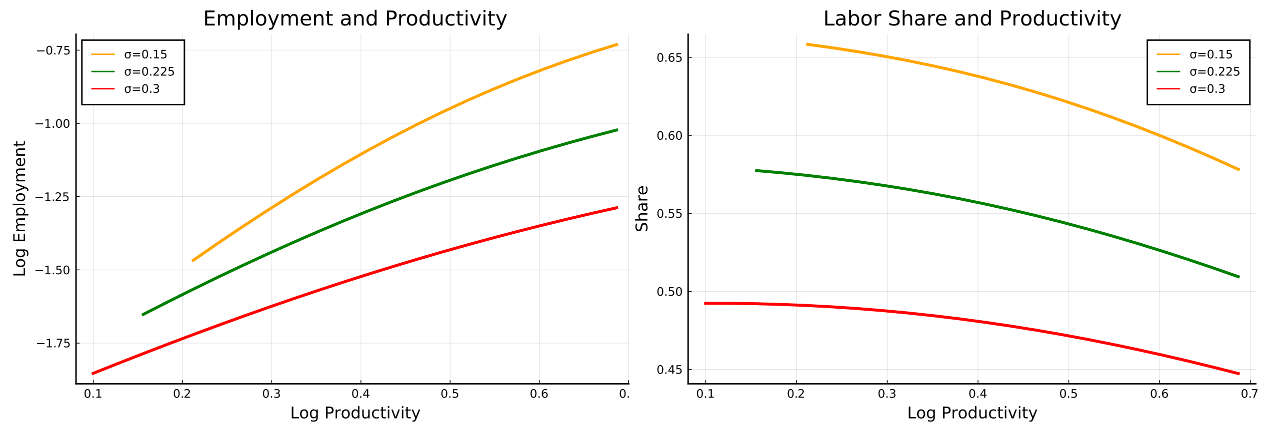
Figure 7: Firm Distribution over Productivity

The implications of a flattening of the wage policy for earnings inequality are immediate:

Inequality drops as labor market competition increases. See Figure 8, which illustrates the standard deviation of log wages as a function of σ. Figure 9 illustrates the comparative statics of employment and the labor share, respectively. As firms' labor market power decreases, the labor share increases – the more so, the lower firm productivity. Interesting, employment – which is kept constant in aggregate – is redistributed from low- to high-productivity firms.

The upshot of this section is that more competitive labor markets reduce mark-downs, increase labor shares, and increase wages across the board. The equilibrium wage-productivity profile becomes flatter, resulting in a reductions in pass-through, a reallocation of employment from low- to high-productivity firms and greater aggregate output.

Figure 8: Earnings Inequality



(a) Employment (b) Labor Share

Figure 9: Employment, Labor Share, and Productivity

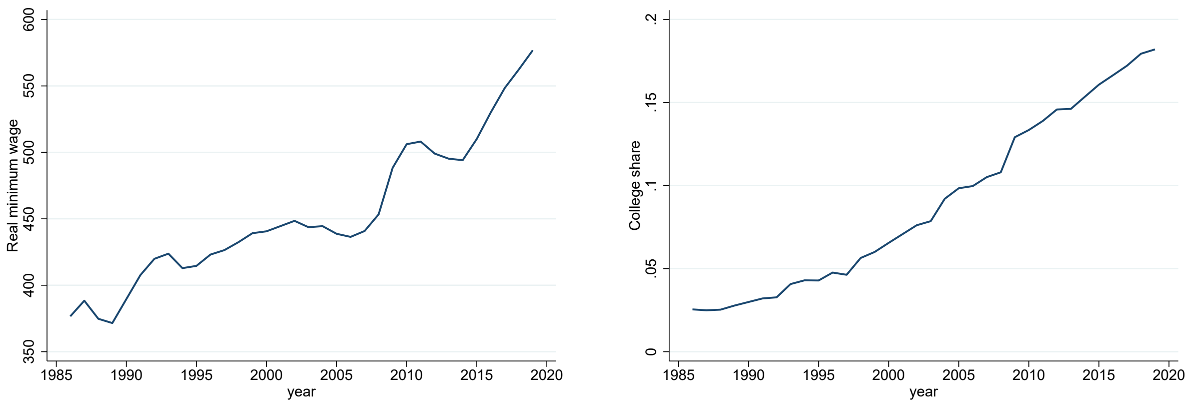
5 Accounting for the dynamics of earnings inequality in Portugal

In this section, we employ our theory to shed light on the determinants of the dynamics of inequality in Portugal. The key feature we need to account for is a compression in the earnings distribution associated with decline in the variance of firm fixed effect an a small increase in the variance of worker fixed effects.

While the decline in earnings inequality documented above is a novel finding, it has long been noted that earnings inequality, after a substantial rise up to the mid 90s, did not change appreciably over the next decade. When engaged with the goal of rationalizing such feature of the data, scholars placed substantial emphasis on policy – the minimum wage, in particular – and the rise in educational attainment.

Figure 10a illustrates the evolution of the general (non-agricultural) real minimum wage since the mid-1980s. In 2019, the minimum wage was 40% higher than in the mid-1990s. Figure 10b shows that the share of college graduates among working individuals has also increased dramatically, from just under 5% in the mid-1990s to 19% in 2019.

Our objective is to gauge whether these two forces alone may have indeed been responsible for the rich dynamics that we have uncovered in Section 2. We begin by generalizing the framework introduced in Section 3.



(a) Real minimum wage

(b) Share of college graduates

Figure 10: Minimum wage and share of college graduates among workers

5.1 A more general theory

Workers are now split between low-skill (type-1), and high-skill (type-2), available in mass N_1 and N_2, respectively. Firms operate the CES production function

$$y = z_i \left[\chi (s_1 l_1)^{\frac{\nu-1}{\nu}} + (1-\chi)(s_2 l_2)^{\frac{\nu-1}{\nu}} \right]^{\frac{\gamma}{\nu-1}}, \tag{13}$$

with $\chi \in (0,1)$, $\nu > 0$, $\nu \neq 1$, $0 < \gamma \leq \nu$. The elasticity of substitution is ν, while returns to scale are γ/ν.

Labor markets are perfectly segmented. Once again, each worker makes contact with two firms, randomly drawn from the distribution. Firms decide how many binding offers to extend in each market.

Wages are subject to the constraint $w \geq \underline{w}$, where $\underline{w}$ denotes the minimum wage. An interesting implication is that in equilibrium low-productivity firms may want to extend less than the maximum number of binding wage offers. For such firms, $m < \bar{m}_i \equiv \frac{2N_i}{N_f}$ for either $i = 1$ or $i = 1, 2$.

Firms above a certain productivity threshold will not be bound by the minimum wage. Their optimality conditions will be the natural extension of (8) to the case with heterogenous workers. Wages will be strictly lower than the marginal product on both labor markets and therefore $m = \bar{m}_i$ for $i = 1, 2$.

Firms bound by the minimum wage can be partitioned in two subsets. In one, characterized by relatively high productivity, firms will feature a marginal product higher than their wage when extending the maximum number of binding offers. Those in the remaining partition will instead choose $m < \bar{m}_i$ for at least $i = 1$ (the low-skill market), thereby equating the marginal product of that factor to the minimum wage.

Consider a firm of productivity z_j that falls in the latter set. By the law of large numbers, only a fraction $m_{ji}/\bar{m}_i$ of the type-i workers paired with it will receive an offer. It follows that the unconditional probability that a type-i worker will accept an offer w is now

$$\Phi_i(w) = \sum_{j=1}^{N} \frac{1}{1+e^{\frac{w_{ji}-w}{\sigma}}} \frac{m_{ji}}{\bar{m}_i}\mu_j + 1 - \sum_{j=1}^{N} \frac{m_{ji}}{\bar{m}_i}\mu_j =$$
$$= 1 - \sum_{j=1}^{N} \frac{e^{\frac{w_{ji}-w}{\sigma}}}{1+e^{\frac{w_{ji}-w}{\sigma}}} \frac{m_{ji}}{\bar{m}_i}\mu_j.$$

5.2 Analysis

Our methodological approach calls for identifying structural parameter values under which the steady-state of our model is consistent with a set of salient statistics for the Portuguese economy in the mid-1990s. We will then ask under what conditions, if any, increases in minimum wage and educational attainment of the magnitude recorded in Portugal over the intervening period, may have been responsible for the evolution of inequality up to 2019.

We begin by setting a number of parameters externally: $\nu = 1.01$, $\gamma = 0.9$, $\rho_z = 0.9$ and $\sigma_z = 0.5$. We then set $N_2 = 0.05$ to reflect the observation that the average fraction of workers with college degree was 5% between 1993 and 1997. Finally, we set the aggregate TFP parameter A, the minimum wage $\underline{w}$, as well as the parameters σ e κ, so that in steady state the ratio of minimum wage to median wage, the labor share, the variance of the worker fixed effect and the variance of the firm fixed effects are as close as possible to the average values recorded in Portugal over the period 1993-97.

We were able to match exactly three of the targets. As in the data, the minimum wage is 57% of the median wage, the variance of the firm fixed effect is 0.081 and the variance of the worker fixed effect is 0.116. However, the aggregate labor share is lower than in the data: 42% instead of 50%.

Next, we turn to Portugal in the period 2014-2019. In such interval, the average real minimum wage was 25% higher than in the mid-90s. The average fraction of college educate workers was 16%, 11 percentage points higher than in the mid-90s. We ask: Are there values of the structural parameters σ and χ such that the steady state of the model with higher minimum wage and higher college attainment generates data-conforming values for the labor share, the ratio of minimum wage to median wage, the variance of worker fixed effect, and the variance of firm fixed effect?

It turns out that by simply lowering the parameter χ from 0.826 to 0.78, the model matches the ratio of minimum wage to median wage, the variance of the worker fixed effect, and the variance of the firm fixed effect. The labor share rises by 3 percentage points, while preliminary calculation place the rise in Portugal's labor share around 10 percentage points.

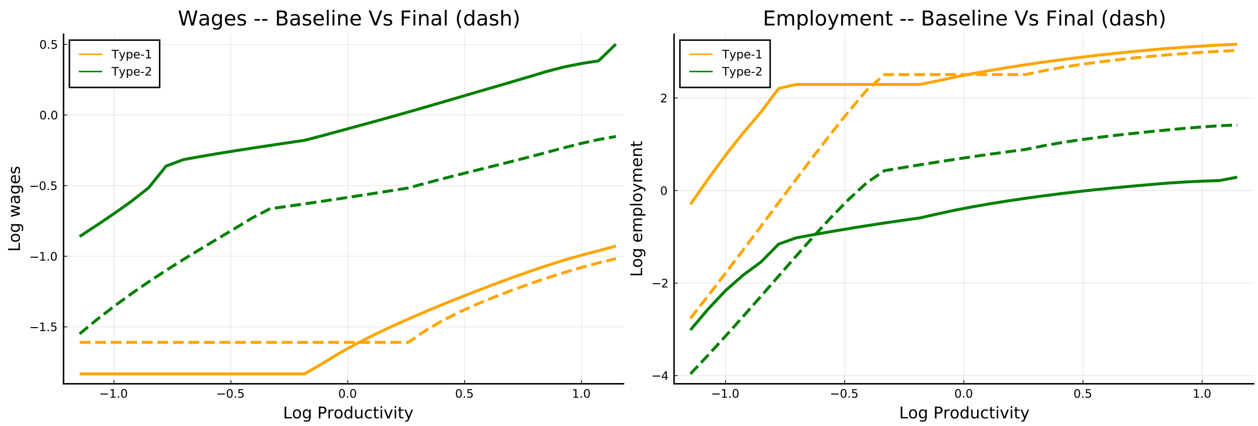
5.3 Assessment

Figure 11 describes firms' equilibrium policies for wages and employment in the initial and final steady states, respectively. For simplicity, the distribution of firms over productivity is kept constant across the two equilibria.

The left panel shows that the wage compression is the composition of three effects. One is the mechanical increase in minimum wage paid by low-productivity firms to low-skill workers. Another is the decline in wages for low-skill workers employed at relatively more productive firms. The third is the rather uniform decline in wages for high-skill workers. All workers except those remunerated at the minimum wage see their salaries decline.

The right panel hints that the decline in the mass of low-skill workers is entirely absorbed by low-productivity firms. In fact, inspection reveals that the drop in employment at low-productivity firms is larger then than the exogenous decline of low-skill workers, leading to a lower employment rate. Low-productivity firms also reduce their demand for high-skill workers. The exogenous increase in the mass of high-skill workers find employment at relatively high-productivity firms.

In aggregate, both employment and value added decline. As already stated above, the labor share increases. Its evolution is driven by two countervailing forces. On the one hand, the hike in the minimum wage raises the labor share at mid-productivity firms for which



(a) Wages (b) Employment

Figure 11: Baseline (solid) Vs Final (dashed)

the minimum did not bind in the initial steady state. On the other hand, value added is reallocated towards larger firms, which have a lower labor share. The former effect dominates.

5.4 Investigating the economic mechanism

Everything else equal, an increase in the fraction of high-skill workers lowers their wages relative to the low-skill's. Consistent with the evidence, this leads to a lower variance of earnings. However, it accomplishes this result by counterfactually reducing the variance of worker fixed effects and by increasing median wages. Recall that the variance of worker fixed effects has increased marginally and that the minimum wage rose as a fraction of median wages.

In the equilibrium under consideration, the minimum wage only applies to low-skill workers. An increase in the minimum wage leads to an expansion of the firms that pay minimum wage. This tends to mechanically reduce the variance of firm fixed effect. The rise in the minimum wage also affects the rest of the distribution. For given quantities, a higher minimum wage is pro-competitive. Higher-productivity firms need to raise their wages if they wish to keep their workforce in the face of a stronger competition. However, a higher minimum wage lowers the number of job offers made by low-productivity firms directly affected by the minimum wage. This effect soften competition for high-productivity firms.

A higher minimum wage also reduces wages for high-skill workers, even though the level of their pay exceeds the mandated minimum. The reason is that, due to complementarity in production, a decline in low-skill employment by low-productivity firms also triggers a decline in their demand for high-skill labor. In turn, this reduces the wages of high-skill workers.

The bottom line is that an increase in the minimum wages leads to a decline in both the variance of firm fixed effect and the variance of worker fixed effect, together with an increase in the ratio of minimum wage to median wage.

A decline in the parameter χ is what is needed for the model to generate a slight increase in the variance of worker fixed effects in spite of a push towards a lower variance induced by the changes in both minimum wage and educational attainment. Such shift in the production function can be thought of as skill-biased technical change, as it raises the relative demand for the services of high-skill workers.

6 Conclusion

TBA

Appendix

A Quadros de Pessoal

We rely upon the following variables:

- Total monthly earnings (*rganho*) is the sum of base earnings, plus overtime pay, plus regular monthly supplements such as productivity and seniority premia, as well as compensating differentials.

- Monthly hours (*hnormais*) are normal working-time paid hours.

- Worker age (*idade_Cod*) is the actual worker age at the time of the survey (bottom-coded at 17 and top-coded at 68). To avoid measurement error, age is recomputed as the difference between the current year and the mode of the year of birth implied by reported age.

- Schooling (*habil*) is the highest degree attended. Based upon this variable, we construct three schooling dummies, for the equivalent of 9th grade or less, secondary or post-secondary non-college, and college and over (including Bachelor). We then take the mode of each indicator for each worker.

- Firm–level sectoral codes are at two–digit level, based upon the *C.A.E. - Classificação Portuguesa das Atividades Económicas* classification system. They have been harmonized over time (C.A.E. was revised in 1995, 2003, and 2007). Within an estimation subperiod, the firm–level code is taken to be mode across years.

- Number of employees (*pemp*) equals all persons that have worked for the firm in October, either remunerated by the firm or not (e.g. owner-managers and family members), and including short-term (less than one month) absentees. This variable is used to select our AKM estimation sample (dropping small firms), our primary measure for the number of employees in the rest of the paper is the analogously-defined variable drawn from SCIE.

Our focus on for-profit non-financial market-oriented sectors entails dropping firms in the financial sector, public administration, education, health, social services, families with domestic workers, and international organizations.

For workers reported to be working at different firms in a given year, we kept a single firm observation based upon the following criteria: (i) the firm where the individual works the highest number of normal hours, otherwise (ii) the firm where the individual earns the highest (straight-time) earnings, otherwise (iii) the firm where the individual has the longest tenure.

B Sistema the Contas Integradas das Empresas (SCIE)

We rely upon the following variables:

- Gross income (*produção*) equals sales revenue plus the change in inventories, plus some additional operating revenue.
- Gross value added (*valor acrescentado bruto a preços de mercado*) equals gross income minus the expenditure on materials and services.
- Materials and services (*consumos intermédios*) equals the cost of materials and services external to the firm.
- Capital (*ativos fixos tangívies + intangíveis + biológicos e propriedades de investimento*) equals the book value of fixed intangible assets, plus intangible assets, plus investment property.
- Number of employees (*pessoal ao serviço*) equals all persons that have worked for the firm during the year, either remunerated by the firm or not (e.g. owner-managers and family members), and including short-term (less than one month) absentees. This is our primary variable for the number of employees, and is used to compute value-added per worker and firm size (whenever missing, the number of employees from Quadros de Pessoal is used instead).
- Total labor cost (*gastos com o pessoal* equals total (fixed plus transitory) payments to employees, inclusive of social insurance, pension, and other mandatory contributions such as private insurance, plus hiring and firing costs.

The basic firm identifier we use in our empirical analysis is the one from Quadros de Pessoal. Firms in Quadros de Pessoal come also with the same fictitious fiscal number that identifies firms in the SCIE. We are therefore able to match each basic firm identifier from Quadros de Pessoal with a SCIE fictitious fiscal number in any given year. We restrict to SCIE firms which are also in Quadros de Pessoal, which for the most part implies dropping independent contractors and firms without paid employees.

C Robustness

We show that our bottom-line results from Section 2 are not very sensitive to alternative sample selection criteria. First, Table 4 displays the basic decomposition without restricting to firms in the market sector. Second, Table 5 contains the basic decomposition when our baseline sample is further restricted to firms with at least 10 workers on average over their entire life-cycle.[9]

Third, Table 6 restricts the baseline sample to worker observations with base earnings above 1.2 of the minimum wage. The wage compression since the mid-1990s becomes muted. However, we still observe the basic dynamics we highlight in the paper, of significant compression in firm pay while inequality in worker characteristics increases. Figure 12 plots the dynamics of the main components of the basic AKM decomposition.

[9] Our motivation is to act conservatively in order to avoid biased estimates of the cross-sectional variances, what the literature refers to as "limited mobility bias" (Bonhomme et al., 2020; Engbom et al., 2021). This problem is magnified by the presence of small firms, whose workers tend to exhibit lower mobility. Our simple approach is therefore to eliminate small firms in a long-run/life-cycle sense.

	Interval 1 (1986-1992)		Interval 2 (1993-1997)		Interval 6 (2014-2019)		Change 1–2		Change 2–6	
	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share
Var(y)	.2	–	.256	–	.217	–	.056	–	-.039	–
Var(WFE)	.07	35.1	.121	47.4	.151	69.6	.051	91.3	.03	-75.7
Var(FFE)	.077	38.4	.094	36.9	.033	15.4	.018	31.4	-.061	155.8
Var(Xb)	.012	6.2	.004	1.5	.007	3.2	-.009	-15.6	.003	-8
Var(ϵ)	.015	7.4	.014	5.7	.01	4.7	0	-.6	-.004	11.2
2Cov(WFE, FFE)	.017	8.5	.021	8.1	.029	13.2	.004	6.8	.008	-20.1
2Cov(WFE, Xb)	.002	1.2	0	-.1	-.012	-5.7	-.003	-4.9	-.012	30.8
2Cov(FFE, Xb)	.006	3.2	.002	.6	-.001	-.3	-.005	-8.4	-.002	6

Table 4: Basic Decomposition (baseline sample plus non-market firms)

	Interval 1 (1986-1992)		Interval 2 (1993-1997)		Interval 6 (2014-2019)		Change 1–2		Change 2–6	
	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share
Var(y)	.205	–	.262	–	.232	–	.057	–	-.03	–
Var(WFE)	.073	35.6	.126	48.3	.168	72.3	.053	93.7	.041	-138.1
Var(FFE)	.073	35.4	.086	32.9	.028	12	.014	23.7	-.058	195.1
Var(Xb)	.013	6.2	.004	1.4	.007	3.2	-.009	-15.6	.004	-12.4
Var(ϵ)	.015	7.4	.015	5.8	.01	4.4	0	0	-.005	17
2Cov(WFE, FFE)	.022	10.8	.029	11.1	.033	14.3	.007	12.2	.004	-13.7
2Cov(WFE, Xb)	.003	1.3	0	-.1	-.014	-5.9	-.003	-4.9	-.014	45.4
2Cov(FFE, Xb)	.007	3.2	.001	.5	-.001	-.3	-.005	-9	-.002	6.7

Table 5: Basic Decomposition (baseline sample minus small firms)

	Interval 1 (1986-1992)		Interval 2 (1993-1997)		Interval 6 (2014-2019)		Change 1–2		Change 2–6	
	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share
Var(y)	.166	–	.204	–	.201	–	.038	–	-.003	–
Var(WFE)	.075	45.3	.127	62.5	.166	82.9	.052	137.2	.039	-1245
Var(FFE)	.056	33.6	.071	34.7	.035	17.2	.015	39.2	-.036	1153.6
Var(Xb)	.013	7.9	.004	1.8	.011	5.6	-.009	-24.8	.007	-239.1
Var(ϵ)	.015	9.3	.015	7.2	.011	5.3	-.001	-1.9	-.004	128.9
2Cov(WFE, FFE)	.003	1.8	-.011	-5.2	-.005	-2.6	-.013	-35.3	.005	-168.2
2Cov(WFE, Xb)	.001	.5	-.002	-1.1	-.017	-8.4	-.003	-8	-.015	466.8
2Cov(FFE, Xb)	.003	1.5	0	.1	0	0	-.002	-6.3	0	3

Table 6: Basic Decomposition (base wages over 1.2 of min wage)

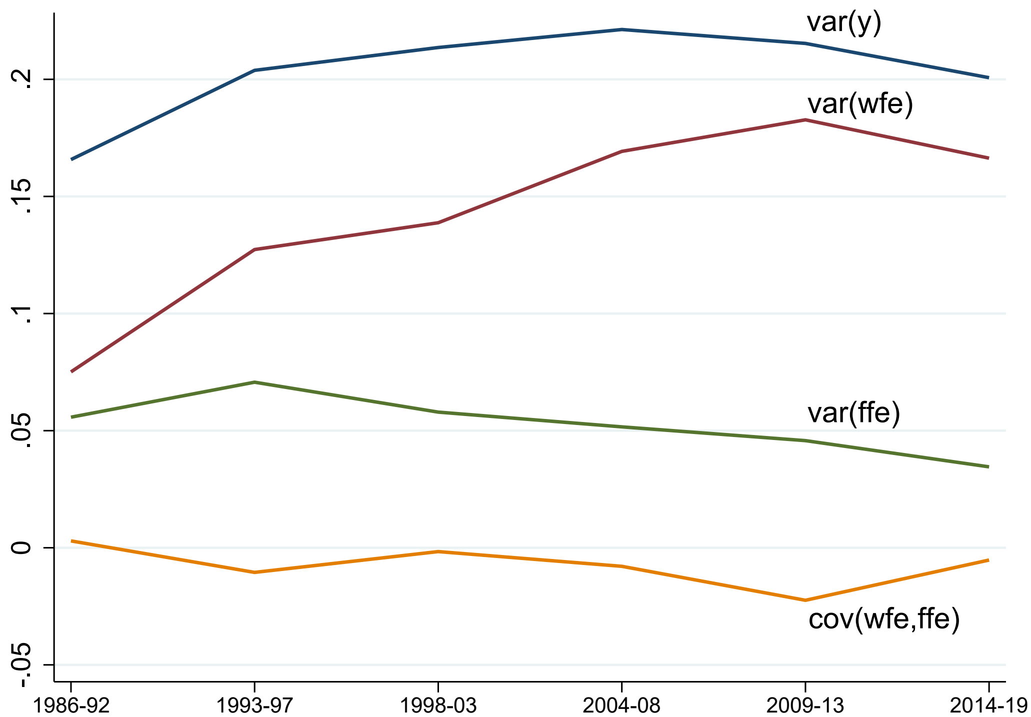


Figure 12: Basic Decomposition (base wages over 1.2 of min wage)

References

Abowd, J. M., F. Kramarz, and D. N. Margolis (1999): "High Wage Workers and High Wage Firms," *Econometrica*, 67, 251–334.

Alvarez, J., F. Benguria, N. Engbom, and C. Moser (2018): "Firms and the Decline in Earnings Inequality in Brazil," *American Economic Journal: Macroeconomics*, 10, 149–189.

Bilal, A. G., N. Engbom, S. Mongey, and G. L. Violante (2021): "Firm and Worker Dynamics in a Frictional Labor Market," *Econometrica*, forthcoming.

Bonhomme, S., K. Holzheu, T. Lamadon, E. Manresa, M. Mogstad, and B. Setzler (2020): "How Much Should we Trust Estimates of Firm Effects and Worker Sorting?" NBER Working Papers 27368, National Bureau of Economic Research, Inc.

Card, D., A. R. Cardoso, J. Heining, and P. Kline (2018): "Firms and Labor Market Inequality: Evidence and Some Theory," *Journal of Labor Economics*, 36, 13–70.

Card, D., J. Heining, and P. Kline (2013): "Workplace Heterogeneity and the Rise of West German Wage Inequality," *The Quarterly Journal of Economics*, 128, 967–1015.

Cardoso, A. R. (1999): "Firms' Wage Policies and the Rise in Labor Market Inequality: The Case of Portugal," *Industrial and Labor Relations Review*, 53, 87–102.

Clementi, G. L. and B. Palazzo (2016): "Entry, Exit, Firm Dynamics, and Aggregate Fluctuations," *American Economic Journal: Macroeconomics*, 8, 1–41.

Engbom, N., C. Moser, and J. Sauermann (2021): "Firm pay dynamics," Working Paper Series 2021:21, IFAU - Institute for Evaluation of Labour Market and Education Policy.

Gouin-Bonenfant, m. (2022): "Productivity Dispersion, Between-Firm Competition,and the Labor Share," Tech. rep., Columbia University.

Hopenhayn, H. A. (1992): "Entry, Exit and Firm Dynamics in Long Run Equilibrium," *Econometrica*, 60, 1127–50.

Messina, J. and J. Silva (2019): "Twenty Years of Wage Inequality in Latin America," *The World Bank Economic Review*, 35, 117–147.

Song, J., D. J. Price, F. Guvenen, N. Bloom, and T. von Wachter (2018): "Firming Up Inequality," *The Quarterly Journal of Economics*, 134, 1–50.