A DYNAMIC EQUILIBRIUM MODEL OF COMMUTING, RESIDENTIAL AND WORK LOCATION CHOICES *

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Abstract

We present a dynamic equilibrium model of joint residential and work location choice in which commuting costs depend on the distance between work and home, and house prices equate supply and demand for housing. We estimate the model using Danish register data for households in Greater Copenhagen area. We then predict the effects of increasing supply of residential housing on house rental prices, job mobility, residential sorting, commuting and welfare. In the second counterfactual simulation we show differential impact of telecommuting which implies substantial welfare gains and increased labor supply on the margin by educated workers whereas low educated workers benefit only slightly from lower housing prices in urban areas caused by out migration.

KEYWORDS: Structural estimation, life cycle models, dynamic equilibrium models, housing, residential choice, work location, labor supply.

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1 Introduction

Like most other countries, Denmark is undergoing a process of urbanization and spatial concentration of economic activity. While this has led to increased productivity in the largest cities through agglomeration effects, it has also resulted in major societal challenges. These include the extensive and systematic flows of people and jobs to urban areas resulting in traffic congestion and significant increases in house prices, thereby altering the demographic composition of cities and increasing regional inequality.

Various policies are considered to deal with the downsides of urbanization and congestion, including infrastructure investments, relocating government jobs from Copenhagen to the rest of Denmark, increasing urban density and housing supply, and promoting telecommuting. However, the dynamic effects of these policies are not well understood due to the complexity of households' joint choice of employment, work and residential location, as well as commuting. A dynamic perspective is required to understand how these decisions vary over the life cycle and are affected by family structure, housing prices, amenities, and uncertainty about future job opportunities. For example, a temporary shock such as increased commuting cost due to an epidemic has a very different dynamic impact than a permanent shock such as expanding the supply of housing via rezoning and urban development.

A dynamic model enables us to predict house price trends. For example, suppose the space in central Copenhagen freed up by older households choosing to retire and move to suburban locations fails to offset the inflow of younger households responding to job opportunities. The equilibrium response to this imbalance increases house prices but also pushes out less well off households further from the center, resulting in longer commutes, increased traffic congestion, and urban gentrification.

Intelligent policy-making requires a dynamic equilibrium model that predicts how housing prices, job opportunities, amenities, and the cost of commuting affects individuals' choices of where to live and work, and the impact on house prices, commuting, residential sorting, and inequality. This is the key challenge we address in this paper.

We develop and estimate a dynamic equilibrium life cycle model of residential and work locations, taking into account households' changing need for housing size, location-specific earning potentials, commuting costs, amenities, and moving costs.

Using the model, we study how these choices affect house prices, commuting patterns, and demographic composition of cities in the short run, that is, treating the location of jobs, housing stock, amenities, and transport infrastructure as given. We then run counterfactual simulations to predict the effects of two policies: i) an increase in housing supply and ii) changing the commuting times via the introduction of telecommuting.

Our model is inspired by Buchinsky, Gotlibovski and Lifshitz (2014), which we extend to a dynamic discrete-continuous choice setting with endogenous house prices and equilibrium constraints. We focus on *equilibrium* location choices of the full regional population rather than those of a specific demographic group of newly immigrated residents. The life cycle framework allows us to account for age-dependent heterogeneity. To reduce the complexity of modeling life cycle consumption, savings, and borrowing decisions, we assume individuals have quasi-linear utility functions and do not face borrowing constraints. Instead we capture these effects via heterogeneity in the marginal utility of money, such that rich households with lower marginal utility of money have a higher demand for housing and sort into more expensive geographic areas. Given the complexity of the model, we also assume that the housing size can be adjusted continuously in each time period without any adjustment costs.

These simplifications provide a computationally tractable framework for studying location decisions in more detail. We can derive a closed-form solution for the optimal level of housing as a static subproblem that can be solved independently of the overall discrete location choice dynamic programming problem. The residential and work location choices are dynamic and subject to high fixed adjustment costs. Even with these simplifications, it is challenging to estimate the model due to the large number of states and choices which depend on the number of work and residence locations. To ameliorate this curse of dimensionality we aggregate location choices to the municipal level and restrict attention to the island of Zealand (which includes Copenhagen and its surroundings). Out of the 98 municipalities in Denmark, we consider the 16 municipalities located in the Greater Copenhagen Area (henceforth GCA), with the surrounding regions of Zealand providing the "outside option".

Using Danish administrative panel data we track all households, their members, jobs, and residential locations for the period 1993-2013. We focus on the years 2000-2004 and 2009-2013 to structurally estimate the model using a nested backward induction maximum likelihood algorithm. Subsequently, we use the model to simulate the counterfactuals mentioned above. This has already found a practical application. Currently, the Danish Ministry of Transportation (DMT) has adopted our analysis of the first counterfactual policy of increased housing supply for urban planning. Specifically, the Danish Parliament has recently approved the construction of a large artificial island, named Lynetteholm, which is located in the harbor of Copenhagen and can be integrated with the existing city via roads, tunnels and metro lines. The island is intended to protect Copenhagen against flooding and has the potential to hold up to 3 million square meters of housing. This would amount to a 12% increase in the stock of housing in central Copenhagen.

In the first counterfactual, we present a stylized version of the counterfactual analysis conducted for the DMT. In particular, we consider a 5% increase in the housing stock in central Copenhagen and find that this policy increases urbanization as households move from the peripheral regions towards the center. Equilibrium prices fall in all locations, especially in the two most central municipalities where the policy is implemented. Five years after this expansion, rental prices are lower by 6.6% in these locations.

In the second counterfactual, we consider the effect of extended use of telecommuting. We treat telecommuting as a feasible option for highly educated workers, though not for the less educated, such as service workers, who must be physically present at their work locations. We find that the educated workers' option to telecommute causes some of them to gradually migrate to more spacious suburban locations while still retaining high-paying jobs in the center of Copenhagen. The reduction in time spent commuting and ability to consume more housing in desirable suburban locations significantly increases welfare of educated workers. Their out migration also increases suburban prices relative to urban center prices, but the reduction in the latter is not sufficiently large to result in a big benefit to less educated workers who face unchanged commuting costs.

Our paper contributes to several strands of the literature. First of all, our model captures a household's trade-off between locating close to a dense labor market while having to pay more for housing in equilibrium, a trade-off originally formulated in the mono-centric city models of Alonso et al. (1964) and Muth (1969). Recent papers have expanded on this framework by modeling the transportation network across the city while tracking the home and work location of inhabitants. See for instance Tsivanidis

(2019), Heblich, Redding and Sturm (2020), Dingel and Tintelnot (2020), Teulings, Ossokina and de Groot (2018), Severen (2021), Chernoff and Craig (2021). Other papers such as Allen and Arkolakis (2019) and Monte, Redding and Rossi-Hansberg (2018) develop tractable models that encompass the flow of labor and goods across regions of an entire country rather than merely within a metropolitan area. The latter show that local employment responses to labor demand shocks hinge on local infrastructure. Our counterfactual that decreases the cost of commuting for educated workers echoes this result, as educated labor flows to the urban labor market while moving their residences out. Yet, our main contribution to this line of research is that we offer a dynamic life cycle model structurally estimated on full population administrative panel data. We can therefore provide a detailed account of the distribution of heterogeneous welfare gains in our counterfactuals.

A further significant contribution is that our model characterizes the dynamic non-stationary response of housing prices and relocation patterns to a policy intervention. Understanding the short-run dynamics of an intervention is, we believe, crucial for policy advice. This also distinguishes our work from the contribution by Ahlfeldt, Redding, Sturm and Wolf (2015). They develop a general equilibrium model of a city where people select a combination of residence and job locations while letting wages and prices of land adjust in response to moving patterns. Their focus is on estimating the extent of agglomeration on productivity, not on the location choice per se. Even though they do study how equilibrium land prices change in response to altered moving patterns, they only estimate the long-run effects in a *static* modeling framework. A dynamic model is crucial in the counterfactuals we consider to fully understand the life cycle-dependent adjustment paths in migration and employment and the evolution of home prices that only *gradually* unfold due to significant fixed costs of moving.

The work by Kennan and Walker (2011), Oswald (2019), Dahl (2002) and Tunali (2000) are methodologically also closely related to ours in that they formulate structural life-cycle models in which economic agents undertake costly moving decisions in order to gain in either income prospects, access to amenities or both. Their scope is on the nationwide allocation of labor and thus focus on larger regions without allowing for commuting. Related is also Diamond (2016) who demonstrates an endogenous relationship between labor market prospects in cities and city amenities, which drive inequality in welfare between high- and low-educated individuals.

An important feature of our model is that urban labor markets are attractive not just because of higher wages but also because distance is costly when searching for jobs. This is a point stressed by Manning and Petrongolo (2017) who create a search model that accounts for commuting, which turns out to be a critical factor for job search behavior. They model the endogenous structure of the local labor market, taking the home location of workers as fixed. We allow for the estimated job arrival probabilities to depend on the urban density, although they are not an equilibrium outcome of our model. There is a substantial literature concerned with estimating the willingness to pay for local non-traded amenities, which may be endogenous to the current spatial sorting of households. See for instance Sieg, Smith, Banzhaf and Walsh (2004), Bayer, McMillan, Murphy and Timmins (2016) and the review by Kuminoff, Smith and Timmins (2013). Our structural estimates include the willingness to pay for local amenities such as access to cafés and bars but we abstract away from amenities that are an outcome of a sorting equilibrium, e.g. the sociodemographic makeup of a neighborhood. However, in line with the sorting literature we note that our counterfactual simulations are based on a housing market equilibrium where local prices must ensure zero excess demand for square meters in each region. Changing local amenities or labor market prospects thus implies changes in housing prices in equilibrium, yet in a non-linear way that depends on transitory demographic shifts.

The rest of the paper is organized as follows: Section 2 outlines the model. Section 3 introduces the algorithm we use to solve and estimate the model and describes how we solve for short-run equilibrium prices. Section 4 presents the empirical results, including parameter estimates and model fit in terms of house prices, residential and work location choices, and the resulting commuting and spatial sorting. In 5 we do the counterfactual simulations discussed above and section 6 concludes.

2 A Dynamic Model of Residential and Work Locations

In this section we formulate an individual level dynamic decision problem of residential and work location taking future prices and job opportunities in different regions as given. In the following section we integrate it into a model of temporary equilibrium where home prices are determined to equate supply and demand for housing region by region.

2.1 Sequential choice of work and residence locations

At each age t of the life cycle, $t \in \{t_0, \ldots, T\}$, individuals discretely choose their work and residence locations (including the option of not working), as well as the size of their residence. We set $T = 76$ such that nearly all individuals are retired and no longer change residence further. Let R denote the number of possible locations. The number of discrete location choices in the model (including non-employment) is then $R \times (R+1)$. Accounting for the cost of moving job and residence requires the same number of location states. Even not counting other state variables, this makes the location choice problem computationally very hard. To tackle this problem we make a number of simplifying assumptions.

First, we assume there are no fixed costs associated with adjusting housing size in the current location. We treat all individuals as renters, and the per period cost of housing equals their choice of square meters times the rental price per square meter. It follows that we can compute the indirect utility of housing given individual characteristics and region of residence. We derive the corresponding static demand for home size in Section 3.3. Thus, our dynamic discrete choice model subsumes the continuous choice of house size in the indirect utility for housing.

Second, we assume that work and residence location choices are made sequentially, namely that work location choice is made first followed by the residence location choice. Even though this assumption does not decrease the number of alternatives in the resulting nested choice model ($R+1$ work location nests by R residential alternatives each), it allows us to introduce a sensible job matching process. Namely, we differentiate between the job transition *choice* that denotes intention, from the job *outcome* that becomes the next period work location. Thus, our model allows for unsuccessful attempts to change work location and involuntary unemployment.

In our computational approach we recognize the fact that the expected future value of the current period choices only depends on the work and residence locations *realized by the end of the period.* We therefore formulate the dynamic programming problem in terms of expected value functions, keeping its dimensionality on the order of R^2

rather than R^4 ($(R+1)\times R$ states by $(R+1)\times R$ choices) as would be required by the traditional solution in the space of choice-specific value functions.

Because "work location" appears in the model in three different forms (existing, intended, and realized work location), we use the following explicit notation to distinguish between them. We denote wl_t the beginning of the period existing work location (*state*) and d_t^w the period t choice of intended work location (*choice*). This may or may not be the same as wl_t. Finally, to denote the *outcome* of the job match process during period t we simply use wl_{t+1}, as the realized in period t work location becomes the existing one in period $t+1$.[1] Similarly, let rl_t denote the period t residential location and d_t^r the choice of new residence location. We assume perfect control over the location of the residence (subject to the equilibrium house prices), and therefore the location of residence in period $t+1$ is given by the choice at period t, i.e. $rl_{t+1}=d_t^r$.

The timing of decisions is as follows. At the start of period t the individuals know their work and residence location and other state variables x_t described below, captured by state vector $s_t=(wl_t,rl_t,x_t)$. Individuals make their work and residential location choices sequentially but instantaneously at the start of each period t, with the intended work location decision made first, followed by the residential location decision made *conditional* on the realization of the employment search, i.e. realized work location wl_{t+1}. Once the intended work location is chosen, the job search outcome is realized, and the residence location choice is made, the household determines the optimal house size depending on their own characteristics and the chosen region of residence. Thereafter, the housing consumption is enjoyed for the rest of the period, and the process transitions to the next period.

Individuals' discrete choices also depend on *IID* generalized extreme value idiosyncratic shocks $\varepsilon_t=(\varepsilon_t^w,\varepsilon_t^r)\in\mathbb{R}^{R+1}\times\mathbb{R}^R$ that can be interpreted as transitory components of the utility that the econometrician does not observe. These stochastic components are revealed to the individual sequentially: at the time of the work location decision d_t^w only the "work location shocks" ε_t^w are known, whereas the residential location shocks ε_t^r are only revealed after the individual learns the outcome of their employment search. In other words we assume that the households find out the idiosyncratic attributes of the

[1] Using notation wl_{t+1} as the realized work location in period t involves a degree of confusion with the time subscripts, but we opt to bear this cost to avoid having an additional outcome variable.

residence locations only once they know where their job takes them. These assumptions lead to the standard nested logit choice structure of the work and residence location decisions.

2.2 Job search transitions

Before deriving the recursive formulation of the model, we specify the possible transitions in the job search process. A spatial model with fixed wages could lead to the outcome where far more people want to move into a high wage region than there are available jobs. We introduce a simplified labor market into the model to avoid this unrealistic scenario.

Let the spatial work region $wl_t = \emptyset$ denote the state of non-employment, which can naturally be combined with any residence region rl_t. We assume that unemployment can be chosen voluntarily, but also allow for involuntary job separations with a certain probability, including the cases when no job transition is intended ($d_t^w = wl_t$).

Let $\pi_t^n(d_t^w, wl_t, x_t)$, $d_t^w \neq wl_t$, denote the probability of finding a new job in the region d_t^w given household characteristics x_t. If $d_t^w = wl_t$, then $\pi_t^n(wl_t, wl_t, x_t) \equiv \pi_t^k(wl_t, x_t)$ denotes the probability of keeping the existing job in location wl_t[2]. If the individual chooses to stop working, $d_t^w = \emptyset$, then $\pi_t^n(\emptyset, wl_t, x_t) = 1$, there is perfect control over this decision. However, if the individual searches for a new job in a different region, then $d_t^w \neq wl_t$ and the transition probability is

$$wl_{t+1} = \begin{cases} d_t^w & \text{with probability } \pi_t^n(d_t^w, wl_t, x_t), \\ wl_t & \text{with probability } \big(1 - \pi_t^n(d_t^w, wl_t, x_t)\big)\pi_t^k(wl_t, x_t), \\ \emptyset & \text{with probability } \big(1 - \pi_t^n(d_t^w, wl_t, x_t)\big)\big(1 - \pi_t^k(wl_t, x_t)\big). \end{cases} \tag{1}$$

Thus, if an individual chooses to search for a job in a new location $d_t^w \neq wl_t$, there are three possible outcomes: i) the individual receives a job offer in this location; ii) the individual does not get a job offer in the location but is able to keep her existing job; or iii) the individual's job search is unsuccessful and she is laid off from her current job.

If the individual does not search for a job in a new location, $d_t^w = wl_t$, we assume they intend to continue working in the same location as before, and the transition probabilities can be computed as the special case of (1) where we set $\pi_t^n(d_t^w, wl_t, x_t) = 0$.

[2] We treat job transitions within the same region as equivalent to staying on the current job.

For unemployed individuals specification (1) can be applied as well, in which case the last two rows collapse into one and we have

$$wl_{t+1} = \begin{cases} d_t^w & \text{with probability } \pi_t^n(d_t^w, \emptyset, x_t), \\ \emptyset & \text{with probability } 1 - \pi_t^n(d_t^w, \emptyset, x_t). \end{cases} \quad (2)$$

This specification allows for differences in the chance of landing a job, which depends on current employment status, but we do not place any ex ante restrictions on the ranking of the job probabilities conditional on (d_t^w, wl_t). We do assume that all unemployed individuals older than 60 retire. More precisely, if $t > 60$ and $wl_t = \emptyset$ the probability of getting a new job is zero. Non-employment thus effectively becomes an absorbing state for the elderly. Note that few individuals actually return to employment after a non-employment spell, but we make no further restrictions on employment transitions. Our assumption helps to identify job finding probabilities; had we classified the large fraction of our population who is de facto retired as job searching, the estimates of $\pi_t^n(d_t^w, \emptyset, x_t)$ would be hugely downward biased. The precise functional forms for these transition probabilities are given in Section 2.4.

2.3 Recursive formulation and Bellman equations

Let $V_t(s_t, \varepsilon_t)$ denote the optimal discounted utility, which is a function of the observed state variables $s_t = (wl_t, rl_t, x_t)$ and unobserved variables ε_t. As mentioned in Section 2.1, we focus on solving for the *expected value function* $EV_t(wl_{t+1}, rl_{t+1}, x_t)$, and then express V_t in terms of current utility and discounted future utility βEV_t. Note that the expected value function at period t depends on the work and residence locations at period $t+1$. Even though this may appear as a type of "clairvoyance" of the decision makers, it is merely the consequence of our timing assumptions. Locations next period (wl_{t+1}, rl_{t+1}) are the result of decisions in the relocation stage at the start of each period.

Unlike the expected value function $EV_t(wl_{t+1}, rl_{t+1}, x_t)$, the period t (deterministic) flow utility accounts for switching costs associated with relocations, and therefore has to depend on both initial locations and the realized location. To allow for maximum flexibility in how switching costs enter the model we use a generic utility function given by $u(wl_t, rl_t, wl_{t+1}, rl_{t+1}, x_t)$. Note that the choice variables enter into the utility function indirectly: choice of work location d_t^w governs the job search process described in previous section, and under assumed perfect control the choice of residence location, we have $d_t^r = rl_{t+1}$.

Given the nested discrete choice structure in the model described in Section 2.1, the extreme value shocks $\varepsilon_t = (\varepsilon_t^w, \varepsilon_t^r)$ enter the Bellman equation in a non-trivial way. We build the Bellman equation in stages following the backward induction over the events within the time period. Let β denote the discount factor of the individual. We assume the discount factor depends on individual survival rates by allowing discount factors to change with age.

Recall that $\varepsilon_t^r \in \mathbb{R}^R$ are the stochastic components of the utility corresponding to the choice of residence location, once the outcome of the job search process is revealed, and the new work location wl_{t+1} is known. Let $\varepsilon_t^r(d_t^r)$ be the idiosyncratic utility costs/benefits of choosing to move to location d_t^r. We assume it is extreme value with scale parameter σ_r. Let $EV_t^r(wl_t, rl_t, wl_{t+1}, x_t)$ be the *ex ante* expected value for individuals who know their employment location outcome wl_{t+1} but have not learned the residential location shocks $\{\varepsilon_t^r(d_t^r)\}$ yet. This is given by the usual log-sum formula

$$
\begin{aligned}
&EV_t^r(wl_t, rl_t, wl_{t+1}, x_t) = \\
&\sigma_r \log\left(\sum_{d^r} \exp\{[u(wl_t, rl_t, wl_{t+1}, d^r, x_t) + \beta EV_t(wl_{t+1}, d^r, x_t)]/\sigma_r\}\right).
\end{aligned}
\tag{3}
$$

The implied residence location choice probabilities are given by the multinomial logit formulas

$$
P_t^r(d_t^r | wl_t, rl_t, wl_{t+1}, x_t) = \frac{\exp\{[u(wl_t, rl_t, wl_{t+1}, d_t^r, x_t) + \beta EV_t(wl_{t+1}, d_t^r, x_t)]/\sigma_r\}}{\sum_{d^r} \exp\{[u_t(wl_t, rl_t, wl_{t+1}, d^r, x_t) + \beta EV_t(wl_{t+1}, d^r, x_t)]/\sigma_r\}}.
\tag{4}
$$

Now consider the choice of the work location at the beginning of period t, d_t^w. Because this choice is moderated by the job search process, we have to take into account the probabilities $\pi_t(d_t^w, wl_t, x_t, wl_{t+1})$ that govern how the intended job location d_t^w translates into the realized one wl_{t+1}. Let $v^w(wl_t, rl_t, x_t, d_t^w)$ denote the expected choice-specific value corresponding to the particular choice of job location d_t^w. We have

$$
v_t^w(wl_t, rl_t, x_t, d_t^w) = \sum_{wl} \pi_t(d_t^w, wl_t, x_t, wl) EV_t^r(wl_t, rl_t, wl, x_t).
\tag{5}
$$

Now recall that $\varepsilon_t^w \in \mathbb{R}^{R+1}$ are the stochastic components corresponding to the choice of work location, with additional voluntary choice of non-employment. Similar to the residential location choice, let $EV_t^w(wl_t, rl_t, x_t)$ be the ex ante expected value for individuals who have not learned the work location shocks $\{\varepsilon_t^w(d_t^w)\}$ yet. Under the assumption that the shocks have an extreme value distribution with scale parameter σ_w, $EV_t^w(wl_t, rl_t, x_t)$ is given by the log-sum formula

$$EV_t^w(wl_t, rl_t, x_t) = \sigma_w \log\left(\sum_{d^w} \exp\left\{\sum_{wl} \pi_t(d^w, wl_t, x_t, wl) EV_t^r(wl_t, rl_t, wl, x_t)/\sigma_w\right\}\right). \tag{6}$$

Similarly, we have the usual multinomial logit choice probability for the choice of work location

$$P_t^w(d_t^w|wl_t, rl_t, x_t) = \frac{\exp\{v_t^w(wl_t, rl_t, x_t, d_t^w)/\sigma_w\}}{\sum_{d^w} \exp\{v_t^w(wl_t, rl_t, x_t, d^w)/\sigma_w\}}. \tag{7}$$

After accounting for the transition probabilities $\pi^x(x_t, x_{t+1})$ of the non-location state variables, which we assume are independent of both the stochastic shocks $\varepsilon_t = (\varepsilon_t^w, \varepsilon_t^r)$ and the labor market probabilities $\pi_t^n(d_t^w, wl_t, x_t)$ and $\pi_t^k(wl_t, x_t)$, we have by the definition of the expected value function

$$EV_t(wl_{t+1}, rl_{t+1}, x_t) = \sum_{x_{t+1}} \pi^x(x_t, x_{t+1}) EV_{t+1}^w(wl_{t+1}, rl_{t+1}, x_{t+1}). \tag{8}$$

Combining equations (3), (5) and (8), we obtain a Bellman operator in expected value function space that maps $EV_{t+1}(wl_{t+2}, rl_{t+2}, x_{t+1})$ in (3) into $EV_t(wl_{t+1}, rl_{t+1}, x_t)$ in (8). We solve the individual's problem by backward induction from the maximum possible age T. For each period t we compute the expected value functions $EV_t(wl_{t+1}, rl_{t+1}, x_t)$, and the corresponding choice probabilities $P_t^w(d_t^w|wl_t, rl_t, x_t)$, and $P_t^r(d_t^r|wl_t, rl_t, wl_{t+1}, x_t)$ that serve as the basis for formulating the likelihood function.

2.4 State space dynamics

Table 1 presents the non-location state variables in the model that we include to control for the heterogeneity among the households. Together with the two location variables (wl_t, rl_t) they form the full vector of state variables. The time-invariant *household type* is given by education (schooling) edu_t, while the combined children and marital status

Table 1: Non-location state variables including household types that enter x_t.

Symbol	Description	Possible Values	
cs_t	Number of children at home	0	no children
		1	1 child
		2	2 or more children
ms_t	Marital status	0	single
		1	married/cohabiting
edu_t	Education (school) type	0	Less than short cycle education
		1	Short cycle education
		2	Long cycle education (BA/master/PhD)

(cs_t, ms_t) evolves as a simultaneous state-dependent Markov process with transition probabilities defined below. The evolution of (cs_t, ms_t) depends on age and schooling. To reduce the computational burden we opt for an assumption that the number of children can maximally change by one every year. Obviously, this fails in case of twin births, couple formation where the spouse has more than one child, or if more than one child moves out of the household, but we believe this assumption is accurate enough to capture the most important dynamics. The evolution of (cs_t, ms_t) is governed by

$$(cs_{it+1}, ms_{it+1}) \sim \mu_{cs,ms}(\cdot | cs_t, ms_t, edu_t, age_t). \tag{9}$$

The transition probabilities of children and marital status are estimated separately in a first step. Given that the education is time invariant, the transition probability of the non-spatial part of the state space vector $\pi^x(x_t, x_{t+1})$ is given by (9). Details on the particular specification are given in the online appendix.

The probability of getting a new job $\pi^n_t(d^w_t, wl_t, x_t)$ and the probability of keeping the existing job $\pi^k_t(wl_t, x_t)$ are defined as follows.

$$\begin{aligned}\pi^n_t(d^w_t, wl_t, x_t) = \Bigg[1 + exp\Bigg(- \Big(\beta_0^{\pi(new)} + \beta_a^{\pi(new)} age_t + \beta_{unemp}^{\pi(new)} \mathbb{1}_{wl_t=\emptyset} \\ + \beta_{jobdensity}^{\pi(new)} jobdensity(d^w_t) + \sum_{k=1}^{2} \left(\beta_s^{\pi(new)}(k) \mathbb{1}_{edu_t=k}\right)\Big)\Bigg)\Bigg]^{-1},\end{aligned} \tag{10}$$

where the term $jobdensity(d^w_t)$ captures the heterogeneity in job moving behavior and is defined as the number of jobs of for the education edu_t jobs in region d^w_t, and $\mathbb{1}$ is the indicator function.

A positive $\beta^{\pi(new)}_{jobdensity}$ increases the individual's probability of receiving a job offer from a region which has more jobs of their education type, adding to its attractiveness. Including a full search equilibrium of the labor market in our model, as in Manning and Petrongolo (2017), would be desirable but is computationally intractable in our setting. Instead we assume the constant demand for labor and proxy job availability using the described count measure. The probability of keeping one's current job is defined by

$$\pi^k_t(wl_t, x_t) = \left[1 + exp\left(-\left(\beta^{\pi(keep)}_0 + \beta^{\pi(keep)}_a age_t + \sum_{k=1}^{2}\left(\beta^{\pi(keep)}_s(k)\mathbb{1}_{edu_t=k}\right)\right)\right)\right]^{-1}. \tag{11}$$

2.5 Specification of the utility function

The utility function $u(wl_t, rl_t, wl_{t+1}, rl_{t+1}, x_t)$ that we introduced in Section 2.3 is the "indirect utility" stemming from the specification which includes preferences for the housing size. We describe it in full detail now.

The utility of any location choice can generally be written as the sum of the following components (suppressing arguments and indices)

$$u = u_m + u_r + u_h + \underbrace{amenities - swcost^p_r - ttimecost}_{u_o}, \tag{12}$$

where u_m is the *monetary utility* (disposable income net of housing expenditures), u_r is *disutility of work* which is equal to zero when $wl_{t+1} = \emptyset$[3], u_h is the *housing utility* obtained from the utilization of a chosen home size, *amenities* reflects the *regional-specific* attractiveness of housing options, $swcost^p_r$ is the *psychological costs* of changing the location of residence, and *ttimecost* is the *cost of commuting* between the chosen locations of work and residence. According to our timing convention, all the house and regional characteristics correspond to the chosen location rl_{t+1}. It is this location that is enjoyed during period t, after the instantaneous moving phase in the beginning of the period.

The u_m component can be expressed as a product of the marginal utility of money

[3]The utility of retirement for the eligible individuals ($t \geq 60$) is instead given by a fixed constant $u_r = c_{work,ra}$.

$\kappa(inc_t)$ which depends on disposable individual income and the consumable earnings

$$u_m = \kappa(inc_t)\big(inc_t - hcost_t\big), \tag{13}$$

where inc_t denotes disposable individual income in period t, and $hcost_t$ is the cost of living in a house of the optimal size in the chosen region. We assume the following functional form for the marginal utility of money

$$\begin{aligned}\kappa(inc_t) = \kappa_0 + \sum_{\tilde{y}=1}^{Y} \kappa_{year,\tilde{y}} \mathbb{1}_{\{year=\tilde{y}\}} + \kappa_y inc_t + \kappa_{ms} ms_t + \\ + \sum_{k=1}^{2} \kappa_{c,k} \mathbb{1}_{\{cs_t=k\}} + \kappa_a age_t + \sum_{j=1}^{2} [(\kappa_{s,j} + \kappa_{as,j} age_t) \mathbb{1}_{\{edu_t=j\}}],\end{aligned} \tag{14}$$

where κ_0 and κ_{year} establish the base level of marginal utility of money in the corresponding calendar years. Assuming $\kappa_y < 0$, we have linearly decreasing marginal utility of money, implying richer households are less sensitive to the costs of housing. In the absence of a wealth state variable and a consumption/savings choice in the model, marginal utility subsumes all effects of the credit constraint or availability of mortgage. The difference in the level of marginal utility of money for couples relative to singles is given by κ_{ms}. Thus, while singles must afford housing solely through their own income, couples may share their expenses on housing which changes their marginal utility of money. To further account for heterogeneity in $\kappa(inc_t)$, we also include effects of children (κ_c), schooling (κ_s), age (κ_a) and interaction between age and schooling (κ_{as}). The age and schooling effects approximate for the potentially increasing disconnect between earned income and wealth as one ages and with higher level of schoolings.

Individual income $inc_t = inc_t(wl_t, wl_{t+1}, x_t)$ is modeled by a collection of Mincer-type equations estimated separately by region, education and calendar year. We also use regional and income-specific tax schedules to transform pre-tax income into disposable post-tax income which the individual consumes including housing and commuting expenses. More details on the specifications for incomes and tax schedules are provided in Section 3 and in the online appendix.

Regional amenities are modeled as a combination of spatial and calendar time specific constants. In particular, we use the number of cafés and bars[4] in the region as proxy

[4]More precisely we use the total number of individuals working in these industries in the region.

for downtown amenities. To reflect individual heterogeneity in tastes for amenities, we interact them with age and number of children as follows

$$amenities(rl_{t+1}) = (\alpha_0^{cafe} + \alpha_a^{cafe} age_t + \sum_{k=1}^{2} \alpha_{c,k}^{cafe} \mathbb{1}_{\{cs_t=k\}}) cafes_{rl_{t+1}} + \sum_{rl=1}^{R} \alpha_{rl} \mathbb{1}_{\{rl_{t+1}=rl\}}, \tag{15}$$

where α^{rl} is a vector of coefficients for each region and $cafes_{rl_{t+1}}$ measures the number of cafès and bars per square kilometer in region rl_{t+1}. Psychological moving costs $swcost_r^p$ are a functions of family characteristics, age and education:

$$swcost_r^p(x_t) = \mathbb{1}_{\{rl_t \neq rl_{t+1}\}} \left[\gamma_0 + \gamma_a age_t + \sum_{k=1}^{2} \gamma_{c,k} \mathbb{1}_{\{cs_t=k\}} + \gamma_{ms} ms_t + \sum_{j=1}^{2} \gamma_{s,j} \mathbb{1}_{\{edu_t=j\}}\right], \tag{16}$$

so the propensity to move changes with age and family size.

The costs of commuting between rl_{t+1} and wl_{t+1} are assumed to be proportional to the exogenous travel time between the work and home locations and allowed to change over calendar time to reflect potential changes in infrastructure and congestion. Hence, we have

$$ttimecost = (\eta_0 + \sum_{y=1}^{Y} \eta_{year,y} \mathbb{1}_{\{year=y\}}) ttime(rl_{t+1}, wl_{t+1}) \tag{17}$$

where the function $ttime(rl_{t+1}, wl_{t+1})$ denotes the travel time between work location wl_{t+1} and residence location rl_{t+1}.

We find that the regional-specific sales price of housing in the data is almost perfectly linear in home size measured in square meters of floor space (results not shown). It is therefore natural to specify housing demand $h(rl_{t+1}, x_{it}; P^h(rl_{t+1}))$ in residential region rl_{t+1} as a function of the *per square meter* regional-specific housing price $P^h(rl_{t+1})$. We can express it as an equivalent annual rental price.

Housing costs $hcost_t$ are then given by a product of size of the house h_{t+1} and equilibrium price $P(rl_{t+1})$

$$hcost_t(rl_{t+1}, h_{t+1}) = \psi_{uc} P(rl_{t+1}) h_{t+1}, \tag{18}$$

where the scale parameter ψ_{uc} reflects mortgage expenses and housing taxes and is allowed to change over calendar time as follows

$$\psi_{uc} = \psi_0 + \sum_{y=1}^{Y} \psi_{year,y} \mathbb{1}_{\{year=y\}}. \tag{19}$$

The demand for housing also depends on individual characteristics such as household size and income. This reflects that richer people can buy relatively more square meters, and that larger families may substitute space for location. We define the utility u_h of living in a house as a quadratic polynomial of its size h_t with heterogeneous coefficients

$$u_h = \Phi(x_t) h_{t+1} + \frac{1}{2} \phi_{h2} h_{t+1}^2, \tag{20}$$

where $\phi_{h2} < 0$ governs the degree of diminishing returns to house size and $\Phi(x_t)$ is a heterogeneous parameter given by

$$\Phi(x_t) = \phi_0 + \sum_{y=1}^{Y} \phi_{year,y} \mathbb{1}_{\{year=y\}} + \phi_a age_t + \phi_{a2} age_t^2 + \phi_{ms} ms_t + \\ + \sum_{k=1}^{2} \phi_{c,k} \mathbb{1}_{\{cs_t=k\}} + \sum_{j=1}^{2} \phi_{s,j} \mathbb{1}_{\{edu_t=j\}} + \sum_{rl=1}^{R} \phi_{rl} \mathbb{1}_{\{rl_{t+1}=rl\}}. \tag{21}$$

Based on the specification of the utility function in (12)-(14) and the housing cost in (18), we form the first order conditions for the optimal amount of housing. The optimal choice of the house size is given by

$$h_{t+1} = [\kappa(inc_t) P(rl_{t+1}) \psi_{uc} - \Phi(x_t)] \phi_{h2}. \tag{22}$$

Substituting expression (22) back into the utility function defined in equations (12)-(19), we obtain the final specification of the indirect utility function $u(wl_t, rl_t, wl_{t+1}, rl_{t+1}, x_t)$.

Therefore, our model assumes that households can freely adjust the size/quality of their home in each period and in every region independent of moving.[5] Moreover, because we abstract away from any savings and home equity, households only consider the ”square meter rental costs” that pertains to homes in each region through local prices. Both of these assumptions allow for the optimal amount of housing to be separable from the dynamic choice of location and be expressed as the solution to a static subproblem that enters into the indirect instantaneous utility. This greatly reduces the computational

[5]This is equivalent to having no cost of moving within the region to the house of optimal size.

burden, effectively allowing the structural estimation of the model to be carried out.

3 Structural Estimation

This section describes the estimation strategy. We estimate the model sequentially in three steps. First, we separately estimate the parameters of the income and tax equations as well as parameters in transition probabilities of the states. Second, we estimate the the housing demand equation to obtain *reduced form* parameters which are rescaled within structural estimation during the next stage. Finally, we use maximum likelihood to estimate the remaining structural parameters.

3.1 Summary of the data sample

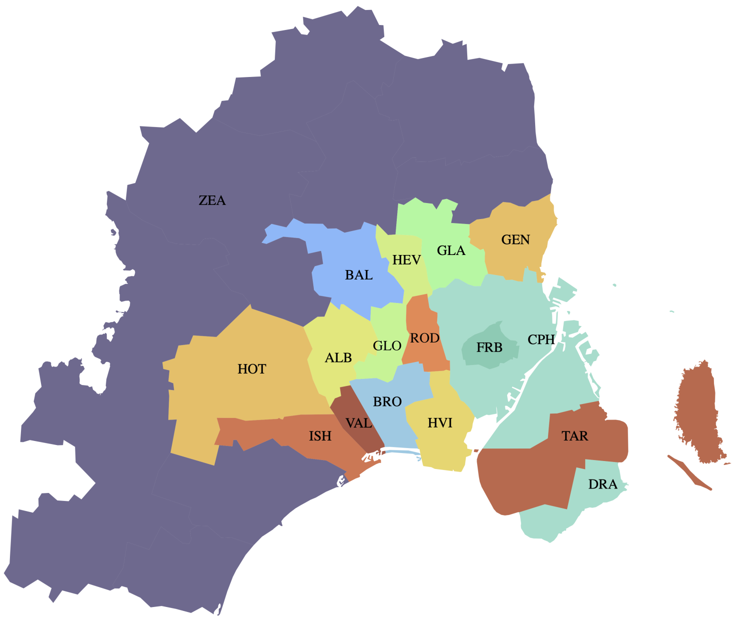
We have access to full population register data for the period 1993-2013. This data combines detailed information on residential and work locations, demographic characteristics, housing size and labor market outcomes. For the estimation we focus on the years 2000-2004 and 2009-2013. We exclude the years around the housing boom in the financial crisis and do not attempt to model the temporary price hike during this period, but rather focus on explaining the increasing spatial variation in house prices characterized by increasingly diverging house prices between urban and rural areas.[6]

We observe each individual's choice $d_{i,t} \equiv \{rl_{i,t+1}, wl_{i,t+1}, h_{i,t+1}\}$ and state $s_{i,t}$ on an annual basis. Further, for each calendar year we observe regional house prices, local amenities and local labor market attributes such as the number of jobs available for individuals with different levels of education. As mentioned earlier, we focus on the municipalities in the GCA. Figure 1 shows the set of regions that we use and their abbreviations.

The data reveal that while 58% of residential moves do not involve a change in work region, a large share of 33% of home location moves involve a change in work region either one year before or after the residential move. This highlights the importance of modeling the dynamics and simultaneity in home and work location choices.

[6]Detailed description of how the sample we use is constructed from individual Danish registers is provided in the online appendix.

Figure 1: Definition of regions in the Greater Copenhagen Area



Note: The abbreviations denote the following regions: Copenhagen (CPH), Frederiksberg (FRB), Ballerup (BAL), Broendby (BRO), Dragoer (DRA), Gentofte (GEN), Gladsaxe (GLA), Glostrup (GLO), Herlev (HEV), Albertslund (ALB), Hvidovre (HVI), Hoeje-Taastrup (HOT), Roedovre (ROD), Ishoej (ISH), Taarnby (TAR), Vallensbaek (VAL), rest of Zealand (ZEA).

3.2 Income equations, tax equations and transition probabilities

The estimation of transition probabilities for children and marital status, $\mu_{cs,ms}$, is performed non-parametrically on the data pooled over age conditional on the level of schooling. Survival probabilities are also estimated non-parametrically over age by marital status and schooling. See the online appendix for further details.

In order to capture regional differences in both income level and its age gradient, we estimate the coefficients of the wage equations separately for each combination of region and education. Unemployment benefits are estimated lumped up with other social security payments.[7] We find significant variation in incomes across regions, education groups and these differences vary over the life cycle, which all contribute to a stronger identification of marginal utility of money.

In addition, we estimate regional-specific tax schedules with three income brackets to resemble the Danish income tax system where municipality taxes in rich municipalities

[7]We do not allow for regional differences in non-employment income, even though these are indeed observed due to differences in savings. Yet, since an individual would not be able to change her savings by moving, we abstract away from differences in average regional savings. The result is of course that the returns to income received while working is downward biased in rich areas and upward biased in poor areas.

tend to be lower than in poor municipalities. Predicted tax payments from the model explain 91% of the variation in actual tax payments.[8]

3.3 Housing demand

The fact that the optimal choice of the housing size characterized by the first order condition (22) is a static sub-problem independent of the dynamic location choices allows us to estimate it in the second step of our three stage estimation sequence. Using the pooled micro data we estimate the following demand for housing equation by OLS

$$\begin{aligned} h_{it+1} = \tilde{\phi}_0 + \sum_{y=1}^{Y} \tilde{\phi}_{year,y} \mathbb{1}_{\{year=y\}} + \tilde{\phi}_a age_t + \tilde{\phi}_{a2} age_t^2 + \tilde{\phi}_{ms} ms_t + \sum_{k=1}^{2} \tilde{\phi}_{c,k} \mathbb{1}_{\{cs_t=k\}} \\ + \sum_{j=1}^{2} \tilde{\phi}_{s,j} \mathbb{1}_{\{edu_t=j\}} + \sum_{rl=1}^{R} \tilde{\phi}_{rl} \mathbb{1}_{\{rl_{t+1}=rl\}} + \tilde{\kappa}_y [inc_t \cdot \psi_{uc} P(rl_{it+1})] \\ - \Bigg(\tilde{\kappa}_0 + \sum_{y=1}^{Y} \tilde{\kappa}_{year,y} \mathbb{1}_{\{year=y\}} + \tilde{\kappa}_{ms} ms_t + \sum_{k=1}^{2} \tilde{\kappa}_{c,k} \mathbb{1}_{\{cs_t=k\}} + \tilde{\kappa}_a age_t \\ + \sum_{j=1}^{2} [(\tilde{\kappa}_{s,j} + \tilde{\kappa}_{as,j} age_t) \mathbb{1}_{\{edu_t=j\}}] \Bigg) \psi_{uc} P(rl_{it+1}) + \rho_{it}, \end{aligned} \tag{23}$$

where ρ_{it} is a random error. Note that the parameters $\tilde{\phi}$ and $\tilde{\kappa}$ in the *reduced-form demand equation* in (23) are proportional to the structural parameters that index marginal utility of money $\kappa(\cdot)$ and heterogeneous housing utility parameters in $\Phi(\cdot)$. The respective scale factors $-1/\phi_{h2} > 0$ and $-\psi_{uc}/\phi_{h2} > 0$ are identified together with the remaining structural parameters on the third stage of our estimation procedure. In other words, the reduced-form estimates are kept fixed during the structural estimation and only *rescaled* using the values of the structural parameters ψ_{uc} and ϕ_{h2}. This approach significantly reduces the dimensionality of the maximum likelihood problem when estimating the full model.

3.4 Maximum likelihood estimation of structural parameters

Having obtained estimates for state transitions, income and tax equations and reduced form (scaled) housing demand parameters, the final step involves estimating structural parameters, θ, by maximum likelihood. Recall that θ includes parameters indexing

[8] Further details are available in the online appendix.

probability of getting a new job, (10), probability of keeping current job, (11), marginal utility of money, (14), housing costs, (18) including user costs in (19), utility values of the amenities, (15), psychological costs of moving residence, (16), travel time costs, (17), the utility of retiring, $c_{work,ra}$, and the degree of diminishing returns to house size, ϕ_{h_2}. We fix the discount factor to $\beta = 0.95$ but multiply it by individual survival rates that we estimate.

The likelihood function is derived from the choice probabilities for work and home location decisions given in (4) and (7). Because we assume perfect control for residential location, the latter can be directly evaluated at the data, giving the likelihood of the observed location of residence. To calculate the likelihood of the observed work location, however, we have to integrate out the likelihood over the possible choices and only write the likelihood in terms of observed *work location transitions*, i.e. as transition probabilities from state wl_t to wl_{t+1}.

Observing a "null" transition wl_t to $wl_{t+1} = wl_t$ could have resulted from both an individual deciding to keep their job, and being successful with probability $\pi_t^k(wl_t, x_t)$, and an individual trying to find a new job d_t^w and being unsuccessful with probability $\left(1 - \pi_t^n(d_t^w, wl_t, x_t)\right)\pi_t^k(wl_t, x_t)$. Observing a transition wl_t to $wl_{t+1} \neq wl_t$ could have resulted only from an individual deciding to move jobs and being successful with probability $\pi_t^n(wl_{t+1}, wl_t, x_t)$.

The above two cases also apply for unemployed, $wl_t = \emptyset$. But transitions from employment, $wl_t \neq \emptyset$, to unemployment, $wl_{t+1} = \emptyset$, may happen in three different ways. With probability $\left(1 - \pi_t^n(d_t^w, wl_t, x_t)\right)\left(1 - \pi_t^k(wl_t, x_t)\right)$ an individual could have unsuccessfully tried to transition to a job d_t^w, and at the same time has been displaced. Or, with probability $1 - \pi_t^k(wl_t, x_t)$ an individual could have tried to keep their job wl_t, but was unsuccessful and laid off. Finally, an individual could have voluntarily chosen to quit working, $d_t^w = \emptyset$.

Recall that $\pi_t(d_t^w, wl_t, x_t, wl_{t+1})$ defined in Section 2.3 summarizes the work transition probabilities as a function of the intended work location. The contribution to the likelihood for an individual who is in *observed* work location wl_t and residential location rl_t at time t and in *observed* work location wl_{t+1} and residential location rl_{t+1} at time

$t+1$ is

$$L_t(wl_t, rl_t, wl_{t+1}, rl_{t+1}, x_t) =$$
$$P^r_t(rl_{t+1}|wl_t, rl_t, wl_{t+1}, x_t) \cdot \sum_{d^w} P^w_t(d^w|wl_t, rl_t, x_t)\pi_t(d^w, wl_t, x_t, wl_{t+1}). \qquad (24)$$

The full log-likelihood is constructed from individual likelihoods in the standard way by collecting the individual likelihood contributions and the objective of the maximum likelihood estimation is thus

$$L(\theta) = \frac{1}{N}\sum_i \sum_t \{\log P^r_t(rl_{it+1}|wl_{it}, rl_{it}, wl_{it+1}, x_{it};\theta)+$$
$$+\log \sum_{d^w} P^w_t(d^w|wl_{it}, rl_{it}, x_{it};\theta)\pi_t(d^w, wl_{it}, x_{it}, wl_{it+1};\theta)\}, \quad (25)$$

where N is the number of individuals. To estimate the structural parameters we proceed in the spirit of the Nested Fixed Point (NFXP) algorithm by Rust (1987) . Hence, for each evaluation of the likelihood function we solve the model via backwards induction in each calendar year.

3.5 Solving for equilibrium house prices

The equilibrium object we solve for is housing/rental prices in each calendar year[9], whereas incomes, job arrival and dismissal rates are taken as given and housing supply is assumed fixed. Thus, we take a short-term perspective and thus abstract from the longer run dynamics where new houses are built in response to changes in house prices. We do not attempt to model equilibrium wage determination in the labor market (or longer run location decisions by employers), and ignore that firms in reality may change labor demand in their locations (and thus the number of jobs offered in different locations) in response to changes in local labor supply.

In equilibrium we assume that the total demand for housing measured in square meters equals the supply in each residential region. Thus, when solving for the housing market equilibrium, the R-dimensional vector of regional square meter prices $P^h = (P^h(1), .., P^h(R))$ is set to equate the inelastic, exogenously fixed supply $S_t(rl)$ of total square meters of housing to the demand for the available square meters $D_t(rl, P^h)$ in each

[9]Where we assume that household hold beliefs that equilibrium prices hold during their lifetime.

residential region $rl = \{1, \dots R\}$. For the supply, we simply aggregate the individual-level demand for observed square meters of housing h_{it} for people who already live in region $rl_{it} = rl$ at the beginning of each period t

$$S_t(rl) = \sum_{i=1}^{N} h_{it} \mathbb{1}(rl_{it} = rl). \tag{26}$$

The regional demand for housing $D_t(rl, P^h)$ is calculated as the *expected demand* by taking a population average of housing demand weighted by choice probabilities of either staying or moving to region rl at the end of period t. To obtain demand, we start by simulating N individual states by drawing from observed states in the dataset with replacement. We then simulate a work location *outcome*, wl_{t+1}, using the decision rule P_t^w and job transition probabilities π_t such that we can condition on these in the computation of demand below:

$$D_t(rl, P^h) = \sum_{i=1}^{N} h(rl, x_{it}; P^h(rl)) \Pi_t(rl | wl_{it+1}, rl_{it}, x_{it}; P^h), \tag{27}$$

where $\Pi_t(rl | wl_{it+1}, rl_{it}, x_{it}; P^h)$ is the probability that an individual in state $s_{it} = (wl_{it+1}, rl_{it}, x_{it})$ chooses to live in region rl given the vector of regional house prices, P^h and simulated work location wl_{it+1}. Π_t is given by the right hand side of (4), but here we have added P^h as an argument to signify its dependence on house prices.

The resulting simulator for demand is in principle not smooth given that we have simulated a work location *outcome*, wl_{t+1} using a simple accept/reject simulator. However, since the conditional demand for residence, $\Pi_t(rl | wl_{it+1}, rl_{it}, x_{it}; P^h)$, is smooth in the vector of housing prices and employment probabilities, we can use gradient-based methods to calculate equilibrium. We calculate the house price equilibrium by stacking the excess demand equations to have a system of R equations (for the housing market) in R unknowns. We then solve for the R-dimensional price vector P^h using Newton's method.

The short run equilibrium concept is imposed for simplicity. To work with a long run equilibrium notion that endogenizes the supply of housing, we would need data on zoning regulations and decisions by home builders and developers where to build more in different regions. Finally, commuting times/costs are potentially something to

Table 2: First Stage Parameter Estimates, Reduced form Housing Demand

	Coeff. Estimates	Standard Error	Z-statistic
Const., $\tilde{\phi}_0$	70.2740	0.16419	428.0
Married, $\tilde{\phi}_{ms}$	27.8578	0.03868	720.2
Children, $\tilde{\phi}_c$ (1)	5.7386	0.05149	111.4
Children, $\tilde{\phi}_c$ (2)	14.6098	0.04911	297.5
Age, $\tilde{\phi}_a$	2.1723	0.00348	624.8
Age^2/1000, $\tilde{\phi}_{a2}$	-19.1718	0.03074	-623.6
Price pr. sqm, $\tilde{\kappa}_0$	-296.2954	0.91043	-325.4
Price pr. sqm $\times$ Income, $\tilde{\kappa}_y$	20.2790	0.07002	289.6
Price pr. sqm $\times$ Age, $\tilde{\kappa}_a$	0.0209	0.00853	2.4
Price pr. sqm $\times$ Age x Schooling, $\tilde{\kappa}_{a,s}$ (1)	1.0073	0.00476	211.5
Price pr. sqm $\times$ Age x Schooling, $\tilde{\kappa}_{a,s}$ (2)	2.9563	0.00529	558.4
Price pr. sqm $\times$ Schooling, $\tilde{\kappa}_s$ (1)	-51.7247	0.24317	-212.7
Price pr. sqm $\times$ Schooling, $\tilde{\kappa}_s$ (2)	-95.3167	0.25080	-380.1
Price pr. sqm $\times$ Children, $\tilde{\kappa}_c$ (1)	0.4389	0.29673	1.5
Price pr. sqm $\times$ Children, $\tilde{\kappa}_c$ (2)	13.4067	0.28757	46.6
Price pr. sqm $\times$ Married, $\tilde{\kappa}_{ms}$	-63.4794	0.22117	-287.0

Dependent variable: House size in square meters
Other controls: Regional dummies, $\tilde{\phi}_{rl}$ and time effects $\tilde{\phi}_{year}$ and $\tilde{\kappa}_{year}$

endogenize too, including in the short run. If the counterfactual equilibrium results in changed location patters, the resulting utilization of the road network will change as well and thereby affect congestion and commuting times. Future work will focus on these more involved specifications.

4 Estimation Results

In this section we present the parameter estimates for the reduced form housing demand equations and the complete dynamic location choice model. We show that the model fits the dynamics of observed location choices, sorting and commute patterns quite well over the life cycle and across space. Using the estimated model to solve for equilibrium house prices, we find that the equilibrium prices implied by our model closely track the increased spatial divergence and year to year local price developments of house prices observed in the data.

4.1 Parameter estimates and model fit

Table 2 presents parameter estimates and Figure 2 shows the corresponding model fit obtained from the reduced form housing demand regression in (23). We regress the size of individuals' homes measured in square meters, against demographic variables as well as price per square meter, P_{it} and its interaction between the two.

The coefficients have reasonable magnitudes and expected signs. We see that demand is increasing as a function of age and household size. For individuals with the same marginal utility of money, those who live in couples have homes that on average measure 27.9 m^2 more than singles. Children living at home is also associated with larger dwellings (5.7 additional m^2 for families with one child and 14.6 m^2 with two or more children). Housing demand is decreasing in prices for all individuals as the combined heterogeneous reduced form coefficient on prices, $\frac{\kappa\psi_{uc}}{\phi_{h2}}$ is negative for all combinations of demographics and predicted income after taxes. As expected, richer individuals are generally less price-responsive (the coefficient on prices interacted with income is positive). We also include an educational-specific linear effect in age to proxy for differences in for example life-cycle wealth. Compared to low-educated individuals, the highly educated are more price sensitive when young and less price sensitive later in life. This contributes to explaining the higher housing demand later in life.

The average price per square meter was $32,145$ DKK In Copenhagen in year 2000 (measured in 2011 consumer prices).[10] Hence, individuals choosing to live in the Copenhagen municipality will on average demand $20.279 \cdot 32,145/100,000 = 6.5$ more square meters of housing for each additional 100,000 DKK of individual annual income after tax. Similarly, a single individual aged 30, with no children, low education, and after-tax income of 300,000 DKK living in Copenhagen demands 38.0 fewer square meters of housing compared to an individual with similar characteristics living outside the capital area (Rest of Zealand) where square meter prices are $15,981$ DKK on average, i.e. around 16,000 DKK lower than in Copenhagen municipality[11].

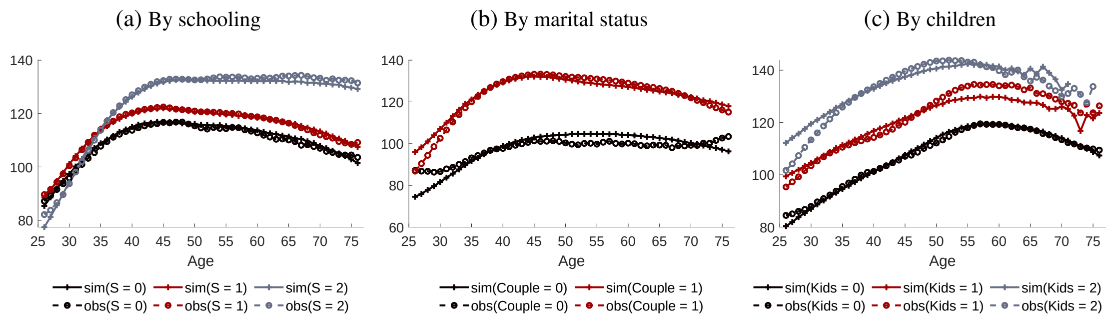
Figure 2 clearly illustrates the demographic differences in housing demand over the life cycle. We also see that the reduced form model for housing demand closely captures the overall differences in the changes in demand over the life cycle for each educational group. Though there are some challenges capturing the differences in demand at the beginning of the life cycle when we separate the age profiles by marital status or children,

[10] Both after-tax income, inc_{it} and house prices, P_{it} are measured in $100,000$ DKK/year. After-tax income and house prices are deflated using consumer prices with the base year 2011 so that for example the implicit willingness to pay for housing, amenities and commuting will be measured in 2011 units of disposable income. We use this unit of measurement throughout.

[11] The difference in housing demand across these two regions is computed as $(-296.2954 + 20.2790 \cdot 3 + 0.0209 \cdot 30) \cdot (0.32145 - 0.15981) = -37.958$.

the model does capture the crucial dependence between household composition and housing demand.

Figure 2: House size in square meters over the life cycle



Recall that our estimation strategy only allowed for identification of scaled parameters in the first step housing demand. The vector of reduced form coefficients $(\tilde{\phi} = -\phi/\phi_{h2}, \tilde{\kappa} = \kappa\psi_{uc}/\phi_{h2})$ in Table 2 are proportional to the structural parameters that index the marginal utility of housing $\phi = (\phi_0, \phi_{ms}, \phi_c, \phi_a, \phi_{a2})$ and the parameters $\kappa = (\kappa_0, \kappa_y, \kappa_c, \kappa_{ms}, \kappa_s, \kappa_{a,s})$ that index marginal utility of money, but scaled with $-\phi_{h2} > 0$ and $\phi_{h2}/\psi_{uc} < 0$ respectively. Holding fixed the reduced form estimates from the first-step demand equation, we estimate $\phi_{h2} < 0$ and $\psi_{uc} > 0$ along with the remaining structural parameters by maximizing the likelihood from the residential and work location choice model.

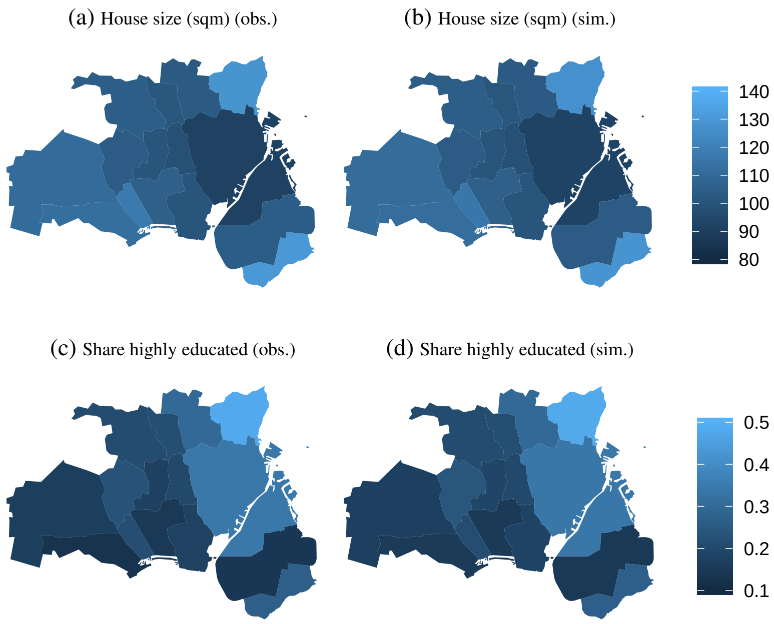
Table 3: User Cost of Housing and Curvature Parameter of Housing Demand

	Coeff. Estimates	Standard Error	Z-statistic
Coef. on $h^2, \phi_{h2} \times 1000$	-0.0465	0.00036	-127.4
Baseline user cost of housing, ψ_0	0.0239	0.00024	99.9
Time effect, ψ_{2001}	-0.0052	0.00015	-34.9
Time effect, ψ_{2002}	-0.0045	0.00016	-28.8
Time effect, ψ_{2003}	-0.0063	0.00015	-41.0
Time effect, ψ_{2004}	-0.0090	0.00016	-57.3
Time effect, ψ_{2009}	-0.0035	0.00015	-23.6
Time effect, ψ_{2010}	-0.0076	0.00015	-50.2
Time effect, ψ_{2011}	-0.0089	0.00015	-57.7
Time effect, ψ_{2012}	-0.0088	0.00015	-57.1
Time effect, ψ_{2014}	-0.0110	0.00016	-67.4

The parameter estimates for ϕ_{h2} and ψ_{uc} are given in Table 3. We estimate the annual user costs of housing to $\hat{\psi}_{uc} = 0.024$, i.e. 2.4% of the house market value. This is definitely on the low side, but there are certain factors that explain it. First, interest payments are deductible in the income tax, so these rates should reflect user costs after taxes. Since the tax value of interest rate deductions is around 35%, the user cost before taxes are correspondingly larger. Furthermore, our estimation period 2000-2004 is mostly characterized by increasing housing prices. In the standard user cost equation for housing, expected discounted capital gains reduce the user cost. If that equation truly lies in the back of people's mind when making housing purchases, then increasing prices and optimistic expectations work to decrease the user costs. This might be what our estimate of ψ_{uc} is picking up.

Using the estimates of $\phi_{h2} = -0.0465/1000$ and $\psi_{uc} = 0.0239$ together with the reduced form estimates in housing demand given in Table 2, we can back out the parameters that index marginal utility of money. As an example, we obtain $\kappa_0 = 0.1525$ and $\kappa_y = -0.0104$. Despite the negative gradient in income (and age for the highly educated), these parameters result in relatively large estimates of marginal utility of money throughout most of the income distribution. The parameters therefore imply a strong trade-off between home size and residential location and a clear sorting by richer individuals into more attractive and expensive regions and larger houses.

Figure 3: Residential sorting and house size by home region



Note: Panels (a) and (b) show the average size of homes in square meters by home region. Panels (c) and (d) show share of highly educated by home region.

Table 4: Taste Variation in Regional Amenities

	Coeff. Estimates	Standard Error	Z-statistic
Taste for cafes and bars, α^{cafe}			
Constant, α_0^{cafe}	0.0118	0.00005	257.9
Age, α_a^{cafe}	-0.0002	0.00000	-257.5
Children, α_c^{cafe} (1)	-0.0047	0.00005	-97.8
Children, α_c^{cafe} (2)	-0.0074	0.00003	-254.4

Other controls: Regional dummies, $\alpha_{rl_{t+1}}$, shown in online appendix.

Figure 3 illustrates the ability of the model to fit housing demand across regions and the sorting of highly educated individuals. The spatial variation in square meter demand and distribution of highly educated are captured well by the model. Residential sorting by age, family composition and marital status match equally well (results not shown). The main channel of educational sorting arises because the income equation is specifically tied to the individual's education, and income predicts the home location through marginal utility of money. For example, the model is able to predict that the share of highly educated is high in Copenhagen, Frederiksberg and Gentofte where square meter prices are high.

Residential sorting is driven mainly by four factors: i) regional variation in house prices and regional-specific amenities, ii) individual differences in housing demand, iii) individual differences in marginal utility of money and iv) distance to local labor markets. The presence of local amenities helps rationalize why individuals prefer to live in regions where prices are high for reasons that are not explained by factors such as better access to local labor markets. To flexibly capture time-constant regional-specific amenities, we include fixed effects for each residential region, $\alpha_{rl_{t+1}}$. Some urban amenities such as parks and green spaces have largely been time constant in the sample period, while others such as cultural centers, restaurants, cafés and bars have changed considerably during the sample period, especially in Copenhagen[12]. To capture this development we include an index based on the number people working in cafés and bars per square kilometer in the set of amenities. The parameter estimates are presented in Table 4.

We allow the taste for these amenities to depend on the demographic variables in the model. The results are in line with our expectations; the taste for cafés and bars declines with age and the number of children. Clearly, such factors are important in explaining

[12]See online appendix for summary statistics of number of cafés and bars in each region.

Table 5: Utility Cost of Moving Residence

	Coeff. Estimates	Standard Error	Z-statistic
Const., γ_0	1.8750	0.00521	360.0
Age, γ_a	0.0579	0.00012	495.1
Children, γ_c (1)	0.4934	0.00382	129.2
Children, γ_c (2)	1.1926	0.00450	265.0
Married, γ_{ms}	-0.0368	0.00291	-12.6
Schooling, γ_s (1)	0.0163	0.00309	5.3
Schooling, γ_s (2)	-0.1803	0.00317	-56.9

the changes in spatial sorting over time, where younger individuals to an increasing extend are willing spend a larger fraction of their income to live in Copenhagen.

Figure 4: Share living in Copenhagen over the life cycle

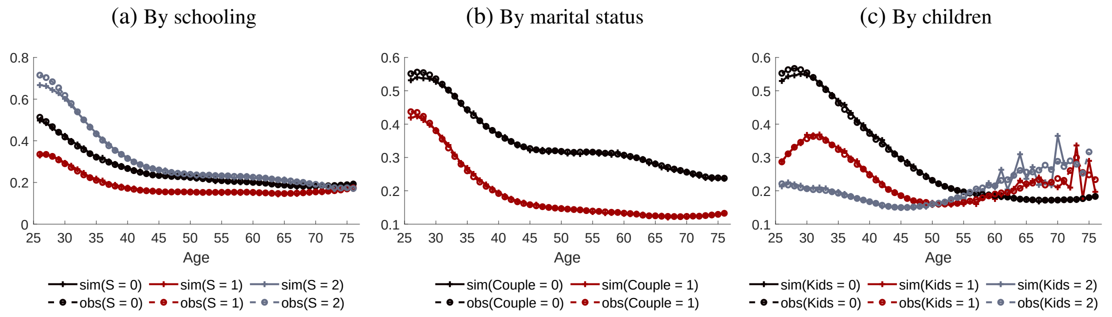
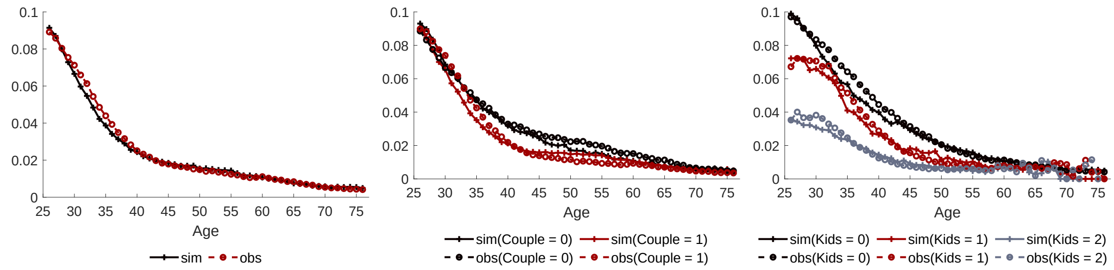


Figure 4 shows the model's overall high ability to fit the probability of living in Copenhagen, but over the life cycle instead of the spatial allocation. Only for the youngest cohorts is there a slight under-prediction. This is partly due to the fact that we do not model educational choice. As many higher educational institutions are located in Copenhagen, this is what attracts many young people to that location. Figure 4a does indeed show that poorer fit is only evident for individuals with high education. It should be noted that these moments are not only driven by the estimates of amenity values, but to some extent by the moving costs that prevent people from moving away from their initial, observed locations. Table 5 displays the estimates for the parameters γ that index these utility costs associated with moving residence. Moving costs increase with age and number of children, reflecting the tendency for one's live to stabilize as one ages (e.g. slower increase in income) and stronger attachment to the local community through kindergartens and schools when children arrive.

Married and highly-educated people are more mobile. The latter reflects the steeper

Figure 5: Share moving residential location over the life cycle

(a) Overall (b) By marital status (c) By children



income profile over the life cycle for highly educated and their more pronounced tendency to reside in Copenhagen when young. Over time, their preferences for living outside of Copenhagen start to kick in, hence prompting a move. The former indicates that, compared to singles, married families are more likely to be prompted to move. In a single-person family, only one person can trigger a move. The extension of the model to explore the intra-household decision making is delegated to future research.

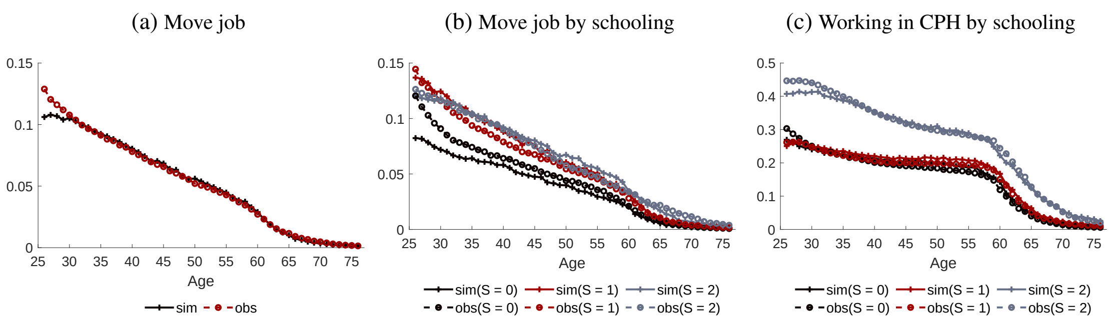
Overall, the model fit in terms of residential moving probabilities is good according to Figure 5a. There is a slight under-prediction in the start of the life cycle, especially for individuals without children and couples. The largest prediction error is found for the probability of moving to and from Copenhagen (results not shown). The general shape of the probability of moving away from Copenhagen (as a share of all individuals in our data) is captured by the model, but it under-predicts the level until the age of 45. A similar pattern is found for the share migrating to Copenhagen. As mentioned above, a key factor left out of the model is that we ignore the obvious fact that Copenhagen is a university city. Without explicitly modeling educational choice and the dynamics of occupational career choice it is hard to explain why younger individuals with low income choose to live in Copenhagen. Other omitted factors are individual taste variation over a more detailed set of regional-specific amenities such as child care and school quality which can readily be included into this model at a low computational cost.

We now move to the ability of the model to predict work location outcomes. Table 6 displays estimates for the parameters for the job arrival and dismissal probabilities $\pi_t^n(d_t^w, wl_t, x_t : \beta^n)$ and $\pi_t^k(wl_t, x_t; \beta^k)$ that determine the work location transition probabilities. They imply a high probability of keeping a job as expected, and there is a positive effect of age and higher levels of schooling. Hence, a 40-year-old individual with low education has a 97.7 percent chance of keeping the job. On the one hand, this high keep probability is an implication of the fact that we focus on inter-regional job moves while most job transitions are within-firm or intra-regional. On the other hand, those types of job moves are of second order when explaining changes in commuting behavior over time and over the life cycle. However, the large regional differences in supply of jobs[13] is strongly reflected in the job probabilities through the job density parameter.

Table 6: Job Arrival and Dismissal

	Coeff. Estimates	Standard Error	Z-statistic
Probability of keeping job: $\pi_t^k(wl_t, x_t; \beta^k)$			
Const., $\beta_0^{\pi(keep)}$	0.3066	0.01085	28.3
Age, $\beta_a^{\pi(keep)}$	0.0558	0.00030	186.7
Schooling, $\beta_s^{\pi(keep)}$ (1)	0.9288	0.00536	173.4
Schooling, $\beta_s^{\pi(keep)}$ (2)	1.0818	0.00575	188.0
Probability of new job: $\pi_t^n(d_t^w, wl_t, x_t : \beta^n)$			
Const., $\beta_0^{\pi(new)}$	-1.0998	0.00466	-235.9
Age, $\beta_a^{\pi(new)}$	-0.0431	0.00010	-415.9
Schooling, $\beta_s^{\pi(new)}$ (1)	0.1980	0.00253	78.3
Schooling, $\beta_s^{\pi(new)}$ (2)	0.2264	0.00278	81.5
Job density $\beta_{jobdensity}^{\pi(new)}$	0.2608	0.00045	583.9
Prev. unempl., $\beta_{unemp}^{\pi(new)}$	1.0474	0.00236	443.9

Figure 6: Job moves



Note: Panels (a) and (b) show the share of all individuals who move work location away from and to Copenhagen, respectively.

Figure 6 shows that the model generally captures the overall age-dependence in regional job transitions well, although there are some challenges of modeling the work transition probabilities for the younger individuals. Concerning the probability of moving

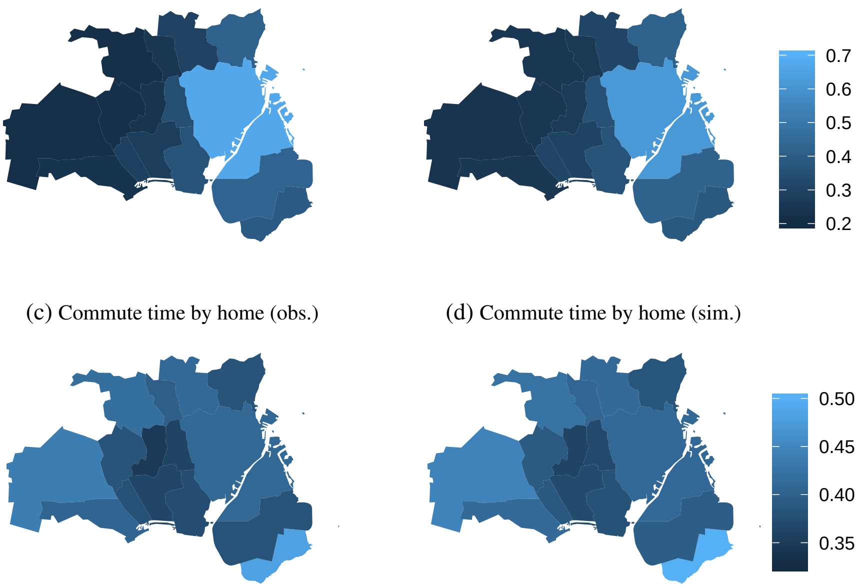
[13] See online appendix for summary statistics on job density by region.

work location to and from Copenhagen, the model predicts that the share moving their job away from Copenhagen (out of all individuals in the data) slightly under-predicts the actual share. The motivation for moving one's job conditional on the home location is shorter commute or higher incomes. Commute distances are exogenous and thus independent of age, while incomes have an age profile. Consequently, incomes may not exhibit enough variation across individuals to perfectly capture the shape. Allowing for unobserved individual heterogeneity in incomes might improve on the fit since we would better capture whether the more mobile individuals are those who have a high unobserved fixed component of incomes that they can bring with them when they move around.

Considering instead the share working in Copenhagen in Figure 6c, the fit looks very good for the individuals older than 35. The heterogeneity across individuals is also reflected in the model predictions. The work location decision is less well-captured for the young highly educated people because we do not model initial conditions or educational choice.

Looking at the share of individuals working in Copenhagen by their home municipality, the top panel of Figure 7 shows how the model captures the spatial distribution quite well. Overall, the model also provides a very good fit of the average commute for each residential region. The estimated commute cost parameter $\eta_0 = 0.179$, with standard error of 0.0015, is an economically and statistically significant parameter, implying substantial disutility of commuting. An average employed person with an after-tax income of 300,000 DKK requires 70,000 DKK additional income to be willing to commute 30 extra minutes per day. Figure 8 illustrates the commute time over the life cycle and across different types of individuals and it is predicted very accurately by the model across several dimensions. For instance, the model captures very well that highly educated commute shorter because they can afford housing close to Copenhagen center, where most of their jobs are, while lower educated workers, whose jobs are less concentrated in Copenhagen, are more likely to live and work outside of the most dense areas implying longer commutes. It is mainly for individuals above age 60 that the model starts to struggle, but there is also a strong selection among working individuals at that age. Thus, it is not surprising that they cannot necessarily be compared to the younger working cohorts.

Figure 7: Work in Copenhagen and commute times (hours) by residential location



(a) Work in Cph. by home (obs.)

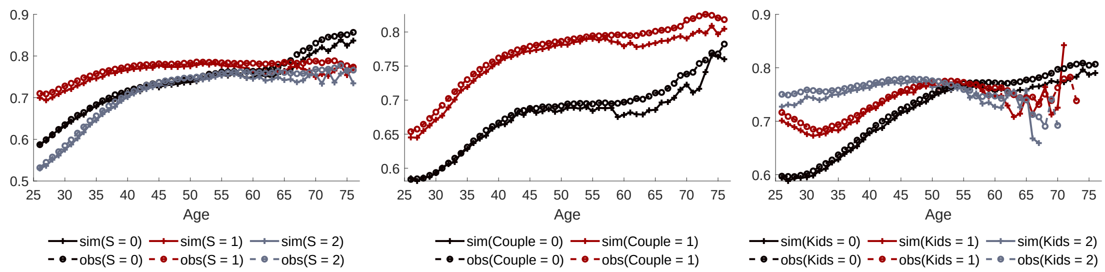
(b) Work in Cph. by home (sim.)

(c) Commute time by home (obs.)

(d) Commute time by home (sim.)

Note: Panels (a) and (b) show the share of individuals in each home region who works in Copenhagen or Frederiksberg. Panels (c) and (d) show average commute time in hours by home region for employed individuals.

Figure 8: Commute time (hours)



(a) By schooling

(b) By marital status

(c) By children

Table 7: Baseline fit: Change in home and work locations (share by schooling) and equilibrium prices (100,000 DKK)

(data - baseline)	$rl(s_0)$	$rl(s_1)$	$rl(s_2)$	$wl(s_0)$	$wl(s_1)$	$wl(s_2)$	P^{eq}
Center of CPH	0.000	0.006	-0.048	0.003	0.033	-0.009	-0.023
West of CPH	-0.005	-0.003	0.012	0.119	0.120	0.111	0.010
North of CPH	-0.004	0.000	0.006	0.019	0.019	0.007	0.000
East of CPH	0.001	-0.001	0.003	0.024	0.025	0.024	0.004
RestOfZealand	0.008	-0.002	0.027	-0.142	-0.222	-0.170	0.012
Unemployment	-	-	-	-0.024	0.024	0.037	-

4.2 Baseline equilibrium

An ultimate test of the goodness of fit of our model is its ability to replicate the house prices observed in the data by direct computation of the equilibrium prices implied by the estimates of the structural parameters, as described in Section 3.5.

Figure 10 plots the computed equilibrium prices against observed price data. The fit appears to be very good both in terms of the price ranking of the different regions as well as the overall levels. Here is it is important to emphasize that the model is estimated without explicitly imposing that the housing market is in equilibrium. The fact that the equilibrium prices predicted from our estimated model closely track the observed house prices in the different regions and across time, provides a good in-sample validation of the many cross-equation restrictions implied by our modeling of location choices and demand for house size.

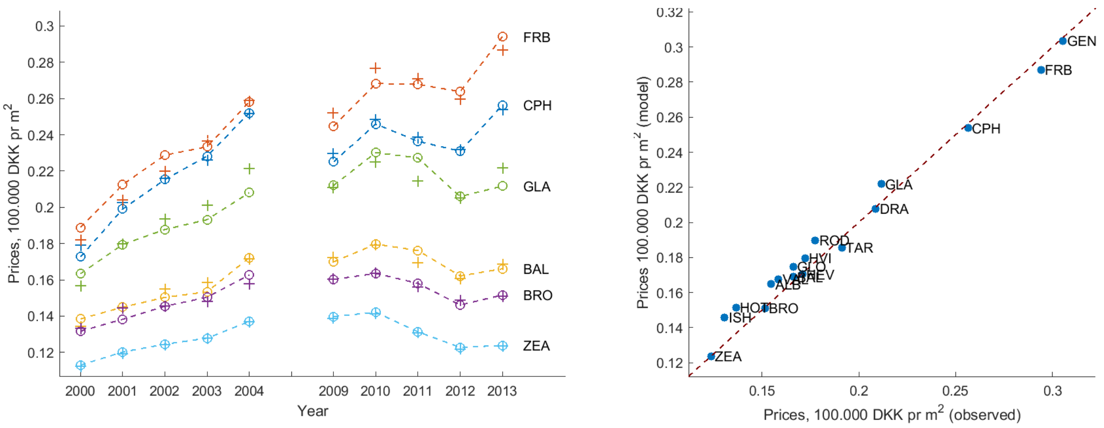
With the overall fit being exceptionally good, there is a slight over-prediction of prices in the cheapest regions and an under-prediction in Frederiksberg, one of the most expensive regions. Our parsimonious modeling of individual income and the lack of savings are again among the potential explanations as to why the model does not fully capture why people are willing to pay such high prices in Frederiksberg, when they can live in Copenhagen at a lower price and with better access to a much higher job density.

To further explore the goodness of fit of the model, we make the comparison between simulated and empirical data for each year in 2009-2013. Thus, we simulate the model forward starting in 2009 and allowing endogenous states to develop over the period. The simulation starts at the empirical data on which the model was estimated. The outcome of simulating the model one period ahead from the empirical data yields the initial state (period 0) of the baseline simulation. Table 7 shows the difference in the distribution of

Figure 9: Observed and predicted equilibrium house prices per m^2

(a) Selected regions, 2000-2004 and 2009-2013

(b) All regions, 2013



Note: Figure 10 show the model fit for equilibrium prices per m^2 (100,000 DKK). All house prices are deflated with consumer prices (base year 2011). Panel (a) plots observed (o) and predicted (+) equilibrium house prices per m^2 over the sample periods 2000-2004 and 2009-2013 (selected regions). Panel (b) plots predicted equilibrium prices (vertical axis) against observed prices (horizontal axis) for all regions in 2013.

residential and work location choices for each education group as well as the difference between empirical and model equilibrium prices in 2013 at an aggregated regional level for expositions. Regions are grouped with other regions that show a similar development. We see that differences in shares across most locations are in the order of 0 to ± 2.5 percentage points, indicating the model's reasonable ability to forecast the locational choices over time. The model has trouble explaining why individuals would work in Rest of Zealand though. This under-prediction mainly implies an over-prediction of working in municipalities West of Copenhagen which are close in space to Rest of Zealand. The inability of the model to predict the desire to work there is not a surprise given the more inaccurate description of this region in terms of travel time and amenities. On the other hand, the model does a good job matching equilibrium prices across regions. The mismatch is in the order of -2,300-1,200 DKK per square meter. Overall, we consider this evidence that the model is able to provide meaningful predictions forward in time and therefore useful for analyzing the implications of counterfactual simulations.

5 Counterfactual Equilibria Simulations

In this section we turn to the evaluation of counterfactual implications of i) an increase in housing supply and ii) a introduction of remote work and telecommuting. We model the latter as a differential reduction of commuting costs for educated workers relative to less educated workers. The counterfactual simulations show strong response in the equilibrium housing prices throughout the country which induce significant relocation of both homes and work.

5.1 Counterfactual I: Increased housing supply

In order to make a valid comparison between the baseline of the model and a counterfactual simulation, we make a series of one period forward simulations and recompute the equilibrium on the housing market at each step similar to how the baseline simulation was done in the end of the previous section. We start the simulation from the empirical data and simulate one period under the baseline policy to form the initial conditions for the counterfactual simulation. The counterfactual policy changes are imposed at the beginning of the next simulated period (period 1). At the end of period 1 it is therefore possible to identify all changes between baseline and counterfactual outcomes at the household level. We run each counterfactual simulation for 5 periods starting in 2009 taking it out to 2013.

The first counterfactual experiment involves a 5 percent exogenous and permanent increase of the housing supply (square meters) in Copenhagen and Frederiksberg which constitute the aggregate region Center of Copenhagen. We interpret this as an illustration of a small-scale version of the planned expansion of the housing supply in central Copenhagen due to the construction of the artificial island Lynetteholm[14]. In general, we find that increasing the supply of housing in a high-density area lowers the prices significantly in this region. This effect spreads to the remaining regions in the economy.

Table 8 summarizes the implications increased housing supply to the location choice for the three levels of schooling. As expected, the share living in Copenhagen increases and it does so by 2.3-2.8% for each education group. The distribution of skills living in the Copenhagen center thus does not change much in equilibrium. This also means that

[14]The official report on the analysis of the Lynetteholm project will be available from the DMT in late 2022.

a policy of simply increasing available square meters does not automatically contribute to fulfilling an agenda of e.g. a more mixed city in terms of socioeconomic background. To achieve such a goal, policy makers would have to ensure these extra square meters are used for construction of smaller size homes to attract low-income individuals. Since our model does not model the supply side and rather allows individuals to scale housing up and down without any adjustment costs, we cannot evaluate a more targeted housing policy. However, the conclusion that urbanization increases is robust.

Those who move in response to the policy change do so differently dependent on their educational level; i.e. low-skilled individuals mainly move out of the municipalities east of Copenhagen while medium-skilled people arrive more uniformly from the other regions. High-skilled individuals primarily move away from the municipalities west of the Copenhagen center. This also means that while the distribution of skills in Copenhagen center itself is not impacted significantly by the policy change, it changes more in the remaining regions. The map in Figure 11a shows the percentage change in the share of highly educated people by home region. We find the largest relative drops in most regions west of Copenhagen though it increases by 1.4% in Glostrup. These different responses to moving away from a certain region points to the heterogeneous trade-offs that are faced by people with various educational background and hence earnings perspectives. A high-skilled worker considers living in regions west of Copenhagen a closer substitute to Copenhagen center than any other region because these two aggregate regions both offer a rather short commute time to high-paying jobs for this group (e.g. in Ballerup which offers one of the highest incomes, all else equal) and closeness to the hub for high-skilled jobs in Copenhagen. Low-skilled workers, on the other hand, have a particularly good job market in Taarnby which belongs to East of Zealand. They would therefore like to live close by this region. In the baseline, however, living in Copenhagen would be too expensive for a low-income individual while the counterfactual changes this trade-off due to changes in equilibrium prices.

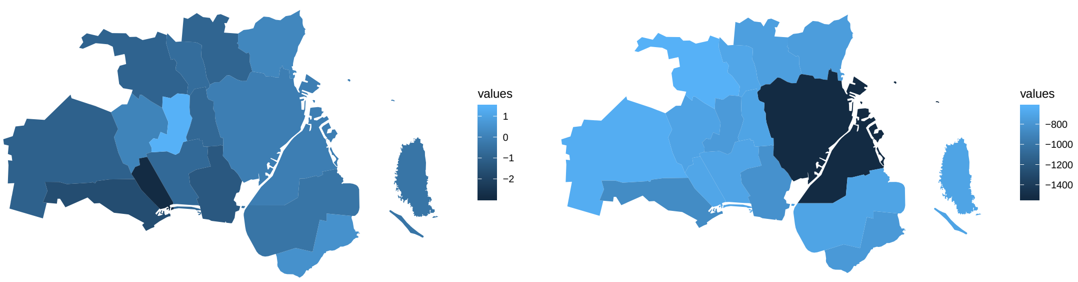
The implications for equilibrium prices are presented in Figure 11b for each region in 2013. The map shows that the supply shock to Copenhagen center is not fully soaked up by an increased number of residents: the equilibrium price per square meter is 1,530 DKK (6.6%) lower in the counterfactual. This is exactly what allows the low-skilled people to reside here while staying close to Taarnby where their high-paying jobs are.

Table 8: Counterfactual I: % change of home and work locations by schooling (2013)

	$rl(s_0)$	$wl(s_0)$	$rl(s_1)$	$wl(s_1)$	$rl(s_2)$	$wl(s_2)$
Center of CPH	2.40	0.27	2.83	0.16	2.32	0.14
West of CPH	-1.06	-0.00	-0.92	-0.20	-2.06	0.18
North of CPH	-1.31	-1.03	-0.55	-0.04	-1.25	0.19
East of CPH	-2.02	0.09	-0.98	-0.12	-1.60	-0.03
RestOfZealand	-0.79	-0.03	-0.54	-0.02	-0.82	-0.51
Unemployment	-	-0.01	-	0.19	-	-0.27

Figure 10: Counterfactual I: Simulated change in sorting and P^{eq} in 2013

(a) % change in share highly educated (b) DKK price change per m^2



At the same time, they are now even closer to the high job density that Copenhagen also offers for this group. The map also illustrates another important aspect of the model; namely that regions are more or less close substitutes. That is, prices also decline in all other regions because the overall supply of housing has risen. The same is true for rest of Zealand which is not included in the map.

5.2 Counterfactual II: increased telecommuting

In the second counterfactual we decrease the commute time by 50% for educated workers in all region pairs and within all regions but leave commuting times for less educated workers unchanged. This is intended to resemble policies that encourage telecommuting that benefit mainly educated workers of the "information sector" but do not benefit less educated workers of "service sector" who must be physically present to perform their jobs.

The policy makes it easier for people with higher level of schooling to keep high paying jobs located in city centers while living in more attractive suburban areas. The overall effect on locational decisions in 2013 are described in Table 9. We find that while highly educated workers move home away the GCA to Rest of Zealand, lower educated

Table 9: Counterfactual II: % change of home and work locations by schooling (2013)

	$rl(s_0)$	$wl(s_0)$	$rl(s_1)$	$wl(s_1)$	$rl(s_2)$	$wl(s_2)$
Center of CPH	3.77	0.44	4.52	0.39	-10.79	3.95
West of CPH	3.46	0.05	4.13	0.01	-13.23	-0.41
North of CPH	7.31	-0.06	7.51	-0.05	-9.62	-0.21
East of CPH	2.08	-0.11	4.81	-0.51	-10.40	1.07
RestOfZealand	-3.95	-0.44	-3.09	-0.76	11.69	7.64
Unemployment	-	-0.27	-	-0.20	-	-8.38

workers make the opposite transition. Consequently, this policy provides lower-income households the opportunity to reside closer to dense labor markets and all regions become more mixed on socioeconomic characteristics.

For the higher educated individuals, both the shares working in Center of Copenhagen and Rest of Zealand have increased. I.e. some of those moving away from the GCA find it worthwhile moving their job there now that they only have to do the relatively long intra-regional commutes or the commute to Copenhagen center half of the week. The main part of these extra workers in Rest of Zealand come from a significant reduction in non-employment though due to the improved trade-off between working and not working when commuting costs are much lower. This is an essential insight: individuals who live in regions with very high commute times will be discouraged to work, all else equal. Improving commute times in rural regions can contribute to reducing the unemployment rates there which are often seen to be higher than in the city centers. For the other education groups, work locations have only changed slightly towards more urbanization despite the relatively large responses on the residential margin. This means these individuals were (more) constrained in baseline as they would prefer living closer to the Copenhagen center but were not able to. All the above relocations result in and from changing equilibrium housing prices. These changes are summarized in Table 10. Looking at the table, the strength of modeling the dynamics of home and work decisions stands out: due to the lower demand for living in the center of Copenhagen for the higher educated as explained above, equilibrium prices fall by 0.53% immediately. However, individuals are reluctant to move instantly due to moving costs so it takes a few years for the population to fully re-locate. By 2013 the prices have fallen by 1.6-2.0% in the GCA regions, making these regions affordable for lower-income households, and increased by 3.0% in Rest of Zealand. This gradual change in relocations and prices underlines the

Table 10: Counterfactual II: % change in equilibrium prices 2009-2013

	2009	2010	2011	2012	2013
Center of CPH	-0.53	-1.01	-1.35	-1.64	-1.95
West of CPH	-0.36	-0.64	-0.93	-1.15	-1.60
North of CPH	-0.57	-1.08	-1.46	-1.58	-1.75
East of CPH	-0.06	-0.10	-0.79	-1.04	-1.66
RestOfZealand	0.79	1.55	2.08	2.45	3.03

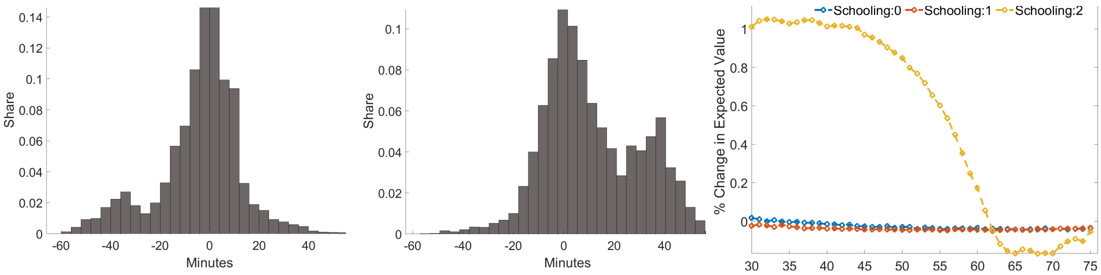
strength of our dynamic model.

To summarize the effects of changes in residential and work locations as well as equilibrium prices, one can study the combined effect, namely commute time. Figure 12a and Figure 12b show the distribution of simulated changes in commute times (conditional on commuting) for low- and high-educated individuals who were working in both baseline and counterfactual and have moved their residence compared to the baseline. We show the conditional effect for exposition, and especially to avoid a large mass point at 0.

While lower educated mainly experience reductions in commute time, the opposite is true for higher educated. This emphasizes the importance of modeling home and work choices simultaneously; whenever commuting incentives are modified, both home and work locations respond. Consequently, a large mass of these higher educated people commute longer *when* they commute, but only do so half of the week. An example is the large probability mass around a commute time change of 30-40 minutes in panel (b). Zooming in on those individuals, we see that they represent people who moved to Rest of Zealand and chose to work in either Rest of Zealand or Copenhagen in particular to benefit from the high employment probabilities. In the baseline, these people lived on the perimeter of the GCA which are all characterized by rather low commute times. Hence, in the new equilibrium, some individuals are willing to endure the higher commute times a few times a week when moving to rural regions in exchange for lower square meter prices, despite the fact that prices across regions started to converge after the intervention. This example demonstrates the complexity of predicting the implications of such a policy and again underlines the importance of applying a rich dynamic model to capture such patterns.

Finally, we compute welfare effects of the travel time reduction. Figure 12c illustrates the average percentage change in the expected value for a given level of schooling over

Figure 11: Counterfactual II: Simulated change in commute time (minutes) and welfare (%)



(a) Δ minutes (low education) (b) Δ minutes (high education) (c) %Δ EV

Note: Changes computed by subtracting baseline from counterfactual. Panels (a) and (b) show changes in commute times conditional on commuting. Panel (c) shows the relative change in welfare.

the life cycle. Highly educated people are the winners as they benefit from having access to the higher-paying jobs in Copenhagen center without paying the high housing prices for this access. Gradually, as these individuals' marginal utility of money decrease due to increasing income with age, the welfare change starts to drop towards 0 around the age of retirement. At that age, most have opted out of the labor force and no longer commute. However, they are facing the increased housing prices in regions where they now live, as indicated by the negative welfare change during the retirement phase. This cost is more than offset by the positive changes to welfare experienced in the younger ages.

For lower educated individuals, the welfare gain is close to zero. This is a combination of them gaining from lower commute times, but only by paying the higher housing prices compared to baseline where they were less likely to live within the expensive GCA. In conclusion, lowering commute times is therefore a welfare-improving policy as it reduces frictions, though the welfare gains are unequally distributed in the population.

6 Conclusion

In this paper we developed a dynamic equilibrium model of joint home and work location decisions as well as housing demand for individuals and estimated its structural parameters using Danish administrative panel data. We found that overall the empirical fit of the model is very good. We focused on the Greater Copenhagen Area (GCA) and analyzed the counterfactual effects of i) increasing the housing supply in the center of Copenhagen and ii) encouraging more telecommuting for highly educated.

Overall, the model developed and estimated in this paper provides valuable insights into our understanding of the location and movement patterns among Danish households. These are driven by the cost of living and commuting and are very heterogeneous in the population. From counterfactual i) we learned that urbanization increased and sorting patterns changed in regions outside the center of Copenhagen. Counterfactual ii) revealed that highly educated would move out of the city to peripheral regions where they could consume larger homes at a reduced price. These relocations freed up space in the center such that lower educated people could afford living closer by their jobs in the new equilibrium. In general, reducing commute times would therefore relax the binding between residential and work locations (here primarily for higher educated) and thereby allow locations to become more specialized in either jobs or residence. As a result, welfare gains were positive in total, but unequally distributed across household types with higher educated being better off and lower educated indifferent.

The implementation of the model is not free of simplifying assumptions, but even in their presence it proves to be a very valuable tool, capable of explaining important variation in the data, and enabling us to undertake interesting counterfactual experiments. Among most significant limitations are the assumptions of temporary equilibrium on the housing market and abstraction from capital gains of housing. Both are computationally infeasible to implement in the current version of the model where we work with a choice set with 18×17 alternatives and a state space of almost 300,000 points. The regions can be less aggregated, and a wider area of the country than the GCA can be used for estimation. Inclusion of the equilibrium wage settlement into the consideration is another obvious dimension for improvement. Even under the assumption of short-term dynamics in the labor market similar to the housing market (so that the supply of jobs is constant) the wages can be treated similarly to house prices and be determined in the spatial equilibrium. All of these latter improvements, although requiring additional work and computational time, are straightforward to implement. We acknowledge their relevance, but leave the implementations for future research.

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