Sovereign Risk and Dutch Disease

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Motivation

- “Natural resource curse”: resource rich economies grow more slowly
- One explanation: the “Dutch disease”
  - growth externality: innovation in manufactring sector
  - natural resource exports depress investment in manufacturing
- Also applies to other inflows (foreign borrowing, foreign aid)
- This paper:
  - Does the Dutch disease affect default risk?
  - Is it a “disease”?

This paper

- Theoretical framework:
  - sectoral allocation of capital affects default risk
  - externality: effect of private capital portfolio on default risk
  - trade-off: future returns to investment vs. present borrowing terms
  - decentralization: tax on returns to non-traded investment
- Quantitative exercise:
  - commodity windfalls amplify externality
  - higher optimal tax (or reserve accumulation) during windfalls

Contribution to literature

- **Sovereign default with natural resources:** López-Martín, Leal, Martínez Fritscher (2019); Hamann, Mendoza, Restrepo-Echavarría (2020); Esquivel (2021)
  - Contribution: decentralization of production
- **Private externality to public debt:** Wright (2006); Kim, Zhang (2012); Arce (2021); Galli (2021); Wu (2021)
  - Contribution: externality in sectoral allocation of capital
- **Dutch disease:** Corden, Neary (1982); Benigno, Fornaro (2013); Alberola, Benigno (2017); Ayres, Hevia, Nicolini (2020)
  - Contribution: study effects of Dutch disease on sovereign default risk

Two-period Model

Environment

- Small-open economy with a continuum of households, competitive firms, and a government
- Households:
  - own firms and capital
  - choose capital allocation
- Benevolent government:
  - issues non-contingent debt in international markets
  - lacks commitment

Preferences and technology

- Household preferences

$$U(c_0, c_1) = u(c_0) + \beta \mathbb{E}_0 [u(c_1)]$$

  where $u' > 0$, $u'' < 0$, and is invertible

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- Consumption good produced with technology

$$c_t = Y(c_{N,t}, c_{T,t}) = \left[\omega^{\frac{1}{\eta}} c_{N,t}^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} c_{T,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where $\eta > 0$ is the elasticity of substitution, $\omega \in (0, 1)$

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- Intermediates produced with technologies:

$$y_N(z_t, K_{N,t}) = z_t K_{N,t}^{\alpha_N}$$
$$y_T(z_t, K_{T,t}) = z_t K_{T,t}^{\alpha_T}$$

where z_0 given, $z_1 \in [\underline{z}, \bar{z}]$ is a shock with CDF $F(z)$

Capital and portfolio allocation

- Fixed amount of capital $\bar{K}$ in economy
- households endowed with $\bar{k} = \bar{K}$, cannot sell to foreigners
- Capital freely allocated in each sector one period in advance:

$$k_{N,t} + k_{T,t} = \bar{k}$$

- Let $k_{T,t} = \lambda_t \bar{k}$ and $K_{T,t} = \Lambda_t \bar{K}$, where $\lambda_t, \Lambda_t \in [0,1]$
  - λ_t is portfolio allocation of a representative household
  - Λ_t is portfolio allocation of the economy
  - Initial $\lambda_0 = \Lambda_0$ given

Debt and default

- Government has legacy B_0, issues B_1
- (z, x) is the aggregate state, $x = (\Lambda, B)$
- In $t = 1$ government observes (z_1, x_1) and makes default decision
  - no default: $C^P(z_1, x_1) = Y\left(y^P_{N,1}, y^P_{T,1} - B_1\right)$
  - default: $C^D(z_1, x_1) = Y\left(y^D_{N,1}, y^D_{T,1}\right)$, productivity is $z_D(z_1) \leq z_1$
- Default set is $\mathcal{D}(x) = [\underline{z}, z^*(x))$, with cutoff $z^*(x)$ such that

$$C^D(z^*(x), x) = C^P(z^*(x), x)$$

Timing

- Period 0:
  1. Government issues B_1 problem
  2. Households observe B_1 and choose λ_1 problem
  3. Foreign lenders observe B_1 and Λ_1 and purchase the debt
  4. Production and consumption occur
- Period 1:
  1. Government observes z_1 and decides to default or repay
  2. Production and consumption occur

Equilibrium and efficiency

- Equilibrium definition is standard
- The Euler equation of a representative household is:

$$0 = \mathbb{E}\left[\beta u'\left(\tilde{C}_1\right) \frac{\left(\tilde{r}_{T,1} - \tilde{r}_{N,1}\right)\bar{K}}{\tilde{P}_1}\right]$$

- The Euler equation of a benevolent planner is:

$$0 = \mathbb{E}\left[\beta u'\left(\hat{C}_1\right) \frac{\left(\hat{r}_{T,1} - \hat{r}_{N,1}\right)\bar{K}}{\hat{P}_1}\right] + u'\left(\hat{C}_0\right) \frac{\partial \hat{\boldsymbol{q}}}{\partial \Lambda} \frac{\hat{B}_1}{\hat{P}_0}$$

Misallocation of capital prices

- **Proposition 1**: If $\eta < 1$, then the default set is shrinking in Λ_1. That is, $\frac{\partial z^*(x)}{\partial \Lambda} \leq 0$. Proof

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from Proposition 1 it follows that q is increasing in Λ, then:

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- Implement efficient allocation with tax:

$$\tau^* = \frac{u'\left(\hat{C}_0\right)\frac{\hat{\partial q}}{\partial \Lambda}\hat{B}_1/\hat{P}_0}{\mathbb{E}\left[\beta u'\left(\tilde{C}_1\right)\tilde{r}_{N,1}/\tilde{P}_1\right]\bar{K}}$$

Infinite horizon

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- Capital for traded k_T and non-traded production k_N
  - stocks depreciate at rate δ
  - adjustment costs $\Psi\left(K_{i,t+1}, K_{i,t}\right)$

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- Timing within period:
  - shocks -> government's decisions -> investment decisions -> debt auction and repayment

Misallocation of capital

- The **household's** no-arbitrage condition is:

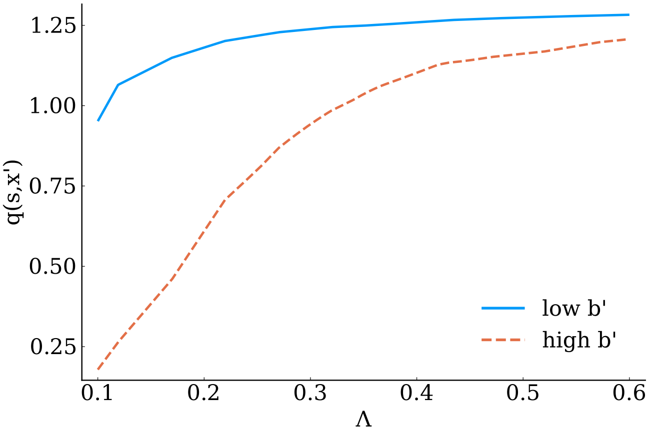
$$0 = \mathbb{E}_t \left[\frac{\beta u'\left(\tilde{c}_{t+1}\right)}{u'\left(\tilde{c}_t\right)} \left(\tilde{R}^D_{T,t+1} - \tilde{R}^D_{N,t+1} \right) \right]$$

- From the **planner's** Euler equation:

$$0 = \mathbb{E}_t \left[\frac{\beta u'\left(\hat{C}_{t+1}\right)}{u'\left(\hat{C}_t\right)} \left(\hat{R}^D_{T,t+1} - \hat{R}^D_{N,t+1} \right) \right] + \left[\frac{\partial \hat{q}}{\partial K_T} - \frac{\partial \hat{q}}{\partial K_N} \right] \frac{\hat{B}' - (1-\gamma) B}{\hat{P}_0}$$

Misallocation of capital calibration

- Recall $q(s, x')$, let $\bar{K} = K_{N,ss} + K_{T,ss}$ and $q(s, \Lambda', B') = q\left(s, (1 - \Lambda')\bar{K}, \Lambda'\bar{K}, B'\right)$



Business Cycle Moments

- Simulate 300 economies of 1050 quarters, drop the first 1000
- Use only samples that start at least 25 quarters after last default

	Planner	Decentralized		Planner	Decentralized
$r - r^*$	7.1%	12.3%	σ_{GDP}	5.8	7.1
$Pr\,(\text{default})$	1.5%	3.0%	σ_c / σ_{GDP}	1.2	1.33
B/GDP	0.30	0.45	$\sigma_{inv} / \sigma_{GDP}$	3.8	4.1
K_N / Y	0.87	1.11	$Cor\,(ca/gdp, gdp)$	-0.44	-0.45
K_T / Y	1.13	1.05	$Cor\,(r - r^*, gdp)$	-0.61	-0.32

Optimal tax and welfare

- Simulate 10,000 quarters, drop the first 1000

	$y_C = y_{C,L}$	$y_C = y_{C,H}$
average τ^*	2.4%	3.0%
Pr (default decentralized)	3.5%	1.5%
Pr (default planner)	2.6%	0.7%

- Welfare computation

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u \left(c^{Pla} \right) \right] = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u \left((1+\chi)\, c^{Dec} \right) \right]$$

yields $\chi = 0.07$

Conclusion

- Sectoral allocation of capital affects default risk:
  - direction of effect driven by complementarity of traded and non-traded goods
  - implies pecuniary externality with private investment
  - natural resources amplify the externality
- Policy implications:
  - strong case for exchange-rate sterilization policies
- In data, resource rich countries: empirical
  - face more stringent borrowing costs
  - accumulate reserves during commodity windfalls

Firms and prices back

- Prices in default are

$$p^D_{N,t} = \left(\frac{\omega}{1-\omega} \frac{y_T\left(z_D\left(z_t\right), \Lambda_t \bar{K}\right) + T^D_t}{y_N\left(z_D\left(z_t\right), \left(1-\Lambda_t\right)\bar{K}\right)} \right)^{\frac{1}{\eta}}$$

$$P^D_t = \left[\omega \left(p^D_{N,t} \right)^{1-\eta} + (1-\omega) \right]^{\frac{1}{1-\eta}}$$

$$r^D_{N,t} = p^D_{N,t} \alpha_N z_D\left(z_t\right) \left(\left(1-\Lambda_t\right)\bar{K}\right)^{\alpha_N - 1}$$

$$r^D_{T,t} = \alpha_T z_D\left(z_t\right) \left(\Lambda_t \bar{K}\right)^{\alpha_T - 1}$$

- Prices in repayment are

$$p^P_{N,1} = \left(\frac{\omega}{1-\omega} \frac{y_T\left(z_1, \Lambda_1 \bar{K}\right) + T^P_t}{y_N\left(z_1, \left(1-\Lambda_1\right)\bar{K}\right)} \right)^{\frac{1}{\eta}}$$

$$P^P_1 = \left[\omega \left(p^P_{N,1} \right)^{1-\eta} + (1-\omega) \right]^{\frac{1}{1-\eta}}$$

$$r^P_{N,1} = p^P_{N,1} \alpha_N z_1 \left(\left(1-\Lambda_1\right)\bar{K}\right)^{\alpha_N - 1}$$

$$r^P_{T,1} = \alpha_T z_1 \left(\Lambda_1 \bar{K}\right)^{\alpha_T - 1}$$

Firms and prices Back

- Prices in default are

$$p_{N,t}^D = \left(\frac{\omega}{1-\omega} \frac{y_T\left(z_D\left(z_t\right), \Lambda_t \bar{K}\right) + T_t^D}{y_N\left(z_D\left(z_t\right), \left(1-\Lambda_t\right)\bar{K}\right)} \right)^{\frac{1}{\eta}}$$

$$P_t^D = \left[\omega \left(p_{N,t}^D \right)^{1-\eta} + (1-\omega) \right]^{\frac{1}{1-\eta}}$$

$$r_{N,t}^D = p_{N,t}^D \alpha_N z_D\left(z_t\right) \left(\left(1-\Lambda_t\right)\bar{K}\right)^{\alpha_N - 1}$$

$$r_{T,t}^D = \alpha_T z_D\left(z_t\right) \left(\Lambda_t \bar{K}\right)^{\alpha_T - 1}$$

- Prices in repayment are

$$p_{N,1}^P = \left(\frac{\omega}{1-\omega} \frac{y_T\left(z_1, \Lambda_1 \bar{K}\right) + T_t^P}{y_N\left(z_1, \left(1-\Lambda_1\right)\bar{K}\right)} \right)^{\frac{1}{\eta}}$$

$$P_1^P = \left[\omega \left(p_{N,1}^P \right)^{1-\eta} + (1-\omega) \right]^{\frac{1}{1-\eta}}$$

$$r_{N,1}^P = p_{N,1}^P \alpha_N z_1 \left(\left(1-\Lambda_1\right)\bar{K}\right)^{\alpha_N - 1}$$

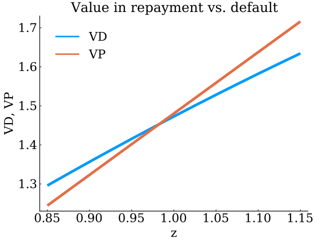
$$r_{T,1}^P = \alpha_T z_1 \left(\Lambda_1 \bar{K}\right)^{\alpha_T - 1}$$

Proof Back

- **Proposition 1**: If $\eta < 1$, then the default set is shrinking in Λ_1. That is, $\frac{\partial z^*(x)}{\partial \Lambda} \leq 0$.
- *Proof*: the derivative of z^* is

$$\frac{\partial z^*(x)}{\partial \Lambda} = -\frac{\frac{\partial C^P(z^*,x)}{\partial \Lambda_1} - \frac{\partial C^D(z^*,x)}{\partial \Lambda_1}}{\frac{\partial C^P(z^*,x)}{\partial z} - \frac{\partial C^D(z^*,x)}{\partial z}}$$

- the denominator is **positive** because C^P and C^D are increasing in z and
  - for $z < z^*(x)$ we have $C^D > C^P$



  - for $z \geq z^*(x)$ we have $C^D \leq C^P$

Proof Back

- the numerator $\frac{\partial V^P(z^*,x)}{\partial \Lambda_1} - \frac{\partial V^D(z^*,x)}{\partial \Lambda_1}$ is **positive** if $\eta > 0$. Note that:

$$\frac{\partial C}{\partial \Lambda} = \underbrace{\frac{\partial Y}{\partial c_T}\frac{\partial y_T}{\partial K_T}\bar{K}}_{MPK \text{ of extra } K_T} - \underbrace{\frac{\partial Y}{\partial c_N}\frac{\partial y_N}{\partial K_N}\bar{K}}_{MPK \text{ of less } K_N}$$

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- so evaluated at (z^*, x) we get the numerator is:

$$\frac{\partial C^P}{\partial \Lambda} - \frac{\partial C^D}{\partial \Lambda} = \left[\frac{\partial Y^P}{\partial c_T}y_T^P - \frac{\partial Y^D}{\partial c_T}y_T^D\right]\frac{\alpha_T}{\Lambda} + \left[\frac{\partial Y^D}{\partial c_N}y_N^D - \frac{\partial Y^P}{\partial c_N}y_N^P\right]\frac{\alpha_N}{1-\Lambda}$$

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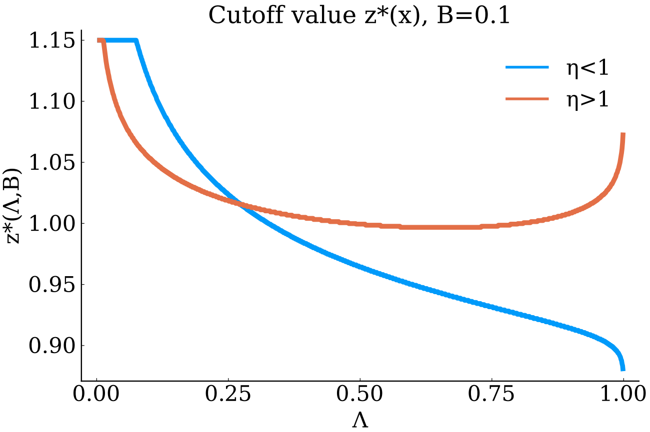
- computing the derivatives of Y and using $y_N^i = c_N^i$:

$$\frac{\partial C^P}{\partial \Lambda} - \frac{\partial C^D}{\partial \Lambda} = \left[\underbrace{\left(\frac{1}{c_T^P}\right)^{\frac{1}{\eta}} z^* - \left(\frac{1}{c_T^D}\right)^{\frac{1}{\eta}} z_D^*}_{>0}\right]\frac{(1-\omega)^{\frac{1}{\eta}}\kappa_T}{\Lambda} + \left[\underbrace{\left(\frac{1}{c_N^D}\right)^{\frac{1-\eta}{\eta}} - \left(\frac{1}{c_N^P}\right)^{\frac{1-\eta}{\eta}}}_{>0 \text{ if } \eta<1}\right]\frac{\omega^{\frac{1}{\eta}}\kappa_N}{1-\Lambda}$$

where the signs follow from $c_N^D \leq c_N^P \implies c_T^D \geq c_T^P$ at (z^*, x).$\square$

Proof



Cutoff value z*(x), B=0.1
1.15
1.10
1.05
1.00
0.95
0.90
z*(Λ,B)
0.00
0.25
0.50
0.75
1.00
Λ
η<1
η>1

Back

Empirical analysis Back

- Want to test three implications of the model about resource rich economies:
  1. face more stringent borrowing terms (higher spreads)
  2. accumulate international reserves during commodity windfalls
  3. exchange rates appreciate during commodity windfalls

Empirical analysis

Back

- Spreads:
  - EMBI spreads: 1993-2015, 37 countries
  - Institutional Investor Index (III): 1979-2015, 184 countries
  - constructed EMBI spreads using III

$$\ln\left(\text{spread}_{i,t}\right) = \gamma_0 + \gamma_1 \ln\left(III_{i,t}\right) + \kappa_i + \mu_t + \epsilon_{i,t}$$

- Natural resource rents as a fraction of GDP from World Development Indicators
- Total external debt stocks and central government debt as a fraction of GDP
- International reserves as a fraction of GDP from IMF
- Real exchange rate calculated as $\xi_{i,t} = \frac{e_{i,t} P_t^{US}}{P_{i,t}}$

Empirical analysis Back

$$s_{i,t} = \beta_0 + \beta_1 \overline{NR}_i + \beta_2 100 * \frac{debt_{i,t}}{GDP_{i,t}} + \beta_3 100 * \frac{reserves_{i,t}}{GDP_{i,t}} + \mu_t + u_{i,t}$$

	(1) EMBI	(2) EMBI	(3) Constructed EMBI	(4) Constructed EMBI
Av (NR rents / GDP)	**0.128****	**0.137**	**0.208****	**0.926*****
	(0.0605)	(0.125)	(0.0804)	(0.281)
Reserves / GDP	-0.124***	-0.132**	-0.360***	-0.0853***
	(0.0375)	(0.0481)	(0.0358)	(0.0285)
Total Debt / GDP	0.0678*		0.167***	
	(0.0332)		(0.0237)	
Gov Debt / GDP		0.0442**		0.122***
		(0.0198)		(0.0380)
Constant	4.330**	3.882***	4.438***	-5.040**
	(1.513)	(0.627)	(0.975)	(1.829)
Year FE	Yes	Yes	Yes	Yes
Observations	520	246	2,645	1,033
Number of countries	43	31	105	84
R-squared	0.267	0.307	0.216	0.292

Clustered standard errors in parenthesis.

Empirical analysis Back

$$\ln\left(100 * \frac{reserves_{i,t}}{GDP_{i,t}}\right) = \chi_0 + \chi_1 \ln\left(100 * \frac{NR_{i,t}}{GDP_{i,t}}\right) + \kappa_i + \mu_t + v_{i,t}$$

	(1) Reserves
$\ln\left(100 * \frac{NR_{i,t}}{GDP_{i,t}}\right)$	**0.117*** **
	(0.0333)
Constant	1.635***
	(0.0380)
Year FE	Yes
Country FE	Yes
Observations	5,044
Number of countries	160
R-squared	0.183

Clustered standard errors in parenthesis.

Empirical analysis Back

$$\ln(rer_{i,t}) = \rho \ln(rer_{i,t-1}) + \phi_1 \left(100 * \frac{NR_{i,t}}{GDP_{i,t}}\right) + \phi_2 \Delta_{t,t-1}\left(100 * \frac{reserves_{i,t}}{GDP_{i,t}}\right) + \kappa_i + \mu_t + \varepsilon_{i,t}$$

	(1) Real Exchange Rate
$\ln(rer_{i,t-1})$	0.909***
	(0.0272)
$\left(100 * \frac{NR_{i,t}}{GDP_{i,t}}\right)$	**-0.00597****
	(0.00284)
$\Delta_{t,t-1}\left(100 * \frac{reserves_{i,t}}{GDP_{i,t}}\right)$	0.00203**
	(0.000833)
Constant	0.280***
	(0.0945)
Year FE	Yes
Country FE	Yes
Observations	3,980
Number of countries	158
R-squared	0.919

Clustered standard errors in parenthesis.

Household problem prices Back

- Since (z_0, x_0) is given, the problem of a representative household is:

$$\begin{aligned} \max_{\lambda_1} \int_{\underline{z}}^{z^*(x_1)} \beta u\left(c_1^D\right) dF\left(z_1\right) + \int_{z^*(x_1)}^{\bar{z}} \beta u\left(c_1^P\right) dF\left(z_1\right) \\ s.t. \quad P_1^D c_1^D = \left[\left(1-\lambda_1\right) r_{N,1}^D + \lambda_1 r_{T,1}^D\right] \bar{k} + \Pi_1^D + T_1^D \\ P_1^P c_1^P = \left[\left(1-\lambda_1\right) r_{N,1}^P + \lambda_1 r_{T,1}^P\right] \bar{k} + \Pi_1^P + T_1^P \\ \Lambda_1 = \Gamma_H\left(B_1\right) \end{aligned}$$

where prices and profits are functions of the state (z_1, x_1)

- Denote the policy function is $\lambda^*\left(B_1\right)$

Government problem Back

- The problem of the government in $t = 0$ is:

$$\max_{B_1} u\left(C_0\right) + \beta \int_{\underline{z}}^{z^*(x_1)} u\left(C_1^D\right) dF\left(z_1\right) + \beta \int_{z^*(x_1)}^{\bar{z}} u\left(C_1^P\right) dF\left(z_1\right)$$
$$s.t. \quad \Lambda_1 = \lambda^*\left(B_1\right)$$

where consumption in $t = 0$ is a function of x_0 (given) and x_1

$$C_0 = C\left(x_0, x_1\right) = Y\left(y_{N,0}^P, y_{T,0}^P + \boldsymbol{q}\left(\boldsymbol{x}_1\right)\boldsymbol{B}_1 - B_0\right)$$

- The solution is B^*

Equilibrium

Back

- **Equilibrium**: policy function $\lambda^*(B)$, debt issuance B^*, price schedule $q(x)$, and beliefs $\Gamma_H(B)$ such that:

  1. given q and Γ_G, B^* solves the government's problem
  2. given Γ_H, λ^* solves the household's problem for any B
  3. beliefs are consistent $\Gamma_H(B) = \lambda^*(B)$
  4. the price q satisfies

$$q(x) = \frac{1 - F(z^*(x))}{1 + r^*}$$

- **Equilibrium allocation**: $\tilde{x} = \left(\tilde{\Lambda}, \tilde{B}\right)$ such that $\tilde{B} = B^*$ and $\tilde{\Lambda} = \lambda^*(B^*)$

Calibration Back

- Parameters from the literature (Bianchi, et.al. (2018); Gordon, et. al. (2018))

Parameter	Value	Parameter	Value
σ	2	β	0.98
r^*	0.01	ϕ	2.5
η	0.83	ω	0.6
α_N	0.33	α_T	0.33
d_0	-0.21	d_1	0.42
ρ	0.94	σ_z	0.027
$y_{C,L}$	0.11	$y_{C,H}$	0.25
Pr (windfall)	0.05	windfall duration	16 quarters
γ	0.05	κ	0.03