Capital Controls and Free-Trade Agreements*

Simon P. Lloyd† Emile A. Marin‡

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Abstract

We study the joint optimal determination of trade tariffs and capital controls in a two-country, two-good model with trade in both goods and assets. Policy is driven by the incentive to manipulate the terms of trade both across goods and over time. When tariffs are ruled out by a free-trade agreement (FTA), capital controls are chosen to trade-off the two margins. Absent a FTA, the planner achieves weakly higher welfare by additionally employing tariffs on goods. However, time-varying tariffs have second-best effects on the cost of borrowing, so the size of optimal capital controls depends on trade policy. Specifically, in response to fluctuations in the endowment of domestic goods capital controls are larger when the optimal tariff is in place. In contrast, faced with fluctuations in the endowment of foreign goods, the optimal time-varying tariff partly substitutes for the use of capital controls, so capital controls are smaller. Our results extend to a Nash equilibrium where countries engage in both capital-control and trade wars.

JEL Codes: F11, F21, F32, F33, F41, F42.

Key Words: Capital Controls; Capital Flows; Free-Trade Agreements; Optimal Policy; Tariffs.

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†Bank of England. Email Address: simon.lloyd@bankofengland.co.uk.

‡University of Cambridge. Email Address: eam65@cam.ac.uk.

1 Introduction

The management of trade and capital flows has been a key pillar of macroeconomic policy over the past century, and has come into sharp focus following global events such as the GFC and the Covid-19 pandemic. As a result, academic and policy debates around both trade tariffs and capital controls have grown in prominence, but for largely independent reasons. Discussions around trade policy often balance economic forces with political factors, while recent debates about capital controls have centred on their role in insulating countries from large and volatile cross-border flows. In this paper, we provide a theory for the joint determination of trade policy and capital controls in a model in which both policy instruments are optimally chosen to act monopolistically in markets and manipulate the terms of trade. Within this framework, we assess how prevailing trade arrangements influence the incentives for, and the size of, optimal capital controls, and we analyse the implications for global welfare.

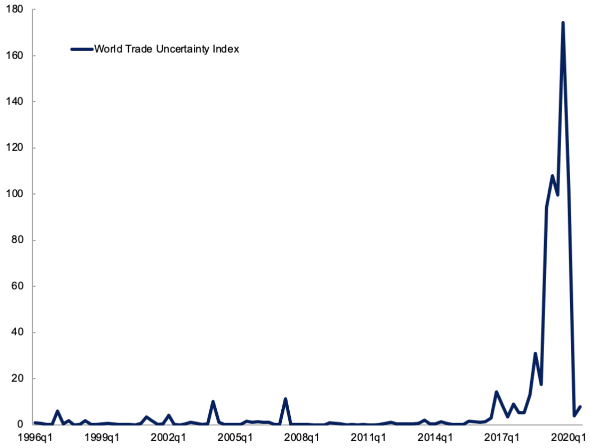
Our analysis is motivated by two observations. First, following at least two decades of growing trade integration (Baier and Bergstrand, 2007), the process of trade liberalisation has stalled in recent years. Perhaps the most notable example is the US-China trade war, one of the first episodes of large-scale tariff increases amongst major world economies since the interwar period (Amiti, Redding, and Weinstein, 2019). This, and other events, have substantially heightened uncertainty around world trade, as the World Trade Uncertainty index (Ahir, Bloom, and Furceri, 2018) in Figure 1 demonstrates. Alongside this, data from the World Trade Organisation shows a declining number of new regional trade agreements since the mid-2010s, and there also is evidence of a deceleration in global value chain integration since the global financial crisis. Against this backdrop, the conduct of trade policy is of renewed academic and policy interest.[1]

Second, international institutions, such as the International Monetary Fund, have revised their view on financial openness and developed a nuanced approach which emphasises a role for capital flow taxation (Qureshi, Ostry, Ghosh, and Chamon, 2011; Basu, Boz, Gopinath, Roch, and Unsal, 2020). Consistent with this, Figure 2 shows the increasing use of macroprudential regulations specifically targeting cross-border flows over time (see Ahnert, Forbes, Friedrich, and Reinhardt, 2020). In support of the growing use of capital controls, academic discourse has gone beyond the canonical 'Mundellian Trilemma'—which prescribes that monetary policy independence can be achieved alongside free capital mobility, as long as exchange rates are flexible (Rey, 2015). Recent contributions emphasise that capital flow management is necessary to support monetary policy transmission in open economies due to terms-of-trade externalities (Farhi and Werning, 2012), financial frictions (Basu et al., 2020) or, specifically for the U.S., dollar scarcity (Marin, 2022).

Building on these observations, we study the interaction between optimal capital flow taxation and trade policy. The starting point for our analysis is a canonical two-country, two-good endowment economy, without nominal or financial frictions. Within the model, a Ramsey plan-

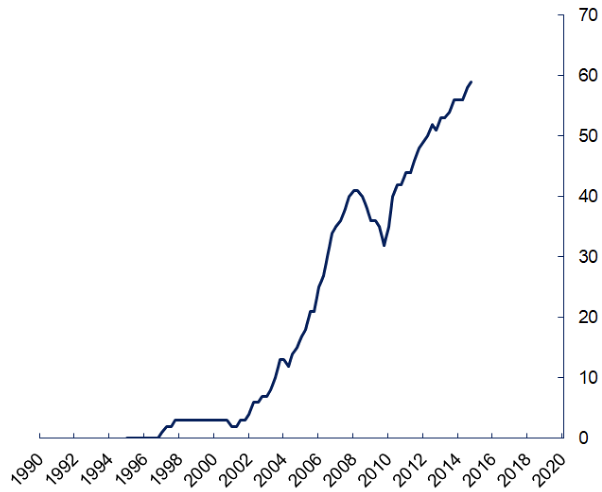
[1] See, for example, Auray, Devereux, and Eyquem (2020) Bergin and Corsetti (2020) and D'Aguanno, Davies, Dogan, Freeman, Lloyd, Reinhardt, Sajedi, and Zymek (2021).

Figure 1: World Trade Uncertainty has Picked Up in Recent Years



Notes: World Trade Uncertainty index constructed for 143 countries starting in 1996, using text analysis on Economist Intelligence Unit country reports. *Sample*: 1996q1-2020q3. *Source*: Ahir et al. (2018).

Figure 2: Growing Role for Capital Controls in Policy Toolkits



Notes: Cumulative number of macroprudential FX regulations, where changes include both loosening (value of -1) and tightening (value of $+1$). *Sample*: 1995q1-2014q4. *Source*: Ahnert et al. (2020).

ner in a large-open economy has incentives to manipulate the terms of trade due to the presence of pecuniary externalities (Geanakoplos and Polemarchakis, 1986). When making their inter-temporal consumption-savings decision and choosing their intra-temporal consumption basket, households do not account for the effect their actions have on relative prices. In contrast, the planner internalises its size in global markets and acts as a monopsonist both for aggregate consumption over time—manipulating the world interest rate—and across goods varieties—

manipulating the relative price of goods statically.[2]

Our point of departure is the analysis of Costinot et al. (2014), who study optimal capital controls in a model with a free-trade agreement (FTA). As such, trade taxes are precluded in their setup. In Costinot et al. (2014), the planner taxes capital inflows at times when the economy is growing faster than the rest of the world. Doing so serves to drive down the world interest rate so that households can borrow more cheaply, but has second-best implications for relative goods prices. In this paper, our main methodological contribution is to relax the constraint imposed on the planner by a FTA, and jointly solve for optimal capital controls and trade policy. We study both the *unilateral* policy equilibrium, where a Ramsey planner maximises domestic welfare while the other country levies no taxes, and the *Nash* equilibrium, where both countries set taxes strategically.

We emphasise four main findings. First, we show that in the absence of a FTA, the planner generally wants to use trade tariffs in addition to capital controls. To isolate the mechanisms at play, we first consider the case where the Home country levies taxes and the Foreign country is passive. In response to changes in the endowment of the good consumed with home bias (good 1), we show that incentives to manipulate the terms of trade inter- and intra-temporally are aligned for the planner. Specifically, if the endowment of good 1 falls, households over-borrow in early periods, failing to internalise that their actions drive up the world interest rate (inter-temporal). In addition, households consume too many domestic goods, driving up their price in a period when they are relatively scarce (intra-temporal). Instead, if the endowment of good 2 falls, inter- and intra-temporl motives move in opposite directions. In both cases, because there are two margins of adjustment, optimal capital controls alone cannot achieve the first-best allocation. Therefore, when a FTA is not in place, the domestic planner sets tariffs on imports to address this.

Second, we show that the optimal determination of capital controls and trade tariffs is interlinked, and the covariance between the two instruments depends on the state of the economy. Our key insight is that tariffs have second-best effects on the path of real exchange rates over time which leads households to borrow inefficiently. When the endowment of good 1 is away from its long-run level (e.g. a domestic downturn), inter- and intra- temporal incentives to manipulate the terms of trade are aligned. We show that in this case, optimal capital flow taxes are larger when there is no FTA and an optimal trade tariff is employed. Instead, when inter- and intra-temporal incentives are not aligned, such as when the endowment of good 2 is away from its long-run level (equivalent to temporarily high trade costs), tariffs generate real exchange rate movements that would—absent further action—incentivise under-borrowing. In this case, the optimal time-varying tariff is a partial substitute for capital controls.

[2]Although each economy is populated by identical agents, policy is driven by the fact that the choice made by an individual agent is inefficient from a social point of view, i.e. different from the choice made if the country was populated by a single agent. This can be interpreted as a macroeconomic approach to the *common-agency* externality (Tirole, 2003), which emphasises lack of coordination in private borrowing. This is a complementary to a large macroeconomic literature which studies the implications of heterogeneity for outcomes and policy (e.g. Bilbiie, 2008; Kaplan, Moll, and Violante, 2018). Marin (2022) studies a TANK model where the two channels interact.

Third, we consider the Nash equilibrium where both countries optimally set capital controls and, in the absence of a FTA, trade tariffs. We verify the mechanisms we analyse in the unilateral equilibrium. Moreover, we show that countries compete using both instruments and follow an *'inverse elasticity rule'*. Capital controls are larger when the elasticity of inter-temporal substitution is low. Similarly, tariffs are more prevalent when the intra-temporal elasticity of substitution between goods is low.[3] In the Nash equilibrium, the total wedge introduced by capital flow taxes and tariffs is larger—consistent with the idea of capital control and tariff wars. In contrast, we show that the cooperative optimal involves no capital control or trade tariffs.

Fourth, we calculate that the costs to global welfare are disproportionately large when trade policy is employed in addition to capital controls. In the unilateral setting, trade policy is not simply redistributive. Domestic welfare gains are small in comparison to Foreign losses, and overall global welfare is lower. When the domestic planner sets tariffs, they push Foreign households away from their efficient allocations, generating costly cross-border spillovers. In a Nash equilibrium, concurrent capital control and trade wars result in larger welfare losses, for each country, than capital control wars alone. These welfare costs predominantly originate from distortions in intra-temporal decisions, since welfare costs fall substantially when the elasticity of substitution between goods rises.

Literature Review. Our work is most closely related to Costinot et al. (2014). They study the role of capital controls as dynamic terms-of-trade manipulation in large-open endowment economies.[4] While policy in our setup is driven by the same underlying pecuniary externality, we study an environment where goods-specific taxes are permitted, departing from their assumption that a FTA is always in place.

The terms-of-trade externality underpinning our optimal policy prescriptions is a key part of the broader literature on capital controls, surveyed in Rebucci and Ma (2019) and Bianchi and Lorenzoni (2021). Mendoza (2002) and Bianchi (2011) study small-open economy models where goods prices appear in borrowing constraints. These models highlight how incentives to manipulate the terms of trade via capital controls can have first-order effects on countries' ability to borrow. Farhi and Werning (2014), Farhi and Werning (2016) and Schmitt-Grohé and Uribe (2016), amongst others, study the use of capital controls to correct aggregate demand externalities in models with nominal rigidities.

Unlike our paper, the literature on trade tariffs has predominantly focused on environments with no trade in assets, albeit with a richer supply-side setup with monopolistic (often heterogeneous) firms. Demidova and Rodriguez-Clare (2009) show that the optimal trade tariff trades off a domestic mark-up distortion and the incentive to increase the number of imported good varieties by spending more on imports. Caliendo, Feenstra, Romalis, and Taylor (2021) revisit

[3] In line with the optimal taxation literature (Atkinson and Stiglitz, 1980; Chari and Kehoe, 1999), the planner taxes inelastic commodities more.

[4] Heathcote and Perri (2016) study capital controls in two-country, two-good model with incomplete markets and capital. But, unlike our paper and Costinot et al. (2014), they do not derive the optimal policy.

the analysis and introduce roundabout production. Using a second-best argument, relying on the double-marginalisation of the domestic mark-up, they show that the optimal tariff is smaller and can even be negative. Our results can be interpreted in a similar vein: relative to the case of financial autarky, tariffs become second-best instruments due to their effects on the cost of borrowing for households. However, in contrast to Caliendo et al. (2021), our results highlight that the second-best tariff can be either larger or smaller depending on the state of the economy.

Our key contribution is to combine analyses of inter-temporal terms-of-trade manipulation with intra-temporal incentives, in order to provide a theory for the joint determination of capital controls and import taxes. There is strong empirical evidence that tariffs are chosen to manipulate the terms of trade. Specifically, Broda, Limao, and Weinstein (2008) show that larger countries face less elastic export supply curves suggesting that,on average, they have more market power and providing a rationale for tariffs in line with our modelling framework.

Our paper also belongs to a new growing literature looking at trade and stabilisation policies jointly. Auray et al. (2020) study the scope for trade wars, modelled via optimal strategic tariffs, and currency wars in a New Keynesian model. But this model features balanced trade, so there is no scope for capital control wars. Bergin and Corsetti (2020) study the optimal response of monetary policy to tariff shocks. They find that the optimal policy response to a symmetric tariff war is generally expansionary, while the response to a unilateral tariff imposed by a trade partner is to engineer a depreciation to offset its effects. Basu et al. (2020) argue that capital controls can form part of an 'integrated policy framework' for optimal macroeconomic stabilisation under certain conditions. However, their paper abstracts from trade policy.

The remainder of the paper is structured as follows. Section 2 describes the two-country, two-good environment and introduces the features of a FTA. Section 3 characterises the optimal Ramsey policy for a unilateral planner. Section 4 considers strategic cross-country interactions between planners, discussing the scope for capital control and trade wars. Section 5 analyses global welfare and cross-border spillovers. Section 6 discusses the generality of our results. Section 7 concludes.

2 Basic Environment

There are two countries, Home H and Foreign F, each populated by a continuum of identical households. In Sections 3-5, countries are assumed to be of equal size. We discuss the implications of country size in Section 6. Time is discrete and infinite, $t = 0, 1, ...$, and there is no uncertainty. The preferences of the representative Home consumer are represented by the additively separable utility function:

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(C_t)$$

where C_t denotes aggregate Home consumption and $u(C)$ is a twice continuously differentiable, strictly increasing and strictly concave function with $\lim_{C \to 0} u'(C) = \infty$. $\beta \in (0, 1)$ is the discount factor. The preferences of the representative Foreign consumer are analogous, with an asterisk denoting Foreign variables.

Consumers in both countries consume two goods, good 1 and good 2. We denote the Home representative consumer's consumption of good 1 and good 2 by $c_{1,t}$ and $c_{2,t}$, respectively, and group them in the vector $\mathbf{c}_t = [c_{1,t}\ c_{2,t}]'$. Home aggregate consumption is defined by the aggregator $C_t \equiv g(\mathbf{c}_t)$, where $g(\cdot)$ is a function that is twice continuously differentiable, strictly increasing, concave and homogeneous of degree one. We define the Jacobian of $g(\mathbf{c}_t)$ by $\nabla g(\mathbf{c}_t) = [g_{1,t}\ g_{2,t}]'$, where $g_i = \frac{\partial g(\mathbf{c}_t)}{\partial c_{i,t}}$ for $i = 1, 2$, while second derivatives are written as $g_{ij} = \frac{\partial^2 g(\mathbf{c}_t)}{\partial c_{i,t} \partial c_{j,t}}$ for $i, j = 1, 2$. The aggregator for the representative Foreign consumer is written as $C_t^* \equiv g^*(\mathbf{c}_t^*)$, with analogous derivatives.

We consider an environment where both countries can be endowed with both goods. Throughout, without loss of generality, we assume that consumers in the Home country have a 'home bias' for good 1, and we describe this as the 'domestic good'. We then label α as the long-run share of good 1 in Home consumption expenditure, where $\alpha > 0.5$, and thus $1 - \alpha < 0.5$ is the long-run share of good 2. Likewise, Foreign consumers prefer good 2 (the 'foreign good'). We assume these preferences are symmetric across countries such that the share of the foreign good in Foreign consumption is $\alpha^* = \alpha$.

Home and Foreign households receive a sequence of endowments of each good. The Home consumer's period-t endowments of goods 1 and 2 are denoted by $y_{1,t}$ and $y_{2,t}$, respectively. Similarly, the Foreign consumer's period-t endowments are $y_{1,t}^*$ and $y_{2,t}^*$. The total world endowment of goods 1 and 2 are $Y_{1,t} \equiv y_{1,t} + y_{1,t}^*$ and $Y_{2,t} \equiv y_{2,t} + y_{2,t}^*$, respectively. Endowments are weakly positive in all periods.

We assume that both countries begin with zero assets in period 0. The Home inter-temporal budget constraint is given by:

$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t - \mathbf{y}_t) \leq 0 \tag{1}$$

where $\mathbf{p}_t = [p_{1,t}\ p_{2,t}]'$ denotes the vector of period-t world goods prices and $\mathbf{y}_t = [y_{1,t}\ y_{2,t}]'$ is the vector of Home endowments. The budget constraint for the representative Foreign consumer is analogous.

We define two additional quantities. First, the terms of trade is given by $S_t = p_{2,t}/p_{1,t}$ and we refer to an increase in S_t as a deterioration of the Home terms of trade. We further assume the law of one price holds, such that this also corresponds to an improvement in the Foreign terms of trade. Second, the real exchange rate is given by the ratio of consumer price indices $Q_t = P_t^*/P_t$, where $P_t^{(*)} \equiv \min_{\mathbf{c}_t^{(*)}} \{\mathbf{p}_t \cdot \mathbf{c}_t^{(*)} : g^{(*)}(\mathbf{c}_t^{(*)}) \geq 1\}$. An increase in Q_t corresponds to a depreciation of the Home real exchange rate.

2.1 Free-Trade Agreements and the Pareto Frontier

In this paper, we study how prevailing trade agreements influence the incentives of a Ramsey planner to levy taxes on capital flows, both unilaterally and when accounting for cross-country strategic interactions. Without a FTA in place, the Ramsey planner can, in effect, seek to set consumption quantities for individual goods $\{c_{1,t}, c_{2,t}\}$ separately. In practice, they can achieve this by setting goods-specific taxes, which we implement as import tariffs. As a result of these tariffs, households' consumption allocations in any period t $(\mathbf{c}_t, \mathbf{c}_t^*)$ need not be *individually* Pareto efficient. In fact, we show that, by exploiting deviations from the Pareto frontier, a unilateral Ramsey planner can achieve weakly higher welfare when there is no FTA, versus a world with a FTA.

In contrast, there are no goods-specific taxes when there is a FTA in place. In this case, households' consumption allocations will be Pareto efficient and can be summarised by:

$$C^*(C_t) = \max_{\mathbf{c}_t, \mathbf{c}_t^*} \{g^*(\mathbf{c}_t^*) \quad \text{s.t.} \quad \mathbf{c}_t + \mathbf{c}_t^* = \mathbf{Y}_t \text{ and } g(\mathbf{c}_t) \geq C_t\} \tag{2}$$

for some C_t, where $\mathbf{Y}_t = [Y_{1,t}\ Y_{2,t}]'$. This problem yields a Pareto Frontier, which summarises efficient combinations of consumption $(c_{1,t}, c_{2,t})$ for a given level of aggregate consumption C.

Definition 1 (Pareto Frontier) *The Home and Foreign Pareto Frontiers for consumption are summarised by* $\mathbf{c}(C)$ *and* $\mathbf{c}^*(C^*)$ *in Appendix A.1.*

When a FTA is in place, once the unilateral Ramsey planner has chosen aggregate consumption C, Home households will choose their consumption basket $\mathbf{c}$ along the static Pareto frontier. In the subsequent sections, we investigate how optimal policy prescriptions differ by comparing environments with and without an FTA, assessing how departures from the Pareto frontier induced by tariffs can influence macroeconomic outcomes.

3 Unilateral Ramsey Planner

We first study an environment in which the Home planner optimally sets capital flow taxes to maximise domestic welfare, while the Foreign planner is assumed to be passive (i.e. does not levy taxes in response to Home policy). This unilateral policy setting helps to isolate the mechanisms at play. Within it, we compare an environment with a FTA in place to one without. The FTA-case corresponds to the two-good environment studied in Costinot et al. (2014). Individual goods allocations are chosen on the Pareto Frontier. In the no-FTA case, the Home planner sets individual consumption allocations $\{c_{1,t}, c_{2,t}\}$, unconstrained by the Pareto frontier. We study how this allocation can be implemented via a combination of capital flow taxes and goods-specific taxes—i.e. trade tariffs.

In the unilateral setup, the equilibrium conditions of the representative Foreign household act as a constraint for the Home Ramsey planning problem. Foreign households undertake a standard optimisation problem, maximising Foreign discounted utility subject to their intertemporal budget constraint at world prices $\mathbf{p}_t$. The first-order conditions of this problem are:[5]

$$\beta^t u^{*\prime}(C_t^*) \nabla g^*(\mathbf{c}_t^*) = \lambda^* \mathbf{p}_t \tag{3}$$

[5]See Appendix A.2 for a statement of the representative Foreign household's optimisation problem.

$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) = 0 \tag{4}$$

where λ^* is the Lagrange multiplier on the Foreign inter-temporal budget constraint.

We use the primal approach to characterise the optimal policy of the Home government. The Home government sets the sequence of Home aggregate consumption $\{C_t\}$ in order to maximise the discounted lifetime utility of the Home representative consumer subject to (i) the representative Foreign consumer's utility maximisation at world prices, (ii) market clearing in each period, and (iii) the prevailing trade agreement.

The first two of these constraints can be summarised in a single implementability condition. As described in the following proposition, this follows from the Foreign optimality conditions, equations (3) and (4), the domestic inter-temporal budget constraint (1), and market clearing, as in Lucas and Stokey (1983).

Proposition 1 (Implementability for Unilateral Planner) *When the Foreign country is passive, an allocation $\{\mathbf{c}_t, \mathbf{c}_t^*\}$, together with world prices $\mathbf{p}_t$, form part of an equilibrium if they satisfy*

$$\sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \tag{IC}$$

where $\boldsymbol{\rho}(C_t) \equiv u^{*\prime}(C^*(C_t))\nabla g^*(\mathbf{c}_t^*(C_t))$ denotes the price of consumption at each t.

3.1 With Free Trade

In the presence of a FTA and using the implementability condition, the Home Ramsey planning problem is given by:

$$\max_{\{C_t\}} \quad \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{P-Unil-FTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \tag{IC}$$

$$\mathbf{c}_t = \mathbf{c}(C_t), \quad \mathbf{c}_t^* = \mathbf{c}^*(C_t) \tag{FTA}$$

where the third line (FTA) summarises the Pareto frontier constraint imposed by the presence of a FTA. After substituting (FTA) into (IC), we assume that $\boldsymbol{\rho}(C_t) \cdot [\mathbf{c}(C_t) - \mathbf{y}_t]$ is a strictly convex function of C_t to guarantee a unique solution to (P-Unil-FTA). Since utility is time-separable and the planner chooses the whole sequence of consumption allocations, the problem can be represented by the following Lagrangian:

$$\mathcal{L} = u(C_t) - \mu \{\boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t(C_t) - \mathbf{y}_t]\}$$

where μ is the multiplier on the implementability constraint.

Optimal Allocation. The first-order condition from the Home planning problem, in the presence of a FTA, can be written as:

$$u'(C_t) = \mu \mathcal{MC}_t^{FTA} \tag{5}$$

where

$$\begin{aligned}\mathcal{MC}_t^{FTA} \equiv & u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*(C_t)) \cdot \mathbf{c}'(C_t) + u^{*\prime\prime}(C_t^*)C^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*(C_t)) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\ & + u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t(C_t))}{\partial C_t} \cdot [\mathbf{c}_t - \mathbf{y}_t]\end{aligned}$$

Equation (5) has the following interpretation. The left-hand side is the marginal utility from one additional unit of aggregate consumption for the representative Home consumer. The right-hand side represents the marginal cost of that unit of consumption, captured by $\mathcal{MC}_t^{FTA}$. The first term in $\mathcal{MC}_t^{FTA}$ is the price of one unit of consumption. It can be shown to be equal to $u^{*\prime}(C_t^*)Q_t^{-1}$. The second term reflects how the inter-temporal price of consumption changes when importing one additional unit of consumption, for given relative goods prices. The final term reflects how relative goods prices change with aggregate consumption.

Notice that if endowments and consumption outcomes coincide, $\mathbf{c}_t = \mathbf{y}_t$, equation (5) collapses to $u'(C_t) = \mu u^{*\prime}(C_t^*)Q_t^{-1}$, which corresponds to the decentralised allocation. Moreover, $\mu = 1$ coincides with perfect risk sharing.

Consider the case where the fraction of good 1 owned by the Home country $y_{1,t}/Y_{1,t}$ temporarily falls today—holding the overall stock of good 1 fixed over time ($Y_{1,t} = Y_1$ for all t). Faced with a higher stream of endowments in the future, Home households will borrow to smooth consumption. However, each additional unit of consumption brought forward raises the cost of borrowing. Additionally, the Home household will buy relatively more units of the the domestic good (good 1) from abroad. Specifically, the fall in y_1 is greater than the fall in c_1 so that $c_1 - y_1$ increases. Home households are buying more of good 1 from abroad at a time when it is relatively more expensive. Both the increase in the cost of borrowing and the increase in the price of good 1 reflect pecuniary externalities, which atomistic households do not internalise.

The planner sets policy to force households to internalise the pecuniary implications of their decisions. From an inter-temporal perspective, they tax capital inflows to delay consumption. Intra-temporally, the planner seeks to decrease the price of good 1. In the presence of a FTA, they achieve this by taxing capital inflows (aggregate consumption). So when the good-1 endowment deviates from its long-run level, the planner's inter- and intra-temporal incentives to manipulate the terms of trade are aligned. Both push the planner to tax capital inflows and delay aggregate consumption.

In contrast, suppose the fraction of the foreign good (good 2) owned by the Home country ($y_{2,t}/Y_2$) temporarily falls. While the planner's inter-temporal incentive to delay consumption is analogous to before, the intra-temporal incentive differs since the Home country will now sell relatively more of good 1 abroad. Specifically, c_1 will fall despite y_1 remaining unchanged so that $c_1 - y_1$ falls. The planner has an incentive to act monopolistically intra-temporally, and drive up the price of good 1. Absent a FTA, this can be achieved by taxing purchases of good 1 from abroad. But, with a FTA in place, the planner will instead subsidise capital inflows, raising C, as a second-best policy. Inter- and intra-temporal incentives are not aligned in this case, and thus optimal capital controls trade off interest rate and terms of trade manipulation. The relative direction of inter- and intra-temporal incentives plays a key role in our subsequent analysis of the relationship between optimal capital controls and trade tariffs.

3.2 Without Free Trade

Without a FTA, the Home planner, unconstrained by the Pareto frontier, directly chooses the allocation of both goods 1 and 2. Aggregate consumption C_t can then be backed out of the consumption aggregator $g(\mathbf{c}_t)$. The Home government's problem is thus:

$$\max_{\{c_{1,t},c_{2,t}\}} \quad \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{P-Unil-nFTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0 \tag{IC}$$

$$C_t = g(\mathbf{c}_t) \tag{nFTA}$$

where the third line (nFTA) reflects the fact the individual consumption allocations $\{c_{1,t}, c_{2,t}\}$ in a given period t are combined to yield aggregate consumption C_t. Notice that the implementability condition is unchanged. As in the FTA-case, we make an assumption—specifically that $\boldsymbol{\rho}(g(\mathbf{c}_t)) \cdot [\mathbf{c}_t - \mathbf{y}_t]$ is strictly convex—to ensure a unique solution to the planning problem.

Optimal Allocation. The first-order conditions of the planning problem—with respect to $c_{1,t}$ and $c_{2,t}$, respectively—are given by:

$$u'(C_t)g_{1,t} = \mu \mathcal{MC}_{1,t}^{nFTA} \tag{6}$$

$$u'(C_t)g_{2,t} = \mu \mathcal{MC}_{2,t}^{nFTA} \tag{7}$$

where μ denotes the Lagrange multiplier on the implementability constraint and:[6]

$$\begin{aligned}\mathcal{MC}_{1,t}^{nFTA} \equiv& u^{*\prime}(C_t^*)g_1^*(\mathbf{c}_t) + u^{*\prime\prime} g_1^*(\mathbf{c}_t^*)\nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\ &+ u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t] \\ \mathcal{MC}_{2,t}^{nFTA} \equiv& u^{*\prime}(C_t^*)g_2^*(\mathbf{c}_t) + u^{*\prime\prime} g_2^*(\mathbf{c}_t^*)\nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\ &+ u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t]\end{aligned}$$

Equations (6) and (7) equate the marginal benefit from a unit of good-specific consumption

[6] For ease of notation, we do not make explicit the dependence of $\mathbf{c}_t^*$ on $\mathbf{c}_t$.

to its marginal cost—for goods 1 and 2, respectively. The planner optimises over the consumption allocation good by good. Take equation (6), for example. As before, the first term on the right-hand reflects the price of one unit of good 1. The next term reflects how the inter-temporal component of that price (i.e. the cost of borrowing) increases. The final term, captures the intra-temporal margin. Specifically, how each good-specific price changes with an additional unit of c_1 consumed.

3.3 Comparing Optimal Allocations

For the Home planner, the first-order condition under a FTA, equation (5), represents a constrained first-best allocation. However, the no-FTA optimality conditions, equations (6) and (7), represent the first-best outcome for the Home country, as the following proposition explains.

Proposition 2 (Optimal Capital Controls without a FTA) *In the absence of a FTA, the unilateral optimal allocation $\mathbf{c}_t$ satisfies equations (6) and (7). Moreover:*

(i) *the level of C achieved in (P-Unil-nFTA) is always weakly higher than that achieved in (P-Unil-FTA);*

(ii) *if the optimal allocation $\mathbf{c}$ in (P-Unil-nFTA) violates the Pareto frontier (2) given by a FTA, then (i) holds strictly; and*

(iii) *the welfare achieved, and corresponding allocation $\mathbf{c}$, in (P-Unil-FTA) and (P-Unil-nFTA) coincide when endowments are proportional to consumer preferences, $y_1 \propto \alpha$, $y_2 \propto 1-\alpha$, $y_1^* \propto 1-\alpha$ and $y_2^* \propto \alpha$.*

Proof: See Appendix A.3. □

The results in Proposition 2 can be understood in Figure 3, which plots the optimal allocations with (blue) and without (green) a FTA alongside the loci of $\{c_1, c_2\}$ which attain different levels of aggregate consumption (grey, and black for $C = 1$). For this, and all subsequent numerical exercises, we use a constant relative risk aversion (CRRA) specification for per-period utility:

$$u(C) \equiv \frac{C^{1-\sigma} - 1}{1-\sigma}$$

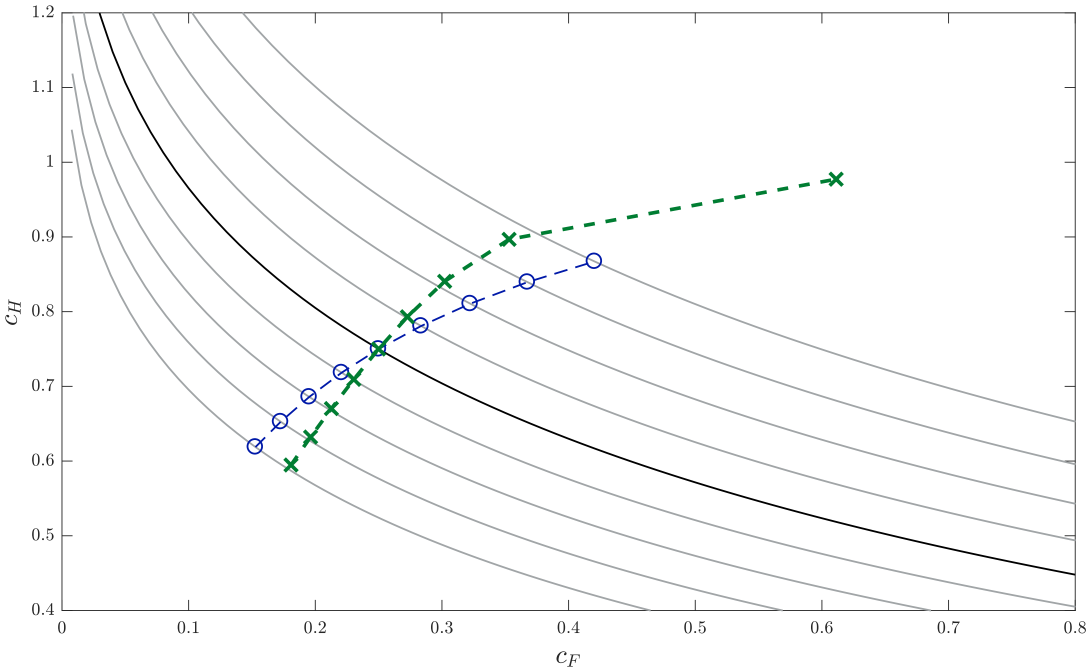
where $\sigma > 0$ denotes the coefficient of relative risk aversion. The aggregate consumption of the representative Home agent is given by the Armington (1969) aggregator:

$$C_t \equiv g(\mathbf{c}_t) = \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{8}$$

where $\phi > 0$ is the elasticity of substitution between good 1 and 2. The Foreign aggregator is analogous.

In the figure, the blue line maps the Pareto frontier: the efficient combinations of $\{c_1, c_2\}$ for different levels of aggregate consumption C, which are consistent with a FTA. But when not

Figure 3: Optimal Allocations and the Pareto Frontier



Notes: Plot of optimal consumption allocations for Home consumer from Ramsey capital flow taxation (i) with a FTA in place (blue circles, i.e. the Pareto frontier) and (ii) absent a FTA, with goods-specific taxation (green crosses) at different Home endowments. Specifically using nine equally-spaced allocations for $y_1 \in [\alpha - 0.25, \alpha + 0.25]$, with $y_1^* = 1 - y_1$, $y_2 = 1 - \alpha$ and $y_2^* = \alpha$. Other model parameters are: $\beta = 0.96$, $\sigma = 2$, $\phi = 1.5$, and $\alpha = 0.75$. Grey/black lines denote loci of $\{c_1, c_2\}$ which attain different levels of aggregate consumption (black for $C = 1$, grey otherwise).

constrained by a FTA, the planner can achieve a higher level of consumption by changing the Home consumption allocation $\{c_1, c_2\}$, as in parts (i) and (ii) of Proposition 2. For $y_1 > \alpha$—the area above the black line—the allocation absent FTA is more biased towards c_1. Whereas for $y_1 < \alpha$—the area below the black line—the allocation is more biased towards c_2. The allocations under a FTA (P-Unil-FTA) and without a FTA (P-Unil-nFTA) coincide in the case $y_1 = y_2^* = \alpha$—part (iii) of Proposition 2. Moving away from the no-trade point the solutions do not generally coincide.

Proposition 2, however, is silent on how the implementation of the allocation differs when the FTA is relaxed, so does not describe how the magnitude of capital controls differs with or without a FTA. Moreover, it does not outline the macroeconomic response to fluctuations in endowments under the optimal policies. To address these questions, we first turn to discuss how the optimal allocation can be implemented using tax instruments.

3.4 Implementation

With a FTA in place, we consider the implementation of the optimal allocation via a capital inflow tax only. In the no-FTA case, two instruments are needed to implement the optimal allocation and we study an implementation with a capital-inflow tax and an import tariff.

While this implementation need not be unique, we choose it because it relies on policy-relevant and observable instruments.[7]

To discuss implementation via a capital-inflow tax, we need to specify the structure of financial markets. We assume households trade in non-contingent bonds, denominated in each good variety. The Home planner imposes the same proportional tax θ_t on the gross return on net lending in all bond markets. So the per-period budget constraint for the Home consumer can be written:

$$\mathbf{p}_{t+1} \cdot \mathbf{a}_{t+1} + \tilde{\mathbf{p}}_t \cdot \mathbf{c}_t = \mathbf{p}_t \cdot \mathbf{y}_t + (1 - \theta_{t-1}) (\mathbf{p}_t \cdot \mathbf{a}_t) - T_t$$

where $\tilde{\mathbf{p}}_t = \mathbf{p}_t$ with a FTA, $\mathbf{a}_t$ denotes the vector of asset positions and T_t is a lump-sum rebate. Given a no-Ponzi condition, $\lim_{t\to\infty} \mathbf{p}_t \cdot \mathbf{a}_t \geq 0$, the first-order conditions associated with Home households' utility maximisation are given by:

$$u'(C_t)g_i(\mathbf{c}_t) = \beta(1 - \theta_t)(1 + r_{i,t})u'(C_{t+1})g_i(\mathbf{c}_{t+1}) \tag{9}$$

for $i = 1, 2$, where $r_{i,t} \equiv \frac{p_{i,t}}{p_{i,t+1}} - 1$ is a good-specific interest rate. Combining this with the analogous Foreign Euler equation, the Home's tax on international capital flows is written as:

$$(1 - \theta_t) = \frac{u'(C_t)}{u'(C_{t+1})} \frac{u^{*\prime}(C^*_{t+1})}{u^{*\prime}(C_t)} \frac{Q_t}{Q_{t+1}} \tag{10}$$

A tax on capital inflows (or a subsidy for capital outflows) is then captured by values of $\theta_t < 0$, interpretable as a tax on current consumption relative to future consumption.

Without a FTA, the Home planner additionally levies a proportional import tariff τ_t, and $\tilde{\mathbf{p}}_t = \boldsymbol{\tau}_t \cdot \mathbf{p}_t$. The representative Home household faces import price $p_{2,t}(1 + \tau_t)$ so the relative demand is given by:

$$\frac{c_{1,t}}{c_{2,t}} = \frac{\alpha}{1 - \alpha} \left(\frac{1}{S_t(1 + \tau_t)} \right)^{-\phi} \tag{11}$$

An import tariff is captured by $\tau_t > 0$.

3.5 Numerical Exercises

Using this implementation, we study two numerical scenarios to illustrate the macroeconomic implications of the optimal policy response to fluctuations. In the first scenario, we investigate the optimal policy response to a departure of the Home endowment of the domestic good (good 1) from its long-run value. In the second, we study the implications of variation in the Home endowment of the foreign good (good 2). For each, we highlight differences in the incentives driving policymaking.

Both experiments are deterministic. We specify initial $y^{(*)}_{i,0}$ and terminal values $\overline{y}^{(*)}_i$ for the endowments $i = 1, 2$, and construct the full sequence of endowments for all periods by assuming

[7] Chari and Kehoe (1999) show that the implementation of the Ramsey optimal allocation via taxation is generally non-unique.

that endowments follow a first-order autoregressive process:

$$y_{i,t+1}^{(*)} = \left(1 - \rho_i^{(*)}\right) \overline{y}_i^{(*)} + \rho_i^{(*)} y_{i,t}^{(*)}, \quad \forall t > 0 \text{ and } i = 1, 2,$$
$$\mathbf{y}_0 = [y_{1,0} \ y_{2,0}]'$$
$$\mathbf{y}_0^* = \left[y_{1,0}^* \ y_{2,0}^*\right]'$$

where, for simplicity, we assume $\rho_1 = \rho_2 = \rho_1^* = \rho_2^*$.

In both experiments, we assume there is no change in the aggregate endowment ($Y_{1,t}$ and $Y_{2,t}$ are constant). As a result, with households able to fully insure their consumption against known changes in their endowment, perfect consumption smoothing is achieved in the decentralised allocation.[8] The planner's allocation contrasts sharply to the decentralised benchmark.

Based on the CRRA per-period utility function and the Armington (1969) specification for aggregate consumption, the model calibration for both experiments is detailed in Table 1. In each experiment, we compare the decentralised allocation, the unilateral Ramsey planning allocation with a FTA in place, and one without a FTA. To compare the dynamic implications of the three policy variants in a consistent manner, we equalise the long-run equilibrium (i.e. 'steady state') of each model by using a steady state import tariff for the Home country.

Table 1: Benchmark Model Calibration

Parameter	**Value**	**Description**
β	0.96	Discount factor, annual frequency
σ	2	Coefficient of relative of risk aversion
ϕ	1.5	Elasticity of substitution between goods 1 and 2
α	0.6	Share of good 1 (good 2) in Home (Foreign) consumption basket
ρ	0.8	Persistence of endowments

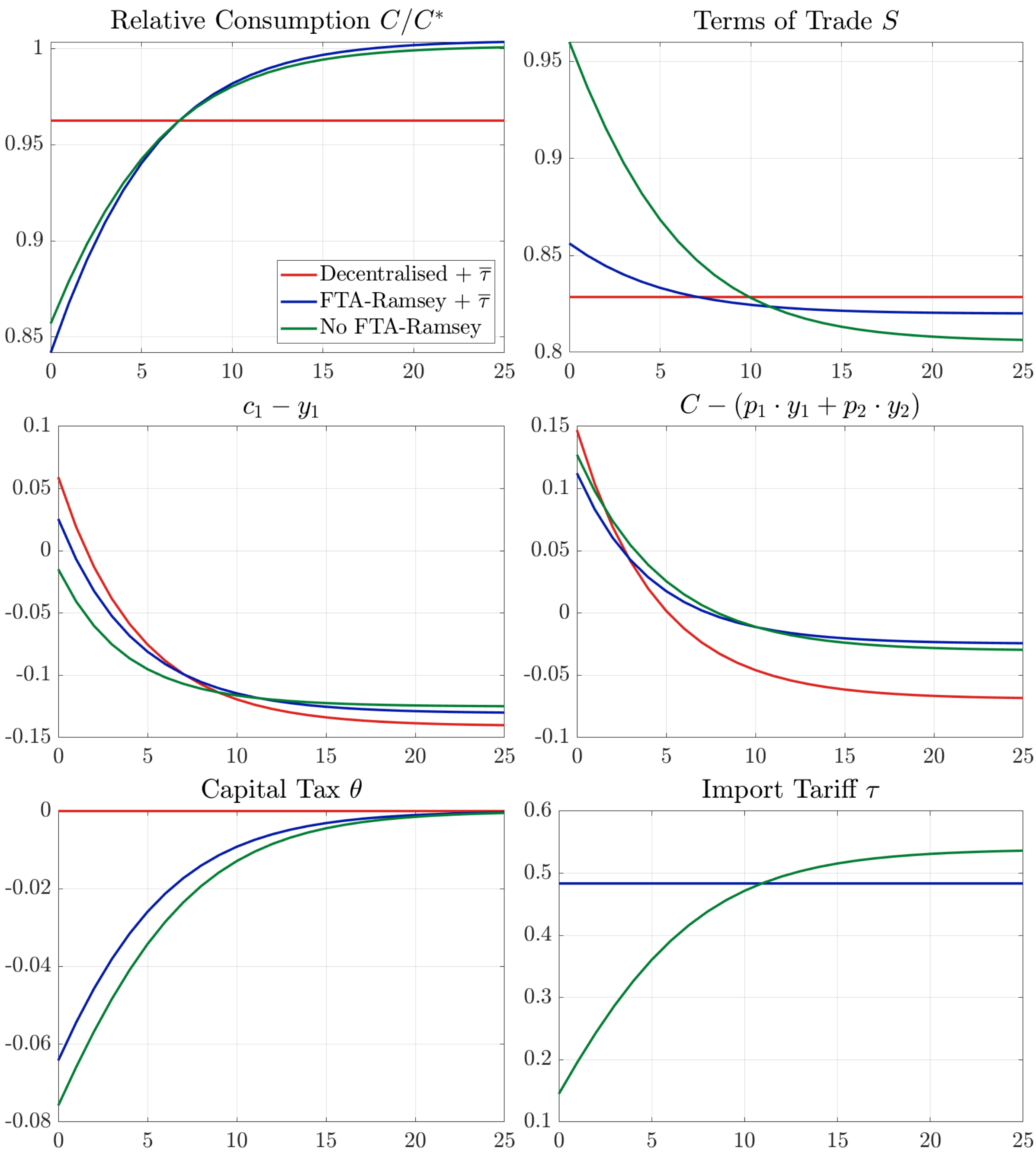
3.5.1 Scenario 1: Temporarily Low Home Endowment of Domestic Good

Our first experiment simulates the Home economy recovering from a domestic downturn, or catching up to the rest of the world. Specifically, the Home country's endowment in good 1 is low in the near term, and grows towards its long-run level. Denoting initial endowment values by $y_{i,0}^{(*)}$ and long-run levels by $\overline{y}_i^{(*)}$ for $i = 1, 2$, we assume that $y_{1,0} = 0.75\overline{y}_1$ and $y_{2,0} = \overline{y}_2$. To ensure there is no aggregate uncertainty: $y_{1,0}^* = 1 - y_{1,0}$ and $y_{2,0}^* = 1 - y_{2,0}$. The resulting time profiles for the allocations are plotted in Figure 4.

Faced with a higher future stream of good-1 endowments, Home households will borrow to smooth consumption in the decentralised allocation. However, since each additional unit of consumption brought forward raises the cost of borrowing, the planner has an incentive to postpone consumption relative to the decentralised allocation. The optimal policy, both with and without a FTA, involves leaning against capital flows. This is demonstrated in the centre-right panel of Figure 4, plotting the evolution of the balance of payments, which varies by less

[8] Our findings are robust to allowing the aggregate endowment to fluctuate.

Figure 4: Time Profile of Optimal Allocations as the Home Endowment of Good 1 Rises in Experiment 1



Notes: Time profile for macroeconomic outcomes in Experiment 1, simulated for 50 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The decentralised and FTA-Ramsey models include a steady-state tariff to ensure their steady-state allocations replicate the No FTA-Ramsey case.

under the two planning solutions relative to the decentralised outcome. Additionally, because the Home endowment of good 1—the good consumed with home bias domestically—is initially lower, the planner has an incentive to restrict the global excess demand for good 1 over and above their endowment y_1. Driving these incentives is the planner's expectation that the future price of c_1 and C will fall. Therefore, the planner taxes aggregate consumption C via a capital inflow tax $\theta < 0$ and, in the absence of a FTA, levies an increasing path for import tariffs.

The main result of this paper is that the planner taxes capital inflows more heavily when the FTA is relaxed. In the presence of a FTA, the planner achieves the desired allocation by choosing a lower level of aggregate consumption C in the near term, which entails a disproportionately lower c_1 on account of Home consumers' preference for good 1. When unconstrained by a FTA, the planner can restrict the net global supply of good 1 via an import tariff, which incentivises Home consumers to purchase a larger fraction of the good-1 endowment on the global market. While the required capital control taxes are generally small—between 6 and 8% on impact for the planner with and without an FTA, respectively—the goods tax is large and variable in the absence of a FTA—over 50% in the long run and increasing from around 15%.

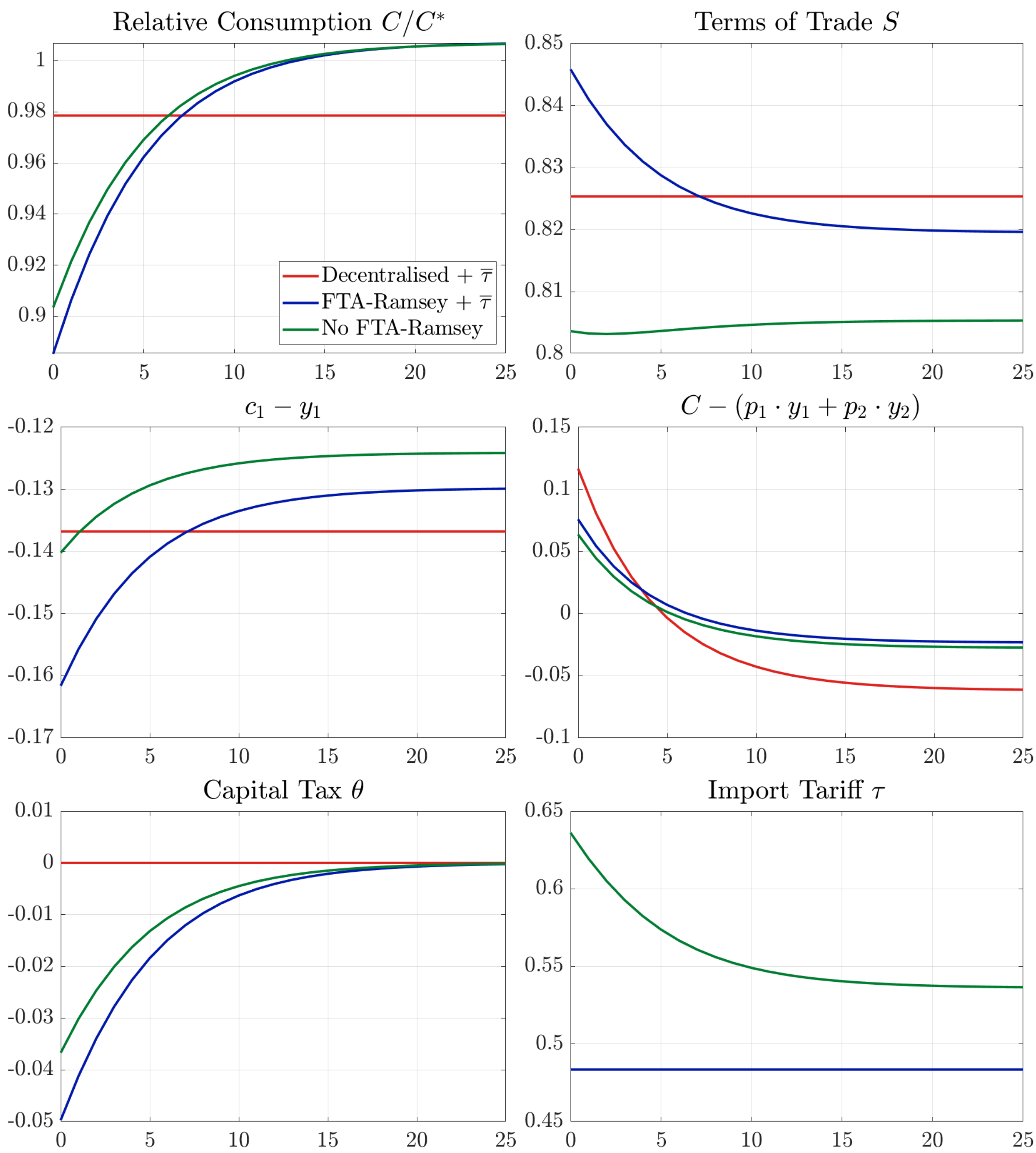
3.5.2 Scenario 2: Temporarily Low Home Endowment of Foreign Good

Our second experiment simulates a scenario in which the Home endowment of the foreign good (good 2) starts at a low value relative to its long-run level. This is akin to a positive Foreign export-sector shock, as the Foreign country's endowment of good 2 is high in the near term, but falls towards its long-run level. We assume that $y^*_{2,0} = 1.25\overline{y}^*_2$ and $y^*_{1,0} = \overline{y}^*_1$. To ensure there is no aggregate uncertainty: $y_{1,0} = 1 - y^*_{1,0}$ and $y_{2,0} = 1 - y^*_{2,0}$. The resulting time profiles for the allocations are plotted in Figure 5.

As in Experiment 1, the Home country borrows in the near term in the decentralised allocation, knowing that their endowment will increase in the future. However, in contrast to Experiment 1, the net supply of good 1 that Home sells abroad rises, because c_1 falls while y_1 is unchanged. The Home planner wants to delay aggregate consumption C inter-temporally, but also has an intra-temporal incentive to drive up the relative price of good 1. Absent a FTA, the planner levies a high import tariff in the near term to increase c_1 and drive up its relative price. In Experiment 2, the optimal capital inflow tax is smaller absent a FTA, as it must strike a balance between restricting C and boosting c_1.

This scenario can also be interpreted through the lens of trade costs. For example, suppose that there are shipping costs associated with transporting the Home country endowment of foreign goods (good 2) to the Home country. Were these to take the form of iceberg trade costs $\hat{\tau} \geq 1$, the Home households would *de facto* be endowed with $y_2/\hat{\tau}$ units of good 2 after trade costs. Home households could consume this amount of good 2 at most without importing more of the good from Foreign. If these are one-directional trade costs, then Experiment 2 can be interpreted as a temporary rise in trade costs for Home. This interpretation leads to an interesting implication: both capital controls and trade tariffs are needed to optimally respond to changes in trade costs.

Figure 5: Time Profile of Optimal Allocations as the Foreign Endowment of Good 2 Falls in Experiment 2



Notes: Time profile for macroeconomic outcomes in Experiment 2, simulated for 50 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The decentralised and FTA-Ramsey models include a steady-state tariff to ensure their steady-state allocations replicate the No FTA-Ramsey case.

3.5.3 Relationship Between Capital Flow and Goods Taxation

We now use the results of Experiments 1 and 2 to analyse the impact of import tariffs on capital flow taxation and ask: does a regime of free trade encourage or discourage the use of capital controls?

Consistent with the determinants of capital flow taxes and goods tariffs depending on the nature of shocks hitting the economy, the relative sign of inter- and intra-temporal incentives differ in the two experiments. In Experiment 1, the planner's inter- and intra-temporal incentives are aligned, such that the planner seeks to move both C and c_1 in the same direction. As a result, capital flow taxes are larger in the absence of a FTA. In contrast, incentives are opposed in Experiment 2 and, consequently, capital flow taxes are smaller in the absence of a FTA.

To investigate the drivers of this, we decompose capital flow taxes θ into two wedges. We do so by taking logs of equation (10):

$$\ln(1-\theta_t) \approx -\theta_t = \underbrace{-\sigma\left(\hat{C}_t - \hat{C}_{t+1} + \hat{C}^*_{t+1} - \hat{C}^*_t\right)}_{\text{Consumption Wedge}} + \underbrace{\left(\hat{Q}_t - \hat{Q}_{t+1}\right)}_{\text{RER Wedge}} \tag{12}$$

where $\hat{x}$ denotes the natural logarithm of x. The 'consumption wedge' component captures incentives to tax capital inflows pertaining to the evolution of relative consumption over time. The 'RER wedge' reflects capital flow taxation incentives linked to the evolution of the real exchange rate Q.

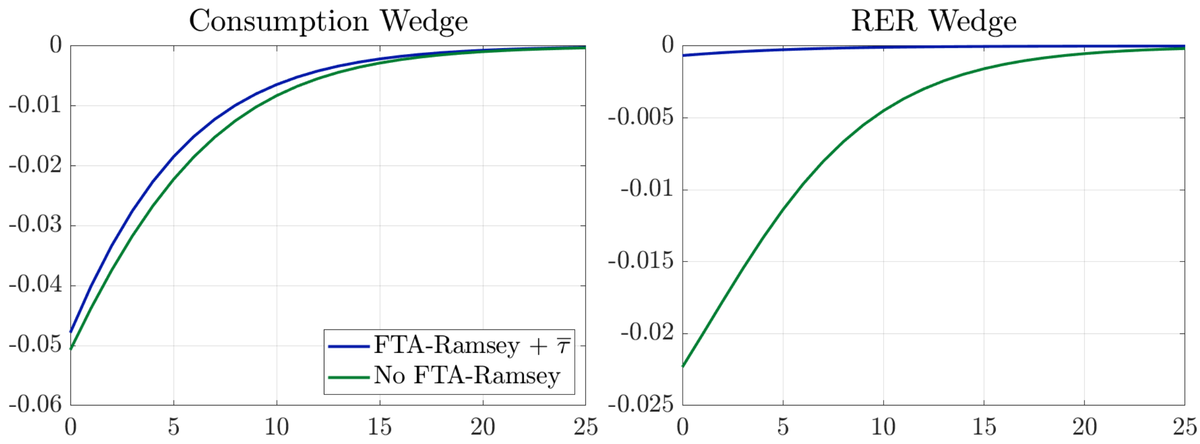
Figures 6 and 7 plot the two wedges for Experiments 1 and 2, respectively. In both, and regardless of whether a FTA is in place or not, the consumption wedge explains the majority of variation in the capital flow tax θ.

However, comparing the sign of the RER wedge across experiments highlights some notable differences. Consistent with inter- and intra-temporal incentives being aligned in Experiment 1, Figure 6 shows that the RER wedge and consumption wedge have the same sign, regardless of the presence of an FTA. In contrast, in Experiment 2 incentives are opposed, and so differences arise in the FTA and no-FTA cases. When the planner has import tariffs available to them, in addition to capital flow taxes, Figure 7 demonstrates that the RER wedge has the opposite sign to both the consumption wedge and the RER wedge for the planner constrained by a FTA. This helps to explain why, in Experiment 2, capital flow taxes are smaller with no FTA. In essence, the planner can levy tariffs that stabilise the terms of trade and, at the same time, offset the need to use capital flow taxes to manipulate relative prices for inter-temporal incentives.

3.5.4 Comparative Statics

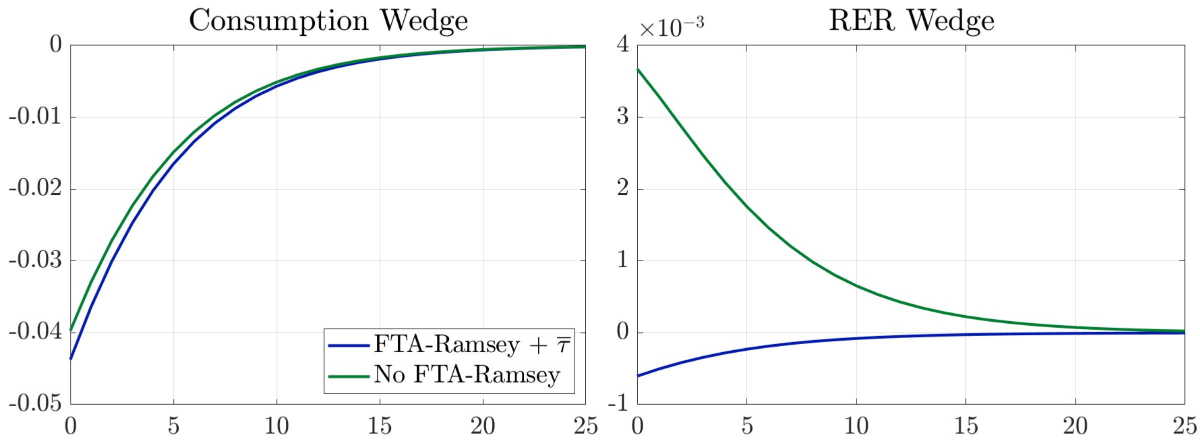
Within the model, two parameters are particularly important for governing the size of the planner's intra- and inter-temporal incentives to manipulate the terms of trade: the intra-temporal elasticity of substitution between goods ϕ (i.e. the trade elasticity) and the coefficient of relative risk aversion σ (i.e. the inverse inter-temporal elasticity of substitution). In doing so, these parameters influence the size of both the optimal capital inflow taxes and optimal

Figure 6: Decomposition of Optimal Capital Flow Taxes for Experiment 1



Notes: Time profile for Home capital flow tax components in Experiment 1, simulated for 50 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The decentralised and FTA-Ramsey models include a steady-state tariff to ensure their steady-state allocations replicate the No FTA-Ramsey case.

Figure 7: Decomposition of Optimal Capital Flow Taxes for Experiment 2



Notes: Time profile for Home capital flow tax components in Experiment 2, simulated for 50 periods. See Table 1 for calibration details. "(No) FTA-Ramsey" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The decentralised and FTA-Ramsey models include a steady-state tariff to ensure their steady-state allocations replicate the No FTA-Ramsey case.

import tariffs. They do so in a manner that is inversely related to the elasticity: the lower the elasticity, the higher the taxes, and vice versa.

Figure 8 demonstrates this for the inter-temporal trade elasticity in the content of Experiment 1—although the 'inverse elasticity rule' holds in both experiments. As the right-hand figure shows, optimal import tariffs are both larger and vary more over time when the trade elasticity is lower. These intra-temporal incentives interact with the optimal capital flow taxes too, which are higher for lower trade elasticities, regardless of the prevailing trade agreement.

Similarly, Figure 9 shows that optimal capital flow taxes are larger when the inter-temporal elasticity of substitution is lower (i.e. higher coefficient of relative risk aversion σ). In turn, variation in import tariffs is larger when σ is high.

4 Strategic Planning Allocation

We now consider the case in which both countries seek to maximise domestic welfare. We denote the Home and Foreign proportional taxes on gross returns by θ_t and θ_t^*, respectively. Likewise, Home and Foreign tariffs are given by τ_t and τ_t^*. We look for a Nash equilibrium, considering each government's optimisation problem and taking the other's tax sequence as given. As in Section 3, we consider the with- and without-FTA cases in turn. In the former case, we can analyse capital control wars. In the latter, we can consider both capital control *and* trade wars.

4.1 With Free Trade

We continue to use the primal approach to characterise the optimal policy. Focusing on the Home planning problem, we can characterise the optimal allocation with a FTA in place, taking the sequence of Foreign capital flow taxes $\{\theta_t^*\}$ as given. Faced with these taxes, the Foreign Euler equations, for $i = 1, 2$ can be written:

$$u^{*\prime}(C_t^*)g_i^*(\mathbf{c}_t^*) = \beta(1-\theta_t^*)(1+r_{i,t})u^{*\prime}(C_{t+1}^*)g_i^*(\mathbf{c}_t^*) \tag{13}$$

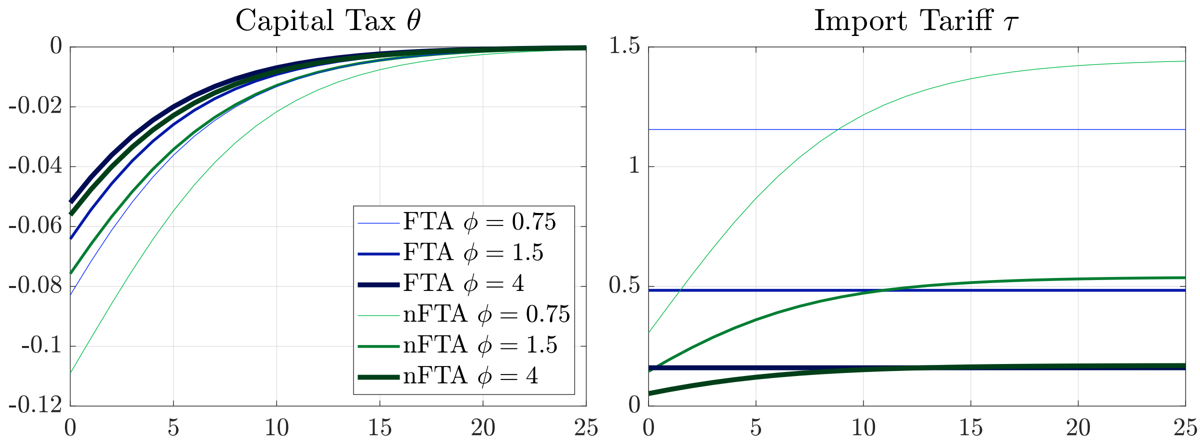
These Foreign optimality conditions, the Home inter-temporal budget constraint and the market clearing conditions yield an implementability condition for the Home planner, which is described in the following proposition.

Proposition 3 (Implementability for Nash Planner with FTA) *Since* $1+r_{i,t} \equiv p_{i,t}/p_{i,t+1}$*, when the Foreign country seeks to set* $\{\mathbf{c}_t^*\}$ *in order maximise domestic welfare, then the Home allocation* $\{\mathbf{c}_t\}$ *forms part of an equilibrium if it satisfies:*

$$\sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t] \leq 0 \tag{IC-Nash-FTA}$$

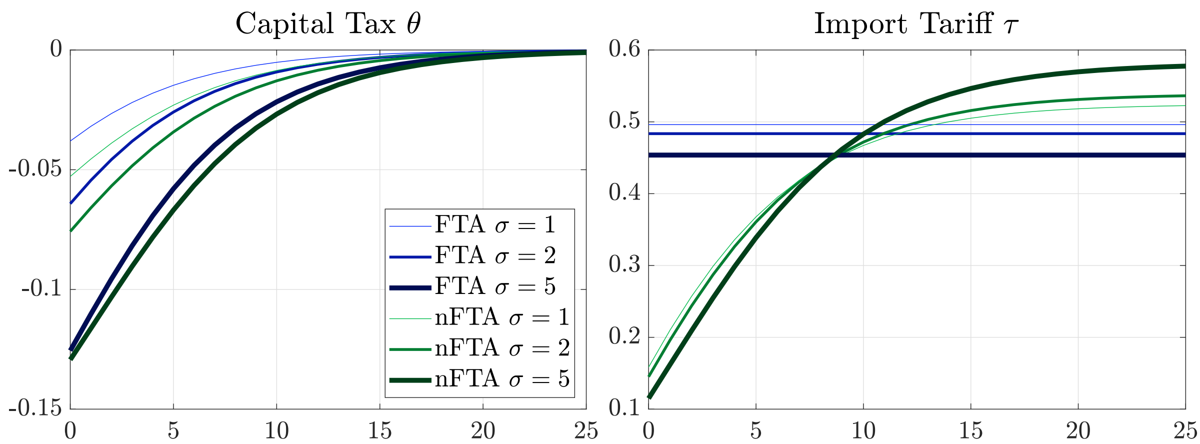
The Home planning problem, accounting for the optimal response by the Foreign planner,

Figure 8: Comparative Statics of Optimal Capital Flow Taxes and Tariffs with Respect to the Intra-temporal Trade Elasticity ϕ in Experiment 1



Notes: Time profile for Home capital flow tax and import tariff in Experiment 1, simulated for 50 periods, with three different values of intra-temporal elasticity of substitution between goods 1 and 2 ϕ. See Table 1 for calibration details. "(n)FTA" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The FTA-Ramsey model includes a steady-state tariff to ensure that the steady-state allocation replicates the nFTA-Ramsey case.

Figure 9: Comparative Statics of Optimal Capital Flow Taxes and Tariffs with Respect to the Coefficient of Relative Risk Aversion σ (Inverse Inter-temporal Elasticity of Substitution) in Experiment 1



Notes: Time profile for Home capital flow tax and import tariff in Experiment 1, simulated for 50 periods, with three different values of the coefficient of relative risk aversion σ (i.e. inverse inter-temporal elasticity of substitution. See Table 1 for calibration details. "(n)FTA" refers to allocation arising from a Home planner acting unilaterally with (without) a FTA in place. The FTA-Ramsey model includes a steady-state tariff to ensure that the steady-state allocation replicates the nFTA-Ramsey case.

is given by:

$$\max_{\{C_t\}} \quad \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{P-Nash-FTA}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right] \beta^t u^{*\prime}(C_t^*) \nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \leq 0 \tag{IC-Nash-FTA}$$

$$\mathbf{c}_t = \mathbf{c}(C_t), \quad \mathbf{c}_t^* = \mathbf{c}^*(C_t) \tag{FTA}$$

which is comparable to the unilateral problem (P-Unil-FTA), albeit with an additional term in the implementability constraint reflecting the Foreign capital flow tax θ_t^*.

Optimal Allocation. Problem (P-Nash-FTA) yields the optimality condition:

$$u'(C_t) = \mu \left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right] \hat{\mathcal{MC}}_t^{FTA} \tag{14}$$

where μ denotes the Lagrange multiplier on the implementability constraint and:

$$\begin{aligned} \hat{\mathcal{MC}}_t^{FTA} \equiv & u^{*\prime}(C_t^*) \nabla g^*(\mathbf{c}_t^*) \cdot \mathbf{c}'(C_t) + u^{*\prime\prime}(C_t^*) C^{*\prime}(C_t) \nabla g^*(\mathbf{c}^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \\ & + u^{*\prime}(C_t^*) \frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial C_t} \cdot [\mathbf{c}_t - \mathbf{y}_t] \end{aligned}$$

Taking the ratio of t and $t+1$ optimality conditions further implies that:

$$\frac{u'(C_t)}{u'(C_{t+1})} = \frac{1}{1-\theta_t^*} \frac{\hat{\mathcal{MC}}_t^{FTA}}{\hat{\mathcal{MC}}_{t+1}^{FTA}} \tag{15}$$

Combining equation (15) with the Foreign Euler equations (13) and the analogous Home Euler equations, yields an expression for $1-\theta_t$. The planning problem of the Foreign government is symmetric, so an analogous expression for $1-\theta_t^*$ can be derived. After some simplification, the combination of these expressions yields a mutual best response function, given by:

$$\frac{\hat{\mathcal{MC}}_t^{FTA}}{\hat{\mathcal{MC}}_t^{*FTA}} = \alpha_0^{FTA} \tag{16}$$

where

$$\alpha_0^{FTA} \equiv \frac{\hat{\mathcal{MC}}_0^{FTA}}{\hat{\mathcal{MC}}_0^{*FTA}}$$

This is the strategic counterpart of equation (5) in Section 3.1. In the Nash bargaining setup, α_0^{FTA} can be interpreted as the bargaining power of the Foreign country relative to the Home.

4.2 Without Free Trade

Absent a FTA, the Home planner must now take both the sequence of Foreign capital flow taxes $\{\theta_t^*\}$ and the sequence of Foreign tariffs $\{\tau_t^*\}$ as given. The Foreign tariff is levied by the Foreign planner on Foreign consumers' purchases of good 1.

As a result of the Foreign tariff, the implementability constraint for the Home planner is different to Proposition 3, equation (IC-Nash-FTA). Defining the vector of inverse Foreign goods-specific tariffs by $\boldsymbol{\tau}_t^* \equiv [(1+\tau_t^*)^{-1}, 1]'$, the following proposition details the implementability for the Home planner when acting strategically in the absence of an FTA.[9]

Proposition 4 (Implementability for Nash Planner without FTA) *The Home allocation forms art of an equilibrium without an FTA if it satisfies:*

$$\sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*)\right]\beta^t u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\leq 0 \qquad \text{(IC-Nash-nFTA)}$$

The Home planning problem is thus given by:

$$\max_{\{\mathbf{c}_t\}} \quad \sum_{t=0}^{\infty} u(C_t) \qquad \text{(P-Nash-nFTA)}$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty}\left[\prod_{s=0}^{t-1}(1-\theta_s^*\right]\beta^t u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t]\leq 0 \qquad \text{(IC-Nash-nFTA)}$$

$$C_t \equiv g(\mathbf{c}_t) \qquad \text{(nFTA)}$$

which is comparable to the unilateral no-FTA problem (P-Unil-nFTA), albeit with additional terms in the implementability constraint reflecting Foreign taxes $\{\theta_t^*, \tau_t^*\}$.

Optimal Allocation. Problem (P-Nash-nFTA) yields the optimality conditions:

$$u'(C_t)g_1(\mathbf{c}_t) = \mu\hat{\mathcal{MC}}_{1,t}^{nFTA} \qquad (17)$$

$$u'(C_t)g_2(\mathbf{c}_t) = \mu\hat{\mathcal{MC}}_{2,t}^{nFTA} \qquad (18)$$

where μ denotes the Lagrange multiplier on the implementability constraint and:

$$\begin{aligned}\hat{MC}_{1,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)(1+\tau_t^*)g_1^*(\mathbf{c}_t^*) + u^{*\prime\prime}g_1^*(\mathbf{c}_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t] \\ & + u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\frac{\partial\nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}}\cdot[\mathbf{c}_t-\mathbf{y}_t] \\ \hat{MC}_{2,t}^{nFTA} \equiv & u^{*\prime}(C_t^*)g_2^*(\mathbf{c}_t^*) + u^{*\prime\prime}g_2^*(\mathbf{c}_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\nabla g^*(\mathbf{c}_t^*)\cdot[\mathbf{c}_t-\mathbf{y}_t] \\ & + u^{*\prime}(C_t^*)\boldsymbol{\tau}_t^{*-1}\cdot\frac{\partial\nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}}\cdot[\mathbf{c}_t-\mathbf{y}_t]\end{aligned}$$

[9]In contrast to the unilateral case, the implementability condition for the Home planner differs in the absence of a FTA because of tariffs set by the Foreign planner.

The Foreign planner undertakes an analogous maximisation. Combining the optimality conditions of the Home and Foreign planners yields the equilibrium allocation, which is summarised in the following proposition.

Proposition 5 (Capital Control and Tariff Wars) *In a Nash equilibrium where each country chooses optimal capital controls* $\{\theta_t, \theta_t^*\}_{t\geq 0}$ *and tariffs* $\{\tau_t, \tau_t^*\}_{t\geq 0}$*, the allocations* $\{\mathbf{c}_t, \mathbf{c}_t^*\}_{t\geq 0}$ *must satisfy*

$$\frac{\hat{\mathcal{MC}}_{1,t}^{nFTA}}{\hat{\mathcal{MC}}_{1,t}^{*nFTA}} = \alpha_{1,0}^{nFTA} \qquad \frac{\hat{\mathcal{MC}}_{2,t}^{nFTA}}{\hat{\mathcal{MC}}_{2,t}^{*nFTA}} = \alpha_{2,0}^{nFTA} \tag{19}$$

where

$$\alpha_{i,0}^{nFTA} \equiv \frac{\hat{\mathcal{MC}}_{i,0}^{nFTA}}{\hat{\mathcal{MC}}_{i,0}^{*nFTA}} \quad \textit{for } i = 1, 2$$

Proof: See Appendix B.1. □

The conditions above are intuitive, reflecting the ratio of the marginal cost of a unit of consumption for the planner across the Home and Foreign country, for each good variety.

4.3 Numerical Exercises

Using these strategic optimality conditions, we revisit our result on the direction and level of capital flow taxes with and without a FTA. Focusing on Experiment 1, we consider the path of capital flow taxes and tariffs when the Home endowment of good 1 is low relative to its long-run value. The key insights from the Nash allocation carry over to Experiment 2 as well.

The left-hand side of Figure 11 illustrates that, in response to a good-1 downturn at Home, capital inflow taxes are larger absent a FTA. Although, for a given trade agreement, Home capital flow taxes are smaller in magnitude in the Nash case, this is due to the presence of Foreign taxes. Moreover, the capital inflow tax in response to a downturn driven by goods-2 (Experiment 2), is also larger in the absence of an FTA in the Nash equilibrium. Our baseline result, namely that optimal capital controls are larger in the absence of a FTA, is therefore stronger when both countries act strategically.

Second, to study the impact of 'policy wars', we define two new quantities which capture the difference in the cost of borrowing in the Home *vis-à-vis* Foreign country, and the relative ratio of tariffs at Home *vis-à-vis* in Foreign:

$$\Delta^R = \frac{1-\theta_t}{1-\theta_t^*}, \quad \Delta^\tau = \frac{1+\tau_t}{1+\tau_t^*} \tag{20}$$

If $\Delta^R > 1$, the cost of borrowing in the Home country is higher *vis-à-vis* the Foreign country, while $\Delta^\tau > 1$ reflects a higher tariffs at Home *vis-à-vis* the Foreign country.

Figure 13 illustrates the evolution of these objects following a domestic downturn driven by good 1 (Experiment 1). Focusing on capital flow taxes, the wedge Δ^θ is larger in all periods in the absence of a FTA. Consistent with optimality conditions, the Home planner taxes capital inflows in an attempt to drive down the price of aggregate consumption and levies a smaller import tariff in the short run, in order to drive down good 1 consumption, while the Foreign planner attempts the opposite. As a result, $\Delta^R > 1$ on impact and approaches 0 as $y_{1,t}$ approaches $\overline{y}_1$ while $\Delta^\tau > 1$ on impact and rises thereafter.

Comparative Statics. At the benchmark calibration ($\sigma = 2, \phi = 1.5$), countries engage in competition over both capital controls and trade tariffs leading to $\Delta^R, \Delta^\tau \neq 1$. We show that, as the elasticity of inter-temporal substitution $\frac{1}{\sigma}$ falls—i.e. σ rises—countries levy larger capital controls, in turn inciting a larger response from each other, in an attempt to reallocate consumption inter-temporally and is reflected in a high $|\Delta^R|$. When σ is high, a representative household is more insensitive to change in the interest rate when choosing to allocate consumption across periods. In contrast, as $\frac{1}{\sigma}$ rises, households are more sensitive to changes in the interest rate and smaller capital controls are levied in the Nash equilibrium.

Conversely, when the trade elasticity ϕ is low, countries engage more in a tariff war leading to a higher $|\Delta^\tau|$. This reflects the well-understood result in public finance that a planner optimally chooses to tax commodities for which demand is price-inelastic. We show the *inverse-elasticity* result extends to a policy war involving competition in capital flow taxes and import tariffs (see, for example, Chari and Kehoe, 1999).[10]

5 Welfare and International Spillovers

Finally, we assess the consequences and spillovers of policy for welfare. Does the optimal policy simply reallocate from the Foreign country to the Home, or does it contribute to increase Home welfare at a disproportional cost to Foreign welfare and therefore world welfare? What are the costs of capital control wars, and are policy wars costlier when a FTA is not in place?

To answer this, we consider the cooperative problem where consumption allocations are chosen to maximise joint (world) welfare as our benchmark. The cooperative planning problem is given by:

$$\max_{\{\mathbf{c}_t\}} \sum_{t=0}^{\infty} \beta^t \Big[u(g(\mathbf{c}_t)) + \kappa u(g^*(\mathbf{c}_t^*)) \Big], \tag{P-Coop}$$

$$\text{s.t} \quad \mathbf{c}_t + \mathbf{c}_t^* = \mathbf{Y}_t, \tag{RC}$$

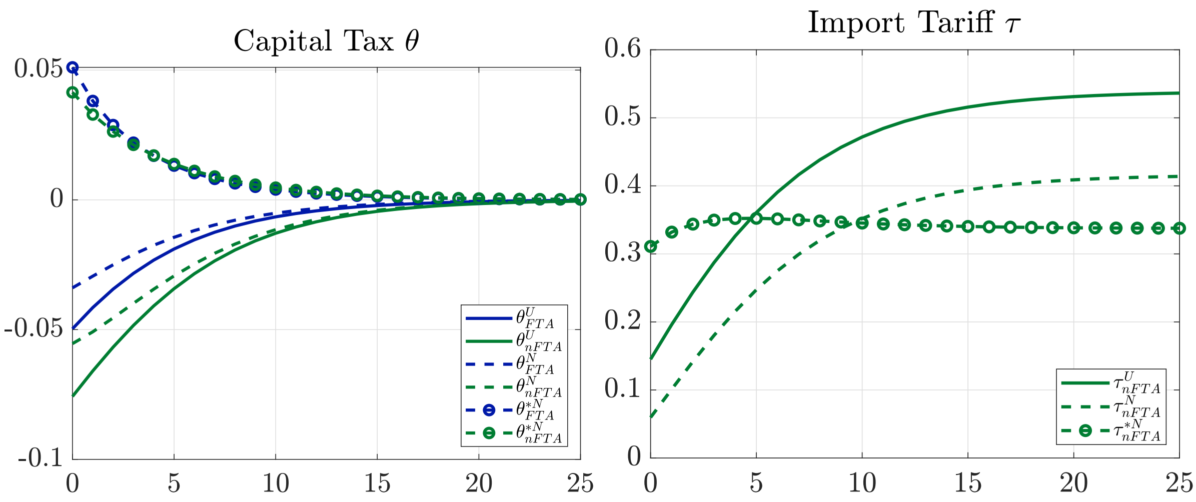
$$\mathbf{c} = \mathbf{c}(C), \quad \mathbf{c}^* = \mathbf{c}^*(C) \tag{FTA}$$

where κ is the weight attributed to Foreign welfare.

The following proposition summarises the key property of the global cooperative problem.

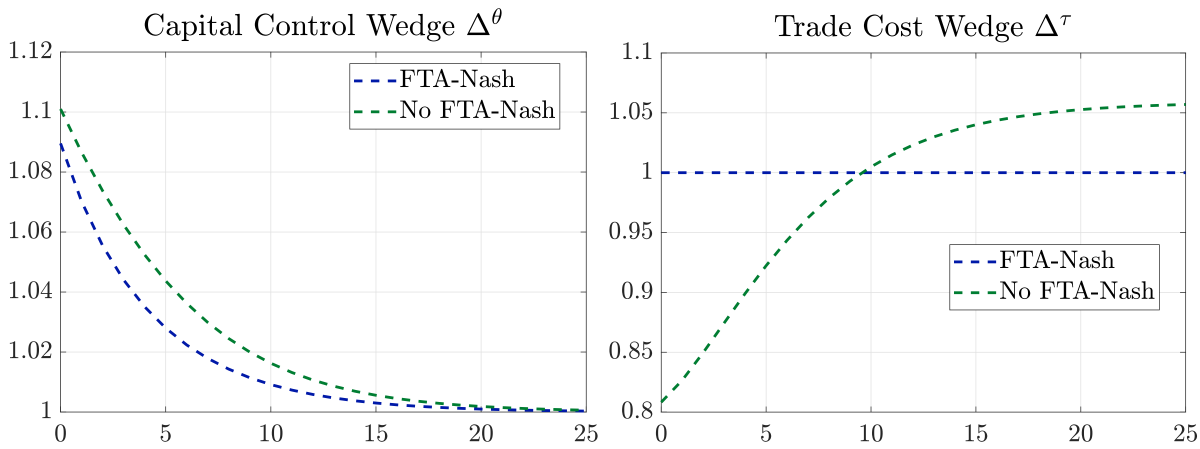
[10] These findings are consistent with the Arrow-Debreu approach of relabelling the future delivery of commodities as a separate good.

Figure 10: Optimal Capital Flow Taxes and Import Tariffs in the Nash Equilibrium in Experiment 1



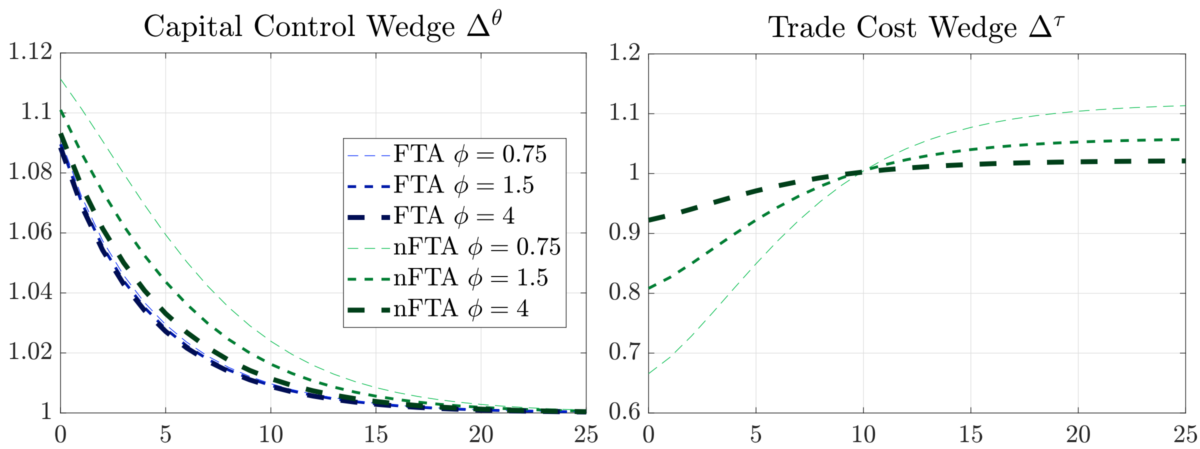
Notes: Optimal capital controls and taxes. 'U' subscript denotes unilateral optimal policy result (for Home). 'N' denotes Nash outcome.

Figure 11: Capital Control and Tariff Weds in the Nash Equilibrium in Experiment 1



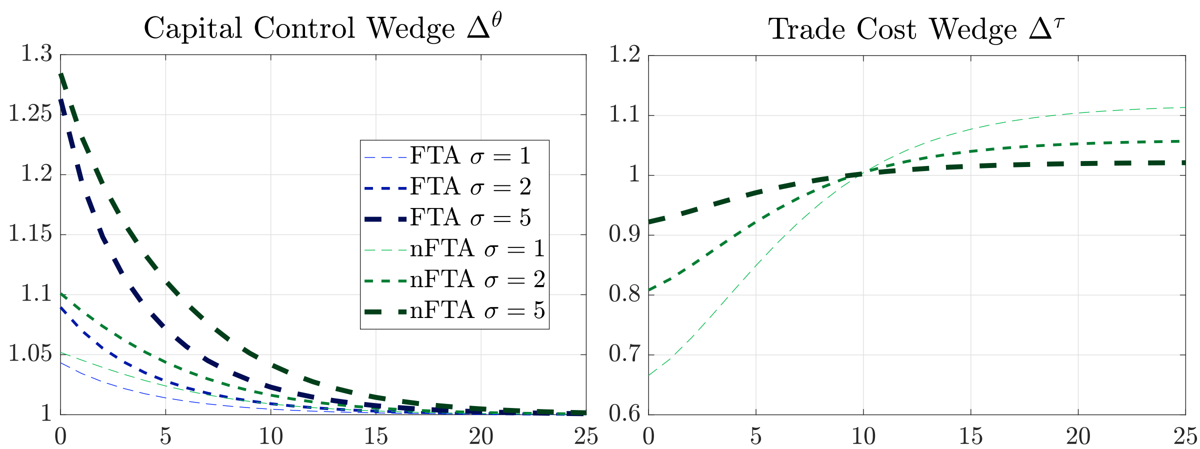
Notes: Difference in cost of borrowing and tariffs across countries.

Figure 12: Experiment 1: Comparative Statics with respect to ϕ



Notes: Optimal capital controls and taxes. ‘U’ subscript denotes unilateral optimal policy result (for Home). ‘N’ denotes Nash outcome.

Figure 13: Experiment 1: Comparative Statics with respect to σ



Notes: Difference in cost of borrowing and tariffs across countries.

Table 2: Welfare and Spillovers. Welfare expressed in terms of % consumption equivalent variation ($-ve$ implies welfare gain).

	H	F	Global $\sum_{H,F}$
Experiment 1			
FTA (Unilateral)	-0.13	0.23	0.050
without FTA (Unilateral)	-0.22	0.27	0.025
with FTA (Nash)	0.068	0.067	0.068
without FTA (Nash)	1.71	1.58	1.65
Experiment 2			
with FTA (Unilateral)	-0.061	0.011	0.0027
without FTA (Unilateral)	-0.082	0.39	0.15
with FTA (Nash)	0.16	-0.0007	0.080
without FTA (Nash)	5.2	0.93	3.1

Proposition 6 (Globally Cooperative Allocation) *In the cooperative allocation, no intervention is optimal such that, if $\kappa = 1$, $\theta_t = \tau_t = 0$.*

Proof: See Appendix C.1. □

Since the optimal cooperative policy coincides with the decentralised allocation, it must be that at the optimal policy—unilateral or Nash, with or absent an FTA—world welfare, as defined in (P-Coop), falls.

Table 2 reports to difference in present discounted welfare in Experiments 1 and 2 under the optimal policy, relative to the decentralised allocation, for the Home and Foreign representative agents respectively. Our results confirm that capital and goods taxation is distortionary and does not simply reallocate consumption across borders. In the case where the Foreign country is passive, the costs to the Foreign country outweigh the gains in the Home country leading to a loss in global welfare. In the presence of a FTA, capital controls change in the path of consumption over time which is inefficient for the Foreign country. In a Nash equilibrium where a FTA holds, the Foreign country benefits relative to the unilateral case by levying taxes itself, but global welfare falls further.

In the absence a FTA, countries levy taxes not only to change the path of consumption over time, but also its composition across goods varieties, so the the welfare costs from policy wars are higher when countries engage in both capital controls and tariff wars. In the working paper version of this chapter, we also investigate the origins of welfare losses. We show that, as the elasticity of inter-temporal substitution rises, welfare costs from capital control wars under a FTA become very small but are almost unchanged absent a FTA. In contrast, costs sharply fall as ϕ rises both with and without a FTA in place. We therefore find evidence that the costs to *both* capital control and tariff wars are predominantly from intra-temporal choice distortions.

6 Discussion: Generality of Results

Throughout this chapter, we have used a comparatively pared back two-country, two-good endowment model. This simplification has enabled us to provide analytical solutions, but has abstracted from many features discussed in the existing literature—for example firm and household heterogeneity, firm entry and exit, roundabout production, borrowing constraints etc. While these features may have additional policy implications, we nevertheless emphasise that the pecuniary externalities at the heart of our analysis are sufficiently general. We detail two dimensions along which our results can be generalised.

Endowment vs. Production Economies. We focus on an endowment economy to abstract from the complexities of price-setting, which have been a key focus of open-economy macroeconomics in past decades (e.g. Devereux and Engel, 2003; Benigno and Benigno, 2003; Corsetti, Dedola, and Leduc, 2010). Nevertheless, with full specialisation assumed ($y_1 = 1$, $y_2 = 0$, $y_1^* = 0$ and $y_2^* = 1$), our endowment setup is isomorphic to one with production subject to technology $y_1 = f(A, L)$, where each country is endowed with a fixed quantity of labour $\overline{L}$. If we further assume that the function f is first-order homogeneous in A, fluctuations in y_1 and y_2^* in our endowment economy can be interpreted as reflecting movements in Home and Foreign productivity—A and A^*, respectively.[11] Moving away from the full specialisation case, we assume labour in each country, is employed in two sectors, one which produces good 1 and another producing good 2. The latter can be interpreted as an 'export' sector and fluctuations in y_2^* can represent fluctuations in export-sector productivity.

Size in Goods and Financial Markets. This chapter considers a two-country model where each country is large in both goods and financial markets. As a consequence, the planner internalises the effect of domestic allocations on both goods prices and the real interest rate, motivating the use of capital controls and tariffs. Similar conclusions can be reached when considering two generalisations of this. First, moving to a small-open economy setting (i.e. with $N \to \infty$ Foreign countries). As detailed in Costinot et al. (2014), countries remain large in goods markets for their *domestic* variety. So, the incentive for intra-temporal manipulation of the terms of trade remains.

Second, for countries to be able manipulate the real (world) interest rate using capital controls or tariffs, countries must be large enough to individually influence world consumption.[12] Arguably, this can only apply to a small set of countries, e.g. U.S. and China. However, this result can be recovered in the presence of financial frictions. A recent literature, including Basu et al. (2020), Bianchi and Lorenzoni (2021) and Marin (2022), consider an upward-sloping supply curve for international borrowing and emphasise that capital controls can be used to manipulate

[11] Alternatively, if technology is linear and productivity is constant, y_1 and y_2^* can reflect exogenous movements in labour supply (L and L^*, respectively) such as those studied in Guerrieri, Lorenzoni, Straub, and Werning (2020).

[12] This is a critique espoused by Rebucci and Ma (2019) amongst others.

interest rates through premia.[13] For example, Marin (2022) models the U.S. as a small-open economy (taking C^* as given) but emphasises it is large in dollar markets. Capital controls or tariffs which reduce the supply of dollar borrowing by the U.S. can reinforce monopolistic rents and have significant welfare implications.

Finally, Egorov and Mukhin (2020) show that in the presence of nominal rigidities and dollar currency pricing, i.e. when world exports are priced in dollars, U.S. prices affect the world stochastic discount factor and the U.S. is able to manipulate the inter-temporal terms of trade even if it is small.

7 Conclusion

In this chapter, we analyse optimal capital flow taxation comparing outcomes with and without a binding FTA. We find that optimal capital controls and trade taxes are interdependent and their relationship depends on the nature of economic fluctuations.

Within a standard two-country, two-good endowment model, trade occurs along two margins—over time and across goods varieties. If, in addition to capital controls, we allow for goods-specific taxation (for instance, in the form of import tariffs), we show that a planner who levies taxes can achieve higher welfare. The departure from an FTA offers the planner a second instrument, such that the first-best allocation can be achieved. The planner taxes capital inflows at times when the Home country is borrowing between two periods, so as to drive down the interest rate and address inter-temporal incentives. In conjunction, the planner uses an import tax to increase the relative demand for Home consumption in periods where the country is exporting Home goods, thus constraining net supply to drive up the price of the Home good and address intra-temporal incentives.

Moving away from an FTA, if the Foreign country is passive, optimal capital flow taxes levied by the Home country are larger if the planner's inter- and intra-temporal terms-of-trade manipulation incentives are aligned. In such cases, optimal import tariffs amplify relative consumption changes and real exchange rate misalignments, thus requiring larger capital flow taxes to implement the optimal allocation. When inter- and intra-temporal incentives are opposed, the resulting capital flow taxes are smaller. In this case, tariffs move the real exchange rate in a direction that supports efficient levels of borrowing so the required of capital controls is smaller.

In a Nash equilibrium, due to competition in capital flow taxes, the level of capital flow taxes rises in both cases when the FTA is relaxed. Allowing countries to engage in policy wars, we show capital control wars dominate when the elasticity of inter-temporal substitution is low, while tariff wars intensify as the trade elasticity falls—consistent with an inverse-elasticity rule. When the Foreign country is passive, the optimal policy brings welfare gains to the Home policymaker, but cross-border spillovers are negative and disproportionately large. Absent an FTA, we show that trade policy is not simply redistributive. By levying trade taxes, the planner

[13] These frictions build on Gabaix and Maggiori (2015) which in turn builds on a large literature on limits to arbitrage, e.g. Gromb and Vayanos (2010).

pushes foreign households away from their efficient allocation of goods varieties, generating further costly cross-border spillovers in addition to inefficient reallocation of consumption over time.

In ongoing research, we extend the environment in a number of dimensions. First,we consider the mapping between pecuniary externalities and financial frictions, such as the case of a borrowing constraint. Prices in the borrowing constraint drive the interaction between optimal capital control and trade policy since,usually, a terms of trade appreciation tends to relax this constraint (see, for example, Bianchi, 2011). This motive to appreciate the terms of trade in periods where the borrowing constraint is most binding is of first-order interest to policymakers. Second, while in this chapter we have focused on the intensive margin– i.e. how large are capital controls, we extend our analysis to the extensive margin– i.e when are capital controls used. We focus on a strategic setting with retaliation and show, amongst other results, that a trade union (such as the European Union) fosters a capital flows union.

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Appendix

A Derivations and Proofs

A.1 Pareto Frontier

This sub-section provides derivations for the Pareto frontier, which is stated in Definition 1 of Section 2.1. The Pareto frontier summarises combinations of consumption allocations $\{c_{1,t}, c_{2,t}\}$ which are Pareto efficient, given a level of aggregate consumption C_t.

The Home representative household chooses their consumption by minimising expenditure, for a given level of aggregate consumption $\overline{C}$:

$$\min_{c_{1,t},c_{2,t}} \quad p_{1,t}c_{1,t} + p_{2,t}c_{2,t} \quad \text{s.t.} \quad \overline{C} = g(\mathbf{c}_t)$$

The first-order conditions for this problem yield the Home relative demand equation:

$$\frac{g_{1,t}}{g_{2,t}} = \frac{p_{1,t}}{p_{2,t}} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\phi}} \left(\frac{c_{2,t}}{c_{1,t}}\right)^{\frac{1}{\phi}} \tag{21}$$

where $p_{1,t}/p_{2,t} \equiv 1/TOT_t$ and TOT_t refers to the terms of trade.

To derive the Pareto frontier, note that the analogous Foreign relative demand curve is:

$$\frac{g^*_{1,t}}{g^*_{2,t}} = \frac{p_{1,t}}{p_{2,t}} = \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\phi}} \left(\frac{c^*_{2,t}}{c^*_{1,t}}\right)^{\frac{1}{\phi}} \tag{22}$$

Equating relative prices across countries, equations (21) and (22) yield:

$$\frac{c^*_{2,t}}{c^*_{1,t}} = \left(\frac{\alpha}{1-\alpha}\right)^2 \frac{c_{2,t}}{c_{1,t}} \tag{23}$$

This expression for optimal relative consumption must be consistent with goods market clearing ($Y_{i,t} = c_{i,t} + c^*_{i,t}$ for $i = 1, 2$). Combining (23) with goods market clearing, we attain the following expressions for consumption:

$$c_{1,t} = \frac{bc_{2,t}Y_{1,t}}{Y_{2,t} - (1-b)c_{2,t}} \tag{24}$$

$$c_{2,t} = \frac{c_{1,t}Y_{2,t}}{bY_{1,t} + (1-b)c_{1,t}} \tag{25}$$

where $b \equiv \left(\frac{\alpha}{1-\alpha}\right)^2$.

Solving for $\mathrm{d}c_i(C)/\mathrm{d}C$ Rearranging the Armington aggregator, we can show that:

$$c_{1,t}(C_t) = \left[\frac{C_t^{\frac{\phi-1}{\phi}} - (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}}{\alpha^{\frac{1}{\phi}}}\right]^{\frac{\phi}{\phi-1}} \tag{26}$$

$$c_{2,t}(C_t) = \left[\frac{C_t^{\frac{\phi-1}{\phi}} - \alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}}}{(1-\alpha)^{\frac{1}{\phi}}}\right]^{\frac{\phi}{\phi-1}} \tag{27}$$

Equating equations (25) with (27) yields:

$$\left[C_t^{\frac{\phi-1}{\phi}} - \alpha^{\frac{1}{\phi}} c_{1,t}(C_t)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} (bY_{1,t} + (1-b)c_{1,t}(C_t)) = c_{1,t}(C_t) Y_{2,t} \, (1-\alpha)^{\frac{1}{\phi-1}}$$

Totally differentiating this expression and rearranging yields:

$$\frac{\mathrm{d}c_{1,t}(C_t)}{\mathrm{d}C_t} = \frac{C_t^{-\frac{1}{\phi}} X_t (bY_{1,t} + (1-b)c_{1,t}(C_t)}{Y_{2,t} - c_{2,t}(C_t)(1-b) + \alpha^{\frac{1}{\phi}} c_{1,t}(C_t)^{-\frac{1}{\phi}} X_t (bY_{1,t} + (1-b)c_{1,t}C_t)}$$

where $X_t \equiv \left[C_t^{\frac{\phi-1}{\phi}} - \alpha^{\frac{1}{\phi}} c_{1,t}(C_t)^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} (1-\alpha)^{-\frac{1}{\phi-1}}$.

The expression for $\mathrm{d}c_{2,t}(C_t)/\mathrm{d}C_t$ can be derived analogously.

A.2 Foreign Household Optimisation

This sub-section details the representative Foreign consumer's optimisation problem, which acts as a constraint for the unilateral Home Ramsey planner in Section 3.

Foreign households maximise their discounted lifetime utility subject to their inter-temporal budget constraint, given world prices $\mathbf{p}_t$:

$$\max_{\{\mathbf{c}_t\}} \quad U_0^* = \sum_{t=0}^{\infty} \beta^t u^*(g^*(\mathbf{c}_t))$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) \leq 0$$

The first-order conditions for this problem are given by (3) and (4) in Section 3, where λ^* is the Lagrange multiplier on the Foreign inter-temporal budget constraint.

A.3 Proof to Proposition 2

First, note that any outcome achievable in (P-Unil-FTA) is achievable in (P-Unil-nFTA). Part (i) follows immediately since (P-Unil-nFTA) is a relaxed version of (P-Unil-FTA) therefore the planner achieves weakly better outcomes when the FTA is relaxed. However, we analyse this further. Equations (5), (6), and (7) satisfy the following total derivative rule:

$$\frac{d\mathcal{L}}{dC} = \frac{\partial \mathcal{L}}{\partial c_1} c_1'(C) + \frac{\partial \mathcal{L}}{\partial c_2} c_2'(C)$$

The solution to (P-Unil-FTA) (when an FTA is in force) satisfies $\frac{d\mathcal{L}}{dC} = 0$ at the (constrained) optimal allocation. Since $c_1'(C), c_2'(C)$ are positive and increasing functions given by Lemma 1, generally $sign(\frac{d\mathcal{L}}{dc_1}) = -sign(\frac{d\mathcal{L}}{dc_2})$ indicating an incentive to adjust consumption across varieties remains at the constrained optimal allocation.

In contrast, the solution to (P-Unil-nFTA) given by (6) and (7) implies $\frac{d\mathcal{L}}{dc_1} = \frac{d\mathcal{L}}{dc_2} = 0$ which necessarily implies aggregate consumption is (unconstrained) optimal as well. Formally, denote,

$$\overline{C} = \{C : \max \quad \mathcal{L}(C) \mid c_1(C), c_2(C) \text{ on Pareto Frontier}\}, \tag{28}$$

where $\overline{C}$ is a scalar because $\mathcal{L}$ is strictly concave in the region of interest. Then note that $\frac{d\mathcal{L}}{dc_1}_{|c_H(\overline{C}),c_2(\overline{C})}, \frac{d\mathcal{L}}{dc_2}_{|c_1(\overline{C}),c_2(\overline{C})} \neq 0$. If, e.g. $\frac{d\mathcal{L}}{dc_1}_{|c_1(\overline{C}),c_2(\overline{C})} > 0$, then $\frac{d\mathcal{L}}{dc_2}_{|c_1(\overline{C}),c_2(\overline{C})} < 0$ and there exists ϵ perturbation such that a $c_1(\overline{C}) \pm \epsilon, c_2(\overline{C}) \pm \epsilon$ are preferred.

Furthermore, (ii) follows since it must be then that $c_1'(C)$, $c_2'(C)$ implied by (6) and (7) violate Lemma 1 (ii) if $\frac{d\mathcal{L}}{dc_1}_{|c_1(\overline{C}),c_2(\overline{C})}, \frac{d\mathcal{L}}{dc_2}_{|c_1(\overline{C}),c_2(\overline{C})} \neq 0$. Conversely, if $\dfrac{d\mathcal{L}}{dC} = 0 \implies \frac{\partial \mathcal{L}}{\partial c_1} = 0, \frac{\partial \mathcal{L}}{\partial c_2} = 0$ if $c_1'(C), c_2'(C)$ are not binding, i.e. the constraints are identical to the correspondence implied by (6) and (7).

(iii) The allocations coincide when there is no trade in goods in equilibrium as the households' choice is optimal for the planner. □

A.4 Derivatives of the Consumption Aggregator

In this sub-section, we define the derivatives of the Armington (1969) aggregator which defines aggregate consumption in our computational experiments. We present the expressions for the representative Home consumer only, but they are analogous for the representative Foreign consumer.

The first derivatives of the Home aggregator are given by:

$$g_1(\mathbf{c}_t) \equiv \frac{\partial g(\mathbf{c}_t)}{\partial c_{1,t}} = \alpha^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} = \alpha^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} C_t^{\frac{1}{\phi}}$$

$$g_2(\mathbf{c}_t) = \frac{\partial g(\mathbf{c}_t)}{\partial c_{2,t}} = (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} = (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} C_t^{\frac{1}{\phi}}$$

The second derivatives are:

$$g_{11}(\mathbf{c}_t) = -\frac{1}{\phi} \alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{-1-\phi}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}}$$

$$
\begin{aligned}
&+ \frac{1}{\phi}\alpha^{\frac{2}{\phi}} c_{1,t}^{-\frac{2}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{2-\phi}{\phi-1}} \\
g_{12}(\mathbf{c}_t) =& \frac{1}{\phi}\alpha^{\frac{1}{\phi}}(1-\alpha)^{\frac{1}{\phi}} c_{1,t}^{-\frac{1}{\phi}} c_{2,t}^{-\frac{1}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{2-\phi}{\phi-1}} \\
g_{21}(\mathbf{c}_t) =& g_{12}(\mathbf{c}_t) \\
g_{22}(\mathbf{c}_t) =& -\frac{1}{\phi}(1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{-1-\phi}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} \\
&+ \frac{1}{\phi}(1-\alpha)^{\frac{2}{\phi}} c_{2,t}^{-\frac{2}{\phi}} \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{2-\phi}{\phi-1}}
\end{aligned}
$$

B Nash Allocation

Consider the problem faced by the Foreign planner,

$$\max_{\{\mathbf{c}_t^*\}} \sum_{t=0}^{\infty} \beta^t \; u\left(g(\mathbf{c}_t^*)\right) \tag{$P1^{*\ Nash}$}$$

$$s.t \quad \sum_{t=0}^{\infty} \left[\Pi_{s=0}^{t-1}(1-\theta_s)\right] \beta^t u'(g(\mathbf{c}_t)) \boldsymbol{\tau}_t^{-1} \boldsymbol{\nabla} g(\mathbf{c}_t) \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) \leq 0 \tag{$IC^{*\ Nash}$}$$

where,

$$\boldsymbol{\tau}_t = \begin{bmatrix} 1 & 0 \\ 0 & (1-\tau_t) \end{bmatrix} \tag{29}$$

The first order conditions for the Foreign country with respect to $c_{H,t}^*$ and $c_{F,t}^*$ are given by,

$$C_t^{*\ -\sigma} g_{1,t}^* = \mu \left[\Pi_{s=0}^{t-1}(1-\theta_s)\right] \left\{ C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t} \begin{bmatrix} g_{1,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{bmatrix} - C_t^{-\sigma} \begin{bmatrix} g_{11,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{21,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{bmatrix} \right\}, \tag{30}$$

$$\implies \quad C_t^{*\ -\sigma} g_{1,t}^* = \mu \; \hat{MC}_{1,t}^*$$

and,

$$C_t^{*\ -\sigma} g_{2,t}^* = \mu \left[\Pi_{s=0}^{t-1}(1-\theta_s)\right] \left\{ C_t^{-\sigma} g_{2,t}(1-\tau_t)^{-1} + \sigma C_t^{-\sigma-1} g_{2,t} \begin{bmatrix} g_{1,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{bmatrix} - C_t^{-\sigma} \begin{bmatrix} g_{12,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{22,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{bmatrix} \right\} \tag{31}$$

$$\implies \quad C_t^{-\sigma} g_{2,t}^* = \mu \; \hat{MC}_{2,t}^*$$

B.1 Proof to Proposition 5

Dividing (17) by its $t+1$ analogue yields,

$$\frac{C_t^{-\sigma} g_{1,t}}{C_{t+1}^{-\sigma} g_{1,t+1}} = \frac{1}{1-\theta_t^*} \frac{\hat{MC}_{1,t}}{\hat{MC}_{1,t+1}} \tag{32}$$

Evaluating the Foreign analogue for $i = 1$, i.e. (30), and using it to substitute out $\frac{1}{1-\theta_t^*}$ above, and using the analogous Home euler to substitute in $1-\theta_t$ yields the expression for the optimal tax on capital flows levied by the Home country:

$$
1-\theta_t = \frac{1+\sigma C_t^{*-1}\left[\begin{array}{c} g^*_{1,t}(c_{1,t}-y_{1,t})+ \\ g^*_{2,t}(1-\tau^*_t)^{-1}(c_{2,t}-y_{2,t}) \end{array}\right] - \dfrac{1}{g^*_{1,t}}\left[\begin{array}{c} g^*_{11,t}(c_{1,t}-y_{1,t})+ \\ g^*_{21,t}(1-\tau^*_t)^{-1}(c_{2,t}-y_{2,t}) \end{array}\right]}{1+\sigma C_{t+1}^{*-1}\left[\begin{array}{c} g^*_{1,t+1}(c_{1,t+1}-y_{1,t+1})+ \\ g^*_{2,t+1}(1-\tau^*_{t+1})^{-1}(c_{2,t+1}-y_{2,t+1}) \end{array}\right] - \dfrac{1}{g^*_{1,t+1}}\left[\begin{array}{c} g^*_{11,t+1}(c_{1,t+1}-y_{1,t+1})+ \\ g^*_{21,t+1}(1-\tau^*_{t+1})^{-1}(c_{2,t+1}-y_{2,t+1}) \end{array}\right]} \tag{33}
$$

Dividing (30) by its $t+1$ analogue yields,

$$
\frac{C_t^{*\;-\sigma} g^*_{1,t}}{C_{t+1}^{*\;-\sigma} g^*_{1,t+1}} = \frac{1}{1-\theta_t}\frac{C_t^{-\sigma}g_{1,t}+\sigma C_t^{-\sigma-1}g_{1,t}\left[\begin{array}{c} g_{1,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{2,t}(1-\tau_t)^{-1}(c^*_{2,t}-y^*_{2,t}) \end{array}\right] - C_t^{-\sigma}\left[\begin{array}{c} g_{11,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{21,t}(1-\tau_t)^{-1}(c^*_{2,t}-y^*_{2,t}) \end{array}\right]}{C_{t+1}^{-\sigma}g_{1,t+1}+\sigma C_{t+1}^{-\sigma-1}g_{1,t+1}\left[\begin{array}{c} g_{1,t+1}(c^*_{1,t+1}-y^*_{1,t+1})+ \\ g_{2,t+1}(1-\tau_{t+1})^{-1}(c^*_{2,t+1}-y^*_{2,t+1}) \end{array}\right] - C_{t+1}^{-\sigma}\left[\begin{array}{c} g_{11,t+1}(c^*_{1,t+1}-y^*_{1,t+1})+ \\ g_{21,t+1}(1-\tau_{t+1})^{-1}(c^*_{2,t+1}-y^*_{2,t+1}) \end{array}\right]}
$$

$$
= \frac{1}{1-\theta_t}\frac{\hat{MC}^*_{1,t}}{\hat{MC}^*_{1,t+1}} \tag{34}
$$

and following the analogous steps as for (32) yields the expression for the optimal tax on capital flows levied by the Foreign country:

$$
1-\theta^*_t = \frac{1+\sigma C_t^{-1}\left[\begin{array}{c} g_{1,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{2,t}(1-\tau_t)^{-1}(c^*_{2,t}-y^*_{2,t}) \end{array}\right] - \dfrac{1}{g_{1,t}}\left[\begin{array}{c} g_{11,t}(c^*_{1,t}-y^*_{1,t})+ \\ g_{21,t}(1-\tau_t)^{-1}(c^*_{2,t}-y^*_{2,t}) \end{array}\right]}{1+\sigma C_{t+1}^{-1}\left[\begin{array}{c} g_{1,t+1}(c^*_{1,t+1}-y^*_{1,t+1})+ \\ g_{2,t+1}(1-\tau_{t+1})^{-1}(c^*_{2,t+1}-y^*_{2,t+1}) \end{array}\right] - \dfrac{1}{g_{1,t+1}}\left[\begin{array}{c} g_{11,t+1}(c^*_{1,t+1}-y^*_{1,t+1})+ \\ g_{21,t+1}(1-\tau_{t+1})^{-1}(c^*_{2,t+1}-y^*_{2,t+1}) \end{array}\right]} \tag{35}
$$

To reach the conditions characterizing allocations in a Nash equilibrium, combine (32) and

(35) yields,

$$\frac{\begin{array}{c} C_t^{*-\sigma} g_{1,t}^* + \sigma C_t^{*-\sigma-1} g_{1,t}^* \left[\begin{array}{c} g_{1,t}^*(c_{1,t} - y_{1,t}) + \\ g_{2,t}^*(1-\tau_t^*)^{-1}(c_{2,t} - y_{2,t}) \end{array} \right] - \\ C_t^{*-\sigma} \left[\begin{array}{c} g_{11,t}^*(c_{1,t} - y_{1,t}) + \\ g_{21,t}^*(1-\tau_t^*)^{-1}(c_{2,t} - y_{2,t}) \end{array} \right] \end{array}}{\begin{array}{c} C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t} \left[\begin{array}{c} g_{1,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array} \right] - \\ C_t^{-\sigma} \left[\begin{array}{c} g_{11,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{21,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array} \right] \end{array}} = \alpha_{1,0},$$

Similarly, combining (34) and (33) yields,

$$\frac{\begin{array}{c} C_t^{*-\sigma} g_{2,t}^*(1-\tau_t^*)^{-1} + \sigma C_t^{*-\sigma-1} g_{2,t}^* \left[\begin{array}{c} g_{1,t}^*(c_{1,t} - y_{1,t}) + \\ g_{2,t}^*(1-\tau_t^*)^{-1}(c_{2,t} - y_{2,t}) \end{array} \right] - \\ C_t^{*-\sigma} \left[\begin{array}{c} g_{21,t}^*(c_{1,t} - y_{1,t}) + \\ g_{22,t}^*(1-\tau_t^*)^{-1}(c_{2,t} - y_{2,t}) \end{array} \right] (1-\tau_t)^{-1} \end{array}}{\begin{array}{c} C_t^{-\sigma} g_{2,t}(1-\tau_t)^{-1} + \sigma C_t^{-\sigma-1} g_{2,t} \left[\begin{array}{c} g_{1,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{2,t}(1-\tau_t)^{-1}(c_{2,t}^* - y_{2,t}^*) \end{array} \right] - \\ C_t^{-\sigma} \left[\begin{array}{c} g_{12,t}(c_{1,t}^* - y_{1,t}^*) + \\ g_{22,t}(1-\tau_t)(c_{2,t}^* - y_{2,t}^*) \end{array} \right] (1-\tau_t^*)^{-1} \end{array}} \frac{1-\tau_t^*}{1-\tau_t} = \alpha_{2,0},$$

The constant $\alpha_{1,0}$ is given by,

$$\alpha_{1,0} = \frac{\begin{array}{c} C_0^{*-\sigma} g_{1,0}^* + \sigma C_0^{*-\sigma-1} g_{1,0}^* \left[\begin{array}{c} g_{1,0}^*(c_{1,0} - y_{1,0}) + \\ g_{2,0}^*(1-\tau_0^*)^{-1}(c_{2,0} - y_{2,0}) \end{array} \right] - \\ C_0^{*-\sigma} \left[\begin{array}{c} g_{11,0}^*(c_{1,0} - y_{1,0}) + \\ g_{21,0}^*(1-\tau_0^*)^{-1}(c_{2,0} - y_{2,0}) \end{array} \right] \end{array}}{\begin{array}{c} C_0^{-\sigma} g_{1,0} + \sigma C_0^{-\sigma-1} g_{1,0} \left[\begin{array}{c} g_{1,0}(c_{1,0}^* - y_{1,0}^*) + \\ g_{2,0}(1-\tau_0)^{-1}(c_{2,0}^* - y_{2,0}^*) \end{array} \right] - \\ C_0^{-\sigma} \left[\begin{array}{c} g_{11,0}(c_{1,0}^* - y_{1,0}^*) + \\ g_{21,0}(1-\tau_0)^{-1}(c_{2,0}^* - y_{2,0}^*) \end{array} \right] \end{array}}$$

and $\alpha_{2,0}$ is given by,

$$\alpha_{2,0} = \frac{1-\tau_0^*}{1-\tau_0} \frac{\begin{array}{c} C_0^{*-\sigma} g_{2,0}^* (1-\tau_0^*)^{-1} + \sigma C_0^{*-\sigma-1} g_{2,0}^* \left[\begin{array}{c} g_{1,0}^*(c_{1,0}-y_{1,0})+ \\ g_{2,0}^*(1-\tau_0^*)^{-1}(c_{2,0}-y_{2,0}) \end{array} \right] - \\ C_0^{*-\sigma} \left[\begin{array}{c} g_{12,0}^*(c_{1,0}-y_{1,0})+ \\ g_{22,0}^*(1-\tau_0^*)^{-1}(c_{2,0}-y_{2,0}) \end{array} \right] (1-\tau_0)^{-1} \end{array}}{\begin{array}{c} C_0^{-\sigma} g_{2,0} (1-\tau_t)^{-1} + \sigma C_0^{-\sigma-1} g_{2,0} \left[\begin{array}{c} g_{1,0}(c_{1,0}^*-y_{1,0}^*)+ \\ g_{2,0}(1-\tau_0)^{-1}(c_{2,0}^*-y_{2,0}^*) \end{array} \right] - \\ C_0^{-\sigma} \left[\begin{array}{c} g_{12,0}(c_{1,0}^*-y_{1,0}^*)+ \\ g_{22,0}(1-\tau_0)(c_{2,0}^*-y_{2,0}^*) \end{array} \right] (1-\tau_0^*)^{-1} \end{array}}$$

Finally, substituting out τ_t and τ_t^* yields,

$$\frac{\begin{array}{c} C_t^{*-\sigma} g_{1,t}^* + \sigma C_t^{*-\sigma-1} g_{1,t}^* \left[\begin{array}{c} g_{1,t}^*(c_{1,t}-y_{1,t})+ \\ g_{1,t}^* S_t (c_{2,t}-y_{2,t}) \end{array} \right] - \\ C_t^{*-\sigma} \left[\begin{array}{c} g_{11,t}^*(c_{1,t}-y_{1,t})+ \\ g_{21,t}^* \frac{g_{1,t}^*}{g_{2,t}^*} S_t (c_{2,t}-y_{2,t}) \end{array} \right] \end{array}}{\begin{array}{c} C_t^{-\sigma} g_{1,t} + \sigma C_t^{-\sigma-1} g_{1,t} \left[\begin{array}{c} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{1,t} S_t (c_{2,t}^*-y_{2,t}^*) \end{array} \right] - \\ C_t^{-\sigma} \left[\begin{array}{c} g_{11,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{21,t} \frac{g_{1,t}}{g_{2,t}} S_t (c_{2,t}^*-y_{2,t}^*) \end{array} \right] \end{array}} = \alpha_{1,0},$$

and,

$$\frac{\begin{array}{c} C_t^{*-\sigma} g_{2,t}^* + \sigma C_t^{*-\sigma-1} g_{2,t}^* \left[\begin{array}{c} g_1^*(c_{1,t}-y_{1,t})+ \\ g_{1,t}^* S_t (c_{2,t}-y_{2,t}) \end{array} \right] - \\ C_t^{*-\sigma} \left[\begin{array}{c} g_{12,t}^*(c_{1,t}-y_{1,t})+ \\ g_{22,t}^* \frac{g_{1,t}^*}{g_{2,t}^*} S_t (c_{2,t}-y_{2,t}) \end{array} \right] \end{array}}{\begin{array}{c} C_t^{-\sigma} g_{2,t} + \sigma C_t^{-\sigma-1} g_{1,t} \left[\begin{array}{c} g_{1,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{1,t} S_t (c_{2,t}^*-y_{2,t}^*) \end{array} \right] - \\ C_t^{-\sigma} \left[\begin{array}{c} g_{12,t}(c_{1,t}^*-y_{1,t}^*)+ \\ g_{22,t} \frac{g_{1,t}}{g_{2,t}} S_t (c_{2,t}^*-y_{2,t}^*) \end{array} \right] \end{array}} \frac{g_{1,t}}{g_{2,t}} \frac{g_{2,t}^*}{g_{1,t}^*} = \alpha_{2,0},$$

which complete the proof. □

To derive the optimal tariffs, divide the Foreign by the Home optimality condition for good 1 and use the Euler to substitute in the Home optimal tariff on the LHS. Use the foreign Euler to substitute out the Foreign optimal tariff:

$$1-\tau_t=\frac{1}{S_t}\frac{\begin{array}{c}C_t^{*-\sigma}g_{1,t}^*S_t+\sigma C_t^{*-\sigma-1}g_{2,t}^*\left[\begin{array}{c}g_{1,t}^*(c_{1,t}-y_{1,t})+\\ g_{1,t}^*S_t(c_{2,t}-y_{2,t})\end{array}\right]-\\ C_t^{*-\sigma}\left[\begin{array}{c}g_{12,t}^*(c_{1,t}-y_{1,t})+\\ g_{22,t}^*\frac{g_{1,t}^*}{g_{2,t}^*}S_t(c_{2,t}-y_{2,t})\end{array}\right]\end{array}}{\begin{array}{c}C_t^{*-\sigma}g_{1,t}^*+\sigma C_t^{*-\sigma-1}g_{1,t}^*\left[\begin{array}{c}g_{1,t}^*(c_{1,t}-y_{1,t})+\\ g_{1,t}^*S_t(c_{2,t}-y_{2,t})\end{array}\right]-\\ C_t^{*-\sigma}\left[\begin{array}{c}g_{11,t}^*(c_{1,t}-y_{1,t})+\\ g_{21,t}^*\frac{g_{1,t}^*}{g_{2,t}^*}S_t(c_{2,t}-y_{2,t})\end{array}\right]\end{array}}$$

and,

$$1-\tau_t^*=\frac{1}{S_t}\frac{\begin{array}{c}C_t^{-\sigma}g_{1,t}S_t+\sigma C_t^{-\sigma-1}g_{2,t}\left[\begin{array}{c}g_{1,t}(c_{1,t}^*-y_{1,t}^*)+\\ g_{1,t}S_t(c_{2,t}^*-y_{2,t}^*)\end{array}\right]-\\ C_t^{-\sigma}\left[\begin{array}{c}g_{12,t}(c_{1,t}^*-y_{1,t}^*)+\\ g_{22,t}\frac{g_{1,t}}{g_{2,t}}S_t(c_{2,t}^*-y_{2,t}^*)\end{array}\right]\end{array}}{\begin{array}{c}C_t^{-\sigma}g_{1,t}+\sigma C_t^{-\sigma-1}g_{1,t}\left[\begin{array}{c}g_{1,t}(c_{1,t}^*-y_{1,t}^*)+\\ g_{1,t}S_t(c_{2,t}^*-y_{2,t}^*)\end{array}\right]-\\ C_t^{-\sigma}\left[\begin{array}{c}g_{11,t}(c_{1,t}^*-y_{1,t}^*)+\\ g_{21,t}\frac{g_{1,t}}{g_{2,t}}S_t(c_{2,t}^*-y_{2,t}^*)\end{array}\right]\end{array}}$$

abroad.

B.2 Nash equilibrium with FTA

Consider the Nash problem when a FTA is in place for both Home and Foreign planners. If a FTA is in place, $\boldsymbol{\tau}_t,\boldsymbol{\tau}_t^*=1$, the Home planner chooses C_t and the Foreign C_t^* and $\mathbf{c}(C_t),\mathbf{c}^*(C_t^*)$ are given by Lemma 1. Then the allocations C_t, C_t^* in a Nash equilibrium must satisfy,

$$\frac{\begin{array}{c}C_t^{*-\sigma}(g_{1,t}^*c_{1,t}'(C_t)+g_{2,t}^*c_{2,t}'(C_t))+\\ \sigma C_t^{*\ -\sigma-1}C_t^{*\prime}(C_t)\left[g_{1,t}^*(c_{1,t}-y_{1,t})+g_{2,t}^*(c_{2,t}-y_{2,t})\right]+\\ C_t^{*-\sigma}\left[(g_{11,t}^*+g_{21,t}^*)c_{1,t}'(C_t)(c_{1,t}-y_{1,t})+(g_{12,t}^*+g_{22,t}^*)c_{2,t}'(C_t)(c_{2,t}-y_{2,t})\right]\end{array}}{\begin{array}{c}C_t^{-\sigma}(g_{1,t}c_{1,t}^{*\prime}(C_t)+g_{2,t}c_{2,t}^{*\prime}(C_t))+\\ \sigma C_t^{-\sigma-1}C_t'(C_t^*)\left[g_{1,t}(c_{1,t}^*-y_{1,t}^*)+g_{2,t}(c_{2,t}^*-y_{2,t}^*)\right]+\\ C_t^{-\sigma}\left[(g_{11,t}+g_{21,t})c_{1,t}^{*\prime}(C_t^*)(c_{1,t}^*-y_{1,t}^*)+(g_{12,t}+g_{22,t})c_{2,t}^{*\prime}(C_t^*)(c_{2,t}^*-y_{2,t}^*)\right]\end{array}}=\alpha_0^{FTA} \tag{36}$$

Optimal capital controls levied by the home country are given by,

$$1-\theta_t = \frac{\begin{gathered}(g^*_{1,t}c'_{1,t}(C_t) + g^*_{2,t}c'_{2,t}(C_t))+ \\ \sigma C_t^{*\ -1} C_t^{*\prime}(C_t)\left[g^*_{1,t}(c_{1,t}-y_{1,t}) + g^*_{2,t}(c_{2,}-y_{2,t})\right] + \\ \left[\begin{array}{c}(g^*_{11,t}+g^*_{21,t})c'_{1,t}(C_t)(c_{1,t}-y_{1,t}) + (g^*_{12,t}+ \\ g^*_{22,t})c'_{2,t}(C_t)(c_{2,t}-y_{2,t})\end{array}\right]\end{gathered}}{\begin{gathered}(g^*_{1,t+1}c'_{1,t+1}(C_{t+1}) + g^*_{2,t+1}c'_{2,t+1}(C_{t+1}))+ \\ \sigma C_{t+1}^{*\ -1} C_{t+1}^{*\prime}(C_{t+1})\left[g^*_{1,t+1}(c_{1,t+1}-y_{1,t+1}) + g^*_{2,t+1}(c_{2,t+1}-y_{2,t+1})\right] + \\ \left[\begin{array}{c}(g^*_{11,t+1}+g^*_{21,t+1})c'_{1,t+1}(C_{t+1})(c_{1,t+1}-y_{1,t+1})+ \\ (g^*_{12,t+1}+g^*_{22,t+1})c'_{2,t+1}(C_{t+1})(c_{2,t+1}-y_{2,t+1})\end{array}\right]\end{gathered}} \tag{37}$$

with an analogous condition for the foreign.

C Cooperative Allocation

C.1 Proof to Proposition 6

When a FTA is in place, the optimal cooperative allocation satisfies,

$$u'(g(\mathbf{c}_t)) + \kappa u'(g(\mathbf{c}_t^*))\frac{dC^*}{dC} = 0 \tag{38}$$

where $\frac{dC_t^*}{dC_t} = -\frac{P_t}{P_t^*}$, yielding the decentralised risk sharing condition (**??**) with $\kappa = \frac{u'(g(\mathbf{c}_{t-1}))}{u'(g(\mathbf{c}_{t-1}^*))}\frac{P_{t-1}^*}{P_{t-1}}$ implying $\theta_t = 0$. Relaxing the FTA does not change the optimal allocation (since goods taxes are zero at the optimal). With FTA, first order condition follows straightforwardly once by substituting $\frac{dC_t^*}{dC_t} = -\frac{P_t}{P_t^*}$.

Relaxing the FTA, we get two first order conditions,

$$u'(g(\mathbf{c}_t))g_1 + \kappa u'(g(\mathbf{c}_t^*))g_1^*\frac{dc_1^*}{dc_1} = 0, \tag{39}$$

$$u'(g(\mathbf{c}_t))g_2 + \kappa u'(g(\mathbf{c}_t^*))g_2^*\frac{dc_2^*}{dc_2} = 0 \tag{40}$$

Note that $g_1/g_1^* = \frac{dC}{dc_1}\frac{dc_1^*}{dC^*} = \frac{dC}{dC^*}\frac{dc_1^*}{dc_1} = -\frac{dC}{dC^*}$, therefore both of the above conditions imply (38), as in the FTA case. □

D Computational Appendix

We consider a setup without uncertainty. Our model is therefore simply solved as the following system of equations. Consider the one good case.

The model has $T+2$ FOCs: $T+1$ w.r.t. c_t and 1 with respect to the multiplier μ_0

$$c_t^{-\sigma} = \mu_0 \left[c_t^{*-\sigma} - \sigma c_t^{*-\sigma-1} \left(y_t - c_t \right) \right] \quad \text{for } t = 0, 1, ..., T$$

$$0 = \sum_{t=0}^{T} \beta^t c_t^{*-\sigma} \left(y_t - c_t \right)$$

In addition we have market clearing in each period:

$$c_t + c_t^* = y_t + y_t^* \quad \text{for } t = 0, 1, ..., T$$

Thus we have $2T+3$ equations in $2T+3$ unknowns: $\{c_t\}_{t=0}^{T}$, $\{c_t^*\}_{t=0}^{T}$ and μ_0.

Using vector notation, taking $\mathbf{y} = [y_0, y_1, ..., y_T]'$ and $\mathbf{y}^* = [y_0^*, y_1^*, ..., y_T^*]'$ as inputs, then solving the following system of equations,

$$\mathbf{c}^{-\sigma} = \mu_0 \left[\mathbf{c}^{*-\sigma} - \sigma \mathbf{c}^{*-\sigma-1} (\mathbf{y} - \mathbf{c}) \right]$$

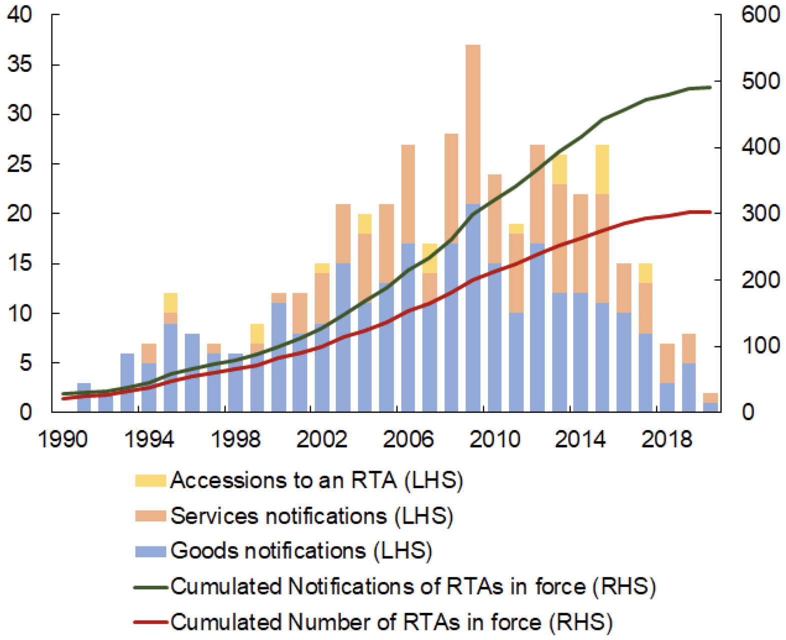
$$\mathbf{c} + \mathbf{c}^* = \mathbf{y} + \mathbf{y}^*$$

$$0 = \mathbf{x}'(\mathbf{y} - \mathbf{c}), \quad \text{where } \mathbf{x} = \mathbf{b} \odot \mathbf{c}^{*-\sigma}$$

where $\mathbf{b} = [\beta^0, \beta^1, ..., \beta^T]'$.

E World Trade Organisation Data

Figure 14: Decline in Regional Trade Agreements in Recent Years



Notes: Number of regional trade agreements per year (bars, left-hand axis). Cumulative number of regional trade agreements, with cumulation beginning in 1948 (lines, right-hand axis). *Source*: WTO.