Capital Controls and Free-Trade Agreements

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The views expressed here do not necessarily reflect the position of the Bank of England.

Trade & Financial Openness Not Always Aligned

- · Bretton Woods: Free trade promoted, but capital controls widely used
- Post-Bretton Woods: Increased trade and more financial openness
- Recent Years:
 - Growing protectionism (China-US trade war; Brexit; export restrictions post-Covid)
 - More sanguine views on capital controls (IMF's Integrated Policy Framework) and increasing 'macroprudential FX regulation'

How does conduct of capital controls change in a world with less free trade?

Trade & Financial Openness Not Always Aligned



Source: Ahir, Bloom and Furceri (2018)

Source: Ahnert, Forbes, Friedrich and Reinhardt (2021)

Lloyd and Marin (BoE and UC Davis)

Capital Controls and Free-Trade Agreements

Two-country endowment economy with **terms-of-trade externalities (ToT)**

[Geanakoplos and Polemarchakis, 1986]

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How does size in financial (goods) markets affect the conduct of trade (financial) policy?

Related Literature

Non-Exhaustive

- **Capital Controls**: Costinot, Lorenzoni and Werning (2014); Bianchi (2011); Farhi and Werning (2016); Bianchi and Lorenzoni (2021); ...
 - · Over-/under-borrowing due to size externalities
- **Trade Policy**: Lerner (1936); Broda, Limão and Weinstein (2008); Costinot and Werning (2019); Corsetti and Bergin (2020); Caliendo, Feenstra, Romalis and Taylor (2021); ...
 - · Optimal trade tariffs usually derived without trade in assets
- **Integrated Policy Analysis**: Ostry et al. (2010); Basu et al. (2020); Auray, Devereux and Eyquem (2020), Jeanne (2022)

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 - · With FTA, capital controls strike compromise across the two margins
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- 4. Domestic **welfare** gains more than offset by losses abroad from **spillovers**

Model-in-a-Slide

- · Countries: Home H and Foreign F (*). Goods: 1 and 2.
- *Time*: $t = 0, 1, ..., \infty$. No uncertainty. Zero assets at t = 0.
- Preferences: $U_0 = \sum_{t=0}^{\infty} \beta^t u(C_t)$, where $\beta \in (0, 1)$, C_t aggregate consumption, and $u(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$ with $\sigma > 0$
- \cdot Households consume both goods 1 and 2:

$$C_{t} \equiv g(\mathbf{c}_{t}) = \left[\alpha_{1}^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha_{1})^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}$$

where $\mathbf{c}_t = [c_{1,t}, c_{2,t}]$, $\alpha_1 \in (0.5, 1]$, and $\phi > 0$ is 'elasticity of trade'

 \cdot Exogenous country endowments: $\mathbf{y}_t^{(*)} = [y_{1,t}^{(*)}, y_{2,t}^{(*)}]$

· Real Exchange Rate $Q = \frac{P^*}{P}$ and Terms of Trade $S = \frac{p_2}{p_1}$

Key Friction: Terms-of-Trade Externality

- · Large countries affect prices when making consumption decisions: $\frac{dC^*}{dC} \neq 0$, $\frac{dc_1^*}{dc_1} \neq 0$
- Planner has incentive to exercise monopoly/monopsony power in goods markets both statically and dynamically [Costinot, Lorenzoni and Werning, 2014]
 - \cdot Inter-temporal: tax capital inflows when borrowing is relatively high $(R\downarrow)$
 - · Intra-temporal: subsidise c_1 when selling relative more of good 1 (p_1 \uparrow)

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- + Terms of trade underpin important frictions in the literature:
 - · Borrowing constraints [Bianchi, 2011]
 - · ELB and other monetary constraints [Farhi and Werning, 2016]
 - · Market segmentation and limits to arbitrage [Fanelli and Straub, 2018; Marin, 2020]
 - · Heterogeneous cons. baskets [Cravino and Levchenko, 2017, Fanelli and Straub, 2018]

Incentives

Optimal Unilateral Policy: Setup

- Home country sets capital flow taxes to maximise welfare of domestic representative agent
- **Primal Approach**: Home planner chooses $\{c_t\}$ in order to maximise welfare of representative agent U_0 , taking as given:
 - 1. Foreign consumer maximising U_0^* subject to intertemporal budget constraint

$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) \le 0$$

where $\mathbf{p}_t = [p_{1,t}, p_{2,t}]$ is vector of world prices

2. Goods market clearing

$$y_{1,t} + y_{1,t}^* = c_{1,t} + c_{1,t}^*$$
 $y_{2,t} + y_{2,t}^* = c_{2,t} + c_{2,t}^*$

Foreign Maximisation

Optimal Allocations with FTA

With FTA [Costinot, Lorenzoni, Werning, 2014]

- \cdot Choose C given FTA
- \cdot 1 FOC + 1 Instrument

$$\underbrace{\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}C}}_{FOC=0} = \frac{\partial\mathcal{L}}{\partial c_1} \underbrace{c_1'(C)}_{FTA} + \frac{\partial\mathcal{L}}{\partial c_2} \underbrace{c_2'(C)}_{FTA}$$

$$\cdot u'(C_t) = \mu \mathcal{M} \mathcal{C}_t^{FTA}$$

 $\Rightarrow \text{ Trade off } \tfrac{\partial \mathcal{L}}{\partial c_1} \text{ and } \tfrac{\partial \mathcal{L}}{\partial c_2} \text{, with } c_1 \text{ and } c_2$ constrained by FTA

Optimal Allocations with and without FTA

With FTA [Costinot, Lorenzoni, Werning, 2014]

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- \cdot 1 FOC + 1 Instrument



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Without FTA

- $\star\,$ Choose c_1 and c_2 , given $C=g(\mathbf{c})$
- \star 2 FOCs + 2 Instruments

$$\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}C} = \underbrace{\frac{\partial\mathcal{L}}{\partial c_1}}_{FOC=0} c_1'(C) + \underbrace{\frac{\partial\mathcal{L}}{\partial c_2}}_{FOC=0} c_2'(C)$$

$$\star \ u'(c_{i,t}) = \mu \mathcal{M} \mathcal{C}_{i,t}^{nFTA}$$
 for $i = 1, 2$

 $\Rightarrow C$ optimal for Home planner and can violate FTA constraint

Relaxing FTA Can Increase Home Welfare

Proposition

Suppose preferences are symmetric, $\alpha_1 = \alpha_2^*$ and $\alpha_2 = \alpha_1^*$, then in general: $C^{nFTA} > C^{FTA}$

- (i) When $C^{nFTA} > C^{FTA}$: optimal nFTA allocation violates Pareto frontier
- (ii) $C^{nFTA} = C^{FTA}$ when endowments are proportional to preferences: $y_1 \propto \alpha_1$, $y_2 \propto \alpha_2$, $y_1^* \propto \alpha_1^*$ and $y_2^* \propto \alpha_2^*$

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Intuition

- Departing from FTA, planner can manipulate relative goods prices favourably (as long as endowments are not already proportional to preferences)
- With two instruments, no need to strike compromise across inter- and intra-temporal margins

Visual Intuition: Allocations with FTA

Feasible combinations of $\{c_1, c_2\}$ given F

FTA \Rightarrow *H* cannot impose good-specific taxes \Rightarrow ($\mathbf{c}_t, \mathbf{c}_t^*$) is Pareto efficient



Note: $\phi = 1.5$, $\alpha_1 = \alpha_2^* = 0.75$, $y_1 = \alpha_1 \pm 0.25$, $y_2 = \alpha_2$, $y_i^* = 1 - y_i$ for i = 1, 2.

Visual Intuition: Relaxing FTA

Feasible combinations of $\{c_1, c_2\}$ given F

No FTA \Rightarrow H sets optimal import tariffs \Rightarrow unconstrained by Pareto frontier



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Instruments

- · Implement allocation with capital inflow tax $\theta < 0$ and import tariff $\tau > 0$
- Equalise steady states (via exo. tax) to focus on welfare gains along transition

$$\sigma=2$$
, $eta=0.96$, $\phi=1.5$, $ho=0.8$, $lpha_1=lpha_2^*=0.6$ and $\overline{y}_1=\overline{y}_2^*=0.8$

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Inter-temporal incentives:

- H endowment low today \Rightarrow Incentive to borrow
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Intra-temporal incentives:

Good 1 endowment low today \Rightarrow Sell less to Foreign

- \Rightarrow Incentive to subsidise imports of good 2 to $\downarrow p_1$
- \Rightarrow Will also dis-incentivise borrowing $\downarrow R$

Growing Endowment of Home-Bias Good



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Capital Controls and Free-Trade Agreements

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- \Rightarrow Reduced borrowing will $\downarrow p_1$

Intra-temporal incentives:

Good 1 endowment relatively high today

- \Rightarrow Sell more to Foreign
- \Rightarrow Incentive to tax imports of good 1 to $\uparrow p_1$
- \Rightarrow But this will incentivise borrowing $\uparrow R$

Growing Endowment of 'Foreign' Good



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Interplay Between Capital Controls and Tariffs

#1: Growing Endowment of Home-Bias Good 1: inter- and intra-incentives aligned

- \cdot Capital-inflow tax heta < 0 drives down borrowing $\downarrow R$ and price of good 1 $\downarrow p_1$
- + Subsidy on imports drives down p_1 and reduces need to borrow $\downarrow R$
- ⇒ Without FTA: higher capital-inflow tax to exploit departures from Pareto frontier

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#2: Growing Endowment of 'Foreign' Good 2: inter- and intra-incentives mis-aligned

- \cdot Capital-inflow tax heta < 0 drives down borrowing $\downarrow R$ and price of good 1 $\downarrow p_1$
- \cdot Tax on imports bids up p_1 and increases incentives to borrow $\uparrow R$
- \Rightarrow Without FTA: optimal tariffs result in lower capital-flow taxes

Country Size and Financial Frictions

Potential critique:

- 1. Few countries are large or strategic enough to manipulate ToT
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But, within simple static model of financial frictions, SOE ($C^* \rightarrow Y_2$) still has incentives to manipulate ToT, and especially so when the frictions are large: [Bianchi, 2011]

$$\sum_{s} \mathbf{p}(s) \cdot \mathbf{a}(s) + \underbrace{\phi(\mathbf{a}(s), Q(s))}_{\text{cost of borrowing}} \leq 0$$
 (ex ante)
$$\mathbf{p}(s) \cdot [\mathbf{c}(s) - \mathbf{y}(s)] \leq \mathbf{p}(s) \cdot \mathbf{a}(s)$$
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Financial cost lower if **either** borrowing lower $(C \downarrow)$ or appreciation $(c_1 \uparrow)$ \Rightarrow Import tariff can substitute for capital inflow tax in SOE

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Capital Controls and Free-Trade Agreements

Capital-Control and Tariff Wars: Welfare and Spillovers

Spillovers Dwarf Domestic Gains, Especially with Tariffs

Proposition

- * Global Cooperative Optimum: No intervention
- \star Unilateral: Welfare gain in H small relative to loss in F, esp. without FTA
- \star Nash: Larger aggregate losses with capital control and tariff wars

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Table: Welfare Gains and Spillovers: expressed in terms of % cons. eq.

Experiment 1	H	F	Global $\sum_{H,F}$
FTA (Unilateral)	+0.13	-0.23	-0.05
without FTA (Unilateral)	+0.22	-0.27	-0.03
with FTA (Nash)	-0.07	-0.07	-0.07
without FTA (Nash)	-1.71	-1.58	-1.65

Conclusion

Cannot separate discussions around capital controls and trade protectionism

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 - 2. When inter-/intra incentives **mis-**aligned capital inflow tax and tariff substitutes

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- Policy prescriptions for trade and financial openness interlinked
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 - 2. When inter-/intra incentives **mis-**aligned capital inflow tax and tariff substitutes
- ► In SOE with financial frictions, tariffs act as a **substitute** for capital-inflow taxes
- > Domestic gains from capital controls and tariffs are small, but spillovers large

Appendix

Foreign Consumer Maximisation

· Representative Foreign consumer problem:

$$\max_{\{\mathbf{c}_t^*\}} \quad U_0^* = \sum_{t=0}^\infty \beta^t U^*(C_t^*) \quad \text{ s.t. } \quad \sum_{t=0}^\infty \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) \le 0$$

 \Rightarrow Optimality conditions:

$$\beta^t U^{*'}(C_t^*) \nabla g_c^*(\mathbf{c}_t^*) = \lambda^* \mathbf{p}_t$$
$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) = 0$$

where
$$\nabla g_c^*(\mathbf{c}_t) = \left[\frac{\partial g^*(\mathbf{c}_t^*)}{\partial c_{1,t}^*}, \frac{\partial g^*(\mathbf{c}_t^*)}{\partial c_{2,t}^*}\right]$$



Unilateral Home Planning Problem

With FTA [Costinot, Lorenzoni & Werning, 2014]

$$\max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t u(C_t)$$
(P-FTA)
s.t.
$$\sum_{t=0}^{\infty} \rho(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0$$
(IC)
$$\mathbf{c}_t = \mathbf{c}_t(C_t), \quad \mathbf{c}_t^* = \mathbf{c}_t^*(C_t)$$
(FTA)

where $\rho(C_t) \equiv \beta^t u^{*'}(C_t^*) \nabla g_c^*(\mathbf{c}_t^*(C_t))$



Unilateral Home Planning Problem

$$\max_{\{\mathbf{c}_t\}} \sum_{t=0}^{\infty} \beta^t u(C_t)$$
 (P-nFTA)
s.t.
$$\sum_{t=0}^{\infty} \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0$$
 (IC)
$$C_t = g(\mathbf{c}_t)$$
 (nFTA)

where $\rho(C_t) \equiv \beta^t u^{*\prime}(C_t^*) \nabla g_c^*(\mathbf{c}_t^*(C_t))$

▶ Back