Heterogeneity, Bubbles and Monetary Policy

Jacopo Bonchi[1] Salvatore Nisticó[2]

[1]LUISS Guido Carli

[2]Sapienza University of Rome

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Motivation: MP, Bubbles and Wealth Inequality

- **Central Banks** (CBs) concerned about distributional aspects
- **BUT** "bubbles" in asset prices can redistribute wealth
- **Research question(s)**
  1. what are the implications of bubble-driven fluctuations for wealth and consumption inequality?
  2. what's the optimal response to bubble-driven fluctuations if the CB takes into account distributional aspects?

What We Do

- We build a **dynamic new Keynesian model** with
  - **Discontinuous Asset-Market Participation** (DAMP)
  - **Endogenous labor supply**
  - **Efficient BGP** (no monopolistic distortions)
- We discuss the **implications** of these assumptions **for existence and magnitude of rational bubbles** (e.g. compared to Galí (AEJ:Macro 2021))
- We derive a **quadratic welfare-based loss function** for the CB, highlighting **the role of wealth inequality** (= consumption dispersion)
- We discuss the ***normative* implications** of **rational bubbles for monetary policy** in a LQ framework

BGP

Related Literature

- **Rational Bubbles in OLG models**

  Samuelson (1958), Tirole (1985), Galí (2014, 2021), Miao et al. (2019)

- **Rational bubbles in infinite horizon models**

  Kocherlakota (1992), Miao and Wang (2012, 2014, 2018) and Hirano and Yanagawa (2017)

- **TANK models**

  Cúrdia and Woodford (2010, 2011, 2016), Bilbiie (2018, 2020), Bilbiie and Ragot (2021)

Our Economy

Extended DAMP economy (Nisticò, JEEA 2016) Graph

Demand-side

- **3 Layers of heterogeneity** Structure Equations
  1. asset-market participation: **particip.** (ϑ) vs **hand-to-mouth** ($1-\vartheta$)
  2. employment status: **employed** (α) vs **unemployed** ($1-\alpha$)
  3. longevity in asset markets: **incumbent** (γ) vs **newcomers** ($1-\gamma$)

- probability of transition out of asset markets: $1-\gamma$
- probability of transition out of employment: $1-\nu$
- **GHH preferences** consistent with a BGP:

$$U_{t|s}^{j} = \log \widetilde{C}_{t|s}^{j},$$

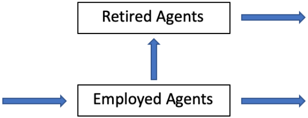
with

$$\widetilde{C}_{t|s}^{j} \equiv C_{t|s}^{j} - V(N_{t|s}^{j}) \qquad V(N_{t|s}^{j}) \equiv \frac{\delta \Gamma^{t}}{1+\varphi} \left(N_{t|s}^{j}\right)^{1+\varphi}$$

and $j \in \mathcal{T} \equiv \{pe, pu, re, ru\}$, $s \in (-\infty, t]$.

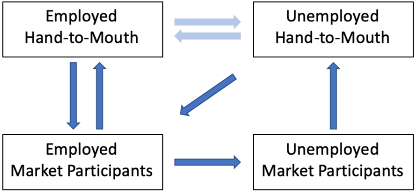
Heterogeneity in Our Economy

Stochastic transition in **Galí (2021)**



Stochastic transition in **Nisticò (2016)**

Stochastic transition in **our economy**



Back

Our Economy (2)

Supply-side: (almost) standard New Keynesian

- Exogenous productivity growth (BGP): $\Gamma \equiv (1+g) \geq 1$
- probability of firm's survival to next period: $\nu\gamma$
- Monopolistic competition and Calvo pricing
- **Employment subsidy** $\tau^F \Rightarrow$ distortions in BGP: $\varpi \equiv 1 - \frac{1}{(1+\mu)(1-\tau^F)}$

Government Govt BC Aggregation

- **Tax Revenues**
  - lump-sum taxes on employed agents
  - dividend tax τ^D
- **Fiscal Spending**
  - employment subsidy to firms
  - transfers to hand-to-mouth agents/rule-of-thumbers (RoT)
  - unemployment benefit for unemployed RoTs
- **Welfare-maximizing CB**

Balanced-Growth Paths

- **Our Baseline**: Back
  - **DAMP**: positive share of hand-to-mouth agents ($\vartheta < 1$)
  - **endogenous labor supply**: $\varphi < \infty$
  - **efficient BGP** (for normative analysis): $\varpi = 0$

$$q^B = \frac{\vartheta\varphi}{1+\varphi}\frac{\gamma(\beta R - \nu)}{(1-\beta\gamma)(R-\gamma\nu)}$$

where $R \equiv \frac{1+r}{1+g}$

  - multiplicity of BGP equilibria (one degree of freedom)

- **Galí (2021)** (nested in our model, $\vartheta = 1$ and $\varphi \to \infty$)

$$q^B = \frac{\gamma(\beta - \Gamma\Lambda\nu)}{(1-\beta\gamma)(1-\Gamma\Lambda\gamma\nu)}$$

Conditions for existence:

1. non-negative new bubbles: $R \leq 1 \quad \Rightarrow r < g$
2. non-negative aggregate bubbles: $R \geq \nu/\beta \quad \Rightarrow \nu < \beta$

Result 1

- **Finite planning horizon isomorphic to finite lives but smaller aggregate bubbles** compared to Galí (2021) due to
  - **DAMP**
  - **Endogenous labor supply**
  - **NO Monopolistic distortions**

Proof

Linear Model

$$\widehat{x}_t = \Phi E_t \widehat{x}_{t+1} - \frac{\varphi}{1+\varphi} \frac{\Phi}{1-\beta\gamma\Phi} \widehat{r}_t + \frac{1-\beta\gamma}{\vartheta\beta\gamma} \widehat{q}_t^B \tag{1}$$

$$\widehat{y}_t = \Theta \left(\frac{\widehat{q}_t^B}{\vartheta} + \widehat{x}_t \right) \tag{2}$$

$$\widehat{q}_t^B = \frac{\beta}{\nu} \Phi E_t \widehat{b}_{t+1} - q^B \widehat{r}_t \tag{3}$$

$$\widehat{q}_t^B = \widehat{b}_t + \widehat{u}_t \tag{4}$$

$$\widehat{\pi}_t = \beta\gamma\Phi E_t \widehat{\pi}_{t+1} + \kappa \widehat{y}_t, \tag{5}$$

in which $\widehat{r}_t \equiv \widehat{\imath}_t - E_t \widehat{\pi}_{t+1}$ and

$$\Phi \equiv \frac{\nu\Gamma\Lambda}{\beta} \qquad \tau \equiv \frac{\tau^D(\varphi - \mu)}{(1-\vartheta)(1+\mu)}$$

$$\Theta \equiv \frac{\vartheta(1-\beta\gamma)}{(1-\vartheta)(\tau-\varphi)} \qquad \kappa \equiv \varphi \frac{(1-\theta)(1-\gamma\nu\Gamma\Lambda\theta)}{\theta}$$

Linear Model (2)

- **Equations:** Result 4

$$\widehat{x}_t = \Phi E_t \widehat{x}_{t+1} - \frac{\varphi}{1+\varphi} \frac{\Phi}{1-\beta\gamma\Phi} \widehat{r}_t + \frac{1-\beta\gamma}{\vartheta\beta\gamma} \widehat{q}_t^B$$

$$\widehat{y}_t = \Theta \left(\frac{\widehat{q}_t^B}{\vartheta} + \widehat{x}_t \right)$$

$$\widehat{q}_t^B = \frac{\beta}{\nu} \Phi E_t \widehat{b}_{t+1} - q^B \widehat{r}_t$$

$$\widehat{q}_t^B = \widehat{b}_t + \widehat{u}_t$$

$$\widehat{\pi}_t = \beta\gamma\Phi E_t \widehat{\pi}_{t+1} + \kappa \widehat{y}_t,$$

- **IS equation** with DAMP and efficient BGP ($\vartheta < 1$, $\varpi = 0$, $\tau^D > 0$):

$$\widehat{y}_t = \Phi E_t \widehat{y}_{t+1} - \frac{\varphi}{1+\varphi} \frac{\Theta\Phi}{1-\beta\gamma\Phi} \widehat{r}_t + \frac{\Theta}{\vartheta\beta\gamma} \left(\widehat{q}_t^B - \beta\gamma\Phi E_t \widehat{q}_{t+1}^B \right)$$

Result 2

- **Endogenous labor supply**:
  1. **implies a lower interest-rate elasticity of equilibrium rational bubbles** (= MP less effective in affecting the bubble size)
  2. **Indeterminacy of output gap w/o DAMP** Proof

The Welfare Criterion

$$\mathcal{L}_{t_0} = \frac{1}{2}\frac{\varepsilon\varphi}{\kappa}E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\widehat{\pi}_t^2 + \alpha_y\widehat{y}_t^2 + \alpha_\omega\widehat{\omega}_t^2\right)\right\}$$

where

$$\widehat{\omega}_t \equiv \frac{1}{\alpha\vartheta}\widehat{q}_t^B - \frac{\widehat{u}_t}{\vartheta(1-\gamma)} - \frac{1-\alpha}{\alpha}\widehat{x}_t = \frac{\gamma}{1-\beta\gamma}\left(\widehat{\widetilde{c}}^p_{t|in} - \widehat{\widetilde{c}}^p_{t|nc}\right)$$

and

$$\alpha_y \equiv \frac{\kappa}{\varphi\varepsilon}\left[\varphi + \left(\frac{1+\varphi}{\varphi}\right)\left(\frac{1-\vartheta}{\vartheta}\right)(\tau-\varphi)^2\right]$$

$$\alpha_\omega \equiv \frac{\kappa\vartheta}{\varepsilon\varphi}\left(\frac{1+\varphi}{\varphi}\right)\frac{(1-\gamma)(1-\beta\gamma)}{\gamma}$$

Derivations

Result 3

- **Endogenous policy trade-off** (= SIT NOT optimal) btw
  1. **inflation/output-gap stability**: $\widehat{\pi}_t$ and $\widehat{y}_t$
  2. **consumption dispersion**: $\widehat{\omega}_t$

Optimal Monetary Policy Problem

$$\max \mathcal{L}_{t_0} = \frac{1}{2}\frac{\varepsilon\varphi}{\kappa}E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\widehat{\pi}_t^2+\alpha_y\widehat{y}_t^2+\alpha_\omega\widehat{\omega}_t^2\right)\right\}$$

such that

$$\widehat{\omega}_t = \frac{1}{\alpha\vartheta}\widehat{q}_t^B - \frac{\widehat{u}_t}{\vartheta(1-\gamma)} - \frac{1-\alpha}{\alpha}\widehat{x}_t \tag{6}$$

$$\widehat{x}_t = \Phi E_t\widehat{x}_{t+1} - \frac{\varphi}{1+\varphi}\frac{\Phi}{1-\beta\gamma\Phi}\widehat{r}_t + \frac{1-\beta\gamma}{\vartheta\beta\gamma}\widehat{q}_t^B \tag{7}$$

$$\widehat{y}_t = \Theta\left(\frac{\widehat{q}_t^B}{\vartheta}+\widehat{x}_t\right) \tag{8}$$

$$\widehat{q}_t^B = \frac{\beta}{\nu}\Phi E_t\widehat{q}_{t+1}^B - q^B\widehat{r}_t - \frac{\beta}{\nu}\Phi E_t\widehat{u}_{t+1} \tag{9}$$

$$\widehat{\pi}_t = \beta\gamma\Phi E_t\widehat{\pi}_{t+1} + \kappa\widehat{y}_t. \tag{10}$$

Derivations

Optimal Monetary Policy Problem: Discretion

$$max \frac{1}{2}\left(\widehat{\pi}_t^2 + \alpha_y \widehat{y}_t^2 + \alpha_\omega \widehat{\omega}_t^2\right),$$

s.t.

$$\widehat{y}_t = \frac{\Theta}{\vartheta}\frac{1+\chi}{\chi}\widehat{q}_t^B + K_{y,t} \qquad \widehat{\pi}_t = \kappa \widehat{y}_t + K_{\pi,t}$$

where $K_{x,t}$ and $K_{\pi,t}$ collect expectational terms and $\chi \equiv (1-\Phi)\,\frac{\beta\gamma}{1-\beta\gamma}$

- **Targeting rule**

$$\Theta \alpha_y \widehat{y}_t + \Theta \kappa \widehat{\pi}_t = \alpha_\omega \frac{1-\alpha(1+\chi)}{\alpha(1+\chi)}\widehat{\omega}_t, \qquad (11)$$

Result 4

- **Policy trade-off more stringent around a bubbleless BGP because MP cannot affect the bubble size directly**

Intuition

Optimal Discretionary Monetary Policy: Bubbleless BGP

- Consider a bubbleless BGP where $q^B = \chi = 0$ and $\Phi = 1$
  - Aggregate bubble dynamics

$$\widehat{q}_t^B = \frac{\beta}{\nu} E_t \widehat{q}_{t+1}^B - \frac{\beta}{\nu} E_t \widehat{u}_{t+1} \tag{12}$$

  - Targeting rule

$$\Theta \alpha_y \widehat{y}_t + \Theta \kappa \widehat{\pi}_t = \alpha_\omega \left(\frac{1-\alpha}{\alpha} \right) \widehat{\omega}_t \tag{13}$$

  - Stationary solutions

$$\widehat{q}_t^B = R_0 \widehat{q}_{t-1}^B + e_t, \tag{14}$$

where $R_0 \equiv \nu/\beta < 1$ and $e_t \equiv \widehat{b}_t - E_{t-1}\{\widehat{b}_t\} + \widehat{u}_t$ is a martingale difference process

Optimal Discretionary Monetary Policy: Bubbleless BGP and Old Bubble Shock

- consider $e_t = e_t^b = \widehat{b}_t > 0$ and $E_{t-1}\{\widehat{b}_t\} = \widehat{u}_t = 0$

$$\widehat{\pi}_t = \frac{\psi_\pi^q R_0}{\vartheta}\widehat{q}_{t-1}^B + \frac{\psi_\pi^b}{\vartheta}e_t^b \tag{15}$$

$$\widehat{y}_t = \frac{\psi_y^q R_0}{\vartheta}\widehat{q}_{t-1}^B + \frac{\psi_y^b}{\vartheta}e_t^b \tag{16}$$

$$\widehat{\omega}_t = \frac{\psi_\omega^q R_0}{\vartheta}\widehat{q}_{t-1}^B + \frac{\psi_\omega^b}{\vartheta}e_t^b \tag{17}$$

Optimal Discretionary Monetary Policy: Bubbleless BGP and New Bubble Shock

- consider $e_t = e_t^u = \widehat{u}_t > 0$ and $E_{t-1}\{\widehat{b}_t\} = \widehat{b}_t = 0$

$$\widehat{\pi}_t = \frac{\psi_\pi^q R_0}{\vartheta}\widehat{q}_{t-1}^B - \frac{\psi_\pi^u}{\vartheta} e_t^u \tag{18}$$

$$\widehat{y}_t = \frac{\psi_y^q R_0}{\vartheta}\widehat{q}_{t-1}^B - \frac{\psi_y^u}{\vartheta} e_t^u \tag{19}$$

$$\widehat{\omega}_t = \frac{\psi_\omega^q R_0}{\vartheta}\widehat{q}_{t-1}^B - \frac{\psi_\omega^u}{\vartheta} e_t^u \tag{20}$$

Result 5

- **The optimal response to to bubbly fluctuations depends on the nature (owner) of the bubbly shock (asset)**

  1. shock to **old bubbles**:
     accommodation of the **expansionary effects**
  2. shock to **new bubbles**:
     leaning against the **expansionary effects** on impact (then accommodation)

To Sum Up

- **Theory**

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- **Theory**
  1. Finite planning horizon isomorphic to finite lives but smaller aggregate bubbles compared to Galí (2021) due to DAMP, endogenous labor supply and an efficient BGP
- **Policy** to dampen fluctuations in wealth (ineq) from bubbles

To Sum Up

- **Theory**
  1. Finite planning horizon isomorphic to finite lives but smaller aggregate bubbles compared to Galí (2021) due to DAMP, endogenous labor supply and an efficient BGP
- **Policy** to dampen fluctuations in wealth (ineq) from bubbles
  1. limited effectiveness of the policy rate as an instrument
  2. SIT in general **NOT** optimal
  3. Policy response depends on the nature of the bubbly shock/owner of the bubble

Thank you for your attention.

Asset-Market Participation

1. **Market Participants**

$$m^p_{t|s} \equiv \vartheta \left(1-\gamma\right) \gamma^{t-s}$$

$$\sum_{s=-\infty}^{t} m^p_{t|s} = \vartheta$$

2. **Hand-to-mouth** agents/**Rule-of-thumbers** (RoT)

$$m^r_{t|k} \equiv \left(1-\vartheta\right) \left(1-\varrho\right) \varrho^{t-k}$$

$$\sum_{k=-\infty}^{t} m^r_{t|k} = 1-\vartheta$$

3. **Outflow = inflow from the asset market** Back

$$\vartheta(1-\gamma) = (1-\vartheta)(1-\varrho)$$

Employment Status

- **Transition INTO employment** when turning participant from RoT
- **Transition OUT of employment** when participant
  - **Employed** participants

$$m_{t|s}^{pe} \equiv \vartheta\left(1-\gamma\right)\left(\gamma\nu\right)^{t-s}$$

$$\sum_{s=-\infty}^{t} m_{t|s}^{pe} = \vartheta\alpha$$

where $\alpha \equiv (1-\gamma)/(1-\gamma\nu)$

  - **Unemployed** participants

$$m_{t|s}^{pu} \equiv \vartheta\left(1-\gamma\right)\gamma^{t-s}\left(1-\nu^{t-s}\right)$$

$$\sum_{s=-\infty}^{t} m_{t|s}^{pu} = \vartheta\left(1-\alpha\right)$$

Back

Employment Status (2)

- **RoTs keep their employment status** until they turn participant

1. **Employed** RoT

$$m_{t|k}^{re} \equiv (1-\vartheta)(1-\varrho)\,\alpha\varrho^{t-k}$$

$$\sum_{k=-\infty}^{t} m_{t|k}^{re} = (1-\vartheta)\,\alpha$$

2. **Unemployed** RoT

$$m_{t|k}^{ru} \equiv (1-\vartheta)(1-\varrho)(1-\alpha)\,\varrho^{t-k}$$

$$\sum_{k=-\infty}^{t} m_{t|k}^{ru} = (1-\vartheta)(1-\alpha)$$

Back

Hand-to-mouth Agents (Rule-of-thumbers)

$$\max \log \left(C_t^j - \frac{\delta \Gamma^t}{1+\varphi} \left(N_t^j \right)^{1+\varphi} \right)$$

s.t.

$$C_t^{rj} = W_t N_t^{rj} - T_t^{rj}$$

where $j \in \{e, u\}$, $N_t^{ru} = 0$ and $(1-\alpha) T_t^{ru} = \alpha T_t^{re} + T_t^r$.

- Consumption of employed RoTs

$$C_t^{re} = W_t N_t^{re} - T_t^{re}$$

- Consumption of unemployed RoTs

$$\widetilde{C}_t^{ru} = C_t^{ru} = T_t^{ru} = \widetilde{C}_t^{re}$$

- Labor supply

$$N_t^{re} = \left(\frac{w_t}{\delta} \right)^{\frac{1}{\varphi}}$$

- Consumption of all RoTs

$$C_t^r = \alpha \delta^{-\frac{1}{\varphi}} w_t^{\frac{1+\varphi}{\varphi}} \Gamma^t + T_t^r$$

Back

Market Participants

- **Finite planning horizon** though infinitely-lived

$$\max E_t \sum_{t=0}^{\infty} (\beta\gamma)^t U_{t|s}^{pj}$$

s.t.

$$C_{t|s}^{pj} + E_t \left\{ \Lambda_{t,t+1} Z_{t+1|s}^{j} \right\} + \int_{i \in \mathcal{F}} \left[Q_t^F(i) - \left(1 - \tau^D\right) D_t(i) \right] Z_{t+1|s}^{Fj}(i)\, di + Q_t^B Z_{t+1|s}^{Bj} = A_{t|s}^{j} + W_t N_{t|s}^{pj} - T_t^{pj},$$

where $j \in \{e, u\}$ and $N_{t|s}^{pu} = T_{t|s}^{pu} = 0$, and

$$A_{t|s}^{j} \equiv \frac{1}{\gamma} \left[Z_{t|s}^{j} + \int_{i \in \mathcal{F}^*} Q_t^F(i)\, Z_{t|s}^{Fj}(i)\, di + B_t Z_{t|s}^{Bj} \right]$$

$$A_{t|t}^{e} \equiv \frac{Q_{t|t}^{F}}{\vartheta} + \frac{U_t}{\vartheta(1-\gamma)}$$

Market Participants: FOCs

Stochastic Discount Factor:

$$\Lambda_{t,t+1} = \beta \frac{\widetilde{C}^{pe}_{t|s}}{\widetilde{C}^{pe}_{t+1|s}} = \beta \frac{C^{pu}_{t|s}}{C^{pu}_{t+1|s}},$$

Equity shares:

$$Q^F_t(i) = \left(1 - \tau^D\right) D_t(i) + \gamma \nu E_t \left\{\Lambda_{t,t+1} Q^F_{t+1}(i)\right\}$$

Bubbles:

$$Q^B_t = E_t \{\Lambda_{t,t+1} B_{t+1}\}$$

Endogenous Labor supply:

$$N^{pe}_t = N^{pe}_{t|s} = \left(\frac{w_t}{\delta}\right)^{\frac{1}{\varphi}}$$

where $w_t \equiv W_t/\Gamma^t$. Back

Market Participants: Aggregation

Consumption:

$$C_t^p = (1-\beta\gamma)\left(\frac{Q_t^F + Q_t^B}{\vartheta} + \alpha H_t\right) + \alpha V\left(N_t^{pe}\right)$$

Aggregate Wealth:

$$\vartheta A_t = \int_{i\in\mathcal{F}^*} Q_t^F(i)di + (1-\gamma)Q_{t|t}^F + B_t + U_t = Q_t^F + Q_t^B$$

Aggregate Bubble:

$$Q_t^B = B_t + U_t$$

Aggregate stock-market value:

$$Q_t^F \equiv \int_{i\in\mathcal{F}^*} Q_t^F(i)di + (1-\gamma)Q_{t|t}^F$$

Back

Market Participants: Incumbents and Newcomers

- **Incumbent** $(s < t)$
  - Individual Wealth $(j \in \{e, u\})$

$$A^{j}_{t|s} \equiv \frac{1}{\gamma}\left[Z^{j}_{t|s} + \int_{i\in\mathcal{F}^*} Q^F_t(i)\, Z^{Fj}_{t|s}(i)\, di + B_t Z^{Bj}_{t|s}\right]$$

  - Aggregate Consumption

$$\gamma C^{p}_{t|in} = (1-\beta\gamma)\left(\frac{\gamma\nu Q^F_t + B_t}{\vartheta} + \alpha\gamma\nu H_t\right) + \alpha\gamma\nu V\left(N^{pe}_t\right)$$

- **Newcomer** $(s = t)$
  - Individual Wealth

$$A^{e}_{t|t} \equiv \frac{Q^F_{t|t}}{\vartheta} + \frac{U_t}{\vartheta(1-\gamma)}$$

  - Aggregate Consumption

$$(1-\gamma)C^{p}_{t|nc} = (1-\beta\gamma)\left[\frac{(1-\gamma)Q^F_t + U_t}{\vartheta} + (1-\gamma)H_t\right] + (1-\gamma)V\left(N^{pe}_t\right)$$

Government Budget Constraint

$$\alpha\vartheta T_t^{pe} + \alpha(1-\vartheta)T_t^{re} + \tau^D D_t = \tau^F W_t N_t + (1-\alpha)(1-\vartheta)T_t^{ru}$$

where

$$(1-\alpha)\, T_t^{ru} = \alpha T_t^{re} + T_t^{r}$$

Back

Aggregation (Productivity-adjusted)

Consumption of participants:

$$\widetilde{c}_t^p = (1-\beta\gamma)\left(\frac{q_t^B}{\vartheta} + x_t\right) + \alpha v\left(N_t^{pe}\right) \tag{21}$$

⇒ Positive complementarity effects of labor on consumption

Fundamental Wealth:

$$x_t \equiv E_t\left\{\sum_{k=0}^{\infty}(\gamma\nu\Gamma)^k \Lambda_{t,t+k}\left[\frac{1-\tau^D}{\vartheta}d_{t+k} + \alpha\left(w_{t+k}N_{t+k}^{pe} - t_{t+k}^{pe} - v(N_{t+k}^{pe})\right)\right]\right\} \tag{22}$$

Aggregate Bubble:

$$q_t^B = b_t + u_t = \Gamma E_t\left\{\Lambda_{t,t+1}q_{t+1}^B\right\} - \Gamma E_t\left\{\Lambda_{t,t+1}u_{t+1}\right\} \tag{23}$$

Aggregate (**endogenous**) labor supply:

$$w_t = \delta\left(\frac{N_t}{\alpha}\right)^{\varphi}. \tag{24}$$

Balanced-Growth Paths (2)

DAMP economy with endogenous labor supply and employment subsidies:

$$q^B = \eta \frac{\gamma(\beta R - \nu)}{(1-\beta\gamma)(R-\gamma\nu)}$$

with $\eta \equiv \left[\varpi + (1-\varpi)\frac{\vartheta\varphi}{1+\varphi}\right]$

Intuition

- spending of participants in the absence of bubbles

$$c^p = \left[\left(\frac{\eta}{\vartheta}\right)\frac{1-\beta\gamma}{1-\gamma\nu/R} + \frac{1-\varpi}{1+\varphi}\right] y$$

- income of participants

$$y^p = \left(1 + \varpi\frac{1-\vartheta}{\vartheta}\right) y$$

- economy-wide excess savings in the absence of bubbles

$$\vartheta(y^p - c^p) = \eta\left(1 - \frac{1-\beta\gamma}{1-\gamma\nu/R}\right) y$$

Linear Model: the role of DAMP

Consider Galí (2021) plus endogenous labor supply ($\vartheta = 1$, $\varpi > 0$, $\tau^D = 0$).

- Equation (2) becomes:

$$\widehat{y}_t = \frac{1-\beta\gamma}{\varpi}\left(\frac{\widehat{q}_t^B}{\vartheta} + \widehat{x}_t\right)$$

- the implied IS-type relation (using the above and the dynamics of fundamental wealth, eq. 1)

$$\widehat{y}_t = \Phi E_t \widehat{y}_{t+1} - \frac{\Phi}{\varpi}\left(\frac{\varphi}{1+\varphi}\right)\left(\frac{1-\beta\gamma}{1-\beta\gamma\Phi}\right)\widehat{r}_t + \frac{1-\beta\gamma}{\varpi\beta\gamma}\left(\widehat{q}_t^B - \beta\gamma\Phi E_t \widehat{q}_{t+1}^B\right)$$

⇒ interest- and bubble-elasticities of output larger the smaller the distortions

⇒ infinite elasticities if BGP is efficient (cfr. fiscal multipliers)

⇒ indeterminate equilibrium output

IS-type equation with DAMP and efficient BGP ($\vartheta < 1$, $\varpi = 0$, $\tau^D > 0$):

$$\widehat{y}_t = \Phi E_t \widehat{y}_{t+1} - \frac{\varphi}{1+\varphi}\frac{\Theta\Phi}{1-\beta\gamma\Phi}\widehat{r}_t + \frac{\Theta}{\vartheta\beta\gamma}\left(\widehat{q}_t^B - \beta\gamma\Phi E_t \widehat{q}_{t+1}^B\right)$$

Back

The Welfare Criterion (2)

Expected social welfare

$$\mathcal{W}_{t_0} \equiv E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} U_t \right\} \tag{25}$$

where the aggregate period-utility U_t is the weighted average

$$U_t \equiv \sum_{j \in \mathcal{T}} \sum_{s=-\infty}^{t} \chi_s^j U_{t|s}^j \tag{26}$$

for a system of Pareto-weights $\{\chi_s^j\}$, with $j \in \mathcal{T} = \{pe, pu, re, ru\}$ and

$$\sum_{j \in \mathcal{T}} \sum_{s=-\infty}^{t} \chi_s^j = 1.$$

Back

The Welfare Criterion (3)

Take a SOA of (25) given (26) around an (limited) efficient BGP

- (limited) efficient BGP solves the Ramsey problem that maximizes (25)

$$\text{s.t.} \qquad \Gamma^t \left[\sum_{s=-\infty}^{t} m_{t|s}^{pe} N_{t|s}^{*pe} + \sum_{s=-\infty}^{t} m_{t|s}^{re} N_{t|s}^{*re} \right] = Y_t^*$$

$$= C_t^* = \sum_{j \in \mathcal{T}} \sum_{s=-\infty}^{t} m_{t|s}^{j} C_{t|s}^{*j}, \qquad (27)$$

with

✓ X_t^*: BGP-level of generic variable X

✓ $m_{t|s}^j$: relative mass of agent-type j, cohort $s \leq t$, s.t. $\sum_{j \in \mathcal{T}} \sum_{s=-\infty}^{t} m_{t|s}^j = 1$

Efficiency of the BGP requires:

1. appropriate system $\{\chi_s^j\}$ supporting a specific cross-sectional distribution of wealth and consumption
2. appropriate employment subsidy offsetting monopolistic distortions ($\varpi = 0$) Back

The Welfare Criterion (4)

⇒ a quadratic Taylor expansion of (25) is a valid SOA of expected social welfare that can be evaluated using only FOA equilibrium conditions:

$$\mathcal{L}_{t_0} = \frac{1}{2}\frac{\varepsilon\varphi}{\kappa}E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\widehat{\pi}_t^2 + \alpha_y\widehat{y}_t^2 + \alpha_\omega\widehat{\omega}_t^2\right)\right\}$$

where

$$\widehat{\omega}_t \equiv \frac{1}{\alpha\vartheta}\widehat{q}_t^B - \frac{\widehat{u}_t}{\vartheta(1-\gamma)} - \frac{1-\alpha}{\alpha}\widehat{x}_t = \frac{\gamma}{1-\beta\gamma}\left(\widehat{\widetilde{c}}_{t|in}^{p} - \widehat{\widetilde{c}}_{t|nc}^{p}\right)$$

and

$$\alpha_y \equiv \frac{\kappa}{\varphi\varepsilon}\left[\varphi + \left(\frac{1+\varphi}{\varphi}\right)\left(\frac{1-\vartheta}{\vartheta}\right)(\tau-\varphi)^2\right] \tag{28}$$

$$\alpha_\omega \equiv \frac{\kappa\vartheta}{\varepsilon\varphi}\left(\frac{1+\varphi}{\varphi}\right)\frac{(1-\gamma)(1-\beta\gamma)}{\gamma} \tag{29}$$

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Optimal Monetary Policy

1. Assume $\widehat{y}_t = \widehat{\pi}_t = 0$ for all t:
   - $\Rightarrow$ if $\widehat{q}_t^B = \frac{\alpha \widehat{u_t}}{1-\gamma}$ for all t then $\widehat{y}_t = \widehat{\pi}_t = \widehat{\omega}_t = 0$ and welfare is max
   - $\Rightarrow$ otherwise: optimal to give up SIT to reduce consumption dispersion
2. Assume $\widehat{u}_t = \widehat{y}_t = \widehat{\pi}_t = 0$ for all t:
   - $\Rightarrow$ equations (7)–(10) imply

$$\widehat{b}_t = \Psi E_t \widehat{b}_{t+1}$$

with $\Psi = \frac{\nu}{\beta R}\left[1 + (1-R)\frac{1-\beta\gamma}{R-\nu\gamma}\right]$

   - $\Rightarrow$ if $R \in [R^*, 1]$ (BGP globally unstable) then $\widehat{b}_t = 0$ and welfare is max
   - $\Rightarrow$ otherwise: optimal to give up SIT to reduce consumption dispersion

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Optimal Monetary Policy (2)

Result 3a.

- **The optimality of the SIT depends on the global stability properties of the BGP**

1. Globally **unstable** **BGP**: **SIT** is **optimal**
2. Globally **stable** **BGP**: **Optimal deviations from SIT**

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