Negative Rates

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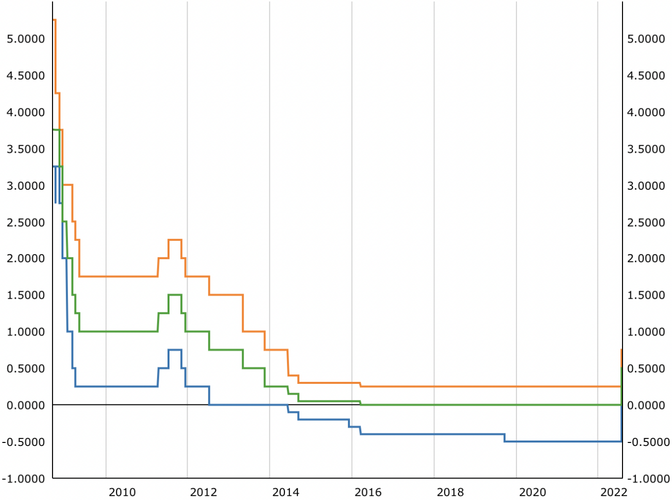
motivation



ECB marginal lending facility rate

ECB main refinancing operations rate

ECB deposit facility rate

goals

- to explain zero/negative rates in an equilibrium set up
- to provide a rationale for bank intermediation starting from "natural" assumptions
- to shed light on policy

what is out there?

Eggertson *et al.* (2019): "the theoretical literature on negative interest rates is perhaps surprisingly somewhat smaller [than the empirical], given the high stakes in the policy debate"

- Rognlie (2015) "integrate[s] cash [...] by including [a] concave utility from real cash balances into household preferences"
- Brunnermeier and Koby (2016) "assume [...] that loans are priced at marginal costs that include costs from leverage"
- Ulate (2019): "deposits and loans have the same duration [which] side-steps maturity transformation as an aspect of banking", [and] "household[s ...] save [only] by depositing their money in [...] banks, or by holding cash"
- Eggerston *et al.* (2019) introduce opaque intermediation costs

set-up

a standard infinite-horizon deterministic neoclassical model with:

- households and firms are constrained by the timing of the availability of their own funds

- factor markets opening in the "morning"
- output market opening in the "evening"
- banks intermediate funds needed/made idle by the time mismatch, and allow for a higher participation in the capital market

- loans to firms have a "long" maturity (2 periods) while household deposits have a "short" maturity (1 period)

results

1. steady state output is higher with banks than (the conterfactual) without —if inflation and lending rates are low, and labor supply sufficiently inelastic
2. the first best steady state cannot (generically) be a market outcome with a passive central bank
3. to implement planner allocations, collateral requirements / leverage bounds —based on expected inflation and lending rates— are needed
4. the first-best steady state requires a zero lending rate from banks to firms and a negative lending rate from the central bank to banks

household

$$\max_{0 \leq c_t, k_{t+1}, h_t, m_t^h, d_t} \sum_{t=1}^{+\infty} (\delta^h)^{t-1} \left[u(c_t) - v(h_t) \right]$$

$$m_t^h \leq r_t k_t + w_t h_t + \pi_t^f + \sum_b \pi_t^b$$

$$c_t + k_{t+1} + \phi(l_t) d_t \leq m_t^h + \frac{r_{t-1}^d}{\rho_t} d_{t-1}$$

firm

$$\max_{0 \leq k_t, h_t, l_t, m_t^f, \pi_t^f} \quad \sum_{t=1}^{+\infty} (\delta^f)^{t-1} \pi_t^f$$

$$r_t k_t + w_t h_t + \pi_t^f + \frac{r_{t-1}^l}{\rho_t} \frac{r_{t-2}^l}{\rho_{t-1}} l_{t-2} \leq l_t + \frac{1}{\rho_t} m_{t-1}^f$$

$$m_t^f \leq f(k_t + e^f, h_t)$$

$$r_t^l l_t \leq f(k_t + e^f, h_t)\theta$$

banks

$$\max_{0 \leq l_t^b, d_t^b, q_t^b, \pi_t^b} \quad \sum_{t=1}^{+\infty} (\delta^b)^{t-1} \pi_t^b$$

$$\pi_t^b + l_t^b - \frac{r_{t-1}^l}{\rho_t} \frac{r_{t-2}^l}{\rho_{t-1}} l_{t-2}^b \leq d_t^b - \frac{r_{t-1}^d}{\rho_t} d_{t-1}^b + q_t^b - \frac{r_{t-1}^q}{\rho_t} q_{t-1}^b$$

$$\frac{r_{t-1}^l}{\rho_t} l_{t-1}^b + l_t^b = e^b + d_t^b + q_t^b$$

$$\eta l_t^b \leq e^b$$

household's optimizing

$$\frac{u'(c_t)}{\delta^h u'(c_{t+1})} = r_{t+1} = \frac{1}{\phi(l_t)} \frac{r_t^d}{\rho_{t+1}}$$

$$\frac{v'(h_t)}{u'(c_t)} = w_t$$

$$m_t^h = r_t k_t + w_t h_t + \pi_t^f + \sum_b \pi_t^b$$

$$c_t + k_{t+1} + \phi(l_t) d_t = m_t^h + \frac{r_{t-1}^d}{\rho_t} d_{t-1}$$

firm's optimizing

$$\frac{r_t}{f_k(k_t + e^f, h_t)} = \frac{1}{r_t^l}\left[\delta^f \frac{r_t^l}{\rho_{t+1}} + \theta\left(1 - \delta^f \frac{r_t^l}{\rho_{t+1}} \cdot \delta^f \frac{r_{t+1}^l}{\rho_{t+2}}\right)\right] = \frac{w_t}{f_h(k_t + e^f, h_t)}$$

$$r_t k_t + w_t h_t + \pi_t^f + \frac{r_{t-2}^l}{\rho_{t-1}} \frac{r_{t-1}^l}{\rho_t} l_{t-2} = \frac{1}{\rho_t} m_{t-1}^f + l_t$$

$$m_t^f = f(k_t + e^f, h_t)$$

$$r_t^l l_t \leq f(k_t + e^f, h_t)\theta$$

$$\left[1 - \delta^f \frac{r_t^l}{\rho_{t+1}} \cdot \delta^f \frac{r_{t+1}^l}{\rho_{t+2}}\right]\left[r_t^l l_t - f(k_t + e^f, h_t)\theta\right] = 0$$

$$\delta^f \frac{r_t^l}{\rho_{t+1}} \cdot \delta^f \frac{r_{t+1}^l}{\rho_{t+2}} \leq 1$$

banks' optimizing

$$r_t^d = r_t^q$$

$$\pi_t^b + l_t^b - \frac{r_{t-1}^l}{\rho_t}\frac{r_{t-2}^l}{\rho_{t-1}} l_{t-2}^b = d_t^b - \frac{r_{t-1}^d}{\rho_t} d_{t-1}^b + q_t^b - \frac{r_{t-1}^q}{\rho_t} q_{t-1}^b$$

$$\frac{r_{t-1}^l}{\rho_t} l_{t-1}^b + l_t^b = e^b + d_t^b + q_t^b$$

$$\eta l_t^b \leq e^b$$

$$\left[\left(\delta^b \frac{r_t^l}{\rho_{t+1}} - \delta^b \frac{r_t^d}{\rho_{t+1}}\right) + \delta^b \frac{r_t^l}{\rho_{t+1}}\left(\delta^b \frac{r_{t+1}^l}{\rho_{t+2}} - \delta^b \frac{r_{t+1}^d}{\rho_{t+2}}\right)\right]\left(\eta l_t^b - e^b\right) = 0$$

$$0 \leq \left(\delta^b \frac{r_t^l}{\rho_{t+1}} - \delta^b \frac{r_t^d}{\rho_{t+1}}\right) + \delta^b \frac{r_t^l}{\rho_{t+1}}\left(\delta^b \frac{r_{t+1}^l}{\rho_{t+2}} - \delta^b \frac{r_{t+1}^d}{\rho_{t+2}}\right)$$

market clearing

$$c_t + k_{t+1} = f(k_t + e^f, h_t)$$

$$l_t = \sum_b l_t^b$$

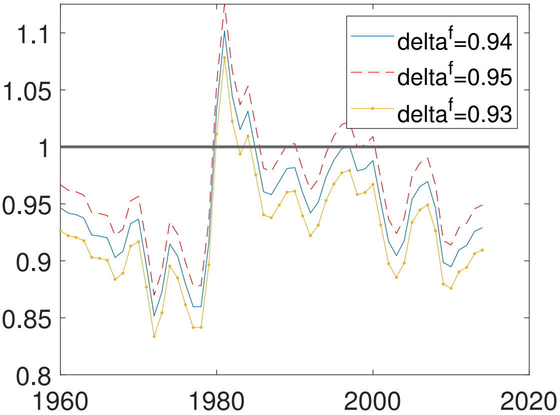
$$d_t = \sum_b d_t^b$$

$$\color{red}{0} = \sum_b q_t^b$$

equilibrium

- household's optimizing
- firm's optimizing
- banks' optimizing
- market clearing

consistent with observations?



$$\delta^f \frac{r_t^l}{\rho_{t+1}} \cdot \delta^f \frac{r_{t+1}^l}{\rho_{t+2}} \leq 1$$

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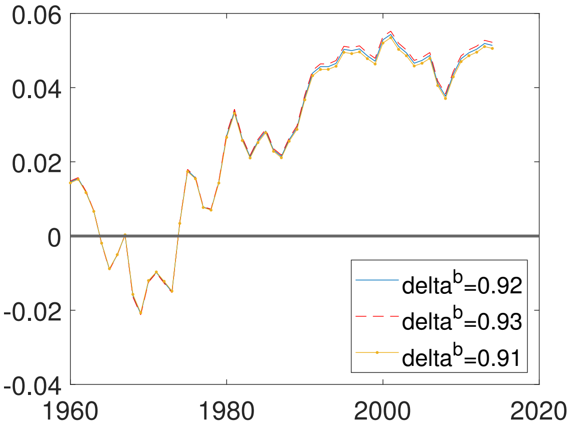

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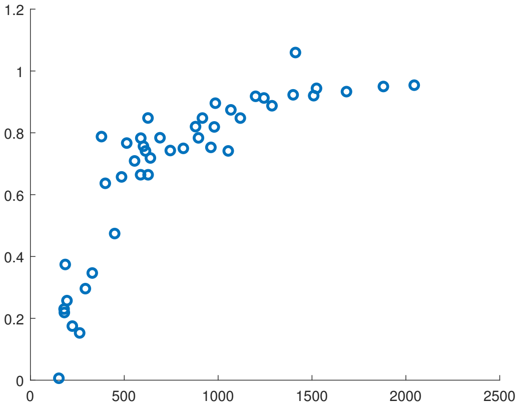

$$f(k_t + e^f, h_t) - \frac{1}{\rho_t} f(k_{t-1} + e^f, h_{t-1}) = [1 - \phi(l_t)] d_t$$

(1) the equilibrium SS with banks has a higher output...

(2) the planner's SS is not a market outcome...

(3) decentralising planner allocations

any equilibrium allocation satisfying

$$\theta_t = \frac{r_t^l - \delta^f \frac{r_t^l}{\rho_{t+1}}}{1 - \delta^f \frac{r_t^l}{\rho_{t+1}} \cdot \delta^f \frac{r_{t+1}^l}{\rho_{t+2}}}$$

is a planner's allocation, why?

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$$1 = \left[\delta^f \frac{r_t^l}{\rho_{t+1}} + \theta_t \left(1 - \delta^f \frac{r_t^l}{\rho_{t+1}} \cdot \delta^f \frac{r_{t+1}^l}{\rho_{t+2}}\right)\right] \frac{1}{r_t^l}$$

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(4) decentralising the planner's SS

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the market implementation of the planner's SS requires

- lending to firms at a zero rate :

$$r^l = 1$$

- lending to banks at a negative rate :

$$r^q < 1$$

whenever $\theta \geq 1$ and $\frac{\delta^h}{\delta^f} > \phi(l)$

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household's FOC

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household's FOC at SS

$$\frac{u'(c\ )}{\delta^h u'(c\quad)} = \frac{1}{\phi(l\ )}\frac{r^d}{\rho}$$

household's FOC at SS

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at SS decentralising the planner's

$$\frac{1}{\delta^h} = \frac{1}{\phi(l)} \frac{r^q}{\delta^f}$$

at SS decentralising the planner's

$$\phi(l)\frac{\delta^f}{\delta^h} = \qquad r^q$$

at SS decentralising the planner's

$$1 > \phi(I)\frac{\delta^f}{\delta^h} = \qquad r^q$$

iff

$$\delta^h > \phi(I)\,\delta^f$$

shortcomings

- no actual borrowing from the central bank
- no room for banks deposits at the central bank either
- reserve requirements seem not to play much of a role
- business cycle aspects are not addressed
- ...

but still...

to take home

- observed zero and negative rates are compatible [even optimal] with an equilibrium model that withstands confronting data
- policy may need to focus not just on rates but on leverage levels —reminiscent of Geanakoplos (2010)
- reserve requirements may not play a role in decentralising planner's allocations because of the required negative lending rate to banks
- the results should be robust given the stripped-down nature of the set-up...