Heteroskedastic Proxy Vector Autoregressions

Testing for Time-Varying Impulse Responses in the Presence of Multiple Proxies

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- Alternative identification schemes available, e.g. exploiting statistical features of the data such as heteroskedasticity

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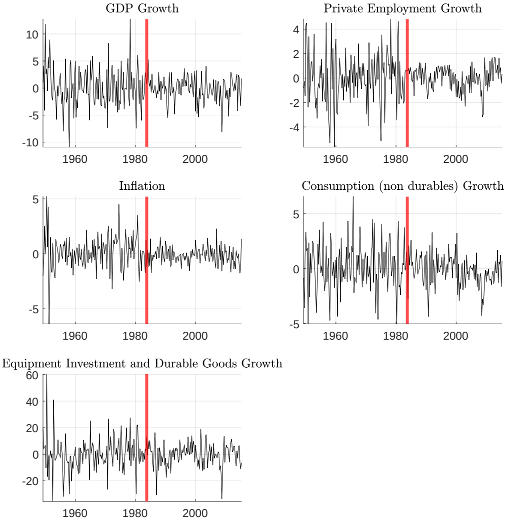
- Premise: Identification of a single shock by one or more proxies
- Unsuitable when shocks are identified collectively

Motivation (cont.)

OLS residuals of a U.S. VAR(4) macro model



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- Test for time-varying impact effects when a set of shock is collectively identified by proxies
- Key insight: impulse response impact effect is time-varying if a linear transformation is time-varying
- Monte Carlo simulation: Stylized and "realistic" setting
- Application to the impact of two total factor productivity shocks in the US (see Lunsford 2015)

Heteroskedastic Proxy VAR Models

Reduced form:

$$y_t = \nu + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t, \tag{1}$$

$$u_t \sim (0, \Sigma_t) \tag{2}$$

$$\mathbb{E}(u_t u_t') = \Sigma_t = \Sigma_u(m) \quad \text{for} \quad t \in \mathcal{T}_m, \quad m = 1, \ldots, M, \tag{3}$$

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- M volatility regimes
- (Known) volatility changes at T_m, where $T_0 = 0$ and $T_M = T$

Heteroskedastic Proxy VAR Models (cont.)

Structural form

$$u_t = B(m)\boldsymbol{w}_t, \quad (4)$$

$$B(m) = [B_1(m) : B_2(m)] \quad (5)$$

$$\Theta_h(m) = \Phi_h B(m) \quad (6)$$

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- K variables, K_1 identified shocks, K_2 non-identified shocks
- $\boldsymbol{w}'_t = (\boldsymbol{w}'_{1t}, \boldsymbol{w}'_{2t})$, $\boldsymbol{w}_{1t} = (w_{1t}, \dots, w_{K_1 t})'$, $\boldsymbol{w}_{2t} = (w_{K_1+1,t}, \dots, w_{Kt})'$, $Var(\boldsymbol{w}_t)$ diagonal
- $\Phi_i = \sum_{j=1}^{i} \Phi_{i-j} A_j, \quad \Phi_0 = I_K$
- $B_i(m)$: impact effects of shocks in $\boldsymbol{w}_{it}$, i=1,2 in volatility regime m
- Structural impulse responses (6) time-varying at all horizons if impact effects (5) time-varying

Heteroskedastic Proxy VAR Models (cont.)

Identification

N proxies

$$z_t = (z_{1t}, \ldots, z_{Nt})', \quad t \in \mathcal{T}_m \tag{7}$$

$$\mathbb{E}(\boldsymbol{w}_{1t} z_t') = C_m \neq 0, \quad C_m \ (K_1 \times N), \quad rk(C_m) = K_1 \quad \text{(relevance)}, \tag{8}$$

$$\mathbb{E}(\boldsymbol{w}_{2t} z_t') = 0 \quad \text{(exogeneity)}. \tag{9}$$

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This implies

$$\mathbb{E}(u_t z_t') = B(m)\mathbb{E}(\boldsymbol{w}_t z_t') = B_1(m) C_m. \tag{10}$$

i.e. z_t contain information to identify the first $K_1 < K$ shocks collectively.

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- Partition $B_1(m)$:

$$B_1(m) = \begin{bmatrix} B_{11}(m) \\ B_{12}(m) \end{bmatrix},$$

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- This transformation can be estimated from the data

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Testing for Time-varying Impact Effects (cont.)

- Instead of testing

$$\mathbb{H}_0 : B_1(m) = B_1(k) \quad \text{versus} \quad \mathbb{H}_1 : B_1(m) \neq B_1(k) \tag{11}$$

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- We are testing

$$\mathbb{H}_0 : B_{12}(m)B_{11}(m)^{-1} = B_{12}(k)B_{11}(k)^{-1}$$

vs.

$$\mathbb{H}_1 : B_{12}(m)B_{11}(m)^{-1} \neq B_{12}(k)B_{11}(k)^{-1} \tag{12}$$

- Test statistic $\eta(m,k) \xrightarrow{d} \chi^2(K_1(K-K_1))$

Testing for Time-varying Impact Effects (cont.)

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- Practical issues:
  - Choice of volatility regimes (pretesting)
  - Sample lengths within the regimes

1. Motivation
2. Heteroskedastic Proxy VAR Models
3. Testing for Time-varying Impact Effects
4. Monte Carlo Simulations
   - DGP1
   - DGP2
5. The Impact of TFP Shocks on the U.S. Economy
6. Conclusions
7. Appendix

DGP1: Setup

- Based on Lütkepohl & Schlaak (2021)
- $M = 3$ volatility regimes (known volatility change points)
- $K = 3$ variables
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$$A_1 = \begin{bmatrix} 0.79 & 0.00 & 0.25 \\ 0.19 & 0.95 & -0.46 \\ 0.12 & 0.00 & 0.62 \end{bmatrix},$$

$B(m) = I_3$ under $\mathbb{H}_0$, and

$$B(1) = I_3, \quad B(2) = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 4 \\ 4 & 6 & 6 \end{bmatrix}, \quad B(3) = \begin{bmatrix} 4 & 2 & 1 \\ -2 & 2 & 8 \\ 2 & 1 & 10 \end{bmatrix}$$

DGP1: Setup (cont.)

- Therefore, under $\mathbb{H}_1$:

$$B_{12}(1)B_{11}(1)^{-1} = [0, 0] \quad (13)$$

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- $\Sigma_u(m) = B(m)\Lambda_m B(m)', \quad m = 1, \dots, M$. with $\Lambda_1 = I_3$, $\Lambda_2 = diag(4, 9, 12)$ and $\Lambda_3 = diag(1, 4, 9)$

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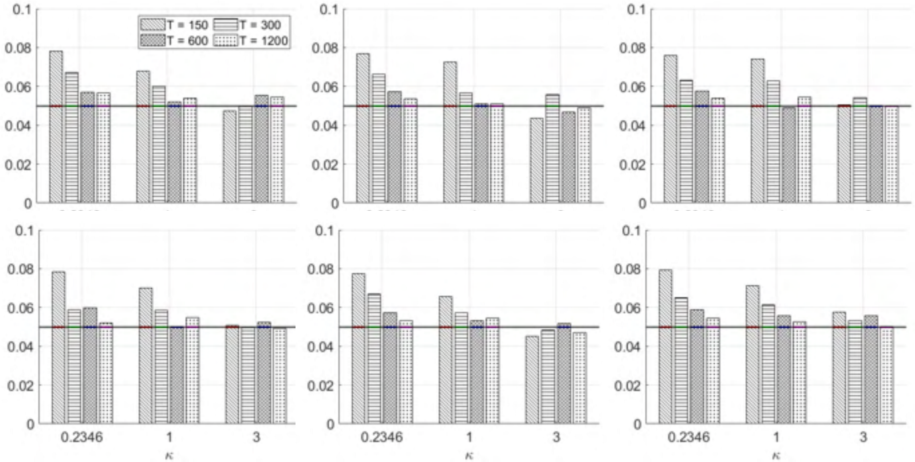
- Estimated models include intercept, 5000 replications
- $T = 150, 300, 600, 1200$

DGP1 Results under $\mathbb{H}_0$: Proxy-shock correlation: $\rho = 0$ (top row) vs $\rho = 0.5$ (bottom row), $p = 1$



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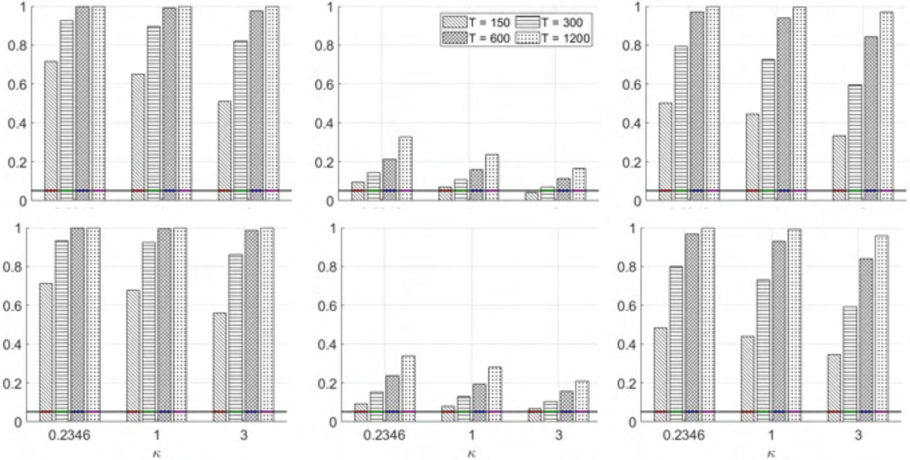

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DGP2: Setup

- Informed by Lunsford (2015)
  - 5 variables
  - 269 observations 1948Q2 = 2015Q2
  - 2 shocks
  - 2 proxies
- We fit VAR(1) model, stable process (max. Eigenvalue 0.7444)
- Search for volatility break using

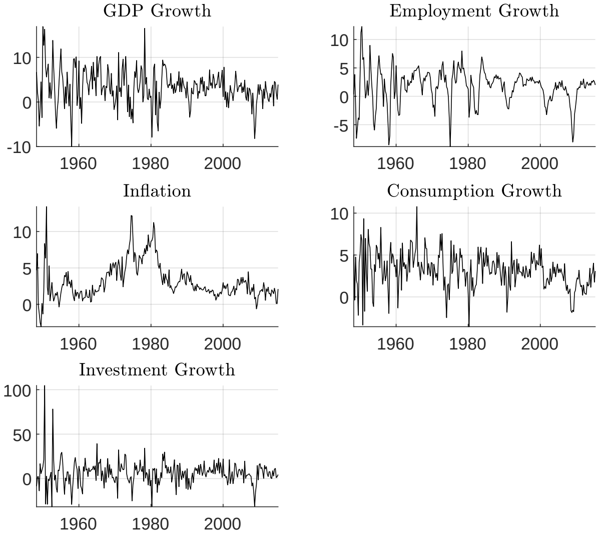
$$\psi(T_1) = T_1 \log\det\hat{\Sigma}_u(1) + (T - T_1)\log\det\hat{\Sigma}_u(2) \qquad (17)$$

over $T_1 \in \{0.15T, \dots, 0.85T\}$

- We find $1982Q4$

Details

DGP2: Setup (cont.)



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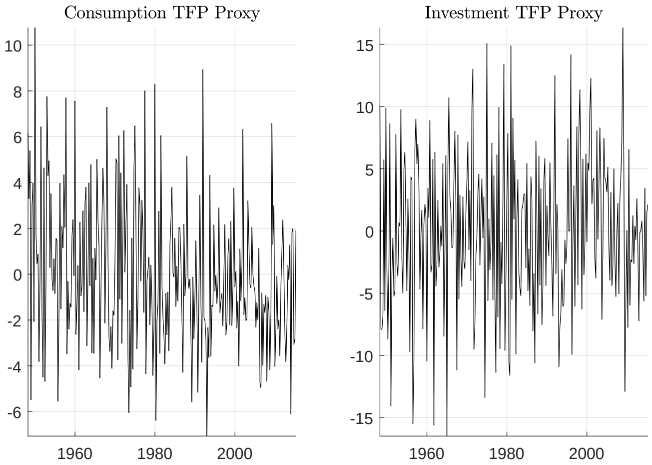
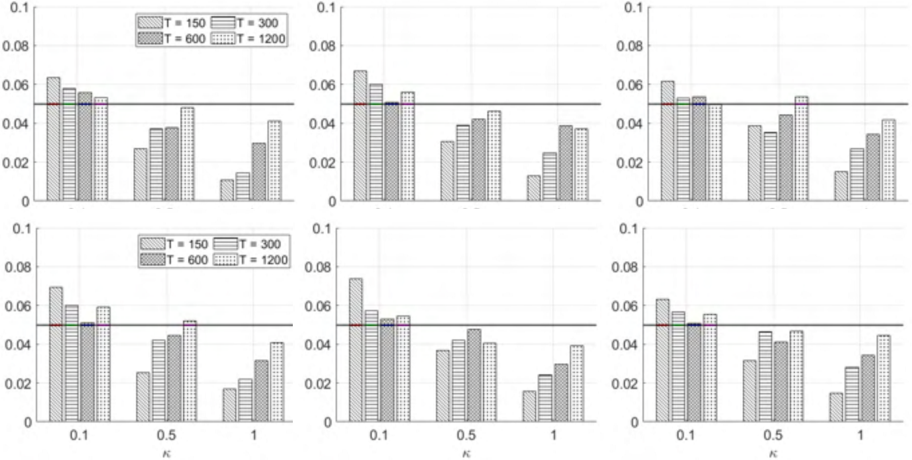


DGP2 Results under $\mathbb{H}_0$: $T_1 = T_{true} = 0.5T$ (top row) vs $T_1 = 0.4T$ (bottom row), $p = 1$



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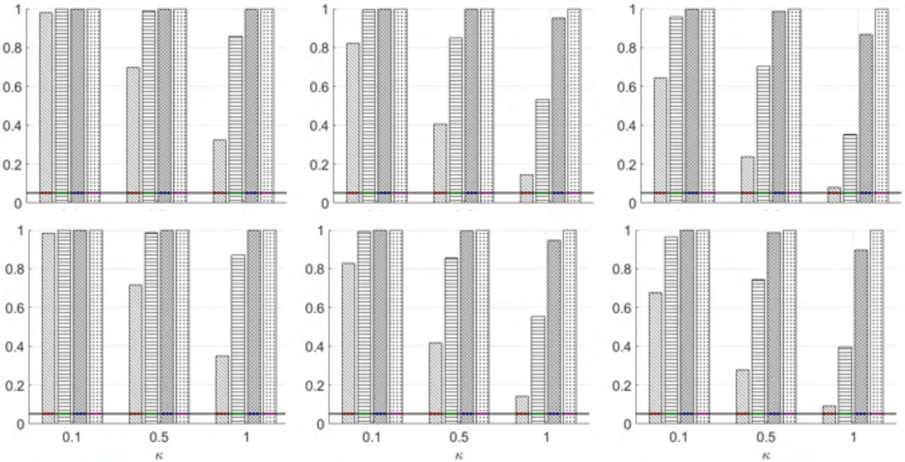

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The Impact of TFP Shocks on the U.S. Economy

- Benchmark study: 5-variate quarterly SVAR by Lunsford (2015)
- GDP growth, employment growth, inflation, consumption growth, investment growth
- Now, following Lunsford (2015), estimate a VAR(4)
- $N = 2$ proxies based on Fernald (2014):
  - Consumption TFP proxy
  - Investment TFP proxy
- Proxy construction:
  1. regress 2 TFP measures (excluding durable goods and for durable goods and equipment investment) on 4 lags of y_t and a constant
  2. use residuals as proxies
- Condition on volatility change in 1983Q4 as start of the Great Moderation (see e.g. McConnell & Perez-Quiros 2000, Galí & Gambetti 2009)

Application: Volatility Change Point Selection

T_1	test statistic	p-value
1982Q3	11.004	0.088
1982Q4	11.282	0.080
1983Q1	10.980	0.089
1983Q2	10.600	0.102
1983Q3	11.953	0.063
1983Q4	12.364	0.054
1984Q1	13.013	0.043
1984Q2	13.730	0.033
1984Q3	13.712	0.033
1984Q4	13.679	0.033
1985Q1	12.987	0.043
1985Q2	12.533	0.051
1985Q3	12.512	0.051
1985Q4	12.234	0.057

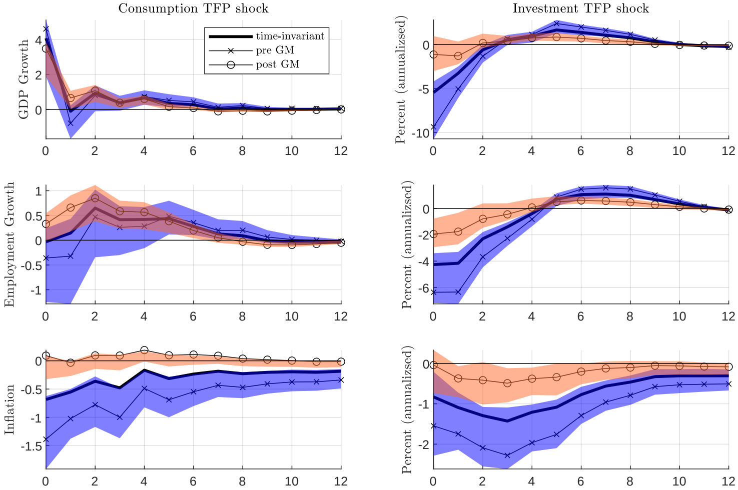
Application: Computing Impulse Responses

- Challenge: separate identification (not needed to execute our test)
- Lunsford (2015) adds proxies separately and empirically finds that each proxy is correlated with only one shock
- Corresponds to setting C_m diagonal, so that columns of $D(m)$ are scalar multiples of the true impact effects
- Distorted impulse response if true C_m not diagonal, but still suggestive evidence for or against a shift in the actual impulse responses
- Regime-specific moving block bootstrap (see e.g. Jentsch & Lunsford 2019, Bruns & Lütkepohl 2020) to compute confidence bands

Application: Impulse Responses

Full IRFs

90% Confidence Bands from regime-dependent MBB



Conclusions

- New test for time-varying impulse responses for heteroskedastic Proxy VARs
- Individual identification not necessary
- Asymptotic properties
- Performance in small sample in various settings
  - Larger samples and stronger proxies improve power
  - Larger lag orders, more variables, more proxies decrease power
  - Limited effect of even substantial volatility change point misspecification
- Application to US TFP shocks
  - Change in the response of some variables in pre- and post-GM period

Thank you for your attention

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Transformed Test Statistic: derivation

$$\begin{bmatrix} I_{K_1} \\ B_{12}(m)B_{11}(m)^{-1} \end{bmatrix} = B_1(m)B_{11}(m)^{-1}$$

$$= B_1(m)C_m Q C_m'(C_m Q C_m')^{-1} B_{11}(m)^{-1}$$

$$= B_1(m)C_m Q C_m' B_{11}(m)'(B_{11}(m)C_m Q C_m' B_{11}(m)')^{-1}$$

for any positive definite $(N \times N)$ matrix Q.
$D_1(m)$ is the upper $(K_1 \times N)$ part of

$$D(m) = \begin{bmatrix} D_1(m) \\ D_2(m) \end{bmatrix}$$

Transformed Test Statistic: derivation (cont.)

We can estimate

$$B_{12}(m)B_{11}(m)^{-1} = D_2(m)QD_1(m)'[D_1(m)QD_1(m)']^{-1}$$

consistently as

$$B_{12}(\widehat{m)B_{11}}(m)^{-1} = \widehat{D}_2(m)Q\widehat{D}_1(m)'[\widehat{D}_1(m)Q\widehat{D}_1(m)']^{-1}$$

where we choose

$$Q = \left(\sum_{t\in\mathcal{T}_m} z_t z_t'\right)^{-1}$$

Transformed Test Statistic: derivation (cont.)

Slutsky's theorem:

$$\sqrt{T}\text{vec}\left(B_{12}(\widehat{m)B_{11}}(m)^{-1} - B_{12}(m)B_{11}(m)^{-1}\right) \xrightarrow{d} \mathcal{N}\left(0, V(m)\right),$$

where

$$V(m) = \frac{1}{\tau_m}\frac{\partial\text{vec}[B_{12}(m)B_{11}(m)^{-1}]}{\partial\text{vec}D(m)'}\Sigma_D(m)\frac{\partial\text{vec}[B_{12}(m)B_{11}(m)^{-1}]'}{\partial\text{vec}D(m)}$$

can be estimated as

$$\widehat{V}(m) = \frac{1}{\tau_m}\frac{\widehat{\partial\beta(m)}}{\partial\text{vec}D(m)'}\widehat{\Sigma}_D(m)\frac{\widehat{\partial\beta(m)'}}{\partial\text{vec}D(m)},$$

with

$$\widehat{\Sigma}_D(m) = \frac{1}{\tau_m T}\sum_{t\in\mathcal{T}_m}\text{vec}(\hat{u}_t z_t' - \widehat{D}(m))[\text{vec}(\hat{u}_t z_t' - \widehat{D}(m))]'$$

Transformed Test Statistic: derivation (cont.)

Define

$$\beta(m) = \text{vec}[B_{12}(m)B_{11}(m)^{-1}]$$

Then we can use the test statistic

$$\begin{aligned} &\eta(m,k) \\ =&T\left(\hat{\beta}(m)-\hat{\beta}(k)\right)'\left(\widehat{V}(m)+\widehat{V}(k)\right)^{-1}\left(\hat{\beta}(m)-\hat{\beta}(k)\right) \\ &\xrightarrow{d} \chi^2(K_1(K-K_1)). \end{aligned}$$

back

DGP2 Setup: Simulation parameters

- Under $\mathbb{H}_0$: We decompose $\Sigma_u(1)$ and $\Sigma_u(2)$ such that

$$\Sigma_u(1) = BB' \quad \text{and} \quad \Sigma_u(2) = B\Lambda_2 B',$$

- $\Lambda_2 = \text{diag}(0.57, 0.15, 0.18, 0.35, 0.39)$

$$z_t = \Phi(m)\boldsymbol{w}_{1t} + v_t$$

with $v_t \sim \mathcal{N}(0, \kappa\Sigma_v)$, $t = 1, \dots, T$. Such that for $\kappa = 1$ the covariance matrix of the proxies,

$$\Sigma_z = \Phi(1)\Phi(1)' + \Sigma_v = \Phi(2)\Lambda_2\Phi(2)' + \Sigma_v$$

is similar to the empirical (Lunsford 2015)

$$T^{-1}\sum_{t=1}^{T} z_t z_t' = \begin{bmatrix} 9.95 & 5.41 \\ 5.41 & 36.88 \end{bmatrix}.$$

- $\Phi(m)$ diagonal

back

DGP2 Setup: Additional proxies

- $K_1 = 3$:

$$\Phi(1) = \begin{bmatrix} 1.70 & 0 & 0 \\ 0 & 2.24 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Phi(2) = \begin{bmatrix} 2.26 & 0 & 0 \\ 0 & 5.83 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\Sigma_v(1) = \Sigma_v(2) = \begin{bmatrix} 7.05 & 5.35 & 0 \\ 5.35 & 31.89 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

- $K_1 = 4$:

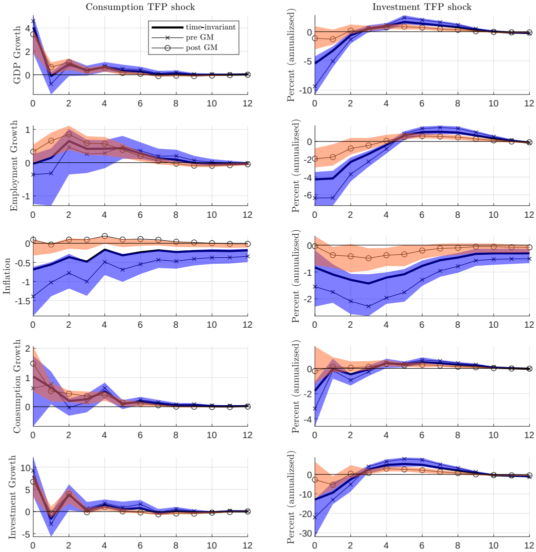
$$\Phi(1) = \begin{bmatrix} 1.70 & 0 & 0 & 0 \\ 0 & 2.24 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \Phi(2) = \begin{bmatrix} 2.26 & 0 & 0 & 0 \\ 0 & 5.83 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\Sigma_v(1) = \Sigma_v(2) = \begin{bmatrix} 7.05 & 5.35 & 0 & 0 \\ 5.35 & 31.89 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

back

Impulse Responses

back



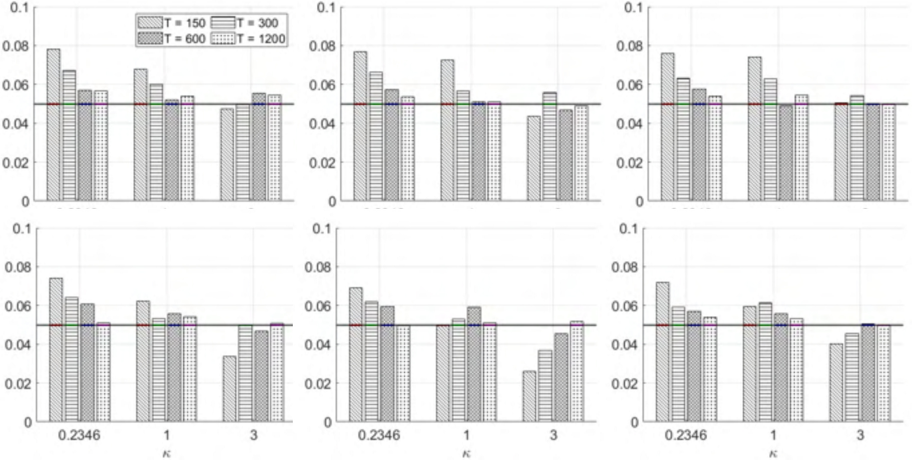
Application: Summary of Findings

1. Inflation response to consumption TFP shock significant only in pre-GM period
2. Initial effect of investment TFP shock on GDP growth and employment growth stronger in pre-GM period
3. Time-invariant IRFs seem to be close to an average across the two regimes
4. As in Lunsford (2015):
   - Consumption TFP shock can be interpreted as a supply shock (in post-GM regime)
   - Investment TFP shock cannot be interpreted as a supply shock but rather negative demand shock

DGP1: Correlations of z_t and w_{1t}

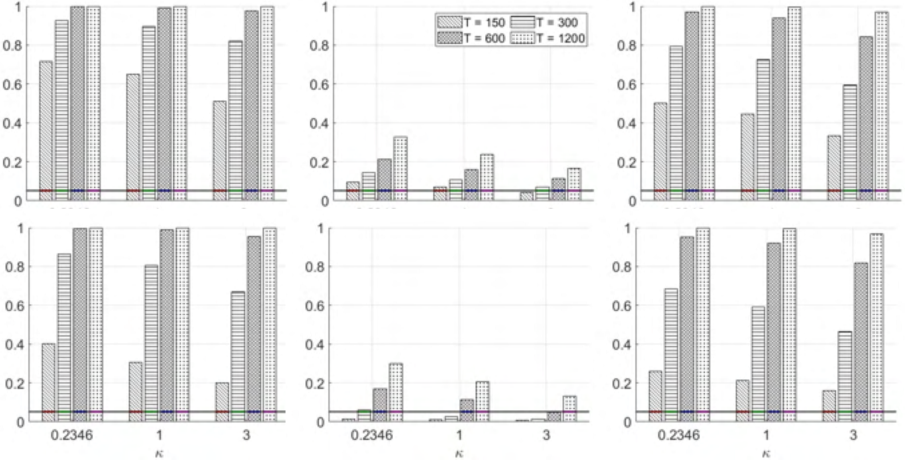
	κ	z_t	$t \in \mathcal{T}_1$		$t \in \mathcal{T}_2$		$t \in \mathcal{T}_3$	
			w_{1t}	w_{2t}	w_{1t}	w_{2t}	w_{1t}	w_{2t}
$\rho = 0$	0.2346	z_{1t}	0.900	0.000	0.972	0.000	0.900	0.000
		z_{2t}	0.000	0.900	0.000	0.987	0.000	0.972
	1	z_{1t}	0.707	0.000	0.894	0.000	0.707	0.000
		z_{2t}	0.000	0.707	0.000	0.949	0.000	0.894
	3	z_{1t}	0.500	0.000	0.756	0.000	0.500	0.000
		z_{2t}	0.000	0.500	0.000	0.866	0.000	0.756
			w_{1t}	w_{2t}	w_{1t}	w_{2t}	w_{1t}	w_{2t}
$\rho = 0.5$	0.2346	z_{1t}	0.900	0.000	0.972	0.000	0.900	0.000
		z_{2t}	0.410	0.821	0.313	0.938	0.236	0.944
	1	z_{1t}	0.707	0.000	0.894	0.000	0.707	0.000
		z_{2t}	0.333	0.667	0.302	0.905	0.218	0.873
	3	z_{1t}	0.500	0.000	0.756	0.000	0.500	0.000
		z_{2t}	0.243	0.485	0.277	0.832	0.186	0.743

DGP1 Results under $\mathbb{H}_0$: Lag order $p = 1$ (top row) vs $p = 12$ (bottom row), $\rho = 0$



$\mathbb{H}_0 : \beta(1) = \beta(2)$ $\mathbb{H}_0 : \beta(1) = \beta(3)$ $\mathbb{H}_0 : \beta(2) = \beta(3)$

DGP1 Results under $\mathbb{H}_1$: Lag order $p = 1$ (top row) vs $p = 12$ (bottom row), $\rho = 0$



$\mathbb{H}_0 : \beta(1) = \beta(2)$ $\mathbb{H}_0 : \beta(1) = \beta(3)$ $\mathbb{H}_0 : \beta(2) = \beta(3)$

DGP2: Setup (cont.)

- We generate proxies

$$z_t = \Phi(m)\boldsymbol{w}_{1t} + v_t, \quad v_t \sim \mathcal{N}(0, \kappa\Sigma_v) \tag{18}$$

$corr(z_t, \boldsymbol{w}_{1t})$

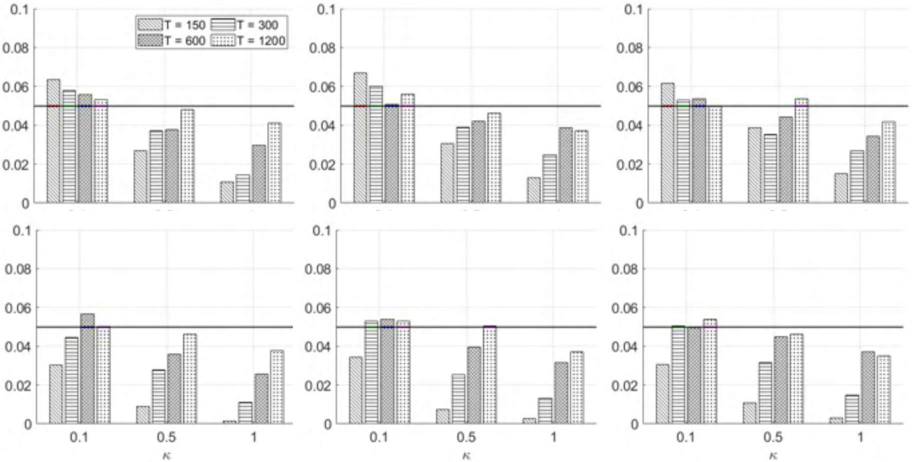
		w_{1t}	w_{2t}
$\kappa = 0.1$	z_{1t}	0.897	0.000
	z_{2t}	0.000	0.782
		w_{1t}	w_{2t}
$\kappa = 0.5$	z_{1t}	0.672	0.000
	z_{2t}	0.000	0.489
		w_{1t}	w_{2t}
$\kappa = 1$	z_{1t}	0.540	0.000
	z_{2t}	0.000	0.368

DGP2: Setup (cont.)

We investigate the test's performance in the following scenarios

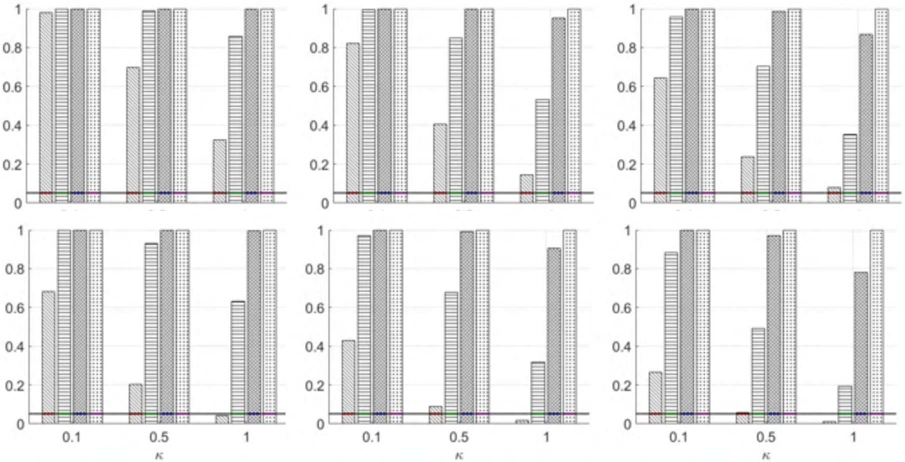
1. Lag order
2. Proxy strength
3. Increase number of identified shocks (and proxies) ▸ Details
4. Misspecifying the volatility change point
5. Searching for the volatility change point

DGP2 Results under $\mathbb{H}_0$: Lag order $p = 1$ (top row) vs $p = 12$ (bottom row), $T_1 = T_{true} = 0.5T$



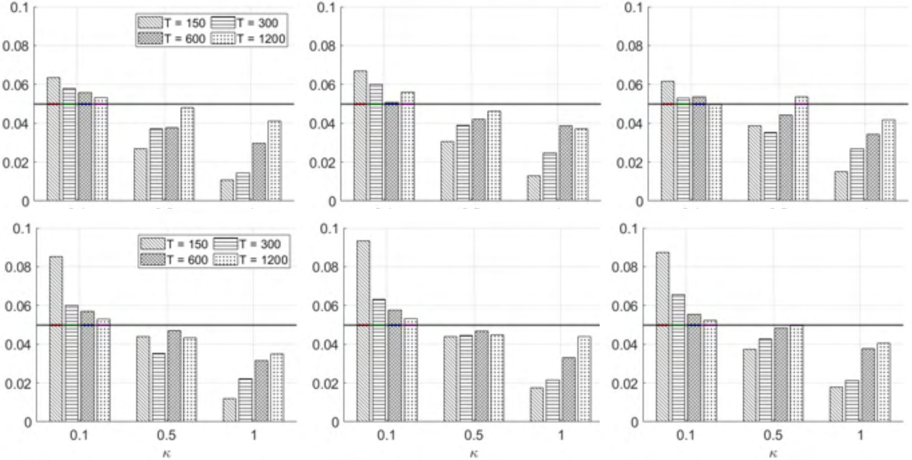
$K_1 = 2$ $K_1 = 3$ $K_1 = 4$

DGP2 Results under $\mathbb{H}_1$: Lag order $p = 1$ (top row) vs $p = 12$ (bottom row), $T_1 = T_{true} = 0.5T$



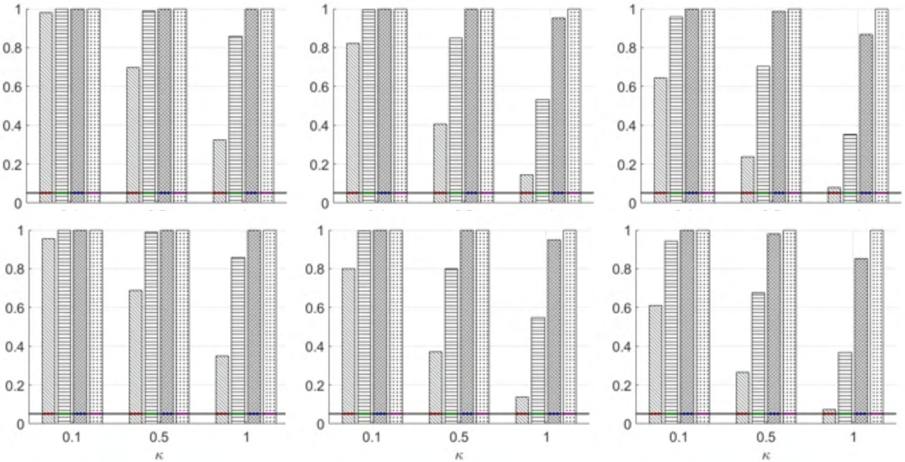
$K_1 = 2$ $K_1 = 3$ $K_1 = 4$

DGP2 Results under $\mathbb{H}_0$: $T_1 = T_{true} = 0.5T$ (top row) vs T_1 via stat. search (bottom row), $p = 1$



$K_1 = 2$ $K_1 = 3$ $K_1 = 4$

DGP2 Results under $\mathbb{H}_1$: $T_1 = T_{true} = 0.5T$ (top row) vs T_1 via stat. search (bottom row), $p = 1$



$K_1 = 2$ $K_1 = 3$ $K_1 = 4$