Impulse response estimation via flexible local projections

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Motivation

Estimating impulse response $x_t \rightarrow y_{t+h}$ via Local Projections
Jordà (2005)

$$y_{t+h} = \alpha^{(h)} x_t + \boldsymbol{\beta}^{(h)\prime} \boldsymbol{z}_t + u_{t+h}^{(h)} \qquad h = 0, 1, ..., H \tag{1}$$

An underlying **linearity assumption** for every h

Motivation

Implications: it cannot study dependence along..

1) state when shock hits

   'Is monetary policy still effective in a deep recession?'

2) size of shocks

   'Do financial shocks disrupt the economy more than proportionally as the size of the shock increases?'

3) sign of shocks

   'Are the effects of positive uncertainty shocks the flipped sign of the effects of negative uncertainty shocks?'

Motivation

Nonlinear extensions usually **assume functional forms**

$$y_{t+h} = \quad \mathrm{F}(g_t)\cdot\left[\alpha_1{}^{(h)} x_t + \boldsymbol{\beta}_1{}^{(h)\prime} \boldsymbol{z}_t\right] + \\ \left(1 - \mathrm{F}(g_t)\right)\cdot\left[\alpha_2{}^{(h)} x_t + \boldsymbol{\beta}_2{}^{(h)\prime} \boldsymbol{z}_t\right] + u_{t+h}^{(h)} \tag{2}$$

with $\mathrm{F}(g_t) \in [0, 1]$

Auerbach & Gorodnichenko (2013), Ramey & Zubairy (2018)

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\end{aligned} \tag{2}
$$

with $\mathrm{F}(g_t) \in [0,1]$

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Limitations

- relies on functional form
- estimation of parameters in $F(.)$ very challenging
- hard to study multiple nonlinearities jointly

Contribution of the paper

We propose a **non-parametric LP** procedure: **BART-LP**

Bayesian Additive Regression Trees

1) take BART from machine learning literature
   Chipman, George & McCulloch (2010), Hill, Linero & Murray (2020),
2) adapt it to Local Projections
3) Monte Carlo simulations
4) application to financial shocks

Contribution of the paper

Filling a gap in the literature

- BART has been applied to Vector Autoregressive models
  Huber & Rossini (2022)
  Huber, Koop, Onorante, Pfarrhofer & Schreiner (2020)
  Clark, Huber, Koop, Marcellino & Pfarrhofer (2021)
- Has not been used yet in Local Projections

Related literature

Bayesian Linear LPs

Miranda-Agrippino & Ricco (2021)

Nonlinear LPs

Ruisi (2019), Inoue, Rossi & Wang (2022)

VARs versus LPs

Kilian & Kim (2011), Alloza, Gonzalo & Sanz (2019), Breitung, Brüggemann et al. (2019) Herbst & Johannsen (2021), and Bruns & Lütkepohl (2022), Stock & Watson (2018) and Plagborg-Møller & Wolf (2021), Lusompa (2021)

IRF approximations

Barnichon & Brownlees (2019)

Plan of the talk

1. **Introduce BART**
2. Applying BART to LP
3. Application to financial shocks

The idea of BART

Some unknown conditional expectation function

$$y_t = E(y_t|\boldsymbol{x}_t) + \epsilon_t \tag{3}$$

$$E(\epsilon_t|\boldsymbol{x}_t) = 0 \tag{4}$$

The idea of BART

Some unknown conditional expectation function

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The parametric approach assumes a functional form

$$E(y_t|\boldsymbol{x}_t) \approx \boldsymbol{\beta}'\boldsymbol{x}_t \tag{5}$$

$$E(y_t|\boldsymbol{x}_t) \approx F(g_t)\boldsymbol{\delta}_1'\boldsymbol{x}_t + (1 - F(g_t))\boldsymbol{\delta}_2'\boldsymbol{x}_t \tag{6}$$

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BART approximates it with a sum of J binary regression trees

$$E(y_t|\boldsymbol{x}_t) \approx \sum_{i=1}^{J} f_i(\boldsymbol{x}_t|\Gamma, \boldsymbol{\mu}) \tag{7}$$

that build on splitting rules on the space of covariates

BART, an introduction

Suppose you have data on y_t and two explanatory variables (x_{1t}, x_{2t}), want to compute prediction $\hat{Y}$ for

$$x_1 = 10 \qquad x_2 = 1.3$$

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The linear approach

- use (y_t, x_{1t}, x_{2t}) to estimate $y = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{1t} + \epsilon_t$
- compute $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 \cdot 10 + \hat{\beta}_2 \cdot 1.3$

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- compute $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 \cdot 10 + \hat{\beta}_2 \cdot 1.3$

BART

- use (y_t, x_{1t}, x_{2t}) to estimate posterior distribution of the tree structure and remaining parameters
- simulate $\hat{Y}$ using the trees

BART, an introduction

$$\hat{Y}_{x_1=10,\ x_2=1.3} = \ ?$$

1) p = probability that a node splits
2) i = index of variable used to assess direction of split
3) c = cut-off value used to assess direction of split
4) μ = terminal node value

X_1 = 10
X_2 = 1.3

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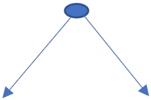

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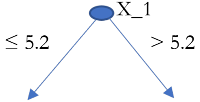

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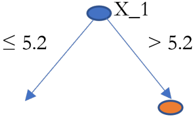

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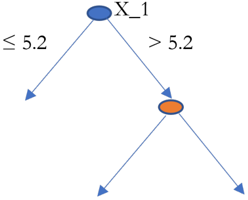

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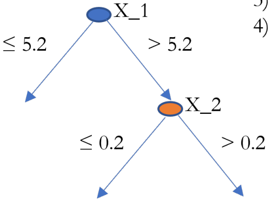

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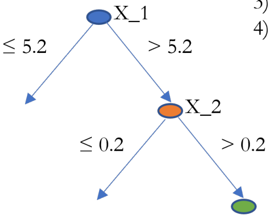

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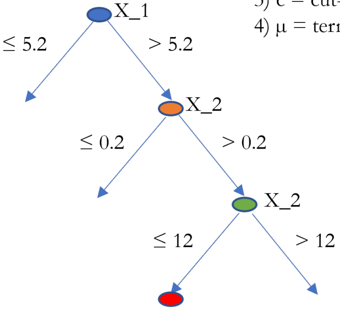

X_1 = 10
X_2 = 1.3

BART, an introduction

$$\hat{Y}_{x_1=10,\ x_2=1.3} = \ 2.22$$

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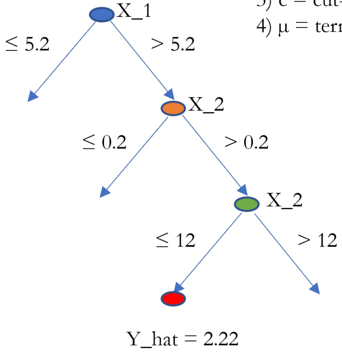

X_1 = 10
X_2 = 1.3

BART, an introduction

$$\hat{Y}_{x_1=10,\ x_2=1.3} = \ -1.12$$

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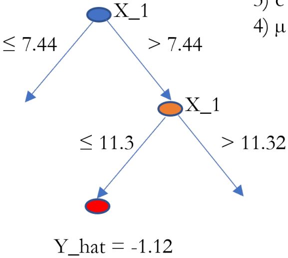

X_1 = 10
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BART, an introduction

$$\hat{Y}_{x_1=10,\ x_2=1.3} = \ 11.052$$

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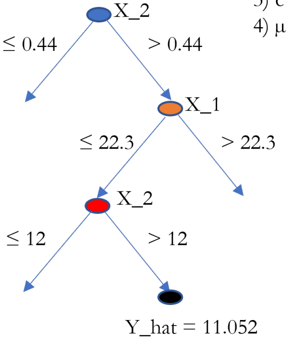


X_1 = 10
X_2 = 1.3

BART, an introduction

BART can treat different parts of the parameter space of $y, \boldsymbol{x}$ differently

A single tree generates

$$\hat{Y} = f_i(\boldsymbol{x}|\Gamma, \boldsymbol{\mu}) \tag{8}$$

Sum of trees

$$E(y_t|\boldsymbol{x}) \approx \sum_{i=1}^{J} f_i(\boldsymbol{x}|\Gamma, \boldsymbol{\mu}) \tag{9}$$

BART, an introduction

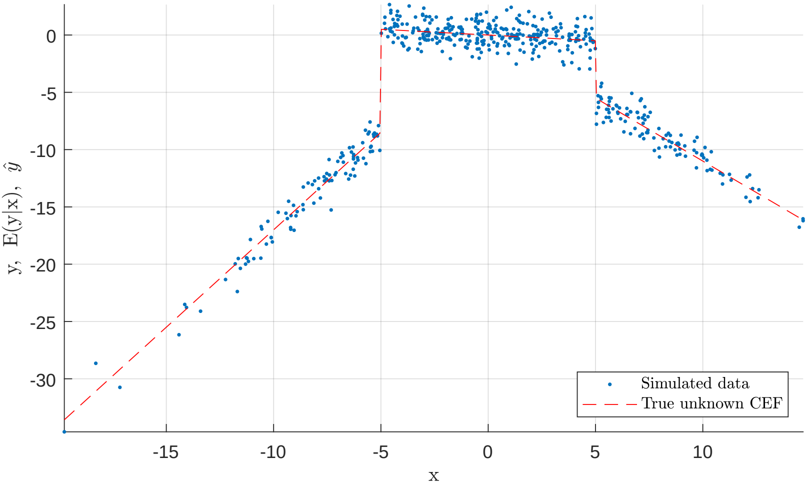
A numerical illustration

$$
\begin{aligned}
y_t = &+1.7 \cdot x_t \cdot \mathrm{I}(x_t < -5) &+ \\
&-0.1 \cdot x_t \cdot \mathrm{I}(-5 \leq x_t < 5) &+ \\
&-1.1 \cdot x_t \cdot \mathrm{I}(x_t \geq 5) &+ \ \epsilon_t
\end{aligned} \tag{10}
$$

$$x_t \sim N(0,1) \tag{11}$$

$$\epsilon_t \sim N(0,1) \tag{12}$$

BART, an introduction



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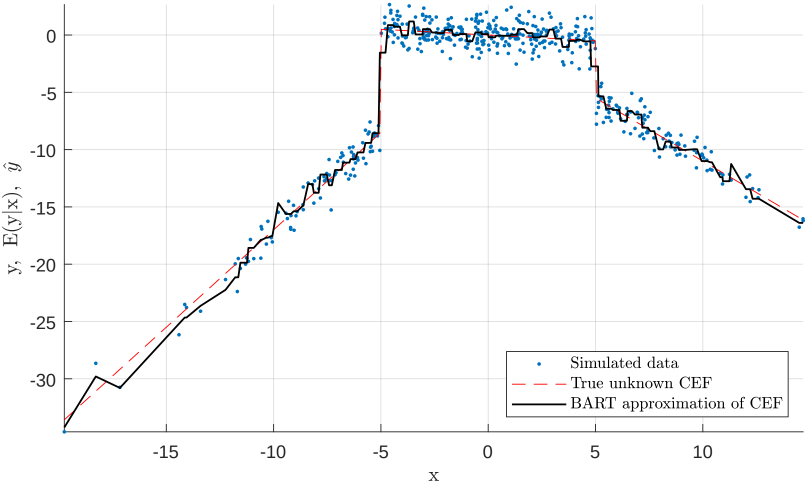

Plan of the talk

1. Introduce BART
2. **Applying BART to LP**
3. Application to financial shocks

Applying BART to LPs

Replace

$$y_{t+h} = \alpha^{(h)} x_t + \boldsymbol{\beta}^{(h)\prime} \boldsymbol{z}_t + u_{t+h}^{(h)} \tag{13}$$

with

$$y_{t+h} = \sum_{j=1}^{J} f_{h,j}(x_t, \boldsymbol{z}_t | \Gamma_j, \boldsymbol{\mu}_j) + \epsilon_{t+h}^{(h)} \tag{14}$$

Applying BART to LPs

Autocorrelation in $\epsilon_{t+h}^{(h)}$, $h \geq 1$

- control for estimated residuals for $h = 0$, Lusompa (2021)

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Identification

- a preliminary SVAR for impulse vector, Jordà (2005)
- controlled observables or instruments, Barnichon & Brownlees (2019), Plagborg-Møller & Wolf (2021)
- true shocks (in simulations)

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Generalized impulse responses, Koop et al. (1996)

- simulate from trees as difference in predictions
- predictions depend on conditioning values (sign, size, history matter)

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3. **Application to financial shocks**

Application to financial shocks

- Financial shocks and the real economy in the US
- Barnichon, Matthes & Ziegenbein (2022) discuss a large disagreement in the empirical effects of financial shocks
  - Large: narrative approach, Romer & Romer (2017) (EBP $\rightarrow$ 1%, GDP -6%, persistent)
  - Small: linear SVAR, Gilchrist & Zakrajšek (2012) (EBP $\rightarrow$ 1%, GDP -2%, temporary)

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Nonlinear model, find *adverse* shocks stronger effects than favourable shocks. Confirmed by Forni et al. (2021)

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- We use BART-LP to revisit their result and extend to size of shock

Application to financial shocks

- 6 variables: growth of
  - industrial production
  - CPI inflation
  - unemployment rate
  - Excess bond premium by Gilchrist & Zakrajšek (2012),
  - stock returns
  - federal funds rate
- Sample size: 1973M1 - 2022M2 (can include Covid)
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Application to financial shocks

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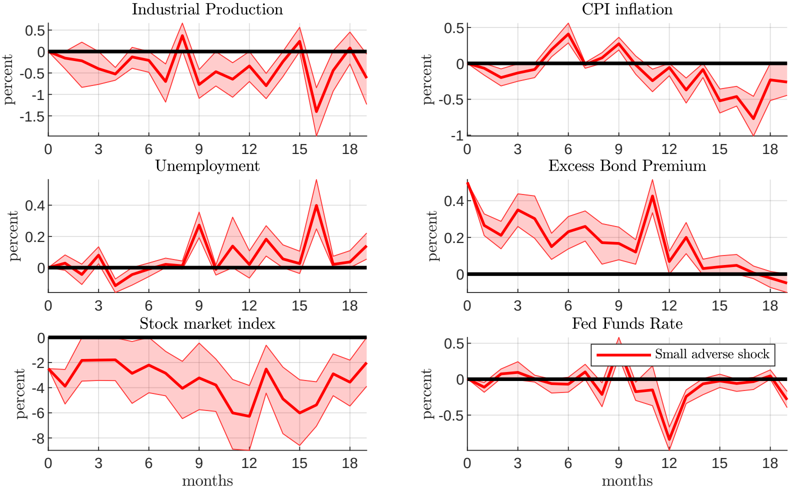
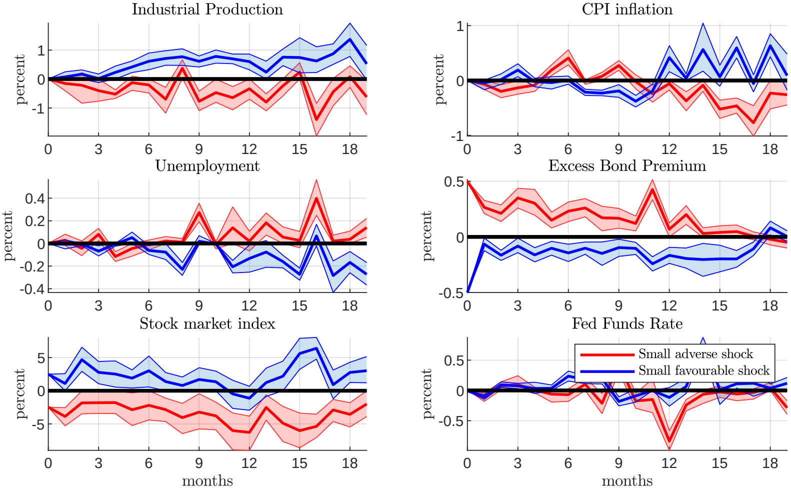
- Sample size: 1973M1 - 2022M2 (can include Covid)
- As in Gilchrist & Zakrajšek (2012): financial shock affects only fast moving variables contemporaneously (linear VAR)
- Two nonlinearities, jointly
  - adverse versus favourable financial shocks
  - small (50 bps ≈ 1 *std*) versus large (100 bps ≈ 2 *std*) shocks

Application to financial shocks



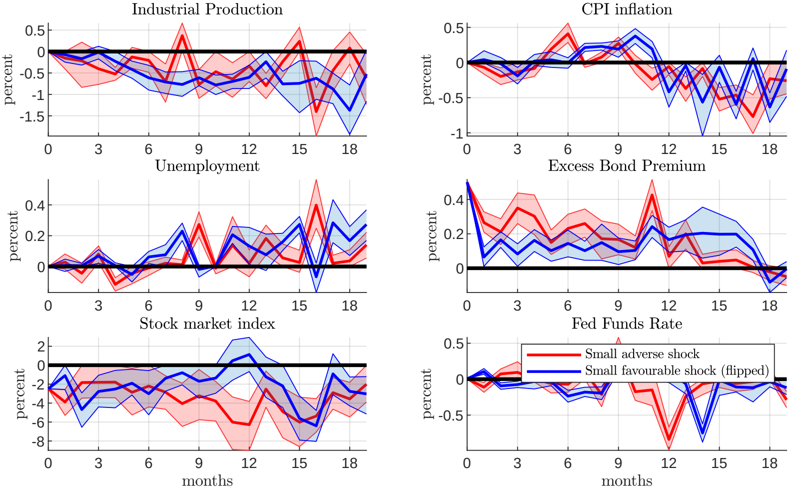
Small adverse shocks are recessionary

Application to financial shocks



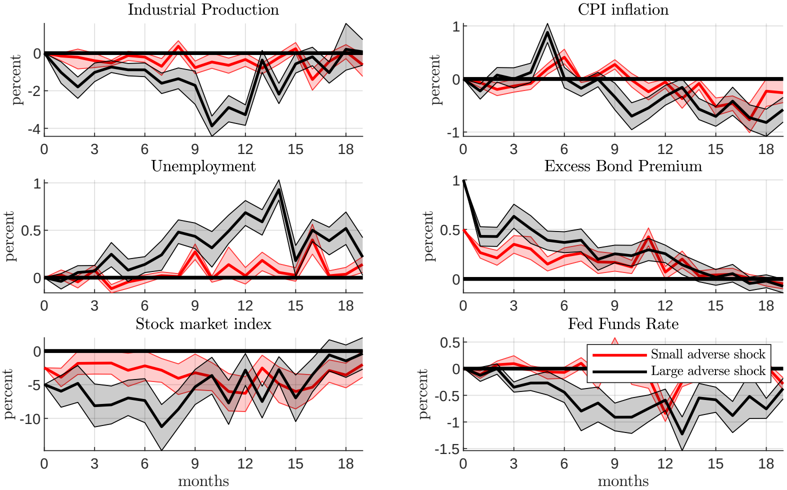
Small adverse vs. favourable shocks

Application to financial shocks



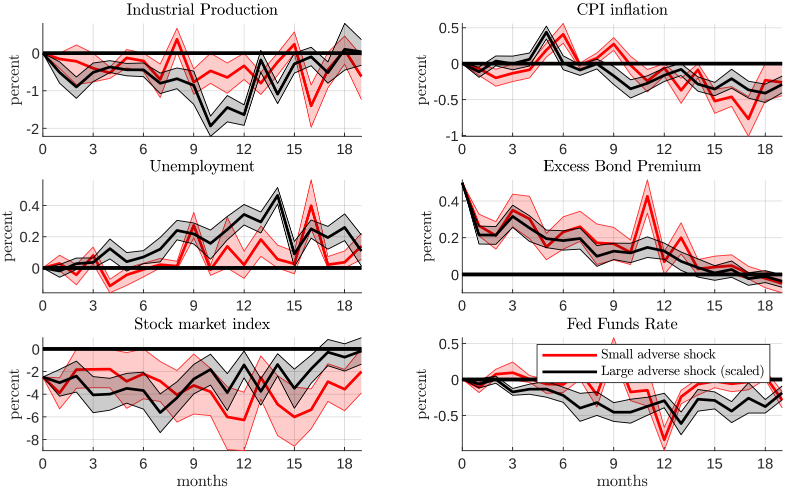
Small adverse vs. favourable shocks have symmetric effects

Application to financial shocks



Effect of adverse shocks increases more than proportionately in size

Application to financial shocks



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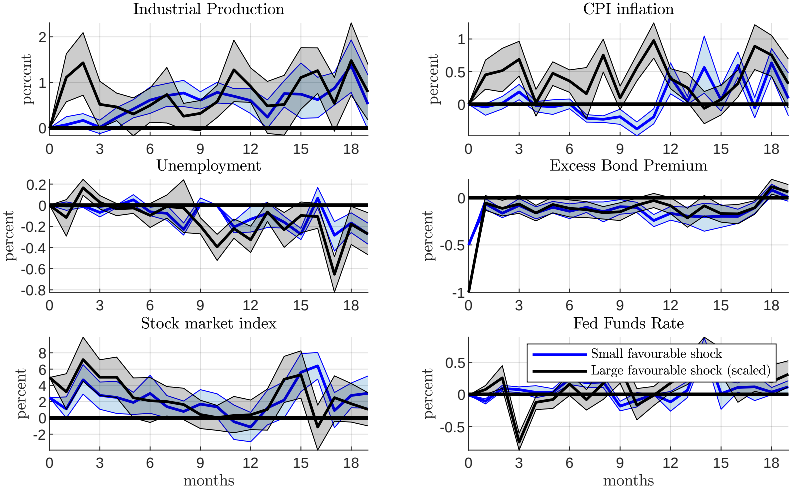
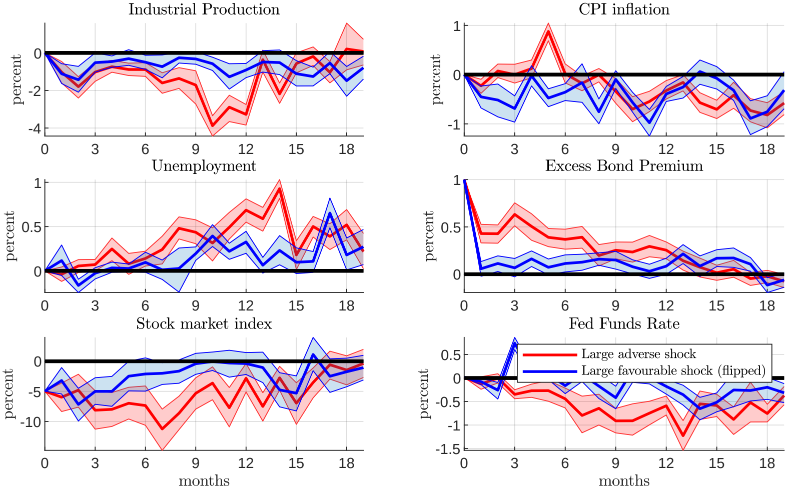


Effect of favourable shocks does not increase much in size

Application to financial shocks



Large adverse shocks more effective than large favourable shocks

Conclusions

Local projections are frequently used to estimate impulse responses

We propose to use BART to estimate LPs non-parametrically

Large financial shocks generate asymmetric effects: stronger if adverse shocks

Alloza, M., Gonzalo, J. & Sanz, C. (2019), 'Dynamic effects of persistent shocks'.

Auerbach, A. J. & Gorodnichenko, Y. (2013), Fiscal multipliers in recession and expansion, *in* A. Alesina & F. Giavazzi, eds, 'Fiscal policy after the financial crisis', University of Chicago Press, pp. 63–98.

Barnichon, R. & Brownlees, C. (2019), 'Impulse Response Estimation by Smooth Local Projections', *The Review of Economics and Statistics* **101**(3), 522–530.

Barnichon, R., Matthes, C. & Ziegenbein, A. (2022), 'Are the effects of financial market disruptions big or small?', *Review of Economics and Statistics* **104**(3), 557–570.

Breitung, J., Brüggemann, R. et al. (2019), 'Projection estimators for structural impulse responses', *University of Konstanz Department of Economics Working Paper Series* **5**.

Bruns, M. & Lütkepohl, H. (2022), 'Comparison of local projection estimators for proxy vector autoregressions', *Journal of Economic Dynamics and Control* **134**, 104277.

Chipman, H. A., George, E. I. & McCulloch, R. E. (2010), 'BART: Bayesian additive regression trees', *The Annals of Applied Statistics* **4**(1), 266–298.

Clark, T. E., Huber, F., Koop, G., Marcellino, M. & Pfarrhofer, M. (2021), 'Tail forecasting with multivariate Bayesian additive regression trees'.

Forni, M., Gambetti, L., Maffei-Faccioli, N. & Sala, L. (2021), 'Nonlinear transmission of financial shocks: Some new evidence'.

Gilchrist, S. & Zakrajšek, E. (2012), 'Credit spreads and business cycle fluctuations', *American Economic Review* **102**(4), 1692–1720.

Herbst, E. & Johannsen, B. K. (2021), 'Bias in local projections'.

Hill, J., Linero, A. & Murray, J. (2020), 'Bayesian additive regression trees: A review and look forward', *Annual Review of Statistics and Its Application* **7**(1), 251–278.

Huber, F., Koop, G., Onorante, L., Pfarrhofer, M. & Schreiner, J. (2020), 'Nowcasting in a pandemic using non-parametric mixed frequency VARs', *Journal of Econometrics* .

Huber, F. & Rossini, L. (2022), 'Inference in bayesian additive vector autoregressive tree models', *The Annals of Applied Statistics* **16**(1), 104–123.

Inoue, A., Rossi, B. & Wang, Y. (2022), 'Local Projections in Unstable Environments'.

Jordà, Ò. (2005), 'Estimation and inference of impulse responses by local projections', *American Economic Review* **95**(1), 161–182.

Kilian, L. & Kim, Y. J. (2011), ‘How reliable are local projection estimators of impulse responses?’, *Review of Economics and Statistics* **93**(4), 1460–1466.

Koop, G., Pesaran, M. H. & Potter, S. M. (1996), ‘Impulse response analysis in nonlinear multivariate models’, *Journal of Econometrics* **74**(1), 119–147.

Lusompa, A. (2021), ‘Local Projections, Autocorrelation, and Efficiency’, (RWP 21-01).

Miranda-Agrippino, S. & Ricco, G. (2021), ‘Bayesian Local Projections’.

Plagborg-Møller, M. & Wolf, C. K. (2021), ‘Local Projections and VARs Estimate the Same Impulse Responses’, *Econometrica* **89**(2), 955–980.

Ramey, V. A. & Zubairy, S. (2018), ‘Government spending multipliers in good times and in bad: Evidence from US historical data’, *Journal of Political Economy* **126**(2), 850–901.

Romer, C. D. & Romer, D. H. (2017), ‘New evidence on the aftermath of financial crises in advanced countries’, *American Economic Review* **107**(10), 3072–3118.

Ruisi, G. (2019), ‘Time-Varying Local Projections’, (891).

Stock, J. H. & Watson, M. W. (2018), ‘Identification and estimation of dynamic causal effects in macroeconomics using external instruments’, *The Economic Journal* **128**(610), 917–948.