

Nonlinear Prices, Homogeneous Goods, Search*

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Abstract

We analyze competition on nonlinear prices in homogeneous goods markets with consumer search. In equilibrium firms offer two-part tariffs consisting of a linear price and lump-sum fee. The equilibrium production is socially efficient as the linear price of equilibrium two-part tariffs equals to the production marginal cost. Firms thus compete in lump-sum fees, which are dispersed in equilibrium. We show that sellers enjoy higher profit, whereas consumers are worse-off with two-part tariffs than with linear prices. The competition softens because with two-part tariffs firms can make effective per-consumer demand less elastic than the actual demand.

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1 Introduction

In this paper we analyze nonlinear prices in homogeneous goods market with imperfect competition. Nonlinear prices are widespread. Examples include markets for utilities, consumer financial credits, and telecommunication services. In these markets goods are fairly homogeneous across sellers, while buyers are usually offered some form of nonlinear prices. For instance, German electricity providers offer two-part tariffs which consists of a linear price and a monthly lump-sum fee. A total price a buyer pays for a credit consists of interest payments and a fixed service fee. A mobile-phone buyer's total payment includes a monthly fee for a fixed data and a lump-sum fee at a time of signing a contract.

Despite prevalence of nonlinear prices in competitive homogeneous goods markets, models of such markets remain understudied. The reason for this is the argument that, in markets where firms compete on prices, equilibrium offers do not depend on the type of pricing scheme firms employ. This is because sellers do not possess any market power. If a seller offers a slightly more expensive deal than its rivals, it loses all its buyers to the competitors. Therefore independent of the pricing scheme, production is socially optimal and consumers receive the entire social surplus.

In real-world markets, however, sellers exercise market power even in homogeneous goods markets as some consumers cannot easily walk away to competing sellers when these consumers are offered a bad deal. The reason for this is that inspecting sellers' offers entails a significant time cost. For some buyers this cost may be so high that they prefer foregoing search and buying from a current seller at a high price (see e.g., [De los Santos et al. \(2012\)](#), [Kaplan and Menzio \(2015\)](#), [Alexandrov and Koulayev \(2017\)](#), [Lach and Moraga-González \(2017\)](#), [Gorodichenko et al. \(2018\)](#), [Gugler et al. \(2018\)](#), [Galenianos and Gavazza \(2020\)](#)). While sellers possess market power over these buyers, sellers also have incentive to compete for consumers who can easily walk away to competitors. These two opposing incentives causes imperfect competition which oftentimes results in price dispersion (e.g., [Varian \(1980\)](#), [Burdett and Judd \(1983\)](#), [Stahl \(1989\)](#)). Although there is a large body of literature studying consumer search markets, they are typically limited to linear prices. There are a few reasons for this. First, linear prices are easy to examine and thus they serve as a natural first step in analyzing search markets. Also in markets with indivisible goods and unit demand, equilibrium outcomes with certain types nonlinear

prices, such as two-part tariffs, coincide with that of linear prices.

In this paper we employ traditional models of sequential consumer search with homogeneous goods to study nonlinear prices. Standard features of such models are as follows. Oligopoly firms supply homogeneous goods to a large number of buyers and compete on prices. Consumers have downward sloping demand and differ in their costs of learning firms' offers. When facing a same offer, a buyer with a low search cost is likely to sample an offer of another firm whereas a buyer with a high search cost is likely to forego search and buy outright. The novel feature of our model is that firms can offer nonlinear prices.

We show that firms compete on two-part tariffs in equilibrium. A two-part tariff consists of a linear price and a fixed fee. The linear price of any equilibrium two-part tariff is equal to the production marginal cost. Therefore production is socially optimal with nonlinear prices. To contrast, if firms can offer linear prices only, the equilibrium prices (which are dispersed) are higher than the production marginal cost (Stahl (1989)). Thus the social surplus with linear prices is lower than that with nonlinear prices.

Fixed fees of equilibrium two-part tariffs are dispersed, meaning that sellers compete in fixed fees to attract buyers with low search costs. The equilibrium profit is higher and consumer surplus is lower with two-part tariffs than with linear prices. To understand the intuition one can think of firms offering two-part tariffs as them offering a unit of a hypothetical product, which contains socially optimal amount of the real product, at a fixed fee. Hence, sellers effectively face inelastic demand per-buyer. In contrast, if firms can only compete in linear prices, they set prices at the elastic part of the demand function. As sellers exercise greater market power with inelastic demand than with elastic one, their profits are higher with nonlinear prices than with linear prices. This in turn harms buyers.

Our paper contributes to two stands of the literature. One is on nonlinear prices. Early studies on nonlinear prices focus predominantly on markets with a monopolist seller (e.g., Oi (1971), Spence (1977), Mussa and Rosen (1978), Stokey (1979)). Later studies extend these models to competitive environment, yet their analysis centers on markets with differentiated products (e.g., Hayes (1987), Armstrong and Vickers (2001), Rochet and Stole (2002), Yin (2004), Garrett et al. (2019), Tamayo and Tan (2021)). In some of these studies it is possible to interpret qualities that firms offer as quantities offered. Doing so one would reformulate a market of vertically differentiated products as a

homogeneous goods market where firms compete in quantities offered, i.e., they compete a la Cournot. This would deviate from our study as we consider competition on prices.

The other branch of literature that is close to our paper is the literature on consumer search. Traditional studies of consumer search with homogeneous product and linear prices include [Salop and Stiglitz \(1977\)](#), [Varian \(1980\)](#), [Burdett and Judd \(1983\)](#), and [Stahl \(1989\)](#). [Shelegia and Wilson \(2021\)](#) is a recent paper that touches the issue of nonlinear prices in markets where buyers have imperfect information on firms' offers. The authors provide a useful method to establish equilibrium with two-part tariffs in a model where consumers differ in the number of offers they observe (as in [Butters \(1977\)](#) and [Varian \(1980\)](#)). One of the main differences of their study is that buyers do not really search, i.e., they passively wait for advertisements to arrive and then decide from which firm to buy goods.¹ In our paper buyers can actively search for different deals. Also whereas the contribution of their paper is largely methodological, we are interested in welfare implications of nonlinear prices. To the best of our knowledge [Janssen and Parakhonyak \(2013\)](#) is the only paper that studies a form of nonlinear prices in homogeneous goods markets with search and undertakes welfare analysis. The authors consider a market where a firm can match its price to a competitor's price if a buyer credibly demonstrates that the competing firm charges a price lower than the price of the firm under question. This form of nonlinear price is optimal only if consumers with high search costs can somehow observe prices of multiple sellers without searching. In our paper a consumer has to actively search to observe a firm's deal.

The rest of the paper is organized as follows. In the section we lay out our main model. In sections 3 and (4) we provide equilibrium analysis and welfare analysis, respectively. In section 5 we extend the main model to allow for a more general distribution of search costs and for a mixture of simultaneous and sequential search. We show that our main result is robust to such extensions. The final section concludes.

¹In the appendix of their paper, [Shelegia and Wilson \(2021\)](#) consider an extension of their main model where buyers who received an ad can search one more firm.

2 Model

There are n identical firms that produce homogeneous goods at a constant marginal cost normalized to zero. Firms compete in prices. A unit mass of buyers, or consumers, wish to consume the product. Each consumer is willing to buy $q(p)$ amount of the product at price p . Before searching, consumers do not know offers of firms and need to engage in sequential search to learn them. To avoid *Diamond-paradox* equilibrium where consumers do not search beyond the first firm (Diamond (1971)), we assume that a positive share of buyers observe offers of multiple firms.² Specifically, we say that $\lambda \in (0, 1)$ share of buyers have zero search cost and thus they observe all prices in the market. The remaining share of buyers incur a positive search cost, denoted by $s > 0$, each time they search a firm. Following a terminology proposed by Stahl (1989) we refer to consumers with zero search cost as *shoppers* and those with a positive search cost as *nonshoppers*. Identity of each consumer is her private knowledge. To ensure that there is some search we assume that searching the first firm is free.

The demand function $q(p)$ is non-increasing in p with $q(\bar{p}) = 0$ for some finite $\bar{p} > 0$. Letting $\pi(p) \equiv q(p)p$ be the revenue per consumer that price p generates, we assume that there exists p^m that maximizes $\pi(p)$. This assumption is justified if the demand elasticity is increasing in price:

$$-\frac{q'(p)p}{q(p)} \text{ strictly increases with } p. \quad (1)$$

We let $\pi^m \equiv \pi(p^m)$ to simplify the notation.

It is useful to define a function $v(\pi)$ to represent a surplus a buyer enjoys when generating revenue π . This function is decreasing in π as per-consumer revenue is increasing in price on $[0, p^m]$ and consumer surplus $\int_p^{\bar{p}} q(z)dz$ decreases with p . Assumption in (1) implies that $v(\pi)$ is strictly concave on $[0, \pi^m]$, which can be seen differentiating $v(\pi(p))$ to obtain

$$v'(\pi(p)) = -\frac{1}{1 + \frac{q'(p)p}{q(p)}}. \quad (2)$$

The timing of the game is as follows. First, firms simultaneously choose their sales strategies. Second, without knowing firms' offers, consumers engage in search. Having

²This is perhaps the most common way of overcoming Diamond-paradox in consumer search literature (e.g., Salop and Stiglitz (1977), Varian (1980), Stahl (1989)).

observed an offer of at least one firm, consumers may make purchases.

The solution concept is Reservation price equilibrium (RPE). An RPE is a perfect Bayesian equilibrium where buyers search according to the reservation price rule. If in a model where firms charge linear price a buyer searched k number of firms, her reservation price is the lowest of those k prices such that she is indifferent between buying at that price and searching further. If the lowest price a buyer has observed is lower than the reservation price, she buys outright. If this price is higher than the reservation price, she searches further. It is typical to assume that buyers have passive off-equilibrium beliefs. It means that if a buyer observes a deal which is not a part of equilibrium, she believes that sellers, whom she has not searched yet, play equilibrium strategy. In a model where firms employ nonlinear prices, we need to replace a linear price with a total price and the above definition of equilibrium goes through. We focus on symmetric equilibria.

3 Equilibrium

We first consider a case where firms can offer nonlinear prices and then a case where firms compete in linear prices. For each case, the analysis consists of two parts. Assuming that there is an equilibrium in nonlinear prices, we characterize this equilibrium. We later verify that the equilibrium indeed exists.

3.1 Nonlinear Prices

We start the analysis by making some important observations. First, it is clear that firms never offer nonlinear prices that yield a negative per-consumer profit. It is also clear that it is sub-optimal to offer a deal that renders negative utility to buyers, as buyers would never accept such a deal and, thus, a firm offering this deal does not make any sales.

To make further observations it is useful to think of firms as offering utilities to buyers. Our next observation is that a symmetric equilibrium strategy of firms must be in mixed-strategies. The reasoning can be established by contradiction. Suppose that there exists a symmetric equilibrium in a pure strategy. Due to symmetry, all firms must offer the same level of utility to buyers in equilibrium and therefore serve equal share of nonshoppers as well as shoppers. However, an individual firm has an incentive to offer a slightly higher

utility to attract all shoppers and increase its profit, a contradiction.

We generalize this result in the following lemma.

Lemma 1. *If there exists a symmetric equilibrium with nonlinear prices, it must be in mixed-strategies and an equilibrium distribution of utilities has no atom.*

The understanding why firms do not offer any utility with a strictly positive probability is similar to the idea why there does not exist a symmetric equilibrium in pure strategies. Suppose for contradiction that a (symmetric) equilibrium distribution of utilities offered contains an atom at some utility level. This means that all firms tie at that utility level with a strictly positive probability, in which case they serve equal share of buyers. This cannot be optimal as an individual firm has an incentive to offer a slightly higher utility to attracts all shoppers and, thus, discontinuously raise its profit. Thus we arrive at a contradiction.

For our next observation we define a reservation utility of a nonshopper at a given search round as the lowest utility at which she buys outright foregoing search. In a symmetric equilibrium a nonshopper samples utility from the same utility distribution in each search round. In this case it is well-established that the reservation utility of a nonshopper is stationary (Kohn and Shavell (1974)), i.e., it is the same at each search round. Clearly, if a nonshopper searches all firms she buys at a firm offering the highest (positive) utility level. If the highest utility level offered is negative she does not make a purchase. This bring us to the next observation which we state in the next lemma.

Lemma 2. *If there exists a symmetric equilibrium with nonlinear prices and a positive reservation utility, no firm offers a utility below the reservation utility.*

The reasoning is by contradiction. Assume that some, or all, firms offer utility levels lower than the reservation utility. All consumers who happen to visit such a firm search further. Therefore this firm makes positive sales only if all the other firms happen to offer a utility lower than the utility of the firm under question. If, however, all firms offer such low utilities, it is optimal to offer the highest of those utilities, which follows from a standard Bertrand-type of argument. This highest utility must be equal to the reservation utility since there is no atom in the utility distribution, a contradiction.

We are now ready to make the final observation about firms' strategies.

Lemma 3. *If there exists a symmetric equilibrium with nonlinear prices and a positive reservation utility, cumulative distribution of utilities does not have a flat region in the support.*

The reasoning is by contradiction, and therefore assume that an equilibrium distribution of utilities has a flat region in the support. A firm expects to sell to the same share of buyers both at the highest utility level and at the lowest utility level in that flat region. In equilibrium an individual firm must be indifferent between offering those two utility levels. This can be the case only if per-consumer revenues generated by these utility levels are the same. These per-consumer revenues are the same only if the social surplus generated by offering the low utility level is lower than that generated by offering the high utility level. However, a firm is better-off if it chooses the high utility level but charges an additional lump-sum fee so that the final utility offered is the same as the low utility level. This leads to a contradiction.

The above three lemmas imply that, given a reservation utility of nonshoppers, the distribution of utilities is continuously increasing in its compact support. We use this fact to determine optimal (nonlinear) pricing strategy that results in such distribution of utilities. Letting (p, t) represent a two-part tariff where p is the linear price and t is a lump-sum fee, we state our next result in the following lemma.

Lemma 4. *If there exists a symmetric equilibrium in nonlinear prices, firms compete in two-part tariffs (p, t) where $p = 0$.*

The reasoning is as follows. Let utility level \tilde{u} be in the support of the utility distribution conditional on the reservation utility. If a nonlinear price that results in an offer of this utility level also generates a social surplus given by S , the associated per-consumer revenue equals to $S - \tilde{u}$. A firm offering utility \tilde{u} then chooses a nonlinear price that maximizes the social surplus. It is easy to see that the social surplus is maximized at a linear price equal to the production marginal cost. Then, a two-part tariff is a part of equilibrium if its linear price equals to the production marginal cost and its lump-sum fee equals to $v(0) - \tilde{u}$, where $v(0) = \int_0^{\bar{p}} q(z) dz$ is the highest social surplus.

Lemma 4 allows us to reformulate firms' problem in terms of two-part tariffs. Importantly, the lemma informs us that firms compete in fixed fees to attract shoppers. We let H represent the distribution of lump-sum fees. Note that a lump-sum fee and a utility

offered are linearly related through the following equation: $t(u) = v(0) - u$. Therefore, lemmas 1-3 apply to characterize lump-sum fees offered in equilibrium. In particular, these lemmas imply that H has a compact support and is continuously increasing in its support. Also the highest fixed-fee in the support must be no greater than the lowest of $v(0)$ and the reservation fee, which we define as follows.

A reservation fee, denoted by t_R , is the highest lump-sum fee at which nonshoppers buy outright, thus terminating search. To determine the reservation fee we equate the expected benefit of search to the cost of search when t_R is the lowest price a nonshopper has observed so far:

$$\int_{\underline{t}}^{t_R} H(t) dt = s. \quad (3)$$

Here \underline{t} is the lowest lump-sum fee in the support of H . Notice that the expected benefit of search, given by the left-hand side of the equation, strictly increases with t_R . Then, there exists a unique reservation fee that solves the equation if a solution exists. A solution clearly exists for small search costs. If a solution does not exist we set $t_R = v(0)$.

While two-part tariffs—with their linear prices equal to the production marginal cost and lump-sum fees distributed according to H —fully characterize firms optimal strategies, the reservation fee characterizes nonshoppers' optimal search strategy. The equilibrium so characterized exists if the firms' (nonlinear) pricing policies and nonshoppers' optimal search rule are consistent. This is indeed the case as shown in the next proposition.

Proposition 1. *For any $\lambda \in (0, 1)$ and $s > 0$, there exists a unique symmetric RPE given by (p, H, t_R) . The equilibrium two-part tariffs are given by $p = 0$ and H , where*

$$H(t) = 1 - \left[\frac{1 - \lambda}{n\lambda} \left(\frac{\bar{t}}{t} - 1 \right) \right]^{\frac{1}{n-1}} \quad \text{with support } [\underline{t}, \bar{t}] \quad (4)$$

and $\bar{t} = \min\{t_R, v(0)\}$. Furthermore, there exists $\bar{s} > 0$ such that t_R is determined by (3) for $s \leq \bar{s}$.

The proof is in the appendix and the intuition is as follows. To determine H for any given t_R we use the fact that an individual firm is indifferent of setting any fixed-fee in the support of H and prefer them to fixed-fees outside the support. (Note that \underline{t} solves $H(\underline{t}) = 0$.) This distribution of fees is then consistent with nonshoppers' optimal search rule in (3) if the expected benefit of search increases with t_R given that H is a function

of t_R too. We show in the appendix that this is indeed the case.

3.2 Linear Prices

We now turn our attention to the case where firm can offer only linear prices, as in [Stahl \(1989\)](#). Since the model with linear prices is a special case of a model with two-part tariffs where lump-sum fees are set equal to zero, similar arguments from the previous subsection apply to determine an equilibrium. To avoid repetition we summarize the equilibrium result with linear prices as follows.

There exists a unique symmetric equilibrium where firms choose a random per-consumer revenue according to F , determined by

$$F(\pi) = 1 - \left[\frac{1 - \lambda}{n\lambda} \left(\frac{\bar{\pi}}{\pi} - 1 \right) \right]^{\frac{1}{n-1}}, \text{ with support } [\underline{\pi}, \bar{\pi}]. \quad (5)$$

Here $\underline{\pi}$ solves $F(\underline{\pi}) = 0$ and $\bar{\pi} = \min \{ \pi^m, \pi_R \}$ where π_R is a reservation revenue which is uniquely determined by

$$\int_{\underline{\pi}}^{\pi_R} (-v'(\pi))F(\pi)d\pi = s \quad (6)$$

if a solution exists. A solution definitely exists for sufficiently small s as the left-hand side of the equation, which represents the benefit of search, is positive and increasing in $\pi_R > 0$. With a slight abuse of notation we let $\bar{s} > 0$ be the cutoff value of the search cost such that the solution to (6) exists for $s \leq \bar{s}$. If a solution does not exist we set $\pi_R = \pi^m$. The equilibrium firm profit is then $(1 - \lambda)\bar{\pi}/n$.

Proposition 2 (Proposition 1 in [Stahl \(1989\)](#)). *For any $0 < \lambda < 1$ and $n \geq 2$, there exists a unique symmetric RPE given by (F, π_R) where F is given by (5) and π_R is determined by (6) for $s \leq \bar{s}$ and set equal to π^m otherwise.*

For the proof we refer to [Stahl \(1989\)](#), although a reader can easily establish the proof by following a line of argument presented for the case of nonlinear prices. The intuition is similar to one behind [Proposition 1](#) and thus we do not discuss it.

4 Welfare Analysis

Our main aim is to compare the equilibrium market outcomes with nonlinear prices to those with linear prices. The following proposition states the result.

Proposition 3. *Total surplus and firm profit are higher, whereas consumers surplus is lower with nonlinear prices than with linear prices.*

The reasoning behind the first part follows from our equilibrium analysis. Recall that the equilibrium production is socially optimal with nonlinear prices as the equilibrium linear price of two-part tariffs equals to the production marginal cost. If firms compete in linear prices only, they choose positive levels of per-consumer revenue in equilibrium, which means that the equilibrium prices are bounded above the production marginal cost. Therefore production is lower with linear prices than with nonlinear prices.

The proof of the last two parts of the proposition is in the appendix. The main intuition lies in the *effective* per-consumer demand firms face under the two different pricing schemes. We know that with linear prices firms price at the elastic part of the demand curve in equilibrium. One can think that with nonlinear prices firms face a unit (i.e., inelastic) demand for a hypothetical product which contains $q(0)$ amount of the real product. Firms' market power is higher with inelastic demand than with elastic demand. Therefore nonlinear prices allow sellers to soften competition by transforming the effective demand. This in turn raises profits and harms buyers.

5 Extensions

In this section we demonstrate that our main result holds in different versions of our model. We consider two extensions. In one of them, a buyer's search cost is a random draw from a distribution function that is continuously increasing in its convex support (e.g., [Stahl \(1996\)](#)). In the other extension we assume homogeneous search cost but heterogeneity in success of observing offers. Specifically a searching buyer obtains offers from an unknown number of firms (e.g., [Wilde \(1977\)](#), [Burdett and Judd \(1983\)](#)). This search protocol, known as *noisy search*, equips sequential search with some properties of nonsequential search.

5.1 Continuous Search Cost Distribution

We first examine a case where each buyer's search cost is a random draw from interval $[0, \bar{c}]$ according to log-concave distribution function G with density g . We assume that \bar{c} is positive and allow it to be infinite.

[Stahl \(1996\)](#) shows that there exist a continuum of symmetric equilibria in linear prices. In all these equilibria consumers do not search beyond the first firm and all firms charge the same price. In terms of per-consumer revenue, any revenue in interval $[\pi^*, \pi^m]$ can be supported in equilibrium, where π^* solves

$$\pi^* = \arg \max_{\pi} (1 - G[v(\pi^*) - v(\pi)]) \pi.$$

Log-concavity of G ensures a unique solution. The reason why π^* can be supported in equilibrium is as follows. If all sellers charge a price to extract π^* , an individual deviation to a lower price is clearly unprofitable. Consider an individual deviation to a slightly higher price that extracts revenue, say, $\tilde{\pi}$. This deviation results in a loss of consumers, as consumers with search costs lower than $v(\pi^*) - v(\tilde{\pi})$ walk away to rival sellers. This loss of consumers prevents firms to individually raise their prices.

The understanding why monopoly price is the same as that of Diamond-paradox. If all sellers are expected to charge the monopoly price, searching an additional firm is not profitable. If however buyers do not search beyond the first firm, it is optimal for firms to offer the monopoly price, which justifies buyers' above expectation.

We now turn our attention to nonlinear prices. As in the case of linear prices, we can apply [Stahl \(1996\)](#)'s argument to solve for equilibria with nonlinear prices. One can easily check that analogous to equilibria with linear prices, buyers do not search beyond the first firm and firms play symmetric pure strategy (nonlinear) pricing in any equilibrium. Moreover, optimal nonlinear price is a two-part tariff. The linear price of a two-part tariff in any equilibrium is equal to the production marginal cost, which is implied by [Lemma 4](#). Any fixed fee in interval $[t^*, v(0)]$ can be supported in equilibrium, where t^* is determined as

$$t^* = \arg \max_t (1 - G[t - t^*]) t.$$

The equilibrium industry profit then lies in $[t^*, v(0)]$, while the consumer surplus lies in $[v(0) - t^*, 0]$.

To compare market outcomes under different pricing regimes one needs to know which equilibria are played. Selection of equilibria is oftentimes either based on refinement rule or one's judgment of which equilibrium fits real-world markets. However, it seems natural to us to focus on the most competitive equilibria, a route which is also taken in the existing literature.³ We state the main result in the following proposition.

Proposition 4. *Consider equilibria characterized by π^* for linear prices and t^* for nonlinear prices. With nonlinear prices sellers' profits and social welfare are higher, but consumer surplus is lower than with linear prices.*

The reasoning behind the first two parts is fairly simple. We can easily solve for π^* and t^* to obtain

$$\begin{aligned}\pi^*(-v'(\pi^*)) &= \frac{1}{g(0)}, \\ t^* &= \frac{1}{g(0)}.\end{aligned}$$

As in equilibrium with linear prices firms always choose prices at the elastic part of the demand, it must be that $-v'(\pi) > 1$ for all $\pi \leq \pi^m$. From this it directly follows that $0 < \pi^* < t^*$ for any $0 < g(0) < \infty$. Note next that the linear price of equilibrium two-part tariffs equals to the production marginal cost, whereas the equilibrium prices are higher than the production marginal cost when firms can offer only linear prices. Therefore the social welfare is higher with two-part tariffs than with linear prices.

We prove in the appendix that consumers are better-off with linear prices than with nonlinear prices. To understand the intuition it is useful to note that if the density of consumers with zero search cost $g(0)$ equals to $1/v(0)$, meaning that $t^* = v(0)$, the consumer surplus is zero with nonlinear prices but positive with linear prices. As the density of consumers with zero search cost increases without bounds, the equilibrium profits converge to zero in both pricing regimes. Therefore buyers receive the entire social surplus in both pricing regimes when the density of consumers with zero search cost becomes very large. The challenge is then to show that the consumer surplus is

³Janssen and Reshidi (2020), for instance, focus on equilibrium with linear prices generating revenue π^* to study price discrimination of retailer sellers by a manufacturer.

increasing in the density of consumers with zero search cost more slowly with linear price than with nonlinear prices. We prove in the appendix that this is the case.

5.2 Noisy Search

We finally turn our attention to markets where buyers employ noisy search, which incorporates some properties of nonsequential search into the model of sequential search. Key features of nonsequential search is that a buyer chooses a number of firms to search after which the search is terminated. Noisy search allows such a consumer to search again if she is not satisfied with offers she received, yet it abstracts from formally modeling nonsequential search. Therefore with noisy search a buyer requests deals from (searches) $m \geq 2$ number of firms at the beginning of each search round and receives responses from an unknown number of them, say k , at the end of a search round. Here k is a random variable with $1 \leq k \leq m$. At the end of a search round, a buyer decides whether to terminate the search or to send requests to m more firms in the next search round. If the search is terminated the buyer can either make a purchase at the lowest observed price or drop out of the market. With a slight abuse of notation we let $s > 0$ represent the cost of searching m firms in each search round.

Noisy search is prevalent in markets where buyers observe search results with a delay. For instance, in mortgage markets a consumer sends applications to several banks and receives replies from them after some time. Some of those banks may not quote a price because they find the consumer too risky to cooperate with. In addition, before contacting banks, the consumer faces uncertainty about which banks will approve her application.⁴

5.2.1 Additional Assumptions

To keep the model tractable and ensure uniqueness of equilibrium, we introduce some additional assumption. Our first assumption is that there are infinitely many firms. This assumption ensures that buyers search rule is stationary. Search strategy of buyers may be nonstationary with finite number of firms. This is because with finite number of firms the expected benefit of search is declining. For instance, suppose that there are ten firms, buyers search four firms in each search round ($m = 4$) and a buyer has observed two firms'

⁴Atayev (2019) analyzes uncertain product availability in search markets.

offers in the first search round. The expected benefit of searching in the second round equals to that of searching in the third search round, as in each of those rounds a buyer may receive up to four new offers. To contrast, assume that the buyer received four offers in the first search round. Then, the expected benefit of searching in the second round may be higher than that searching in the third search round. This is because a buyer may receive four new offers in the second search round in which case she can receive at most two offers in the third search round.

Our next assumption is that a strictly positive (equal) share of buyers observe offers of each firm at the first search round. We also let $\mu(k)$ represent the probability with which a buyer receives $k(\leq m)$ number of responses, where $\mu(0) = 0$. Note that if $\mu(1) = 1$ the unique equilibrium is Diamond-paradox. Conversely, if $\mu(1) = 0$ the unique equilibrium is a textbook Bertrand equilibrium. To rule out these trivial equilibria we assume that $0 < \mu(1) < 1$, and to ensure uniqueness of equilibrium we set $0 < \mu(2) < 1$.

The game unfolds as follows. First, firms simultaneously choose their offers. Second, without knowing offers, buyers engage in the first round of noisy search. After observing offers, buyers may make purchase, drop out of the market, or search in the second search round. The cycle continues until all buyers either make a purchase or drop out of the market. The solution concept is an RPE.

5.2.2 Analysis

With only linear prices allowed our model boils down to the model of [Burdett and Judd \(1983\)](#). The authors show existence of a unique equilibrium which is characterized by a symmetric mixed-strategy pricing. In terms of per-consumer revenue, firms randomly draw a revenue π according to distribution F , which is strictly increasing in its support and solves

$$\sum_{k=1}^m k\mu(k)(1 - F(\pi))^{k-1}\pi = \mu(1)\bar{\pi} \quad (7)$$

and has support $[\underline{\pi}, \bar{\pi}]$. Here $\underline{\pi}$ solves $F(\underline{\pi}) = 0$ and $\bar{\pi} = \min\{\pi_R, \pi^m\}$, where π_R is implicitly determined by

$$\int_{\underline{\pi}}^{\pi_R} (-v'(\pi)) \sum_{k=1}^m \mu(k)(1 - F(\pi))^{k-1} d\pi = s. \quad (8)$$

If a solution to the equation does not exist, we set $\pi_R = \pi^m$.

Some observations are in order. As in our main model, buyers do not search beyond the first search round in equilibrium. We can hence consider buyers who observe only one price as nonshoppers. Also the equilibrium prices are dispersed and the distribution of per-consumer revenue is strictly and continuously increasing in its compact support. The only difference is that the equilibrium is unique with noisy search, whereas with the sequential search there exist asymmetric equilibria for $n \geq 3$.⁵

Next, the equilibrium with noisy search also resembles that with nonsequential search. Particularly, the search is terminated after the first search round, which is exogenously imposed in non-sequential search markets. There is however an important difference. In nonsequential search models the highest price in the support of equilibrium price distribution equals to the monopoly price. This highest price does not depend on the search cost. In our model with noisy search, the search cost is a key determinant of the highest price in the support of the equilibrium price distribution.

The equilibrium with nonlinear prices is also similar to the one in our main model. To avoid repetition we state the result outright. In equilibrium firms offer two-part tariffs with $p = 0$ and t distributed according to H that is strictly increasing in its support and solves

$$\sum_{k=1}^m k\mu(k)(1 - H(t))^{k-1}t = \mu(1)t_R. \quad (9)$$

The support of H is $[\underline{t}, t_R]$ which are determined by $H(\underline{t}) = 0$ and

$$\sum_{k=1}^m \mu(k) \int_{\underline{t}}^{t_R} (1 - H(t))^{k-1} dt = s. \quad (10)$$

If a solution to the equation does not exist or $t_R > v(0)$, we set $t_R = v(0)$.

We now turn to welfare analysis. As the equilibria under the two pricing regimes with noisy search are qualitatively the same as the respective symmetric equilibria with sequential search in our main model, the welfare analysis with noisy search is qualitatively the same as that with the sequential search. Specifically, we have the following result.

⁵Baye et al. (1992) show existence of asymmetric equilibria where the share of buyers who observe exactly two prices is zero, and Johnen and Ronayne (forthcoming) show uniqueness of symmetric equilibrium where that share of buyers is positive.

Proposition 5. *With noisy search, social welfare and firm profits are higher, whereas consumer surplus is lower with nonlinear price than with linear prices.*

The reasoning and the intuition behind this result is similar to one behind Proposition 3. Therefore we omit them.

6 Conclusion

We see our paper as the first one to examine welfare effects of nonlinear prices in homogeneous goods markets with search. We demonstrate that traditional models which consider linear price understate firms' market power. With nonlinear prices available in their hands, sellers can make effective per-buyer demand less elastic and soften competition.

A Proofs

A.1 Proof of Proposition 1

We start with existence. Consider firms pricing strategy for any given t_R . Suppose that an individual firm deviates to linear pricing strategy and the deviation price is p_d . The expected profit of the deviating firm equals to

$$\left(\frac{1-\lambda}{n} + \lambda \left[1 - H \left(\int_0^{p_d} q(z) dz \right) \right]^{n-1} \right) q(p_d) p_d = \frac{1-\lambda}{n} \left(\frac{q(p)p}{\int_0^p q(z) dz} \right) \bar{t},$$

where we obtained the equality using (4) and recall that $\bar{t} = \min\{t_R, v(0)\}$. The optimal p_d either solves

$$\frac{\partial}{\partial p} \left(\frac{q(p)p}{\int_0^p q(z) dz} \right) \Big|_{p=p_d} = 0$$

or satisfies $H(\int_0^{p_d} q(z) dz) = 0$. In the former case, the equation determining p_d is

$$q'(p_d)p_d + q(p_d) - \frac{q(p_d)^2 p_d}{\int_0^{p_d} q(z) dz} = 0. \quad (11)$$

As the denominator of the third term on the LHS of the equation are positive, it must be $p_d < p^m$ since p^m solves $q'(p)p + q(p) = 0$. However, a firm can offer a two-part tariff ($p = 0, t = \int_0^{p_d} q(z) dz$) that yields the same utility to buyers as the deviation price p_d and increase its profit, as $\int_0^{p_d} q(z) dz > q(p_d)p_d$. In the case where p_d satisfies $H(\int_0^{p_d} q(z) dz) = 0$, we have $\int_{p_d}^{\bar{p}} q(z) dz = \underline{t}$. As in the previous case, offering the two-part tariff that gives buyers utility $\int_0^{\bar{p}} q(z) dz - \underline{t}$ yields a higher expected profit than offering linear price p_d , as per-consumer revenue with p_d equals to $q(p_d)p_d$ and that with the two-part tariff equals to \underline{t} where $\underline{t} = \int_0^{p_d} q(z) dz > q(p_d)p_d$. In either case, we arrive at a contradiction. This means that there is no profitable deviation to linear pricing.

To complete the proof of existence it is left to show that, given H , there exists a unique t_R that solves (3) for $s \leq \bar{s}$. To do that we need to show that the LHS of (3), which is positive, strictly increases with t_R . The derivative of the LHS of (3) w.r.t. t_R is $1 - \int_{\underline{t}}^{t_R} \partial H(t) / \partial t_R dt$. As

$$\frac{\partial H(t)}{\partial t_R} = \frac{t}{t_R} \times \frac{\partial H(t)}{\partial t} \leq \frac{\partial H(t)}{\partial t} \quad \text{for any } t \leq t_R,$$

we have $1 - \int_{\underline{t}}^{t_R} \partial H(t) / \partial t_R dt > 1 - \int_{\underline{t}}^{t_R} \partial H(t) / \partial t dt = 0$. This means that the LHS of (3) is indeed strictly increasing in t_R . Then, there must exist a unique t_R solving (3) for $s \leq \bar{s}$.

We next prove that this is a unique symmetric equilibrium. Clearly, there cannot be other symmetric equilibria in two-part tariffs as t_R and H are uniquely determined by (3) and (4), respectively, for $p = 0$. Then, it suffices to show that there does not exist another symmetric equilibrium in linear prices. For contradiction, assume that there exists such an equilibrium with some price \tilde{p} in the support of equilibrium price distribution (in the there is a pure-strategy symmetric equilibrium \tilde{p} is the equilibrium price) and the per-consumer revenue $q(\tilde{p})\tilde{p}/n$. However, the firm can offer a two-part tariff with $p = 0$ and

$t = \int_0^{\bar{p}} q(z)dz$ which would yield the same utility to consumers but improve the firm's per-consumer revenue. The last point can be seen as follows: $\int_0^{\bar{p}} q(z)dz > q(\bar{p})\bar{p}$. This contradicts to our assumption that there exists a symmetric equilibrium in linear prices. This establishes uniqueness of the symmetric equilibrium with two-part tariffs.

The proof of the proposition is now complete.

A.2 Proof of Proposition 3

For the proof it suffices to compare profits and consumer surpluses under two pricing regimes. The industry profit with nonlinear prices, which is $(1 - \lambda)\bar{t}$, is higher than that with linear prices, which is $(1 - \lambda)\bar{\pi}$, if $\bar{t} > \bar{\pi}$. To see that this is the case, first recall that F and H are similar and the only difference is their supports. Using formulas which determine these distributions, observe that $\underline{\pi}/\bar{\pi} = (1 - \lambda)/(1 + (n - 1)\lambda)$ and $\underline{t}/\bar{t} = (1 - \lambda)/(1 + (n - 1)\lambda)$, which implies

$$\frac{\bar{\pi}}{\underline{\pi}} = \frac{\bar{t}}{\underline{t}}. \quad (12)$$

Second, recall that the upper bound of F is determined by (6) and that of H is determined by (3) for small c (for high c we know that $\bar{\pi} < \bar{t}$ from Proposition 3). As firms price at the elastic part of the demand meaning that $-v'(\pi) > 1$ for all $\pi \leq \pi^m$ in (6), it must be that $\bar{\pi} - \underline{\pi} < \bar{t} - \underline{t}$. Divide both sides of the inequality by $\bar{\pi}\bar{t}$ to obtain

$$\frac{1}{\bar{t}} - \frac{1 - \lambda}{1 + (n - 1)\lambda} \times \frac{1}{\bar{t}} < \frac{1}{\bar{\pi}} - \frac{1 - \lambda}{1 + (n - 1)\lambda} \times \frac{1}{\bar{\pi}},$$

which can be reduced to $\bar{t} > \bar{\pi}$, which was required to prove.

To compare consumer surplus under different pricing regimes we start noting that the consumer surplus generating the equilibrium profit with linear prices is not less than

$$\frac{n - 1}{n}(1 - \lambda)v(\bar{\pi}) + \left(1 - \frac{n - 1}{n}(1 - \lambda)\right)v(\underline{\pi}).$$

With two-part tariffs the consumer surplus equals to

$$\lambda v(0) + (1 - \lambda) \left(\int_0^{\bar{p}} q(z)dz - \bar{t} \right).$$

The former is greater than the latter if

$$v(\underline{\pi}) \geq \frac{\lambda}{1 - \frac{n-1}{n}(1 - \lambda)}v(0) + \frac{1 - \lambda}{1 - \frac{n-1}{n}(1 - \lambda)}(v(0) - \bar{t}) - \frac{\frac{n-1}{n}(1 - \lambda)}{1 - \frac{n-1}{n}(1 - \lambda)}v(\bar{\pi}).$$

Since $2(n - 1)/n \geq 1$ for any $n \geq 2$, the inequality certainly holds if

$$v(\underline{\pi}) \geq \frac{\lambda}{1 - \frac{n-1}{n}(1 - \lambda)}v(0) + \frac{\frac{n-1}{n}(1 - \lambda)}{1 - \frac{n-1}{n}(1 - \lambda)}v(\bar{\pi}) - \frac{1 - \lambda}{1 - \frac{n-1}{n}(1 - \lambda)}(v(\bar{\pi}) - v(0) - \bar{t}). \quad (13)$$

As $v(\cdot)$ is a concave function, the LHS of the inequality is greater than the sum of the first

two-terms on the RHS. Then, the inequality certainly holds if $v(\bar{\pi}) \geq v(0) - \bar{t}$, which we prove as follows. Note that both sides of this inequality converge to $v(0)$ as $s \downarrow 0$. Since both sides of the inequality are decreasing in s , the inequality holds if the derivative of the LHS is less negative than that of the RHS. The derivative of the LHS is $v'(\bar{\pi})\partial\bar{\pi}/\partial s$ and that of the RHS is $-\partial\bar{t}/\partial s$. To derive these derivatives we differentiate (3) and (6) w.r.t. s to obtain

$$\begin{aligned} \frac{\partial\bar{\pi}}{\partial s} \left(-v'(\bar{\pi}) - \frac{1}{n-1} \left(\frac{1-\lambda}{1+(n-1)\lambda} \right)^{\frac{1}{n-1}} \int_{\underline{\pi}}^{\bar{\pi}} (-v'(\pi)) \left(\frac{\bar{\pi}}{\pi} - 1 \right)^{-\frac{n-2}{n-1}} \frac{d\pi}{\pi} \right) &= 1, \\ \frac{\partial\bar{t}}{\partial s} \left(1 - \frac{1}{n-1} \left(\frac{1-\lambda}{1+(n-1)\lambda} \right)^{\frac{1}{n-1}} \int_{\underline{t}}^{\bar{t}} \left(\frac{\bar{t}}{t} - 1 \right)^{-\frac{n-2}{n-1}} \frac{dt}{t} \right) &= 1. \end{aligned}$$

Then the derivatives of the LHS and the RHS of our inequality are, respectively,

$$\frac{1}{-1 + \frac{1}{n-1} \left(\frac{1-\lambda}{1+(n-1)\lambda} \right)^{\frac{1}{n-1}} \int_{\underline{\pi}}^{\bar{\pi}} \frac{v'(\pi)}{v'(\bar{\pi})} \left(\frac{\bar{\pi}}{\pi} - 1 \right)^{-\frac{n-2}{n-1}} \frac{d\pi}{\pi}},$$

$$\frac{1}{-1 + \frac{1}{n-1} \left(\frac{1-\lambda}{1+(n-1)\lambda} \right)^{\frac{1}{n-1}} \int_{\underline{t}}^{\bar{t}} \left(\frac{\bar{t}}{t} - 1 \right)^{-\frac{n-2}{n-1}} \frac{dt}{t}}.$$

The former is less negative than the latter if

$$\int_{\underline{t}}^{\bar{t}} \left(\frac{\bar{t}}{t} - 1 \right)^{-\frac{n-2}{n-1}} \frac{dt}{t} \geq \int_{\underline{\pi}}^{\bar{\pi}} \frac{v'(\pi)}{v'(\bar{\pi})} \left(\frac{\bar{\pi}}{\pi} - 1 \right)^{-\frac{n-2}{n-1}} \frac{d\pi}{\pi},$$

which is true as (i) $v'(\pi) < 0$ and $v'(\cdot)$ is a decreasing function (recall concavity of $v(\cdot)$), meaning that $v'(\pi)/v'(\bar{\pi}) < 1$ for any $0 \leq \pi \leq \bar{\pi}$, and (ii) we know from the first part of the proof that $\bar{t} - \underline{t} \geq \bar{\pi} - \underline{\pi}$, $\bar{t} \geq \bar{\pi}$ and $\underline{t} \geq \underline{\pi}$. This proves that the derivative of $v(\bar{\pi})$ w.r.t. s is less negative than the derivative of $v(0) - \bar{t}$. Then (recalling the limiting results when $s \downarrow 0$) it is indeed true that $v(\bar{\pi}) \geq v(0) - \bar{t}$ for any $s > 0$.

This inequality proves in its turn that the inequality in (13) is true. Then, the consumer surplus is indeed lower with nonlinear prices than with linear prices. The proof of the proposition is now complete.

A.3 Proof of Proposition 4

To compare the consumer surpluses, we first note that the equilibrium consumer surplus with linear prices is always positive. We next note that the consumer surplus increases with $g(0)$ in both pricing regimes. Moreover, the consumer surplus with nonlinear prices equals to zero if $g(0) = 1/v(0)$, whereas the consumer surplus under both pricing regimes converges to $v(0)$ as $g(0) \rightarrow \infty$. Then, consumers are worse-off with nonlinear prices than with linear prices if consumer surplus is increasing in $g(0)$ ($> 1/v(0)$) more slowly with linear prices than with nonlinear prices. The condition holds if the derivative of the consumers surplus with respect to $g(0)$ is lower with linear prices than with nonlinear

prices. These derivatives are respectively,

$$\begin{aligned}\frac{dv(\pi^*)}{dg(0)} &= v'(\pi^*)\frac{d\pi^*}{dg(0)}, \\ \frac{d(v(0) - t^*)}{dg(0)} &= -\frac{dt^*}{dg(0)},\end{aligned}$$

where $v(\pi^*)$ is the equilibrium consumers surplus with linear prices.

To evaluate these derivatives, we differentiate the equations solving for π^* and t^* with respect to $g(0)$ to obtain

$$\begin{aligned}\frac{d\pi^*}{dg(0)} &= \frac{1}{[g(0)]^2 (v'(\pi^*) + v''(\pi^*)\pi^*)}, \\ \frac{dt^*}{dg(0)} &= -\frac{1}{[g(0)]^2}.\end{aligned}$$

Then, the derivative of the consumers surplus with linear prices is lower than that with nonlinear prices if

$$\frac{1}{1 + \frac{v''(\pi^*)}{v'(\pi^*)}\pi^*} \leq 1.$$

This inequality holds since $v''(\pi^*)/v'(\pi^*) > 0$, which follows from the fact that $v(\cdot)$ is a decreasing concave function. This shows that the consumer surplus is rising more slowly in $g(0)$ with linear prices than with nonlinear prices for all $g(0) > 1/v(0)$. This fact—along with the two facts mentioned in the previous paragraph—proves that the consumer surplus is higher with linear prices than with nonlinear prices.

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