The Dynamics of Firm-level Pay: Theory and Evidence from Portugal.*

Rui Castro† Gian Luca Clementi‡

February 2021

Preliminary and incomplete - Please do not circulate.

Abstract

Recent empirical work has emphasized the role played by firms in shaping earnings inequality. In this paper, we build a tractable model of firm pay heterogeneity, by introducing labor market's monopsonistic power in a Hopenhayn-style firm dynamics framework. The model naturally generates wage-size premia. We use our theory to help understand the dynamics of earnings inequality in Portugal. Like in other developed economies, earnings inequality in Portugal has increased sharply from the mid-1980s until the mid-1990s. However, it has been steeply decreasing ever since. We trace this unique dynamics to firm-level changes. The sole factor pushing inequality downwards has been a larger compression in firm pay, mostly driven by a decline in the pass-through from firm-level productivity to wages. Our model suggests that a decrease in firms' labor market power may have been a chief determinant of such decline.

Keywords: Firm Dynamics, Earnings Inequality, Labor Share.

JEL Codes: D33, E24, E25, L11, L25.

*We thank Sílvia Gonçalves, Pedro Raposo, and seminar participants at Universidade Católica Portuguesa for helpful comments. All errors are ours. Rui Castro acknowledges financial support from SSHRC and the generous hospitality of Universidade Católica Portuguesa while working on this project.

†Department of Economics and CIREQ, McGill University. Email: rui.castro@mcgill.ca. Web: rui-castro.research.mcgill.ca.

‡Department of Economics, Stern School of Business, New York University and NBER. Email: clem@nyu.edu. Web: http://people.stern.nyu.edu/gclement/.

1 Introduction

Recent empirical work made possible by the availability of employer-employee matched datasets has shown that firms may have played an important role in shaping the dynamics of earnings inequality in both developed and developing countries.[1] In this paper, we build a tractable theory of firm-level pay heterogeneity, by introducing labor market's monopsonistic power in the familiar firm dynamics model due to Hopenhayn (1992).

Our model generates a monotone-increasing map from productivity into wages, the shape of which has equilibrium implications for earnings inequality. We use our theory to shed light on the evolution of earnings inequality in Portugal over the last 25 years.

Using high-quality administrative data, we document that, after a fast-paced increase between the mid-80s and the mid-90s, earnings inequality has been steeply declining, driven exclusively by a compression in firm pay. In fact, other components such as cross-worker inequality and assortative matching between workers and firms have pushed inequality in the opposite direction.

When looking for a rationalization of these facts, our evidence suggests a limited role for lower heterogeneity in observable firm–level characteristics, such as productivity. The key driver of lower firm pay dispersion appears to have been a decline in the pass-through from firm-level productivity to wages.

Our theory suggests that a decline in firms labor market power may have been the reason why the pass-through declined. According to our model, a drop in firms' labor market power reduces earnings inequality. The thrust of the analysis will consist in exploiting a series of over-identifying restrictions on the data for the purpose of identification. We find that lower market power is associated with a generalized increase in wages and in the labor share. While wage markdowns drop across the board, they fall by more at low-productivity firms. This induces a redistribution of employment from low- to high-productivity firms and an improvement in allocative efficiency.

The remainder of this paper is structured as follows. In Section 2, we carefully document the evolution of earnings inequality in Portugal and we highlight the role played by firms.

[1] See for example (Abowd, Kramarz, and Margolis, 1999; Card, Heining, and Kline, 2013; Song, Price, Guvenen, Bloom, and von Wachter, 2018; Alvarez, Benguria, Engbom, and Moser, 2018).

In Section 3 we introduce our model and define the equilibrium. In Section 4 we draw the implications of decline market power for all variables of interest.

2 Earnings Inequality in Portugal: Evidence

2.1 Quadros de Pessoal

The earnings data is from Quadros de Pessoal (QP), an administrative data set covering all private-sector firms employing at least one salaried worker, including those owned by the government. The data is collected annually by the Ministry of Labor to ensure compliance with labor laws. The reporting is for the month of March until 1993 and for October since. The data is unavailable for 1990 and 2001.

QP covers all workers employed in Portugal, except civil servants and independent contractors, as well as temporary rural and domestic workers. It is a matched employer-employee data set, providing comprehensive information on workers and some basic information on firms. We focus on workers aged 18–65, working full-time (at least 140 normal monthly hours), and earning at least the minimum wage as base pay. We exclude firms in farming and fishing, since earnings in these sectors are often not reported on a monthly basis.

Our favored measure of compensation is total monthly earnings, i.e. base salary plus overtime and regular monthly supplements such as productivity and seniority premia, as well as compensating differentials.[2] To avoid outliers, we trim the top and bottom 1% earnings observations per year. Further details on the data are in Appendix A.

2.2 Earnings Inequality Dynamics

Figure 1 plots the variance of log earnings since 1986, the year Portugal joined the European Union (European Economic Community then).

Inequality has increased dramatically from 1986 until the mid-1990s. The variance grew by nearly 60% variance over just a decade. From the mid-1990s onwards, however, inequality has first plateaued and then briskly declined since the financial and sovereign debt crisis of

[2] When running our analysis using hourly wages instead, the magnitudes change slightly, but the message does not.

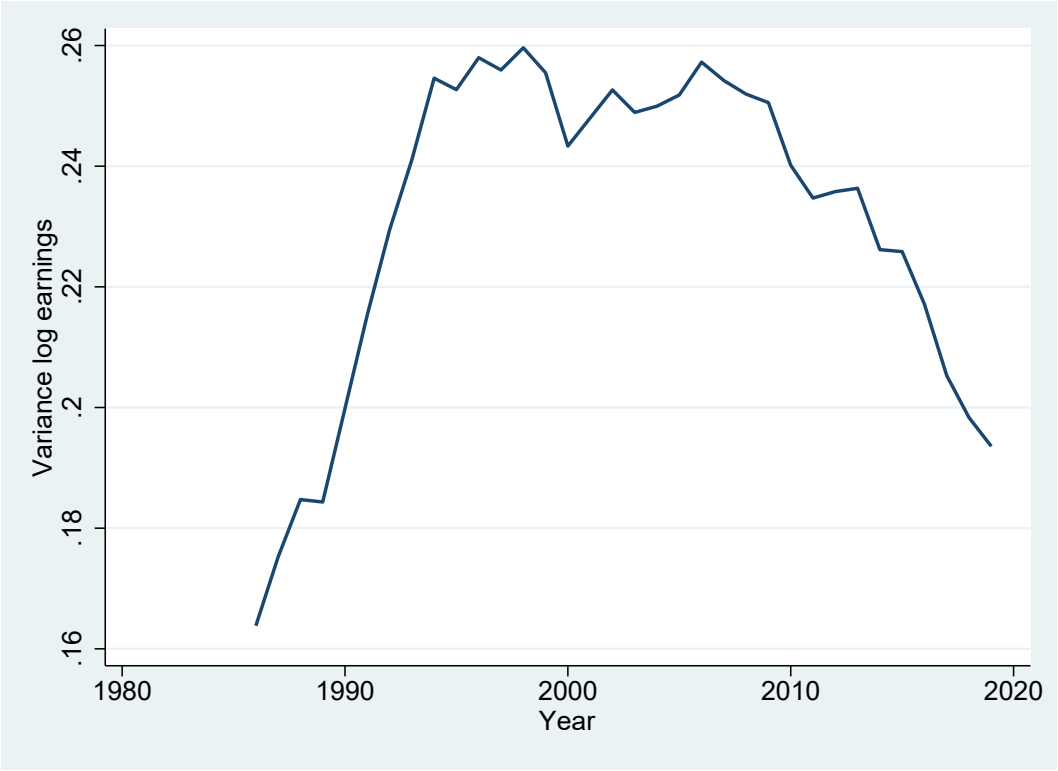


Figure 1: Quadros de Pessoal, 1986-2019

2008-9. The variance of earnings dropped by about 25% from the mid-1990s' peak to 2019. An yearly rate of decline of about 1% is economically significant and comparable to the rate of increase in the same measure of inequality reported by Song, Price, Guvenen, Bloom, and von Wachter (2018) for the United States between 1985 and 2009.

As far as we know, this non-monotonic dynamics is unique in the developed world. The sharp increase in inequality from the mid-1980s until the mid-1990s was also documented by Cardoso (1999), using the same data. This paper focuses on inequality past the mid-90s, a period during which, thanks to the reduction in informality, Quadros de Pessoal has significantly increased worker coverage.

Inspired by the recent empirical literature highlighting the key role that firms have played in shaping inequality in Germany (Card, Heining, and Kline, 2013), the U.S. (Song, Price, Guvenen, Bloom, and von Wachter, 2018) and Brazil (Alvarez, Benguria, Engbom, and Moser, 2018), we ask whether employers have also played a substantial role in Portugal.

2.2.1 The Role of firms: A first look

Figure 2 displays a decomposition of the variance of log earnings into the variance of average earnings between firms and the employment-weighted average of the within-firm earnings variances.

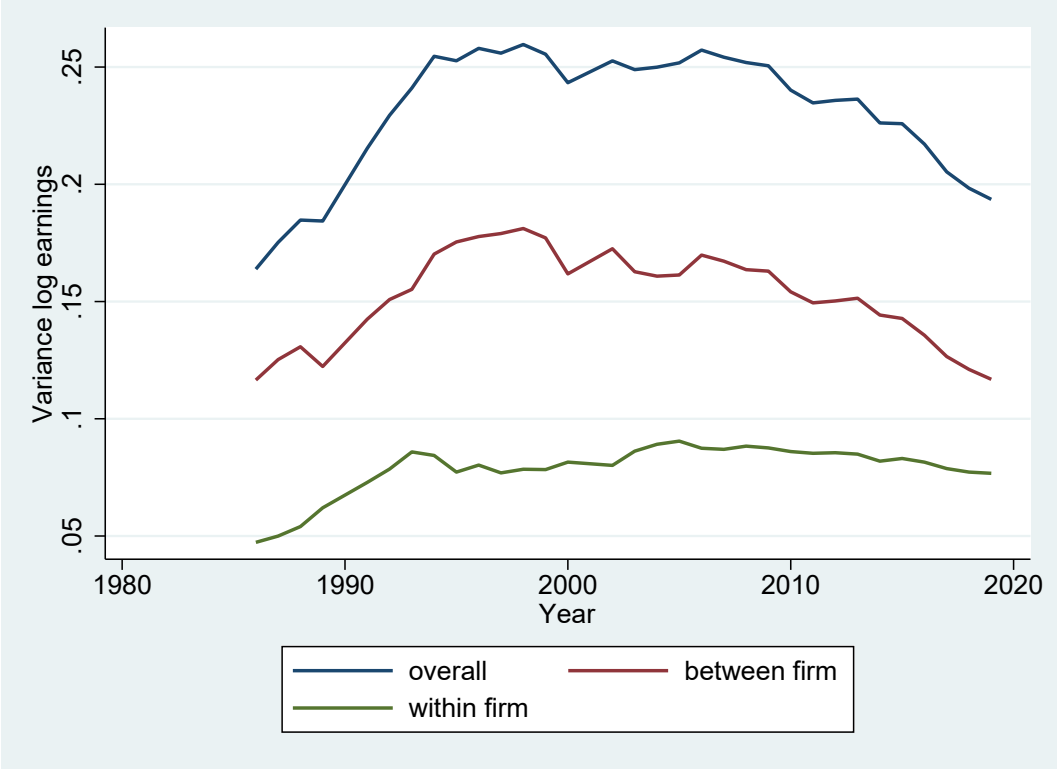


Figure 2: Between-within firm decomposition

The decomposition gives a first indication of the importance of firms in the Portuguese context. The between-firm component accounts for between 50% and 60% of the level. Most importantly for our purpose, it is also the main component behind the dynamics, accounting for about 2/3 of the increase until the mid-1990s and for about 120% of the decline since the mid-1990s.

A natural question is whether the evolution of the between-firm component is driven by changes over time in the composition of the workforce at the firm level or by variation in intrinsic firm–level characteristics.

2.2.2 AKM regression

To isolate the role played by intrinsic firm–level characteristics, we now switch over to the AKM regression framework due to Abowd, Kramarz, and Margolis (1999). The AKM approach extends the prototypical Mincer wage regression framework by adding firm fixed effects to control for observed and unobserved systematic firm-level characteristics, in addition to worker fixed effects and a set of time-varying observables. Our regression equation for the two-way fixed effect (AKM) model is

$$y_{ijt} = \text{WFE}_i + \text{FFE}_j + \mathbf{X}'_{ijt}\beta + \varepsilon_{ijt}. \tag{1}$$

For the time-varying observables we follow Card, Heining, and Kline (2013) and include a full set of year dummies and a worker age cubic polynomial, both fully interacted with schooling attainment.[3] As is standard in the literature, we estimate (1) over six non-overlapping subperiods p of four/five years each. This is a simple way to capture variation over time in the estimated coefficients.[4] We therefore estimate cross-sectional distributions of WFE_i^p and FFE_j^p per subperiod p, plus the coefficients in β^p.

For estimation purposes, we need to restrict our sample in a number or ways. First, we focus on the largest connected set of all firms (and their workers) for which every worker in the set has ever worked for. Second, we discard singletons (unique firm-worker observations). Third, we focus on firms with at least 10 workers on average over their entire life-cycle.[5]

[3] As in Card, Heining, and Kline (2013), age is normalized (by deviations from the age of 40) so that the polynomial implies flat experience profiles at age 40. The linear effect of age is dropped as it is captured by the worker fixed effects together with time.

[4] Engbom, Moser, and Sauermann (2021) consider an alternative approach by extending the original AKM's identification requirement. They show that firm-specific time trends are identified within a set of workers which are mobile not just across firms, but across firm-years.

[5] Our motivation is to act conservatively in order to avoid biased estimates of the cross-sectional variances, what the literature refers to as "limited mobility bias" (Bonhomme, Holzheu, Lamadon, Manresa, Mogstad, and Setzler, 2020; Engbom, Moser, and Sauermann, 2021). This problem is magnified by the presence of small firms, whose workers tend to exhibit lower mobility. Our simple approach is therefore to eliminate small firms in a long-run/life-cycle sense. Our main results about inequality trends are not very sensitive to eliminating these firms.

2.2.3 Basic variance decomposition

Ignoring the time-varying observables for now, the AKM regression delivers the following basic variance decomposition by subperiod p:

$$\text{var}\left(y_{ijt}\right) = \underbrace{\underbrace{\text{var}\left(\text{WFE}_i\right)}_{\text{fixed}} + \underbrace{\text{var}\left(\varepsilon_{ijt}\right)}_{\text{random}}}_{\text{workers}} + \underbrace{\underbrace{\text{var}\left(\text{FFE}_j\right)}_{\text{firm pay}} + \underbrace{2\text{cov}\left(\text{WFE}_i, \text{FFE}_j\right)}_{\text{assortative matching}}}_{\text{firms}}. \tag{2}$$

The first two components are attributed to workers. var(WFE$_i$) is the dispersion in the systematic component that captures observable features such as schooling and gender, as well as unobservable characteristics such as motivation and inter-personal traits. var(ε_{ijt}) is the dispersion in the random component capturing permanent or transitory earnings changes due to shocks to, say, health or training. The two remaining components are attributed to firms. First is the dispersion in the systematic firm-level component capturing characteristics such as productivity, size, or management quality. This is the firm pay component, enjoyed by any worker employed by a specific firm. Second is an assortative matching component conveying the extent to which high-wage workers are employed by high-wage firms.

The dispersion in worker fixed effects could vary across subperiods due to either persistent changes in the systematic components of continuing workers, including changes in the minimum wage, or to cohort effects induced by the characteristics of either labor force entrants or exiters. In turn, the latter could stem from the significant increases in college graduation and female labor force participation which have taken place since the mid-1980s. The dispersion in firm pay may have changed due to cross-period changes in incumbents' pay policies, or may be due to cohort effects induced by the specific characteristics of entrants and exiters within each subperiod.

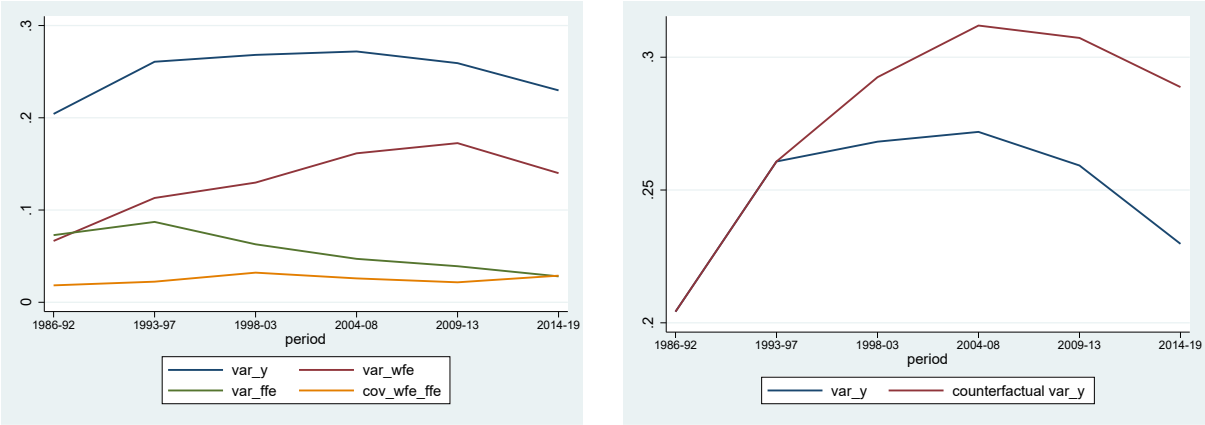
The results of this decomposition exercise are in Table 1. We emphasize two main trends across three subperiods: The increase in inequality from the mid-1980s until the mid-1990s and the decline since the mid-1990s. The table contains also the time-varying observables (Xb) that were omitted in (2).

Figure 3a shows the period-by-period dynamics of the most significant components from Table 1. The variance of firm pay is a significant driver of the increase in overall earnings

	Interval 1 (1986-1992)		Interval 2 (1993-1997)		Interval 6 (2014-2018)		Change 1–2		Change 2–6	
	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share
Var(y)	.204	–	.261	–	.23	–	.057	–	-.031	–
Var(WFE)	.066	32.6	.113	43.4	.14	60.9	.047	82.4	.027	-86.7
Var(FFE)	.073	35.7	.087	33.4	.028	12.2	.014	25.3	-.059	190.3
Var(Xb)	.014	7	.004	1.6	.008	3.6	-.01	-18	.004	-13.5
Var(ϵ)	.015	7.4	.015	5.8	.01	4.5	0	0	-.005	16.1
2Cov(WFE, FFE)	.018	9	.022	8.6	.029	12.6	.004	7.1	.007	-21
2Cov(WFE, Xb)	.007	3.5	.012	4.5	.011	4.8	.005	8.1	-.001	2.5
2Cov(FFE, Xb)	.01	4.9	.007	2.8	.003	1.5	-.003	-4.9	-.004	12.2

Table 1: Basic Decomposition

inequality until the mid-1990s and it's by far the key force behind the decline since then. In fact, it is the only major force pushing for earnings compression. It accounts for 190% of the overall decline in inequality and outmatches the impact of greater worker heterogeneity, responsible for a negative contribution of 86.7%.



(a) Main components

(b) Counterfactual earnings variance

Figure 3: Basic variance decomposition

An alternative way to appreciate the importance of the compression in firm pay for the overall inequality decline is to evaluate (2) while counterfactually holding the variance of firm fixed effects constant at the mid-1990s level. Figure 3b shows that earnings inequality would have increased rather than declined.

Such outsize role of firm pay as a driver of earnings inequality is in common with the experiences of Brazil and Ecuador, where inequality has steadily declined, and with those of U.S. and Germany, where inequality has steadily increased.[6]

2.2.4 Detailed variance decomposition

A more detailed decomposition, along the lines of Song, Price, Guvenen, Bloom, and von Wachter (2018), splits the overall variance in worker fixed effects into a within-firm and a

[6]See Alvarez, Benguria, Engbom, and Moser (2018) for Brazil, Messina and Silva (2019) for Ecuador, Card, Heining, and Kline (2013) for Germany, and Song, Price, Guvenen, Bloom, and von Wachter (2018) for the United States

between-firm component. Once again abstracting from the time-varying observables:

$$\text{var}\left(y_{ijt}\right) = \underbrace{\text{var}\left(\text{WFE}_i - \overline{\text{WFE}}_j\right) + \text{var}\left(\varepsilon_{ijt}\right)}_{\text{within-firm}} + \underbrace{\underbrace{\text{var}\left(\text{FFE}_j\right)}_{\text{firm pay}} + \underbrace{2\text{cov}\left(\overline{\text{WFE}}_j, \text{FFE}_j\right)}_{\text{sorting}} + \underbrace{\text{var}\left(\overline{\text{WFE}}_j\right)}_{\text{segregation}}}_{\text{between-firm}}. \quad (3)$$

The various components add up to the between and within firm components of Figure 2. The payoff consists in gaining further insight into the worker-firm matching patterns. We now have a sorting component reflecting the tendency for high-wage firms to attract high-wage workers and a segregation component reflecting the tendency for firms to employ similar workers.

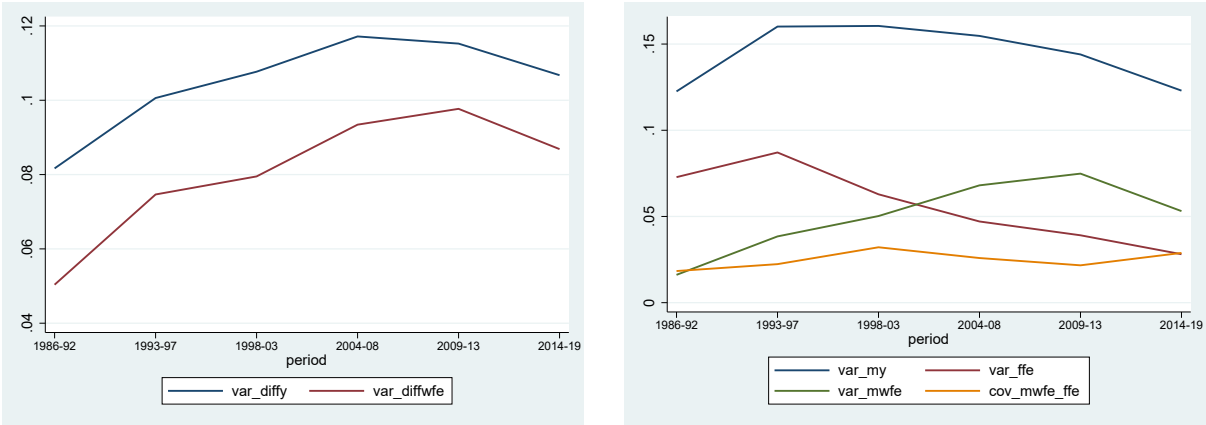
Table 2 provides a comprehensive account of the results, while Figure 4 zeroes in on the main components. Figure 4a suggests that the overall increase in the variance of worker fixed effects is mirrored within Portuguese firms. Within-firm inequality rose simply because the dispersion in relevant worker-level characteristics rose in Portugal. Figure 4b confirms that the compression in firm pay since the 1990s has been the chief driver of earnings inequality. Whereas sorting exerts only a slight force towards an increase in inequality (-12.8% of the total variance since the mid-1990s), Portuguese workers have become significantly more segregated, a powerful force leading to higher inequality (-39.4%). The bottom-line is that compression in firm pay has counteracted both of these forces and has single-handedly brought earnings inequality down.

2.3 Determinants of compression in firm pay since the mid-1990s

What are the economic forces primarily responsible for the greater compression in firm pay? The literature has suggested that firm-level heterogeneity in pay may be intimately tied to the variation in firm-level productivity in a non-competitive labor market. This motivates us to investigate the association between firm pay and firm-level productivity. We postulate the following log-linear specification for firm j's pay policy in subperiod p:

	Interval 1 (1986-1992)		Interval 2 (1993-1997)		Interval 6 (2014-2018)		Change 1–2		Change 2–6	
	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share	Comp.	Share
Var(y)	.204	–	.261	–	.23	–	.057	–	-.031	–
Var(m_y)	.123	60	.16	61.4	.123	53.5	.038	66.5	-.037	119.8
Var(m_WFE)	.016	7.9	.038	14.7	.053	23.1	.022	39.4	.015	-47.3
Var(FFE)	.073	35.7	.087	33.4	.028	12.2	.014	25.3	-.059	190.3
Var(m_Xb)	.002	1	.001	.2	.001	.5	-.001	-2.5	.001	-1.6
2Cov(m_WFE, FFE)	.018	9	.022	8.6	.029	12.6	.004	7.1	.007	-21
2Cov(m_WFE, m_Xb)	.003	1.6	.004	1.7	.008	3.7	.001	2.1	.004	-12.8
2Cov(FFE, m_Xb)	.01	4.9	.007	2.8	.003	1.5	-.003	-4.9	-.004	12.2
Var(diff_y)	.082	40	.101	38.6	.107	46.5	.019	33.5	.006	-19.8
Var(diff_WFE)	.05	24.7	.075	28.6	.087	37.8	.024	42.9	.012	-39.4
Var(diff_Xb)	.012	6	.003	1.3	.007	3.1	-.009	-15.4	.004	-11.9
Var(ϵ)	.015	7.4	.015	5.8	.01	4.5	0	0	-.005	16.1
2Cov(diff_WFE, diff_Xb)	.004	1.9	.007	2.8	.003	1.1	.003	5.9	-.005	15.3

Table 2: Detailed Decomposition



(a) Within-firm

(b) Between-firm

Figure 4: Detailed Decomposition

$$\text{FFE}_{jp} = \alpha_p + \gamma_p \log z_{jp}, \tag{4}$$

where z_{jp} is the firm's within-period systematic productivity level, and γ_p is the pass-through from productivity to the firm-level component of wages. From the firm pay policy (4) we obtain

$$\text{var}\left(\text{FFE}_{jp}\right) = \gamma_p^2 \text{var}\left(\log z_{jp}\right), \tag{5}$$

which indicates two possible sources for the compression in firm pay: (i) A compression in the distribution of firm–level productivity and (ii) a decline in the pass-through.

We first ask whether the Portuguese data is consistent with a specification such as (4). We do this by regressing the estimated AKM firm fixed effects on measures of firm–level productivity, following Alvarez, Benguria, Engbom, and Moser (2018) very closely.

We link the Quadros de Pessoal with the Sistema de Contas Integradas das Empresas (SCIE), which allows us to compute measures of firm-level productivity. The SCIE provides a very broad coverage of private-sector non-financial and for-profit firms operating in Portugal. However, it is only available since 1996. Between 1996 and 2003, the information is drawn from the Inquérito à Empresa Harmonizado (IEH), a survey compulsory for all private-sector firms. In 2004 there was a major overall of the data reporting and data quality, which became based upon administrative balance-sheet information (Informação Empresarial Simplificada, IES) used by the Public Administration and the Bank of Portugal. The coverage was also

extended to include entrepreneurs and independent contractors. Further details can be found in Appendix B.

We compute two alternative measures of firm-level productivity. Our favorite is firm–level TFP, computed for firm j in sector s and year t as

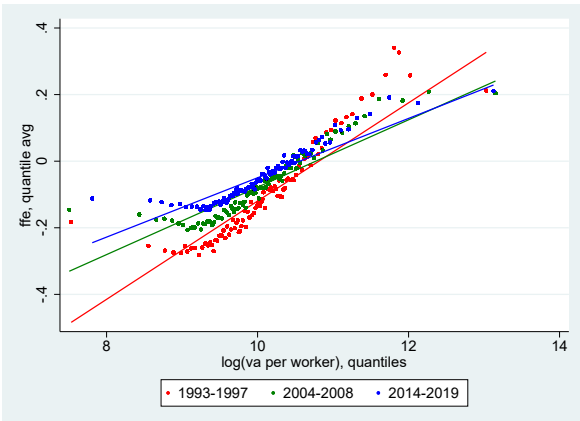
$$z_{jt} = \frac{y_{jt}}{n_{jt}^{\alpha_n^s} m_{jt}^{\alpha_m^s} k_{jt}^{1-\alpha_n^s-\alpha_m^s}},$$

where y is real sales (CPI deflated), n is the number of employees, m is the real value of intermediates, k is the real book value of capital, and the α's are the sector-specific averages of the firm–level revenue shares of total expenditures in labor and materials. Unfortunately information on the book value of capital is only available since 2010, and so is firm-level TFP. We therefore rely mostly on real value-added per worker (CPI deflated), which is available since 1996 and hence covers the whole period of firm pay compression. We average each measure of productivity over each subperiod.

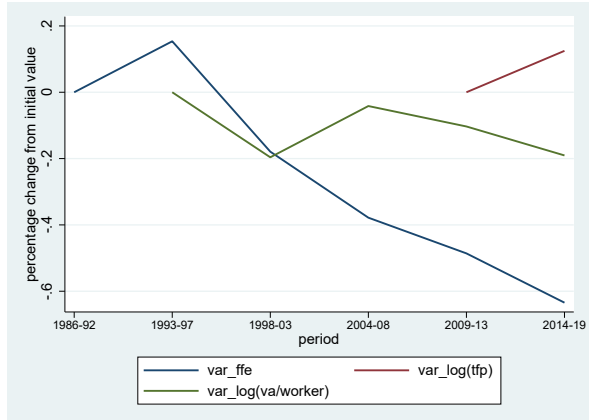
Figure 5 displays the evolution of the two potential determinants of firm pay compression. Figure 5a illustrates the relationship between firm pay and value-added per worker across different subperiods. It plots the average log value-added per worker quantiles against the corresponding firm fixed effects, averaged over each quantile. Quite clearly, high-productivity firms pay more. However, it is also quite clear that the pass-through has weakened over time, especially from the mid-1990s to the mid-2000s.

Figure 5b displays the evolution of firm–level productivity dispersion against the dispersion in firm fixed effects which we wish to explain. Value-added per worker did become less dispersed over time, however the figure suggests that it did so at a significantly lower pace than firm pay. Firm–level TFP actually became more dispersed, working against firm pay compression. However our sample of TFP is unfortunately very short as explained above. Taken together, these observations suggest the weakening of the pass-through from productivity to wages played a major role in the compression of firm pay.

To provide a quantitative assessment of the contribution of the pass-through dynamics, in each subperiod we regress the firm fixed effects on log value-added per worker (regression weighted by worker-years, for comparability with the variance of firm fixed effects computed



(a) Firm pay and value added per worker



(b) Variance of firm pay, value-added per worker, and TFP

Figure 5: Determinants of Firm Pay Compression

	1993-97			1998-03			2004-08			2009-13			2014-19		
log va per worker	.177	.150	.112	.165	.129	.125	.118	.110	.100	.107	.103	.094	.103	.102	.094
log size	–	.049	.039	–	.031	.032	–	.028	.023	–	.016	.014	–	.014	.012
sector dummies	no	no	yes	no	no	yes	no	no	yes	no	no	yes	no	no	yes
R^2	.301	.404	.489	.290	.391	.447	.208	.271	.299	.193	.223	.240	.144	.261	.283

Notes: All coefficients are significant at 1%. Weighted by worker-years.

Table 3: Regression of firm pay on firm characteristics

out of the AKM regression). We also consider sector and firm size controls, since these are likely co-determinants of firm pay. The results, reported in Table 3, show a reduction in the pass-through. The baseline regression without controls displays a decline from 0.177 to 0.103. These figures are somewhat lower but yet comparable to those estimated by Alvarez, Benguria, Engbom, and Moser (2018) for Brazil – a decline from 0.26 to 0.14 between 1996-2000 and 2008-2012.[7]

Differently from Brazil, the dispersion in value-added per worker in Portugal has declined as well. In order to quantify the contribution of the decline in the pass-through in each of the regression scenarios of Table 3, we compute share of the change in the firm pay variance explained by our regression model that is accounted for by the change in the pass-through alone.[8] We find that the decline in the pass-through contributed to about 84% in

[7] Our figures are also very much in line with the 0.1-0.15 range reported by Card, Cardoso, Heining, and Kline (2018) for Portugal, although they restrict to male workers in the late 2000s and employ a different data set for firm–level characteristics.

[8] We perform the following exercise. In each regression scenario of Table 3, equation (5) is the expression for the conditional variance of firm pay in subperiod p. We then compute $\Delta \log \gamma_p^2 / \Delta \log \text{var}\left(\text{FFE}_{jp}\right)$, where

the case without further controls, 83% when controlling for size, and 44% when controlling for both size and sector. We conclude that the decline in the pass-through has been a major contributor for the compression in firm pay.

3 Model

Our model nests a monopsonistic labor market in a off-the-shelf theory of firm dynamics along the lines of Hopenhayn (1992). The framework has clear similarities with those introduced in Gouin-Bonenfant (2020) and Bilal, Engbom, Mongey, and Violante (2021), respectively. Our environment is more parsimonious and closer in spirit to the firm dynamics literature. We concentrate our attention on the implications for wages.

Firms produce an homogeneous good by means of the production function $y = zl^{\gamma}$, where l is labor, z is a firm-level (idiosyncratic) productivity index, and $0 < \gamma \leq 1$. Productivity is stochastic. It takes value on a discrete support, i.e. $z \in Z = \{z_1, \ldots, z_N\}$, and its evolution is traced by the Markov chain Π with typical element $\pi_{ij} \equiv \Pr(z_{t+1} = z_j | z_t = z_i)$.

For simplicity, we assume that the distribution of entrants E over Z is time-invariant and that firms exit with a constant probability $\delta \in (0, 1)$.

Let $N_w > 0$ denote the mass or workers. The value to a worker of a wage offer w is

$$u(w) = w + \varepsilon,$$

where ε is a match-specific amenity shock distributed according to the Gumbel distribution with location parameter equal to 0 and scale parameter $\sigma \geq 0$. In other words, ε proxies for non-pecuniary aspects of the job offer that the worker may value, such as proximity, colleagues' characteristics, and required effort.

In every period, each worker receives offers from two firms only, drawn randomly from the population. For any pair of wage offers $\{w_i, w_j\}$, her optimization problem is simply

$$\max \ \{w_i + \varepsilon_i, w_j + \varepsilon_i\}.$$

the time differences are from the 1993-97 period to the 2014-19 period.

Workers' optimization, jointly with firms' wage offers, induce a labor supply curve faced by each firm $mN_w\Phi(w)$, where $m > 0$ denotes the meeting rate and $\Phi(w)$ is the probability that a wage offer w is accepted. The firm's value contingent on productivity realization z_i is

$$v(z_i, \Phi_t) = \max_w \; z_i[l(w)]^\gamma - wl(w) + \beta\delta \sum_{j=1}^{N} \pi_{ij} v(z_j, \Phi_{t+1})$$
$$\text{s.t. } \; l(w) = mN_w\Phi_t(w),$$

where $\beta \in (0, 1)$ is the discount factor.

Notice that the firm's optimization problem is static. Necessary condition for optimality is

$$z\gamma l^{\gamma-1} - w = \frac{\Phi(w)}{\phi(w)}, \tag{6}$$

where $\phi(w) = \Phi'(w)$. Since the elasticity of labor supply is $\epsilon_s(w) = \frac{\phi(w)w}{\Phi(w)}$, equation (6) can be rewritten as the familiar condition relating a monopsonistic firm's wage markdown to the labor supply elasticity:

$$\frac{z\gamma l^{\gamma-1} - w}{w} = \frac{1}{\epsilon_s(w)}.$$

The wage is pushed up to the point where the marginal gain deriving from adding to labor equals the marginal cost from adding to the wage bill of inframarginal workers.

3.1 Stationary Recursive Equilibrium

Given our simplifying assumptions, the evolution of the firm distribution over the productivity space Z is independent of wages. Letting $\hat{\mu}_t$ denote the measure of firms at time t,

$$\hat{\mu}_t = (1 - \delta)\Pi'\hat{\mu}_{t-1} + E.$$

It follows that the stationary measure of firms $\hat{\mu}$ satisfies

$$\hat{\mu} = (I - (1 - \delta)\Pi')^{-1}E$$

and that the mass of firms is $N_f = ||\hat{\mu}||_1$. From now on, let $\mu_i = \mu(z_i) = \hat{\mu}(z_i)/N_f$. The assumption that each workers gets exactly two wage offers implies that each firm's meeting rate is $m = 2/N_f$.

Notice now that, when comparing two wage offers $\{w_i, w_j\}$, the probability that a worker accepts job i is

$$\Pr(w_i + \varepsilon_i \geq w_j + \varepsilon_j) = \frac{1}{1 + e^{\frac{w_j - w_i}{\sigma}}}.$$

Restricting firms to pure strategies, the share of workers a firm is able to attract by offering a wage w when firms's wage policy is given by $w_j \equiv w(z_j)$, is

$$\Phi(w) = \sum_{j=1}^{N} \frac{1}{1 + e^{\frac{w_j - w}{\sigma}}} \mu_j.$$

In turn, this means that the vector of equilibrium wage policies $w_i = w(z_i)$ is uniquely determined by the system of equations

$$\left[z_i \gamma \left(m N_w \sum_j \frac{1}{1 + e^{\frac{w_j - w_i}{\sigma}}} \mu_j \right)^{\gamma - 1} - w_i \right] \sum_j \frac{1}{\sigma} \frac{e^{\frac{w_j - w_i}{\sigma}}}{\left[1 + e^{\frac{w_j - w_i}{\sigma}} \right]^2} \mu_j - \sum_j \frac{1}{1 + e^{\frac{w_j - w_i}{\sigma}}} \mu_j = 0, \tag{7}$$

for $i = 1, 2, ..., N$.

We close this section by providing our definition of stationary equilibrium.

Definition. A stationary equilibrium consists of a time-invariant firm distribution μ, a mass of firms N_f, an individual wage function w, and a meeting rate m such that

- $\mu = \hat{\mu}/N_f$, where $\hat{\mu} = (I - (1 - \delta)\Pi')^{-1} E$ and $N_f = ||\hat{\mu}||_1$
- the vector w solves the system of equations (7) for $m = 2/N_f$.

4 Labor market power and the distribution of firm-level pay

A main implication of our theory is a map from productivity into wages. In this section, we study how this map depends on σ. Along with the meeting rate m, the parameter σ has an obvious effect on firms' labor market power, as it controls the importance of job amenities

relative to wages in workers' choice. For $\sigma = 0$, workers will simply choose the highest salary. This is the scenario where firms enjoy the least labor market power. For σ arbitrary large, wages essentially don't matter. The probability of the worker accepting a job is 1/2 no matter what. This is the least competitive scenario.

We solve for the equilibrium wages choices in a parameterized version of the model. We then perform a comparative statics exercise by looking at the steady-state implications of changes in the value of σ.

Figure 6 displays the firm distribution given our specification for the Markov chain Π and the distribution of entrants E. We made assumptions so that firms start relatively small and therefore tend to grow over time.

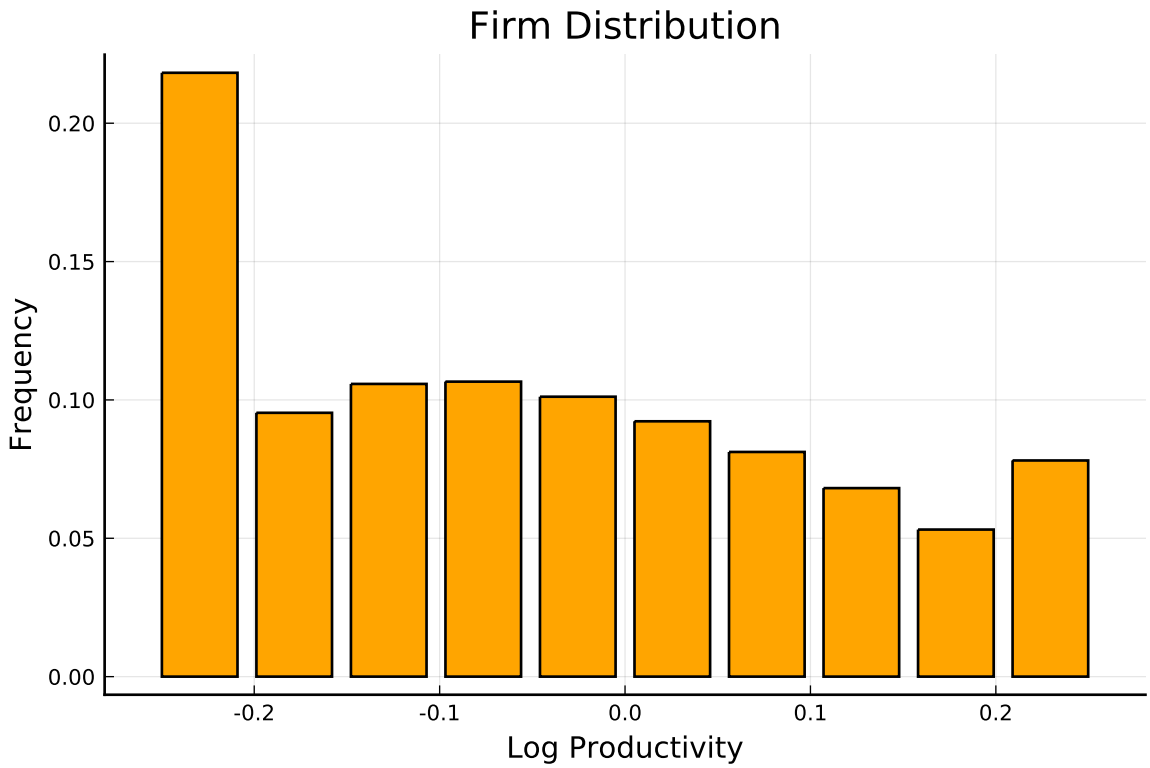
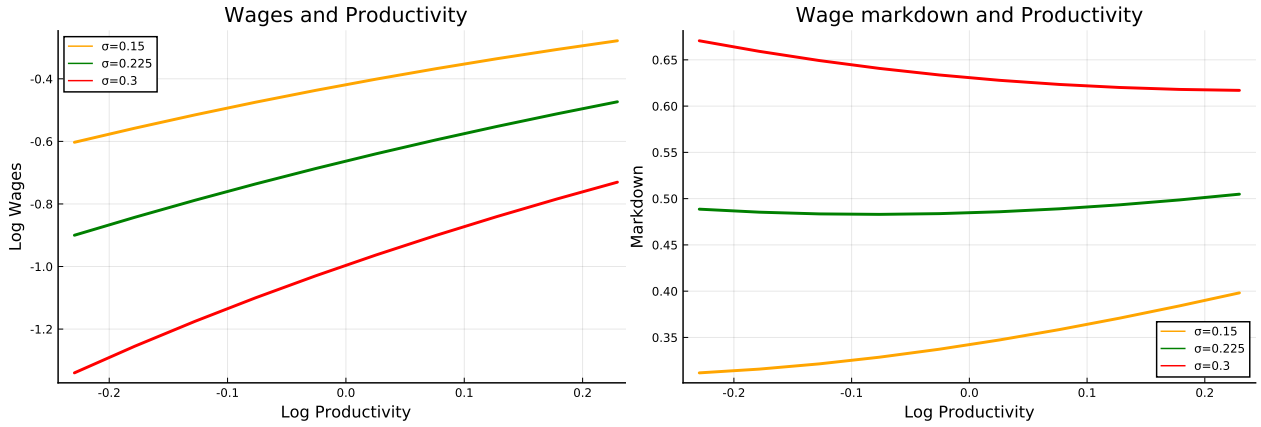


Figure 6: Firm Distribution over Productivity

Figure 7a illustrates the equilibrium pay policy for three different levels of σ. Wages are strictly increasing in productivity and therefore in size. The reason is simply that more productive firms have a higher marginal product of labor and therefore a greater marginal gain of raising salaries.



(a) Wages

(b) Markdown

Figure 7: Wages, Markdown, and Productivity

As competition increases, i.e. as σ declines, wages increase too. This is the case because, for low σ, a marginal increase in wages has a greater yield in terms of hires. Notice however that this effect is proportionally larger for low-productivity firms, leading to a flatter wage-productivity profile. This is confirmed by the observation that the wage markdown, illustrated in Figure 7b, grows with σ – at a faster rate for low-productivity firms.

The implications of a flattening of the wage policy for earnings inequality are immediate: Inequality drops as labor market competition increases. See Figure 8, which illustrates the standard deviation of log wages as a function of the parameter σ. This suggests a possible rationalization of the evidence on the dynamics of Portugal's firm-level pay illustrated in Section 2.

Recall that our empirical findings support an important decline in the pass-through of productivity to wages, accompanied by a less significant compression in firm–level productivity. Figure 7a hints that such decline in pass-through may be the result of a decrease in Portuguese firms' labor market power.

Portugal has indeed been experiencing a significant increase in urbanization and migration towards the coastal regions, as well as a significant improvement in road infrastructure. This would be consistent with the decrease of geographical as well as informational obstacles to competition, possibly affecting low-productivity firms the most. Analogously, E.U. membership may have also increased competitive pressure. The resulting increase in wage competition might have therefore generated the compression in firm pay. We shall investigate

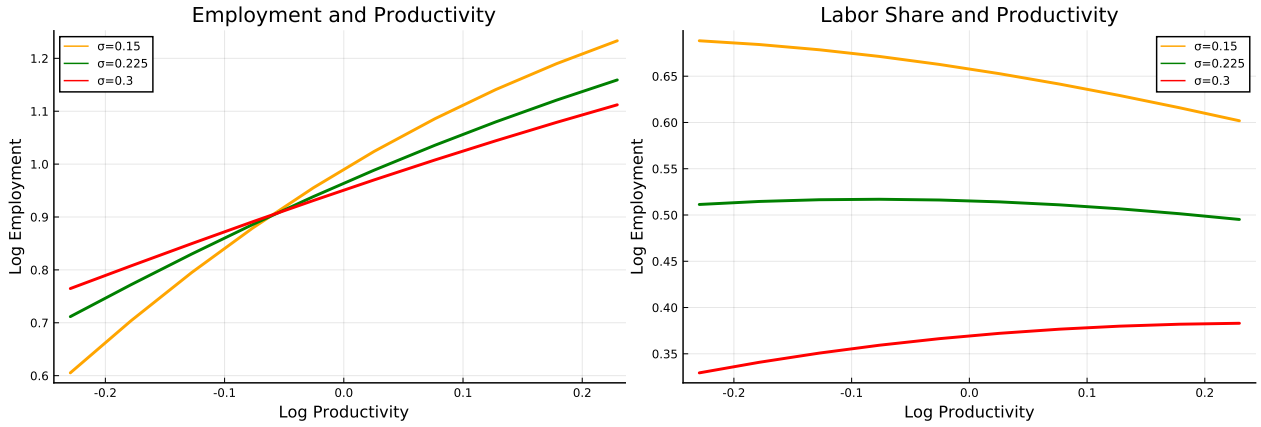
this possibility carefully.

Figure 9 illustrates the comparative statics of employment and the labor share, respectively. As firms' labor market power decreases, the labor share increases – the more so, the lower firm productivity. Interesting, employment – which is kept constant in aggregate – is redistributed from low- to high-productivity firms.

The upshot is that more competitive labor markets reduce mark-downs, increase labor shares, and increase wages across the board. The equilibrium wage-productivity profile becomes flatter, resulting in a reductions in pass-through, a reallocation of employment from low- to high-productivity firms and greater aggregate output.

According to our theory, a decline in the pass-through from productivity to wages such as that documented in Section 2 is not necessarily associated with a decline in workers' welfare. To the extent that a flatter wage-productivity profile is caused by a lower σ, it will be accompanied by higher wages ad a higher labor share. In other words, a lower σ affects non only the slope by also the intercept of the relation between wages and productivity. In the scenario we have examined, the intercept rises.

Figure 8: Earnings Inequality



(a) Employment

(b) Labor Share

Figure 9: Employment, Labor Share, and Productivity

5 Conclusion

TBA

Appendix

A Quadros de Pessoal

Details here.

B Sistema the Contas Integradas das Empresas

Details here.

References

Abowd, J. M., F. Kramarz, and D. N. Margolis (1999): "High Wage Workers and High Wage Firms," *Econometrica*, 67(2), 251–334.

Alvarez, J., F. Benguria, N. Engbom, and C. Moser (2018): "Firms and the Decline in Earnings Inequality in Brazil," *American Economic Journal: Macroeconomics*, 10(1), 149–189.

Bilal, A. G., N. Engbom, S. Mongey, and G. L. Violante (2021): "Firm and Worker Dynamics in a Frictional Labor Market," *Econometrica*, forthcoming.

Bonhomme, S., K. Holzheu, T. Lamadon, E. Manresa, M. Mogstad, and B. Setzler (2020): "How Much Should we Trust Estimates of Firm Effects and Worker Sorting?," NBER Working Papers 27368, National Bureau of Economic Research, Inc.

Card, D., A. R. Cardoso, J. Heining, and P. Kline (2018): "Firms and Labor Market Inequality: Evidence and Some Theory," *Journal of Labor Economics*, 36(S1), 13–70.

Card, D., J. Heining, and P. Kline (2013): "Workplace Heterogeneity and the Rise of West German Wage Inequality," *The Quarterly Journal of Economics*, 128(3), 967–1015.

Cardoso, A. R. (1999): "Firms' Wage Policies and the Rise in Labor Market Inequality: The Case of Portugal," *Industrial and Labor Relations Review*, 53(1), 87–102.

Engbom, N., C. Moser, and J. Sauermann (2021): "Firm pay dynamics," Working Paper Series 2021:21, IFAU - Institute for Evaluation of Labour Market and Education Policy.

Gouin-Bonenfant, É. (2020): "Productivity Dispersion, Between-Firm Competition,and the Labor Share," Discussion paper, Columbia University.

Hopenhayn, H. A. (1992): "Entry, Exit and Firm Dynamics in Long Run Equilibrium," *Econometrica*, 60, 1127–50.

Messina, J., and J. Silva (2019): "Twenty Years of Wage Inequality in Latin America," *The World Bank Economic Review*, 35(1), 117–147.

Song, J., D. J. Price, F. Guvenen, N. Bloom, and T. von Wachter (2018): "Firming Up Inequality," *The Quarterly Journal of Economics*, 134(1), 1–50.