GDP Solera

The Ideal Vintage Mix[*]

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Abstract

We exploit the information in all the vintages of GDE and GDI measurements from a given comprehensive revision to obtain a better measurement of aggregate economic activity by exploiting cointegration between the different measures and taking seriously the vintage release calendar. We also combine overlapping comprehensive revisions to improve our measurement of the most recent observations, with particular attention to the Great Recession and the pandemic. We use the values of the estimated parameters of our dynamic state space model to assess whether comprehensive revisions induce changes in the long-run growth rate and the persistence of shocks to economic activity.

Keywords: Cointegration, Comprehensive revisions, Signal extraction, Vintages.

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1 Introduction

Despite the recent interest in alternative measures, such as the Human Development Index or the different Gross National Happiness measures, Gross Domestic Product (GDP) remains the dominant concept to gauge the aggregate performance of an economy over a given period of time. In the United States of America, the estimates of aggregate economic activity that the Bureau of Economic Analysis (BEA) publishes are used not only by policy makers and research economists, but also by private sector agents, including households and companies, in making their production and consumption decisions, as well as their financial plans.

The BEA uses a mixture of survey, tax and other business and administrative data, as well as various indicators, which are subject to sampling errors and biases that cannot be directly assessed. As time goes by, though, the BEA acquires more and better information, and for that reason it systematically updates its measures, which results in a sequence of estimates for a given quarter known as revisions. In fact, the whole revision process is rather elaborate, and it is important to distinguish between three types: (i) subsequent releases for a given quarter, usually called the "advance", "second" and "third" estimates; (ii) annual (or "final") revisions, which simultaneously update all the quarters of the three previous calendar years; and (iii) occasional comprehensive revisions, which recompute the entire history of the series after a major methodological change that effectively modifies its definition. The importance of revisions should not be underestimated. For example, Orphanides (2001) convincingly argues that the use of preliminary versus final revisions can lead to different monetary policy recommendations.

While in the last two decades there has been considerable progress in jointly modeling the different vintages of data (see, for example, Aruoba (2008), Jacobs and van Norden (2011) and the references therein), some of these studies have ignored a second important consideration: the BEA produces not just one but two different measures of aggregate output and income: Gross Domestic Expenditure (GDE) and Gross Domestic Income (GDI). GDE measures activity as the sum of all final expenditures in the economy, which is reflected in the output side of the national income and product accounts. In turn, GDI measures activity as the sum of all income generated in production, and is therefore captured on the income side of the national accounts (NIPAs). In theory, the flows of income and expenditure should be equal, and thus, GDE and GDI should yield the same measure of economic activity. In practice, though, they differ not only because of the revisions, but because each is calculated from data from completely different sources (see Landefeld, Seskin, and Fraumeni (2008) for a review). The systematic, and at times noticeable, deviation between them (officially known as *statistical discrepancy*)[1] was traditionally regarded by many academic economists as a curiosity in the NIPAs. However, the Great Recession led to substantially renewed interest in academic and policy circles about the possibility of obtaining more reliable economic activity figures by combining the two measures, and various proposals for improved combinations have been discussed (see, e.g. Nalewaik (2010), Nalewaik (2011), Greenaway-McGrevy (2011), Aruoba, Diebold, Nalewaik, Schorfheide, and Song (2016) and Jacobs, Sarferaz, Sturm, and van Norden (2020)). For example, the *GDPplus* measure of Aruoba et al. (2016) is currently released on a monthly schedule by the Federal Reserve Bank of Philadelphia.

The purpose of our paper is to simultaneously tackle all these measurement issues within a single, internally coherent, signal extraction framework.[2] Intuitively, given that GDE and GDI are based on different sources, and that these are subject to successive systematic revisions, one would expect that a more accurate estimate of the underlying economic activity can be obtained by exploiting the dynamic and static recurrent patterns in the observed series. In some respects, the recurrent updating of our signal-extraction process can be regarded as analogous to the criaderas and soleras system of sherry wine aging, whereby the final product is obtained by fractional blending inputs from different vintages over a perennial dynamic procedure that gives sherry its distinctive character.[3]

Despite involving a moderately large number of both latent and observed variables, our model is both flexible and parsimonious thanks to the economic and statistical discipline that we impose on the measurement errors. Our crucial point of departure from the previous literature is that we follow Almuzara, Amengual, and Sentana (2019) and Almuzara, Fiorentini, and Sentana (2021) in imposing that (i) any two aggregate output and income measures (in logs) are cointegrated, with cointegrating vector (1,-1); and that (ii) measurement errors are mean-reverting and stationary, although they may be serially correlated. Thus, we are able to focus not only in quarterly growth rates, but also look at the level of output, which is of considerable interest in itself, particularly in regional or cross-country comparisons.

In addition, the data release calendar is at the core of our model. Specifically, we explicitly take into account that the "advance" GDE estimate is published one month after the end of the quarter, and that the "second" and "third" estimates are published two and three months

[1] See Grimm (2007) for a detailed methodological insight.

[2] Stone, Champernowne, and Meade (1942) is the first known reference to the signal-extraction framework of our paper. Early literature is surveyed in Weale (1992). See also Smith, Weale, and Satchell (1998).

[3] As explained by agent 007 to M in the 1971 James Bond film Diamonds are forever.

after the end of the quarter, respectively. We also acknowledge the fact that the timing of the quarterly releases for GDI is slightly different, as it incorporates information from the quarterly census of employment and wages. Importantly, we also consider the annual data revisions of both series that are published in the summer of the following year, and which typically affect the values for all the quarters of the previous three years.

The final novel ingredient of our model is the combination of data from different comprehensive revisions, which take place approximately every five years. These revisions incorporate changes in definitions, classifications, and statistical methodology. The most recent comprehensive revision was published in July 2018, with a detailed analysis in a BEA paper dated August 2019. In that report, the U.S. statistical office presented revised annual estimates for 1929-2017 and revised quarterly estimates for 1947-2017.[4] Often, comprehensive revisions reflect either improved or totally new surveys on sectors of the economy that have become increasingly important. Despite these systematic differences, the joint modeling of multiple comprehensive revisions is particularly relevant at the time when a new one is released, which is precisely when there is very little information about the statistical properties of its successive vintages and annual revisions.

The closest paper to ours is Jacobs et al. (2020) who also exploit the release process of GDP and GDI to obtain improved real-time estimates of economic activity. Nevertheless, they focus on growth rates and abstract from comprehensive redefinitions of GDP.[5]

From the point of view of implementation, our model can be cast in linear state-space form and is amenable to the use of Bayesian methods of inference for both parameters and latent variables. In particular, we develop a Gibbs sampling algorithm that tackles estimation and signal-extraction simultaneously, allowing for an efficient and conceptually simple integration of uncertainty coming from different sources. Moreover, our strategy to model comprehensive revisions is easily adaptable to cover a wide range of potential applications (e.g., price data) and extensions of the basic dynamic model (e.g., nonnormality or stochastic volatility).

After estimating our model exploiting all the available US data, we use it to answer a number of empirically relevant questions. First, do comprehensive revisions modify the descriptive characteristics of economic growth, such as its mean and persistence? Second, what is the contribution of the different estimates (i.e., advance, second, third, etc.) to the precision of signal

[4] The next comprehensive revision is expected in July 2023.

[5] One additional difference is that Jacobs et al. (2020) propose a framework to separate news from noise in the revision process along the lines of Jacobs and van Norden (2011). Extending our model to incorporate a distinction between news and noise is feasible and constitutes a promising avenue for future research. See appendix E for more information.

extraction about economic activity? Importantly, our estimates suggest that (i) comprehensive revisions have not led to appreciable changes in the properties of growth rates, and that (ii) most of the precision gains in signal extraction takes place by the time the third estimates of GDE and GDI become available. Finally, we provide several additional empirical exercises, including an assessment of our improved estimate of economic activity during the COVID-19 pandemic.

The rest of the document is organized as follows. We begin with a detailed description of the data in Section 2. Section 3 introduces the model, while in Section 4, we give the details of the estimation and filtering algorithms. Section 5 reports the empirical analysis, including the improved "GDP solera" measure of economic activity produced by our method. Finally, Section 6 concludes.

2 Data background

Our empirical analysis uses data on all the GDE and GDI vintages from the BEA. To get a better sense of the data, it is instructive to review the timing of the release process as it happens regularly over a typical year. Table 1 exemplifies the process in a recent period. Estimates for quarterly GDP are released in the following order:[6]

(A) Advance estimate, based on source data incomplete or subject to further revision by the source agency, and released near the end of the first month after the end of the quarter,

(B) Second/third estimates, which use broader and more detailed data, and are released near the end of the second and third months, respectively,

(C) Latest estimates, which reflect the results of both annual and comprehensive updates.

For GDI only second, third and latest estimates are prepared because of data availability, except for the fourth quarter of each year, for which only third and latest estimates are released.

Normally, a single estimate for the latest quarter is added to the GDE/GDI series at a time, but there are two kinds of updates where multiple quarters are simultaneously updated:

(a) Annual updates, usually done in July, which cover at least the most recent three calendar years (e.g. July 2017's annual update revised 2017Q1, and all quarters from 2016, 2015 and

[6]Before 2009Q2, the BEA used the terminology "advance", "preliminary" and "final" for what it now calls "advance", "second" and "third", respectively.

2014). They incorporate newly available annual source data, and minor methodological changes.

(b) Comprehensive (or benchmark) updates, which are done approximately every 5 years (the last updates were in December 2003, July 2009, July 2013 and July 2018). They incorporate major periodic source data (for example data released at frequencies lower than 1 year), and some major methodological changes. Real GDP is usually rebased, with the reference year chosen such that it will remain fixed during the subsequent annual updates.[7]

TABLE 1. GDE and GDI release schedule for the period 2016Q1-2018Q2.

Release Month	**Estimate**	**GDE**		**GDI**	
		New	**Updated**	**New**	**Updated**
Jan 2017	Advance	2016Q4			
Feb 2017	Second		2016Q4		
Mar 2017	Third		2016Q4	2016Q4	
Apr 2017	Advance	2017Q1			
May 2017	Second		2017Q1	2017Q1	
Jun 2017	Third		2017Q1		2017Q1
Jul 2017	Advance*	2017Q2	2014Q1-2017Q1†		2014Q1-2017Q1†
Aug 2017	Second		2017Q2	2017Q2	
Sep 2017	Third		2017Q2		2017Q2
Oct 2017	Advance	2017Q3			
Nov 2017	Second		2017Q3	2017Q3	
Dec 2017	Third		2017Q3		2017Q3
Jan 2018	Advance	2017Q4			
Feb 2018	Second		2017Q4		
Mar 2018	Third		2017Q4	2017Q4	
Apr 2018	Advance	2018Q1			
May 2018	Second		2018Q1	2018Q1	
Jun 2018	Third		2018Q1		2018Q1
Jul 2018	Advance**	2018Q2	1947Q1-2018Q1	2018Q2	1947Q1-2018Q1

NOTES. [*] Annual update, [**] Comprehensive update, [†] 13 quarters, i.e. last 3 years

We will use all avaliable GDE and GDI vintages over the period 1984Q1-2020Q4 for our main empirical analysis. Specifically, we will account for five versions of economic activity — the result of 4 comprehensive revisions in 2003, 2008, 2013 and 2018.

The series (in levels) of different comprehensive revision releases are depicted in Figure 1 where we also plot data produced by early and annual revisions for the periods between

[7] Vintages Y2011Q2E1 and Y2014Q2E1 are exceptions because the reference year was also revised. This resulted in change of the GDP deflator and, in turn, a change of real GDP for the whole series since 1947.

two consecutive comprehensive revisions. Figure 2 zooms on two subperiods to illustrate the different measures of economic activity.

3 Model

Let x_t be an aggregate quantity of interest — in our empirical analysis, US economic output (in logs) during quarter t. As most of the literature that followed Stone et al. (1942), we treat x_t as a latent variable of which only noisy measurements y_t are available. The task is to construct rules mapping measurements into inferences about the latent x_t.[8]

This section develops a framework that allows us to combine multiples y_t's for the purposes of obtaining an improved estimate of economic activity. For clarity, we begin in subsection 3.1 with a version of our model that has no comprehensive revisions, adding them in subsection 3.2.

3.1 Modeling early and annual estimates

Let y_{it}^m be a noisy measurement of x_t, where, broadly speaking, the index i denotes type (e.g., GDE and GDI estimates) and the index m denotes release (e.g., early and annual estimates). This distinction is important because we will assume orthogonality of measurement errors along i but we will permit correlation over m for measurements with the same i. Orthogonality across expenditure and income measures is useful to achieve identification of the serial dependence in x_t, while correlation between the measurement errors of different releases of the same measure is to be expected.

The model is given by the set of measurement equations

$$y_{it}^m = x_t + v_{it}^m, \;\; m = 1, \ldots, M_i, \quad i = 1, \ldots, N,$$

where v_{it}^m is the measurement error in y_{it}^m. For each i, collect $y_{it}^1, \ldots, y_{it}^{M_i}$ into the vector y_{it} and stack $y_{1t}, \ldots, y_{Nt}$ into y_t. Defining v_{it}, for each i, and v_t likewise, we obtain,

$$y_t = 1_{M\times 1} x_t + v_t, \tag{1}$$

where $M = \sum_{i=1}^N M_i$ and $1_{M\times 1}$ is an M-dimensional vector of ones.

In this context, we assume that the following conditions hold:

[8] For background on output measurements, see Landefeld et al. (2008), Nalewaik (2010), and Nalewaik (2011).

Assumption 1.

(a) Δx_t *is I(0);*

(b) $v_{1t}, \ldots, v_{Nt}$ *are I(0);*

(c) $\Delta x_t, v_{1t}, \ldots, v_{Nt}$ *are mutually orthogonal at all lags and leads.*

Assumption 1(a) is made because y_t measures economic activity in levels.[9] Together with assumption 1(b), it implies that y_t is cointegrated with cointegration rank $M - 1$.[10] Cointegration is a feature that matters for our empirical analysis and a very plausible assumption for aggregate measurement problems (see, e.g., Almuzara et al. (2021) for discussion). Assumption 1(c), on the other hand, is key for identification as asserted in the following proposition, whose proof can be found in appendix A:

Proposition 1. *Under assumption 1, if* $N > 1$*, the autocovariances of* $\Delta x_t, v_{1t}, \ldots, v_{Nt}$ *are nonparametrically identified from the autocovariances of* Δy_t*.*

Our empirical analysis features $N = 2$, as we use GDE and GDI measurements of output.[11]

3.2 Modeling comprehensive revisions

Our approach to modeling comprehensive revisions is to treat each version of the variable of interest introduced by the revision process as a different latent variable, while at the same time allowing for strong dependence among them.

Let C be the number of versions. Rather than a single variable, our extended model makes x_t a vector,

$$x_t = \begin{pmatrix} x_{1t} \\ \vdots \\ x_{Ct} \end{pmatrix}. \tag{2}$$

Here x_{ct}, for $c = 1, \ldots, C$, represents the hypothetical value of economic output that could be measured with the definitions and methods introduced by revision c if data sources and the

[9] We take the definition of I(0) process from the multivariate generalization of that in Stock (1994). Consider a time series $\omega_t = \sum_{\ell=0}^{\infty} \Theta_\ell \varepsilon_{t-\ell}$ with Θ_ℓ an $n \times n$ matrix and ε_t and n-dimensional vector. Then, ω_t is I(0) if (i) ε_t is a weakly stationary vector m.d.s., (ii) $\sum_{\ell=0}^{\infty} \Theta_\ell \Theta_\ell'$ is nonsingular and (iii) $\sum_{\ell=0}^{\infty} \ell \|\Theta_\ell\| < \infty$.

[10] Any set consisting of $M - 1$ pairwise differences among the y_{it}^m is a basis for the cointegration space.

[11] $N = 1$ may be relevant for other applications. In those cases, identification can be achieved by imposing restrictions on the cross-dependence among $v_{1t}^1, \ldots, v_{1t}^{M_1}$ (e.g., assuming $v_{1t}^{m_1}$ and $v_{1t}^{m_2}$ orthogonal at all lags and leads), or by a sufficiently tight parametric structure.

measuring tools were perfect. Analysts and policy makers typically focus on the latest version x_{Ct}. However, there are important reasons for jointly modeling $x_{1t}, \ldots, x_{Ct}$: first, there is interest in older definitions of economic activity from a historical perspective, since after all, those were the only ones available at the time; second, there is also considerable interest in understanding the impact of comprehensive revisions on the static and dynamic characteristics of the growth rates in aggregate economic activity; finally, there is also substantial interest in quickly learning about the dynamics of the measurement errors in the most recent version, which might lead to improved inferences about x_{Ct} itself.

Measurement equation. Let δ_{it}^m be a $1 \times C$ array that has 1 in entry c if y_{it}^m measures x_{ct} and 0 otherwise. The array δ_{it}^m is known since it can be easily computed by comparing the date of comprehensive revisions and the release date of y_{it}^m. Our model postulates that

$$y_{it}^m = \delta_{it}^m x_t + v_{it}^m, \quad i = 1, \ldots, N, \ m = 1, \ldots, M_i.$$

Concatenating δ_{it}^m vertically to conform with y_{it} and y_t, we obtain the $M_i \times C$ array δ_{it} and the $M \times C$ array δ_t, which lead to the measurement equation

$$y_t = \delta_t x_t + v_t. \tag{3}$$

Equation (3) generalizes (1) into a deterministically time-varying measurement equation. We also note that some of the entries of y_t may be missing, e.g., because old methods are not applied to the computation of new estimates or because the release protocol stipulates so.

Assumption 1 is adopted without change (except that Δx_t is a vector process now). This way, our framework generalizes naturally the multiple measurements single latent variable models of the literature (e.g., Weale (1992), Smith et al. (1998), Aruoba et al. (2016), Almuzara et al. (2019), and Almuzara et al. (2021)) to a situation in which there are multiple latent variables of interest.

Identification revisited. Because the measurement equation is time-varying, the spectrum of y_t depends on t. However, given that the time-variation is deterministic, this entails a trivial form of non-stationarity from the point of view of identification. In our empirical analysis, moreover, there is a subvector of y_t that is stationary since there is a time-invariant block in δ_t. This allows us to establish identification exploiting a generalization of proposition 1 applied to the time-invariant block. We state sufficient conditions for non-parametric identification in proposition 2 (its proof is in appendix A).

Proposition 2. *Suppose there are indices i_1, i_2 ($i_1 \neq i_2$) and matrices E_{i_1}, E_{i_2} such that (i) $E_{i_1} y_t$ and $E_{i_2} y_t$ are nonempty subvectors of $y_{i_1,t}$ and $y_{i_2,t}$, respectively, (ii) $E_{i_1}\delta_t$ and $E_{i_2}\delta_t$ are time-invariant, and (iii) $\operatorname{rank}(E_{i_1}\delta_t) = \operatorname{rank}(E_{i_2}\delta_t) = C$. Then, under assumption 1, the autocovariances of $\Delta x_t, v_{1t}, \ldots, v_{Nt}$ are nonparametrically identified from those of Δy_t.*

As an example, consider a model with $C = 2$ versions of economic activity. Suppose $N = 2$ with $M_1 = M_2 = 2$ and $\delta_t = \begin{pmatrix} I_2 & I_2 \end{pmatrix}'$ for all t. The measurement equation is

$$\begin{pmatrix} y_{1t}^1 \\ y_{1t}^2 \\ y_{2t}^1 \\ y_{2t}^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix} + \begin{pmatrix} v_{1t}^1 \\ v_{1t}^2 \\ v_{2t}^1 \\ v_{2t}^2 \end{pmatrix}.$$

This setup clearly satisfies the conditions of proposition 2 with $i_1 = 1, i_2 = 2$, $E_{i_1} = \begin{pmatrix} I_2 & 0_{2\times 2} \end{pmatrix}$, and $E_{i_2} = \begin{pmatrix} 0_{2\times 2} & I_2 \end{pmatrix}$. The autocovariances of $\Delta x_t, v_{1t}, v_{2t}$ are, consequently, identified from the autocovariances of Δy_t. Some intuition can be gained by first considering the measurement sub-systems

$$\begin{pmatrix} y_{1t}^c \\ y_{2t}^c \end{pmatrix} = 1_{2\times 1} x_{ct} + \begin{pmatrix} v_{1t}^c \\ v_{2t}^c \end{pmatrix}, \quad c = 1, 2.$$

Proposition 1 can be applied and immediately delivers the marginal serial dependence structure of the processes $\Delta x_{1t}, \Delta x_{2t}, v_{1t}^1, v_{1t}^2, v_{2t}^1, v_{2t}^2$. Next, it is possible to recover the cross-autocovariances of the two signals by observing that

$$\operatorname{Cov}\left(\Delta x_{1t}, \Delta x_{2,t-\ell}\right) = \operatorname{Cov}\left(\Delta y_{1t}^c, \Delta y_{2,t-\ell}^c\right)$$

holds for $c = 1, 2$ and all ℓ. Finally, for $i = 1, 2$ and all ℓ, we have

$$\operatorname{Cov}\left(\Delta v_{it}^1, \Delta v_{i,t-\ell}^2\right) = \operatorname{Cov}\left(\Delta y_{it}^1, \Delta y_{i,t-\ell}^2\right) - \operatorname{Cov}\left(\Delta x_{1t}, \Delta x_{2,t-\ell}\right).$$

In our empirical analysis we rely on $C = 5$ versions of both GDE and GDI, in addition to early and latest estimates. This implies that, for all t, δ_t contains two distinct blocks which are equal to I_C, one corresponding to GDE measurements and another to GDI, and the conditions in proposition 2 are automatically satisfied. Hence, the joint dynamics of Δx_t are non-parametrically identified.[12]

Transition equation. Although the spectrum of x_t is non-parametrically identified, to implement our empirical analysis we specify a parametric model for $f_{\Delta x}, f_{v_1}, \ldots, f_{v_N}$ that satisfies assumption 1 and, at the same time, is amenable to estimation by Bayesian methods. We adopt a Bayesian approach because it allows us to easily integrate estimation and filtering uncertainty when performing signal extraction, our main objective.

Specifically, we model Δx_t as a restricted VAR with a factor structure in the error term,

$$\begin{aligned} \Delta x_t &= \mu_x + \operatorname{diag}(\rho_x)\left(\Delta x_{t-1} - \mu_x\right) + \left(\lambda_x \eta_{xt} + \operatorname{diag}(\sigma_x)\varepsilon_{xt}\right), \\ \eta_{xt} &\overset{iid}{\sim} N(0,1) \text{ independent of } \varepsilon_{xt} \overset{iid}{\sim} N(0_{C\times 1}, I_C). \end{aligned} \tag{4}$$

We collect the unknown parameters of the Δx_t process into $\theta_x = (\mu_x, \rho_x, \lambda_x, \sigma_x)$. In principle, we allow for differences in the mean, persistence and variance of economic growth across versions. In fact, estimating θ_x will allow us to empirically test whether comprehensive revisions had any impact on the implied dynamic properties of economic activity.

The initial condition for the level is modeled as independent of $\eta_{xt}, \varepsilon_{xt}$ for all t, and

$$x_1 \sim N(\mu_1, \Sigma_1).$$

This accommodates potential differences in levels between versions x_t (because, e.g., different series use different base periods as deflator).[13]

For the measurement errors of type i we postulate a restricted VAR(1) model with a factor structure in the error too, i.e.,

$$\begin{aligned} v_{it} &= \operatorname{diag}(\rho_i) v_{i,t-1} + \left(\lambda_i \eta_{it} + \operatorname{diag}(\sigma_i)\varepsilon_{it}\right), \\ \eta_{it} &\overset{iid}{\sim} N(0,1) \text{ independent of } \varepsilon_{it} \overset{iid}{\sim} N(0_{M_i\times 1}, I_{M_i}). \end{aligned} \tag{5}$$

[12] One qualification worth making is that because past versions are discontinued, we are truly learning about the joint autocorrelation structure of x_t within the period in which they overlap. This amounts to a long period in our sample, spanning 1947Q1 to 2003Q2 (the time of the first comprehensive revision), yet a period that excludes the instabilities originating with, e.g., the Subprime Crisis and the Pandemic.

[13] We will treat μ_1 and Σ_1 as known and take Σ_1 to reflect a diffuse prior over x_1. A relatively easy-to-implement alternative would be to estimate μ_1.

We collect the unknown parameters into $\theta_i = (\rho_i, \lambda_i, \sigma_i)$. Note that serial correlation of measurement errors in levels is permitted in our model, and in fact, the analysis of statistical discrepancies highlights it as a relevant empirical feature. We also allow for variation in the autocorrelations and volatilities across different releases.

State-space representation. The parameter vector of the model is $\theta = (\theta_x, \theta_1, \ldots, \theta_N)$. Given θ, we can cast equations (3), (4) and (5) ($i = 1, \ldots, N$) in state-space form in a number of ways,

$$\begin{aligned} y_t &= H_t X_t, \\ X_t &= C(\theta) + F(\theta) X_{t-1} + G(\theta) U_t, \\ U_t &\overset{iid}{\sim} N(0_{(C+M+N+1)\times 1}, I_{C+M+N+1}). \end{aligned}$$

Here we have defined

$$X_t = \begin{pmatrix} x_t \\ x_{t-1} \\ v_{1t} \\ \vdots \\ v_{Nt} \end{pmatrix} \text{ and } U_t = \begin{pmatrix} \eta_{xt} \\ \varepsilon_{xt} \\ \eta_{1t} \\ \varepsilon_{1t} \\ \vdots \\ \eta_{Nt} \\ \varepsilon_{Nt} \end{pmatrix},$$

together with

$$\begin{aligned} H_t &= \begin{pmatrix} \delta_t & 0_{M\times C} & I_M \end{pmatrix}, \\ C(\theta) &= \begin{pmatrix} (I_C - \mathrm{diag}(\rho_x))\mu_x \\ 0_{C\times 1} \\ 0_{M\times 1} \end{pmatrix}, \\ F(\theta) &= \mathrm{diag}\left(\begin{pmatrix} I_C + \mathrm{diag}(\rho_x) & -\mathrm{diag}(\rho_x) \\ I_C & 0_{C\times C} \end{pmatrix}, \mathrm{diag}(\rho_1), \ldots, \mathrm{diag}(\rho_N) \right) \text{ and} \\ G(\theta) &= \mathrm{diag}\left(\begin{pmatrix} \lambda_x & \mathrm{diag}(\rho_x) \end{pmatrix}, \begin{pmatrix} \lambda_1 & \mathrm{diag}(\rho_1) \end{pmatrix}, \ldots, \begin{pmatrix} \lambda_N & \mathrm{diag}(\rho_N) \end{pmatrix} \right). \end{aligned}$$

For the initial condition we have $X_1 \sim N(\tilde{\mu}_1, \tilde{\Sigma}_1)$ where $\tilde{\mu}_1, \tilde{\Sigma}_1$ are made compatible with μ_1, Σ_1 and covariance-stationarity of $v_{1t}, \ldots, v_{Nt}$.

That the model admits a linear state-space representation with Gaussian errors is important because it implies that $X_{1:T}, U_{1:T}$ are jointly normally distributed conditioned on $y_{1:T}, \theta$, and the algorithm of Durbin and Koopman (2002) can be used to efficiently simulate that distribution (see subsection 4.2).

3.3 Some objects of interest

Our framework delivers inference about a number of empirically interesting objects that arise in a wide range of applications. In this aside we develop a few of them that we report in our empirical analysis in Section 5. An additional object that measures the L^2-optimality of estimates is discussed in appendix D.

GDP solera. Of course, the GDP solera $\hat{x}_t = \mathbb{E}\left[x_t | y_{1:T}\right]$ (and, more generally, the conditional distribution of $x_{1:T}$ given $y_{1:T}$) is a key output from our model.

The impact of comprehensive revisions. A relevant empirical question that can be answered within our framework is whether comprehensive revisions modify the dynamic properties of economic activity. We do so by looking at the posterior distribution of $\mu_x, \rho_x, \lambda_x, \sigma_x$.

Measures of real-time precision gains. The release process for economic activity implies a mapping between each month τ and the available measurements (for all periods). Let $\mathcal{Y}^\tau$ be the σ-algebra generated by all such measurements. Fix c and t, and consider the quantity

$$V_t^\tau = \text{Var}\left(\mathbb{E}\left[x_{ct} | \mathcal{Y}^\tau\right] - x_{ct}\right),$$

as τ increases. This gives a measure of the precision gains from signal extraction in real time.

4 Inference for parameters and latent variables

4.1 Estimation

Our objective is to conduct inference on parameters θ and latent variables $x_{1:T}$. A Bayesian approach offers a convenient option to carry on both tasks, integrating estimation and signal-extraction uncertainties in a unified, conceptually natural way. Moreover, the model lends itself to stable and efficient algorithms, exploiting a Gibbs sampler for estimation and the Durbin and

Koopman (2002) algorithm for signal extraction. Let $p(\cdot)$ denote a generic density (with respect to an appropriate dominating measure).

Prior. We will specify a prior for θ by proposing $N+1$ independent priors for $\theta_x, \theta_1, \ldots, \theta_N$. The family of priors we describe is fairly standard and permits a simple implementation of the Gibbs sampler (as the priors are conjugate conditional on latent variables). It can also accommodate a flat prior for certain values of the hyperparameters.

For the parameters of the signal we use

- $\pi_x = 1/\sigma_x^2 \sim \Gamma_C(d_x/2, p_x/d_x)$ (here divisions are made elementwise and Γ_C represents a vector of independent gamma-distributed random variables), and
- $\beta_x = ((I_C - \text{diag}(\rho_x)\mu_x, \rho_x, \lambda_x)|\sigma_x \sim N(b_x, R_x \otimes \text{diag}(\sigma_x^2))$.

The hyperparameters p_x and b_x control the prior mean of π_x and β_x, while d_x and R_x govern the informativeness of the prior distributions — higher d_x and R_x produce tighter priors while $d_x = 0_{C\times 1}$ and $R_x = 0_{3\times 3}$ give a flat prior over π_x and β_x (which is not necessarily flat for θ_x).

Likewise, for the parameters of measurement error we use for each $i = 1, \ldots, N$

- $\pi_i = 1/\sigma_i^2 \sim \Gamma_{M_i}(\nu_i/2, p_i/\nu_i)$, and
- $\beta_i = (\rho_i, \lambda_i)|\sigma_i \sim N(b_i, R_i \otimes \text{diag}(\sigma_i^2))$.

Of course, the same considerations made for p_x, b_x, d_x, R_x apply to p_i, b_i, d_i, R_i.

Gibbs sampler. Although the prior $p(\theta)$ and the likelihood $p(y_{1:T}|\theta)$ are readily available (the likelihood is an output of the Kalman filter applied to the state-space representation of the model), the posterior $p(\theta|y_{1:T})$ is not. Bayesian estimation can instead be performed via MCMC, i.e., drawing a Markov chain $\{\theta^s\}_{s\geq 1}$ that has by invariant distribution the posterior.

A convenient approach to MCMC in our model is Gibbs sampling which, as a matter of fact, tackles estimation and filtering at the same time by targeting $p(\theta, X_{1:T}|y_{1:T})$ (assume $X_{1:T}$ is expanded to include $\eta_{x,1:T}, \eta_{1,1:T}, \ldots, \eta_{N,1:T}$). The algorithm is described in detail in appendix B.

4.2 Filtering

Signal extraction of x_t is a by-product of estimation. The latent variable draws obtained in step (1) from iteration over the Gibbs sampler algorithm $(x_{0:T}^s)_{s\geq 1}$ have the desired distribution

$p(x_{0:T}|y_{1:T})$. Moreover, the Gibbs sampler already integrates estimation uncertainty since

$$p(x_{0:T}|y_{1:T}) = \int_{\Theta} p(x_{0:T}|\theta, y_{1:T})\ p(\theta|y_{1:T})\ d\theta.$$

with Θ the parameter space.

It is worth noting that while $p(x_{0:T}|\theta, y_{1:T})$ is a normal density, $x_{1:T}$ need not be normal given $y_{1:T}$ once θ is integrated out. In particular, Var $(x_t|y_{1:T})$ may depend on the data through the posterior density of θ in contrast to Var $(x_t|\theta, y_{1:T})$ which is constant in $y_{1:T}$.

Finally, the Markov chain $(x^s_{0:T}, \theta^s)_{s\geq 1}$ is all that is needed to approximate by simulation the posterior distribution of the objects of interest listed in subsection 3.3. Our empirically analysis heavily relies on that technique.

5 GDP Solera: empirical analysis

To be completed

We estimate our model using a flat prior as described in Section 4, running the Gibbs sampler for 55,000 iterations with a burn-in of 5,000 and a thinning of 1 every 5 iterations. The result is a Markov chain $(X^s_{1:T}, \theta^s)^S_{s=1}$, $S = 10,000$, that by all accounts appears well-converged with low autocorrelation across draws. Our analysis is based on it.

Posterior distributions of parameters are reported in table C.1 in Appendix C. It is noteworthy that the unconditional means, autoregressive coefficients, loadings and standard deviations of shocks for the different version of economic activity are very similar. This seems to suggest that the comprehensive revision process does not modify either the static or the dynamic properties of the object being measured, although it affects smoothed values. Moreover, common shocks to the different elements of x_t seem to be more important than individual shocks.

Figure 3 reports the smoothed estimates for GDP growth from six different GDP solera releases, which are estimated every 18 months as follows: The first vintage uses data until July of 2013 to provide estimates up to the 2013Q2. Similarly, the second one provides estimates up to 2014Q4 using data until January of 2015; following in that manner until the sixth one which, using data until January of 2021, delivers estimates of GDP growth until 2020Q4. As can be seen in panel (a) of Figure 3, which depicts the six series starting from 2004Q1, all estimates display very close paths until 2010Q1. Notice that the growth rates estimated for a few quarters near the end of the series are somewhat different from the corresponding estimates obtained with the next release, an effect that it is very likely due to smoothing. Additionally, in the second quarter of both 2011 and 2012, the two most recent vintages (green) present a different pattern than the others. This could be explained by the fact that the vintages 2004Q1-2019Q2 and 2004Q1-2020Q4 incorporate modifications on the GDP definition due to the comprehensive revision that took place in July of 2018. Indeed, panel (b), which only reports the two most recent releases, shows an extremely similar pattern between them, although 2004Q1-2020Q4 include data from the pandemic. The post pandemic estimates for the pre-pandemic period are remarkably stable to the inclusion of the large outliers in 2020 data.

To assess the impact of using data from all comprehensive revisions, we have also estimated the model of Section 3 using only the most recent comprehensive revision data (i.e. July 2018). Figure 4 reports the posterior medians of GDP growth and their point-wise 90% credible sets based on both datasets for the period 2017Q1 to 2019Q4. As can be seen from comparing panels (a) and (b), the importance of using all comprehensive data becomes evident as its inclusion is associated with significantly tighter bands around the estimates of economic activity. In addition, using all comprehensive seems to deliver a smoother pattern for the dynamics of the signal. In turn, Figures 5 and 6 reports the mean-square error $\sqrt{\text{Var}\left(\Delta x_{ct} | \mathcal{Y}^{\tau}\right)}$ for a sequence of 24 months τ starting in April of the year corresponding to x_{ct} (when the advance estimate for the first quarter becomes available). The main pattern is that most of the precision gains occur when the third estimates are released, followed by a slight further gain from the annual estimates. For the smoothed signal in 2017Q1 as new data arrives when using all comprehensive data and the most recent one, respectively, we again offer a comparison of Figures 5 and 6. The different panels of these figures clearly show how uncertainty about the signal gets reduced as new releases arrive.

Next, we conduct a couple of exercises aimed to shed light on the effect of data revisions as well as the arrival of information on successive quarters on the estimates of a given quarter. The first example of concurrent and revised estimates -up to 24 months- focuses on 2008Q4, that is the worst quarter of the so-called Great Recession, while the second one deals with 2019Q2, which is supposed to be a relatively normal quarter which, however, includes the effect of the pandemic data on parameter estimates towards the end of the period. Figures 8 and 9 report the results as well as the credible sets from recursive estimation of the model on a monthly basis. As in the previous figures, solid lines are the posterior medians and the shaded areas represent 90% point-wise credible bands. Moreover, additional data releases for those specific quarters are displayed too (GDE: blue crosses and GDI: red diamonds).

Regarding 2008Q4, it is interesting to notice in Figure 8 the fast adjustment of the posterior median of GDP growth to a -6% as early as the third releases are included in the exercise. It is also noteworthy, the strong effect that the comprehensive revision corresponding to July of 2009 has on the precision of the estimates, reducing dramatically the length of the credible sets.

Similar features are also present when repeating the exercise but for 2019Q2 as Figure 9 displays, though mostly related to the annual update (covering the most recent calendar years) that took place in July of 2019. Importantly, the effect of the pandemic data on the uncertainty about the model parameters can be seen from the July of 2020's annual update.

We finally compare our measure of economic activity GDP solera with the GDPplus of Aruoba et al. (2016). First we look at estimates of GDP between the first quarter of 1984 and the second quarter of 2021. For GDP solera we use all vintages of data available until September 2021 and, for comparison, we take the end of September release of GDPplus.[14] The estimated annualized growth rates are plotted in panel (a) of Figure 10, the two series are quite close to each other with a contemporaneous correlation at 0.86, the average annualized growth over the sample period is 2.64% for GDPplus and 2.59% for GDPsolera. The GDP solera estimates appears to be more volatile with a standard deviation which is 40% larger than the GDPplus one. The smoothness of GDPplus results in relatively conservative estimates of the large fall and rise of economic activity after the start of the Covid 19 outbreak. To shed light on this we have performed the exercise reported in panel (b) of Figure 10, where the two real time estimates of economic activity for 2020Q1 and 2020Q2 are reported using real time data. Both estimators of GDP for 2020Q1 are in agreement and quite stable as new information become available. On the contrary, the estimators for 2020Q2 are very different and this difference arises when the October 2020 releases of advance GDE for 2020Q3 is published, so that the big rise in growth rate estimates with respect to the September estimates is probably due to the weight assigned to this observation by the optimal GDPplus filter. The difference between GDPplus and GDP solera growth rates estimates in the four quarters of 2020 are strikingly large. The most recent figures produced by the BEA for the pandemic recession are closer to the GDP solera series.

In the last exercise, we look at timely concurrent online estimates of GDP growth rates. First we consider estimates for a given quarter based on the information available up to one month after the end of that quarter, when only the "advance" GDE estimates is released. The results are reported in panel (a) of Figure 11. Panel (b) displays the estimates of GDP growth rates obtained three months after the end of the quarter when the figures for "third" releases of GDE

[14] GDPplus uses data sources from 1960Q1 but this should not affect too much estimates, at least for more recent years

and GDI are published. Real time estimates appear to be more similar than the corresponding historical ones, however, in panel (a) a few differences are observed in the first two quarters of 2015 and at the end of the series after the 2020Q2 drop due to the pandemic, while the two series in panel (b) are remarkably similar.

6 Conclusion

To be completed

We exploit the information in all the vintages of GDE and GDI measurements from a given comprehensive revision to obtain a better measurement of aggregate economic activity by exploiting cointegration between the different measures and taking seriously the vintage release calendar. We also combine overlapping comprehensive revisions to improve our measurements of the most recent observations, with particular attention to the great recession and the pandemic. We use the values of the estimated parameters of our dynamic state space model to assess whether comprehensive revisions induce changes in the long-run growth rate and the persistence of shocks to economic activity.

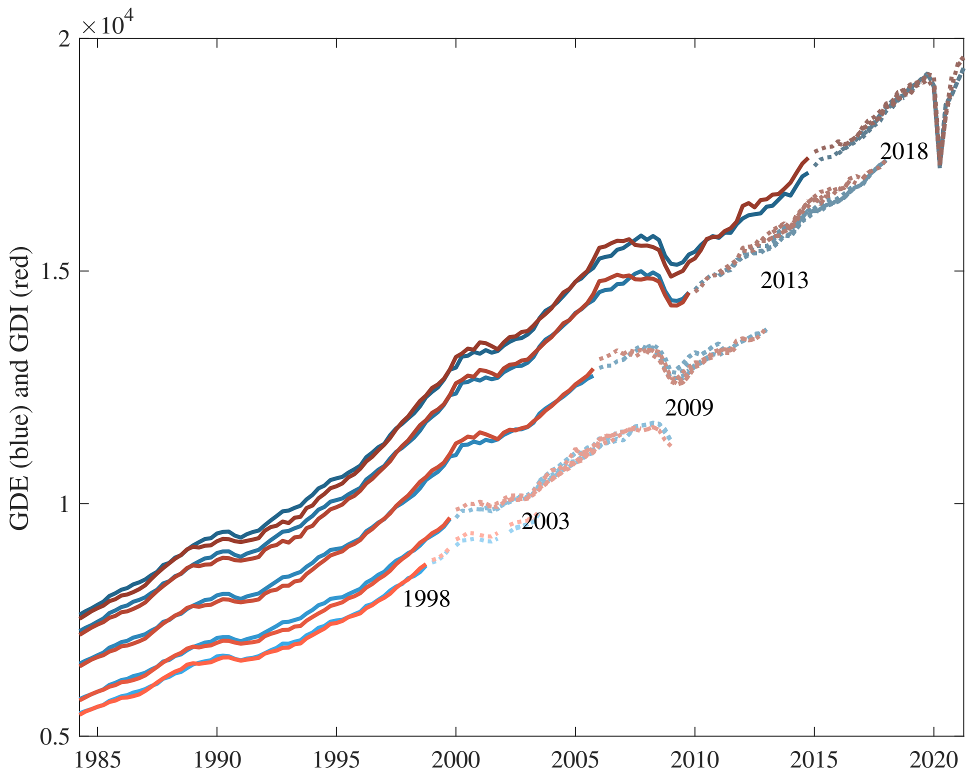
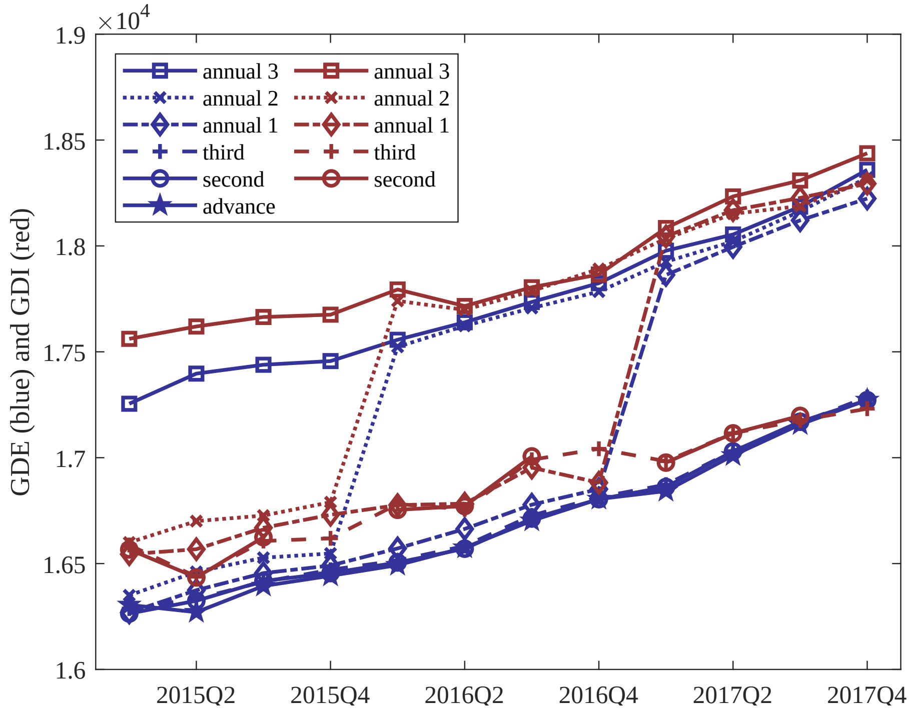
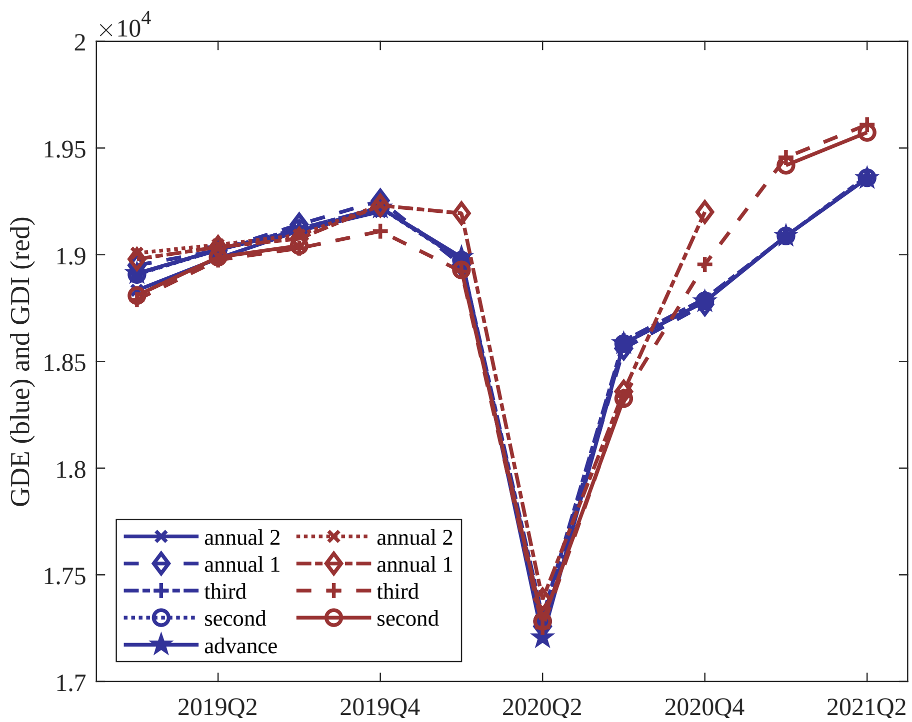


FIGURE 1. **GDE and GDI data from the BEA.** Solid lines represent data released under comprehensive revisions while dashed lines represent data produced by early and annual revisions.

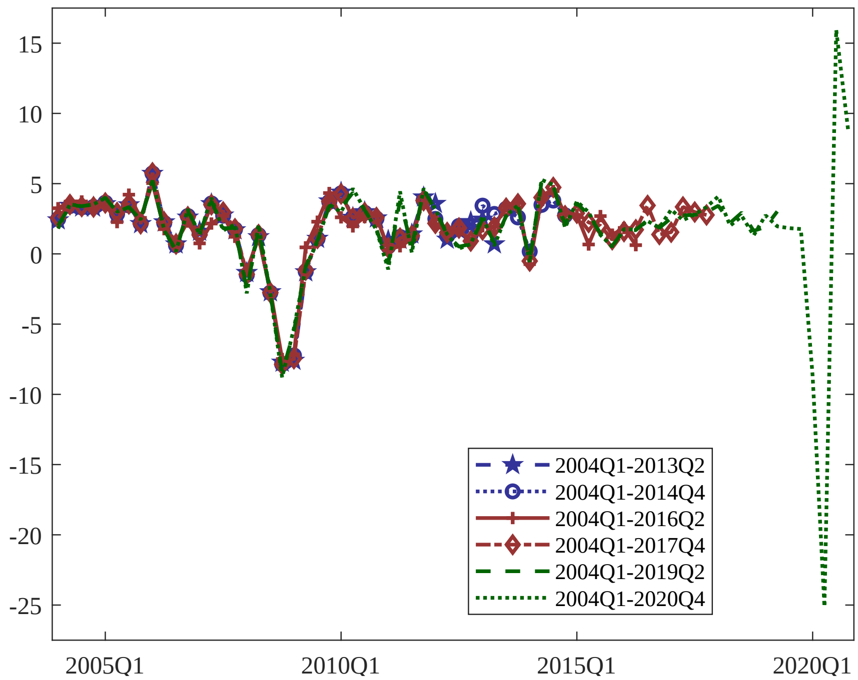


(a) Measurements 2015Q1-2017Q4

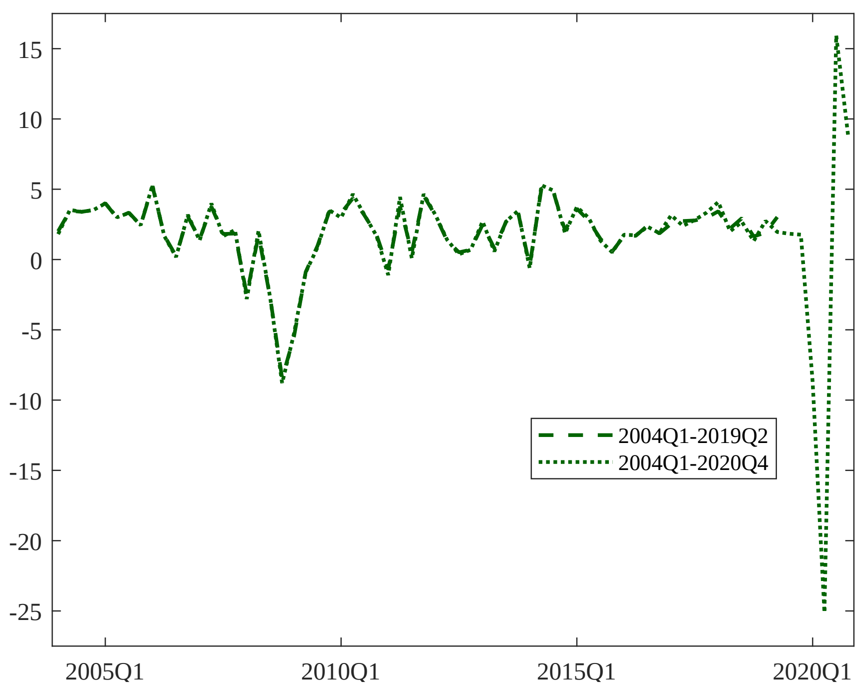


(b) Measurements 2019Q1-2021Q2

FIGURE 2. **GDE and GDI data from the BEA.** Each subplot reports levels for a different version of economic activity.

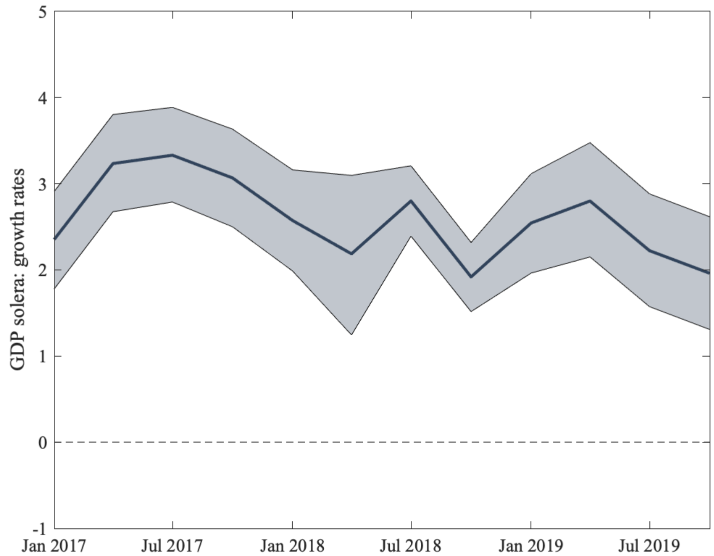


(a) Stability with respect to 2018 comprehensive revision and pandemic data

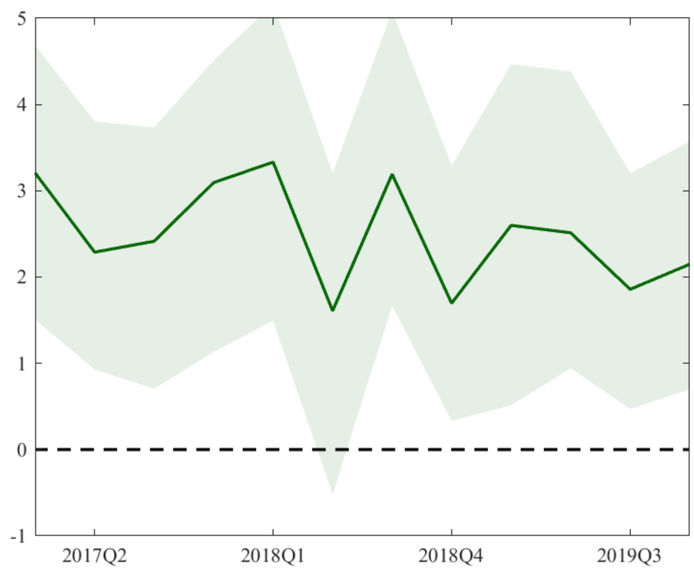


(b) Stability with respect to pandemic data

FIGURE 3. **GDP solera releases.** The first release uses data until July of 2013 to provide estimates up to the 2013Q2. Similarly, the second one provides estimates up to 2014Q4 using data until January of 2015; following in that manner until the sixth one which, using data until January of 2021, delivers estimates of GDP growth until 2020Q4.

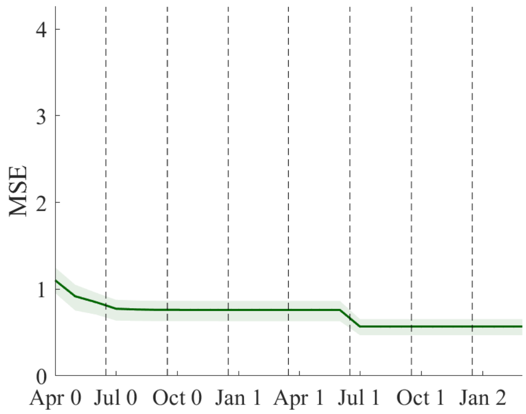


(a) Using all comprehensive revisions (five signals)

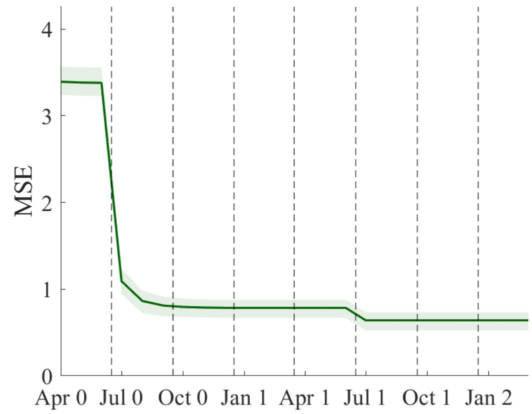


(b) Using the most recent comprehensive revision (one signal)

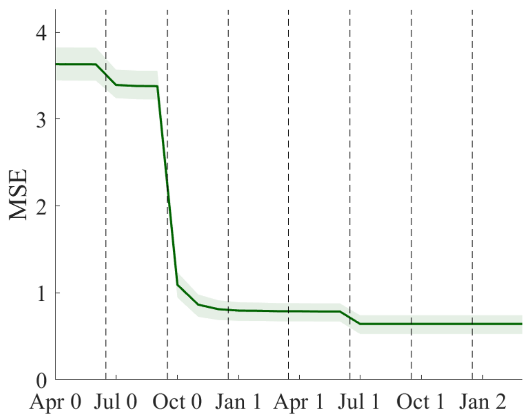
FIGURE 4. **Signal extraction** for Δx_{Ct}. The indigo line is the median of Δx_{Ct} given $y_{1:T}$ and the shaded area represents t-wise 90%-probability intervals.



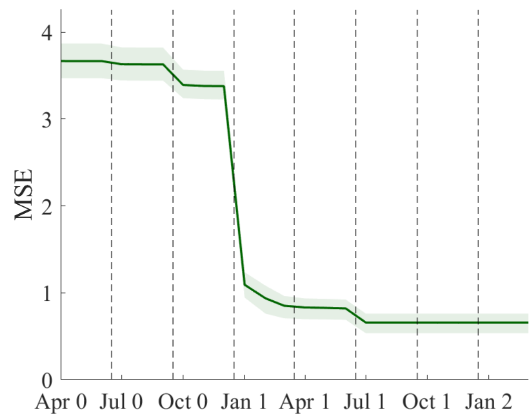
(a) Year 0 / Quarter 1



(b) Year 0 / Quarter 2

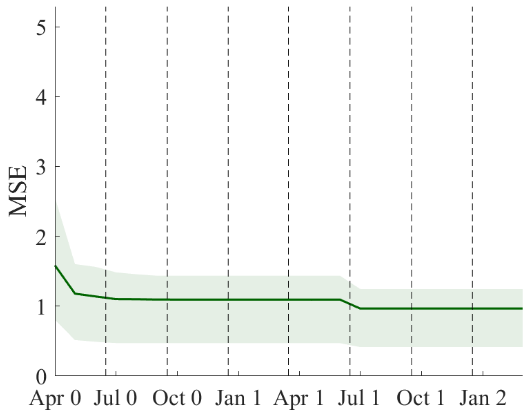


(c) Year 0 / Quarter 3

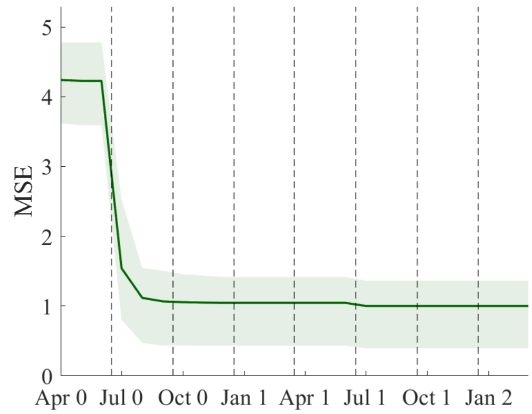


(d) Year 0 / Quarter 4

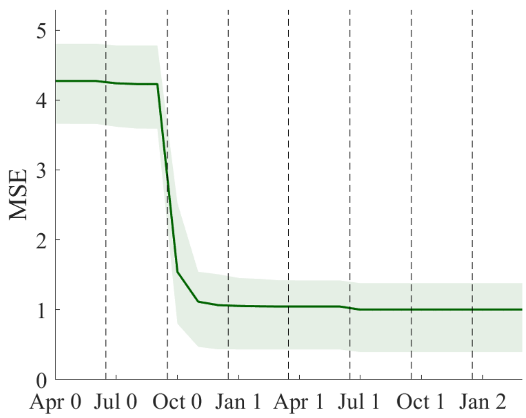
FIGURE 5. $\sqrt{\text{Var}\left(x_{ct}|\mathcal{Y}^{\tau}\right)}$ **for a sequence of months using all comprehensive revisions (five signals).** The solid green line is the posterior median of the root-MSEs while the shaded areas are month-wise 90% probability intervals.



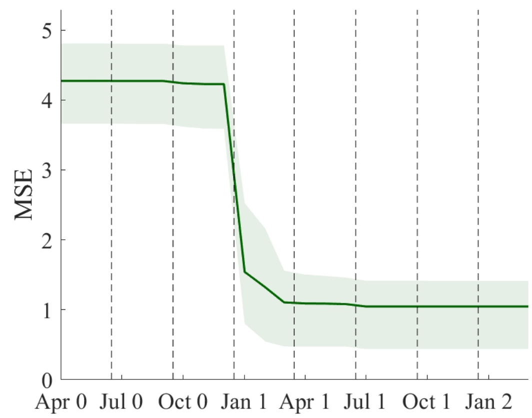
(a) Year 0 / Quarter 1



(b) Year 0 / Quarter 2



(c) Year 0 / Quarter 3



(d) Year 0 / Quarter 4

FIGURE 6. $\sqrt{\mathrm{Var}\left(x_{ct}|\mathcal{Y}^{\tau}\right)}$ **for a sequence of months using the most recent comprehensive revision (one signal).** The solid green line is the posterior median of the root-MSEs while the shaded areas are month-wise 90% probability intervals.

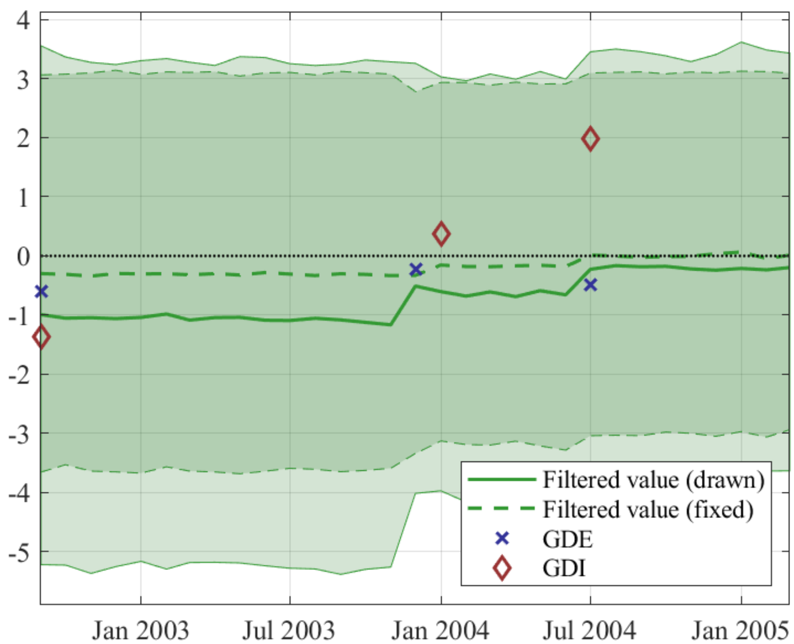


FIGURE 7. **Real time filtering of** Δx_{2001Q1}. The solid line is the posterior median and the shaded area is a 90%-pointwise credible band. Data releases for GDE (blue crosses) and GDI (red diamonds) are displayed too.

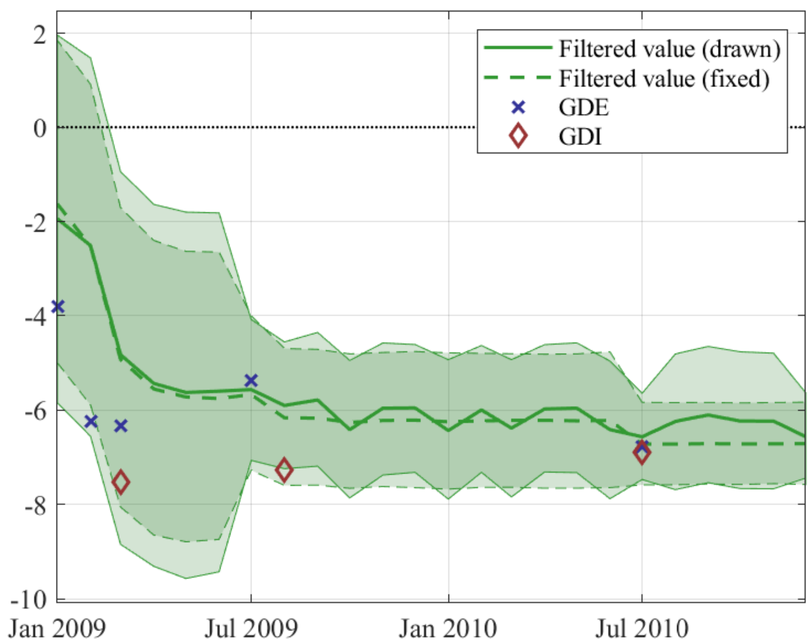


FIGURE 8. **Real time filtering of Δx_{2008Q4}.** The solid line is the posterior median and the shaded area is a 90%-pointwise credible band. Data releases for GDE (blue crosses) and GDI (red diamonds) are displayed too.

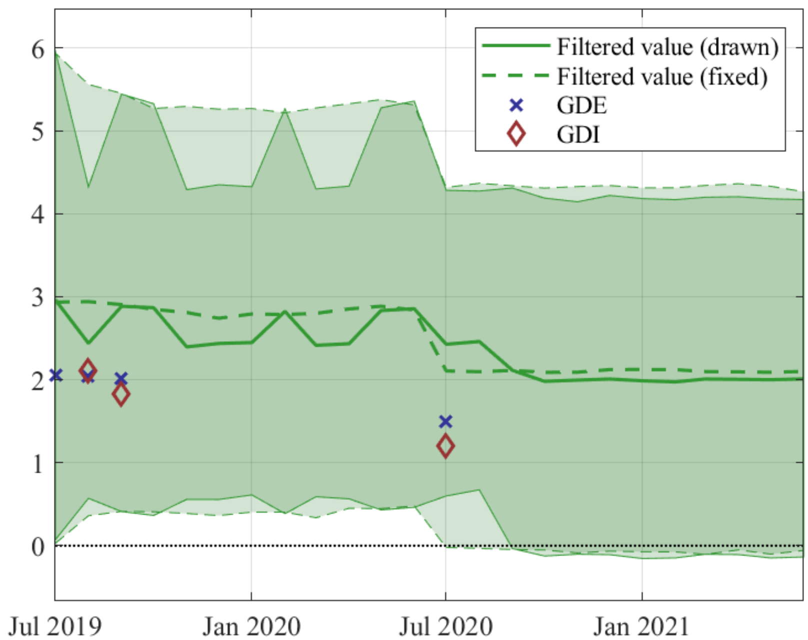
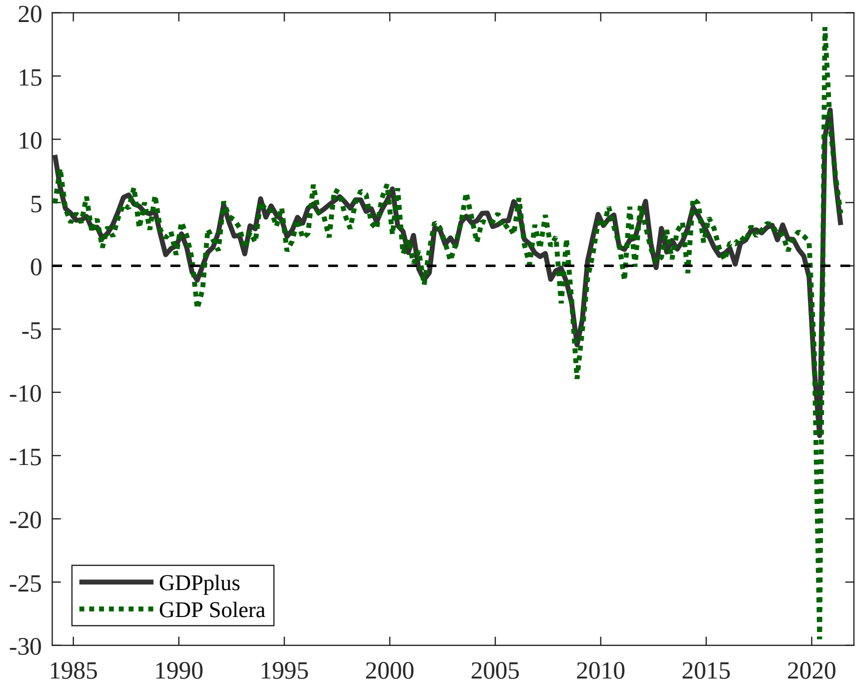
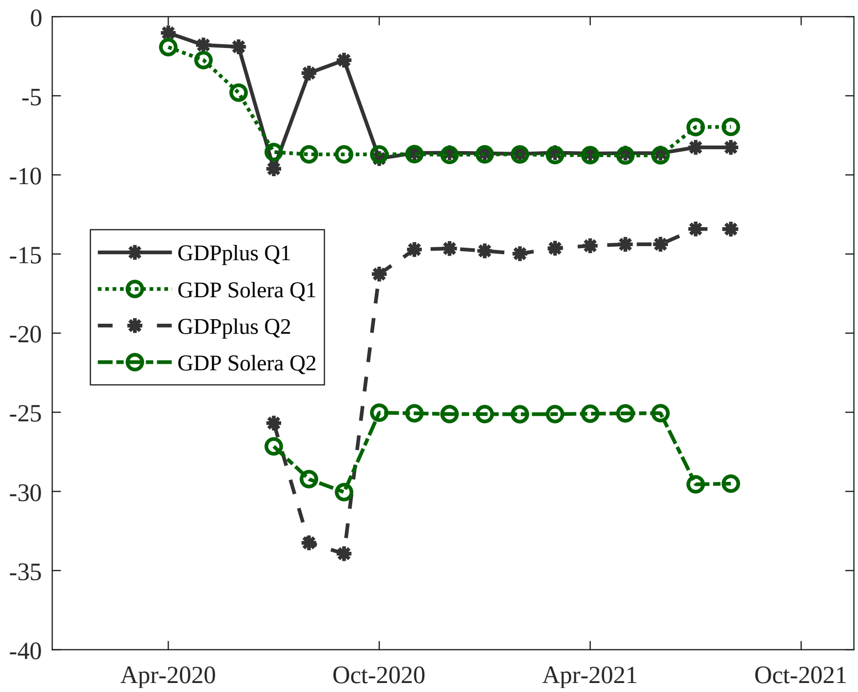


FIGURE 9. **Real time filtering of** Δx_{2019Q2}. The solid line is the posterior median and the shaded area is a 90%-pointwise credible band. Data releases for GDE (blue crosses) and GDI (red diamonds) are displayed too.

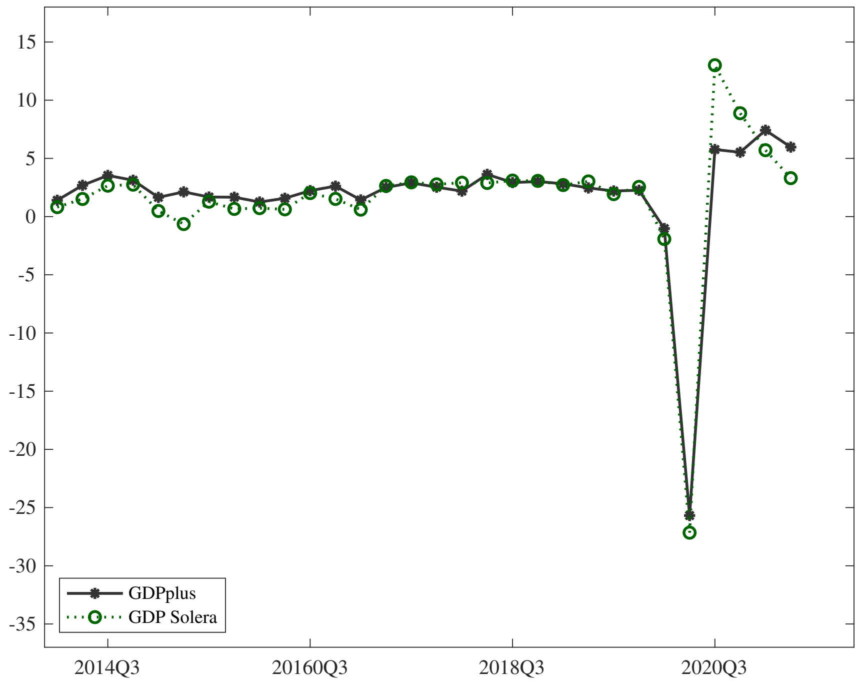


(a) Comparison of most recent historical estimates

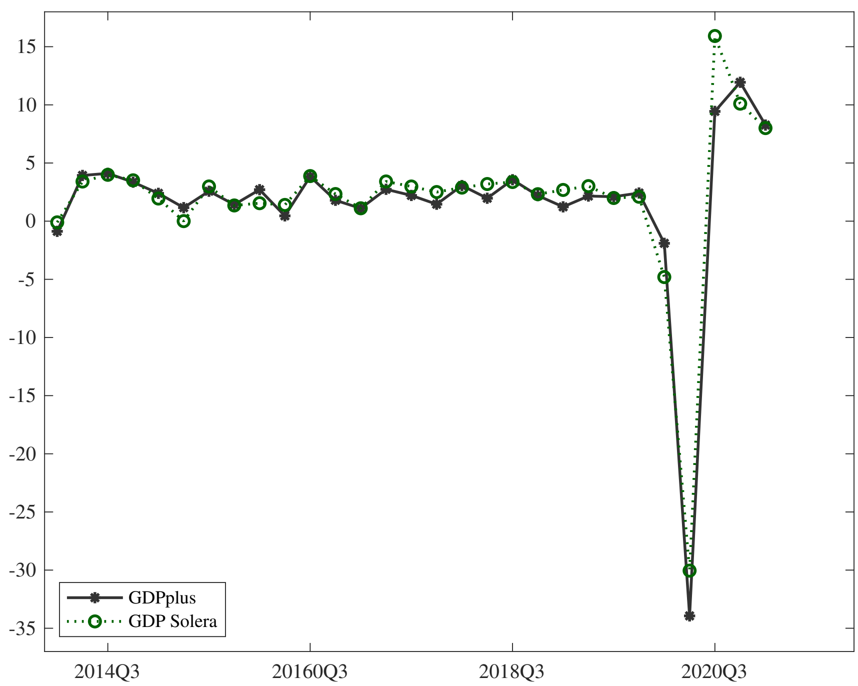


(b) First two quarters of 2020 estimates

FIGURE 10. **GDPplus versus GDP Solera**. Panel (a) displays GDPplus and GDP Solera series estimated using data until August 2021. Panel (b) displays GDP revised estimates at the beginning of the Covid-19 outbreak across April 2020 to August 2021.



(a) GDP growth estimates one month after the end of the quarter



(b) GDP growth estimates three months after the end of the quarter

FIGURE 11. **Nowcast: GDPplus versus GDP Solera**. Panel (a) displays GDPplus and GDP Solera series estimated with information available one month after the end of the quarter when only advance of GDE is available for the most recent quarter. GDPplus for 2018Q4 was released in February 2019. Panel (b) displays GDPplus and GDP Solera series estimated with information available three months after the end of the quarter.

Appendix A Identification

A.1 Proof of proposition 1

Let f_ω denote the spectrum of a time series $\{\omega_t\}$. Of course, identification of the autocovariance function of $\{\omega_t\}$ is equivalent to identification of f_ω. Hence, an alternative statement to proposition 1 is that under assumption 1, if $N > 1$, $f_{\Delta x}$ and $f_{v_1}, \ldots, f_{v_I}$ are nonparametrically identified from $f_{\Delta y}$.

To see why, let us write

$$f_{\Delta y}(\lambda) = 1_{M\times M} f_{\Delta x}(\lambda) + |1 - e^{i\lambda}|^2 \operatorname{diag}\Big(f_{v_1}(\lambda), \ldots, f_{v_N}(\lambda)\Big), \quad 0 \le \lambda \le 2\pi.$$

If E_i is the $M_i \times M$ matrix such that $y_{it} = E_i y_t$, we get $E_{i_1} f_{\Delta y}(\lambda) E'_{i_2} = 1_{M_{i_1}\times M_{i_2}} f_{\Delta x}(\lambda)$ for $i_1 \neq i_2$—such a pair i_1, i_2 exists only if $N > 1$. With $f_{\Delta x}$ pinned down, one then recovers

$$f_{v_i}(\lambda) = |1 - e^{i\lambda}|^{-2} E_i \left(f_{Dy}(\lambda) - 1_{M\times M} f_{Dx}(\lambda) \right) E'_i,$$

dealing with the removable singularity at $\lambda = 0$ by using that each entry f_{v_i} is holomorphic over the unit circle. ■

It follows from the proof of proposition 1 that if in addition to $N > 1$ we have $M_i > 1$ for at least one i, the model imposes overidentifying restrictions and is, therefore, testable. This is the case in our empirical analysis, although we do not pursue such tests. If the spectra $f_{\Delta x}, f_{v_1}, \ldots, f_{v_N}$ belong to a particular parametric class, an indirect approach to testing the overidentifying restrictions is to use dynamic specification tests as in Fiorentini and Sentana (2019).

A.2 Proof of proposition 2

By condition (ii) in the proposition, $D_{i_1} = E_{i_1}\delta_t$ and $D_{i_2} = E_{i_2}\delta_t$ are time-invariant. By assumption 1 and condition (i), moreover, $E_{i_1} v_t$ and $E_{i_2} v_t$ are uncorrelated at all lags and leads. Ergo,

$$E_{i_1} f_{\Delta y}(\lambda) E'_{i_2} = D_{i_1} f_{\Delta x}(\lambda) D'_{i_2}, \quad 0 \le \lambda \le 2\pi.$$

Now, by condition (iii), $\operatorname{rank}(D_{i_1}) = \operatorname{rank}(D_{i_2}) = C$. In that case,

$$f_{\Delta x} = (D'_{i_1} D_{i_1})^{-1} D'_{i_1} f_{\Delta y} D_{i_2} (D'_{i_2} D_{i_2})^{-1}.$$

Identification of $f_{v_1}, \ldots, f_{v_N}$ then follows by an analogous argument to that in proposition 1. ∎

Appendix B Details of estimation algorithm

The algorithm updates unknowns by drawing iteratively from the following distributions:

(1) $p(X_{1:T}|\theta, y_{1:T})$: using the state-space representation of the model, $X_{1:T}$ is obtained from the simulation smoother proposed by Durbin and Koopman (2002).

(2) $p(\theta_x|\theta_1, \ldots, \theta_N, X_{1:T}, y_{1:T})$: first notice that $(\Delta x_{1:T}, \eta_{x,1:T})$ are sufficient for θ_x, i.e.,

$$p(\theta_x|\theta_1, \ldots, \theta_N, X_{1:T}, y_{1:T}) = p\left(\theta_x \middle| \Delta x_{1:T}, \eta_{x,1:T}\right),$$

and because of the conjugacy of the prior we recover $\mu_x, \rho_x, \lambda_x, \sigma_x$ from

(i) $\pi_x = 1/\sigma_x^2 | \Delta x_{1:T}, \eta_{x,1:T} \sim \Gamma_C(\tilde{d}_x/2, \tilde{p}_x/\tilde{d}_x)$ where

$$\begin{aligned} \tilde{d}_x &= d_x + T - 1, \\ \frac{\tilde{d}_x}{\tilde{p}_x} &= \frac{d_x}{p_x} + \sum_{t=2}^{T} (\Delta x_t - \mu_x - \operatorname{diag}(\rho_x)(\Delta x_{t-1} - \mu_x) - \lambda_x \eta_{xt})^2; \end{aligned}$$

(ii) $\beta_x = ((I_C - \operatorname{diag}(\rho_x)\mu_x, \rho_x, \lambda_x)|\sigma_x, \Delta x_{1:T}, \eta_{x,1:T} \sim N(\tilde{b}_x, \tilde{R}_x \otimes \operatorname{diag}(\sigma_x^2))$ where

$$\tilde{R}_x = R_x + \sum_{t=2}^{T} \begin{pmatrix} 1 & \Delta x_{t-1} & \eta_{xt} \\ \Delta x_{t-1} & \Delta x_{t-1}^2 & \Delta x_{t-1}\eta_{xt} \\ \eta_{xt} & \Delta x_{t-1}\eta_{xt} & \eta_{xt}^2 \end{pmatrix},$$

$$\tilde{R}_x \tilde{b}_x = R_x b_x + \sum_{t=2}^{T} \begin{pmatrix} \Delta x_t \\ \Delta x_{t-1}\Delta x_t \\ \eta_{xt}\Delta x_t \end{pmatrix}.$$

(3) $p\left(\theta_i \middle| \theta_x, (\theta_j)_{j \neq i}, X_{1:T}, y_{1:T}\right)$ for each i: first notice that $(v_{i,1:T}, \eta_{i,1:T})$ are sufficient for θ_i, i.e.,

$$p\left(\theta_i \middle| \theta_x, (\theta_j)_{j \neq i}, X_{1:T}, y_{1:T}\right) = p\left(\theta_i | v_{i,1:T}, \eta_{i,1:T}\right),$$

and because of the conjugacy of the prior we recover $\rho_i, \lambda_i, \sigma_i$ from

(i) $\pi_i = 1/\sigma_i^2 | v_{i,1:T}, \eta_{i,1:T} \sim \Gamma_{M_i}(\tilde{d}_i/2, \tilde{p}_i/\tilde{d}_i)$ where

$$\tilde{d}_i = d_i + T - 1,$$
$$\frac{\tilde{d}_i}{\tilde{p}_i} = \frac{d_i}{p_i} + \sum_{t=2}^{T} \left(v_{it} - \mathrm{diag}(\rho_i) v_{i,t-1} - \lambda_i \eta_{it}\right)^2;$$

(ii) $\beta_i = (\rho_i, \lambda_i) | \sigma_i, v_{i,1:T}, \eta_{i,1:T} \sim N(\tilde{b}_i, \tilde{R}_i \otimes \mathrm{diag}(\sigma_i^2))$ where

$$\tilde{R}_i = R_i + \sum_{t=2}^{T} \begin{pmatrix} v_{i,t-1}^2 & v_{i,t-1}\eta_{it} \\ v_{i,t-1}\eta_{it} & \eta_{it}^2 \end{pmatrix},$$
$$\tilde{R}_i \tilde{b}_i = R_i b_i + \sum_{t=2}^{T} \begin{pmatrix} v_{i,t-1} v_{it} \\ \eta_{it} v_{it} \end{pmatrix}.$$

A small comment is that the choice of hyperparameters $d_x = 0_{C\times 1}$, $R_x = 0_{3\times 3}$, $d_i = 0_{M_i \times 1}$, and $R_i = 0_{2\times 2}$, despite implying improper priors, still leads to a well-defined algorithm and a proper posterior distribution.

Appendix C Posterior distributions

Parameter	Post. mean	90%-CI	MC stderr	Autocorr
${\mu_x}^{(1)}$	2.234	[1.879, 2.617]	0.0142	0.78
${\mu_x}^{(2)}$	2.324	[2.012, 2.666]	0.0134	0.89
${\mu_x}^{(3)}$	2.307	[1.978, 2.664]	0.0145	0.92
${\mu_x}^{(4)}$	2.394	[2.077, 2.747]	0.0142	0.93
${\mu_x}^{(5)}$	2.437	[2.121, 2.791]	0.0143	0.92
${\rho_x}^{(1)}$	0.444	[0.395, 0.495]	0.0013	0.66
${\rho_x}^{(2)}$	0.439	[0.397, 0.481]	0.0013	0.79
${\rho_x}^{(3)}$	0.439	[0.397, 0.48]	0.0014	0.83
${\rho_x}^{(4)}$	0.43	[0.39, 0.471]	0.0014	0.86
${\rho_x}^{(5)}$	0.397	[0.354, 0.439]	0.0014	0.85
${\lambda_x}^{(1)}$	3.079	[2.819, 3.332]	0.0074	0.84
${\lambda_x}^{(2)}$	3.014	[2.841, 3.199]	0.0065	0.88
${\lambda_x}^{(3)}$	3.208	[3.036, 3.387]	0.007	0.89
${\lambda_x}^{(4)}$	3.199	[3.037, 3.375]	0.0066	0.9
${\lambda_x}^{(5)}$	3.356	[3.188, 3.534]	0.0065	0.88
${\sigma_x}^{(1)}$	0.464	[0.376, 0.573]	0.0011	0.37
${\sigma_x}^{(2)}$	0.299	[0.248, 0.36]	0.0004	0.23
${\sigma_x}^{(3)}$	0.276	[0.234, 0.331]	0.0004	0.18
${\sigma_x}^{(4)}$	0.25	[0.213, 0.296]	0.0003	0.14

${\sigma_x}^{(5)}$	0.284	[0.238, 0.346]	0.0006	0.3
${\rho_{\mathrm{GDE}}^{\mathrm{nc}}}^{(1)}$	0.043	[-0.058, 0.144]	0.0007	0.07
${\rho_{\mathrm{GDE}}^{\mathrm{nc}}}^{(2)}$	0.027	[-0.035, 0.088]	0.0007	0.35
${\rho_{\mathrm{GDE}}^{\mathrm{nc}}}^{(3)}$	0.047	[-0.032, 0.126]	0.0007	0.21
${\rho_{\mathrm{GDE}}^{\mathrm{nc}}}^{(4)}$	0.522	[0.374, 0.661]	0.001	0.04
${\rho_{\mathrm{GDE}}^{\mathrm{nc}}}^{(5)}$	0.479	[0.376, 0.576]	0.001	0.17
${\rho_{\mathrm{GDE}}^{\mathrm{nc}}}^{(6)}$	0.322	[0.232, 0.418]	0.0017	0.6
${\rho_{\mathrm{GDE}}^{\mathrm{c}}}^{(1)}$	0.106	[0.038, 0.181]	0.0012	0.67
${\rho_{\mathrm{GDE}}^{\mathrm{c}}}^{(2)}$	0.019	[-0.024, 0.062]	0.0007	0.59
${\rho_{\mathrm{GDE}}^{\mathrm{c}}}^{(3)}$	0.028	[-0.011, 0.067]	0.0006	0.56
${\rho_{\mathrm{GDE}}^{\mathrm{c}}}^{(4)}$	0.057	[0.02, 0.098]	0.0006	0.56
${\rho_{\mathrm{GDE}}^{\mathrm{c}}}^{(5)}$	-0.002	[-0.043, 0.039]	0.0005	0.46
${\lambda_{\mathrm{GDE}}^{\mathrm{nc}}}^{(1)}$	-0.749	[-0.923, -0.561]	0.0057	0.74
${\lambda_{\mathrm{GDE}}^{\mathrm{nc}}}^{(2)}$	-0.727	[-0.878, -0.564]	0.0054	0.92
${\lambda_{\mathrm{GDE}}^{\mathrm{nc}}}^{(3)}$	-0.684	[-0.84, -0.515]	0.0055	0.86
${\lambda_{\mathrm{GDE}}^{\mathrm{nc}}}^{(4)}$	0.377	[0.144, 0.618]	0.0047	0.29
${\lambda_{\mathrm{GDE}}^{\mathrm{nc}}}^{(5)}$	1.013	[0.815, 1.215]	0.0032	0.27
${\lambda_{\mathrm{GDE}}^{\mathrm{nc}}}^{(6)}$	1.309	[1.166, 1.453]	0.0033	0.49
${\lambda_{\mathrm{GDE}}^{\mathrm{c}}}^{(1)}$	1.593	[1.442, 1.743]	0.0033	0.74
${\lambda_{\mathrm{GDE}}^{\mathrm{c}}}^{(2)}$	1.182	[1.079, 1.28]	0.0034	0.87
${\lambda_{\mathrm{GDE}}^{\mathrm{c}}}^{(3)}$	1.178	[1.083, 1.271]	0.0035	0.87
${\lambda_{\mathrm{GDE}}^{\mathrm{c}}}^{(4)}$	1.171	[1.08, 1.266]	0.004	0.87
${\lambda_{\mathrm{GDE}}^{\mathrm{c}}}^{(5)}$	1.093	[0.996, 1.189]	0.0038	0.86
${\sigma_{\mathrm{GDE}}^{\mathrm{nc}}}^{(1)}$	0.637	[0.551, 0.743]	0.0006	0.1
${\sigma_{\mathrm{GDE}}^{\mathrm{nc}}}^{(2)}$	0.204	[0.15, 0.272]	0.0007	0.58
${\sigma_{\mathrm{GDE}}^{\mathrm{nc}}}^{(3)}$	0.34	[0.28, 0.407]	0.0005	0.26
${\sigma_{\mathrm{GDE}}^{\mathrm{nc}}}^{(4)}$	1.471	[1.262, 1.727]	0.0028	0.16
${\sigma_{\mathrm{GDE}}^{\mathrm{nc}}}^{(5)}$	0.892	[0.772, 1.039]	0.001	0.07
${\sigma_{\mathrm{GDE}}^{\mathrm{nc}}}^{(6)}$	0.464	[0.355, 0.605]	0.002	0.54
${\sigma_{\mathrm{GDE}}^{\mathrm{c}}}^{(1)}$	0.288	[0.188, 0.419]	0.0019	0.74
${\sigma_{\mathrm{GDE}}^{\mathrm{c}}}^{(2)}$	0.145	[0.113, 0.191]	0.0004	0.51
${\sigma_{\mathrm{GDE}}^{\mathrm{c}}}^{(3)}$	0.14	[0.111, 0.178]	0.0003	0.43
${\sigma_{\mathrm{GDE}}^{\mathrm{c}}}^{(4)}$	0.143	[0.113, 0.184]	0.0004	0.46
${\sigma_{\mathrm{GDE}}^{\mathrm{c}}}^{(5)}$	0.16	[0.125, 0.206]	0.0005	0.47
${\rho_{\mathrm{GDI}}^{\mathrm{nc}}}^{(1)}$	0.671	[0.541, 0.788]	0.0009	0.11
${\rho_{\mathrm{GDI}}^{\mathrm{nc}}}^{(2)}$	0.674	[0.552, 0.792]	0.0008	0.06
${\rho_{\mathrm{GDI}}^{\mathrm{nc}}}^{(3)}$	0.718	[0.622, 0.812]	0.0008	0.17
${\rho_{\mathrm{GDI}}^{\mathrm{nc}}}^{(4)}$	0.741	[0.672, 0.808]	0.0007	0.19
${\rho_{\mathrm{GDI}}^{\mathrm{nc}}}^{(5)}$	0.707	[0.646, 0.768]	0.0008	0.27
${\rho_{\mathrm{GDI}}^{\mathrm{c}}}^{(1)}$	0.106	[0.009, 0.256]	0.003	0.72
${\rho_{\mathrm{GDI}}^{\mathrm{c}}}^{(2)}$	0.743	[0.696, 0.786]	0.0005	0.36
${\rho_{\mathrm{GDI}}^{\mathrm{c}}}^{(3)}$	0.577	[0.522, 0.625]	0.0008	0.51
${\rho_{\mathrm{GDI}}^{\mathrm{c}}}^{(4)}$	0.568	[0.519, 0.612]	0.001	0.81
${\rho_{\mathrm{GDI}}^{\mathrm{c}}}^{(5)}$	0.597	[0.55, 0.639]	0.0009	0.77
${\lambda_{\mathrm{GDI}}^{\mathrm{nc}}}^{(1)}$	0.905	[0.588, 1.248]	0.0034	0.11
${\lambda_{\mathrm{GDI}}^{\mathrm{nc}}}^{(2)}$	0.826	[0.5, 1.15]	0.0029	0.08
${\lambda_{\mathrm{GDI}}^{\mathrm{nc}}}^{(3)}$	1.253	[0.956, 1.542]	0.0031	0.14
${\lambda_{\mathrm{GDI}}^{\mathrm{nc}}}^{(4)}$	1.322	[1.071, 1.586]	0.0026	0.14
${\lambda_{\mathrm{GDI}}^{\mathrm{nc}}}^{(5)}$	1.535	[1.315, 1.757]	0.0029	0.25

${\lambda^{c}_{GDI}}^{(1)}$	0.907	[0.742, 1.093]	0.0046	0.82
${\lambda^{c}_{GDI}}^{(2)}$	1.073	[0.892, 1.257]	0.0036	0.56
${\lambda^{c}_{GDI}}^{(3)}$	1.732	[1.557, 1.917]	0.0035	0.52
${\lambda^{c}_{GDI}}^{(4)}$	1.929	[1.778, 2.086]	0.0036	0.8
${\lambda^{c}_{GDI}}^{(5)}$	2.034	[1.878, 2.201]	0.0037	0.74
${\sigma^{nc}_{GDI}}^{(1)}$	1.738	[1.478, 2.077]	0.0021	0.17
${\sigma^{nc}_{GDI}}^{(2)}$	1.821	[1.587, 2.103]	0.0016	0.05
${\sigma^{nc}_{GDI}}^{(3)}$	1.693	[1.464, 1.973]	0.0019	0.11
${\sigma^{nc}_{GDI}}^{(4)}$	1.268	[1.101, 1.47]	0.0014	0.09
${\sigma^{nc}_{GDI}}^{(5)}$	1.001	[0.856, 1.181]	0.0016	0.16
${\sigma^{c}_{GDI}}^{(1)}$	0.275	[0.185, 0.412]	0.0023	0.75
${\sigma^{c}_{GDI}}^{(2)}$	0.498	[0.408, 0.606]	0.001	0.31
${\sigma^{c}_{GDI}}^{(3)}$	0.533	[0.456, 0.625]	0.0009	0.21
${\sigma^{c}_{GDI}}^{(4)}$	0.272	[0.201, 0.35]	0.0008	0.52
${\sigma^{c}_{GDI}}^{(5)}$	0.351	[0.272, 0.432]	0.0009	0.46

TABLE C.1. Posterior distribution of parameters of the model

NOTES. Unconditional means μ_x, loadings $\lambda_x, \lambda_{GDP}, \lambda_{GDI}$ and standard deviations, $\sigma_x, \sigma_{GDP}, \sigma_{GDI}$ are annualized.

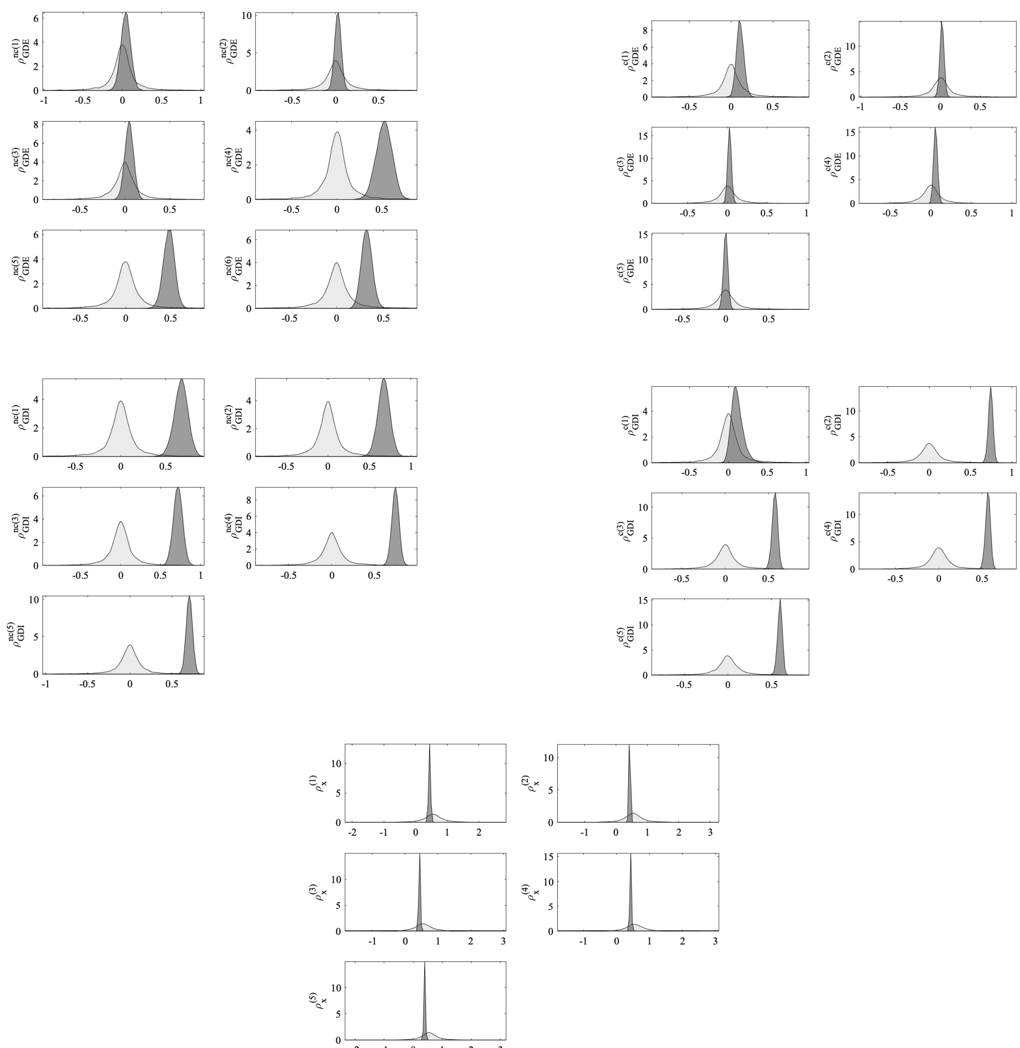


FIGURE C.1. **Parameter estimation.** Priors (light area) and posteriors (dark area) distributions of the parameters.

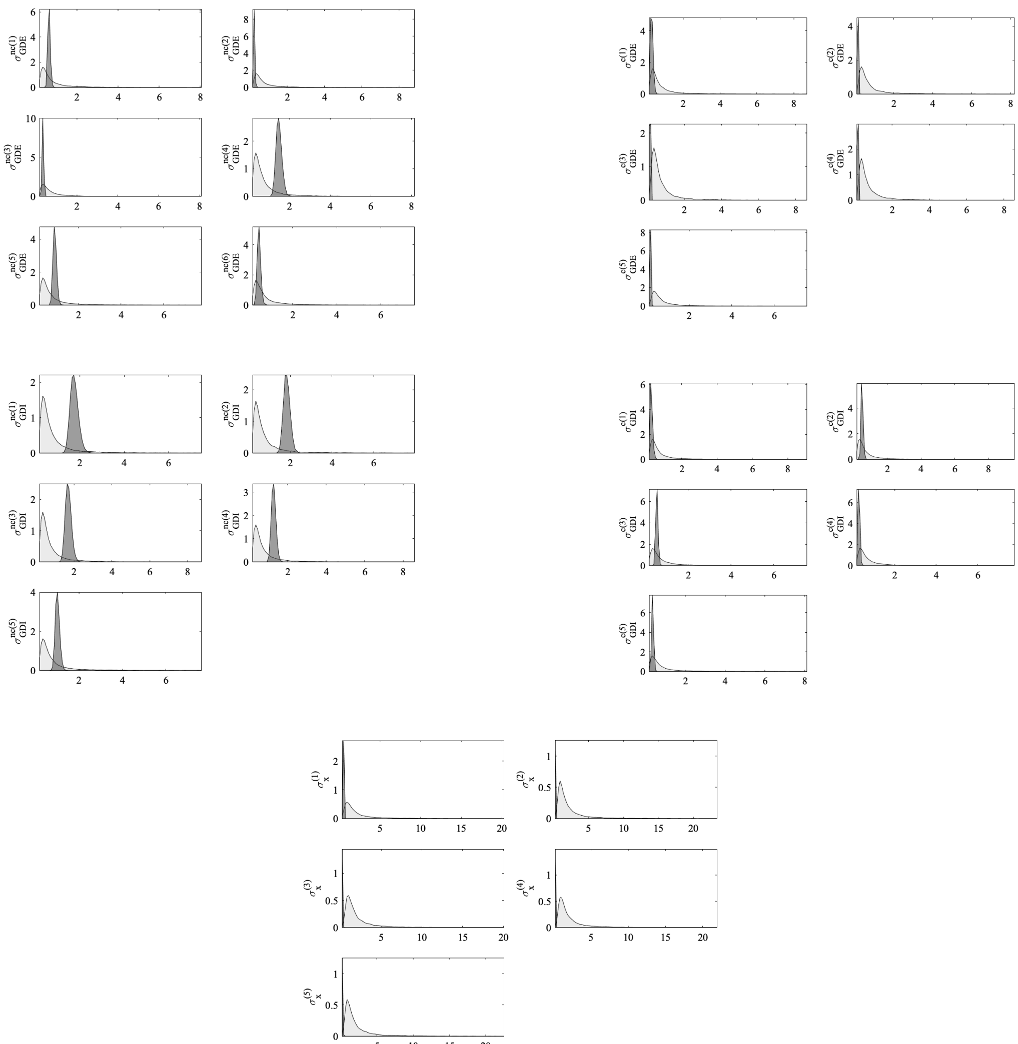


FIGURE C.2. **Parameter estimation.** Priors (light area) and posteriors (dark area) distributions of the parameters.

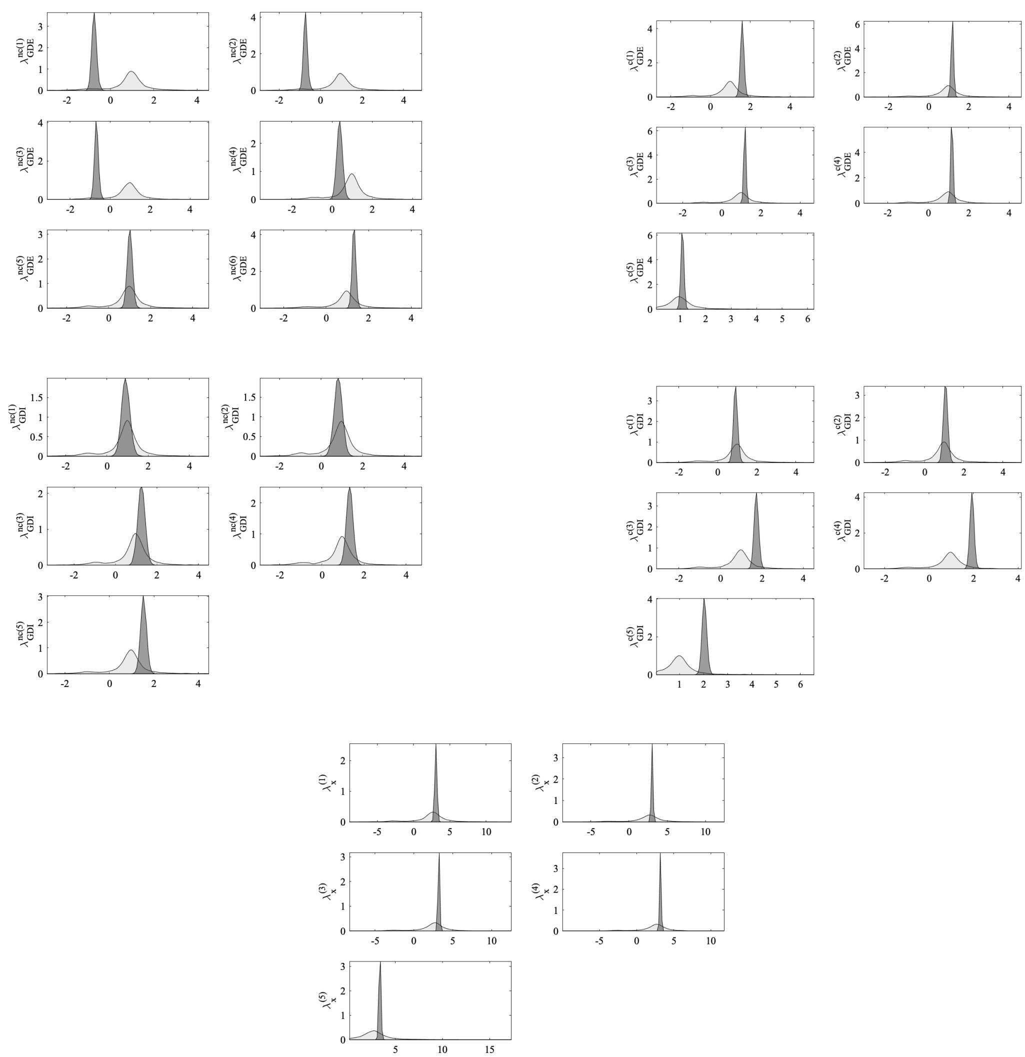


FIGURE C.3. **Parameter estimation.** Priors (light area) and posteriors (dark area) distributions of the parameters.

Appendix D Implications of L^2-optimality

Consider the following model for the release process. For each type of estimate i and quarter t, the statistical office collects inputs $\iota_{it}^1, \dots, \iota_{it}^{J_{it}}$ on which the estimates y_{it}^m are based — e.g., sectoral surveys. The point we want to make is that if estimates are produced to minimize expected square loss (i.e., the L^2-distance between the estimate and x_t), the optimal signal-extraction rule maps x_t to its most recent release. For ease of exposition, let $C = 1$ (which gives the model with no comprehensive revisions).

Fix i and t and let $\sigma(\cdot)$ denote generated σ-algebra. We will assume that (i) there are integers $\{J_{it}^m\}_{m=1}^{M_i}$ such that $J_{it}^m \leq J_{it}^{m+1}$ and y_{it}^m is $\mathcal{I}_{it}^m$-measurable with $\mathcal{I}_{it}^m = \sigma\{\iota_{it}^1, \dots, \iota_{it}^{J_{it}^m}\}$ for all m, and (ii) the statistical office minimizes $L^2(y_{it}^m - x_t) = \mathbb{E}\left[|y_{it}^m - x_t|^2\right]$.[15] We also assume x_t has finite variance (by appropriate choice of initial conditions). From (i) we obtain $\mathcal{I}_{it}^m \subset \mathcal{I}_{it}^{m+1}$ for all m, and from (ii),

$$y_{it}^m = \mathbb{E}\left[x_t \middle| \mathcal{I}_{it}^m\right], \quad m = 1, \dots, M_i.$$

Let $\tilde{\mathcal{I}}_{it}$ be a σ-algebra such that $\tilde{\mathcal{I}}_{it} \subset \mathcal{I}_{it}^m$ for all m.[16] With a slight abuse of notation,

$$\mathbb{E}\left[x_t \middle| y_{it}^1, \dots, y_{it}^m, \tilde{\mathcal{I}}_{it}\right] = \mathbb{E}\left[\mathbb{E}\left[x_t \middle| \mathcal{I}_{it}^m\right] \middle| y_{it}^1, \dots, y_{it}^m, \tilde{\mathcal{I}}_{it}\right] = \mathbb{E}\left[y_{it}^m \middle| y_{it}^1, \dots, y_{it}^m, \tilde{\mathcal{I}}_{it}\right] = y_{it}^m,$$

by an application of the law of iterated expectations.

In words, if measurements minimize expected square loss, all measurements of x_t but the most recent one contain no useful information to extract x_t. A reasonable situation is one where the statistical office computes y_{it}^m using input data corresponding only to quarter-t economic activity. A measure that captures L^2-optimality in that context would compare the expected loss of y_{it}^m with that of $\mathbb{E}\left[x_t \middle| y_{it}^1, \dots, y_{it}^m\right]$ (i.e., taking $\tilde{\mathcal{I}}_{it} = \varnothing$). For example,

$$D_{it}^m = \mathrm{Var}\left(\mathbb{E}\left[x_t \middle| y_{it}^1, \dots, y_{it}^m\right] - x_t\right) \Big/ \mathrm{Var}\left(v_{it}^m\right).$$

We have $0 \leq D_{it}^m \leq 1$ with $D_{it}^m = 1$ indicating full L^2-optimality. That $D_{it}^m < 1$ may be evidence that, e.g., the measurements optimize a different loss function or the weights given to the inputs do not exploit the dynamic model.

[15] Assumption (i) allows for data on past and future periods to be included among the time-t inputs.

[16] For example, if the time-t inputs include all the data needed to construct past measurements, $\tilde{\mathcal{I}}_{it}$ may be the σ-algebra generated by all past measurements.

Appendix E News and noise model

Consider a setup in which $N = 1$ (we will omit the subindex indicating type which would be 1), $M = M_1 = 3$, and there is a single comprehensive version of GDP (i.e., $C = 1$). Suppose the data follows the news-noise model of Jacobs and van Norden (2011) and Jacobs et al. (2020):

$$\Delta y_t = \begin{pmatrix} \Delta y_t^1 \\ \Delta y_t^2 \\ \Delta y_t^3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Delta \tilde{y}_t + \begin{pmatrix} \nu_t^1 \\ \nu_t^2 \\ \nu_t^3 \end{pmatrix} + \begin{pmatrix} \zeta_t^1 \\ \zeta_t^2 \\ \zeta_t^3 \end{pmatrix} = 1_{3\times 1} \Delta \tilde{y}_t + \nu_t + \zeta_t,$$

where ν_t^m and ζ_t^m are news and noise components. News are defined by the condition that $\mathrm{Cov}\left(\nu_t^m, \Delta \tilde{y}_t + \nu_t^{m'}\right) = 0$ for all $m' \leq m$, while noise must satisfy $\mathrm{Cov}\left(\zeta_t^m, \Delta \tilde{y}_t + \nu_t^m\right) = 0$. These, however, are not enough to pin down a unique decomposition of y_t in terms of $\tilde{y}_t, \nu_t, \zeta_t$ and we will further impose $\zeta_t^1, \zeta_t^2, \zeta_t^3$ are uncorrelated to each other.

To simplify the argument, we will assume that (i) $\Delta \tilde{y}_t + \nu_t^3$ follows an AR(1) process and (ii) ν_t and ζ_t are uncorrelated over time. Moreover, we note that the news-noise model is typically applied to measurements of GDP growth, as opposed to our model which focuses on the level.

The goal is to understand how the news-noise model maps to ours,

$$\begin{pmatrix} y_t^1 \\ y_t^2 \\ y_t^2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} x_t + \begin{pmatrix} v_t^1 \\ v_t^2 \\ v_t^3 \end{pmatrix} = 1_{3\times 1} x_t + v_t.$$

We can write

$$\Delta y_t = \begin{pmatrix} \Delta y_t^1 \\ \Delta y_t^2 \\ \Delta y_t^3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} (\Delta \tilde{y}_t + \nu_t^3) + \begin{pmatrix} (\nu_t^2 - \nu_t^3) + (\nu_t^1 - \nu_t^2) \\ (\nu_t^2 - \nu_t^3) \\ 0 \end{pmatrix} + \begin{pmatrix} \zeta_t^1 \\ \zeta_t^2 \\ \zeta_t^3 \end{pmatrix},$$

where $\nu_t^1 - \nu_t^2, \nu_t^2 - \nu_t^3, \zeta_t^1, \zeta_t^2, \zeta_t^3$ are mutually orthogonal white noise processes. If we set

$$\begin{aligned}
\Delta x_t &= \Delta \tilde{y}_t + \nu_t^3, \\
\Delta v_t^1 &= (\nu_t^2 - \nu_t^3) + (\nu_t^1 - \nu_t^2) + \zeta_t^1, \\
\Delta v_t^2 &= (\nu_t^2 - \nu_t^3) + \zeta_t^2, \\
\Delta v_t^3 &= \zeta_t^3,
\end{aligned}$$

we obtain a particular case of our model in which, not surprisingly, $\rho = 1_{3\times1}$. Measurement error are therefore white noise in first differences with a particular variance matrix,

$$\text{Var}\left(\Delta v_t\right) = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} & 0 \\ \Sigma_{12} & \Sigma_{22} & 0 \\ 0 & 0 & \Sigma_{33} \end{pmatrix}.$$

If we give Δv_t the factor structure in (5) (again maintaining $\rho = 1_{M\times1}$),

$$\Delta v_t = \begin{pmatrix} \Delta v_t^1 \\ \Delta v_t^2 \\ \Delta v_t^3 \end{pmatrix} = \begin{pmatrix} \lambda^1 \\ \lambda^2 \\ \lambda^3 \end{pmatrix} \eta_t + \begin{pmatrix} \sigma^1 \varepsilon_t^1 \\ \sigma^2 \varepsilon_t^2 \\ \sigma^3 \varepsilon_t^3 \end{pmatrix} = \lambda \eta_t + \text{diag}(\sigma)\varepsilon_t,$$

with $\eta_t \overset{iid}{\sim} N(0,1)$, $\varepsilon_t \overset{iid}{\sim} N(0_{3\times1}, I_3)$ and η_t independent of ε_t, the news-noise model implies the restriction $\lambda_3 = 0$. The rest of the parameters, $\lambda^1, \lambda^2, \sigma^1, \sigma^2, \sigma^3$, can be recovered from $\text{Var}\left(\Delta v_t\right)$.

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