Optimal Monetary Policy in a Two-Sector Environmental DSGE Model

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Abstract

Climate change has become central to the global economic policy agenda. Monetary authorities have begun to express concern about the economic risks posed by climate change, requiring central banks to review their policy. Also the emerging "Greenflation", as a consequence of the energy transition, has raised the question of whether and how the authorities should react to aggregate and relative price developments. In this paper, we discuss the possible impact, coming from the inclusion of climate change in a two-sector dynamic stochastic general equilibrium model, on the conduct of monetary policy. We investigate the optimal mix of environmental and macroeconomic stabilization policies. In particular, we examine the optimal response of the interest rate to sector-specific price changes. We show that optimal policy rule parameters vary significantly between the models, are affected by the type of environmental policy implemented and depend on the specific shock occurring.

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Keywords: Climate change, inflation, macroeconomic stabilization, environmental policy, greenflation.

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1 Introduction

Since the Paris Agreement of 2015, climate change has become central to the global economic policy agenda. Besides governments and (supra)national public bodies, also monetary authorities have begun to express concern about the economic risks posed by climate change and see the need for their proactive involvement. Examples are the speech given in 2015 by former governor of the Bank of England Mark Carney, who warned that climate change poses severe risks to economic development and financial stability; or more recently (July 2021), the new monetary policy strategy approved by the European Central Bank (ECB), in which it is claimed that central banks should commit to include the impact of climate change in their policy framework and be supportive to policy initiatives addressing environmental issues. The recent abnormal increase in commodity and energy prices has also become a source of concern for monetary authorities. The ongoing energy transition contributes to imbalances between supply and demand for these resources, leading to questions as to whether or not the current "Greenflation" is inevitable and how the authorities should react to price developments (Blas 2022). But how does climate change really affect the conduct of monetary policy? Is central banks' mandate of aggregate price stability still optimal while targeting environmental issues? The goal of this paper is to discuss the possible impact coming from the inclusion of climate change in a standard macroeconomic model on the optimal monetary policy reaction function, and to investigate the optimal mix of environmental and macroeconomic stabilization policies from a welfare perspective.

To achieve this goal, we make use of the DSGE modelling approach to compare the effect of different environmental-monetary policy mixes on the business cycle and welfare. As conventional DSGE models, employed to design macroeconomic policies, often neglect the natural environment, we develop an economic model augmented with a climate module, featuring two sectors: a *green* "clean" sector and a *brown* "polluting" sector; the purpose is to study how the distinction between a polluting and a non-polluting industry affects the efficient design of monetary policy when in combination with different climate policies addressing the environmental externalities. In particular, in our framework monetary policy controls the nominal interest rate, which affects consumption, output and prices. Since climate change and environmental policies can severely affect the economic development, they cannot be deemed to be unrelated with monetary policy, as it cares about growth and stabilization. In this work we focus our attention on the price stability target in a Taylor-rule based policy and the trade off between stabilizing output, inflation and limiting climate change. Our approach is to look at sectoral rather than general level of prices, assuming that the central bank can differentiate between green and brown goods' prices when setting its policy. In the main simulation we test the response of the economy to exogenous sector-specific technology and cost-push shocks. To assess the role of environmental damage, we also compare economies both with and without pollution externalities. We show that optimal policy rule parameters vary significantly between the models, are affected by the type of environmental policy implemented and depend on the specific shock occurring.

The bridge in the literature between general equilibrium models and environment comes from *Integrated Assessment Models* (IAM). The pioneering work of William Nordhaus (Nordhaus 1977, 2010; Nordhaus and Sztorc 2013), with his *Dynamic Integrated model of Climate and the Economy* (DICE), can be considered the forerunner of the wide strand of literature developed in recent decades around IAM. DICE is an analytical model, designed as a policy optimization tool, trying to represent the interconnection between climate and global economic system. In addition to the neoclassical economic growth theory, on which it is grounded, here the "negative natural capital" of carbon concentrations is included (Nordhaus 2010). Environmental policies aiming at reducing anthropic emissions are therefore intended as investments to reduce this negative capital. On the early work of Nordhaus is also based a strand of literature combining environmental issues with the real business cycle theory. Among these, Heutel (2012) formally stresses the importance of business cycles in driving public policies: his idea is that, in order to design focused interventions to address climate change, it is important to build a model in which climate policies are explicitly integrated with macroeconomic fluctuations. He develops a DSGE model with random exogenous shocks, in which pollution appears as a stock variable that negatively affects the economy. This kind of integrated models has been defined environmental DSGE (E-DSGE).

Early E-DSGE literature has focused primarily on the different effects of specific public environmental policies on the business cycle. Extension of the model by Heutel (2012) have been provided by adding uncertainty (Hassler and Krusell 2018), New Keynesian nominal rigidities (Annicchiarico and Di Dio 2015, 2017), financial (Carattini et al. 2021) or labor-market frictions and migration (Chan 2019; Gibson and Heutel 2020). More recently, the attention has shifted to the interaction between environmental and monetary policies and on possible extension of the tools available to central banks. In this respect, Chen et al. (2020) and Chan (2020) develop a climate-augmented monetary policy rule, by adding an emission target in the standard Taylor equation. They find that such a kind of monetary policy can create a conflict between welfare and climate objectives. Other scholars instead drive the attention on central banks' balance sheets composition, developing models with financial frictions in which green-quantitative easing programs are enforced (Diluiso et al. 2020; Ferrari and Nispi Landi 2021). We contribute to this literature by asking whether central banks should react differently to price changes in the green and the brown sector.

The paper is organized as follows: in section 2, we set up the notation and define optimal monetary policy both in a simple one-sector New Keynesian DSGE model, and in a two-sector DSGE, in which firms at the intermediate-level are split into two identical sectors. In section 3, first we conduct a broad analysis of impulse response functions in the E-DSGE, shedding light on the effect of monetary and climate policies mixes on the business cycle; then we show how optimal monetary policy is affected by the inclusion of greenhouse gas emissions and environmental policy in the standard two-sector DSGE model. In section 4 we look at the optimal monetary rule and welfare loss when asymmetric shocks hit the economy. Section 5 concludes.

2 Optimal Monetary Policy in a Simple Framework without Climate Change

In this section we first start developing a *plain-vanilla* New Keynesian-DSGE model (with price and investment rigidities); then we amend it by splitting the economy into two-symmetric productive sectors. This will serve as a benchmark to design our main E-DSGE model, which adds a climate module and environmental externalities to the two-sector DSGE model. The purpose here is to show how optimal simple-rule policy parameters vary when revising the model structure (from 1 to 2 sectors) and introducing new frictions in the model (environmental damage). The calibration for common parameters is the same for all three models. For a detailed description see section 3.4.

2.1 One-Sector DSGE model

In our baseline model, firms produce a non-differentiated final consumption good, using intermediate goods as input and operating in a perfectly competitive market. A continuum of firms producing differentiated goods operate in a monopolistically competitive regime and employ capital and labor as inputs in a Cobb-Douglas production function. Besides firms, other actors operating in the economy are a central monetary authority (e.g. the central bank), a central fiscal authority (e.g. the government) and households/labor force.

2.1.1 Households

The intertemporal utility function of the representative infinitively-lived household can be formulated as follows:

$$U = \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\varphi_c} - 1}{1-\varphi_c} - \psi \frac{l_t^{1+\varphi_l}}{1+\varphi_l} \right) \tag{1}$$

where l_t represents working hours and c_t per-capita consumption (at time t); period utility is characterized by a constant relative risk aversion (CRRA), where φ_c is the inverse elasticity of intertemporal substitution[1], ψ weights the disutility of working and φ_l is the inverse of the Frisch elasticity[2]; β is the intertemporal discount factor. Households solve the following optimization problem:

$$\max_{c_t, l_t, b_t, i_t} \quad \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\varphi_c} - 1}{1-\varphi_c} - \psi \frac{l_t^{1+\varphi_l}}{1+\varphi_l} \right] \tag{2}$$

$$\text{s.t.} \quad b_t + c_t + i_t = b_{t-1} \frac{r_{t-1}}{\pi_t} + k_{t-1} r_t^k + w_t l_t + - t_t + \mathcal{T}_t \tag{3}$$

Here, b_t is bond, i_t represents investment, $b_{t-1} r_{t-1}$ denotes revenues from holding bonds, π_t is CPI inflation rate; $k_{t-1} r_t^k$ is the income from capital service and $w_t l_t$ income from labor; t_t is a lump-sum tax and $\mathcal{T}_t$ represents profits from final goods firms ownership (equal to zero because of the perfect competition regime in which they operate). Following the example of Christiano et al. (2005), we also consider the existence of implicit adjustment costs in investment[3], which makes adjusting the investment level in response to a departure of capital from its optimal level costly. In so doing, investment is smoothed over time. The law of motion of capital with quadratic adjustment costs is then:

$$k_t = (1-\delta) k_{t-1} + i_t \left[1 - \frac{\phi_i}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] \tag{4}$$

where $\phi_i > 0$ denotes the investment cost parameter. Households utility maximization problem yields the following first order conditions (FOC) with respect to (w.r.t.) consumption c_t:

$$c_t^{-\varphi_c} = \lambda_t; \tag{5}$$

FOC w.r.t. labor l_t:

$$w_t = \frac{\psi l_t^{\varphi_l}}{\lambda_t}; \tag{6}$$

FOC w.r.t. capital k_t:

$$q_t = \frac{\lambda_{t+1}}{\lambda_t} \beta^t \left((1-\delta) q_{t+1} + r_{t+1}^k \right); \tag{7}$$

where q_t measures the marginal value of capital w.r.t. consumption and is known as Tobin's q; FOC w.r.t. bonds b_t:

$$r_t = \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta^t} \pi_{t+1}, \tag{8}$$

where inflation π_{t+1} is defined as

$$\pi_{t+1} = \frac{p_{t+1}}{p_t}; \tag{9}$$

[1]For $\varphi_c = 1$, we use $log(c_t)$ for consumption in the utility function.

[2]For example, choosing a value equal to 2, means that an increase of 1% in wage rate increases hours work by 0.5%.

[3]In their work, Christiano et al. (2005) do not specify a functional form for the investment adjustment cost but rather some properties, which are perfectly met with our choice.

FOC w.r.t. investment i_t:

$$\begin{aligned}1 =& q_t \left[1 - \frac{\phi_i}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 - \phi_i \left(\frac{i_t}{i_{t-1}} - 1\right)\frac{i_t}{i_{t-1}}\right] + \\ & \beta q_{t+1} \left[\frac{\lambda_{t+1}}{\lambda_t}\phi_i \left(\frac{i_{t+1}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right]\end{aligned} \tag{10}$$

From equation (8) we can write real interest rate r_t^r as

$$r_t^r = \frac{r_t}{\pi_{t+1}} \tag{11}$$

2.1.2 Firms

Firms producing final good y_t operate in a perfectly competitive market and use a combination of intermediate goods $y_{i,t}$ from producers i in production function with constant elasticity of substitution (CES):

$$y_t = \left(\int_0^1 y_{i,t}^{\frac{\xi-1}{\xi}} di\right)^{\frac{\xi}{\xi-1}} \tag{12}$$

where ξ is the elasticity of substitution parameter and y_t is known as the *Dixit-Stiglitz aggregator*. Because we are in perfect competition, prices are given; combining equation (12) with FOC of intratemporal profit maximization w.r.t. the intermediate good, we obtain the demand function for $y_{i,t}$:

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t}\right)^{-\xi} y_t \tag{13}$$

Zero profit condition for final goods firms requires then the aggregate price index to be

$$p_t = \left(\int_0^1 p_{i,t}^{1-\xi} di\right)^{\frac{1}{1-\xi}} \tag{14}$$

Intermediate firms i employ labor l and capital k as input in a standard Cobb-Douglas production function, as follows:

$$y_{i,t} = A_{i,t}\left(k_{i,t-1}\right)^{\alpha}\left(l_{i,t}\right)^{1-\alpha} \tag{15}$$

where A_t is the total factor productivity and α is the input share parameter. Nominal rigidities typical of the New Keynesian framework are modeled by introducing quadratic adjustment costs ($AC_{i,t}$) à la Rotemberg (1983)[4], which intermediate firms pay whenever they adjust their price w.r.t. the steady state inflation level:

$$AC_{i,t} = \frac{\phi_p}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - \bar{\pi}\right) \tag{16}$$

Profits are defined as

$$\Pi_{i,t} = p_{i,t}y_{i,t} - w_{i,t}l_{i,t} - r_{i,t}^k k_{i,t-1} \tag{17}$$

Intermediate firms sell their own output at price p_i and pay production factors price (wage w_i and capital interest rate r_i^k) to labor and capital issuers (households). Since in symmetric

[4] As an alternative, we could have employed Calvo (1983) model of pricing, but we opted for Rotemberg as a more parsimonious model. Moreover, Rotemberg and Calvo models deliver equivalent dynamics when log-linearized around a zero inflation steady state (Ascari and Rossi 2012).

equilibrium all firms within the same sector choose the same price, we can drop the index i. From intermediate firm's profits maximization, FOC w.r.t. capital k_t yields:

$$r_t^k = mc_t A_t \alpha \left(k_{t-1}\right)^{\alpha-1} \left(l_t\right)^{1-\alpha}; \tag{18}$$

where $mc_{i,t}$ is the Lagrangian multiplier related to marginal costs; FOC w.r.t. labor l_t:

$$w_t = mc_t A_t (1-\alpha) \left(k_{t-1}\right)^{\alpha} \left(l_t\right)^{-\alpha}; \tag{19}$$

FOC w.r.t. price p_t:

$$\pi_t \left(\pi_t - \bar{\pi}\right) = \beta \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1}}{y_t} \pi_{t+1} \left(\pi_{t+1} - \bar{\pi}\right)\right] + \frac{\xi}{\phi_p} \left[mc_t - \frac{\xi - 1}{\xi}\right] \tag{20}$$

Total factor productivity evolves following an AR(1) process:

$$log(A_t) = (1-\rho_a) log(\bar{A}) + \rho_a log(A_{t-1}) + e_a \tag{21}$$

where $\bar{A}$ is the steady state level of technology and e_a is an exogenous productivity shock.

2.1.3 Monetary authority and public sector

The monetary authority follows a simple feedback rule of the Taylor (1993) rule class to set nominal interest rate[5]. It is assumed to have a dual mandate of business cycle and price stability:

$$\frac{r_t}{\bar{r}} = \left(\frac{r_{t-1}}{\bar{r}}\right)^{\rho_m} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} \left(\frac{y_t}{\bar{y}}\right)^{\phi_y}\right]^{1-\rho_m} exp(e_m) \tag{22}$$

where $\bar{r}$, $\bar{\pi}$ and $\bar{y}$ represent the corresponding Ramsey steady state of nominal interest rate, inflation and aggregate demand[6]; ϕ_π and ϕ_y are standard policy parameters determining the response of nominal interest rate to variation in inflation and output; ρ_m denotes the degree of monetary policy inertia; e_m is the exogenous monetary policy shock. Public sector expenditure g_t is completely financed by an income tax t_t:

$$g_t = t_t + b_t - \left(1 + r_{t-1}\right) b_{t-1} \tag{23}$$

$$\text{where} \quad t_t = \omega y_t \tag{24}$$

t_t is defined as a fixed share of income y_t, determined by the parameter ω. To simplify, we assume net supply of bonds to be zero. Public expenditure also follows an AR(1) process as TFP:

$$log(g_t) = (1-\rho_g) log(\bar{g}) + \rho_g log(g_{t-1}) + e_g \tag{25}$$

where $\bar{g}$ is the usual steady state level and e_g is an exogenous fiscal policy shock. The good-market clearing condition implies:

$$y_t = c_t + g_t + i_t + \frac{\phi_p}{2} y_t (\pi_t - \bar{\pi}_t)^2 \tag{26}$$

Finally, the bond-market clears at $b_t = 0$.

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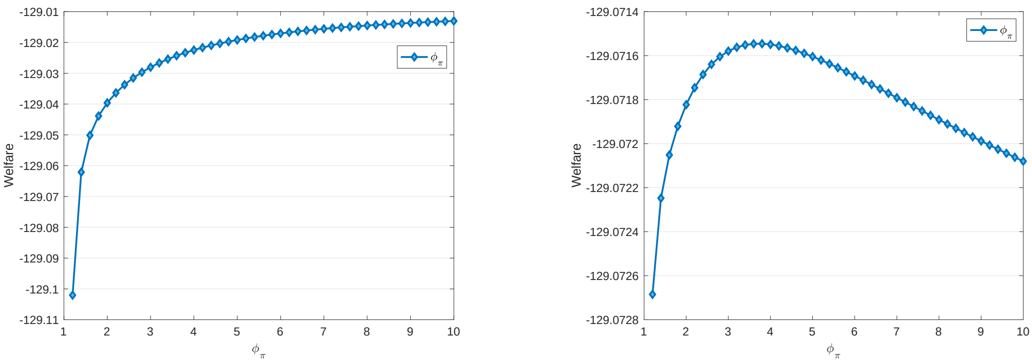
[5]The choice of a simple rule comes from the fact that this class of rule-based policy is readily implementable and easy to communicate to the public. In contrast, a socially optimal policy designed by a Ramsey planner is not directly operational. Nevertheless, Schmitt-Grohé and Uribe (2007) have shown that the outcomes, in terms of welfare maximization, are extremely close between the two approaches.

[6]Alternatively we could have set the output gap in the Taylor rule as the deviation of output from its natural level. As in Smets and Wouters (2003), natural or potential output is defined as the one that would be produced in a completely frictionless economy with full price-flexibility and no adjustment costs. Nevertheless, as Schmitt-Grohé and Uribe (2007) point out, it is quite unrealistic for the policymaker to have all the information required for computing the flexible-price aggregate output (such as the joint distribution of shocks and their occurrence). For robustness check, we tested our model with such a kind of output gap: the results are not far removed from those shown in the following sections.

2.1.4 Optimal monetary policy

Following the approach proposed by Schmitt-Grohé and Uribe (2007), we compute optimal monetary policy rule maximizing the welfare of the representative agent. We distinguish two main scenarios: in the first, the economy is hit by a technology shock; in the second, only a cost-push shock occurs. In both cases welfare is maximal for a muted response to output gap ($\phi_y = 0$); conversely, the optimal inflation gap parameter is maximal for $\phi_\pi = 10$ in a TFP shock scenario and for $\phi_\pi = 3.8$ (interior solution)[7], in a cost-push shock scenario. Note that, in our setup, a TFP shock brings no trade-off in the monetary policy, and the central bank can freely set its policy rate to the natural level. This looks numerically like an extremely large reaction coefficient to the inflation gap [8]. The two cases are displayed in figure 1.



(a) TFP shock

(b) Cost-push shock

Figure 1: Optimal monetary policy, one-sector DSGE model

What emerges from the policy parameters optimization is that closing the inflation gap is the major concern for the monetary authority. Clearly the negative inflation caused by a TFP shock induces central banks to cut their nominal interest rate, as an further stimulus to consumption and hence welfare. While the positive price variation, due to the rising production costs, calls the monetary authority to increase interest rate so as to reduce aggregate demand (and consequently prices) up to a specific level. An excessive reaction of monetary policy, instead, would have the effect of excessively weaken consumption and cut the welfare level.

2.2 Optimal Monetary Policy in a Two-Sector Model

A first extension to the standard one-sector DSGE model, previously described, consists of splitting firms at the intermediate level into two symmetric sectors (which we call G and B for reasons that will become clear in next section), featuring same production function and costs. Those intermediate goods are aggregated within their sector and then bundled together to form the final good.

[7] We restrict policy coefficients to lie in a relatively large interval $[0, 10]$.

[8] Schmitt-Grohé and Uribe (2007) show that in an unconstrained optimization, the inflation coefficient would exceed 300.

2.2.1 Households

For what concerns households' budget, we differentiate capital, labor and wage by sector[9], such that

$$b_t + c_t + i_t = b_{t-1}\frac{r_{t-1}}{\pi_t} + r_t^k k_{t-1} + w_t^G l_t^G + w_t^B l_t^B - t_t + \mathcal{T}_t \tag{27}$$

Capital and labor supply for the two sectors aggregate according to the following equations:

$$l_t = \left[\left(l_t^B\right)^{1+\varphi_h} + \left(l_t^G\right)^{1+\varphi_h}\right]^{\frac{1}{1+\varphi_h}}; \tag{28}$$

$$k_t = \left[k_t^B + k_t^G\right] \tag{29}$$

where $\varphi_h > 0$ represents the willingness of households to substitute labor between sectors. By setting this parameter higher than zero, we open to imperfect labor mobility across sectors. Again, from utility maximization (2), FOC w.r.t. labor l_t^G and l_t^B read:

$$w_t^G = \frac{\psi\left(l_t^G\right)^{\varphi_h}(l_t)^{\varphi_l-\varphi_h}}{\lambda_t}; \tag{30}$$

$$w_t^B = \frac{\psi\left(l_t^B\right)^{\varphi_h}(l_t)^{\varphi_l-\varphi_h}}{\lambda_t}; \tag{31}$$

2.2.2 Firms

Firms producing the final good employ a linear function using a constant elasticity of substitution (CES) aggregator[10], combining the two sectors' intermediate goods:

$$y_t = \left[\left(y_t^G\right)^{\frac{\epsilon-1}{\epsilon}} + \left(y_t^B\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{32}$$

where y_t^G and y_t^B are respectively goods from industry G and B, while ϵ represents the elasticity of substitution between the two. The final good firm solves an intratemporal maximization problem to determine the optimal input combination:

$$\max_{y_t^G, y_t^B} \quad p_t y_t - \left[p_t^G y_t^G + p_t^B y_t^B\right]$$

where p_t^G and p_t^B are sector-specific prices and p_t represents the aggregate price index. This problem yields the following demand functions:

$$y_t^G = y_t\left(\frac{p_t^G}{p_t}\right)^{-\epsilon} \tag{33}$$

$$y_t^B = y_t\left(\frac{p_t^B}{p_t}\right)^{-\epsilon} \tag{34}$$

The aggregate price can be written as

$$p_t = \left[\left(p_t^G\right)^{1-\epsilon} + \left(p_t^B\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{35}$$

[9] We avoid setting a specific return on capital for each sector first, because we assume households can easily switch their investments from one sector to the other; second, as we do not want to focus on capital market frictions here.

[10] Unitary elasticity of substitution helds the classic Cobb-Douglas function; instead we choose a value $\frac{\epsilon-1}{\epsilon} < 1$, which makes the two inputs imperfect substitutes.

Clearly, both demand functions are increasing in the final good production and price and decreasing in their own price. Similarly to equation (12), we define intermediate goods G and B as an aggregation of intermediate inputs $y_{i,t}^{G,B}$ produced by monopolistically competitive firms i, with constant elasticity of substitution:

$$y_t^j = \left(\int_0^1 \left(y_{i,t}^j \right)^{\frac{\xi-1}{\xi}} di \right)^{\frac{\xi}{\xi-1}}, \quad \text{where} \quad j = \{G, B\} \tag{36}$$

Intermediate input demand reads:

$$y_{i,t}^j = \left(\frac{p_{i,t}^j}{p_t^j} \right)^{-\xi} y_t^j \tag{37}$$

By combining the latter with the demand function of aggregate intermediate good (33 - 34), we get:

$$y_{i,t}^j = \left(\frac{p_{i,t}^j}{p_t^j} \right)^{-\xi} y_t \left(\frac{p_t^j}{p_t} \right)^{-\epsilon} \tag{38}$$

Zero profit condition for intermediate goods firms requires then the aggregate intermediate price index to be

$$p_t^j = \left(\int_0^1 p_{i,t}^{1-\xi} di \right)^{\frac{1}{1-\xi}} \tag{39}$$

Intermediate inputs producers, operating in a monopolistically competitive market, employ a Cobb-Douglas function as in 15, with sector-specific production factors:

$$y_{i,t}^j = \quad A_{i,t}^j \left(k_{i,t-1}^j \right)^{\alpha} \left(l_{i,t}^j \right)^{1-\alpha} \tag{40}$$

In both sectors firms pay a price adjustment cost (see equation (16)) and maximize their profits, defined as

$$\Pi_{i,t}^j = p_{i,t}^j y_{i,t}^j - w_{i,t}^j l_{i,t}^j - r_{i,t}^k k_{i,t-1}^j \tag{41}$$

Firms maximize profits w.r.t. their factors of production. The resulting FOC are the same as in the one-sector case (18 - 19) and are identical across industries G and B. Firms choose also the optimal price for their products; in this way we can compute two distinct inflation rates for sectors G and B:

$$\pi_t^j \left(\pi_t^j - \bar{\pi} \right) = \beta \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1}^j}{y_t^j} \pi_{t+1}^j \left(\pi_{t+1}^j - \bar{\pi} \right) \right] + \frac{\xi}{\phi_p} \left[mc_t^j - \frac{\xi - 1}{\xi} \right] \tag{42}$$

Marginal costs are obviously identical and depend on the degree of elasticity between intermediate goods within the same sector.

2.2.3 Central banks and market clearing

The central bank employs a non-standard Taylor Rule to set its policy rate: while maintaining the usual mandate of price stability, it takes into account the relative change of p_t^G and p_t^B instead of the general level of price p_t. The Taylor rule in equation (22) is then augmented with two new inflation parameters, as follows:

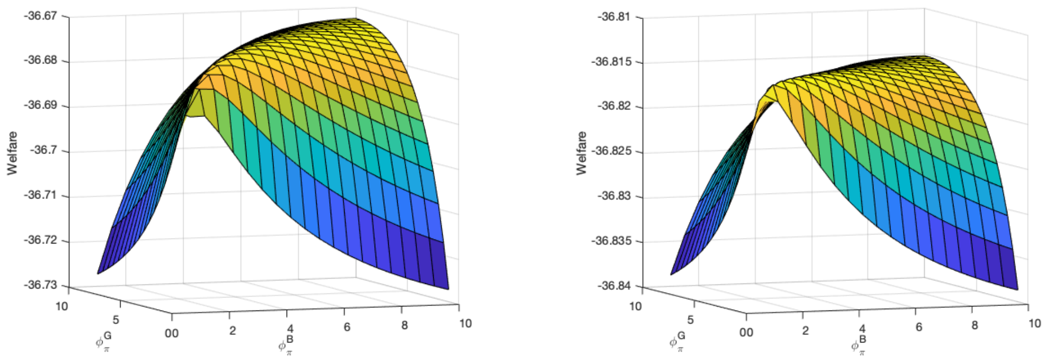
$$\frac{r_t}{\bar{r}} = \left(\frac{r_{t-1}}{\bar{r}} \right)^{\rho_m} \left[\left(\frac{\pi_t^G}{\bar{\pi}} \right)^{\phi_\pi^G} \left(\frac{\pi_t^B}{\bar{\pi}} \right)^{\phi_\pi^B} \left(\frac{y_t}{\bar{y}} \right)^{\phi_y} \right]^{1-\rho_m} exp(e_m) \tag{43}$$

where ϕ_π^G and ϕ_π^B denote the response of interest rate to the sector-specific inflation variation. Public expenditure is defined according to equation (23). The good-market clearing condition implies:

$$y_t = c_t + g_t + i_t + \frac{\phi_p}{2} y_t^G (\pi_t^G - \bar{\pi}_t)^2 + \frac{\phi_p}{2} y_t^B (\pi_t^B - \bar{\pi}_t)^2 \tag{44}$$

2.2.4 Optimal monetary policy

Once again, we optimize monetary policy parameters of our new Taylor rule 43 maximizing the households' welfare. This time the response parameter to output gap is set by default to $\phi_y = 0$. Welfare maximizing parameters for the inflation gap are $\phi_\pi^G = \phi_\pi^B = 3$ in the case of a simultaneous cost-push shock in the two sectors (panel b of figure 2); while with a TFP-shock welfare is maximal for reaction coefficients on inflation equal to 10 (as before, see panel a in figure 2). These results replicate those for the one-sector DSGE (with slightly lower reaction of interest rate to inflation gap when cost-push shock occurs) given the (almost) identical structure of the two models (besides the introduction of frictional labor mobility and imperfect substitution between the two sectors).



(a) TFP shock

(b) Cost-push shock

Figure 2: Optimal monetary policy, two-sector DSGE

3 Environmental-DSGE

In this section we extend the two-sector DSGE by introducing environmental externalities in the model. Building on the work of Annicchiarico and Di Dio (Annicchiarico and Di Dio 2015, 2017) and Hassler and Krusell (2018), we integrate the Environmental-DSGE model by Heutel (2012) with New Keynesian nominal rigidities and an intermediate firm level. As before, in this model three kind of firms operate in the economy: a final consumption good producer, bundling "green" and "brown" output and operating in a perfectly competitive market; two intermediate firms aggregating respectively "green" and "brown" differentiated goods using a CES aggregator function; and the "brown" polluting firms[11] and the "green" clean firms operating in a monopolistically competitive regime. These latter both employ capital and labor as production factors.

3.1 Firms

3.1.1 Final-good firms

Similarly to Acemoglu et al. (2012), firms producing final good employ a linear function as follows:

$$y_{i,t} = y_t^E \tag{45}$$

[11] While in Heutel (2012) all firms pollutes, in our model only brown firms do.

where y_t^E is a CES aggregator combining green and brown intermediate goods as inputs:

$$y_t^E = \left[(1-\Delta)^{\frac{1}{\epsilon}} \left(y_t^G\right)^{\frac{\epsilon-1}{\epsilon}} + \Delta^{\frac{1}{\epsilon}} \left(y_t^B\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{46}$$

where y_t^G and y_t^B are respectively green and brown goods, Δ is a weighting parameter[12] and ϵ represents the elasticity of substitution between the two inputs. Intermediate firm i solves an intratemporal maximization problem to determine the optimal input combination:

$$\max_{y_t^G, y_t^B} \quad p_t^E y_t^E - \left[p_t^G y_t^G + p_t^B y_t^B\right]$$

where p_t^G and p_t^B are respectively green and brown goods' prices; p_t^E represents the aggregate price index. As in previous section, this problem yields the demand functions for the two sectors:

$$y_t^G = y_t^E (1-\Delta) \left(\frac{p_t^G}{p_t^E}\right)^{-\epsilon} \tag{47}$$

$$y_t^B = y_t^E \Delta \left(\frac{p_t^B}{p_t^E}\right)^{-\epsilon} \tag{48}$$

Aggregate price reads

$$p_t^E = \left[(1-\Delta)\left(p_t^G\right)^{1-\epsilon} + \Delta\left(p_t^B\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{49}$$

In this case both demand functions depend on the composition of y^E, given by the endogenous value of Δ.

3.1.2 Green and Brown intermediate input firms

Aggregate goods G and B are again defined as a combination of intermediate inputs $y_{i,t}^{G,B}$ produced by a continuum of firms indexed with i (see equation (37)) By substituting the value of $y_t^{G,B}$ from 47 - 48, into the intermediate input demand function 37, we get:

$$y_{i,t}^G = \left(\frac{p_{i,t}^G}{p_t^G}\right)^{-\xi} y_t^E (1-\Delta) \left(\frac{p_t^G}{p_t^E}\right)^{-\epsilon} \tag{50}$$

$$y_{i,t}^B = \left(\frac{p_{i,t}^B}{p_t^B}\right)^{-\xi} y_t^E \Delta \left(\frac{p_t^B}{p_t^E}\right)^{-\epsilon} \tag{51}$$

Green and brown firms employ sector-specific labor and capital in their production; also for both TFP is negatively affected by pollution. Brown-good firms differ from green-good producers in that they face a trade off between paying a tax on their negative externalities and employing a fraction of their production to abate their polluting emissions. The brown production function is

$$y_{i,t}^B = A_{i,t}^B \left(k_{i,t-1}^B\right)^{\alpha} \left(l_{i,t}^B\right)^{1-\alpha}, \tag{52}$$

$$\text{where} \quad A_t^B = (1 - D_t(x_t))\, a_t^B \tag{53}$$

[12] Following Ferrari and Nispi Landi (2021), the value of Δ is calculated by fixing the steady state amount of atmospheric carbon stock, such that it represents the intermediate input composition accounting for the current level of air pollution.

Similarly for green producers

$$y^G_{i,t} = \quad A^G_{i,t}\left(k^G_{i,t-1}\right)^{\alpha}\left(l^G_{i,t}\right)^{1-\alpha} \tag{54}$$

where $A^{G,B}_t$ is the total factor productivity net of the damage function D_t, and $a^{G,B}_t$ is the industry-specific technology factor.

The linkage between production and climate is expressed via the damage function and the abatement spending. Let's here define the functions that make up the climate module:

$$D_t(x_t) = d_0 + d_1 x_t + d_2 x_t^2 \tag{55}$$
$$x_t = \eta x_{t-1} + e_t + e^{ROW}_t \tag{56}$$
$$e_t = (1-\mu_t)h(y^B_{i,t}), \quad \text{with} \quad \mu_t \in [0,1] \tag{57}$$
$$z_t = g(\mu_t)y^B_{i,t} \tag{58}$$

Damage D_t is a quadratic function of pollution stock x_t[13]. Pollution stock is a function of domestic emissions e_t and emissions from the rest of the world e^{ROW}_t; η is a parameter describing the decay rate of atmospheric pollution. Domestic emissions are a function of polluting firms' production and abatement; μ_t is the fraction of emissions abated; z_t represents the total abatement spending. In addition we define two auxiliary function:

$$h\left(y^B_{i,t}\right) = \gamma_1\left(y^B_{i,t}\right)^{1-\gamma_2}, \quad \text{with} \quad 0 < \gamma_1, \gamma_2 \le 1 \tag{59}$$
$$g(\mu_t) = \theta_1 \mu_t^{\theta_2} \tag{60}$$

such that industrial emissions are increasing concave function of output. The resource constraint of brown-good producers can be expressed as follows:

$$\Pi^B_{i,t} = p^B_{i,t}y^B_{i,t} - \tau^E_{i,t}e_{i,t} - z_{i,t} - w^B_{i,t}L^B_{i,t} - r^{k,B}_{i,t}k^B_{i,t-1} \tag{61}$$

where $\Pi^B_{i,t}$ is brown firm profit and τ^E_t is the *carbon tax* on industrial emissions that polluting firms pay to the central fiscal authority. Profit maximization problem for polluting firms yields the following Lagrangian function:

$$\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty}\beta^t\lambda_t \Bigg\{ & y^E_t\Delta\left(\frac{p^B_t}{p^E_t}\right)^{-\epsilon}\left(\frac{p^B_{i,t}}{p^B_t}\right)^{-\xi} \\
& \left[\left(\frac{p^B_{i,t}}{p^B_t}\right) - \tau^E_{i,t}\left(1-\mu_{i,t}\right)\gamma_1\left[y^E_t\Delta\left(\frac{p^B_t}{p^E_t}\right)^{-\epsilon}\left(\frac{p^B_{i,t}}{p^B_t}\right)^{-\xi}\right]^{-\gamma_2} - \theta_1\mu^{\theta_2}_{i,t}\right] + \\
& -w^B_{i,t}l^B_{i,t} - r^{k,B}_{i,t}k^B_{i,t-1} - \frac{\phi_p}{2}\left(\frac{p^B_{i,t}}{p^B_{i,t-1}} - \bar{\pi}\right)^2 y^B_t + \\
& +mc^B_{i,t}\left[A^B_{i,t}\left(k^B_{i,t-1}\right)^{\alpha}\left(l^B_{i,t}\right)^{1-\alpha} - y^E_t\Delta\left(\frac{p^B_t}{p^E_t}\right)^{-\epsilon}\left(\frac{p^B_{i,t}}{p^B_t}\right)^{-\xi}\right]\Bigg\}
\end{aligned}$$

[13]There is an extensive discussion in the literature about the real magnitude of damage. According to some experts, Nordhaus' damage function underestimates the impact of the stock of pollution on the economy (Howard and Sterner 2017). In addition, temperature increases due to environmental degradation would affect different parts of the world asymmetrically, and would even benefit some countries in the northern part of the globe (Kalkuhl and Wenz 2020). Nevertheless, investigating the nature of the damage function is beyond the scope of our work, so we follow the standard approach in the E-DSGE literature.

where $mc_{i,t}^{G,B}$ is the sectorial Lagrangian multiplier related to marginal costs. FOC w.r.t. capital k_t^B:

$$r_t^{k,B} = mc_t^B A_t^B \alpha \left(k_{t-1}^B\right)^{\alpha-1} (l_t^B)^{1-\alpha}; \tag{62}$$

FOC w.r.t. labor l_t^B:

$$w_t^B = mc_t^B A_t^B (1-\alpha) \left(k_{t-1}^B\right)^{\alpha} (l_t^B)^{-\alpha}; \tag{63}$$

FOC w.r.t. abatement μ_t:

$$\tau_t^E \gamma_1 \left[y_t^E \Delta \left(\frac{p_t^B}{p_t^E} \right)^{-\epsilon} \right]^{-\gamma_2} = \theta_1 \theta_2 \mu_t^{\theta_2 - 1} \tag{64}$$

FOC w.r.t. price p_t^B:

$$\begin{aligned} \pi_t^B \left(\pi_t^B - \bar{\pi}\right) =& \beta \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1}^B}{y_t^B} \pi_{t+1}^B \left(\pi_{t+1}^B - \bar{\pi}\right) \right] + \\ &+ \frac{\xi}{\phi_p} \left[mc_t^B - \frac{\xi - 1}{\xi} + \tau_t^E (1-\mu_t) \gamma_1 (1-\gamma_2) \left[y_t^E \Delta \left(\frac{p_t^B}{p_t^E} \right)^{-\epsilon} \right]^{-\gamma_2} + \theta_1 \mu_t^{\theta_2} \right] \end{aligned} \tag{65}$$

Note that since brown and green firms incur different costs, they will not set same price even in equilibrium; hence, we cannot simplify price variables p_t^B and p_t^E. For green-good producers instead, the resource constraint is

$$\Pi_{i,t}^G = p_{i,t}^G y_{i,t}^G - w_{i,t}^G L_{i,t}^G - r_{i,t}^{k,G} k_{i,t-1}^G \tag{66}$$

As before, profit maximization problem yields the following Lagrangian function:

$$\begin{aligned} \mathcal{L} =& \sum_{t=0}^{\infty} \beta^t \lambda_t \Bigg\{ y_t^E (1-\Delta) \left(\frac{p_t^G}{p_t^E} \right)^{-\epsilon} \left(\frac{p_{i,t}^G}{p_t^G} \right)^{1-\xi} - w_{i,t}^G l_{i,t}^G - r_{i,t}^{k,G} k_{i,t-1}^G - \frac{\phi_p}{2} \left(\frac{p_{i,t}^G}{p_{i,t-1}^G} - \bar{\pi} \right)^2 y_t^G + \\ &+ mc_{i,t}^G \left[A_{i,t}^G \left(k_{i,t-1}^G\right)^{\alpha} \left(l_{i,t}^G\right)^{1-\alpha} - y_t^E (1-\Delta) \left(\frac{p_t^G}{p_t^E} \right)^{-\epsilon} \left(\frac{p_{i,t}^G}{p_t^G} \right)^{-\xi} \right] \Bigg\} \end{aligned}$$

Again, FOC w.r.t. capital k_t^G yields:

$$r_t^{k,G} = mc_t^G A_t^G \alpha \left(k_{t-1}^G\right)^{\alpha-1} (l_t^G)^{1-\alpha}; \tag{67}$$

FOC w.r.t. labor l_t^G:

$$w_t^G = mc_t^G A_t^G (1-\alpha) \left(k_{t-1}^G\right)^{\alpha} (l_t^G)^{-\alpha}; \tag{68}$$

FOC w.r.t. price p_t^G:

$$\pi_t^G \left(\pi_t^G - \bar{\pi}\right) = \beta \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1}^G}{y_t^G} \pi_{t+1}^G \left(\pi_{t+1}^G - \bar{\pi}\right) \right] + \frac{\xi}{\phi_p} \left[mc_t^G - \frac{\xi - 1}{\xi} \right] \tag{69}$$

Technology in both sectors evolves following an AR(1) process:

$$log(a_t^j) = (1-\rho_a) log(\bar{a}^j) + \rho_a log(a_{t-1}^j) + e_a^j \tag{70}$$

where $\bar{a}^j$ is the steady state level of technology and e_a^j is a sector-specific exogenous productivity shock.

3.2 Monetary authority and public sector

The monetary authority follows the same unconventional Taylor Rule as before (see equation (43)), accounting for the relative variation of green-good and brown-good prices. Public sector expenditure g_t is financed both by lump-sum and emissions taxes, following the usual AR(1) process:

$$g_t = t_t + \tau_t^E e_t + b_t - (1 + r_{t-1})\, b_{t-1} \tag{71}$$

Finally, the good-market clearing condition implies:

$$y_t = c_t + g_t + i_t + z_t + \frac{\phi_p}{2} y_t^G (\pi_t^G - \pi_t)^2 + \frac{\phi_p}{2} y_t^B (\pi_t^B - \pi_t)^2 \tag{72}$$

Once again, the bond-market clears at $b_t = 0$.

3.3 Environmental dynamics and policies

With regards to environmental regimes, three different policies are taken into consideration:

1. fixing the emissions tax (and as a consequence the abatement effort) to a constant value (hereby Tax policy): $\tau_t = \bar{\tau}$;
2. choosing an emission intensity target linked to output (Target policy): $e_t = t_e y_t$;
3. defining a limit on emissions based on a fixed amount of pollution stock (Cap policy): $e_t = \bar{x}(1 - \eta) - e^{row}$.

In all the three cases, the emission tax represents the major instruments to offset the environmental friction[14].

3.4 Calibration

The model is calibrated at the quarterly frequency on the US, mainly relying on the parametrization already in use in the standard RBC and New Keynesian literature; climate module parameters and steady-state values (such as the current atmospheric pollution stock, which amounts to an average of 880 $GtCO_2$ in 2021) are updated to the most recent available data following Gibson and Heutel (2020)[15] and Carattini et al. (2021). Calibrated parameters are reported in table 1.

3.5 Impulse response functions in the E-DSGE model

The graphs below (from figure 3 to 6) show the responses of the endogenous variables to various shocks when a standard Taylor is in place. For example, with a green technology shock (figure 3) the green output's price dynamic entails a delayed tightening monetary policy: after an initial settlement, the slow return of both green and brown inflation to their steady state boosts the increase in nominal interest rate. The consequence is a reduction

[14] When no climate policy is set instead, we simply shut down the environmental tax and the abatement effort, such that $\tau_t^E = \mu_t = 0$. In so doing, the public sector is not endowed with any policy instruments to correct the distortion, and brown firms do not internalize their negative externalities, keeping producing at the same pace regardless the amount of CO_2 they emit. However, setting the emission tax equal to 0 distorts the composition of the economy, as the value of the weighting parameter Δ is affected by the value of the tax, with a significant impact on the coefficients of the optimized monetary policy.

[15] In their work, Gibson and Heutel (2020) employ the updated version of Nordhaus' DICE 2016R2 model to estimate the parameters of damage, emissions and abatement cost functions.

Table 1: Parameters value

Parameter	Description	Value	Source
β	Discount factor	0.995	RBC literature
φ_c	Inverse elasticity of intertemporal sub.	1	
φ_l	Inverse Frish elasticity	2	Hassler and Krusell (2018)
φ_h	Inverse elasticity of labor sub. btw sectors	2	
ψ	Disutility of work	20	
δ	Capital depreciation	0.025	
ϕ_i	Investment adjustment cost	2.48	Christiano et al. (2005)
ξ	Elasticity of sub. btw intermediate goods	6	NK-DSGE literature
ϵ	Elasticity of sub. btw brown and green goods	2	Carattini et al. (2021)
Δ	Weight of brown good	0.5095	(to get $\bar{x} = 880\ GtCO_2$)
ϕ_p	Price adjustment cost	58.2524	Ascari and Rossi (2012)
α	Share of capital in production	1/3	
d_0	Damage function constant	-0.0076	Gibson and Heutel (2020)
d_1	Damage function linear parameter	8.1e-6	
d_2	Damage function quadratic parameter	1.05e-8	
η	Pollution decay rate	0.9965	Allen et al. (2018)
γ_1	Shifter of emissions function	1	
γ_2	Emissions elasticity	0.4	
θ_1	Abatement cost function coefficient	0.074	
θ_2	Abatement cost function exponent	2.6	
$\phi_\pi^{B,G}$	Mon. pol. response to green/brown inflation	1.5	
ϕ_π	Mon. pol. response to inflation	1.5	
ϕ_y	Mon. pol. response to output	0	
ρ_a	Persistence of TFP/cost-push shock	0.95	
ρ_c	Persistence of cost-push shock	0.8	
ρ_g	Public spending persistence	0.9	
ρ_m	Mon. pol. inertia	0.2	
$\sigma_a^{B,G}$	SD of sector-specific TFP shock	0.01	
$\sigma_c^{B,G}$	SD of sector-specific cost-push shock	0.01	
σ_g	SD of government expenditure shock	0.01	
σ_m	SD of monetary policy shock	0.01	

in output growth (and more heavily in brown production) and a decrease in emissions; this is stronger with an emission target policy, as this entails the highest increase in abatement effort and emission tax. Similarly, with a brown technology shock (figure 4) the excess supply of brown good -because of the more efficient production- boosts emissions and pollution stock growth. The cap policy is the most beneficial in terms of reducing pollution but it is also the most detrimental w.r.t. consumption.

Similar dynamics can be detected in the cost-push shock scenario: when the shock hits the green sector (figure 5), green price increases and the clean production falls; since the two inputs are not perfect substitutes, final good producers require also less of the brown input, which in turns makes p_t^B decreasing. Brown production increases and consequently also emissions; this leads in the cap regime to an increment of environmental tax and abatement effort. The initial response of the central bank is to tighten its policy by increasing r_t, which is reflected in consumption trends and aggregate demand. Since policy coefficients here are not optimized, but rather follow the standard symmetric response to price variation, the reaction of monetary policy to a cost-push shock in the brown sector (figure 6) remains unchanged. What varies instead is the emissions, as a shock in the polluting industry increases the sectorial price, lowers the production and brings about lower emissions. Here again, as in the case of a TFP shock in the green sector, a target policy is the most efficient from the environmental perspective and the most expensive in terms of consumption (and welfare).

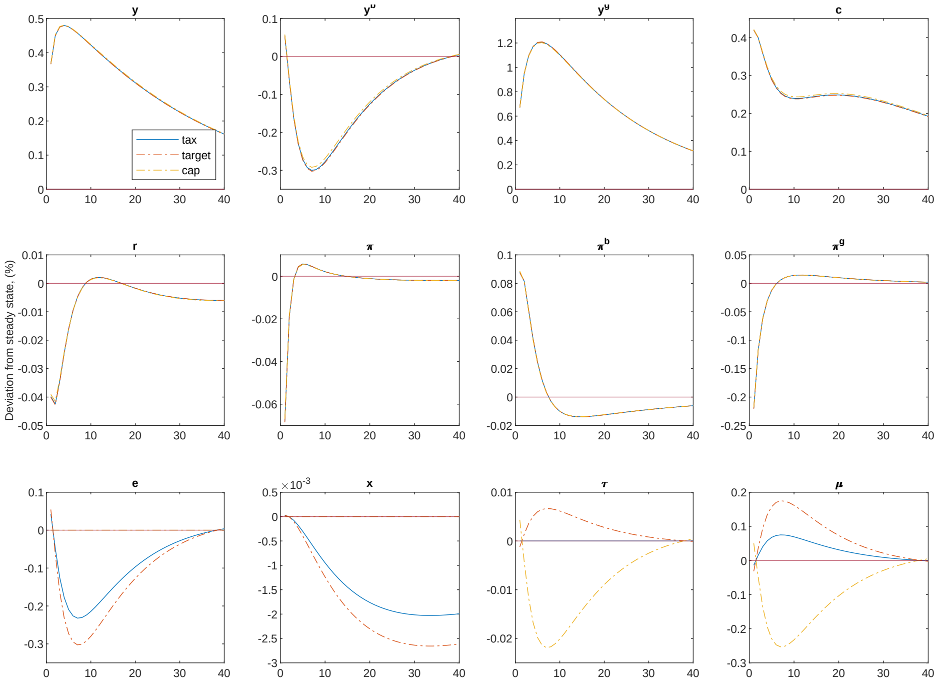


Figure 3: Impulse response function to green TFP shock under different environmental policies.

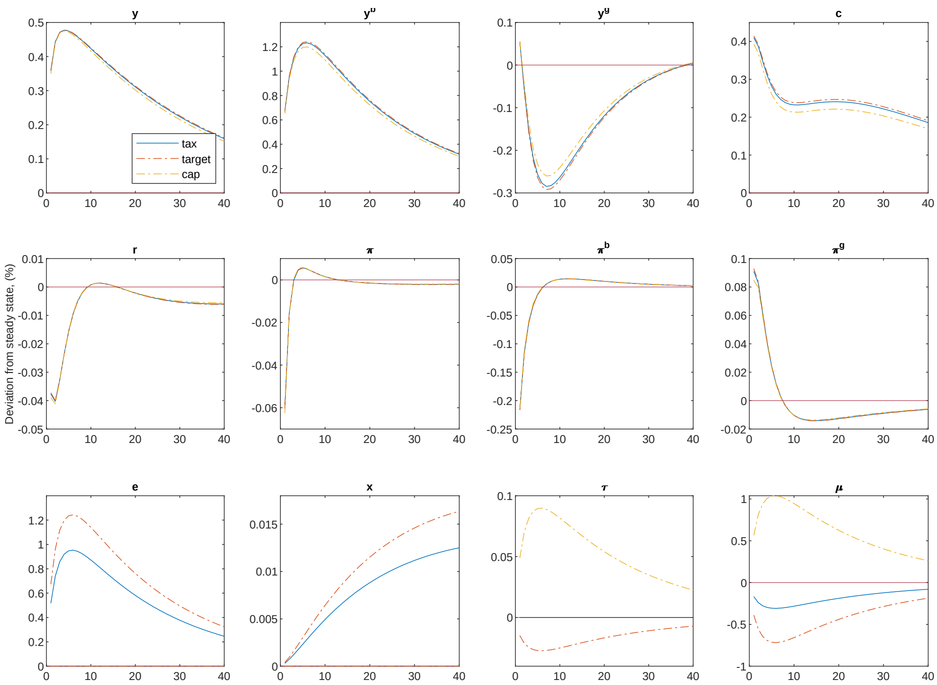


Figure 4: Impulse response function to brown TFP shock under different environmental policies.

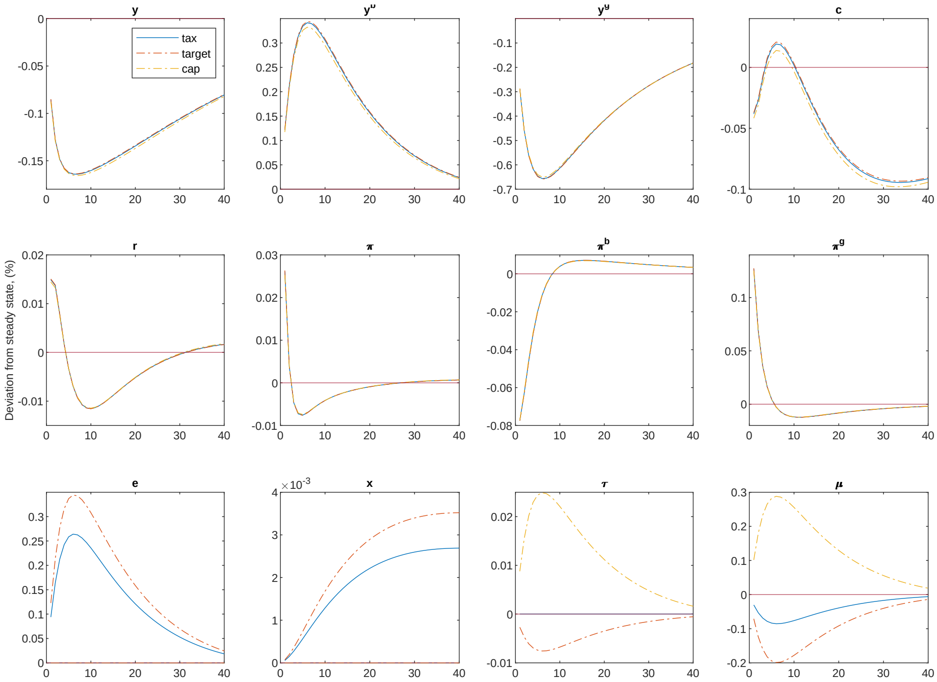


Figure 5: Impulse response function to green cost-push shock under different environmental policies.

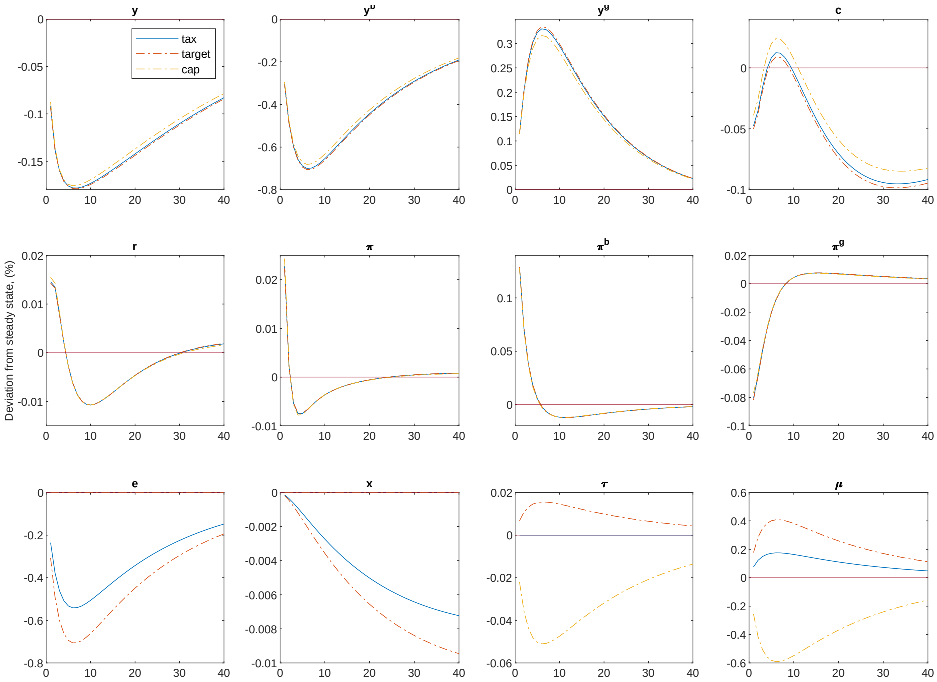
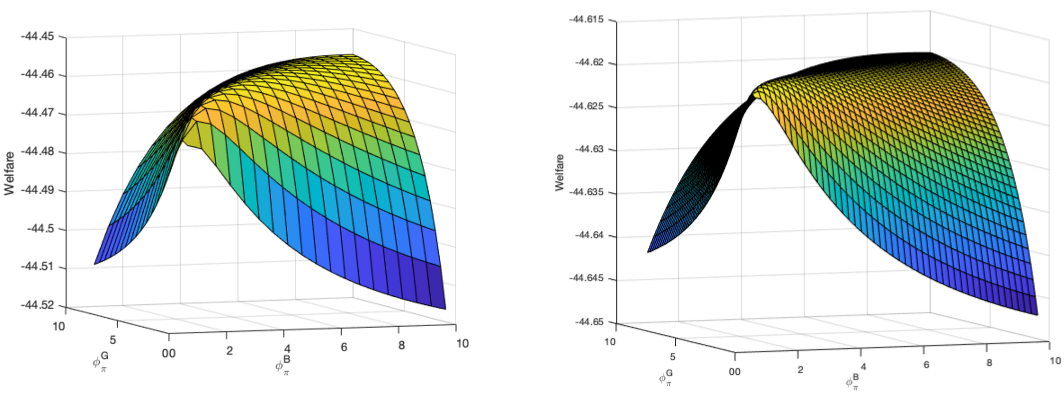


Figure 6: Impulse response function to brown cost-push shock under different environmental policies.

3.6 Optimal monetary policy

We now derive the value of Taylor rule parameters that maximizes households' welfare. In this section we focus our attention on an economy with a fixed emissions tax (Tax policy regime). Once again, even with the introduction of environmental externalities and climate policies in the model, optimal interest rate does not react to output gap variation $\phi_y = 0$. For what concerns inflation coefficients, the results here mirror those of the simple two-sector DSGE model: very large value with a TFP shock, interior solution with a cost-push shock. In the second scenario the optimal values of inflation gap reaction coefficients are now different from those computed for the two-sector model. The introduction of the environmental friction and climate policies induces an asymmetric response of monetary policy to the inflation variation in the two sectors. The resulting new optimal coefficients are $\phi_\pi^G = 2.8$ and $\phi_\pi^B = 2.6$, against a value of 3 for both parameters in the simpler DSGE model. Why are the optimal reaction coefficients now different for the two sectorial inflation rates? The results suggest that inflation in the brown sector and inflation in the green sector should be treated differently by monetary policy, because their impact on income and households' welfare is different. An increase in the brown inflation rate – as a results of an expansion of costs – brings a contraction in the demand for brown goods and a shift to the green production; this is of course beneficial in terms of pollution and helps reducing the environmental damage. Because of this indirect effect on the extent of damage, the monetary policy reaction is then more accommodative with brown inflation than with green inflation, when both sectors are hit by the cost-push shock. An important role here is also played by the degree of substitutability between the two intermediate input. Given they are imperfect substitute and share the same weight in the final good production function, their demand is not perfectly flexible with respect to price variation. A price increase in one of the two sectors then, does not bring a sufficient compensation in terms of production in the other sector. The result is a contraction also in the final good production, labor and capital demand and consumption. Given the existence of this trade off between reducing the emissions and preventing the aggregate income not to fall too much, the reaction of monetary policy to the brown inflation increase cannot be extremely accommodative.

This result suggests that the environmental friction in the E-DSGE model cannot be completely offset by the employment of the emissions tax, but requires an additional policy instrument, that is monetary policy[16]. Which can bring about a trade off between price stability and environmental target. Results of this simulation are displayed in figure 7 below.



(a) TFP shock

(b) Cost-push shock

Figure 7: Optimal monetary policy, E-DSGE.

[16]This would be in line also with the *Tinbergen rule*, named after the economist Jan Tinbergen, according to which policymakers trying to achieve multiple economic targets need at least one policy tool for each of them. In this case there would be a multitude of instruments -monetary policy and emission tax- working in tandem to achieve the environmental target of reducing polluting emissions.

3.7 Sensitivity analysis

In this section, we investigate how the optimal monetary policy coefficients vary with the value of important parameters in the model. The optimal coefficients are collected in table 2. First, we analyze the sensitivity to the elasticity of substitution between brown and green goods ϵ. For the baseline case, ϵ equals 2 (low substitutability). When increasing this value to 10 (highly substitutability), we observe an increase in the optimal value of the monetary policy reaction to green inflation rate ϕ_π^G (see figure 8). When increasing the substitution rate, a positive variation of brown good price due to a cost-push shock, for example, translates into a higher variation of green good demand (w.r.t. the baseline case) and a less significant drop in the aggregate production. Higher demand for the green input means also higher price, requiring a more robust response to green inflation to stabilize price volatility.

The variation associated to a change in the weight parameter Δ is instead more sizable (see figure 9). With a very low weight on brown input ($\Delta = 0.2$), the value of ϕ_π^B is about twice that of ϕ_π^G. The reason is that the effect of a price increase in the dirty sector does not induce a significant growth neither in the demand for the clean good, nor in the pollution stock. So the trade off between stabilizing prices and limiting emissions is mitigated if not entirely eliminated. The effect is reversed when brown input is predominant in the production of the final good ($\Delta = 0.8$). The only difference is that the overall reaction of monetary policy to price variation in both sectors (given by the sum of ϕ_π^G and ϕ_π^B) is lower with respect to the previous case. Again here the different result is due to the trade off environmental damage-price, which induces a more accommodative reaction of monetary policy to brown inflation.

We also note that reducing the utility discount factor β (from 0.995 to 0.90) breaks the asymmetry observed between the two inflation coefficients in the optimal monetary policy reaction function. A reduction in β implies a higher utility associated to present consumption (and leisure) w.r.t. future one. Thus, the long-term effect of environmental degradation loses its relevance.

Lastly, we explore how the Taylor rule coefficients vary with the (inverse) elasticity of intertemporal substitution ϕ_c. What we observe is that both the coefficients associated to the inflation rates increase together with the value of the parameter ϕ_c, maintaining also a certain degree of asymmetry ($\phi_\pi^G > \phi_\pi^B$). A higher value of ϕ_c (lower intertemporal elasticity) means that consumption growth is not very sensitive to changes in the real interest rate set by the monetary authority. Hence the reaction of monetary policy to the inflation gap volatility is generally stronger, as it affects less the consumption habits (and so the welfare) of agents.

Table 2: Sensitivity analysis of optimal monetary policy coefficients, cost-push shock

Parameter	Value	ϕ_π^G	ϕ_π^B
ϵ	2.0	2.8	2.6
–	6.0	2.8	2.6
–	10.0	3.0	2.6
Δ	0.5	2.8	2.6
–	0.2	2.6	5.0
–	0.8	5.0	2.2
β	0.995	2.8	2.6
–	0.90	4.2	4.2
–	0.999	2.8	2.6
ϕ_c	1.0	2.8	2.6
–	2.0	3.6	3.4
–	3.0	4.4	4.0

Figure 8: Optimal monetary policy with high elasticity of substitution, cost-push shock

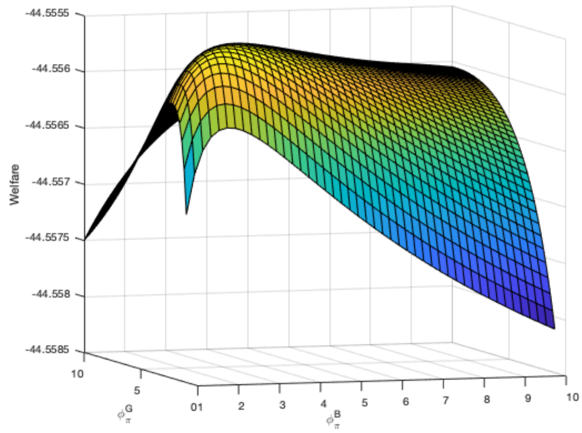
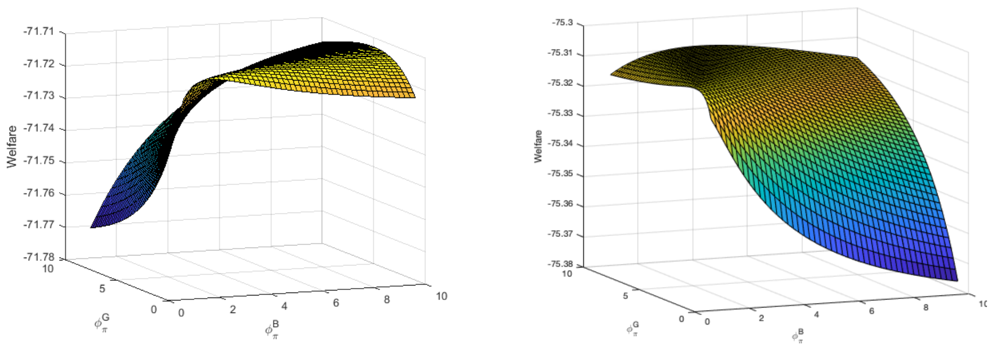


Figure 9: Optimal monetary policy with different intermediate inputs composition, cost-push shock

(a) $\Delta = 0.2$

(b) $\Delta = 0.8$



4 Optimal monetary rule and welfare loss with asymmetric shocks

What happens to the optimal Taylor rule coefficients when shocks hit asymmetrically the two intermediate sectors? In order to assess the existence of differences in the monetary policy reaction function between the two-sector DSGE and the E-DSGE, we assign the 2 shocks a parameter $\Omega \in [0, 1]$ that identifies their weight in the green and brown sectors: with a value equal to 0 the shock affects only the green sector, with a value of 1 only the brown sector. The results of this simulation are displayed in figure 10 as follows: we plot the difference between the policy coefficients $\phi_\pi^G - \phi_\pi^B$ against the weight parameter Ω. As expected, in the two-sector DSGE (solid blue line), the optimized parameters vary symmetrically with respect to the shock weight and are equal ($\phi_\pi^G - \phi_\pi^B = 0$) when the shock hits both sectors with the same intensity ($\Omega = 0.5$). This is valid both with a technology (panel a) and a cost-push shock (panel b). With an E-DSGE model, instead, the optimal response of monetary policy is asymmetric, and the degree of asymmetry varies with the environmental regime in place.

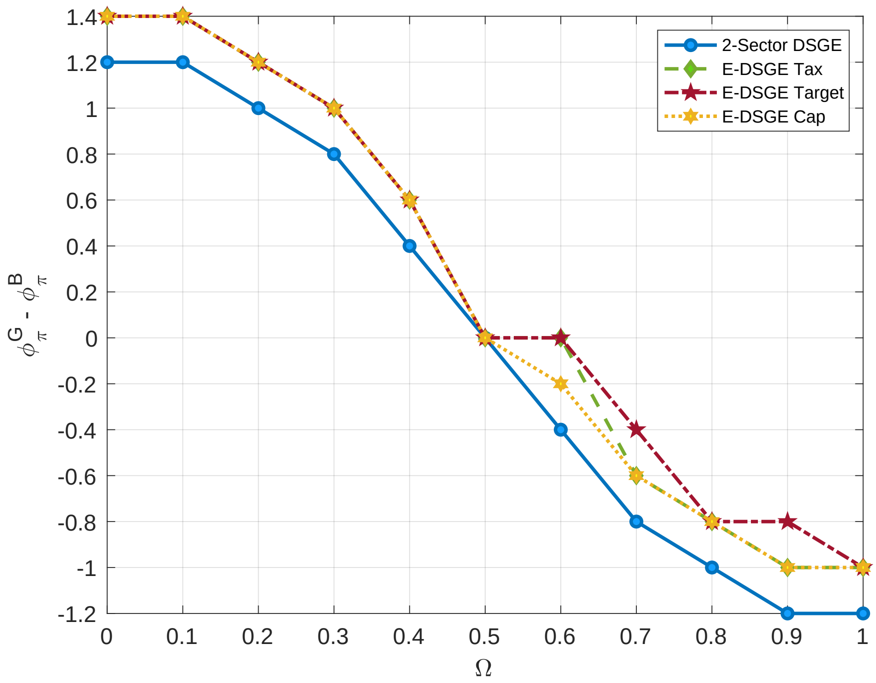
In the event of a cost-push shock, the strongest asymmetry can be detected for the Target policy case (dash-dotted red line). When a TFP shock occurs, all the three regimes show a similar behavior in magnitude. Only the case of the Cap policy with a cost-push shock represents an exception: the Taylor rule coefficients are perfectly symmetric with respect to Ω, although when the shock is polarized to one of the two sectors, the difference between the coefficients is more subtle than that obtained with the 2-sector DSGE model. The reason might be related to the fact that since such an environmental policy forces emissions' growth not to deviate from their steady state, the emission tax becomes sufficient as an instrument to offset the environmental friction, such that it does not require an additional "boost" from monetary policy.

In order to derive also a direct and meaningful interpretation of the results, we compute households' welfare variation expressed in percentage terms of steady state consumption variation, or consumption equivalent (CE). The analysis is conducted comparing the welfare gain/cost derived from the implementation of the optimized policy rule, in comparison to a predetermined baseline policy (see Appendix A. for a full derivation). Both in the two-sector DSGE and in the E-DSGE case, the baseline coefficients for inflation gap are set, in accordance with the literature, equal to $\phi_\pi^G = \phi_\pi^B = 1.5$ and $\phi_y = 0$. For the E-DSGE model, we also compute the optimal monetary policy for each environmental regime, so as to account for possible interaction between monetary and climate policies. The resulting optimized parameters and welfare cost are displayed in table 3. Here we focus on the extreme cases where the shock hits only one of the two intermediate sectors. The degree of asymmetry in the optimal policy coefficients is almost unaltered when switching from a Tax to a Target environmental regime. The only exception, as already explained before, is represented by the case of a Cap policy with cost-push shock: here the optimal Taylor rule coefficients are perfectly symmetric when the shock affects only one of the two sectors.

We can detect a common pattern in the two shock scenarios for the 3 environmental regimes: the reaction of the interest rate to prices variation is stronger in the case where a technology (cost-push) shock occurs in the brown (green) sector w.r.t. the green (brown) one. So a weaker (stronger) reaction to positive (negative) inflation in the brown sector -when asymmetric shocks hit the economy- is welfare optimal. In a broad sense, what emerges is that when the shock induces an increase in the production and demand for brown goods (e.g. TFP shock in the brown sector), the optimal reaction of monetary policy is to offset it by reducing households willingness of consumption with a higher interest rate. Conversely, a shock that favors the demand for green goods (e.g. a cost-push shock in the polluting industry) brings a weaker reaction of interest rate. Why is this the case? Because a relative increase (reduction) in the demand for green (brown) goods induces a reduction of polluting emissions and environmental damage; which increases the net income left at disposal for consumption and investment and, indirectly, contributes to the growth of households' welfare level.

Figure 10: Optimal monetary policy, shock decomposition, two-sector DSGE vs E-DSGE

(a) TFP Shock



(b) Cost-push Shock

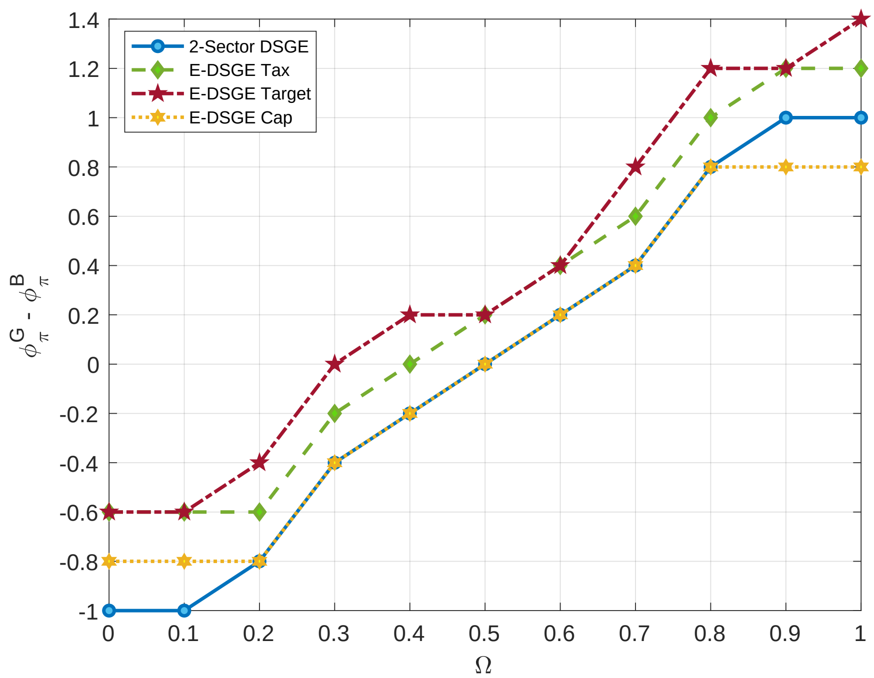


Table 3: Policy parameters and welfare loss, sector-specific shocks, two-sector DSGE vs E-DSGE

Model	Shock	ϕ_π^G	ϕ_π^B	Welfare cost (%)
Two-sector	G-TFP	5.0	3.8	-2.3751e-05
–	B-TFP	3.8	5.0	-2.3751e-05
–	G-Cost push	4.0	5.0	-1.6799e-06
–	B-Cost push	5.0	4.0	-1.6799e-06
E-DSGE Tax policy	G-TFP	5.0	3.6	-2.8752e-05
–	B-TFP	4.0	5.0	-2.1098e-06
–	G-Cost push	4.4	5.0	-1.7932e-06
–	B-Cost push	5.0	3.8	-2.2329e-06
E-DSGE Target policy	G-TFP	5.0	3.6	-2.8968e-05
–	B-TFP	4.0	5.0	-2.0887e-05
–	G-Cost push	4.4	5.0	-1.8043e-06
–	B-Cost push	4.8	3.4	-2.4248e-06
E-DSGE Cap policy	G-TFP	5.0	3.6	-2.7981e-05
–	B-TFP	4.0	5.0	-2.2308e-05
–	G-Cost push	4.2	5.0	-1.8674e-06
–	B-Cost push	5.0	4.2	-1.9359e-06

5 Conclusions

The relevance of climate change to economic stability has prompted all actors, including central banks, to review their policies aimed at growth and stabilizing the business cycle. In this paper we have investigated how environmental degradation and public policies aimed at counteracting this phenomenon influence the conduct of monetary policy. In particular, we have examined how the response of the interest rate (set by the central bank through a simple rule-based policy) to sectorial price variations changes. To do this, we have developed a two-sector New Keynesian DSGE model, to which we have added an environmental friction and a climate module. Differentiating sectors with respect to their production costs and introducing environmental elements into the model leads the interest rate to respond asymmetrically, depending on which sector is hit by and the kind of shock (technological or cost). The optimal Taylor rule parameters (in terms of welfare maximization), depend also on whether the inflation variation is associated to a shortage in production – because of an increase in production cost – or to an excess supply of a specific intermediate good -because of a more efficient production-. An important policy implications for central banks is that an asymmetric response to sectoral inflation can be welfare optimal, if this induces a relative lower level of polluting emissions. Such is the case for example with the economy hit by a cost-push shock: the shock hitting predominantly the green sector leads to an increase in the demand for the brown good (given they are substitute) and emissions; the optimal reaction of monetary policy is then to increase more than proportionally (with respect to the same policy when the shock affects the brown industry) the nominal interest rate, so as to impede an excessive growth of polluting production. In this way monetary policy not only targets inflation but also to some extent the emissions fluctuations.

The feasibility of such an asymmetric reaction by the central bank hinges on its ability to identify the cause associated with the price change and to distinguish its sector of origin. As stated in a recent speech by the economic historian Harold James, "not all price increases are the same, and some are desirable (...)." (Harold 2021); this somehow would corroborate the idea of a central bank supporting the ecological transition of the economy by accommodating (more) the variation of "certain" prices, while impeding that of "others". While it may not be easy to apply, such a non-standard monetary rule would provide additional support for at least stabilizing emissions within the business cycle. This meets the need, identified by the ECB, to include the impact of climate change in monetary policymaking and to revise such policies with the intent of contributing to the ecological transition.

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Appendix

A. Optimal monetary policy: consumption equivalent variation

Optimal monetary policy is computed maximizing households' utility function, and the welfare variation is measured in terms of consumption equivalent, as follows:

$$W_t^o = E_t \sum_{t=0}^{\infty} \beta^t U((1-\Upsilon)c_t, l_t) \tag{73}$$

$$W_t^o - W_t^b = E_t \sum_{t=0}^{\infty} \beta^t \left[U((1-\Upsilon)c_t, l_t) - U(c_t, l_t)\right]$$

$$W_t^o - W_t^b = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{((1-\Upsilon)c_t)^{1-\varphi_c} - 1}{1-\varphi_c} - \psi \frac{l_t^{1+\varphi_l}}{1+\varphi_l} - \frac{c_t^{1-\varphi_c} - 1}{1-\varphi_c} + \psi \frac{l_t^{1+\varphi_l}}{1+\varphi_l}\right]$$

$$\Upsilon = 1 - \left[\left(W_t^o - W_t^b\right)(1-\varphi_c)(1-\beta)(c)^{\varphi_c - 1} + 1\right]^{\frac{1}{1-\varphi_c}} \tag{74}$$

Here Υ stands for the welfare cost of implementing a specific policy rule -denoted as *optimal* (o)- vs the *baseline* (b) policy, in terms of CE.

B. E-DSGE steady state

$$a^{G,B} = \bar{a} \tag{75}$$

$$g = \bar{g} \tag{76}$$

$$\tau^E = \bar{g} - t \tag{77}$$

$$\pi = \bar{\pi} \tag{78}$$

$$r = \frac{\bar{\pi}}{\beta} \tag{79}$$

$$r^r = \frac{1}{\beta} \tag{80}$$

$$q = 1 \tag{81}$$

$$r^k = \frac{1}{\beta} - (1-\delta) \tag{82}$$

$$p^E = 1 \tag{83}$$

$$x = 880 \quad GtCO_2 \tag{84}$$

$$e = \frac{x(1-\eta)}{6} \tag{85}$$

$$\tau^E = \bar{\tau} \tag{86}$$

$$\mu = \left[\frac{\tau^E \gamma_1 \left(\Delta y^E \left(\frac{p^B}{p^E}\right)^{-\epsilon}\right)^{-\gamma_2}}{\theta_1 \theta_2}\right]^{\frac{1}{\theta_2 - 1}} \tag{87}$$

$$y^B = \left[\frac{e}{\gamma_1(1-\mu)}\right]^{\frac{1}{1-\gamma_2}} \tag{88}$$

$$D = \left[d_2(x)^2 + d_1(x) + d_0\right] \tag{89}$$

$$A^G = (1-D)a^G \tag{90}$$

$$A^B = (1-D)a^B \tag{91}$$

$$\Delta = \frac{y^B}{y^E}\left(\frac{p^B}{p^E}\right)^{\xi} \tag{92}$$

$$p^G = \left[\frac{(p^E)^{1-\xi} - \Delta(p^B)^{1-\xi}}{1-\Delta}\right]^{\frac{1}{1-\xi}} \tag{93}$$

$$y^G = y^E(1-\Delta)\left(\frac{p^{G}}{p^E}^{-\epsilon}\right) \tag{94}$$

$$mc^B = \frac{\xi - 1}{\xi} - \theta_1\mu^{\theta_2} - \gamma_1\tau^E\left[y^E\Delta\left(\frac{p^B}{p^E}\right)^{-\epsilon}\right]^{-\gamma_2}(1-\gamma_2)(1-\mu) \tag{95}$$

$$mc^G = \frac{\xi - 1}{\xi} \tag{96}$$

$$k^B = \frac{\alpha y^B}{r_k^B}mc^B \tag{97}$$

$$k^G = \frac{\alpha y^G}{r_k^G}mc^G \tag{98}$$

$$l^B = \left(\frac{y^B}{A^B(k^B)^{\alpha}}\right)^{\frac{1}{1-\alpha}} \tag{99}$$

$$l^G = \left(\frac{y^G}{A^G(k^G)^{\alpha}}\right)^{\frac{1}{1-\alpha}} \tag{100}$$

$$w^B = \frac{(1-\alpha)y^B}{l^B}mc^B \tag{101}$$

$$w^G = \frac{(1-\alpha)y^G}{l^G}mc^G \tag{102}$$

$$k = k^G + k^B \tag{103}$$

$$l = \left[(l^G)^{(1+\phi_h)} + (l^B)^{(1+\phi_h)}\right]^{\frac{1}{1+\phi_h}} \tag{104}$$

$$i = \delta k \tag{105}$$

$$z = y^B\theta_1\mu^{\theta_2} \tag{106}$$

$$c = y - i - \bar{g} - z \tag{107}$$

C. IRF to public expenditure and monetary shocks

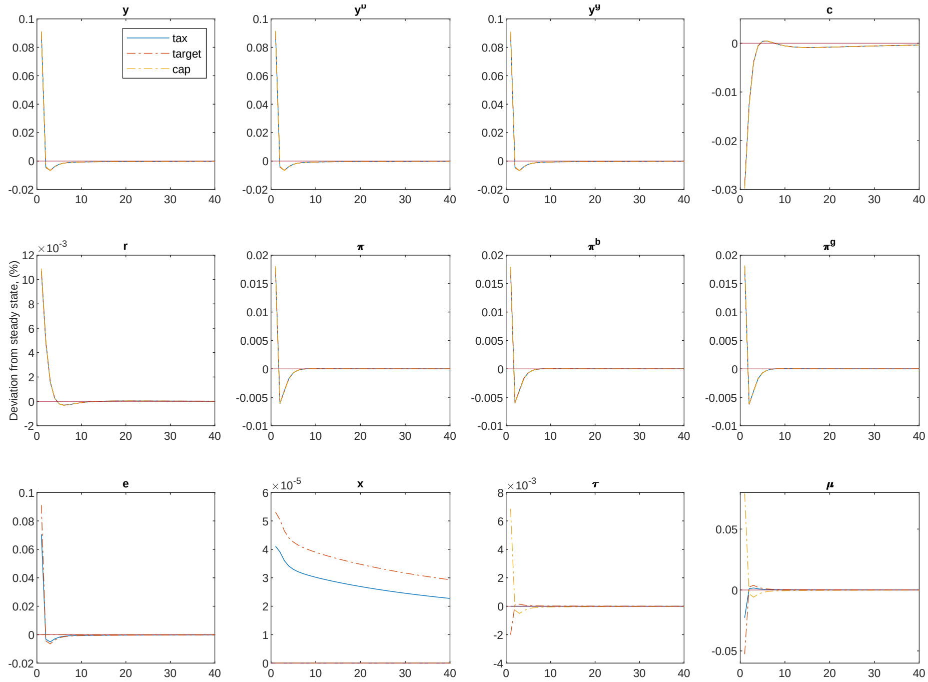

Figure 11: Impulse response function to public expenditure shock under different environmental policies.

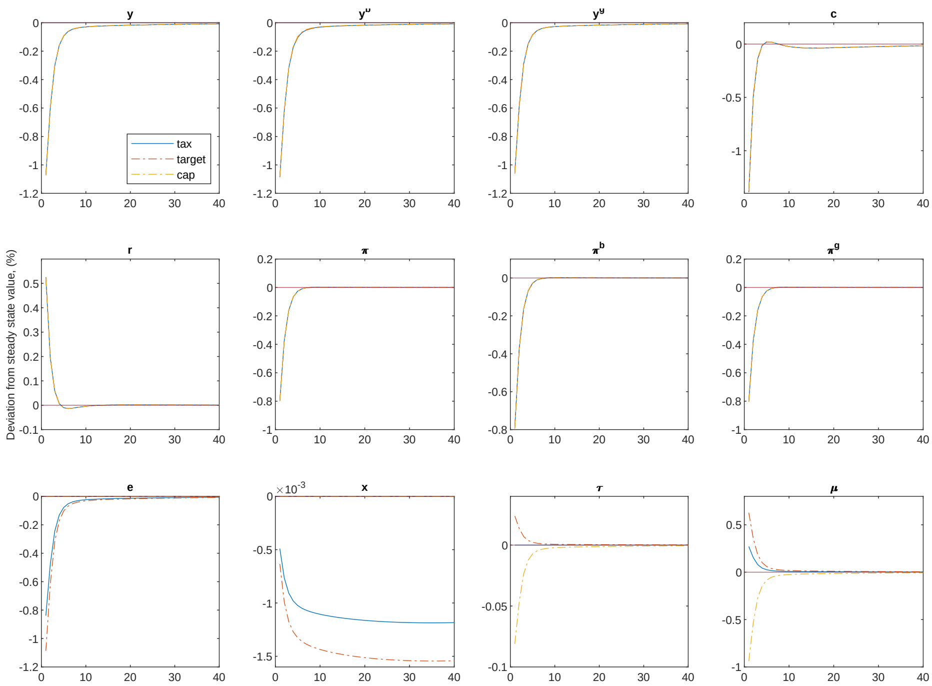

Figure 12: Impulse response function to monetary shock under different environmental policies.