Time–Frequency Analysis of Cryptocurrency Attention

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Abstract

We present a wavelet analysis of the daily returns of Bitcoin, Ethereum, and Litecoin at five selected crypto exchanges that identifies the fractal dynamics of the short- and long-term persistent processes. The investors' attention is proxied by the Search Volume Index provided by Google. We detect significant temporal cyclical movements and coherence between cryptocurrency returns and retail investor attention at long investment horizons: from the beginning of 2017 to the middle of 2018 and, to a lesser degree, in 2019. Investment horizons that dominated in 2017 and 2018 were mainly driven by retail investor attention rather than by uncertainty, risk, or stock markets. Therefore, we do not confirm that cryptocurrencies can be considered a safe-haven asset in times of crisis because there is no significant negative comovement between the returns of cryptocurrencies and stock returns or economic uncertainty, contrary to popular belief. Furthermore, the phase shift analysis indicates that attention can serve as a leading indicator for the cryptocurrency returns, particularly in 2017 and 2018. Therefore, retail investors are encouraged to use the Search Volume Index as an early warning indicator in case of sudden changes in the cryptocurrency returns to maximize their profits or minimize losses

Keywords**:** Cryptocurrencies, Attention, Wavelet Analysis.

JEL Classifications**:** G14, C14, C58.

1. Introduction

Cryptocurrencies have emerged as innovative and disruptive assets that have been used mainly for speculative purposes and, to a lesser degree, trading in specialized platforms, since the financial crisis. Due to the novelty of cryptocurrencies and investors' behavior, the cryptocurrency markets are not effective with the boom–bust cycle of cryptocurrency values characterized by the periods of exponential rise, as well as the sudden corrections (Garcia et al., 2014; Bouoiyour and Selmi, 2015) and by a high level of uncertainty and excessive short-term volatility which disappears in the long horizon (Corbet et al., 2018b). As such, cryptocurrencies cannot be treated as regular currencies. This fact has attracted significant attention in economic research, the financial world, as well as by the general public and the media.

Cryptocurrencies have also attracted a lot of new retail investors who do not trust traditional financial products and traditional brokerage companies and use the Internet to look for new investments opportunities, higher yields, lower transaction costs or greater autonomy which sometimes leads to overconfidence ((Barber and Odean, 2001). Recently, retail investors have used alternative information sources, e.g. social networks, to form more sophisticated and coordinated investment strategies. Moreover, retail investors often apply technical trading based on online data and producing cyclical behavior.

In our paper, we study the cyclical behavior of cryptocurrency returns and retail investor attention using a wavelet analysis. There is a growing body of literature that uses wavelet analysis to study the dynamics of cryptocurrency prices. This method is appropriate for economic developments which are subject to steady structural changes, such as those observed at crypto exchanges. Moreover, it allows the identification of specific investment time horizons denoted as market fractions (Peters, 1994). It allows us to identify different types of investors according these investment horizons. Kristoufek (2018) and Kristoufek and Vosvrda (2019) employ time–frequency analysis and measure long-range dependence, fractal dimension, and entropy. Celeste et al. (2019) also study the price volatility of cryptocurrencies in the context of fractal dynamics and come to the conclusion that Bitcoin prices exhibit long-term memory, with a weakening trend in 2016 and 2017, while Ethereum and Ripple perform contrastingly with a growing memory.

We present the wavelet analysis of cyclical behavior that identifies the fractal dynamics of the short- and long-term persistent processes. Fractal dynamics are here synonymous with the heterogeneity of the investment horizons identified by different frequencies (In et al., 2011, Chakrabarty et al., 2015). The cryptocurrency markets are characterized by extreme events when buying and selling orders are not efficiently cleared and investment horizons prevail.

These events were represented not only by the cryptocurrency price bubble in 2017 and the subsequent fall of cryptocurrency prices in the first half of 2018, but also by their revival in the middle of 2019 and the price correction at the beginning of 2020.

We make the following three main contributions to this growing stream of literature. First, we use a time-frequency approach and identify significant dynamic cyclical behavior and comovement at low frequencies (long investment horizons) from the beginning of 2017 to the middle of 2018 and, to a lesser degree, in 2019. Thus, we extend the results of Kristoufek (2013b) for Bitcoin and Celeste et al. (2019) for Bitcoin, Ethereum, and Ripple, who confirm the existence of fractal dynamics in cryptocurrency markets. We show that the previous results are also robust to the inclusion of attention factors of investors as proxied by the search frequency in Google (Da et al., 2011).

Second, we use unique daily data set on Google trends, which extends previous analysis based only on a weekly frequency (e.g., Kristoufek, 2015). Daily data enables us to perform a deep analysis of investment horizons in daily frequency. Moreover, daily data is more appropriate for the analysis of cryptocurrency returns due to low liquidity and high volatility of these markets (see Karaa et al., 2021). We prove a stable comovement of retail investor attention and cryptocurrency returns for cycles of lengths over 64 or 128 days (long investment horizon) of the three main cryptocurrencies (Bitcoin, Ethereum, and Litecoin) at six separate crypto-exchanges in the US as well as the UK. From this perspective, we show that investment horizons that dominated in 2017 and 2018 were driven mainly by retail investor attention and not by uncertainty, risk or stock markets. As such, cryptocurrencies do not serve as safe-haven assets. Moreover, we contribute to the empirical literature on cryptocurrencies and uncertainty comovement (e.g. Al-Yahyaee et al., 2019, Zhang et al., 2020).

Third, we analyze coherence and phase shift to explore the lags in comovements between the retail investor attention (proxied by the Search Volume Index, SVI) and cryptocurrency returns. We show that the SVI precedes the cryptocurrencies returns, particularly in 2017 and 2018. Therefore, the SVI is a leading indicator for returns, with a lag of up to 5 days in the cases of Bitcoin and Litecoin, or a few days longer in the case of Ethereum. Thus, the SVI may serve the investors as an early indicator of movement in cryptocurrency returns in future leading to lesser uncertainty and higher profits.

The paper is organized as follows. Section 2 contains the literature review. A detailed overview of data and methods is provided in Sections 3 and 4. Section 5 presents the results of the continuous wavelet transform (CWT). Analysis of the retail investor attention and cryptocurrency return coherence is provided in Section 6. Section 7 provides robustness analysis, and Section 8 concludes.

2. Literature Review

2.1 Cryptocurrencies and its Functions

Since their creation more than a decade ago, cryptocurrencies have been confronted with a high degree of skepticism. Bitcoin and a significant number of other cryptocurrencies are created through mining operations; most market participants purchase them and pay regular fiat currencies (Li and Wang, 2017). Following general discussion (Nakamoto, 2008), we use the term cryptocurrency, although the decentralized nature of cryptocurrencies and their high volatility have led to extensive discussion of whether they can be classified as currency in the sense of a transaction medium (Li and Wang, 2017). Their high volatility, in particular, makes cryptocurrencies less attractive for regular transactions than fiat money. Alvarez-Ramirez et al. (2018) document the risks for users engaging with cryptocurrencies as a transaction medium with such a significant price fluctuation. There is general agreement that cryptocurrencies hardly fulfill the traditional characteristics of the exchange tool we commonly refer to as currency (Bariviera et al., 2017). However, cryptocurrencies can serve as a new financial asset, and as such they can also be classified as a store of value.

In our paper, we also address the question whether cryptocurrencies represent new alternative (safe-haven) assets. While the Bitcoin is now commonly believed to be the 'New Gold', this view is largely rejected by academic research. Bouri et al. (2017a) find that Bitcoins act mainly as a diversifier but not as a hedge tool or a safe-haven asset. Furthermore, Bouri et al. (2017b) prove that Bitcoin serves as a hedge only in times of extreme (either high or low) uncertainty, however, it holds only at shorter investment horizons. Yermack (2015) argues that Bitcoin prices are disconnected from gold and the main international currencies. Similarly, Corbet et al. (2018a) show that Bitcoin, Ripple, and Litecoin are not related to other financial assets such as gold or stocks, although cryptocurrencies may serve mainly for diversification purposes for investors with short investment horizons. Pele et al. (2021) show that due to special statistical features of cryptocurrencies they can be classified as a separate kind of classical financial assets.

2.2 Investor Attention Measurement

In our paper, we follow the growing body of literature employing Search Volume Index provided by Google. As online sources provide a variety of data, it can be a really difficult task to gain appropriate data. As a result, economic agents often rely on search engines such as Google because it covers more than 70% of worldwide searches[1].

The way of using Google search data for economic series forecasting was initially paved by Choi and Varian (2009a; 2009b). Suhoy (2009) finds that search data can help identify inferences in economic growth before official data are released. Data on Google searches are often correlated with various economic indicators and help to make short-term predictions (Choi and Varian, 2012). Additional information contained in Google searches increase the forecasting performance of conventional models using a conventional set of predictors (Koop and Onorante, 2016). Various authors use Google search data for the nowcasting or forecasting of exchange rate behavior (Smith, 2012; Seabold and Coppola, 2015; Bulut, 2018), stock returns (Preis et al., 2010, 2013; Kristoufek, 2013a; Challet and Ahmed, 2015, Bijl et al., 2016), returns of precious metals (Salisu et al., 2020), unemployment rate (Askitas and Zimmermann, 2009; Pavlicek and Kristoufek, 2015; Chadwick and Şengül, 2015; Tuhkuri, 2016; D'Amuri and Marcucci, 2017), or private consumption (Vosen and Schmidt, 2011; Vosen and Schmidt, 2012; Carriere-Swallow and Labbe, 2013).

Retail investors prefer seeking data on the Internet when they plan to take their investment decisions (Barber and Odean, 2001; Salisu et al., 2020). Google Search Volume Index is often used as a measure of attention (or information demand) of economic agents in financial markets as originally proposed by Da et al. (2011) and confirmed by numerous authors (Vlastakis and Markellos, 2012; Goddard et al., 2015; Urquhart, 2018; Heyman et al., 2019; Hsieh et al.; 2020; Ramos et al., 2020).

Thus far, Google search data has rarely been applied to cryptocurrencies. Kristoufek (2015) study the relationship between the price of Bitcoin and search queries on Google and Wikipedia. They find a strong bidirectional correlation between these variables, i.e. the search queries have an impact on the prices of Bitcoin and the prices of Bitcoin have an impact on the search queries. His interpretation implies that the bidirectional relationship can easily produce frequent bubbles connected with the movement of the price of the Bitcoin. Urquhart (2018) shows that realized volatility and trading volume are significant drivers of investor attention measured by Google search data but investor attention does not predict the realized volatility, trading volume or returns. Aalborg et al. (2019) study the return, volatility and trading volume of Bitcoin and conclude that the trading volume of Bitcoin can be predicted using data from Google searches.

[1] Source: https://www.netmarketshare.com/search-engine-market-share.aspx?qprid=4&qpcustomd=0.

The previous literature, however, used generally available weekly data while we obtained Google search data on daily frequency.

3. Data

We use a unique daily data on retail investor attention, which is based on the Search Volume Indices (Google Searches), providing information about search intensity of selected phrases (volume index of internet search queries in range from 0–100).[2] We obtained daily data on google searches from the Google Trends API since 2004 as monthly subsamples. The data for different months are chained using weights by relative searches of specific month in order to compare relative popularity during the whole analyzed period.

We analyze fluctuations in the returns of the main three cryptocurrencies (Bitcoin, Ethereum, and Litecoin). We use data[3] from the five largest cryptocurrency exchanges: Bitfinex, Bitstamp, Bittrex, Coinbase, and Kraken, between October 6, 2013 (August 7, 2015, for Ethereum and October 24, 2013, for Litecoin) and March 31, 2020. Basic statistics of returns provides evidence of negative skewness of Bitcoin and Ethereum returns which denotes higher frequency of positive returns with the occurrence of greater than average losses. On the contrary, positive skewness of Litecoin returns shows very unlikely extreme downside (Table A1 in the Appendix).

The data for these cryptocurrencies have only been available since the end of 2013.[4] The market capitalization has increased from several thousand USD to the maximum value of up to several hundred million for some crypto exchanges (e.g. Bitcoin volume at Bitfinex, and significantly lower volumes for the other cryptocurrencies) in 2018, while it declined again more recently (see Table A2 in the Appendix). As of September 2019, the volume was between 6 million USD (Bitrex) and 100 million USD (Coinbase) for Bitcoin. Although other cryptocurrencies based on the blockchain aimed to improve the properties of Bitcoin, they have always remained in its shadow, and their values have represented only a fraction of that of Bitcoin: (USD 1 million, Bitrex) and 30 million (Bitfinex) for Etherum, and finally between USD 0.4–20 million USD (Bitrex) for Litecoin in September 2019.

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[2] The normalized search query index at a given point in time is a ratio of the total search volume for each query to the total number of all search queries. In the robustness analysis, we also use the ASVI (Da et al., 2011). Our keywords merge the name of a selected cryptocurrency with the name of a cryptocurrency exchange to proxy the demand for analyzed cryptocurrencies.

[3] Due to low liquidity and long transaction periods, daily data are likely to be more appropriate for the presented analysis than high-frequency data.

[4] The data are obtained from the free database http://www.CryptoDataDownload.com. Detailed data and figures on development of prices and volumes are available from the authors.

Our data set covers the period of unprecedented rise in the value of all the analyzed cryptocurrencies, as well as its subsequent correction and corresponding portfolio rebalancing. Interestingly, the markets recovered again during 2019 but experienced a new correction at the end of the sample. According to the available data, the value of Bitcoin started at slightly more than 100 USD (available only from Kraken) in 2013. The highest closing price of Bitcoin of nearly 20,000 USD per token was achieved on December 16, 2017. It is interesting to note that the crashes of 2018 and 2019 occurred simultaneously on all major crypto exchanges.[5] This fact confirms the synchronic behavior of cryptocurrencies (Pele et al., 2021). Moreover, we use the global stock market (S&P 500 index), and the Economic Policy Uncertainty Index (Baker et al., 2016).

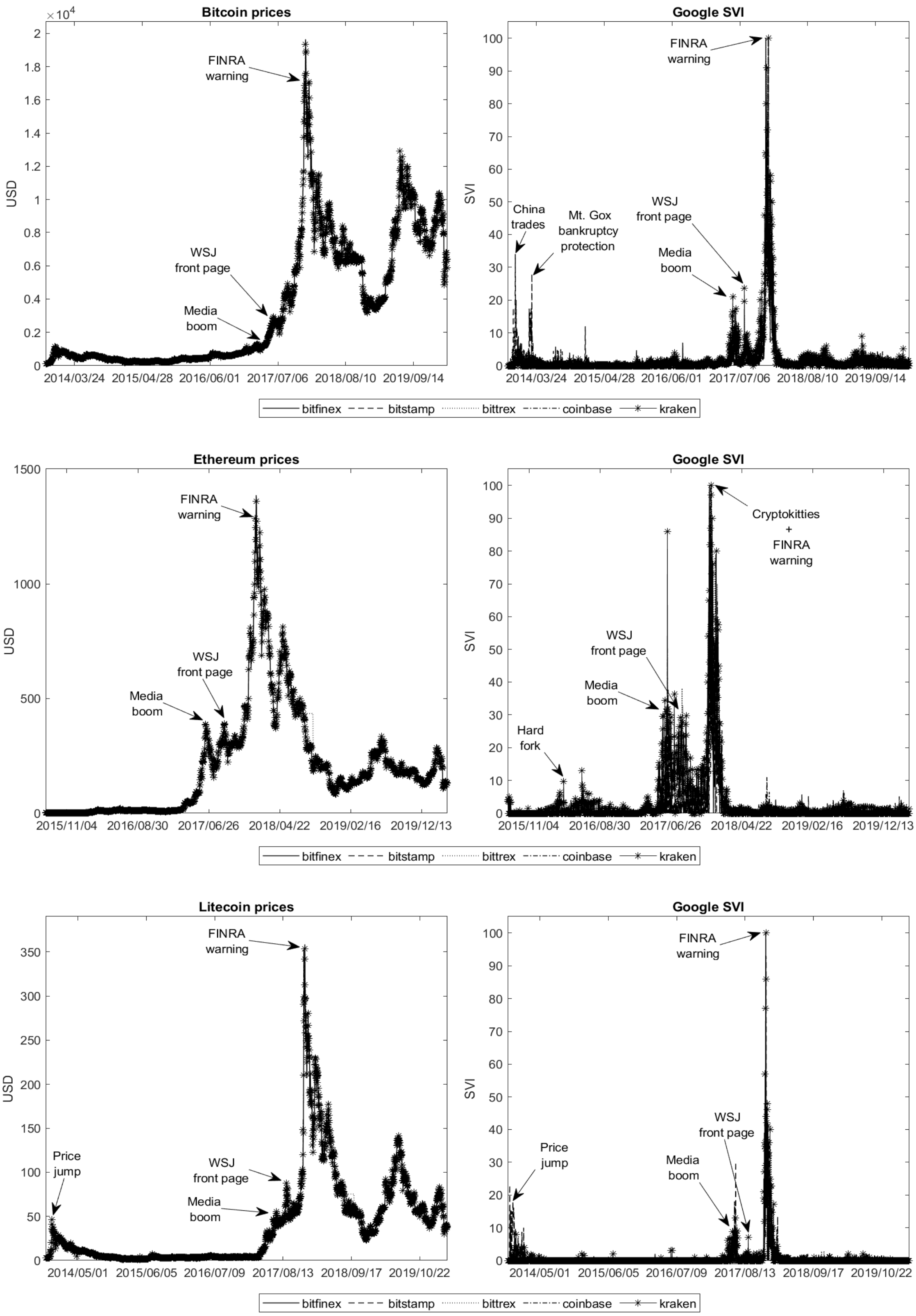
In Figure 1, we present time a domain representation of cryptocurrency prices and attention measured by SVI. With all three cryptocurrencies, there is a strong but highly volatile appreciation trend in 2017. Within this period, we can see a marked increase in the number of companies accepting Bitcoin payments and a jump in trading volumes, when mostly inexperienced investors expected immediate gains. Moreover, some countries (e.g. Russia and Japan) legalized cryptocurrency transactions, which also became popular in China. However, regulatory warnings concerning companies focusing on cryptocurrencies and blockchain technology were also common: Examples of these were from the Securities and Exchange Commission (SEC), in August 2017, and, in particular, from the Financial Industry Regulatory Authority (FINRA), in December 2017. Indeed, the data document extensive speculative price bubble that peaked in December 2017 and January 2018. After its burst, China announced that it would limit crypto mining in 2018, and South Korea banned anonymous cryptocurrency trading (22 January 2018). Some companies announced that they would stop accepting Bitcoin. Facebook banned users from advertising cryptocurrencies on 30 January 2018. Subsequently, the Securities and Exchange Commission issued another warning on 16 February 2018. In March 2018, both Google and Twitter prohibited online advertising of cryptocurrencies. Goldman Sachs started Bitcoin trading operations as the first Wall Street bank, on 2 May 2018. However, the US Department of Justice opened an investigation into Bitcoin traders in Britain because of possible criminal price manipulation in the digital currency markets; this was followed by the U.S. Commodity Futures Trading Commission’s warning in July 2018, concerning speculative purchases and a high risk of hacking, fraud and theft. Prices of

[5] Some smaller crypto-markets (e.g. QuadrigaCX and Coinfloor, which are not analyzed here) experienced the crash up to two days later in 2018.

cryptocurrencies fell until November or December 2018, and the next few months can be described as the Crypto winter, when the prices reached their lowest. The investors that left the market were mostly small and less experienced, while stable investors awaited new opportunities. In April 2019, an unknown buyer in Asia purchased a large number of Bitcoin, leading to positive sentiment and price rises in the markets.

The exchange rate of cryptocurrencies has probably been increasingly influenced by the COVID-19 outbreak since the beginning of 2020. The news at the beginning of March caused a sharp decline that was followed by a continuous price rise in the second half of March. The exchange rate cryptocurrencies possibly benefited from fears concerning the market of traditional financial assets.

Figure 1: Time Domain Representation of the Cryptocurrency Prices and Retail Investor attention



Note: FINRA (Financial Industry Regulatory Authority, WSJ (Wall Street Journal), SVI (Search Volume Index).

As far as the attention concerning Bitcoin is concerned, the first smaller peak was seen in November 2013 in Bitstamp, when the BTC China exchange overtook the Japan-based Mt. Gox and the Europe-based Bitstamp, and became the largest Bitcoin trading exchange. The second peak in February 2014 was documented when Mt. Gox suspended withdrawals; this was explained by technical reasons. By the end of the month, Mt. Gox had filed for bankruptcy protection in Japan, and this influenced other crypto exchanges. As far as Litecoin was concerned, there was a small jump in searches in November 2013; this was characterized as a period of intense Litecoin price rise. In June 2016, Ethereum's hard fork was completed, which attracted people's attention. However, much larger jumps of attention were documented in June 2017, after months of extensive media coverage (the news was published mainly in May and June 2017) focusing on the cryptocurrencies' success that ended on 7 June, 2017, when the Wall Street Journal presented news about the future of cryptocurrencies on its front page, prompting a marked jump in attention regarding all three cryptocurrencies. However, the absolute peak was reached in December 2017, just before the burst of the speculative price bubble in January 2018 and the subsequent price fall in 2018. Sometimes, attention may be caused by a reason other than investment, such as in the case of Ethereum at the beginning of December 2017, when an unexpected demand for this cryptocurrency was caused by collectors of the digital cartoons as part of the virtual game, CryptoKitties; this fever slowed down trade and delayed transactions in this market.

4. Methods

Our empirical strategy consists of three steps. First, we employ continuous wavelet transformation of cryptocurrency returns using the Morlet wavelet (function $\psi(t)$)[6] and report the fractal dynamics of cryptocurrency daily returns at different frequencies that emphasize events when investment horizons prevail. Let us consider wavelet domain as $\psi(.)$, to be defined as the inverse wavelet transform:

$$C_{\psi} = \int_{0}^{\infty} \frac{|\Psi(f)|^2}{f} df < \infty, \tag{1}$$

[6] The Morlet wavelet provides optimal trade-off between both time and frequency localization in the financial time series (Crowley, 2007, Rua, 2010). The oscillation is regulated by the parameter ω_0, leading to improved scale localization but decreased time localization, and vice-versa. For this analysis a $\omega_0 = 6$ is chosen, as it exhibits strong similarities to the Fourier period, leading to an improved interpretation of the result, in accordance with earlier wavelet studies conducted in the economic field (Rua, 2010).

where $\Psi(f)$ represents the Fourier transform of the wavelet $\psi(.)$, defined in this analysis as $\Psi(f) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi(t)e^{ift}dt$. Thus, for all finite functions this leads to zero mean wavelets, so $\psi(0) = 0$, formally written as:

$$\int_{-\infty}^{\infty} \psi(t)dt = 0. \tag{2}$$

Based on a given time series x(t), we define the continuous wavelet transform with the function ψ as:

$$W_{x;\psi}(p,q) = \int_{-\infty}^{\infty} x(t)\frac{1}{\sqrt{p}}\,\psi\left(\frac{t-q}{p}\right)dt. \tag{3}$$

Second, we identify significant leading indicators of cryptocurrency returns at different investment horizons and time periods. We employ wavelet coherence (WTC) to detect a comovement at different frequency scales, interpreted as correlation of cryptocurrency returns, attention, stock returns, and economic policy uncertainty. Technically, we identify common time-localized oscillation in the nonstationary time series:

$$R^2(p,q) = \frac{\left|S(p^{-1}W_{xy}(p,q)\right|^2}{S(p^2|W_x(p,q)|^2)S(p^2\left|W_y(p,q)\right|^2)}, \tag{4}$$

where the numerator represents the smoothed cross-wavelet spectra squared, and the denominator the power spectrum of both individual signals. The smoothing operator S is defined as $S(W) = S_{scale}(S_{time}\big(W_x(p)\big)$, with S_{scale} representing the smoothing in the wavelet scale and S_{time} (Torrence and Webster, 1999). The results of the coherency are normalized and can range from $0 \leq R^2(p,q) \leq 1$, where a small value suggests a weak correlation and values close to 1 indicate a strong correlation between the signals.

To test the significance of the wavelet coherence analysis, the simulated values are compared with Monte Carlo simulations. Throughout the paper, we use a five percent significance level to identify the significant areas in the presented figures. Additionally, we apply the definition of Baruník et al. (2013) to obtain details about the possible delays between the cycles of the two signals:

$$\phi_{xy}(p,q) = tan^{-1}\left(\frac{\Im\left\{S(p^{-1}W_{xy}(p,q))\right\}}{\Re\left\{S(p^{-1}W_{xy}(p,q))\right\}}\right) \tag{5}$$

with $\mathfrak{I}$ as the imaginary part of the complex number and $\mathfrak{R}$ representing the real part.[7] Phase shifts can vary between 0° (zero-phase) and 360°, where the signal is retaining the original wavelet. In our analysis, however, only shifts of up to 180°, with a change in polarity, are relevant. Arrows indicate the direction of the lag between the signals. Thus, the phase shift confirms the leading indicators at different investment horizons (frequency ranges) and time periods, when significant correlation is identified.

Third, we analyze the detailed lags between the time series at selected investment horizons, employing cross-correlation sequence estimates and the maximal overlap discrete wavelet transform (MODWT). Thus, this numerical analysis also shows short delays that are hard to detect visually because the change of a phase arrow is very small. In this step, we decompose all the analyzed time series employing a discrete wavelet transform (DWT). DWT is a discretized version of CWT, where the scale parameter j is discretized to integer powers of 2^j, $j=1, 2, 3, \ldots$, and the wavelet takes the form $\frac{1}{\sqrt{2^j}}\psi(\frac{1}{2^j}(n-2^j m))$, where n represents a signal length and m is a number of scales. The MODWT is a filtering operation that transforms the time series into coefficients using variation over the set of scales. We follow Gencay et al. (2002) and define the variance $\tilde{\sigma}_l^2(j)$ of the time series x at scale j as:

$$\tilde{\sigma}_x^2(j) = \frac{1}{\tilde{N}_J}\sum_{t=X_j-1}^{N-1} \tilde{d}_{j,t}^x, \tag{6}$$

where $\tilde{d}_{j,t}^x$ represents the variance at scale j and $\tilde{N}$ is the number of non-boundary coefficients. The MODWT correlation $\tilde{\rho}_{x,y}(j)$ of two time series x and y for scale j is obtained by wavelet covariance and the square root of wavelet variances:

$$\tilde{\rho}_{x,y}(j) = \frac{\tilde{\sigma}_{x,y}(j)}{\tilde{\sigma}_x\ (j)\tilde{\sigma}_y\ (j)}. \tag{7}$$

Lags for the cross-correlation sequence are maximized at 30 days, where attention, stock returns, and economic policy uncertainty are lagged with respect to the cryptocurrency returns.

5. Continuous Wavelet Transform and Market Dynamics

We analyze the individual exchange rates of cryptocurrencies using the CWT to identify optically cyclical persistence in specific periods and frequencies. This approach allows us both

[7] Note that the smoothing operator S is used again, but in this case to reduce possible noise in the data.

to identify periods with significant regularities and comovements with possible determinants of cryptocurrency prices. Cycle frequencies (expressed in days) could serve as an indicator of investment horizons (short- vs. long-term).

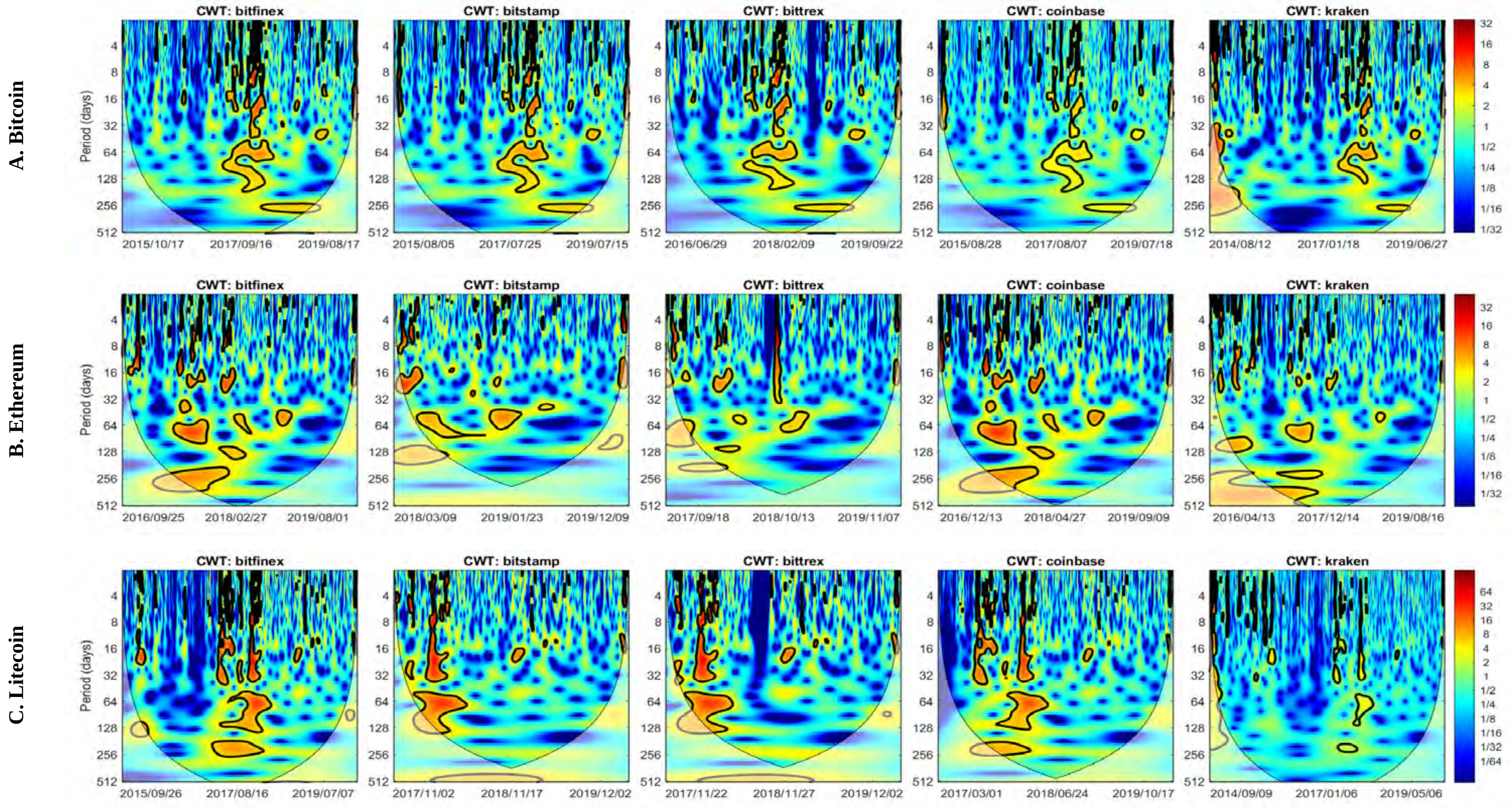
Figure 2 presents the returns of analyzed cryptocurrencies and crypto exchanges in the frequency domain. The areas surrounded by black lines display the significance of the cycles on a five percent level, as tested against red noise using Monte Carlo simulations. The coned line in the bottom half of the graphic displays the regions that are influenced by the edge effects. These effects occur when the wavelet is centered near the beginning or the end of the time series, which can potentially disturb the results for these periods. Data outside of this line cannot be statistically inferred in the analysis. The affected areas are defined as the cone of influence (Torrence and Compo, 1998). The color intensity represents the power spectrum of the results. This spectrum can range from dark blue for low-power areas up to bright yellow for high-power spectra.

In the case of Bitcoin (Figure 2, Block A), initially, there were nearly no or only very short and nearly randomly distributed areas of significant cycles for all the crypto exchanges. However, the picture completely changed from the beginning of 2017 to the middle of 2018, and again in 2019, when significant areas emerged, especially around the investment horizon of approximately 48–192 days. These periods were characterized by relatively high price movements (first the speculative bubble in 2017, its burst in January 2018, and its subsequent price fall; then, in 2019, the price's recovery once more). Furthermore, we could see increasing similarities between the individual crypto exchanges.

This pattern is slightly different from the cycles found for the other two alternative cryptocurrencies. Both Ethereum (Figure 2, block B) and Litecoin (Figure 2, block C) were characterized by occasionally significant cycles during nearly the whole of the analyzed period and for nearly all frequencies, i.e. the cycle persistence is documented here. In general, mainly very short cycles (below 32 days) were significant for these currencies. However, the periods with significant cycles at specific frequencies were relatively short, usually not much longer than one or two full cycles of these frequencies, which made it more difficult for investors to exploit them for profitable trading and created mostly selling signals, as a result of the increasing uncertainty. The importance of these regularities also declined towards the end of the sample as trading in these currencies also lost its importance. Furthermore, the significance of the results in the cases of all three cryptocurrencies (mainly Litecoin) and the Kraken crypto exchange is extremely low. Finally, we can see somewhat larger differences between the patterns found for different crypto exchanges.

To summarize, erratic movements along a trend initially characterized the analyzed crypto exchanges. As the markets have become more mature, the importance of the deterministic component has increased as well.

Figure 2: Continuous Wavelet Transform of Selected Cryptocurrency Returns and Crypto Exchanges



Note: Color scales represent wavelet power using the Morlet wavelet; the areas surrounded by black lines denote the results of the Monte Carlo significance test, and the light shading shows the region influenced by the edge effects. Source: own estimations.

6. Retail Investor attention and Cryptocurrency Return Coherence

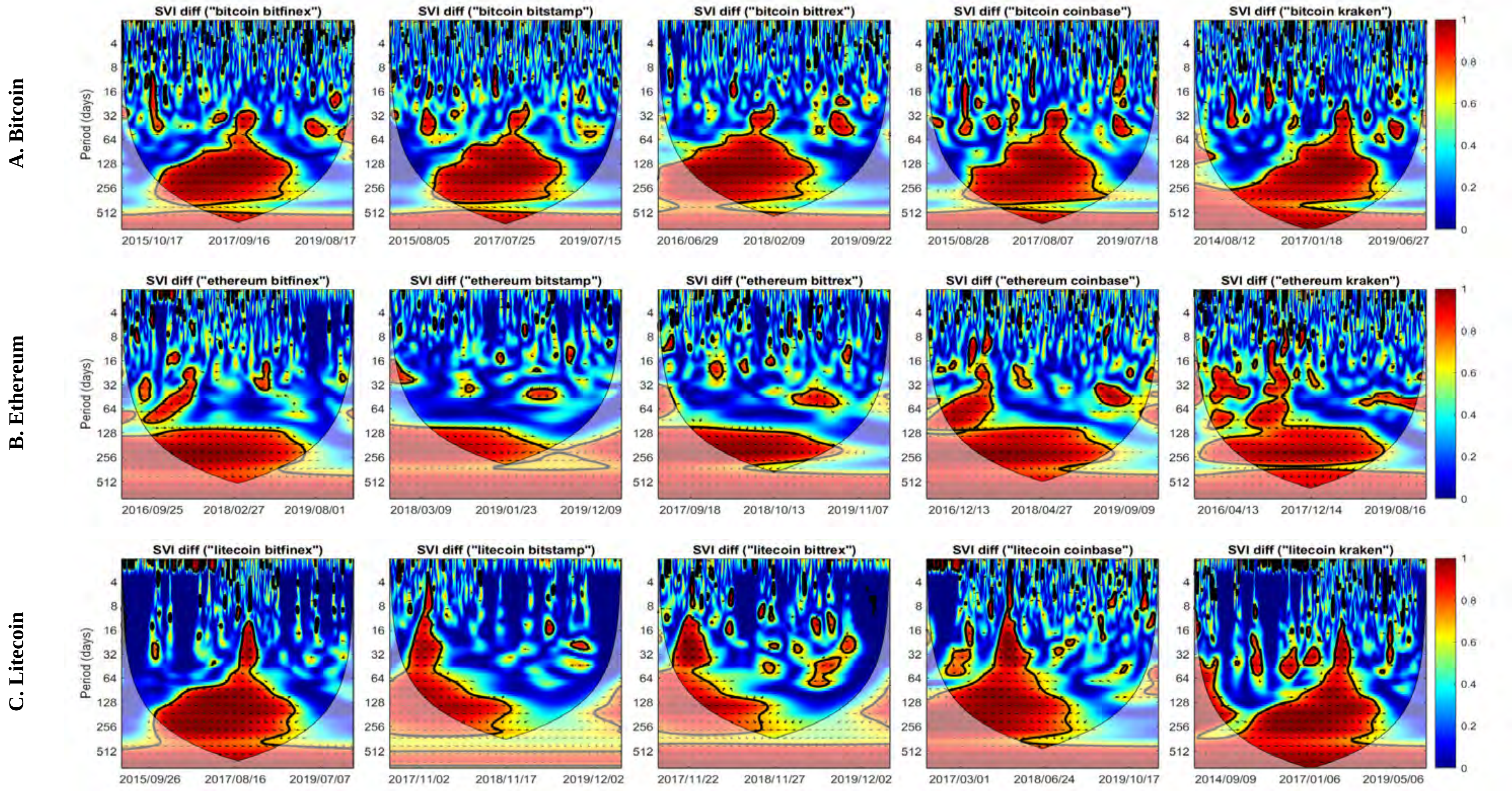
In the next step, we provide an analysis of comovements in the time–frequency domain represented by the wavelet coherence. The significant areas of comovements are identified at low-frequency scales represented by high coherence (the red areas bordered by a black line).[8] Figure 3 presents retail investor attention (represented by the SVI) comovement with cryptocurrency returns for the five analyzed crypto exchanges. Our results show significant and stable comovements between these two variables, particularly for cycles of lengths over 64 or 128 days, which mostly represent institutional investors or individuals with longer time horizons.

In the of case of Bitcoin, the significant comovements expanded to cycle lengths between 32 and 64 days in the second half of 2017 and first half of 2018, i.e. during and after the price bubble period, and at the beginning of 2020, when markets faced price and search drops (particularly in the case of Bitfinex and, to some extent, the Kraken exchanges). The coherence pattern appears to be largely similar for the remaining crypto exchanges. However, the significant area is slightly larger for Bittrex and somewhat smaller for the Kraken exchanges. Overall, the period of strong coherence in these medium-term cycles is somewhat shorter and seems to be strongly related to the price bubble period. The second block of Figure 3 illustrates the SVI comovement with Ethereum returns in the analyzed crypto exchanges. In this case, the results are relatively more stable when compared with the results for Bitcoin and are significant for the long cycles (above approximately 128 days) for the whole analyzed period. Finally, in the last block, we show the Litecoin returns coherence with the SVI. These results are partially different from the results presented for the previous cryptocurrencies. The areas of coherence are somewhat turbulent and differ for the individual crypto exchanges. Furthermore, the area of significant coherence for short and long cycles is associated with the price bubble at the end of 2017 and at the beginning of 2018, in the cases of all crypto exchanges.

We also verify the robustness of our results using Abnormal Search Volume Index (ASVI) (Da et al., 2011). Our results (Figure A1 in the Appendix) confirm the stable comovement at low frequencies (long investment horizons); however, in the case of Bitcoin, the comovement is more volatile at the beginning and at the end of the analyzed period (biased by edge effects).

[8] As before, the statistical significance at five percent against white noise is estimated using Monte Carlo simulations.

Figure 3: Wavelet Coherence of Retail Investor Attention and Cryptocurrency Returns



Note: The color scales represent wavelet coherencies; the black contours denote insignificance at five percent against red noise, and the light shading shows the regions probably influenced by the edge effects. The direction of the relationship (the leading indicator) is represented by arrows (a left arrow denotes antiphase (180°) while a right arrow denotes inphase (0° or 360°). A downward pointing arrow indicates SVI as a leading indicator of cryptocurrency returns. Source: own estimations.

Furthermore, we find that retail investor attention represented by the SVI can be considered to be a leading indicator for cryptocurrency returns. The results of phase shifts at different frequencies and periods are represented by phase arrows.[9] In the case of Bitcoin, phase arrows pointing right denote inphase comovement between the SVI and Bitcoin returns and do not imply significant lead or lag behavior of the cyclical movements but , rather, positive comovement for most of the analyzed period. However, our results confirm that the SVI indicator is a leading indicator for Ethereum and Litecoin returns. Phase arrows pointing down at 45° at a frequency of 128 days denote that Ethereum returns lag after the SVI indicators by 16 days. For Litecoin, we confirm the existence of a slightly shorter lag. At the frequency between 16–32 days, the lag between the SVI indicator and Litecoin returns is from 2–4 days at most of the crypto exchanges (denoted by a 45° phase arrow) at the beginning of 2018. This could be viewed as a signal of short-term activity by inexperienced investors during the cryptocurrency price bubble and holds for all the analyzed cryptocurrencies in most crypto exchanges.

Finally, we find robust evidence of a stable phase shift between the SVI indicator and the cryptocurrencies returns at long investment horizons (long cycles at frequencies below 256 days) that points to a lag exceeding 14 days (phase arrows between 10° and 45°). However, these results can be partly affected by edge effects and should be interpreted with a high degree of caution.

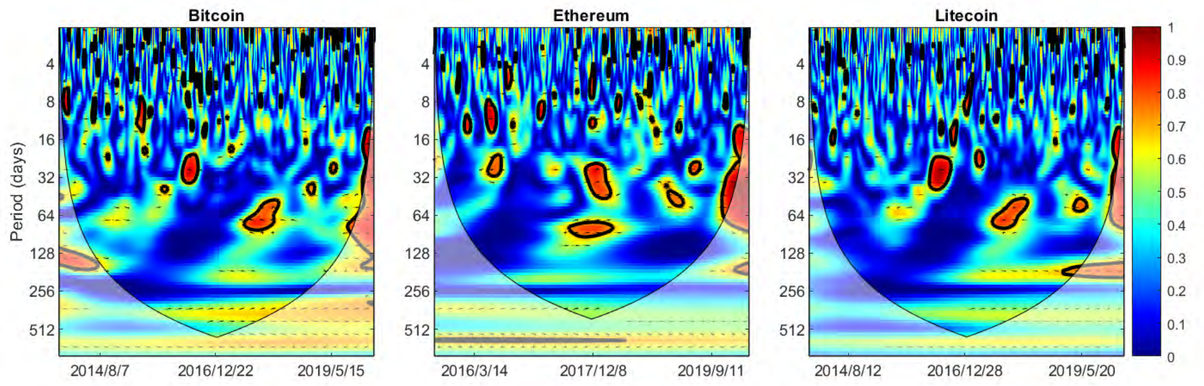
7. Robustness Analysis

We examine the sensitivity of our analysis in three ways. First, we check whether cryptocurrency returns are positively or negatively correlated with the main stock markets. Second, we check whether the cryptocurrencies can be considered as a safe-haven asset, which is demanded in periods of high uncertainty. Our results do not show any important relationship to traditional or alternative market factors as we do not find any robust evidence of such comovement. The results in Figures 4 and 5 do not show any significant coherence for cryptocurrencies returns and main stock market indices, as well as the economic policy

[9] The direction of the leading or lagging time series is represented by arrows (a left arrow denotes antiphase (180°), while a right arrow denotes inphase (0° or 360°). However, the interpreting of the phase as a lead or a lag has to be done relative to the antiphase, because a lead of 90° is also a lag of 270°.

uncertainty index.[10] The only exception is the first quarter of 2020, when we detect coherence for stock and the returns of selected cryptocurrencies, signaling a joint drop in returns of both variables and coherence for uncertainty and the returns of selected cryptocurrencies showing negative correlation. This confirms that selected cryptocurrencies do not serve as a safe-haven asset as their returns fall together with falling stock returns and rising levels of economic policy uncertainty.

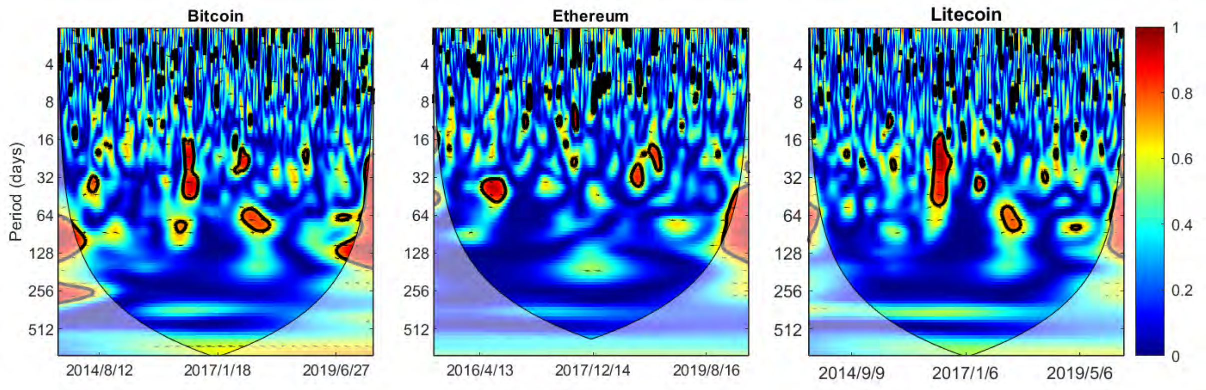
Figure 4: Wavelet Coherence of Stock and Cryptocurrency Returns (Kraken)



Note: The color scales represent wavelet coherencies; the black contours denote insignificance at five percent against red noise, and the cone lines and the light shading show regions influenced by edge effects. The direction of the relationship (leading indicator) is represented by arrows (a left arrow denotes antiphase (180°), while a right arrow denotes inphase (0° or 360°). Downward pointing arrow indicates stock returns as a leading indicator of cryptocurrency returns. Source: own estimations.

[10] We present only results for one selected crypto exchange (Kraken) in Figures 4–6: these are representative for all analyzed crypto exchanges. The additional results are available in Tables A2 and A3 in the Appendix.

Figure 5: Wavelet Coherence of Economic Policy Uncertainty and Cryptocurrency Returns (Kraken)



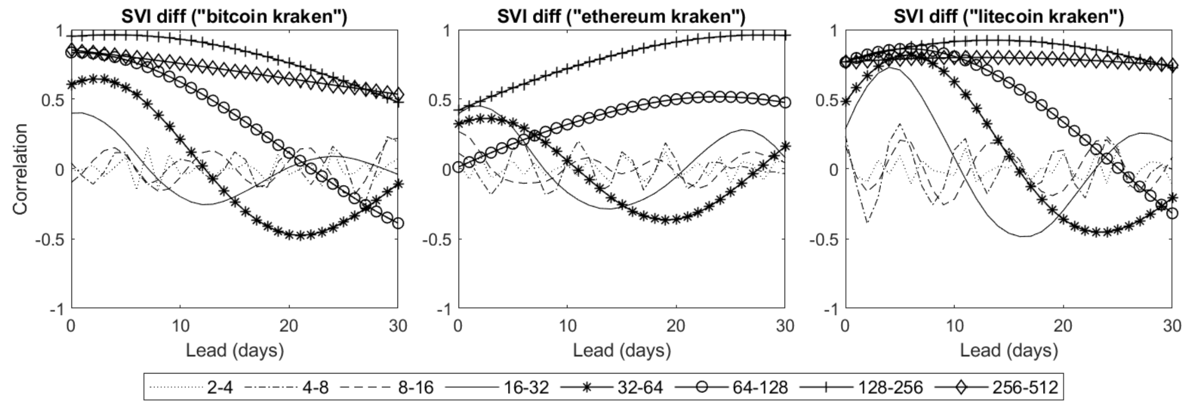
Note: The color scales represent wavelet coherencies; the black contours denote insignificance at five percent against red noise, and the cone lines and light shading show regions influenced by edge effects. The direction of the relationship (leading indicator) is represented by arrows (a left arrow denotes antiphase (180°), while a right arrow denotes inphase (0° or 360°). Downward pointing arrow indicates Economic Policy Uncertainty as a leading indicator of cryptocurrency returns. Source: own estimations.

Another limitation of the interpretation of the phase arrows in the previous section is that the visual analysis can barely detect negligible changes in the slope of the phase arrows, which, however, may be highly important for a proper identification, particularly at low frequencies. It is especially important for the Bitcoin, where only a negligible downward change of a phase arrow implies a lag between 2–8 days. These shortcomings are solved in the following analysis. In particular, we use multiscale correlation employing the MODWT and show that cryptocurrency returns and the SVI change at different frequencies (see Table A3 in the Appendix). Results presented in Table A3, block A for Bitcoin, confirm our results discussed in the previous section, i.e. there is a significant comovement at longer frequencies of between 32–256 days. Table A3, block B, presenting correlations of the SVI and price returns of Ethereum, gives ambiguous results. We only identify the existence of correlation at both short and long frequencies in the cases of Coinbase and Kraken exchanges, as these results do not contain any lags, significantly identified in Figure 3, block B. Finally, we present multiscale correlation of the SVI and Litecoin returns in Table A3, block C, which confirms comovements for all frequencies and crypto exchanges.

Due to the limitations of the standard visual analysis, the next important step of the robustness analysis is the numerical identification of cross-correlation sequences as a leading indicator of cryptocurrency returns (Figure 6). We examine the cross-correlation sequence corresponding to all the analyzed frequency scales and lags from 0–30 days. Cross-correlation sequences are insignificant for short investment horizons (frequency scales shorter than 32 days) that do not imply any significant lead between the cyclical movement of the analyzed time series.

In Figure 6, we can see an increasing comovement of the SVI and cryptocurrency returns with a lag of up to 30 days (see also Figure A4, block A). We can see that the SVI is a leading indicator, with a lead of up to 10 days at frequencies of 32–256 days, in the cases of almost all the crypto exchanges. These results confirm our main findings, i.e. insignificant comovement of long cycles. The results in Figure 6 (see also Figure A4, block B) for Ethereum currency show a relatively long lag of up to 30 days at frequencies of 64–256 days, and in some cases of 32–64 days. We must be cautious when interpreting these results because of the relatively short length of the analyzed time series. Finally, we present results for Litecoin currency in Figure 6 (see also Figure A4, block C). In this case, the lag between the changes of the SVI and the changes of Litecoin returns is from 5–10 days for all frequencies from 32–256 days: this corresponds largely to the results presented in Figure 3, where the phase shift points to the existence of a delay corresponding to a lead of 10 days. In contrast to Dyhrberg (2016), our findings imply that that cryptocurrency returns are weakly or not correlated with stock markets and economic policy uncertainty. Thus, cryptocurrencies do not serve as a safe-haven asset in times of crisis. These conclusions are also confirmed by Bouri et al. (2017a), Corbet et al. (2018a), Shahzad et al. (2019), and Yermack (2015).

Figure 6: Wavelet Cross-Correlation of Retail Investor Attention and Cryptocurrency Returns (Kraken)



Note: The wavelet cross-correlation sequence shows changes in correlation using different lags in days (x-axis) at different frequency scales (different line styles). Increasing correlation denotes the existence of lag. Source: own estimations.

8. Conclusions

Cryptocurrencies are still novel in the history of financial markets. In spite of this, they have already attracted strong public attention. Numerous programmers and investors have created a unique laboratory with parallel universes generating plentiful data that can be used for economic analysis of investor behavior. This special framework can be used to discuss, e.g. the importance of retail investor attention, economic uncertainty, or general capital market developments.

In our paper, we extend the existing research by a deep wavelet analysis that explores the dynamics of attention and cryptocurrencies' returns. We also employ continuous wavelet transformation of attention data and cryptocurrency returns and report fractal dynamics of cryptocurrency daily returns at different frequencies, which emphasizes periods when investment horizons prevail. Moreover, we use the wavelet coherence to identify comovements at different frequencies that are interpreted as the correlation of cryptocurrency returns, attention, stock returns and economic policy uncertainty. Then, we analyze lags between our time series at selected investment horizons, employing cross-correlation sequence estimates. Finally, yet importantly, we use a longer period of daily data than in earlier studies (Peters, 1994), including a highly dynamic period between October 2013 and September 2019 and the data from the first quarter of 2020. We find what investment horizons are prevailing during the times of crises.

Our findings show that prices of the analyzed cryptocurrencies are not directly interconnected with stock prices or macroeconomic conditions and that cryptocurrency markets are not effective as they are influenced mainly by retail investor attention. The only period characterized by the existence of coherence is the first quarter of 2020, when we find coherence of both stock returns and the Economic Policy Uncertainty Index and the returns of cryptocurrencies. However, as the returns of cryptocurrencies fall together with falling stock returns and a rising level of economic policy uncertainty, our findings imply, similarly to Bouri et al. (2017a), Yermack (2015), and Corbet et al. (2018a), that cryptocurrencies cannot be considered to be alternative assets (e.g. stocks).

Our research makes three main contributions to the existing discussion on cryptocurrencies. First, using a wavelet coherence approach, our results reveal significant cyclical behavior of cryptocurrencies prices in the long investment horizon, after the crash of the cryptocurrencies at the end of 2017, although this cyclicality largely disappeared again in 2019. We prove the importance of the fractal dynamics that represent investment horizons in cryptocurrency prices.

Second, we detect a stable comovement of retail investor attention and returns of the analyzed cryptocurrencies, mainly for long cycles (with a length of greater than 64, and in some cases greater than 128 days). As such, retail investor attention, proxied by the Google search frequency, (Da et al., 2011) comoves with the prices of cryptocurrencies, as was particularly seen during the turbulent times in 2007 and 2008.

Last, we investigate the coherence and its phase shift at different investment horizons and time periods to examine the possible lags in comovements between the selected cryptocurrencies returns and retail investor attention and show that the Search Volume Index (provided by Google) is a leading indicator for the cryptocurrency returns particularly in 2017 and 2018. The length of the identified lag is up to 5 days in the cases of Bitcoin and Litecoin, or a few days longer in the case of Ethereum. In the light of these results, investors are encouraged to use the Search Volume Index as an indicator of early warning in case of possible sudden changes of cryptocurrency returns as it can help them to maximize their profits or minimize losses.

Appendix

Table A1: Cryptocurrency Returns

Variables	Mean	Std. Dev.	Quantiles					Skewness	Kurtosis
			Min	0.25	Mdn	0.75	Max		
Bitcoin returns									
- Bitfinex	3.32	297.73	-3096.50	-21.36	1.17	36.26	3090.00	-0.45	29.04
- Bitstamp	3.12	284.97	-3092.90	-18.72	1.07	31.94	2976.49	-0.22	27.59
- Bittrex	3.82	319.04	-3055.49	-37.75	0.00	53.65	2904.00	-0.42	23.35
- Coinbase	3.10	288.26	-3080.95	-19.30	0.81	31.68	3300.01	0.15	29.75
- Kraken	2.66	254.46	-3093.30	-15.88	0.97	22.32	2161.30	-0.48	29.85
Ethereum returns									
- Bitfinex	0.08	21.39	-257.30	-2.78	0.00	3.40	154.80	-1.04	30.12
- Bitstamp	-0.22	26.01	-218.83	-5.55	-0.06	6.08	157.47	-0.42	16.89
- Bittrex	0.06	25.81	-250.71	-4.97	0.00	6.40	156.85	-1.44	25.48
- Coinbase	0.09	21.31	-198.63	-3.22	0.00	3.61	168.82	-0.17	22.36
- Kraken	0.08	19.08	-209.88	-1.96	0.00	2.14	165.82	-0.39	28.18
Litecoin returns									
- Bitfinex	0.02	5.41	-58.26	-0.54	0.00	0.40	81.93	2.42	64.41
- Bitstamp	0.00	7.39	-43.39	-1.91	-0.18	1.86	99.35	3.19	48.40
- Bittrex	0.00	7.28	-49.64	-1.77	0.00	1.61	82.25	2.06	35.04
- Coinbase	0.03	7.07	-45.20	-1.40	-0.02	1.28	133.48	5.68	113.14
- Kraken	0.02	4.78	-43.64	-0.47	0.00	0.33	79.85	3.46	73.97

Table A2: Volumes of Selected Markets

		2014	2015	2016	2017	2018	2019	2020
Bitcoin	Bitfinex		7 656	9 286	158 504	269 184	80 442	84 148
	Bitstamp	2 844	4 032	2 980	59 307	84 110	66 135	69 422
	Bittrex			1	25 393	18 457	6 145	7 133
	Coinbase		2 340	3 357	83 262	98 755	105 106	124 537
	Kraken	8	23	763	19 666	45 235	49 565	57 274
Ethereum	Bitfinex			777	47 091	109 453	25 898	23 939
	Bitstamp				27 376	17 798	8 219	8 872
	Bittrex				6 602	4 092	801	1 188
	Coinbase			465	46 997	65 452	22 482	35 255
	Kraken		2	406	13 226	26 304	14 002	16 986
Litecoin	Bitfinex		9 310	94	20 831	21 844	9 495	4 364
	Bitstamp				6 858	4 772	3 707	2 834
	Bittrex				4 565	1 356	301	187
	Coinbase			4	38 928	34 537	16 633	13 658
	Kraken	1	1	4	2 058	2 244	2 130	1 870

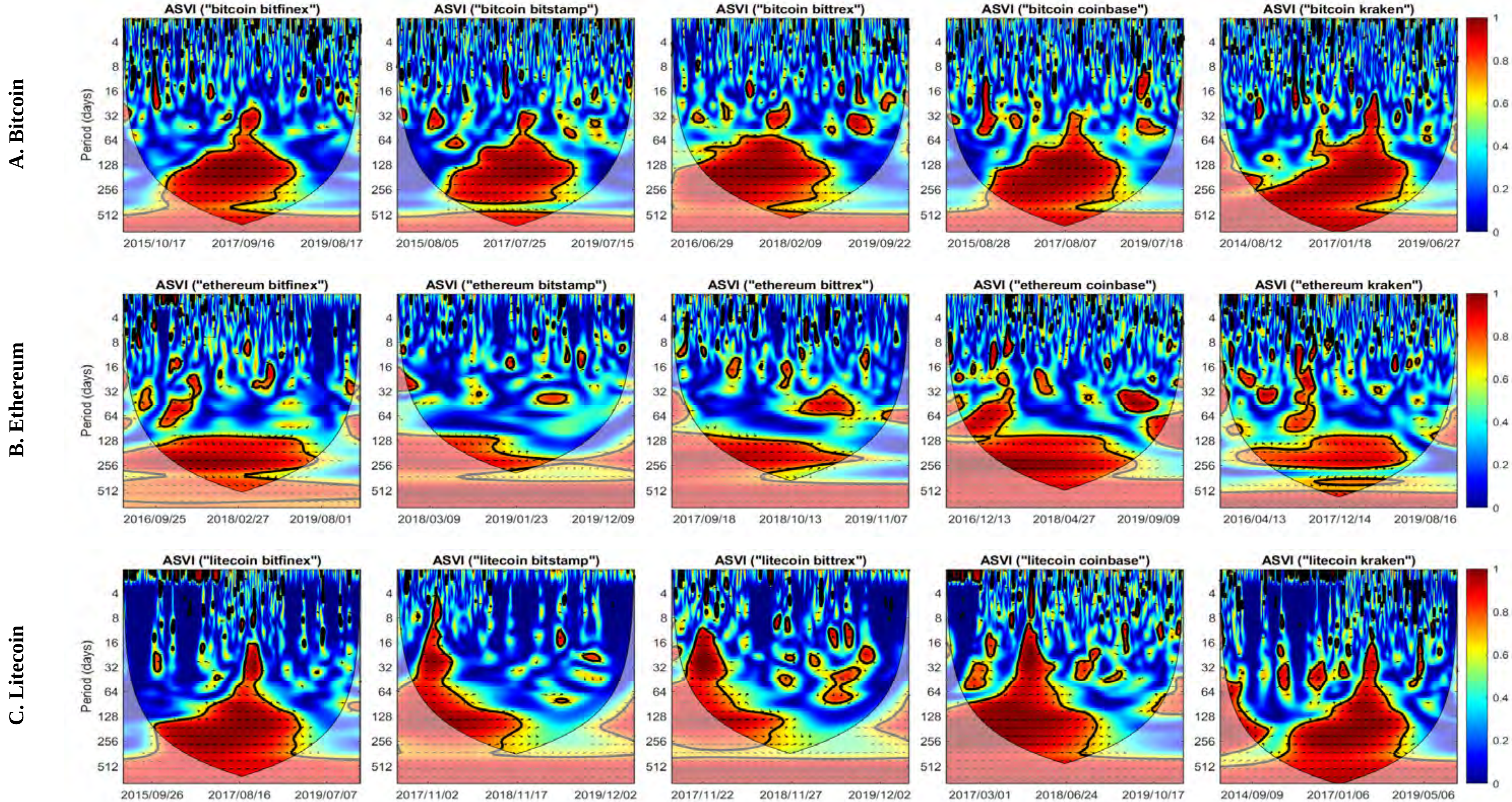
Note: Daily averages in thousand USD, datasource: www.CryptoDataDownload.com

Table A3: Multiscale Comovement Employing the MODWT

		Frequency scales (days)							
	Markets	2-4	4-8	8-16	16-32	32-64	64-128	128-256	All
Bitcoin	Bitfinex	-0.0742*	-0.1359**	-0.0209	0.0898	0.5412***	0.7158***	0.9459***	-0.0487**
	Bitstamp	-0.0951**	-0.0593	-0.1361	0.3727***	0.5397***	0.8191***	0.9376***	-0.0593***
	Bittrex	-0.0269	0.1590***	0.0593	0.3201***	0.5709***	0.8592***	0.9200***	0.0610**
	Coinbase	0.0339	0.2703***	-0.0059	0.2832***	0.6800***	0.8368***	0.9567***	0.1059***
	Kraken	-0.0126	0.0443	-0.0972	0.4004***	0.6079***	0.8359***	0.9508***	0.0189
Ethereum	Bitfinex	0.2377***	0.2026***	0.0142	-0.1412	0.1927	-0.1881	0.3148	0.1790***
	Bitstamp	0.1875***	0.1333	0.0273	-0.0199	0.0211	NA	NA	0.1519***
	Bittrex	0.1180*	0.1406	-0.1287	0.3887**	0.0878	-0.204	NA	0.1074***
	Coinbase	0.0483	0.2827***	0.1375	0.3490***	0.3880*	0.1794	NA	0.1447***
	Kraken	0.0431	0.1145*	0.2696***	0.3965***	0.3239*	0.014	NA	0.0820***
Litecoin	Bitfinex	0.0462	0.3704***	0.3289***	0.0211	0.1754	0.4274	0.576	0.1246***
	Bitstamp	0.1010*	0.4483***	0.4047***	0.4621***	0.6132***	0.5005	NA	0.2560***
	Bittrex	0.0739	0.4417***	0.4081***	0.5207***	0.5370**	0.3666	NA	0.1901***
	Coinbase	0.4406***	0.6470***	0.4467***	0.4321***	0.4988***	0.7845***	NA	0.4884***
	Kraken	0.1624***	0.2370***	0.1681**	0.2880***	0.4816***	0.7660***	0.7696***	0.1961***

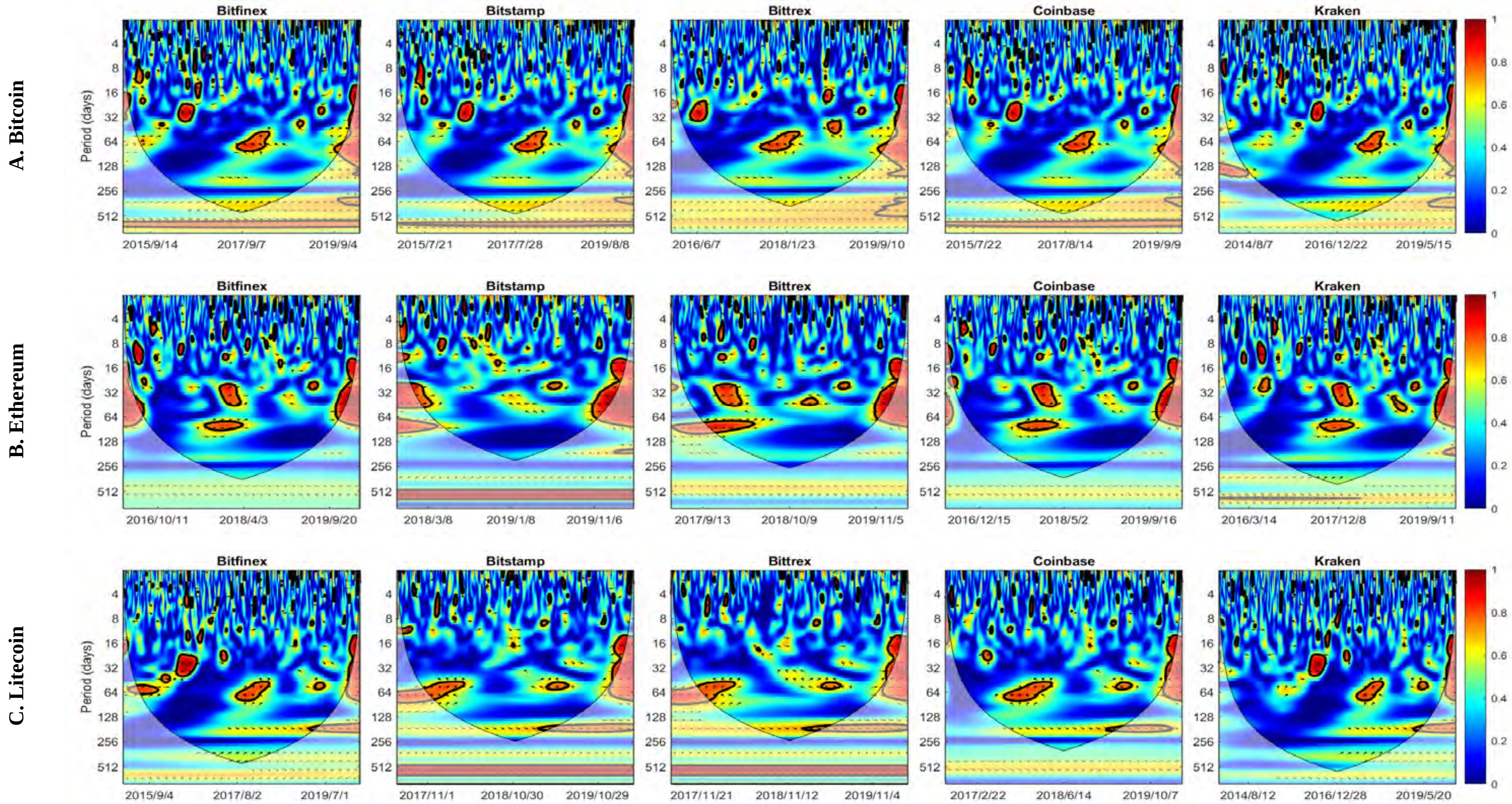
Note: Correlation using MODWT at different frequency scales (in days). The last column (All) reports correlation at all frequency scales (identical to the time series correlation). Source: own estimations.

Figure A1: Wavelet Coherence of the Abnormal Search Volume Index and Cryptocurrency Returns



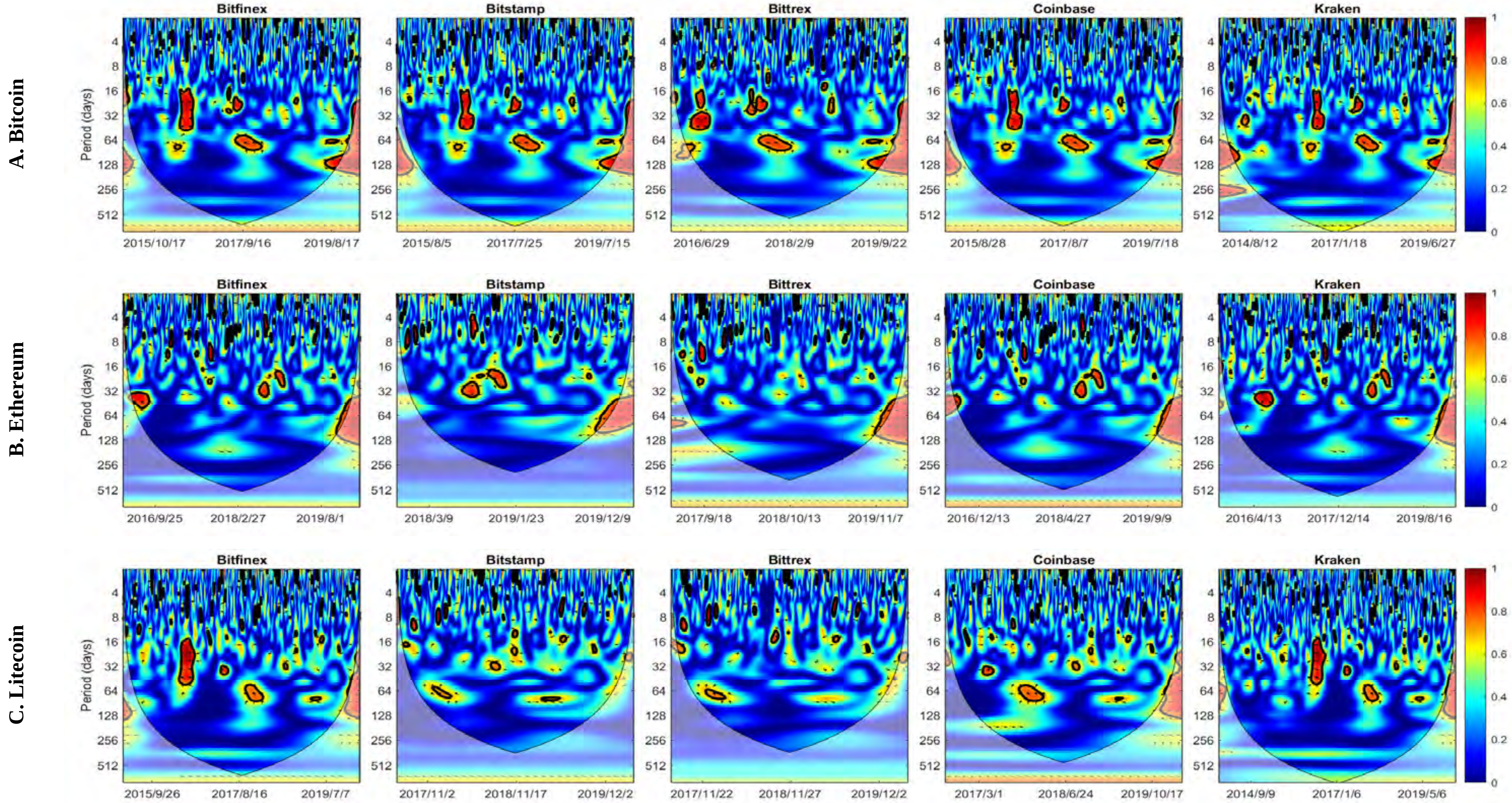
Note: The color scales represent wavelet coherencies; the black contours denote insignificance at five percent against red noise, and the light shading shows regions probably influenced by edge effects. The direction of the relationship (leading indicator) is represented by arrows (a left arrow denotes antiphase (180°), while a right arrow denotes inphase (0° or 360°). Downward pointing arrow indicates ASVI as a leading indicator of Cryptocurrency Returns. Source: own estimations.

Figure A2: Wavelet Coherence of S&P 500 and Cryptocurrency Returns



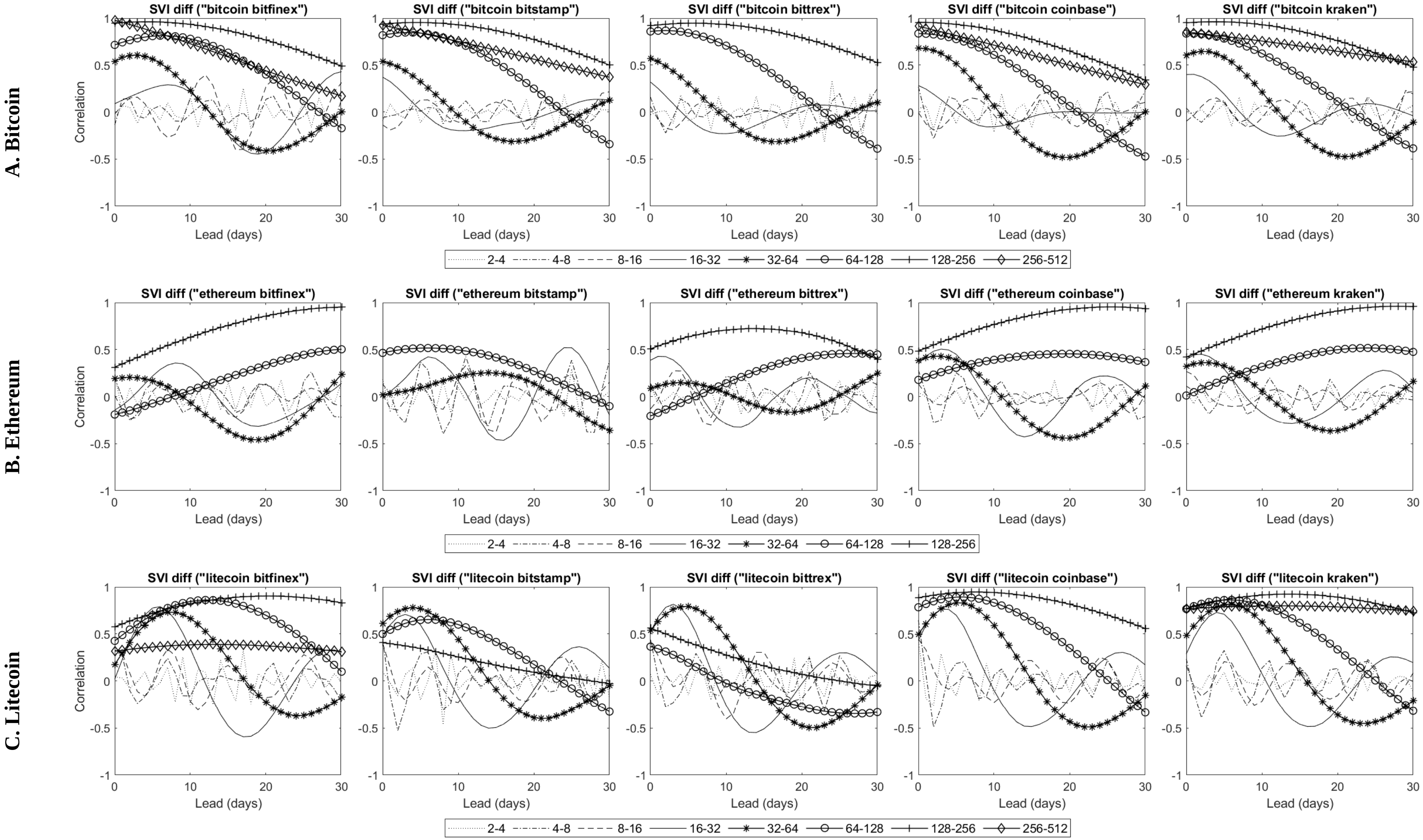
Note: The color scales represent wavelet coherencies, the black contours denote insignificance at 5% against red noise, and the light shading shows regions probably influenced by edge effects. The direction of the relationship (leading indicator) is represented by arrows (a left arrow denotes anti-phase (180°) while a right arrow denotes in-phase (0° or 360°). Downward pointing arrow indicates stock returns as a leading indicator of Cryptocurrency Returns. Source: own estimations.

Figure A3: Wavelet Coherence of Economic Policy Uncertainty Index and Cryptocurrency Returns



Note: The color scales represent wavelet coherencies, the black contours denote insignificance at 5% against red noise, and the light shading shows regions probably influenced by edge effects. The direction of the relationship (leading indicator) is represented by arrows (a left arrow denotes anti-phase (180°) while a right arrow denotes in-phase (0° or 360°). Downward pointing arrow indicates stock returns as a leading indicator of Cryptocurrency Returns. Source: own estimations.

Figure A4: Wavelet Cross-correlation of Search Volume Index and Cryptocurrency Returns



Note: The wavelet cross-correlation sequence shows changes in correlation using different lags in days (x-axis) at different frequency scales (different line styles). Increasing correlation denotes existence of lag. Source: own estimations.

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