# Wealth and Directed Search with Heterogeneous Layoff Risk

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**Abstract**

How does wealth affect the incentive to search for a new job when job-to-job moves are risky? We answer this question by incorporating incomplete markets and learning about match quality into a directed-search job ladder model. In our model, risk averse agents search for new jobs when employed, conditional on their accumulated wealth and estimated quality of their current match. Making a job-to-job transition comes with a wage increase. However, it also comes with the downside risk that the worker may be incompatible with the new firm and face a higher risk of unemployment, while they know that they are relatively safe in their current job. Relative to existing mechanisms relating wealth to search behaviour, we show that agents with lower wealth now experiment less, and are more likely to stay in low-paying jobs. We solve our "risky job ladder" model numerically, calibrating the increased risk from changing jobs using worker micro data. We show that accounting for the risk workers face when changing jobs has aggregate implications for the labour share, unemployment, and rates of reallocation in the economy.

**Keywords:** Incomplete Markets, Heterogeneous Agents, Job Ladder, Unemployment, Directed Search

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# 1 Introduction

Every year, a significant fraction of workers change jobs, either voluntarily or involuntarily.[1] While at the aggregate level, this process of worker reallocation may be productivity enhancing, at the individual level it entails significant amounts of risk. In this paper, we study how *experimentation* and the inability to fully insure against *risk* shape worker incentives to search for new jobs, and study their aggregate implications.

Specifically, in this paper we introduce uninsurable risk into a learning-based job ladder model. In our model, agents gradually learn if they are suited for their current job, and therefore their "job safety". If an agent perceives themselves to be in a safe job, moving job increases the risk of being in a bad match, and hence being fired. Our main result is that when asset markets are incomplete, low wealth agents may therefore prefer to stay in low paying jobs which they have learned to be safe, rather than experimenting with a new job, even if it comes with a pay rise. We show that this implies a highly non-monotonic relationship between wealth and job-to-job mobility (which we confirm in the data), differently from the existing literature, and has aggregate implications for the labour share, unemployment, and rates of reallocation.

We build a continuous-time heterogeneous agent model where risk-averse agents learn about the quality of their current job (a.k.a. "match") over time while accumulating assets to smooth uninsurable income risk. Agents are allowed to search both on and off the job for new matches, and can direct their search towards jobs of different wages and (hence) job finding probabilities. We choose a parsimonious learning structure, which can be directly disciplined using micro data on how quickly workers firing probabilities decline with their tenure in their current match. Within a given match, a worker may turn out to be a "bad match", which is initially unobservable to both the worker and firm. If they are a bad match, their productivity will immediately drop to zero at some known rate, on top of the usual exogenous separation rate common to both good and bad matches. Thus, Bayesian learning naturally implies that the worker infers that her probability of being in a good match constantly increases over time if she has not yet been laid off. Borrowing terminology from the literature on multi-armed bandits and learning, here "no news is good news". This learning process implies each newly climbed rung of the ladder is initially more slippery than the previous rung. Hence, we call

[1] For example, Davis et al. (2012) show average quit and layoff rates in the order of 7% per quarter since the early 2000s in US data. Monthly job-to-job flow rates are estimated to be between 2.2% to 3.2% (see Hornstein et al. (2011) for a recent survey). Kambourov and Manovskii (2009) show that more than 10 percent of workers change occupations on a 1-digit level every year.

our model a "risky job ladder" model.

By taking seriously the notion of job-specific risk, our model enriches standard job-ladder models in a novel way relative to the existing literature. Abstracting from effects such as tenure wage premia, in most job ladder models, workers would in principle move job for any wage increase, no matter how small. This is because there is typically no cost of moving job, and all jobs are equally risky. Therefore, a move to a job which pays a marginally higher wage will always be worthwhile. In our model, climbing the job ladder is risky, since all new jobs carry a higher risk of being a bad match than the current job.

We solve our model numerically, using the continuous-time heterogeneous-agent methods recently developed by Achdou, Han, Lasry, Lions, and Moll (2017). We calibrate the model to standard moments, augmented with micro-data from the Survey of Income and Program Participation (SIPP) on EU-tenure hazards, which we use to discipline our learning process.

At the individual level, our main result is to show that experimentation behaviour is non-monotone in wealth, and depends crucially on the level of risk an agent perceives in their current match. In our directed search framework, the amount of experimentation is simply measured by the wage of new job that a worker targets. We say that agents who search for low wage jobs with high job finding rate experiment more, while agents who search for higher wage jobs with lower job finding rates experiment less.

For agents who perceive themselves to be in safe jobs, we show that having low wealth will make them experiment *less*. This is because accepting an outside job offer carries a much higher risk of unemployment for these agents than their current job. Therefore, since low wealth agents are effectively more risk averse, only wealthy agents would take the risk of leaving a safe job for a risky new job. According to this effect, which we dub "experimentation risk", low wealth agents make fewer job switches with larger pay rises.

On the other hand, for agents who perceive themselves to be in risky jobs, we show that having low wealth will make them experiment *more*. This effect, already shown by Chaumont and Shi (2017) in a model where all jobs carry the same risk, is because low wealth agents are effectively more risk averse and worry about becoming unemployed with such little wealth. They would rather have a small pay rise with certainty than a larger pay rise with lower probability in order not to further deplete their precautionary savings. Low wealth agents thus make more job switches with smaller pay rises. According to this effect, which we dub "waiting risk", low wealth agents make more job switches with smaller pay rises.

These results have important implications for empirical work investigating the relationship between wealth and the search behaviour of employed agents. We show that simple average relationships between wealth and search behaviour (e.g. EE switch probability or reservation wage) hides a rich distributional variation. We furthermore show that the true relationships are non-monotonic, and highly dependent on the agents wealth level, perceived risk, and current wage.

Additionally, our model implies that the effect of asset holdings on search behaviour is stronger for the unemployed than for employees, once other factors are controlled for. Unemployed workers, unlike employees, don't trade off the safety of their current job against temporary increase in riskiness of the new job when accepting a job offer. For unemployed workers, lower wealth will always encourage them to search faster, consistent with existing theoretical results and the empirical work of Herkenhoff et al. (2015), Algan et al. (2003), Bloemen and Stancenelli (2001), Lentz and Tranbaes (2005), and Griffy (2017).

Having explored the effects of learning and incomplete markets on worker behaviour, we embed our workers into a closed economy general equilibrium model, allowing us to study both the individual level and aggregate level effects. Calibrating our learning process to micro data from the Survey of Income and Program Participation (SIPP) shows that the learning process is important enough to give a quantitative bite. In our calibration, 24% of matches are bad matches. If you are in a bad match, this is revealed at a speed which implies an extra 10% probability of being fired in the first month of a job, and 72% probability of being fired within the first year. Thus, for agents who have learned that their job is safe, making a job to job move entails a significant rise in risk.

In equilibrium, the model features a highly unequal wealth distribution, as employed agents understand that they must accumulate extra precautionary savings if they want to take the risk of making job to job transitions. We also compute the distribution of beliefs across employed agents, and find that a large fraction of workers are in jobs which they belief to be safe, meaning that they have chosen to stay in them long enough to learn that they are most likely in a good match. This is because incomplete markets means that workers cannot insure against the risk that changing job will lead them to arrive in a bad match. Thus, workers choose to experiment less, and remain in jobs longer rather than making risky job-to-job moves.

To investigate the role of learning in driving changes in aggregate outcomes, we compare our model to one where changing jobs entails a smaller rise in risks. We recalibrate the learning process so that the implied EU rate is higher in all jobs, and rises less in the first months of a job, while holding other parameters at their baseline calibrated values. This experiment brings our model closer to the seminal model of Chaumont and Shi (2017), which also features incomplete markets and a directed-search job ladder, but no learning process, and jobs which have constant EU separation risk.

This experiment shows that increasing the importance about learning about job risk has meaningful effects on the aggregate economy. We find that learning leads to more wealth inequality, as workers must accumulate larger buffer stocks to self-insure and allow themselves the risk of making job-to-job switches. This is due to a dynamic effect: in models where unemployment risk is constant over the life of a job, agents are able to use their wage to accumulate savings during their job against the risk of being fired. However, in reality (and our model) unemployment risk is concentrated in the early months and years of a job, meaning that agents do not have time to use the wage from the job itself to self insure against the risk of being fired.

Finally, we compare equilibrium unemployment, average wage, and EE switching rate in our baseline model to the recalibrated model where learning is less important. Several interesting differences emerge. The labour share is higher in the baseline model than in a model with less learning. The EE rate is 30% lower when learning is increased, which shows the key idea of the paper, that learning risk reduces the incentive to change jobs. Finally, the unemployment rate is 9% higher when learning is increased. This shows that learning and incomplete markets have important implications not only for the job ladder and reallocation of already employed workers, but for the overall unemployment rate as well.

**Related literature.** Our paper is related to Lise (2013), Eeckhout and Sepahsalari (2015), Hawkins and Mustre-del Río (2017), and Chaumont and Shi (2017), who also consider search models with incomplete markets, asset accumulation, and heterogeneous matches.

Our paper is most closely related to Chaumont and Shi (2017). As in their model, we consider directed search. In their model, firms strictly prefer to hire workers with higher wealth as such workers apply to new jobs with high wages which are harder to get. As a consequence, such workers stay with their current employer longer. In our model new jobs are initially more risky which discourages low wealth agents from applying to new jobs. They work with a small open economy assumption, in our model the endogenous interest rate adjusts to clear the asset market which creates additional feedback between the asset and the labour market.

In Eeckhout and Sepahsalari (2015) agents with lower wealth self-select via directed search into low paying jobs to get a job faster. They consider an economy with heterogeneous firms and fixed interest rate while we have homogenous firms and endogenous interest rate. Hawkins and Mustre-del Río (2017) introduce occupational mobility into an incomplete markets model via occupation-specific human capital accumulation and productivity shocks. Our paper is different from theirs in that we focus on learning and experimentation within matches. Lise (2013) studies asset accumulation in a model of on the job search. We additionally consider learning, and its implications for risk, on agents saving and search choices. Hubmer (2018) builds on Lise (2013) to study implications of the job ladder for earning's risk in a model with partial insurance through asset accumulation. Larkin (2019) considers a model where jobs are characterised by both a wage and a "riskiness" modelled as an exogenous probability of separation. We also have jobs with two dimensional characteristics, but differ from his model in that we model risk through an explicit learning structure. These papers and our own build on earlier work by, for example, Krusell et al. (2010), who introduce incomplete markets and asset accumulation into fully-fledged search and matching models.

Recent work has demonstrated that the ability to borrow is important for the unemployed and affects their search behaviour. Braxton et al. (2019) find that there is significant heterogeneity in the ability of the unemployed to borrow. This is important as Herkenhoff et al. (2016) document that the unemployed with better access to credit search longer for a new job but at the same time, get higher wages. This has general equilibrium implications, as in Krusell et al. (2010).

More generally, we build on the large search and matching literature, especially the literature related to job-to-job transitions and worker flows as in (Burdett and Mortensen, 1998, Delacroix and Shi, 2006) and occupational mobility, that has been studied, among others, by Carrillo-Tudela et al. (2016) and Kambourov and Manovskii (2008). These strands of literature we complement by considering asset accumulation and incomplete markets. As a result, worker's movement on the ladder does not only depend on her individual personal characteristics and current wage level, but asset holdings as well.

The literature on learning in the labour market (McCall, 1990, Miller, 1984, Papageorgiou, 2013) assumes risk neutral preferences. We compare the predictions of our model to the risk-neutral benchmark and find that market incompleteness yields less experimentation.

We make a contribution relative to incomplete markets models in the Aiyagari (1994)-Huggett (1993) tradition, by considering the feedback from wealth into income shocks. These

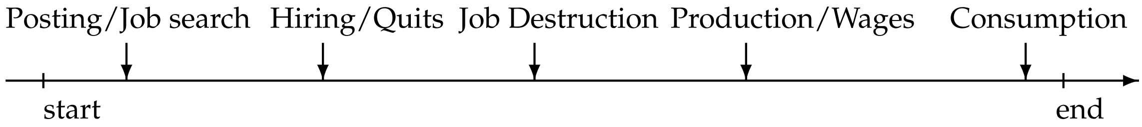



Figure 1: Timing of events

models typically treat income as exogenous and study the effects of income shocks on the wealth distribution and other endogenous objects. In our model, income is partly endogenous because it depends on the experimentation choices of households, meaning that past shocks and choices feed into current and future income realisations.

Finally, the model is solved numerically using the continuous-time heterogeneous agent modelling techniques of Achdou et al. (2017), allowing us to numerically solve Bayesian learning models in an incomplete markets setting, where closed form solutions are not available.

## 2 Simple model

To highlight the main features of our approach, we consider a one-period economy. There is a unit mass of risk averse agents in the economy at the beginning of the period. The preferences of agents over consumption$c$ are represented by a strictly increasing and strictly concave utility function $u(c)$. At the beginning of the period, all agents are either unemployed or employed. The timing of the events is depicted on Figure 1.

First, all agents can search for new jobs for free, sending one application to a vacancy that posts a wage with commitment. There is directed search, in each submarket indexed with $(\theta_i, w_i)$ the matches between searchers $S_i$ and vacancies $V_i$ are given by a matching function $M(S_i, V_i)$ satisfying standard assumptions. Given submarket tightness $\theta_i = \frac{V_i}{S_i}$ a probability of finding a job for a worker searching in this submarket is $\mu(\theta_i(w_i)) = \frac{M(S_i,V_i)}{S_i}$ The probability a vacancy hires a worker is $\zeta(\theta_i(w_i)) = \frac{M(S_i,V_i)}{V_i}$. There is free entry in posting a vacancy at a cost $\kappa$ and each job produces $y$. In equilibrium, for active submarkets the following condition must hold:

$$\kappa = \zeta(\theta(w))(y - w). \tag{1}$$

Hence, no jobs with $w > y - \kappa$ are offered in equilibrium. Agents who have been matched with a vacancy can accept a new job. To do so, employed workers must first quit their current jobs. Then, after the acceptance/quit decision takes place, a fraction of active jobs (those newly formed and old jobs as well) is destroyed. This timing is designed to reflect a risk in moving to a new job which we specify next.

The unemployed earn $b$ while workers earn $w$ and $w > b$. All agents enter with net assets $A \geq -b$. All new jobs are destroyed with probability $\sigma_h$, while the probability of job destruction in ''old'' jobs is $\sigma_l = \sigma_h - \delta_\sigma$ with $\sigma_h \geq \delta_\sigma \geq 0$. Hence, leaving an old job for a new job may entail an increase in unemployment risk if $\delta_\sigma > 0$. In the final stage agents consume their savings and their earnings. Thus, the only problem all agents face is which job to apply to. The unemployed solve:

$$\max_{w^u} \mu\left(\theta(w^u)\right)(1-\sigma_h)u(A+w^u) + \Big(1-\mu(\theta(w^u))(1-\sigma_h)\Big)u(A+b),$$

while the employed solve:

$$\begin{aligned}\max_{w^e} \mu(\theta(w^e))\left[(1-\sigma_h)u(A+w^e)+\sigma_h u(A+b)\right]\\ +\Big(1-\mu(\theta^e)\Big)\left[(1-\sigma_l)u(A+w)+\sigma_l u(A+b)\right].\end{aligned}$$

Note, everyone can choose not to search by choosing $\mu(\theta(w)) = 0$, choosing any arbitrarily high wage that violates the free entry condition. The optimal job search decisions are encapsulated in the following result.

**Lemma 1 (Optimal targeted wages)** *Let $\psi(w) \equiv -\frac{\mu(\theta)}{\frac{\partial\mu(\theta)}{\partial\theta}\frac{\partial\theta}{\partial w}}$, a continuous, differentiable and strictly decreasing function. The optimal job search decisions satisfy:*

$$\theta(w^u) > 0 \text{ and } \frac{u(A+w^u)-u(A+b)}{u'(A+w^u)} = \psi(w^u), \tag{2}$$

$$\theta(w^e) = 0 \text{ or } \theta(w^e) > 0 \text{ and } \frac{u(A+w^e)-u(A+w)}{u'(A+w^e)} + \frac{\sigma_h-\sigma_l}{1-\sigma_h}\frac{u(A+b)-u(A+w)}{u'(A+w^e)} = \psi(w^e). \tag{3}$$

*The wage targeted by the unemployed increases with unemployment benefit, $\frac{\partial w^u}{\partial b} > 0$. For the employees, when $\theta(w^e) > 0$, the targeted wage increases in old job wage $\frac{\partial w^e}{\partial w} > 0$, but it decreases in unemployment benefit $\frac{\partial w^e}{\partial b} < 0$.*

Observe that job destruction parameters are irrelevant for the choice of $w^u$, it's only the individual wealth and the value of unemployment benefits that matters. For employees, when there is a difference in job destruction risk between old and new jobs, $\sigma_h > \sigma_l$, there is an additional negative term $\frac{\sigma_h-\sigma_l}{1-\sigma_h}\frac{u(A+b)-u(A+w)}{u'(A+w^e)}$ showing up. As this additional term decreases the left hand side and on the right hand side we have a decreasing function of the target wage, whenever $\sigma_h > \sigma_l$ with all else kept equal, the targeted wage is *larger* than for $\sigma_h = \sigma_l$. We label this result as the *compensation for increased risk* effect. This compensation might require a wage that exceeds the largest wage the firms are ready to post, hence it's also possible that the employees don't search and set $\theta(w^e) = 0$. This is also the reason for the targeted wage $w^e$ to decrease in unemployment benefits, as the more generous the benefits are, the smaller decrease in income following an unsuccessful job change.

Regarding the effect of $A$ on target wages, it is useful to stress that for it to matter, preferences must exhibit a wealth effect.

**Example 1 (CARA utility)** *Let $u(c) = 1 - e^{-\alpha c}$ with $\alpha$ the risk aversion parameter. Then, the targeted wages are independent of individual asset levels.*

Assuming no wealth effect away, we rearrange (3) and differentiate it with respect to $A$ to arrive at:

$$\frac{\frac{\sigma_h-\sigma_l}{1-\sigma_h}u'(A+b) - \frac{1-\sigma_l}{1-\sigma_h}u'(A+w)}{\left(\psi'(w^e)-1\right)u'(A+w^e) + \psi(w^e)u''(A+w^e)} = 1 + \frac{\partial w^e}{\partial A}. \tag{4}$$

The denominator on the left hand side is always negative but the sign of the numerator can vary, depending on the job destruction risk parameters. In fact, as we demonstrate it in the following example for logarithmic utility, the effect of wealth on job search behaviour can be of *arbitrary* sign and magnitude. Note, below we only report sufficient conditions that are independent of $w^e$. Straightforwardly from (4), when the left hand side is positive, but less than one, the partial derivative is negative and less than one in absolute value etc., however, $w^e$ features there.

**Example 2 (Logarithmic utility)** *Let $u(c) = \log(c)$, then:*

1. $\frac{\partial w^u}{\partial A} > 0$*, and if* $\sigma_l = \sigma_h$*, then also* $\frac{\partial w^e}{\partial A} > 0$,
2. *when* $\sigma_l < \sigma_h$*, then* $\frac{\sigma_h-\sigma_l}{1-\sigma_h}w - b > A \implies \frac{\partial w^e}{\partial A} < -1$.

To begin with, wages that the unemployed seek increase in individual wealth. Next, when there is no difference in job destruction risk between old and new jobs, more wealth implies higher target wages for employees. However, when changing a job entails an increase in the

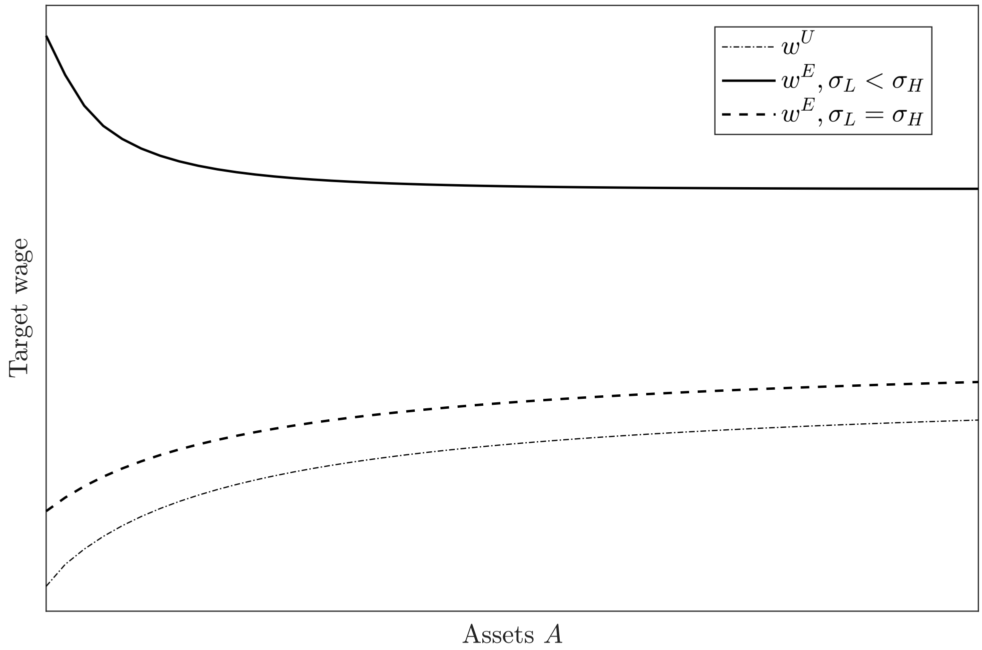



Figure 2: Illustration of dependence of target wages on individual assets, logarithmic utility

likelihood of becoming unemployed, the picture changes and because of the compensation for risk effect, workers target high wages which *decrease* in wealth. We illustrate these results on Figure 2.

The second inequality in Example 2 tells us that when individual assets are sufficiently low (lower than the difference between a fraction of the current wage, and the unemployment benefit), the response of target wage to assets is negative, we can even say it is more than proportional.

In reality, we can expect that individual assets, current wage and safety of the current job to be correlated. For example, agents who spent a long time in unemployment may settle for a low wage, but their assets might have been depleted as well. For those agents, the response of target wage to individual assets can be negative. On the other hand, for employees at the top of the wage ladder, the size of drop of income following an unsuccessful job change is sizeable. Therefore, for those agents the effect of individual wealth on target wage can be negative as well, if they didn't have enough time to accumulate assets. Hence, a simple unconditional correlation between individual assets and mobility might be masking a rich heterogeneity in their relationship. Therefore, a joint distribution of wages, job safety and wealth must be taken into account. In the following section we propose a dynamic model capable of capturing possibly non-monotone effects of wealth on mobility across a *risky* job ladder.

# 3 Quantitative model

We present a model of learning and incomplete markets, focusing on the experimentation and asset accumulation decisions of the worker. The model is in continuous time, and is populated by a continuum of ex ante identical workers, an endogenously determined mass of identical single-worker firms, and a government. Matches have an idiosyncratic quality component, which is unobserved to both the worker and the firm, and which creates the learning structure. The model features a job ladder, and worker will use directed search both on and off the job to search for higher paid employment. We model a closed economy, where the demand and supply of savings must equate, and the equilibrium interest rate is endogenous. We focus our exposition on stationary equilibrium where the interest rate and policy functions are constant over time, but also consider deterministic transition experiments in later sections.

## 3.1 Matching market

There are search frictions in the labour market. Workers can direct their search towards firms posting different wages, so that different workers may face different job offer arrival rates. In particular, the labour market consists of a continuum of submarkets, which are indexed by the wage offered by firms in that submarket, $w_t$, and the wealth of workers searching in that submarket, $a_t$. Thus, workers of different wealth levels are separated into different submarkets, which simplifies the analysis.[2] Firms commit to paying the posted wage throughout the life of the match, and do not alter wages in response to outside offers by other firms.

Within each submarket, denote by $\theta(a_t, w_t)$ market tightness, the ratio of vacancies to (effective search units of) worker applications in that market. Workers and firms take the tightness in each submarket as given when choosing where to search. Matching is random within each submarket, with the total number of matches determined by a standard constant returns to

[2] In our model agent's behaviour will also depend on their belief about their safety in their current job, which will be defined as $p_t$. We do not index submarkets by this belief, but this is without generality since in equilibrium in each submarket free entry in firm vacancy posting means that tightness will only depend on $(a_t, w_t)$. Requiring agents of different wealth levels to search in different submarkets is a restriction on the model, since wealth affects future quit probabilities and hence the incentives of firms to post vacancies. Chaumont and Shi (2017) require agents with different wealth levels to search in different submarkets to ensure their model is block recursive. Since our model, with a closed economy and endogenous interest rate, is not block recursive in any case, we do not need to assume that workers are separated into submarkets by wealth, but do so to keep the model closer to Chaumont and Shi (2017).

scale matching function. Each search-unit of worker search effort receives a match with probability $\mu(\theta_t)$ and each vacancy a match with probability $q(\theta_t) = \mu(\theta_t)/\theta_t$, where $\mu(\theta_t)$ is a known function.

There is an infinite mass of firms willing to post vacancies, such that vacancies will be posted to any active submarkets until firms make zero profit ex-ante. Search effort is exogenous, and unemployed workers exert one unit of search effort per unit of time. Employed workers are assumed to exert $s_e < 1$ units of search effort per unit of time, in order to empirically match their lower job finding rates. An unemployed worker searching in submarket $(a, w)$ therefore receives offers at rate $\mu(\theta(a_t, w_t))$, while an employed worker does at rate $s_e\mu(\theta(a_t, w_t))$.

### 3.2 Workers

Workers discount the future at rate $\rho$, and also die at rate $\zeta$. New agents are born at the same rate and enter into unemployment, to keep the mass of workers fixed at 1. In what follows, we suppress the individual agent index $i$ for notational convenience. While not required theoretically, adding lifecycle dynamics is helpful quantitatively for matching wealth distributions. The total discounting rate is thus $\rho + \zeta$. We consider standard time-separable preferences,

$$V_0 = \mathrm{E}_0 \int_0^\infty e^{-(\rho+\zeta)t} u(c_t) dt, \tag{5}$$

where $c_t$ is the instantaneous consumption rate. Leisure is unvalued, and workers either work full time or are unemployed. The agents can save and borrow using only risk-free assets which pay interest rate $r_t$. Assets consist of non-defaultable claims made by households and governments, as well as the equity issued by firms. We consider either steady states or deterministic transitions, so there is no aggregate risk and debt and equity claims are both riskless and consider part of the generic asset stock.

In our baseline model, households are born unemployed with zero assets. There is no family structure or inheritance, so if workers die with positive assets these are collected by the government as a 100% inheritance tax.[3]

Let an individual's stock of savings at $t$ be denoted $a_t$. The supply of savings will come

[3] If a worker dies with negative assets, rather than assuming that lenders absorb the loss of non-repayment, we assume that the government pays off the worker's debts for them. We could alternatively assume that workers must partake in annuity markets to insure themselves against the risk of being unable to repay due to death.

from claims issued by households and ownership of firms, which pay out profits as dividends. Agents' consumption rate is $c_t$. If an agent currently earns income, from either a wage or benefits, of $z_t$, then saving evolves according to the budget constraint

$$\dot{a}_t \equiv \frac{da_t}{dt} = z_t + ra_t - c_t. \tag{6}$$

Importantly, the agent faces a borrowing constraint

$$a_t \geq \underline{a} \tag{7}$$

which states that their borrowing cannot exceed $-\underline{a}$. The non-contingent nature of the return on capital and limit on total borrowing implies that agents will be unable to perfectly insure risks, in particular their idiosyncratic risk coming from employment and wage dynamics. As discussed in Achdou et al. (2017), in continuous time, as long as consumption and income rates are not infinite (which is the case here due to risk aversion and lack of infinite arbitrage opportunities) this borrowing constraint only binds when the agent's wealth is already exactly at the borrowing constraint. That is, when $a_t > \underline{a}$ then the borrowing constraint will never bind in the next instant of time. However, imposing the constraint that $a_{t+dt} \geq \underline{a}$ whenever $a_t = \underline{a}$ implies that the borrowing constraint boils down to imposing $\dot{a}_t \geq 0$ at this state. Rearranging the budget constraint, we thus have the constraint that

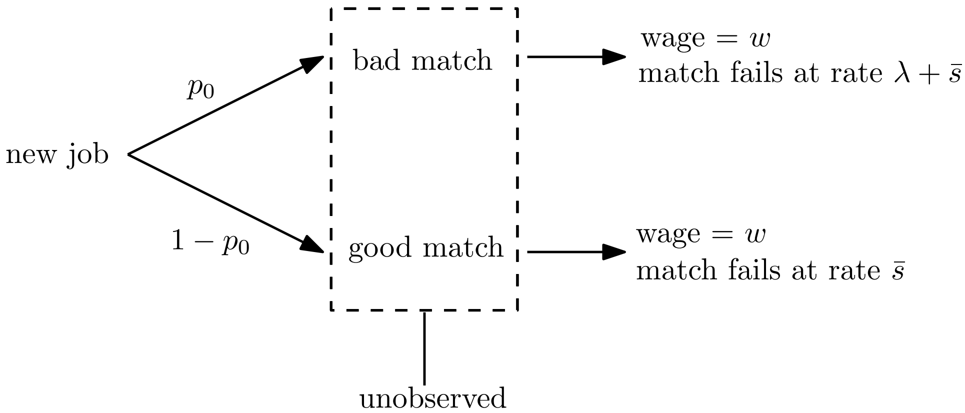
$$c_t \leq z_t + r\underline{a} \tag{8}$$

when $a_t = \underline{a}$.

#### 3.2.1 Learning and income structure

If unemployed, agents simply receive income $z_t = b_t$, where $b_t$ are unemployment benefits paid by the government. If employed, the agents earn a wage and receive income $z_t = (1 - \tau_t)w_t$, where $\tau_t$ is a proportional labour income tax. All worker-firm matches produce $y$ units of output each unit of time, and thus their productivity (measured in output) is identical. However, matches will have an idiosyncratic quality component, which manifests only in the probability of the match breaking down. This quality is unobserved to both the worker and the firm, and both learn symmetrically about the match quality by observing whether the match has broken down. Specifically, a match can be one of two types. If it is a "bad" match, then at some instant the productivity of the match will drop permanently to zero and the worker will therefore be immediately fired. If it is a "good" match, then the worker only faces a common component of the layoff risk, but otherwise maintains positive productivity within a match. Since type is match specific, if a worker suspects that she is a bad type in the current match, she is always free to switch to a new firm, if the opportunity arises, where she might be a better match.

Figure 3: Learning process




Information about the quality of the match is revealed as follows. Before having any information, the agent has a prior belief $p_0$ that the match is a bad match. Her current estimate of being in a bad match is denoted $p_t$, and is updated through Bayesian learning. If the current match is bad, then eventually the worker (and employer) will receive a signal that the match is bad. This signal arrives according to a Poisson process with rate $\lambda$. If the current match is good, then a signal that the match is bad will never arrive. If the signal is revealed, we assume this amounts to the productivity of the match dropping to zero so that the worker is instantly fired. Hence, being in a bad match is equivalent to facing additional layoff risk. This learning structure is very simple, and has the appealing property that the longer the agent is employed at a particular match, the lower they estimate the probability of them being unsuitable. Thus, with time passing, the perceived and actual job safety increase. Bayesian learning means the estimated probability of being in a bad match evolves according to

$$dp_t = -\lambda p_t(1 - p_t)dt \tag{9}$$

If an agent observes the signal that she is in a bad match then $p_t$ jumps to 1. While not required, it is instructive to solve explicitly for the time path of $p_t$ in the absence of shocks:

$$p_t = \frac{1}{1 + \left(\frac{1}{p_0} - 1\right) e^{\lambda t}} \tag{10}$$

Note that $p_t$ starts at $p_0$ at the beginning of a match, and then declines towards 0. Additionally, all matches may exogenously separate for other reasons at common rate $s$. Since matches additionally collapse at rate $\lambda$ if a match is bad, the total (perceived and actual) match separation rate is $\lambda p + s$, and is declining in match tenure.

### 3.2.2 Value and policy functions

The unemployed agents' state variable is just $a$, their current wealth. We denote their value function by $v^u(a)$. This is defined recursively by

$$(\rho + \zeta)v^u(a) = \max_{c,w'} u(c) + v_a^u(a)(b + ra - c) + \mu(\theta(a, w')) \left(v^e(a, p_0, w') - v^u(a)\right) \tag{11}$$

subject to the borrowing constraint $c \leq b + r\underline{a}$ when $a = \underline{a}$. The various terms capture the agent's consumption and saving decision, and the change in value when searching for a job at wage $w'$. $v^e(a, p_0, w')$ denotes the value of starting a new job with wage $w'$. Since there has been no learning at the beginning of a new match, all new jobs have the same prior belief $p_0$ that the match is a bad type. Let $c^u(a)$ denote the optimal consumption policy function for unemployed workers, and $\dot{a}^u(a)$ their optimal asset drift. These solve the first order condition $v_a^u(a) = u'(c)$ unless the borrowing constraint binds. Let $w^{*u}(a)$ denote their optimal choice of $w'$, the wage submarket they search in, and let $\theta^u(a) \equiv \theta(a, w^{*u}(a))$ denote market tightness in their chosen submarket.

For any wage $w$ and generic belief $p$ about the quality of the match, we define the value function of employed agents as $v^e(a, p, w)$. This is defined as:

$$\begin{aligned}(\rho + \zeta)v^e(a, p, w) = &\max_{c,w'} u(c) + v_a^e(a, p, w)((1 - \tau)w + ra - c) - \lambda p(1 - p)v_p^e(a, p, w) + ... \\ &+ s_e\mu(\theta(a, w')) \left(v^e(a, p_0, w') - v^e(a, p, w)\right) + (\lambda p + s)\left(v^u(a) - v^e(a, p, w)\right)\end{aligned} \tag{12}$$

subject to the borrowing constraint $c \leq (1 - \tau)w + r\underline{a}$ when $a = \underline{a}$. Terms on the first line correspond to consumption and saving decisions, plus the change in value as beliefs evolve.

The term $-\lambda p(1-p)v^e_p(a,p,w)$ denotes the change in expected utility over time as the estimated probability of being in a bad match falls as the separation shock fails to materialise. On the next line, $s_e\mu(\theta(a,w'))$ denotes the rate at which new job offers arrive if the worker searches in the submarket with wage $w'$. If the worker receives and accepts such an offer, they trade the current value $v^e(a,p,w)$ for value in the new job $v^e(a,p_0,w')$. This new job has the higher wage $w'$, but resets the belief about match quality to the prior, $p_0$. If the worker has been in the current match for any amount of time, the current belief of quality will be $p < p_0$, meaning that moving job must worsen the belief of match quality, and increase the (correctly) estimated probability of being fired. The final term gives the probability that the match terminates and the worker moves to unemployment. This is the sum of the exogenous separation rate $s$ and $\lambda p$, which is the additional probability of being fired in a bad match multiplied by the current estimated probability of being in a bad match. As with unemployed workers, the consumption first order condition gives $v^w_a(a,p,w) = u'(c)$ unless the borrowing constraint binds. Let denote the employed worker policy functions by $c^e(a,p,w)$, $\dot{a}^e(a,p,w)$, $w^{*e}(a,p,w)$, and $\theta^e(a,p,w) \equiv \theta(a,w^{*e}(a,p,w))$, with the only difference being that these policy functions also depend on beliefs and current wages.

### 3.2.3 Characterisation of optimal search behaviour

How does a worker's wealth affect the type of jobs they are willing to apply for? This is the key question we use our model to understand, and here we explain the main ideas in the paper using the solution to the worker's search problem.

In this directed search environment, workers face a simple tradeoff when choosing which submarket to direct their search towards. If they to look for jobs with a high wage, $w'$, they face a lower probability that they will find such a job, $\mu(\theta(a,w'))$, and will have to wait longer for a successful match. This is because (conditional on productivity) firms make less profit in higher wage matches, and so have less incentive to post vacancies, meaning that market tightness is typically decreasing in the posted wage: $\theta_{w'}(a,w') < 0$.[4]

All workers face this same tradeoff, but will differ in how much they value a higher wage versus the speed at which they can obtain a new job. The insight of the existing literature is that agents with low wealth will prefer to search for jobs with a higher acceptance rate but a lower

[4] This is simple to prove analytically in a model without on the job search, but is more challenging here. It is always true in equilibrium in our numerical work.

wage, in order to gain a (smaller) pay rise more quickly. This is because low wealth agents are at risk of hitting their borrowing constraints, making their value more sensitive to wealth, and so desire a quick pay rise to help them build up a buffer stock of savings. This mechanism holds in our model, but will be dampened and potentially overturned for employed agents due to our new learning mechanism.

The existing intuition can be seen from the solution to the unemployed worker's search problem. Their first order condition for the optimal targeted $w'$ is

$$\mu(\theta(a,w'))v^e_{w'}(a,p_0,w') = -\theta_{w'}(a,w')\mu'(\theta(a,w'))\left(v^e(a,p_0,w') - v^u(a)\right) \tag{13}$$

Here, the left hand side gives the marginal benefit of searching for a higher wage job, which is that conditional on finding a job the wage, and hence value, will be higher: $v^e_{w'}(a,p_0,w') > 0$.[5] Recall that all new jobs start with prior belief $p_0$, since no learning has occurred. The right hand side gives the marginal cost, which is that it will take longer to find a job. $-\theta_{w'}(a,w')\mu'(\theta(a,w'))$ is the decrease in the job finding rate as the wage is increased, and $v^e(a,p_0,w') - v^u(a)$ is therefore the lost value from remaining unemployed longer relative to the value of employment at that wage.

The standard intuition is that for low wealth agents, this marginal cost is higher, because it is more painful to remain unemployed with low wealth. Mathematically, this manifests in the difference in values, $v^e(a,p_0,w') - v^u(a)$, being larger for low wealth agents, because the value of being unemployed is more sensitive to wealth than the value of being employed. This means that unemployed agents would rather find a job quickly, by directing their search towards low wage jobs.

The first order condition for optimal search for employed workers is

$$\mu(\theta(a,w'))v^e_{w'}(a,p_0,w') = -\theta_{w'}(a,w')\mu'(\theta(a,w'))\left(v^e(a,p_0,w') - v^e(a,p,w)\right) \tag{14}$$

This condition is identical to the condition for unemployed workers, except that employed workers forgo their current value, $v^e(a,p,w)$, when they accept a new job. In a model without learning, employed workers face exactly the same tradeoff as the unemployed. The model without learning can be found by assuming there are no bad matches, so that $p_0 = p = 0$ in all matches. In this case, the cost of waiting in the current job is $v^e(a,0,w') - v^e(a,0,w)$. Exactly

[5] This is easily proved using standard envelope arguments.

the same logic applies as with unemployed workers, which is that value in the current (lower wage) job is more sensitive to wealth than in the new (higher wage) job. Accordingly, low wealth agents have a higher cost of waiting in the current job, and search for a new job faster by accepting smaller pay rises.

However, in our full model with learning, this need not be the case. With learning, when an agent switches to a new job their wage will rise from $w$ to $w'$, but the new job will be riskier than the old job since the belief of being a bad match rises from current $p$ to $p_0 \geq p$. Since the new job is risky and carries more unemployment risk, the value of the new (higher paying) job may actually be more sensitive to wealth than the value of the old (lower paying) job. If this is the case, high wealth agents, who are better self-insured, will instead have the lower cost of waiting, and will search for a new job faster by accepting smaller pay rises.

Thus, the central insight of our model is that the effect of wealth on the incentive to change jobs depends crucially on assumptions about risk. In the standard model, where changing jobs entails no increase in risk and only a rise in pay, high wealth agents have *less need* to change job, and search less. In our learning model, where changing jobs also entails a temporary increase in unemployment risk, high wealth agents are *better insured* and hence more willing change job, and search more.

It is important to note that our risk mechanism only applies to already employed agents, since the unemployed do not have jobs and therefore do not increase the risk of unemployment by changing jobs. Therefore, our model is fully consistent with existing empirical work showing that unemployment agents search less when their wealth is high. Instead, by introducing a new risk channel unique to employed agents, our model allows the effect of wealth on search behaviour to differ between the employed and unemployed.

### 3.3 Firms and equilibrium market tightness

An infinite mass of potential single worker firms can post vacancies at flow cost $\kappa$. All vacancies lead to matches which produce $y$ units per unit of time, but whether the match with the eventual worker is good or bad is not known ex ante, and only gradually learned during the match. Firms can post vacancies to any submarket, and will be indifferent about which submarket to post to in equilibrium, since all will give the firm zero expected profits. Firms rebate profits to their owners, who are workers who save, and hence discount the future at the endogenous interest rate $r$.

A firm which pays it's worker $w$ receives $y - w$ as instantaneous profits. The value of the firm is these profits, discounted at rate $r$ and accounting for the possibility that the match might fail, learning, and that the worker might die or quit for another firm. In order to calculate these effects, the value of the firm will depend on the worker's assets, $a$, and the current estimated probability that the match is bad, $p$. Call $J(a, p, w)$ the value to a firm of a filled job with wage $w$, current $p$ and worker assets $a$. This solves the following HJB functional equation:

$$(r+\zeta)J(a,p,w) = y - w + J_a(a,p,w)\dot{a} - \lambda p(1-p)J_p(a,p,w) - \left(\lambda p + s + s_e\mu(\theta(a,w'))\right)J(a,p,w) \tag{15}$$

where it is understood that $\dot{a} = \dot{a}^e(a,p,w)$ and $w' = w^{*e}(a,p,w)$ are the worker's policy functions. Firm value depends on worker wealth $a$ because it affects the probability that a worker will quit, to a firm with higher wage $w' > w$, and dissolve the match. It depends on the estimated probability the match is bad, $p$, because it affects the probability the match fails, as well as worker quit behaviour.

Before meeting a worker, the firm posts a vacancy at flow cost $\kappa$. If it chooses to post in the submarket $(a, w)$, it fills the vacancy at rate $q(\theta(a, w))$, which it takes as given. When it meets a worker, neither the worker nor firm knows if the match is good or bad, and so they assign the prior belief $p_0$ to this probability, meaning that the value of the job once it is filled as known to be $J(a, p_0, w)$. Let $V(a, w)$ denote the value of a vacancy in a given submarket. This therefore satisfies the HJB $rV(a,w) = -\kappa + q(\theta(a,w))(J(a,p_0,w) - V(a,w))$.

Since there are infinite potential entrants, in equilibrium in each submarket either no firms choose to post a vacancy, if $V(a, w) < 0$, or firms post vacancies until the value of a vacancy drops to zero, $V(a, w) = 0$. Using the HJB, this gives the standard complementary slackness conditions $q(\theta(a,w))J(a,p_0,w) \leq \kappa$ and $\theta(a,w) \geq 0$. Inverting the matching function, this allows us to solve for equilibrium tightness in each submarket as

$$\theta(a,w) = \begin{cases} q^{-1}\left(\frac{\kappa}{J(a,p_0,w)}\right) & q(\theta(a,w))J(a,p_0,w) \geq \kappa \\ 0 & else \end{cases} \tag{16}$$

For any active submarkets this gives the familiar free entry condition.

### 3.4 Worker distributions and evolution of unemployment

Denote by $n_t$ total employment at time $t$. Since there is a unit mass of workers, total unemployment is $u_t = 1 - n_t$, which also equals the unemployment rate. Let $g^e(a, p, w)$ denote the density of employed workers with state $(a, p, w)$, such that $n_t = \int_{a,p,w} g^e(a, p, w) dw dp da$, and similarly define $g^u(a)$ as the distribution across unemployed workers.

Notice that the distribution over employed workers is defined over the agents' beliefs about whether they are in a good or bad match. Even though it is unobserved by workers and firms, one can also think about the distribution of workers across good and bad matches. That is, what is the density of workers with state $(a, p, w)$ who are in a good match, and what is the density of workers with state $(a, p, w)$ who are in a bad match? Luckily, since agents are Bayesian learners and we can apply the law of large numbers, we do not need to consider these more complicated distributions. In fact, we know that within any group of workers with belief $p$, exactly the fraction $p$ will be in bad matches, and $1 - p$ will be in good matches.

Using agents optimal decisions we can construct a transition function for the worker distributions: $[\dot{g}^e_t, \dot{g}^u_t] = f(g^e_t, g^u_t)$. This transition takes into account that workers die at rate $\zeta$, and are new workers are born into unemployment with zero assets. In steady state we have $\dot{g}^e_t = \dot{g}^u_t = 0$. Knowing these distributions tells us the unemployment rate, but it is also instructive to express the evolution for unemployed using a simpler expression. Let $\bar{p}_t \equiv (\int_{a,p,w} p g^e(a, p, w) dw dp da)/n_t$ denote the average belief of employed workers. Let $\bar{\mu}^u_t \equiv (\int_a \mu(\theta^u(a)) g^u(a) da)/u_t$ denote the average job finding rate of unemployed workers. Then unemployment evolves according to

$$\dot{u}_t = -\bar{\mu}^u_t u_t + (\lambda \bar{p}_t + s) n_t + \zeta(1 - u_t) \tag{17}$$

This expression is the familiar accounting of unemployment flows, additionally accounting for birth and death. $\bar{\mu}^u_t u_t$ gives the flow of unemployed workers finding a job, which is the average job finding rate multiplied by the number of unemployed workers. $(\lambda \bar{p}_t + s) n_t$ gives the flow of workers losing their jobs, which is the average separation rate, $(\lambda \bar{p}_t + s)$, multiplied by the number of employed workers. Recall that all matches fail at rate $s$, and that bad matches, of which there are a fraction $\bar{p}_t$ in the population, additionally fail at rate $\lambda$. Finally, a flow $\zeta$ workers are born into unemployment, and unemployed workers die at rate $\zeta$. In steady state we have $\dot{u}_t = 0$ and unemployment is given by $u = (\lambda \bar{p} + s + \zeta)/(\lambda \bar{p} + s + \bar{\mu}^u + \zeta)$.

This equation makes clear how assets and learning affect equilibrium unemployment. Firstly, the wealth distribution of unemployed workers affects their incentives to search and hence job finding rate. In this class of models, low wealth leads unemployed workers to accept lower wage job offers in exchange for a higher job finding rate, meaning that if average wealth of the unemployed falls their job finding rate should rise. Secondly, the average job separation rate depends on the distribution of matches across good and bad types. If workers are mostly in good matches, the separation rate will be low. Wealth affects the average quality of matches indirectly, by changing the incentive of workers to make job to job moves, where they trade higher wages for an increased probability of being in a bad match relative to their existing job.

### 3.5 Government

The government collect taxes, and uses them to fund unemployment benefits, as well as general government spending, $G$. The government may also borrow and save using the risk free asset. In all experiments, we assume that the government's policies are exogenous, and hence treated as parameters. General government spending is created from the output good, is unvalued, and included only to allow the model to be calibrated to a sensible tax rate, given that governments in reality spend money not only on unemployment benefits.

Let $\bar{w}_t \equiv (\int_{a,p,w} w g^e(a,p,w) dw dp da)/n_t$ denote the average wage of employed workers, implying total tax revenues of $\tau_t \bar{w}_t n_t$. Total spending on unemployment benefits is $bu_t$. Let $a_t^h \equiv \int_{a,p,w} a g^e(a,p,w) dw dp da + \int_a a g^u(a) da$ denote total household wealth. Since death is random, the average wealth of people who die is just the average wealth of the population, implying that the government receives revenues from inheritance taxation of $\zeta a_t^h$.

Let $a_t^g$ denote the government's total assets (positive being saving) at time $t$. This evolves according to $\dot{a}_t^g = \tau_t \bar{w}_t n_t + \zeta a_t^h - bu_t - G + r_t a_t^g$. If the government chose to ran a balanced budget ($a_t^g = 0$) then we would have $\tau_t \bar{w}_t n_t + \zeta a_t^h = bu_t + G$, meaning that revenues and spending are exactly balance. In our baseline model we assume that the government holds its saving constant at some $a^g$ which does not change over time ($\dot{a}_t^g = 0$), giving the government budget constraint

$$\tau_t \bar{w}_t n_t + \zeta a_t^h + r_t a^g = bu_t + G \tag{18}$$

This is similar to the case of a balanced budget, except that interest on government saving (debt) serves as an additional source of revenue (cost) to the government.

### 3.6 Market clearing

We have already described the evolution of the labour market, and the underlying competitive search equilibrium concept. Since we assume a closed economy, we must also impose goods and asset market clearing and solve for the equilibrium interest rate, $r_t$. It is worth noting that the endogenous interest rate creates interactions between agents' decisions which break the usual block-recursivity results in this model. That is, agents need to know the interest rate to make their decisions, but the equilibrium interest rate will depend on the whole distribution of assets, wages, and beliefs across agents.

Goods market clearing requires that all output is either consumed, used to pay vacancy posting costs, or used in general government spending, $G$:

$$\begin{aligned} n_t y = &\int_{a,p,w} c^e(a,p,w) g^e(a,p,w) dw dp da + \int_a c^u(a) g^u(a) da + \\ &\kappa \left( \int_{a,p,w} \theta^e(a,p,w) g^e(a,p,w) dw dp da + \int_a \theta^u(a) g^u(a) da \right) + G \end{aligned} \tag{19}$$

Total output is $n_t y$, since all employed workers produce output $y$, and the various forms of spending are given on the right hand side. The top row gives total consumption of employed and unemployed agents. The bottom row gives government spending and the spending on vacancy posting costs. To see this, note that the numbers of vacancies in each submarket is the mass of workers searching multiplied by the market tightness.

In our closed economy, asset market clearing requires that net saving equals the supply of assets in the economy. Asset demand comes from households and the government, who can both borrow and save by issuing risk-free claims or purchasing firm equity. The only asset in positive net supply in the economy is firm equity, which is claims to the streams of profits made by firms. Defining $A_t$ as the endogenous total value of asset supply in the economy at time $t$, asset market clearing states that $A_t = a_t^h + a_t^g$. By Walras' Law, asset market clearing is implied by goods market clearing and the other equilibrium conditions of the model.

## 4 Calibration and steady state results

In this section we present numerical results from our calibrated model. We focus on how incomplete markets and learning affect worker behaviour, and in particular on how a worker's wealth affects their incentive to experiment using the job ladder.

## 4.1 Calibration

We calibrate the model to US data. We calibrate the model to a monthly frequency, so that one unit of time corresponds to one month. We specialise to CRRA utility, $u(c) = c^{1-\sigma}/(1-\sigma)$ with coefficient of relative risk aversion of $\sigma = 2$. The discount rate is set to $\rho = -(1/12)\log(1 - 0.05)$ implying a 5% annual rate, and the rate of death is set to $\zeta = 1/(12 \times 45)$, so that agents live for 45 years on average, implying a 45 year working life.

The learning process is chosen to match the data on job separation rates (EU rates) by job tenure. The downside risk of changing job in this model comes the fact that, as in the data, the probability of an employed worker transitioning to unemployment is higher the shorter the worker's tenure on that job. We choose $p_0$, $\lambda$, and $s$ to match the EU-tenure hazard for workers who did not start the current job after an unemployment spell as measured in SIPP data, as reported by Mercan and Schoefer (2019). We choose the baseline separation rate, $s$ such that the equilibrium unemployment rate in the model is 5%. $s$ determines the separation probability for a worker with very long tenure, and we choose $p_0$ and $\lambda$ such that the ratio of the monthly separation probability in month 1 (month 12) to the separation probability for workers with very long tenure is 4:1 (2:1) as in the data.

We normalise match output to $y = 1$. The vacancy posting cost, $\kappa$, is chosen to influence firm job creation incentives so that unemployed workers find a job within one month with probability 25%, following data from Hornstein et al. (2011). The relative search intensity of employed workers, $s_e$ is chosen to match a monthly job-to-job transition probability of 2.5%. We assume a Cobb-Douglas matching function with elasticity $\psi$ of matches to searching workers, giving $\mu(\theta) = \theta^{1-\psi}$ and $q(\theta) = \theta^{-\psi}$, and set $\psi = 0.5$.

The wage distribution is endogenous, and we choose unemployment benefits to be equal to 40% of the average wage. The borrowing constraint, $\underline{a}$ is set so that agents can borrow up to one month of the average wage. We target a 3% annual equilibrium interest rate, $r = -(1/12)\log(1 - 0.03)$. This is achieved by choosing the government saving position, $a^g$ to clear the asset market. Labour taxes are set at 20%, giving $\tau = 0.2$. Unvalued government spending, $G$, is chosen to balance the government budget given its tax receipts and spending on benefits.

## 4.2 Worker policy functions: experimentation and switching behaviour

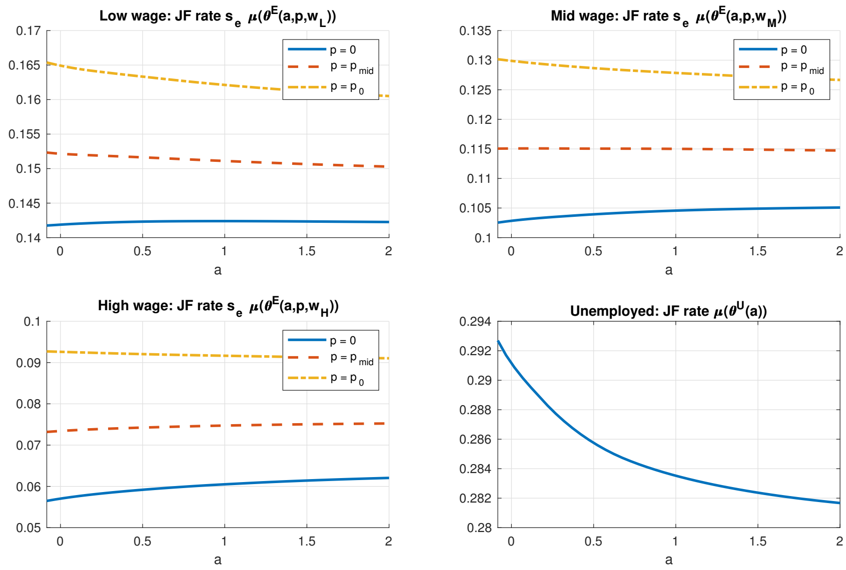
In order to illustrate our main result, we begin with a description of agents policy functions in our calibrated equilibrium. Figure 4 plots various slices of the job finding rate and targeted wage policy functions. Panel (a) shows the optimal job finding rate, with the first three panels for employed agents and the bottom for unemployed agents. All figures plot the policy function across assets, $a$, with slices taken across other variables where relevant.

The bottom right panel gives the job finding rate of unemployed agents across wealth: $\mu(\theta^u(a))$. Consistent with previous models (e.g. Chaumont and Shi (2017)) less wealthy unemployed agents search for jobs faster, and have higher job finding rates than high wealth unemployed agents. The same panel in subfigure (b) shows how: low wealth agents direct their search towards lower paying jobs. This result is well known, and reflects the standard intuition about how incomplete markets affect search. Low wealth unemployed agents are at risk of hitting their borrowing constraints, and so desire a pay rise fast. They achieve this by accepting lower wage offers, while wealthier agents are happier to wait and direct towards higher wage jobs with lower job finding rates.

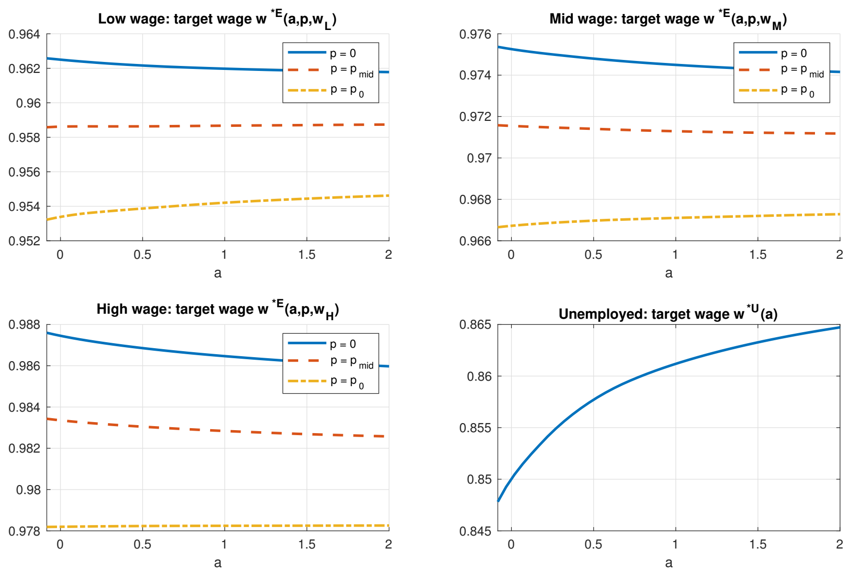
Our new results come when looking at the behaviour of employed agents in the remaining panels. Interestingly, we see that their search behaviour does not have a simple monotonic relationship with wealth. This effect is clearest when looking at the job finding rates of high wage agents, $\mu(\theta^e(a, p, w))$, in the bottom left panel of subfigure (a). These are plotted for three levels of beliefs. The dash-dotted yellow line gives the policy for $p = p_0$, which corresponds to agents who have just started their current job, and hence have not learned at all whether the job is safe or not, maintaining their belief at the prior. The blue line corresponds to $p = 0$, giving agents who have been in their current job for a long time, and have learned that they are definitely a good match. The dashed red line gives an intermediate belief.

Our main result is that the effect of wealth on search behaviour is not the same for all workers, and depends crucially on beliefs about the safety of the current job. For workers who have just joined their job, they perceive their job to be just as risky as any other job offer they could receive. For these workers, the logic applied to unemployed workers applies, and search intensity is declining in wealth, as shown in the dash-dotted yellow line. This is the result of Chaumont and Shi (2017). Our new result is that this effect is completely reversed for workers in safe enough jobs. For these workers, changing job is risky, since they know they have a low

Figure 4: Directed search behaviour: worker policy functions



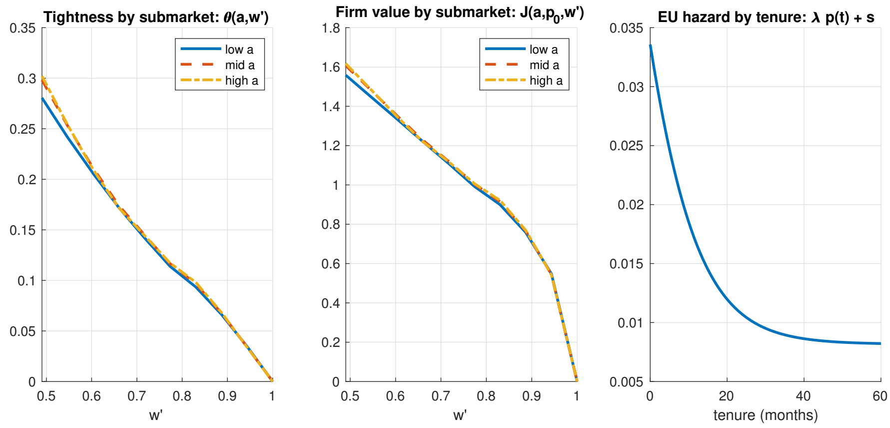

(a) Job finding rate by assets, employment status, wage, and beliefs




(b) New wage target by assets, employment status, wage, and beliefs

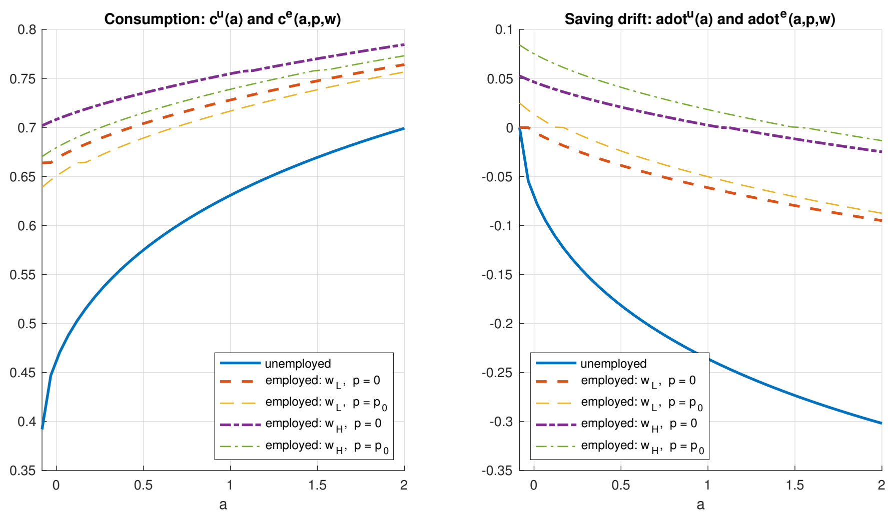
Within each subfigure, the first three panels give slices of employed workers policy functions, and the bottom right panel gives the same for unemployed workers. Low, middle, and high wage are equally spaced across the equilibrium wage distribution.

Figure 5: Market tightness and firm value by submarket. Learning Process.




Left and centre panels plot equilibrium tightness and firm value in each $(a, w')$ submarket. Right panel plots the agents monthly expected EU rate by months of tenure on the current job.

Figure 6: Consumption and saving policy functions




Panels plot consumption (left) and change in assets (right) by assets for various agents. The blue line plots the policy functions for unemployed agents, with the remainder giving slices of the employed policy functions. Low and high wage span the observed wage distribution, and $p = 0$ and $p = p_0$ span the range of beliefs.

probability of fired in their current job, while this probability is definitely higher in a new job. Thus, these workers are only willing to change job if they have enough wealth to tolerate the increased risk of unemployment this brings. Thus, for workers in safe jobs, job finding rates are increasing in wealth, not decreasing. As shown in the equivalent panel in subfigure (b), this is achieved by higher wealth agents searching in higher wage submarkets while their jobs are risky, while the opposite is true once they learn that their jobs are safe.

The remaining panels show that this non-monotonicity in the relationship between wealth and search behaviour holds across the wage distribution. This is the key idea in our model. In a world with incomplete markets, changing jobs can be risky. If this is perceived to be sufficiently risky, low wage agents will experiment less, resulting in lower job finding and EE transition rates. This confounds the typical narrative about the effect of wealth on search behaviour, creating interesting non-monotone dynamics which have implications for how we expect incomplete markets to affect the reallocation and the income distribution.

To understand the source of this result further, Figure 5 and Figure 6 plot further features of the equilibrium. The first two panels of Figure 5 plot firm-side policy functions. The centre panel plots firm value in each $(a, w')$ submarket, given by $J(a, p_0, w')$. This shows that firms face lower value in higher wage submarkets, since in these submarkets workers by construction take more of the period profits as wage payments. This encourages lower vacancy posting in high wage submarkets, and hence lower market tightness, as shown in the left panel. The exact firm value and tightness differ in submarkets indexed with different wealth levels, $a$, since wealth affects the probability a worker will quit and dissolve the match, but this effect is relatively small compared to the direct impact of wage on tightness.

In the right panel we plot the estimated EU hazard by workers tenure on their current job. This follows directly from the learning process: if agents are in a good match their EU rate is $s$, and in a bad match it is $\lambda + s$. Their estimated probability of being in a bad match is $p$, giving total EU rate $\lambda p + s$. Their Bayesian learning process implies that beliefs evolve as known function of their time on the job, giving a deterministic relationship between EU hazard and job tenure. This learning process is the main deviation of our model from standard job ladder models, and generates the excess risk from changing job when you have learned that your job is safe.

In Figure 6 we plot the consumption and saving policy functions across wealth, employment status, and wage and beliefs. Unemployed workers, who have the lowest income, have the lowest consumption (left panel) and consequently dissave all the way to the borrowing constraint (right panel). Employed agents have higher consumption, which is higher when their wage is higher, and when they perceive themselves to be safe. Agents in less safe jobs do extra precautionary saving, and hence lower consumption in response to their higher unemployment risk. The right panel plots the change in assets, meaning that the target saving level for each state is where the line crosses zero. Here we see that higher wage agents save more, doing precautionary saving against possible future wage cuts. Most importantly in this plot, we see the severe dependence of consumption when unemployed on wealth: $c^u(a)$ is declining sharply as wealth falls when we approach the borrowing constraint. This is because unemployed agents are borrowing and further dissaving as they remain unemployed. Thus, as their wealth is exhausted they must sharply cut their consumption. This is not true for employed agents, who are better able to smooth their consumption. This behaviour of consumption is why low wealth agents fear unemployment, and drives the precautionary search behaviour of employed agents.

The forces shown across these figures combine to generate our main result. The fact that the EU rate is declining in tenure generates the increase in risk necessitated by making a job to job transition. The fact that consumption is so sensitive to wealth when unemployed explains why low wealth agents fear unemployment. Agents then use directed search to tradeoff job finding rate and wage when searching for a job, and do so differentially depending on their fear of unemployment. In existing models, low wealth agents fear spending more time with low wealth, in case they become unemployed later, and hence prefer to take a small pay rise and high job finding rate. The novelty of our analysis is that changing jobs may directly increase unemployment risk, meaning that low wealth agents may instead be afraid of making the job switch at all, preferring to take a low job finding rate and changing job only for a sufficiently high pay rise.

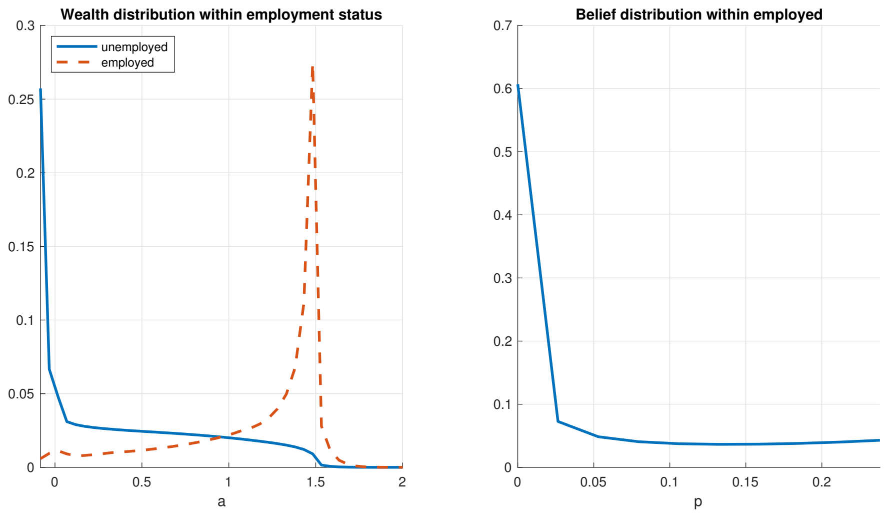
### 4.3 Aggregates and distributions

Having discussed the forces shaping individual behaviour, we now turn to aggregates. In Figure 7 we plot the wealth distribution (left panel) and distribution of beliefs amongst employed agents (right panel). We split the wealth distribution into employed and unemployed agents, and normalise each distribution to integrate to one in the plots.

The wealth distribution illustrates how agents use saving in the non-contingent bond as a form of self insurance in this model. Unemployed agents have lower wealth on average, with a discrete spike of agents stuck up against their borrowing limit. Employed agents, anticipating the risk of unemployment, save.

The belief distribution gives the density of employed agents with belief $p$ that their job is a bad match. While this declines continuously from $p_0$ to 0 with job tenure, as shown in Figure 5, most agents are bunched in jobs with belief close to $p = 0$. This shows that agents prefer to wait in safe jobs, thus distorting the distribution of beliefs. Therefore, the individual preference for safety, and unwillingness to to switch, discussed from individual policy functions, manifests in the aggregate. We discuss this further in the next section, where we compare the model to one where changing jobs entails a smaller rise in risk.

Figure 7: Marginal asset and belief distributions




Left panel plots the asset distributions for employed and unemployed workers. Distributions are normalised to integrate to one within each group. Right panel plots the distribution of beliefs across employed agents.

### 4.4 The role of learning

To illustrate how the extra risk induced by our learning process affects the job ladder, we compare our model to one where most jobs are good matches, meaning that agents are less afraid of experimentation, since it leads to a smaller increase in risk. We call this the "lower learning" model.

Specifically, we recalibrate our model, holding most parameters at their calibrated values from the baseline model. We now target that the EU rate in the first month of a job is only 2 times higher than the baseline EU rate, and declines almost entirely to baseline within one year. We recalibrate the baseline separation rate, $s$, so that the average EU rate is the same as in the baseline model. Thus, in this alternative model all parameters are identical, except that the EU tenure profile is flatter meaning that unemployment risk does not increase as much when starting a new job, and remain riskier throughout the life of a job.[6]

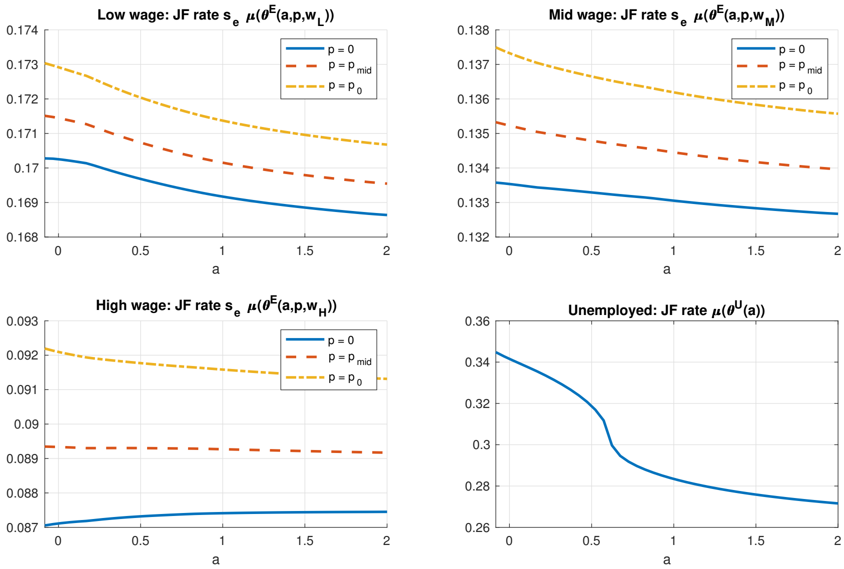
The results are given in Figure 8. Subfigure (a) repeats Figure 4 in the lower learning model. The difference from the baseline model is clear: when learning is removed, job finding rates are now decreasing in wealth across most of the employment distribution. This is because making an EE switch does not raise the probability of being in a bad match and hence being fired. Thus, agents do not fear EE switches, and Chaumont and Shi (2017) result is the main force relating wealth and the EE rate. As with the unemployed, agents search faster for jobs when they are low wealth, in order to guarantee a (smaller) pay rise faster.

Subfigure (b) repeats the asset and belief distribution in the lower learning model. Interestingly, in this version of the model there is less wealth inequality, as evidenced by the smaller spike at high asset levels for employed agents. While the risk of becoming unemployed is the same as in the baseline model, since it is more evenly spread out across the tenure of a job, it is easier for agents to deal with. Intuitively, in our model changing job when your wealth is low is very risky. If you get fired, it is likely to be in the first year of the job, before you have had a chance to use the wealth you can accumulate on the job to self insure yourself. In the lower learning model, while you are working you can always accumulate wealth to improve your buffer stock of savings. Since unemployment risk is evenly spread over the life of a job, agents have more chance to accumulate buffer stock savings, and so can self insure better. This reduces the need to dramatically accumulate wealth, leading to lower wealth inequality.

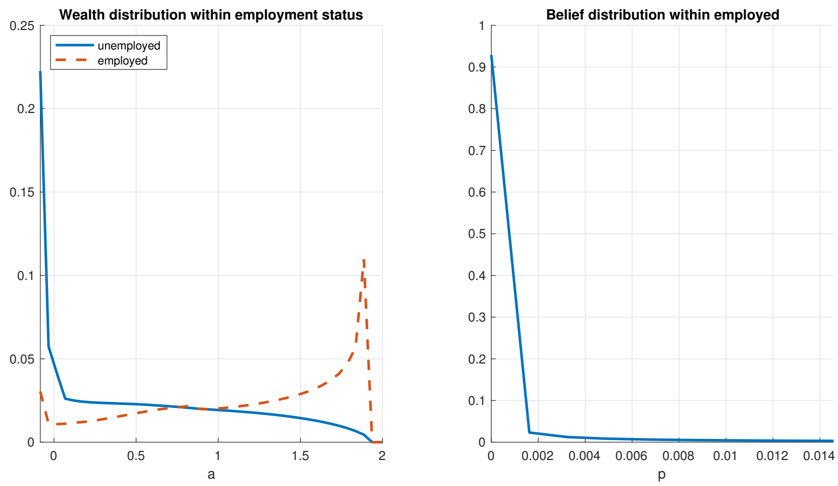
Finally, we compare equilibrium unemployment, average wage, and EE switching rate in our baseline model to the recalibrated model. Several interesting differences emerge. The labour share is higher in the baseline model than in a model with less learning. The EE rate is 30% lower when learning is increased, which shows the key idea of the paper, that learning risk reduces the incentive to change jobs. Finally, the unemployment rate is 9% higher when learning is increased. This shows that learning and incomplete markets have important implications not only for the job ladder and reallocation of already employed workers, but for the overall unemployment rate as well.

[6] We continue to target the same equilibrium interest rate of 3%, meaning that we do not change the equilibrium in the asset market, and only study the effect on the labour market for a given level of the interest rate.

Figure 8: Results from model with less learning




(a) Job finding rate by assets, employment status, wage, and beliefs




(b) New wage target by assets, employment status, wage, and beliefs

Top subfigure replicates Figure 4 for the model recalibrated to reduce the importance of learning, showing how job finding rates vary across agents. Bottom subfigure does the same for Figure 7, showing the marginal asset and belief distributions.

## 5 Empirical Evidence

## 6 Conclusions

How does wealth affect the incentive to search for a new job when job-to-job moves are risky? In this paper, we answered this question by incorporating incomplete markets and learning about downside risk into a directed-search job ladder model. We developed a new "risky job ladder" model, showing that the interaction between the increase in risk from changing jobs and incomplete markets has effects on both individual job search and savings behaviour, and on the aggregate economy.

We built a continuous-time heterogeneous agent model where risk-averse agents learn about the quality of their current job (a.k.a. "match") over time while accumulating assets to smooth uninsurable income risk. Agents are allowed to search both on and off the job for new matches, and can direct their search towards jobs of different wages and (hence) job finding probabilities. In our model, agents gradually learn if they are suited for their current job, and therefore their "safety". If an agent perceives themselves to be in a safe job, moving job increases the risk of being in a bad match, and hence being fired.

Our main result is that when asset markets are incomplete, low wealth agents may therefore prefer to stay in low paying jobs which they have learned to be safe, rather than experimenting with a new job, even if it comes with a pay rise. We show that this implies a highly non-monotonic relationship between wealth and job-to-job mobility, differently from the existing literature, which we confirm using worker micro data. In the aggregate, relative to a model without learning, this difference in worker behaviour leads to more wealth inequality, a higher labour share, less job-to-job mobility, and higher unemployment.

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# A Proofs and derivations

## A.1 Proof of Lemma 1

The definition of $\psi(w)$ follows from straightforward differentiation of the expected utility with respect to the targeted wages. The derivative of tightness with respect to wage follows from the free entry condition:

$$\kappa = \zeta(\theta(w))(y-w) \implies 0 = \frac{\partial \zeta(\theta(w))}{\partial \theta}\frac{\partial \theta}{\partial w}(y-w) - \zeta(\theta(w)) \implies \frac{\partial \theta}{\partial w} = \frac{\zeta(\theta(w))}{\frac{\partial \zeta(\theta(w))}{\partial \theta}}\frac{1}{y-w} < 0,$$

as $\frac{\partial \zeta(\theta)}{\partial \theta} < 0$. We scrap dependence of $\theta$ on $w$ for notational convenience and arrive at:

$$\psi(w) = -(y-w)\frac{\mu(\theta)}{\mu'(\theta)}\frac{\zeta'(\theta)}{\zeta(\theta)} = -(y-w)\frac{\mu(\theta)}{\theta\mu'(\theta)}\frac{\theta\zeta'(\theta)}{\zeta(\theta)} = (y-w)\frac{\phi(\theta)}{1-\phi(\theta)},$$

where $\phi(\theta) = -\frac{\theta\zeta'(\theta)}{\zeta(\theta)}$ is the elasticity of the job filling rate with respect to market tightness[7], so that $1-\phi(\theta) = \frac{\theta\mu'(\theta)}{\mu(\theta)}$. It holds that $\phi'(\theta) \geq 0$ with equality for constant elasticity of substitution matching functions. Finally:

$$\psi'(w) = -\frac{\phi(\theta)}{1-\phi(\theta)} + (y-w)\frac{\phi'(\theta)}{[1-\phi(\theta)]^2}\frac{\partial \theta}{\partial w} < 0.$$

Regarding monotonicity, it's enough to focus on the employees, as they nest the problem of the unemployed for $w = b$ and $\sigma_l = \sigma_h$. Therefore, we rearrange the optimality condition for employees to read:

$$-u(A+w) + \frac{\sigma_h - \sigma_l}{1-\sigma_h}\left(u(A+b) - u(A+w)\right) = \psi(w^e)u'(A+w^e) - u(A+w^e). \tag{20}$$

The differentiation with respect to current wage yields:

$$\underbrace{-\frac{1-\sigma_l}{1-\sigma_h}u'(A+w)}_{<0} = \underbrace{\left[\left(\psi'(w^e)-1\right)u'(A+w^e) + \psi(w^e)u''(A+w^e)\right]}_{<0}\frac{\partial w^e}{\partial w} \iff \frac{\partial w^e}{\partial w} > 0,$$

while the differentiation with respect to unemployment benefit $b$ yields:

$$\underbrace{\frac{\sigma_h - \sigma_l}{1-\sigma_h}u'(A+w)}_{>0} = \underbrace{\left[\left(\psi'(w^e)-1\right)u'(A+w^e) + \psi(w^e)u''(A+w^e)\right]}_{<0}\frac{\partial w^e}{\partial b} \iff \frac{\partial w^e}{\partial b} < 0.$$

### A.2 Proof of Example 2

For the effects of individual wealth on target wage it's enough to focus on the unemployed, as the only difference between them and the employees for $\sigma_h = \sigma_l$ is in $b \neq w$. Rearranging and

[7] It is also the surplus splitting rule in competitive search models, $\phi(\theta) \in (0,1)$.

differentiating with respect to $A$ yields:

$$\frac{\left[u'(A+w^u)(1+\frac{\partial w^u}{\partial A}) - u'(A+b)\right] u'(A+w^u)}{[u'(A+w^u)]^2}$$
$$-\frac{[u(A+w^u)-u(A+b)]\, u''(A+w^u)}{[u'(A+w^u)]^2}(1+\frac{\partial w^u}{\partial A}) = \psi'(w^u)\frac{\partial w^u}{\partial A}$$

so that:

$$\frac{1-\frac{u'(A+b)}{u'(A+w^u)} - \frac{u''(A+w^u)[u(A+w^u)-u(A+b)]}{[u'(A+w^u)]^2}}{\psi'(w^u) - 1 + \frac{u''(A+w^u)[u(A+w^u)-u(A+b)]}{[u'(A+w^u)]^2}} = \frac{\partial w^u}{\partial A} \tag{21}$$

The denominator of the ratio on the left hand side is negative, hence to determine the sign of the partial derivative we focus solely on the numerator. As $u(c)$ is strictly concave, hence the standard result on the function being strictly bounded from above by first order Taylor approximation for $w^u > b$:

$$\begin{aligned}
&u(A+w^u) - u(A+b) < u'(A+b)\,(w^u - b) \implies \\
&1 - \frac{u'(A+b)}{u'(A+w^u)} - \frac{u''(A+w^u)\,[u(A+w^u)-u(A+b)]}{[u'(A+w^u)]^2} < \\
&1 - \frac{u'(A+b)}{u'(A+w^u)} - \frac{u''(A+w^u)u'(A+b)\,(w^u-b)}{[u'(A+w^u)]^2} = \\
&1 - \frac{u'(A+b)}{u'(A+w^u)}\left(1 + \frac{u''(A+w^u)\,(w^u-b)}{u'(A+w^u)}\right).
\end{aligned}$$

Now, let's assume $u(c) = \log(c)$:

$$1 - \frac{u'(A+b)}{u'(A+w^u)}\left(1+\frac{u''(A+w^u)\,(w^u-b)}{u'(A+w^u)}\right) = 1 - \frac{u'(A+b)}{u'(A+w^u)}\left(1 - \frac{w^u-b}{A+w^u}\right),$$

and

$$1 - \frac{u'(A+b)}{u'(A+w^u)}\left(1-\frac{w^u-b}{A+w^u}\right) = 1 - \frac{A+w^u}{A+b}\frac{A+b}{A+w^u} = 0,$$

hence, as 0 is a strict upper bound for the numerator, we have shown that $\frac{\partial w^u}{\partial A} > 0$. Note that fully analogous derivations are applicable for $\frac{\partial w^e}{\partial A} > 0$ when $\sigma_h = \sigma_l$ with the sole difference being that $A+b$ is replaced by $A+w$. The second result in this Example is a straightforward outcome of manipulation of the numerator in condition (4) assuming it's greater than zero:

$$\frac{\sigma_h - \sigma_l}{1 - \sigma_h} u'(A + b) - \frac{1 - \sigma_l}{1 - \sigma_h} u'(A + w) > 0.$$