Asset Purchases, Limited Asset Markets Participation and Inequality

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Abstract

I provide novel evidence that quantitative easing (QE) reduces inequality in the Euro Area. Using a SVAR with high frequency identification I show that an identified QE shock for the Euro Area is redistributive and expansionary. Then, I build a New Keynesian DSGE model with household heterogeneity, financial frictions and nominal rigidities to rationalize the empirical findings. Bond purchases increase aggregate demand and benefit financially restricted households more than investors, due to the dominance of QE's indirect effects. Furthermore, the foregoing term spread between bonds and reserves harms the investors and thus reduces income and consumption inequality in line with my empirical findings. Finally, in a normative exercise, I show that this result can change when considering a subset of Euro Area members with low asset markets participation and flexible wage setting. QE can be contractionary and increase inequality when these two conditions are met. I show analytically that this result is a product of the profit generated income effects which are active in a flexible wage setting.

JEL classification: E44; E52; E58; E61

Keywords: Quantitative Easing; Inequality; Financial Participation; Heterogeneity; DSGE; SVAR; External Instrument

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1. Introduction

Asset purchase programmes following the Great Recession and the ongoing Covid-19 pandemic have been extensively employed to hold down long-term interest rates and stimulate aggregate demand. A question posed by policymakers (Yellen, 2016; Bernanke, 2015; Draghi, 2016) and which has gained the media attention is whether and to what extent asset purchase programmes have contributed to the increase of inequality.[1] In this paper, I analyse the impact of QE on households' inequality empirically and also using a state of the art quantitative macro model with heterogeneous households in their financial holdings. Both the empirical and the theoretical model show that QE in the Euro Area reduces inequality and produce the same responses. I finally explore the effects of QE on inequality in a subset of countries with low asset markets participation. This is particularly interesting for some EA member states that share this characteristic. Low participation leads to an inequality increase and a contractionary QE.

Figure 1 shows the income inequality index for the the Euro Area using data from the Standardized World Income Inequality Database (henceforth SWIID), Solt (2020).[2] Income inequality in the Euro Area as proxied by the Gini index, has been growing steadily from the beginning of the EA until the end of 2016. In 2016, a year after the Asset Purchase Programme by the ECB had been initiated, started to decline. In this paper I am looking on whether the ECB's QE programme initiated in 2015, has helped to reduce inequality.

I develop a New Keynesian DSGE model calibrated for the Euro Area with nominal rigidities, financial intermediaries, financially constrained and unconstrained households and a central bank that can purchase assets from banks and unconstrained households by issuing reserves. The QE framework follows Gertler and Karadi (2013) but a formal representation of reserves and the asset swap mechanism induced by the QE is developed. Household Two Agents New Keynesian (TANK) specification borrows from Galí, López-Salido, and Vallés (2007) (GLV hereafter) and banks are modelled similarly to Gertler and Kiyotaki (2010): a costly enforcement problem creates a leverage constraint on the intermediaries. QE induces more lending through relaxing the leverage constraints of the financial intermediaries, similarly to Sims and Wu (2021), by the exchange of banks' government bonds with reserves and thus stimulates aggregate demand. A setting is developed where central bank purchases of government bonds financed by reserves create direct and indirect effects on the real economy affecting differently those with and without access to financial markets. QE in the model works as a credit stimulating mechanism to the real economy. QE-induced effects are possible due to the existence of the banker's leverage constraint which eliminates the perfect substitutability of assets and leads to money non-neutrality.[3]

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[1]Does Quantitative Easing Mainly Help the Rich? (CNBC), Debate rages on quantitative easing's effect on inequality (Financial Times), Quantitative easing helped vulnerable more than rich, says ECB (Financial Times).

[2]A detailed explanation of how I construct the index is provided in Section 2.

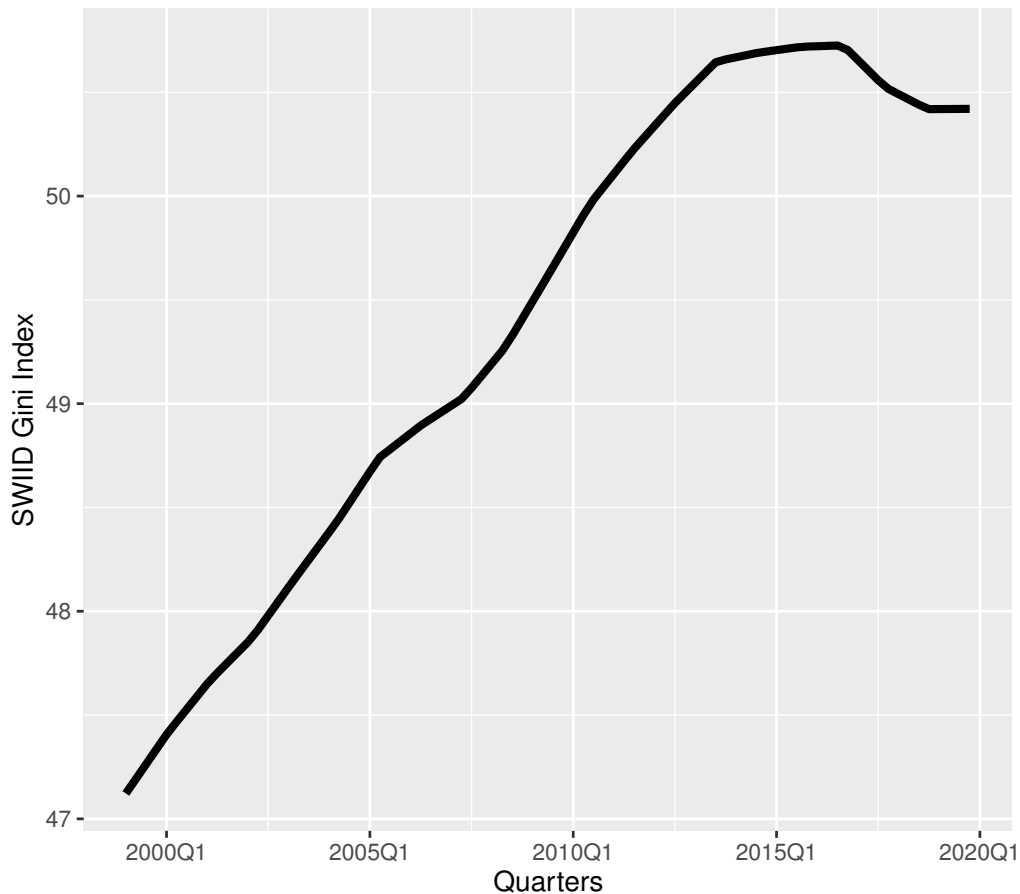[3]Studies that show the neutrality of open market operations in frictionless settings originate from

Fig. 1. Income Inequality Index for the the Euro Area

I show that the indirect (i.e general equilibrium) effects outweigh the direct effects (i.e. asset price increase and reduction in credit and bonds spreads) leading to a reduction in income and consumption inequality. Consider an increase in the bond holdings of the central bank in an economy with two types of consumers, agents with a multi-asset portfolio and hand to mouth, financially constraint, consumers. Asset prices increases benefit the bond holders but the reduced interest returns harm their portfolios. Furthermore, reserves' interest rate receivable would be much lower than the risky assets they held. I show that the interest rate differential channel dominates and savers' income falls after a QE. The indirect effects from wage increases post-QE benefit the hand to mouth consumers whose income is their labour compensation. This leads to a reduction in income and consumption inequality between the two income groups. Bernanke (2020) provides a similar argument: the potential benefits to asset holders of higher asset prices are partially offset by the lower returns they can earn on their wealth.

Wallace (1981). He was the first to show that open market operations are not effective under the assumption of the same return between money and assets purchased and a fixed fiscal policy stance. The result remains the same in future studies built on more relaxed assumptions. See also Sargent and Smith (1987); Chamley and Polemarchakis (1984); Eggertsson and Woodford (2003); Curdia and Woodford (2010).

Here, I show that the benefits are not only offset but outweighed by the lower returns. Lastly, I show that the inequality response after a QE programme is countercyclical, similar to that of a policy rate cut which has been empirically examined by Coibion, Gorodnichenko, Kueng, and Silvia (2017) for the US and Samarina and Nguyen (2019) for the Euro Area. Bilbiie, Kanzig, and Surico (2021) in a simple model with heterogeneity and capital agrees with these findings.

Limited asset markets participation modelling incorporates two relatively recently explored data facts on the asset composition and the consumption smoothing of the households. This heterogeneity observed in the data, is what creates changes in inequality between the two groups due to monetary policy. Empirical literature shows that a large fraction of households consumption plans can be tracked almost exclusively by their income.[4] In the Euro Area, about 20 % of the population can be characterized as hand to mouth consumers; they hold a total value of financial assets that is close to zero.[5]

To motivate my research question I investigate the relationship between QE and inequality empirically. This is done by the use of an external instrument SVAR with high frequency identification based on the work of Mertens and Ravn (2013) and Gertler and Karadi (2015). To identify QE policy surprises I make use of the Euro Area Monetary Policy Event Study Database by Altavilla, Brugnolini, Gürkaynak, Motto, and Ragusa (2019) and I develop and use the QE factor as external instrument. Using Euro Area data I show that a QE shock is expansionary and redistributive.

Given the fact that the quantitative model mimics well the findings of the empirical model, in the final part of the paper, I proceed to a normative exercise. I show that expansionary and distributive impact of the QE can vanish when the asset markets participation level is low and the wages are flexible. Specifically, consumption and income inequality, if these two conditions hold, increase after a QE shock and QE becomes a contractionary monetary policy. This is particularly interesting for some Euro Area economies that have both low asset market participation and a high degree of wage flexibility. As can be seen in Figure 5 provided in Section 5, Estonia, Latvia, Malta and Ireland are countries that fulfil these two conditions.

The economic intuition of this result is as follows. After a QE shock, when labour markets are competitive, an increase in marginal costs leads to a fall in profits, as in the class of NK models after a monetary easing (see Christiano, Eichenbaum, and Evans (2005)). The former effect, combined with low asset markets participation leads to a negative response of output after QE due to the substantially negative income effect the savers suffer from countercyclical profits. This leads to an increase in consumption and income inequality, in contrast with the baseline model in which the asset market participation is set at the EA average. In an economy with wage rigidities, marginal

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[4]Recent studies on the households' wealth distribution, originating from Mankiw (2000) in the US, have shown the existence of a population share of 30-40% that does not smooth consumption across states and time. See also Krueger, Mitman, and Perri (2016). The authors find that 40% oh the households hold almost no net worth.

[5]See Appendix A, I show using household-level data, the Eurosystem Household Finance and Consumption Survey (HFCS), the households' financial assets distribution.

costs do not increase as much and profits are pro-cyclical leading to a positive output's response for any level of asset market participation. This result complements Broer, Harbo Hansen, Krusell, and Öberg (2020) findings on the countercyclical behaviour of profits in the flexible wage NK model and extends it to a financial frictions model with heterogeneity focusing on the effect of QE on inequality.

Related Literature. This study contributes to the fast-growing literature of unconventional monetary policy; precisely to three strands: on models of households heterogeneity with financial frictions, on impact effects of QE to inequality and on the external instrument SVARs.

Galí et al. (2007) (GLV) firstly introduced a two-agents NK framework to study the effects of government spending on consumption. More recently, Kaplan, Moll, and Violante (2018), develop Aiyagari-type heterogeneous agent frameworks with New Keynesian nominal rigidities (HANK) making the characterization and study of the full income and wealth distribution feasible.[6] As shown by Debortoli and Galí (2017), a two agents framework is able to identify differences in average consumption *between* the constrained and unconstrained agents but is less effective in characterising consumption heterogeneity *within* the subset of unconstrained households. Since the main focus of this paper is on the interactions between the two types of agents, it suffices to use a less rich setting of heterogeneity and build on the GLV framework.[7]

The paper borrows also from the financial frictions literature. This addition to macroeconomic models goes back to Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999) and a recent post-crisis revival originated by Gertler and Kiyotaki (2010).[8] A costly enforcement problem limits the ability to arbitrage across the deposit, bond and credit markets.[9] This paper, to my knowledge, is the first one that adds financial fictions à-la Gertler and Karadi (2011) in a model with limited asset markets participation and analyses the real effects of QE on inequality and aggregate demand.

The emphasis on the heterogeneity in financial asset holdings relates this paper also to Bilbiie (2008) who uses a simplified version of GLV and shows that a change in the level of asset market participation can alter the sing on the monetary policy shock on aggregate demand: an interest rate hike can have stimulative effects when asset markets participation is low. Colciago (2011) shows that when wages are rigid this result does not hold due to the procyclicality of profits. Broer et al. (2020) identify as well the importance of wage rigidities on the cyclicality of profits in a two agent model while Cantore and Freund (2021) propose a novel TANK variation where the savers receive only dividends and no labour income. Their model also cancels out the large profit income effect existing in textbook NK models. The present paper follows this reasoning and extends the result to a QE framework with a financial frictions model.

This paper is also closely related to the literature on the distributional effects of mon-

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[6]See also McKay and Reis (2016), Acharya and Dogra (2020) and Ravn and Sterk (2021).

[7]Studies using a TANK framework also include Monacelli (2009), Bilbiie, Monacelli, and Perotti (2013), Colciago (2011) and Galí, López-Salido, and Vallés (2004).

[8]For a comprehensive literature review see Brunnermeier, Eisenbach, and Sannikov (2013).

[9]For a similar financial frictions setting see also Carlstrom, Fuerst, and Paustian (2017) and Sims and Wu (2021) among others.

etary policy. The seminal paper in this fast growing literature, Coibion et al. (2017), focuses on the impact of conventional monetary policy in the US and shows that inequality decreases after an interest rate cut. Most empirical studies agree that the QE effects benefit mostly the lower end of income distribution in line with this paper's results. Lenza and Slacalek (2018) employ a Bayesian VAR for the EA to identify the effects of asset purchases showing that QE reduces income and wealth inequality. Ampudia, Georgarakos, Slacalek, Tristani, Vermeulen, and Violante (2018) similarly show that the indirect effects of monetary policy outweigh the direct ones. Bilbiie et al. (2021) in a HANK model with capital, but without a banking sector, find also that income and consumption inequality are countercyclical. Empirical studies by Bunn, Pugh, Yeates et al. (2018) using UK data and Bivens (2015) also concur on the relatively greater effect on lower income households.[10]

Cui and Sterk (2021) and Hohberger, Priftis, and Vogel (2019a) are the studies this paper is more closely related to. Hohberger et al. (2019a) in parallel work, conduct a similar study where they evaluate the effects of QE on consumption and income inequality in a standard NK setting with two agents. They show that consumption and income inequality fall after a QE policy, in line with this paper's results. Cui and Sterk (2021) use heterogeneous-agents model with liquid and partially liquid wealth, and nominal rigidities. They find that a QE policy reduces consumption and income inequality initially but increases both later on. This comes mainly from the fact the inequality in liquid assets increases. Here, this effect is absent since the rule of thumb income group holds zero liquid or illiquid assets, making inequality decrease after a QE.

The aforementioned papers differ in many characteristics with the present study. This study is enriched with a financial sector that is the key to the QE pass-through to the real economy and to inequality. QE relaxes the leverage constraint of the financial intermediaries. Furthermore it is what eliminates the perfect substitutability of assets and the QE neutrality in contrast with the above papers which this occurs through exogenous portfolio costs. Lastly, this study explores the effects asset market participation heterogeneity on QE effects in two different labour market settings. To my knowledge there is no other study employing a TANK model with financial frictions and an explicit framework for asset purchases by the central bank that measures the QE effects on consumption and income inequality and the impact financial asset holdings heterogeneity has on the QE.

Finally, related to this paper's empirical specification there is a vast literature on SVARs using a set of different identification methods. A very comprehensive summary is included in Ramey (2016). The present study is the first one to employ an external SVAR with the monetary policy surprises as in Altavilla et al. (2019) to identify the QE's impact in the Euro Area.

The plan of the paper is as follows. Section 3 describes the model. In Section 4, I show the first result of the paper: how the QE calibrated to mimic the Asset Purchase Programme affects the Euro Area economy and consumption and income inequality.

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[10]For a literature review on macroeconomic models with heterogeneity and their distributional effects see Colciago, Samarina, and de Haan (2018).

In Section 5 it is shown in an analytical and quantitative framework that QE can have a negative impact conditional to asset markets participation level and the wage setting scheme. Section **??** verifies the findings of the structural model findings using an external instrument SVAR. Finally, Section 6 concludes.

2.   Quantitative Easing and Inequality: SVAR Evidence

I provide novel empirical evidence on identifying the impact of an expansionary unconventional monetary policy shock on inequality and the macroeconomy in an external instrument SVAR model. Results show that QE is stimulative and redistributive by decreasing income inequality. The data is for the Euro Area aggregate level and an I construct an income inequality index for the EA using the Gini coefficient reported in the Standardized World Income Inequality Database (henceforth SWIID) Solt (2020). I employ the proxy-SVAR approach as introduced by Stock and Watson (2012) and Mertens and Ravn (2013). Due to the difficulty of identifying monetary shocks in the data as elaborated in Ramey (2016), this approach provides a novel way that makes use of external instruments for the structural shocks of interest. The method I use is most closely related to Gertler and Karadi (2015) high-frequency identification (HFI) approach.

To identify external instruments for the QE shock I use the Euro Area Monetary Policy Event Study Database (EA-MPD) constructed in Altavilla et al. (2019) (ABGMR hereafter), together with their methodology to extract the factors. The novelty of this approach is that a QE factor can be extracted from the data and be used directly as an instrument.

Data. I analyse quarterly data from 1999Q1 to 2019Q4, starting at the starting year of the Euro Area and leaving out of my sample the current pandemic. The baseline VAR has ten variables, including two policy indicators, a Euro Area income inequality index, the 10 year Euro Area benchmark bond rate and the the 3 month rate and seven economic and financial variables: the CPI, the real GDP, a EA stock prices index, the employment level, a measure for the wages, real consumption and real profits. This follows a similar specification by Slacalek, Tristani, and Violante (2020). The VAR has two lags based on the AIC criterion.

The income inequality data comes from the Gini coefficient reported in the SWIID. I take the Gini of equivalized household market (pre-tax, pre-transfer) income. Results hold with also the post-tax, post-transfer specification. I construct an income inequality index for the Euro Area by combining all Euro Area countries' inequality data from the SWIID dataset and weighting them by the respective country GDP weights.

Data for the macro variables comes -mainly- from the Area Wide Model dataset originally constructed by Fagan, Henry, and Mestre (2001). The updated AWM database starts in 1970Q1 (for most variables) and is available until 2017Q4. To update the data further, I make use of publicly updated data from Eurostat, ECB and the OECD.

Given that the analysis is focused on the Quantitative Easing, the instrument is used for the period 2014 to the end of the sample, which is the period QE took place in the

Euro Area. I thus use the full sample 1999Q1 to 2019Q4 to estimate the lag coefficients and obtain the reduced form residuals in equation (1). Then I use the reduced form residuals for the period 2014 to 2019 to identify the impact of QE surprises (i.e the vector **S**).

Estimation Methodology. The VAR has the following general structural form:

$$\mathbf{A}\mathbf{V_t} = c + \sum_{i=1}^{p} \mathbf{C_j}\mathbf{V_{t-1}} + \epsilon_\mathbf{t},$$

where $\mathbf{V_t}$ is a vector of the n economic and financial variables included in the estimation, $\mathbf{C_j}$ for $j = 1...p$ are $n \times n$ coefficient matrices and $\epsilon_\mathbf{t}$ is a $n \times 1$ vector of structural white noise shocks. Multiplying each side by $\mathbf{A}^{-1}$ we get the reduced form VAR:

$$\mathbf{V_t} = c + \sum_{i=1}^{p} \mathbf{B_j}\mathbf{V_{t-1}} + \mathbf{u_t}, \tag{1}$$

$u_t = \mathbf{S}\epsilon_\mathbf{t}$ is the reduced form residuals, a function of the structural shocks $\epsilon_\mathbf{t}$. Also, $\mathbf{B_j} = \mathbf{A}^{-1}\mathbf{C_j}$ and $\mathbf{S} = \mathbf{A}^{-1}$.

We can partition the vector of structural shocks according to the structural shock of interest, in this case the QE shock, and the rest. That is

$$\epsilon_\mathbf{t} = \begin{bmatrix} \epsilon_t^{QE} & \epsilon_t^{R} \end{bmatrix}$$

Let s denote the column matrix of **S** which is associated with the impact of the reduced form residuals $\mathbf{u_t}$ of the structural shock of interest ϵ_t^{QE}. To compute the impulse responses of the system to this shock we have to estimate:

$$\mathbf{V_t} = c + \sum_{i=1}^{p} \mathbf{B_j}\mathbf{V_{t-1}} + \mathbf{s}\epsilon_t^{QE}$$

At this stage we could proceed by applying the widely used timing or coefficient restrictions as is common in the SVAR literature (see for example the coefficient restrictions in Blanchard and Perotti (2002) or sign restrictions in Mountford and Uhlig (2009)) in order to identify the elements in s. In a study with similar scope to the present, Lenza and Slacalek (2018) use a combination of zero and sign restrictions. They make the identifying assumption that an expansionary asset purchase shock decreases the term spread (defined as long-term minus short-term interest rate) and has a positive impact on the real economy of the four countries under analysis. As mentioned in Gertler and Karadi (2015), this is problematic for a VAR that includes financial variables, like the present one, in which the policy indicator has contemporaneous effect on financial variables. Therefore, I follow the work of Mertens and Ravn (2013) and Stock and Watson (2012) and use the proxy-SVAR method to obtain covariance restrictions which allows for no direct, hard-wired assumptions on the elements of **S**.

Let $\mathbf{Z_t}$ be the instrument of interest that is correlated with the shock of interest, in

this instance the QE shock ϵ_t^{QE}, but is also orthogonal to all the rest of the shocks ϵ_t^R, that is:

$$\mathbb{E}_t[\mathbf{Z_t}\epsilon_t^{QE}] = \Phi \tag{2}$$

$$\mathbb{E}_t[\mathbf{Z_t}\epsilon_t^R] = 0. \tag{3}$$

Condition (2) is the relevance condition that states that the correlation between the instrument and the structural policy shock must be different from zero where Φ is a scalar. Condition (3) is the exogeneity condition that implies the instrument is uncorrelated with any other structural shock. When these two conditions are met the instrument can be used as a proxy for the structural shock ϵ_t^{QE}. These two assumptions are the key identifying assumptions which add restrictions to the matrix $\mathbf{S}$.

The estimation proceeds in the three following steps: First I estimate the reduced form VAR (1) with least squares to obtain estimates of the reduced form residuals vector $\mathbf{u_t}$. We can partition the vector $\mathbf{u_t}$ into residual from the policy indicator equation and from the rest of the variables different from the policy indicator which yields $\mathbf{u_t} = [\mathbf{u_t^{QE}} \mathbf{u_t^R}]$. In order to isolate the variation in $\mathbf{u_t^{QE}}$ that is due to the structural monetary policy shock ϵ_t^{QE} only, we regress the former on the vector of instrumental variables Z_t and a constant:

$$\mathbf{u_t^{QE}} = \alpha + \beta Z_t + \psi_t.$$

The fitted value that yields from the regression $(\mathbf{u_t^{QE}})^f$ can be used to estimate the ratio of $\mathbf{s^R}/s^{QE}$:

$$\mathbf{u_t^R} = \frac{\mathbf{s^R}}{s^{QE}}(\mathbf{u_t^{QE}})^f + \theta_t.$$

This yields and unbiased ratio of $\mathbf{s^R}/s^{QE}$.

Identifying the QE surprises. I construct the instrument used for the QE shock using changes in the yields of risk-free rates at different maturities, spanning one-month to ten-years, around the EA policy meetings. Data comes from the Altavilla et al. (2019) EA-MPD dataset that is continuously updated and covers the period 2002 to 2020. The EA-MPD dataset reports median price changes around the time interval of past ECB monetary policy meetings for a broad class of assets and various maturities, including Overnight Index Swaps (OIS), sovereign yields, stock prices, and exchange rates. ECB monetary meetings have a distinct sequence, firstly there is the press release at at 13.45 Central European Time where a policy decision in announced without further elaboration followed by the press conference at 14.30 where the monetary policy strategy and its details are explained more broadly.

Using tick data, they document the price changes about 10 minutes before and after the meeting and they estimate by principal components the factors that yield from the monetary policy changes. To extract monetary policy surprises that are economically interpretable the factors are rotated as in Gürkaynak (2005). The rotation is made such that the QE factor has no impact in the 1 month OIS rates and also has no impact in the pre-crisis period of the dataset 2002-2008 (the factor is restricted to have the smallest variance in that period). Based on the risk free assets' maturity type those factors load,

four factors are identified: the "Target" that loads only on the short rates, "Timing", "Forward Guidance" and the "QE" factor that loads only in the longer-term rates.

ABGMR have estimated the factors up to 2018. I proceed by updating the monetary policy factors until 2020 using the up to date EA-MPD dataset and following the work of Gürkaynak, Sack, and Swanson (2007) and the procedures of ABGMR described above, I estimate and rotate the latent factors in the same fashion. Naturally, in my VAR exercise I use the QE factor as an external instrument for the QE surprises. Given that the rest of the dataset is in quarterly frequency, I follow Slacalek et al. (2020) and sum all the intra-day surprises of the QE factor that occur in a quarter.
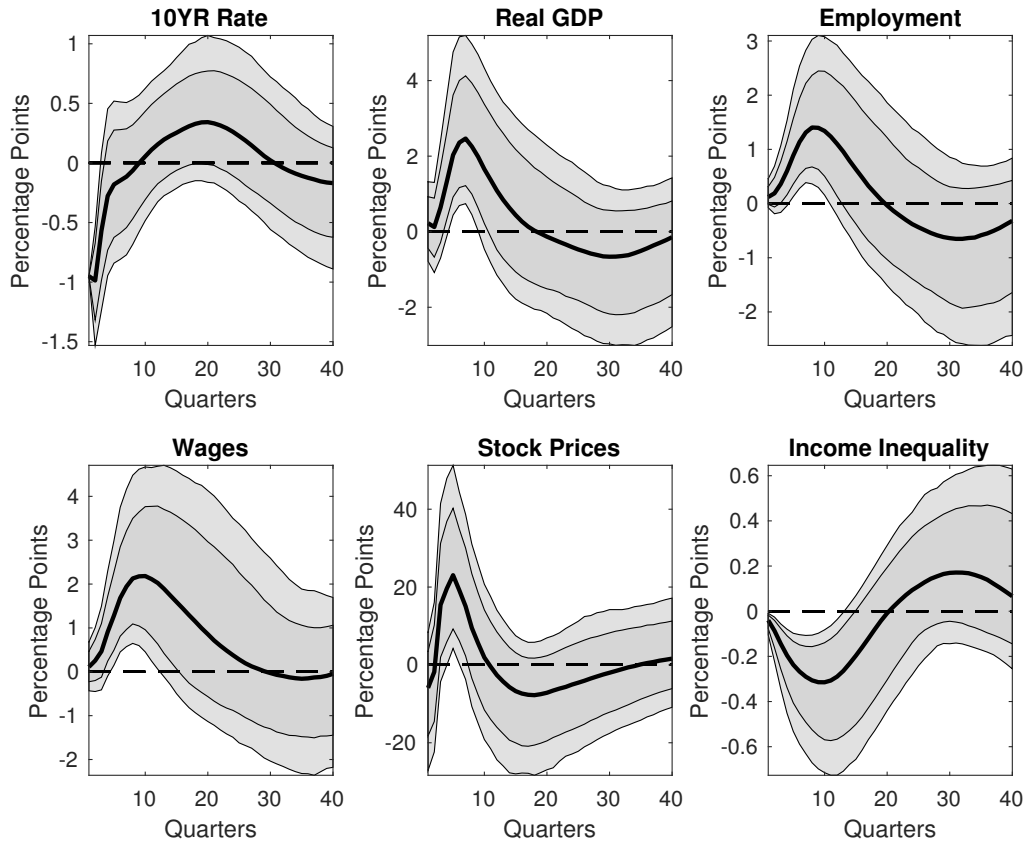

Fig. 2. Impulse Responses to a QE Shock. *Notes: The solid line shows the median responses after 50000 draws. The darker bands span the 16-84 percentiles of the draws distribution while the lighter band the 9-95 percentiles. The X axis shows the quarters while the Y axis the percent change.*

Results. Figure 2 shows the impulse responses of the 10 year benchmark rate of the Euro Area and the income inequality index, after a QE shock. The shock is scaled such that it reduces the ten-year rate by 95 bps on impact as has been documented the

APP did. [11] The solid line shows the median responses after 50000 draws. The darker bands span the 16-84 percentiles of the draws distribution while the lighter band the 9-95 percentiles. Figure 8 showing the impulse responses of all the variables used in the SVAR estimation, is in Appendix B. For robustness, the same exercise using the Cholesky identification has been performed and is also presented in Appendix B.

The responses of macroeconomic variables to a quantitative easing shock are in line with the evidence on the topic. Following a reduction of the ten-year rate by 95 bps output increases at its peak at 2 percentage points. Output's response gradually dies out after 10 quarters. A similar pattern is followed by the responses of wages, employment and stock prices; they peak at 1,2 and 20 percentage points respectively after the QE shock. Inflation's response (shown in Appendix) in positive but insignificant while consumption and profits, also shown in the Appendix, are impacted positively by the shock.

Income inequality is significantly affected by the unconventional monetary policy. After the shock, it decreases significantly by 0.3 percentage points. The response remains negative and statistically significant for two years. Then the response becomes positive but is insignificant. In terms of magnitude, the impact on income inequality is small relative to the other macroeconomic variable responses but nevertheless statistically significant.

3.   The Model

The economy is populated by two types of households: Rule of thumb and optimising households that differ in their ability to participate in the assets market. A continuum of firms and financial intermediaries owned by the optimizers, labour wide unions that set the wages, capital goods producers and retailers, a monetary authority and the treasury complete the model economy. There is a moral hazard problem between the savers and the banks. Banks can steal a fraction of their funds and return them to their families. This problem introduces an incentive constraint to the model to be followed by the banks. Finally, the central bank performs its conventional monetary policy under a Taylor rule, but can also engage in asset purchases and pay the investors back the same value in newly created reserves.

3.1.   Households - The Two Agents Framework

All households are assumed to have identical preferences, given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i [\ln(C_{t+i}^s) - \frac{\chi}{1+\epsilon} L_{t+i}^{1+\epsilon,s}], \tag{4}$$

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[11] Given the recent analysis of Eser, Lemke, Nyholm, Radde, and Vladu (2019) on the APP's impact on the yield curve: "A 10 year term premium compression of around 50 bps was associated with the initial APP announcement in January 2015. With the expansion of the programme the yield curve impact has become more marked and is estimated to be around 95 bps in June 2018."

C_{t+i}^s denotes the per capita consumption of the household members and L_{t+i}^s the supply of labour. The super-index $s \in [o, r]$ specifies the household type (o for "optimizers" or r for "rule of thumb"). $\beta \in [0, 1]$ is the discount factor. Due to the stochastic setting, households make expectations for the future based on what they know in time t and $\mathbb{E}_t$ is the expectation operator at time t. Finally, ϵ is the inverse Frisch elasticity of labour supply and χ is the relative utility weight of labour.

Optimizers. Optimizers account to a measure of $(1-\lambda)$ of the economy's population. Their portfolio includes one period government bonds B_t^o, bank deposits D_t^o and firm shares S_t^o. They can freely adjust their deposit holdings. However, they are not experts in trading bonds and shares. Transactions above or below a frictionless level $\bar{S}_t^o$ and $\bar{B}_t^o$ for shares and bonds respectively require broker expertise and this induces costs. Costs equal to $\frac{1}{2}\kappa(S_t^o - \bar{S}_t^o)^2$ for shares and $\frac{1}{2}\kappa(B_t^o - \bar{B}_t^o)^2$ for bonds deviating from their respective frictionless level.[12]

Optimizing households budget constraint then is

$$C_t^o + T_t^o + D_t^o + q_t[B_t^o + \frac{1}{2}\kappa(B_t^o - \bar{B}^o)^2] + Q_t[S_t^o + \frac{1}{2}\kappa(S_t^o - \bar{S}^o)^2]$$
$$= W_t L_t^o + \Pi_t + R_{d,t}D_{t-1}^o + R_{b,t}q_{t-1}B_{t-1}^o + R_{k,t}Q_{t-1}S_{t-1}^o, \quad (5)$$

Total deposits D_t^o are the sum of households' private deposits and deposits created by the exchange of securities with reserves when the central bank purchases those during a QE. They are remunerated at the risk-free rate $R_{d,t}$. $R_{b,t}$ and $R_{k,t}$ are the gross returns for the bonds and shares respectively in period t. W_t is the real wage that both types of households take as given. T_t^o are taxes (or transfers if negative) that optimizing households pay every period. Finally, optimizers receive income Π_t from the ownership of both non-financial firms and financial intermediaries.

The problem of the optimizing household is to choose $C_t^o, L_t^o, D_t^o, B_t^o, S_t^o$ in order to maximize its expected utility (4) subject to the budget constraint (5) at every period. Let $u_{c,t}^o$ denote the marginal utility of consumption and $\Lambda_{t,t+1}$ denote the optimizing household's stochastic discount factor (the intertemporal marginal rate of substitution)

$$\Lambda_{t,t+1} \equiv \beta \frac{u_{c^o,t+1}}{u_{c^o,t}}. \quad (6)$$

Maximizing optimizers' utility with respect to deposits yields their intertemporal optimality condition

$$\mathbb{E}_t \Lambda_{t,t+1} R_{d,t+1} = 1. \quad (7)$$

The choices for private securities and long-term government bonds are given by:

$$S_t^o = \bar{S}_t^o + \frac{\mathbb{E}_t \Lambda_{t,t+1}(R_{k,t+1} - R_{t+1})}{\kappa}$$

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[12]This is similar to Gertler and Karadi (2013). Another interpretation following Kaplan et al. (2018) is that bonds and stocks are illiquid assets and thus have adjustment costs while deposits are liquid assets.

$$B_t^o = \bar{B}_t^o + \frac{\mathbb{E}_t \, \Lambda_{t,t+1}(R_{b,t+1} - R_{t+1})}{\kappa} \tag{8}$$

It follows that households always hold the frictionless amount of each asset. Their demand for extra units is increasing in the excess returns relative to the respective curvature parameter that governs the marginal transaction cost κ. As marginal transaction costs go to zero, excess returns disappear: there is frictionless arbitrage between the two assets and all assets' interest rates are equalized. On the other hand, when marginal transaction costs go to infinity, households' asset demands go to their respective frictionless capacity values.

I consider two labour market specifications. Under the first setting the labour market is competitive and each household chooses the quantity of hours supplied given the market wage W_t. In the second case wages are set by a labour union. Hours are demand driven by firms taking the wages as given by the union, households are ready to supply as many hours as required by the firms given the wage. Both wage specifications are analysed in section 3.2.

Rule of Thumb. Rule of thumb households account for a λ measure of households. Their participation in financial markets is restricted. They cannot smooth consumption either by trading securities or by acquiring bank deposits. They consume their net income at every period which is their labour income net of taxes. Their budget constraint is:

$$P_t C_t^r = P_t W_t L_t^r + P_t T_t^r. \tag{9}$$

C_t^r, L_t^r, T_t^r denote, respectively, consumption, hours worked and taxes (or transfers).

Rule of thumb agents maximize their utility subject to their budget constraint. Accordingly, the level of consumption will equate labour income specified by (9).

Rule of thumb agents' taxation is the only fiscal variable that matters for the model's fiscal allocation as is shown in Proposition 2. Optimizing agents internalize the government budget constraint through their government bond holdings. On the other hand, a change in the tax rate (or transfer) of the rule of thumb consumers implies a change in their taxes today or in the future.[13] I study two transfer schemes for the rule of thumb consumers: a no-redistribution scheme where transfers to rule of thumb agents are zero and a fiscal rule that taxes the profits of the optimizing households and rebates them to hand to mouth consumers.

3.2. Wage Setting

Here I develop the two wage setting schemes of the model: perfectly competitive labour markets and wage-setting by unions.

3.2.1. Perfectly Competitive Labour Markets

In the case of perfect competition in labour markets, households choose optimally their labour supply taking wages as given. The optimality condition with respect to

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[13]Similar results are obtained for the TANK model in Bilbiie et al. (2013).

hours worked for a household of type s is

$$u_{c,t}^s W_t = \chi (L_t^s)^\epsilon. \tag{10}$$

In the case of the rule of thumb consumers, due to the very form of the logarithmic utility function, combining (9) and (10) we find an analytical expression of hours that the rule of thumb agents optimally supply:

$$L_t^r = \left(\frac{1 - \frac{T_t^r}{C_t^r}}{\chi} \right)^{(\frac{1}{1+\epsilon})}. \tag{11}$$

3.2.2. Wage Setting by Unions

In the second case it is assumed that wage decisions are delegated to a continuum of labour unions. Hours are determined by firms taking the wages set by unions as given.[14] Households supply the hours required by the firms given the wage set by unions. Firms are also indifferent to the type of household they employ. Therefore, all households types supply the same working hours $L_t^o = L_t^r = L_t$.

Labour supply L_t is a composite of heterogeneous labour services

$$L_t = \left(\int_0^1 L_{h,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} dh \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{12}$$

where $L_{h,t}$ is the supply of labour service h and ϵ_w is the elasticity of substitution between labour and consumption across household types.

At each period there is a probability $1 - \xi_\omega$ that the wage for each particular labour service $W_{h,t}$ is set optimally. The union buys homogeneous labour at nominal price $W_{h,t}$, repackages it by adding a mark-up and chooses the optimal wage W_t^* to maximize the objective function where labour income of the two types is weighed by their marginal utilities of consumption.

$$\lambda \left[u_{c,t}^r W_{h,t} L_{h,t} - \frac{\chi}{1+\epsilon} L_t^{1+\epsilon} \right] + (1 - \lambda) \left[u_{c,t}^o W_{h,t} L_{h,t} - \frac{\chi}{1+\epsilon} L_t^{1+\epsilon} \right] \tag{13}$$

.

Aggregation. Aggregate variables are given by the population weighted average of the corresponding variables of each household type.

$$C_t \equiv (1 - \lambda)C_t^o + \lambda C_t^r \tag{14}$$

$$L_t \equiv (1 - \lambda)L_t^o + \lambda L_t^r \tag{15}$$

$$T_t \equiv (1 - \lambda)T_t^o + \lambda T_t^r \tag{16}$$

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[14]For a detailed exposition on wage setting see Appendix C.

The H superscript denotes the total asset holdings of households.

$$S_t^H \equiv (1-\lambda)S_t^o$$
$$B_t^H \equiv (1-\lambda)B_t^o$$
$$D_t^H \equiv (1-\lambda)D_t^o$$

3.3. Financial Frictions

Banks. Banks are funded with deposits, extend credit to non-financial firms and buy bonds from the government. At QE they exchange the asset purchased by the central bank with reserves. Each bank j allocates its funds to buying a quantity $s_{j,t}$ of financial claims on non-financial firms at price Q_t and government bonds $b_{j,t+1}^B$ at price q_t. Banks' liabilities are made up from households' deposits $d_{j,t+1}^H$. When the central bank proceeds in securities' purchases ($Q_t s_t$ or $q_t b_t$) it pays back the bank with an equivalent value of reserves $m_{j,t}$.[15] Finally, $n_{j,t+1}$ is the capital equity accumulated. Formally, the bank's balance sheet is:

$$Q_t s_{j,t}^B + q_t b_{j,t}^B + m_{j,t}^B = n_{j,t} + d_{j,t}^H. \tag{17}$$

To limit bankers ability to save and eventually overcome their financial constraint by using own funds, following Gertler and Kiyotaki (2010) we assume the following:

Assumption 1. (Entry and Exit). Each period, a fraction $1-\sigma_B$ of bankers, exit and give retain earnings to their household. An equal number of new bankers enter at the same time. They begin with a start up fund of ξ given to them by their household.

The bank's net worth evolves as the difference between interest gains on assets and interest payments on liabilities.

$$n_{j,t+1} = R_{k,t}Q_{t-1}s_{j,t-1}^B + R_{b,t}q_{t-1}b_{j,t-1}^B + R_{m,t}m_{j,t}^B - R_t d_{j,t}^H.$$

Let Z_t be the net period income flow to the bank from a loan that is financing to a firm and δ the depreciation rate of capital being financed. Then the rate of return to the bank on the loan, $R_{k,t+1}$, is given by:

$$R_{k,t+1} = \frac{Z_t + (1-\delta)Q_{t+1}}{Q_t}. \tag{18}$$

Long-term bond is a perpetuity that pays one euro per period indefinitely. The real rate

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[15]We can think $m_{j,t}^B$ as the sum of reserves a bank receives from the purchases not only of its own securities but also from the ones the households listed to the bank hold. The bank will transfer the exact same amount to the household's deposit account (see McLeay, Radia, and Thomas (2014)), keeping the balance sheet constraint intact.

of return on the bond $R_{b,t+1}$ is given by:

$$R_{b,t+1} = \frac{1/P_t + q_{t+1}}{q_t}.$$

Central bank reserves bear a zero weight in the banks' constraint and, as it will be shown momentarily, have a gross return $R_{m,t}$ equal to the risk-free rate R_t. It follows that banks have no inventive to hold reserves in equilibrium.

The bankers' objective at the end of period t, is to maximize the expected present value of future dividends. Since the banks are owned by the optimizing households, their stochastic discount factor Λ_{t+1} is used as the discounting measure.

$$V_{j,t} = \mathbb{E}_t \sum_{j=1}^{\infty} (1 - \sigma_B)\sigma_B^{j-1}\Lambda_{t+1}n_{j,t+1}. \tag{19}$$

To motivate a limit on the banks' ability to obtain deposits, a costly enforcement constraint is introduced in the same fashion as in Gertler and Kiyotaki (2010).

Assumption 2. (Costly Enforcement Constraint). A banker can abscond a fraction of her assets and transfer them back to her household members, depositors can force the bank into bankruptcy and get the remaining fraction of assets. It is assumed that the banker can divert loans easier than diverting bonds and reserves.

The depositors continue providing funds to the bank as long as the following incentive constraint is not violated:

$$V_{j,t} \geqslant \theta[Q_t s_{j,t}^B + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B]. \tag{20}$$

where θ is the fraction of assets that the banker may divert and $\Delta \in (0,1)$ and $\omega \in (0,1)$ are the ratios of how many bonds and how much reserves the banker can divert. On the left of (20) is the franchise value of the banker, which is what the banker would lose from diverting, while on the right are the banker's gains from diverting, which is a fraction θ of her assets.

The value of the bank at the end of period $t - 1$ must satisfy the Bellman equation:

$$V_{j,t-1}(s_{j,t-1}^B, b_{j,t-1}^B, m_{j,t}^B, d_{j,t}^H) = E_{t-1}\Lambda_{t-1,t}\sum_{i=1}^{\infty}\{(1 - \sigma_B)n_{j,t}$$
$$+ \sigma_B \max_{d_{j,t}}[\max_{s_{j,t}^B, b_{j,t}^B, m_{j,t}^B} V_t(s_{j,t}^B, b_{j,t}^B, m_{j,t}^B, d_{j,t}^H)]\}. \tag{21}$$

Banker's problem is to maximize (19) subject to the balance sheet (17) and their constraint (20).

Proposition 1. A solution to the banker's dynamic program is

$$V_{j,t}(s_{j,t}^B, b_{j,t}^B, d_{j,t}^H, m_{j,t}^B) = A_{j,t}^B n_t;$$

and the marginal value of the banker's net worth A^B solves:

$$A^B_{j,t} = \mu^s_t \phi_t + \nu_{d,j,t}.$$

μ^s_t is the stochastic spread between the loan and the deposit rates, ϕ_t is the maximum leverage and $\nu_{d,j,t}$ is the marginal loss from deposits.

Proof. See Appendix D.

The proposition clarifies the role of the bank's net worth in the model. We can rewrite the incentive constraint using the linearity of the value function as:

$$\frac{A^B_{j,t}}{\theta} \geqslant \frac{[Q_t s^B_{j,t} + \Delta q_t b^B_{j,t} + \omega m^B_{j,t}]}{n_t} = \phi_t. \tag{22}$$

The adjusted leverage of a banker cannot be greater than $A^B_{j,t}/\theta$. The right hand side shows that as the net worth of the banker decreases the constraint is more likely to bind.

The maximum adjusted leverage ratio of the bank after the solution of the bank's problem (see Appendix D) yields:

$$\phi_t = \frac{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}}{\theta - \mathbb{E}_t \Lambda_{t,t+1}(R_{k,t+1} - R_{t+1})}. \tag{23}$$

Maximum adjusted leverage ratio depends positively on the marginal cost of the deposits and on the excess value of bank assets. As the credit spread increases, banks' franchise value V_t increases and the probability of a bank diverting its funds declines. On the other hand, as the proportion of assets that a bank can divert, θ increases, the constraint binds more.

Importantly, the maximum adjusted leverage ratio does not depend on any individual bank characteristics, therefore the heterogeneity in the bankers' holdings and net worth, does not affect aggregate dynamics. Hence, it is straightforward to express individual financial sector variables in aggregate form.

Aggregation. Let S^B_t be the total quantity of loans that banks intermediate, B^B_t the total number of government bonds they hold, M^B_t the total quantity of reserves and N_t their total net worth. Furthermore, by definition, total deposits acquired by the households D^H_t are equal with the total deposits of the banking sector. Using capital letters for the aggregate variables, the banks' aggregate balance sheet becomes

$$Q_t S^B_t + q_t B^B_t + M^B_t = N_t + D^H_t. \tag{24}$$

Since the leverage ratio (23) does not depend on factors associated with an individual bank's characteristics we can sum up across banks and get the aggregate bank constraint in terms of the total net worth in the economy:

$$Q_t S^B_t + \Delta q_t B^B_t + \omega M^B_t = \phi_t N_t. \tag{25}$$

16

The above equation gives the overall demand for loans $Q_t S_t$. When the incentive constraint is binding, the demand for assets is constrained by the net worth of the bank adjusted by the leverage. We can get some intuition here for what changes in the bank's constraint during the QE. No matter the security the central bank purchases, since their weights are higher than the weight of reserves ($1 > \Delta > \omega$), the exchange of securities with reserves relaxes the constraint and stimulates lending to the non-financial sector. If the constraint does not bind, then all three types of assets would have the same returns, equal to the deposit rate and QE would be ineffective.

Aggregate net worth is the sum of the new bankers' and the existing bankers' equity: $N_{t+1} = N_{y,t+1} + N_{o,t+1}$. Young bankers' net worth is the earnings from loans multiplied by ξ_B which is the fraction of asset gains that being transferred from households to the new bankers

$$N_{y,t+1} = \xi[R_{k,t} Q_{t-1} S^B_{t-1} + R_{b,t} q_{t-1} B^B_{t-1} + R_{m,t} M^B_{t-1}]$$

and the net worth of the old is the probability of survival for an existing banker multiplied by the net earnings from assets and liabilities

$$N_{o,t+1} = \sigma[R_{k,t} Q_{t-1} S^B_{t-1} + R_{b,t} q_{t-1} B^B_{t-1} + R_{m,t} M^B_{t-1} - R_t D^H_t].$$

3.4. Central Bank, Asset Purchases and the Treasury

Central Bank. The central bank uses two policy tools. Firstly, it adjusts the policy rate according to the Taylor rule specified here below. Secondly, it can engage in risky asset purchases from households and banks. When balance sheet constraints are tight, excess returns rise. Central bank purchases relax the incentive constraint of the banks and increase aggregate demand, thus driving up asset prices.[16]

Under a QE operation, the central bank buys securities from banks and households. These can be either private assets S^G_t or bonds B^G_t. It does this by paying the assets purchased by their respective price Q_t and q_t. To finance those purchases it creates electronically reserves M_t that pay back purchases from households and banks:

$$Q_t S^G_t + q_t B^G_t = M_t.$$

It is assumed that the central bank turns over any profits to the treasury and receives transfers to cover any losses. The central bank's budget constraint is:

$$T^{CB}_t + R_t M_{t-1} + Q_t S^G_t + q_t B^G_t = R_{b,t} q_{t-1} B^G_{t-1} + R_{s,t} Q_{t-1} S^G_{t-1} + M_t \tag{26}$$

where T^{CB}_t are transfers of the central bank to the treasury.

Monetary policy is also characterised by a simple Taylor rule. It sets the nominal interest rate i_t such as to respond to deviations of inflation and output from its flexible price equilibrium level Y^*:

$$i_t = i + \kappa_\pi \pi + \kappa_y (Y - Y^*) + \epsilon_{m,t},$$

---

[16]See Araújo, Schommer, and Woodford (2015) for a same intuition under a different setting.

where i is the steady state level of the nominal interest rate and $\epsilon_{m,t}$ an exogenous monetary policy shock. The relation between nominal and real interest rates is given by the Fisher equation:

$$1 + i_t = R_{t+1} \frac{P_{t+1}}{P_t}.$$

With the addition of the central bank in the model, three agents can hold assets or bonds: Optimizing households, banks and the central bank. The total quantity of loans therefore is decomposed as:

$$S_t = S_t^B + S_t^H + S_t^G \tag{27}$$

and for the bonds:

$$B_t = B_t^B + B_t^H + B_t^G. \tag{28}$$

If we combine these identities and insert them into the balance sheet constraint of the banks we have:

$$Q_t S_t \leqslant \phi N_t + Q_t S_t^H + Q_t S_t^G + \Delta(q_t B_t^G + q_t B_t^H - q_t B_t) \tag{29}$$

The above constraint implies that when government purchases either loans or bonds it relaxes the balance sheet constraint of the banking sector. This can, in financial stress periods, reduce the excess returns and stimulate the economy. When this constraint does not bind and the inequality holds, asset or bond purchases made by the government are neutral. This happens due to frictionless arbitrage that characterizes the economy when the banks has no binding constraint. Wallace (1981) in his seminal paper has made use of that assumption for the neutrality theorem of the open market operations.

Equation (29) gives another insight into the asset purchase mechanism. Buying loans or bonds does not have the same impact to the loosening of the banks' balance sheet constraint. In fact, since loans have an absconding fraction of 100%, purchases of loans by the central bank relaxes the constraint more than the purchase of bonds with a coefficient $\Delta < 100\%$. Intuitively, the central bank acquiring government bonds frees up less bank capital than does the acquisition of a similar amount of private loans.

It is now easier to understand when the irrelevance theorem holds. Since the government creates as many reserves as the value of the assets purchased ($M_t = q_t B_t^G + Q_t S_t^G$), then in the case of frictionless arbitrage between the existing assets ($R_{s,t} = R_{b,t} = R_t$), the market operations are indeed irrelevant. But since the financial frictions included in the model disrupt the frictionless arbitrage, asset purchases have an effect on the real economy.

The share of the total assets that is purchased by the government follows a second order stochastic process.[17] Specifically,

$$S_t^G = \phi_{s,t} S_t,$$

$$B_t^G = \phi_{b,t} B_t.$$

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[17]As is shown in the calibration section, an AR(2) is the best way to simulate the ECB's Asset Purchase Program schedule.

$\phi_{b,t}$ and $\phi_{s,t}$ obey a second order stochastic process.

Treasury. The treasury collects lump sum taxes $T_t = \lambda T_t^r + (1 - \lambda)T_t^o$ to finance its public expenditures which are fixed relative to output, $\bar{G} = \gamma^G Y^{ss}$. It also targets a constant real level of long-term debt, denoted by $\bar{B}$. It collects taxes at rate t_{pr} from non-financial firms' profits and redistribute them back to the hand to mouth households, $T_t^r = t_{pr}Prof_t$.

The treasury's budget constraint is:

$$\bar{G} + q_{t-1}R_{b,t}\bar{B} = q_t\bar{B} + T_t + T_t^{CB}. \tag{30}$$

.

Proposition 2. Fiscal policy matters only through the impact of taxes (transfers) on hand to mouth agents. Therefore, the only fiscal variable that needs to be defined is the hand to mouth transfers (or taxes).

Proof. I make use of the optimizers budget constraint (5), the bank's -owned by optimizing agents- balance sheet (17), the taxes aggregator and the treasury and central bank's budget constraints (26), (30). Substituting the latter four equations in the optimizers' budget constraint and using the financial variables aggregator, the aggregate resource constraint yields:

$$C_t^R + \frac{\bar{G}}{1 - \lambda} - \frac{\lambda}{1 - \lambda}T_t^K + adj\{B, S\} = W_tL_t^R. \tag{31}$$

Where $adj\{B, S\}$ are the adjustment costs for bonds and shares that households have to pay, defined in (5).

Taxes on optimizers and any short of government bond decision do not matter for the allocation. $\qquad\square$

3.5. Non-Financial Firms and Nominal Price Rigidities

The non-financial firms are separated into three types: intermediate, final goods firms (retailers) and capital goods producers. To allow for nominal price rigidities, I assume that the differentiated intermediate goods i produced by a continuum of monopolistically competitive intermediate goods firms are subject to Calvo price stickiness.

The final output composite is a CES composite of all indeterminate goods i: $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}$ where ζ denotes the elasticity of substitution across intermediate goods. Each period there is a fixed probability $1 - \gamma$ that a firm will adjust its price. Each firm chooses the reset price P_t^* subject to the price adjustment frequency constraint. Firms can also index their price to the lagged rate of inflation with a price indexation parameter γ_p. The goods are then sold and used as inputs by a perfectly competitive firm producing the final good. Finally, the capital goods producers create new capital under investment adjustment costs and sell it to goods producers at a price Q_t. The non-financial sector problem is described in detail in Appendix E.

Capital stock evolves according to the law of motion of capital

$$K_{t+1} = I_t + (1 - \delta)K_t. \tag{32}$$

The intermediate good $i \in [0, 1]$ is produced by a monopolist who uses a constant returns to scale production function combining capital and labour:

$$Y_t(i) = A_t K_t(i)^\alpha L_t(i)^{1-\alpha}. \tag{33}$$

A_t is the total factor productivity. It finances its capital needs each period by obtaining funds from banks and households. To acquire the funds to buy capital, the firm issues $S_t(i)$ claims equal to the number of units of capital acquired $K_{t+1}(i)$ and prices each claim at the price of a unit of capital $Q_t(i)$. Then by arbitrage: $Q_t(i)S_t(i) = Q_t(i)K_{t+1}(i)$. The funds acquisition between goods firms and its lenders is under no friction. Firm's lenders can perfectly monitor the firms and there is perfect information.

Resource Constraint. Final output may be either transformed into consumption good, invested or used by the government for government spending:

$$Y_t = C_t + I_t[1 + \tilde{f}\left(\frac{I_t}{I_{t-1}}\right)] + G.$$

4. Quantitative Analysis

In this section I present the model's calibration and the first set of results of the paper: the impact of the quantitative easing on inequality.

4.1. Calibration

The model's calibration is performed in order to match Euro Area stylized facts and is divided in conventional and banking parameters. It follows broadly the calibration of the updated version of the New Area-Wide Model (NAWM), (Christoffel, Coenen, and Warne (2008), Coenen, Karadi, Schmidt, and Warne (2018)), the DSGE model of the ECB. Parameters in the NAWM are estimated by the use of Bayesian methods in the time span of 1985Q1-2014Q4 using times series for 18 macroeconomic variables which feature prominently in the ECB/Eurosystem staff projections. One period in the model is one quarter. All the calibrated values are presented in Table 1.

Financial parameter values are chosen in order to match specific Euro Area banking characteristics namely the banks' average leverage, lending spread and planning horizon. There are three parameters that characterise the behaviour of the financial sector in the model. This is the absconding rate θ, the fraction of entering bankers initial capital fund ξ_B, and the steady-state value of the survival rate, σ_B. I calibrate these parameters to match certain steady-state moments following the moments reported in Coenen et al. (2018). The steady-state leverage of the banks is set equal to 6, which corresponds to the average asset-over-equity ratio of monetary and other financial institutions as well

Parameters	Value	Definition
		Households
β	0.998	Discount rate
χ	4.152	Relative utility weight of labour
λ	0.20	Share of rule of thumb agents
ϵ	2	Inverse Frisch elasticity of labour supply
$\bar{S}^R/S$	0.500	Proportion of shares of the optimizers
$\bar{B}^R/B$	0.750	Proportion of bond holdings of the optimizers
κ	1	Portfolio adjustment cost parameter
		Banks
θ	0.20	Absconding rate
Δ	0.5	Absconding fraction for bonds
ω	0	Absconding fraction for reserves
ξ_B	0.0014	Entering bankers initial capital
σ_B	0.950	Bankers' survival rate
		Intermediate and Capital Goods Firms
δ	0.025	Depreciation of capital
α	0.36	Capital share
η	5.77	Inverse elasticity of net investment to the price of capital
		Wage and Price Setting
ζ	4.340	Elasticity of labour substitution
ξ_w	0.890	Probability of keeping the price constant
γ_w	0.417	Wage Indexation parameter
ζ	2.540	Elasticity of substitution between goods
γ	0.720	Probability of keeping the wages constant
γ_p	0.480	Indexation parameter
		Treasury Policy
γ^G	0.20	Steady state fraction of government expenditures to output
t_{pr}	0%-40%	Optimizers' profit tax rate
		Monetary Policy
κ_π	1.860	Inflation coefficient in the Taylor rule
κ_y	0.147	Output gap coefficient in the Taylor rule
ρ_m	0.860	Interest-rate smoothing
ρ_1	1.700	First AR coefficient of the bond purchase shock
ρ_2	-0.730	Second AR coefficient of the bond purchase shock
ψ	0.015	Initial asset purchase shock

Table 1: Parameter Values

as non-financial corporations, with weights equal to their share of assets in total assets between 1999Q1 and 2014Q4 according to the Euro Area sectoral accounts. Second, the steady-state spread of the lending rate over the risk-free rate, $R_t^k - R_t$ is set to 2.17 percentage points at the steady state, which is the average spread between the long-term cost of private-sector borrowing and the EONIA rate from 2003Q1 to 2014Q4. The banks planning horizon is set equal to 5 years. These parameters are also in line with the related studies in the literature. Finally I set the fraction of bonds that can be absconded Δ to 50% targeting a steady state bond spread half to the lending spread. The absconding rate of reserves ω is set to zero. Similarly to Sims and Wu (2021) I assume that they are fully recoverable by depositors in the event of bankruptcy. Since they are in essence central bank money, the central bank has full control on them.

Regarding the bond market, the long term target of the real bonds supply by the treasury equals 70% of GDP. The fraction of long-term bonds held by banks is 25% which is consistent with the sovereign debt holdings of the banking sector according to EA data. This leaves the rest 75% of the bond holdings to the optimizers' portfolio. The fraction of shares held by optimizing households is 50%.

The values for the share of capital α and the depreciation rate δ are chosen to 0.36 and 0.025 respectively following the estimation results of Christoffel et al. (2008). Similarly, the value of β is assigned to 0.998, chosen to be consistent with an annualised equilibrium real interest rate of 2%. The relative utility weight of labour χ is chosen to ensure a level of labour close to $1/3$ in steady state, a fairly common benchmark in the literature (see Corsetti, Kuester, Meier, and Müller (2014)). The parameter of the inverse Frisch elasticity of labour supply ϵ is one difficult to identify. In the NAWM, this parameter is not estimated and is set a to 2 which is the one I employ here as well. ϵ has a crucial role on the IADL results of the paper following next. I provide additional robustness checks in the Appendix I for a range of ϵ starting from 0.5 to 2. Results of the paper hold for all these values.

The elasticity of substitution between goods ζ and the capital adjustment costs also follow the NAWM and set to 2.54 and 5.77 respectively. The same holds for the wage setting parameters. The government spending as a fraction of the GDP is set to 20% also following other studies for the Euro Area. Retail firms parameters: the elasticity of substitution between goods, the Calvo probability and the price indexation parameter are set to the value estimated in the NAWM. The same holds for the monetary policy parameters: the inflation and output gap coefficients in the Taylor rule and the interest rate smoothing parameter.

The share of rule of thumb consumers is chosen to be $\lambda = 0.20$. Using the data from the Eurosystem Household Finance and Consumption Survey, as explained in Section 1, almost the bottom 20% of the Euro Area households hold essentially no net worth at all. This is also in line with the estimates of Slacalek et al. (2020). The same value is also used by a similar study for the EA with LAMP Hohberger, Priftis, and Vogel (2019b). The profits' tax rate used in the IADL results of the paper takes values from 0% to 40% in the exercises performed.[18]

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[18]Results remain qualitatively similar under any reasonable tax rate.

The bond purchase shock is modelled as an AR(2) process.[19] The AR(2) process in contrast with an AR(1) captures the expectation of the further expansion of central bank purchases in the future, which is the case in the ECB's APP started in 2015Q1. Purchases for the first year are constant to 60 billion euro, then in 2016 increase to 80 billion for four quarters to eventually go back to 60 billion and fade out. Relative to 2015 GDP purchases increase from a 2% to almost 4% at their peak. To illustrate this pattern, the first AR coefficient is chosen to 1.700 and the second being -0.730 while the initial shock is chosen to 0.015. For an easy comparison between the QE and the conventional monetary policy shock, I calibrate the magnitude of the latter such as it provides the same increase in GDP with the one induced by the QE shock.

4.2. Impulse Response Analysis

I proceed with a quantitative exercise on identifying i) what was the impact of the ECB's APP programme on the macroeconomy, ii) its impact on consumption and income inequality and iii) what is the difference with an accommodative monetary policy shock, assuming that the economy is not at the effective lower bound. I present the results of the model with sticky wages. For high levels of asset markets participation, as it occurs in the calibrated model, the two specifications offer qualitatively similar results. The model is solved non-linearly following Lindé and Trabandt (2019).

Central Bank Bond Purchases and Conventional Monetary Policy. How do a bond purchasing programme similar to the APP and an expansionary monetary policy shock that produces a similar output increase affect the main macro variables? Figure 3 plots these dynamics for output, investment, inflation, hours worked and a series of interest rates and spreads. The bond purchase shock is a second-order autoregressive process similarly to Gertler and Karadi (2013) calibrated to mimic the APP programme of the ECB. The monetary policy shock is set such that it produces the same increase in output of about 2.9% which is translated to a 80 bps reduction in the policy rate. In the case of the QE shock the nominal interest rate is set constant for the first four quarters and then it is let to follow the Taylor rule of the central bank. This simulates the inability of the central bank to use conventional monetary at times is forced to use unconventional measures.[20] In Appendix I provide robustness analysis which shows that results hold for the case of i) fully flexible and ii) fully constant policy rate. In bold lines, the responses of a bond shock reflect the responses of a conventional interest rate reduction.

Bond purchases stimulate the economy and increase output as Figure 3 shows. The current calibration of the rule of thumb agents' measure to the EA average ($\lambda = 0.20$) leads to the case that both MP and QE shock increase aggregate demand.

In the case of QE the main mechanism works through the loosening of the banks' constraint. Central bank intermediation increases asset prices Q_t and this leads to an

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[19]This follows similar studies that conclude that the ECB's QE program is characterised by a AR(2) process (see Andrade, Breckenfelder, De Fiore, Karadi, and Tristani (2016), Hohberger et al. (2019b)).

[20]ECB after the initiation of its APP programme in 2015Q1 kept its main refinancing operations interest rate constant for a year.
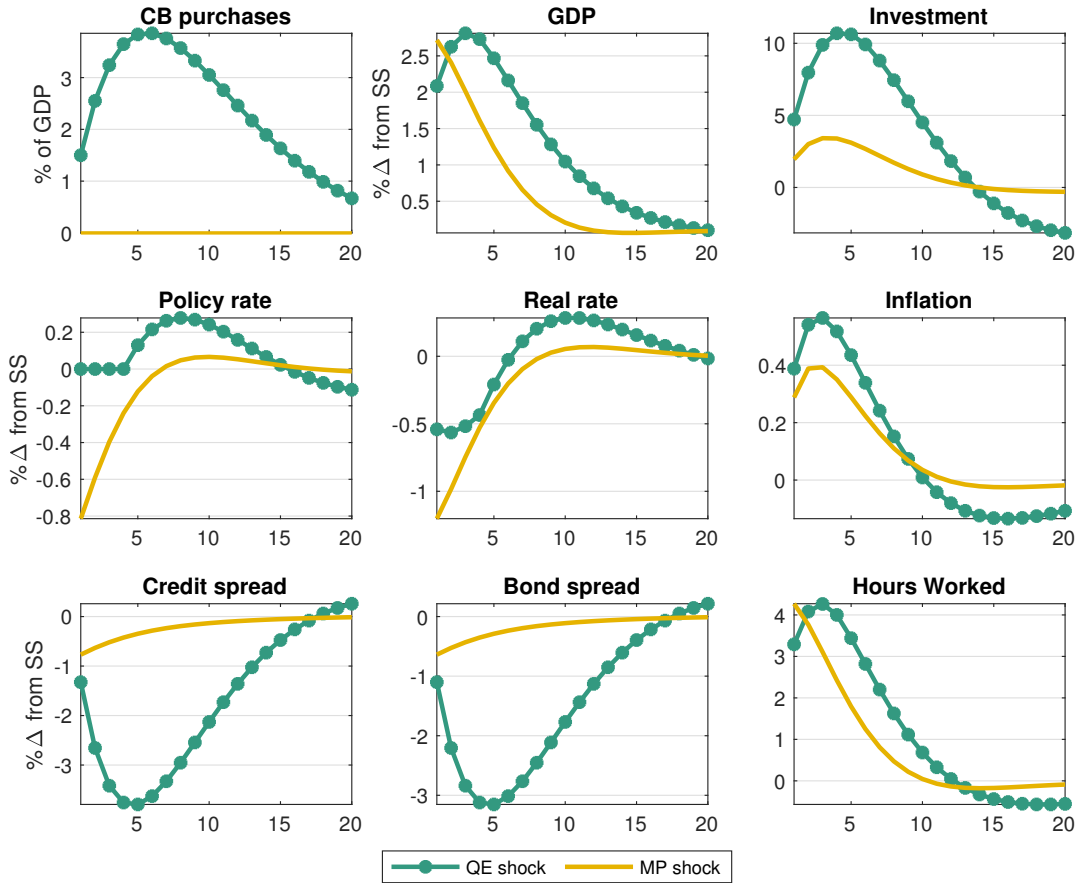
Fig. 3. Government Asset Purchase Shock and Conventional Monetary Policy Shock

increase in banks' valuation (net worth). Standard financial accelerator effects lead to a further increase of capital price and an economic upturn. An increase in the bonds' prices drives banks to buy more assets which leads to an increase in assets' prices. Excess returns reduce for both securities as can be seen in the graph and it is much more substantial for the QE case. This is because it directly affects the banks' financial constraint. The economic upturn also affects the real economy due to the higher demand for employment and wage and hours worked increase. Finally, in both cases, the real rate drops while in the monetary policy shock case it does by much more.

Overall, responses are qualitatively similar for both conventional and unconventional monetary policy shocks. Excess returns of the bonds and loans decrease more in the QE and this has an impact on the inequality measures shown next.

Income and Consumption Inequality. Figure 4 shows the responses of income and consumption of the two income groups as well as the relative inequality measure for both variables after a QE and MP shock. Consumption (income) inequality are defined as the optimizers' consumption (income) over aggregate consumption (income).
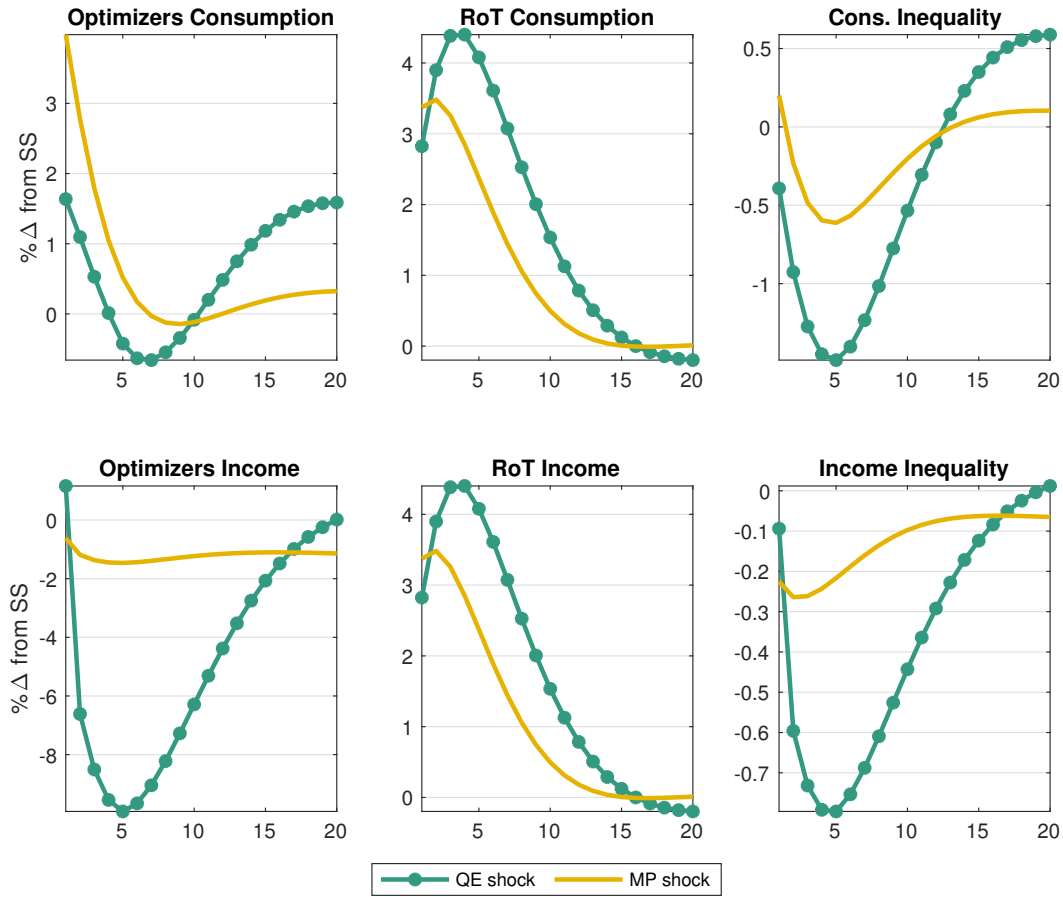
Fig. 4. Government Asset Purchase Shock and Conventional Monetary Policy Shock: Inequality

After an expansionary monetary shock, consumption increases for both agents. Rule of thumb agents' consumption strictly follows the real wage path which in both cases it goes up due to the increased demand for labour. Optimizer's consumption increases on impact but responds also through the intertemporal substitution channel. Notice that were the nominal interest not constant for the first four periods, consumption of the optimizing agents would have been decreasing (see Appendix) since nominal interest rates would go up due to the Taylor rule specification. Through the standard intertemporal substitution mechanism the optimizing agents would have lowered their consumption. Both consumption responses lead to a reduction in consumption inequality between the two income groups. This is in line with the well established fact that hand to mouth consumers have a higher marginal propensity to consume than the financially unconstrained agents (Auclert (2019), Kaplan et al. (2018) among others).

The second row of Figure 4 shows the income responses of the two agents. Rule of thumb income, following the real wage path increases while optimizers' income de-

cline. The reasoning is the following: after a QE shock, optimizers reduce their bond holdings as shown in their demand function for bonds (8) as excess bond returns fall. This has a negative impact on their balance sheet since they lose from the *interest rate differential* between the bond and the risk-free rate together with the real rate reduction that they receive in their deposits and reserves. Due to the exchange of bonds to reserves, the income reduction is much more amplified at the QE shock. The two agents' income responses lead to a fall in income inequality for both accommodative policies while it is more amplified in the QE case due to the higher excess returns loss.

5.   QE, Asset Markets Participation and Wage Rigidities

In this section I provide a normative exercise regarding the changing behaviour of the impact of QE when asset markets participation is low and the wage setting is flexible. From now on, I focus on a subset of EA countries that share these characteristics. Having the model at hand and given that it mimics well the empirical findings, I show that in a country with low level of financial inclusion and a flexible wage setting QE can be contractionary and inequality increases. This is in contrast with the benchmark results I showed in the previous two sections regarding the EA average.
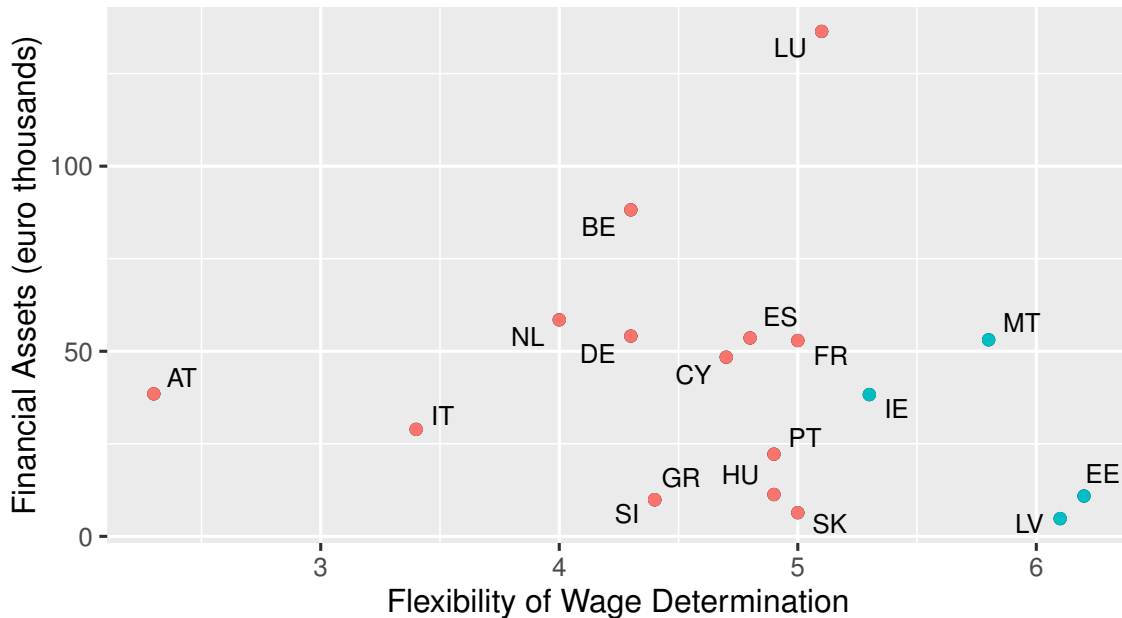

Fig. 5. Total Financial Assets among EA Households and Flexibility of Wage Determination

A subset of EA countries fulfils these two characteristics. Figure 5 shows the degree of wage flexibility against the level of financial assets held by an average household for nineteen European countries. Data for the financial assets' level comes from the

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Household Finance and Consumption Survey. The data for the wage flexibility determination comes from the Global Competitiveness Index of the World Economic Forum. As shown in the figure (in the coloured version in cyan), Estonia, Latvia, Malta and Ireland are countries that fulfil these two conditions. I show, with the use of the quantitative model, that these countries can instead experience negative effects from the QE, leading to an increase in inequality.

I examine analytically and quantitatively the existence of a region where accommodative monetary policy, when asset markets participation is limited and wages flexible, can have contractionary effects instead of stimulating aggregate demand and decreasing inequality. Bilbiie (2008) names this Inverted Aggregate Demand Logic (IADL).[21]

When wages are flexible, QE can be contractionary and reduce inequality for low levels of asset market participation, while when wages are sticky this result is muted. I also show that the contractionary effects, if present, can be avoided by fiscal redistribution of a portion of profits from the firm owners to the hand to mouth consumers. I provide analytical expressions that show the *direct effect* of interest rate reduction and quantitative easing on output. Then, I show the fraction of constrained agents that pushes the model into the IADL area, that is making the total effect of the accommodative policy contractionary. To pursue this, due to the high dimensionality of the model, I solve the model numerically.

In order to derive analytical results, I make the following, not distorting, assumptions: Consumption and hours worked are equal among all the members in steady state.[22] Therefore in steady state: $L = L^r = L^o$ and $C = C^r = C^o$. The first assumption can be implemented by a particular choice of χ, whereas the second by introducing a tax level that makes optimizers' consumption equal to that of the rule of thumb agents. Furthermore, due to no-redistribution, I assume that rule of thumb agents taxation is zero: $T_t^r = 0$. Under these assumptions, we can express the consumption and labour aggregators (14), (15) as $l_t = \lambda l_t^r + (1 - \lambda) l_t^o$ and $c_t = \lambda c_t^r + (1 - \lambda) c_t^o$ respectively, where lower case letters denote log deviations from the non-stochastic steady state.

The optimality condition (11) without including any tax (or transfer) rule dictates that the labour supply of the rule of thumb agents in levels is always constant, therefore $l_t^r = 0$. The labour consumption optimality conditions are in log-linear terms: $c_t^r = w_t + l_t^r$ and $c_t^o = w_t - \epsilon l_t^o$. Using the aggregate consumption, labour consumption optimal choices, and the hours worked aggregator we get:[23]

$$w_t = c_t + \epsilon l_t. \tag{34}$$

Note that the above relation holds for both labour market settings, given that both agents have equal consumption and work the same hours in steady state. Substitut-

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[21]A key departure from Bilbiie's work is that the present model includes banks and capital.

[22]The latter holds without any further arrangement for the centralised wage setting market where firms choose uniformly the labour required given the wage set by the unions.

[23]The derivations of the main equations of this chapter are presented in Appendix H

ing (34) in the labour optimality condition of the optimizing agents:

$$c_t^o = c_t - \epsilon \left(\frac{\lambda}{1 - \lambda} \right) l_t. \tag{35}$$

Trivially with no hand to mouth consumers $\lambda = 0$, c_t^o follow the aggregate consumption schedule. Introducing limited asset market participation in the model makes optimizers' consumption reacting negatively to an increase of the *aggregate* employment. This is due to the wage being the rule of thumb agents' only source of income.

Doing the same exercise for the rule of thumb agents:

$$c_t^r = c_t + \epsilon l_t.$$

Rule of thumb agents' consumption schedule reacts positively in changes of aggregate consumption and employment with elasticity ϵ. Having the above relations in hand I proceed with the derivation of the aggregate Euler equation.

The log-linearised versions of the production function and resource constraint are $y_t = \alpha k_t + (1 - \alpha) l_t$ and $y_t = c_t s_c + i_t s_i + s_g$ respectively. Inserting both equations in the optimizing agents' consumption function (35) and substituting the result to the optimizers' Euler equation $c_t^o = \mathbb{E}_t \{c_{t+1}^o\} + [\mathbb{E}_t \{\pi_{t+1}\} - r_t]$ we arrive to the aggregate Euler equation or IS curve:

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\delta}[r_t - \mathbb{E}_t\{\pi_{t+1}\}] - \frac{1}{\delta}\frac{s_i}{s_c}\Delta i_{t+1} + \frac{1}{\delta}\frac{\epsilon\lambda}{(1-\lambda)(1-\alpha)}[\alpha\Delta k_{t+1}]. \tag{36}$$

where

$$\delta = \frac{1}{s_c} - \epsilon\frac{\lambda}{(1-\lambda)(1-\alpha)}$$

and $s_c = C^{ss}/Y^{ss}$, $s_i = I^{ss}/Y^{ss}$, $s_g = G^{ss}/Y^{ss}$.

Profits.— Profits play a crucial role in the analysis. As it will be shown below, it is the primary reason for the IADL existence. Profits from non-financial corporations are given by $Prof_t = Y_t - W_t L_t - Z_t K_t$. Log-linearising it around the steady state (with $d_t = ln((Prof_t - Prof)/Y))$ we get:

$$d_t = y_t - (w_t + l_t) - (z_t + k_t). \tag{37}$$

Profits move counter-cyclically in response to demand shocks, a standard feature of the NK models.

The bond buying programme in the present setting is an one time increase in the government bond holdings and a simultaneous reduction of the holdings of banks and households accompanied with an increase in the banks' reserves. Finding the *direct effect* of QE on output is a tedious process. For the interested reader the derivations are in Appendix H. As presented in the Appendix after some algebra manipulation the

direct effect of QE on output using the IS equation is:

$$-\frac{1}{\delta}\frac{\epsilon\lambda\alpha}{(1-\lambda)(1-\alpha)}\frac{\Delta B^G}{S}b_t^G.$$

Using the fact that $b_t^G = \frac{B_t^G - B^G}{B^G}$, $B^G = 0$ central bank bonds at steady state are zero, and after some algebra manipulation the above equation becomes:

$$\frac{1}{\frac{(1-\lambda)(1-\alpha)K}{s_c\alpha\epsilon\lambda\Delta} - \frac{\epsilon\lambda K}{\alpha\epsilon\lambda\Delta}}B_t^G.$$

Setting the above expression equal to zero, we can find the threshold value λ^* that makes the direct effect of the quantitative easing policy ineffective.

For a low λ below the threshold value or equivalently when financial participation is high, output reacts inversely to a QE shock. As we move to higher values of λ this effect it becomes even stronger. When $\lambda > \lambda^*$, and the fraction of hand to mouth consumers is big enough, δ becomes negative and distorts the well known stimulating effect of accommodative monetary policy using the policy rate. In that region, lower interest rates restrain aggregate demand and we enter the Inverse Aggregate Demand Logic region. Finally, as λ reaches its upper bound of 1 where no agent hold assets, $1/\delta$ decreases towards zero; QE as a monetary policy tool becomes irrelevant.

Feeding the model with the parameter values from the model's calibration shown in Section 4.1, I show the *total* impact effect of a QE shock to main macro variables as a function of rule of thumb agents, where $\lambda \in [0, 0.9]$. The top row of Figure 6 shows the *total* impact effect on aggregate output and profits of a QE shock conditional on different fractions of rule of thumb agents. The bottom row of the figure shows the *total* impact on consumption and income inequality. This is performed for two cases: perfect labour market and imperfect labour markets (the sticky wages case). This distinction is important, as I will explain momentarily, the wage stickiness neutralises the countercyclical behaviour of profits, which is the main factor that drags down aggregate demand and inequality goes up after a QE shock.

As the rule of thumb fraction increases this shifts the value of output upwards. This continues up to a point where aggregate demand reaches its maximum. When λ is over the threshold of $\lambda^* = 0.30$, then the QE shock has the opposite effect on the aggregate variables; expansionary monetary policy generates contractionary effects. As λ reaches its upper limit, and agents cannot have intertemporal decisions, monetary policy becomes ineffective.

To understand the reasoning behind the IADL it is useful to first focus on the region where there is restricted limited participation: $\lambda < \lambda^*$. A QE shock leads to an increase in aggregate demand. Wage increases from the intertemporal substitution of asset holders and this wage increase translates to a further increase in demand, since non-asset holders consume their wage income (assuming no transfers). This generates a shift in labour demand upwards. As Figure 6 shows this effect is not constant across the domain of λ values. To understand why this is the case it is important to focus on
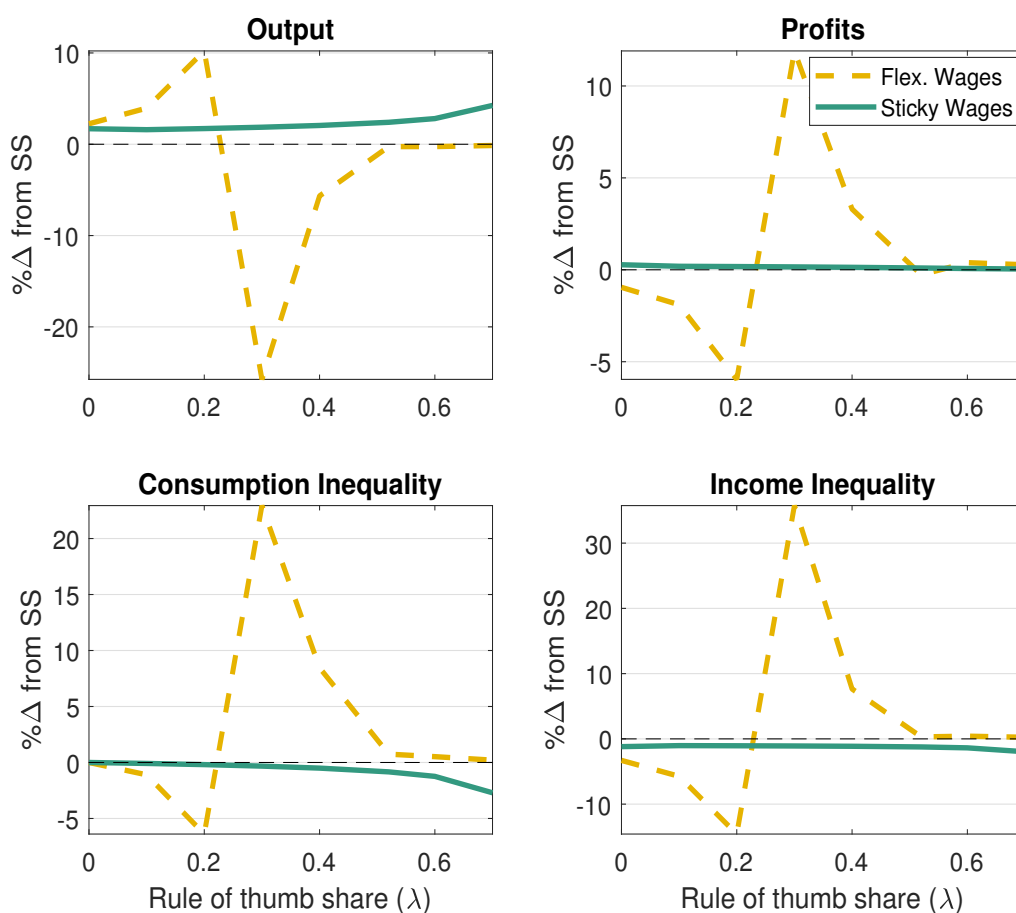
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Fig. 6. Impact Effects Conditional on Asset Market Participation: QE Shock

the role of profits. Profits as shown above analytically and in the bottom panel of Figure 6 are countercyclical. Consequently, as the asset market participation lowers, the less the negative consequences of the profits experienced by the majority of population, the non-asset holders. Therefore, as λ increases and until it reaches λ^* aggregate demand increases continuously. The counter cyclicality of profits will induce aggregate demand to drop and there is a new equilibrium with lower output, consumption and wages. Finally, reaching the end of the λ domain, at $\lambda = 0.6$ almost no agent holds assets and the interest rate policy is ineffective.

When we introduce labour unions that set the wages and thus sticky wages in the model, results change. After an accommodative monetary policy shock of the same magnitude as before, we see that for all levels of asset market participation the impact effect of output never turns negative. The introduction of sticky wages manages to keep marginal costs stable and therefore the impact effect of profits is always pro-cyclical. Consequently, profits no longer drag aggregate demand down and output's response

is always positive for the λ domain.

Due to the potentially important role the inverse Frisch elasticity of labour supply, ϵ, could have on the results, I provide additional additional checks in the Appendix I for a range of the parameter starting from 0.5 to 2. Results of the paper hold for all these values.

5.1. *Redistribution of Profits*

When labour markets are perfectly competitive, accommodative monetary policy can have negative effects. In the Appendix J I focus on the frictionless labour market setting and provide results under the assumption that taxation is redistributive. Taxation is following a simple fiscal rule of redistribution of profits to the hand to mouth consumers defined as:

$$T_t^r = t_{pr} Prof_t.$$

That is, a percentage of the profits is allocated to the hand to mouth consumers who were entitled zero transfers under the baseline scenario examined before. What changes is as the rule of thumb consumers' profits receivable profits increases, the IADL region shifts to the right. The negative effects of profits are shared between the two groups which alleviates the contractionary effects of the negative income shock.

6. Conclusion

Asset purchase programmes following the Great Recession were employed extensively, while the recent pandemic urged central banks to expand further their balance sheets to ease the market turmoil. In this paper, I provide novel evidence, using both a theoretical and an empirical specification, on how QE affects inequality and the macroeconomy in the Euro Area. Results show that QE reduces inequality and it is expansionary. Overall, the indirect effects of QE outweigh the direct effects and agents that do not own financial assets benefit relatively more, leading to a reduction in inequality. Given that the model is in line with the empirical findings, I finally proceed to a normative exercise and show that QE can be contractionary when asset markets participation is low. This is particularly important for a subset of EA countries with low asset markets participation rates.

I build and calibrate a state of the art New Keynesian DSGE model for the Euro Area economy with limited assets market participation, financially constrained banks and price and wage rigidities. To motivate my research question I employ an external instrument SVAR using a QE shock identified through a high frequency approach. Results from the data specification and the DSGE model show that QE is stimulative and reduces income and consumption inequality when the assets market participation level is set to the Euro Area average. Quantitative easing can be contractionary for low levels of financial participation when wages are fully flexible, while sticky wages mute the contractionary effects.

Arguably, a substantial limitation of this model is the absence of housing, which has been left out to reduce the model's complexity. Slacalek et al. (2020) provide a characterization of Euro Area households based on their holdings of liquid and illiquid assets. They can be broadly divided into optimizers, wealthy hand to mouth and poor hand to mouth. Differently to this model, optimizers and wealthy hand to mouth hold housing on top of their other assets which, importantly, are very similar in volume. Therefore, including housing in the present study, accommodative monetary policy would have had the same positive price to income effect of the wealthy hand to mouth and the optimizers, leaving, at least qualitatively, the inequality results of this paper between the two groups intact.

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Appendix A   Hand to Mouth Evidence

I restrict attention to the first wave of the HFCS data conducted mainly in 2009 and 2010 in order to abstract from the effects of the 2008 financial crisis. Also, the survey was performed well before the start of ECB's QE in March 2015. Nevertheless, the results are similar for the subsequent waves. The data has been collected from 15 Euro Area member states for a sample of more than 62,000 households.

Figure 7 reports the distribution of financial and real asset holdings of the Euro Area residents.[24] As the Figure shows, 20-30% of the Euro Area households hold a total value of financial assets that is close to zero (green bar). In comparison all percentiles of Euro Area households hold substantial values of real assets (yellow bar). Naturally, this leads to the question on how QE will affect those that have financial asset holdings in comparison with those that do not.

Fig. 7. Total Financial and Real Assets among EA Households

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[24]Financial assets include deposits (sight and saving accounts), mutual funds, bonds, shares, money owed to the households, value of voluntary pension plans and whole life insurance policies of household members and other financial assets item - which includes private non-self-employment businesses, assets in managed accounts and other types of financial assets. Real assets include the value of household's main residence.

Appendix B   SVAR Figures and Cholesky Identification

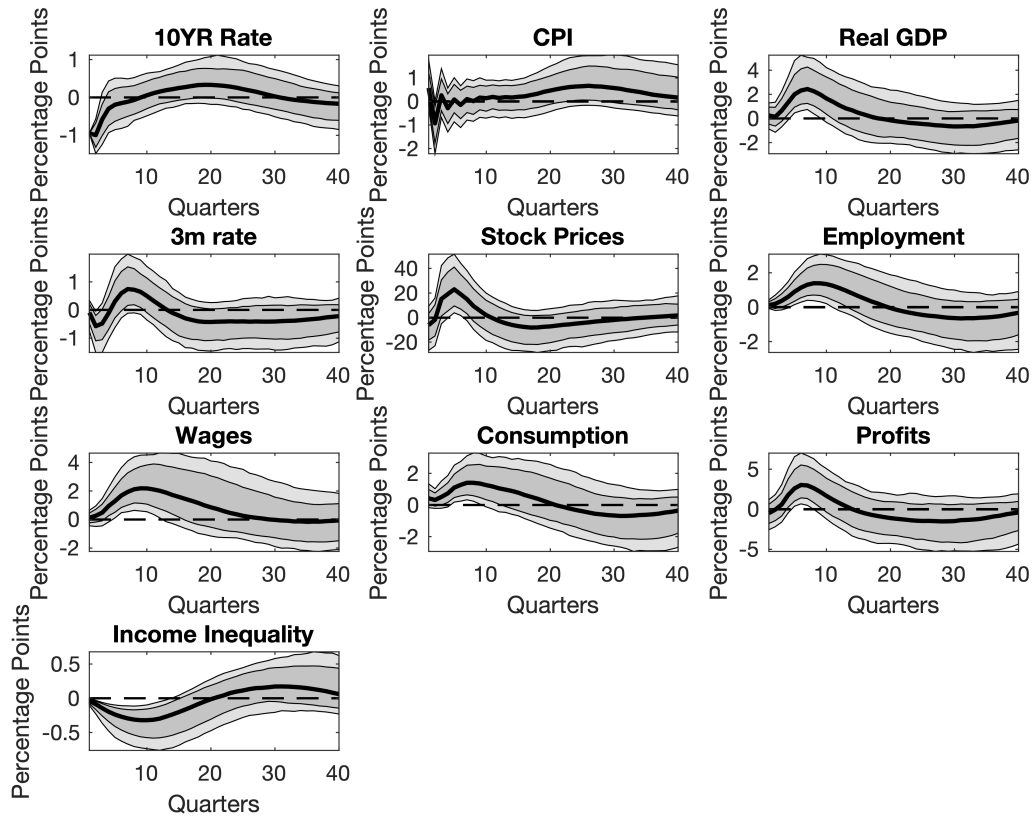Figure 8 shows the impulse responses to a QE shock of all variables used in the SVAR estimation.

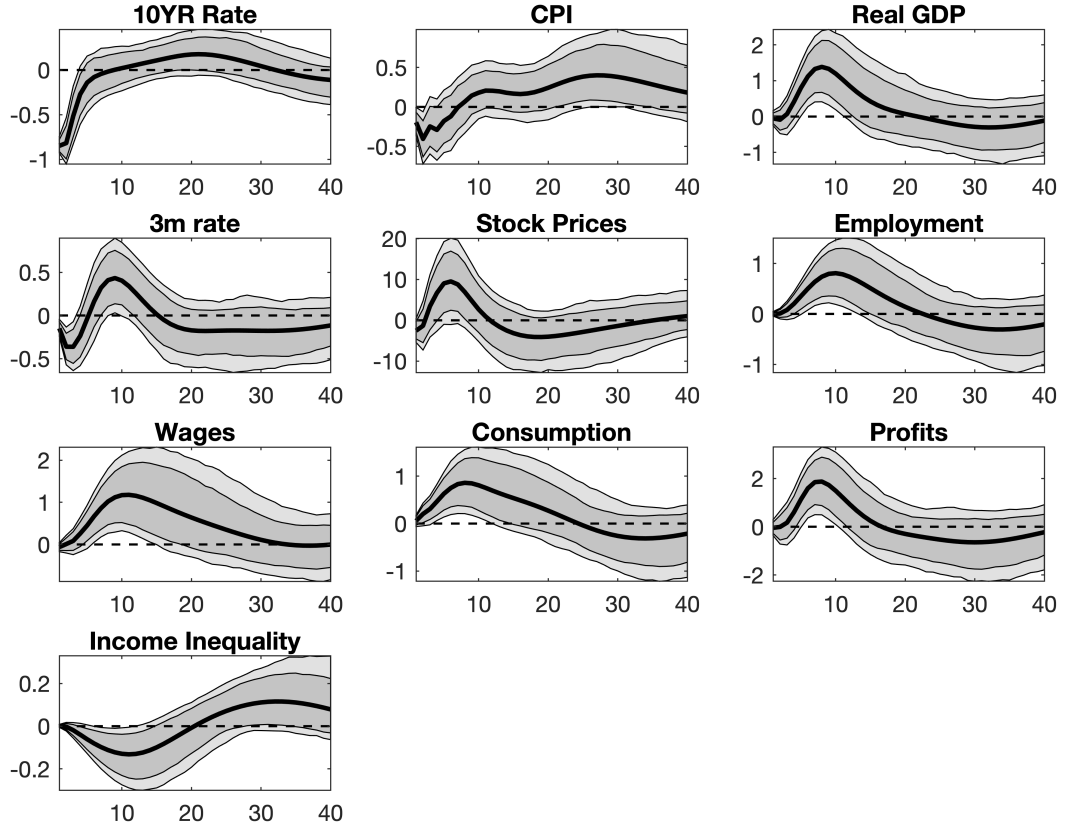Fig. 8. Impulse Responses to a QE Shock: All Variables.
Notes: The solid line shows the median responses after 50000 draws. The darker bands span the 16-84 percentiles of the draws distribution while the lighter band the 9-95 percentiles. The X axis shows the quarters while the Y axis the percent change.

Figure 9 shows the impulse responses to a QE shock using the standard Cholesky identification. The shock is normalised such as to produce a 95 bps drop in the ten-year rate. In comparison with the impulse responses using the external instrument approach, the two methods give similar results. All variables are responding as expected after an accommodative monetary shock with the exception of the price index. The CPI drops for the first 10 quarters and then increases though insignificantly.

Fig. 9. Impulse Responses to a QE Shock with Cholesky Identification.
Notes: The solid line shows the median responses after 50000 draws. The darker bands span the 16-84 percentiles of the draws distribution while the lighter band the 9-95 percentiles. The X axis shows the quarters while the Y axis the percent change.

Appendix C   Wage-Setting by Unions

The problem of the union is to maximize its objective function (in the main text).

$$\lambda \left[u_{c,t}^r W_{h,t} L_{h,t} - \frac{\chi}{1+\epsilon} L_t^{1+\epsilon} \right] + (1-\lambda) \left[u_{c,t}^o W_{h,t} L_{h,t} - \frac{\chi}{1+\epsilon} L_t^{1+\epsilon} \right]$$

.

subject to

$$L_{h,t} = \left(\frac{W_{f,t}}{W_t^*} \right)^{-\epsilon_w} L_t$$

The first order condition yields:

$$\left(\frac{\lambda}{u_{c,t}^r u_{l,t}^r} + \frac{1-\lambda}{u_{c,t}^o u_{l,t}^o}\right) W_t = \mu^W$$

where $\mu^W = \frac{\epsilon_w}{\epsilon_w - 1}$ and $u_{c,t}^j u_{l,t}^j$ is the marginal rate of substitution of agent of type j.

Appendix D   Bank's Problem

This appendix describes the method used for solving the banker's problem. I solve this, with the method of undetermined coefficient in the same fashion as in Gertler and Kiyotaki (2010). I conjecture that a value function has the following linear form:

$$V_t(s_{j,t}, d_{j,t}, b_{j,t}^B, m_{j,t}^B) = \nu_{l,j,t} s_{j,t} + \nu_{b,j,t} b_{j,t}^B + \nu_{m^B,j,t} m_{j,t}^B - \nu_{d,j,t} d_{j,t} \tag{D.1}$$

where $\nu_{s,j,t}$ is the marginal value from credit for bank j, $\nu_{d,t}$ the marginal cost of deposits, $\nu_{m^B,j,t}$ the marginal value from the central bank reserves and $\nu_{b^B,j,t}$ the marginal value from purchasing one extra unit of sovereign bonds. The banker's decision problem is to choose $s_{j,t}, b_{j,t}^B, m_{j,t}^B, d_{j,t}$ to maximize $V_{j,t}$ subject to the incentive constraint (20) and the balance sheet constraint (17). Using (17) we can eliminate $d_{j,t}$ from the value function. This yields:

$$V_{j,t} = s_{j,t}(\nu_{s,t} - \nu_{d,t}Q_t) + b_{j,t}^B(\nu_{b,j,t} - \nu_{d,j,t}q_t) + m_{j,t}^B(\nu_{m,j,t} - \nu_{d,j,t}) + \nu_{d,t}n_{j,t}^B.$$

Let $\mathcal{L}$ be the Lagrangian of the maximization problem and λ_t the Lagrange multiplier.

$$\mathcal{L} = V_t + \lambda_t[V_t - \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B)] = (1 + \lambda_t)V_t - \lambda_t \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B).$$

The first order and Kuhn-Tucker conditions for the maximization problem are:

$$\frac{\theta \mathcal{L}}{\theta s_{j,t}} : (1 + \lambda_t)(\frac{\nu_{s,j,t}}{Q_t} - \nu_{d,j,t}) = \lambda_t \theta \tag{D.2}$$

$$\frac{\theta \mathcal{L}}{\theta b_{j,t}^B} : (1 + \lambda_t)(\frac{\nu_{b^B,t}}{q_t} - \nu_{d,j,t}) = \Delta \lambda_t \theta \tag{D.3}$$

$$\frac{\theta \mathcal{L}}{\theta m_{j,t}^B} : (1 + \lambda_t)(\nu_{m^B,t} - \nu_{d,j,t}) = \omega \lambda_t \theta \tag{D.4}$$

The Kuhn-Tucker condition yields:

$$KT : \lambda_t[s_{j,t}(\nu_{s,j,t} - \nu_{d,t}Q_t) + b_{j,t}^B(\nu_{b^B,j,t} - \nu_{d,j,t}q_t) + m_{j,t}^B(\nu_{m^B,j,t} - \nu_{d,j,t})$$
$$+ \nu_{d,j,t}n_{j,t}^B - \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B)] = 0. \tag{D.5}$$

I define the excess value of bank's financial claim holdings as

$$\mu_t^s = \frac{\nu_{s,j,t}}{Q_t} - \nu_{d,j,t}. \tag{D.6}$$

The excess value of bank's bond holdings relative to deposits

$$\mu_t^b = \frac{\nu_{b^B,t}}{q_t} - \nu_{d,j,t},$$

and the excess value of bank's reserve holdings relative to deposits

$$\mu_t^m = \nu_{m^B,j,t} - \nu_{d,j,t}.$$

Then from the first order conditions we have:

$$\mu_t^b = \Delta \mu_t^s. \tag{D.7}$$

Setting the fraction of the absconding rate for reserves ω to 0%, the reserves first order condition (D.4) implies that

$$\nu_{m^B,t} = \nu_{d,j,t}. \tag{D.8}$$

This relationship implies that the gain from one extra unit of reserves is exactly the same with the cost of raising one extra unit of deposits. This helps us to show that when reserves is a strictly riskless asset, the bank is not taking them into account when the optimization problem is formulated. From (D.5) and (D.7) when the constraint is binding ($\lambda_t > 0$) we get:

$$s_{j,t}(\nu_{s,t} - \nu_{d,t}Q_t) + b_{j,t}^B(\nu_{b^B,j,t} - \nu_{d,j,t}q_t) + m_{j,t}^B(\nu_{m,j,t}^B - \nu_{d,j,t}) + \nu_{d,t}n_{j,t} = \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B)$$

$$s_{j,t}(\mu_t^s Q_t) + b_{j,t}^B(\mu_t^b q_t) + m_{j,t}^B(\mu_t^m) + \nu_{d,t}n_{j,t} = \theta(Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B)$$

$$Q_t s_{j,t}(\mu_t^s - \theta) + q_t b_{j,t}^B(\Delta \mu_t^s - \Delta \theta) + m_{j,t}^B(\omega \mu_t^s - \omega \theta) + \nu_{d,t}n_{j,t} = 0$$

$$Q_t s_{j,t}(\mu_t^s - \theta) + \Delta q_t b_{j,t}^B(\mu_t^s - \theta) + \omega m_{j,t}^B(\mu_t^s - \theta) + \nu_{d,t}n_{j,t} = 0$$

and by rearranging terms, we get equation the adjusted leverage constraint:

$$Q_t s_{j,t} + \Delta q_t b_{j,t}^B + \omega m_{j,t}^B = \frac{\nu_{d,t}n_{j,t}}{\theta - \mu_t^s} \tag{D.9}$$

which gives the bank asset funding. It is given by the constraint at equality, where ϕ_t is the maximum leverage allowed for the bank. The constraint limits the portfolio size to the point where the bank's required capital is exactly balanced by the fraction of the weighted measure of its assets. Hence, in times of crisis, where a deterioration of banks' net worth takes place, supply for assets will decline.

Now, in order to find the unknown coefficients I return to the guessed value function

$$V_{j,t} = Q_t s_{j,t}(\mu_t^s) + q_t b_{j,t}^B(\mu_t^b) + m_{j,t}^B(\mu_t^m) + \nu_{d,t} n_{j,t}^B. \tag{D.10}$$

Substituting (D.9) into the guessed value function yields:

$$V_t = (n_{j,t}\phi_t - \Delta q_t b_{j,t}^B - \omega m_{j,t}^B)\mu_t^s + q_t b_{j,t}^B(\mu_t^b) + m_{j,t}^B(\mu_t^m) + \nu_{d,t} n_{j,t}^B \Leftrightarrow \tag{D.11}$$
$$V_t = (n_{j,t}\phi_t)\mu_t^s + q_t b_{j,t}^B(\mu_t^b - \Delta\mu_t^s) + m_{j,t}^B(\mu_t^m - \omega\mu_t^s) + \nu_{d,t} n_{j,t}^B$$

and by (D.7) the guessed value function (D.11) becomes:

$$V_t = \phi_t \mu_t^s n_{j,t} + \nu_{d,j,t} n_{j,t}$$

Given the linearity of the value function we get that

$$A_t^B = \phi_t \mu_t^s + \nu_{d,j,t}.$$

The Bellman equation (21) now is:

$$V_{j,t-1}(s_{j,t-1}, x_{j,t-1}, d_{j,t}, m_{j,t-1}) = \mathbb{E}_{t-1} \Lambda_{t-1,t} \sum_{i=1}^{\infty} \{(1-\sigma_B) n_{j,t}^B$$
$$+ \sigma_B(\phi_t \mu_t^s + \nu_{d,j,t}) n_{j,t}^B\}. \tag{D.12}$$

By collecting terms with $n_{j,t}$ the common factor and defining the variable Ω_t as the marginal value of net worth:

$$\Omega_{t+1} = (1 - \sigma_B) + \sigma_B(\mu_{t+1}^s \phi_{t+1} + \nu_{d,t+1}). \tag{D.13}$$

The Bellman equation becomes:

$$V_{j,t}(s_{j,t}, b_{j,t}^B, m_{j,t}^B, d_{j,t}) = E_t \Lambda_{t,t+1} \Omega_{t+1} n_{t+1}^B =$$
$$= E_t \Lambda_{t,t+1} \Omega_{t+1}[R_{k,t} Q_{t-1} s_{j,t-1} + R_{b,t} q_{t-1} b_{j,t-1}^B + R_t m_{j,t}^B - R_t d_{j,t}]. \tag{D.14}$$

The marginal value of net worth implies the following: Bankers who exit with probability $(1-\sigma_B)$ have a marginal net worth value of 1. Bankers who survive and continue with probability σ_B, by gaining one more unit of net worth, they can increase their assets by ϕ_t and have a net profit of μ_t per assets. By this action they acquire also the marginal cost of deposits $\nu_{d,t}$ which is saved by the extra amount of net worth instead of an additional unit of deposits. Using the method of undetermined coefficients and

comparing (D.1) with (D.14) we have the final solutions for the coefficients:

$$\nu_{s,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{k,t+1} Q_t$$
$$\nu_{b^B,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{b,t+1} q_t$$
$$\nu_{m^B,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1}$$
$$\nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1}$$
$$\mu_t^s = \frac{\nu_{s,j,t}}{Q_t} - \nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{k,t+1} - R_{t+1}]$$
$$\mu_t^b = \frac{\nu_{b,j,t}}{Q_t} - \nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{b,t+1} - R_{t+1}] \tag{D.15}$$
$$\mu_t^m = \nu_{m,j,t} - \nu_{d,j,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{t+1} - R_{t+1}] = 0 \tag{D.16}$$

Appendix E   Price Setting

Final-Good Firms.— The profit maximization problem of the retail firm is:

$$\max_{Y_t(j)} P_t \left(\int_0^1 Y_t(i)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} - \int_0^1 P_t(i) Y_t(i) di.$$

The first order condition of the problem yields:

$$P_t \frac{\zeta}{\zeta-1} \left(\int_0^1 Y_t(i)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}-1} \frac{\zeta-1}{\zeta} Y_t(i)^{\frac{\zeta-1}{\zeta}-1} = P_t(i).$$

Combining the previous FOC with the definition of the aggregate final good we get:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\zeta} Y_t.$$

Nominal output is the sum of prices times quantities across all retail firms i:

$$P_t Y_t = \int_0^1 P_t(i) Y_t(i) di.$$

Using the demand for each retailer we get the aggregate price level:

$$P_t = \left(\int_0^1 P_t(i)^{1-\zeta} di \right)^{\frac{1}{1-\zeta}}.$$

Intermediate-Good Firms.— Intermediate good firms are not freely able to change prices each period. Following the Calvo price updating specification each period there is a fixed probability $1 - \gamma$ that a firm will be able to adjust its price.

The problem of the firm can be decomposed in two stages. Firstly, the firm hires

labour and rents capital to minimize production costs subject to the technology constraint (33). Thus, it is optimal to minimize their costs which are the rental rate to capital and the wage rate for labour:

$$\min_{K_t(i),L_t(i)} P_t W_t l_t(i) + P_t Z_t K_t(i)$$

subject to

$$A_t K_t(i)^\alpha L_t(i)^{1-\alpha} \geqslant \left(\frac{P_t(i)}{P_t}\right)^{-\zeta} Y_t.$$

The problem's first order conditions are:

$$W_t = \frac{P^{nom}_{m,t}(i)}{P_t}(1-\alpha)A_t\frac{Y_t(i)}{L_t(i)}, \tag{E.1}$$

$$Z_t = \frac{P^{nom}_{m,t}(i)}{P_t}\alpha A_t\frac{Y_t(i)}{K_t(i)}. \tag{E.2}$$

$P^{nom}_{m,t}$ is the Lagrange multiplier of the minimization problem and the marginal cost of the firms with $P_{m,t} = \frac{P^{nom}_{m,t}(i)}{P_t}$ being the real marginal cost. Standard arguments lead to that marginal cost is equal across firms. Solving together the above equations we find an expression for the real marginal cost $P_{m,t}$ which is independent of each specific variety:

$$P_{m,t} = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^\alpha W_t^{1-\alpha} Z_t^\alpha.$$

In the second stage of the firm's problem, given nominal marginal costs, the firm chooses its price to maximize profits. Firms are not freely able to change prices each period. Each period there is a fixed probability $1-\gamma$ that a firm will adjust its price. Each firm chooses the reset price P_t^* subject to the price adjustment frequency constraint. Firms can also index their price to the lagged rate of inflation with a price indexation parameter γ_p. They discount profits s periods in the future by the stochastic discount factor $\Lambda_{t,t+s}$ and the probability that a price price chosen at t will remain the same for some periods γ^s. The second stage of the updating firm at time t us to choose $P_t^*(i)$ to maximize discounted real profits:

$$\max_{P_t^*(i)} \mathbb{E}_t \sum_{s=0}^\infty \gamma^s \Lambda_{t,t+1}\left(\frac{P_t^*(i)}{P_{t+s}} - P_{m,t+s}\right)Y_{t+s}(i)$$

subject to

$$Y_{t+s}(i) = \left(\frac{P_t^*(i)}{P_{t+s}}\prod_{\kappa=1}^s(1+\pi_{\tau+\kappa-1})^{\gamma_p}\right)^{-\zeta}Y_{t+s}.$$

where π_t is the rate of inflation from $t-i$ to t. The first order condition of the problem

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is:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} \left(\frac{P_t^*(i)}{P_{t+s}} \prod_{\kappa=1}^{s} (1 + \pi_{\tau+\kappa-1})^{\gamma_p} - P_{m,t+s} \frac{\zeta}{\zeta-1} \right) Y_{t+s}(i) = 0.$$

Using the constraint and rearranging we get:

$$P_t^*(i) = \frac{\zeta}{\zeta-1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} P_{m,t+s} P_{t+s}^{\zeta} Y_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \gamma^s \Lambda_{t,t+1} P_{t+s}^{\zeta-1} \prod_{\kappa=1}^{s} (1 + \pi_{\tau+\kappa-1})^{\gamma_p} Y_{t+s}}.$$

Since nothing on the right hand side depends on each firm i, all updating firms will update to the same reset price, P_t^*. By the law of large numbers the evolution of the price index is given by:

$$P_t = [(1-\gamma)(P_t^*)^{1-\zeta} + \gamma(\Pi_{t-1}^{\gamma_p} P_{t-1})^{1-\zeta}]^{\frac{1}{1-\zeta}}.$$

Capital Goods Producers.— Capital goods producers produce new capital and sell it to goods producers at a price Q_t. Investment on capital (I_t) is subject to adjustment costs. Their objective is to choose $\{I_t\}_{t=0}^{\infty}$ to solve:

$$\max_{I_\tau} \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_t I_t - [1 + \tilde{f}\left(\frac{I_\tau}{I_{\tau-1}}\right) I_\tau] \right\}.$$

where the adjustment cost function $\tilde{f}$ captures the cost of investors to increase their capital stock:

$$\tilde{f}\left(\frac{I_\tau}{I_{\tau-1}}\right) = \frac{\eta}{2}\left(\frac{I_\tau}{I_{\tau-1}} - 1\right)^2 I_\tau.$$

η is the inverse elasticity of net investment to the price of capital. The solution to the decision problem of the investors yields the competitive price of capital:

$$Q_t = 1 + \left(\eta \frac{I_\tau}{I_{\tau-1}}\left(\frac{I_\tau}{I_{\tau-1}} - 1\right) + \frac{\eta}{2}\left(\frac{I_\tau}{I_{\tau-1}} - 1\right)^2 - \eta \Lambda_{t,\tau} \frac{I_{\tau+1}^2}{I_\tau^2}\left(\frac{I_\tau}{I_{\tau-1}} - 1\right) \right).$$

Profits. Firms' nominal profits are: $Prof_t(i) = P_t(i)Y_t(i) - W_t P_t L_t(i) - Z_t P_t K_t(i)$. Using (E.1) and (E.2) we get $W_t P_t L_t(i) = P_{m,t}^{nom}(i)(1-\alpha)A_t Y_t(i)$ and $Z_t P_t K_t(i) = P_{m,t}^{nom}(i)\alpha A_t Y_t(i)$, We then can write real profits as: $\frac{Prof_t(i)}{P_t} = \frac{P_t(i)}{P_t}Y_t(i) - P_{m,t}Y_t(i)$.[25]

Aggregation. Total profits of non financial firms are equal to the sum of profits earned

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[25]In Gertler and Karadi (2011) firms derive revenues from selling their good and selling the undepreciated portion of the physical capital back to the capital producers. Therefore profits are $Prof_t = P_t(i)Y_t(i) + Q_t(i)(1-\delta)K_t(i) - W_t P_t L_t(i) - R_{k,t}Q_{t-1}(i)K_t(i)$. Substituting $R_{k,t}$ from (18) we get the same equation for aggregate real profits as in (E.3).

by intermediate good firms:

$$Prof_t = \int_0^1 Prof_t(i)di.$$

Under standard arguments and using that supply should equal demand in all markets: $\int_0^1 N_t(i)di = N_t$, $\int_0^1 K_t(i)di = K_t$, we get that total profits of the firms are:

$$Prof_t = Y_t - W_t L_t - Z_t K_t. \tag{E.3}$$

Appendix F Steady State

As it is shown on the main text, the rule of thumb agents will always supply constant labour hours equal to and the first order condition for labour supply the rule of thumb agents:

$$L^r = \left(\frac{1}{\chi}\right)^{\left(\frac{1}{1+\epsilon}\right)}$$

From labour hours the aggregator (15) we get the labour hours supplied by the optimizing agents:

$$L^o = \frac{L - \lambda L^r}{1 - \lambda}.$$

Rearranging the optimizing agents' first order condition for labour, utilizing the fact that $U_c^o = 1/C^o$, we can get an expression between consumption of the agents and labour supply:

$$C^o = \frac{W}{\chi(L^o)^\epsilon}.$$

Utilizing the above relation and the optimal consumption path of the rule of thumb agents, the consumption aggregator (14) becomes

$$C = \lambda W L^r + (1 - \lambda)\frac{W}{\chi(L^o)^\epsilon}.$$

After some algebraic manipulation we end up to the total consumption coming from the demand side of the economy:

$$C = W\left[\lambda L^r + (1 - \lambda)^{1+\epsilon}\frac{(L - \lambda L^r)^{-\epsilon}}{\chi}\right] \tag{F.1}$$

In addition, from the resource constraint we have:

$$C = Y - I - G - \tau_b B^G - \tau_s S^G,$$

where in a steady state $B^G = S^G = 0$. Therefore:

$$C = L \left[(1 - \gamma) \left(\frac{K}{L} \right)^\alpha - \delta \frac{K}{N} \right] \tag{F.2}$$

To get an expression of K/L we make use of the marginal product of capital (E.2):

$$\frac{L}{K} = (\frac{Z}{A\alpha})^{\frac{1}{1-\alpha}},$$

yielding

$$\frac{K}{L} = \left(\frac{\alpha \left(\frac{\epsilon - 1}{\epsilon} \right)}{R_k - (1 - \delta)} \right)^{\frac{1}{1-\alpha}}. \tag{F.3}$$

Thus, combining the expressions (F.1), (F.2), (F.3) we obtain an equation depending only on parameters, calibrated values ($spread_{Rk}$) and L^k and determines steady state hours L. Having found L, using the labour hours aggregator (15) we can easily find the labour hours worked by the rule of thumb agents L^o. Thus, consumption of the optimizing agents can be pinned down. Notice that an equation between optimizers' consumption and aggregate consumption can be found by combining the first order condition for labour supply and the demand side aggregate consumption equation(F.1) and solving for W. Then:

$$C^o = \frac{C(1 - \lambda)^\epsilon / \chi}{\lambda L^o (L - \lambda L^o)^\epsilon + (1 - \lambda)^{1+\epsilon} / \chi} \tag{F.4}$$

Appendix G   Robustness: Effect of Policy Rate Specification

In this section I provide robustness of the main text results' on unconventional monetary policy and inequality presented in Section 4.2. Specifically, I show that results hold for any of the three interest rate specifications considered. These include: a constant rate for four periods (main text specification), a free floating rate and a fully constant rate. In the main text Figures 3 and 4 plot the response of aggregate variables and inequality measures when the policy rate remains constant for four periods. This is a reduced form way to mimic the fact that the central banks do not change the policy rate when engaging in QE due to the effective lower bound. Due to intertemporal substitution effects arising from this specification optimizers' consumption increases on impact after a QE shock.

Figure 10 shows the responses of the same variables shown in the main text, when the policy rate is let to follow the Taylor rule from period one. In contrast with the main text, policy rate increases as it is imposed by the inflation and output increase. Qualitatively the directions of the responses remain the same with Figure 3. Nevertheless, due to the rise of the policy rate output peaks at a 1.2% increase instead of 3% for the same

amount of bonds purchases by the central bank. Responses of the remaining variables are also more moderate in comparison with the policy rate specification in the main text.
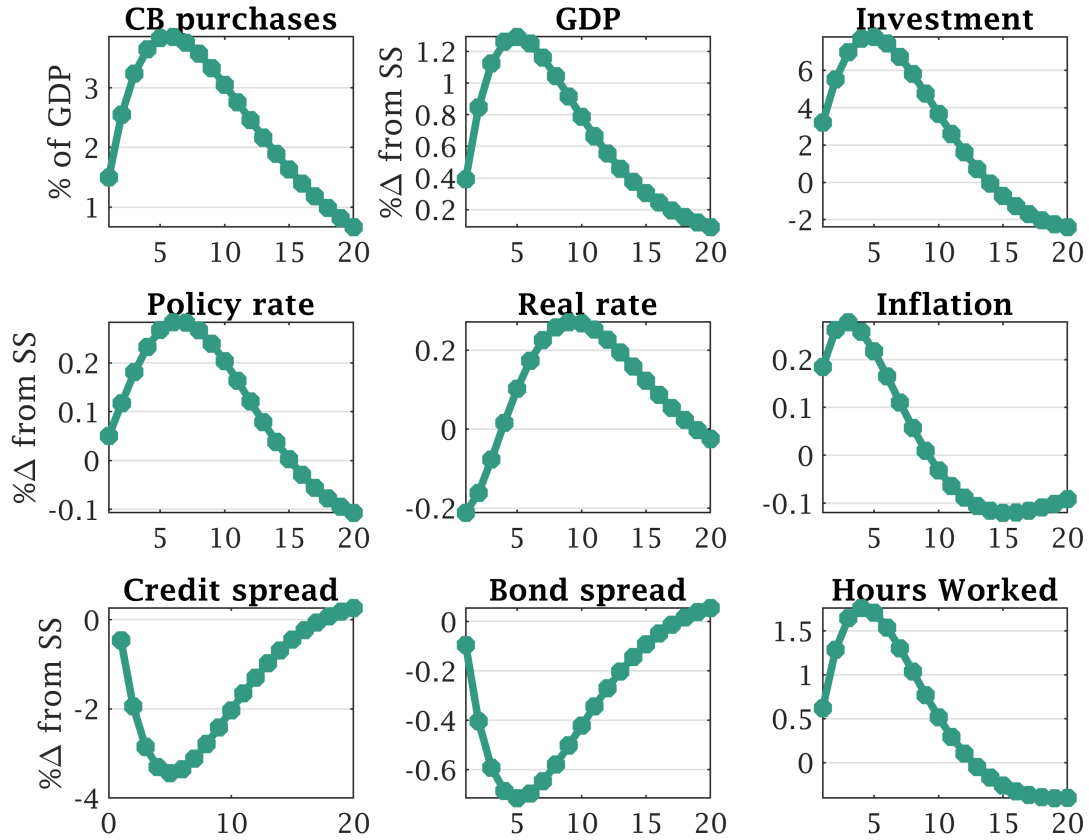

Fig. 10. Government Asset Purchase Shock. Flexible Policy Rate

Figure 11 describes the behaviour of the two agents' consumption and income responses for the case of a free floating rate and it is a complement to Figure 4 in the main text. The main difference with the main text's case is that due to the increase of the policy rate, optimizers' consumption now decreases on impact. This is due to intertemporal substitution effect. The lower savers' consumption would amplify further the reduction of consumption inequality, but the increase in the rule of thumb agents' consumption is smaller than in the main text leading to a similar magnitude of consumption inequality decrease.
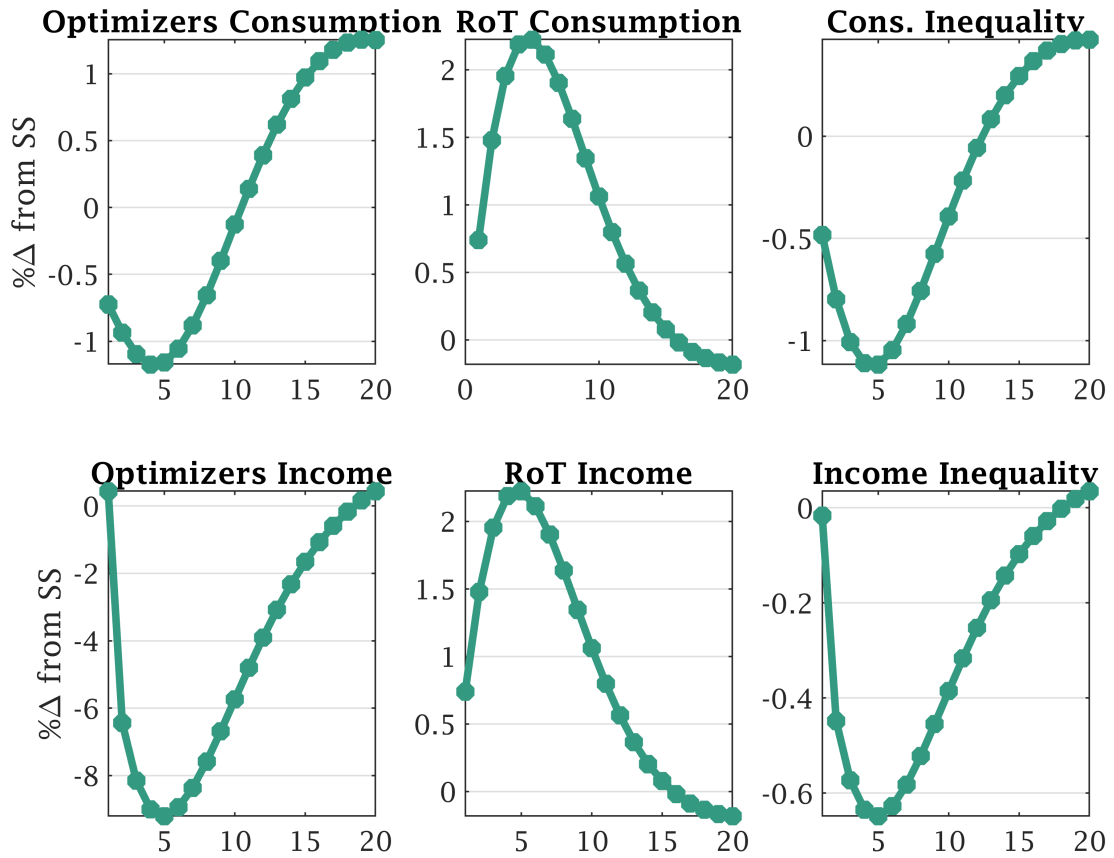
Fig. 11. Government Asset Purchase Shock. Constant Policy Rate

Finally, I perform the same exercise in Figures 12 and 13 for the third interest rate specification where it stays constant for all the simulation periods. The fact that the policy rate does not follow the Taylor rule reinforces the stimulative effects of the QE as is evident by the output and investment increase after the QE shock.
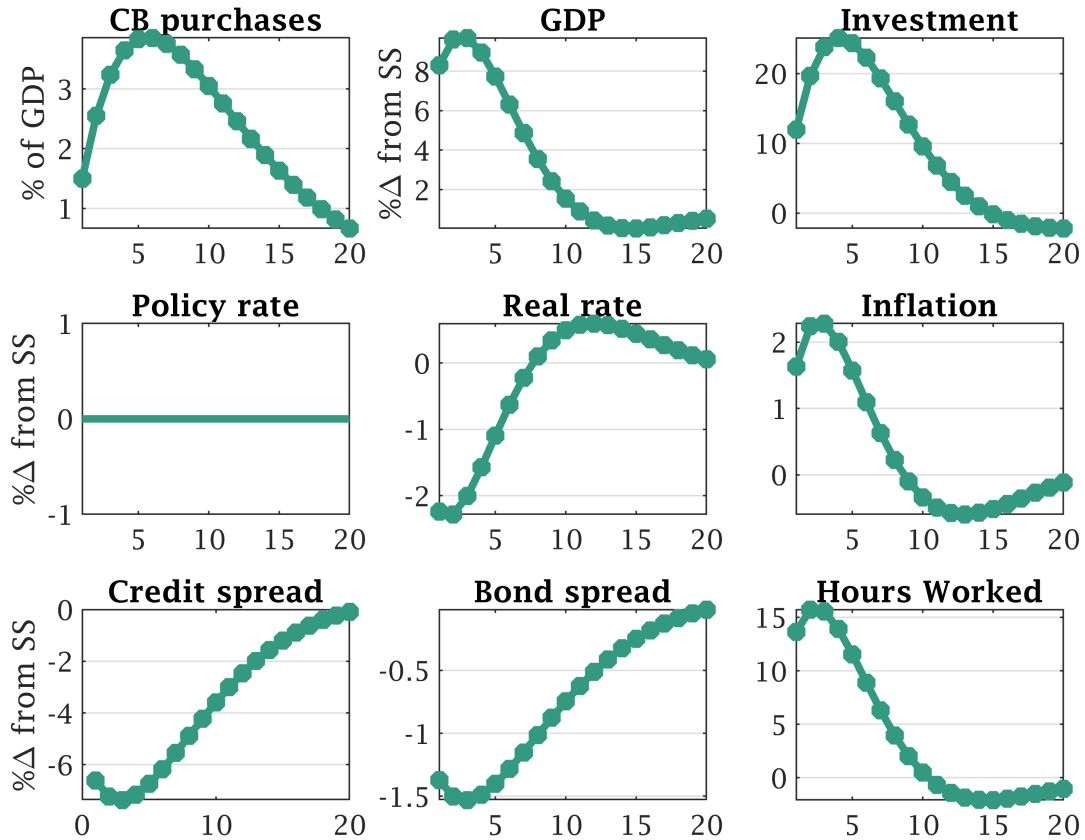

Fig. 12. Government Asset Purchase Shock. Flexible Policy Rate

On the consumption and income inequality, again the responses have the same sign as in the main text. Due to the stimulative effects of a constant rate, consumption and income increase for more leading to a higher fall of the consumption and income inequality.
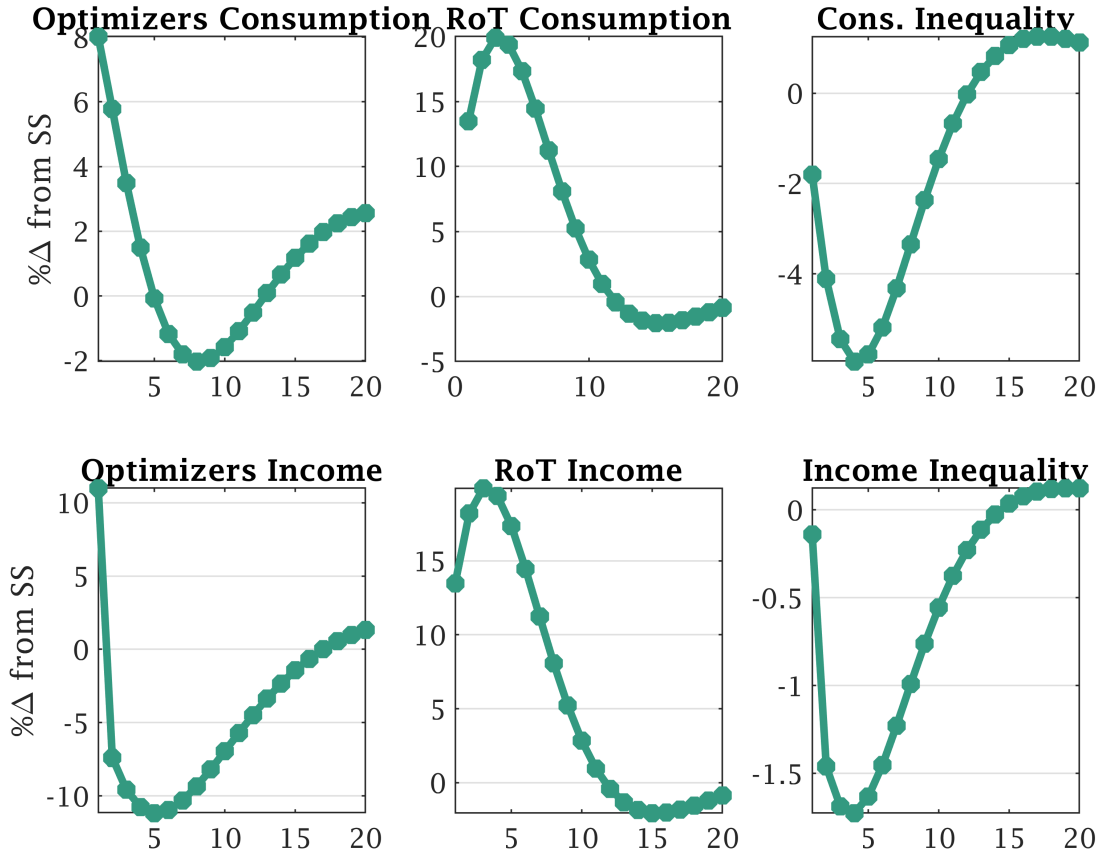
Fig. 13. Government Asset Purchase Shock. Constant Policy Rate

Appendix H Derivations for Section 4

Proof of (34) **for the case of perfect labour market:**

$$c_t = \lambda c_t^r + (1-\lambda)c_t^o$$
$$c_t = \lambda w_t + (1-\lambda)(w_t - \epsilon l_t^o)$$
$$c_t = \lambda w_t + (1-\lambda)(w_t - \epsilon \frac{l_t}{1-\lambda})$$

therefore

$$w_t = c_t + \epsilon l_t.$$

Proof of (34) **for the case of wage setting by unions:** The first order condition for

the wage setting problem yields:

$$\left(\frac{\lambda}{MRS_t^r} + \frac{1-\lambda}{MRS_t^o}\right) W_t = \mu^W$$

where $MRS_t^o = u_{c,t}^o u_{l,t}^o$ and $MRS_t^r = u_{c,t}^r u_{l,t}^r$. Log-linearising this around the steady state yields:

$$w_t = \psi_r c_t^r + \psi_o c_t^o + \epsilon(\psi_r + \psi_o)l_t \tag{H.1}$$

where $\psi_r = \mu^W \frac{\lambda W}{MRS^r}$ and $\psi_o = \mu^W \frac{(1-\lambda)W}{MRS^o}$. Since both agents provide the same labour hours at any time and consumption in steady state is equalized between both agents, in steady state $MRS^o = MRS^r$. Therefore we can write (H.1) as:

$$w_t = c_t + \epsilon l_t.$$

Proof of IS equation (36): Assuming no TFP process in the production function, its log-linearised form is: $y_t = \alpha k_t + (1-\alpha)n_t$. Solving for n_t and substituting to (35) we get

$$c_t^o = c_t - \left(\frac{\lambda}{1-\lambda}\right)\left[\frac{y_t - \alpha k_t}{1-\alpha}\right]$$

Log-linearising the resource constraint we get $y_t = c_t s_c + i_t s_i + s_g$ since the proportion of government bond and shares are zero in the steady state. Solving for $c_t = \frac{y_t - i_t s_i - s_g}{s_c}$ and inserting the resource constraint we have:

$$c_t^o = y_t\left(\frac{1}{s_c} - \epsilon\frac{\lambda}{(1-\lambda)}\frac{1}{(1-\alpha)}\right) - i_t\frac{s_i}{s_c} - \frac{s_g}{s_c} + \epsilon\frac{\lambda}{(1-\lambda)}\frac{1}{(1-\alpha)}\alpha k_t.$$

Inserting the above into the optimizers Euler equation $c_t^o = \mathbb{E}_t\{c_{t+1}^o\} + [\mathbb{E}_t\{\pi_{t+1}\} - r_t]$, we get

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\delta}[r_t - \mathbb{E}_t\{\pi_{t+1}\}] - \frac{1}{\delta}\frac{s_i}{s_c}\Delta i_{t+1} + \frac{1}{\delta}\frac{\epsilon\lambda}{(1-\lambda)(1-\alpha)}[\alpha\Delta k_{t+1}].$$

where

$$\delta = \frac{1}{s_c} - \epsilon\frac{\lambda}{(1-\lambda)(1-\alpha)}$$

and $s_c = C^{ss}/Y^{ss}$, $s_i = I^{ss}/Y^{ss}$ and $s_g = G^{ss}/Y^{ss}$.

H.1   Derivations for the Direct Effect of QE on Output

To introduce government bonds in the IS equation derived above, I use the capital market clearing. From the capital market clearing (27) we have $K_t = K_t^B + K_t^H + K_t^G$. Log-linearising it around the steady-state yields:

$$k_t = s_k^H k_t^H + s_k^B k_t^B + s_k^G k_t^G, \tag{H.2}$$

where $s_k^H = K^H/K, s_k^B = K^B/K, s_k^G = K^G/K$. Log-linearising the aggregate incentive constraint of the bank around the steady state:

$$QS^B(\hat{Q}_t + k_t^B) + \Delta qB^B(\hat{q}_t + b_t^B) = \phi N(\hat{\phi} + n_t).$$

The small letters are the log-deviations of the variables from their steady state. $\hat{Q}_t$ is the corresponding value for the price of capital and $\hat{q}_t$ for the price of bonds. Solving for the bankers' capital holdings:

$$k_t^B = -\frac{\Delta qB^B}{QS^B}b_t^B - \frac{\Delta qB^B}{QS^B}\hat{q}_t + \frac{\phi N}{QS^B}(\hat{\phi} + n_t) - \hat{Q}_t. \tag{H.3}$$

Taking the log deviations of the capital market clearing (28) and solving for the banks' bond holdings:

$$b_t^B = -\frac{s_b^G}{s_b^B}b_t^G + \frac{b_t}{s_b^B} - \frac{s_b^H}{s_b^B}b_t^H, \tag{H.4}$$

where $s_b^H = B^H/B, s_b^B = B^B/B, s_b^G = B^G/B$.

Plugging (H.3),(H.4) into (H.2):

$$k_t = s_k^H k_t^H + s_k^B[-\frac{\Delta qB^B}{QS^B}\left(-\frac{s_b^G}{s_b^B}b_t^G + \frac{b_t}{s_b^B} - \frac{s_b^H}{s_b^B}b_t^H\right)$$

$$-\frac{\Delta qB^B}{QS^B}\hat{q}_t + \frac{\phi N}{QS^B}(\hat{\phi} + n_t) - \hat{Q}_t] + s_k^G k_t^G.$$

Since we are interested on the direct effect of government bond purchases (assuming everything else remains constant) we are interested in

$$k_t = s_k^B \frac{\Delta qB^B}{QS^B}\frac{B^G}{B^B}b_t^G = \frac{\Delta B^G}{S}b_t^G.$$

The *direct effect* on output using the IS equation is:

$$-\frac{1}{\delta}\frac{\epsilon\lambda\alpha}{(1-\lambda)(1-\alpha)}\frac{\Delta B^G}{S}b_t^G.$$

Using the fact that $b_t^G = \frac{B_t^G - B^G}{B^G}, B^G = 0$, and after some algebra manipulation the above equation becomes:

$$\frac{1}{\frac{(1-\lambda)(1-\alpha)K}{s_c\alpha\epsilon\lambda\Delta} - \frac{\epsilon\lambda K}{\alpha\epsilon\lambda\Delta}}B_t^G.$$

Appendix I   Robustness: Inverted Aggregate Demand Logic

In this Appendix, I show that the IADL for the case of QE and a monetary policy shock holds for any reasonable parametrization of the inverse Frisch elasticity of labour supply. Figures 14 and 15 show the impact effect of output after a monetary policy and QE shock conditional on the degree of asset markets participation. This is repeated for four different values for the inverse Frisch elasticity: 0.5, 1, 1.5 and 2. What is clear, is that under every parametrization of the Frisch elasticity IADL remains valid. In all cases, impact effect on output grows as the asset markets participation is decreasing. This holds until a threshold value of λ, different for each case, which makes the impact effect negative until it reaches again a value close to zero.
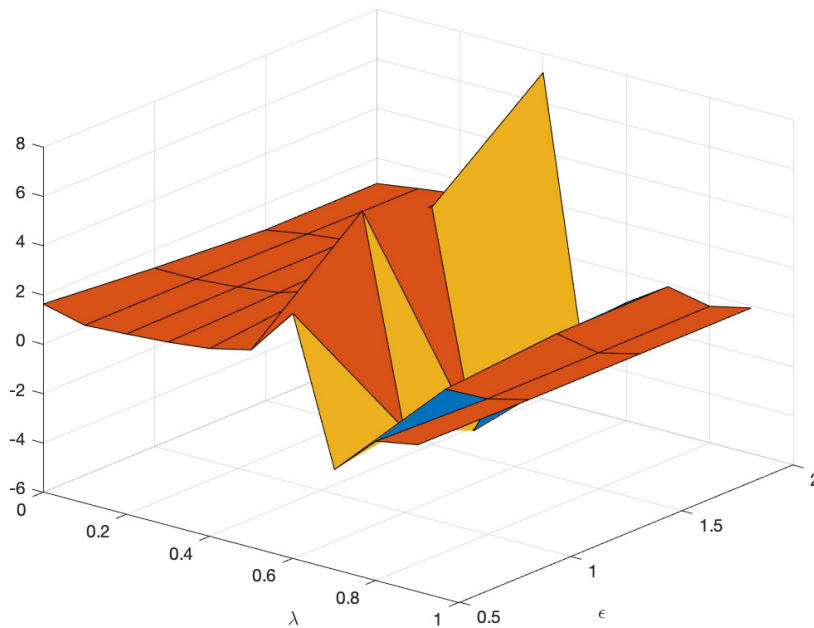

Fig. 14. Sensitivity to Inverse Frisch Elasticity Values: MP Shock

Appendix J   Profit Redistribution and Limited Asset Markets Participation

When labour markets are perfectly competitive, accommodative conventional and unconventional monetary policy can have negative effects. In this Appendix I show the results of an expansionary monetary policy for different levels of asset markets participation under the assumption that taxation is redistributive.
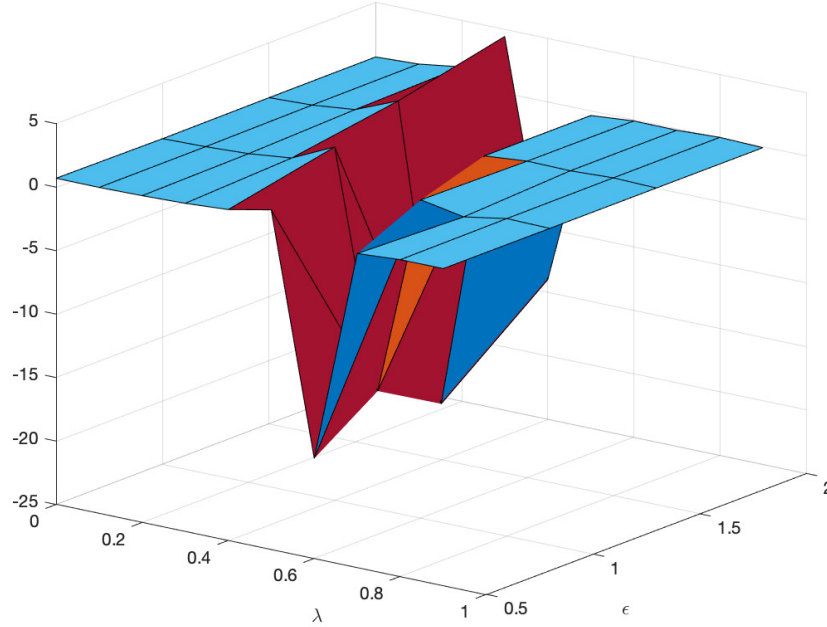
Fig. 15. Sensitivity to Inverse Frisch Elasticity Values: QE Shock

Taxation is following a simple fiscal rule of redistribution of profits to the hand to mouth consumers defined as:

$$T_t^r = t_{pr} Prof_t.$$

That is, a percentage of the profits is allocated to the hand to mouth consumers who were entitled zero transfers under the baseline scenario examined before. What changes is as the rule of thumb consumers' profits receivable profits increases, the IADL region shifts to the right. The negative effects of profits are shared between the two groups which alleviates the contractionary effects of the negative income shock.

I assume three different taxation parameter values: 0% (baseline scenario), 20% and 40%. It's important to note that this is an ad-hoc choice for the profit tax parameter values. Since the purpose of this exercise is to identify the changes when transfers to rule of thumb agents are non-zero, the choice of a data driven parameter is not crucial. Due to the complexity of the model I abstract from the analytical solution of this case and I show numerically what is the *total* impact effect of a monetary policy shock and a QE shock to output. To show the counter- or procyclicality of both policies under the taxation regime I focus on the impact effect of both policies on output.

Figure 16 shows the paths of the impact effect of output after both a conventional accommodative monetary policy shock (on the top panel) and a bond purchase shock (on the bottom panel). Both impact effects are plotted as a function of λ. Shocks follow the process specified in the calibration section. The yellow line corresponds to the baseline scenario of no redistribution, while the green line to a tax rate of 20% and the
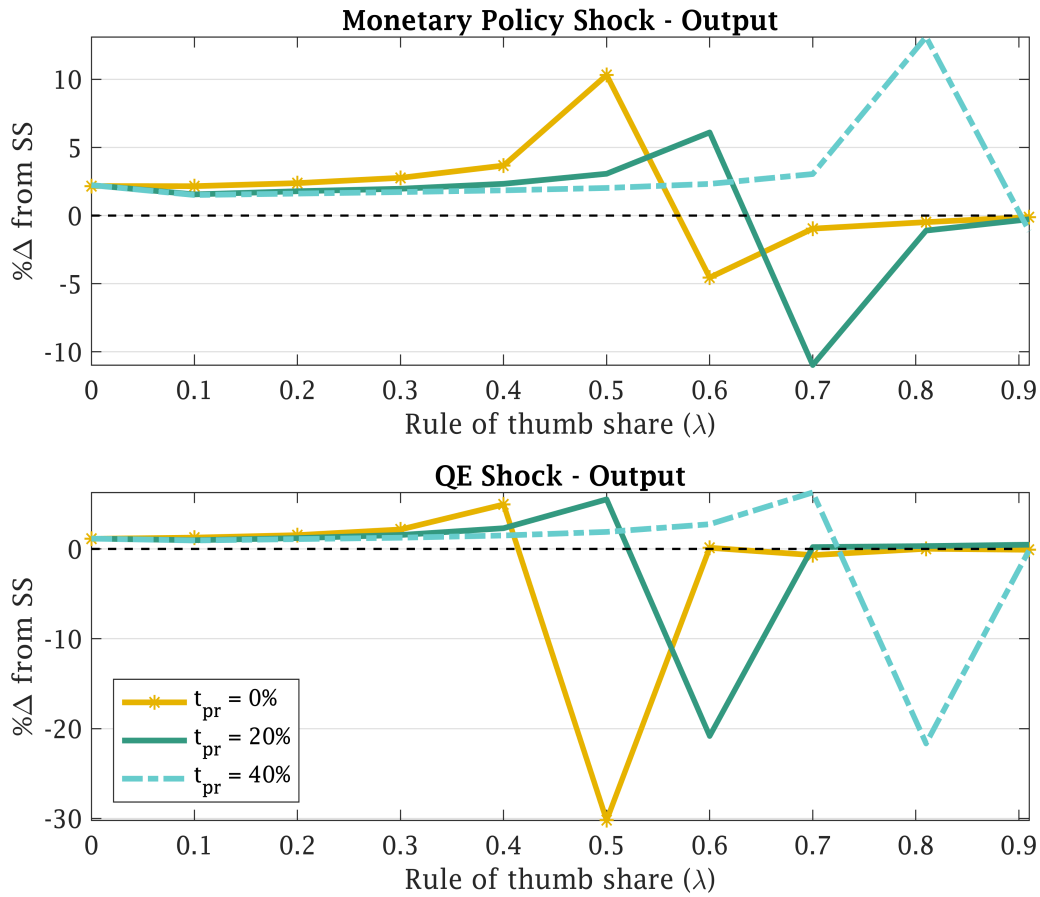
Fig. 16. Inverted Aggregate Demand Logic (Profit Redistribution)

cyan line to a tax rate of 40%. What changes in comparison with the no redistribution case is that as the tax rate increases, the threshold of λ that makes both monetary policy tools contractionary shifts to the right. At the same time, the impact effect of output is milder for both cases of fiscal redistribution compared to the benchmark for reasonable values of λ (up to 0.7).[26]

Under fiscal redistribution, the rule of thumb agents share partially the negative effects of profits. As the financial participation level goes down, profits' role in output becomes limited. Opposed to the benchmark case, now rule of thumb agents internalize partially the adverse effects and thus aggregate demand does not increase as much as in the benchmark case. On the other hand, the impact effect of output remains positive for most of the domain λ, especially in the high taxation case. This stops at a threshold level of λ where profits have been decreased by so much that they induce a

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[26]Note that the impact effect is plotted until $\lambda = 0.90$ since the analysis is restricted to the range of λ values consistent with a unique equilibrium.

drop in aggregate demand. Redistributive fiscal policy preserves the procyclicality of accommodative monetary policy tools.