The Coase Conjecture and Agreement Rules in Policy Bargaining

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Abstract

An agenda-setter proposes a spatial policy to voters and can revise the initial proposal if it gets rejected. Voters can communicate with each other and have distinct but correlated preferences, which the agenda-setter is uncertain about. I investigate whether the ability to make a revised proposal is valuable to the agenda-setter. When a single acceptance is required to pass a policy, the equilibrium outcome is unique and has a screening structure. Because the preferences of voters are single-peaked, the Coase conjecture is violated and the ability to make a revised proposal is valuable. When two or more acceptances are required to pass a policy, there is an interval of the agenda-setter's equilibrium expected payoffs. The endpoints have a screening structure, leading to the same conclusions as in the case of a single acceptance. Interestingly, an increase in the required quota q may allow the agenda-setter to extract more surplus from voters. An application to spending referenda suggests that the expected budget may increase in response to allowing the bureaucrat to make a revised proposal and/or an increase in the number of voters whose acceptance is required.

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1 Introduction

Motivation Many collective decision-making situations involve an agenda-setter with the sole power to make proposals, a group of voters with a collective power to veto proposals, and the possibility of a revised proposal if the initial proposal is rejected. For example, about 750 school budget elections are conducted every year in the State of New York. In every school district, officials put forward a budget proposal and qualified voters decide in an election whether to pass it. Occasionally, initial proposals are rejected, in which case a revised proposal is made and a second election is held.[1] If the revised budget is also defeated, the school district implements a contingency plan which does not allow to increase spending compared to the previous year.[2] Other examples include: the nomination and confirmation of presidential appointees in the United States; the decision-making in committees and boards; the proposal of and voting on public spending measures.

When the agenda-setter is uncertain about the preferences of voters, she has an opportunity to learn from rejected proposals and use this information when making revised proposals. In light of the Coase (1972) conjecture, it is not clear if there is an agenda-setting advantage in such a setting. Famously, the Coase conjecture claims that a monopolist selling a durable good on a market would not be able to charge a price above the cost of production because of the competition with the future self. The Coase conjecture holds in veto bargaining over price (Fudenberg, Levine, and Tirole, 1985; Gul, Sonnenschein, and Wilson, 1986), but the examples of decision-making situations given above do not fit this description. In these examples, the bargaining is not over price but over policy, and the power to approve and veto proposals is not individual but collective. When bargaining over policy, the agenda-setter and voters can agree about the direction in which the status quo should be changed but disagree about the magnitude of the change. And since the approval and veto power is collective, the incentives of players can be influenced by the conflict of interest between voters, the ability of voters to communicate with each other, and the voting rule.

In this paper, I study a model of policy-making in which an agenda-setter makes

[1] Based on data from the New York State Education Department, in the five years from 2015 to 2019, the rate of rejection varied between 0.7% and 2.4%. There are five school districts in which the initially proposed budgets were rejected on two occasions between 2015 and 2019.

[2] From 2015 to 2019, the rate of defeat of the revised budgets varied between 16.7% and 33.3%.

a policy proposal that must be approved by voters and makes a revised proposal if the initial proposal is rejected. I focus on situations in which the agenda-setter is uncertain about the preferences of voters and learns from rejected proposals. Uncertainty about the preferences of voters can explain why initial proposals are sometimes rejected. I investigate how the voting rule affects equilibrium outcomes and whether the ability to make a revised proposal is valuable to the agenda-setter. I also highlight the role played by the conflict in preferences among voters and the ability of voters to communicate with each other.

Model In the baseline model, described in Section 2, the agenda-setter and n voters indexed by i bargain over a one-dimensional policy $x \in \mathbb{R}_+$. When $n = 2$, the voters may represent two political parties or two groups of individual voters acting as voting blocks. The policy space $\mathbb{R}_+$ may represent an ideological inclination of a nominee for a federal court, an issue size of a bond offering, or a level of expenditure on a public project (Romer and Rosenthal, 1979). The status quo policy, assumed to be 0, remains in effect until another policy passes. The agenda-setter has two attempts at passing a policy proposal. In each of two periods, until some policy passes, the agenda-setter makes a proposal, and voters cast their votes. If the proposed policy is approved by the required number q of voters, it replaces the status quo and remains in effect indefinitely. If the proposed policy is the first attempt to replace the status quo and rejected, the agenda-setter makes a revised proposal.

Tensions in this model arise from the differences in goals, the asymmetric information about the effects of policies, and impatience. The agenda-setter is risk-neutral and maximizes the implemented policy. Voters have quadratic preferences with state-dependent ideal policies that are strictly positive.[3] Voters privately observe the state of the world which can be either high h or low ℓ. The agenda-setter is uninformed about the state of the world. For each required number q of voters, the q-th highest ideal policy is strictly positive in each state and is higher in state h compared to state ℓ. Finally, the agenda-setter and voters prefer earlier agreement and discount future payoffs at a common discount rate $\delta < 1$. This baseline model is tractable yet sufficiently rich to provide insights into the effects of the voting rule on the incentives of players and the implemented policies.

[3]These assumptions can be relaxed without affecting the qualitative results. For example, players can have any (continuous) quasi-concave utility function that is single-peaked. The only restriction is that the agenda-setter's ideal policy is the largest among all players.

By communication I mean the ability of voters to discuss their strategies with each other coupled with the inability to write binding agreements. Non-binding communication among voters leads to an equilibrium refinement in the spirit of Bernheim, Peleg, and Whinston (1987) called *coalition-proofness*, which requires that the equilibrium strategies are robust to joint deviations by voters that are *improving* and *self-enforcing*. This model-free approach to communication is analytically convenient because it does not require a formal extension of the baseline model. Nonetheless, coalition-proof equilibria can be shown to be outcome-equivalent to some equilibria in a model with pre-vote (but post-proposal) round of simultaneous cheap-talk among voters.

Questions (1) The ability to make a revised proposal if the initial proposal is rejected captures the agenda-setter's lack of commitment to a single proposal. This lack of commitment has been long recognized as a potential detriment for the agenda-setter and is a subject of the conjecture by Coase (1972) that the agenda-setting advantage disappears when players become perfectly patient. The intuition behind this conjecture, originally formulated for a durable-good monopolist but later shown to hold in bilateral bargaining over price (Fudenberg et al., 1985), is that the veto player evaluates the current proposals not with respect to the status quo but with respect to the anticipated revisions. As a consequence, the agenda-setter competes with the future self and this competition completely washes away the agenda-setting advantage. Does the agenda-setter value the ability to make a revised proposal in case the initial proposal is rejected?

There are two features of my model that make this question interesting. First, it is not obvious that the Coase conjecture holds in bargaining over policy (as opposed to bargaining over price). Since the ideal policies of voters are strictly positive, the anticipated revisions can be worse for voters than the current proposals. In this case, the agenda-setter does not compete with the future self and may preserve the agenda-setting advantage. Second, the forces behind the Coase conjecture may be weakened by a q-majority voting rule and the conflict of interest among voters. When implementing a policy requires approval from more than a single voter (i.e., when the required quota is $q \geq 2$), the initial proposal can be rejected with multiple levels of support. The agenda-setter's belief, and therefore the revised proposal, can depend on the level of support for the initial proposal. Therefore, the q-majority voting rule when $q \geq 2$ allows greater flexibility in providing incentives for voters in the first period compared to the case when $q = 1$.

(2) A variety of voting rules are used in practice, including majority rule used for the confirmation of presidential appointees and unanimity rule used by corporate boards for the approval of actions by the "unanimous written consent." How does an increase in the required quota q affect the implemented policies and the agenda-setter's expected payoff?

From the agenda-setter's perspective, there is a tradeoff between the voting rules. On the one hand, a smaller required quota allows the agenda-setter to target fewer voters who are more aligned with the agenda-setter, while a larger required quota forces the agenda-setter to secure approval from more voters. On the other hand, a larger required quota moves the set of credible revised proposals towards the status-quo policy since the agenda-setter always targets a voter with the q-th highest ideal policy. Because of the single-peaked preferences of voters, this effect of a larger required quota may allow the agenda-setter to extract more surplus from voters using the initial proposal.

Main results First, I show that the Coase conjecture is violated (under some conditions on the primitives of the model) when the bargaining is over a spatial policy. When $q = 1$, there is a unique equilibrium path which has a screening structure. The agenda-setter makes an initial proposal that can pass only when the ideal policies of voters are high, i.e., in state h, and revises the initial proposal to be closer to the status quo policy 0 if the initial proposal is rejected. The screening proposal makes a voter with the highest ideal policy in state h, say voter j, indifferent between accepting the initial proposal in the first period and the revised proposal in the second period. When the revised proposal is closer to the status-quo policy 0 than the ideal policy of voter j in state h (which happens when the difference in ideal policies between states ℓ and h is relatively large), the screening proposal converges (as players become arbitrarily patient) to a policy that is symmetric to the revised proposal around that ideal policy. This means that the agenda-setting advantage does not vanish thanks to the single-peaked preferences of voters with strictly positive ideal policies.

When $q \geq 2$, the flexibility in using revisions to provide incentives for voters results in a multiplicity of equilibrium paths. The agenda-setter's expected payoff is no longer unique. Instead, there is an interval of expected payoffs that can be supported in equilibrium. Nonetheless, the endpoints of this interval correspond to equilibrium paths that have a screening structure similar to the case $q = 1$. As a result, the Coase conjecture can be violated because of the single-peaked preferences of voters. The forces behind these violations of the Coase conjecture make the ability to make a revised proposal valuable to the agenda-setter for any required quota $q \in N$.

Second, I show that increasing the number q of voters required to pass a policy can raise the expected policy implemented in equilibrium and the agenda-setter's equilibrium expected payoff. The screening structure of equilibrium paths and single-peaked preferences of players play a crucial role. In the second period, the agenda-setter always makes a revised proposal that sets a voter with the q-th highest ideal policy in either state ℓ or h (depending on the posterior belief) to the status-quo level of payoff. Therefore, the required quota q affects the anticipated revised proposals which serve as threats for rejecting the initial proposal. I provide conditions under which an increase in the required quota q moves the anticipated revised proposal closer to the status-quo policy 0 and further away from the ideal policy of a voter targeted by the initial proposal, giving a higher expected payoff to the agenda-setter.

Implications/predictions The results in this paper suggest that the average size of adopted policy can (all else being held constant) be higher in settings in which the agenda-setter is allowed to make a revised proposal compared to settings in which the agenda-setter is not. For example, one could expect higher average school budgets in New York, where a second school budget election is held if the initial budget gets rejected, than in New Jersey, where a contingency plan goes into effect if the proposed budget gets rejected. The results also suggest that states that require a 60 percent supermajority to pass school budgets, as some states do, may have higher average school budgets than states that require a simple 50 percent majority.

Approach My analysis relies on a complete characterization of the implemented policies and the agenda-setter's expected payoffs under minimal assumptions on the voting strategies of voters. I only assume that voters use weakly undominated voting strategies. For instance, voting strategies are not necessarily symmetric and are not required to be threshold in policy proposals.

I show that the equilibrium path is unique when $q = 1$, implying that the agenda-setter's expected payoff is also unique. When $q \geq 2$, there is a multiplicity of equilibrium paths, which results in a multiplicity of the agenda-setter's expected payoffs. It does not seem feasible to provide a complete characterization of equilibrium voting strategies without additional restrictions. Instead, I provide a complete characterization of the agenda-setter's expected payoffs that can be supported in equilibrium. I derive two bounds and show that any value outside these bounds *cannot* be supported as the agenda-setter's expected payoff in equilibrium no matter how the off-path beliefs are specified. In turn, I show that any value inside these bounds *can* be supported as the agenda-setter's expected payoff in equilibrium by explicitly constructing such equilibrium. The main results in this paper are based on the comparison of the unique expected payoff when $q = 1$, the payoff bounds when $q \geq 2$, and the unique payoff in the benchmark case when the agenda-setter can commit to a single proposal when $q \in N$.

Because of the multiplicity of expected payoffs when the voting rule requires an agreement of more than a single voter, $q \geq 2$, the comparative statics with respect to the required quota must rely on some form of equilibrium payoff selection. To circumvent this problem, I provide the necessary and sufficient conditions for some equilibrium payoff selection to be non-monotone decreasing in the required quota q and for each equilibrium payoff selection to be non-monotone.

Outline I present the model in Section 2. I consider the case $q = 1$ in Section 3 and the case $q \geq 2$ in Section 4, which also contains the comparative statics with respect to the required quota. Section 5 concludes.

Related literature

Repeated referenda. Thematically, this paper fits within the literature on repeated referenda originated from Romer and Rosenthal (1979).[4] The first paper to introduce strategic voting in the model of repeated referenda was Morton (1988).[5] More recent work on repeated referenda includes Rosenthal and Zame (2019) and Chen (2020, 2022).

[4]Romer and Rosenthal (1979) builds on a single-period model of Romer and Rosenthal (1978). Denzau and Mackay (1983) is another influential paper with an uniformed agenda-setter making a single proposal to privately informed voters.

[5]Even though Morton (1988) considers strategic voters and discusses the potential conflict between the signaling and pivotal incentives, that paper eventually assumes that the pivotal incentives dominate.

Rosenthal and Zame (2019) evaluate the role of voter sophistication on the agenda-setter's ability to benefit from being able to make a revised proposal. In the most interesting case of sophisticated voters, the analysis in Rosenthal and Zame (2019) is limited to a single voter, making it closely related to the analysis of the q-majority voting rule with $q = 1$ and the comparison with commitment benchmark in Section 3.

Chen (2020) studies whether the ability to make a revised proposal is valuable to the agenda-setter and, in particular, focuses on the tradeoff between signaling and pivotal incentives of voters. The model is similar to the one considered here and features two periods and multiple privately informed voters with quadratic preferences. The key difference is that Chen (2020) assumes that the voting strategies in the initial period are threshold in the ideal policies given that the initial proposal is on path of play. In contrast, I only assume that the voting strategies are weakly undominated. I provide a sharp characterization of the expected payoffs that the agenda-setter can achieve in equilibrium, which allows me to perform an exhaustive comparison of equilibrium outcomes with the commitment benchmark. My results also complement Chen (2020) by showing how the following characteristics of the environment affect the adopted policies: (i) the conflict in preferences among voters, (ii) the voting rule, (iii) the prior belief about the state, and (iv) the ability of voters to communicate with each other.

Both Rosenthal and Zame (2019) and Chen (2020) assume that players give equal weight to both periods of policy-making, limiting the analysis of forces behind the Coase conjecture. In contrast, I characterize the agenda-setter's expected payoffs for a given discount factor and then derive the limit of this set as players become perfectly patient. As a result, I am able to show that the Coase conjecture may be violated in bargaining over policy due to single-peaked preferences.

Chen (2022) compares a straw poll to a binding referendum under different voting rules. Besides the differences in substantive question and explicit comparison of voting rules, Chen (2022) uses an approach similar to Chen (2020) and focuses on the voting strategies in the straw poll period that are threshold in ideal policies given that the initial proposal is on the path of play. The approach to communication in Chen (2022) is distinct from the one adopted here. Chen (2022) models communication using a straw poll followed by a binding vote on a single proposal. In contrast, I allow voters to discuss their strategies with each other and study equilibria that are robust to such communication.

Bargaining. This paper also contributes to the literature on bargaining with asymmetric information and the Coase conjecture (Fudenberg, Levine, and Tirole, 1985; Gul, Sonnenschein, and Wilson, 1986).[6] The main features that differentiate this paper from the literature on the Coase conjecture are: (i) the bargaining is over a spatial policy and not over a distributive policy (price); and (ii) the approval and veto rights are not individual but collective. I show that the Coase conjecture may be violated in settings with single-peaked preferences. Using an alternative set of assumptions on the ideal policies and the determination of status-quo payoffs, the methods used in this paper can be applied to the analysis of price bargaining between an informed seller and an informed "buyer" represented by n players with competing preferences, for example, when $n = 2$ the "buyer" could represent a couple.

Signaling through voting. Since the voting record can influence the agenda-setter's beliefs and the revised proposal, in this paper voters face both pivotal and signaling incentives when choosing how to vote on the initial proposal. Signaling incentives of voters have been previously studied in a variety of contexts (Piketty (2000), Razin (2003), Meirowitz (2005), Shotts (2006), Meirowitz and Tucker (2007), Meirowitz and Shotts (2009), and McMurray (2017)). These papers focus on determining which incentive, pivotal or signaling, drives the voting behavior in *large* elections. In my model, voters also face the tradeoff between pivotal and signaling incentives, as captured by equation (3).

Endogenous proposals. This paper contributes to the literature that studies the role of voting mechanisms in the presence of asymmetric information and agenda control (Austen-Smith, 1987). Bond and Eraslan (2010) study the effect of the voting rule on the efficiency of an adopted policy when the agenda-setter proposes the policy that is voted on. Bouton, Llorente-Saguer, Macé, and Xefteris (2021) compare the efficiency of "voting mechanisms" when the agenda-setter serves as a "gatekeeper" and decides whether the vote takes place. Both papers focus on the aggregation of information

[6] Earlier work on the Coase conjecture includes Stokey (1981) and Bulow (1982). More recent papers are (among others): Deneckere and Liang (2006), who assume that the seller is privately informed about her cost; Ortner (2017) who allows the seller's cost to stochastically evolve over time; and Doval and Skreta (2020) who follow a mechanism design approach. In a setting with spatial policies, Kartik, Kleiner, and Van Weelden (2021) derive conditions for interval delegation to be an optimal mechanism without transfers from the agenda-setter's perspective.

dispersed among voters and allow the agenda-setter to be privately informed, but assume that no revisions take place when a policy does not gather enough support. In contrast, the central features of my analysis are the agenda-setter's ability to revise a rejected proposal and the associated changes in the incentives of players. Moreover, both papers consider common value environments,[7] while I emphasize the role of heterogeneity in preferences among voters. Henry (2008) studies bargaining over a distributive policy and shows that the agenda-setter may offer positive transfers to more voters than the required quota. I focus on a one-dimensional policy space that does not allow the agenda-setter to make targeted transfers to voters.

Sequential voting. Finally, this paper is related to the literature on the role of voting rule in collective search (Albrecht, Anderson, and Vroman, 2010; Compte and Jehiel, 2010; Moldovanu and Shi, 2013), collective experimentation (Strulovici, 2010), and sequential voting with private information (Ordeshook and Palfrey, 1988; Kleiner and Moldovanu, 2017). In these papers, alternatives arrive exogenously until the committee collectively decides to stop the search and accept the current proposal. In contrast, I assume that the alternatives are endogenously selected by the agenda-setter.

2 The model

The agenda-setter A and n voters indexed by $i \in N = \{1, \dots, n\}$ bargain over a one-dimensional policy $x \in \mathbb{R}_+$ using a q-majority voting rule with $q \in N$. The status-quo policy is 0. Policy x may represent a level of public spending or an increase in capital stock of a company. In every period $t \in \{1, 2\}$ some policy $x_t \in \mathbb{R}_+$ is implemented. Policy x_t is the status-quo policy 0 until another policy $p \in \mathbb{R}_+$ passes and gets implemented in the remaining periods.

In period $t = 1$, the agenda-setter makes a policy proposal p_1. After voters observe p_1, they simultaneously cast their votes. The action set of each voter i is $A_i = \{0, 1\}$ with a generic element $a_{i,1}$, where $a_{i,1} = 0$ if voter i rejects the initial proposal and $a_{i,1} = 1$ if voter i accepts the initial proposal.[8] Proposal p_1 passes when

[7] More precisely, Bond and Eraslan (2010) assume that the preferences of voters are almost perfectly aligned and Bouton, Llorente-Saguer, Macé, and Xefteris (2021) allow partisan voters whose preferences do not depend on the underlying state of the world.

[8] This assumption rules out the possibility of abstention.

the required quota q of voters accepts it. In case p_1 passes, it is implemented in both periods, $x_1 = x_2 = p_1$, and the game ends. In case p_1 is rejected, the status quo is implemented in the first period, $x_1 = 0$, and the same process is repeated in period $t = 2$ after the voting record is revealed.

The agenda-setter is maximizing the policy – the period utility of the agenda-setter from implementing policy $x \in \mathbb{R}_+$ is $u_A(x) = x$. Voters have quadratic preferences over $\mathbb{R}_+$ with the ideal policies that depend on state $\omega \in \Omega$. The period utility of voter $i \in N$ from implementing policy $x \in \mathbb{R}_+$ in state $\omega \in \Omega$ is $u_i(x;\omega) = -\left(\frac{1}{2}y_i^\omega - x\right)^2$, where $\frac{1}{2}y_i^\omega > 0$ is the ideal policy of voter i in state ω. Future payoffs are discounted at rate $\delta \in (0,1)$, so the total payoffs of the agenda-setter and voter $i \in N$ from a policy sequence $(x_1, x_2) \in \mathbb{R}_+^2$ in state $\omega \in \Omega$ are

$$U_A(x_1, x_2) = (1-\delta)u_A(x_1) + \delta u_A(x_2);$$
$$U_i(x_1, x_2; \omega) = (1-\delta)u_i(x_1;\omega) + \delta u_i(x_2;\omega).$$

The information about the state $\omega \in \Omega$ is asymmetric. There are two possible states, $\Omega = \{\ell, h\}$. Voters observe the state ω, but the agenda-setter does not. The state is drawn and observed by voters at the beginning of the game.

For each $q \in N$, let $q(\cdot) : \Omega \to N$ be a mapping from the state space Ω to the set of players N such that player $q(\omega)$ has the q-th highest ideal policy in state ω, i.e.:

$$\#\{j \in N \mid y_{q(\omega)}^\omega < y_j^\omega\} < q(\omega) \leq \#\{j \in N \mid y_{q(\omega)}^\omega \leq y_j^\omega\}.$$

Mapping $q(\cdot)$ defines a "synthetic voter" whose ideal policy is q-th highest in each state $\omega \in \Omega$. I assume that every synthetic voter is monotone in the sense that for each $q \in N$ we have $0 < y_{q(\ell)}^\ell < y_{q(h)}^h$. Therefore, state ℓ can be interpreted as being "low" and state h as being "high."

Remark. A trivial case when every synthetic voter $q(\cdot)$ is monotone is when every voter is monotone, that is, $y_i^\ell < y_i^h$ for all $i \in N$. However, every synthetic voter $q(\cdot)$ can be monotone even if some voter i is not. For example, let $N = \{1, 2\}$ and assume that $y_1^\ell = 1 < 4 = y_1^h$ and $y_2^\ell = 3 \not< 2 = y_2^h$. Voter 2 is not monotone yet both $y_{1(\ell)}^\ell = 3 < 4 = y_{1(h)}^h$ and $y_{2(\ell)}^\ell = 1 < 2 = y_{2(h)}^h$ hold.

2.1 Strategies and beliefs

Let $\hbar_t$ denote the public history up to period $t \in \{1, 2\}$, including the proposed policies and the voting record, and ending right before the proposal in period t. Thus, the public history in period $t = 1$ is empty, and the public history in period $t = 2$ consists of the proposed policy and the choices of voters in the previous period.

Let H be the set of non-empty public histories at which the agenda-setter gets to make a proposal. Public history $\hbar \in H$ captures all of the information available to the agenda-setter when making a revised policy proposal in period 2. A (behavioral) strategy of the agenda-setter is $\pi = (\pi_1, \pi_2)$, where $\pi_1 \in \Delta(\mathbb{R}_+)$ is her proposal strategy in the first period and $\pi_2 : H \to \Delta(\mathbb{R}_+)$ is her proposal strategy in the second period. In particular, π_1 is the distribution of proposed policies in the first period, and, for each public history $\hbar \in H$, $\pi_2^{\hbar}$ is the distribution of proposed policies in the second period.

In addition to a (possibly empty) public history $\hbar_t$, each voter $i \in N$ observes the state ω before choosing whether to accept or reject a policy proposal. A (behavioral) strategy of voter $i \in N$ is $\alpha_i = (\alpha_{i,1}, \alpha_{i,2})$, where $\alpha_{i,1} : \mathbb{R}_+ \times \Omega \to [0, 1]$ is the acceptance strategy of voter i in the first period and $\alpha_{i,2} : \mathbb{R}_+ \times \Omega \times H \to [0, 1]$ is the acceptance strategy of voter i in the second period. In particular, the probability with which voter i in state $\omega \in \Omega$ accepts proposal $p_1 \in \mathbb{R}_+$ in the first period is $\alpha_{i,1}^{\omega}(p_1) \in [0, 1]$. The probability with which voter i in state ω and history $\hbar$ accepts proposal p_2 in the second period is $\alpha_{i,2}^{\omega}(p_2; \hbar)$.

The common prior belief over the state space Ω is $\hat{\mu}(\cdot) \in \mathcal{F}$, where $\mathcal{F} = \Delta(\Omega)$ is the set of probability measures over Ω. To simplify notation, I often write $\hat{\mu}$ instead of $\hat{\mu}(h)$ and refer to $\hat{\mu} \in [0, 1]$ as a prior belief. The period-2 belief generally depends on what has transpired in the game prior to the second period. Let $\mathcal{M} = \{\mu^{\hbar}(\cdot)\}_{\hbar \in H}$ be a belief system, where $\mu^{\hbar} \in \mathcal{F}$ for each $h \bar{\in} H$. For instance, $\mu^{\hbar}(\cdot)$ is the agenda-setter's belief over Ω when the history is $\hbar \in H$. Similar to the prior belief $\hat{\mu}(\cdot)$, I often write $\mu^{\hbar}$ instead of $\mu^{\hbar}(h)$ and refer to $\mu^{\hbar}$ as a posterior belief in history $\hbar$.

2.2 Induced outcomes

Given a prior belief $\hat{\mu}$, each assessment $(\sigma, \mathcal{M})$ induces a distribution $F_{\sigma,\mathcal{M}}$ over the outcome space $\Omega \times \mathbb{R}_+ \times \mathbb{R}_+$. In turn, each state $\omega \in \Omega$ induces a marginal distribution $F_{\sigma,\mathcal{M}}^{\omega}$ over the space of policy sequences $\mathbb{R}_+ \times \mathbb{R}_+$. Each player's ex-ante expected payoff for a given assessment $(\sigma, \mathcal{M})$ can be written as follows:

$$\begin{aligned}
V_A(\sigma, \mathcal{M}) &= \int \{(1-\delta)x_1 + \delta x_2\} \, \mathrm{d}F_{\sigma,\mathcal{M}}(\omega, x_1, x_2) \\
&= \sum_{\omega \in \Omega} \hat{\mu}(\omega) \left\{ \int \{(1-\delta)x_1 + \delta x_2\} \, \mathrm{d}F^{\omega}_{\sigma,\mathcal{M}}(x_1, x_2) \right\} \\
V_i(\sigma, \mathcal{M}) &= \int \{(1-\delta)u_i(x_1;\omega) + \delta u_i(x_2;\omega)\} \, \mathrm{d}F_{\sigma,\mathcal{M}}(\omega, x_1, x_2) \\
&= \sum_{\omega \in \Omega} \hat{\mu}(\omega) \left\{ \int \{(1-\delta)u_i(x_1;\omega) + \delta u_i(x_2;\omega)\} \, \mathrm{d}F^{\omega}_{\sigma,\mathcal{M}}(x_1, x_2) \right\}, \quad i \in N.
\end{aligned}$$

Given an assessment $(\sigma, \mathcal{M})$, a policy sequence $(x_1, x_2) \in \mathbb{R}_+ \times \mathbb{R}_+$ is *on path* if there exists state $\omega \in \Omega$ such that (x_1, x_2) is in the support of $F^{\omega}_{\sigma,\mathcal{M}}$. The bargaining protocol restricts the supports of marginal distributions $F^{\ell}_{\sigma,\mathcal{M}}$ and $F^{h}_{\sigma,\mathcal{M}}$ over $\mathbb{R}_+ \times \mathbb{R}_+$ that can be induced by assessment $(\sigma, \mathcal{M})$. Consider a policy sequence (x_1, x_2). If $x_1 > 0$, then (x_1, x_2) can be on path only if $x_1 = x_2$ because $x_1 > 0$ implies that x_1 was proposed in the first period, defeated the status-quo policy 0, and was implemented in both periods. This fact reduces the number of cases one needs to consider when analyzing policy sequences that are on path of play and therefore contribute to the agenda-setter's expected payoff $V_A(\sigma, \mathcal{M})$.

2.3 Equilibrium concept

The equilibrium concept is a *coalition-proof Perfect Bayesian Equilibrium* (coalition-proof PBE). An assessment $(\sigma, \mathcal{M})$ consisting of a (behavioral) strategy profile $\sigma = (\pi, \alpha)$ and a belief system $\mathcal{M}$ is a PBE if the strategies of players are sequentially rational, that is, players maximize their expected payoffs every time they move, and the beliefs are consistent, that is, for each initial proposal $p_1 \in \mathbb{R}_+$ the posterior belief $\mu^{\hbar}$ is derived from the prior $\hat{\mu}$ using the Bayesian rule on path of play induced by p_1. This definition implies that the agenda-setter cannot "signal" any information using the initial proposal.[9] In particular, this definition requires that players continue to use the Bayesian rule even when the initial proposal is not on path of play, i.e., is caused by an agenda-setter's deviation.[10]

[9]This assumption is sometimes referred to as "not-signaling-what-you-don't-know."

[10]It follows that the concept of Perfect Bayesian Equilibrium used here can be more precisely described as an *almost* PBE rather than a *weak* PBE in the terminology of Mailath (2019).

In many settings, it is natural to assume that voters can discuss their strategies with each other. Such communication can take many forms: voters participating in public and private discussions, members of interest groups using media outlets to further their agenda, and countless ways to interact using social media platforms. Some sort of communication is particularly likely when voters have similar preferences and information. Although voters may be able and willing to discuss their plans, it is much less likely that they can write binding agreements specifying how they should vote in every situation. To account for these possibilities, I assume that voters engage into direct unmediated communication and consider a refinement of PBE inspired by Bernheim, Peleg, and Whinston (1987).

Fix a coalition $C \subseteq N$ and an assessment $(\sigma, \mathcal{M})$, where $\sigma = (\pi_1, \alpha_1, \pi_2, \alpha_2)$. A period-1 voting strategy profile $\tilde{\alpha}_1 = (\tilde{\alpha}_C, \alpha_{N \setminus C})$ is *improving for C with respect to* $(\sigma, \mathcal{M})$ if for each $i \in C$ we have $V_i(\tilde{\sigma}, \mathcal{M}) > V_i(\sigma, \mathcal{M})$ where $\tilde{\sigma} = (\pi_1, \tilde{\alpha}_1, \pi_2, \alpha_2)$. An existence of improving deviation for some set of voters C does not immediately imply that the equilibrium under consideration is unreasonable. Since binding agreements are generally not possible, an improving deviation upsets an equilibrium only if the improving deviation itself is robust to further deviations by some members of C.

Fix a coalition $C \subseteq N$ and an assessment $(\sigma, \mathcal{M})$, where $\sigma = (\pi_1, \alpha_1, \pi_2, \alpha_2)$. A period-1 voting strategy profile $\alpha_1 = (\alpha_{i,1})_{i \in N}$ is *self-enforcing for C with respect to* $(\sigma, \mathcal{M})$ if there does not exist a period-1 voting strategy profile $\tilde{\alpha}_1$ which is improving for some $\tilde{C} \subset C$ with respect to $(\sigma, \mathcal{M})$. Of course, when $(\sigma, \mathcal{M})$ is an equilibrium, the voting strategies in σ are self-enforcing for singletons with respect to $(\sigma, \mathcal{M})$. In other words, PBE requires (among other restrictions) that the equilibrium strategies are robust to unilateral deviations by voters. Coalition-proofness places an additional restriction on $(\sigma, \mathcal{M})$ that the equilibrium strategies are robust to joint deviations by voters that are self-enforcing.

A perfect Bayesian equilibrium $(\sigma, \mathcal{M})$ is *coalition-proof* if there does not exist $\tilde{\alpha}_1$ which is improving for some $C \subseteq N$ with respect to $(\sigma, \mathcal{M})$ and self-enforcing for C with respect to $(\tilde{\sigma}, \mathcal{M})$, where $\tilde{\sigma} = (\pi_1, \tilde{\alpha}_1, \pi_2, \alpha_2)$. Under the q-majority voting rule with $q = 1$, coalition-proofness has no bite and therefore does not affect the set of equilibria. When $q \geq 2$, Examples 1 and 2 in Online Appendix B demonstrate equilibria that do not pass the criteria of coalition-proofness.

2.4 Coasian equilibria

The original Coase conjecture (Coase, 1972) states that a monopolist facing a downward sloping demand curve for a durable good would not be able to price-discriminate buyers.[11] The argument goes like this. As long as there are consumers willing to pay above the marginal cost, the monopolist would find it profitable to reduce the price in order to reach some of them. Expecting a lower price, consumers would refuse to buy at a higher price, effectively making the demand curve infinitely elastic. As a result the monopolist would not be able to charge a price above the competitive level.

Bilateral models of bargaining usually focus on whether equilibria are not Coasian in the sense that the limit of the agenda-setter's expected payoff exceeds $y^{\ell}_{q(\ell)}$, which means that the agenda-setting advantage does not disappear as players become perfectly patient. I call such sequences of equilibria non-Coasian.[12] Formally, fix a required quota $q \in N$ and let $\{\delta_k\}_{k=1}^{\infty}$ be a sequence of discount factors converging to 1. A corresponding sequence of equilibria $\{(\sigma_k, \mathcal{M}_k)\}_{k=1}^{\infty}$ is called:

(i) *Coasian* if the agenda-setter's expected payoff converges to $y^{\ell}_{q(\ell)}$;

(ii) *non-Coasian* if the agenda-setter's expected payoff converges to $v > y^{\ell}_{q(\ell)}$.

Theorems 3 and 7 provide the necessary and sufficient conditions for the existence of Coasian equilibria in my model.

2.5 Equilibrium definition

Under the q-majority voting rule, a policy proposal p_t passes in period t if and only if at least q voters accept p_t. The public belief $\mu^{\hbar}$ in the second period generally depends on the number and the identities of voters who rejected the initial proposal p_1. Accordingly, in the first period, voters must take into account the effect of their vote on the public belief in the second period.

Each history $\hbar \in H$ can be described by an initial policy proposal $p_1 \in \mathbb{R}_+$ and a (possibly empty) coalition of players C who voted to accept it. For each $\kappa \in N \cup \{0\}$,

[11] A textbook treatment of the Coase conjecture can be found in Fudenberg and Tirole (1991).

[12] In multilateral bargaining over spatial policy, there may exist equilibria in which the agenda-setter's expected payoff is below $y^{\ell}_{q(\ell)}$ but such equilibria require that voters cannot communicate with each other (see Example 2 with $q = n = 2$ in Online Appendix B). In such cases, the agenda-setting advantage is actually a disadvantage. When this disadvantage does not disappear as players become perfectly patient, I call such sequences of equilibria sub-Coasian.

let $\mathcal{C}_\kappa$ be the collection of coalitions of size κ. The set of histories H can be written as $H = \{\hbar = (p_1, C) \mid p_1 \in \mathbb{R}_+,\ C \in \bigcup_{\kappa=0}^{q-1} \mathcal{C}_\kappa\}$. Given a period-1 strategy profile $\sigma_1 \in \Sigma$, we can use the Bayesian rule to obtain

$$\mu^{p_1,C}(h) = \frac{\left(\prod_{i\in N\setminus C}(1-\alpha_{i,1}^h(p_1))\right)\left(\prod_{j\in C}\alpha_{j,1}^h(p_1)\right)\hat{\mu}(h)}{\sum_{\omega=\ell,h}\left(\prod_{i\in N\setminus C}(1-\alpha_{i,1}^\omega(p_1))\right)\left(\prod_{j\in C}\alpha_{j,1}^\omega(p_1)\right)\hat{\mu}(\omega)}.$$

Let $V_{i,2}^q(\omega;\mu^\hbar)$ be the expected period-2 payoff of voter i in state ω if the period-2 belief is $\mu^\hbar$:

$$V_{i,2}^q(\omega;\mu^\hbar) = \int_{p_2\in\mathbb{R}_+}\left\{\max_{a_{i,2}\in\{0,1\}} U_{i,2}^q(p_2;\omega,(a_{i,2},\alpha_{-i,2}))\right\}\mathrm{d}\pi_2(\mu^\hbar)(p_2), \tag{1}$$

where $U_{i,2}^q$ is the period-2 expected payoff of voter i and $\mu^\hbar$ is the agenda-setter's period-2 belief.[13]

When voter i accepts an initial policy proposal p_1, it passes only when at least $q-1$ other voters also accept p_1. If less than $q-1$ other voters accept p_1 and the set of voters who accept it is C, then the revised belief is $\mu^{p_1,C}$ and a revised proposal is made in the next period. Combining these possibilities, the expected payoff of voter i from accepting policy p_1 is

$$\begin{aligned}
&U_{i,1}(p_1;\omega,(a_{i,1},\alpha_{-i,1})) = \\
&= \text{Prob}\left(a_{i,1}+\sum_{j\neq i}a_{j,1}\geq q \mid \omega,\alpha_{-i,1}\right)u_i(p_1;\omega) \\
&\quad + \text{Prob}\left(a_{i,1}+\sum_{j\neq i}a_{j,1}< q \mid \omega,\alpha_{-i,1}\right)\left[(1-\delta)u_i(0;\omega)+\delta V_{i,2}^q(\omega;\mu^{p_1,C})\right] \\
&= \text{Prob}\left(a_{i,1}+\sum_{j\neq i}a_{j,1}\geq q \mid \omega,\alpha_{-i,1}\right)\left[-\left(\frac{1}{2}y_i^\omega - p_1\right)^2\right] \\
&\quad + \text{Prob}\left(a_{i,1}+\sum_{j\neq i}a_{j,1}< q \mid \omega,\alpha_{-i,1}\right)\left[(1-\delta)\left(-\frac{1}{4}(y_i^\omega)^2\right)+\delta V_{i,2}^q(\omega;\mu^{p_1,C})\right].
\end{aligned} \tag{2}$$

In equilibrium, voter i in state ω chooses an action with the highest expected

[13]The analysis of the second period is standard and presented in Online Appendix A.

payoff. Consider the difference between the expected payoffs in (2) for $a_{i,1} = 1$ and $a_{i,1} = 0$:

$$
\begin{aligned}
&U^q_{i,1}(p_1; \omega, (1, \alpha_{-i,1})) - U^q_{i,1}(p_1; \omega, (0, \alpha_{-i,1})) = \\
&= \sum_{C \in \mathcal{C}_{q-1}} \text{Prob}\left(\sum_{j \neq i} a_{j,2} = q - 1 \mid \omega, \alpha_{-i,2}\right) \left[(y^\omega_i - p_1)p_1 - \delta V^q_{i,2}(\omega; \mu^{p_1,C}) - \delta \frac{1}{4}(y^\omega_i)^2\right] \\
&\quad + \mathbf{1}(q \geq 2) \sum_{\kappa=0}^{q-2} \left\{ \sum_{C \in \mathcal{C}_\kappa} \text{Prob}\left(\sum_{j \neq i} a_{j,2} = \kappa \mid \omega, \alpha_{-i,2}\right) \left[\delta V^q_{i,2}(\omega; \mu^{p_1,C+i}) - \delta V^q_{i,2}(\omega; \mu^{p_1,C})\right] \right\}.
\end{aligned} \tag{3}
$$

The first term in (3) captures the incentive of voter i arising from policy considerations. In other words, it captures the tradeoff between implementing the initial policy proposal p_1 in period 1 and a revised policy proposal in period 2. This tradeoff arises only when voter i is pivotal, i.e., exactly $q - 1$ other voters accept p_1. The second term in (3) captures the incentive of voter i arising from signaling considerations. When the initial policy proposal p_1 gets rejected, a revised proposal depends on the agenda-setter's posterior belief about the state. Therefore, voter i can use her vote to influence the revised proposal through the posterior belief. The signaling incentive is present only when less than $q - 1$ other voters accept p_1.

Unless voter i believes that she is never pivotal or always pivotal, i.e., unless we have $\sum_{C \in \mathcal{C}_{q-1}} \text{Prob}\left(\sum_{j \neq i} a_{j,2} = q - 1 \mid \omega, \alpha_{-i,2}\right) \in \{0, 1\}$, voter i must take into account both policy and signaling considerations. Voter i's period-1 acceptance probability $\alpha^\omega_{i,1}(p_1)$ must satisfy

$$
\alpha^\omega_{i,1}(p_1) = \begin{cases} 1 & \text{if } U^q_{i,1}(p_1; \omega, (1, \alpha_{-i,1})) > U^q_{i,1}(p_1; \omega, (0, \alpha_{-i,1})), \\ 0 & \text{if } U^q_{i,1}(p_1; \omega, (1, \alpha_{-i,1})) < U^q_{i,1}(p_1; \omega, (0, \alpha_{-i,1})). \end{cases} \tag{4}
$$

A period-1 voting strategy profile $\alpha_1 = (\alpha_{i,1})_{i \in N}$ must satisfy (4) in every PBE, even when it is not coalition-proof; in a coalition-proof PBE, α_1 must also be such that there does not exist coalition $C \subseteq N$ and strategy profile $\tilde{\alpha}_1 = (\tilde{\alpha}_C, \alpha_{N \setminus C})$ which is improving and self-enforcing.

From the agenda-setter's perspective, each initial policy proposal $p_1 \in \mathbb{R}_+$ is associated with a collection of events. The probability that only voters in set $C \subseteq N$

accept policy p_1 is given by:

$$\hat{W}(C,p_1)=\sum_{\omega\in\Omega}\hat{\mu}(\omega)\left(\prod_{i\in C}\alpha^{\omega}_{i,1}(p_1)\right)\left(\prod_{i\in N\setminus C}(1-\alpha^{\omega}_{i,1}(p_1))\right).$$

Therefore, the probability $\hat{W}^q(p_1)$ that policy p_1 passes under the q-majority voting rule can be written as:

$$\hat{W}^q(p_1)=\sum_{\kappa=q}^{n}\sum_{C\in\mathcal{C}_\kappa}\hat{W}(C,p_1).$$

The agenda-setter's expected payoff from making an initial policy proposal p_1 is

$$U^q_{A,1}(p_1)=\hat{W}^q(p_1)p_1+\sum_{\kappa=0}^{q-1}\sum_{C\in\mathcal{C}_\kappa}\hat{W}(C,p_1)\delta V^q_{A,2}(\mu^{p_1,C}). \tag{5}$$

In equilibrium, the agenda-setter proposes a policy that maximizes her expected payoff (5), that is, she solves the following problem:

$$\max_{p_1\in\mathbb{R}_+}U^q_{A,1}(p_1); \tag{6}$$

and she proposes policy p_1 with positive probability only when p_1 is a solution to (6):

$$\pi_1(p_1)>0 \text{ implies } p_1\in\arg\max_{p'_1\in\mathbb{R}_+}U^q_{A,1}(p'_1).$$

In the remainder of the paper, I characterize the coaltion-proof equilibria under the q-majority voting rule and analyze their properties. I begin by considering case $q=1$ in Section 3 and move to case $q\geq 2$ in Section 4.

3 "Bilateral" bargaining over policy ($q=1$)

The case when a single acceptance is required to pass a policy proposal, $q=1$, is special because the agenda-setter makes the revised proposal only when the initial proposal is rejected unanimously. This feature prevents the agenda-setter from using revised proposals as punishments for deviations. Nonetheless, this case is important for multiple reasons. First, Theorem 1 shows that the equilibrium predictions when $q=1$ are as if the the agenda-setter was engaged in bilateral bargaining with the synthetic voter $1(\cdot)$, allowing me to discuss the differences between the bilateral bargaining over a distributive policy (e.g., Fudenberg, Levine, and Tirole (1985)) and bilateral bargaining over a spatial policy. In particular, I describe the possible violations of the Coase conjecture when the policy is spatial (Theorems 3 and 4). Second, the characterization of the agenda-setter's equilibrium expected payoffs when $q \geq 2$ is closely related to the bilateral bargaining case, as shown in Section 4.[14]

3.1 Characterization: Unique equilibrium path

Lemma 1 asserts that for any prior belief $\hat{\mu}$, the policy space $\mathbb{R}_+$ can be partitioned into up to four regions, each corresponding to a different probability that a policy passes if proposed in the first period. Similar to the second period (see Online Appendix A), the initial proposal p_1 passes in both states if it is sufficiently small, $p_1 < y^{\ell}_{1(\ell)}$, and gets rejected in both states if it is sufficiently large, $p_1 > y^{h}_{1(h)}$. Unlike in the second period, the initial proposal p_1 between $y^{\ell}_{1(\ell)}$ and $y^{h}_{1(h)}$ may also get rejected in both states, reflecting the fact that voters take into account the possibility of revision.

No matter what the prior belief $\hat{\mu}$ is, we can identify proposals that pass only in high state h. Define z^1 as follows:

$$z^1 = \max\{p_1 \in \mathbb{R}_+ \ \mid \ (y^{h}_{1(h)} - p_1)p_1 \geq \delta(y^{h}_{1(h)} - y^{\ell}_{1(\ell)})y^{\ell}_{1(\ell)}\}. \tag{7}$$

By definition, policy z^1 satisfies $y^{\ell}_{1(\ell)} < z^1 < y^{h}_{1(h)}$ and is the agenda-setter's preferred policy among those that are acceptable to voter $1(\cdot)$ in state h whenever this voter expects a revised proposal to be $y^{\ell}_{1(\ell)}$. Each initial proposal p_1 in $(y^{\ell}_{1(\ell)}, z^1)$ passes with certainty in state h and gets rejected in state ℓ. The prior belief $\hat{\mu}$ determines what happens to an initial proposal p_1 in $(z^1, y^{h}_{1(h)})$. Define $m^1 = \frac{y^{\ell}_{1(\ell)}}{y^{h}_{1(h)}}$. If the prior belief that the state is h is sufficiently high, $\hat{\mu} > m^1$, then each initial proposal p_1 in $(z^1, y^{h}_{1(h)})$ passes in state h with probability $\hat{\alpha}^1 \in (0, 1)$ and gets rejected in state ℓ. Probability $\hat{\alpha}^1$ is derived from a condition that the posterior belief that the state is h equals m^1 after an initial proposal gets rejected, $\hat{\alpha}^1 = \frac{\hat{\mu}-m^1}{\hat{\mu}(1-m^1)}$. And if the prior belief that the state is h is sufficiently low, $\hat{\mu} \leq m^1$, then each initial proposal $p_1 \in (z^1, y^{h}_{1(h)})$ gets rejected in both states. Having defined z^1 and $\hat{\alpha}^1$, we can state

[14] In the special case when the voters with q highest ideal policies have identical preferences and can communicate, the equilibrium predictions are exactly the same as when the agenda-setter is engaged in bilateral bargaining with one of these voters (Theorem 6).

the formal result.[15]

Lemma 1. *Consider a q-majority voting rule with $q = 1$. In every PBE, the probability $W^1(p_1)$ that an initial proposal $p_1 \in \mathbb{R}_+$ passes equals:*

$$\hat{W}^1(p_1) = \begin{cases} 1 & \text{if } 0 < p_1 < y^{\ell}_{1(\ell)}, \\ \hat{\mu} & \text{if } y^{\ell}_{1(\ell)} < p_1 < z^1, \\ \hat{\mu}\hat{\alpha}^1 & \text{if } z^1 < p_1 < y^h_{1(h)} \text{ and } \hat{\mu} > m^1, \\ 0 & \text{if } z^1 < p_1 < y^h_{1(h)} \text{ and } \hat{\mu} \leq m^1, \text{ or } y^h_{1(h)} < p_1. \end{cases} \tag{8}$$

Given the probabilities with which the initial proposals pass, we can find the posterior belief after the initial proposal gets rejected and find the continuation payoffs of the agenda-setter.[16] When an initial proposal $p_1 \in (y^{\ell}_{1(\ell)}, z^1)$ gets rejected, the posterior belief $\mu^{\hbar}$ assigns probability 1 to state ℓ and the agenda-setter's continuation payoff is $y^{\ell}_{1(\ell)}$. If the prior belief that the state is h is sufficiently high, $\hat{\mu} > m^1$, then when the initial proposal $p_1 \in (z^1, y^h_{1(h)})$ gets rejected, the posterior belief $\mu^{\hbar}$ is such that $\mu^{\hbar} = m^1$ by the choice of $\hat{\alpha}^1$ and the agenda-setter's continuation payoff is $y^{\ell}_{1(\ell)}$ again. If the prior belief that the state is h is sufficiently low, $\hat{\mu} \leq m^1$, then when the initial proposal $p_1 \in (z^1, y^h_{1(h)})$ gets rejected, the posterior belief $\mu^{\hbar}$ equals the prior $\hat{\mu}$ and the agenda-setter's continuation payoff is $y^{\ell}_{1(\ell)}$ yet again. Finally, when the initial proposal $p_1 > y^h_{1(h)}$ gets rejected, the posterior belief $\mu^{\hbar}$ equals the prior $\hat{\mu}$ and the agenda-setter's continuation payoffs is $V^1_{A,2}(\hat{\mu}) = \max\{y^{\ell}_{1(\ell)}, \hat{\mu}y^h_{1(h)}\}$. We can write the the agenda-setter's expected payoff as follows:

$$U^1_{A,1}(p_1) = \begin{cases} p_1 & \text{if } p_1 < y^{\ell}_{1(\ell)}, \\ \hat{\mu}p_1 + (1-\hat{\mu})\delta y^{\ell}_{1(\ell)} & \text{if } y^{\ell}_{1(\ell)} < p_1 < z^1, \\ \mathbf{1}(\hat{\mu} \leq m^1)\delta y^{\ell}_{1(\ell)} \\ \quad + \mathbf{1}(\hat{\mu} > m^1)(\hat{\mu}\hat{\alpha}^1 p_1 + (1-\hat{\mu}\hat{\alpha}^1)\delta y^{\ell}_{1(\ell)}) & \text{if } z^1 < p_1 < y^h_{1(h)}, \\ \delta \max\{y^{\ell}_{1(\ell)}, \hat{\mu}y^h_{1(h)}\} & \text{if } y^h_{1(h)} < p_1. \end{cases} \tag{9}$$

In equilibrium, the agenda-setter proposes a policy that maximizes her expected payoff (9).

[15] All proofs, except the proof of Theorem 5 in Appendix C, can be found in Appendix D.
[16] See Lemma A.3 in Online Appendix A.

The existence of PBE places restrictions on the probabilities that the initial proposals $y^\ell_{1(\ell)}$, z^1, and $y^h_{1(h)}$ pass if proposed.[17] In the discussion that follows, I assume that $y^\ell_{1(\ell)}$ passes in both states, z^1 passes with certainty in state h, and $y^h_{1(h)}$ passes in state h with probability $\hat{\alpha}^1$.[18] There are three initial proposals that the agenda-setter can make in equilibrium, depending on the prior belief $\hat{\mu}$. Each of these initial proposals gives rise to a distinct equilibrium outcome. Define the following belief thresholds, the purpose of which will become apparent shortly:

$$m^\ell = \frac{(1-\delta)y^\ell_{1(\ell)}}{z^1 - \delta y^\ell_{1(\ell)}}, \tag{10}$$

$$m^h = \frac{m^1(y^h_{1(h)} - \delta y^\ell_{1(\ell)})}{y^h_{1(h)} - z^1 + m^1(z^1 - \delta y^\ell_{1(\ell)})}. \tag{11}$$

In the first possible equilibrium outcome, the initial proposal is $y^\ell_{1(\ell)}$, which is made when the agenda-setter is sufficiently pessimistic about the state being h, $\hat{\mu} < m^\ell$. For $y^\ell_{1(\ell)}$ to be the unique optimal proposal, we must have

$$y^\ell_{1(\ell)} > \hat{\mu} z^1 + (1-\hat{\mu})\delta y^\ell_{1(\ell)},$$

which is equivalent to $\hat{\mu} < m^\ell$. Note that the definition of z^1 in (7) and the strict concavity of $(y^h_{1(h)} - p_1)p_1$ in p_1 imply $z^1 > (1-\delta)y^h_{1(h)} + \delta y^\ell_{1(\ell)}$ and therefore $m^\ell < m^1$. It follows that whenever $\hat{\mu} < m^\ell$ we also have

$$y^\ell_{1(\ell)} > \hat{\mu}\hat{\alpha}^1 y^h_{1(h)} + (1-\hat{\mu}\hat{\alpha}^1)\delta y^\ell_{1(\ell)},$$

because, on the one hand, the latter inequality is equivalent to $\hat{\mu} < \frac{m^1(1-\delta+1-m^1)}{1-\delta m^1}$ and, on the other hand, we have $m^1 < \frac{m^1(1-\delta+1-m^1)}{1-\delta m^1}$. The initial proposal $y^\ell_{1(\ell)}$ passes in both states, so the game effectively ends in the first period with certainty.

In the second possible equilibrium outcome, the initial proposal is z^1, which is made when the agenda-setter is moderately optimistic about the state being h, $m^\ell <$

[17] See the analysis of the second period in Online Appendix A for a detailed discussion of the associated issues.

[18] This assumption has no effect on the agenda-setter's expected payoff but simplifies exposition. The statement and proof of Theorem 1 are completely general.

$\hat{\mu} < m^h$. For z^1 to be the unique optimal proposal, we must have

$$\hat{\mu} z^1 + (1 - \hat{\mu})\delta y^\ell_{1(\ell)} > y^\ell_{1(\ell)},$$

which is equivalent to $\hat{\mu} > m^\ell$. If the prior belief is not too high, $\hat{\mu} \le m^1$, then z^1 is indeed the unique optimal proposal in the first period. However, if $\hat{\mu} > m^1$, then for z^1 to be the unique optimal proposal, we must also have

$$\hat{\mu} z^1 + \delta(1 - \hat{\mu}) y^\ell_{1(\ell)} > \hat{\mu}\hat{\alpha}^1 y^h_{1(h)} + (1 - \hat{\mu}\hat{\alpha}^1)\delta y^\ell_{1(\ell)},$$

which is equivalent to $\hat{\mu} < m^h$. Straightforward algebra shows that $m^1 < m^h$. The initial proposal z^1 passes in state h but gets rejected in state ℓ, in which case the posterior belief $\mu^{\hbar}$ assigns probability 0 to state h and the revised proposal is $y^\ell_{1(\ell)}$.

In the third (and final) possible equilibrium outcome, the initial proposal is $y^h_{1(h)}$, which is made when the agenda-setter is very optimistic about the state being h, $m^h < \hat{\mu}$. For $y^h_{1(h)}$ to be the unique optimal proposal, we must have

$$\hat{\mu}\hat{\alpha}^1 y^h_{1(h)} + (1 - \hat{\mu}\hat{\alpha}^1)\delta y^\ell_{1(\ell)} > \hat{\mu} z^1 + \delta(1 - \hat{\mu}) y^\ell_{1(\ell)},$$

which is equivalent to $m^h < \hat{\mu}$. Inequalities $m^1 < m^h$ and $m^\ell < m^h$ imply $m^\ell < m^h$. Therefore, we also have

$$\hat{\mu}\hat{\alpha}^1 y^h_{1(h)} + (1 - \hat{\mu}\hat{\alpha}^1)\delta y^\ell_{1(\ell)} > y^\ell_{1(\ell)}.$$

The initial proposal $y^h_{1(h)}$ passes in state h with probability $\hat{\alpha}^1$ and gets rejected in state ℓ, in which case the posterior belief $\mu^{\hbar}$ assigns probability m^1 to state h and the agenda-setter makes the proposal $y^h_{1(h)}$ again in the second period.

Theorem 1. *Under the q-majority voting rule with $q = 1$, there exists a unique PBE path, such that:*

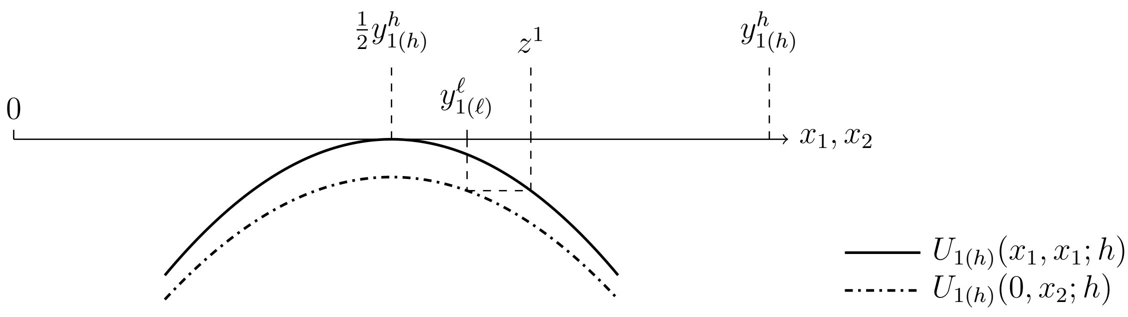
(i) If $\hat{\mu} < m^\ell$, the initial proposal is $y^\ell_{1(\ell)}$ which passes.

(ii) If $m^\ell < \hat{\mu} < \mu^h$, the initial proposal is $z^1 > y^\ell_{1(\ell)}$ which passes in state h and gets rejected in state ℓ. The revised proposal is $y^\ell_{1(\ell)}$ which passes.

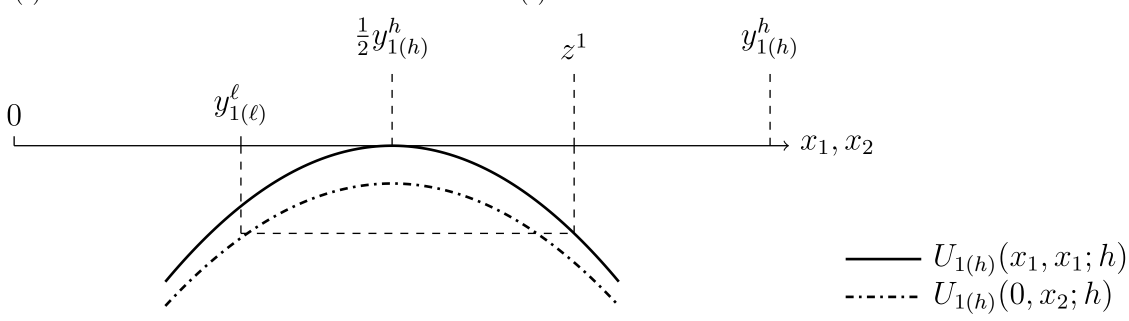
(iii) If $\hat{\mu} > m^h$, the initial proposal is $y^h_{1(h)} > z^1$ which gets rejected in state ℓ and passes in state h with probability $\hat{\alpha}^1$. The revised proposal is $y^h_{1(h)}$ which gets

rejected in state ℓ and passes in state h.

(iv) In case $\hat{\mu} = m^{\ell}$, the agenda-setter randomizes between $y^{\ell}_{1(\ell)}$ and z^1, and in case $\hat{\mu} = m^h$, the agenda-setter randomizes between z^1 and $y^h_{1(h)}$. The rest of the equilibrium path is described as in cases (i)-(iii).



(a) Case $y^h_{1(h)} \leq 2y^{\ell}_{1(\ell)}$. Voter $1(h)$ is indifferent between policy z^1 without delay and policy $y^{\ell}_{1(\ell)}$ with delay. Policy z^1 converges to $y^{\ell}_{1(\ell)}$ as $\delta \to 1$.



(b) Case $y^h_{1(h)} > 2y^{\ell}_{1(\ell)}$. Voter $1(h)$ is indifferent between policy z^1 without delay and policy $y^{\ell}_{1(\ell)}$ with delay. Policy z^1 converges to $y^h_{1(h)} - y^{\ell}_{1(\ell)}$ as $\delta \to 1$.

Figure 1: The determination of policy z^1 in Theorem 1. The ideal policy of voter $1(\cdot)$ in state $\omega \in \Omega$ is $\frac{1}{2}y^{\omega}_{1(\omega)}$.

Remark. The unique equilibrium path described in Theorem 1 is analogous to the one arising in bilateral bargaining over a distributive policy with two possible valuations of the buyer, for example, the models in Chapter 10 of Fudenberg and Tirole (1991) and Section 3 of Deneckere and Liang (2006).

Since there is a unique equilibrium path, we can define the agenda-setter's expected payoff $V^1_{A,1}$ under the q-majority voting rule with $q = 1$ and derive its limit as players become arbitrarily patient. The following result will allow me to compare the agenda-setter's expected payoff with the Coasian bound $y^{\ell}_{1(\ell)}$ and the commitment benchmark derived in Appendix A.

Theorem 2. *Under the q-majority voting rule with $q = 1$, as players become perfectly patient, $\delta \to 1$, we have:*

$$\lim_{\delta\to 1} V^1_{A,1} = \begin{cases} \hat{\mu} y^h_{1(h)} & \text{if } y^h_{1(h)} > 2y^{\ell}_{1(\ell)} \text{ and } \hat{\mu} > \frac{1}{2}, \\ & \text{or } y^h_{1(h)} \leq 2y^{\ell}_{1(\ell)} \text{ and } \hat{\mu} > \frac{y^{\ell}_{1(\ell)}}{y^h_{1(h)}}, \\ \hat{\mu}(y^h_{1(h)} - y^{\ell}_{1(\ell)}) + (1-\hat{\mu})y^{\ell}_{1(\ell)} & \text{if } y^h_{1(h)} > 2y^{\ell}_{1(\ell)} \text{ and } \hat{\mu} \leq \frac{1}{2}, \\ y^{\ell}_{1(\ell)} & \text{if } y^h_{1(h)} \leq 2y^{\ell}_{1(\ell)} \text{ and } \hat{\mu} \leq \frac{y^{\ell}_{1(\ell)}}{y^h_{1(h)}}. \end{cases}$$

3.2 Coasian equilibria and the value of commitment

Theorem 2 implies that the limit of the agenda-setter's expected payoff cannot be smaller than the Coasian bound $y^{\ell}_{1(\ell)}$ but can be larger, which implies that the Coase conjecture can be violated. The first reason why the Coase conjecture can be violated is because the agenda-setter may prefer to make the same proposal $y^h_{1(h)}$ in both periods, which happens when the prior belief that the state is h is sufficiently large, $\hat{\mu} > m^h$. This violation is caused by the short bargaining horizon and the resulting commitment power in the second period.[19]

The second (and more interesting) reason why the Coase conjecture can be violated is related to the single-peaked preferences of voters. If the ideal policies of voter $1(\cdot)$ in different states are sufficiently far apart, $y^h_{1(h)} > 2y^{\ell}_{1(\ell)}$, then the threat of proposing $y^{\ell}_{1(\ell)}$ in the second period gives an incentive to voter $1(h)$ in state h to accept policies that are preferred by the agenda-setter to $y^{\ell}_{1(\ell)}$ even when players are perfectly patient. As a result, the initial policy proposal converges to $y^h_{1(h)} - y^{\ell}_{1(\ell)} > y^{\ell}_{1(\ell)}$ (compare Figures 1a and 1b). When the bargaining is over a distributive policy (Fudenberg, Levine, and Tirole, 1985), the revised proposal is always closer to the ideal policy of the "voter" which is 0, and therefore such violation of the Coase conjecture is not possible.[20]

[19]In the models of bilateral bargaining over a distributive policy, and specifically when the "gap" assumption holds, the bargaining ends in finitely many periods and the final period effectively features commitment. However, the bargaining horizon is determined endogenously. See Fudenberg, Levine, and Tirole (1985) for details.

[20]However, the Coase conjecture can be violated in bilateral bargaining over price when the buyer's valuation and the seller's cost are correlated, see Deneckere and Liang (2006).

The following result focuses on the violation of the Coase conjecture caused by the single-peaked preferences of voter $1(\cdot)$. I assume that the prior belief that the state is h is such that $\hat{\mu} \leq \frac{y^{\ell}_{1(\ell)}}{y^{h}_{1(h)}}$, which implies that the agenda-setter's expected payoff under the commitment benchmark equals the Coasian bound $y^{\ell}_{1(\ell)}$. Nonetheless, even in this case the agenda-setter can exploit the single-peaked preferences of voter $1(h)$ and reduce this voter's information rent.

Theorem 3. *Consider a q-majority rule with $q = 1$ and suppose that the prior belief that the state is h satisfies $\hat{\mu} \leq \frac{y^{\ell}_{1(\ell)}}{y^{h}_{1(h)}}$. Then, every sequence of equilibria is non-Coasian if the ideal policies of synthetic voter $1(\cdot)$ are such that $y^{h}_{1(h)} > 2y^{\ell}_{1(\ell)}$ and Coasian otherwise.*

Theorem 2 also implies that the ability to revise the initial proposal is valuable to the agenda-setter. The following result provides the conditions when the ability to revise the initial proposal is strictly valuable.

Theorem 4. *Consider a q-majority rule with $q = 1$. When players become perfectly patient, $\delta \to 1$, the agenda-setter's expected payoff is weakly greater than the commitment benchmark, and it is strictly greater if and only if $y^{h}_{1(h)} > 2y^{\ell}_{1(\ell)}$ and $\hat{\mu} < \frac{1}{2}$.*

Not surprisingly, the key ingredient in this result is the single-peaked preferences of voter $1(\cdot)$; but unlike in Theorem 3, the agenda-setter sometimes can reduce voter $1(h)$'s information rent even when the prior belief that the state is h if such that $\hat{\mu} > \frac{y^{\ell}_{1(\ell)}}{y^{h}_{1(h)}}$ and the agenda-setter's expected payoff under the commitment benchmark is above the Coasian bound $y^{\ell}_{1(\ell)}$.

There are at least two other papers that show that the agenda-setter may exploit a sequence of elections to receive a higher expected payoff than the commitment benchmarks. However, the forces behind the Coase conjecture (or its violation documented in Theorems 3 and 4) are absent in both. In Romer and Rosenthal (1979), voters are myopic and the ability to revise the initial proposal does not affect the voting behavior. In Rosenthal and Zame (2019), players do not discount future payoffs so the fundamental tradeoffs are different.

4 "Multilateral" bargaining over policy ($q \geq 2$)

In this section, I consider the case when more than a single acceptance is required to pass a policy proposal, $q \geq 2$. A notable distinction from the previous case $q = 1$ is that the agenda-setter can make different revised proposals depending on the number and identities of voters who accepted the initial proposal that got rejected. As a result, there is a multiplicity of equilibrium outcomes. Theorem 5 shows that the set of agenda-setter's expected payoffs that can be supported in equilibrium is an interval. The endpoints $\underline{w}_q$ and $\overline{w}_q$ of this interval are determined similarly to the agenda-setter's expected payoff $V^1_{A,1}$ from Theorem 2 and converge to it when the preferences of voters $\{1(\cdot), \ldots, q(\cdot)\}$ become perfectly aligned (Theorem 6). Naturally, the Coase conjecture can be violated for the same reason as before, namely, the single-peaked preferences of voters (Theorems 7 and 8). The screening nature of the endpoints $\underline{w}_q$ and $\overline{w}_q$ along with the possible violation of the Coase conjecture lead to another unexpected result: the agenda-setter's expected payoff can increase when the required quota is increased from $q \geq 2$ even though the agenda-setter must seek acceptance from more voters (Theorem 9).

4.1 Characterization: Multiplicity of equilibrium paths

The following result provides a sharp characterization of the agenda-setter's expected payoffs that can be achieved in equilibrium when voters are allowed to communicate with each other.

Theorem 5. *Under the q-majority rule with $q \geq 2$, the agenda-setter's coalition-proof equilibrium expected payoff equals v if and only if $v \in [\underline{w}_q, \overline{w}_q]$.*

As shown in Appendix B, the lower bound $\underline{w}_q$ is equivalent to the agenda-setter's expected payoff when bargaining against a single voter $q(\cdot)$. However, the shape of the limit set depends not only on the ideal policies of voter $q(\cdot)$ but also on the ideal policies of voter $1(\cdot)$. In part, the upper bound $\overline{w}_q$ is the same as when bargaining against a single voter with ideal policies $y^{\ell}_{q(\ell)}$ and $y^{h}_{1(h)}$ in states ℓ and h. These observations suggest that when the preferences of voters $1(\cdot)$ and $q(\cdot)$ coincide, the agenda-setter's expected payoff can be described as arising from bilateral bargaining with any voter $j(\cdot)$ where $j \in \{1, \ldots, q\}$. The following result focuses on the case when players become perfectly patient.

Theorem 6. *Consider a q-majority voting rule with $q \geq 2$ and suppose the preferences of voters $1(\cdot)$ through $q(\cdot)$ become perfectly aligned, $\max_{\omega\in\Omega}\{1(\omega) - q(\omega)\} \to 0$. When players become perfectly patient, $\delta \to 1$, the agenda-setter's expected payoff is unique and coincides with the case $q = 1$.*

4.2 Coasian equilibria and the value of commitment

Similar to the case $q = 1$, the Coase conjecture may be violated for $q \geq 2$ either because of the agenda-setter's ability to commit in the second period or the single-peaked preferences of voters. Analogous to Theorem 3, the following result focuses on the case when the prior belief that the state is h is such that $\hat{\mu} \leq \frac{y^{\ell}_{q(\ell)}}{y^{h}_{q(h)}}$, which implies that the agenda-setter's expected payoff under the commitment benchmark equals the Coasian bound $y^{\ell}_{q(\ell)}$.

Theorem 7. *Consider a q-majority voting rule with $q \geq 2$ and suppose that the prior belief that the state is h satisfies $\hat{\mu} \leq \frac{y^{\ell}_{q(\ell)}}{y^{h}_{q(h)}}$. Then:*

(i) When $y^{h}_{q(h)} > 2y^{\ell}_{q(\ell)}$, every sequence of equilibria is non-Coasian.

(ii) When $y^{h}_{q(h)} \leq 2y^{\ell}_{q(\ell)}$, if either $y^{h}_{1(h)} - y^{\ell}_{q(\ell)} \geq y^{h}_{q(h)}$ or both $y^{\ell}_{q(\ell)} < y^{h}_{1(h)} - y^{\ell}_{q(\ell)} < y^{h}_{q(h)}$ and $\hat{\mu} \leq \frac{y^{\ell}_{q(\ell)}}{2y^{\ell}_{q(\ell)} - y^{h}_{1(h)} + y^{h}_{q(h)}}$, almost every sequence of equilibria is non-Coasian.

(iii) Otherwise, every sequence of equilibria is Coasian.

Is the ability to commit to a single proposal valuable to the agenda-setter under the q-majority voting rule with $q \geq 2$? By comparing the commitment benchmark $V^{C}_{q} = \max\{y^{\ell}_{q(\ell)}, \hat{\mu}y^{h}_{q(h)}\}$ derived in Appendix A with the limit of the lower bound $\underline{w}_q$ as players become perfectly patient derived in Appendix B (equation (B.7)), we can see that every expected payoff under the unanimity rule when players are perfectly patient is at least as high as V^{C}_{q}.

Theorem 8. *Consider the q-majority voting rule with $q \geq 2$. When players become perfectly patient, $\delta \to 1$:*

(i) When both $y^{h}_{q(h)} > 2y^{\ell}_{q(\ell)}$ and $\hat{\mu} \leq \frac{1}{2}$, the agenda-setter's expected payoff is strictly greater than the commitment benchmark in every equilibrium.

(ii) When $y^h_{q(h)} \leq 2y^\ell_{q(\ell)}$ or $\hat{\mu} > \frac{1}{2}$, if either $y^h_{1(h)} - y^\ell_{q(\ell)} \geq y^h_{q(h)}$ or both $y^\ell_q < y^h_{1(h)} - y^\ell_{q(\ell)} < y^h_{q(h)}$ and $\hat{\mu} \leq \frac{y^\ell_{q(\ell)}}{2y^\ell_{q(\ell)} - y^h_{1(h)} + y^h_{q(h)}}$, the agenda-setter's expected payoff is strictly greater than the commitment benchmark in almost every equilibrium.

(iii) Otherwise, the agenda-setter's expected payoff equals the commitment benchmark in every equilibrium.

These results imply that in settings with single-peaked preferences, such as referenda on public spending, allowing the agenda-setter to maker a revised proposal on average leads to a higher level of implemented policy. Intuitively, the anticipated revised proposal serves as a threat for voters when the state is high, allowing the agenda-setter to limit the information rent that the voters can receive. Single-peaked preferences of voters play a crucial role, because the anticipated revised proposal is a threat only when it is smaller than the ideal level of spending of the target voter.

4.3 An increase in the required quota

A variety of required quotas are used in practice, ranging from a simple majority, $q = \frac{n}{2}$, to unanimity ($q = n$). Some of the more prominent examples include: voting by a supermajority in United Nations Security Council, voting by a qualified majority in the Council of the European Union, and voting by a supermajority in the United States Congress required in some circumstances (overriding a presidential veto, ratifying a treaty, removing a federal official from office, etc.). Occasionally, reforms to change the required quotas are proposed and even succeed, as was the case in the United States Senate when the required number of votes to end the debate by invoking cloture was reduced from $\frac{2}{3}$ to $\frac{3}{5}$ in 1970 and to $\frac{1}{2}$ on certain issues in 2013 and 2017.

Because of the multiplicity of the expected payoffs that can be supported in equilibrium, the comparative statics with respect to quota q must rely on equilibrium selection except in extreme cases. A mapping $\phi(\cdot) : \{2, \ldots, n\} \to \mathbb{R}_+$ is an *equilibrium payoff selection* if for each $q \in \{2, \ldots, n\}$ we have $\lim_{\delta \to 1} \underline{w}_q \leq \phi(q) \leq \lim_{\delta \to 1} \overline{w}_q$. An equilibrium payoff selection $\phi(\cdot)$ is *monotone* if it is weakly decreasing on $\{2, \ldots, n\}$.

The following result provides conditions for the existence of a non-monotone equilibrium payoff selection or, in other words, for an existence of $q \in \{2, \ldots, n-1\}$ such that the agenda-setter's expected payoff under the larger quota $q + 1$ is strictly greater than the expected payoff under the smaller quota q. The question boils down to finding q such that $\overline{w}_{q+1} > \underline{w}_q$.

Theorem 9. *There exists an equilibrium payoff selection $\phi(\cdot)$ that is non-monotone if and only if there exists $q \in \{2, \dots, n-1\}$ such that $y^h_{1(h)} > 2y^\ell_{(q+1)(\ell)}$ and one of the following conditions holds:*

(i) $\max\left\{\frac{1}{2}, \frac{y^\ell_{q(\ell)}}{y^h_{q(h)}}\right\} < \hat{\mu} < \frac{y^\ell_{(q+1)(\ell)}}{2y^\ell_{(q+1)(\ell)} - y^* + y^h_{q(h)}}$;

(ii) $y^h_{q(h)} \leq 2y^\ell_{q(\ell)}$ *and* $\frac{y^\ell_{q(\ell)} - y^\ell_{(q+1)(\ell)}}{y^* - 2y^\ell_{(q+1)(\ell)}} < \hat{\mu} \leq \min\left\{\frac{y^\ell_{q(\ell)}}{y^h_{q(h)}}, \frac{y^\ell_{(q+1)(\ell)}}{2y^\ell_{(q+1)(\ell)} - y^* + y^h_{(q+1)(h)}}\right\}$;

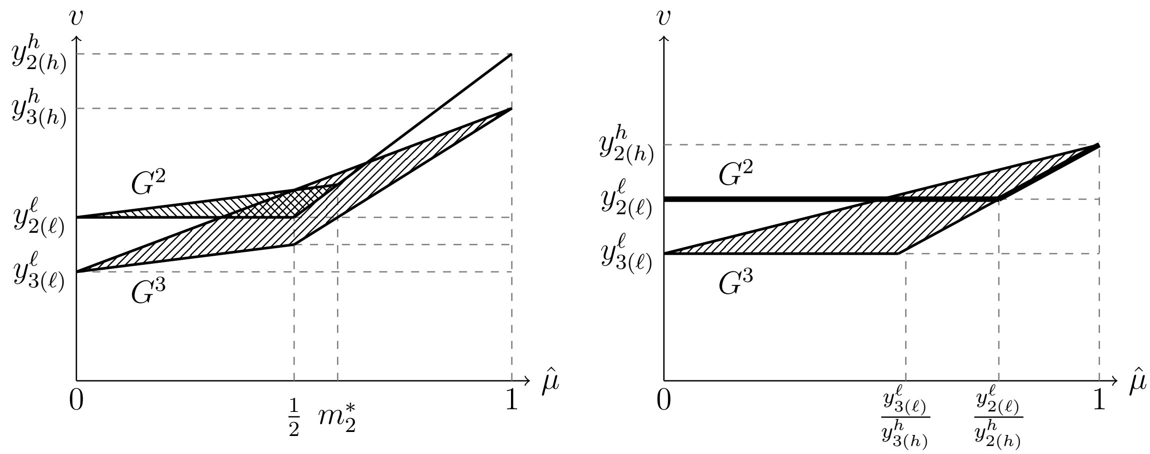
(iii) $y^h_{q(h)} > 2y^\ell_{q(\ell)}$ *and* $\frac{y^\ell_{q(\ell)} - y^\ell_{(q+1)(\ell)}}{y^* - 2y^\ell_{(q+1)(\ell)} - y^h_{q(h)} + 2y^\ell_{q(\ell)}} < \hat{\mu} \leq \min\left\{\frac{1}{2}, \frac{y^\ell_{(q+1)(\ell)}}{2y^\ell_{(q+1)(\ell)} - y^* + y^h_{(q+1)(h)}}\right\}$,

where $y^* = \min\{y^h_{1(h)}, y^\ell_{(q+1)(\ell)} + y^h_{(q+1)(h)}\}$.

Theorem 9 implies that in settings with single-peaked preferences, e.g., public spending referenda, a higher required quota can be associated with a higher (on average) implemented policy. This result is surprising because a higher required quota gives collective veto power to smaller coalitions of voters and thus implies that the policies that pass give at least the status-quo level of payoff to a larger number of voters. The screening structure of equilibrium paths reconciles these facts. Intuitively, an increase in required quota lowers the anticipated revised proposals, allowing the agenda-setter to limit the information rent that the voters can receive when the state is high.

It is worth nothing that in some cases almost every equilibrium payoff selection is non-monotone. Figure 2b provides an example in which not only the upper bound $\overline{w}_{q+1}$ of the agenda-setter's expected payoff when the quota is $q+1$ is strictly greater than the lower bound $\underline{w}_q$ when the quota is q, but also the lower bound $\underline{w}_{q+1}$ when the quota is $q+1$ equals the upper bound $\overline{w}_q$ when the quota is q. The example in Figure 2b is especially interesting because the agenda-setter's expected payoff is unique and equals the commitment benchmark when the quota is q, but any expected payoff between the commitment and full information benchmarks can be supported in a coalition-proof PBE when the quota is $q+1$.

Theorem 9 also implies that every equilibrium payoff selection is monotone when for each $q \in \{2, \dots, n-1\}$ either $y^h_{1(h)} \leq 2y^\ell_{(q+1)(\ell)}$ or the conditions *(i)*-*(iii)* are violated. Intuitively, this is the case when "voters heterogeneity," by which I mean the



(a) There exists a non-monotone equilibrium payoff selection when the prior belief $\hat{\mu}$ is not too extreme.

(b) Almost every equilibrium payoff selection is non-monotone for $\hat{\mu} \geq m_2$.

Figure 2: The limit sets of the agenda-setter's expected payoffs under the q-majority voting rule. There are $n = 3$ voters, limit sets for $q = 2$ and $q = 3$ are depicted. For any $q \geq 2$, G^q is a graph of the correspondence that maps the prior belief $\hat{\mu}$ to the interval $[\underline{w}_q, \overline{w}_q]$.

differences between the ideal policies of synthetic voters in a given state, is sufficiently greater than "state heterogeneity," by which I mean the differences between the ideal policies of each synthetic voter in separate states.

The following corollary to Theorems 6 and 9 provides a comparison of the agenda-setter's expected payoffs in the "bilateral" and "multilateral" cases. It turns out that the agenda-setter cannot have a greater expected payoff when the required quota is $q \geq 2$ than when $q = 1$. For instance, with only two voters the agenda-setter would always prefer the majority rule to the unanimity rule.[21]

Corollary 1. *The agenda-setters expected payoff under the q-majority voting rule with $q = 1$ cannot be smaller than under the q-majority voting rule with $q \geq 2$.*

[21] This is not necessarily true when voters cannot communicate with each other, see Online Appendix C.

5 Conclusion

In this paper, I have analyzed a model of bargaining of over a spatial policy between an agenda-setter and n voters in which the agenda-setter is uncertain the preferences of voters. I provided a full characterization of the agenda-setter's equilibrium expected payoffs under the q-majority voting rule with $1 \leq q \leq n$ assuming that voters use weakly undominated voting strategies and can communicate with each other. Using this characterization, I demonstrated that the Coase conjecture can be violated owing to the single-peaked preferences of voters. As a consequence, the ability to make a revised proposal can be strictly valuable to the agenda-setter. I also showed that the agenda-setter's expected payoff can increase in response to an increase in the required number q of voters whose approval is required to pass a policy.

There are multiple avenues for future work on multilateral models of bargaining over spatial policy. It would be interesting to see if the results in this paper are robust to the assumption that voters observe the state without noise.[22] The analysis can be easily extended to any finite number of periods and states and to the case when the utilities of voters are not quadratic.[23] The techniques in this paper can also be used to analyze a model in which the agenda-setter's utility is state-dependent (Deneckere and Liang, 2006). A vital but more challenging extension is to allow players with private information make proposals.[24]

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[22] In a companion paper, I analyze the ability of large elections to aggregate information when the information about the common state is dispersed among voters and the agenda-setter can make a revised proposal if the initial proposal is rejected.

[23] See footnote 3.

[24] Banks (1990, 1993) and Lupia (1992) study single-period models of monopoly agenda control with a privately informed agenda-setter and focus on the signaling role of proposals.

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Appendix A: Benchmarks

I consider two versions of the model that serve as useful benchmarks for the discussion of main results. For a given required quota $q \in N$ and state $\omega \in \Omega$, voter i is the *target voter in state* ω if the ideal policy of voter i is q-th highest in state ω. Note that the identity of the target voter generally depends on the required quota q and state ω. For a given quota $q \in N$, the identity of the target voter in each state is given by the synthetic-voter mapping $q(\cdot)$ defined in Section 2.

A.1 Complete information

Which policies are proposed and implemented when the agenda-setter knows the preferences of voters? Since the proposal power is concentrated at the hands of the agenda-setter, when the required quota is $q \in N$ and the state is $\omega \in \Omega$ we are effectively looking at the ultimatum bargaining game between the agenda-setter and the target voter, namely, synthetic voter $q(\omega)$. By a standard argument, in any equilibrium with weakly undominated voting strategies each voter $i \in N$ in state ω accepts proposal $p_1 \in \mathbb{R}_+$ if and only if $p_1 \leq y_i^\omega$. In turn, the agenda-setter makes proposal $y^\omega_{q(\omega)}$ and this proposal passes.

Benchmark 1 (Full information)**.** For a given quota q, when the information is complete, the agenda-setter's expected payoff equals

$$V_q^F = \hat{\mu} y^h_{q(h)} + (1 - \hat{\mu}) y^\ell_{q(\ell)}.$$

The full-information benchmark provides an upper bound for the agenda-setter's expected payoff. In Appendix B, I show that the agenda-setter can approximate this upper bound V_q^F under some conditions. Note that this benchmark is the worst outcome for a target voter because she receives her status-quo payoff in every state. Under informational asymmetry, the target voter receives an information rent in state h in most equilibria.

A.2 Commitment (take-it-or-leave-it offer)

Which policies are proposed when the agenda-setter commits to make a single proposal? The incentives of players in this benchmark are exactly the same as in the second (last) period of the baseline model. Therefore, the analysis in Online Appendix A continues to hold after replacing the posterior belief μ^h with the prior belief $\hat{\mu}$. When voting strategies are weakly undominated, voter $i \in N$ in state $\omega \in \Omega$ accepts proposal p_1 if and only if $p_1 \leq y_i^\omega$. However, the agenda-setter does not observe the state ω and therefore cannot use it to tailor the proposal to the target voter. The agenda-setter can either propose $y_{q(\ell)}^\ell$, which passes in both states, or propose $y_{q(h)}^h$, which passes only in state h.

Benchmark 2 (Commitment)**.** For a given quota q, when there is a single offer, the agenda-setter's expected payoff equals

$$V_q^C = \max\{y_{q(\ell)}^\ell,\ \hat{\mu} y_{q(h)}^h\}.$$

The commitment benchmark allows us to find out whether the ability to make a revised proposal is beneficial to the agenda-setter.

Appendix B: Payoff bounds

In this section, I derive the bounds of the agenda-setter's expected payoffs in coalition-proof equilibria and the limits of these bounds as players become perfectly patient. For each $q \in N$ and $\omega \in \Omega$, let:

$$G_q = \{p_1 \in \mathbb{R}_+ \ \mid \ (y_j^h - p_1)p_1 \geq \delta(y_j^h - y_{q(\ell)}^\ell)y_{q(\ell)}^\ell \text{ for at least 1 voter}\},$$
$$L_q^\omega = \{p_1 \in \mathbb{R}_+ \ \mid \ (y_j^h - p_1)p_1 \leq \delta(y_j^h - y_{q(\omega)}^\omega)y_{q(\omega)}^\omega \text{ for at least } n-q+1 \text{ voter(s)}\}.$$

Define $p_q^* = \min\{y^h_{q(h)}, \max G_q\}$ and $p_q^\ell = \min L_q^\ell$, and notice that we have $p_q^\ell \leq p_q^* \leq y^h_{q(h)}$. In addition, define $p_q^h = \min L_q^h \cap [y^h_{q(h)}, \infty)$ if $q < \frac{n+1}{2}$ and $p_q^h = \max L_q^h \cap [0, y^h_{q(h)}]$ if $q \geq \frac{n+1}{2}$. Finally, let $m_q = \frac{y^\ell_{q(\ell)}}{y^h_{q(h)}}$ and $\hat{\alpha}^q = \frac{\hat{\mu} - m_q}{\hat{\mu}(1-m_q)}$. The bounds for the agenda-setter's expected payoff in coalition-proof equilibria are given by:

$$\begin{aligned}
\overline{w}_q = \max\{ & y^\ell_{q(\ell)}, \hat{\mu} p_q^* + \delta(1-\hat{\mu}) y^\ell_{q(\ell)}, \mathbf{1}(\hat{\mu} > m_q)(\hat{\mu}\hat{\alpha}^q y^h_{q(h)} + \delta(1-\hat{\mu}\hat{\alpha}^q) y^\ell_{q(\ell)}), \\
& \mathbf{1}(p_q^h \leq y^\ell_{q(\ell)} \leq y^\ell_{(n-q+1)(\ell)})(\delta\hat{\mu} y^h_{q(h)} + (1-\hat{\mu}) y^\ell_{q(\ell)})\}, \qquad \text{(B.1)} \\
\underline{w}_q = \max\{ & y^\ell_{q(\ell)}, \delta\hat{\mu} y^h_{q(h)}, \mathbf{1}(\hat{\mu} > m_q)(\hat{\mu}\hat{\alpha}^q y^h_{q(h)} + (1-\hat{\mu}\hat{\alpha}^q)\delta y^\ell_{q(\ell)}), \\
& \mathbf{1}(\hat{\mu} \leq m_q)(\hat{\mu} p_q^\ell + \delta(1-\hat{\mu}) y^\ell_{q(\ell)}), \\
& \mathbf{1}(\hat{\mu} > m_q)\mathbf{1}(y^\ell_{q(\ell)} < \min\{p_q^\ell, p_q^h\})(\hat{\mu} p_q^h + \delta(1-\hat{\mu}) y^\ell_{q(\ell)})\}. \qquad \text{(B.2)}
\end{aligned}$$

I derive the limits of policies p_q^* and p_q^ω for $\omega \in \Omega$ as players become perfectly patient. Consider p_q^* first. Notice that $y_q^\ell \in G_q$ (for instance, let $j = q(h)$) and, for each $p_1 > y_q^\ell$, $(y_j^h - p_1)p_1 < \delta(y_j^h - y^\ell_{q(\ell)}) y^\ell_{q(\ell)}$ implies that $(y^h_{i(h)} - p_1)p_1 < \delta(y^h_{i(h)} - y^\ell_{q(\ell)}) y^\ell_{q(\ell)}$ for all $i(h) \geq j$. If follow that p_h^* equals either $y^h_{q(h)}$ or the largest root of the quadratic equation $(p_1)^2 - y^h_{1(h)} p_1 + \delta(y^h_{1(h)} - y^\ell_{q(\ell)}) y^\ell_{q(\ell)}$, whichever is smaller. The roots are $\frac{1}{2}(y^h_{1(h)} + [(y^h_{1(h)})^2 - 4\delta(y^h_{1(h)} - y^\ell_{q(\ell)}) y^\ell_{q(\ell)}]^{\frac{1}{2}})$ and $\frac{1}{2}(y^h_{1(h)} - [(y^h_{1(h)})^2 - 4\delta(y^h_{1(h)} - y^\ell_{q(\ell)}) y^\ell_{q(\ell)}]^{\frac{1}{2}})$. As $\delta \to 1$, the limit of the expression under the radical equals $(y^h_{1(h)} - 2y^\ell_{q(\ell)})^2$, so the limit of the largest root equals $\frac{1}{2}(y^h_{1(h)} - (y^h_{1(h)} - 2y^\ell_{q(\ell)})) = y^\ell_{q(\ell)}$ if $y^h_{1(h)} \leq 2y^\ell_{q(\ell)}$ and equals $\frac{1}{2}(y^h_{1(h)} + (y^h_{1(h)} - 2y^\ell_{q(\ell)})) = y^h_{1(h)} - y^\ell_{q(\ell)}$ if $y^h_{1(h)} > 2y^\ell_{q(\ell)}$. Thus, we have:

$$\lim_{\delta \to 1} p_q^* = \begin{cases} y^\ell_{q(\ell)} & \text{if } y^h_{1(h)} \leq 2y^\ell_{q(\ell)}, \\ \min\{y^h_{1(h)} - y^\ell_{q(\ell)}, y^h_{q(h)}\} & \text{if } y^h_{1(h)} > 2y^\ell_{q(\ell)}; \end{cases}. \qquad \text{(B.3)}$$

Next, consider p_q^ℓ. Notice that $y^\ell_{q(\ell)} \notin L_q^\ell$ and $y^h_{q(h)} \in L_q^\ell$ (for instance, look at voters in $\{1(h), \ldots, q(h)\}$) and, for each $p_1 > y^\ell$, we have $(y_j^h - p_1)p_1 > \delta(y_j^h - y^\ell_{q(\ell)}) y^\ell_{q(\ell)}$ implies that $(y^h_{i(h)} - p_1)p_1 > \delta(y^h_{i(h)} - y^\ell_{q(\ell)}) y^\ell_{q(\ell)}$ for all $i(h) < j$. If follow that p_h^ℓ is the largest root of the quadratic equation $(p_1)^2 - y^h_{q(h)} p_1 + \delta(y^h_{q(h)} - y^\ell_{q(\ell)}) y^\ell_{q(\ell)}$. Proceeding as above, we obtain:

$$\lim_{\delta \to 1} p_q^\ell = \begin{cases} y^\ell_{q(\ell)} & \text{if } y^h_{q(h)} \leq 2y^\ell_{q(\ell)}, \\ y^h_{q(h)} - y^\ell_{q(\ell)} & \text{if } y^h_{q(h)} > 2y^\ell_{q(\ell)}. \end{cases} \qquad \text{(B.4)}$$

Finally, consider p_q^h. If $q < \frac{n+1}{2}$, which is equivalent to $y^h_{q(h)} > y^h_{(n-q+1)(h)}$, p_q^h converges to $y^h_{(n-q+1)(h)}$ from above when players become perfectly patient. And if $q \geq \frac{n+1}{2}$, p_q^h is the smallest root of the quadratic equation $(p_1)^2 - y^h_{(n-q+1)(h)} p_1 + \delta(y^h_{(n-q+1)(h)} - y^h_{q(h)}) y^h_{q(h)}$. Proceeding as above, we obtain:

$$\lim_{\delta \to 1} p_q^h = \begin{cases} y^h_{(n-q+1)(h)} & \text{if } y^h_{q(h)} > y^h_{(n-q+1)(h)}, \\ y^h_{(n-q+1)(h)} - y^h_{q(h)} & \text{if } y^h_{q(h)} \leq y^h_{(n-q+1)(h)} < 2y^h_{q(h)}, \\ y^h_{q(h)} & \text{if } 2y^h_{q(h)} \leq y^h_{(n-q+1)(h)}. \end{cases} \tag{B.5}$$

Now, I derive the limits of upper and lower bounds in (B.1) and (B.2) as players become perfectly patient. Consider the upper bound $\overline{w}_q$. Notice that $y^h_{(n-q+1)(h)} - y^h_{q(h)} \geq y^\ell_{q(\ell)}$ implies $y^h_{1(h)} - y^h_{q(h)} \geq y^\ell_{q(\ell)}$ and the reverse implication holds after reversing the inequalities. Therefore, the agenda-setter's expected payoff can approximate the full information benchmark $V_q^F = \hat{\mu} y^h_{q(h)} + (1-\hat{\mu}) y^\ell_{q(\ell)}$ if and only if $y^h_{1(h)} - y^h_{q(h)} \geq y^\ell_{q(\ell)}$ which implies $\lim_{\delta \to 1} p_q^* = y^h_{q(h)}$. If $y^h_{1(h)} - y^h_{q(h)} < y^\ell_{q(\ell)}$, the limit of $\overline{w}_q$ is at least $\hat{\mu} \lim_{\delta \to 1} p_q^* + (1-\hat{\mu}) y^\ell_{q(\ell)} \geq y^\ell_{q(\ell)}$, where the inequality holds since $\lim_{\delta \to 1} p_q^* \geq y^\ell$. Moreover, $\hat{\mu} \lim_{\delta \to 1} p_q^* + (1-\hat{\mu}) y^\ell_{q(\ell)}$ equals $y^\ell_{q(\ell)}$ when $y^h_{1(h)} \leq 2y^\ell_{q(\ell)}$ and $\hat{\mu}(y^h_{1(h)} - y^\ell_{q(\ell)}) + (1-\hat{\mu}) y^\ell_{q(\ell)}$ when $y^h_{1(h)} > 2y^\ell_{q(\ell)}$. Finally, it is straightforward to verify that $\hat{\mu}\hat{\alpha}^q y^h_{q(h)} + (1-\hat{\mu}\hat{\alpha}^q) y^\ell_{q(\ell)} = \hat{\mu} y^h_{q(h)}$ and check that $\hat{\mu} y^h_{q(h)} > \hat{\mu} \lim_{\delta \to 1} p_q^* + (1-\hat{\mu}) y^\ell_{q(\ell)}$ if and only if $\hat{\mu} > m_q^* \geq \frac{y^\ell_{q(\ell)}}{y^h_{q(h)}}$ where $m_q^* = \frac{y^\ell_{q(\ell)}}{y^h_{q(h)} - \lim_{\delta \to 1} p_q^* + y^\ell_{q(\ell)}}$. Therefore, we have:

$$\lim_{\delta \to 1} \overline{w}_q = \begin{cases} \hat{\mu} y^h_{q(h)} + (1-\hat{\mu}) y^\ell_{q(\ell)} & \text{if } y^h_{1(h)} - y^\ell_{q(\ell)} \geq y^h_{q(h)}, \\ \hat{\mu} y^h_{q(h)} & \text{if } y^\ell_{q(\ell)} < y^h_{1(h)} - y^\ell_{q(\ell)} < y^h_{q(h)} \text{ and } \hat{\mu} > m_q^*, \\ & \quad \text{or } y^h_{1(h)} - y^\ell_{q(\ell)} \leq y^\ell_{q(\ell)} \text{ and } \hat{\mu} > m_q, \\ \hat{\mu}(y^h_{1(h)} - y^\ell_{q(\ell)}) + (1-\hat{\mu}) y^\ell_{q(\ell)} & \text{if } y^\ell_{q(\ell)} < y^h_{1(h)} - y^\ell_{q(\ell)} < y^h_{q(h)} \text{ and } \hat{\mu} \leq m_q^*, \\ y^\ell_{q(\ell)} & \text{if } y^h_{1(h)} - y^\ell_{q(\ell)} \leq y^\ell_{q(\ell)} \text{ and } \hat{\mu} \leq m_q. \end{cases} \tag{B.6}$$

Next, consider the lower bound $\underline{w}_q$. When the prior belief is low, $\hat{\mu} \leq \frac{y^\ell_{q(\ell)}}{y^h_{q(h)}}$, we have $y^\ell_{q(\ell)} \geq \hat{\mu} y^h_{q(h)}$. In addition, $\hat{\mu} \lim_{\delta \to 1} p_q^\ell + (1-\hat{\mu}) y^\ell_{q(\ell)} = y^\ell_{q(\ell)}$ when $y^h_{q(h)} \leq 2y^\ell_{q(\ell)}$. When $y^h_{q(h)} > 2y^\ell_{q(\ell)}$, we have $\lim_{\delta \to 1} p_q^\ell = y^h_{q(h)} - y^\ell_{q(\ell)}$ and $\hat{\mu} \lim_{\delta \to 1} p_q^\ell + (1-\hat{\mu}) y^\ell_{q(\ell)} > y^\ell_{q(\ell)}$ since $\hat{\mu} < \frac{1}{2}$. When the prior belief is high, $\hat{\mu} > \frac{y^\ell_{q(\ell)}}{y^h_{q(h)}}$, we have $\hat{\mu} y^h_{q(h)} > y^\ell_{q(\ell)}$. It is straightforward to verify that $\hat{\mu}\hat{\alpha}^q y^h_{q(h)} + (1-\hat{\mu}\hat{\alpha}^q) y^\ell_{q(\ell)} = \hat{\mu} y^h_{q(h)}$. Finally, we have

$\hat{\mu}y^h_{q(h)} > \hat{\mu}\lim_{\delta\to 1} p^\ell_q + (1-\hat{\mu})y^\ell_{q(\ell)} = y^\ell_{q(\ell)}$ when $y^h_{q(h)} \le 2y^\ell_{q(\ell)}$ and $\hat{\mu}y^h_{q(h)} > \hat{\mu}\lim_{\delta\to 1} p^\ell_q + (1-\hat{\mu})y^\ell_{q(\ell)} = \hat{\mu}(y^h_{q(h)} - y^\ell_{q(\ell)}) + (1-\hat{\mu})y^\ell_{q(\ell)}$ when $y^h_{q(h)} > 2y^\ell_{q(\ell)}$ if and only if $\hat{\mu} > \frac{1}{2}$. Letting $p^q = \lim_{\delta\to 1}\min\{p^\ell_q, p^h_q\}$ and $m^h_q = \frac{y^\ell_{q(\ell)}}{y^h_{q(h)} - p^q + y^\ell_{q(\ell)}}$, we obtain:

$$\lim_{\delta\to 1}\underline{w}_q = \begin{cases} y^\ell_{q(\ell)} & \text{if } y^h_{q(h)} - y^\ell_{q(\ell)} \le y^\ell_{q(\ell)} \text{ and } \hat{\mu} \le m_q, \\ \hat{\mu}(y^h_{q(h)} - y^\ell_{q(\ell)}) + (1-\hat{\mu})y^\ell_{q(\ell)} & \text{if } y^h_{q(h)} - y^\ell_{q(\ell)} > y^\ell_{q(\ell)} \text{ and } \hat{\mu} \le m_q, \\ \hat{\mu}p^q + (1-\hat{\mu})y^\ell_{q(\ell)} & \text{if } y^h_{1(h)} - y^\ell_{q(\ell)} > y^\ell_{q(\ell)} \text{ and } \frac{y^\ell_{q(\ell)}}{y^h_{q(h)}} < \hat{\mu} \le m^h_q, \\ \hat{\mu}y^h_{q(h)} & \text{if } y^h_{q(h)} - y^\ell_{q(\ell)} \le y^\ell_{q(\ell)} \text{ and } \hat{\mu} > m_q, \\ & \quad\text{or } y^h_{q(h)} - y^\ell_{q(\ell)} > y^\ell_{q(\ell)} \text{ and } \hat{\mu} > m^h_q. \end{cases} \tag{B.7}$$

These limits are used in Figure 2 to plot the limit set of the agenda-setter's expected payoffs and in the proofs of Theorems 6, 7, 8, and 9.

Appendix C: Proof of Theorem 5

Proof of Theorem 5.

Step 1: *The agenda-setter's expected payoff cannot be smaller than $\underline{w}_q$ in any coalition-proof PBE.*

Since the agenda-setter cannot do worse than cause delay without revising the prior belief $\hat{\mu}$, her expected payoff cannot be smaller than $\delta\max\{y^\ell_{q(\ell)}, \hat{\mu}y^h_{q(h)}\}$. Moreover, the agenda-setter's expected payoff cannot be smaller than $y^\ell_{q(\ell)}$, because in every coalition-proof PBE there exists a sequence of initial proposals converging to $y^\ell_{q(\ell)}$ such that the limit of the agenda-setter's expected payoff is at least y^ℓ. To show this, fix a coalition-proof PBE and suppose there exists $\varepsilon > 0$ such that each initial proposal $p_1 \in (y^\ell_{q(\ell)} - \varepsilon, y^\ell_{q(\ell)})$ gets rejected in state $\omega = \ell$ with a positive probability. The revised proposal is either $y^\ell_{q(\ell)}$, which passes, or $y^h_{q(h)}$, which gets rejected. Since for p_1 sufficiently close to $y^\ell_{q(\ell)}$ we have $(y^\ell_{q(\ell)} - p_1)p_1 > \delta(y^\ell_{q(\ell)} - y^\ell_{q(\ell)})y^\ell_{q(\ell)}$ and $p_1 > \delta y^\ell_{q(\ell)}$, a deviation by voters in N^ℓ_q to pass p_1 is improving and self-enforcing. Now, let the state be $\omega = h$ and consider a sequence of proposals converging to $y^\ell_{q(\ell)}$ from below such that each proposal along the sequence passes with certainty in state $\omega = \ell$. If $\delta < m_q$, a similar argument implies that each initial proposal sufficiently far into the sequence passes with certainty in state $\omega = h$. And if $\delta \ge m_q$, the agenda-setter's expected payoff is bounded below by a convex combination of $y^\ell_{q(\ell)}$ and $\delta y^h_{q(h)}$, which is weakly greater than $y^{\ell}_{q(\ell)}$.

To show that the agenda-setter's expected payoff is at least $\hat{\mu}p^{\ell}_q + (1-\hat{\mu})\delta y^{\ell}_{q(\ell)}$ when $\hat{\mu} \leq m_q$, first note that $p^{\ell}_q > y^{\ell}_{q(\ell)}$ since $y^{\ell}_{q(\ell)} \notin L^{\ell}_q$. Therefore, each initial proposal close to p^{ℓ}_q gets rejected with certainty in state $\omega = \ell$. I will show that there exists a sequence of initial proposals converging to p^{ℓ}_q from below such that every proposal in this sequence passes with certainty in state h. Suppose there exists $\varepsilon > 0$ such that each initial proposal $p_1 \in (p^h_q - \varepsilon, p^h_q)$ gets rejected with positive probability in state h. Fix $p_1 \in (p^{\ell}_q - \varepsilon, p^{\ell}_q)$. Since $\hat{\mu} \leq m_q$, the revised proposal is $y^{\ell}_{q(\ell)}$, and $(y^h_{q(h)} - p_1)p_1 > \delta(y^h_{q(h)} - y^{\ell}_{q(\ell)})y^{\ell}_{q(\ell)}$ implies that the joint deviation by voters in N^h_q to pass p_1 is profitable and self-enforcing.

When $\hat{\mu} > m_q$, each policy $p_1 \in (y^h_{q(h)} - \varepsilon, y^h_{q(h)})$, where $\varepsilon > 0$ is sufficiently small, gets rejected with certainty in state ℓ and passes with probability $\hat{\alpha}^q$ in state h. By the definition of $\hat{\alpha}^q$, the agenda-setter's posterior belief is m_q if p_1 gets rejected. Unless voter $q(h)$ is indifferent between accepting and rejecting p_1, there exists a profitable and self-enforcing deviation either to veto p_1 by voters in $N \setminus N^h_q$ or pass it by voters in N^h_q. It follows that the agenda-setter's expected payoff is at least $\hat{\mu}\hat{\alpha}^q y^h_{q(h)} + (1 - \hat{\mu}\hat{\alpha}^q)\delta y^{\ell}_{q(\ell)}$.

Finally, suppose $\hat{\mu} > m_q$ and $y^{\ell}_{q(\ell)} < \min\{p^{\ell}_q, p^h_q\}$. Each policy $p_1 \in (\min\{p^{\ell}_q, p^h_q\} - \varepsilon, \min\{p^{\ell}_q, p^h_q\})$ is rejected with certainty in state ℓ when $\varepsilon > 0$ is sufficiently small. Suppose p_1 is rejected with positive probability in state h. If the revised proposal is mixed, p_1 must pass with probability α^h and make voter $q(h)$ indifferent between accepting and rejecting p_1. This case has been considered above. If the revised proposal is $y^{\ell}_{q(\ell)}$ or $y^h_{q(h)}$, there exists a profitable and self-enforcing deviation to pass p_1 by the definition of p^{ω}_q, $\omega \in \Omega$. In this case, the agenda-setter's expected payoff is at least $\hat{\mu}p^q + \delta(1-\hat{\mu})y^{\ell}_{q(\ell)}$.

Step 2: *The agenda-setter's expected payoff v cannot be greater than $\overline{w}_q$ in any coalition-proof PBE.*

Fix a required quota $q \geq 2$ and an equilibrium $(\sigma, \mathcal{M})$. Since any $n - q + 1$ voters have collective power to veto any policy, then a policy that passes in state ℓ cannot exceed $y^{\ell}_{q(\ell)}$, and a policy that passes in state h cannot exceed $y^h_{q(h)}$.

Suppose that the agenda-setter's expected payoff $V_A(\sigma, \mathcal{M})$ is above $\overline{w}_q$. This implies that there exists a policy $p_1 \in \text{supp } \pi_1$ such that $\eta(p_1) > \overline{w}_q$ where $\eta(p_1) = \sum_{\omega\in\Omega} \hat{\mu}(\omega)[\int\{(1-\delta)x_1 + \delta x_2\}\mathrm{d}F^{p_1,\omega}_{\sigma,\mathcal{M}}(x_1, x_2)]$. There are three cases, each resulting in a contradiction.

Suppose $\text{supp}F^{p_1,\ell}_{\sigma,\mathcal{M}} = \text{supp}F^{p_1,h}_{\sigma,\mathcal{M}}$. Then, $(1-\delta)x_1 + \delta x_2 \leq y^{\ell}_{q(\ell)}$ for all (x_1, x_2) in these supports and $\eta(p_1) \leq y^{\ell}_{q(\ell)}$.

Suppose there exists $(x_1, x_2) \in \text{supp}F^{p_1,h}_{\sigma,\mathcal{M}}$ such that $(x_1, x_2) \notin \text{supp}F^{p_1,\ell}_{\sigma,\mathcal{M}}$. If $p_1 = 0$, $\eta(p_1) \leq \hat{\mu}\delta y^h_{q(h)}$. If $p_1 > 0$, then p_1 gets rejected in state ℓ with certainty. Rejection of p_1 leads to a revised proposal that is either $y^{\ell}_{q(\ell)}$ or $y^h_{q(h)}$. If the revised proposal is $y^{\ell}_{q(\ell)}$, then we must have $p_1 \leq p^*_q$, otherwise there is a joint deviation by all voters in state h to reject p_1 that is improving and self-enforcing. This gives us $(1-\delta)x_1 + \delta x_2 \leq p^*_q$ and therefore $\eta(p_1) \leq \hat{\mu} p^*_q + (1-\hat{\mu})\delta y^{\ell}_{q(\ell)}$. If the revised proposal is $y^h_{q(h)}$, then p_1 cannot be accepted with certainty in state h and the prior belief must be sufficiently high, $\hat{\mu} > m^q$. The highest initial proposal that can win in state h is $y^h_{q(h)}$ and this initial proposal must win with probability $\hat{\alpha}^q$ which makes the agenda-setter indifferent between revised proposals $y^h_{q(h)}$ and $y^{\ell}_{q(\ell)}$. This gives us $\eta(p_1) \leq \hat{\mu}\hat{\alpha}^q y^h_{q(h)} + (1-\hat{\mu}\hat{\alpha}^q)\delta y^{\ell}_{q(\ell)}$ when $\hat{\mu} > m^q$.

Suppose there exists $(x_1, x_2) \in \text{supp}F^{p_1,\ell}_{\sigma,\mathcal{M}}$ such that $(x_1, x_2) \notin \text{supp}F^{p_1,h}_{\sigma,\mathcal{M}}$. If $p_1 = 0$, then $\eta(p_1) \leq (1-\hat{\mu})\delta y^{\ell}_{q(\ell)}$. If $p_1 > 0$, then $p_1 \leq y^{\ell}_{q(\ell)}$ and is rejected in state h with certainty. Rejection of p_1 leads to a revised proposal that is either $y^{\ell}_{q(\ell)}$ or $y^h_{q(h)}$. If the revised proposal is $y^h_{q(h)}$, we must have $p^h_q \leq p_1$, otherwise there exists an improving and self-enforcing deviation by q voters to pass p_1 in state h. This inequality implies $p^q_q \leq y^{\ell}_{q(\ell)}$, which, in turn, implies $y^{\ell}_{q(\ell)} \leq y^{\ell}_{(n-q+1)(\ell)}$. It follows that $(1-\delta)x_1 + \delta x_2 \leq y^{\ell}_{q(\ell)}$ and $(1-\delta)\tilde{x}_1 + \delta\tilde{x}_2 \leq \delta y^h_{q(h)}$ for all $(\tilde{x}_1, \tilde{x}_2) \in \mathcal{F}^{p_1,h}_{\sigma,\mathcal{M}}$ and therefore $\eta(p_1) \leq \hat{\mu}\delta y^h_{q(h)} + (1-\hat{\mu})y^{\ell}_{q(\ell)}$ when $p^h_q \leq y^{\ell}_{q(\ell)} \leq y^{\ell}_{(n-q+1)(\ell)}$.

Step 3: *For each $v \in [\underline{w}_q, \overline{w}_q]$, there exists a coalition-proof PBE such that the agenda-setter's expected payoff equals v.*

For each $\omega \in \Omega$, define $\underline{z}^\omega = \min\{p_1 \in \mathbb{R}_+ \mid (y^h_{(n-q+1)(h)} - p_1)p_1 \geq \delta(y^h_{(n-q+1)(h)} - y^\omega_{q(\omega)})y^\omega_{q(\omega)}\}$. If $y^h_{(n-q+1)(h)} \leq y^\omega_{q(\omega)}$, then $\underline{z}^\omega = 0$; and if $y^h_{(n-q+1)(h)} > y^\omega_{q(\omega)}$, then the inequality is strict for $p_1 = \delta y^\omega_{q(\omega)}$. Therefore, $\underline{z}^\omega < \delta y^\omega_{q(\omega)}$. This definition implies that for all $p_1 \in [\underline{z}^\omega, y^\omega_{q(\omega)}]$ we have $(y - p_1)p_1 \geq \delta(y - y^\omega_{q(\omega)})y^\omega_{q(\omega)}$ whenever $y \leq y^h_{(n-q+1)(h)}$.

Fix $v \in [\underline{w}_q, \overline{w}_q]$ and let $z = \min\{p^*_q, \frac{v - \delta(1-\hat{\mu})y^{\ell}_{q(\ell)}}{\hat{\mu}}\}$ and $\beta = \min\{1, \frac{v - \delta\hat{\mu}y^h_{q(h)}}{(1-\hat{\mu})y^{\ell}_{q(\ell)}}\}$.

Define $\sigma_1 = (\pi_1, (\alpha_{i,1})_{i\in N})$ as follows. For all $p_1 \in \mathbb{R}_+$ let:

$$\alpha_{i,1}(p_1;\ell) = \begin{cases} 1 & \begin{aligned} &\text{if } i \in N_q^\ell,\ p_1 \in [\underline{z}^\ell, y^\ell_{q(\ell)}),\ \text{and } \hat{\mu} \le m_q,\\ &\quad \text{or } i \in N_q^\ell,\ p_1 \in [\underline{z}^h, y^\ell_{q(\ell)}),\ \text{and } \hat{\mu} > m_q,\\ &\quad \text{or } i \in N_q^\ell,\ p_1 = y^\ell_{q(\ell)} \text{ and } \mathbf{1}(p_q^h \le y^\ell_{q(\ell)} \le y^\ell_{(n-q+1)(\ell)}) = 0,\\ &\quad \text{or } i \in N_q^\ell \setminus \{q(\ell)\},\ p_1 = y^\ell_{q(\ell)} \text{ and } \mathbf{1}(p_q^h \le y^\ell_{q(\ell)} \le y^\ell_{(n-q+1)(\ell)}) = 1,\\ &\quad \text{or } i \in N_q^h \setminus \{q(h)\},\ p_1 \in (z, y^h_{q(h)}] \text{ and } \hat{\mu} > m_q, \end{aligned}\\ \beta & \text{if } i = q(\ell),\ p_1 = y^\ell_{q(\ell)} \text{ and } \mathbf{1}(p_q^h \le y^\ell_{q(\ell)} \le y^\ell_{(n-q+1)(\ell)}) = 1,\\ 0 & \text{otherwise,} \end{cases}$$

$$\alpha_{i,1}(p_1;h) = \begin{cases} 1 & \begin{aligned} &\text{if } i \in N \setminus N^h_{n-q+1},\ p_1 \in [\underline{z}^\ell, y^\ell_{q(\ell)}),\ \text{and } \hat{\mu} \le m_q,\\ &\quad \text{or } i \in N \setminus N^h_{n-q+1},\ p_1 \in [\underline{z}^h, y^\ell_{q(\ell)}),\ \text{and } \hat{\mu} > m_q,\\ &\quad \text{or } i \in N_q^h \text{ and } p_1 \in (y^\ell_{q(\ell)}, z],\\ &\quad \text{or } i \in N_q^h,\ p_1 = y^\ell_{q(\ell)} \text{ and } \mathbf{1}(p_q^h \le y^\ell_{q(\ell)} \le y^\ell_{(n-q+1)(\ell)}) = 0,\\ &\quad \text{or } i \in N_q^\ell \setminus \{q(\ell)\},\ p_1 = y^\ell_{q(\ell)} \text{ and } \mathbf{1}(p_q^h \le y^\ell_{q(\ell)} \le y^\ell_{(n-q+1)(\ell)}) = 1,\\ &\quad \text{or } i \in N_q^h \setminus \{q(h)\},\ p_1 \in (z, y^h_{q(h)}] \text{ and } \hat{\mu} > m_q, \end{aligned}\\ \hat{\alpha}^q & \text{if } i = q(h),\ p_1 \in (z, y^h_{q(h)}] \text{ and } \hat{\mu} > m_q,\\ 0 & \text{otherwise,} \end{cases}$$

$$\begin{aligned} \operatorname{supp} \pi_1 &= \arg\max_{p_1 \in [z, z+1]} G(p_1),\ \text{where } G : \{y^\ell_{q(\ell)}, z, y^h_{q(h)}, y^h_{q(h)} + 1\} \to \mathbb{R}_+ \text{ is defined as}\\ G(y^\ell_{q(\ell)}) &= \mathbf{1}(p_q^h \le y^\ell_{q(\ell)} \le y^\ell_{(n-q+1)(\ell)})(\delta\hat{\mu} y^h_{q(h)} + (1-\hat{\mu})\beta y^\ell_{q(\ell)})\\ &\quad + (1 - \mathbf{1}(p_q^h \le y^\ell_{q(\ell)} \le y^\ell_{(n-q+1)(\ell)}))y^\ell_{q(\ell)},\\ G(z) &= \hat{\mu} z + \delta(1-\hat{\mu}) y^\ell_{q(\ell)},\\ G(y^h_{q(h)}) &= \mathbf{1}(\hat{\mu} > m_q)\hat{\mu}\hat{\alpha}^q y^h_{q(h)} + \delta(1 - \hat{\mu}\hat{\alpha}^q) y^\ell_{q(\ell)},\\ G(y^h_{q(h)} + 1) &= \delta \max\{\hat{\mu} y^h_{q(h)}, y^\ell_{q(\ell)}\}. \end{aligned}$$

Define $\mathcal{M} = \{\mu^\hbar\}_{\hbar \in H}$ as follows. Let $\mu^\hbar$ be such that:

$$\mu^\hbar = \begin{cases} 0 & \text{if } C \in \mathcal{C}_0,\\ 1 & \text{if } C \in \mathcal{C}_1,\\ \hat{\mu} & \text{otherwise,} \end{cases}$$

where collections $\mathcal{C}_0$ and $\mathcal{C}_1$ of coalitions depend on the initial proposal p_1 and the prior belief $\hat{\mu}$. If both $p_1 \in [\underline{z}^\ell, y^\ell_{q(\ell)})$ and $\hat{\mu} \le m_q$, or both $p_1 \in [\underline{z}^h, y^\ell_{q(\ell)})$ and $\hat{\mu} > m_q$, or both $p_1 = y^\ell_{q(\ell)}$ and $\mathbf{1}(p^h_q \le y^\ell_{q(\ell)} \le y^\ell_{(n-q+1)(\ell)}) = 0$, then:

$$\mathcal{C}_0 = \{C \subseteq N \mid y^h_j - y^h_{q(h)} > y^\ell_{q(\ell)} \text{ for } j = \min\{(N \setminus N^h_{n-q+1}) \cap C\}\},$$
$$\mathcal{C}_1 = \{C \subseteq N \mid y^h_j - y^h_{q(h)} \le y^\ell_{q(\ell)} \text{ for } j = \min\{(N \setminus N^h_{n-q+1}) \cap C\}\}.$$

If $p_1 = y^\ell_{q(\ell)}$ and $\mathbf{1}(p^h_q \le y^\ell_{q(\ell)} \le y^\ell_{(n-q+1)(\ell)}) = 1$, then:

$$\mathcal{C}_0 = \{C \subseteq N \mid y^h_j - y^h_{q(h)} > y^\ell_{q(\ell)} \text{ for } j = \min\{(N^\ell_q \setminus \{q(\ell)\}) \cap C\}\},$$
$$\mathcal{C}_0 = \{C \subseteq N \mid y^h_j - y^h_{q(h)} \le y^\ell_{q(\ell)} \text{ for } j = \min\{(N^\ell_q \setminus \{q(\ell)\}) \cap C\}\}.$$

If $p_1 \in (y^\ell_{q(\ell)}, z]$, then:

$$\mathcal{C}_0 = \{C \subseteq N \mid C = N \text{ or } y^h_j - y^h_{q(h)} > y^\ell_{q(\ell)} \text{ for } j = \min\{N^h_q \cap C\}\},$$
$$\mathcal{C}_0 = \{C \subseteq N \mid y^h_j - y^h_{q(h)} \le y^\ell_{q(\ell)} \text{ for } j = \min\{N^h_q \cap C\}\}.$$

If both $p_1 = (z, y^h_{q(h)}]$ and $\hat{\mu} \le m_q$, then $\mathcal{C}_0 = \mathcal{C}_1 = \varnothing$, and if both $p_1 = (z, y^h_{q(h)}]$ and $\hat{\mu} > m_q$, then:

$$\mathcal{C}_0 = \{C \subseteq N \mid y^h_j - y^h_{q(h)} > y^\ell_{q(\ell)} \text{ for } j = \min\{(N^h_q \setminus \{q(h)\}) \cap C\}\},$$
$$\mathcal{C}_0 = \{C \subseteq N \mid y^h_j - y^h_{q(h)} \le y^\ell_{q(\ell)} \text{ for } j = \min\{(N^h_q \setminus \{q(h)\}) \cap C\}\}.$$

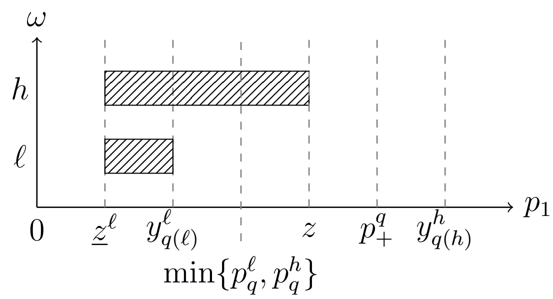
The conditional probability that policy $p_1 \in \mathbb{R}_+$ passes if proposed in the first period is depicted in Figure 3. I will verify that $(\sigma^I, \mathcal{M})$ is a coalition-proof PBE and induces the agenda-setter's expected payoff v.

Step 3a. In this step, I prove that the agenda-setter's proposal strategy π_1 is sequentially rational. Notice that there are only a few candidate policies for the agenda-setter's initial proposal. The agenda-setter can: propose $y^\ell_{q(\ell)}$ and receive an expected payoff $y^\ell_{q(\ell)}$ (when $p^h_q \le y^h_{q(h)} \le y^h_{(n-q+1)(h)}$ does not hold) or $\delta\hat{\mu}y^h_{q(h)} + (1-\hat{\mu})\beta y^\ell_{q(\ell)}$ (when $p^h_q \le y^h_{q(h)} \le y^h_{(n-q+1)(h)}$ holds), propose z and receive $\hat{\mu}z + \delta(1-\hat{\mu})y^\ell_{q(\ell)}$, propose $y^h_{q(h)}$ and receive $\hat{\mu}\hat{\alpha}^q y^h_{q(h)} + \delta(1-\hat{\mu}\hat{\alpha}^q)y^\ell_{q(\ell)}$ (when $\hat{\mu} > m_q$), or propose any policy not in $[\underline{z}^\ell, y^h_{q(h)}]$, say $y^h_{q(h)} + 1$, and receive $\delta \max\{y^\ell_{q(\ell)}, \hat{\mu}y^h_{q(h)}\}$.

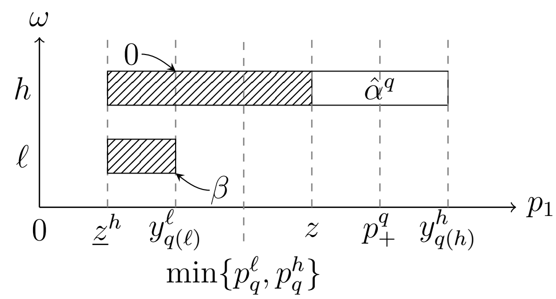
Since $v \ge \underline{w}_q$, we have $v \ge \hat{\mu}p^\ell_q + \delta(1-\hat{\mu})y^\ell_{q(\ell)}$. If also $v \le \hat{\mu}p^*_q + (1-\hat{\mu})\delta y^\ell_{q(\ell)}$, then $v = \hat{\mu}z + \delta(1-\hat{\mu})y^\ell_{q(\ell)}$ by the definition of z and making an initial proposal z is optimal for the agenda-setter. Proposing $y^\ell_{q(\ell)}$ is not optimal when $y^h_{1(h)} - y^h_{q(h)} < B^1_q$

since $v \geq \underline{w}_q \geq y^\ell_{q(\ell)}$. When $y^h_{1(h)} - y^h_{q(h)} \geq B^1_q$, if proposing $y^\ell_{q(\ell)}$ is optimal, then the definition of β implies that the agenda-setter is indifferent between proposing p^*_q and $y^\ell_{q(\ell)}$. Finally, proposing $y^h_{q(h)} + 1$ is not optimal since $v \geq \underline{w}_q \geq \delta \max\{y^\ell_{q(\ell)}, \hat{\mu} y^h_{q(h)}\}$.

Consider the complementary case $v > \hat{\mu} p^*_q + (1-\hat{\mu})\delta y^\ell_{q(\ell)}$, which implies $z = p^*_q$. If $v = y^\ell_{q(\ell)}$ then either $\delta y^h_{q(h)} \geq y^\ell_{q(\ell)}$ which implies that proposing $y^\ell_{q(\ell)}$ is optimal and $v = \delta \hat{\mu} y^h_{q(h)} + (1-\hat{\mu})\beta y^\ell_{q(\ell)} = y^\ell_{q(\ell)}$, or $\delta y^h_{q(h)} < y^\ell_{q(\ell)}$ which implies that $y^h_{1(h)} - y^h_{q(h)} < B^1_q$, proposing $y^\ell_{q(\ell)}$ is optimal, and $v = y^\ell_{q(\ell)}$. If $v = \delta \hat{\mu} y^h_{q(h)}$, then $\beta = 0$ and making an initial proposal $y^h_{q(h)} + 1$ is optimal. If $v = \hat{\mu}\hat{\alpha}^q y^h_{q(h)} + \delta(1-\hat{\mu}\hat{\alpha}^q) y^\ell_{q(\ell)}$, then making an initial proposal $y^h_{q(h)}$ is optimal because $\delta\hat{\mu} y^h + q(h) + (1-\hat{\mu})\beta y^\ell_{q(\ell)} > v$ implies $\frac{v - \delta\hat{\mu} y^h_{q(h)}}{(1-\hat{\mu}) y^\ell_{q(\ell)}} < \beta$ which is a contradiction. Finally, if $v > \max\{y^\ell_{q(\ell)}, \delta\hat{\mu} y^h_{q(h)}, \hat{\mu}\hat{\alpha}^q y^h_{q(h)} + \delta(1-\hat{\mu}\hat{\alpha}^q) y^\ell_{q(\ell)}, \hat{\mu} p^+ + \delta(1-\hat{\mu}) y^\ell_{q(\ell)}\}$, making an initial proposal $y^\ell_{q(\ell)}$ is optimal and gives the agendas-setter an expected payoff $v = \delta\hat{\mu} y^h_{q(h)} + (1-\hat{\mu})\beta y^\ell_{q(\ell)}$.



(a) $\hat{\mu} \leq m_q$, $\mathbf{1}(p^h_q \leq y^\ell_{q(\ell)} \leq y^\ell_{(n-q+1)(\ell)}) = 0$



(b) $\hat{\mu} > m_q$, $\mathbf{1}(p^h_q \leq y^\ell_{q(\ell)} \leq y^\ell_{(n-q+1)(\ell)}) = 1$

Figure 3: Probability that $p_1 \in \mathbb{R}_+$ passes, conditional on state $\omega \in \Omega$, if proposed given assessment $(\sigma, \mathcal{M})$. This probability equals 1 in shaded regions and 0 unless specified otherwise.

Step 3b. In this step, I prove that the voting strategy profile $\sigma_1 = (\sigma_{1,i})_{i\in N}$ is sequentially rational and there does not exist an improving and self-enforcing joint deviation.

Assume $\omega = \ell$. Suppose $p_1 \in [\underline{z}, y^\ell_{q(\ell)}]$. On path, voters in N^ℓ_q accept p_1 and voters in $N \setminus N^\ell_q$ reject it. Consider a deviation by some coalition $\tilde{C} \subseteq N$, where $\tilde{C}$ can be a singleton. If the initial proposal p_1 still passes, the deviation is not improving. Suppose that p_1 gets rejected, which implies that $N^\ell_q \cap \tilde{C}$ is non-empty. Let $j = \min\{N^\ell_q \cap \tilde{C}\}$. If $y^\ell_j - y^\ell_{q(\ell)} \leq y^h_{q(h)}$, the revised proposal is $y^h_{q(h)}$ which gets rejected. Since $p_1 \leq y^\ell_{q(\ell)}$, the deviation does not increase voter j's expected payoff. If $y^\ell_j - y^\ell_{q(\ell)} > y^h_{q(h)}$, the revised proposal is $y^\ell_{q(\ell)}$ which passes. Since $y^\ell_j \leq y^\ell_{1(\ell)}$, the definition of $\underline{z}$ implies that $(y_j^\ell - p_1)p_1 \geq \delta(y_j^\ell - y_{q(\ell)}^\ell)y_{q(\ell)}^\ell$ for all $p_1 \in [\underline{z}, y_{q(\ell)}^\ell]$, and therefore the deviation does not increase voter j's expected payoff.

Continue assuming $\omega = \ell$ and suppose $p_1 \in (y^\ell, z]$. On path, voters in N reject p_1 and the revised proposal is $y_{q(\ell)}^\ell$. Consider a deviation by some coalition $\tilde{C} \subseteq N$. The deviation is not improving if p_1 passes because $n - q$ players in $\{q(\ell)\} \cup (N \setminus N_q^\ell)$ prefer $y_{q(\ell)}^\ell$ after delay to p_1 without delay. Suppose p_1 gets rejected, which implies that $N_q^\ell \cap C$ is non-empty. The deviation is not improving if the revised proposal is $y_{q(\ell)}^\ell$, so suppose the revised proposal is $y_{q(h)}^h$ which gets rejected. The deviation is not improving since every voter in N_q^ℓ prefers $y_{q(\ell)}^\ell$ to the status-quo policy 0.

Continue assuming $\omega = \ell$ and suppose $p_1 \notin [\underline{z}, z]$. On path, voters in N reject p_1 and the revised proposal is $y_{q(\ell)}^\ell$ since $\hat{\mu} \leq \frac{y_{q(\ell)}^\ell}{y_{q(h)}^h}$. The analysis above continues to hold.

Assume $\omega = h$ and suppose $p_1 \in [\underline{z}, y_{q(\ell)}^\ell]$. On path, voters in N_q^h accept p_1 and voters in $N \setminus N_q^h$ reject it. Consider a deviation by some coalition $\tilde{C} \subseteq N$. If the initial proposal p_1 still passes, the deviation is not improving. Suppose that p_1 gets rejected, which implies that $N_q^h \cap \tilde{C}$ is non-empty. Let $j = \min\{N_q^h \cap \tilde{C}\}$. If $y_j^h - y_{q(h)}^h \leq y_{q(h)}^h$, the revised proposal is $y_{q(h)}^h$ which passes. The definition of $\underline{z}$ implies that $(y_j^h - p_1)p_1 > \delta(y_j^h - y_{q(\ell)}^\ell)y_{q(\ell)}^\ell$. In turn, $y_j^h - y_{q(h)}^h \leq y_{q(h)}^h$ implies that $(y_j^h - y_{q(\ell)}^\ell)y_{q(\ell)}^\ell \geq (y_j^h - y_{q(h)}^h)y_{q(h)}^h$. Therefore, the deviation does not increase voter j's expected payoff.

Continue assuming $\omega = h$ and suppose $p_1 \in (y^\ell, z]$. On path, voters in N_q^h accept p_1 and voters in $N \setminus N_q^h$ reject it. Consider a deviation by some coalition $\tilde{C} \subseteq N$. If the initial proposal p_1 still passes, the deviation is not improving. Suppose that p_1 gets rejected, which implies that $N_q^h \cap \tilde{C}$ is non-empty. Let $j = \min\{N_q^h \cap \tilde{C}\}$. Since $\hat{\mu} \leq \frac{y_{q(\ell)}^\ell}{y_{q(h)}^h}$, the revised proposal is $y_{q(\ell)}^\ell$ which passes. If the deviation increases voter j's expected payoff, then $(y_j^h - p_1)p_1 < \delta(y_j^h - y_{q(\ell)}^\ell)y_{q(\ell)}^\ell$. But this inequality implies $(y_{1(h)}^h - p_1)p_1 < \delta(y_{1(h)}^h - y_{q(\ell)}^\ell)y_{q(\ell)}^\ell$ and therefore $z > p_q^*$, which is a contradiction. It follows that the deviation does not increase voter j's expected payoff.

Continue assuming $\omega = h$ and suppose $p_1 \notin [\underline{z}, z]$. On path, voters in N reject p_1 and the revised proposal is $y_{q(\ell)}^\ell$ since $\hat{\mu} \leq \frac{y_{q(\ell)}^\ell}{y_{q(h)}^h}$. Consider a deviation by some coalition $\tilde{C} \subseteq N$. If the initial proposal p_1 still gets rejected, the deviation is not improving. Suppose that p_1 passes. If the deviation is improving and $p_1 < \underline{z}$, we must have $(y_j^h - p_1)p_1 > \delta(y_j^h - y_{q(\ell)}^\ell)y_{q(\ell)}^\ell$ for at least q voters. And since $\underline{z} < \delta y_{q(\ell)}^\ell$, in particular we must have $(y_{(n-q+1)(h)}^h - p_1)p_1 > \delta(y_{(n-q+1)(h)}^h - y_{q(\ell)}^\ell)y_{q(\ell)}^\ell$. But this inequality contradicts the definition of $\underline{z}$. If $p_1 > z$, the definition of p_q^ℓ and $z \geq p_q^\ell$ imply $(y^h_{q(h)} - p_1) < \delta(y^h_{q(h)} - y^\ell_{q(\ell)})y^\ell_{q(\ell)}$ and thus also $(y^h_{j(h)} - p_1) < \delta(y^h_{j(h)} - y^\ell_{q(\ell)})y^\ell_{q(\ell)}$ for all $j \in q+1, \ldots, n$. Therefore, the deviation cannot be improving for any coalition $\tilde{C}$ of at least q voters. ■

Appendix D: Remaining proofs

Proof of Lemma 1. Under the q-majority voting rule with $q = 1$, there is no signaling incentive and (4) simplifies to

$$\alpha^\omega_{i,1}(p_1) = \begin{cases} 1 & \text{if } \; (y^\omega_i - p_1)p_1 > \delta(V^q_{i,2}(\omega; \mu^{p_1,\varnothing}) + \frac{1}{4}(y^\omega_i)^2), \\ 0 & \text{if } \; (y^\omega_i - p_1)p_1 < \delta(V^q_{i,2}(\omega; \mu^{p_1,\varnothing}) + \frac{1}{4}(y^\omega_i)^2). \end{cases} \tag{D.1}$$

Let us begin by making (D.1) more amenable to analysis. First, consider term $V^q_{i,2}(\omega; \mu^{p_1,\varnothing})$ defined in (1). For each revised policy p_2, we can write

$$\begin{aligned} &\max_{a_{i,2} \in \{0,1\}} U^q_{i,2}(p_2; \omega, (a_{i,2}, \alpha_{-i,2})) = \\ &\quad = -\frac{1}{4}(y^\omega_i)^2 + \max\left\{(y^\omega_i - p_2)p_2, \left(1 - \prod_{j \neq i}(1 - \alpha^\omega_{j,2})(p_2)\right)(y^\omega_i - p_2)p_2\right\}. \end{aligned} \tag{D.2}$$

For each proposal p_1, define

$$Y_i(p_1; \omega) = \int_{\mathbb{R}_+} \max\left\{(y^\omega_i - p_2)p_2, \left(1 - \prod_{j \neq i}(1 - \alpha^\omega_{j,2})(p_2)\right)(y^\omega_i - p_2)p_2\right\} \mathrm{d}\pi_2(\mu^{p_1,\varnothing})(p_2). \tag{D.3}$$

Then, using (D.1), (D.2), and (D.3), voter i in state ω votes to accept p_1 if

$$(y^\omega_i - p_1)p_1 > \delta Y_i(p_1; \omega) \tag{D.4}$$

and votes to reject p_1 if this inequality is reversed.

I prove each line in (8) separately. The first line follows from Steps 1 and 2 below. The second through fifth lines are verified using Step 3 in combination with Steps 4 through 7.

Step 1: Each proposal $0 < p_1 < y^\ell_{1(\ell)}$ *passes with certainty in state* ℓ. If $\alpha_{i,1}(p_1; \ell) = 1$ for some $i \neq 1(\ell)$, then there is nothing to prove, so suppose that $\alpha_{i,1}(p_1; \ell) < 1$ for all $i \neq 1(\ell)$. Then, voter $1(\ell)$ in state ℓ must accept policy p_1 because $(y^\ell_{1(\ell)} - p_1)p_1 > 0 \geq \delta Y_{1(\ell)}(p_1; \ell)$, there the second inequality follows from part (ii) of Lemma A.3.

Step 2: *Each proposal* $0 < p_1 < y^\ell_{1(\ell)}$ *passes with certainty in state* h. Suppose $\alpha_{i,1}(p_1; h) < 1$ for all $i \neq 1(h)$, because otherwise there is nothing to prove. If also $\alpha_{1(h),1}(p_1; h) < 1$, then Step 1 implies that when p_1 gets rejected, the posterior belief μ^{p_1} assigns probability 1 to state h. This implies $Y_{1(h)}(p_1; h) = 0$ and therefore $(y^h_{1(h)} - p_1)p_1 > 0 = \delta Y_{1(h)}(p_1; h)$, which contradicts $\alpha_{1(h),1}(p_1; h) < 1$.

Step 3: *Each proposal* $p_1 > y^\ell_{1(\ell)}$ *gets rejected with certainty in state* ℓ. In state ℓ, each voter $i \in N$ strictly prefers to reject policy p_1 because $(y^\ell_i - p_1)p_1 < 0 \leq \delta Y_i(p_1; \ell)$, where the second inequality follows from Lemma A.1 and part (ii) of Lemma A.3.

Step 4: Each proposal $y^\ell_{1(\ell)} < p_1 < z^1$ *passes with certainty in state* h. Suppose $\alpha_{i,1}(p_1; h) < 1$ for all $i \neq 1(h)$. By (D.3) and part (ii) of Lemma A.3, we have $Y_{1(h)}(p_1; h) \leq (y^h_{1(h)} - y^\ell_{1(\ell)})y^\ell_{1(\ell)}$. Then, the definition of z^1 implies $(y^h_{1(h)} - z^1)z^1 > \delta(y^h_{1(h)} - y^\ell_{1(\ell)})y^\ell_{1(\ell)} \geq \delta Y_{1(h)}(p_1; h)$ and thus $\alpha_{1(h),1}(p_1; h) = 1$.

Step 5: If $\hat{\mu} > m^1$*, each proposal* $z^1 < p_1 < y^h_{1(h)}$ *passes with probability* $\hat{\alpha}^1$ *in state* h. First, note that we can let $\alpha_{i,1}(p_1; h) = 0$ for all $i \neq 1(h)$ without loss of generality since we are only interested in the probability that p_1 passes.

Next, I show that $\alpha_{1(h),1}(p_1; h) \in (0, 1)$. If $\alpha_{1(h),1}(p_1; h) = 1$, then when p_1 gets rejected, the posterior belief μ^{p_1} assigns probability 1 to state ℓ and the agenda-setter makes proposal $y^\ell_{1(\ell)}$ in the second period, implying $Y_{1(h)}(p_1; h) = (y^h_{1(h)} - y^\ell_{1(\ell)})y^\ell_{1(\ell)}$. But the definition (7) of z^1 implies that $(y^h_{1(h)} - p_1)p_1 < \delta Y_{1(h)}(p_1; h)$, which contradicts $\alpha_{1(h),1}(p_1; h) = 1$. If $\alpha_{1(h),1}(p_1; h) = 0$, then the posterior μ^{p_1} equals the prior $\hat{\mu}$ and the agenda-setter makes proposal $y^h_{1(h)}$ in the second period, implying $Y_{1(h)}(p_1; h) = 0$. But $(y^h_{1(h)} - p_1)p_1 > 0 = \delta Y_{1(h)}(p_1; h)$, which contradicts $\alpha_{1(h),1}(p_1; h) = 0$.

It follows that voter $1(h)$ must be indifferent, i.e., $(y^h_{1(h)} - p_1)p_1 = \delta Y_{1(h)}(p_1; h)$. Let $\gamma(p_1)$ be the probability that the agenda-setter makes proposal $y^\ell_{1(\ell)}$ in the second period. Then, we can write $Y_{1(\ell)}(p_1, h) = \gamma(p_1)(y^h_{1(h)} - y^\ell_{1(\ell)})y^\ell_{1(\ell)}$ and $(y^h_{1(h)} - p_1)p_1 = \delta\gamma(p_1)(y^h_{1(h)} - y^\ell_{1(\ell)})y^\ell_{1(\ell)}$. Since $p_1 \in (z^1, y^h_{1(h)})$, it must be the case that $\gamma(p_1) \in (0, 1)$, implying that the posterior belief $\mu^{\hbar}$ must make the agenda-setter indifferent between proposing $y^\ell_{1(\ell)}$ and $y^h_{1(h)}$ in the second period, i.e., $\mu^{\hbar} = m^1$. Therefore, we must have $\alpha_{1(h),1}(p_1; h) = \hat{\alpha}^1$.

Step 6: If $\hat{\mu} \leq m^1$*, each proposal* $z^1 < p_1 < y^h_{1(h)}$ *passes with certainty in state* h. Since $z^1 \geq y^\ell_{1(\ell)}$, Step 3 implies that p_1 gets rejected with certainty in state ℓ. Consider state h. Since $\hat{\mu} \leq m^1$, $\alpha_{i,1}(p_1; h) > 0$ for any $i \in N$ implies that the agenda-setter proposes $y^\ell_{1(\ell)}$ when p_1 gets rejected. Therefore, $Y_{1(h)}(p_1;h) = (y^h_{1(h)} - y^\ell_{1(\ell)})y^\ell_{1(\ell)} > 0$ since $y^h_{1(h)} > y^\ell_{1(\ell)}$, and $Y_i(p_1;h) = (y^h_i - y^\ell_{1(\ell)}))y^\ell_{1(\ell)}$, since $\alpha_{1(h),2}(y^\ell_{1(\ell)};h) = 1$ by Lemma A.1. By the definition of z^1 in (7), all voters strictly prefer to reject p_1, in particular, $(y^h_i - p_1)p_1 < \delta Y_i(p_1;h)$, which contradicts $\alpha_{i,1}(p_1;h) > 0$.

Step 7: Each proposal $p_1 > y^h_{1(h)}$ gets rejected with certainty in state h. Trivial. ■

Proof of Theorem 1. The proof relies on the expressions in (8) and (9). Denote $R^1 = \{p_1 \mid p_1 < y^\ell_{1(\ell)}\}$, $R^2 = \{p_1 \mid y^\ell_{1(\ell)} < p_1 < z^1\}$, $R^3 = \{p_1 \mid z^1 < p_1 < y^h_{1(h)}\}$, and $R^4 = \{p_1 \mid y^h_{1(h)} < p_1\}$. Using (9), we have

$$\sup_{p_1 \in R} U_{A,1}(p_1) = \begin{cases} y^\ell_{1(\ell)} & \text{if } R = R^1, \\ \hat{\mu} z^1 + (1-\hat{\mu})\delta y^\ell_{1(\ell)} & \text{if } R = R^2, \\ \mathbf{1}(\hat{\mu} \le m^1)\delta y^\ell_{1(\ell)} & \\ \quad + \mathbf{1}(\hat{\mu} > m^1)(\hat{\mu}\hat{\alpha}^1 y^h_{1(h)} + (1-\hat{\mu}\hat{\alpha}^1)\delta y^\ell_{1(\ell)}) & \text{if } R = R^3, \\ \delta \max\{y^\ell_{1(\ell)},\ \hat{\mu} y^h_{1(b)}\} & \text{if } R = R^4. \end{cases}$$

(i) Let $\hat{\mu} < m^\ell$ where m^ℓ is defined in (10). Then, $\sup_{R^1} U_{A,1} = y^\ell_{1(\ell)} > \sup_{R^2} U_{A,1}$. Since $m^\ell < m^1$, then $\hat{\mu} < m^1$ and thus $y^\ell_{1(\ell)} > \sup_{R^3} U_{A,1} = \sup_{R^4} U_{A,1} = \delta y^\ell_{1(\ell)}$. These inequalities imply that for any $p_1 \in \mathbb{R}_+$, there exists $\tilde{p}_1 < y^\ell_{1(\ell)}$ such that $U_{A,1}(\tilde{p}_1) > U_{A,1}(p_1)$. For an equilibrium to exist, we must have $U_{A,1}(y^\ell_{1(\ell)}) = \sup_{p_1 \in R^1} U_{A,1}(p_1)$ and therefore $W^1(y^\ell_{1(\ell)}) = 1$.

(ii) Let $m^\ell < \hat{\mu} \le m^1$. Then, $\sup_{R^2} U_{A,1} = \hat{\mu} z^1 + (1-\hat{\mu})\delta y^\ell_{1(\ell)} > \sup_{R^1} U_{A,1} = y^\ell_{1(\ell)} > \sup_{R^3} U_{A,1} = \sup_{R^4} U_{A,1} = \delta y^\ell_{1(\ell)} = \hat{\mu} y^h_{1(h)}$. Just like in part (i), we must have $U_{A,1}(z^1) = \sup_{p_1 \in R^2} U_{A,1}(p_1)$ and therefore $W^1(z^1) = \hat{\mu}$.

Now, let $m^1 < \hat{\mu} < m^h$ where m^h is defined in (11). Then, $\sup_{R^2} U_{A,1} = \hat{\mu} z^1 + (1-\hat{\mu})\delta y^\ell_{1(\ell)} > \sup_{R^3} U_{A,1} = \hat{\mu}\hat{\alpha}^1 y^h_{1(h)} + (1-\hat{\mu}\hat{\alpha}^1)\delta y^\ell_{1(\ell)}$. Since $m^1 < \hat{\mu}$ and $m^\ell < m^R$, also $\sup_{R^2} U_{A,1} = \hat{\mu} z^1 + (1-\hat{\mu})\delta y^\ell_{1(\ell)} > \sup_{R^1} U_{A,1} = y^\ell_{1(\ell)}$. And since $m^1 < \hat{\mu}$, also $\sup_{R^2} U_{A,1} = \hat{\mu} z^1 + (1-\hat{\mu})\delta y^\ell_{1(\ell)} > \sup_{R^4} U_{A,1} = \delta\hat{\mu} y^h_{1(h)}$. Again, we must have $U_{A,1}(z^1) = \sup_{p_1 \in R^2} U_{A,1}(p_1)$ and therefore $W^1(z^1) = \hat{\mu}$.

(iii) Let $m^h < \hat{\mu}$. Since $m^1 < m^h$, we have $\sup_{R^3} U_{A,1} = \hat{\mu}\hat{\alpha}^1 y^h_{1(h)} + (1-\hat{\mu}\hat{\alpha}^1)\delta y^\ell_{1(\ell)} > \sup_{R^2} U_{A,1} = \hat{\mu} z^1 + (1-\hat{\mu})\delta y^\ell_{1(\ell)} > \sup_{R^4} U_{A,1} = \delta\hat{\mu} y^h_{1(h)}$. And since $m^\ell < m^h$, also $\sup_{R^3} U_{A,1} = \hat{\mu}\hat{\alpha}^1 y^h_{1(h)} + (1-\hat{\mu}\hat{\alpha}^1)\delta y^\ell_{1(\ell)} > \sup_{R^2} U_{A,1} = \hat{\mu} z^1 + (1-\hat{\mu})\delta y^\ell_{1(\ell)} >$

$\sup_{R^1} U_{A,1} = y^\ell_{1(\ell)}$. We must have $U_{A,1}(y^h_{1(h)}) = \sup_{p_1 \in R^3} U_{A,1}(p_1)$ and therefore $W^1(z^1) = \hat{\mu}\hat{\alpha}^1$.

(iv) Let $\hat{\mu} = m^\ell$. Then, $\sup_{R^1} U_{A,1} = \sup_{R^2} U_{A,1}$. For an equilibrium to exist, we must have either $U_{A,1}(y^\ell_{1(\ell)}) = \sup_{p_1 \in R^1} U_{A,1}(p_1)$ or $U_{A,1}(z^1) = \sup_{p_1 \in R^2} U_{A,1}(p_1)$ or both. Therefore, we must have either $W^1(y^\ell_{1(\ell)}) = 1$ or $W^1(z^1) = \hat{\mu}$ or both.

(v) This case is analogous to (iv).

■

Proof of Theorem 2. Theorem 1 implies that the agenda-setter's expected payoff can be written as follows:

$$V^1_{A,1} = \begin{cases} y^\ell_{1(\ell)} & \text{if } \hat{\mu} \leq m^\ell, \\ \hat{\mu} z^1 + (1-\hat{\mu})\delta y^\ell_{1(\ell)} & \text{if } m^\ell < \hat{\mu} \leq m^h, \\ \hat{\mu}\hat{\alpha}^1 y^h_{1(h)} + (1-\hat{\mu}\hat{\alpha}^1)\delta y^\ell_{1(\ell)} & \text{if } m^h < \hat{\mu}. \end{cases} \tag{D.5}$$

We need to derive the limits of policy z^1 and belief thresholds m^ℓ and m^h as the discount factor δ converges to 1. We have:

(A) if $y^h_{1(h)} \leq 2y^\ell_{q(\ell)}$, then $(z^1,\ m^\ell,\ m^h)$ converges to $\left(y^\ell_{1(\ell)},\ \frac{2y^\ell_{1(\ell)} - y^h_{1(h)}}{y^\ell_{1(\ell)}},\ \frac{y^\ell_{1(\ell)}}{y^h_{1(h)}}\right)$;

(B) if $y^h_{1(h)} > 2y^\ell_{1(\ell)}$, then $(z^1,\ m^\ell,\ m^h)$ converges to $\left(y^h_{1(h)} - y^\ell_{1(\ell)},\ 0,\ \frac{1}{2}\right)$.

By (7), policy z^1 is the largest root of quadratic equation $(p_1)^2 - y^h_{1(h)} p_1 + \delta(y^h_{1(h)} - y^\ell_{1(\ell)}) y^\ell_{1(\ell)}$. The roots are $\frac{1}{2}(y^h_{1(h)} + [(y^h_{1(h)})^2 - 4\delta(y^h_{1(h)} - y^\ell_{1(\ell)}) y^\ell_{1(\ell)}]^{\frac{1}{2}})$ and $\frac{1}{2}(y^h_{1(h)} - [(y^h_{1(h)})^2 - 4\delta(y^h_{1(h)} - y^\ell_{1(\ell)}) y^\ell_{1(\ell)}]^{\frac{1}{2}})$. As δ goes to 1, the limit of the expression under the radical equals $(y^h_{1(h)} - 2y^\ell_{1(\ell)})^2$, so the limit of the largest root equals $\frac{1}{2}(y^h_{1(h)} - (y^h_{1(h)} - 2y^\ell_{1(\ell)})) = y^\ell_{1(\ell)}$ if $y^h_{1(h)} \leq 2y^\ell_{1(\ell)}$ and equals $\frac{1}{2}(y^h_{1(h)} + (y^h_{1(h)} - 2y^\ell_{1(\ell)})) = y^h_{1(h)} - y^\ell_{1(\ell)}$ if $y^h_{1(h)} > 2y^\ell_{1(\ell)}$, giving us the limits of z^1.

By definition (10), we have $m^\ell = \frac{y^\ell_{1(\ell)}(1-\delta)}{z^1 - \delta y^\ell_{1(\ell)}}$. From above, if $y^h_{1(h)} > 2y^\ell_{1(\ell)}$, we have $\lim_{\delta \to 1} z^1 = y^h_{1(h)} - y^\ell_{1(\ell)}$, which implies $\lim_{\delta \to 1} m^\ell = 0$. If $y^h_{1(h)} \leq 2y^\ell_{1(\ell)}$, we have $\lim_{\delta \to 1} z^1 = y^\ell_{1(\ell)}$ and taking the limit of m^ℓ as δ goes to 1 results in $\frac{0}{0}$ indeterminacy. Applying the L'Hôpital's rule, we obtain $\lim_{\delta \to 1} m^\ell = \frac{2y^\ell_{1(\ell)} - y^h_{1(h)}}{y^\ell_{1(\ell)}}$. This gives us the limits of m^ℓ. The limits of m^h follow directly from its definition (11) and the limits of z^1.

We are ready to prove Theorem 3. First, let $y^h_{1(h)} \le 2y^\ell_{1(\ell)}$. By part (A), policy z^1 converges to $y^\ell_{1(\ell)}$ and threshold m^h converges to $\frac{y^\ell_{1(\ell)}}{y^h_{1(h)}}$. By (D.5), the agenda-setter's expected payoff converges to $y^\ell_{1(\ell)}$ when $\hat{\mu} \le \frac{y^\ell_{1(\ell)}}{y^h_{1(h)}}$ and $\hat{\mu} y^h_{1(h)} > y^\ell_{1(\ell)}$ when $\hat{\mu} > \frac{y^\ell_{1(\ell)}}{y^h_{1(h)}}$.

Second, let $y^h_{1(h)} > 2y^\ell_{1(\ell)}$. By part (B), policy z^1 converges to $y^h_{1(h)} - y^\ell_{1(\ell)}$ and thresholds m^ℓ and m^h converge to 0 and $\frac{1}{2}$. By (D.5), the agenda-setter's expected payoff converges to $\hat{\mu}(y^h_{1(h)} - y^\ell_{1(\ell)}) + (1-\hat{\mu}) y^\ell_{1(\ell)} > y^\ell_{1(\ell)}$ when $\hat{\mu} \le \frac{1}{2}$ and to $\hat{\mu} y^h_{1(h)} > y^\ell_{1(\ell)}$ when $\hat{\mu} > \frac{1}{2}$. ■

Proof of Theorem 3. This follows directly from Theorem 2. ■

Theorem 4 (Formal statement)**.** *We have:*

(i) If $y^h_{1(h)} \le 2y^\ell_{1(\ell)}$, then $\lim_{\delta\to 1} V^1_{A,1} = V^C_1$ for all $\hat{\mu} \in (0,1)$.

(ii) If $y^h_{1(h)} > 2y^\ell_{1(\ell)}$, then $\lim_{\delta\to 1} V^1_{A,1} = V^C_1$ when $\hat{\mu} \ge \frac{1}{2}$ and $\lim_{\delta\to 1} V^1_{A,1} > V^C_1$ when $\hat{\mu} < \frac{1}{2}$.

Proof of Theorem 4. Benchmark 2 implies that $V^C_1 = \max\{y^\ell_{1(\ell)}, \hat{\mu} y^h_{1(h)}\}$.

(i) Let $y^h_{1(h)} \le 2y^\ell_{1(\ell)}$. The conclusion follows directly from Theorem 2 and Benchmark 2.

(ii) Let $y^h_{1(h)} > 2y^\ell_{1(\ell)}$, which is equivalent to $m^1 = \frac{y^\ell_{1(\ell)}}{y^h_{1(h)}} < \frac{1}{2}$. Let $\hat{\mu} \ge \frac{1}{2}$. Since $m^1 < \frac{1}{2}$, Benchmark 2 implies $V^C_1 = \hat{\mu} y^h_{1(h)}$. At the same time, Theorem 2 implies $\lim_{\delta\to 1} V^1_{A,1} = \hat{\mu} y^h_{1(b)}$. Next, let $\hat{\mu} < \frac{1}{2}$. If also $\hat{\mu} \le m^1$, then $V^C_1 = y^\ell_{1(\ell)}$ and $\lim_{\delta\to 1} V^1_{A,1} = \hat{\mu}(y^h_{1(h)} - y^\ell_{1(\ell)}) + \hat{\mu} y^\ell_{1(\ell)} > y^\ell_{1(\ell)}$. And if $\hat{\mu} \in (m^1, \frac{1}{2})$, then $V^C_1 = \hat{\mu} y^h_{1(h)}$ and $\lim_{\delta\to 1} V^1_{A,1} = \hat{\mu}(y^h_{1(h)} - y^\ell_{1(\ell)}) + \hat{\mu} y^\ell_{1(\ell)} > \hat{\mu} y^h_{1(h)}$.

■

Theorem 6 (Formal statement)**.**
Fix the preferences of voter $1(\cdot)$. Letting $\Delta_q = \max_{\omega\in\Omega}\{1(\omega) - q(\omega)\}$, we have $\lim_{\substack{\Delta_q\to 0\\ \delta\to 1}} \overline{w}_q = \lim_{\substack{\Delta_q\to 0\\ \delta\to 1}} \underline{w}_q = \lim_{\delta\to 1} V^1_{A,1}$.

Proof of Theorem 6. The limit of $V^1_{A,1}$ is derived in Theorem 2. The limits of $\overline{w}_q$ and $\underline{w}_q$ are given in (B.6) and (B.7) and clearly continuous at $\Delta_q = 0$. Plugging $q(\omega) = 1(\omega)$ for $\omega \in \Omega$ into (B.6) and (B.7), we immediately reach the required conclusion. ■

Proof of Theorem 7. We have $\underline{w}_q \geq y^{\ell}_{q(\ell)}$ by the definition in (B.2). The condition in part *(i)* is equivalent to $\lim_{\delta\to 1} \underline{w}_q > y^{\ell}_{q(\ell)}$ and follows from (B.7). The conditions in part *(ii)* are equivalent to $\lim_{\delta\to 1} \underline{w}_q = y^{\ell}_{q(\ell)} < \lim_{\delta\to 1} \overline{w}_q$ and follow from (B.6) and (B.7). The remaining case is when $\lim_{\delta\to 1} \underline{w}_q = y^{\ell}_{q(\ell)} = \lim_{\delta\to 1} \overline{w}_q$, giving us part *(iii)*. ■

Proof of Theorem 8. The conditions in part *(i)* are equivalent to $\lim_{\delta\to 1} \underline{w}_q > V^C_q$. The conditions in part *(ii)* are equivalent to $\lim_{\delta\to 1} \underline{w}_q = V^C_q < \lim_{\delta\to 1} \overline{w}_q$. The remaining case is when $\lim_{\delta\to 1} \underline{w}_q = V^C_q = \lim_{\delta\to 1} \overline{w}_q$. ■

Proof of Theorem 9. Note that $\underline{w}_q < \overline{w}_{q+1}$ cannot hold when either $\overline{w}_{q+1} = y^{\ell}_{(q+1)(\ell)}$ or $\overline{w}_{q+1} = \hat{\mu} y^{h}_{(q+1)(h)}$. The conditions follows directly from the comparison of (B.6) for $q+1$ and (B.7) for q. Case *(i)* corresponds to $\lim_{\delta\to 1} \underline{w}_q = \hat{\mu} y^{h}_{q(h)}$, case *(ii)* to $\lim_{\delta\to 1} \underline{w}_q = y^{\ell}_{q(\ell)}$, and case *(iii)* to $\lim_{\delta\to 1} \underline{w}_q = \hat{\mu}(y^{h}_{q(h)} - y^{\ell}_{q(\ell)}) + (1-\hat{\mu}) y^{\ell}_{q(\ell)}$. The value of y^* distinguishes between $\lim_{\delta\to 1} \overline{w}_{q+1} = \hat{\mu} y^{h}_{(q+1)(h)} + (1-\hat{\mu}) y^{\ell}_{(q+1)(\ell)}$ and $\lim_{\delta\to 1} \overline{w}_{q+1} = \hat{\mu}(y^{h}_{1(h)} - y^{\ell}_{(q+1)(\ell)}) + (1-\hat{\mu}) y^{\ell}_{(q+1)(\ell)}$. ■

Online Appendix A: Second period: Equilibrium definition and properties

The analysis of the second period is fairly simple but contains a number of insights that can help better understand the proposal and voting behavior in the first period. I show that irrespective of the voting rule, provided that voters use undominated strategies, they vote to accept policies that give them a higher period utility than the status quo and reject policies that give them a lower period utility than the status quo. Facing such behavior, the agenda-setter chooses between a "risky" proposal $y^h_{q(h)}$ that sets the target voter $q(\cdot)$ to the status-quo payoff in state h and a "guaranteed" proposal $y^\ell_{q(\ell)}$ that sets the target voter to the status-quo payoff in state ℓ.

Fix a public history $\hbar \in H$ and voter $i \in N$ in state $\omega \in \Omega$. Under a q-majority voting rule, the status-quo policy 0 remains in effect when less than q voters accept a revised proposal $p_2 \in \mathbb{R}_+$, and p_2 passes when at least q voters accept it. Given a period-2 voting profile $a_2 \in \{0,1\}^n$, the period-2 payoff of voter i in state ω is given by

$$U_{i,2}(p_2;\omega,a_2) = \begin{cases} u_i(p_2;\omega) & \text{if } a_{i,2} + \sum_{j\neq i} a_{j,2} \geq q, \\ u_i(0;\omega) & \text{if } a_{i,2} + \sum_{j\neq i} a_{j,2} < q. \end{cases}$$

For each period-2 voting strategy profile $\alpha_{-i,2} \in \Sigma_{-i}$ of voters other than i, we can write the period-2 expected payoff of voter i in state ω as follows:

$$\begin{aligned} U_{i,2}(p_2;\omega,(a_{i,2},\alpha_{-i,2})) = u_i(p_2;\omega)\text{Prob}\left(a_{i,2} + \sum_{j\neq i} a_{j,2} \geq q\right) \\ + u_i(0;\omega)\text{Prob}\left(a_{i,2} + \sum_{j\neq i} a_{j,2} < q\right), \quad a_{i,2} = 0,1. \end{aligned}$$

Let $\mathcal{A}^\omega_q(\alpha_{-i,2}(p_2))$ be the probability that voter i is pivotal for p_2 in state ω:

$$\mathcal{A}^\omega_q(\alpha_{-i,2}(p_2)) = \text{Prob}\left(\sum_{j\neq i} a_{j,2} = q-1 \mid \alpha_{-i,2}(p_2)\right).$$

In equilibrium, each voter in each state chooses an action with the highest expected payoff. The difference between the period-2 expected payoffs of voter i in state ω from accepting and rejecting a revised proposal p_2 equals

$$U_{i,2}(p_2;\omega,(1,\alpha_{-i,2}))-U_{i,2}(p_2;\omega,(0,\alpha_{-i,2}))=\mathcal{A}_q^\omega(\alpha_{-i,2}(p_2))\left(-\left(\frac{1}{2}y_i^\omega-p_2\right)^2+\frac{1}{4}(y_i^\omega)^2\right)$$
$$=\mathcal{A}_q^\omega(\alpha_{-i,2}(p_2))(y_i^\omega-p_2)p_2. \qquad \text{(A.1)}$$

Voter i in state ω accepts a revised proposal p_2 if this difference is strictly positive, and rejects p_2 if the difference is strictly negative. In case (A.1) equals zero, voter i is indifferent between accepting and rejecting p_2. The indifference captured by $\mathcal{A}_q^\omega(\alpha_{-i,2}(p_2))(y_i^\omega-p_2)p_2=0$ can be of two kinds. When $\mathcal{A}_q^\omega(\alpha_{-i,2}(p_2))>0$, the indifference is driven by the policy effects of the revised proposal p_2 and the status-quo policy 0. And when $\mathcal{A}_q^\omega(\alpha_{-i,2}(p_2))=0$, the indifference is driven by the inability of voter i in state ω to affect an outcome of the vote, i.e., she is not pivotal. Whenever $\mathcal{A}_q^\omega(\alpha_{-i,2}(p_2))=0$, I require that voter i in state ω accepts p_2 if it gives her a higher period payoff than the status-quo policy 0. In other words, I assume that voters use weakly undominated voting strategies. Therefore, we can write the equilibrium conditions on the period-2 voting strategy of voter i in state ω as follows:

$$\alpha_{i,2}(p_2;\omega)=\begin{cases}1 & \text{if } (y_i^\omega-p_2)p_2>0,\\ 0 & \text{if } (y_i^\omega-p_2)p_2<0.\end{cases} \qquad \text{(A.2)}$$

The optimal voting strategy in (A.2) allows us to draw some conclusions about the voting behavior in the second period. Notice that the expression $(y_i^\omega-p_2)p_2$ is symmetric in p_2 around $\frac{1}{2}y_i^\omega$. Therefore, (A.2) implies that voter i in state ω accepts a revised proposal p_2 if it is closer to voter i's ideal policy $\frac{1}{2}y_i^\omega$ than the status-quo policy 0, and rejects p_2 if it is more distant from i's ideal policy than the status-quo policy.

Lemma A.1. *Consider a q-majority voting rule. Fix a period-2 policy proposal $p_2\in\mathbb{R}_+$ and voter $i\in N$ in state $\omega\in\Omega$. Then, voter i accepts p_2 if $0<p_2<y_i^\omega$ and rejects it if $p_2>y_i^\omega$.*

Under a q-majority voting rule, for a revised proposal p_2 to pass at least q voters must accept it. Recall that $q(\omega)\in N$ is a voter with q-th highest ideal policy in state $\omega\in\Omega$. Lemma A.1 implies that when a revised proposal p_2 is made, it passes if $0<p_2<y_{q(\omega)}^\omega$ and the status-quo policy 0 remains in effect if $p_2>y_{q(\omega)}^\omega$.

Lemma A.2. *Consider a q-majority voting rule. Fix a period-2 policy proposal* $p_2 \in \mathbb{R}_+$ *and state* $\omega \in \Omega$. *If policy* p_2 *is proposed in period 2, then* p_2 *passes if* $0 < p_2 < y^{\omega}_{q(\omega)}$ *and gets rejected if* $p_2 > y^{\omega}_{q(\omega)}$.

Lemma A.2 does not say what happens when proposal p_2 coincides with the status-quo policy 0 or the policy $y^{\omega}_{q(\omega)}$ which gives the target vote in state ω the same payoff as the status quo. We will return to this question after we characterize an optimal period-2 proposal strategy of the agenda-setter.

Given Lemma A.2, to compute her expected payoff from making a revised proposal p_2, the agenda-setter needs to have a belief over the state space Ω. For any history $\hbar \in H$, the agenda-setter's belief is $\mu^{\hbar} \in \mathcal{F}$. Given $\hbar$, for any revised proposal $p_2 \in \mathbb{R}_+$ the agenda-setter assigns probability $W^q(\mu^{\hbar}, p_2)$ that p_2 passes under the q-majority voting rule. Using Lemma A.2, we can write the probability $W^q(\mu^{\hbar}, p_2)$ that proposal p_2 passes given belief $\mu^{\hbar}$ as follows:

$$W^q(\mu^{\hbar}, p_2) = \begin{cases} 1 & \text{if } 0 < p_2 < y^{\ell}_{q(\ell)}, \\ \mu^{\hbar} & \text{if } y^{\ell}_{q(\ell)} < p_2 < y^{h}_{q(h)}, \\ 0 & \text{if } y^{h}_{q(h)} < p_2. \end{cases} \tag{A.3}$$

This expression is easy to understand. State ℓ is interpreted as low because the ideal policy of the target voter $q(\omega)$ in state ℓ is lower than in state h. This is captured by the assumption that $y^{\ell}_{q(\ell)} < y^{h}_{q(h)}$. So, whenever policy p_2 is acceptable to voter $q(\ell)$ in low state ℓ, $0 < p_2 < y^{\ell}_{q(\ell)}$, it is also acceptable to voter $q(h)$ in high state h, meaning that policy p_2 passes in every state, that is, $W^q(\mu^{\hbar}, p_2) = 1$. Similarly, if policy p_2 is not acceptable to voter $q(h)$ in high state h, $p_2 > y^{h}_{q(h)}$, then it is also not acceptable to voter $q(\ell)$ in low state ℓ, meaning that policy p_2 gets rejected in every state, that is, $W^q(\mu^{\hbar}, p_2) = 0$. Finally, if policy p_2 is acceptable to voter $q(h)$ in high state h but not in low state ℓ, $y^{\ell}_{q(\ell)} < p_2 < y^{h}_{q(h)}$, then policy p_2 passes only in state h, that is $W^q(\mu^{\hbar}, p_2) = \mu^{\hbar}$.

Recall that the agenda-setter's period payoff from implementing a policy $x \in \mathbb{R}_+$ is $u_A(x) = x$. Therefore, the agenda-setter receives payoff p_2 if policy p_2 passes, and the status-quo payoff 0 if policy p_2 gets rejected. We can write the period-2 expected payoff of the agenda-setter as follows:

$$U^q_{A,2}(\mu^{\hbar})(p_2) = W^q(\mu^{\hbar}, p_2)p_2, \quad p_2 \in \mathbb{R}_+. \tag{A.4}$$

In equilibrium, the agenda-setter proposes a policy that maximizes her expected payoff (A.4). The agenda-setter solves the following problem:

$$\max_{p_2 \in \mathbb{R}_+} U^q_{A,2}(\mu^\hbar)(p_2); \tag{A.5}$$

and she proposes policy p_2 with positive probability only when p_2 is a solution to (A.5):

$$\pi^q_2(\mu^\hbar)(p_2) > 0 \text{ implies } p_2 \in \arg\max_{p'_2 \in \mathbb{R}_+} U^q_{A,2}(\mu^\hbar)(p'_2). \tag{A.6}$$

Using (A.3), we can write down the expected period-2 payoff of the agenda setter given in (A.4) as follows:

$$U^q_{A,2}(\mu^\hbar)(p_2) = \begin{cases} p_2 & \text{if } 0 \le p_2 < y^\ell_{q(\ell)}, \\ \mu^\hbar p_2 & \text{if } y^\ell_{q(\ell)} < p_2 < y^h_{q(h)}, \\ 0 & \text{if } y^h_{q(h)} < p_2. \end{cases}$$

Let m_q denote the belief that the state is h which makes the agenda-setter indifferent between the period-2 proposals $y^\ell_{q(\ell)}$ and $y^h_{q(h)}$ assuming that these proposals are certain to win in corresponding states, $m_q = \frac{y^\ell_{q(\ell)}}{y^h_{q(h)}}$.

Figure 4 depicts $U^q_{A,2}(\mu^\hbar)(\cdot)$ in the special case when $y^\ell_{q(\ell)} = \mu^\hbar y^h_{q(h)}$. Generally, $y^\ell_{q(\ell)}$ is greater or smaller than $\mu^\hbar y^h_{q(h)}$, depending in the agenda-setter's belief $\mu^\hbar$ and the ideal policies of the target voter $q(\cdot)$ in different states. The agenda-setter proposes policy $y^\ell_{q(\ell)}$ when she is pessimistic, that is $\mu^\hbar < m_q$, and proposes policy $y^h_{q(h)}$ when she is optimistic, $\mu^\hbar > m_q$. Since $y^\ell_{q(\ell)}$ passes in both states, the agenda-setter's expected payoff equals $y^\ell_{q(\ell)}$ when $\mu^\hbar < m_q$. And since y^h passes only in state h, the agenda-setter's expected payoff equals $\mu^\hbar y^h_{q(h)}$ when $\mu^\hbar > m_q$.

So far we could not say anything about the equilibrium actions of voters when they are indifferent between accepting and rejecting policy p_2, but now we can. Since the proposal policy of the agenda-setter must be optimal, (A.6) requires that $U^q_{A,2}(\mu^\hbar)$ has a maximum on $\mathbb{R}_+$. From figure (4), it is clear that a maximum does not exist if both $U^q_{A,2}(\mu^\hbar)y^\ell_{q(\ell)}$ and $U^q_{A,2}(\mu^\hbar)y^h_{q(h)}$ are smaller than $y^\ell_{q(\ell)}$.

This issue is familiar from the models of bilateral bargaining, in which proposals must be accepted in equilibrium whenever the veto player is indifferent between accepting and rejecting. Here, a similar requirement must hold in equilibrium. First,

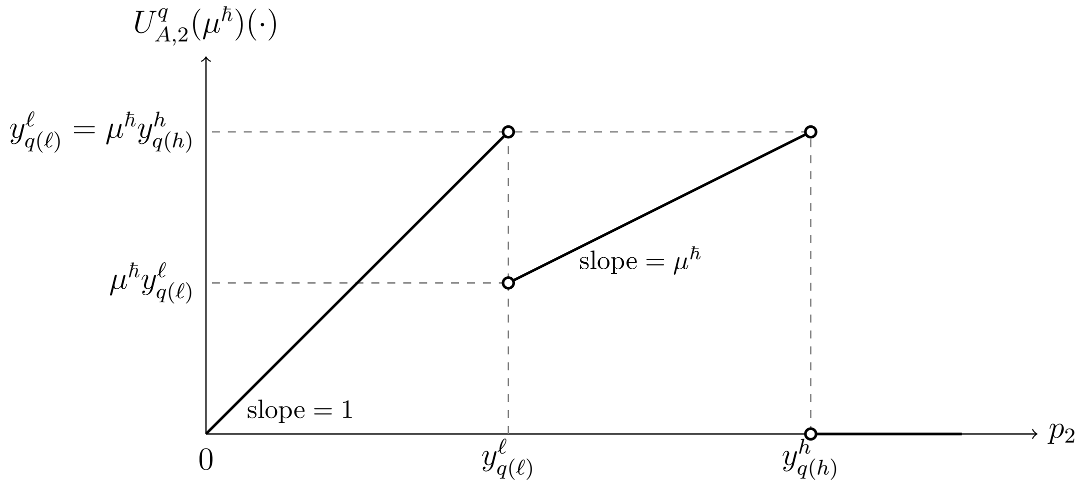


Figure 4: The expected period-2 payoff of the agenda-setter when voters have the same information. The depicted case, $\mu^\hbar = m_q$, is knife-edge but important in certain equilibria.

consider proposal $y^\ell_{q(\ell)}$. Since this proposal is smaller than $(y^h + b)$, the target voter $q(\cdot)$ accepts $y^\ell_{q(\ell)}$ in the high state h but is indifferent in the low state ℓ. Therefore, depending in the probability with which the target voter $q(\cdot)$ in state ℓ accepts p_2, the expected payoff of the agenda-setter varies between $\mu^\hbar y^\ell_{q(\ell)}$, which corresponds to the probability that the target voter $q(\cdot)$ in state ℓ rejects proposal $y^h_{q(h)}$, and $y^\ell_{q(\ell)}$, which corresponds to the probability that the target voter $q(\cdot)$ in state ℓ accepts proposal $y^h_{q(h)}$. Second, consider proposal $y^h_{q(h)}$, which turns out to be a mirror case. The target voter $q(\cdot)$ rejects this proposal in low state ℓ but is indifferent in high state h, and therefore the expected payoff of the agenda-setter varies between 0 and $\mu^\hbar y^h_{q(h)}$ as the probability that the target voter $q(\cdot)$ in state h accepts proposal $y^h_{q(h)}$ varies between 1 and 0. Overall, in equilibrium the target voter $q(\cdot)$ must either accept $y^\ell_{q(\ell)}$ in state ℓ or accept $y^h_{q(h)}$ in state h, or both.

There are two important takeaways that we can draw about the equilibrium strategy profile in the second period. First, there may be a multiplicity of PBE arising from our inability to pin down the acceptance strategies of voters. Second, there is only a handful of proposals that the agenda-setter may conceivably make, although she may be indifferent between multiple proposals. The proof of Lemma A.3 relies on the same arguments as given above for the knife-edge case and is therefore omitted.

Lemma A.3. *Consider a q-majority voting rule. In every PBE, if the agenda-setter's* *belief is* $\mu^{\hbar} \in \mathcal{F}$*, then:*

(i) the agenda-setter's expected payoff in the second period equals

$$V_{A,2}^{q}(\mu^{\hbar}) = \begin{cases} y_{q(\ell)}^{\ell} & \text{if } \mu^{\hbar} \leq m_q, \\ \mu^{\hbar} y_{q(h)}^{h} & \text{if } \mu^{\hbar} > m_q; \end{cases}$$

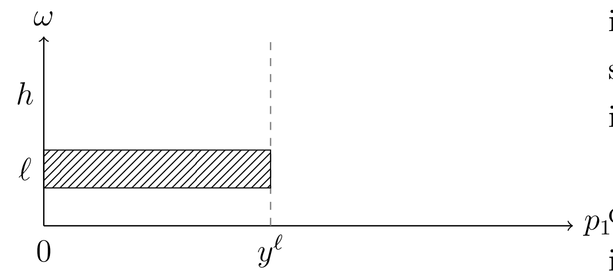
(ii) the agenda-setter's proposal strategy $\pi_2(\mu^{\hbar})$ *is such that*

$$\pi_2(\mu^{\hbar})(p_2) = \begin{cases} 1 & \text{if } \mu^{\hbar} < m_q \text{ and } p_2 = y_{q(\ell)}^{\ell}, \text{ or } \mu^{\hbar} > m_q \text{ and } p_2 = y_{q(h)}^{h}, \\ 0 & \text{if } p_2 \neq y_{q(\ell)}^{\ell} \text{ and } p_2 \neq y_{q(h)}^{h}. \end{cases}$$

The predictions from the analysis of the second period are analogous to the a single-period model in Romer and Rosenthal (1978, 1979). Even though voters are strategic in my model, the assumption that voters use weakly undominated voting strategies implies that the voting strategies are sincere in the second period. Romer and Rosenthal show that when the status-quo policy is located to the left of the fixed q-th ideal policy, the agenda-setter's expected payoff increases as the status-quo policy decreases. It is trivial to check that for a given posterior belief $\mu^{\hbar}$, the agenda-setter's expected payoff $V_{A,2}^{q}$ increases when the ideal policies of voter $q(\cdot)$ increase.

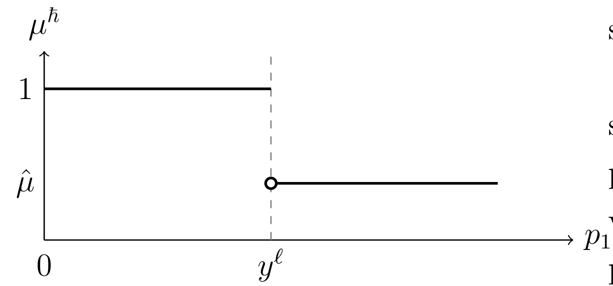
Online Appendix B: Increasing sequence of proposals under unanimity rule

B.1 Example 1: Reverse screening



(a) Period 1 symmetric voting strategy.

Accept Reject



(b) Posterior belief that the state is h following a split vote.

Figure 5: Period 1 voting strategies and posterior beliefs.

This is an example of a "reverse screening" equilibrium in which the agenda-setter becomes more optimistic after an initial proposal gets rejected.

Voter $i \in \{L, R\}$ in state $\omega \in \Omega$ accepts a revised policy $p_2 \in \mathbb{R}_+$ if and only if $p_2 \leq y_i^\omega$. The agenda-setter proposes policy y^h if the belief is $\hat{\mu}^\hbar \geq m_L$ and policy y^ℓ otherwise. It follows from Lemma A.1 and Lemma A.3 that these period-2 strategies are sequentially rational.

In period 1, voters use threshold strategies: in state ℓ both voters accept policy p_1 iff $p_1 \leq y^\ell$ and in state h both voters reject every policy. If an initial proposal $p_1 \leq y^\ell$ is rejected by both voters, the revised belief $\mu^\hbar$ assigns probability 1 to state h. If an initial proposal $p_1 > y^\ell$ is rejected by both voters, the revised belief $\mu^\hbar$ is the same as the prior belief $\hat{\mu}$. At every other history $\hbar$ with $p_1 \leq y^\ell$, the revised belief $\mu^\hbar$ assigns probability 1 to state h, and at every other history $\hbar$ with $p_1 > y^\ell$, the revised belief $\mu^\hbar$ is the same as the prior belief $\hat{\mu}$. The agenda-setter makes an initial proposal y^ℓ.

The voting strategies in period 1 are sequentially rational. In state h, unilateral deviations do not affect the agenda-setter's belief and therefore lead to the same revised proposal. Likewise, for any proposal $p_1 > y^\ell$ in state ℓ unilateral deviations do not affect the posterior belief. At the same time, in state ℓ both voters prefer any policy $p_1 \leq y^\ell$ to the breakdown in negotiations caused by a unilateral deviation from voting to accept p_1. For the initial proposal, the agenda-setter can either propose $p_1 \leq y^\ell$, which has an expected payoff $(1-\hat{\mu})p_1 + \delta\hat{\mu}y^h$, or $p_1 > y^\ell$ which has an expected payoff $\max\{\delta y^\ell,\ \delta\hat{\mu}y^h\}$. It is easy to see that y^ℓ is an optimal proposal.

In this example, the agenda-setter starts with a prior belief $\hat{\mu}$ and makes a policy proposal y^ℓ. This proposal passes only if the state is ℓ, so the agenda-setter concludes that the state must be h when y^ℓ gets rejected. Since the agenda-setter (correctly) believes that the state is h, she makes a revised proposal y^h which passes in period 2. This equilibrium can be interpreted as a reverse-screening because the initial proposal screens out (the less desirable) state ℓ , contrary to the usual screening that screens out (the more desirable) state h.

Another implication of the freedom to choose off-path beliefs following deviations by voters is the existence of equilibria in which the agenda-setter's expected payoff is below y^ℓ_d.

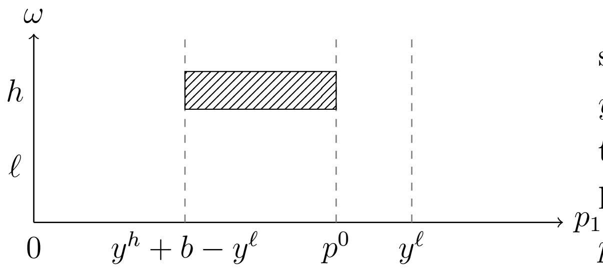
B.2 Example 2

This is an example of an equilibrium in which the agenda-setter's expected payoff is strictly below y^ℓ_L. Let $\hat{\mu} \leq \frac{y^\ell}{y^h}$ and let p^0 be any policy such that $y^h + b - y^\ell < p^0 < y^\ell$ and $p^0 \geq \delta y^\ell$.[25]

Voter $i \in \{L, R\}$ in state $\omega \in \Omega$ accepts a revised policy $p_2 \in \mathbb{R}_+$ if and only if $p_2 \leq y^\omega_i$. The agenda-setter proposes policy y^h if the belief is $\hat{\mu}^\hbar \geq m_L$ and policy y^ℓ otherwise. It follows from Lemma A.1 and Lemma A.3 that these period-2 strategies are sequentially rational.

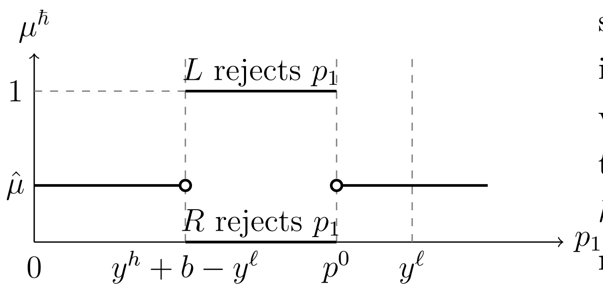
In period 1, voters use the following strategies: in state ℓ both voters reject every policy and in state h both voters accept policy p_1 iff $p_1 \in [y^h + b - y^\ell, p^0]$. If an initial proposal $p_1 \in [y^h + b - y^\ell, p^0]$ is rejected by both voters, the revised belief $\mu^\hbar$ assigns probability 0 to state h. If an initial proposal $p_1 \notin [y^h + b - y^\ell, p^0]$ is rejected by both voters, the revised belief $\mu^\hbar$ is the same as the prior $\hat{\mu}$. The revised beliefs after split votes on $p_1 \notin [y^h + b - y^\ell, p^0]$ remain the same as the prior belief. The revised beliefs after split votes on $p_1 \in [y^h + b - y^\ell, p^0]$ are tailored: if voter R rejects p_1 then the revised belief assigns probability 0 to state h, and if voter L rejects p_1 then the revised belief assigns probability 1 to state h. The agenda-setter makes an initial proposal p^0.

[25] One possible vector of values is: $\hat{\mu} = \frac{3}{4}$, $p^0 = \frac{5}{2}$, $y^\ell = 3$, $y^h = 4$, $b = 1$, and $\delta = \frac{5}{6}$.



(a) Period 1 symmetric voting strategy.

Accept Reject



(b) Posterior belief that the state is h following a split vote.

Figure 6: Period 1 voting strategies and posterior beliefs in Example 2.

The voting strategies in period 1 are sequentially rational. For $p_1 \notin [y^h + b - y^\ell, p^0]$, unilateral deviations do not affect the agenda-setter's belief and therefore lead to the same revised proposal. For $p_1 \in [y^h + b - y^\ell, p^0]$, consider state ℓ first. Whether or not voter L deviates, the agenda-setter believes that the state is ℓ. If voter R deviates, then the agenda-setter believes that the state is h leading to a breakdown of negotiations; but voter R in state ℓ prefers policy y^ℓ to the status-quo policy 0. Consider state h next. If voter L deviates, then the revised proposal is y^h and voter L gets the status-quo payoff; but voter L prefers any policy smaller than y^h to the status-quo payoff. If voter R deviates, then the revised proposal is y^ℓ. The assumption $y^h + b - y^\ell < p^0 < y^\ell$ guarantees that voter R prefers p_1 to a period of delay followed by policy y^ℓ. For the initial proposal, the agenda-setter can either propose $p_1 \in [y^h + b - y^\ell, p^0]$, which has an expected payoff $\hat{\mu} p_1 + \delta(1-\hat{\mu})y^\ell$, or $p_1 \notin [y^h + b - y^\ell, p^0]$ which has an expected payoff $\max\{\delta y^\ell,\ \delta \hat{\mu} y^h\}$. By the choice of $\hat{\mu}$, we have $\max\{\delta y^\ell,\ \delta \hat{\mu} y^h\} = \delta y^\ell$. And since $p^0 \geq \delta y^\ell$, we can see that p^0 is an optimal proposal.

In Example 2, the agenda-setter's expected payoff is $\hat{\mu} p^0 + \delta(1 - \hat{\mu})y^\ell$ which is strictly smaller than y^ℓ by the choice of p^0. It is worth noting that since $\delta y^\ell \leq p^0 < y^\ell$ policy p^0 converges to y^ℓ as players become perfectly patient. Therefore, the agenda-setter's expected payoff converges (from below) to y^ℓ. Theorem C.2 shows that there does not exist a sequence of PBE in which the agenda-setter's expected payoff converges to a value strictly below y^ℓ.

B.3 Coalition-proofness

Example 1 (continued)**.** The PBE in Example 1 is not coalition-proof if $b < y^\ell$. Consider the following joint deviation: in state h both voters accept proposal $p_1 = y^\ell$. This deviation is improving and self-enforcing. In particular, voter R in state h prefers policy y^ℓ to a period of delay followed by policy y^h provided that $b < y^\ell$.

Example 2 (continued)**.** The PBE in Example 2 is not coalition-proof. Consider the following joint deviation: in state ℓ both voters accept proposal $p_1 = p^0$. This deviation is improving and self-enforcing. In particular, voter R in state ℓ prefers policy p^0 to a period of delay followed by policy y^ℓ under the maintained assumption that $\delta y^\ell \leq p^0 < y^\ell$.

Online Appendix C: Absence of communication

In this section, I show that under the unanimity rule there is a severe multiplicity of equilibrium paths. The set of agenda-setter's expected payoffs that can be supported in equilibrium is a closed interval. Coasian equilibria exist only when the agenda-setter is pessimistic about the state. There always exist non-Coasian equilibria and never exist sub-Coasian equilibria. As a consequence, the agenda-setter always values the ability to make a revised proposal under the q-majority voting rule.

C.1 Characterization: Payoff bounds

The freedom to choose off-path beliefs leads to a continuum of possible equilibrium outcomes, some of which may feature rather surprising dynamics. In particular, under the q-majority voting rule with $q \geq 2$, the sequence of policy proposals on equilibrium path may be increasing (see Online Appendix B). As Theorem 1 suggests, this is not possible when bargaining the is "bilateral," that is, $q = 1$.

I begin by characterizing the ex ante expected payoffs which cannot be supported in any PBE. Define $\overline{v}_q$ as follows:

$$\overline{v}_q = \max\{y^{\ell}_{q(\ell)},\ \hat{\mu} y^{h}_{q(h)} + \delta(1-\hat{\mu}) y^{\ell}_{q(\ell)},\ \delta\hat{\mu} y^{h}_{q(h)} + (1-\hat{\mu}) y^{\ell}_{q(\ell)}\}. \tag{C.1}$$

Lemma C.1. *Under the q-majority voting rule with $q \geq 2$, in every PBE the agenda-setter's ex ante expected payoff is at most $\overline{v}_q$.*

Under the q-majority voting rule, sufficiently large groups of voters have power to veto any policy. If policy p_1 passes in state ω, then this policy must be at least as good as the status-quo policy 0 for at least q voters. It follows that the agenda-setter's expected payoff is bounded above by $\hat{\mu} y^{h}_{q(h)} + (1-\hat{\mu}) y^{\ell}_{q(\ell)}$, which is the expectation of $y^{\omega}_{q(\omega)}$ under the prior belief $\hat{\mu}$. Lemma C.1 provides an upper bound $\overline{v}_q$ that is strictly smaller than $\hat{\mu} y^{h}_{q(h)} + (1-\hat{\mu}) y^{\ell}_{q(\ell)}$. Since the agenda-setter does not observe the state, there must be a period of delay if distinct policies are implemented in distinct states. Theorem C.1 shows that $\overline{v}_q$ is a tight upper bound, meaning that there exists an equilibrium in which the agenda-setter's expected payoff equals $\overline{v}_q$.

To complete the characterization of the ex ante expected payoffs which cannot be supported in any PBE, define $\underline{v}_q$ as follows:

$$\underline{v}_q = \max\{\delta y^{\ell}_{q(\ell)},\ \delta\hat{\mu} y^{h}_{q(h)}\}. \tag{C.2}$$

Intuitively, the agenda-setter cannot do worse than cause a period of delay without learning anything and therefore can guarantee an expected payoff which is at least $\delta \max\{y^{\ell}_{q(\ell)},\ \hat{\mu} y^{h}_{q(h)}\}$.

Lemma C.2. *Under the q-majority voting rule with $q \geq 2$, in every PBE the agenda-setter's ex ante expected payoff is at least $\underline{v}_q$.*

It turns out that every value between $\underline{v}_q$ and $\overline{v}_q$ can be supported in equilibrium as the agenda-setter's ex ante expected payoff. Theorem C.1 provides a sharp characterization of the agenda-setter's expected payoffs that can be supported in equilibrium.

Theorem C.1. *Under the q-majority voting rule with $q \geq 2$, the agenda-setter's ex ante expected payoff equals v in some PBE if and only if $v \in [\underline{v}_q, \overline{v}_q]$.*

The proof of Theorem C.1 uses a constructive argument. For each $v \in [\underline{v}_q, \overline{v}_q]$, I define an assessment $(\sigma, \mathcal{M})$ and then prove it to be an equilibrium that induces an agenda-setter's ex-ante expected payoff that equals v. The proof of Theorem C.1 also reveals that not every agenda-setter's expected payoff can be supported using threshold voting strategies. Overall, there are three classes of equilibrium strategies and belief systems that are sufficient for the construction. These classes are defined in Section C.4. The equilibrium outcomes associated with each class are:

(I) The initial proposal p_1 is strictly greater than $y^{\ell}_{q(\ell)}$ and passes only in state h. If it gets rejected, then the agenda-setter is certain that the state is ℓ and offers $y^{\ell}_{q(\ell)}$ in the second period. This equilibrium outcome is similar to the screening equilibrium in Theorem 1.

(II) The initial proposal p_1 is strictly smaller than $y^{\ell}_{q(\ell)}$ and passes only in state ℓ. If it gets rejected, then the agenda-setter is certain that the state is h and offers $y^{h}_{q(h)}$ in the second period. This equilibrium outcome is similar to the reverse screening equilibrium in the Online Appendix B.

(III) The initial proposal p_1 is greater than 0 and is only accepted in state h. If it gets rejected, then the agenda-setter is certain that the state is ℓ and offers $y^{\ell}_{q(\ell)}$

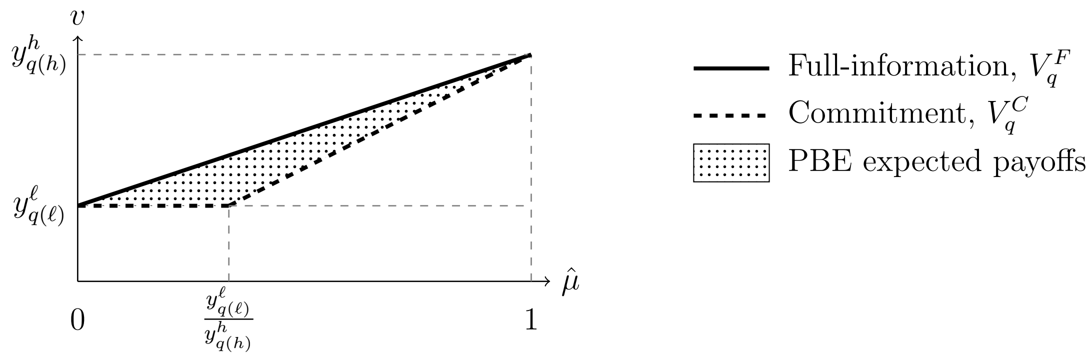


Figure 7: The limit set of the agenda-setter's expected payoffs under the q-majority voting rule and the benchmarks as players become perfectly patient.

in the second period. This equilibrium outcome is similar to class (I) except the initial proposal p_1 can be smaller than $y^\ell_{q(\ell)}$ yet get rejected in state ℓ.

C.2 Coasian equilibria and the value of commitment

Since $\overline{v}_q$ converges to the expected value of $y^\omega_{q(\omega)}$ as the discount factor δ converges to 1,

$$\lim_{\delta\to 1} \overline{v}_q = \hat{\mu} y^h_{q(h)} + (1-\hat{\mu}) y^\ell_{q(\ell)} = V^F_q, \tag{C.3}$$

the statement of Theorem C.1 implies that we can always construct a non-Coasian sequence of equilibria in which the limit of the agenda-setter's expected payoff exceeds $y^\ell_{q(\ell)}$. The limit of $\underline{v}_q$ as the discount factor δ converges to 1 is given by

$$\lim_{\delta\to 1} \underline{v}_q = \max\{y^\ell_{q(\ell)}, \hat{\mu} y^h_{q(h)}\} = V^C_q, \tag{C.4}$$

and Theorem C.1 implies that we cannot construct a sub-Coasian sequence of equilibria in which the limit of the agenda-setter's expected payoff is below $y^\ell_{q(\ell)}$. Figure 7 shows the graph of correspondence that maps the prior belief $\hat{\mu}$ to set $[\lim_{\delta\to 1} \underline{v}_q, \lim_{\delta\to 1} \overline{v}_q]$.

Analogous to Theorem 3, the following result focuses on the case when the prior belief that the state is h is such that $\hat{\mu} \le \frac{y^\ell_{q(\ell)}}{y^h_{q(h)}}$, which implies that the agenda-setter's expected payoff under the commitment benchmark equals the Coasian bound $y^\ell_{q(\ell)}$.

Theorem C.2. *Suppose that the prior belief that the state is h satisfies $\hat{\mu} \le \frac{y^\ell_{q(\ell)}}{y^h_{q(h)}}$. Then, under the q-majority voting rule with $q \ge 2$, almost every sequence of equilibria is non-Coasian.*

Is the ability to commit to a single proposal valuable to the agenda-setter under the unanimity rule? By comparing the expected payoff $V_q^C = \max\{y^\ell_{q(\ell)}, \hat{\mu} y^h_{q(h)}\}$ for the commitment benchmark derived in Appendix A with the limit of the lower bound $\underline{v}_q$ as players become perfectly patient given in (C.4), we can see that every expected payoff under the unanimity rule when players are perfectly patient is at least as high as V_q^C.

Theorem C.3. *When players become perfectly patient, the agenda-setter's expected payoff under the q-majority voting rule with $q \geq 2$ is weakly greater than under the commitment benchmark, and it is strictly greater in almost every equilibrium.*

Remark. While the above result holds in the limit as $\delta \to 1$, for any $\delta < 1$ we can construct a PBE in which the agenda-setters expected payoff equals $\delta \max\{y^\ell_{q(\ell)}, \hat{\mu} y^h_{q(h)}\}$ which is strictly smaller than the commitment benchmark V_q^C.

C.3 An increase in required quota q

In this section, I consider the effects of an increase in the number of voters required to pass a policy, i.e., quota q. I focus on cases when the agenda-setter's expected payoffs are characterized by Theorem C.1. The effect of an increase in required quota from $q = 1$ to $q \geq 2$ is a corollary to Theorem 5 in Section 4. The main result is Theorem C.4 which provides conditions for the existence of equilibria such that the agenda-setter's expected payoff increases in response to an increase in required quota q.

Because of the multiplicity of the expected payoffs that can be supported in equilibrium, the comparative statics with respect to quota q has to rely on equilibrium selection except in extreme cases. A mapping $\phi(\cdot) : \{2, \dots, n\} \to \mathbb{R}_+$ is an *equilibrium payoff selection* if for each $q \in \{2, \dots, n\}$ we have $\lim_{\delta \to 1} \underline{v}_q \leq \phi(q) \leq \lim_{\delta \to 1} \overline{v}_q$. An equilibrium payoff selection $\phi(\cdot)$ is *monotone* if it is weakly decreasing on $\{2, \dots, n\}$.

The following result provides conditions for the existence of a non-monotone equilibrium payoff selection or, in other words, for an existence of $q \in \{2, \dots, n-1\}$ such that the agenda-setter's expected payoff under the larger quota $q+1$ is strictly greater than the expected payoff under the smaller quota q. The argument relies on the comparison of bounds $\overline{v}_{q+1}$ and $\underline{v}_q$ given in Theorem C.1.

Theorem C.4. *There exists an equilibrium payoff selection $\phi(\cdot)$ that is non-monotone if and only if there exists $q \in \{2, \ldots, n-1\}$ such that one of the following conditions hold:*

(i) $\dfrac{y^\ell_{q(\ell)} - y^\ell_{(q+1)(\ell)}}{y^h_{(q+1)(h)} - y^\ell_{(q+1)(\ell)}} < \hat{\mu} \le \dfrac{y^\ell_{q(\ell)}}{y^h_{q(h)}}$;

(ii) $\dfrac{y^\ell_{q(\ell)}}{y^h_{q(h)}} \le \hat{\mu} < \dfrac{y^\ell_{(q+1)(\ell)}}{y^h_{q(h)} - \left(y^h_{(q+1)(h)} - y^\ell_{(q+1)(\ell)}\right)}$.

Theorem C.4 also implies that when the conditions *(i)* and *(ii)* are violated for each $q \in \{2, \ldots, n-1\}$, then any equilibrium payoff selection is monotone. To illustrate, consider a special case when the ideal policies of all synthetic voters are equally spaced in state ℓ and a change in state from ℓ to h shifts the ideal policies of all synthetic voters by an equal amount. More precisely, assume that $y^\ell_{1(\ell)} = b_v$, $y^\ell_{q(\ell)} = \frac{n-q+1}{n} b_v$ for $q = 2, \ldots, n$, and $y^h_{q(h)} - y^\ell_{q(\ell)} = b_s$ for $q \in N$. I refer to b_v as "voter heterogeneity" and to b_s as "state heterogeneity." Theorem C.4 implies that any equilibrium payoff selection is monotone when

$$\frac{1}{n} b_v > \frac{n-2}{n-1} b_s, \tag{C.5}$$

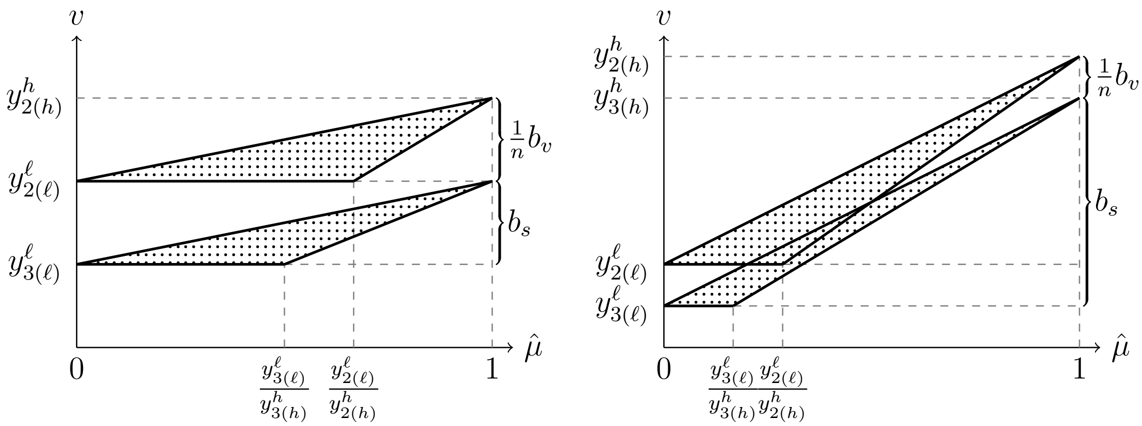
that is, when voter heterogeneity is sufficiently large relative to state heterogeneity (see Figure 8).

C.4 Proofs

Proof of Lemma C.1. Fix a required quota $q \ge 2$ and an equilibrium $(\sigma, \mathcal{M})$. Since any $n-q+1$ voters have collective power to veto any policy, then a policy that passes in state ℓ cannot exceed $y^\ell_{q(\ell)}$, and a policy that passes in state h cannot exceed $y^h_{q(h)}$.

Suppose that the agenda-setter's expected payoff $V_A(\sigma, \mathcal{M})$ is above $\overline{v}_q$. This implies that there exists a policy $p_1 \in \text{supp } \pi_1$ such that $\eta(p_1) > \overline{v}_q$ where $\eta(p_1) = \sum_{\omega \in \Omega} \hat{\mu}(\omega) [\int \{(1-\delta)x_1 + \delta x_2\} \, \mathrm{d}F^{p_1,\omega}_{\sigma,\mathcal{M}}(x_1, x_2)]$. There are three cases, each resulting in a contradiction.

Suppose $\text{supp} F^{p_1,\ell}_{\sigma,\mathcal{M}} = \text{supp} F^{p_1,h}_{\sigma,\mathcal{M}}$. Then, $(1-\delta)x_1 + \delta x_2 \le y^\ell_{q(\ell)}$ for all (x_1, x_2) in these supports and $\eta(p_1) \le y^\ell_{q(\ell)}$.



(a) Parameters are $\frac{1}{n}b_v = b_s = 2$. Any equilibrium payoff selection is monotone.

(b) Parameters are $\frac{1}{n}b_v = 1$ and $b_s = 5$. There exists a non-monotone equilibrium payoff selection.

Figure 8: The limit sets of the agenda-setter's expected payoffs under the q-majority voting rule. There are $n = 3$ voters, limit sets for $q = 2$ and $q = 3$ are depicted.

Suppose there exists $(x_1, x_2) \in \text{supp}F^{p_1,h}_{\sigma,\mathcal{M}}$ such that $(x_1, x_2) \notin \text{supp}F^{p_1,\ell}_{\sigma,\mathcal{M}}$. If $p_1 = 0$, then $(1-\delta)x_1+\delta x_2 \leq \delta y^h_{q(h)}$ and for all $(\tilde{x}_1, \tilde{x}_2) \in \text{supp}F^{p_1,\ell}_{\sigma,\mathcal{M}}$ we have $(1-\delta)\tilde{x}_1+\delta\tilde{x}_2 \leq \delta y^\ell_{q(\ell)}$; therefore, $\eta(p_1) \leq \hat{\mu}\delta y^h_{q(h)} + (1-\hat{\mu})\delta y^\ell_{q(\ell)}$. If $p_1 > 0$, then $(1-\delta)x_1 + \delta x_2 \leq y^h$ and for all $(\tilde{x}_1, \tilde{x}_2) \in \text{supp}F^{p_1,\ell}_{\sigma,\mathcal{M}}$ we have $(1-\delta)\tilde{x}_1 + \delta\tilde{x}_2 \leq \delta y^\ell_{q(\ell)}$; therefore, $\eta(p_1) \leq \hat{\mu} y^h_{q(h)} + (1-\hat{\mu})\delta y^\ell_{q(\ell)}$.

Suppose there exists $(x_1, x_2) \in \text{supp}F^{p_1,\ell}_{\sigma,\mathcal{M}}$ such that $(x_1, x_2) \notin \text{supp}F^{p_1,h}_{\sigma,\mathcal{M}}$. If $p_1 = 0$, then $(1-\delta)x_1 + \delta x_2 \leq \delta y^\ell$ and for all $(\tilde{x}_1, \tilde{x}_2) \in \text{supp}F^{p_1,h}_{\sigma,\mathcal{M}}$ we have $(1-\delta)\tilde{x}_1 + \delta\tilde{x}_2 \leq \delta y^h_{q(h)}$; therefore, $\eta(p_1) \leq \hat{\mu}\delta y^h_{q(h)} + (1-\hat{\mu})\delta y^\ell_{q(\ell)}$. If $p_1 > 0$, then $(1-\delta)x_1 + \delta x_2 \leq y^\ell$ and for all $(\tilde{x}_1, \tilde{x}_2) \in \text{supp}F^{p_1,h}_{\sigma,\mathcal{M}}$ we have $(1-\delta)\tilde{x}_1 + \delta\tilde{x}_2 \leq \delta y^h_{q(h)}$; therefore, $\eta(p_1) \leq \hat{\mu}\delta y^h_{q(h)} + (1-\hat{\mu}) y^\ell_{q(\ell)}$. ■

Proof of Lemma C.2. Fix an equilibrium $(\sigma, \mathcal{M})$ and suppose that the agenda-setter's expected payoff $V_A(\sigma, \mathcal{M})$ is below $\underline{v}_q$. This implies that for each policy $p_1 \in \mathbb{R}_+$ we have

$$\eta(p_1) = \sum_{\omega\in\Omega} \hat{\mu}(\omega) \left\{ \int \{(1-\delta)x_1 + \delta x_2\} \, \mathrm{d}F^{p_1,\omega}_{\sigma,\mathcal{M}}(x_1, x_2) \right\} < \underline{v}_q$$

where $\underline{v} = \max\{\delta y^\ell_{q(\ell)},\ \delta\hat{\mu} y^h_{q(h)}\}$.

Pick any $p_1 > y^h_{q(h)}$ and let $(x_1, x_2) \in \text{supp}\ F^{p_1,\omega}_{\sigma,\mathcal{M}}$ for some $\omega \in \Omega$. Then, $x_1 = 0$

since player L strictly prefers to veto p_1 in every state. Since proposal p_1 is defeated in every state, it induces (together with the prior $\hat{\mu}$ and the period-1 voting strategies of voters) a distribution G of posterior beliefs. Given every belief $\mu^{\hbar}$ in the support of G, the agenda-setter maximizes her expected payoff in the second period. Ex ante, the agenda-setter cannot do worse using experiment G than using her prior belief $\hat{\mu}$, so the agenda-setter's expected payoff in the second period is at least $\max\{y^{\ell}_{q(\ell)},\ \hat{\mu} y^{h}_{q(h)}\}$. ■

There are two classes of period-1 strategies and belief systems that are used in the proof of Theorem C.1. Each class depends on a parameter $z \in \mathbb{R}_+$, although this dependence is suppressed in the notation. For each state $\omega \in \Omega$ and $q \in \{2, \ldots, n\}$, let N^{ω}_q be the set of players with q highest ideal policies in state ω. For each $\hbar \in H$, write $\hbar = (p_1, C)$ where p_1 is the initial proposal and C is the set of voters who rejected it

For $z > \delta y^{\ell}_{q(\ell)}$ and small $\varepsilon > 0$, define $\sigma^I_1 = (\pi^I_1, (\alpha^I_{i,1})_{i \in N})$ as follows. For all $p_1 \in \mathbb{R}_+$ let:

$$\alpha^I_{i,1}(p_1;\ell) = 0, \quad i \in N, \qquad \alpha^I_{i,1}(p_1;h) = 0, \quad i \notin N^h_q,$$

$$\alpha^I_{i,1}(p_1;h) = \begin{cases} 1 & \text{if } p_1 \in [z-\varepsilon, z], \\ 0 & \text{if } p_1 \notin [z-\varepsilon, z], \end{cases} \quad i \in N^h_q,$$

$$\operatorname{supp} \pi^I_1 = \arg\max_{p_1 \in [z, z+1]} G^I(p_1), \text{ where } G^I : \{z, z+1\} \to \mathbb{R}_+ \text{ is defined as}$$

$$G^I(z) = \hat{\mu} z + \delta(1-\hat{\mu}) y^{\ell}_{q(\ell)}, \quad G^I(z+1) = \underline{v}_q.$$

Define $\mathcal{M}^I = \{\mu^{\hbar}\}_{\hbar \in H}$ as follows. If $p_1 \notin [z-\varepsilon, z]$, let $\mu^{\hbar} = \hat{\mu}$ for any $C \subseteq N$. And if $p_1 \in [z-\varepsilon, z]$, let $\mu^{\hbar}$ be such that $\mu^{\hbar} = \begin{cases} 0 & \text{if } C \in \mathcal{C}_0, \\ 1 & \text{if } C \in \mathcal{C}_1, \\ \hat{\mu} & \text{otherwise,} \end{cases}$ where

$$\mathcal{C}_0 = \left\{ C \subseteq N \;\middle|\; \begin{array}{l} C = N, \\ \text{or } N^h_q \cap C = \{j\}, \text{ and } y^h_j - y^h_{q(h)} > y^{\ell}_{q(\ell)}, \\ \text{or } N^{\ell}_q \cap C = \{j\}, \text{ and } y^{\ell}_j - y^{\ell}_{q(\ell)} > y^h_{q(h)}, \\ \text{or } N^{\ell}_q \setminus C = \{j\}, \text{ and } y^{\ell}_j - y^{\ell}_{q(\ell)} > y^h_{q(h)}, \end{array} \right\}$$

$$
\mathcal{C}_1 = \left\{ C \subseteq N \;\middle|\; \begin{array}{l} N_q^h \cap C = \{j\}, \text{ and } y_j^h - y_{q(h)}^h \leq y_{q(\ell)}^\ell, \\ \text{or } N_q^\ell \cap C = \{j\}, \text{ and } y_j^\ell - y_{q(\ell)}^\ell \leq y_{q(h)}^h, \\ \text{or } N_q^\ell \setminus C = \{j\}, \text{ and } y_j^\ell - y_{q(\ell)}^\ell \leq y_{q(h)}^h. \end{array} \right\}
$$

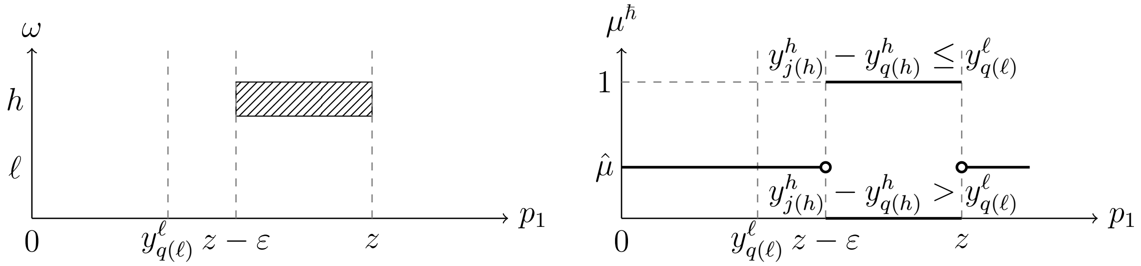


Figure 9: Depiction of $(\sigma^I, \mathcal{M}^I)$. Only the posterior beliefs following a unilateral deviation by some player $j \in N_q^h$ are shown. Here, $\varepsilon > 0$ is small and either $z > y^\ell_{q(\ell)}$ or $z \in (\delta y^\ell_{q(\ell)}, y^\ell_{q(\ell)}]$.

For $z < y^\ell_{q(\ell)}$, define $\sigma_1^{II} = (\pi_1^{II}, (\alpha_{i,1}^{II})_{i\in N})$ as follows. For all $p_1 \in \mathbb{R}_+$ let:

$$
\alpha_{i,1}^{II}(p_1; h) = 0, \quad i \in N, \qquad \alpha_{i,1}^{II}(p_1; \ell) = 0 \quad i \notin N_h^\ell,
$$

$$
\alpha_{i,1}^{II}(p_1; \ell) = \begin{cases} 1 & \text{if } p_1 \leq z, \\ 0 & \text{if } p_1 > z, \end{cases} \qquad i \in N_q^\ell,
$$

$$
\operatorname{supp} \pi_1^{II} = \arg\max_{p_1 \in [z, z+1]} G^{II}(p_1), \text{ where } G^{II} : \{z, z+1\} \to \mathbb{R}_+ \text{ is defined as}
$$

$$
G^{II}(z) = (1-\hat{\mu})z + \delta\hat{\mu} y^h_{q(h)}, \quad G^{II}(z+1) = \underline{v}_q.
$$

Define $\mathcal{M}^{II} = \{\mu^\hbar\}_{\hbar \in H}$ by letting $\mu^\hbar = \begin{cases} \hat{\mu} & \text{if } p_1 > z, \\ 1 & \text{if } p_1 \leq z. \end{cases}$

Proof of Theorem C.1. By Lemma C.1 and Lemma C.2, if v is the agenda-setter's expected payoff then $v \in [\underline{v}_q, \overline{v}_q]$. For the other direction, fix $v \in [\underline{v}_q, \overline{v}_q]$. I will construct an equilibrium $(\sigma, \mathcal{M})$ such that $V_A(\sigma, \mathcal{M}) = v$.

The period-2 strategy profile used in the construction does not depend on v. Define $\sigma_2 = (\pi_2, (\alpha_{i,2})_{i\in N})$ as follows:

$$
\alpha_{i,2}(p_2; \omega) = \begin{cases} 1 & \text{if } (y_i^\omega - p_2)p_2 \geq 0, \\ 0 & \text{if } (y_i^\omega - p_2)p_2 < 0, \end{cases} \qquad p_2 \in \mathbb{R}_+, \ \omega \in \Omega, \ i \in N; \tag{C.6}
$$

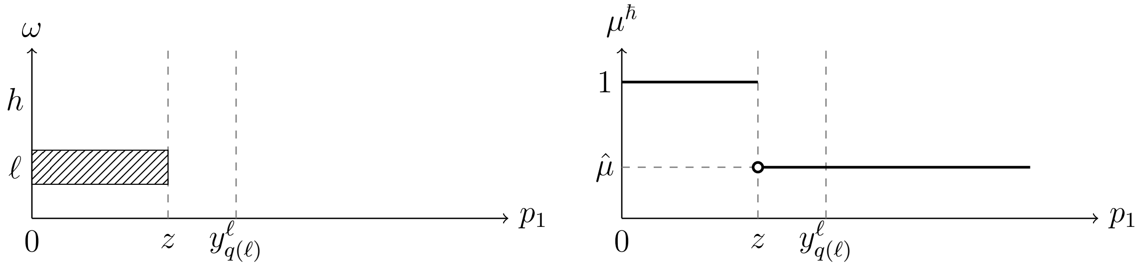


Figure 10: Depiction of $(\sigma^{II}, \mathcal{M}^{II})$. Only the posterior beliefs following a unilateral deviation by some player $j \in N^h_q$ are shown. Here, $z < y^\ell_{q(\ell)}$.

$$\pi_2(\mu^\hbar)(p_2) = \begin{cases} 1 & \text{if } \mu^\hbar < m_q \text{ and } p_2 = y^\ell_{q(\ell)}, \\ 1 & \text{if } \mu^\hbar \geq m_q \text{ and } p_2 = y^h_{q(h)}, \\ 0 & \text{if } p_2 \neq y^\ell_{q(\ell)} \text{ and } p_2 \neq y^h_{q(h)}, \end{cases} \qquad \mu^\hbar \in \mathcal{F}. \tag{C.7}$$

The period-1 strategy profile and the belief system depend on $v \in [\underline{v}_q, \overline{v}_q]$ and belong to one of two classes defined above.

If $v \geq (1-\hat{\mu})y^\ell_{q(\ell)} + \delta\hat{\mu}y^h_{q(h)}$, let $\sigma = (\sigma^I_1, \sigma_2)$ with $z = \frac{v - \delta(1-\hat{\mu})y^\ell_{q(\ell)}}{\hat{\mu}}$ and $\mathcal{M} = \mathcal{M}^I$. Since $v \geq (1-\hat{\mu})y^\ell_{q(\ell)} + \delta\hat{\mu}y^h_{q(h)}$ we have $z > \delta y^\ell_{q(\ell)}$, so the strategy profile σ^I is well-defined.

Period-1 voting strategies are sequentially rational. When $p_1 \notin [z-\varepsilon, z]$, unilateral deviations from unanimously rejecting p_1 do not change the posterior belief.

Suppose $p_1 \in [z-\varepsilon, z]$ and the state is ℓ. On path, we have $C = N$, $\mu^\hbar = 0$, and the revised proposal is $y^\ell_{q(\ell)}$ which passes. A unilateral deviation by voter $j \notin N^\ell_q$ does not affect the posterior belief. Consider a unilateral deviation by voter $j \in N^\ell_q$. If $y^\ell_j - y^\ell_{q(\ell)} > y^h_{q(h)}$, then the posterior belief remains $\hat{\mu}^\hbar = 0$. And if $y^\ell_j - y^\ell_{q(\ell)} \leq y^h_{q(h)}$, the posterior belief is $\mu^\hbar = 1$, implying that the revised proposal is $y^h_{q(h)}$ which gets rejected since the state is ℓ. But any voter $j \in N^\ell_q$ prefers $y^\ell_{q(\ell)}$ to the status-quo 0.

Suppose $p_1 \in [z-\varepsilon, z]$ and the state is h. On path, we have $C = N \setminus N^h_q$ so p_1 passes. A unilateral deviation by voter $j \notin N^h_q$ does not affect this outcome. Consider a unilateral deviation by voter $j \in N^h_q$. If $y^h_j - y^h_{q(h)} > y^\ell_{q(\ell)}$, then the posterior belief is $\hat{\mu}^\hbar = 0$, implying that the revised proposal is $y^\ell_{q(\ell)}$ which passes. Since $\delta y^\ell_{q(\ell)} < p_1 \leq y^h_{q(h)}$ for small $\varepsilon > 0$ and $y^h_j - y^h_{q(h)} > y^\ell_{q(\ell)}$, such deviation cannot increase the expected payoff of voter j. If $y^h_j - y^h_{q(h)} \leq y^\ell_{q(\ell)}$, then the posterior belief is $\mu^\hbar = 1$, implying that the revised proposal is $y^h_{q(h)}$ which passes since the state is h. Since $\delta y^\ell_{q(\ell)} < p_1 \leq y^h_{q(h)}$ for small $\varepsilon > 0$ and $y^h_j - y^h_{q(h)} \leq y^\ell_{q(\ell)}$, such deviation cannot increase the expected payoff of voter j.

The period-1 proposal z is optimal because the value of function G^I at $z = \frac{v-\delta(1-\hat{\mu})y^\ell_{q(\ell)}}{\hat{\mu}}$ equals $v \geq \underline{v}_q$. The agenda-setter's expected payoff from making proposal z equals $\hat{\mu}z + (1-\hat{\mu})\delta y^\ell_{q(\ell)} = v$.

If $v < (1-\hat{\mu})y^\ell_{q(\ell)} + \delta\hat{\mu}y^h_{q(h)}$, let $\sigma = (\sigma_1^{II}, \sigma_2)$ with $z = \frac{v-\delta\hat{\mu}y^h}{1-\hat{\mu}}$ and $\mathcal{M} = \mathcal{M}^{II}$. Since $v < (1-\hat{\mu})y^\ell_{q(\ell)} + \delta\hat{\mu}y^h_{q(h)}$ we have $z < y^\ell_{q(\ell)}$, so the strategy profile σ^{II} is well-defined.

Period-1 voting strategies are sequentially rational. When $p_1 > z$, unilateral deviations by voters in N^ω_q, $\omega \in \Omega$, from rejecting p_1 do not affect the posterior belief.

Suppose $p_1 \leq z$ and the state is h. On path, we have $C = N$, $\mu^\hbar = 1$, and the revised proposal is $y^h_{q(h)}$ which passes since the state is h. A unilateral deviation by any voter $i \in N$ does not affect the posterior belief.

Suppose $p_1 \leq z$ and the state is ℓ. On path, we have $C = N \setminus N^\ell_q$ so p_1 passes. A unilateral deviation by voter $j \notin N^\ell_q$ does not affect this outcome. A unilateral deviation by voter $j \in N^\ell_q$ induces a posterior belief $\mu^\hbar = 1$, implying that the revised proposal is $y^h_{q(h)}$ which gets rejected since the state is ℓ. But since $z < y^\ell_{q(\ell)}$, any voter $j \in N^\ell_q$ prefers $p_1 \leq z$ to the status-quo 0.

The period-1 proposal z is optimal because the value of function G^{II} at $z = \frac{v-\delta\hat{\mu}y^h_{q(h)}}{1-\hat{\mu}}$ equals $v \geq \underline{v}_q$. The agenda-setter's expected payoff from making proposal z equals $(1-\hat{\mu})z + \delta\hat{\mu}y^h_{q(h)} = v$. ■

Proof of Theorem C.2. Theorem C.1 and the limits of $\underline{v}_q$ and $\overline{v}_q$ in (C.3) and (C.4) imply that $\max\{y^\ell_{q(\ell)}, \hat{\mu}y^h_{q(h)}\} \leq v \leq \hat{\mu}y^h_{q(h)} + (1-\hat{\mu})y^\ell_{q(\ell)}$ if and only if v is the limit agenda-setter's expected payoff along a sequence of equilibria as players become perfectly patient. Moreover, we have $\max\{y^\ell_{q(\ell)}, \hat{\mu}y^h_{q(h)}\} = y^\ell_{q(\ell)} < \hat{\mu}y^h_{q(h)} + (1-\hat{\mu})y^\ell_{q(\ell)}$ when $0 < \hat{\mu} \leq \frac{y^\ell_{q(\ell)}}{y^h_{q(h)}}$. ■

Proof of Theorem C.3. Follows immediately from Benchmark 2 and the limit of $\underline{v}_q$ given in (C.4). ■