Tail Risk and Expectations*

Yeow Hwee Chua† Zu Yao Hong‡

This draft: January 2022
First draft: November 2020

Abstract

We incorporate tail risk in a Bayesian learning framework with information frictions and study how individuals' expectations respond to first and second moment shocks. In our model, we retain "rational news" through Bayesian learning and abstract from any behavioral biases. First, we show that individuals overreact under tail risk, that is, individuals are excessively optimistic and pessimistic as compared to a Bayesian learning framework without tail risk. Second, uncertainty shocks lead to more pessimistic forecasts. Third, the magnitude of overreaction depends on the level of uncertainty in the economy. We provide empirical support for our theoretical findings using quantile regressions and event studies. Our findings shed light on factors driving overreaction in expectations and highlight the importance of uncertainty shocks in propagating macroeconomic stability.

JEL classification: E7, E32, E44, D84

Keywords: Tail Risk, Uncertainty, Overreaction

*We thank Sumit Agarwal, Pierre De Leo, Allan Drazen, Nuno Limão, John Shea, Luminita Stevens and various participants at the University of Maryland Brownbag session, and conference participants at the Western Economic Association International Virtual Conference, Royal Economic Society Conference, Fifth PKU-NUS Annual International Conference on Quantitative Finance and Economics, Spring Meeting of Young Economists 2021, Asian Meeting of Econometric Society, and the Money, Macro and Finance Conference for helpful comments. We also thank Ayse Sapci and Marco Brianti for their insightful discussions.

†Department of Economics, Nanyang Technological University, (yeowhwee@gmail.com).
‡Department of Economics, University of Maryland, (zhong1@umd.edu).

1 Introduction

Tail risks refer to the probability of extreme negative losses and play an important role in the financial stability and resilience of the financial system (Adrian et al. (2019)). There has been a growing interest in the study of tail risks as policymakers highlighted how changes in financial conditions could shift the distribution of future GDP (Duprey and Ueberfeldt (2020), Aikman et al. (2018), IMF (2017)), resulting in larger effects for downside risk. Recent studies have highlighted that tail risks in the economy will significantly impact the growth of real economic activity (Loria et al. (2019)) and can endogenously generate persistent changes in beliefs and economic activity (Kozlowski et al. (2019)). In this paper, we build on the vast literature studying Bayesian learning in macroeconomics (Baley and Veldkamp (2021)) and study how Bayesian learning with tail risks could impact the formation of macroeconomic expectations.

The expectations of individuals play a central role in the study of macroeconomics. In particular, studies have shown that individuals systematically respond to information given (Coibion and Gorodnichenko (2015)) and that there exists overreaction in macroeconomic expectations (Bordalo et al. (2020), Angeletos et al. (2020), Afrouzi et al. (2020)). By incorporating tail risks in a Bayesian learning framework and studying how expectations of economic agents change in response to first and second moment shocks in our model, we take a complementary approach to the existing work. In our model, we retain "rational news" through Bayesian learning and abstract from any behavioral biases. Relative to a model without tail risks, we find that individuals overreact under tail risk and that uncertainty shocks lead to increased pessimism. Much work has been done in examining the psychology of tail events (Barberis (2013)). Earlier studies have also shown that overreaction in expectations and increased pessimism, due to first and second moment shocks, can occur under additional behavioral assumption, such as diagnostic expectations (Bordalo et al. (2020)) and loss aversion (Chatterjee and Milani (2020)). The key contribution of this paper is to provide theoretical and empirical results that highlight the relationship between tail risk and expectations of individuals without imposing additional restrictions. We rely primarily on imperfect information in the form of a signal extraction problem in a Bayesian learning environment. Consequently, they are portable and can be applied in other settings, such as the stock markets and credit markets.

To study tail risk and expectations, we rely on noisy information models (Mackowiak and Wiederholt (2009), Sims (2003)) when there is a constraint on information flows. Just like the real world, economic agents cannot observe a hidden state, such as GDP growth, perfectly. Instead, they observe an imperfect signal of the hidden state and form expectations from a known distribution. In our model, we follow Gourio (2012) and use disaster risk (with time-varying risk premia) to incorporate tail risk. The presence of disaster risk implies that the distribution of the hidden state can shift downwards with some probability p. Disaster risk has been shown to account for puzzles in stock market prices (Wachter (2013), Gabaix (2012), Barro (2006)), bond prices (Singleton (2021)) and exchange rate dynamics (Gabaix and Farhi (2016)). In estimating the time-varying disaster probability, Barro and Liao (2021) show that the disaster probability is highly correlated across different countries and is able to forecast economic growth. Recent studies have also incorporated information frictions and investors' learning in disaster risk models to study asset prices (Ghaderi et al. (2021)). Here, we focus on the direct interaction of first and second moments shock on expectations when there are information frictions and disaster risks.

In our model, when forecasters observe a signal from the economy, they cannot tell if the signal they received is due to variation in the noise of the signal, the actual distribution itself or disaster risk. Consequently, they behave differently when faced with first and second moment shocks. We have three main findings.

First, we show in our model that individuals overreact when there is a first moment shock, that is, individuals are excessively optimistic and pessimistic compared to a Bayesian learning framework without tail risk. The underlying mechanisms for overreaction with tail risks are as follows: When forecasters receive a bad signal, they cannot tell if the bad signal is due to the disaster shock. As such, they overweigh the bad signal to the disaster shock and become overly pessimistic. On the other hand, when forecasters receive a favorable signal, they are confident of the absence of disaster risk. As such, they underweigh the disaster shock and become overly optimistic.

Second, a positive shock to uncertainty (defined as the variance of noise in the signal) leads to a decrease in posterior expectations (defined as expectations of the hidden state conditional on signals received) in a model with tail risk. This implies more pessimistic forecasts in an environment with higher uncertainty. When there is an increase in the variance of noise in the signal, due to downside risk, forecasters perceive an increase in the probability of a low hidden state, even though they receive a neutral signal. Consequently, forecasters attribute a higher weight to disaster risk. Hence, this lowers their expectations of the hidden state. In comparison, in the absence of tail risks, uncertainty shocks do not influence posterior expectations as there is no disaster risk.

Third, the magnitude of overreaction under tail risk depends on the level of uncertainty in the economy. Consider a positive shock to the hidden state. With high uncertainty from the signal, the forecasters will still attribute a higher weight to the disaster risk unconditionally and lower their expectations. At the same time, the forecasters understand that the probability of receiving a good signal, given that the actual hidden state is high, increases due to higher uncertainty. Hence, forecasters become relatively more optimistic in an environment of high uncertainty compared to low uncertainty. As such, this generates larger optimistic overreaction behavior when uncertainty is high. In contrast, when there is a negative shock to the hidden state, forecasters understand that the probability of obtaining a bad signal, given that the actual hidden state is low, will increase. As such, forecasters attribute an even higher weight to the disaster risk. This suggests that bad shocks to the hidden state generate larger pessimistic overreaction behavior under a high uncertainty environment.

We then take our model predictions to the data. We use a combination of data to test our theoretical predictions. To measure posterior expectations, we use data from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters (SPF) and the University of Michigan's Consumer Sentiment Index. We then use the VIX index traded in the Chicago Board Options Exchange to measure uncertainty. To measure overreaction behavior, we use data from the SPF and follow the approach in Bordalo et al. (2020) to construct time-varying overreaction measures by regressing forecast errors on forecast revisions across all forecasters at a time period.

In this paper, we identify tail risk episodes by following the approach of Adrian et al. (2019) in studying the distribution of macroeconomic risk through quantile regressions. This is done by estimating the distribution of annualized GDP growth and retrieving the fitted values of quantiles. We then define tail risk as the difference between the median and the 5th percentile of GDP 4 quarters ahead. Tail risk episodes are subsequently defined as events when the measure of tail risk is greater than its 75th percentile.[1]

Our empirical findings are consistent with our theoretical results. Using an event study approach and the constructed overreaction measures, we first show more overreaction under tail risk episodes. We then show in a regression that an increase in uncertainty is associated with a larger decrease in expectations when the economy exhibits tail risk. Finally, we find that the magnitude of overreaction is higher when the level of uncertainty in the economy is higher. Moreover, we did a series of robustness tests. For instance, one might be concerned that our results are driven by recessions and not tail risk events. To allay these concerns, we only consider tail risk episodes which do not coincide with recessions and find similar results. We also provided alternative specifications, such as focusing on OLS regressions (instead of event study) and using alternative definitions of uncertainty, such as the macroeconomic uncertainty based on Jurado et al. (2015). Our findings remain unchanged.

Our theoretical and empirical findings show that tail risks could potentially be a key driver for overreaction behavior. There are several implications. First, we are able to relate our findings to emerging literature of the drivers and implications of diagnostic

[1]Our results are robust if we consider other thresholds. This is shown in Section 4.3.

expectations (Bianchi et al. (2021), Bordalo et al. (2018)). Leveraging on a psychologically founded non-Bayesian model of belief formation, diagnostic expectations is seen to provide much tractability in explaining macroeconomic outcomes such as overreaction in expectations. A back of envelope exercise shows that our tail risk model is able to explain 14 to 40% of overreaction in the model with diagnostic expectations. In addition, our results highlight the importance of tail risk in propagating macroeconomic stability. As highlighted by Simsek (2021), changes in the beliefs of tail risks can explain asset bubbles. In our model, we find that when there is a decrease in tail risk, uncertainty shock leads to a smaller fall in expectations and individuals overreact less. This suggests that macroprudential policies (Galati and Moessner (2013) and Claessens (2015)) that reduce downside risk can dampen the effect of uncertainty shocks on the economy through expectations and volatile macroeconomic moments caused by overreaction in individuals.

Related Literature. Our findings combine several strands of innovations in the research of modern business cycles: tail risk, overreaction, and uncertainty. First, this paper directly contributes to the literature on tail risk. In asset pricing, tail risk has shown to have strong predictive power for aggregate market returns (Kelly and Jiang (2014)) and generate higher aggregate risky premia (Ai and Bhandari (2021)). Several studies have also embarked on a non-parametric approach to estimate the distribution faced by economic agents. These studies show that the presence of tail risks generate pessimistic beliefs which are persistent and stagnant, leading to slow recoveries from recessions (Kozlowski et al. (2020)), large spikes in uncertainty (Orlik and Veldkamp (2014)), and increased stock market volatility (Wachter and Zhu (2019)). Our paper contributes to the literature by showing that this behavior of persistent and stagnant pessimistic beliefs can be attributed to the interaction between tail risks and uncertainty. In contrast to Orlik and Veldkamp (2014), which study tail risks as a cause of uncertainty, we highlight the effect of uncertainty on beliefs and expectations in the environment of tail risk.

Earlier studies have shown that economic policies are able to manage tail risk (Hattori et al. (2016), Brunnermeier and Sannikov (2014)). In this paper, we seek to highlight the importance of tail risks and expectations in economic policies. While central banks have long weighed downside risks in managing the economy (Kilian and Manganelli (2008)), recent work have underscored the importance of macroprudential policies to ensure stability in the economy. For instance, Aikman et al. (2019) documented that the negative impact of the global financial crisis could have been reduced significantly by the use of macroprudential policies. In addition, Cerutti et al. (2017) used a cross-country sample to show that macroprudential policies are effective in reducing credit growth, and Claessens et al. (2013) used data from individual banks to show that macroprudential policies reduce leverage and asset prices. Our paper adds to the literature by studying the role of expectations, financial stability, and tail risk.

Second, we contribute to the literature studying the behavior of overreaction. Among others, it has been shown that individuals extrapolate their expectations in stock markets (Greenwood and Shleifer (2014)), bank stocks (Baron and Xiong (2017)), house prices (Kuchler and Zafar (2019)) and bond yields (Brooks et al. (2018)). To reconcile these findings, many studies have departed from the assumption of rational expectations and rationalized the results with different behavioral biases such as diagnostic expectations (Bordalo et al. (2019)) and natural expectations (Fuster et al. (2010)). This provides a more realistic account of the business cycle. For instance, diagnostic expectations have shown to be able to account for credit cycle features, such as counter-cyclical credit spreads (Bordalo et al. (2018)). Through diagnostic expectations, perceived productivity distribution shifts to the left (right) when there is a bad (good) news shock, leading to overreaction in macroeconomic expectations. In contrast, our paper does not rely on behavioral biases to generate overreaction. Using Bayesian learning, our model shows that tail risks with information frictions could be a potential cause of overreaction behavior in individuals. In addition, we show different magnitudes of overreaction over the business cycle. One possible reason is uncertainty.

Finally, we seek to contribute to the study of uncertainty in the macroeconomy. Since Bloom (2009)'s seminal work, many studies have documented the importance of uncertainty shocks in influencing business cycles, financial crisis and asset price fluctuations (Bloom et al. (2018), Ordonez (2013), Pastor and Veronesi (2012)). Much work has also shown that sentiments, defined as exogenous variation in expectations, have led to changes in business cycle fluctuations and credit cycles (Angeletos and La'O (2013), Benhabib et al. (2019), Bordalo et al. (2018)). Chatterjee and Milani (2020) used an asymmetric loss function to highlight the role of *perceived* uncertainty shocks in affecting expectations in business cycles.[2] In contrast, we abstract from behavioral biases, such as loss aversion, and show that *actual* uncertainty shocks can lead to a decrease in expectations under tail risk. Also, unlike Bloom (2009), who relied on real frictions in generating effects of uncertainty on output, we show that expectations can be an additional channel of the propagation of uncertainty shocks to the economy. In doing so, we hope to shed light on another dimension in the literature.

The remainder of the paper is organised as follows. Section 2 presents the model. Section 3 presents the empirical results. Section 4 provides some policy and quantitative implications, as well as robustness tests for our empirical results. Section 5 concludes. All proofs are relegated to the online appendix.

[2] In Chatterjee and Milani (2020), an asymmetric loss function implies that individuals exhibit loss aversion.

2 Model

In forming their expectations, economic agents are subject to constraints in information flows. For instance, in Lucas (1972)'s islands model, agents cannot observe all the prices in the economy. In Kydland and Prescott (1982)'s business cycle model, agents cannot distinguish between transitory and permanent productivity shocks. In this paper, we incorporate tail risks in a Bayesian learning framework with information frictions and use a signal extraction problem to show that tail risk matters.

We denote our main model as the tail risk model. In comparison, the model without any tail risk is denoted as the benchmark model. We then incorporate first and second moment shocks, and compare the dynamics of individual behavior between the tail risk model and the benchmark model.

2.1 Set-up

Consider N forecasters. Each forecaster forms expectations of a hidden state Z_t.[3] The hidden state can be GDP growth, inflation rate, unemployment rate etc. All forecasters face an identical process of Z_t. Denote $z_t = \log Z_t$. There are two periods in the model.

1. *Period 0.* Each forecaster starts with a belief about the hidden state Z_t. This is denoted by the prior distribution of Z_t: $f(z_t)$.

2. *Period 1.* Each forecaster then observes a signal $s^z_{i,t}$. The signal $s^z_{i,t}$ is given by:

$$s^z_{i,t} = z_t + e_{i,t} \tag{1}$$

   where $e_{i,t} \sim N(0, \sigma^{e2}_t)$. The noise $e_{i,t}$ and signal $s^z_{i,t}$ are heterogeneous across each forecaster. However, we assume that σ^e_t is common across forecasters as each forecaster faces the same level of uncertainty. This follows the process:

$$\log \sigma^e_t = v_t \tag{2}$$

   where $v_t \sim N(0, \sigma^2_v)$.

   Based on their signals received, they update their beliefs about the hidden state Z_t. This is denoted by the posterior distribution of Z_t: $f(z_t | s^z_{i,t})$.

[3] In standard real business cycle models, Z_t refers to the total productivity process.

We begin with some definitions:

Definition 1. *Prior expectations are defined as the unconditional expected value of the hidden state z_t: $E_i(z_t)$. Posterior expectations are defined as the expected value of the hidden state z_t, conditional on realized signal $s^z_{i,t}$: $E_i(z_t|s^z_{i,t})$.*

Forecasters start with their prior expectations of z_t, which is defined to be $E(z_t)$. The prior expectations are derived from each forecaster's prior beliefs of the distribution $f(z_t)$. After each forecaster observes the signal $s^z_{i,t}$, they will form expectations of z_t through Bayesian learning. They update their expectations to $E_i(z_t|s^z_{i,t})$ after observing $s^z_{i,t}$. This is derived from each forecaster's posterior beliefs of the distribution $f(z_t|s^z_{i,t})$. As signals are heterogeneous across each forecaster, updated expectations differ across each forecaster. This captures the cross-sectional heterogeneity in forecasts. Prior expectations do not differ across forecasters and do not depend on the signals received.[4] In contrast, posterior expectations depend on the signals received, and hence, they can differ across forecasters.

Definition 2. *Prior Uncertainty is defined as $Var(e_{i,t})$. Posterior Uncertainty is defined as the variance of z_t, conditional on realized signal $s^z_{i,t}$: $Var_i(z_t|s^z_{i,t})$.*

Definition 2 states that prior uncertainty is defined as the unconditional variance of the exogenous process of the noise in the signal.[5] Posterior uncertainty is defined as the variance of the productivity process, conditional on receiving the signal. In short, posterior uncertainty captures the beliefs and perception of the hidden state for each forecaster after receiving the signal. Also, an increase in prior uncertainty in the benchmark and tail risk models leads to a rise in posterior uncertainty. This shows that the general definition of uncertainty is consistent across prior and posterior uncertainty.

We now present the benchmark and tail risk model.

A. The Benchmark Model.

In the benchmark model, Z_t follows the following process:

$$\log Z_t = u_t \tag{3}$$

where $u_t \sim N(0, \sigma_z^2)$.

In the presence of information frictions, each forecaster i cannot observe Z_t directly. Instead, they observe an imperfect signal of the hidden state, denoted by $s^z_{i,t}$. Hence, $s^z_{i,t}$

[4] This holds in a static model where there is no persistence of the evolution of z_t.

[5] The results of this paper also hold if prior uncertainty is defined as the variance of the actual hidden state itself, $Var(z_t)$. We assume our current definition of prior uncertainty for analytical tractability and to keep the economic intuition simple.

is given by

$$s_{i,t}^{z} = u_t + e_{i,t} \tag{4}$$

Before forecasters observe the signal, they believe that the prior distribution of z_t in the benchmark model is given by:

$$f(z_t) = \phi\Big(\frac{z_t}{\sigma_z}\Big) \tag{5}$$

After observing the signal $s_{i,t}^{z}$, forecasters generate a posterior distribution of z_t, which is given by:

$$f(z_t|s_{i,t}^{z}) \propto f(s_{i,t}^{z}|z_t) \cdot f(z_t) = \phi\Big(\frac{s_{i,t}^{z} - z_t}{\sigma_t^{e}}\Big) \cdot \phi\Big(\frac{z_t}{\sigma_z}\Big) \tag{6}$$

B. The Tail Risk Model.

We now turn to our tail risk model. Here, we follow Gourio (2012) and use disaster risk to model tail risk. This builds on studies such as Kozlowski et al. (2019) which associates tail risk with rare disasters. In particular, the hidden state follows the process:

$$\log Z_t = -I_t\gamma + u_t \tag{7}$$

where I_t is an indicator that equals 1 if the economy is in a disaster state, and equals 0 if the economy is not in a disaster state. Hence, I_t follows a Bernoulli distribution with $Pr(I_t = 1) = p$. As such, $\log Z_t$ follows a normal distribution of mean 0 with probability $1 - p$, and a normal distribution of mean $-\gamma$ with probability p. Consequently, the prior distribution of the hidden state z_t is given by:

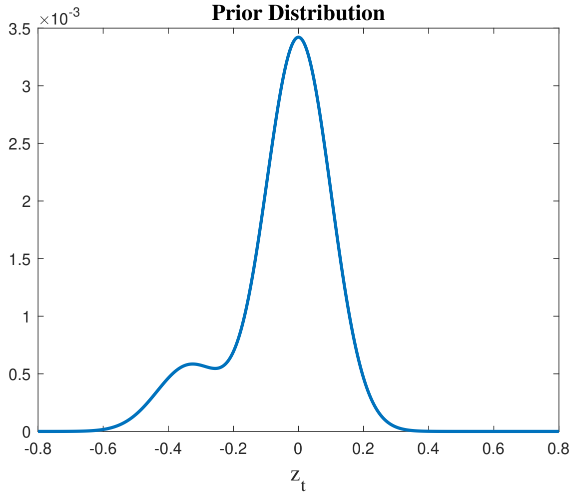
$$f(z_t) = (1 - p)\phi\Big(\frac{z_t}{\sigma_z}\Big) + p\phi\Big(\frac{z_t + \gamma}{\sigma_z}\Big) \tag{8}$$

where $\phi(\cdot)$ is the probability density function (PDF) of a standard normal distribution. Unlike the benchmark model, the probability density function of z_t is not a normal distribution. It is now a mixture of normal distributions, in which the mean of one of the normal distributions is lower than the other corresponding normal distribution by a fixed amount γ due to the disaster risk.

We provide a concrete example in Figure 1 based on the parameter values used in the signal extraction problem in Table 1. We want to highlight two parameters in this example. First, we match the decline of the mean of the disaster state, γ, with the decline in the mean of the GDP distribution (relative to its standard deviation) during a recession in tail risk episodes.[6] Second, our choice of the probability of the disaster state, p, matches the skewness of the GDP distribution in 2008Q4 generated by quantile regressions.[7]

Figure 1 plots the corresponding distribution of prior beliefs for all forecasters, given by the PDF of z_t. As depicted in Figure 1, the prior distribution is not symmetric around the mean and exhibits "tail risk", where the left tail exhibits more mass than the right tail. The behavior of tail risk has been emphasized in Kozlowski et al. (2019). This can be due to the occurrence of extreme events that are rare and unusual. As such, business cycle dynamics are asymmetric and left-skewed.

Figure 1: Distribution of z_t



Notes: This figure presents the distribution of prior beliefs for all forecasters based on the parameters in Table 1 for the tail risk model.

Signals observed by each forecaster i at each time period t under the tail risk model can then be written as follows:

$$s^z_{i,t} = -I_t\gamma + u_t + e_{i,t} \tag{9}$$

In the benchmark model, the signal is a sum of normal variables (Equation (4)). Hence, the posterior beliefs of each forecaster can be written as a normal random variable. However, in this case, the posterior distribution does not follow a normal distribution and

[6]The measure of tail risk used in this paper refers to the difference between the 5th percentile and the median of the GDP distribution estimated by quantile regressions. The quantile regression is specified in Section 3.1, Equation (28), where we regress average GDP growth four quarters ahead on current GDP growth. Tail risk episodes are defined as the time period in which the measure of tail risk is greater than its 75th percentile. This is specified in Section 3.1, Definition 5.

[7]We choose to match the skewness of the GDP distribution in 2008Q4, which has the largest tail risk measure across the sample period from 1978 to 2016.

Table 1: Parameters Used in the Signal Extraction Problem

Mean of Hidden State	μ_z	0
SD of Hidden State	σ_z	0.1
Autocorrelation of Hidden State	ρ_z	0
Mean of Uncertainty Process	μ_σ	0.1
Autocorrelation of Uncertainty Process	ρ_σ	0
Probability of Disaster State	p	0.143
Shift of mean in the Disaster State	γ	0.335

Notes: This table shows the parameter values used for the calibration of Figure 1.

can be written as:

$$f(z_t|s^z_{i,t}) \propto f(s^z_{i,t}|z_t) \cdot f(z_t) = \phi\Big(\frac{s^z_{i,t} - z_t}{\sigma^e_t}\Big) \cdot \Big[(1-p)\phi\Big(\frac{z_t}{\sigma_z}\Big) + p\phi\Big(\frac{z_t + \gamma}{\sigma_z}\Big)\Big] \tag{10}$$

In the next section, we analyze how expectations respond to first and second moment shocks in the tail risk model compared to the benchmark model. We want to highlight that we induce first and second moment shocks in our model while conditioning on the absence of an actual disaster shock. This implies that the actual value of I_t will always be equal to 0. However, from the lens of the forecaster, I_t takes a value of 1 with probability p as the forecaster does not observe the actual occurance of a disaster shock.

2.2 Theoretical Results

In this section, we state the key propositions and the relevant results. All the proofs are available in the Online Appendix.

Proposition 1. *In the benchmark model, posterior expectations are given by:*

$$E(z_t|s^z_{i,t}) = r_t\mu_z + (1-r_t)s^z_{i,t} \tag{11}$$

In the tail risk model, posterior expectations are given by

$$E(z_t|s^z_{i,t}) = r_t\mu_z + (1-r_t)s^z_{i,t} - B(p, \gamma, \sigma^e_t, s^z_{i,t}) \tag{12}$$

where $\mu_z = 0$ and,

$$r_t = \frac{\sigma^{e2}_t}{\sigma^{e2}_t + \sigma^2_z} \tag{13}$$

$$B(p,\gamma,\sigma_t^e,s_{i,t}^z) = \frac{r_t \gamma p e^{A_{12}+A_{13}}}{p e^{A_{12}+A_{13}} + (1-p) e^{A_{22}+A_{23}}} \tag{14}$$

such that

$$A_{12} = -\frac{1}{2} \frac{(\mu_z - \gamma)^2 \sigma_t^{e2} + \sigma_z^2 s_{i,t}^{z2}}{\sigma_t^{e2} \sigma_z^2} \tag{15}$$

$$A_{13} = \frac{1}{2} \frac{\left(r_t(\mu_z - \gamma) + (1 - r_t) s_{i,t}^z\right)^2}{\sigma_{z|s_{i,t}}^2} \tag{16}$$

$$A_{22} = -\frac{1}{2} \frac{\mu_z^2 \sigma_t^{e2} + \sigma_z^2 s_{i,t}^{z2}}{\sigma_t^{e2} \sigma_z^2} \tag{17}$$

$$A_{23} = \frac{1}{2} \frac{\left(r_t \mu_z + (1 - r_t) s_{i,t}^z\right)^2}{\sigma_{z|s_{i,t}}^2} \tag{18}$$

In the benchmark model, posterior expectations are a result of standard Bayesian learning of normal random variables. However, in the tail risk model, posterior expectations include an additional term: $B(p,\gamma,\sigma_t^e,s_{i,t}^z)$. Unsurprisingly, $B(p,\gamma,\sigma_t^e,s_{i,t}^z)$ depends on the parameters of tail risk p and γ. When $p = 0$ and $\gamma = 0$, then posterior expectations in the tail risk model collapse to the posterior expectations in the benchmark model.

Suppose a rare disaster did not occur, and the forecaster receives a signal $s_{i,t}^z = 0$. Even though $\Pr(z_t = 0)$ is the local maximum, due to tail risk, the forecaster's prior expectation of the hidden state is less than 0. As such, the forecaster revises his expectations downward. This implies that in the presence of tail risks, posterior expectations exhibit downward bias (less than 0) compared to their counterpart in the benchmark model (equal to 0). Our findings are consistent with Orlik and Veldkamp (2014) who find that the presence of tail risks explains downward forecast bias present in the data.

We now consider the median forecaster in our analysis.

Definition 3. *A median forecaster receives a signal where* $e_{i,t} = 0$.

The cross-section heterogeneity in forecasts is driven by $e_{i,t}$. Low (high) forecasts are a consequence of receiving a low (high) value of $e_{i,t}$. Since $e_{i,t}$ is a normal random variable with zero mean, hence, the median forecaster receives a signal where $e_{i,t} = 0$.

2.2.1 Forecasters Overreact to First Moment Shocks

In this section, we analyze how posterior expectations change in response to a first moment shock. Recall that in the absence of any shock, the median forecaster will exhibit downward forecast bias in the tail risk model compared to the baseline model. We account for the downward forecast bias by considering the change in expectations when there is a shock to the hidden state in both the benchmark and tail risk models. This implies that in the tail risk model:

$$E(\Delta z_{i,t}^{TR}|s_{i,t}) = (1 - r_t)(s_{i,t}^z - \mu_z) - B(p, \gamma, \sigma_t^e, s_{i,t}^z) + B(p, \gamma, \sigma_t^e, \mu_z) \tag{19}$$

The change in expectations in the benchmark model is given by:

$$E(\Delta z_{i,t}^{N}|s_{i,t}) = (1 - r_t)(s_{i,t}^z - \mu_z) \tag{20}$$

Proposition 2. *The median forecaster ($e_{i,t} = 0$) overreacts to news, that is,*

$$E(\Delta z_{i,t}^{TR}|s_{i,t}) > E(\Delta z_{i,t}^{N}|s_{i,t}), \text{ if } u_{i,t} > 0 \tag{21}$$

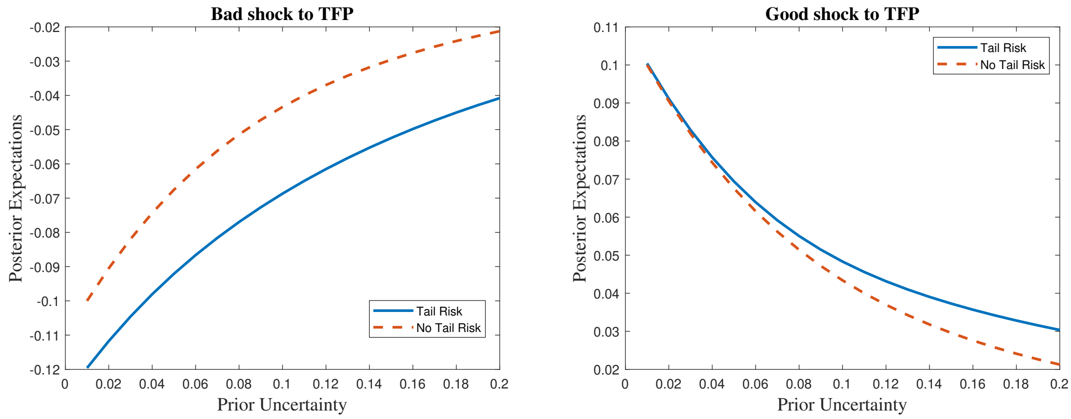
$$E(\Delta z_{i,t}^{TR}|s_{i,t}) < E(\Delta z_{i,t}^{N}|s_{i,t}), \text{ if } u_{i,t} < 0 \tag{22}$$

Consider a positive shock to the hidden state, z_t. Proposition 2 shows that individuals overreact to news under the model with tail risk. In Figure 2, we plot the dynamics of the forecasts due to shocks to the hidden state and study the outcomes in both the benchmark and tail risk models. Figures 2 (a) and 2 (b) examine the impact of a decrease and increase of the hidden state, z_t by one standard deviation, respectively. In Figure 2 (a), we find that forecasts are overly pessimistic when there is a negative shock to the hidden state ($u_{i,t} < 0$). The posterior expectations in the tail risk model are systematically lower than that of the benchmark model (no tail risk) for all levels of prior uncertainty. In comparison, we find that forecast revisions are overly optimistic when there is a positive shock to the hidden state ($u_{i,t} > 0$) in Figure 2(b). Here, posterior expectations are higher in the tail risk model than the benchmark model. Indeed, overreaction occurs because

$$\frac{\partial B(p, \gamma, \sigma_t^e, s_{i,t}^z)}{\partial s_{i,t}^z} < 0 \tag{23}$$

What drives overreaction? When there is a negative shock to the hidden state, the median forecaster receives a bad signal in our model. The forecaster then mistakenly attributes it to disaster risk in the model with tail risk. As such, the forecaster becomes excessively pessimistic compared to the benchmark model. This leads to pessimistic overreaction, which implies a larger decrease in expectations. Figure 3 plots the posterior distributions for the benchmark model and the model with tail risk when there is a

Figure 2: Overreaction and Posterior Expectations



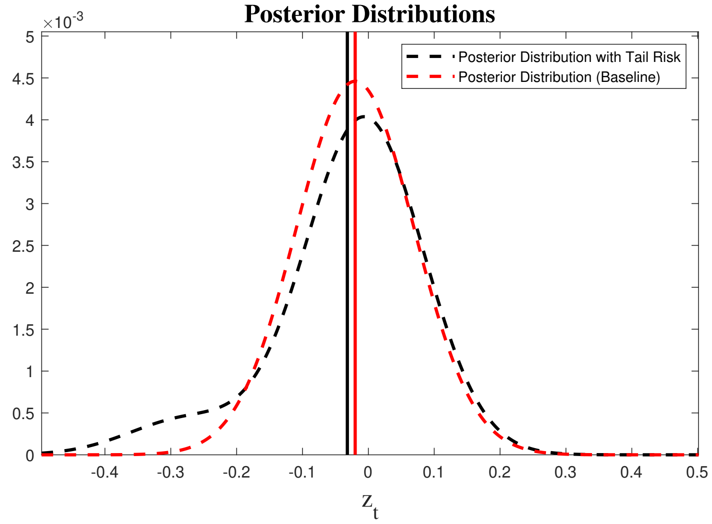
(a) Negative TFP shock (b) Positive TFP shock

Notes: This figure presents the impact of a negative and positive hidden state shock to posterior expectations in panels (a) and (b) respectively. The vertical axis in both panels denote changes in posterior expectations due to a hidden state shock. This results in overreaction behavior.

negative shock. The vertical lines are the conditional mean of the corresponding posterior distributions. As highlighted earlier, the posterior distribution in the benchmark model resembles a normal distribution. However, the posterior distribution with tail risk exhibits significant mass on its left tail, as forecasters assign a higher probability to the presence of a disaster. When we compare the peak of the distribution between the benchmark model and the model with tail risk, Figure 3 shows that the posterior distribution with tail risk is positioned slightly to the right of the posterior distribution of the benchmark model. This should increase posterior expectations in the model with tail risk compared to the benchmark model. However, this effect is dominated by the increased mass of the left tail in the posterior distribution with tail risk. Hence, overall, forecasters are overly pessimistic.

In the case of a positive shock to the hidden state, the median forecaster receives a favorable signal. Due to the good signal, the forecaster is more optimistic about the absence of a disaster shock. As such, the forecaster becomes excessively optimistic, as compared to the benchmark model. This leads to optimistic overreaction, which implies a larger increase in expectations. Figure 4 plots the posterior distributions in both models when the forecaster encounters a positive hidden state shock. The posterior distribution in the benchmark model shifts to the right of the unconditional mean at 0 and retains its shape of a normal distribution. In the model with tail risk, the posterior distribution exhibits a lesser left tail risk than the case with a negative hidden state shock, as shown in Figure 3. Nonetheless, the increased mass in the left tail is still present due to the possibility of disaster risk. However, the mass in the left tail is significantly reduced in the case of a positive hidden state shock, and the posterior distribution resembles a normal distribution. This implies that the median forecaster is relatively more confident about

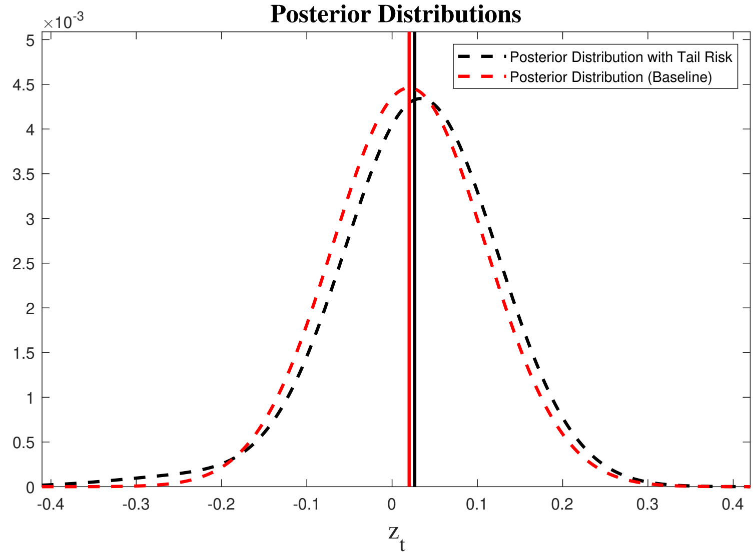
Figure 3: Overreaction with negative hidden state shock (Posterior Distributions)



Notes: This figure presents the impact of a negative hidden state shock on the posterior distribution.

the absence of a disaster shock, given a favorable signal. Similar to the previous analysis of a negative hidden state shock, the posterior distribution with tail risk is positioned to the right of the posterior distribution of the benchmark model. This effect dominates the left tail risk, and as such, forecasters are excessively optimistic in the case of a positive hidden state shock.

Figure 4: Overreaction with positive hidden state shock (Posterior Distributions)



Notes: This figure presents the impact of a positive hidden state shock on the posterior distribution.

Our findings are consistent with the evidence of overreaction in the literature. The main difference between our model and that of the literature is that we retain "rational news" in the form of Bayesian learning and generate overreaction in expectations by incorporating tail risks. We do not rely on behavioral biases or other type of expectations (such as diagnostic expectations) to generate such an effect. Since our model with tail risk in a Bayesian learning environment can generate overreaction in expectations, it is hence possible to map our model into the framework of other expectation channels such as diagnostic expectations (Bordalo et al. (2020)). This suggests that a framework with tail risks will be able to generate more volatile credit cycles due to overreaction behavior. We further discuss this in Chapter 4.2.

So far, we have only considered the median forecaster. The effects of overreaction will vary when we consider heterogeneous signals and forecasters. When the noise of the signal is positive ($e_{i,t} > 0$), forecasters will be even more optimistic when there is a positive shock to the hidden state. In contrast, when the noise of the signal is negative ($e_{i,t} < 0$), and when there is a positive shock to the hidden state, the possibility of overreaction depends on the magnitude of $e_{i,t}$ and u_t. More precisely, overreaction and excessive optimism will only occur if $e_{i,t} + u_t > 0$. Moreover, since $e_{i,t} < 0$ works against the effects of the positive hidden state shock, the magnitude of the overreaction will be lower.

When there is a negative shock to the hidden state, coupled with a favorable signal $e_{i,t} > 0$, forecasters will be less pessimistic. In this case, whether there will be overreaction or underreaction depends on the magnitudes of $e_{i,t} > 0$ and $u_t < 0$ as they work in opposite directions. If $e_{i,t} + u_t < 0$, then forecasters will still overreact and be excessively pessimistic. However, if $e_{i,t} + u_t > 0$, then forecasters will be optimistic even if there is a negative shock to the hidden state.

2.2.2 Second Moment Shocks Lead to More Pessimism

After studying the impact of a first moment shock, we turn to the impact of second moment shocks.

Proposition 3. *Consider a median forecaster. In the tail risk model, an increase in prior uncertainty decreases posterior expectations, that is,*

$$\frac{\partial E(z_t|s^z_{i,t})}{\partial \sigma^e_t} = -\frac{\partial B(p, \gamma, \sigma^e_t, s^z_{i,t})}{\partial \sigma^e_t} < 0 \tag{24}$$

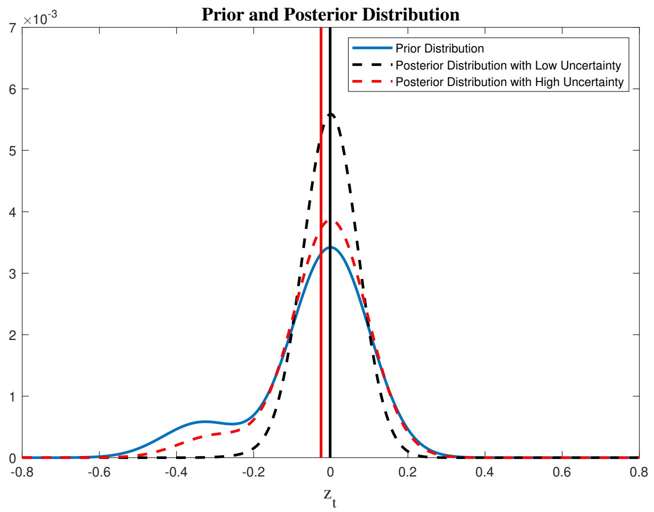
Proposition 3 shows that an increase in uncertainty, denoted by σ^e_t, decreases posterior expectations and consequently, forecasts. This has been highlighted by Chatterjee and Milani (2020) whichi rely on an asymmetric loss function to generate an adverse effect of uncertainty shocks on expectations. Nonetheless, Chatterjee and Milani (2020) did not find evidence favoring an asymmetric loss function for real GDP. In our model with tail risks, we do not rely on such an assumption. The presence of tail risks, as documented in

Adrian et al. (2019), can generate a negative effect of uncertainty shocks on expectations.

In the tail risk model, when forecasters observe a signal of the hidden state, they cannot tell whether the realized signal is due to variation in the signal's noise or the hidden state itself. As forecasters cannot differentiate between the noise of the signal and the hidden state, they also cannot differentiate if the realized signal is due to disaster risk, or a first moment shock.

When there is an increase in prior uncertainty, the variance of noise in the signal increases. Forecasters understand that the probability of receiving the current signal given that the actual value of z_t is low, that is, $\Pr(s^z_{i,t} = \mu_z | z^L_{i,t})$ increases. Hence, since $f(z_t|s^z_{i,t}) \propto f(s^z_{i,t}|z_t) \cdot f(z_t)$, this implies that forecasters perceive that the probability of a actual low hidden state, given the realized signal, will be higher. As such, forecasters attribute a higher weight to disaster risk and think that the distribution of the hidden state might have shifted leftwards. This lowers their forecasts of the hidden state, conditional on receiving the signal.

Figure 5: Relationship between uncertainty shocks and posterior expectations for median forecaster



Notes: This figure presents the posterior distribution of the forecaster's beliefs about the hidden state with low and high uncertainty by decreasing (increasing) the standard deviation by half its original value.

To study how changes in uncertainty impact the posterior distribution, we decrease (increase) the standard deviation by half its original value to simulate low (high) uncertainty respectively. This is shown in Figure 5. Figure 5 plots the posterior distribution of the forecaster's beliefs about the hidden state and compares the posterior distribution between low and high uncertainty. The corresponding vertical line shows the mean of the distribution. As shown in Figure 5, the posterior distribution with low uncertainty resembles a normal distribution. In contrast, the posterior distribution with high uncertainty exhibits left tail risk, which is closer to the shape of the prior distribution.

Unlike the model with tail risk, an increase in σ_t^e does not affect posterior expectations in the benchmark model. This is evident in Equation (11) as $B(p, \gamma, \sigma_t^e, s_{i,t}^z) = 0$ in the benchmark model. Intuitively, after receiving the signal, the median forecaster realizes that an increase in prior uncertainty increases the probability that the actual hidden state z_t can be high or low equally due to the symmetry of the normal prior distribution. Even though this increases posterior uncertainty, it has no effect on posterior expectations as the increase in optimism and pessimism exactly cancel each other out.

Corollary 1. *A forecaster who receives relatively bad (good) news receive a signal with* $e_{i,t} < 0$, ($e_{i,t} > 0$)*. An increase in prior uncertainty decreases posterior expectations to a larger (smaller) extent for a forecaster that receives relatively bad (good) news. Mathematically,*

$$\left|\frac{\partial B(p, \gamma, \sigma_t^e, s_{i,t}^z)}{\partial \sigma_t^e}\right|_{e_{i,t}<0} > \left|\frac{\partial B(p, \gamma, \sigma_t^e, s_{i,t}^z)}{\partial \sigma_t^e}\right|_{e_{i,t}>0} \tag{25}$$

A natural question is whether the magnitude of the decrease in posterior expectations due to a positive uncertainty shock differs over forecasters with different news flows. Corollary 1 shows that if a forecaster receives a bad signal, the forecaster decreases the forecasts more when prior uncertainty rises.[8] The opposite holds for a forecaster who receives a good signal.

When the forecaster receives a bad signal ($e_{i,t} < 0$), the forecaster attributes an even higher weight to the disaster risk. Hence, posterior expectations decrease by a larger extent for a given increase in prior uncertainty. In the contrary, when the forecaster receives a good signal ($e_{i,t} > 0$), due to the increase in prior uncertainty, the forecaster still attributes a higher weight to the disaster risk unconditionally. However, the forecaster realizes that the probability of obtaining a good signal given a high hidden state ($\Pr(s_{i,t}^z > \mu_z | z_{i,t}^H)$) increases. Therefore, forecasters who receive a good signal are relatively more optimistic than forecasters who receive a neutral or bad signal. Figure B.1 in the Online Appendix shows the responses of posterior expectations to prior uncertainty in the case of forecasters who receive heterogeneous signals.[9]

[8] In Corollary 1, when there is a signal with non-zero noise, we account for the effects of prior uncertainty shocks on forecasts under the benchmark model and isolate the effect of prior uncertainty shocks on forecasts that is due to tail risk.

[9] In Figure B.1, when there is a signal with non-zero noise, we account for the effects of prior uncertainty shocks on forecasts under the benchmark model and isolate the effect of prior uncertainty shocks on forecasts that is due to tail risk.

2.2.3 First and Second Moment Shocks and Larger Overreaction

Proposition 4. *The magnitude of overreaction depends on prior uncertainty σ_t^e, that is*

$$\frac{\partial |E(\Delta z_{i,t}^{TR}|s_{i,t}) - E(\Delta z_{i,t}^{N}|s_{i,t})|}{\partial \sigma_t^e} = \frac{\partial |B(p,\gamma,\sigma_t^e, s_{i,t}^z) - B(p,\gamma,\sigma_t^e,\mu_z)|}{\partial \sigma_t^e} > 0 \tag{26}$$

Proposition 4 states that individuals overreact more when prior uncertainty is higher. Consider a positive shock to u_t. Under high prior uncertainty, the forecaster will still attribute a higher weight to the disaster risk unconditionally. Therefore, this will lead to lower posterior expectations. However, at the same time, when the forecaster receives a good signal, the probability of obtaining a good signal given that the actual value of productivity is high ($\Pr(s_{i,t}^z > \mu_z | z_{i,t}^H)$), increases due to higher prior uncertainty. Hence, forecasters become relatively more optimistic in the case of high prior uncertainty compared to the case of low prior uncertainty. This generates larger optimistic overreaction behavior in the case of high prior uncertainty.

In contrast, consider a negative shock to the hidden state. When prior uncertainty is high, forecasters understand that the probability of obtaining a bad signal given that actual productivity is low ($\Pr(s_{i,t}^z < \mu_z | z_{i,t}^L)$), will be relatively higher. Hence, forecasters become more pessimistic and attribute an even higher weight to the disaster risk. Therefore, under high prior uncertainty, bad shocks to the hidden state generate larger pessimistic overreaction than lower prior uncertainty.

These findings are also evident in Figure 2. In Figure 2(a), in which we consider a negative shock to u_t, the percentage difference in posterior expectations between the benchmark model and the model with tail risk is more considerable when the level of prior uncertainty increases. Figure 2(b) illustrates the same observation in the case of a positive shock to the hidden state.

Proposition 4 implies time-varying overreaction coefficients. This is consistent with the findings of Bordalo et al. (2020), in which there is larger magnitudes of overreaction in high levels of prior uncertainty. Consequently, Proposition 4 can also explain overreaction cycles, in which following periods of large magnitudes of overreaction, forecasters exhibit lower magnitudes of overreaction. Our explanation hinges on the combination of sizeable prior uncertainty shocks and shocks to the hidden state, which generates large magnitudes of overreaction at that particular point in time. This is followed by relatively lower uncertainty and hence lower magnitudes of overreaction. This is also consistent with Orlik and Veldkamp (2014), which highlight that the magnitude of uncertainty shocks tends to spike up at a point in time before returning to lower levels.

3 Empirical Findings

Having established our theoretical results, we now turn to our empirical analysis. After we identify tail risk episodes, we use a combination of regression analysis and event studies to test our theoretical findings.

3.1 Data

The key insight of this paper requires data of expectations, uncertainty, measures of overreaction behavior, and tail risk. We consider the following data relating to the U.S economy from 1978 to 2016. To ensure consistency across the different data sets, we use quarterly data.

First, we use the University of Michigan's Consumer Sentiment Index, denoted as sentiments in this section, to measure expectations. The Consumer Sentiment Index is derived from asking consumers in the survey whether they are better or worse off financially and their perception of the economy in the future. The index is then calculated by aggregating across consumers at each point in time. The University of Michigan's Consumer Sentiment Index is a prominent indicator in public discourse (Dominitz and Manski (2003)) and is a leading indicator of consumption growth (Souleles (2004)).

For uncertainty, we use the VIX index. The VIX index is an implied stock market volatility index traded in the Chicago Board Options Exchange. It is a leading barometer of economic uncertainty (Kozeniauskas et al. (2018)). For robustness, we include other measures of uncertainty such as Jurado et al. (2015) which make use of econometric methods to measure the conditional volatility of the purely unforecastable component of the future value of the series.

To measure overreaction behavior and expectations, we use the Survey of Professional Forecasters (SPF) run by the Federal Reserve Bank of Philadelphia. It is conducted quarterly, with around 40 professional forecasters being surveyed at each point in time. The forecasters report their forecasts for the current and next four quarters for several macroeconomic outcomes. These include GDP, price indices, consumption, investment, unemployment, government consumption, housing, and financial variables such as government bonds and corporate AAA bonds (See Table 2 for the complete list). In this paper, we follow Bordalo et al. (2020) and run the following regression for each macroeconomic outcome in the survey.

$$x_{t+h} - x^i_{j,t+h|t} = \beta^i_0 + \beta^i_1 FR^i_{j,t,h} + \epsilon^i_{j,t,h} \tag{27}$$

where $x^i_{j,t+h|t}$ denote individual j's forecast of macroeconomic variable i at time t

about the future value x_{t+h} of a variable. Individual forecast revisions are denoted by $FR^i_{j,t,h} = (x_{j,t+h|t} - x^i_{j,t+h|t-1})$. Bordalo et al. (2020) pool forecasters and estimate a common coefficient β_1. If $\beta_1 < 0$, this implies that the average forecaster overreacts as upward forecast revisions predict lower realizations relative to the forecasts. Similarly, if $\beta_1 > 0$, this implies that the average forecaster underreacts to his own information and upward forecast revisions predict higher realizations relative to the forecasts. In this paper, we run the same regression for each quarter and each macroeconomic variable while pooling all forecasters. Hence, we extract $\beta^i_{1,t}$ where t refers to each quarter and i denotes each macroeconomic variable.

Table 2 provides summary statistics of the overreaction coefficients derived from the regression in Equation (27). We consider 15 macroeconomic variables from the SPF. We find that the measures of overreaction constructed are mostly negative across all variables and time, suggesting the presence of overreaction behavior among individuals. [10]

Table 2: Descriptive Statistics of Overreaction Coefficients

Variable:	Obs. (1)	Mean (2)	St. Dev. (3)	Min. (4)	Median. (5)	Max. (6)
Real GDP	39	-0.42	0.22	-1.00	-0.42	0.21
Nominal GDP	39	-0.44	0.34	-1.80	-0.44	0.42
Industrial Production	39	-0.45	0.28	-1.11	-0.45	0.17
Real Consumption	36	-0.46	0.27	-1.25	-0.43	-0.03
GDP Price Index Inflation	39	-0.44	0.13	-0.74	-0.42	-0.21
Consumer Price Index	36	-0.52	0.28	-1.38	-0.47	0.02
Real Nonresidential Investment	39	-0.41	0.25	-0.80	-0.44	0.20
Real Residential Investment	39	-0.22	0.32	-1.03	-0.25	0.87
Real Federal Government Consumption	36	-0.51	0.26	-1.10	-0.50	0.20
Real State and Local Government Consumption	36	-0.43	0.21	-0.79	-0.40	0.03
Housing start	36	-0.56	0.29	-1.28	-0.56	-0.08
Unemployment	36	-0.21	0.38	-1.04	-0.22	1.04
Three-month Treasury Rate	25	-0.39	0.23	-0.83	-0.38	0.08
Ten-year Treasury Rate	36	-0.40	0.24	-0.78	-0.36	0.24
AAA Corporate Bond Rate	36	-0.54	0.22	-1.03	-0.55	0.04

Notes: Columns (1) to (6) show the summary statistics for overreaction coefficients. Each overreaction coefficient at time t for variable i is obtained from running the regression of Equation (27) across all forecasters.

Finally, we obtain GDP data from the US Bureau of Economic Analysis to construct measures of tail risk. In order to identify tail risks, we follow Adrian et al. (2019) and estimate the distribution of GDP growth semi-parametrically by running quantile regres-

[10] Note that we are unable to retrieve some of the overreaction coefficients as some observations of forecasts are missing.

sions. We run the following quantile regression.

$$\hat{Q}_{y_{t+h}|x_t}(\tau|x_t) = x_t\hat{\beta}_\tau^Q \tag{28}$$

where x_t refers to the annualized growth rate of GDP at time t, y_{t+h} the annualized average growth rate of GDP four quarters ahead and τ refers to the percentile of the distribution.

We then extract measures of skewness in the quantile regression. In particular, we extract $\hat{Q}_{\tau=0.50} - \hat{Q}_{\tau=0.05}$ and $\hat{Q}_{\tau=0.95} - \hat{Q}_{\tau=0.50}$ to measure the left tail mass and right tail mass of the GDP growth distribution respectively. Table 3 shows summary statistics of these measures.

Table 3: Descriptive Statistics of Tail Risks

Variable:	Obs. (1)	Mean (2)	St. Dev. (3)	75th (4)	90th (5)
$\hat{Q}_{\tau=0.50} - \hat{Q}_{\tau=0.05}$	156	3.27	0.31	3.43	3.63
$\hat{Q}_{\tau=0.95} - \hat{Q}_{\tau=0.50}$	156	1.89	0.09	1.94	1.99

Notes: Columns (1) to (5) show statistics for the corresponding variables. $\hat{Q}_\tau$ refers to the estimated value of average real GDP at quantile τ from the regression in Equation (28).

We observe that in general, $\hat{Q}_{\tau=0.50} - \hat{Q}_{\tau=0.05} > \hat{Q}_{\tau=0.95} - \hat{Q}_{\tau=0.50}$, which shows that the GDP distribution is highly skewed with fatter tails to the left of the distribution. Also, right tail measures are fairly stable with a low standard deviation and interquartile range. In contrast, left tail measures are highly volatile, suggesting time-varying downside tail risks. These facts are consistent with Adrian et al. (2019). In this paper, we define tail risk as the left tail measure calculated from quantile regressions. This is similar to the concept of growth-at-risk that has been adopted in recent papers that study GDP tail risks (Adrian et al. (Forthcoming), Duprey and Ueberfeldt (2020)).

Definition 4. *Tail risk is defined as the difference between the median and the 5th percentile of GDP 4 quarters ahead, that is* $\hat{Q}_{y_{t+h}|x_t}(\tau = 0.50|x_t) - \hat{Q}_{y_{t+h}|x_t}(\tau = 0.05|x_t)$.

We then define tail risk episodes as the time period in which tail risk is greater than its 75th percentile. Empirical results remain robust if we consider other thresholds.

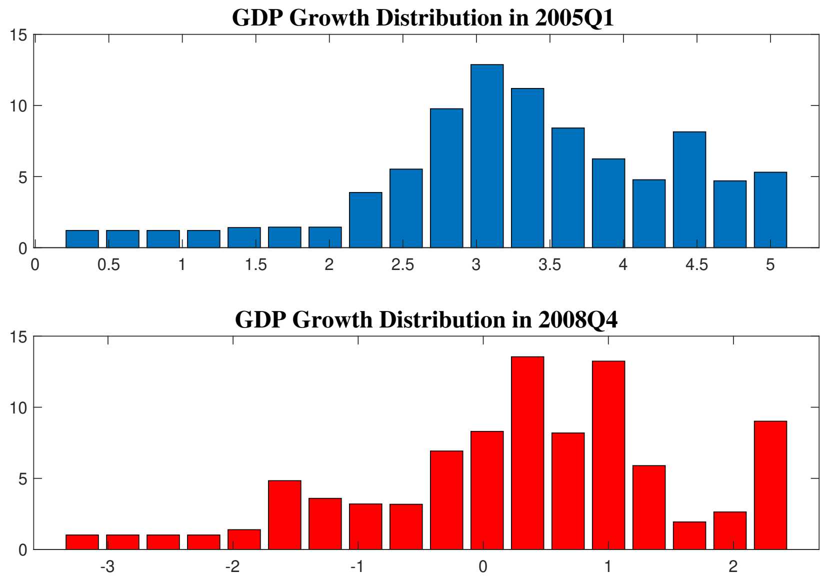
Definition 5. *Tail risk episode is defined as the time period in which tail risk is greater than its 75th percentile.*

We would like to highlight two points with regard to tail risks.

First, even though the measure of tail risk used in this paper is a measure of skewness, skewness of the GDP growth distribution does not imply tail risk. To give a concrete example, we highlight two time periods: 2005Q1 and 2008Q4. By fitting different values of percentiles (with a grid interval of five percentiles) with our quantile regression, we can estimate the GDP growth distribution at each point in time. Figure 6 plots the estimated GDP growth distribution in 2005Q1 and 2008Q4. Based on our definition, 2008Q4 is in a period with tail risk episode, while 2005Q1 is not.

Although both distributions exhibit a left tail which implies negative skewness, the distribution in 2005Q1 is closer to a normal distribution.[11] When we consider the GDP growth distribution in 2008Q4, the distribution display fatter left tails. In particular, as the distribution flattens to the left of its peak, the probability density function rises up again as the values of GDP growth rates diminish. This resembles the theoretical distribution depicted in Figure 1 of the paper. Comparing between other periods of tail risk and non-tail risk episodes yield distributions similar to the ones depicted in Figure 6.

Figure 6: GDP Distributions



Notes: This figure shows the estimated GDP growth distribution in 2005Q1 and 2008Q4.

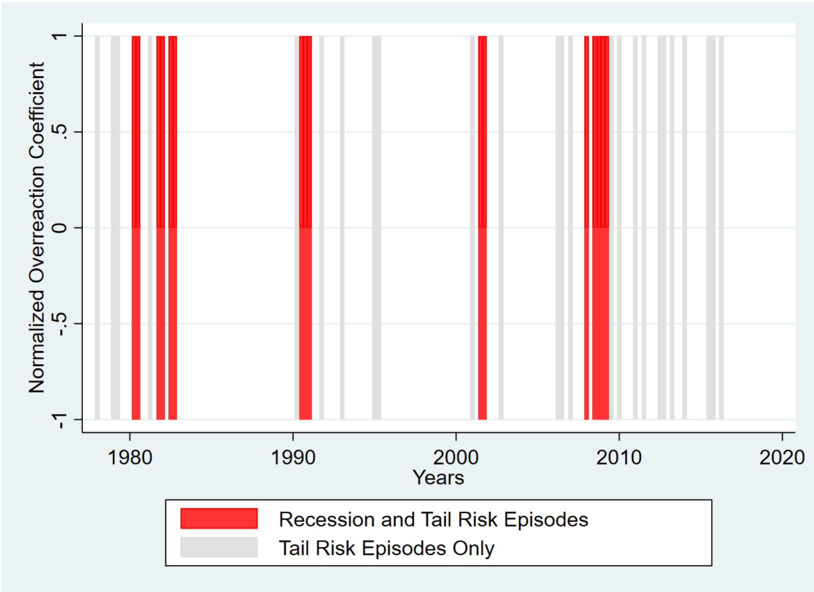
Second, we would like to distinguish between tail risk episodes and recessions as the former may not imply the latter. Figure 7 plots tail risk episodes across time. The red shaded bars denote tail risk episodes when the economy is in a recession, while the grey shaded bars denote tail risk episodes when the economy is not in a recession.[12] We find that tail risk episodes can happen even under normal times. Intuitively, even when the economy is not in a crisis, the probability of an occurrence of rare events can be

[11]GDP growth distributions during normal times exhibit a left tail as summary statistics of GDP growth show that its distribution is left-skewed, even in normal times.

[12]We use the NBER recession indicators to denote that the economy is in a recession.

higher than usual. For instance, after the Great Financial Crisis in 2008, policymakers are persistently pessimistic about the economy as fears of an additional crisis after 2008 continue to emerge. Tail risk episodes can also happen before a crisis (such as before the 1990 recessions).

Figure 7: Tail Risk Episodes



Notes: This figure highlights tail risk episodes (including those that coincide with recessions).

Table 4 shows summary statistics for all variables involved and compares the whole sample and tail risk episodes. The measures of VIX and sentiments do not exhibit much variation compared to GDP and the constructed overreaction measure. When comparing the whole sample and tail risk episodes, we expect tail risk episodes to be associated with poorer economic performance. Table 4 shows that about a third of the sample constitutes tail risk episodes. In the sample of tail risk episodes, the average value of VIX is larger, consistent with counter-cyclical uncertainty. In addition, the average values of GDP and the sentiment index are lower in tail risk episodes, as expected. Also, Table 4 shows that the mean of the overreaction coefficient is lower during tail risk episodes, which suggests larger magnitudes of overreaction in times of distress.

3.2 Empirical Analysis

In this section, we provide empirical support for the predictions in our model.

3.2.1 Overreaction in tail risk events

To test Proposition 2, in which forecasters overreact under tail risk events, we rely on event study methodologies by studying how the overreaction coefficients β_t^i behave around

Table 4: Summary Statistics

Variable: (Whole Sample)	Obs. (1)	Mean (2)	St. Dev. (3)	Min. (4)	Median. (5)	Max. (6)
$\log VIX_t$	156	1.28	0.12	1.04	1.28	1.77
$\log SENT_t$	156	1.93	0.07	1.74	1.95	2.04
GDP_t	156	2.75	3.07	-8.40	3.00	16.40
Overreaction Coefficient	2168	-0.42	0.37	-2.03	-0.42	1.67
Variable: (Tail Risk Episodes Only)	Obs. (1)	Mean (2)	St. Dev. (3)	Min. (4)	Median. (5)	Max. (6)
$\log VIX_t$	46	1.31	0.14	1.08	1.30	1.77
$\log SENT_t$	46	1.88	0.07	1.74	1.87	2.04
GDP_t	46	-0.55	2.44	-8.40	0.50	1.50
Overreaction Coefficient	551	-0.46	0.36	-2.03	-0.45	0.77

Notes: Columns (1) to (6) show statistics for the corresponding variables. Each overreaction coefficient at time t for variable i is obtained from running the regression of Equation (27) across all forecasters. Tail Risk Episodes occur when the tail risk measure exceeds its 75th percentile.

tail risk episodes. Here, we define each event as episodes of tail risk and conduct event studies using β_t^i independently across the 15 macroeconomic variables. We first retrieve the implied β_t^i denoted by $\hat{\beta}_t^i$ under normal times by running the following regression:

$$\beta_t^i = \alpha + \gamma \times \text{Controls} + \text{error} \tag{29}$$

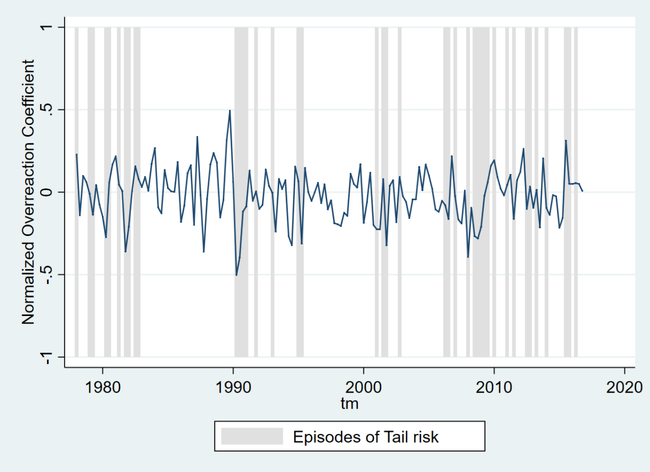
To ensure that our results are not driven by recessions, crises, or any other aspects of the business cycle, we control for macroeconomic variables such as GDP growth, inflation, and unemployment. We then retrieve $\hat{\beta}_t^i$, which is the overreaction coefficient implied by normal times. We proceed to construct abnormal returns defined as the difference between the actual value of β_t^i and $\hat{\beta}_t^i$

$$AR_t^i = \beta_t^i - \hat{\beta}_t^i \tag{30}$$

where AR_t^i denote the abnormal returns for macroeconomic variable i at time t, which is filtered after controlling for major macroeconomic variables which may drive our event study results. We then pool AR_t^i across all the variables in the SPF to obtain AR_t.

Figure 8 plots AR_t over the period from 1978 to 2016. As shown in Figure 8, there is suggestive evidence that abnormal returns tend to spike downwards during tail risk episodes. We proceed to analyze the behavior of AR_t during periods around tail risk episodes. We consider the event window three periods before and after tail risk episodes and conduct event study analysis. In particular, we pool and average estimates of AR_t in each period during the event window.

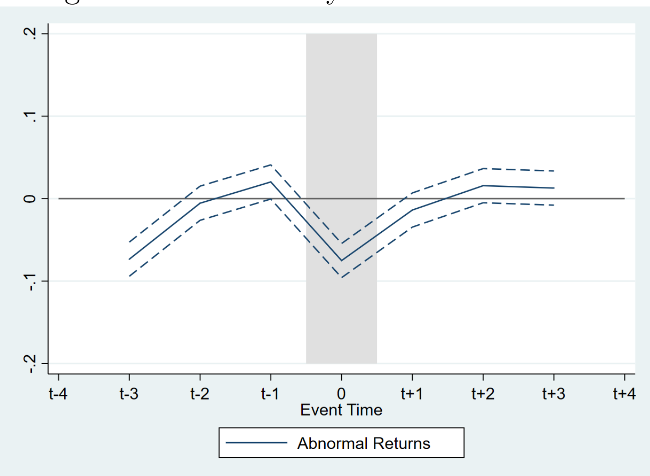
Figure 8: Abnormal Returns over the Business Cycle



Notes: This figure presents the abnormal returns over the time period 1978 to 2016.

Figure 9 plots the abnormal returns during the event window. We consider all abnormal returns around all tail risk episodes, in which period t refers to the tail risk episode, $t-i$ refers to i periods before the tail risk episode and $t+i$ refers to i periods after the tail risk episode. As shown in Figure 9, there is a sharp decline in abnormal returns at time t. This implies that the overreaction coefficient β_t is more negative during the tail risk episodes at time t. Figure 10 plots abnormal returns at different recession periods since tail risk episodes are correlated with the occurrence of recessions. In all recessions dated 1980, 1990, 2001, and 2008, we find that the abnormal returns exhibit sharp declines, consistent with Figure 9.

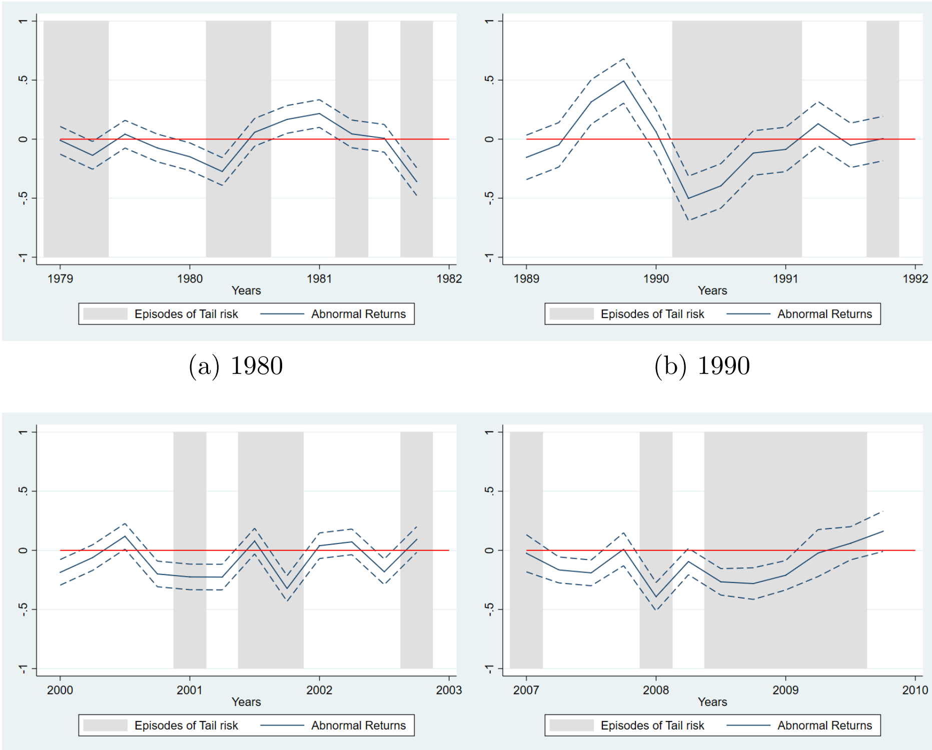
Figure 9: Event Study Abnormal Returns.



Notes: This figure presents the abnormal returns before and after the tail risk episodes (t=0). $t-i$ refers to i periods before the tail risk episode and $t+i$ refers to i periods after the tail risk episode. The dotted lines denote 95 percent confidence interval bands.

When we consider tail risk episodes that are not driven by major recessions, we find

Figure 10: Event Study of Recession Episodes



(a) 1980 (b) 1990

(c) 2001 (d) 2008

Notes: This figure presents the abnormal returns at different time periods. The shaded grey area represents tail risk episodes. The dotted lines denote 95 percent confidence interval bands.

a similar pattern. Even though the abnormal returns average to 0, the mean of the overreaction coefficient β_t remains negative outside tail risk episodes. This does not mean that tail risks are a necessary ingredient to overreaction, but rather, a sufficient condition for overreaction to occur. When overreaction occurs even outside of tail risk episodes, we attribute this to other factors such as diagnostic expectations as in Bordalo et al. (2020).

3.2.2 Expectations and Uncertainty Shocks

We now examine Proposition 3 which highlight that an increase in uncertainty decreases expectations in the event of tail risks. To test this prediction, we run the following regression.

$$Y_{t|t+1} = \alpha_0 + \alpha_1 TR_t + \alpha_2 \log VIX_t + \alpha_3 \log VIX_t \times TR_t + v_t \tag{31}$$

where $Y_{t|t+1}$ refers to median forecasts of GDP growth from the SPF, $\log VIX_t$ refers to the logarithm of the VIX index, GDP_t refers to GDP growth rate and TR_t is a dummy variable equal to 1 in a tail risk episode. For the dependent variable, we also consider the University of Michigan's Sentiment index (denoted by $\log S_t$) in place of $Y_{t|t+1}$. To allay concerns that our result is driven by periods of recession rather than episodes of tail risk, we further control for the interaction between recession episodes and uncertainty. Our key focus is on α_3. If α_3 is negative, there will be lower expectations when there is high uncertainty in tail risk events. The results of Equation (31) are presented in Table 5.

Table 5: Regression Results between Expectations and Uncertainty

Dependent Variable:	$Y_{t+1\|t}$	$Y_{t+1\|t}$	$Y_{t+1\|t}$	$\log S_t$	$\log S_t$	$\log S_t$
	(1)	(2)	(3)	(4)	(5)	(6)
$\log VIX_t$	-5.30***	-0.923	-0.857	-0.106**	0.034	0.043
	(1.71)	(1.13)	(1.55)	(0.045)	(0.041)	(0.042)
TR_t		12.38***	5.12		0.305***	0.222
		(3.85)	(3.44)		(0.071)	(0.138)
$\log VIX_t \times TR_t$		-10.19***	-4.27*		-0.279***	-0.204*
		(2.96)	(2.58)		(0.058)	(0.115)
REC_t			15.33***			-0.208
			(4.77)			(0.204)
$\log VIX_t \times REC_t$			-11.71***			0.108*
			(3.43)			(0.156)
Observations	156	156	156	156	156	156
R^2	0.08	0.17	0.32	0.04	0.25	0.41

Notes: Newey-West standard errors with a lag length of 4 quarters are in parenthesis. *, ** and *** denotes significance level at 10%, 5% and 1% respectively. REC_t is a recession dummy equal to 1 if the economy is in a NBER-dated recession.

Columns 1 and 4 show that an increase in the VIX Index by 1 percent relates to a decrease in the median forecasts of GDP growth by 5.3 percent (significant at 1%) and a decrease in sentiments by 0.1 percent (significant at 5%) respectively. Hence, there is a negative relationship between uncertainty and expectations. Nonetheless, Columns 2 and 5 find that the relationship between expectations and uncertainty is no longer statistically significant after we account for tail risk episodes. This is consistent with Proposition 3, in which uncertainty shocks do not influence expectations outside of tail risk events. Moreover, Columns 2 and 5 show that an increase in the VIX Index by 1 percent during tail risk events corresponds to a decrease in the median forecasts of GDP growth by 10.2 percent (significant at 1%) and a decrease in sentiments by 0.28 percent (significant at 1%) respectively.

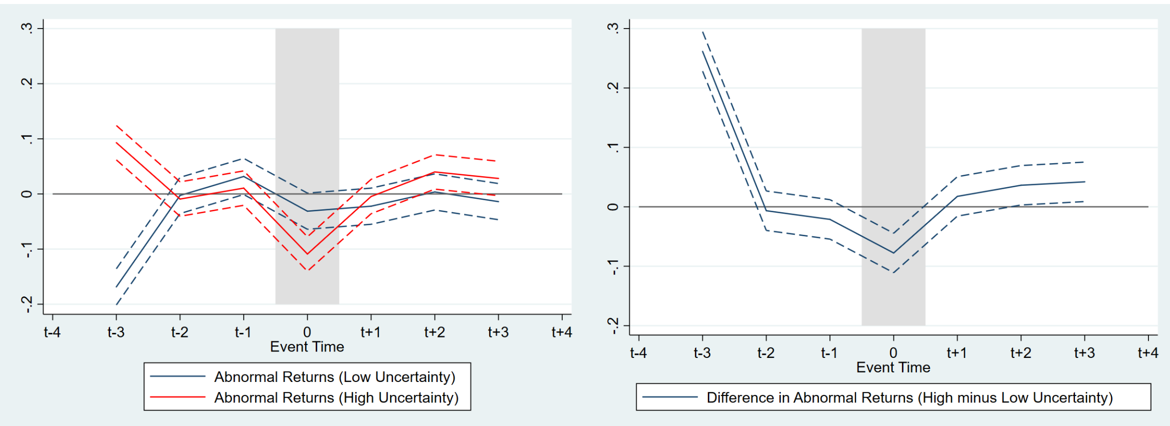
We also find that our findings are robust to recessionary events. Columns 3 and 6 show that after we account for recessions, an increase in the VIX Index by 1 percent during tail risk events relates to a decrease in the median forecasts of GDP growth by 4.3 percent (significant at 10%) and a decrease in sentiments by 0.2 percent (significant at 10%) respectively. This implies that the transmission of uncertainty shocks to expectations depends on tail risk episodes and not due to the recessionary effects. Consequently, we find that tail risk episodes explain the transmission of uncertainty shocks to expectations.

Our findings have business cycle implications. In Bloom (2009), uncertainty shocks depress output because of real frictions such as non-convex adjustment costs. This works through the channel of real options effect. Firms adopt the "wait and see" approach during periods of high uncertainty. As such, they act cautiously and pause investment. In this paper, we find that uncertainty shocks can cause a fall in output through expectations. Increases in uncertainty decrease expectations, and as such, since economic agents' actions take into account their expectations of the future, this decreases economic activity.

3.2.3 Overreaction and Uncertainty Shocks

Lastly, to test Proposition 4, we examine how the magnitude of overreaction changes with uncertainty. Proposition 4 states that when uncertainty is higher, the magnitude of overreaction will be higher. This implies that the overreaction coefficient β_t should be larger in absolute terms during high periods of uncertainty. Hence, calculated abnormal returns should be larger in absolute terms during high periods of uncertainty. Here, we categorize periods of high (low) uncertainty as the event in which values of the VIX index are above (below) the median. We then conduct separate and independent event studies for the periods of high and low uncertainty, while keeping the same methodology in deriving the normal returns.

Figure 11: Event Study Abnormal Returns (Comparing Uncertainty)



(a) Abnormal Returns (By Uncertainty) (b) Difference in Abnormal Returns

Notes: This figure presents the abnormal returns before and after the tail risk episodes (t=0). $t-i$ refers to i periods before the tail risk episode and $t+i$ refers to i periods after the tail risk episode. The dotted lines denote 95 percent confidence interval bands.

Figure 11 compares the abnormal returns between different event studies for different magnitudes of uncertainty. $t = 0$ refers to the tail risk event, while $t - i$ and $t + i$ refers to i periods before and after the tail risk event respectively. In Figure 11 (a), the abnormal return in blue refers to the analysis conducted considering periods of low uncertainty, while the abnormal return in red refers to analysis conducted only considering periods of high uncertainty. As shown in Figure 11 (a), the decline in abnormal returns is more pronounced during periods of high uncertainty. This implies that the overreaction coefficients are larger in absolute terms when the level of uncertainty is high. We then compare the difference in abnormal returns between high and low uncertainty in Figure 11 (b). Here, we find that the difference is negative and statistically significant only during the tail risk event. This suggests that uncertainty plays an important role in the behavior of economic agents during episodes of tail risk. This is consistent with Proposition 4 in our model.

4 Discussion and Robustness

This section discusses the implications of our findings and presents the robustness checks of our empirical results. First, we relate our results to the literature of diagnostic expectations. Second, we provide some policy implications based on our results. Finally, we conducted additional robustness tests.

4.1 Relation to Diagnostic Expectations

Our paper speaks directly to diagnostic expectations, a psychologically micro-founded approach that exploits behavioral biases to reconcile empirical findings of overreaction behavior in individuals. By incorporating Kahneman and Tversky (1972) representativeness heuristic into a signal extraction problem, diagnostic expectations depart from rational expectations as individuals follow the "kernel of truth". Each forecaster overweighs the probability of the hidden state that are representative of recent news. Denoting beliefs formed from diagnostic expectations to be $E_t^\theta(z_{t+1})$ and beliefs formed from rational expectations as $E_t(z_{t+1})$, Bordalo et al. (2020) show that

$$E_t^\theta(z_{t+1}) = E_t(z_{t+1}) + \theta_t[E_t(z_{t+1}) - E_{t-1}(z_{t+1})] \tag{32}$$

where $\theta_t \geq 0$ is the diagnosticity parameter that measures the extent of overreaction to news. We denote θ_t to be time-varying in this paper as we consider different magnitudes of overreaction over time. In Bordalo et al. (2020), θ is estimated to be ≈ 0.5. To benchmark our tail risk model with diagnostic expectations, we conduct a back of envelope exercise by mapping the magnitude of overreaction in our model to θ. Since $\rho_z = 0$, $E_{t-1}(z_{t+1}) = \mu_z = 0$ and

$$E_t(z_{t+1}) = (1 - r_t)s_{i,t}^z \tag{33}$$

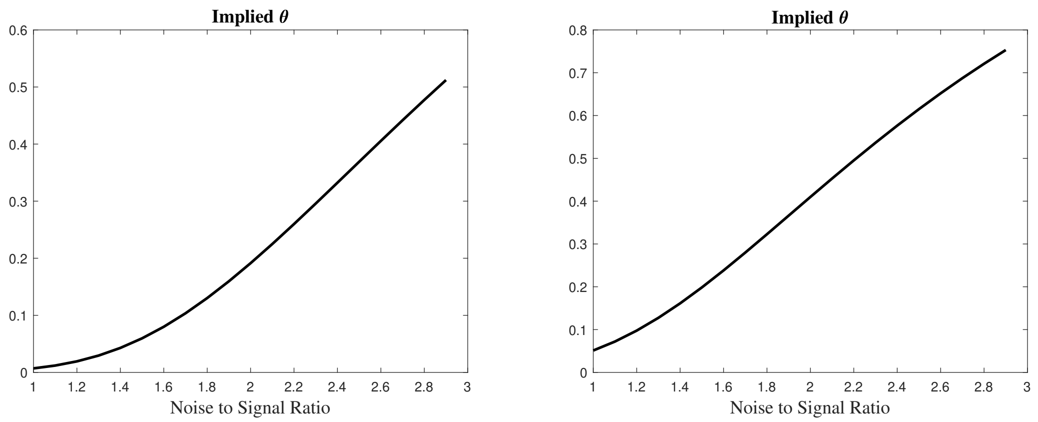
where $r_t = \frac{\sigma_t^{e2}}{\sigma_t^{e2}+\sigma_z^2}$. Then it is straightforward to show that

$$\theta_t = \frac{B(p,\gamma,\sigma_t^e,\mu_z) - B(p,\gamma,\sigma_t^e,s_{i,t}^z)}{(1-r_t)(s_{i,t}^z)} \tag{34}$$

Using parameter values in Table 1 and a noise to signal ratio of 1.5, a positive one standard deviation shock to the hidden state corresponds to an implied θ_t value of 0.07.[13] In contrast, a negative one standard deviation shock to the hidden state corresponds to an implied value of 0.20.

Figure 12 plots the implied diagnosticity parameter θ_t which varies with the noise to signal ratio, in which the noise to signal ratio is defined as σ_t^e/σ_z. Since Proposition 4 states that the magnitude of overreaction increases with prior uncertainty, it is not a surprise that the implied diagnositicity parameter increases with the noise to signal ratio. From Proposition 4, we show that the numerator of Equation (34) increases with prior uncertainty, that is, the *level* of overreaction is positively associated with prior uncertainty. As prior uncertainty increases, Bayesian learning implies that individuals will choose to rely less on the signal. This is evident in the denominator, which falls as the noise to signal ratio increases. Hence, the effects of uncertainty in the noise of the signal unambiguously lead to an increase in the diagnosticity parameter.

Figure 12: Implied θ Varying with Noise to Signal Ratio



(a) 1 SD Positive Shock (b) 1 SD Negative Shock

Notes: This figure presents the relationship of the implied θ_t value with the noise to signal ratio based on 1 SD of positive shock (Panel A) and 1 SD of negative shock (Panel B).

From Figure 12, we see that the noise to signal ratio needs to be sufficiently large to generate an implied θ of 0.5 that is highlighted in Bordalo et al. (2020). When we consider a one standard deviation positive shock to the hidden state, the noise to signal

[13]We consider a noise to signal ratio of 1.5 as this is equivalent to the average estimated noise to signal ratio in Bordalo et al. (2020). Noise to signal ratio is defined as $\frac{\sigma_t^e}{\sigma_z}$.

ratio needs to exceed 3 in order to generate an implied θ of 0.5. In contrast, under a one standard deviation negative shock to the hidden state, the noise to signal ratio needs to exceed 2.2 in order to generate an implied θ of 0.5. In the case of $\rho_z = 0$, the coefficient β_1^i in equation (27) satisfies

$$\beta_1^i = \frac{\theta(1+\theta)}{(1+\theta)^2} \tag{35}$$

In this case, our average implied value of $\theta = 0.14$ corresponds to an implied value of $\beta_1^i = 0.12$. This is consistent with the empirical counterpart shown in Figure 9, where β_1^i is lower by about 0.09 units in a tail risk episode. This accounts for about 20 % in variation of β_1^i since the average value of β_1^i is -0.42 (Table 4, Top Panel, Column 2).

Since the implied values of θ in our tail risk model ranges from 0.07 to 0.20, our tail risk model can explain about 14 to 40 % of overreaction that is estimated in Bordalo et al. (2020). We view our work as a complementary explanation to overreaction behavior observed in individuals and leave it for future work to understand other drivers of diagnostic expectations.

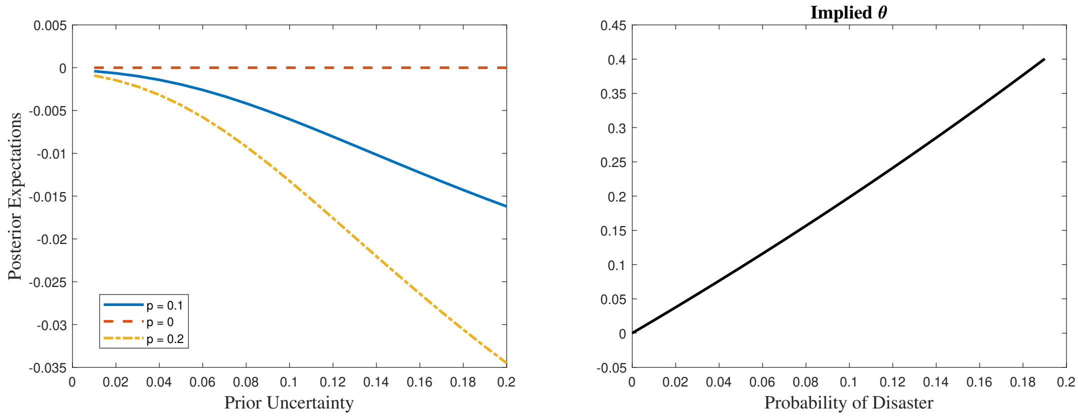
4.2 Policy Implications

Since tail risk has an impact on expectations, policies that can manage tail risk will be able to affect expectations. Consequently, policies that can reduce p, the probability of the occurrence of a disaster in our model can reduce the magnitude of overreaction. This reduces the mass of the left tail, and thus tail risk. When $p = 0$, the asymmetric mass of the left tail disappears, and as such, the economy exhibits no tail risk. We consider the effects of a reduction in p on the transmission of uncertainty shocks to expectations, and the magnitude of overreaction.

Figure 13 (a) illustrates the effects of a reduction in p on the transmission of uncertainty shocks to expectations. When a decrease in tail risk is captured by a decrease of p from 0.2 to 0.1, a given sized positive uncertainty shock leads to a lower decrease in posterior expectations. Moreover, under a perfect scenario of no tail risk, in which p is equal to 0, uncertainty shocks do not affect posterior expectations. Given that uncertainty is counter-cyclical and exhibits large fluctuations in recessions, macroprudential policies that reduce downside risk can dampen the effect of uncertainty shocks on the economy through expectations.

Figure 13 (b) shows the effect of a reduction in downside risk on the magnitude of overreaction. In this exercise, we use Equation (34) to quantify the extent of overreaction. The implied θ increases with the probability of disaster risk. Moreover, when there is no downside risk ($p = 0$), our model implies that there will be no overreaction in

Figure 13: Effects of a reduction in p



(a) Prior Uncertainty and Posterior Expectations (b) Implied θ

Notes: This figure shows how changing the probability of disaster (p) will impact the relationship between posterior expectations and uncertainty (Panel A) and implied θ (Panel B).

expectations. However, as our model explains only 14 to 40 % of overreaction behavior in the data, as such, even though in a perfect setting where macroprudential policies reduces p down to zero, we expect that overreaction will still be present in agents' expectations due to other factors such as behavioral biases. This is not to say that macroprudential policies will be ineffective. Our model predicts that the magnitudes of overreaction behavior will decrease with macroprudential policies that reduce downside risk. A reduction in the magnitude of overreaction behavior leads to less volatile macroeconomic moments.

4.3 Robustness

For robustness, we conducted additional tests. First, we seek to allay concerns that the threshold of the tail risk episodes could drive our results. Our main analysis identified tail risk episodes as the time period in which tail risk exceeds its 75th percentile (Definition 5). It is possible that changes in the thresholds for the tail risk episodes could impact our main results. We address this concern by varying the tail risk episodes across all thresholds from 50th to 90th percentile. Our main results remain unchanged. For instance, the regression coefficient α_3 in equation (31) remains economically and statistically significant. Figure B.2 in the online appendix plots regression coefficient α_3 in equation (30) when the measure of tail risk exceeds different thresholds. The coefficients are all negative and statistically significant at the 5 % level, and are all similar in magnitude.

We also consider different thresholds for our event study analysis. The results continue to show overreaction in tail risk episodes. Nonetheless, it is worth noting that when we consider higher thresholds, there will be larger magnitudes of overreaction. This is expected as when the tail risk measures exceed a higher threshold, this captures tail risk episodes that contain greater tail risk. We provide an example in the online appendix. Figure B.3 (a) considers tail risk episodes in which the tail risk measures exceed the 90th percentile, while Figure B.3 (b) highlights tail risk episodes in which the measures of tail risk exceed the 50th percentile. It is evident that there is a larger overreaction when the tail risk measures exceed the 90th percentile in Figure B.3 (a) compared to that of the 50th percentile in Figure B.3 (b). Greater tail risk leads to larger overreaction, as implied by our model.

To further support our event study analysis, we exploit the panel data and conduct the following OLS regression to examine whether there is overreaction.

$$\beta_{i,t} = \delta_0 + \delta_1 TR_t + \mathbf{X'_{i,t}} \mathbf{\Gamma} + \lambda_i + \epsilon_{i,t} \tag{36}$$

where $\beta_{i,t}$ denotes the overreaction coefficient, TR_t is a dummy equal to 1 in tail risk episodes, $\mathbf{X'_{i,t}}$ is a vector of controls used in the event study, and λ_i is the variable-specific fixed effect. Our variable of concern is δ_1. Should δ_1 be negative, there will be larger magnitudes of overreaction in tail risk episodes.

Next, we run the following OLS regression to examine the role of uncertainty:

$$\beta_{i,t} = \delta_0 + \delta_1 TR_t + \delta_2 \log VIX_t + \delta_3 TR_t \times \log VIX_t + \mathbf{X'_{i,t}} \mathbf{\Gamma} + \lambda_i + \epsilon_{i,t} \tag{37}$$

where the variables are similar to that of Equation (36). Here we include an additional variable, $\log VIX_t$, which relates to the logarithm of the VIX index. Our variable of concern is δ_3, which highlights that the magnitude of overreaction in tail risk episodes will depend on the level of uncertainty in the economy.

Table 6 presents the results. Based on the results of Equation (36), we find that during tail risk episodes, the overreaction coefficients are lower by 0.068 (statistically significant at 1 % level). This suggests that the magnitude of overreaction is larger in tail risk episodes. Based on the results of Equation (36), we find that a percentage increase in the VIX index during tail risk episodes corresponds to a decrease in the overreaction coefficients by 0.364 (statistically significant at 1 % level). This provides evidence of larger magnitudes of overreaction when there is high uncertainty in tail risk episodes.

We want to highlight that our estimates in the regression imply that the relationship between overreaction and tail risk episodes is weaker than estimates in the event study approach. This is because we control for first and second-moment shocks (such as GDP and VIX) outside of tail risk episodes by extracting the abnormal returns of the overreaction coefficient in the event study approach. Consequently, we allow first and second-moment shocks to be present during tail risk episodes in our event study. In contrast, the OLS panel regressions imply that we eliminate all first and second moment shocks in both

Table 6: Regression Results between Overreaction and Tail Risk Episodes

Dependent Variable:	$\beta_{i,t}$ (1)	$\beta_{i,t}$ (2)
TR_t	-0.068***	0.387**
	(0.026)	(0.171)
$\log VIX_t$	-0.194**	0.141
	(0.083)	(0.086)
$\log VIX_t \times TR_t$		-0.364***
		(0.137)
Variable-specific Fixed Effects	Yes	Yes
Controls	Yes	Yes
Observations	2000	2000
Adjusted R^2	0.077	0.080

Notes: Robust standard errors are in parenthesis. *, ** and *** denotes significance level at 10%, 5% and 1% respectively.

tail risk and non-tail risk episodes by controlling for macroeconomic variables that might drive tail risk. Naturally, our theoretical model implies that the estimates of overreaction in tail risk episodes will be lower. We view the OLS approach as setting a lower bound (in absolute terms) on the magnitude of overreaction in tail risk episodes.

Next, to ensure that our event study results rely on tail risk episodes and not recessionary events, we focus on non-recession episodes. We obtain similar findings with our main results. Based on the event study when there are no recessions during tail risk events, Figure B.4 in the online appendix continues to provide evidence of overreaction behavior. In addition, Figure B.5 in the online appendix plots abnormal returns during different non-recessionary time periods. These provide further evidence that overreaction is driven by tail risk episodes and not economic crisis per se.

We also show that our findings are robust to our choice of data. Here, we move away from VIX and consider an alternative measure of macroeconomic uncertainty based on Jurado et al. (2015), which is constructed using a set of predictors in a factor augmented vector autoregression.[14] Table B.1 shows the regression in Equation (31), where we test whether uncertainty shocks lead to a decrease in expectations during tail risk episodes. The coefficient of the interaction term between uncertainty and tail risk episodes α_3 remains economically and statistically significant. This suggests that our evidence is robust to the use of VIX (a measure of financial uncertainty) or a broad-based measure of macroeconomic uncertainty (Jurado et al. (2015)).

[14] Since uncertainty is only relevant for Proposition 3 and 4, we do not conduct robustness exercises related to alternative uncertainty measures for Proposition 2.

5 Conclusion

In conclusion, we find that tail risk matters for expectations. By incorporating tail risks in a Bayesian learning framework with information frictions, we have three main findings. First, we show that individuals overreact under tail risk, that is, individuals are excessively optimistic and pessimistic compared to a Bayesian learning framework without tail risk. We also show that our model can explain part of behavioral approaches such as diagnostic expectations. Hence, we view our work as a complementary explanation to overreaction observed in individuals. Since overreaction implies more volatile business cycles and expectations, we highlight that we can dampen overreaction behavior by reducing tail risk.

Second, under tail risk, uncertainty shocks lead to a decrease in expectations, which implies more pessimistic forecasts. In comparison, uncertainty shocks do not influence expectations in the absence of tail risks. Since economic agents rely on their expectations in making decisions, changes in expectations due to uncertainty shocks could potentially lead to suboptimal actions in economic agents. Our finding sheds light on an additional channel in which uncertainty shocks are transmitted to the economy.

Third, we find that the magnitude of overreaction under tail risk depends on the level of uncertainty in the economy. In particular, we attempt to explain overreaction cycles, in which large magnitudes of overreaction are followed by overreaction that is smaller in magnitude. Our explanation hinges on large surges in uncertainty during tail risk episodes, which leads to an increase in overreaction.

Our theoretical and empirical findings have implications for policymakers. We highlight the importance of reducing tail risks in managing expectations and show that tail risks can partially explain behavioral biases, such as overreaction and extrapolative behavior. We could study how augmenting tail risk in theoretical models can explain behavioral puzzles in macroeconomics for future work. This will tighten the relationship between rational learning and behavioral biases. Also, more work can be done to evaluate the effect of macroprudential policies that reduce tail risk and uncertainty through the expectations channel. We leave this for future work.

References

Adrian, T., N. Boyarchenko, and D. Giannone (2019): "Vulnerable growth," *American Economic Review*, 109, 1263–89.

Adrian, T., F. Grinberg, N. Liang, and S. Malik (Forthcoming): "The term structure of growth-at-risk," *American Economic Journal: Macroeconomics*.

Afrouzi, H., S. Y. Kwon, A. Landier, Y. Ma, and D. Thesmar (2020): "Overreaction and working memory," *National Bureau of Economic Research Working Paper*.

Ai, H. and A. Bhandari (2021): "Asset pricing with endogenously uninsurable tail risk," *Econometrica*, 89, 1471–1505.

Aikman, D., J. Bridges, S. Burgess, R. Galletly, I. Levina, C. O'Neill, A. Varadi, et al. (2018): "Measuring risks to UK financial stability," Tech. rep., Bank of England.

Aikman, D., J. Bridges, A. Kashyap, and C. Siegert (2019): "Would macroprudential regulation have prevented the last crisis?" *Journal of Economic Perspectives*, 33, 107–30.

Angeletos, G.-M., Z. Huo, and K. A. Sastry (2020): "Imperfect Macroeconomic Expectations: Evidence and Theory," *National Bureau of Economic Research Working Paper*.

Angeletos, G.-M. and J. La'O (2013): "Sentiments," *Econometrica*, 81, 739–779.

Baley, I. and L. Veldkamp (2021): "Bayesian learning," *CEPR Discussion Paper No. DP16377*.

Barberis, N. (2013): "The psychology of tail events: progress and challenges," *American Economic Review*, 103, 611–16.

Baron, M. and W. Xiong (2017): "Credit expansion and neglected crash risk," *The Quarterly Journal of Economics*, 132, 713–764.

Barro, R. J. (2006): "Rare disasters and asset markets in the twentieth century," *The Quarterly Journal of Economics*, 121, 823–866.

Barro, R. J. and G. Y. Liao (2021): "Rare disaster probability and options pricing," *Journal of Financial Economics*, 139, 750–769.

Benhabib, J., X. Liu, and P. Wang (2019): "Financial markets, the real economy, and self-fulfilling uncertainties," *The Journal of Finance*, 74, 1503–1557.

Bianchi, F., C. L. Ilut, and H. Saijo (2021): "Implications of Diagnostic Expectations: Theory and Applications," Tech. rep., National Bureau of Economic Research.

Bloom, N. (2009): "The impact of uncertainty shocks," *Econometrica*, 77, 623–685.

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2018): "Really uncertain business cycles," *Econometrica*, 86, 1031–1065.

Bordalo, P., N. Gennaioli, Y. Ma, and A. Shleifer (2020): "Over-reaction in macroeconomic expectations," *American Economic Review*, 110, 2748–82.

Bordalo, P., N. Gennaioli, R. L. Porta, and A. Shleifer (2019): "Diagnostic expectations and stock returns," *The Journal of Finance*, 74, 2839–2874.

Bordalo, P., N. Gennaioli, and A. Shleifer (2018): "Diagnostic expectations and credit cycles," *The Journal of Finance*, 73, 199–227.

Brooks, J., M. Katz, and H. Lustig (2018): "Post-FOMC announcement drift in US bond markets," *National Bureau of Economic Research Working Paper.*

Brunnermeier, M. K. and Y. Sannikov (2014): "A macroeconomic model with a financial sector," *American Economic Review*, 104, 379–421.

Cerutti, E., S. Claessens, and L. Laeven (2017): "The use and effectiveness of macroprudential policies: New evidence," *Journal of Financial Stability*, 28, 203–224.

Chatterjee, P. and F. Milani (2020): "Perceived uncertainty shocks, excess optimism-pessimism, and learning in the business cycle," *Journal of Economic Behavior & Organization*, 179, 342–360.

Claessens, S. (2015): "An overview of macroprudential policy tools," *Annual Review of Financial Economics*, 7, 397–422.

Claessens, S., S. R. Ghosh, and R. Mihet (2013): "Macro-prudential policies to mitigate financial system vulnerabilities," *Journal of International Money and Finance*, 39, 153–185.

Coibion, O. and Y. Gorodnichenko (2015): "Information rigidity and the expectations formation process: A simple framework and new facts," *American Economic Review*, 105, 2644–78.

Dominitz, J. and C. F. Manski (2003): "How should we measure consumer confidence (sentiment)? evidence from the michigan survey of consumers," *National Bureau of Economic Research Working Paper.*

DupreY, T. and A. Ueberfeldt (2020): "Managing GDP Tail Risk," Tech. rep., Bank of Canada.

Fuster, A., D. Laibson, and B. Mendel (2010): "Natural expectations and macroeconomic fluctuations," *Journal of Economic Perspectives*, 24, 67–84.

Gabaix, X. (2012): "Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance," *The Quarterly Journal of Economics*, 127, 645–700.

Gabaix, X. and E. Farhi (2016): "Rare disasters and exchange rates," *The Quarterly Journal of Economics*, 131, 1–52.

Galati, G. and R. Moessner (2013): "Macroprudential policy–a literature review," *Journal of Economic Surveys*, 27, 846–878.

Ghaderi, M., M. Kilic, and S. B. Seo (2021): "Learning, Slowly Unfolding Disasters, and Asset Prices," *Journal of Financial Economics (JFE), Forthcoming*.

Gourio, F. (2012): "Disaster risk and business cycles," *American Economic Review*, 102, 2734–66.

Greenwood, R. and A. Shleifer (2014): "Expectations of returns and expected returns," *The Review of Financial Studies*, 27, 714–746.

Hattori, M., A. Schrimpf, and V. Sushko (2016): "The response of tail risk perceptions to unconventional monetary policy," *American Economic Journal: Macroeconomics*, 8, 111–36.

IMF (2017): "Financial Conditions and Growth at Risk," *Chapter 3 of the Global Financial Stability Report*.

Jurado, K., S. C. Ludvigson, and S. Ng (2015): "Measuring uncertainty," *American Economic Review*, 105, 1177–1216.

Kahneman, D. and A. Tversky (1972): "Subjective probability: A judgment of representativeness," *Cognitive psychology*, 3, 430–454.

Kelly, B. and H. Jiang (2014): "Tail risk and asset prices," *The Review of Financial Studies*, 27, 2841–2871.

Kilian, L. and S. Manganelli (2008): "The central banker as a risk manager: Estimating the Federal Reserve's preferences under Greenspan," *Journal of Money, Credit and Banking*, 40, 1103–1129.

Kozeniauskas, N., A. Orlik, and L. Veldkamp (2018): "What are uncertainty shocks?" *Journal of Monetary Economics*, 100, 1–15.

Kozlowski, J., L. Veldkamp, and V. Venkateswaran (2019): "The tail that keeps the riskless rate low," *NBER Macroeconomics Annual*, 33, 253–283.

——— (2020): "The tail that wags the economy: Beliefs and persistent stagnation," *Journal of Political Economy*, 128, 2839–2879.

Kuchler, T. and B. Zafar (2019): "Personal experiences and expectations about aggregate outcomes," *The Journal of Finance*, 74, 2491–2542.

Kydland, F. E. and E. C. Prescott (1982): "Time to build and aggregate fluctuations," *Econometrica: Journal of the Econometric Society*, 1345–1370.

Loria, F., C. Matthes, and D. Zhang (2019): "Assessing macroeconomic tail risk," *FRB Richmond Working Paper*.

Lucas, R. E. (1972): "Expectations and the Neutrality of Money," *Journal of economic theory*, 4, 103–124.

Mackowiak, B. and M. Wiederholt (2009): "Optimal sticky prices under rational inattention," *American Economic Review*, 99, 769–803.

Ordonez, G. (2013): "The asymmetric effects of financial frictions," *Journal of Political Economy*, 121, 844–895.

Orlik, A. and L. Veldkamp (2014): "Understanding uncertainty shocks and the role of black swans," *National Bureau of Economic Research Working Paper*.

Pastor, L. and P. Veronesi (2012): "Uncertainty about government policy and stock prices," *The Journal of Finance*, 67, 1219–1264.

Sims, C. A. (2003): "Implications of rational inattention," *Journal of Monetary Economics*, 50, 665–690.

Simsek, A. (2021): "The macroeconomics of financial speculation," *Annual Review of Economics*, 13.

Singleton, K. J. (2021): "Presidential Address: How Much "Rationality" Is There in Bond-Market Risk Premiums?" *The Journal of Finance*, 76, 1611–1654.

Souleles, N. S. (2004): "Expectations, heterogeneous forecast errors, and consumption: Micro evidence from the Michigan consumer sentiment surveys," *Journal of Money, Credit and Banking*, 39–72.

Wachter, J. A. (2013): "Can time-varying risk of rare disasters explain aggregate stock market volatility?" *The Journal of Finance*, 68, 987–1035.

Wachter, J. A. and Y. Zhu (2019): "Learning with rare disasters," *Working Paper*.

Online Appendix

A Proofs of Propositions

Proposition 1. *In the baseline model, posterior expectations are given by:*

$$E(z_t|s_{i,t}^z) = r_t\mu_z + (1-r_t)s_{i,t}^z \tag{38}$$

In the extended model, posterior expectations are given by

$$E(z_t|s_{i,t}^z) = r_t\mu_z + (1-r_t)s_{i,t}^z - B(p,\gamma,\sigma_t^e,s_{i,t}^z) \tag{39}$$

$$r_t = \frac{\sigma_t^{e2}}{\sigma_t^{e2}+\sigma_z^2} \tag{40}$$

$$B(p,\gamma,\sigma_t^e,s_{i,t}^z) = \frac{r_t\gamma p e^{A_{12}+A_{13}}}{pe^{A_{12}+A13}+(1-p)e^{A_{22}+A_{23}}} \tag{41}$$

Proof.

$$f(z_t|s_{i,t}^z) = \frac{f(s_{i,t}^z|z_t)f(z_t)}{\int f(s_{i,t}^z|z_t)f(z_t)\mathrm{dz}} \tag{42}$$

$$\begin{aligned} f(s_{i,t}^z|z_t)f(z_t) &= \frac{1}{\sqrt{2\pi\sigma_t^{e2}}}\left[e^{-\frac{1}{2}(\frac{s_{i,t}^z-z_t}{\sigma_t^e})^2}\right]\cdot\frac{1}{\sqrt{2\pi\sigma_z^2}}\left[pe^{-\frac{1}{2}(\frac{z_t-\mu_z+\gamma}{\sigma_z})^2}+(1-p)e^{-\frac{1}{2}(\frac{z_t-\mu_z}{\sigma_z})^2}\right] \\ &= \frac{1}{\sqrt{2\pi\sigma_t^{e2}}}\frac{1}{\sqrt{2\pi\sigma_z^2}}\cdot\left[pe^{-\frac{1}{2}(\frac{z_t-\mu_z}{\sigma_z})^2}e^{-\frac{1}{2}(\frac{s_{i,t}^z-z_t}{\sigma_t^e})^2}+(1-p)e^{-\frac{1}{2}(\frac{z_t-\mu_z}{\sigma_z})^2}e^{-\frac{1}{2}(\frac{s_{i,t}^z-z_t}{\sigma_t^e})^2}\right] \end{aligned} \tag{43}$$

Let $A_1 = pe^{-\frac{1}{2}(\frac{z_t-\mu_z}{\sigma_z})^2}e^{-\frac{1}{2}(\frac{s_{i,t}^z-z_t}{\sigma_t^e})^2}$ and $A_2 = (1-p)e^{-\frac{1}{2}(\frac{z_t-\mu_z}{\sigma_z})^2}e^{-\frac{1}{2}(\frac{s_{i,t}^z-z_t}{\sigma_t^e})^2}$.

Denote

$$A_1 = pe^{A_{11}+A_{12}+A_{13}} \tag{44}$$

$$A_2 = (1-p)e^{A_{21}+A_{22}+A_{23}} \tag{45}$$

where

$$A_{11} = -\frac{1}{2}\frac{(z_t-\frac{\sigma_z^2 s_{i,t}^z+\sigma_t^{e2}(\mu_z-\gamma)}{\sigma_z^2+\sigma_t^{e2}})^2}{\frac{\sigma_t^{e2}\sigma_z^2}{\sigma_t^{e2}+\sigma_z^2}} \tag{46}$$

$$A_{12} = -\frac{1}{2}\frac{(\mu_z-\gamma)^2\sigma_t^{e2}+\sigma_z^2 s_{i,t}^{z2}}{\sigma_t^{e2}\sigma_z^2} \tag{47}$$

$$A_{13} = \frac{1}{2}\frac{\left(\frac{(\mu_z-\gamma)\sigma_t^{e2}+\sigma_z^2 s_{i,t}^z}{\sigma_t^{e2}+\sigma_z^2}\right)^2}{\frac{\sigma_t^{e2}\sigma_z^2}{\sigma_t^{e2}+\sigma_z^2}} \tag{48}$$

$$A_{21} = -\frac{1}{2}\frac{(z_t - \frac{\sigma_z^2 s_{i,t}^z+\sigma_t^{e2}\mu_z}{\sigma_z^2+\sigma_t^{e2}})^2}{\frac{\sigma_t^{e2}\sigma_z^2}{\sigma_t^{e2}+\sigma_z^2}} \tag{49}$$

$$A_{22} = -\frac{1}{2}\frac{\mu_z^2\sigma_t^{e2} + \sigma_z^2 s_{i,t}^{z2}}{\sigma_t^{e2}\sigma_z^2} \tag{50}$$

$$A_{23} = \frac{1}{2}\frac{\left(\frac{\mu_z\sigma_t^{e2}+\sigma_z^2 s_{i,t}^z}{\sigma_t^{e2}+\sigma_z^2}\right)^2}{\frac{\sigma_t^{e2}\sigma_z^2}{\sigma_t^{e2}+\sigma_z^2}} \tag{51}$$

Integrating (38) and (39), we obtain

$$\begin{aligned}\int A_1 \mathrm{dz} &= \int p e^{A_{11}+A_{12}+A_{13}}\mathrm{dz} \\ &= p e^{A_{12}+A_{13}} \int e^{A_{11}}\mathrm{dz} \\ &= p e^{A_{12}+A_{13}} \sqrt{2\pi\sigma_{z|s^z}^2}\end{aligned} \tag{52}$$

$$\begin{aligned}\int A_2 \mathrm{dz} &= \int (1-p) e^{A_{21}+A_{22}+A_{23}}\mathrm{dz} \\ &= (1-p) e^{A_{22}+A_{23}} \int e^{A_{21}}\mathrm{dz} \\ &= (1-p) e^{A_{22}+A_{23}} \sqrt{2\pi\sigma_{z|s^z}^2}\end{aligned} \tag{53}$$

where $\sigma_{z|s^z}^2 = \frac{\sigma_t^{e2}\sigma_z^2}{\sigma_t^{e2}+\sigma_z^2}$

Integrating (37), we obtain

$$\begin{aligned}\int f(s_{i,t}^z|z_t)f(z_t)\mathrm{dz} &= \int \frac{1}{\sqrt{2\pi\sigma_t^{e2}}}\frac{1}{\sqrt{2\pi\sigma_z^2}} \cdot \left[p e^{-\frac{1}{2}(\frac{z_t-\mu_z}{\sigma_z})^2} e^{-\frac{1}{2}(\frac{s_{i,t}^z - z_t}{\sigma_t^e})^2} + (1-p) e^{-\frac{1}{2}(\frac{z_t-\mu_z}{\sigma_z})^2} e^{-\frac{1}{2}(\frac{s_{i,t}^z - z_t}{\sigma_t^e})^2}\right]\mathrm{dz} \\ &= \frac{1}{\sqrt{2\pi\sigma_t^{e2}}}\frac{1}{\sqrt{2\pi\sigma_z^2}} \int \left[p e^{-\frac{1}{2}(\frac{z_t-\mu_z}{\sigma_z})^2} e^{-\frac{1}{2}(\frac{s_{i,t}^z - z_t}{\sigma_t^e})^2} + (1-p) e^{-\frac{1}{2}(\frac{z_t-\mu_z}{\sigma_z})^2} e^{-\frac{1}{2}(\frac{s_{i,t}^z - z_t}{\sigma_t^e})^2}\right]\mathrm{dz} \\ &= \frac{1}{\sqrt{2\pi\sigma_t^{e2}}}\frac{1}{\sqrt{2\pi\sigma_z^2}} \int (A_1 + A_2)\mathrm{dz} \\ &= \frac{1}{\sqrt{2\pi\sigma_t^{e2}}}\frac{1}{\sqrt{2\pi\sigma_z^2}} \sqrt{2\pi\sigma_{z|s^z}^2}[p e^{A_{12}+A_{13}} + (1-p) e^{A_{22}+A_{23}}]\end{aligned} \tag{54}$$

Substituting (46), (47) and (48) into (36),

$$f(z_t|s^z_{i,t}) = \frac{f(s^z_{i,t}|z_t)f(z_t)}{\int f(s^z_{i,t}|z_t)f(z_t)\mathrm{dz}} = \frac{[pe^{A_{11}+A_{12}+A_{13}} + (1-p)e^{A_{21}+A_{22}+A_{23}}]}{\sqrt{2\pi\sigma^2_{z|s^z}}[pe^{A_{12}+A_{13}} + (1-p)e^{A_{22}+A_{23}}]} \tag{55}$$

$$\begin{aligned}
E(z_t|s^z_{i,t}) &= \int z f(z_t|s^z_{i,t})\mathrm{dz} \\
&= \int z \frac{[pe^{A_{11}+A_{12}+A_{13}} + (1-p)e^{A_{21}+A_{22}+A_{23}}]}{\sqrt{2\pi\sigma^2_{z|s^z}}[pe^{A_{12}+A_{13}} + (1-p)e^{A_{22}+A_{23}}]}\mathrm{dz} \\
&= \frac{pe^{A_{12}+A_{13}}}{[pe^{A_{12}+A_{13}} + (1-p)e^{A_{22}+A_{23}}]} \int z \frac{1}{\sqrt{2\pi\sigma^2_{z|s^z}}} e^{A_{11}}\mathrm{dz} \\
&+ \frac{(1-p)e^{A_{22}+A_{23}}}{[pe^{A_{12}+A_{13}} + (1-p)e^{A_{22}+A_{23}}]} \int z \frac{1}{\sqrt{2\pi\sigma^2_{z|s^z}}} e^{A_{21}}\mathrm{dz} \\
&= \frac{pe^{A_{12}+A_{13}}}{[pe^{A_{12}+A_{13}} + (1-p)e^{A_{22}+A_{23}}]} \int z \frac{1}{\sqrt{2\pi\sigma^2_{z|s^z}}} e^{-\frac{1}{2}\frac{(z_t - \frac{\sigma^2_z s^z_{i,t} + \sigma^{e2}_t(\mu_z - \gamma)}{\sigma^2_z + \sigma^{e2}_t})^2}{\frac{\sigma^{e2}_t\sigma^2_z}{\sigma^{e2}_t + \sigma^2_z}}}\mathrm{dz} \\
&+ \frac{(1-p)e^{A_{22}+A_{23}}}{[pe^{A_{12}+A_{13}} + (1-p)e^{A_{22}+A_{23}}]} \int z \frac{1}{\sqrt{2\pi\sigma^2_{z|s^z}}} e^{-\frac{1}{2}\frac{(z_t - \frac{\sigma^2_z s^z_{i,t} + \sigma^{e2}_t\mu_z}{\sigma^2_z + \sigma^{e2}_t})^2}{\frac{\sigma^{e2}_t\sigma^2_z}{\sigma^{e2}_t + \sigma^2_z}}}\mathrm{dz} \\
&= \frac{pe^{A_{12}+A_{13}}}{[pe^{A_{12}+A_{13}} + (1-p)e^{A_{22}+A_{23}}]} \Big[\frac{\sigma^2_z s^z_{i,t} + \sigma^{e2}_t(\mu_z - \gamma)}{\sigma^2_z + \sigma^{e2}_t}\Big] \\
&+ \frac{(1-p)e^{A_{22}+A_{23}}}{[pe^{A_{12}+A_{13}} + (1-p)e^{A_{22}+A_{23}}]} \Big[\frac{\sigma^2_z s^z_{i,t} + \sigma^{e2}_t\mu_z}{\sigma^2_z + \sigma^{e2}_t}\Big] \\
&= \frac{\sigma^2_z s^z_{i,t} + \sigma^{e2}_t\mu_z}{\sigma^2_z + \sigma^{e2}_t} - \frac{p\gamma r_t e^{A_{12}+A_{13}}}{[pe^{A_{12}+A_{13}} + (1-p)e^{A_{22}+A_{23}}]} \\
&= r_t\mu_z + (1-r_t)s^z_{i,t} - B(p,\gamma,\sigma^e_t, s^z_{i,t})
\end{aligned} \tag{56}$$

where $r_t = \frac{\sigma^{e2}_t}{\sigma^2_z + \sigma^{e2}_t}$ and $B(p,\gamma,\sigma^e_t, s^z_{i,t}) = \frac{p\gamma r_t e^{A_{12}+A_{13}}}{[pe^{A_{12}+A_{13}}+(1-p)e^{A_{22}+A_{23}}]}$

□

Proposition 2. *After accounting for the effects of uncertainty shocks on $E(z_{i,t}|s_{i,t})$, the median forecaster ($e_{i,t} = 0$) overreacts to news, that is,*

$$E(z_{i,t}^{TR}|s_{i,t}) > E(z_{i,t}^{N}|s_{i,t}), \text{ if } u_{i,t} > 0 \tag{57}$$

$$E(z_{i,t}^{TR}|s_{i,t}) < E(z_{i,t}^{N}|s_{i,t}), \text{ if } u_{i,t} < 0 \tag{58}$$

Proof.

$$B(p, \gamma, \sigma_t^e, s_{i,t}^z) = \frac{r_t\gamma}{1 + \left(\frac{1-p}{p}\right)e^C} \tag{59}$$

where $C = C_2 + C_3 = A_{22} + A_{23} - A_{12} - A_{22}$, $C_2 = A_{22} - A_{12}$ and $C_3 = A_{23} - A_{13}$

$$C_2 = = -\frac{1}{2\sigma_z^2}\gamma(2\mu_z - \gamma) \tag{60}$$

C_2 is independent of $s_{i,t}^z$ and σ_t^e

$$C_3 = \frac{1}{2\sigma_{z|s^z}^2}(\sigma_t^{e2}\gamma)(2\sigma_z^2 s_{i,t}^z) + \frac{\sigma_t^{e2} \cdot \sigma_t^{e2}\gamma}{2\sigma_{z|s^z}^2}(2\mu_z - \gamma) \tag{61}$$

$$\frac{\partial C_3}{\partial s_{i,t}^z} = \frac{1}{2\sigma_{z|s^z}^2}(\sigma_t^{e2}\gamma)(2\sigma_z^2) > 0 \tag{62}$$

Consequently, $B(p, \gamma, \sigma_t^e, s_{i,t}^z)$ is decreasing in $s_{i,t}^z$.

$$E(z_{i,t}^{TR}|s_{i,t}) - E(z_{i,t}^{N}|s_{i,t}) = B(p, \gamma, \sigma_t^e, \mu_z) - B(p, \gamma, \sigma_t^e, s_{i,t}^z) \tag{63}$$

Since $B(p, \gamma, \sigma_t^e, s_{i,t}^z)$ is decreasing in $s_{i,t}^z$ everywhere, then

$$E(z_{i,t}^{TR}|s_{i,t}) - E(z_{i,t}^{N}|s_{i,t}) = B(p, \gamma, \sigma_t^e, \mu_z) - B(p, \gamma, \sigma_t^e, s_{i,t}^z) > 0 \text{ if } u_{i,t} > 0 \tag{64}$$

$$E(z_{i,t}^{TR}|s_{i,t}) - E(z_{i,t}^{N}|s_{i,t}) = B(p, \gamma, \sigma_t^e, \mu_z) - B(p, \gamma, \sigma_t^e, s_{i,t}^z) < 0 \text{ if } u_{i,t} < 0 \tag{65}$$

□

Proposition 3. *Consider a median forecaster. An increase in prior uncertainty decreases posterior expectations, that is,*

$$\frac{\partial E(z_t|s^z_{i,t})}{\partial \sigma^e_t} = -\frac{\partial B(p,\gamma,\sigma^e_t,s^z_{i,t})}{\partial \sigma^e_t} < 0 \tag{66}$$

Proof.

$$\begin{aligned} B(p,\gamma,\sigma^e_t,s^z_{i,t}) &= \frac{p\gamma r_t e^{A_{12}+A_{13}}}{[pe^{A_{12}+A_{13}} + (1-p)e^{A_{22}+A_{23}}]} \\ &= \frac{r_t\gamma}{1+\left(\frac{1-p}{p}\right)e^{A_{22}+A_{23}-A_{12}-A_{22}}} \\ &= \frac{r_t\gamma}{1+\left(\frac{1-p}{p}\right)e^{C_2+C_3}} \\ &= \frac{r_t\gamma}{1+\left(\frac{1-p}{p}\right)e^{C}} \end{aligned} \tag{67}$$

where $C = C_2 + C_3 = A_{22} + A_{23} - A_{12} - A_{22}$, $C_2 = A_{22} - A_{12}$ and $C_3 = A_{23} - A_{13}$

$$\begin{aligned} C_2 = A_{22} - A_{12} &= -\frac{1}{2}\frac{\mu_z^2\sigma_t^{e2} + \sigma_z^2 s_{i,t}^{z2}}{\sigma_t^{e2}\sigma_z^2} + \frac{1}{2}\frac{(\mu_z-\gamma)^2\sigma_t^{e2} + \sigma_z^2 s_{i,t}^{z2}}{\sigma_t^{e2}\sigma_z^2} \\ &= -\frac{1}{2\sigma_z^2}\gamma(2\mu_z - \gamma) \end{aligned} \tag{68}$$

C_2 is independent of $s^z_{i,t}$ and σ^e_t

$$\begin{aligned} C_3 = A_{23} - A_{13} &= \frac{1}{2\sigma^2_{z|s^z}}\left[(\sigma_t^{e2}\mu_z + \sigma_z^2 s^z_{i,t})^2 - (\sigma_t^{e2}(\mu_z-\gamma) + \sigma_z^2 s^z_{i,t})^2\right] \\ &= \frac{1}{2\sigma^2_{z|s^z}}\left[\sigma_t^{e2}\gamma(2\sigma_t^{e2}\mu_z + 2\sigma_z^2 s^z_{i,t} - \sigma_t^{e2}\gamma)\right] \\ &= \frac{1}{2\sigma^2_{z|s^z}}(\sigma_t^{e2}\gamma)(2\sigma_z^2 s^z_{i,t}) + \frac{\sigma_t^{e2}\cdot\sigma_t^{e2}\gamma}{2\sigma^2_{z|s^z}}(2\mu_z - \gamma) \end{aligned} \tag{69}$$

Differentiating C with respect to σ_t^{e2},

$$\begin{aligned} \frac{\partial C}{\partial \sigma_t^{e2}} &= \frac{\partial C_3}{\partial \sigma_t^{e2}} \\ &= \gamma s^z_{i,t} + \frac{\gamma(2\mu_z-\gamma)(2\sigma_t^{e2} + \sigma_z^2)}{2\sigma_z^2} \end{aligned} \tag{70}$$

Sufficient condition: $\mu_z \leq 0$. In the case of a median forecaster, $s^z_{i,t} = \mu_z \leq 0$.

Then $\frac{\partial C}{\partial \sigma_t^{e2}} < 0 \implies \frac{\partial B(p,\gamma,\sigma_t^e,s^z_{i,t})}{\partial \sigma_t^{e2}} > 0$

□

Corollary 1. *Under the condition* $\frac{1-p}{p}e^C - 1 > 0$*, a forecaster who receives relatively bad (good) news receive a signal with* $e_{i,t} < 0$*,* $(e_{i,t} > 0)$*. An increase in prior uncertainty decreases posterior expectations by more (by less) for a forecaster that receives relatively bad (good) news. Mathematically,*

$$\left|\frac{\partial B(p,\gamma,\sigma_t^e,s_{i,t}^z)}{\partial \sigma_t^e}\right|_{e_{i,t}<0} > \left|\frac{\partial B(p,\gamma,\sigma_t^e,s_{i,t}^z)}{\partial \sigma_t^e}\right|_{e_{i,t}>0} \tag{71}$$

Proof.

$$\frac{\partial B(p,\gamma,\sigma_t^e,.)}{\partial C} = -\frac{r_t\gamma}{[1+\left(\frac{1-p}{p}\right)e^C]^2}\cdot\left(\frac{1-p}{p}\right)e^C \tag{72}$$

$$\begin{aligned}\frac{\partial^2 B(p,\gamma,\sigma_t^e,.)}{\partial C^2} &= \frac{r_t\gamma}{[1+\left(\frac{1-p}{p}\right)e^C]^3}\cdot\left[\left(\frac{1-p}{p}\right)e^C\right]^2 - \frac{r_t\gamma}{[1+\left(\frac{1-p}{p}\right)e^C]^2}\cdot\left(\frac{1-p}{p}\right)e^C \\ &= \left(\frac{1-p}{p}e^C - 1\right)\left(\frac{1-p}{p}e^C\right)\frac{1-\gamma}{(1+\frac{1-p}{p}e^C)^3} > 0\end{aligned} \tag{73}$$

$$\frac{\partial B(p,\gamma,\sigma_t^e,.)}{\partial \sigma_t^e} = \frac{\partial B(p,\gamma,\sigma_t^e,s_{i,t}^z)}{\partial C(\sigma_t^e,s_{i,t}^z)}\cdot\frac{\partial C(\sigma_t^e,s_{i,t}^z)}{\partial \sigma_t^e} > 0 \tag{74}$$

$$\frac{\partial^2 C(\sigma_t^e,s_{i,t}^z)}{\partial \sigma_t^e \partial s_{i,t}^z} = \gamma > 0 \tag{75}$$

$$\begin{aligned}\frac{\partial \frac{\partial B(p,\gamma,\sigma_t^e,s_{i,t}^z)}{\partial \sigma_t^e}}{\partial s_{i,t}^z} &= \frac{\partial^2 B(p,\gamma,\sigma_t^e,s_{i,t}^z)}{\partial s_{i,t}^z \partial \sigma_t^e} \\ &= \underbrace{\frac{\partial^2 B(p,\gamma,\sigma_t^e,s_{i,t}^z)}{\partial C(\sigma_t^e,s_{i,t}^z)^2}}_{>0}\underbrace{\frac{\partial C(\sigma_t^e,s_{i,t}^z)}{\partial s_{i,t}^z}}_{>0}\cdot\underbrace{\frac{\partial C(\sigma_t^e,s_{i,t}^z)}{\partial \sigma_t^e}}_{<0} + \underbrace{\frac{\partial B(p,\gamma,\sigma_t^e,s_{i,t}^z)}{\partial C(\sigma_t^e,s_{i,t}^z)}}_{<0}\cdot\underbrace{\frac{\partial^2 C(\sigma_t^e,s_{i,t}^z)}{\partial \sigma_t^e \partial s_{i,t}^z}}_{>0} < 0\end{aligned} \tag{76}$$

□

Proposition 4. *Under the condition* $\frac{1-p}{p}e^C - 1 > 0$*, the magnitude of overreaction increases in prior uncertainty* σ_t^e*, that is*

$$\frac{\partial |E(z_{i,t}^{TR}|s_{i,t}) - E(z_{i,t}^{N}|s_{i,t})|}{\partial \sigma_t^e} > 0 \tag{77}$$

Proof.

$$\frac{\partial B(p,\gamma,\sigma_t^e,.)}{\partial C} = -\frac{r_t\gamma}{[1+\left(\frac{1-p}{p}\right)e^C]^2}\cdot(\frac{1-p}{p})e^C \tag{78}$$

$$\frac{\partial B(p,\gamma,\sigma_t^e,.)}{\partial \sigma_t^e} = \underbrace{\frac{\partial B(p,\gamma,\sigma_t^e,s_{i,t}^z)}{\partial C(\sigma_t^e,s_{i,t}^z)}}_{<0}\cdot\underbrace{\frac{\partial C(\sigma_t^e,s_{i,t}^z)}{\partial \sigma_t^e}}_{<0} > 0 \tag{79}$$

$$\begin{aligned}\frac{\partial^2 B(p,\gamma,\sigma_t^e,.)}{\partial C^2} &= \frac{r_t\gamma}{[1+\left(\frac{1-p}{p}\right)e^C]^3}\cdot\big[(\frac{1-p}{p})e^C]^2 - \frac{r_t\gamma}{[1+\left(\frac{1-p}{p}\right)e^C]^2}\cdot(\frac{1-p}{p})e^C \\ &= \Big(\frac{1-p}{p}e^C - 1\Big)\Big(\frac{1-p}{p}e^C\Big)\frac{1-\gamma}{(1+\frac{1-p}{p}e^C)^3} > 0\end{aligned} \tag{80}$$

$$\frac{\partial^2 C(\sigma_t^e,s_{i,t}^z)}{\partial \sigma_t^e \partial s_{i,t}^z} = \gamma > 0 \tag{81}$$

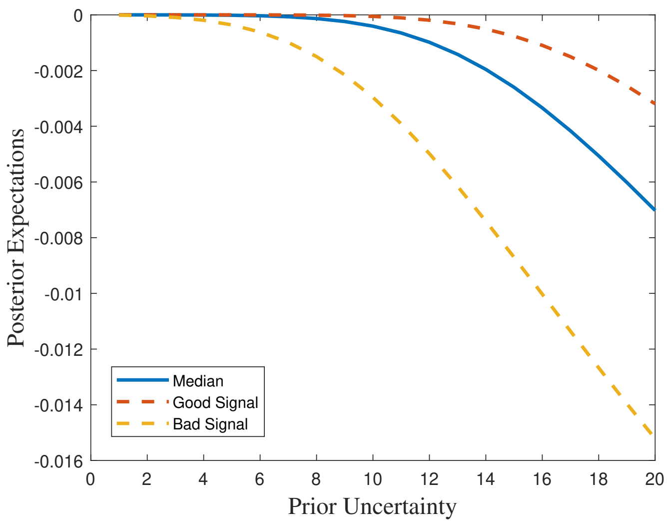
$$\begin{aligned}\frac{\partial \frac{\partial B(p,\gamma,\sigma_t^e,s_{i,t}^z)}{\partial s_{i,t}^z}}{\partial \sigma_t^e} &= \frac{\partial^2 B(p,\gamma,\sigma_t^e,s_{i,t}^z)}{\partial s_{i,t}^z \partial \sigma_t^e} \\ &= \underbrace{\frac{\partial^2 B(p,\gamma,\sigma_t^e,s_{i,t}^z)}{\partial C(\sigma_t^e,s_{i,t}^z)^2}}_{>0}\underbrace{\frac{\partial C(\sigma_t^e,s_{i,t}^z)}{\partial s_{i,t}^z}}_{>0}\cdot\underbrace{\frac{\partial C(\sigma_t^e,s_{i,t}^z)}{\partial \sigma_t^e}}_{<0} + \underbrace{\frac{\partial B(p,\gamma,\sigma_t^e,s_{i,t}^z)}{\partial C(\sigma_t^e,s_{i,t}^z)}}_{<0}\cdot\underbrace{\frac{\partial^2 C(\sigma_t^e,s_{i,t}^z)}{\partial \sigma_t^e \partial s_{i,t}^z}}_{>0} < 0\end{aligned} \tag{82}$$

Consequently,

$B(p,\gamma,\sigma_t^e,0) - B(p,\gamma,\sigma_t^e,s_{i,t}^z)$ is increasing in σ_t^e if $s_{i,t}^z > 0$.
$B(p,\gamma,\sigma_t^e,0) - B(p,\gamma,\sigma_t^e,s_{i,t}^z)$ is decreasing in σ_t^e if $s_{i,t}^z < 0$. □

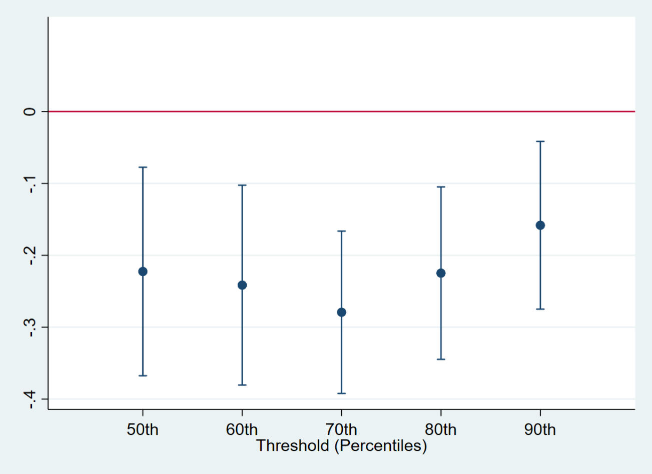
B Additional Figures and Tables

Figure B.1: Relationship between uncertainty shocks and posterior expectations based on type of signal



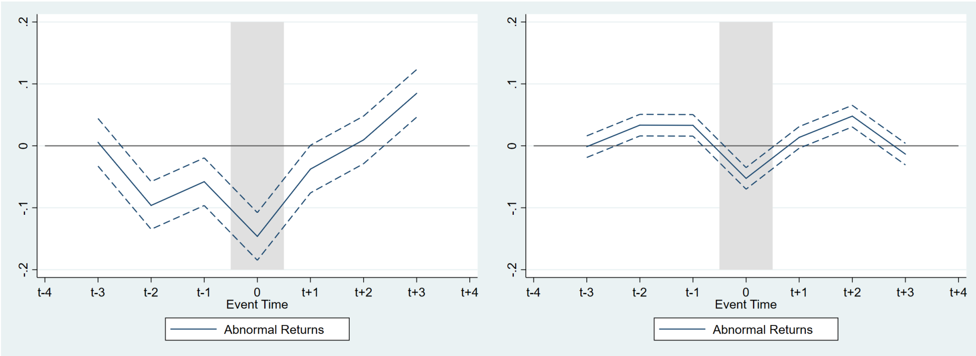
Notes: This figure presents the relationship between uncertainty shocks and posterior expectations based on type of signal by increasing (decreasing) the signal by 1 unit.

Figure B.2: Robustness Test for Tail Risk Episodes Threshold



Notes: This figure presents the 95 percent confidence interval bands of the estimates of α_3 used in Equation (31) with different tail risk episodes threshold.

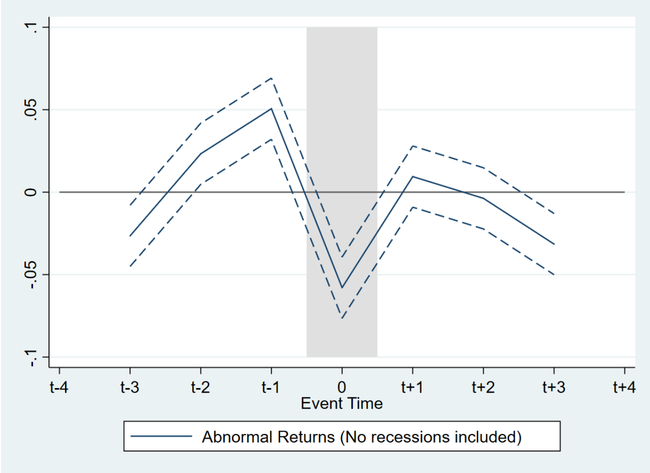
Figure B.3: Robustness Test for Event Study Abnormal Returns



(a) Tail Risk Threshold: > 90th Percentile (b) Tail Risk Threshold: > 50th Percentile

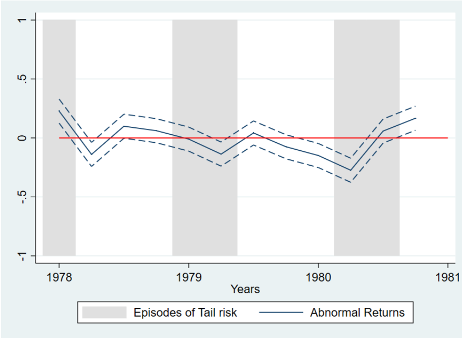
Notes: This figure presents the robustness test for the event study abnormal returns by varying the tail risk threshold to 90th percentile (Panel a) and 50th percentile (Panel b)

Figure B.4: Event Study Abnormal Returns during Non-Recession Periods

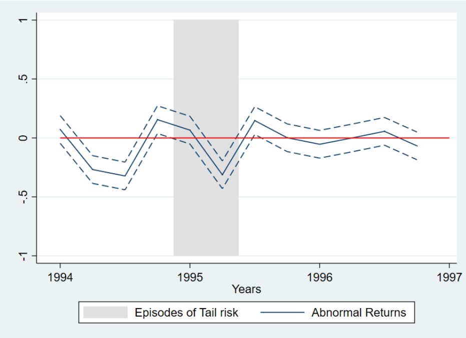


Notes: This figure presents the robustness test for the event study abnormal returns by focusing on tail risk episodes that do not coincide with recessions.

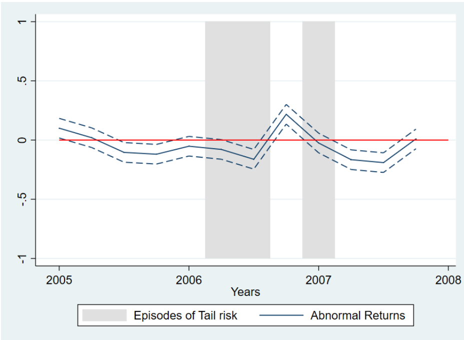
Figure B.5: Event Study of Non-Recession Episodes



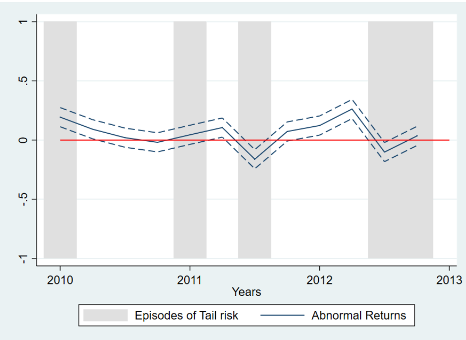
(a) 1979



(b) 1995



(c) 2006



(d) 2011

Notes: This figure presents the abnormal returns at different time periods. The shaded grey area represents tail risk episodes. The dotted lines denote 95 percent confidence interval bands.

Table B.1: Regression Results between Sentiments and Uncertainty

Dependent Variable:	$\log S_t$	$\log S_t$
	(1)	(2)
$\log UNC_t$	-0.746**	0.462***
	(0.059)	(0.013)
TR_t		-0.077***
		(0.028)
$\log UNC_t \times TR_t$		-0.277**
		(0.141)
Observations	156	156
R^2	0.91	0.91

Notes: Newey-West standard errors with a lag length of 4 quarters are in parenthesis. *, ** and *** denotes significance level at 10%, 5% and 1% respectively. Uncertainty measures are from Jurado et al. (2015).