Signaling in Dynamic Markets with Adverse Selection*

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Abstract

We study trade in dynamic decentralized markets with adverse selection. In contrast with the literature on the topic so far, we assume that the informed sellers make the offers and signaling through prices is possible. We establish basic properties of equilibria and discuss the standard two-type case and separating equilibria in detail. We prove that market efficiency, measured by the maximum gains from trade possible in equilibrium, is *invariant* to trading frictions. Our analysis shows that screening and signaling lead to markedly different trading outcomes in dynamic decentralized markets with adverse selection.

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1 Introduction

Adverse selection is a feature of many dynamic decentralized markets, such as housing markets and over-the-counter markets for certain types of financial products. When sellers in such markets have both price setting ability and private information about the quality of the products they sell, prices can be used to signal quality (see, e.g., [Bagwell and Riordan, 1991] and [Wolinsky, 1983]). Despite this possibility, the literature on dynamic decentralized markets with adverse selection has focused on screening by mostly considering models in which the uninformed buyers make the offers. Such focus is restrictive, though, as it is known at least since Wilson [1980] that the outcome of trade in markets with adverse selection can be sensitive to the price setting mechanism.

In this paper, we study signaling through prices in a dynamic decentralized market with adverse selection. In our setting, informed sellers randomly match with and make price offers to uninformed buyers. We provide a partial characterization of the equilibrium set and analyze the standard two-type case and separating equilibria in detail. Similarly to a screening setting, delay in trade takes place in equilibrium if adverse selection is severe enough to prevent trade from taking place at a single price. The intuition for this result is standard: since owners of higher quality goods are endogenously more patient, delay in trade restores trade by ensuring that owners of lower quality goods do not want to pool with owners of higher quality goods and take longer to trade.[1] In screening models of trade this implies that a reduction in trading frictions, by reducing the opportunity cost of not trading, has a negative impact on market efficiency. We show that, in the presence of signaling through prices, trading frictions do not affect market efficiency. Overall, our results show that in dynamic decentralized markets with adverse selection, market outcomes in the presence of signaling differ substantially from market outcomes in the presence of screening.

We start in Section 2 by introducing the environment and defining equilibria. We consider trade in a dynamic decentralized market with a constant inflow of buyers and sellers.

[1] The idea that delay restores trade under adverse selection goes back to Wilson [1980].

Each seller can produce one unit of an indivisible good that is of one of finitely many types. Sellers are privately informed about the type of the good they produce. Both the value of the good to buyers and the cost of producing the good to sellers are strictly increasing in the type of the good. Moreover, gains from trade are positive for each type of the good. At each point in time, buyers and sellers in the market are randomly and anonymously matched in pairs. Once matched to a buyer, a seller posts a price that the buyer either accepts or rejects. If the buyer accepts, then trade takes place and the agents leave the market. Otherwise, the match is dissolved and the agents remain in the market. Trading frictions are captured by the agents' discount factor, which determines the opportunity cost of not trading at a point in time. We consider stationary equilibria in which behavior is time-invariant and the inflow of agents into the market matches the outflow. We do not impose restrictions on the beliefs that buyers can have about the type of the good sold at off-equilibrium prices.

Signaling naturally leads to multiple equilibria. Nevertheless, we show in Section 3 that all equilibria share certain features. A key property of all equilibria is that prices offered by sellers are nondecreasing in the type of the good they produce. A consequence of this result is that the number of prices at which trade can take place is finite. Another consequence of the first result is that if adverse selection is severe enough to prevent sellers from posting the same price, then the lowest type of seller posts a lower price than the highest type of seller. In this case there will necessarily be delay in trade. We also establish in Section 3 that the set of equilibrium payoff vectors for equilibria in which the buyers' payoff is zero is invariant to trading frictions. This result, which we discuss in detail in the text, is at the heart of our result that market efficiency is invariant to trading frictions.[2]

In Section 4, we analyze the two-type case when gains from trade are strictly increasing in the type of the good and adverse selection is severe enough to prevent the (efficient) pooling outcome — this is the case typically considered in the literature. We completely determine the set of equilibrium payoff vectors and show that gains from trade are max-

[2]As it turns out, there exist equilibria in which the buyers' payoff is positive. We briefly discuss their properties at the end of Section 3.

imized in the most efficient separating equilibrium, which features no delay in the trade of the low-type good and a discount-factor independent delay in the trade of the high-type good. This stands in sharp contrast to the case in which buyers make take-it-or-leave-it offers to sellers. In the latter case, the equilibrium is such that gains from trade for both types of the good decrease with a reduction in trading frictions and only gains from trade of the low-type good are realized in the limit as trading frictions vanish.[3]

Separating equilibria capture in an intuitive way the idea that prices signal quality. In Section 5, we characterize separating equilibria, which always exist, and show that in this class of equilibria both the set of equilibrium payoff vectors and the set of possible values of equilibrium gains from trade are invariant to trading frictions. Only considering separating equilibria can be restrictive, though. We also show in Section 5 that with three or more types of the good there can exist non-separating equilibria that Pareto dominate every separating equilibrium even when adverse selection is severe enough to prevent pooling in equilibrium. So, unlike in the two-type case, gains from trade need not be maximized in the most efficient separating equilibrium. This begs the question of what can be said about the efficiency properties of non-separating equilibria, a topic we consider in Section 6

Section 6 considers the general finite-type case. There, we show that market efficiency, measured by the maximum gains from trade possible in equilibrium, is invariant to trading frictions. The proof of this result has several parts. First, we show that gains from trade are maximized in equilibria in which the buyers' payoff is zero. This occurs as reducing buyer payoffs increases the prices at which trade can trade place, relaxing the seller incentive-compatibility constraints and allowing for a greater probability of trade for all types of the good. Then, we show that randomization by sellers hurts gains from trade. Finally, we show that gains from trade in equilibria in which the buyers' payoff is zero and sellers do not randomize equals average seller payoffs in the population. Our efficiency result then follows from the result that the set of equilibrium payoff vectors for equilibria in which the buyers' payoff is zero is invariant to trading frictions.

[3]See Moreno and Wooders [2010] for a derivation of these results.

Our analysis focuses on stationary equilibria. These equilibria implicitly assume that the initial market configuration is such that the outflow of each type of seller matches the inflow given equilibrium behavior. This begs the question of whether the outcomes of a stationary equilibrium, i.e., the prices at which each type of good trades, the discounted probabilities of trade at each of these prices, and the masses of each type of seller in the market, are long-run outcomes of an equilibrium in which one does not impose stationarity to begin with. In Section 7, we show that the answer to this question is positive for a class of stationary equilibria that play a central role in our analysis, namely, the equilibria in which sellers do not randomize and buyer payoffs are zero.[4]

Section 7 also discusses the robustness of our results to alternative specifications of the environment. First, we discuss the timing of trade. In our environment, agents match at the beginning of each period. We show that if new agents in the market have to wait one period before getting the chance to trade, then a reduction in trading frictions linearly increases gains from trade. This effect is due to a mechanical reduction in the cost of waiting for the first trading opportunity. Then, we consider other notions of trading frictions. We show that our main results survive when instead of discounting future payoffs agents exogenously leave the market over time. We also consider within-period matching frictions by assuming that in every period there exists a positive probability that agents in the market do not meet a trading partner. We show that equilibrium gains from trade increase with a decrease in trading frictions in the presence of within-period matching frictions. This is a consequence of an easing of a technological constraint and is unrelated to adverse selection.

Section 8 concludes the paper and the Appendix contains omitted proofs and details.

Related Literature. The idea that prices can signal quality in the presence of adverse selection is old. Wilson [1980], Wolinsky [1983], Milgrom and Roberts [1986], and Bagwell and Riordan [1991] are seminal references. The literature on signaling through prices has mostly considered static settings, though.[5]

[4] This class of equilibria includes separating equilibria as a special case.

[5] Ellingsen [1997] analyzes signaling through prices in a static adverse-selection market in which agents trade an indivisible good. The static version of our setting includes Ellingsen's setting as a special case.

Equilibrium refinements have been developed for signaling games to reduce the equilibrium multiplicity typical of these games by placing restrictions on the beliefs that agents can form off the path of play. The most prominent of these refinements are the Intuitive Criterion of Cho and Kreps [1987] and the D1 refinement of Banks and Sobel [1987]. We take an agnostic view on the belief formation process off the path of play by not imposing any equilibrium refinement in our analysis.[6] As it turns out, the Intuitive Criterion does not refine our equilibrium set while D1 reduces it to the set of separating equilibria.[7]

Existing models of dynamic decentralized trade in the presence of adverse selection, see, e.g., Blouin [2003], Moreno and Wooders [2010], Camargo and Lester [2014], Chiu and Koeppl [2016], Kim [2017], Choi [2018], and Kaya and Roy [2020] abstract from signaling through prices by assuming that buyers make the offers.[8] Our results show that signaling through prices can increase market efficiency relative to screening and leads to different implications for the relationship between market efficiency and trading frictions.

Many of the papers in the literature on dynamic centralized trading in the presence of adverse selection, see, e.g., Hendel and Lizzeri [1999], Janssen and Roy [2002], Kurlat [2013], and Fuchs and Skrzypacz [2015], assume that in every period all trades must take place at the same price, thus ruling out *a priori* the possibility of a relationship between prices and quality within a period. Janssen and Roy [2002] considers a related setting in which a continuum of buyers and sellers trade one unit of an indivisible good in a dynamic centralized market. In equilibrium, lower qualities trade first and all qualities eventually trade. A key result in the paper is that as trading frictions vanish, the number of rounds of trading necessary to trade all qualities of the good except the lowest diverge to infinity.

Models of competitive search with adverse selection differ from the above-mentioned models of centralized trade with adverse selection by allowing for a contemporaneous rela-

[6]See Riley [2001] for a discussion of the different refinements used in the signaling literature and Mailath et al. [1993] for a criticism of belief-based refinements in signaling games.

[7]Both results are straightforward extensions to our dynamic environment of results derived in Ellingsen [1997]; see the Supplementary Appendix for a proof of these results.

[8]An exception is Inderst [2005], which considers trade of Rothschild-Stiglitz contracts in a stationary market in which both sides of the market have the chance to propose a contract. It shows that the least-cost separating contracts are always supported as an equilibrium outcome in the limit as trading frictions vanish.

tionship between prices and quality in equilibrium.[9] Guerrieri and Shimer [2014] considers a dynamic competitive search model with adverse selection in which agents trade assets of different qualities. In equilibrium, sellers of higher-quality assets signal quality by accepting a lower trade probabilities. Chang [2018] considers a dynamic competitive search model with adverse selection and shows that the ability of prices to signal quality is reduced if sellers differ both in the quality of their goods and in their selling needs. We differ from the literature on competitive search with adverse selection by not only considering decentralized trade but also by not imposing belief refinements to constrain equilibria.[10]

Models of bargaining with common-value uncertainty also typically assume that the uninformed party makes the offers.[11] In such bargaining models, a reduction in discounting reduces gains from trade by reducing the effectiveness of delay as a screening device for the buyer. Gerardi et al. [2014] considers bargaining between a long-lived buyer and a long-lived seller when the latter makes the offers. It shows that in the limit as discounting vanishes the outcome of bargaining can be more efficient than when the buyer makes the offers. So, as in our market setting, signaling through prices can increase efficiency.

2 Environment and Equilibria

We first describe the environment and then define equilibria.

2.1 Environment

Time is discrete and the horizon is infinite. There is a single indivisible good, which can be of one of finitely many types. Let $\mathcal{I} = \{1, \dots, N\}$ with $N \geq 2$ be the set of possible

[9] Guerrieri et al. [2010] is the seminal reference in the literature on competitive search models with adverse selection. Gale [1992, 1996] develop static models of Walrasian trade with adverse selection in which a relationship between prices and quality is possible in equilibrium.

[10] Guerrieri and Shimer [2014] and Chang [2018] impose the same refinement as Guerrieri et al. [2010]. In the one-dimensional case of Guerrieri and Shimer [2014], this refinement reduces the equilibrium set to separating equilibria.

[11] See, e.g., Vincent [1989], Evans [1989], and Deneckere and Liang [2006] for models of bargaining between long-lived agents, and Horner and Vieille [2009], Daley and Green [2012], Kaya and Kim [2018], and Hwang [2018] for models of bargaining between a long-lived seller and a sequence of short-lived buyers.

types of the good and denote a typical element of $\mathcal{I}$ by i. In each period, a mass one of anonymous and infinitely-lived buyers and an equal mass of anonymous and infinitely-lived sellers enter the market. The sellers can produce one unit of an indivisible good and are privately informed about the type of the good that they can produce. Let f_i be the fraction of type i sellers in the set of sellers entering the market at each date, i.e., the fraction of sellers who can produce the good of type i. The payoff to a buyer who buys the type-i good at price $p \geq 0$ is $v_i - p$ with $v_i \geq 0$. The payoff to a type-i seller who sells the good at price p is $p - c_i$ with $c_i \geq 0$. The valuations v_i and the (opportunity) costs c_i are strictly increasing in i. So, higher types of the good are associated with greater quality in the sense that they are more desirable for both buyers and sellers. We assume that $v_i > c_i$ for all i, so that gains from trading are positive for all types of the good.[12] Gains from trade need not be increasing in the type of the good, though.

Trade takes place as follows. In each period, the buyers and sellers in the market are randomly and anonymously matched in pairs. In each buyer-seller match, the seller posts a price, which the buyer either accepts or rejects. If the buyer accepts, then trade takes place at the posted price and the agents leave the market. Otherwise, the match is dissolved and the agents remain in the market. All agents have a common discount factor $\delta \in (0, 1)$. This discount factor represents the opportunity cost of not trading at a given point in time and is a measure of trading frictions.

2.2 Equilibria

Given the anonymity and continuum-population assumptions, it suffices to describe aggregate behavior and ensure that this behavior is consistent with individual rationality. We thus follow Mas-Colell [1984] and define strategy profiles in terms of distributions of individual behavior. We consider stationary equilibria in which the behavior of buyers and sellers is time-invariant and the masses of buyers and each type of seller in the market are constant

[12]We can extend our analysis to the case in which gains from trading one or more types of the good are not positive. In this case, case separating equilibria need not exist. We derive necessary and sufficient conditions for equilibrium existence in our environment in the Supplementary Appendix.

over time. In what follows, any measure on $\mathbb{R}_+$ is understood to be Borel, and so is any subset of $\mathbb{R}_+$.

A strategy profile for the sellers is a list $\mu = (\mu_1, \ldots, \mu_N)$ of probability measures on $\mathbb{R}_+$ such that $\mu_i(P)$ is the mass of type-i sellers in the market who post a price p in the set $P \subseteq \mathbb{R}_+$ once matched to a buyer. A strategy profile for the buyers is a Borel-measurable function $\sigma : \mathbb{R}_+ \to [0, 1]$ such that $\sigma(p)$ is the probability that a price p is accepted by a buyer. A belief system for the buyers is a Borel-measurable function $\pi : \mathbb{R}_+ \to \Delta^N$, where Δ^N is the unit simplex in $\mathbb{R}^N$, such that $\pi_i(p)$ is the probability that buyers attach to the event that they purchase a type-i good should they trade at price p.

Given a strategy profile σ for the buyers, the present-discounted expected lifetime payoff to the type-i sellers who post price $p \geq 0$ as long as they are in the market is

$$U_i(p) = \frac{\sigma(p)}{1 - \delta(1 - \sigma(p))}(p - c_i) = \theta(p)(p - c_i); \tag{1}$$

the term $\theta(p)$ is the discounted probability of trade at price p. The anonymity of sellers implies that in equilibrium all type-i sellers must be indifferent between posting any price in the support of the probability measure describing their price-posting behavior.

The average present-discounted expected lifetime payoff V to buyers depends not only on the aggregate behavior of buyers and sellers, described by the strategy profile (μ, σ), but also on the buyer's belief system π and on the distribution of seller types in the market. Let g_i be the fraction of type-i sellers in the market and $\overline{\mu}$ be the probability measure on $\mathbb{R}_+$ such that $\overline{\mu} = \sum_{i=1}^N g_i \mu_i$; by construction, $\overline{\mu}(P)$ is the probability that a buyer receives a price offer in the set $P \subseteq \mathbb{R}$. Then

$$V = \int_{\mathbb{R}_+} \left(\sum_{i=1}^N \pi_i(p)(v_i - p) \right) \sigma(p) d\overline{\mu}(p) + \left(\int_{\mathbb{R}_+} (1 - \sigma(p)) d\overline{\mu}(p) \right) \delta V. \tag{2}$$

Since buyers are anonymous, their equilibrium payoffs are the same and equal to V.

Finally, given the pair (μ, σ) of strategy profiles for sellers and buyers, the probability that a type-i seller in the market trades in a given period is $\mathbb{E}_{\mu_i}[\sigma] = \int_{\mathbb{R}_+} \sigma(p) d\mu_i(p)$. So, in a stationary equilibrium the mass of type-i sellers in the market must be M_i such that

$$M_i \mathbb{E}_{\mu_i}[\sigma] = f_i. \tag{3}$$

The fraction g_i of type-i sellers in the market is then equal to $M_i / \sum_{j=1}^N M_j$.

We can now define stationary equilibria. Except in Section 7.1, when we allow non-stationary behavior, we refer to stationary equilibria simply as equilibria.

Definition. *A (stationary) equilibrium is a list* $\left(\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}}\right)$ *satisfying the following properties.*

1. ***Seller Optimality.*** *For all* $i \in \mathcal{I}$*, every price* p *in the support* S_i *of* μ_i *maximizes*

$$\sigma(p)(p - c_i) + (1 - \sigma(p))\delta U_i.$$

2. ***Buyer Optimality.*** *For all* $p \geq 0$*, the probability of trade* $\sigma(p)$ *maximizes*

$$\sigma\left(\sum_{i=1}^N \pi_i(p) v_i - p\right) + (1 - \sigma)\delta V.$$

3. ***Rational Beliefs.*** *The belief* $\pi(p)$ *is given by Bayes' rule for all* $p \in S = \bigcup_{i=1}^N S_i$.

4. ***Consistency of Payoffs.*** *The payoffs* $(U_i)_{i\in\mathcal{I}}$ *are such* $U_i = U_i(p)$ *for all* $p \in S_i$ *with* $U_i(p)$ *given by* (1) *and the payoff* V *satisfies* (2) *with* $g_i = M_i / \sum_{j=1}^N M_j$.

5. ***Stationarity.*** *The vector of masses* $(M_i)_{i\in\mathcal{I}}$ *satisfy the stationarity condition* (3).

Buyer and seller optimality require that buyers and sellers behave optimally given their continuation payoffs. Payoff consistency implies that these payoffs are consistent with the behavior of agents. Since agents have the option of not trading, it follows that $V \geq 0$ and $U_i \geq 0$ for all $i \in \mathcal{I}$ in any equilibrium. Stationarity implies that at any point in time, the inflow of each type of seller in the market equals the outflow. Thus, all types of good trade in equilibrium.[13] Stationary also implies that the masses M_i are finite, so that

[13]In other words, for each $i \in \mathcal{I}$, the measure μ_i assigns positive probability to the set $\{p \geq 0 : \sigma(p) > 0\}$.

in equilibrium the fraction of each type of seller in the market is positive. Rationality of beliefs requires that buyer beliefs are given by Bayes's rule for all prices in S, the set of prices that sellers post in equilibrium. We impose no further restrictions on beliefs. By being agnostic about the belief formation process for off-equilibrium prices, we place no restrictions on the equilibrium set other than stationarity and Bayes' rule.

For each $\delta \in (0,1)$, the map $\sigma \mapsto \sigma/[1-\delta(1-\sigma)]$ taking probabilities of trade into discounted probabilities of trade is strictly increasing, and so invertible. Thus, we can also describe buyer behavior by means of a Borel-measurable function $\theta : \mathbb{R}_+ \to [0,1]$ such that $\theta(p)$ is the discounted probability of trade at price p. This fact is useful for our analysis.

An equilibrium is *pooling* if there exists $p \geq 0$ such that $S_i \equiv \{p\}$, i.e., all types of seller post the same price in equilibrium. The case of interest is when adverse selection prevents the pooling outcome. An equilibrium is *separating* if the sets S_1 to S_N are mutually disjoint. We say that an equilibrium is *pure* if the sets S_1 to S_N are singletons; separating equilibria are a special case of pure equilibria. Pure equilibria play an important role in our analysis of market efficiency.

We now define gains from trade. Given a strategy profile (μ, σ), gains from trade are

$$G = \sum_{i=1}^{N} f_i \frac{\mathbb{E}_{\mu_i}[\sigma]}{1-\delta\big(1-\mathbb{E}_{\mu_i}[\sigma]\big)}(v_i - c_i),$$

the average expected gain from trade in the entering population. The quantity G captures the expected amount of time it takes for a good to trade and measures market efficiency.[14]

3 Basic Results

In this section, we first prove some useful auxiliary results, then establish basic properties of equilibria, and conclude by showing that the set of equilibrium payoff vectors of equilibria in which the buyers' payoff is zero is invariant to trading frictions. This last result is key

[14]A conjecture given the quasi-linearity of preferences is that gains from trade equal $V + \sum_{i=1}^{N} f_i U_i$, the average payoff of agents in the entering population. As we show in Section 6, this is not true in general.

for our proof that trading frictions do not affect market efficiency.

3.1 Auxiliary Results

Consider a list $\left(\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}}\right)$ satisfying payoff consistency. Since an option for type-i sellers is to offer the same price $p' \geq 0$ as long as they remain in the market, a lower bound for their payoff is $\theta(p')(p' - c_i)$. Given that $U_i = \theta(p)(p - c_i)$ for all $p \in S_i$ by payoff consistency, it then follows that a necessary condition for seller optimality is that $\theta(p)(p - c_i) \geq \theta(p')(p' - c_i)$ for all $p \geq S_i$ and $p' \geq 0$. The first result we establish is that this condition is sufficient as well. The proof of Lemma 1 is in the Appendix.

Lemma 1. *Suppose that the list $\left(\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}}\right)$ satisfies payoff consistency. Then, seller optimality for type-i sellers is equivalent to $\theta(p)(p - c_i) \geq \theta(p')(p' - c_i)$ for all $p \in S_i$ and $p' \geq 0$.*

Equilibria in which buyer payoffs are zero are such that any price at which trade takes place equals the expected quality of the good trading at this price. These equilibria play a prominent role in our analysis. The next result is useful for establishing their existence.

Lemma 2. *Suppose that $E = \left(\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}}\right)$ satisfies rationality of beliefs, payoff consistency, and stationarity. Moreover, suppose that $p = \sum_{i=1}^N \pi_i(p)v_i$ for all $p \in S$ with $\sigma(p) > 0$ and that for each $i \in \mathcal{I}$, $\theta(p)(p - c_i) \geq \theta(p')(p' - c_i)$ for all $p \in S_i$ and $p' \in S$. Then, $\sigma(p)$ and $\pi(p)$ can be modified for $p \notin S$ so that E is an equilibrium.*

Proof. Suppose that σ and π satisfy the assumptions in the statement; note that rationality of beliefs, payoff consistency and stationarity only place restrictions on $\pi(p)$ and $\sigma(p)$ for $p \in S$. Now set $\pi(p)$ to be such that $p = \sum_{i=1}^N \pi_i(p)v_i$ for all $p \notin S$. Then, buyer optimality holds regardless of σ. This, in turn, implies that for each $p \notin S$, we can set $\sigma(p)$ to be such that no type of seller finds it optimal to post p. □

We now establish a sufficient condition for seller optimality. Consider an equilibrium with finite S. For each $p \in S$, let $\mathcal{I}(p) = \{i \in \mathcal{I} : p \in S_i\}$ be the set of seller types that offer p. Now for each $p \in S$ with $p < p^{\max} = \max\{p' : p' \in S\}$, let p^+ be the successor of p in S. Seller optimality implies that if $p \in S$ is such that $p < p^{\max}$, then

$$\theta(p^+)(p^+ - c_i) \leq \theta(p)(p - c_i) \text{ for all } i \in \mathcal{I}(p).$$

The next result we establish shows that if S is finite and certain conditions hold, then no type of seller has an incentive to mimic the behavior of the other types of seller if for all $p \in S$ with $p < p^{\max}$, the above local upward incentive-compatibility constraint is satisfied with equality for the greatest element of $\mathcal{I}(p)$. This result is a consequence of the fact that seller costs are increasing in their types. So, since $(p - c)/(p^+ - c)$ is strictly decreasing in c for all $c < p$, the type of seller posting $p < p^{\max}$ that has the greatest incentive to deviate and post p^+ is the highest-type seller.[15] The proof of Lemma 3 is in the Appendix.

Lemma 3. *Consider a list* $\left(\mu, \sigma, \pi, (U_i)_{i \in \mathcal{I}}, V, (M_i)_{i \in \mathcal{I}}\right)$ *with* S *finite such that: (i)* $p \in S$ *and* $i \in \mathcal{I}(p)$ *imply that* $p \geq c_i$*, with strict inequality if* $p < p^{\max}$*; and (ii)* $j \geq i$ *for all* $i \in \mathcal{I}(p)$ *and* $j \in \mathcal{I}(p')$ *with* $p' > p$*. Now suppose that for all* $p \in S$ *with* $p < p^{\max}$*,*

$$\theta(p^+) = \theta(p) \frac{p - \max\{c_i : i \in \mathcal{I}(p)\}}{p^+ - \max\{c_i : i \in \mathcal{I}(p)\}}. \tag{4}$$

For each $i \in \mathcal{I}$*, it follows that* $\theta(p)(p - c_i) \geq \theta(p')(p' - c_i)$ *for all* $p \in S_i$ *and* $p' \in S$*.*

The condition that $p \in S$ and $i \in \mathcal{I}(p)$ imply that $p \geq c_i$, with strict inequality if $i < N$, is satisfied if all types of seller except possibly the highest obtain positive payoff. The condition that $j \geq i$ for all $i \in \mathcal{I}(p)$ and $j \in \mathcal{I}(p')$ with $p' > p$ implies that prices offered by sellers cannot decrease with their types. As it turns out, both conditions and the condition that S is finite hold in equilibrium. The next result follows immediately.

Corollary 1. *Suppose that* $E = \left(\mu, \sigma, \pi, (U_i)_{i \in \mathcal{I}}, V, (M_i)_{i \in \mathcal{I}}\right)$ *satisfies rationality of beliefs, payoff consistency, stationarity, and the assumptions of Lemma 3. Moreover, suppose that* π *is such that* $p = \sum_{i=1}^{N} \pi_i(p) v_i$ *for all* $p \in S$ *with* $\sigma(p) > 0$*. Then,* $\pi(p)$ *and* $\sigma(p)$ *can be set for* $p \notin S$ *so that* E *is an equilibrium.*

[15] Standard arguments show that local upward incentive compatibility is sufficient for seller optimality.

For each $i \in \mathcal{I}$, let η_i be the measure on $\mathbb{R}_+$ such that $\eta_i(P) = \int_P \sigma(p) d\mu_i(p)$ for all $P \subseteq \mathbb{R}_+$. By construction, $\eta_i(P)$ is the probability that type-i sellers trade at a price in the set $P \subseteq \mathbb{R}_+$ in a given period. The support $S_i^* \subseteq S_i$ of η_i is the set of prices at which the type-i good can trade in equilibrium. The last auxiliary result we establish is relevant for the two-type case and the case of separating equilibria.

Lemma 4. *An equilibrium with $p \in S_1^* \setminus \bigcup_{j=2}^N S_j^*$ satisfies $p = v_1$, $\theta(v_1) = 1$, and $V = 0$.*

The proof of Lemma 4 is in the Appendix. A sketch of the proof is as follows. Consider an equilibrium with a price p that only type-1 sellers offer in equilibrium. First note that the buyers' payoff cannot be positive in such an equilibrium, otherwise type-1 sellers could profitably deviate by posting a price slightly higher than p. Indeed, Bayes' rule implies that the expected value of the good to a buyer who purchases it at price p is v_1, while for any $p' > p$ the expected value of the good to a buyer who purchases it price p' is at least v_1. So, because of discounting, buyers would be willing to trade at a price slightly higher than p instead of having to wait for one more period to trade. Clearly, $p = v_1$ if $V = 0$. Finally, note that $\theta(v_1) = 1$, otherwise type-1 sellers could increase their payoffs by offering a price slightly higher than v_1, increasing their probability of trade discontinuously.

3.2 Basic Properties of Equilibria

As is well-know, signaling leads to multiple equilibria. Nevertheless, as we now show, all equilibria share some common properties. The first of such properties is that seller payoffs are strictly decreasing in seller types. The proof of Lemma 5 is in the Appendix.

Lemma 5. *Consider an equilibrium. For all $i, j \in \mathcal{I}$, $j > i$ implies that $U_i > U_j$.*

The intuition for Lemma 5 is standard in models of dynamic trade with adverse selection. Given that lower-type sellers have a lower opportunity cost of trading, they are able to extract greater informational rents from buyers and obtain higher payoffs.[16] Note from

[16]One can show that seller reservation values, given by $r_i = c_i + \delta U_i$, are strictly increasing in i. Since higher-type sellers have a greater opportunity cost of trading, they are only willing to trade at higher prices.

Lemma 5 that $U_i > 0$ for all $i < N$. So, $\theta(p) > 0$ for all $p \in S_i$ when $i < N$. Thus, except possibly for the type-N sellers, in equilibrium all sellers make offers that are accepted with positive probability by buyers, i.e., $S_i = S_i^*$ for all $i < N$. Type-N sellers can make offers that are rejected with probability one by buyers only if $U_N = 0$.

We now use a standard incentive-compatibility argument to establish our first main property of equilibria, namely, that the prices that the different types of seller post in equilibrium are nondecreasing in their types.

Proposition 1. *Consider an equilibrium. For all $i, j \in \mathcal{I}$, $j > i$ implies that $p \leq p'$ for all $p \in S_i$ and $p' \in S_j$.*

Proof. Let $i, j \in \mathcal{I}$ be such that $i < j$. Seller optimality implies that

$$\theta(p)(p - c_i) \geq \theta(p')(p' - c_i) \text{ and } \theta(p')(p' - c_j) \geq \theta(p)(p - c_j)$$

for all $p \in S_i$ and $p' \in S_j$. Summing the two inequalities, we obtain that

$$\big(\theta(p') - \theta(p)\big)(c_i - c_j) \geq 0.$$

Since $c_j > c_i$, it follows that $\theta(p) \geq \theta(p')$ for all $p \in S_i$ and $p' \in S_j$. We claim that $p' \geq p$ for all $p \in S_i$ and $p' \in S_j$. Let $p \in S_i$ and $p' \in S_j$. Then either $\theta(p') > 0$ or $\theta(p') = 0$. Suppose first that $\theta(p') > 0$. Then $p' \geq c_j$ as $U_j \geq 0$. This, in turn, implies that $p' \geq p$, otherwise $\theta(p)(p - c_j) > \theta(p')(p' - c_j)$, contradicting seller optimality. Suppose now that $\theta(p') = 0$, which is possible only if $j = N$. We claim that $p' \geq v_N - \delta V$. Indeed, since $p' \notin S_k$ for all $k < N$, as $\theta(p) > 0$ for all $p \in \bigcup_{k=1}^{N-1} S_k$, Bayes' rule implies that $\pi_N(p') = 1$. So, buyers reject p' only if $p' \geq v_N - \delta V$. Now observe that since $\theta(p) > 0$ for all $p \in S_i$, buyer optimality implies that $p \leq \sum_{k=1}^{N} \pi_k(p) v_k - \delta V \leq v_N - \delta V \leq p'$. $\square$

A consequence of seller optimality is that $\theta(p)$ is strictly decreasing in p for $p \in S^*$. Since Proposition 1 implies that higher-type sellers trade at higher prices, it then follows that such sellers trade less frequently, remaining longer in the market. Hence, the steady-state distribution of seller types in the market dominates the distribution of seller types in the population in the likelihood-ratio order. The proof of Corollary 2 is in the Appendix.

Corollary 2. *In any equilibrium, the ratio g_i/f_i is nondecreasing in i.*

Proposition 1 also implies that the sets S_i^* are finite. Indeed, if S_i^* is infinite, then there exist two prices p and p' with $\theta(p), \theta(p') \in (0,1)$ at which only the type-i good trades. Bayes' rule then implies that a buyer who purchases the good at either of these two prices assigns probability one to the good being of type i. However, since $\theta(p)$ and $\theta(p')$ are interior, the buyer must be indifferent between trading and not trading at these two prices, which is possible only if $p = p'$. So, the set $S^* = \bigcup_{i=1}^N S_i^*$ of serious price offers is finite. In fact, one can show that for each $i \in \mathcal{I}$, there exists at most one price $p \in S_i^*$ that is offered only by type-i sellers. From this it follows that S^* can have at most $2N-1$ elements. The next result summarizes this discussion. Its proof is in the Appendix.

Corollary 3. *In any equilibrium, the set S^* is finite and has at most $2N-1$ elements.*

A third consequence of Proposition 1 is that prices are non-negatively related to the type of the good. We say that adverse selection is *severe* if $\overline{v} = \sum_{i=1}^N f_i v_i < c_N$. It is easy to see that if adverse selection is not severe, then all sellers pooling at a single price is an equilibrium outcome.[17] The next result shows that pooling is not an equilibrium outcome when adverse selection is severe. In this case, the prices offered by sellers in equilibrium necessarily reveal some information about the type of the good that they produce.

Proposition 2. *Suppose that adverse selection is severe. In any equilibrium there exists $p \in S_1^*$ such that $p \notin S_N^*$.*

Proof. Suppose, by contradiction, that $S_1^* \subseteq S_N^*$. By Proposition 1, there exists $p \in S_N^*$ with $\sigma(p) > 0$ such that $S_i^* = \{p\}$ if $i < N$. Bayes's rule then implies that for all $i < N$,

$$\pi_i(p) = \frac{M_i}{\sum_{j=1}^{N-1} M_j + \alpha_N M_N},$$

[17]The list $E = (\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}})$ such that $\mu_i(\{\overline{v}\}) = 1$, $\sigma(p) = 1$ if $p \leq \overline{v}$ and $\sigma(p) = 0$ otherwise, $\pi_i(p) \equiv f_i$, $U_i = \overline{v} - c_i$, $V = 0$, and $M_i = f_i$ is an equilibrium if $\overline{v} \geq c_N$.

where α_N is the probability that a type-N seller posts p. Now note from the stationarity condition (3) that $M_i = f_i/\sigma(p)$ if $i < N$ and $M_N = f_N/[\alpha_N\sigma(p) + (1-\alpha_N)\sigma']$, where $\sigma' < \sigma(p)$ is the probability of trade conditional on posting a price in $S_N^* \setminus \{p\}$. So,

$$\pi_i(p) = \frac{f_i}{\sum_{j=1}^{N-1} f_j + \alpha_N\sigma(p) f_N/[\alpha_N\sigma(p) + (1-\alpha_N)\sigma']}$$

for all $i < N$. Since $\pi_i(p)$ is strictly decreasing in α_N for all $i < N$, it then follows that

$$\sum_{i=1}^{N} \pi_i(p) v_i = v_N + \sum_{i=1}^{N-1} \pi_i(p)(v_i - v_N) \leq \sum_{i=1}^{N} f_i v_i,$$

as $v_i < v_N$ for all $i < N$. On the other hand, $p \in S_N^*$ only if $p \geq c_N$. Thus,

$$\sum_{i=1}^{N} \pi_i(p) v_i - p \leq \sum_{i=1}^{N} f_i v_i - p < c_N - p \leq 0,$$

and buyers do not find it optimal to accept an offer of p, a contradiction. □

Seller optimality implies that delay in trade happens if S^* has two or more elements. So, there exists delay in trade if adverse selection is severe. The intuition for this result is standard. Since higher-type sellers are endogenously more patient than lower-type sellers, given their greater opportunity cost of trading, a lower rate of trade at higher prices makes trade at different prices incentive feasible: lower-type sellers prefer trading faster at a lower price, making trade at a higher price attractive to buyers by implying that the quality of the good is positively related to its price.

The above discussion suggests that, as in the case in which buyers make the offers, reducing trading frictions should decrease equilibrium gains from trade. Indeed, by making delay in trade less costly for all types of seller, a reduction in trading frictions should, in principle, make delay in trade less effective in inducing lower-type sellers to trade at lower prices. We will see that this is not the case: reducing trading frictions does not affect gains from trade when sellers make the offers. This shows that screening and signaling have quite different implications for the relationship between trading frictions and market efficiency.

3.3 Equilibrium Payoff Vectors and Trading Frictions

We conclude this section by discussing the relationship between equilibrium payoff vectors and trading frictions for equilibria in which the buyers' payoffs are zero. As we later show, these equilibria maximize market efficiency.

First note that Lemma 1 implies that the discount factor does not *directly* affect seller optimality, as it depends only on discounted probabilities of trade. The discount factor also does not directly affect buyer optimality in equilibria in which the buyers' payoff is zero. Intuitively, zero payoff for buyers implies that they are indifferent between trading at any point in time (and not trading as well). Now observe that for each $\delta, \delta' \in (0,1)$ with $\delta \neq \delta'$ and $\sigma \in [0,1]$, there exists a unique $\sigma' \in [0,1]$ with $\sigma/[1-\delta(1-\sigma)] = \sigma'/[1-\delta(1-\sigma')]$. Together, these facts suggest that if for some discount factor δ there exists an equilibrium with $V = 0$, then, by adjusting the probabilities of trade appropriately, for any discount factor $\delta' \neq \delta$ there exists an equilibrium with $V = 0$ in which the prices at which trade takes place and the discounted probabilities of trade at these prices are the same as in the original equilibrium, implying the same seller payoffs.

The problem with the above reasoning is that by changing trade probabilities to preserve discounted probabilities of trade, the masses of each type of seller in the market change, affecting buyer beliefs. Proposition 3 below shows that seller behavior can be adjusted while keeping the prices at which trade takes place the same so as to keep buyer beliefs unchanged. Proposition 3 shows that trading frictions do not affect the ability of sellers to signal quality through prices in the sense that a finite set of expected qualities of the good is a set of prices at which trade can take place for some discount factor if, and only if, it is a set of prices at which trade can take place for all discount factors.

Proposition 3. *Let* $E = \big(\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}}\big)$ *be an equilibrium with* $V = 0$ *when the discount is* δ*. For any* $\delta' \in (0,1)$ *different from* δ*, there exists an equilibrium* E' *with the same payoffs as* E *when the discount factor is* δ'*. Hence, the set of equilibrium payoff vectors for equilibria with* $V = 0$ *is invariant to trading frictions.*

Proof (sketch). Let $E = \big(\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}}\big)$ be an equilibrium with $V = 0$ when the discount factor is δ and let p^1 to p^K be the prices at which trade takes place in E. Now let the discount factor be $\delta' \neq \delta$ and consider $E' = \big(\mu', \sigma', \pi', (U_i')_{i\in\mathcal{I}}, V', (M_i')_{i\in\mathcal{I}}\big)$ such that the prices at which trade takes place are p^1 to p^K and the strategy profile (μ', σ') satisfies

$$\frac{\sigma'(p^k)}{1-\delta'(1-\sigma'(p^k))} = \frac{\sigma(p^k)}{1-\delta(1-\sigma(p^k))}$$

for all $k \in \{1, \ldots, K\}$ and

$$\frac{\mu_i'(\{p^k\})\sigma'(p^k)}{\mathbb{E}_{\mu_i'}[\sigma']} = \frac{\mu_i(\{p^k\})\sigma(p^k)}{\mathbb{E}_{\mu_i}[\sigma]} \tag{5}$$

for all $i \in \mathcal{I}$ and $k \in \{1, \ldots, K\}$; note that $\mu_i'(\{p^k\}) > 0$ if, and only if, $\mu_i(\{p^k\}) > 0$, so that μ_i' and μ_i have the same support for each $i \in \mathcal{I}$. We show in the Appendix that we can choose μ' in this way. Finally, suppose that $U_i' = U_i$ for all $i \in \mathcal{I}$, $V' = 0$, the masses $(M_i')_{i\in\mathcal{I}}$ satisfy stationary, and $\pi'(p)$ satisfies Bayes' rule for all $p \in \{p^1, \ldots, p^K\}$. Since

$$\frac{M_i'\mu_i'(\{p^k\})}{M_i\mu_i(\{p^k\})} = \frac{\sigma(p^k)}{\sigma'(p^k)}$$

for all $i \in \mathcal{I}$ and $k \in \{1, \ldots, K\}$ by stationarity and (5), it follows that

$$\pi_i'(p^k) = \frac{M_i'\mu_i'(\{p^k\})}{\sum_{j=1}^N M_j'\mu_j'(\{p^k\})} = \frac{M_i\mu_i(\{p^k\})}{\sum_{j=1}^N M_j\mu_j(\{p^k\})} = \pi_i(p^k)$$

for all $i \in \mathcal{I}$ and $k \in \{1, \ldots, K\}$. Given that $p^k = \sum_{i=1}^N \pi_i(p^k)v_i$ for all $k \in \{1, \ldots, K\}$, we then have that $p^k = \sum_{i=1}^N \pi_i'(p^k)v_i$ for all $k \in \{1, \ldots, K\}$ as well. Hence, E' satisfies rationality of beliefs, payoff consistency, and stationarity. By Lemma 2, we can then choose σ' and π' for off-equilibrium prices so that E' is an equilibrium. $\square$

Proposition 3 does not apply to equilibria in which the buyers' payoff is positive, as in these equilibria at least one price at which trade takes place differs from the expected quality of the good traded at this price. It turns out that such equilibria do exist.[18] In the Appendix,

[18]Pooling equilibria with $V > 0$ trivially exist if adverse selection is not severe. In the Supplementary Appendix, we provide an example of an equilibrium with $V > 0$ when adverse selection is severe.

we show that for any equilibrium with $V > 0$ there exist a price $p \in S$, $\delta > 0$, and $\varepsilon > 0$ such that if $p < p' < p + \delta$, then $\sum_{i=1}^{N} \pi_i(p')v_i + \varepsilon < \sum_{i=1}^{N} \pi(p)v_i$; that is, an increase in the price of the good above the price p leads to a *discontinuous* drop in its expected quality. We also show in the Appendix that for any equilibrium with $V = 0$, there exists an equilibrium with the same payoffs and gains from trade for which such discontinuous drops in the expected quality of the good do not exist. Hence, the set of equilibrium payoff vectors for equilibria in which increases in price do not lead to discontinuous drops in the expected quality of the good coincides with the set of equilibrium payoff vectors for equilibria with $V = 0$, which is invariant to trading frictions.

4 Two-Type Case

Here, we study the case typically considered in the literature: the two-type case when gains from trade are strictly increasing in the type of the good and adverse selection is severe. In this case, we are able to determine the set of equilibrium payoff vectors and the most efficient equilibrium.

Suppose that $\mathcal{I} = \{1, 2\}$, $v_i > c_i$ for all i, $v_2 - c_2 > v_1 - c_1$, and $f_1 v_1 + f_2 v_2 < c_2$; note that $v_1 < c_2$ *a fortiori*. By Proposition 2, there exists a price that is offered only by type-1 sellers. Lemma 4 then implies that $S_1^* \setminus S_2^* = \{v_1\}$, $\theta(v_1) = 1$, and $V = 0$. Thus, by Proposition 3, the set of equilibrium payoff vectors is invariant to trading frictions. We determine this set explicitly in what follows.[19] Since $S_1^* \setminus S_2^* = \{v_1\}$ and $\theta(v_1) = 1$ imply that $U_1 = v_1 - c_1$, it suffices to determine the possible values for U_2. By Proposition 1, there are two cases to consider: $S_1^* \cap S_2^*$ empty or $S_1^* \cap S_2^*$ a singleton.

Consider first the case in which $S_1^* \cap S_2^*$ is empty. The proof of Corollary 3 shows that S_1^* and S_2^* are singletons, and so $S_1^* = \{v_1\}$. We claim that $S_2^* = \{v_2\}$. Indeed, if $S_2^* = \{p'\}$ for some $p' \geq 0$, then $\pi_2(p') = 1$ by Bayes' rule. So, $v_2 - p' = \delta V = 0$ by buyer optimality, as $\theta(p') < 1$ by seller optimality. Note that $S_2^* = \{v_2\}$ and $v_2 > c_2$ imply that

[19] As it turns out, the assumption that $v_2 - c_2 > v_1 - c_1$ does not matter for the results in this section. We maintain it since it is typical in the literature.

$U_2 > 0$. Hence, $\theta(v_2) > 0$ and $S_2 = S_2^*$. Since $S_1 = S_1^*$, the equilibria with $S_1^* \cap S_2^* = \emptyset$ then correspond to separating equilibria. Now note that a type-2 seller has no incentive to post v_1 since $v_1 < c_2$. A type-1 seller has no incentive to post v_2 if, and only if,

$$\theta(v_2) \leq \frac{v_1 - c_1}{v_2 - c_1}. \tag{6}$$

Condition (6) is necessary for a separating equilibrium. Lemma 2 implies that (6) is also sufficient.[20] So, the set of equilibrium payoff vectors when $S_1^* \cap S_2^* = \emptyset$ is the set of vectors (V, U_1, U_2) with $V = 0$, $U_1 = v_1 - c_1$, and $U_2 \in (0, \overline{U}_2]$, where

$$\overline{U}_2 = \frac{v_1 - c_1}{v_2 - c_1}(v_2 - c_2).$$

The separating equilibrium in which the incentive-compatibility constraint (6) holds with equality Pareto-dominates all other separating equilibria.

Now consider the case in which $S_1^* \cap S_2^*$ is a singleton. In this case, type-1 sellers randomize between offering v_1 and offering a higher price $p \in [c_2, v_2)$ that is also offered by type-2 sellers and accepted by buyers with positive probability; $p = v_2$ is not possible since, by Bayes' rule, the expected quality of the good sold at price v_2 would be smaller than v_2. We claim that type-2 sellers are worse off in this case. Indeed,

$$\theta(p) = \frac{v_1 - c_1}{p - c_1},$$

as type-1 sellers are indifferent between offering p and v_1. Then

$$U_2 = \theta(p)(p - c_2) = \frac{v_1 - c_1}{p - c_1}(p - c_2).$$

Since U_2 is strictly increasing in p, we have that $U_2 < \overline{U}_2$ and the payoff of type-2 sellers in an equilibrium with $S_1^* \cap S_2^*$ a singleton is smaller than their payoff in the most efficient separating equilibrium. It follows that for all $U_2 \in [0, \overline{U}_2)$, there exists an equilibrium with $S_1^* \cap S_2^*$ a singleton in which the seller's payoff is U_2.[21] The following result summarizes.

[20] The equilibrium masses of type-1 and type-2 sellers are $M_1 = f_1$ and $M_2 = f_2/\sigma(v_2)$, respectively.
[21] See the Supplementary Appendix for a proof.

Proposition 4. *The separating equilibrium with* (6) *holding with equality Pareto-dominates all other equilibria. The set of equilibrium payoff vectors is invariant to δ and given by $V = 0$, $U_1 = v_1 - c_1$, and $U_2 \in [0, \overline{U}_2]$.*

We now discuss equilibrium gains from trade. Gains from trade in a separating equilibrium are pinned down by the discounted probability of trade $\theta(v_2)$ for type-2 sellers and given by

$$G = f_1(v_1 - c_1) + f_2\theta(v_2)(v_2 - c_2),$$

which is the average population payoff $f_1U_1 + f_2U_2$. So, the set of possible values for gains from trade in separating equilibria is invariant to δ and equal to $(f_1(v_1 - c_1), \overline{G}]$, where

$$\overline{G} = \left(f_1 + f_2\frac{v_2 - c_2}{v_2 - c_1}\right)(v_1 - c_1).$$

The most efficient separating equilibrium is the one maximizing the discounted probability of trade for type-2 sellers.

It follows from Lemma 6 in Section 6 that equilibrium gains from trade are *smaller* than the average population payoff when $S_1^* \cap S_2^*$ is a singleton and at least the type-1 sellers randomize. The intuition for this result is as follows. By randomizing, type-1 sellers allow type-2 sellers to trade at a higher rate than in a separating equilibrium. This happens at the expense of a lower rate of trade for type-1 sellers, though. The resulting decrease in gains from trade for the type-1 good more than offsets the increase in gains from trade for the type-2 good. The wedge between gains from trade and average seller payoffs happens because randomization by sellers does not reduce their payoffs.

Since any equilibrium in which $S_1^* \cap S_2^*$ is a singleton is Pareto-dominated by the most efficient separating equilibrium, it follows from the results in the above paragraph that gains from trade in any such equilibrium are smaller than the highest gains from trade possible in separating equilibria. We then have the following result.

Proposition 5. *The maximum equilibrium gains from trade are invariant to trading frictions and equal to the maximum gains from trade in a separating equilibrium.*

To summarize, we established the following results in the two-type case when gains from trade are positive and increasing in the type of the good and adverse selection is severe: (*i*) the set of equilibrium payoff vectors is invariant to trading frictions; (*ii*) the most efficient separating equilibrium Pareto-dominates and has higher gains from trade than any other equilibrium; and (*iii*) the maximum equilibrium gains from trade are invariant to trading frictions. The first result follows from the fact severe adverse selection implies that $V = 0$. It turns out that this is not true with three or more types of the good.[22] Regarding the second result, we show in the next section that with three or more types of the good, there exist non-separating equilibria that Pareto-dominate *any* separating equilibrium and lead to higher gains from trade. In Section 6, we extend the last result to the N-type case.

Comparison with Buyer Take-it-or-Leave-it Offers. Moreno and Wooders [2010] characterizes stationary equilibria in the case considered in this section when buyers make take-it-or-leave-it offers to sellers.[23] It shows that a unique equilibrium exists if δ is close enough to one. In this equilibrium, expected gains from trade for both types of good decrease with δ, with the expected gain from trade for the type-2 good decreasing to zero as trading frictions vanish ($\delta \to 1$). This is in stark contrast to the most efficient separating equilibrium with seller offers. Moreover, given that the discounted probability of trade for type-2 sellers is positive in any separating equilibrium with seller offers, any such equilibrium has the feature that gains from trade for both types of the good are higher than in the unique equilibrium with buyer offers when δ is sufficiently close to one.

5 Separating Equilibria

In this section, we characterize separating equilibria and show that for such equilibria both the set of equilibrium payoff vectors and the set of values of equilibrium gains from trade

[22] Lemma 4 implies that $V = 0$ in equilibria in which only type-1 sellers trade at the lowest price possible. So, a necessary condition for equilibria with $V > 0$ is that type-2 sellers offer the lowest price at which trade can take place with positive probability. Proposition 2 implies that this is not possible in the two-type case when adverse selection is severe. In the Supplementary Appendix, we show that this is possible when $N \geq 3$.

[23] Moreno and Wooders [2010] assumes that in any period buyers and sellers in the market match with probability $\alpha \in (0, 1)$. Its equilibrium characterization extends to the case in which $\alpha = 1$ without change.

are invariant to trading frictions. We conclude by showing that separating equilibria can be Pareto-dominated by and have lower gains from trade than non-separating equilibria when there are three or more types of the good.

Our first result provides necessary conditions for a separating equilibrium. It extends the characterization of separating equilibria in the two-type case to the general N-type case.

Proposition 6. *The following holds in separating equilibria: (i)* $V = 0$*; (ii)* $S_i = \{v_i\}$ *for* $i < N$ *and* $v_N \in S_N$*; (iii)* $\theta(v_1) = 1$ *and* $\theta(v_{i+1}) \leq \theta(v_i)(v_i - c_i)/(v_{i+1} - c_i)$ *for* $i < N$*.*

Proof. By assumption, $S_i^* \cap S_j^* = \emptyset$ for all $i \neq j$ in $\mathcal{I}$. The proof of Corollary 3 then implies that S_i^* is a singleton for all $i \in \mathcal{I}$. Since $S_1^* \setminus \bigcup_{j=2}^N S_j^* = S_1^*$, it follows from Lemma 4 that $S_1^* = \{v_1\}$, $\theta(v_1) = 1$, and $V = 0$. Given that $V = 0$, it then follows from Bayes' rule and buyer optimality that $S_i^* = \{v_i\}$ for all $i > 1$. Property (*ii*) is a consequence of the fact that $S_i = S_i^*$ for all $i < N$, while (*iii*) is an immediate consequence of seller optimality. $\square$

Separating equilibria always exist. Indeed, consider $E = \big(\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}}\big)$ such that: (*i*) $\mu_i(\{v_i\}) = 1$; (*ii*) $\theta(v_1) = 1$ and $\theta(v_{i+1}) = \theta(v_i)(v_i - c_i)/(v_{i+1} - c_i)$ for all $i < N$; (*iii*) $\pi_i(v_i) = 1$; (*iv*) $U_i = \theta(v_i)(v_i - c_i)$; (*v*) $V = 0$; and (*vi*) $M_i = f_i/\sigma(v_i)$. By Corollary 1, we can choose $\pi(p)$ and $\sigma(p)$ for $p \notin \{v_1, \ldots, v_N\}$ so that E is an equilibrium.

Since gains from trade are equal to $\sum_{i=1}^N f_i U_i$ in separating equilibria, it follows from Proposition 3 that for such equilibria both the set of equilibrium payoff vectors and the set of possible values of equilibrium gains from trade are invariant to δ.

Corollary 4. *The set of equilibrium payoff vectors and the set of values of equilibrium gains from trade for separating equilibria are invariant to trading frictions.*

We conclude this section by showing that when there are at least three types of the good, there exist distributions of seller types in the entering population for which adverse selection is severe, so that pooling equilibria do not exist, but for which all separating equilibria are Pareto-dominated by and lead to lower gains from trade than a non-separating equilibrium. Suppose that $N = 3$, $(c_1, c_2, c_3) = (0, 1, 4)$, $(v_1, v_2, v_3) = (1, 2, 5)$, and

$(f_1, f_2, f_3) = (1/20, 9/20, 1/2)$; note that adverse selection is severe. It follows from (*iii*) in Proposition 6 that the separating equilibrium with $\theta(v_1) = 1$,

$$\theta(v_2) = \frac{v_2 - c_2}{v_2 - c_1} = \frac{1}{2}, \text{ and } \theta(v_3) = \theta(v_2)\frac{v_3 - c_3}{v_3 - c_2} = \frac{1}{8}$$

Pareto-dominates all other separating equilibria. Seller payoffs are $U_1 = 1$, $U_2 = 1/2$, and $U_3 = 1/8$. Now consider the alternative equilibrium in which type-1 and type-2 sellers pool by offering

$$p_1 = \frac{f_1}{f_1 + f_2}v_1 + \frac{f_2}{f_1 + f_2}v_2 = \frac{19}{10}$$

and type-3 sellers offer v_3. Note that $V = 0$ in this new equilibrium. Set $\theta(p_1) = 1$ and

$$\theta(v_3) = \theta(p_1)\frac{p_1 - c_2}{v_3 - c_2} = \frac{9}{40},$$

so that type-2 sellers are indifferent between offering p_1 and v_3 and type-1 sellers prefer offering p_1. Seller payoffs are now $U_1 = 19/10 > 1$, $U_2 = 9/10 > 1/2$, and $U_3 = 9/40 > 1/8$. Gains from trade are higher as well, as the discounted probabilities of trade for all goods are greater.

The intuition for why the above non-separating equilibrium Pareto-dominates all separating equilibria is as follows. In order to induce separation between the type-1 and type-2 sellers, the discounted probability of trade for the type-2 seller has to be sufficiently small. This, in turn, implies an even smaller discounted probability of trade for the type-3 seller. However, when the distribution of seller types in the population is such that the fraction of type-1 sellers is small enough, pooling them with the type-2 sellers benefits both types of seller by allowing them to trade immediately at a price marginally smaller than v_2. This also benefits type-3 sellers by allowing a higher discounted probability of trade at price v_3 without destroying the incentive of type-2 buyers to trade at the lower, pooling, price.[24]

[24]The logic of the above example can be generalized. In the Supplementary Appendix, we show that with three or more seller types there exists an open set of distributions of seller types for which all separating equilibria are Pareto-dominated by and have lower gains from trade than a non-separating equilibrium. This result resembles the criticism of equilibrium refinements for signaling games by Mailath et al. [1993], which points out that refinements often select Pareto-dominated separating equilibria.

6 General Case

So far, we only considered the two-type case and separating equilibria. The maximum gains from trade achieved by equilibria in these two cases is invariant to trading frictions. These cases are restrictive, though. Indeed, except in the two-type case, there may be non-separating equilibria that Pareto-dominate all separating equilibria and lead to higher gains from trade. We now show that market efficiency is invariant to trading frictions also in the general case. We do so by deriving a sequence of lemmas describing efficiency properties of equilibria. The proofs of all these lemmas are in the Appendix.

First, we show that gains from trade in any equilibrium in which the buyers' payoff is zero is bounded above by average seller payoffs in the entering population, with equality if, and only if, the equilibrium is pure.

Lemma 6. *Consider an equilibrium with $V = 0$. Then $G \leq \sum_{i=1}^{N} f_i U_i$, with equality if, and only if, the equilibrium is pure.*

The next result shows that for any equilibrium with $V > 0$ there exists a more efficient one with $V = 0$. To understand why, consider a non-pooling equilibrium with $V > 0$.[25] Then, $p = \sum_{i=1}^{N} \pi_i(p) v_i - \delta V < \sum_{i=1}^{N} \pi_i(p) v_i$ for every price p at which trade happens. Since the maps $p \mapsto (p - c_i)/(p - c_{i+1})$ are strictly increasing in p for $p \geq c_i$, increasing the prices at which trade takes place, and so reducing buyer payoffs, relaxes the local upward incentive-compatibility constraints for all types of seller. This, in turn, allows for higher discounted probabilities of trade for all types of the good that trade at a price greater than the smallest price at which trade takes place, leading to higher gains from trade.

Lemma 7. *For any equilibrium with $V > 0$, there exists an equilibrium with $V = 0$ that results in higher gains from trade.*

Finally, we show that randomization by sellers hurts gains from trade if $V = 0$.

[25] The result is trivially true if the equilibrium with $V > 0$ is pooling.

Lemma 8. *For any equilibrium with $V = 0$, there exists a pure equilibrium with $V = 0$ that results in higher gains from trade.*

We can now prove our main result.

Proposition 7. *Let $W_{\max}(\delta)$ be the maximum equilibrium gains from trade when the discount factor is δ. Then $W_{\max}(\delta)$ is invariant to δ.*

Proof. Lemmas 7 and 8 imply that gains from trade are maximized when $V = 0$ and the equilibrium is pure. Let $\mathcal{E}^0$ be the set of pure equilibria with $V = 0$. The set $\mathcal{E}^0$ is non-empty since separating equilibria always exist. Now let $\mathcal{G}^0$ be the set of possible values of equilibrium gains from trade for equilibria in $\mathcal{E}^0$. Since, by Lemma 6, $G = \sum_{i=1}^{N} f_i U_i$ for any equilibrium in $\mathcal{E}^0$, Proposition 3 implies that $\mathcal{G}^0$ is invariant to δ. Finally, for any equilibrium, seller payoffs are $U_i = \theta_i(v_i - c_i)$, where θ_i is the discounted probability of trade of the type-i good. Since the set Θ of possible vectors $\theta = (\theta_1, \dots, \theta_N)$ of discounted probabilities of trade is defined by a list of weak inequalities derived from the sellers' incentive-compatibility constraints for trade, the set Θ is closed, and thus compact. So, $W_{\max}(\delta)$ exists and is the same for every value of δ. □

7 Stationarity Assumption and Alternative Specifications of the Environment

In this section, we first address the question of whether the outcomes of stationary equilibria are the long-run outcomes of equilibria in which one does not impose stationarity to begin with. We then discuss the robustness of our results to alternative assumptions about the timing of trade and to alternative forms of trading frictions.

7.1 Stationarity

The stationarity condition (3) assumes that for each $i \in \mathcal{I}$, the initial mass of type-i sellers in the market is the steady-state mass of type-i sellers. This begs the question of whether the outcomes of a stationary equilibrium—the prices at which trade takes place, the discounted probabilities of trade at these prices, and the steady-state masses of each type of seller—are the long-run outcomes of an equilibrium, in which behavior need not be time-invariant, when the economy starts in some initial period with the entry of a first cohort of buyers and sellers in the market. A stationary equilibrium for which this is not the case is not credible.

The notion of equilibrium extends naturally to the case in which one does not impose stationary behavior and assumes that the economy starts in some initial period, which we set to zero without loss, with the entry of a first cohort of buyers and sellers in the market. We refer to such an equilibrium as a non-stationary equilibrium to distinguish it from a stationary equilibrium. Informally, a non-stationary equilibrium is a sequence of strategy profiles, buyer beliefs, payoff vectors, and vectors of seller masses such that: (*i*) in each period $t \geq 0$ sellers behave optimally given the acceptance behavior of buyers in period t and seller continuation payoffs in period $t+1$; (*ii*) in each period $t \geq 0$ buyers behave optimality given their beliefs in period t and their continuation payoffs in period $t+1$; (*iii*) buyer beliefs satisfy Bayes' rule for the prices offered on the path of play; (*iv*) payoffs are consistent with individual behavior; and (*v*) the evolution of the mass of each type of seller in the market is the one implied by the aggregate behavior of buyers and sellers when for each $i \in \mathcal{I}$ the initial mass of type-i sellers is f_i. A formal definition is in the Appendix.

Consider a stationary separating equilibrium E. Since in this equilibrium the masses of each type of seller in the market do not matter for the determination of buyer beliefs, and hence for buyer optimality, it is easy to see that E remains an equilibrium if one replaces the stationary condition (3) with the law of motion for the masses of each type of seller in the market implied by the behavior of buyers and sellers in this equilibrium when for each $i \in \mathcal{I}$ the initial mass of type-i sellers is f_i. Let p_i and σ be, respectively, the price that type-i sellers offer and the strategy profile for buyers in E. Moreover, let M_{it} be the mass of type-i sellers in the market in period $t \geq 0$. Then, $M_{i0} = f_i$ and $M_{it+1} = f_i + (1 - \sigma(p_i))M_{it}$ for all $t \geq 0$. Notice that $\lim_{t\to\infty} M_{it} = f_i/\sigma(p_i)$, which is the steady-state mass of type-i sellers in E.

The argument in the previous paragraph shows that for every stationary separating equilibrium there exists a non-stationary separating equilibrium whose outcomes approach the outcomes of the stationary separating equilibrium in the long-run; in fact, the non-stationary equilibrium is such in every period the prices at which trade takes place and the discounted probabilities of trade at these prices are the same as in the stationary equilibrium. In the Appendix, we show that the same result holds for every pure stationary equilibrium in which buyer payoffs are zero. In particular, the outcomes in the most efficient stationary equilibria are the long-run outcomes of non-stationary equilibria.

7.2 Timing of Trade

In our environment, agents who enter the market get an opportunity to trade immediately. Suppose, instead, that new agents in the market have to wait one period before they are matched to a trading partner. This corresponds to the timing assumptions in continuous-time models of trade; see, e.g., Kaya and Kim [2018] and Kim [2017]. It is immediate to see that the definition of an equilibrium does not change. However, gains from trade are now multiplied by δ. Thus, Proposition 7 implies that maximum equilibrium gains from trade are *linearly* increasing in δ. The negative relationship between maximum gains from trade and trading frictions is a direct consequence of the fact that new agents in the market have to wait one period before getting the opportunity to trade.

The above result should be contrasted with corresponding results in Kaya and Kim [2018] and Kim [2017], which consider continuous-time models of trade with adverse selection in which buyers make take-it-or-leave it offers. They show that a reduction in trading frictions has two opposing effects on market efficiency. The positive effect is the direct, mechanical, effect discussed above. The negative effect is the indirect effect coming from a reduction in the ability of buyers to screen sellers. As it turns out, the direct effect dominates the indirect effect, and a reduction in trading frictions increases equilibrium gains from trade, albeit *sublinearly*. The linear relationship between trading frictions and market efficiency in our setting is due to the absence of the indirect effect.

7.3 Trading Frictions as Probability of Exit

An alternative way of describing trading frictions in our setting is to assume that instead of discounting the future, agents who do not trade in a given period exogenously leave the market with probability $1-\delta \in (0,1)$.[26] As in the baseline environment, an increase in δ reduces trading frictions by reducing the opportunity cost of not trading.

The definition of an equilibrium is the same except that now the stationarity condition is given by

$$M_i\left(1-\delta+\delta\int_{\mathbb{R}_+}\sigma(p)d\mu_i(p)\right)=f_i \tag{7}$$

for all $i \in \mathcal{I}$. Indeed, now a seller exits the market if either the seller trades, which happens with probability $\int_{\mathbb{R}_+}\sigma(p)d\mu_i(p)$, or the seller does not trade but exits exogenously, which happens with probability $1-\delta$ conditional on the seller not trading.

It is immediate to see that the characterization of seller optimality given by Lemma 1 remains valid, and so do the other results in Section 3.1. However, the monotonicity of seller payoffs is weak instead of strict. Indeed, as sellers can now exit the market without trading, sellers of type $i < N$ can obtain zero payoff. In the Supplementary Appendix, we show that $U_i \geq U_j$ if $j > i$ with $U_i > U_j$ when $S_j^* \neq \emptyset$ and that $S_j^* = \emptyset$ when $U_i = 0$.

Since payoffs for sellers of type $i < N$ can now be zero, two or more types of seller can make offers that are rejected in equilibrium. This, in turn, implies that monotonicity of prices in seller types holds only for prices in S^*. Nevertheless, Corollaries 2 and 3 remain true as their proofs rely only on the monotonicity of prices in S^*. Proposition 2 holds as well. However, one cannot conclude from it that there are at least two prices at which trade takes place when adverse selection is severe, as now type-N sellers need not trade in equilibrium. Still, severe adverse selection rules out pooling and leads to delay in trade.

We now show that maximum equilibrium gains from trade are invariant to trading frictions. We first show that Proposition 3 still holds. Let $E = \big(\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}}\big)$ be an equilibrium with $V = 0$ when the discount factor is δ. We show in the Supplementary

[26] For papers that model trading frictions in this way, see McAfee [1993] and Lauermann [2013].

Appendix that one can assume that the set S of prices that sellers post in E is finite. Now let the discount factor be $\delta' \neq \delta$ and consider the list $E' = \big(\mu', \sigma', \pi', (U_i')_{i\in\mathcal{I}}, V', (M_i')_{i\in\mathcal{I}}\big)$ in the proof of Proposition 3 except that now the measures μ_i' are such that $p \in S$,

$$\frac{\mu_i'(\{p\})\big(1-\delta'+\delta'\sigma'(p)\big)}{1-\delta'+\delta'\mathbb{E}_{\mu_i'}[\sigma']} = \frac{\mu_i(\{p\})\big(1-\delta+\delta\sigma(p)\big)}{1-\delta+\delta\mathbb{E}_{\mu_i}[\sigma]}; \tag{8}$$

we show in the Supplementary Appendix that such measures exist. Then, $\pi'(p) = \pi(p)$ for all $p \in S$ by (7). The rest of the proof is the same as in Section 3.

To finish, we show that we can adapt the proof of Proposition 7 to this case. As it turns out, gains from trade in equilibria with $V = 0$ now always equal average seller payoffs in the population; see the Supplementary Appendix for a proof of this. The proof that for every equilibrium with $V > 0$ there exists an equilibrium with $V = 0$ with weakly higher gains from trade is the same as before. Proposition 7 follows from these two observations.

Summarizing, modelling trading frictions as an exogenous exit probability alters some of our results, but does not change the qualitative properties of the equilibrium set, including the fact that market efficiency is invariant to trading frictions.

7.4 Within-Period Matching Frictions

We finish this section by discussing the case in which within-period matching is frictional in the sense that in every period agents in the market are matched to a trading partner with probability $\alpha \in (0,1)$; now an increase in either α or δ reduces trading frictions. The baseline environment with frictionless within-period matching corresponds to the case in which $\alpha = 1$.

The definition of an equilibrium remains the same except that agent payoffs are computed differently and stationarity needs reformulation. The payoff to a type-i seller who posts price $p \geq 0$ in every period now equals

$$U_i(p) = \frac{\alpha\sigma(p)}{1-\delta(1-\alpha\sigma(p))}(p-c_i) = \theta(p,\alpha)(p-c_i).$$

The term $\theta(p, \alpha)$ is the discounted probability of trade for this seller, which is strictly increasing in both α and $\sigma(p)$. The payoff to buyers is now defined recursively by

$$V = \int_{\mathbb{R}_+} \left(\sum_{i=1}^{N} \pi_i(p)(v_i - p) \right) \alpha\sigma(p) d\overline{\mu}(p) + \left(\int_{\mathbb{R}_+} (1 - \alpha\sigma(p)) d\overline{\mu}(p) \right) \delta V.$$

Finally, the stationarity condition (3) is now given by

$$M_i \alpha \int_{\mathbb{R}_+} \sigma(p) d\mu_i(p) = f_i \text{ for all } i \in \mathcal{I}. \tag{9}$$

We first show that the basic properties of equilibria in the baseline environment extend to the case considered here without any change. The characterization of seller optimality given by Lemma 1 remains valid with $\theta(p, \alpha)$ in place of $\theta(p)$. From this, it follows that Lemmas 2 and 3 and Corollary 1 remain valid. It is easy to see that Lemma 4 now holds with $\theta(v_1, \alpha) = (1-\delta+\delta\alpha)^{-1}\alpha$, the highest discounted probability of trade possible. Since the stationarity condition (9) implies that $\int_{\mathbb{R}_+} \sigma(p) d\mu_i(p)$ is positive for all $i \in \mathcal{I}$, so that every type of seller trades in equilibrium, we have that Lemma 5 and Proposition 1 also hold in this case. Likewise, Corollaries 2 and 3 and Proposition 2 remain valid.

We now consider the two-type case of Section 4 and show that a reduction in trading frictions *increases* market efficiency. Lemma 4 and Proposition 2 imply that $V = 0$, $p = v_1$ and $\theta(v_1, \alpha) = (1 - \delta + \delta\alpha)^{-1}\alpha$, so that $U_1 = (1 - \delta + \delta\alpha)^{-1}\alpha(v_1 - c_1)$. As in the baseline environment, there are two cases to consider: either $S_1^* \cap S_2^*$ is empty or $S_1^* \cap S_2^*$ is a singleton.

Consider first equilibria in which $S_1^* \cap S_2^* = \emptyset$. It follows from the proof of Corollary 3 that both S_1^* and S_2^* are singletons, and so $S_1 = S_1^* = \{v_1\}$. The same argument as when $\alpha = 1$ shows that $S_2^* = \{v_2\}$. Since $v_2 > c_2$ implies that $U_2 > 0$, it then follows that $S_2 = S_2^*$. Thus, equilibria with $S_1^* \cap S_2^* = \emptyset$ again correspond to the separating equilibria. Now note that a necessary condition for a separating equilibrium is that

$$\theta(v_2, \alpha) \leq \frac{\alpha}{1 - \delta + \delta\alpha} \frac{v_1 - c_1}{v_2 - c_2}.$$

As type-2 sellers have no incentive to post v_1, the above condition on $\theta(v_2, \alpha)$ is also sufficient by Lemma 2. So, the set of payoff vectors for separating equilibria is the set of vectors (V, U_1, U_2) with $V = 0$, $U_1 = (1 - \delta + \delta\alpha)^{-1}\alpha(v_1 - c_1)$, and $U_2 \in \big(0, \overline{U}_2(\alpha, \delta)\big]$, where

$$\overline{U}_2(\alpha, \delta) = \frac{\alpha}{1 - \delta + \delta\alpha} \frac{v_1 - c_1}{v_2 - c_2}(v_2 - c_2).$$

The maximum equilibrium gains from trade for such equilibria are

$$\overline{G}(\alpha, \delta) = \frac{\alpha}{1 - \delta + \delta\alpha}\left(f_1 + f_2 \frac{v_2 - c_2}{v_2 - c_1}\right)(v_1 - c_1).$$

To conclude, an argument similar to the one of Section 4 shows that gains from trade in equilibria in which $S_1^* \cap S_2^*$ is a singleton are smaller than in the most efficient separating equilibrium.[27] Thus, $\overline{G}(\alpha, \delta)$ is the maximum equilibrium gains from trade.

Note that now the maximum equilibrium gains from trade are strictly increasing in δ. They are also strictly increasing in α. So, lowering trading frictions increases market efficiency. The positive relationship between trading frictions and the maximum equilibrium gains from trade is simply a consequence of an easing of a technological constraint, having nothing to do with adverse selection per se. By assuming frictionless matching within a period, we ignore this mechanical effect of a reduction in trading frictions, allowing us to focus on the interaction between trading frictions and signaling.

8 Conclusion

We study trade in dynamic decentralized markets with adverse selection. In contrast with the literature on the topic so far, we assume that the informed sellers make the offers, so that signaling through prices is possible. We establish a partial characterization of the equilibrium set and discuss the standard two-type case and separating equilibria in detail. We also show that the maximum equilibrium gains from trade are invariant to trading frictions. Overall, our results show that the trading protocol in dynamic decentralized markets with

[27] See the Supplementary Appendix for a proof of this.

adverse selection has a substantive impact on their functioning, including their efficiency. This calls for a better understanding of how the nature of the trading process in dynamic decentralized markets with adverse selection affects trading outcomes in such markets.

Our analysis is fairly general in a number of ways. Unlike the environments typically considered in the literature, we allow for any finite number of types of the good, as opposed to just two, and do not impose that gains from trade are increasing in the type of the good; as we discuss in the Supplementary Appendix, we can even account for the case in which gains from trade are not necessarily positive for all types of the good. We also do not make use of any refinements to constrain the equilibrium set. We, however, restrict attention to stationary equilibria. The literature on dynamic trading with adverse selection that considers non-stationary equilibria typically does so in settings with one-time entry of buyers and sellers, which amounts to assuming that the vintages of the goods being traded are observable.[28] Allowing for non-stationary equilibria in our setting significantly increases the scope for equilibrium behavior. The analysis of non-stationary equilibria in the presence of signaling through prices is left for future research.

References

Kyle Bagwell and Michael H. Riordan. High and declining prices signal product quality. *The American Economic Review*, 81(1):224–239, 1991.

Jeffrey S. Banks and Joel Sobel. Equilibrium selection in signaling games. *Econometrica*, 55(3):647–661, 1987.

Max R. Blouin. Equilibrium in a decentralized market with adverse selection. *Economic Theory*, 22(2):245–262, 2003.

Braz Camargo and Benjamin Lester. Trading dynamics in decentralized markets with adverse selection. *Journal of Economic Theory*, 153:534–568, 2014.

[28] A notable exception is Janssen and Roy [2004]

Briana Chang. Adverse Selection and Liquidity Distortion. *The Review of Economic Studies*, 85(1):275–306, 04 2018.

Jonathan Chiu and Thorsten V. Koeppl. Trading Dynamics with Adverse Selection and Search: Market Freeze,Intervention and Recovery. *The Review of Economic Studies*, 83 (3):969–1000, 2016.

In-Koo Cho and David M. Kreps. Signaling Games and Stable Equilibria. *The Quarterly Journal of Economics*, 102(2):179–221, 1987.

Michael Choi. Imperfect information transmission and adverse selection in asset markets. *Journal of Economic Theory*, 176:619–649, 2018.

Brendan Daley and Brett Green. Waiting for news in the market for lemons. *Econometrica*, 80(4):1433–1504, 2012.

Raymond Deneckere and Meng-Yu Liang. Bargaining with interdependent values. *Econometrica*, 74(5):1309–1364, 2006.

Tore Ellingsen. Price signals quality: The case of perfectly inelastic demand. *International Journal of Industrial Organization*, 16(1):43–61, 1997.

Robert Evans. Sequential Bargaining with Correlated Values. *The Review of Economic Studies*, 56(4):499–510, 10 1989.

William Fuchs and Andrzej Skrzypacz. Government interventions in a dynamic market with adverse selection. *Journal of Economic Theory*, 158:371–406, 2015.

Douglas Gale. A walrasian theory of markets with adverse selection. *The Review of Economic Studies*, 59(2):229–255, 1992.

Douglas Gale. Equilibria and pareto optima of markets with adverse selection. *Economic Theory*, 7(2):207–235, 1996.

Dino Gerardi, Johannes Horner, and Lucas Maestri. The role of commitment in bilateral trade. *Journal of Economic Theory*, 154:578–603, 2014.

Veronica Guerrieri and Robert Shimer. Dynamic adverse selection: A theory of illiquidity, fire sales, and flight to quality. *American Economic Review*, 104(7):1875–1908, 2014.

Veronica Guerrieri, Robert Shimer, and Randall Wright. Adverse selection in competitive search equilibrium. *Econometrica*, 78(6):1823–1862, 2010.

Igal Hendel and Alessandro Lizzeri. Adverse selection in durable goods markets. *The American Economic Review*, 89(5):1097–1115, 1999.

Johannes Horner and Nicolas Vieille. Public vs. private offers in the market for lemons. *Econometrica*, 77(1):29–69, 2009.

Ilwoo Hwang. Dynamic trading with developing adverse selection. *Journal of Economic Theory*, 176:761–802, 2018.

Roman Inderst. Matching markets with adverse selection. *Journal of Economic Theory*, 121(2):145–166, 2005.

Maarten Janssen and Santanu Roy. On durable goods markets with entry and adverse selection. *Canadian Journal of Economics/Revue canadienne d'Ã©conomique*, 37(3): 552–589, 2004.

Maarten C. W. Janssen and Santanu Roy. Dynamic trading in a durable good market with asymmetric information. *International Economic Review*, 43(1):257–282, 2002.

Ayca Kaya and Kyungmin Kim. Trading Dynamics with Private Buyer Signals in the Market for Lemons. *The Review of Economic Studies*, 85(4):2318–2352, 2018.

Ayca Kaya and Santanu Roy. Market screening with limited records. *SSRN Working Paper*, 2020.

Kyungmin Kim. Information about sellers' past behavior in the market for lemons. *Journal of Economic Theory*, 169:365–399, 2017.

Pablo Kurlat. Lemons markets and the transmission of aggregate shocks. *American Economic Review*, 103(4):1463–89, 2013.

Stephan Lauermann. Dynamic matching and bargaining games: A general approach. *American Economic Review*, 103(2):663–689, 2013.

George J. Mailath, Masahiro Okuno-Fujiwara, and Andrew Postlewaite. Belief-based refinements in signalling games. *Journal of Economic Theory*, 60(2):241–276, 1993.

Andreu Mas-Colell. On a theorem of schmeidler. *Journal of Mathematical Economics*, 13 (3):201–206, 1984.

R. Preston McAfee. Mechanism design by competing sellers. *Econometrica*, 61(6):1281–1312, 1993.

Paul Milgrom and John Roberts. Price and advertising signals of product quality. *Journal of Political Economy*, 94(4):796–821, 1986.

Diego Moreno and John Wooders. Decentralized trade mitigates the lemons problem*. *International Economic Review*, 51(2):383–399, 2010.

John G. Riley. Silver signals: Twenty-five years of screening and signaling. *Journal of Economic Literature*, 39(2):432–478, 2001.

Daniel R. Vincent. Bargaining with common values. *Journal of Economic Theory*, 48(1): 47–62, 1989.

Charles Wilson. The nature of equilibrium in markets with adverse selection. *The Bell Journal of Economics*, 11(1):108–130, 1980.

Asher Wolinsky. Prices as Signals of Product Quality. *The Review of Economic Studies*, 50(4):647–658, 1983.

A Appendix

A.1 Proof of Lemma 1

Consider a list $(\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}})$ satisfying payoff consistency. Fix $i \in \mathcal{I}$ and suppose that seller optimality holds for type-i sellers. Since $U_i = \theta(p)(p - c_i)$ for $p \in S_i$ by payoff consistency and $U_i \geq \sigma(p')(p - c_i) + (1 - \sigma(p'))\delta U_i$ for all $p' \geq 0$ by seller optimality, it follows that if $p \in S_i$ and $p' \geq 0$, then

$$\begin{aligned}\theta(p)(p - c_i) &\geq \sigma(p')(p' - c_i) + (1 - \sigma(p'))\delta U_i \\ &\geq \sigma(p')(p' - c_i) + (1 - \sigma(p'))\delta\big[\sigma(p') + (1 - \sigma(p'))\delta U_i\big] \\ &\geq \sum_{k=0}^{\infty}(1 - \sigma(p'))^k\delta^k\sigma(p')(p' - c_i) = \theta(p')(p - c_i).\end{aligned}$$

Now suppose that $\theta(p)(p - c_i) \geq \theta(p')(p' - c_i)$ for all $p \in S_i$ and $p' \geq 0$ and fix $p \in S_i$. Then, since $U_i = \theta(p)(p - c_i) = \sigma(p)(p - c_i) + (1 - \sigma(p))\delta U_i$ by payoff consistency,

$$\sigma(p)(p - c_i) + (1 - \sigma(p))\delta U_i \geq \sigma(p')(p' - c_i) + (1 - \sigma(p'))\delta U_i$$

for all $p' \geq 0$ if, and only if, $U_i[1 - \delta(1 - \sigma(p'))] \geq \sigma(p')(p' - c_i)$ for all $p' \geq 0$, which is true by hypothesis. So, seller optimality holds for type-i sellers.

A.2 Proof Lemma 3

Let $p^{\min} = \min\{p' : p' \in S\}$. It follows from (4) that $\theta(p) = 0$ for all $p \in S$ if $\theta(p^{\min}) = 0$, in which case the conclusion of the lemma holds trivially. Assume then that $\theta(p^{\min}) > 0$. In this case, assumption (*i*) and (4) together imply that $\theta(p) > 0$ for all $p \in S$ and $\theta(p^+) < \theta(p)$ for all $p \in S$ with $p < p^{\max}$.

First, we show that if $p < p^{\max}$, then a type-i seller with $i \in \mathcal{I}(p)$ has no incentive to post any price $p' > p$ in S. Fix $p < p^{\max}$, $i \in \mathcal{I}(p)$, and let $S_+(p) = \{p^1, \ldots, p^K\}$, with $K \geq 1$, be the set of prices in S that are greater than p, ordered from lowest to highest.

Suppose that there exists $k \in \{1, \ldots, K\}$ with $\theta(p^k)(p^k - c_i) \leq \theta(p)(p - c_i)$. Since

$$\theta(p^1) = \theta(p)\frac{p - \max\{c_j : j \in \mathcal{I}(p)\}}{p^1 - \max\{c_j : j \in \mathcal{I}(p)\}} \leq \theta(p)\frac{p - c_i}{p^1 - c_i}$$

by (4) and the fact that $(p-c)/(p^1-c)$ is strictly decreasing in c for all $c < p$, the induction hypothesis is true when $k = 1$. Now observe that

$$\begin{aligned}
&\theta(p^{k+1})(p^{k+1} - c_i) \\
&\qquad = \theta(p^{k+1})(p^{k+1} - \max\{c_j : j \in \mathcal{I}(p^k)\}) + \theta(p^{k+1})(\max\{c_j : j \in \mathcal{I}(p^k)\} - c_i) \\
&\qquad \leq \theta(p^{k+1})(p^{k+1} - \max\{c_j : j \in \mathcal{I}(p^k)\}) + \theta(p^k)(\max\{c_j : j \in \mathcal{I}(p^k)\} - c_i) \\
&\qquad = \theta(p^k)(p^k - \max\{c_j : j \in \mathcal{I}(p^k)\}) + \theta(p^k)(\max\{c_j : j \in \mathcal{I}(p^k)\} - c_i) \\
&\qquad = \theta(p^k)(p^k - c_i);
\end{aligned}$$

the first inequality follows since $\theta(p^{k+1}) < \theta(p^k)$ and $\max\{c_j : j \in \mathcal{I}(p^k)\} \geq c_i$ by (*ii*) whereas the second equality follows from (4). Thus, $\theta(p^{k+1})(p^{k+1} - c_i) \leq \theta(p)(p - c_i)$ by the induction hypothesis, from which be obtain the desired result.

Now we show that if $p > p^{\min}$, then a type-i seller with $i \in \mathcal{I}(p)$ has no incentive to post any price $p' < p$ in S. Fix $p > p_{\min}$ and $i \in \mathcal{I}(p)$. First note that if $p = p^{\max} = c_N$ and $i = N$, then $\theta(p)(p - c_N) \geq \theta(p')(p' - c_N)$ for all $p \in S_N = \{c_N\}$ and $p' \in S$. So, assume that either $p^{\max} > c_N$ or $i < N$ and let $S_-(p) = \{p^1, \ldots, p^K\}$, with $K \geq 1$, be the set of prices in S that are smaller than p, ordered from lowest to highest. Note that $p > c_j$ for all $i \in \mathcal{I}$ with $j \in \mathcal{I}(p)$. Suppose that there exists $k \in \{1, \ldots, K\}$ with $\theta(p^k)(p^k - c_i) \leq \theta(p)(p - c_i)$. Given that

$$\theta(p) = \theta(p^K)\frac{p^K - \max\{c_i : i \in \mathcal{I}(p)\}}{p - \max\{c_i : i \in \mathcal{I}(p)\}} \geq \theta(p^K)\frac{p^K - c_i}{p - c_i},$$

the induction hypothesis is true when $k = K$. Moreover, since $\theta(p^{k-1}) > \theta(p^k)$ and $\max\{c_j : j \in \mathcal{I}(p^{k-1})\} \leq c_i$, a straightforward modification of the argument in the previous paragraph shows that $\theta(p^{k-1})(p^{k-1} - c_i) \leq \theta(p^k)(p^k - c_i)$. So, $\theta(p^{k-1})(p^{k-1} - c_i) \leq \theta(p)(p - c_i)$ by the induction hypothesis, and the desired result follows.

A.3 Proof of Lemma 4

Consider an equilibrium with $S_1^* \setminus \bigcup_{j=2}^N S_j^* \neq \emptyset$ and let p be a price in this set. Then $\pi_1(p) = 1$ by Bayes' rule, so that $v_1 - p \geq \delta V$ by buyer optimality. Let $w_1 = v_1 - \delta V$. We claim that $p = w_1$. Indeed, if $p < w_1$, then $\sum_{j=1}^N \pi(p')v_j - p' \geq v_1 - p' > \delta V$ for all $p' \in (p, w_1)$, in which case type-1 sellers can profitably deviate by offering $p' \in (p, w_1)$ and increasing their payoff to $p' - c_1 > p - c_1 \geq U_1$. Now note that if $p = w_1$, then buyer optimality implies that $V = 0$, so that $p = v_1$. To conclude, observe that if $\theta(v_1) < 1$, then type-1 sellers could profitably deviate by offering $v_1 - \varepsilon$ with $0 < \varepsilon < (1 - \theta(v_1))(v_1 - c_1)$, as such an offer would be accepted with probability one by a buyer.

A.4 Proof of Lemma 5

First note that $U_i > 0$ for all $i < N$, so that $\theta(p) > 0$ and $p > c_i$ for all $p \in S_i$ if $i < N$. Indeed, stationarity and $U_N \geq 0$ imply that there exists $p' \geq c_N$ in S_N with $\theta(p') > 0$. Then $U_i \geq \theta(p')(p' - c_i) > \theta(p')(p' - c_N) \geq 0$ by seller optimality and $c_N > c_i$. Now let $i, j \in \mathcal{I}$ be such that $j > i$. We are done if $j = N$ and $U_N = 0$. So assume that $U_N > 0$ and $\theta(p) > 0$ and $p > c_N$ if $p \in S_N$. Fix $p \in S_i$ and $p' \in S_j$. Since $\theta(p') > 0$ and $p' > c_j$,

$$U_i = \theta(p)(p - c_i) \geq \theta(p')(p' - c_i) > \theta(p')(p' - c_j) = U_j;$$

the first inequality follows from seller optimality whereas the second inequality follows from the fact that $\theta(p') > 0$ and $c_i < c_j$. This concludes the proof.

A.5 Proof of Corollary 2

Let $i, j \in \mathcal{I}$ be such that $i < j$. Since the map $\theta \mapsto \theta(1 - \delta)/(1 - \delta\theta)$ taking discounted probabilities of trade into probabilities of trade is strictly increasing and Proposition 1 implies that $p' \geq p$ for all $p \in S_i^*$ and $p' \in S_j^*$, it follows from (3) and $g_i/g_j = M_i/M_j$ that

$$\frac{g_i}{g_j} = \frac{f_i}{f_j} \frac{\int_{\mathbb{R}_+} \sigma(p) d\mu_j(p)}{\int_{\mathbb{R}_+} \sigma(p) d\mu_i(p)} \leq \frac{f_i}{f_j}.$$

A.6 Proof of Corollary 3

We claim that the sets S_i^* contain at most one element not in $S^* \setminus S_i^*$. This is true for S_1^* by Lemma 4. Let $1 < i \le N$ and suppose, by contradiction, that there exist $p, p' \in S_i^*$ with $p < p'$ and $\{p, p'\} \cap S_j^* = \emptyset$ for all $j \neq i$. Then $\theta(p) > \theta(p') > 0$. Moreover, $\theta(p) < 1$ since $p > p_1$ for some $p_1 \in S_1^*$ by Proposition 1. Now note that $\pi_i(p) = \pi_i(p') = 1$ by Bayes' rule. However, as $\theta(p)$ and $\theta(p')$ are interior, buyer optimality implies that

$$v_i - p = \sum_{k=1}^{N} \pi_k(p) v_k - p = \delta V = \sum_{k=1}^{N} \pi_k(p') v_k - p' = v_i - p',$$

a contradiction. The result that S^* has at most $2N - 1$ elements now follows immediately from the fact that $S_i^* \cap S_{i+1}^*$ with $i < N$ has at most one element by Proposition 1.

A.7 Proof of Proposition 3

We need to show that for each $i \in \mathcal{I}$, there exists μ_i such that

$$\frac{\mu_i'(\{p^k\})\sigma'(p^k)}{\mathbb{E}_{\mu_i'}[\sigma']} = \frac{\mu_i(\{p^k\})\sigma(p^k)}{\mathbb{E}_{\mu_i}[\sigma]} \text{ for all } k \in \{1, \ldots, K\}. \tag{A.1}$$

Fix $i \in \mathcal{I}$ and let $\gamma, \alpha \in \mathbb{R}^K$ be such that $\gamma_k = \sigma'(p^k)$ and $\alpha_k = \mu_i(\{p^k\})\sigma(p^k)/\mathbb{E}_{\mu_i}[\sigma]$ for each $k \in \{1, \ldots, K\}$; note that $\alpha_1 + \cdots + \alpha_K = 1$. Now let $T : \mathbb{R}^K \to \mathbb{R}^K$ be such that

$$T(x) = \left(\frac{\alpha_1}{\gamma_1} \langle x, \gamma \rangle, \ldots, \frac{\alpha_K}{\gamma_K} \langle x, \gamma \rangle \right),$$

where $\langle a, b \rangle$ is the scalar product of $a, b \in \mathbb{R}^K$. Then, $T(x) = Ax$, where A is the matrix

$$A = \begin{pmatrix} \frac{\alpha_1}{\gamma_1}\gamma_1 & \cdots & \frac{\alpha_1}{\gamma_1}\gamma_K \\ \vdots & \ddots & \vdots \\ \frac{\alpha_K}{\gamma_K}\gamma_1 & \cdots & \frac{\alpha_K}{\gamma_K}\gamma_K \end{pmatrix}.$$

We are done if we show that there exists $x^* = (x_1^*, \ldots, x_K^*) \in \mathbb{R}_+^K$ with $x_1^* + \cdots + x_K^* = 1$ such that $T(x^*) = x^*$. Indeed, if μ_i' is such that $\mu_i'(\{p^k\}) = x_k^*$ for $k \in \{1, \ldots, K\}$, then μ_i'

satisfies (A.1). Since T is linear, it suffices to show that $\lambda = 1$ is an eigenvalue of A, and so there exists $x = (x_1, \ldots, x_K) \in \mathbb{R}^K_+ \setminus \{0\}$ with $T(x) = x$, for $x^* = (x_1 + \cdots + x_K)^{-1}x$ is also such that $T(x^*) = x^*$.

Let I be the $K \times K$ identity matrix and A^0 be $K \times K$ matrix given by

$$A^0 = \begin{pmatrix} \alpha_1 & \cdots & \alpha_1 \\ \vdots & \ddots & \vdots \\ \alpha_k & \cdots & \alpha_k \end{pmatrix} - I.$$

Since the determinant is a multi-linear operator defined on the rows and columns of a matrix, the determinants of $A - I$ and A^0 coincide. Now let $e = (1, \ldots, 1) \in \mathbb{R}^K$, e^i be the ith element of the canonical basis of $\mathbb{R}^K$, and $v^i = \alpha_i e - e^i$. By construction, v^i is the ith row of A^0. Given that $v^1 + \cdots + v^K = (\alpha^1 + \cdots + \alpha^K)e - e = 0$, the rows of A^0 are linearly dependent. So, its determinant is zero. This concludes the proof.

A.8 Equilibria with $V > 0$

Consider an equilibrium with belief system π. We say that the expected quality of the good drops discontinuously with an increase in prices at $p \geq 0$ if there exist $\delta > 0$ and $\varepsilon > 0$ such that $\sum_{i=1}^N \pi_i(p)v_i - \sum_{i=1}^N \pi_i(p')v_i > \varepsilon$ for all $p' \in (p, p + \delta)$. We say that an equilibrium has a discontinuous drop in the expected quality of the good if there exists a price p such that the expected quality of the good drops discontinuously with an increase in prices at p.

Proposition 8. *An equilibrium with $V > 0$ has a discontinuous drop in the expected quality of the good. Moreover, for any equilibrium with $V = 0$, there exists an equilibrium without a discontinuous drop in the expected quality of the good resulting in the same payoffs and gains from trade.*

Proof. Suppose that $E = (\mu, \sigma, \pi, (U_i)_{i \in \mathcal{I}}, V, (M_i)_{i \in \mathcal{I}})$ is an equilibrium with $V > 0$. By assumption, there exists $p \in S^*$ with $\sum_{i=1}^N \pi_i(p)v_i + \delta V > p$. Now note that $\sigma(p') < 1$ for all $p' > p$, otherwise seller optimality would be violated. Thus, $p' \geq \sum_{i=1}^N \pi_i(p')v_i + \delta V$ for all $p' > p$, otherwise buyer optimality would be violated. Let then $\varepsilon > 0$ be such that

$2\varepsilon = \sum_{i=1}^N \pi_i(p)v_i + \delta V - p$ and set $\delta = \varepsilon$. Then, $p < p' < p + \delta$ implies that

$$\sum_{i=1}^{N} \pi_i(p)v_i + \delta V > p' + \varepsilon \geq \sum_{i=1}^{N} \pi_i(p')v_i + \delta V + \varepsilon.$$

Now suppose that $E = (\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}})$ is an equilibrium with $V = 0$. We show that we can change σ and π for off-equilibrium prices so that E remains an equilibrium and there exists no discontinuous drop in the expected quality of the good; such changes do not change payoffs and discounted probabilities of trade. It follows from Corollary 3 that $S^* = \{p^1, \ldots, p^K\}$, with p^k strictly increasing in k. Also note that $p^1 \geq v^1$, since $V = 0$ implies that $p^1 = \sum_{i=1}^N \pi_i(p^1)v_i \geq v_1$. Now redefine $\sigma(p)$ and $\pi(p)$ for $p \notin S^*$ as follows. Let $p^0 = v_1$; note that $p^0 = p^1$ if $p^1 = v^1$ and that $p^1 > p^0$ otherwise. For $p < p^0$, let $\pi_1(p) = \sigma(p) = 1$. For $p \in (p^k, p^{k+1})$, with $k \in \{0, \ldots, N-1\}$, let

$$\pi_i(p) = \frac{p - p^k}{p^{k+1} - p^k}\pi_i(p^k) + \frac{p^{k+1} - p}{p^{k+1} - p^k}\pi_i(p^{k+1}) \text{ and } \sigma(p) = 0.$$

Finally, for $p > p^K$, let $\pi_i(p) = \pi_i(p^K)$ and $\sigma(p) = 0$. By construction, $\sum_{i=1}^N \pi_i(p)v_i$ is continuous in p. It is straightforward to verify that buyer and seller optimality hold. $\square$

A.9 Proof of Lemma 6

Consider an equilibrium with $V = 0$. Since $U_i = \theta(p)(p - c_i)$ for all $p \in S_i^*$,

$$\begin{aligned}\sum_{i=1}^{N} f_i U_i &= \sum_{i=1}^{N} f_i \sum_{p\in S^*} \frac{\mu_i(\{p\})\sigma(p)}{\mathbb{E}_{\mu_i}[\sigma]}\theta(p)(p - c_i) \\ &= \sum_{i=1}^{N} f_i \sum_{p\in S^*} \frac{\mu_i(\{p\})\sigma(p)}{\mathbb{E}_{\mu_i}[\sigma]}\theta(p)(v_i - c_i) + \sum_{p\in S^*} \theta(p)\sigma(p) \sum_{i=1}^{N} f_i \frac{\mu_i(\{p\})}{\mathbb{E}_{\mu_i}[\sigma]}(p - v_i).\end{aligned}$$

Now note that since $p = \sum_{i=1}^N \pi_i(p)v_i$ for all $p \in S^*$ by buyer optimality, stationarity and Bayes' rule together imply that

$$\sum_{i=1}^{N} f_i \frac{\mu_i(\{p\})}{\mathbb{E}_{\mu_i}[\sigma]}(p - v_i) = \sum_{i=1}^{N} M_i \mu_i(\{p\})(p - v_i) = 0.$$

Moreover, as $x \mapsto x^2/[1-\delta(1-x)]$ is strictly convex, Jensen's inequality implies that

$$\sum_{p \in S^*} \frac{\mu_i(\{p\})\sigma(p)}{\mathbb{E}_{\mu_i}[\sigma]}\theta(p)(v_i - c_i) \geq \frac{\mathbb{E}_{\mu_i}[\sigma]}{1-\delta(1-\mathbb{E}_{\mu_i}[\sigma])}(v_i - c_i)$$

for each $i \in \mathcal{I}$, with equality if, only if, S_i^* is a singleton. The desired result now follows.

A.10 Proof of Lemma 7

Let $E = \big(\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}}\big)$ be an equilibrium with $V > 0$ and $\{p^1, \ldots, p^K\}$ with $K \geq 1$ and $p^1 < \cdots < p^K$ be the set of prices at which trade takes place in E. For each $k \in \mathcal{K} = \{1, \ldots, K\}$, let $q^k = \sum_{i=1}^N \pi_i(p^k)v_i$; note that q^k is strictly increasing in k by Proposition 1. Seller optimality implies that $p^k \leq q^k - \delta V$ for all $k \in \mathcal{K}$, with equality for $k > 1$ since $\theta(p^k) < 1$ for such values of k. Given that seller payoffs are nonnegative, $q^k > \max\{c_i : \mu_i(\{p^k\}) > 0\}$ for all $k \in \mathcal{K}$.

Consider now the candidate equilibrium $\widehat{E} = \big(\widehat{\mu}, \widehat{\sigma}, \widehat{\pi}, (\widehat{U}_i)_{i\in\mathcal{I}}, \widehat{V}, (\widehat{M}_i)_{i\in\mathcal{I}}\big)$ in which the set of prices at which trade takes place is $\{q^1, \ldots, q^k\}$, with $\widehat{\theta}(q^1) = 1$ and

$$\widehat{\theta}(q^{k+1}) = \widehat{\theta}(q^k)\frac{q^k - \max\{c_i : \mu_i(\{p^k\}) > 0\}}{q^{k+1} - \max\{c_i : \mu_i(\{p^k\}) > 0\}}$$

for all $k < K$. For each $i \in \mathcal{I}$, let $\widehat{\mu}_i$ be such that

$$\frac{\widehat{\mu}_i(\{q^k\})\widehat{\sigma}(q^k)}{\mathbb{E}_{\widehat{\mu}_i}[\widehat{\sigma}]} = \frac{\mu_i(\{p^k\})\sigma(p^k)}{\mathbb{E}_{\mu_i}[\sigma]} \text{ for all } k \in \mathcal{K}. \tag{A.2}$$

A straightforward adaptation of the proof of Proposition 3 shows that these measures exist. Moreover, for each $i \in \mathcal{I}$, let $\widehat{U}_i = \widehat{\theta}(q^k)(q^k - c_i)$ for the values of k with $\widehat{\mu}_i(q^k) > 0$; the payoffs $\widehat{U}_i$ are well-defined given the definitions $\widehat{\theta}$ and $(\widehat{\mu}_i)_{i\in\mathcal{I}}$. Finally, let $\widehat{V} = 0$ and suppose that the masses $(\widehat{M}_i)_{i\in\mathcal{I}}$ satisfy stationarity and the belief $\widehat{\pi}(p)$ satisfies Bayes' rule for all $p \in \{q^1, \ldots, q^k\}$. The same argument used in the proof of Proposition 3 shows that $\widehat{\pi}_i(q^k) = \pi_i(p^k)$ for all $i \in \mathcal{I}$ and $k \in \mathcal{K}$, so that $q^k = \sum_{i=1}^N \widehat{\pi}_i(q^k)v_i$ for all $k \in \mathcal{K}$. Thus, $\widehat{E}$ satisfies rationality of beliefs, payoff consistency, and stationarity. By Corollary 1, we can choose $\widehat{\sigma}$ and $\widehat{\pi}$ for off-equilibrium prices so that $\widehat{E}$ is an equilibrium.

We claim that $\widehat{\theta}(q^k) \geq \theta(p^k)$ for all $k \in \mathcal{K}$. Clearly, $\widehat{\theta}(q^1) \geq \theta(p^1)$. Now suppose, by induction, that $\widehat{\theta}(q^k) \geq \theta(p^k)$ for some $k < K$. We claim that $\widehat{\theta}(q^{k+1}) \geq \theta(p^{k+1})$, from which the desired result holds. Indeed, note that

$$\begin{aligned}
\theta(p^{k+1}) &\leq \theta(p^k)\frac{p^k - \max\{c_i : \mu_i(\{p^k\}) > 0\}}{p^{k+1} - \max\{c_i : \mu_i(\{p^k\}) > 0\}} \\
&\leq \widehat{\theta}(q^k)\frac{p^k - \max\{c_i : \mu_i(\{p^k\}) > 0\}}{p^{k+1} - \max\{c_i : \mu_i(\{p^k\}) > 0\}} \\
&\leq \widehat{\theta}(q^k)\frac{q^k - \delta V - \max\{c_i : \mu_i(\{p^k\}) > 0\}}{q^{k+1} - \delta V - \max\{c_i : \mu_i(\{p^k\}) > 0\}} \\
&\leq \widehat{\theta}(q^k)\frac{q^k - \max\{c_i : \mu_i(\{p^k\}) > 0\}}{q^{k+1} - \max\{c_i : \mu_i(\{p^k\}) > 0\}} = \widehat{\theta}(q^{k+1});
\end{aligned}$$

the first inequality follows from seller optimality in E, the second inequality follows from the induction hypothesis, the third inequality follows since $p^k \leq q^k - \delta V$ for all $k \in \mathcal{K}$ with equality if $k > 1$, and the last inequality follows since $q^{k+1} > q^k$.

To conclude, note that since $\widehat{\sigma}(q^k) \geq \sigma(p^k)$ for all $k \in \mathcal{K}$, equation (A.2) implies that $\mathbb{E}_{\widehat{\mu}_i}[\widehat{\sigma}]\mu_i(\{p^k\}) \geq \mathbb{E}_{\mu_i}[\sigma]\widehat{\mu}_i(q^k)$ for all $i \in \mathcal{I}$ and $k \in \mathcal{K}$. Hence,

$$\mathbb{E}_{\widehat{\mu}_i}[\widehat{\sigma}] = \sum_{k=1}^{K} \mathbb{E}_{\widehat{\mu}_i}[\widehat{\sigma}]\mu_i(\{p^k\}) \geq \sum_{k=1}^{K} \mathbb{E}_{\mu_i}[\sigma]\widehat{\mu}_i(q^k) = \mathbb{E}_{\mu_i}[\sigma]$$

for all $i \in \mathcal{I}$, and gains from trade in $\widehat{E}$ are higher than in E.

A.11 Proof of Lemma 8

We first prove the following auxiliary result.

Claim 1. *Consider an equilibrium $E = \big(\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}}\big)$ and let $\mathcal{I}(p)$ be the set of seller types that offer $p \in S^*$. If $\mu_i(\{p\}) = 1$ for the lowest type i in $\mathcal{I}(p)$, then*

$$\sum_{i\in\mathcal{I}(p)} \pi_i(p)v_i \geq \Big(\sum_{i\in\mathcal{I}(p)} f_i\Big)^{-1} \sum_{i\in\mathcal{I}(p)} f_i v_i.$$

Proof. Fix $p \in S^*$ and let $\mu_i(\{p\}) = 1$ for $i = i_{\min} = \min\{i' : i' \in \mathcal{I}(p)\}$. Given that Proposition 1 implies that $\mu_j(\{p\}) = 1$ for all $i_{\min} < j < i_{\max} = \max\{i' : i' \in \mathcal{I}(p)\}$,

$$\pi_i(p) = \frac{M_i \mu_i(\{p\})}{\sum_{j \in \mathcal{I}(p)} M_j \mu_j(\{p\})} \geq \frac{M_i}{\sum_{j \in \mathcal{I}(p)} M_j}$$

for all $i \in \mathcal{I}(p)$ with $i < i_{\max}$. Since, by stationarity, $M_i \leq f_i/\sigma(p)$ for all $i \in \mathcal{I}(p)$ with equality if $i < i_{\max}$,

$$\pi_i(p) \geq \frac{f_i}{\sum_{j \in \mathcal{I}(p)} f_j}$$

for all $i \in \mathcal{I}(p)$ such that $i < i_{\max}$. The desired result follows as

$$\sum_{i \in \mathcal{I}(p)} \pi_i(p) v_i = v_{i_{\max}} + \sum_{j=i_{\min}}^{i_{\max}-1} \pi_j(p)(v_j - v_{i_{\max}}). \qquad \square$$

Consider an equilibrium $E = \big(\mu, \sigma, \pi, (U_i)_{i \in \mathcal{I}}, V, (M_i)_{i \in \mathcal{I}}\big)$ with $V = 0$. Buyer and seller optimality imply that $p = \sum_{i=1}^N \pi_i(p) v_i \geq \max\{c_i : p \in S_i^*\}$ for all $p \in S^*$, with strict inequality if p is smaller than the maximum of S^*. For each $i \in \mathcal{I}$, let $\underline{p}_i$ be the lowest price at which type-i sellers trade in E. Now let $S^*_{\min} = \{p \in S^* : p = \underline{p}_i \text{ for some } i \in \mathcal{I}\}$ and write $S^*_{\min} = \{p^1, \ldots, p^M\}$, with $p^1 < \cdots < p^M$. Consider then the partition $\{\widehat{\mathcal{I}}^m\}_{m=1}^M$ of $\mathcal{I}$ such that $\widehat{\mathcal{I}}^m = \{i \in \mathcal{I} : \underline{p}_i = p^m\}$. By construction, $\widehat{\mathcal{I}}^m$ is the set of seller types for which the lowest price at which they trade in E is the mth lowest element of $S^*_{\min}$. Note that $\max\{c_i : i \in \widehat{\mathcal{I}}^m\} \leq p^m$ for all $m \in \{1, \ldots, M\}$, with strict inequality if $m < M$. Proposition 1 implies that if $i, k \in \widehat{\mathcal{I}}^m$, then all $j \in \mathcal{I}$ with $i \leq j \leq k$ is also an element of $\widehat{\mathcal{I}}^m$. Finally, let

$$q^m = \left(\sum_{i \in \widehat{\mathcal{I}}^m} f_i\right)^{-1} \sum_{i \in \widehat{\mathcal{I}}^m} f_i v_i.$$

The above claim implies that $q^m \geq p^m$ for all $m \in \{1, \ldots, M\}$.

Consider now the list $\widehat{E} = \big(\widehat{\mu}, \widehat{\sigma}, \widehat{\pi}, (\widehat{U}_i)_{i \in \mathcal{I}}, \widehat{V}, (\widehat{M}_i)_{i \in \mathcal{I}}\big)$ such that: (*i*) $\widehat{\mu}_i(\{q^m\}) = 1$ if $i \in \widehat{\mathcal{I}}^m$; (*ii*) $\widehat{\theta}(q^1) = 1$ and

$$\widehat{\theta}(q^{m+1}) = \widehat{\theta}(q^m) \frac{q^m - \max\{c_i : i \in \widehat{\mathcal{I}}^m\}}{q^{m+1} - \max\{c_i : i \in \widehat{\mathcal{I}}^m\}} \text{ for all } m \in \{1, \ldots, M-1\};$$

and (*iii*) rationality of beliefs, payoff consistency, and stationarity hold. Note that $\widehat{V} = 0$. Thus, Corollary 1 implies that we can choose $\widehat{\sigma}$ and $\widehat{\pi}$ for off-equilibrium prices so that $\widehat{E}$ is an equilibrium.

We claim that $\widehat{U}_i \geq U_i$ for all $i \in \mathcal{I}$. The result is true for $i \in \widehat{\mathcal{I}}^1$ since $\widehat{U}_i = q^1 - c_i \geq p^1 - c_i \geq U_i$. Now suppose, by induction, that there exists $m \in \{1, \ldots, M-1\}$ with $\widehat{U}_i \geq U_i$ for all $i \in \widehat{\mathcal{I}}^m$ and let $i_{\max}(m) = \max\{i : i \in \widehat{\mathcal{I}}^m\}$. Then $i \in \widehat{\mathcal{I}}^{m+1}$ implies that

$$\begin{aligned}
U_i &= \theta(p^{m+1})(p^{m+1} - c_i) \\
&= \theta(p^{m+1})(p^{m+1} - c_i)\frac{p^{m+1} - \max\{c_i : i \in \widehat{\mathcal{I}}^m\}}{p^{m+1} - \max\{c_i : i \in \widehat{\mathcal{I}}^m\}} \\
&\leq U_{i_{\max}(m)}\frac{p^{m+1} - c_i}{p^{m+1} - \max\{c_i : i \in \widehat{\mathcal{I}}^m\}} \\
&\leq \widehat{U}_{i_{\max}(m)}\frac{p^{m+1} - c_i}{p^{m+1} - \max\{c_i : i \in \widehat{\mathcal{I}}^m\}} \\
&= \widehat{\theta}(q^m)(q^m - \max\{c_i : i \in \widehat{\mathcal{I}}^m\})\frac{p^{m+1} - c_i}{p^{m+1} - \max\{c_i : i \in \widehat{\mathcal{I}}^m\}} \\
&= \widehat{\theta}(q^{m+1})(q^{m+1} - \max\{c_i : i \in \widehat{\mathcal{I}}^m\})\frac{p^{m+1} - c_i}{p^{m+1} - \max\{c_i : i \in \widehat{\mathcal{I}}^m\}} \\
&= \widehat{\theta}(q^{m+1})\frac{q^{m+1} - \max\{c_i : i \in \widehat{\mathcal{I}}^m\}}{p^{m+1} - \max\{c_i : i \in \widehat{\mathcal{I}}^m\}}(p^{m+1} - c_i) \\
&\leq \widehat{\theta}(q^{m+1})(q^{m+1} - c_i); \\
&= \widehat{U}_i;
\end{aligned}$$

the first inequality is a consequence of seller optimality in E, the second inequality follows from the induction hypothesis, and the third inequality follows since $p^{m+1} \leq q^{m+1}$ and the map $p \mapsto (p - c_i)/(p - \max\{c_i : i \in \widehat{\mathcal{I}}^m\})$, with $i \in \widehat{\mathcal{I}}^m$, is strictly increasing for $p \geq \max\{c_i : i \in \widehat{\mathcal{I}}^m\}$. This establishes the claim.

The desired result now follows from the fact that Lemma 6 implies that gains from trade in E are bounded above by $\sum_{i=1}^N f_i U_i$, whereas gains from trade in $\widehat{E}$ are $\sum_{i=1}^N f_i \widehat{U}_i$.

A.12 Stationarity

In order to distinguish between stationary and non-stationary equilibria, here we use the subscript ∞ to denote stationary equilibria. This choice of notation is consistent with the idea of considering the outcomes of stationary equilibria as long-run outcomes of non-stationary equilibria. We first present the definition of a non-stationary equilibrium.

Definition. *Let $E = \big((\mu_t)_{t\geq 0}, (\sigma_t)_{t\geq 0}, (\pi_t)_{t\geq 0}, (U_{it})_{i\in\mathcal{I}, t\geq 0}, (V_t)_{t\geq 0}, (M_{it})_{i\in\mathcal{I}, t\geq 0}\big)$ be a list where: (i) μ_t and σ_t are, respectively, strategy profiles for sellers and buyers in period t; (ii) π_t is a belief system for buyers in period t; (iii) U_{it} and V_t are, respectively, present-discounted expected lifetime payoffs for type-i sellers and buyers in period t; and (iv) M_{it} is the mass of type-i sellers in the market in period t. Then E is a non-stationary equilibrium if it satisfies the following properties.*

1. ***Seller Optimality.*** *For all $t \geq 0$ and $i \in \mathcal{I}$, any p in the support S_{it} of μ_{it} maximizes*

$$\sigma(p)(p - c_i) + (1 - \sigma(p))\delta U_{it+1}.$$

2. ***Buyer Optimality.*** *For all $t \geq 0$ and $p \geq 0$, the trade probability $\sigma_t(p)$ maximizes*

$$\sigma\left(\sum_{i=1}^{N} \pi_i(p)v_i - p\right) + (1 - \sigma)\delta V_{t+1}.$$

3. ***Rational Beliefs.*** *For all $t \geq 0$, the belief $\pi_t(p)$ satisfies Bayes' rule for $p \in \bigcup_{i=1}^{N} S_{it}$.*

4. ***Consistency of Payoffs.*** *Seller payoffs are such that for all $i \in \mathcal{I}$ and $t \geq 0$,*

$$U_{it} = \sigma_t(p)(p - c) + (1 - \delta_t(p))\delta U_{it+1} \text{ for all } p \in S_{it}.$$

Buyer payoffs are such that for all $t \geq 0$,

$$V_t = \int_{\mathbb{R}_+} \left(\sum_{i=1}^{N} \pi_{it}(p)(v_i - p)\right)\sigma_t(p) d\overline{\mu}_t(p) + \left(\int_{\mathbb{R}_+} (1 - \sigma_t(p)) d\overline{\mu}_t(p)\right)\delta V_{t+1},$$

where $\overline{\mu}_t = \sum_{i=1}^{N} g_{it}\mu_{it}$ and $g_{it} = M_{it}/\sum_{j=1}^{N} M_{jt}$.

5. ***Evolution of Seller Masses.*** *The sequence* $(M_{it})_{i\in\mathcal{I},t\geq 0}$ *satisfies the law of motion*

$$M_{i0} = f_i \text{ and } M_{it+1} = f_i + \left(1 - \int_{\mathbb{R}_+} \sigma_t(p) d\mu_{it}(p)\right) M_{it} \text{ for all } t \geq 0.$$

We now define what it means for the outcomes of a stationary equilibrium to be the long-run outcomes of a non-stationary equilibrium. For any stationary equilibrium E_∞, let $S^*_{i\infty}$ be the set of prices at which type-i sellers trade and $\theta_{i\infty}$ be the discounted probability of trade for such sellers. Furthermore, for any non-stationary equilibrium E, let S^*_{it} be the set of prices at which type-i sellers trade in period t and θ_{it} be the discounted probability of trade for type-i sellers who are in the market in period t.[29] Finally, let $d_H(S, S')$ be the Hausdorff distance of two subsets S and S' of $\mathbb{R}_+$.[30]

Definition. *Let* $E_\infty = (\mu_\infty, \sigma_\infty, \pi_\infty, (U_{i\infty})_{i\in\mathcal{I}}, V_\infty, (M_{i\infty})_{i\in\mathcal{I}})$ *be a stationary equilibrium and* $E = \big((\mu_t)_{t\geq 0}, (\sigma_t)_{t\geq 0}, (\pi_t)_{t\geq 0}, (U_{it})_{i\in\mathcal{I},t\geq 0}, (V_t)_{t\geq 0}, (M_{it})_{i\in\mathcal{I},t\geq 0}\big)$ *be a non-stationary equilibrium. The outcomes of* E_∞ *are the long-run outcomes of* E *if for all* $i \in \mathcal{I}$*, we have that* $\lim_{t\to\infty} d_H(S^*_{it}, S^*_{i\infty}) = 0$, $\lim_{t\to\infty} \theta_{it} = \theta_{i\infty}$*, and* $\lim_{t\to\infty} M_{it} = M_{i\infty}$.

We can now prove the following result, which generalizes the result derived in the main text to the case of pure stationary equilibria in which the buyers' payoff is zero.

Proposition 9. *Let* $E_\infty = (\mu_\infty, \sigma_\infty, \pi_\infty, (U_{i\infty})_{i\in\mathcal{I}}, V_\infty, (M_{i\infty})_{i\in\mathcal{I}})$ *be pure stationary equilibrium with* $V_\infty = 0$*. There exists a non-stationary equilibrium* E *such that the outcomes of* E_∞ *are the long-run outcomes of* E*.*

Proof. Let $E_\infty = (\mu_\infty, \sigma_\infty, \pi_\infty, (U_{i\infty})_{i\in\mathcal{I}}, V_\infty, (M_{i\infty})_{i\in\mathcal{I}})$ be a pure stationary equilibrium with $V_\infty = 0$. Now let $E = \big((\mu_t)_{t\geq 0}, (\sigma_t)_{t\geq 0}, (\pi_t)_{t\geq 0}, (U_{it})_{i\in\mathcal{I},t\geq 0}, (V_t)_{t\geq 0}, (M_{it})_{i\in\mathcal{I},t\geq 0}\big)$ be such that $\mu_t \equiv \mu_\infty$, $\sigma_t \equiv \sigma_\infty$, $\pi_t \equiv \pi_\infty$, $U_{it} \equiv U_{i\infty}$ for all $i \in \mathcal{I}$, $V_t \equiv 0$, and $(M_{it})_{i\in\mathcal{I},t\geq 0}$ is such that $M_{i0} = f_i$ for all $i \in \mathcal{I}$ and

$$M_{it+1} = f_i + (1 - \sigma_\infty(p_{i\infty}))M_{it}$$

[29] $\theta_{it} = \sum_{s=0}^{\infty} \delta^s \prod_{k=0}^{s-1}(1 - \sigma_{it+k})\sigma_{it+s}$, where σ_{it} is the trade probability for type-i sellers in period t.
[30] By definition, $d_H(S, S') = \max\{\sup_{x\in S} \inf_{x'\in S'} |x - x'|, \sup_{x'\in S'} \inf_{x\in S} |x - x'|\}$.

for all $i \in \mathcal{I}$ and $t \geq 0$, where $p_{i\infty}$ is the price that type-i sellers offer in E_∞, and so is also the price that type-i sellers offer in E. We claim that E is a non-stationary equilibrium. Since $p_{i\infty} = \sum_{j=1}^{N} \pi_{j\infty}(p_{i\infty})v_j = \sum_{j=1}^{N} \pi_{jt}(p_{i\infty})v_j$, we are done if we show that for all $i \in \mathcal{I}$ and $t \geq 0$,

$$\pi_{it}(p_{i\infty}) = \frac{M_{it}}{\sum_{j \in \mathcal{I}_\infty(p_{i\infty})} M_{jt}},$$

where $\mathcal{I}_\infty(p_{i\infty})$ is the set of seller types offering $p_{i\infty}$ in E_∞ and in E as well. Note that if $j \in \mathcal{I}_\infty(p_{i\infty})$, then

$$\frac{M_{jt+1}}{M_{it+1}} = \frac{f_j + (1 - \sigma_\infty(p_{i\infty}))M_{jt}}{f_i + (1 - \sigma_\infty(p_{i\infty}))M_{it}}$$

for all $t \geq 0$. Given that $M_{j0}/M_{i0} = f_j/f_i$, it then follows that $M_{jt}/M_{it} = f_j/f_i$ for all $t \geq 0$. So,

$$\pi_{it}(p_{i\infty}) = \pi_{i\infty}(p_{i\infty}) = \frac{f_i}{\sum_{j \in \mathcal{I}(p_{i\infty})} f_j} = \frac{M_{it}}{\sum_{j \in \mathcal{I}_\infty(p_{i\infty})} M_{jt}}.$$

Since $S^*_{it} \equiv S^*_{i\infty}$ and $\theta_{it} \equiv \theta_{i\infty}$ for all $i \in \mathcal{I}$, we are done if $\lim_{t\to\infty} M_{it} = M_{i\infty}$ for all $i \in \mathcal{I}$. Note that for each $i \in \mathcal{I}$, $\lim_{t\to\infty} M_{it} = f_i/\sigma_\infty(p_{i\infty})$. Also note that the stationarity condition (3) implies that $M_{i\infty} = f_i/\sigma_\infty(p_{i\infty})$. This concludes the proof. $\square$

B Supplementary Appendix (Not for Publication)

B.1 Equilibrium Refinements

Here we show that every equilibrium satisfies the intuitive criterion and that the set of equilibrium payoff vectors for separating equilibria coincides with the set of equilibrium payoff vectors for D1 equilibria. Although originally designed for static signaling games, both refinements extend naturally to our dynamic environment. We begin by presenting these refinements in our setting.

We start with some preliminary definitions. Given a belief π, a price p, and a payoff V, let

$$\Sigma(\pi, p, V) = \mathrm{argmax}_{\sigma\in[0,1]}\left(\sum_{i\in\mathcal{I}} \pi_i v_i - p - \delta V\right)$$

be the set of best replies for a buyer with belief π when the price is p and the buyer's continuation payoff is V. Now for each $\mathcal{I}' \subseteq \mathcal{I}$ let $\Delta(\mathcal{I}')$ be the set of buyer beliefs that assign probability one to the event that the sellers' type is in $\mathcal{I}'$ and define $\Sigma(\mathcal{I}', p, V)$ to be such that

$$\Sigma(\mathcal{I}', p, V) = \bigcup_{\pi\in\Delta(\mathcal{I}')} \Sigma(\pi, p, V).$$

By definition, $\Sigma(\mathcal{I}', p, V)$ is the set of possible best replies for a buyer when the buyer's belief has support in $\mathcal{I}'$, the price is p, and the buyer's continuation payoff is V. Finally, let $\Theta(\mathcal{I}', p, V)$ be the image of $\Sigma(\mathcal{I}', p, V)$ under the map $\sigma \mapsto \sigma/(1 + \delta(1 - \sigma))$. Note that $\Theta(\mathcal{I}', p, V) = \{0\}$ if $p > \max_{i\in\mathcal{I}'} v_i - \delta V$, $\Theta(\mathcal{I}', p, V) = \{1\}$ if $p < \min_{i\in\mathcal{I}'} v_i - \delta V$, and $\Theta(\mathcal{I}', p, V) = [0, 1]$ otherwise.

We now present the refinements in our setting. Given an equilibrium and $p \notin S$, let

$$\mathcal{I}^*(p) = \left\{i \in \mathcal{I} : U_i \leq \max_{\theta\in\Theta(\mathcal{I},p,V)} \theta(p - c_i)\right\}$$

be the set of seller types that could gain by deviating to p. Also, for each $i \in \mathcal{I}^*(p)$, let

$$D_i(p) = \left\{\theta \in \Theta(\mathcal{I}^*(p), p, V) : U_i \leq \theta(p - c_i)\right\}.$$

and $D_i^+(p)$ be the corresponding set when the inequality is strict. By definition, $D_i(p)$ is the set of buyer best replies when buyer beliefs have support in $\mathcal{I}^*(p)$ that make a deviation to p attractive to a type-i seller with $i \in \mathcal{I}^*(p)$, whereas $D_i^+(p) \subset D_i(p)$ is the subset of buyer best replies in $D_i(p)$ that make a type-i buyer with $i \in \mathcal{I}^*(p)$ strictly better off when the buyer deviates to p.

Definition. *The equilibrium E violates the intuitive criterion if there exists $p \notin S$ and $i \in \mathcal{I}^*(p)$ such that $U_i < \min_{\theta \in \Theta(\mathcal{I}^*(p),p,V)} \theta(p - c_i)$. The equilibrium E satisfies D1 if $\pi_i(p) = 0$ for every $p \notin S$ and $i \in \mathcal{I}^*(p)$ for which there exists $j \neq i$ with $j \in \mathcal{I}^*(p)$ and $D_i(p) \subseteq D_j^+(p)$.*

We first show that the intuitive criterion does not refine the equilibrium set.

Proposition B.1. *Every equilibrium satisfies the intuitive criterion.*

Proof. Consider an equilibrium and suppose, by contradiction, that it fails the intuitive criterion. Then there exist $p \notin S$ and $i \in \mathcal{I}^*(p)$ with $U_i < \min_{\theta \in \Theta(\mathcal{I}^*(p),p,V)} \theta(p - c_i)$. Since $U_i \geq 0$, it follows that $\Theta(\mathcal{I}^*(p), p, V) = \{1\}$, otherwise $0 \in \Theta(\mathcal{I}^*(p), p, V)$ and $\min_{\theta \in \Theta(\mathcal{I}^*(p),p,V)} \theta(p - c_i) = 0$. Hence, $v_i - \delta V > p$. We claim that $1 \in \mathcal{I}^*(p)$, so that $v_1 - \delta V > p$. For this, consider $p' \in S_1$ and $p'' \in S_i$. First note that $\theta(p'')(p'' - c_i) = U_i < p - c_i$ by assumption. From type-i seller optimality, it also follows that $\theta(p')(p' - c_i) \leq \theta(p'')(p'' - c_i) < p - c_i$. Given that $c_i > c_1$, we then have that $U_1 = \theta(p')(p' - c_1) = \theta(p')(p' - c_i) + \theta(p')(c_i - c_1) < p - c_1$, which implies the desired result. Now observe that $v_1 - \delta V > p$ implies that $U_1 < v_1 - \delta V - c_1$. This, however, contradicts type-1 seller optimality, as buyers accept $v_1 - \delta V - \varepsilon$ for all $\varepsilon > 0$, and so $U_1 \geq v_1 - \delta V - c_1$. $\square$

We now show that the set of equilibrium payoff vectors for separating equilibria coincide with the set of equilibrium payoff vectors for equilibria that satisfy D1. We start with a useful result.

Lemma B.1. *For any equilibrium, $D_i(p) \subseteq D_j^+(p)$ if $p \notin S$, $i, j \in \mathcal{I}^*(p)$, and $i < j$.*

Proof. Proposition 1 implies that there exist $p^i \leq p^j$ with $U_i = \theta(p^i)(p^i - c_i)$ and $U_j = \theta(p^j)(p^j - c_j)$. Note that $p > p^j$. Let $\theta \in D_i(p)$, so that $\theta(p^i)(p^i - c_i) \leq \theta(p - c_i)$. The desired result follows if we show that $\theta \in D_j^+(p)$. There are two cases to consider. Either $\theta < \theta(p^j)$ or $\theta \geq \theta(p^j)$. In the second case, $\theta(p - c_j) \geq \theta(p^j)(p - c_j) > \theta(p^j)(p^j - c_j) = U_j$. So assume that $\theta < \theta(p^j)$. Then,

$$
\begin{aligned}
\theta(p - c_j) &= \theta(p - c_i) + \theta(c_i - c_j) \\
&\geq \theta(p^i)(p^i - c_i) + \theta(c_i - c_j) \\
&\geq \theta(p^j)(p^j - c_j) + (\theta(p^j) - \theta)(c_j - c_i) > U_j;
\end{aligned}
$$

the second inequality follows from type-i seller optimality. This concludes the proof. $\square$

The next result shows that only separating equilibria can satisfy D1.

Lemma B.2. *An equilibrium satisfies D1 only if it is separating.*

Proof. Consider a non-separating equilibrium. By assumption there exists $p^* \in S$ such that at least two types of seller offer p^* with positive probability. Let $k > 1$ be the highest type of seller that offers p^*. Now consider $p'' \notin S$ with $p^* < p'' < v_k - \delta V$; such a price exists since buyer optimality implies that $\delta V \leq \sum_{i \in \mathcal{I}} \pi_i(p^*) v_i - p^* < v_k - p^*$. Note that $k \in \mathcal{I}^*(p'')$ as a buyer with belief π such that $\pi_k(p'') = 1$ accepts p'' with probability one. This, in turn, implies that $i \in \mathcal{I}(p'')$ for all type-i sellers with $i < k$. Now note that since type-k seller optimality requires that $\theta(p'') < 1$, it must be that $\sum_{i \in \mathcal{I}} \pi_i(p'') v_i - \delta V \leq p''$. Hence, $\sum_{i \in \mathcal{I}} \pi_i(p'') v_i < v_k$, and so there exists $j < k$ such that $\pi_j(p'') > 0$. Since $D_j(p'') \subseteq D_k^+(p'')$, the equilibrium violates D1. $\square$

We can now prove our main result concerning D1.

Proposition B.2. *The set of equilibrium payoff vectors for separating equilibria coincide with the set of equilibrium payoff vectors for equilibria that satisfy D1.*

Proof. Lemma B.2 implies that it is sufficient to prove that for any separating equilibrium there exists a separating equilibrium satisfying D1 that has the same payoffs. Let $E = \big(\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}}\big)$ be a separating equilibrium. By Proposition 6, $V = 0$, $\mu_i(\{v_i\}) = 1$ for $i < N$, and $\mu_N(\{v_N\}) > 0$. Now let $E' = \big(\mu', \sigma', \pi', (U_i')_{i\in\mathcal{I}}, V', (M_i')_{i\in\mathcal{I}}\big)$ be such that $V' = 0$ and $\mu_i'(\{v_i\}) = 1$, $\sigma'(v_i) = \sigma(v_i)$, $\pi'(v_i) = \pi(v_i)$, $U_i' = U_i$, and $M_i' = f_i/\sigma'(v_i)$ for all i. Then, E' satisfies rationality of beliefs, payoff consistency, and stationarity. Also, no type of seller has an incentive to mimic the behavior of another type of seller. By Lemma 2, we can then choose σ' and π' for off-equilibrium prices so that E' is an equilibrium. To conclude, we show that we can do so in such a way that E' satisfies D1. For each $p \notin \{v_1, \ldots, v_N\}$ let $\sigma'(p) = 1$ if $p < v_1$ and $\sigma'(p) = 0$ if $p > v_1$ and define $\pi'(p)$ to be such that $\pi_1'(p) = 1$ if $p < v_1$ and $\pi_i(p) = 1$ if $p > v_1$ and $i = \max\{i \in \mathcal{I} : v_i < p\}$. Clearly, buyer optimality holds for off-equilibrium prices. Moreover, since $\mathcal{I}^*(p) = \max\{i \in \mathcal{I} : v_i < p\}$ for each $p \notin S'$ with $p > v_1$, Lemma B.1 implies that E satisfies D1. $\square$

B.2 Equilibria with $V > 0$

Here, we provide an example of an equilibrium with $V > 0$ when $N = 3$ and adverse selection is severe. Let $f_1 = f_2 = f_3 = 1/3$ and suppose that $c_1 < v_1 = 2$, $c_2 = 2$, $v_2 = 3$, and $3 < c_3 < v_3 = 4$, so that adverse selection is severe. Consider a candidate equilibrium with $\mu_1(\{p^1\}) = \mu_2(\{p^1\}) = 1$, $\mu_3(\{p^2\}) = 1$, $\theta(p^1) = 1$, and $\theta(p^2) = (p^1 - 2)/(p^2 - 2)$, where $p^1 < p^2$; we determine p^1 and p^2 below. Since $\sigma(p^2) < 1$, buyer optimality implies that $p^2 = 4 - \delta V$. Thus, $V > 0$ only if

$$p^1 < \sum_{i=1}^{3} \pi_i(p^1)v_i - \delta V = \frac{1}{2}(v_1 + v_2) - \delta V = \frac{5}{2} - \delta V.$$

Let then $p^1 = 5/2 - \delta V - \xi$, with $\xi < 1/2$ (otherwise $p^1 < 2$). Straightforward algebra shows that $V = (1-\delta)^{-1}(g_1 + g_2)\xi$, where g_i is the fraction of type-i sellers in the market. Since stationarity implies that $M_i = f_i/\sigma(p^1) = f_i$ for $i \in \{1, 2\}$, it follows that $g_1 = g_2$.

Let g denote the common value of g_1 and g_2. Given that stationarity also implies that

$$M_3 = \frac{f_3}{\sigma(p^2)} = \frac{f_3(1-\delta\theta(p^2))}{(1-\delta)\theta(p^2)} = \frac{f_3(p^2-2-\delta(p^1-2))}{(1-\delta)(p^1-2)},$$

we then have that

$$g = \frac{(1-\delta)(p^1-2)}{2(1-\delta)(p^1-2)+p^2-2-\delta(p^1-2)} = \frac{(1-\delta)(p^1-2)}{3(1-\delta)(p^1-2)+p^2-p^1}. \tag{B.3}$$

The right-hand side of (B.3) depends on g through the dependence of p^1 and p^2 on V.

We claim that for all $\xi < 1/2$, equation (B.3) has a unique solution in $(0, 1/2)$ regardless of δ. Indeed, since $p^2 - p^1 = 3/2 + \xi$, $p^1 - 2 = 1/2 - \delta V - \xi$, and $V = (1-\delta)^{-1}2g\xi$, we can rewrite (B.3) as

$$\frac{1}{g} = 3 + \frac{3/2+\xi}{1/2-\xi(1-\delta+2\delta g)}$$

Given that the right-hand side of the above equation is strictly increasing in g, this equation has at most one solution. The desired result follows since the right-hand side of the above equation evaluated at $g = 1/2$ is greater than 2 for all $\delta \in (0,1)$.

To finish, note that for all $\delta \in (0,1)$, we can take ξ sufficiently close to zero for p^1 and p^2 to be such that $p^1 \in (2, 5/2)$ and $p^2 \geq 3$. By choosing buyer beliefs for off-equilibrium prices appropriately, we can ensure that buyers find it optimal to reject offers in (p^1, p^2). Also note that buyers reject offers greater than p^2. So, seller optimality holds, as type-1 sellers have no incentive to post p^2 and type-3 sellers have no incentive to post p^1.

B.3 Equilibrium Payoffs in Two-Type Case for $S_1^* \cap S_2^*$ a Singleton

Here, we show that for all $U_2 \in [0, \overline{U}_2)$, there exists an equilibrium with $S_1^* \cap S_2^*$ a singleton in which the type-2 sellers' payoff is U_2. Suppose that $S_1^* \cap S_2^* = \{p\}$ for some $p \in [c_2, v_2)$. Since the proof of Corollary 3 implies that S_1^* and S_2^* have at most two elements, it follows that $S_1^* = \{v_1, p\}$ and either $S_2^* = \{p\}$ or $S_2^* = \{p, v_2\}$. Indeed, if $S_2^* = \{p, p'\}$ with $p < p'$, then $\pi_2(p') = 1$ by Bayes' rule, in which case $p' = v_2$ by buyer optimality. Given that S_2^* collapses to $\{p\}$ when $\mu_2(\{p\}) = 1$, we can treat the case in which $S_2^* = \{p\}$ as a special case of the case in which $S_2^* = \{p, v_2\}$.

Since a type-1 seller must be indifferent between posting v_1 and p, it must be that

$$\theta(p) = \frac{v_1 - c_1}{p - c_1}. \tag{B.4}$$

In turn, this implies that if $\mu_2(\{v_2\}) > 0$, then

$$\theta(v_2) = \theta(p)\frac{p - c_2}{v_2 - c_2} = \frac{v_1 - c_1}{p - c_1}\frac{p - c_2}{v_2 - c_2}, \tag{B.5}$$

for a type-2 seller must be indifferent between posting any price in S_2^*. Given that $c_2 > c_1$, the ratio $(p - c_2)/(p - c_1)$ is strictly increasing in p. So, $\theta(v_2)$ given by (B.5) satisfies (6), and type-1 sellers have no incentive to deviate and post v_2. Note that we need $p > c_2$ when $S_2^* = \{p, v_2\}$, otherwise $\theta(v_2) = 0$. So, $U_2 > 0$ and $S_2 = S_2^*$ when S_2^* is not a singleton. However, we can have $U_2 = 0$ and S_2^* a strict subset of S_2 when S_2^* is a singleton. Indeed, $U_2 = 0$ when $S_2^* = \{c_2\}$, in which case the type-2 sellers can make offers that are rejected in equilibrium. Since such equilibria are payoff equivalent to the equilibrium in which the type-2 sellers offer c_2 with probability one, we can assume that $S_2 = S_2^*$ without any loss.

Now observe that buyer optimality and $V = 0$ imply that $\mu_1(\{p\})$ and $\mu_2(\{p\})$ must satisfy

$$p = \frac{M_1\mu_1(\{p\})}{M_1\mu_1(\{p\}) + M_2\mu_2(\{p\})}v_1 + \frac{M_2\mu_2(\{p\})}{M_1\mu_1(\{p\}) + M_2\mu_2(\{p\})}v_2; \tag{B.6}$$

stationarity implies that M_1 and M_2 depend on p through $\sigma(p) = (1 - \delta)\theta(p)/(1 - \delta\theta(p))$ and $\sigma(v_2) = (1-\delta)\theta(v_2)/(1-\delta\theta(v_2))$ (the latter only when $\mu_2(\{p\}) < 1$). Below, we show that for each $p \in [c_2, v_2)$ and $\mu_2(\{p\}) = 1 - \mu_2(\{v_2\}) \in (0, 1]$, there exists a unique value of $\mu_1(\{p\})$ for which the pair $(\mu_1(\{p\}), \mu_2(\{p\}))$ solves (B.6) and provide an expression for this value.

For each $\mu \in [0, 1]$, let

$$\alpha(\mu) = \mu\left(\mu + (1 - \mu)\frac{\sigma(v_2)}{\sigma(p)}\right)^{-1}.$$

Note that $\alpha(\mu) = 1$ when $\mu = 1$, in which case the value of $\sigma(v_2)$ does not matter for determining $\alpha(\mu)$. We claim that

$$\mu_1(\{p\}) = \left(1 + \alpha(\mu_2(\{p\}))\frac{f_2}{f_1}\frac{v_2 - p}{v_1 - r_1}\right)^{-1} \alpha(\mu_2(\{p\}))\frac{f_2}{f_1}\frac{v_2 - p}{v_1 - r_1}\left(1 + \frac{v_1 - r_1}{p - v_1}\right) \quad \text{(B.7)}$$

is the unique value of $\mu_1(\{p\})$ for which the pair $(\mu_1(\{p\}), \mu_2(\{p\})$ satisfies (B.6). First note that we can re-write (B.6) as

$$\mu_1(\{p\}) = \mu_2(\{p\})\frac{M_2(v_2 - p)}{M_1(p - v_1)}.$$

Since stationarity implies that

$$M_1 = \frac{f_1}{\mu_1(\{p\})\sigma(p) + 1 - \mu_1(\{p\})} \text{ and } M_2 = \frac{f_2}{\sigma(p)\mu_2(\{p\}) + (1 - \mu_2(\{p\}))\sigma(v_2)},$$

it follows from straightforward algebra that

$$\mu_1(\{p\}) = \mu_2(\{p\})\frac{M_2(v_2 - p)}{M_1(p - v_1)} = \alpha(\mu_2(\{p\}))\frac{f_2}{f_1}\frac{v_2 - p}{p - v_1}\left[\mu_1(\{p\}) + (1 - \mu_1(\{p\}))\frac{p - r_1}{v_1 - r_1}\right],$$

where $r_1 = c_1 + \delta(v_1 - c_1)$. Solving the above equation for $\mu_1(\{p\})$, we obtain (B.7).

The above argument shows that $\theta(p)$ and $\theta(v_2)$ given by (B.4) and (B.5), respectively, and $\mu_1(\{p\})$ and $\mu_2(\{p\})$ satisfying (B.6) with $\mu_2(\{p\}) \in (0, 1]$ are necessary for an equilibrium with $S_1^* \cap S_2^* = \{p\}$. By Lemma 2, these conditions are also sufficient. From this it follows that the equilibrium payoff for the type-2 sellers when $S_1^* \cap S_2^* = \{p\}$ with $p \in [c_2, v_2)$ is

$$U_2 = \theta(p)(p - c_2) = \frac{v_1 - c_1}{p - c_1}(p - c_2).$$

Since $(p - c_2)/(p - c_1)$ is strictly increasing in p, we then have that $U_2 \in [0, \overline{U}_2)$. This establishes the desired result.

B.4 General Results on Equilibrium Existence

We know from Section 2 that a pooling equilibrium exists if adverse selection is not severe. We also know from the main text that separating equilibria always exist when gains from trading are strictly positive for all types. Here, we relax this assumption and present necessary and sufficient conditions for equilibrium existence that hold regardless of whether adverse selection is severe or not. In what follows, let $\mathcal{I}^0 = \{i \in \mathcal{I} : v_i < c_i\}$ be the set of types of the good for which gains from trading are negative.

First note that the results in Section 3 depend only on the assumption that v_i and c_i are strictly increasing in i, and so remain valid. We now state and prove our existence result.

Proposition B.3. *An equilibrium exists if, and only if, $v_N \geq c_N$ and for all $i \in \mathcal{I}^0$, there exists $k > i$ such that $\sum_{j=i}^k f_j v_j \geq \sum_{j=i}^k f_j c_k$, with strictly inequality if $k < N$.*

The proof of the sufficiency part is constructive. In order to understand necessity, first note that the type-N good must trade at price at least c_N. Given that buyers do not trade a price greater than v_N, the type-N good cannot trade in equilibrium if $v_N < c_N$, which violates stationarity. Now suppose that $\mathcal{I}^0$ is nonempty and consider a type $i < N$ of the good with $v_i < c_i$. Since the type-i good must trade at price c_i or higher, this type of good can only trade at some price p if higher types of the good also trade at price p. Let $k > i$ be the highest type of the good that can trade at price p. Then $p \geq c_k$ and $p > c_k$ if $k < N$, as all types of seller except, possibly, the highest obtain positive payoff in equilibrium. On the other hand, by Proposition 1, all types of the good between i and k must trade at price p. So, the expected value of the good to a buyer who purchases it a price p is at most $\big(\sum_{j=i}^k f_j\big)^{-1} \sum_{j=i}^k f_j v_j$, the average quality of the good conditional on its type being in the set $\{i, \ldots, k\}$. Hence, the type-i good can trade only if $\sum_{j=i}^k f_j v_j \geq \sum_{j=i}^k f_j c_k$, with strict inequality when $k < N$.

Proof of Proposition B.3. We first prove necessity. We know from above that $v_N \geq c_N$ is necessary for existence. Now suppose that $i < N$ belongs to $\mathcal{I}^0$ and consider an equilibrium. Let p be a price at which type-i sellers trade in this equilibrium. We can assume that i is the lowest seller type in $\mathcal{I}(p)$, as the inclusion of lower types of seller in $\mathcal{I}(p)$ lowers the expected value of the good to a buyer who purchases it at price p. Buyer and seller optimality imply that $\sum_{j \in \mathcal{I}(p)} \pi_j(p) v_j \geq p > c^i$, where the strict inequality follows since

$U_i > 0$. So, $\sum_{j\in\mathcal{I}(p)} \pi_j(p)v_j \geq p > v^i$, and there exists $k > i$ with $k \in \mathcal{I}(p)$. Now note that $\mu_i(\{p\}) = 1$, as Proposition 1 implies that type-i sellers can pool with higher types only at a single price. The desired result follows from Claim 1 in the proof of Lemma 8 and the fact that $p \geq c_k$, with strict inequality if $k < N$.

We now prove sufficiency. We know that there exists a partition $\{\mathcal{I}^s\}_{s=1}^S$ of $\mathcal{I}$ such that

$$p^s := \left(\sum_{j\in\mathcal{I}^s} f_j\right)^{-1} \sum_{j\in\mathcal{I}^s} f_j v_j \geq \max\{c_i : i \in \mathcal{I}^s\}$$

for each $s \in \{1, \ldots, S\}$, with strict inequality for all $s < S$. Consider the candidate equilibrium $E = \big(\mu, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}}\big)$ such that: (*i*) $\mu_i(\{p^s\}) = 1$ if $i \in \mathcal{I}^s$; (*ii*) $\theta(p^1) = 1$ and

$$\theta(p^{s+1}) = \theta(p^s)\frac{p^s - \max\{c_i : i \in \mathcal{I}^s\}}{p^{s+1} - \max\{c_i : i \in \mathcal{I}^s\}}$$

for all $s \in \{1, \ldots, S-1\}$; and (*iii*) rationality of beliefs, payoff consistency, and stationarity hold. Note that $V = 0$. By Corollary 1, we can then choose σ and π for off-equilibrium prices so that E is an equilibrium. This concludes the proof of the proposition. $\square$

B.5 General Result for the Example from Section 5

Proposition B.4. *Let $N \geq 3$. There exists an open set of distributions of seller types in the population for which adverse selection is severe and all separating equilibria are Pareto-dominated by and have lower gains from trade than a non-separating equilibrium.*

Proof. Consider a separating equilibrium with $\theta(v_{i+1}) = \theta(v_i)(v_i - c_i)/(v_{i+1} - c_i)$ for all $i < N$ and $\theta(v_1) = 1$ and let U_i be the seller payoffs in this equilibrium. Such equilibria exist regardless of the distribution of seller types in the population and, by Proposition 6, Pareto-dominate all other separating equilibria. Now consider a distribution of seller types $(f_1, \ldots, f_N)$ such that $f_i > 0$ for all $i \in \mathcal{I}$ and $(f_1 + f_2)v_2 + f_3v_3 + \cdots + f_Nv_N < c_N$, and assume that the distribution of seller types in the population is $(f_1', \ldots, f_N')$ with $f_1' = \alpha f_1$, $f_2' = \beta f_2$, $f_i' = f_i$ for $i \geq 3$, $\alpha \in (0, 1)$, and $\beta = 1 + (1 - \alpha)(f_1/f_2)$. Notice that $f_1'v_1 + f_2'v_2 < (f_1 + f_2)v_2$, and so adverse selection is severe.

Construct a new equilibrium as follows. Let $p = (f_1' v_1 + f_2' v_2)/(f_1' + f_2') > v_1$ and consider an equilibrium such that $S_1 = S_2 = \{p\}$, $S_i = \{v_i\}$ for $i > 2$, $\theta(p) = 1$, $\theta(v_3) = (p - c_2)/(v_3 - c_2)$, and $\theta(v_{i+1}) = \theta(v_i)(v_i - c_i)/(v_{i+1} - c_i)$ for all $i > 2$. Since $V = 0$ in this new equilibrium, its existence is ensured by Corollary 1. Seller payoffs are $U_1' = p - c_1$, $U_2' = p - c_2$ and $U_i' = \theta(v_i)(v_i - c_i)$ for $i \geq 3$. Notice that $U_1' > U_1$. Also, given that $\lim_{\alpha \to 0} p = v_2$, there exits $\overline{\alpha} \in (0, 1)$ such that if $\alpha \in (0, \overline{\alpha})$, then $U_2' > U_2$ and $\theta(v_3)$ is higher in the new equilibrium. This, in turn, implies that $\theta(v_i)$ is higher in the new equilibrium for all $i \in \{3, \ldots, N\}$, implying $U_i' > U_i$ for $i \geq 3$. So, as long as α is sufficiently close to zero, the new equilibrium Pareto-dominates the original separating equilibrium. It also has higher equilibrium gains from trade, as the discounted probabilities of trade are higher for all types of the good. Clearly, the equilibrium construction is robust to changes in $(f_1, \ldots, f_N)$. □

B.6 Omitted Details from Section 7

We begin by providing the omitted details from the discussion of trading frictions as probability of exit. We then show that with within-period matching frictions it is still true that in the two-type case with severe adverse selection gains from trade in equilibria with $S_1^* \cap S_2^*$ a singleton are smaller than gains from trade in the most efficient separating equilibrium.

Trading Frictions as Probability of Exit. We first show if $j > i$, then $U_i \geq U_j$ with $U_i > U_j$ if $S_j^* \neq \emptyset$ and $S_j^* = \emptyset$ if $U_i = 0$. Fix $i, j \in \mathcal{I}$ with $j > i$. Then $U_j = \theta(p)(p - c_j)$ with $p \in S_j$. Since $U_i \geq \theta(p)(p - c_i)$ and $c_j > c_i$, we have that $U_i \geq \theta(p)(p - c_j) = U_j$, with strict inequality if $\theta(p) > 0$. So, $U_i > U_j$ if $S_j^* \neq \emptyset$ and $U_i = 0$ implies that $S_j^* = \emptyset$.

We now show for any equilibrium with S infinite there exists a payoff-equivalent equilibrium with S finite. Consider an equilibrium $E = \big(\mu, \sigma, \pi, (U_i)_{i \in \mathcal{I}}, V, (M_i)_{i \in \mathcal{I}}\big)$ in which S is infinite. Let $\widetilde{\mathcal{I}} = \{i \in \mathcal{I} : S_i \text{ is infinite}\}$. This set is nonempty by assumption. Since S_i^* is finite for all $i \in \mathcal{I}$, the set $S_i \setminus S_i^*$ is infinite for all $i \in \widetilde{\mathcal{I}}$, so that $U_i = 0$ for all $i \in \widetilde{\mathcal{I}}$. For each $i \in \widetilde{\mathcal{I}}$ consider a probability measure $\widetilde{\mu}_i$ such that $\widetilde{\mu}_i(\{p\}) = \mu_i(\{p\})$ for all $p \in S_i^*$ and $\widetilde{\mu}_i(\{p'\}) = 1 - \mu_i(S_i \setminus S_i^*)$ for some $p' > v_N$. Now consider the candidate equilibrium $\widetilde{E} = \big(\widetilde{\mu}, \sigma, \pi, (U_i)_{i\in\mathcal{I}}, V, (M_i)_{i\in\mathcal{I}}\big)$. Clearly, buyer and seller optimality hold in $\widetilde{E}$. Likewise, rationality of beliefs hold in $\widetilde{E}$ since the set of prices at which trade takes place is the same as in E. Given that buyer optimality implies that $\sigma(p') = 0$ for all $p' > v_N$, it then follows that $E_{\mu_i}[\sigma] = E_{\widetilde{\mu}_i}[\sigma]$ for all $i \in \widetilde{\mathcal{I}}$. Thus, stationarity also holds, so that $\widetilde{E}$ is an equilibrium. By construction, payoffs in E and $\widetilde{E}$ are the same.

Next, we show that for each $i \in \mathcal{I}$ one can construct a measure μ_i' with support in the finite set S satisfying (8). Let $S = \{p^1, ..., p^K\}$ and define $\alpha, \gamma \in \mathbb{R}^K$ to be such that

$$\alpha_k = \frac{\mu_i(\{p^k\})\big(1 - \delta + \delta\sigma(p^k)\big)}{1 - \delta + \delta\mathbb{E}_{\mu_i}[\sigma]}$$

and $\gamma_k = 1 - \delta' + \delta'\sigma'(p^k)$; note that $\sum_{i=1}^K \alpha_i = 1$. Now let $T : \mathbb{R}^K \to \mathbb{R}^K$ be such that

$$T(x) = \left(\frac{\alpha_1}{\gamma_1}\langle x, \gamma\rangle, \ldots, \frac{\alpha_K}{\gamma_K}\langle x, \gamma\rangle\right).$$

It is clear from the proof of Proposition 3 that the values of α and γ in T do not matter for the proof that T has a fixed point in the unit simplex. This establishes the desired result.

Finally, we show that gains from trade in equilibria with $V = 0$ equal average seller payoffs. Indeed, since $V = 0$ and (7) imply that

$$\sum_{i=1}^N f_i \sum_{p\in S^*} \mu_i(\{p\}) \frac{\sigma(p)}{1 - \delta + \delta\mathbb{E}_{\mu_i}[\sigma]}(v_i - p) = 0$$

and $(1 - \delta + \delta\sigma(p))U_i = \sigma(p)(p - c_i)$ for all $p \in S_i^*$, we then have that

$$\begin{aligned}
\sum_{i=1}^N f_i G_i &= \sum_{i=1}^N f_i \sum_{p\in S^*} \mu_i(\{p\}) \frac{\sigma(p)}{1 - \delta + \delta\mathbb{E}_{\mu_i}[\sigma]}(p - c_i) \\
&= \sum_{i=1}^N f_i \sum_{p\in S^*} \frac{\mu_i(\{p\})\big(1 - \delta + \delta\sigma(p)\big)}{1 - \delta + \delta\mathbb{E}_{\mu_i}[\sigma]} U_i \\
&= \sum_{i=1}^N f_i U_i.
\end{aligned}$$

Within-Period Matching Frictions. We show that gains from trade in any equilibrium in which $S_1^* \cap S_2^* = \{p'\}$ with $p' \in [c_2, v_2)$ are smaller than gains from trade in the most efficient separating equilibrium. The same argument of Section 4 shows that $S_1^* = \{v_1, p'\}$ and either $S_2^* = \{p'\}$ or $S_2^* = \{p', v_2\}$. Since the first case is a special case of the second when $p' = v_2$, we assume that $S_2^* = \{p', v_2\}$ in what follows. Type-1 seller optimality implies that

$$\theta(p', \alpha) = \theta(v_1, \alpha) \frac{(v_1 - c_1)}{p' - c_1}.$$

So, the payoff of type-2 sellers is

$$U_2 = \theta(v_1, \alpha) \frac{(v_1 - c_1)}{p' - c_1} (p' - c_2).$$

Note that U_2 is strictly increasing in p', and so U_2 is bounded above by $\overline{U}_2(\alpha, \delta)$, the highest payoff possible for type-2 sellers in a separating equilibrium. Now consider equilibrium gains from trade. The same argument as in the proof of Lemma 6 shows that

$$f_1 U_1 + f_2 U_2 = f_1 \left[\frac{(1 - \mu_1(\{p'\}))}{E_{\mu_1}[\sigma]} \theta(v_1, \alpha)(v_1 - c_1) + \frac{\mu_1(\{p'\})\sigma(p')}{E_{\mu_1}[\sigma]} \theta(p', \alpha)(v_1 - c_1) \right]$$
$$+ f_2 \left[\frac{\mu_2(\{p'\})\sigma(p')}{E_{\mu_2}[\sigma]} \theta(p', \alpha)(v_2 - c_2) + \frac{(1 - \mu_2(\{p'\}))\sigma(v_2)}{E_{\mu_2}[\sigma]} \theta(v_2, \alpha)(v_2 - c_2) \right].$$

Given that the map $\sigma \to \alpha\sigma^2(1 - \delta + \delta\alpha\sigma)^{-1}$ is strictly convex, Jensen's inequality then implies that $f_1 U_1 + f_2 U_2 > G$. Thus, $W < f_1 U_1 + f_2 \overline{U}_2(\alpha, \delta) = \overline{G}(\alpha, \delta)$.