Searching Online and Product Returns*

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Abstract

The steady growth of e-commerce has led to a surge in products being returned after purchase. We extend the seminal consumer search paper by Wolinsky (1986) and analyze product returns as resulting from a trade-off between the social waste of returns and the search efficiency arising from the relative ease to assess one's value for a good after purchase compared to brick-and-mortar shopping. The model gives clear predictions when consumers inspect before or after purchase in terms of the social costs and benefits of product returns. We compare these market outcomes with what is socially optimal and find intuitive regulatory prescriptions for improving market efficiency and demonstrate how other natural, but ill-formed policies can harm consumers.

Keywords: product returns; consumer search; search efficiencies, product matches **JEL codes:** D40, D83, L10

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1 Introduction

E-commerce has come with a sharp increase in products being returned after purchase. The National Retail Federation estimates that across different retail channels \$428 billion of merchandise value is returned in 2020, which is around 10% of total retail sales. Focusing on online retail only, these numbers are approximately \$102 billion and 18% of online sales. They also mention that "online returns more than doubled and are a major driver of the overall growth of returns".¹ Given this development, retail firms start to treat product returns strategically by considering optimal return policies. Moreover, product returns are a social concern in that they potentially point to a large social cost of e-commerce.

An important source for higher return rates in online markets, we argue, is that online consumer search differs from that in traditional markets along two dimensions. First, search for objective information is much easier online. With only a few clicks, consumers can get access to prices and other objective product features. However, and second, it is certainly not that easy to learn subjective features of product match online: How will a certain pair of glasses fit, or, how easy will is it to operate a digital camera? To overcome this latter aspect of online search, firms may choose lenient return policies providing incentives for consumers to buy the product without spending effort *before* purchase to determine whether the product fits their needs. Instead, consumers may check product fit in a more comfortable home environment and return the product if they learn the fit is not good enough.

Product returns are also an important topic for regulatory agencies and policies towards returns differ across the world. In the USA, there is no general regulation in place, but some individual states mandate that retailers give refunds on all products that are returned within 20 or 30 days of purchase if they themselves do not provide a clearly stated return & refund policy.² In the European Union, refunds on online purchases are mandatory.³ Even though full refunds seem to be the norm (either because they are mandated or because of a

¹See, https://9d4f6e00179f3c3b57f1-4eec5353d4ae74185076baef01cb1fa1.ssl.cf5. rackcdn.com/Customer%20Returns%20in%20the%20Retail%20Industry%20-%20short%20pdf. pdf. This tendency is confirmed by other sources, see e.g., https://www.invespcro. com/blog/ecommerce-product-return-rate-statistics/ and https://www.nosto.com/ ecommerce-statistics/return-rate/.

²See, e.g., https://www.findlaw.com/consumer/consumer-transactions/customer-returns-and-refund-laws-by-state.html

³"If you bought a product or a service online or outside of a shop..., you have the right to cancel and return your order within 14 days, for any reason and without a justification" See, https://europa.eu/youreurope/citizens/consumers/shopping/guarantees-returns.

firm's voluntary, strategic choice) small print may make all the difference as refund policies differ according to (i) who should pay for the return cost and (ii) whether firms charge a so-called restocking fee (which can be up to 20% of the purchase price) and these aspects are typically not regulated.

This leads us to the following questions. In the absence of regulations, is it optimal for firms to stimulate product returns? If so, do consumers or firms benefit from product returns? What are the typical features of product markets where products are returned? Are product returns to be avoided as they are socially wasteful or do they, on the contrary, provide an efficient solution to the trade-off between search cost reduction and cost increases due to returns? Are regulations redundant if firms voluntarily offer product returns?

To answer these questions, we build on the seminal consumer search paper by Wolinsky (1986). Our methodological contribution consists of extending it in a number of fundamental ways. First, we divide the search cost into two parts: a very small cost of learning price and a much larger inspection cost to learn the personal match value. Second, we allow consumers to purchase a product without inspecting its match value before purchase. If they do so, they can inspect the product after purchase at a lower than before-purchase inspection cost or not inspect at all. Third, firms choose two prices: the price at which consumers purchase the product and the refund consumers get when they return the product, i.e., firms think strategically about their return policy. Fourth, if consumers return the product, it has a salvage value for the firm that is typically smaller than the production cost. Thus, there are two important aspects of offering product returns: (i) the difference between production cost and salvage value represents the (social) cost of product returns, while (ii) the difference in search costs before and after purchase represents a potential (social) benefit.⁴

We have several sets of results. In the absence of regulation, we first characterize candidate equilibria where firms offer return policies and consumers inspect the product *after* purchase. In all these equilibria, firms offer a refund that is larger than the salvage value and in this sense stimulate returns. Importantly, firms can still make positive gains on products that are returned, namely when the difference between the price and refund they choose is larger than the difference between the production cost and the salvage value.

⁴There is one important real-world aspect regarding product returns that we abstract away from, namely fraud. Especially in online transactions, fraud may come from both sides of the market: firms may ship broken or otherwise non-functional products, while consumers may buy products for a certain occasion and then return them. Our model deals with firms that care about their reputation and who keep track of -and banconsumers who seem to be engaged in fraudulent behavior.

Moreover, the price and return policy are always such that consumers are indifferent between inspecting before and after purchase. Thus, even if consumers only inspect after purchase, the threat of inspecting before is important in characterizing the equilibrium.

This characterization of the strategic price and refund policies, leads to the following comparative static results. First, the social cost of returns is completely absorbed by the firms in the sense that if the production cost increases relative to the salvage value, firms stick to their strategy choices. This is because in a refund equilibrium the production cost acts as a fixed cost, and on the margin, firms' strategy choices only affect whether consumers who bought the product will keep it. Thus, firms' optimal strategy choices depend on the salvage value, but not on the production cost. Second, in equilibria with returns social welfare is higher in markets where it is relatively easier for consumers to inspect after purchase. Interestingly, depending on the match value distribution, it may well be that only firms benefit from this increase in search efficiency, while consumers become worse off. An important reason for this is that, in a refund equilibrium, the option to inspect before purchase acts as a binding outside option and this option does not become more attractive if inspecting after purchase is easier. Finally, if the cost of inspecting before purchase is relatively large, while the cost of inspecting after is negligible, then the refund equilibrium is characterized by firms extracting (almost) all surplus. This result follows from the fact that firms can exploit consumers having a very unattractive outside option by offering a return policy that is only used by consumers with a very low match value, while setting a price that is equal to the average match value.

Next, we analyze when different types of equilibria exist. It is clear that if the inspection cost is too large, only no trade equilibria exist. More importantly, we show that for smaller inspection costs, a unique equilibrium exists and that, depending on the parameter values, it is such that either (*i*) all firms offer a refund resulting in consumers inspecting after purchase, or (*ii*) all firms effectively incentivize consumers to inspect before purchase (which results in the Wolinsky equilibrium), or (*iii*) some firms offer an effective return policy, while others do not. Refund equilibria exist when there are large efficiency gains of inspecting at home, and the social waste of returns (measured as the difference between production cost and salvage value) is relatively small. The Wolinsky equilibrium exists in the opposite case where the efficiency gains of inspecting at home are small, while the social waste of returns is large. In between these two parameter regions, the equilibrium has some firms

offering a return policy, whereas others do not. This equilibrium characterization partially explains why one observes more product returns in some industries than in others. The National Retail Federation reports that retail categories such as apparel and footwear have a relatively high product return rate, whereas the product return rate for other categories such as beauty and health care is relatively low.⁵ For the first two categories product match is important and large efficiencies can be gained by inspecting at home, while the salvage value is reasonably close to the production cost. For the second set of categories the salvage value is almost zero and return policies are therefore unattractive.

Finally, we have a set of welfare results. We first consider that a social planner can choose the price and refund policy firms offer. Here, we have two results. First, it is optimal for the social planner to either choose to have all or no firms offering refund contracts. All firms offering refunds is socially optimal if, and only if, inspecting after purchase comes at relatively low cost and the difference between production cost and salvage value is relatively small. Broadly speaking, refunds are socially efficient when the market outcome is such that firms offer refunds. Second, and more surprisingly, in a market equilibrium firms never offer the optimal refund policy. The socially optimal outcome is such that the difference between production cost and salvage value, i.e., it is socially optimal to let consumers pay for transportation cost. The equilibrium refund policy that firms choose is generally too low from a social planner's perspective.

We also consider the more practical, regulatory issue of whether refunds should be mandated by law, i.e., we consider that a regulator can mandate a minimal threshold percentage of the sales price that firms should offer as a refund, while leaving the pricing decision to the discretion of firms. Clearly, if it is socially optimal not to have refunds, then such a policy is not optimal. When we restrict attention to the more interesting case where the market offers refunds that are too small from a social welfare perspective, we have two sets of results. First, the threshold harms consumers and benefits firms, while leaving social welfare unaffected, if it is chosen just marginally above the market equilibrium outcome. The reason is that firms react to the regulation by increasing prices. Second, if the regulation is more bold and the threshold is substantially above the refund percentage that the market would choose, then the regulator can achieve the first-best.

The above results indicate that inspection after purchase and product returns are key

⁵+See, https://9d4f6e00179f3c3b57f1-4eec5353d4ae74185076baef01cb1fa1.ssl.cf5. rackcdn.com/Customer%20Returns%20in%20the%20Retail.

elements to understand the functioning of online markets. The results explain that product returns are an integral part of online markets and that despite the appearance of inefficiency, they can be efficient in reducing search/inspection cost and improving match values. However, the firms' incentives of providing product returns are not aligned with the socially optimal trade-off and depending on the market conditions, there can be too many or too few returns.

Our paper is related to several branches of literature. First, a recent branch in the consumer search literature builds on the Wolinsky (1986) equilibrium, but allows for consumers to direct their search based on the prices firms charge (see, e.g., Armstrong (2017), Choi et al. (2018), Haan et al. (2018), among others). This literature is inspired by the observation that in online markets price information is more easy to acquire than other types of information. These papers stick, however, to the standard consumer search set-up that consumers cannot buy without inspecting before purchase. Doval (2018) is the first paper in this literature introducing the option of buying without inspection, but she only considers the optimal consumer search problem and studies how the optimal stopping rule differs from the classic Weitzman (1979) rule, while Chen et al. (2021) introduces this option in the Choi et al. (2018) model.⁶ None of these papers studies product returns, however, or the question whether the market stimulates firms to offer lenient return policies leading to product returns. Petrikaitė (2018) introduces the option of product returns in a model where consumers learn one component of their match value after purchase, but the key issues of this paper, namely whether firms stimulate product returns and whether this enhances market efficiency, are not addressed in this paper as firms cannot choose their return policy and the salvage value is assumed to be equal to the production cost.

Outside of the consumer search literature there is a literature solving the "excess refund puzzle." The unifying thread throughout this literature is that consumers can only learn how much they like a product after purchasing it. This research typically addresses returns in a monopoly context where consumers do not have an outside option and learn the product's match value after purchase at no cost. In Che (1996) appears to be the first paper to consider customer return policies, but firms can only choose whether or not to allow for product returns and if they do so, they should give a full refund. Matthews and Persico (2007) allow consumers the option to become informed before purchase at a cost and, similar to our paper,

⁶Fishman and Lubensky (2016) also introduces the option of purchasing without inspection in a Wolinskytype model where the first search is free, but like Chen et al. (2021) they also do not study product returns.

also find that firms may offer a refund that exceeds the salvage value. They do not study a market context, however, and the trade-off between reducing consumers' inspection/search cost and the inefficiency of product returns is not studied. Moreover, an important element in our welfare analysis is that firms benefit from product returns as consumers who return their purchase continue searching another firm, while they pay a price that is higher than the refund they get. This aspect of product returns is absent in a monopoly market.⁷

The rest of the paper is organized as follows. The next section introduces the model. Section 3 focuses on market equilibria where firms offer a refund that incentivizes consumers to inspect the product only after purchase. Section 4 investigates what type of market outcomes can be sustained as equilibria and when these different types of equilibria exist, while Section 5 investigates the welfare properties of these equilibria. Section 6 concludes with a discussion, while proofs withheld from the text are given in the Appendix.

2 Model and Preliminary Results

The market is comprised of a unit mass of consumers with a unit demand for a product and a unit mass of firms who supply the product. Following the consumer search literature based on Wolinsky (1986), a consumer's match value with a firm is drawn from a distribution G with support $[\underline{v}, \overline{v}] \subset \mathbb{R}$, density g, with both G and 1 - G logconcave. Match values are independent across firms and consumers. At the start of the game, firms simultaneously set not only their price $p \ge 0$, but also their **refund** $\tau \ge 0$, which is the amount of money given back to a consumer who purchases the product and then decides to return it. We often refer to the pair (p, τ) as the firm's "contract." Firms face a constant marginal cost of production $c \ge 0$ and a **salvage value** $\eta \in [0, c]$ for items that have been purchased and subsequently returned. The difference $c - \eta$ captures the cost of product returns.

Consumers are uncertain of their match values as well as firms' contracts, but they can learn these through costly search. A consumer starts out by incurring a small but positive search cost $\epsilon > 0$ to visit a firm to learn the price it charges and the return policy it offers. The consumer then has three options: He can incur an **inspection cost** s > 0 to learn his

⁷Other papers in this literature include Inderst and Ottaviani (2013) and Hinnosaar and Kawai (2020). Inderst and Tirosh (2015) show that committing to a refund policy can help a seller provide useful advice to a consumer. Hinnosaar and Kawai (2020) explore robust pricing with refunds, where consumers' value either match or do not match with the firm's product.

match value with the firm, he can buy the product without inspecting it, or he can decide to leave and go visit another firm (or leave the market altogether). If the consumer buys the product without first inspecting it, he has the option to incur an inspection cost of βs , with $\beta \in [0, 1]$, to learn his match value after purchase. If the consumer learns the match value after purchase, he can decide whether to keep the product or to return it to the firm receiving τ and then possibly continue his search at another firm. The term $(1 - \beta)s$ measures the search efficiency of inspecting after purchase and captures the reduction in inspection cost if the consumer inspects after purchase relative to inspecting before purchase. Let $\Omega \subset \mathbb{R}^5_+$ denote the set of parameter values $\omega = (c, \beta, \eta, s, \epsilon)$.

Throughout the paper we focus on Perfect Bayesian Equilibria where firms choose their strategies to maximize expected profits given their information and consumers choose an optimal sequential search strategy. Consumers hold passive beliefs, meaning that if they observe a firm's deviation from equilibrium behavior, they continue to believe that other firms stick to their equilibrium decisions.

2.1 Some comments on the Interpretation of the Model

We now discuss different aspects of our model. First, the difference in inspection costs before and after purchase can be interpreted in different ways. For example, one may interpret *s* as the cost of going to a brick-and-mortar store and βs as the cost of inspecting at home after having purchased the product online. Arguably, the inspection cost at home is (much) lower, but may still be positive given that a product typically has to be returned within a certain narrow time period. Depending on the product, β ranges between 0 and 1. Another interpretation is that consumers can inspect all properties of the product online, but it is still more cumbersome to do so than after it has been shipped home.

Second, an implication of the assumption that consumers incur an arbitrarily small but positive search cost to learn a firm's price and return policy is that they cannot direct their search to firms with lower prices and/or a more favorable return policy. We know from the search literature that small search costs may have very different implications from search costs being equal to zero and we think this assumption is appropriate in many markets, and even applies to many online markets.

Third, the model does not explicitly define transportation costs, which are clearly important for online purchases. It also does not explicitly address who pays for transportation costs. These aspects are, however, easily interpreted in our model as follows. First, the transportation cost itself is another reason why the salvage value η may be smaller than the production cost c. Second, the difference between p and τ is a measure how these transportation costs are divided between firms and consumers. If, for example, $p = \tau$, then firms pay for the transportation cost, while if $p - \tau = c - \eta$, then consumers pay. As firms choose p and τ , they implicitly determine who pays for the transportation costs.

Fourth, the model is flexible enough to also cover environments where consumers learn the match value after purchase at no cost and only incur a cost when returning the product. If we denote this cost of returning the product by γ , then our model is isomorphic to this new model, by letting $\beta = 0$ and defining an adjusted refund as $\hat{\tau} = \tau - \gamma$ and having the salvage value be equal to $\eta - \gamma$.

2.2 Consumers

We now present some preliminary findings starting with the consumer side. Consider the consumer's problem at the moment he has incurred the search cost ϵ to visit a firm and learned its price and refund (p, τ) . Denote the consumer's outside option by a. The outside option is endogeneous to the model and depends on the equilibrium contracts offered by other firms. The consumer has four possible paths of play. (*i*) If he decides to leave the current firm, he gets a. (*ii*) He may decide to buy the product and neither inspect it before nor after purchasing it, yielding the payoff v - p. (*iii*) If he decides to inspect the product before purchase, his payoff is v - p - s if the net value exceeds the outside option $v - p \ge a$ and otherwise the payoff is a - s. (*iv*) If he buys the product and inspects afterwards, his payoff is $v - p - \beta s$ if his match value exceeds the payoff from returning the product $v \ge a + \tau$, and otherwise his payoff is $a + \tau - p - \beta s$. Thus, the expected utility from each of these options is given by the following expressions.⁸

$$\begin{cases} \text{never inspect} & U_N = E(v) - p \\ \text{inspect after} & U_A = E(\max\{v, \tau + a\}) - p - \beta s \\ \text{inspect before} & U_B = E(\max\{v - p, a\}) - s \\ \text{leave} & U_L = a \end{cases}$$
(1)

⁸The expressions for utility in (1) are functions of the price, refund, outside option, and model primitives: $U_i(p, \tau, a, \omega)$ for $i \in \{N, A, B, L\}$. Throughout the paper, we suppress these arguments when convenient.



Figure 1: Inspection choices for a given price p and return policy τ .

When presented a price and return policy, the consumer selects the inspection option yielding the highest expected payoff. Figure 1 illustrates how the optimal choice depends on the price and return policy of the firm that is visited. When the return policy is unfavorable and the price is moderate, consumers adopt the usual search strategy of inspecting before purchase. When the return policy is unfavorable and the observed price is too high, they leave the firm without inspecting the product, while if the product is sufficiently cheap they buy without ever intending to inspect the good. When the return policy is more favorable, the consumer will opt to first buy the product and then inspect it, returning it if it turns out that his match value is low. As we shall discuss, the sizes of the different regions depend on the underlying search parameters *s* and β as well as on the outside option *a*.

Importantly, the figure shows that that there exists a region "Inspect Before" where firms offer a positive refund that is never used by consumers as it is too low. Thus, offering such a contract results in the same market outcome as not offering a refund. In the rest of the paper, we refer to policies where firms offer a refund as those that induce consumers to inspect after purchase and there is a positive probability that the consumer returns the product. For the region "Inspect Before" we say that no refund is given.

To characterize consumer behavior, it is useful to let $S \equiv E(v - \underline{v})$ and introduce the *reservation price* $r : [0, S) \rightarrow [\underline{v}, \overline{v}]$, implicitly defined by

$$E(\max\{v - r(x), 0\}) = x.$$
 (2)

Intuitively, r(x) is the price at which consumers are indifferent between incurring the

inspection cost *x* and taking the outside option of zero. Letting $S' = E(\bar{v} - v)$, we follow Doval (2018) and introduce the *backup price* $b : [0, S') \rightarrow [\underline{v}, \overline{v}]$, implicitly defined by

$$E(\max\{b(x) - v, 0\}) = x.$$
 (3)

Similarly, b(x) corresponds to the price making consumers indifferent between incurring the cost x to inspect a firm's product and buying it without inspection when the outside option is zero. Throughout the paper, we maintain that the sum of the search and inspection costs is less than the unique value s^* equating the reservation and backup prices $r(s^*) = b(s^*)$, *i.e.*, $0 < s + \epsilon < s^*$.

2.3 Firms

Given the inspection strategy adopted by consumers, we now turn to the firms' problem. Denote the probability a consumer continues to search after visiting another, randomly drawn, firm by q. Then, if the consumer's outside option is a in each round of search, a firm's expected profit from offering a particular price and refund is determined by the consumers' inspection decisions as follows.

$$\begin{cases} \text{never inspect} & \pi_N = \frac{p-c}{1-q} \\ \text{inspect after} & \pi_A = \frac{p-c+(\eta-\tau)G(a+\tau)}{1-q} \\ \text{inspect before} & \pi_B = \frac{(p-c)(1-G(a+p))}{1-q} \\ \text{leave} & \pi_L = 0 \end{cases}$$
(4)

If the consumer never inspects, the firm's profit is simply equal to p - c over all consumers that visit him. If consumers inspect afterwards, the firm has to give the refund τ back to all consumers who return the product in exchange for the salvage value η . From the consumer's problem it is clear that a fraction $G(a + \tau)$ returns the product. Finally, if consumers inspect before purchase, the firm's profit is simply the Wolinsky profit.

The terms 1-q in the different profit expressions may appear to be relatively unimportant scaling factors. That appearance is false, however, as we shall show in the next sections. The probability q endogenously depends on contracts that firms offer and thereby on the

⁹Appendix D.1 verifies the existence of a unique $0 < s^* < \min\{S, S'\}$ equating $r(s^*) = b(s^*)$. Thus, our analysis is restricted to the set $\Omega = \{(c, \beta, \eta, s, \epsilon) \in \mathbb{R}^5_+ : 0 \le \beta \le 1, 0 \le \eta \le c, 0 < s + \epsilon < s^*\}$

parameters in Ω . In particular, some of the comparative statics effects on profits and on welfare arise, because the market generates more product returns. For example, firms may make more profits overall despite the fact that on each direct sale they generate less profits, simply because they also make money over products that are returned as $p > \tau$.

To ensure the firm's best response is well-defined, assume that whenever the consumer is indifferent between multiple inspection options, he takes the one yielding the highest profit. To find possible best replies, we should examine all points of discontinuity whereby $U_j(p,\tau) = U_k(p,\tau)$ for $j, k \in \{N, A, B, L\}$ and $k \neq j$ and also consider interior optima, where the consumer strictly prefers an inspection option j and $\nabla \pi_j = 0$. In principle, this gives 6 possible classes of best responses at boundary regions where consumers are indifferent between at least two options and 4 possible classes of interior solutions. It is clear, however, that there cannot be an equilibrium where some consumers leave the firm upon learning its price and refund policy.

3 Refund Equilibria

In this section, we characterize properties of candidate equilibria where firms offer a refund such that consumers inspect the product after purchase and claim the refund if their match value is relatively small. We use these results in the next section to show when refund equilibria exist and in Section 5 to characterize the efficiency of market outcomes.

Allowing purchased items to be returned leads to a simple trade-off. Consumers benefit from waiting to inspect a product until after buying it, when it is easier to do so, and returning it when dissatisfied (i.e. when $\beta < 1$). Firms absorb a loss when returned products lose value (i.e. when the salvage value is less than the production $\cos \eta < c$). In this section, we detail how this trade-off determines the return policies that are offered in the market place.

We refer to symmetric candidate refund equilibria as candidate equilibria where all firms offer the same contract (p, τ) , firms do not have an incentive to offer a different refund policy (that induces consumers to inspect after purchase), consumers' most preferred inspection option is to buy the good and inspect after purchase, and the outside option is $a = U_A - \epsilon$. Any refund equilibrium has to satisfy these conditions. These candidate equilibria are not necessarily equilibria of the game, however, as in this section, we do not check whether firms prefer to offer contracts that induce different behavior on the part of consumers. Our first result characterizes refund equilibria and shows that they are unique if they exist.

Theorem 1. If a symmetric refund equilibrium exists, it is unique and has the following properties: (i) the price and refund are set so that consumers are indifferent between inspecting products before and after purchasing them, i.e. $U_A(p, \tau, a) = U_B(p, a)$, and (ii) the refund exceeds the salvage value, i.e., $\tau > \eta$.

The first property is easily understood. In symmetric refund equilibria, a consumer buys the product before inspecting its match value. A more generous refund policy makes consumers not only return the product more often, it also transfers more money to consumers who make a return, reducing profit. Thus, a firm would not want to offer a more generous refund than is strictly necessary for consumers to be willing to inspect afterwards. As Figure 1 illustrates, this means that equilibrium contracts must lie along one of the boundary regions equating $U_A = U_L$, $U_A = U_B$, or $U_A = U_N$. Reasoning along the lines of Diamond (1971), as consumers do not yet know their match value before buying, there cannot be an active market with consumers being indifferent with leaving. Likewise, an argument akin to Diamond's rules out an active market with consumers being indifferent between inspecting after purchase and not inspecting at all (see Lemma B.5). Thus, the only symmetric refund equilibrium is one in which consumers are indifferent between inspecting before and after making a purchase and also strictly prefer these to the other options.

The second property importantly shows that firms incentivize consumers to return products by offering a refund that is higher than their salvage value of returned products. This can be understood by examining a firm's optimal choice of refund, letting price be a function $p(\tau)$ such that the consumer participation constraint $U_A = U_B$ binds. Along $U_A = U_B$, an increase in the refund leads to an increase in price, with a rate of change $\frac{dp(\tau)}{d\tau} = G(a + \tau)/G(a + p)$. Differentiating profit yields

$$\frac{d\pi_A}{d\tau} \propto \frac{G(a+\tau)}{G(a+p)} - G(a+\tau) - (\tau - \eta)g(a+\tau).$$
(5)

The first two terms capture the net change in consumer expenditures, holding the frequency with which returns are made fixed. The last term captures the effect of changing the frequency of returns. Moving up along the curve $p(\tau)$ leads to a decline in consumer utility as the price goes up and U_B is decreasing in price. By the Envelope Theorem, the reduction in utility equals to the change in their net expenditures.¹⁰ Plugging this into (5), yields

$$\frac{d\pi_A}{d\tau} \propto \frac{dU_A}{d\tau} - (\tau - \eta)g(a + \tau).$$
(6)

Thus, at an optimum, increasing the frequency of returns must be costly, i.e. the refund must exceed the salvage value $\tau - \eta = -\frac{dU_A}{d\tau}/g(a+\tau) > 0$.

Another result supplied by the theorem, and one that proves useful later, is that symmetric refund equilibria are unique. The argument supporting this result is instructive. In equilibrium, consumer surplus is equal to the expected utility from inspecting after purchasing a good net the search cost $a = U_A - \epsilon$. As consumers are made indifferent with inspecting before purchase, surplus also satisfies $a = U_B - \epsilon$. From (1), these equalities are equivalent to $E(\max\{v - \tau - a, 0\}) = p - \tau + \beta s + \epsilon$ and $E(\max\{v - p - a, 0\}) = s + \epsilon$. By the definition of the reservation price (2), the expressions simplify to

$$a + \tau = r(p - \tau + \beta s + \epsilon) \text{ and } a + p = r(s + \epsilon)$$
 (7)

Taken together, these conditions implicitly define the equilibrium difference between the price and refund by $\delta = p - \tau$ by

$$r(\delta + \beta s + \epsilon) + \delta = r(s + \epsilon). \tag{8}$$

The appendix verifies that a unique δ solves this equation. Substituting δ along with both conditions in (7) into (5), a unique refund τ is seen to solve the firm's first order conditions. Solving for the equilibrium price and consumer surplus is then immediate.

We next present three propositions characterizing important comparative statics results. First, we consider changes in the firm's production technology and the return technology, measured by changes in c and η .

Proposition 1. In market equilibria with returns: (i) the price p and refund τ are unaffected by the production cost c so that profit is decreasing in c, consumers gain the same surplus, and total welfare is smaller, and (ii) a higher salvage value η leads to higher profits, less consumers surplus, and larger total welfare.

That a firm fully absorbs increases in the production cost is a consequence of the fact ¹⁰This is verified by differentiating $U_A(p(\tau), \tau) = \int_{a+\tau}^{\overline{v}} (1 - G(v)) dv + \tau - p(\tau) - \beta s + a \text{ in } \tau$. that the production cost enters additively into the firm's profit function, while the consumers indifference condition is unaffected. This is not the case for the salvage value η , as this value is only of relevance for returned products and its impact therefore depends also on the firms' choice of τ . A larger salvage value leads to an increase in both the price and refund, while leaving their difference unchanged. Such a shift is harmful for consumers and beneficial to firms for the simple reason that the price is paid for each item sold, while the refund is only given back to the fraction who do not like the product. Combining the two effects, an increase in the salvage value improves welfare.

Next, we consider how market outcomes are affected by changes in the search technology affecting the ease of inspection after purchase as measured by the parameter β .¹¹

Proposition 2. In markets with returns a smaller β leads to: (i) higher total welfare, (ii) larger difference between price and refund $p - \tau$, while the refund is smaller, and (iii) higher consumer surplus if and only if the price is lower.

It will not come as a surprise that overall welfare improves when post-purchase inspection is relatively less costly. After all, in a refund equilibrium, it is this inspection cost that determines consumer behaviour. What is probably more surprising is how this increase in total welfare is distributed between the two sides of the market. In equilibrium, a firm is kept from raising its price or lowering its refund by the threat of a consumer inspecting its product before buying it, resulting in a discontinuous drop in profit. When β is smaller, the threat to inspect before purchase is less attractive so that firms can introduce a larger wedge between the price and refund and also reduce the refund itself. If this implies that the price increases, consumers are worse off.

To compute the effects, differentiating (8) yields $\frac{d\delta}{d\beta} = -s/G(a+\tau) < 0$. Using this and (7), reducing β is seen to reduce the frequency of returns since $\frac{d(a+\tau)}{d\beta} = s/G(a+\tau) > 0$. From the firm's first order conditions and the logconcavity of *G*, the change in the refund is

$$\frac{d\tau}{d\beta} = \frac{1 - G(a + p)}{G(a + p)} \cdot \frac{\partial}{\partial \tau} \left(\frac{G(a + \tau)}{g(a + \tau)} \right) \cdot \frac{s}{G(a + \tau)} > 0.$$

Turning to consumer surplus, because the right side of the second condition in (7) is constant in β , whatever the effect that reducing β has on the price, the effect on consumer

¹¹The comparative statics with respect to *s* are more difficult to determine for general values of β , but for $\beta = 0$ they are exactly the opposite to the ones reported here for β .

surplus is the opposite. Using the identity $p = \delta + \tau$, it follows that consumers benefit from a marginal reduction in the post-purchase inspection cost, if and only if,

$$\frac{1-G(a+p)}{G(a+p)}\frac{\partial}{\partial\tau}\left(\frac{G(a+\tau)}{g(a+\tau)}\right)>1.$$

Whether this condition holds depends on the properties of the match value distribution. That consumer harm is a real possibility (and that the above condition fails to hold) follows when at the end of this section we consider that the match values are uniformly distributed between zero and one and show in that case that $\frac{da}{d\beta} > 0$. Figure 2 confirms this finding for the uniform distribution by displaying how consumer surplus and profits are affected by a change in β . One can see that changes in β have quantitatively significant effects. One can also easily verify from the figure that total surplus is decreasing in β .



Figure 2: Consumers Surplus *a* and profit π_A as functions of the relative inspection cost β in a refund equilibrium. Values are uniformly distributed with s = 0.07 and $\epsilon = 0.001$

Another way to consider the implications of the difference in the cost of inspecting before and after purchase is to explore what the equilibrium contracts look like if the relative ease of inspecting after purchase is large, i.e., if β is close to 0 and s is relatively large, but not too large so that consumers are still active in the market. This case is interesting as it nicely illustrates the importance of the consumers' "outside option" of threatening to continue searching before purchase in shaping equilibrium outcomes. When s is relatively large, this threat is almost non-existent as it is too costly for consumers to inspect before. Thus, firms could abuse this situation by offering almost no refund, while keeping a high price and consumers will still only inspect after purchase.



Figure 3: Price p and refund τ , consumer surplus a and profit π_A as functions of s in a refund equilibrium. Values are uniformly distributed and β , c, η , $\epsilon \rightarrow 0$.

The next result formalizes this insight by showing that in a candidate refund equilibrium, firms can expropriate almost all ex-ante surplus by setting prices close to E(v), while offering almost no refund.

Proposition 3. Consider $\beta = \eta = \underline{v} = 0$. When $s \to s^*$ and $\epsilon \to 0$, we have that: (i) consumer surplus vanishes, i.e., $a \to 0$, (ii) the refund vanishes, i.e., $\tau \to 0$, and (iii) firms capture the ex ante surplus, i.e., $p \to E(v)$.

What is interesting about this result is that in the limit refund equilibria have firms not offering refunds. This is in sharp contrast to what we will show in the next section, namely that equilibria where the market is active and consumers never inspect cannot exist. In a refund equilibrium for any $s < s^*$ firms do offer positive refunds, but as it becomes increasingly unattractive for consumers to inspect before purchase as *s* increases, consumers do not have a credible alternative. Firms can exploit that fact by offering contracts where consumers will only return the product if their match value is very low.

Note that the proposition does not state that an equilibrium exists (as existence will be explored more generally in the next section). However, certainly for the uniform distribution a unique refund equilibrium exists in a left neighborhood of s^* and if c is small enough as can be concluded from the analysis at the end of this section.

Note also that even though the equilibrium generates almost full *ex-ante* surplus, it is not the case that the first-best optimum is achieved. In particular, if β is small, it is socially efficient for consumers to inspect the products and return them even if their match value is not very low. In Section 5 we consider the welfare properties of the different equilibria in a more general context and show that if it is socially optimal to have returns,

it is generally optimal to have a more generous refund policy than what the firms would provide in equilibrium. Figure 3 confirms this for the uniform distribution: it shows that if starting from s^* one decreases the inspection cost s, both firms' profits and consumer surplus are increasing. The main reason is displayed in the Left part of the Figure: firms start offering a much more generous refund policy so that many more consumers return their purchase. As firm still make a profit of $p - \tau$ on returned items, they benefit overall if more consumers return their products. Moreover, consumers benefit as their match values tend to be much higher. Thus, this analysis shows the importance of the probability q in deriving the effects of a decrease in s on firms' profits and overall welfare.

Example (Uniformly Distributed Values). To conclude the section, we explicitly solve for the equilibrium expressions of the candidate refund equilibrium when the match value follows the uniform distribution on [0, 1]. From Theorem 1 it follows that $\int_{a+\tau}^{a+p} v dv = (1-\beta)s$ and $\tau - \eta = \frac{(1-(a+p))(a+\tau)}{a+p}$. Thus, a refund equilibrium is described by these two equations along with the condition that $a = r(p - \tau + \beta s + \epsilon) - \tau$. Letting ϵ tend to zero, these equations can be easily solved for a, p and τ as¹²

$$a = \frac{2\left(1 - \sqrt{8s}\right)}{2 - \sqrt{8s}}\sqrt{1 - \sqrt{8s} + 2\beta s} - \eta$$
$$p = \frac{2 - \sqrt{8s}}{2} - \frac{2\left(1 - \sqrt{8s}\right)}{2 - \sqrt{8s}}\sqrt{1 - \sqrt{8s} + 2\beta s} + \eta$$

and

$$\tau = \frac{\sqrt{8s}}{2 - \sqrt{8s}}\sqrt{1 - \sqrt{8s} + 2\beta s} + \eta$$

These equations are also used to compute Figures 2 and 3. It is straightforward to confirm that the refund equilibrium indeed exists for the constellation that is considered in Proposition 3 (see Online Appendix D.2).

4 The Structure of Equilibrium Contracts

In this section, we characterize what type of equilibria exist and for which parameter values they exist. To this end, let us define for each value a, $\mathcal{R}(a) \equiv \{(p, \tau) \in \mathbb{R}^2_+ :$

¹²The expressions for $\epsilon > 0$ are somewhat more involved and are available upon request.

 $U_A(p, \tau, a) = U_B(p, a) \ge U_N(p), U_L(a)$ to be the set of contracts making consumers indifferent between inspecting before and after and preferring these to the other inspection options. Similarly, define $\mathcal{W}(a) = \{(p, \tau) \in \mathbb{R}^2_+ : U_B(p, a) \ge U_A(p, \tau, a), U_N(p), U_L(a)\}$ as the set of contracts where consumers inspect before purchase and that they prefer this to other inspection options. For each τ , let $p(\tau)$ denote the price in the pair $(p(\tau), \tau) \in \mathcal{R}(a)$. The lowest $\underline{\tau}$ and highest $\overline{\tau}$ values the refund takes in the set satisfy $p(\underline{\tau}) = (b(s) - a)_+$ and $p(\overline{\tau}) = (r(s) - a)_+$.¹³ Throughout this section we maintain the following assumption.

Assumption. $\pi_A(p(\tau), \tau, a)$ is quasiconcave in τ when $\eta < \tau < \overline{\tau}$ and $0 \le a$.¹⁴

Lemma B.1 shows that under this assumption *if* for a given *a* firms find it optimal to offer a refund inducing consumers to inspect after purchasing the product, there is always a unique way to do so. This goes beyond Theorem 1 as the latter only shows that if a *symmetric* refund equilibrium exists it is unique, but it does not guarantee that individual firms would not find it profitable to offer a different refund contract. We denote the *refund contract* by $(p_R(a), \tau_R(a))$ and the profit it yields by $\Pi_R(a)$. Define a *candidate refund equilibrium* to be the unique triple $(p_R(a_R), \tau_R(a_R), a_R)$ for which consumer surplus satisfies $a_R = U_A(p_R(a_R), \tau_R(a_R), a_R) - \epsilon$.

Within the set of contracts inducing inspection before purchase W(a), the logconcavity of 1 - G guarantees that firms have a unique optimal price. As indicated in Section 2, consumers will inspect before purchase if a firm selects a contract within W(a) and therefore will not use the refund option in this case. As the refund policy is redundant, we assume that within this set the firm chooses the refund to be equal to zero. Refer to the contract charging this optimal price as the *Wolinsky price*, and we denote it by $p_W(a)$. Similarly, denote the profit from charging the Wolinsky price by $\Pi_W(a)$. By standard results, there is a unique consumer surplus solving $a_W = U_B(p_W(a_W), a_W) - \epsilon$. Refer to the pair $(p_W(a_W), a_W)$ as the *candidate Wolinsky equilibrium*.

We focus on the subset of parameter values $\Omega^* \subset \Omega$ where markets are active and consumers get positive utility from inspecting before purchase when all firms charge the Wolinsky price and from inspecting after purchase when all firms play their part in a

¹³The set $\mathcal{R}(a)$ is nonempty since r(s) > b(s) for all $s < s^*$. Consequently, the set $\mathcal{W}(a)$ is also nonempty as it includes all contracts with a refund of zero and a price between $(b(s) - a)_+$ and $(r(s) - a)_+$.

¹⁴Because $p(\tau)$ is only implicitly defined, it is very difficult to analytically show that $\pi_A(p(\tau), \tau, a)$ is quasiconcave in τ for commonly employed search cost distributions. Numerical analysis for the uniform distribution shows that it is.

candidate symmetric refund equilibrium. Lemmas C.4 - C.5 in the Appendix formally show active markets with positive consumer utility exist if s, ϵ , c and η are small enough. Lemma C.3 shows that for these parameters, if all firms charge the Wolinsky price and consumers inspect before purchase, no firm has an incentive to deviate to cut the price to such an extent that it induces consumers to buy without inspection.

To keep the following discussion clear, we proceed by varying the production cost c and post-purchase inspection parameter β while holding the other parameters constant. These two parameters essentially measure the trade-off between inspecting before and after purchase. Relative to η , c is a measure of the inefficiency dimension related to inspection after purchase due to the possibility of product returns, while β is a measure of the efficiency dimension due to the lower inspection costs. We proceed with a series of claims leading up to the main characterization result, while a more formal treatment is given in the Appendix.

We start off by considering the special case where $\beta = 1$ and $c = \eta$. In this case, there is no inspection efficiency of product returns, but also no cost inefficiencies. We show that a refund equilibrium and a Wolinsky equilibrium co-exist and that they are equivalent in the sense that they yield the same profit and deliver the same consumer surplus. When $\beta = 1$, for a contract to induce returns (i.e. $(p, \tau) \in \mathcal{R}(a)$), it must offer a full refund $\tau = p$. When also $c = \eta$, then granting a full refund yields profit

$$(1-q)\pi_A(p,\tau,a) = p - c - (\tau - \eta)G(a+\tau) = p - c - (p-c)G(a+p)$$
$$= (p-c)(1 - G(a+p)) = (1-q)\pi_B(p,a).$$

Thus, $p_R(a) = p_W(a)$ and $\Pi_R(a) = \Pi_W(a)$ for all *a* and also $a_R = a_W$. Writing ρ_R for the fraction of firms offering a return policy and $\rho_W = 1 - \rho_R$, it is clear that any $\rho_R \in [0, 1]$ constitutes an equilibrium yielding surplus $a^* = a_R = a_W$.

Next, consider the case where $\beta = 1$ and $\eta < c \leq c^*$. As above, any contract in $\mathcal{R}(a)$ must offer a full refund as $\beta = 1$. For any price and a full refund, firms would strictly prefer, however, that consumers inspect before purchase as

$$(1-q)\pi_A(p,\tau,a) = p - c - (\tau - \eta)G(a+\tau) = p - c - (p-\eta)G(a+p)$$

< $(p-c)(1 - G(a+p)) = (1-q)\pi_B(p,a)$

so that firms always strictly prefer the Wolinsky price to the refund contract $\Pi_W(a) > \Pi_R(a)$.

Hence, only a Wolinsky equilibrium exists and $a^* = a_W$.

Third, for the case where $\beta < 1$ and $c = \eta$, consumers are willing to take less than a full refund to inspect after purchase: $(p, \tau) \in \mathcal{R}(a)$ implies $\tau < p$. Thus, when $c = \eta$

$$\max_{(p',\tau')\in\mathcal{R}(a)}\pi_A(p',\tau',a) > \max_{p'\in [(b(s)-a)_+,r(s)-a]}\pi_A(p',p',a) = \max_{p'\in [(b(s)-a)_+,r(s)-a]}\pi_B(p',a)$$

so that firms yield strictly higher profit from the refund contract than the Wolinsky price. Thus, in this case there only exists a refund equilibrium and $a^* = a_R$.

An important part of the argument leading up to the above two claims is that for the equilibrium analysis we only need to focus on firms offering a Wolinsky price or a refund contract. That is, not only do equilibria not exist where some firms offer contracts that differ from one of these two contract types, deviations to other contract types are also never optimal. Thus, only three types of equilibria may exist: (i) all firms offer the Wolinsky price or (ii) all firms offer a refund contract where consumers inspect after purchase, or (iii) an asymmetric equilibrium where some firms offer the Wolinsky price and consumers inspect before purchase, while other firms offer a refund inducing consumers to inspect after.

The formal analysis of this argument is in Lemmas C.6-C.10 and an intuitive argument runs along the following lines. First, Lemma C.6 shows that an equilibrium where all firms offer a contract where consumers do not inspect at all does not exist. Part of the argument is akin to a Diamond-type of argument (Diamond, 1971), namely that in the interior of the "Never Inspect" region of Figure 1 consumers would continue to buy if the visited firm charges a price that is marginally higher than the equilibrium price they expected and, hence, it is optimal for firms to deviate.¹⁵ Inspection of Figure 1 reveals, however, that if consumers are indifferent between buying without inspection and inspecting before purchase, firms cannot raise their price as consumers would then inspect before purchase. Lemma C6 shows that in this case a "reverse" Diamond argument can be used as a firm must cut its price below that of the competition to ensure consumers buy without inspection.

Second, for any combination of parameter values, if there exists an equilibrium where at all firms consumers either inspect before or after purchase and get a utility of a^* , then it must be that if firms weakly prefer offering a refund contract with consumers inspecting

¹⁵Chen et al. (2021) allow for buying without inspection in a model that is otherwise as in Wolinsky (1986), but assume that $\epsilon = 0$. Because of this assumption in their model, an equilibrium without inspection does exist.

after purchase, prices at firms offering a refund contract are strictly lower than the prices at other firms, i.e., $p_R(a^*) < p_W(a^*)$. Lemma C.8 shows that this is a consequence of the logconcavity of *G* and 1 - G and the properties, listed in Theorem 1, a candidate refund equilibrium has to satisfy. Thus, the utility at a refund contract is larger than at the Wolinsky price.

Third, Lemma C.10 shows that in characterizing equilibria, we can confine attention to deviations where firms either offer the Wolinsky price or offer a refund contract. The Lemma uses the results discussed in the previous paragraph and Lemma C.9 that shows that if consumers get a marginally better outside option a, the maximal profit a firm can get from offering a price where consumers do not inspect at all is decreasing faster than the decrease in maximal profit of a firm that either offers a Wolinsky price or a refund contract. This is quite intuitive as consumers always pay the price when they buy and do not inspect at all, whereas in the other two options, consumers only pay the price if they keep the product. As for any given a, firms offer a contract that is optimal for them, it is never optimal to offer a contract where consumers never inspect.

Fourth, for any given $\beta < 1$ if one increases *c*, starting from $c = \eta$, then at some point a refund equilibrium stops existing as it is simply too costly for a firm to induce consumers to buy the product without inspection before purchase. This is stated more formally in Claim **B.1**. The main question is what happens around this transition? Here, the second step turns out to be useful. That step effectively implies that if at the boundary of the region where a refund equilibrium exists firms are indifferent between offering a refund contract or a Wolinsky price, consumers prefer the refund contract to a Wolinsky price. So, if at the boundary all firms would offer a Wolinsky price, an individual firm would be better off offering a somewhat more profitable refund contract as consumers would be happy to accept such a contract. Thus, if *c* is so large that a refund equilibrium does not exist anymore, it must be that the market transits to an equilibrium where some firms offer returns and others offer a Wolinsky price. This is the content of Claim **B.2**.

Fifth, if an active market still exists when the cost c increases further, at some point all firms offer a Wolinsky price and the market is characterized by the Wolinsky equilibrium. This is formally stated in Claim **B**.3.

The above argumentation line is summarized in the main result of this section.

Theorem 2. Equilibria can be characterized by two continuous functions $\underline{c}(\beta)$ and $\overline{c}(\beta)$



Figure 4: Equilibrium Characterization

and a constant $c^* > \eta$ whereby $\eta < \underline{c} < \overline{c}$ for all $0 \le \beta < 1$. For each point with $\beta < 1$ and $c \le \max{\overline{c}, c^*}$ there is a unique equilibrium with trade:

- *1.* For $\eta \leq c \leq \underline{c}$, all firms offer a refund contract.
- 2. For $\underline{c} < c < \overline{c}$, a fraction of firms offer a refund contract.
- 3. For $\overline{c} \leq c \leq c^*$, no firm offers a refund contract.

The Theorem is depicted in Figure 4. At $\beta = 1$ and $c = \eta$ one clearly sees that all three types of equilibria come together as the optimal refund contract has full refunds and mimics market behaviour in a Wolinsky equilibrium. Moving horizontally or vertical only a refund equilibrium, respectively a Wolinsky equilibrium exists. Finally, for any $\beta < 1$ an asymmetric equilibrium with some, but not all, firms offering a refund contract mitigates between the parameter regions where a refund equilibrium or a Wolinsky equilibrium exist.

The equilibrium characterization shows when +the market generates product returns in the absence of regulation and fraud. Despite that regulation and fraud exist in real world markets, we believe that our equilibrium characterization reveals some important underlying forces that explain why product returns are more prevalent in some industries than in others. In particular, while product match is important for apparel and footwear where large efficiencies can be gained by inspecting at home, and returned products can be resold, our model predicts that these markets feature product returns. On the other hand, for categories such as beauty and health care, our model predicts that offering return policies is unattractive as the salvage value is too small relative to the production cost.

5 Efficiency

This section explores how refunds affect social welfare and whether there is room for a social planner to improve market outcomes by controlling the contracts offered by firms. We characterize the efficient outcome, finding that it is achieved when contracts are set so that consumers internalize the social costs and benefits of returns. Comparing these results to the equilibrium characterization, the market performs well in a broad sense, while leaving room for improvement by regulation. When the social costs are relatively low or high relative to their benefits, both the social planner and the market stimulate and do not stimulate returns, respectively. However, if the market equilibrium features returns, it generically stimulates either too many or too few returns (Section 5.1), suggesting that the market may benefit from regulation. Our results also suggest caution: even in settings where the market would benefit from more returns, requiring firms to offer a more generous return policy might result in a reduction in consumers surplus (Section 5.2).

Suppose a social planner selects a distribution $\mu \in \Delta(\mathbb{R}^2_+)$ over the contracts offered by firms with the goal of maximizing social welfare, defined as the sum of industry profit and consumer surplus. We start the welfare analysis by considering a relaxed optimization problem where the social planner not only selects the firms' contracts, but also selects the consumers' inspection option. We then verify that consumers intrinsically prefer to follow the planner's ascribed inspection option at the optimum.

Let π and *a* be the profit and consumer surplus induced by a contract distribution and inspection strategy. When a consumer visits a firm, the planner's continuation value, or the *continuation welfare*, is given by ¹⁶

$$\max \Big\{ \max_{(p,\tau)} (p-c)(1-G(a+p)) + G(a+p)\pi + \int_{a+p}^{\bar{\nu}} (1-G(\nu))d\nu + a - s, \\ \max_{(p,\tau)} p - c - (\tau - \eta)G(a+\tau) + G(a+\tau)\pi + \int_{a+\tau}^{\bar{\nu}} (1-G(\nu))d\nu + \tau - p + a - \beta s, \\ \max_{(p,\tau)} p - c + E(\nu) - p \Big\}.$$

The outer maximization reflects the planner's choice over inspection options and the inner

¹⁶The planner's problem can be solved directly. We take the more instructive approach and solve it as a dynamic program using the Principle of Optimality.

maximization the choice over contracts. Computing the optimal contracts, the optimal price is $\pi + c$ when goods are inspected before purchase and the optimal refund is $\pi + \eta$ when goods are inspected after purchase. The price falls out of the second and third expressions as it only serves as a transfer from consumers to firms and does not affect consumers' propensity to continue their search. Likewise, the refund is ineffective in the first and third expressions as no returns are made. Inputting the optimal contracts and simplifying terms, the continuation welfare becomes

$$\max\left\{\pi + \int_{a+\pi+c}^{\bar{v}} (1 - G(v)) dv + a - s, \pi + \int_{a+\pi+\eta}^{\bar{v}} (1 - G(v)) dv + a - \beta s - c + \eta, E(v) - c\right\}$$
(9)

Given the bound on how large the search and inspection costs can be, it is never optimal for consumers to forgo inspection altogether. This is verified by first noting that the maximal welfare is at least the amount gained by asking all firms to charge the Wolinsky price, $a + \pi \ge r(s + \epsilon) - c$. With this bound, the continuation welfare from inspecting before purchase (the first entry in (9)) is at least $r(s + \epsilon) - c + \epsilon$ and therefore strictly exceeds to the continuation welfare from not inspecting at all E(v) - c.

Thus, goods are always inspected at the social optimum. An optimal contract distribution requires firms whose goods are inspected before purchase to all charge the same price, say \hat{p} , the remaining firms to offer the same refund $\hat{\tau}$, and for these values to satisfy $\hat{p} - c = \hat{\tau} - \eta = \pi$. Consequently, these two equalities imply that the expected price charged by the firms stimulating returns is equal to the price charged by the others. To see this, let $R \subset \mathbb{R}^2_+$ be the event in which a contract stimulates returns and $\rho = \mu(R)$ the fraction of firms who do so. Industry profit can be expressed as

$$\begin{aligned} \pi &= \rho \frac{E(p|R) - c - (\hat{\tau} - \eta)G(a + \hat{\tau})}{1 - q} + (1 - \rho) \frac{(\hat{p} - c)(1 - G(a + \hat{p}))}{1 - q} \\ &= \frac{\rho \left(E(p|R) - c\right) + \pi \cdot (1 - q - \rho)}{1 - q} = \rho \frac{E(p|R) - c - \pi}{1 - q} + \pi \end{aligned}$$

implying $E(p|R) = \pi + c$. A further implication is that industry profit does not depend on the fraction of firms the planner allocates to stimulate returns. To verify this, suppose μ and μ' are distributions over the same optimal contracts, only differing in the rate with which firms offer refund contracts: $\rho' = \mu'(R)$.¹⁷ Then industry profit is the same under both distributions because

$$\begin{aligned} \pi' &= \rho' \frac{E(p|R) - c - (\hat{\tau} - \eta)G(a' + \hat{\tau})}{1 - q'} + (1 - \rho') \frac{(\hat{p} - c)(1 - G(a' + \hat{p}))}{1 - q'} \\ &= \pi \cdot \frac{\rho'(1 - G(a' + \hat{\tau})) + (1 - \rho')(1 - G(a' + \hat{p}))}{1 - q'} = \pi. \end{aligned}$$

As expected profit is not affected by the inspection choices, it is natural that the option delivering the highest continuation welfare coincides with the one that is best for consumers. In other words, $E(U_A(p, \tau, a)|R) = U_A(\hat{p}, \hat{\tau}, a) \ge U_B(\hat{p}, a)$, if and only if, inspecting after purchase maximizes (9), i.e.

$$\int_{a+\pi+\eta}^{a+\pi+c} G(v) \mathrm{d}v \le (1-\beta)s. \tag{10}$$

If either (10) or its reverse hold strictly at a social optimum, then it is socially efficient for all or no firms to offer refunds, respectively. If instead (10) holds with equality, then both consumers and firms are indifferent between the two inspection options, hence, allowing any fraction of firms to offer a refund contract achieves maximal welfare. To determine what is optimal, we only need to compare the two extreme cases. Consumer surplus equals $r(c - \eta + \beta s + \epsilon) - \hat{\tau}$ if all firms stimulate returns and it equals $r(s + \epsilon) - \hat{p}$ if no firm offers a refund.¹⁸ Thus, defining $\psi(\omega) = r(c - \eta + \beta s + \epsilon) + c - \eta - r(s + \epsilon)$, stimulating returns is socially optimal if and only if $\psi(\omega) \ge 0$.

For the final step, we need to ensure that both sides are willing to participate in the market. Of the contract distributions solving the relaxed problem, let the planner choose a simple one in which all firms offer the same contract $(\hat{p}, \hat{\tau})$ equating $\hat{p} - \hat{\tau} = c - \eta$. Profit must be nonnegative for firms to remain in the market, requiring $\hat{p} \ge c$. To produce nonnegative consumer surplus, the price cannot be too large, giving a bound that can be expressed compactly as $\hat{p} \le \max{\{\psi(\omega), 0\}} + r(s + \epsilon)$. Finally, to verify that consumers prefer the socially optimal inspection option to forgoing inspection altogether, the latter option is only preferred when the price is less than b(s) - a and the price we have specified is equal to $r(s + \epsilon) - a > b(s) - a$. The following theorem summarizes our conclusions.

¹⁷In essence, $\mu'(\cdot|R) = \mu(\cdot|R)$ and $\mu'(\cdot|R^C) = \mu(\cdot|R^C)$.

¹⁸These are the solutions to $\hat{a} = U_A(p, \hat{\tau}, \hat{a}) - \epsilon$ when $E(p) - c = \hat{\tau} - \eta$ and $\hat{a} = U_B(p, \hat{a}) - \epsilon$ respectively.

Theorem 3. The social optimum is achieved by having all firms offer a contract $(\hat{p}, \hat{\tau})$ with $\hat{p} - \hat{\tau} = c - \eta$ and $c \le \hat{p} \le \max\{\psi(\omega), 0\} + r(s + \epsilon)$. At the social optimum, (i) If $\psi(\omega) > 0$, consumers inspect a good after purchasing it, (ii) If $\psi(\omega) < 0$, consumers inspect a good before purchasing it, and (iii) If $\psi(\omega) = 0$, consumers either inspect a good before or after purchasing it.

This result shows that if we interpret $c - \eta$ as the transportation cost related to product returns, then if it is socially optimal to have product returns it is socially optimal to let consumers pay for this cost.

It is useful to render this result in graphical terms in order to relate it to the societal costs and benefits of returns as well as to the equilibrium characterization given in Section 4.

Corollary 1. Let $f(\beta)$ be the strictly decreasing function solving $\psi(f(\beta), \beta, \cdot) = 0$. At the social optimum, (i) If (β, c) lies below the graph of f, all firms stimulate returns, (ii) If (β, c) lies above the graph of f, no firms stimulate returns, and (iii) If (β, c) lies on the graph of f, any fraction of firms may stimulate returns.



Figure 5: The curves $f(\beta)$ and $\underline{c}(\beta)$ when match values are uniformly distributed, with $\eta = 0, s = 0.07$, and $\epsilon = 0.001$.

Figure 5 compares for the uniform distribution when the socially efficient outcome involves all firms offering a refund and when the market offers refunds. Recall from Theorem 2 that the equilibrium of the market is a refund equilibrium, if and only if, the cost lies below \underline{c} . If the production cost is close enough to the salvage value, specifically $\eta \le c \le \min\{f(\beta), \underline{c}(\beta)\}$, the market has firms offering refunds just as the planner would do; albeit, with inefficient contracts. On the other extreme, if the market is active and production costs exceed max{ $f(\beta), \bar{c}(\beta)$ }, then the market equilibrium achieves efficiency by not stimulating returns.

Inefficiencies are more apparent for intermediate product costs. For one, an asymmetric equilibrium is evidently nearly always inefficient as the planner would have all or no firms offering refunds for all production costs, aside from the nongeneric case where $c = f(\beta)$. Figure 5 also depicts how there can be costs $f(\beta) < c < c(\beta)$ for which the market stimulates firms offering refunds and the planner would opt for consumers to inspect goods before making a purchase.

5.1 MARKET INEFFICIENCIES

So far, we have determined which contracts are socially optimal and shown that the market broadly performs well, though imperfectly, in matching what the planner would choose in terms of whether contracts induce returns or not. In this section, we analyze the efficiency of equilibrium contracts in the context where both the market and the planner would have firms offering refunds.

Welfare in the Wolinsky equilibrium is $S_W = p_W - c + a_W$ where the sum of the price and consumer surplus satisfy $a_W + p_W = r(s + \epsilon)$. Welfare in the refund equilibrium is

$$\mathbf{S}_{R} = \frac{p_{R} - c}{1 - G(a_{R} + \tau_{R})} \cdot (1 - G(a_{R} + \tau_{R})) + \frac{p_{R} - c - \tau_{R} + \eta}{1 - G(a_{R} + \tau_{R})} \cdot G(a_{R} + \tau_{R}) + a_{R}.$$
 (11)

The first term captures the profit from those consumers who buy and do not make a return, the second term is the profit from those who buy the good and return it, and the final term is consumer surplus.

An important observation from condition (7) is that the sum of the price and consumer surplus are the same in both the refund and Wolinsky equilibria. Rearranging the terms in (11), the welfare expressions are related by

$$\mathbf{S}_R = \mathbf{S}_W + \frac{p_R - c - \tau_R + \eta}{1 - G(a_R + \tau_R)} \cdot G(a_R + \tau_R).$$

Thus, the welfare in a refund equilibrium can be understood as the Wolinsky welfare plus the expected profit made from returns. As a candidate refund equilibrium always admits a positive probability of returns (Theorem 1, Claim A.3), we record the following result.

Proposition 4. Welfare in a candidate refund equilibrium strictly exceeds that in the can-

didate Wolinsky equilibrium if and only if firms generate a positive profit on items that are returned, i.e. $\mathbf{S}_R > \mathbf{S}_W \iff p_R - \tau_R > c - \eta$.

The following is an immediate corollary.

Corollary 2. At a point $\omega \in \Omega$ where the candidate refund equilibrium is strictly more efficient than the candidate Wolinsky, the social planner can improve welfare by reducing $p - \tau$ to equal $c - \eta$, thereby stimulating more returns.

The intuition for this result is found by considering the exercise of starting from a refund equilibrium and gradually increasing the value of the refund. For items that are bought and kept despite the more generous refund, there is no effect. For items that would have already been returned, the increased refund amounts to a simple transfer with a net zero effect on welfare. Where the increased refund is effective is on the margin of changing search behavior: consumers become more likely to make a return. While the Envelope Theorem provides that the change in search behavior does not itself affect consumer surplus, the second term in (11) reveals how firms are affected. Increasing the refund both increases the chance the consumers make a return and the number of consumers who visit and make a purchase. If items that are bought and returned generate a positive profit, then the increasing refund boosts profit and thus welfare.

5.2 REGULATING REFUNDS

We conclude the analysis of efficiency by addressing the practical regulatory question of whether market outcomes can be improved by requiring firms to offer a more generous return policy, but letting them set their own prices. Regulators may mandate firms to offer consumers the possibility to return their purchased item and get a refund, but they (typically) do not have a mandate to intervene with firms' pricing decisions. Our first result suggests regulators should proceed with caution in mandating product returns. Requiring firms to offer a refund above but close to the market level can be welfare neutral while resulting in a reduction in consumer surplus. Our second and more optimistic result shows that, when the societal benefits from returns strongly outweigh their costs, regulators can achieve the first best with a sufficiently bold policy.

Reconsider the social planner's problem, assuming now that he chooses the minimum (threshold) fraction of the price that firms must offer back as a refund. Specifically, when

the planner selects the refund threshold θ , firms are only permitted to offer contracts in the set $X(\theta) = \{(p, \tau) \in \mathbb{R}^2_+ : \tau \ge \theta \cdot p\}$. Consider the interesting case where the unique equilibrium in the market is a refund equilibrium and the social planner can improve welfare by stimulating more returns.¹⁹

For a given θ , we analyze the *constrained equilibrium* in which firms and consumers both play best replies subject to the constraint that contracts belong to $X(\theta)$. Given *a* and $\theta < 1$, the region of permitted contracts can be represented in Figure 1 by the region to the right of the function $p = \frac{1}{\theta}\tau$ beginning at the origin and cutting through the curve $\mathcal{R}(a)$ at most once. There are two plausible symmetric constrained equilibria: one in which contracts continue to lie on the boundary $\mathcal{R}(a)$ and the other where contracts lie within the interior of the region in which consumers prefer to inspect after purchase.

If contracts lie on the boundary in a constrained equilibrium, then they must satisfy $(1 - \theta)p = p_R - \tau_R$ as there is a unique difference δ between the price and refund making consumers indifferent between inspecting before and after purchase when firms offer the same contract (Theorem 1). Thus, given the threshold θ , the price in a boundary equilibrium can be expressed as the function $p(\theta) = \frac{\delta}{1-\theta}$. From the indifference condition and (7), consumer surplus can likewise be expressed as the function $a(\theta) = r(s + \epsilon) - p(\theta)$ and is decreasing in θ . Therefore, if contracts remain on the boundary in a constrained equilibrium, lifting the mandatory refund rate is harmful for consumers. As argued in the following proposition, whenever θ is above but close enough to the equilibrium ratio τ_R/p_R , such an equilibrium exists.

Proposition 5. Consider a refund equilibrium and a regulation imposing a refund threshold that is marginally higher than the equilibrium refund. Then there is a constrained equilibrium generating higher profits and lower consumer surplus, while keeping social welfare unchanged.

Thus, for a refund threshold to offer any improvement on market outcomes, consumers must be made to *strictly* prefer inspection after purchase. We find that as long as the difference $c - \eta$ is not too large, there is a refund threshold the planner can set so that the constrained equilibrium price achieves the social optimum $(1 - \theta)p = c - \eta$.

¹⁹For a given $\omega \in \Omega^*$, the parameter lies in $\{\omega' \in \Omega : (\eta', s', \epsilon') = (\eta, s, \epsilon), \beta < 1, c < \underline{c}(\beta), 0 < \psi(\omega')\}$.

Proposition 6. If the difference between the production cost and salvage value is not too large, then there is a refund threshold $\hat{\theta}$ such that there is a constrained equilibrium that achieves maximal social welfare.

In other words, as long as society's benefits from returns sufficiently outweigh its costs, a planner can achieve efficiency by requiring firms to offer a generous refund.

6 Conclusion

In this paper, we have addressed the trade-off that arises when firms offer consumers the possibility to receive a refund when they return the product after they have purchased it. The return option allows consumers to more easily evaluate whether their purchase satisfies their preferences. However, product returns also come at a cost as the salvage value is typically lower than the production cost. To study this trade-off, we have made a methodological contribution to the consumer search literature by augmenting the seminal search model by Wolinsky (1986) in several dimensions. We have characterized the equilibrium outcomes and have shown that the equilibrium is always unique and is either one where all firms offer a refund policy, or where no firm offers refunds (and the equilibrium is as in Wolinsky (1986), or where some firms do, while others do not. We also have analyzed whether the market generates efficient outcomes and whether a regulator can improve upon the market outcome.

We think our paper opens several directions for future research, especially related to the consumer search process. First, throughout the paper we have maintained the assumption that consumers do not learn any product features unless they incur a more substantial inspection cost. Alternatively, one could make a distinction between objective product features that, certainly in online markets, are (like price) quite readily available to consumers, and other more subjective features that require a consumer to inspect more thoroughly. Consumers may then learn already part of their match value without incurring the inspection cost and this will affect their decision whether to continue to inspect the product beforehand or to buy and inspect afterwards. Second, the current paper has maintained the assumption of the consumer search literature that consumers know their match value after incurring the inspection cost. Alternatively, one may think that consumers learn part of their match value by incurring an inspecting cost and that they learn their match value better (relative

to inspecting before purchase) if they inspect afterwards or if they inspect multiple times.

These alternative ways of modeling the consumer search process may affect the tradeoff that are studied in this paper, but the underlying theme will remain the same: there are costs and benefits of offering refund policies and for a proper understanding of especially online markets, it is important to know whether the market provides proper incentives for an efficient resolution of this trade-off.

A Appendix: Equilibria with Returns

Proof of Theorem 1. As the text argues, in a symmetric equilibrium with returns, consumers are indifferent between inspecting goods before and after purchasing them and strictly prefer these to the other inspection options, i.e. $U_A = U_B > U_L, U_N$. Claims A.1 and A.2 below prove that a unique price, refund, and consumer surplus satisfies the first order conditions to be an optimal contract in this region, and thus a symmetric equilibrium with returns is unique. Examining the firm's first order conditions, Claim A.3 verifies $\underline{y} < a + \tau$ and $a + p < \overline{y}$, thus an equilibrium requires the refund to exceed the salvage value $\tau > \eta$.

Claim A.1. There exists a unique triple $(p, \tau, a) \in \mathbb{R}^3$ satisfying the first order conditions for an interior maximum to

$$\max_{(p,\tau)} \pi_A(p,\tau,a) \text{ s.t. } U_A(p,\tau,a) = U_B(p,a)$$

with consumer's expected utility being $a = U_A(p, \tau, a) - \epsilon$. Moreover, the solution (p, τ, a) is continuously differentiable in all parameters.

To complete the proof developed in the text, we need to show that there exists a unique δ solving (8). Differentiating, the left side is found to be strictly decreasing in δ , is equal to $r(\beta s + \epsilon) \ge r(s + \epsilon)$ when $\delta = 0$, and by Lemma D.2 converges to $E(v) - \beta s - \epsilon < r(s + \epsilon)$ for any increasing sequence $(\delta_n)_{n \in \mathbb{N}}$ converging to $S - \beta s - \epsilon$. Thus a unique δ solves (8). From the firm's first order conditions, a unique refund τ solves

$$\tau - \eta = \frac{1 - G(r(s + \epsilon))}{G(r(s + \epsilon))} \frac{G(r(\delta + \beta s + \epsilon))}{g(r(\delta + \beta s + \epsilon))}$$

Given (δ, τ) , there is a unique price satisfying $p = \delta + \tau$ and unique $a = r(\delta + \beta s + \epsilon) - \tau$.

Continuous differentiability follows immediately from the Implicit Function Theorem.

Claim A.2. Given the solution (p, τ, a) , we have $U_A > U_L$ and $U_A > U_N$.

The first inequality holds trivially whenever $\epsilon > 0$. By writing the utility functions explicitly, the second inequality is seen to hold if and only if $b(s) < a + p = r(s + \epsilon)$ which must hold since $s + \epsilon < s^*$ implies $b(s) < E(v) < r(s + \epsilon)$.

Claim A.3. The solution (p, τ, a) admits a positive probability of returns $G(a + \tau) > 0$.

Having shown in the first claim that $\delta + \beta s + \epsilon < S$ in equilibrium, we have $a + \tau = r(\delta + \beta s + \epsilon) > v$, and thus $G(a + \tau) > 0$.

Proof of Proposition 1. For the first part, as the solution (p, τ, a) in Theorem 1 is not determined by the production cost and thus firms absorb the entire increase in the cost. For the second part, notice that the value for δ solving (8) does not depend on the salvage value, but for a fixed δ , the solution for τ increases $(d\tau/d\eta = 1)$; hence, the solution for p likewise increases. By (7), consumer surplus must decline. From the expression for profit, $\frac{p-c-(\tau-\eta)G(a+\tau)}{1-G(a+\tau)}$, as $a + \tau$ and $\tau - \eta$ are constant and the price is increasing in η , profit itself is increasing.

Proof of Proposition 2. The third part follows immediately from Condition (7), noting that the rightmost term is independent of β and thus a + p is constant in β . For the second part, implicitly differentiating (8) in β obtains $\frac{d\delta}{d\beta} = -\frac{s}{G(a+\tau)} < 0$. Note from (7), that $a + \tau = r(\delta + \beta s + \epsilon)$ is increasing in β with derivative $\frac{d(a+\tau)}{d\beta} = \frac{s}{G(a+\tau)}$. Thus, from the firm's first order conditions and the logconcavity of *G*

$$\frac{d\tau}{d\beta} = \frac{1 - G(a+p)}{G(a+p)} \frac{\partial}{\partial \tau} \cdot \left(\frac{G(a+\tau)}{g(a+\tau)}\right) \cdot \frac{s}{G(a+\tau)} > 0.$$

For the first part, as welfare is equal to $a + \frac{p-c-(\tau-\eta)G(a+\tau)}{1-G(a+\tau)}$, differentiating in β obtains $-\frac{s}{(1-G(a+\tau))G(a+p)} < 0.$

Proof of Proposition 3. Given that $\lim_{(s,\epsilon)\to(s^*,0)} r(s+\epsilon) = E(v)$, (8) and Lemma D.2 provide that $\delta \to S$ and thus (7) gives $a + \tau \to \underline{v}$. From the firm's optimal refund condition $\tau \to \eta$. Finally $a + p = r(s+\epsilon)$ and thus $p \to E(v) - \underline{v} + \eta$.

B Appendix: Structure of Equilibria

This appendix characterizes the model's equilibria.

Lemma B.1. There is a unique best reply in the set of points inducing inspection after purchase and it lies in $\mathcal{R}(a)$.

Proof. Firstly, the fact that $\mathcal{R}(a)$ can admit at most one best reply follows from the objective π_A being strictly increasing in the refund when $\tau \leq \eta$ and quasiconcave thereafter.

A best reply must lie on either the boundary $\mathcal{R}(a)$ or the set making consumers indifferent with inspecting after purchase and leaving $[U_A = U_L] = \{(p, \tau) \in \mathbb{R}^2_+ : U_A(p, \tau, a) = U_L(a)\}$, otherwise a firm can increase its profit by raising its price by a small amount. In the region $[U_A = U_L]$, there are only two potential critical values for firms: One interior with $\tau = \eta$ and the other at the boundary point equating $U_A = U_B = U_L$. The second critical value belongs to $\mathcal{R}(a)$. As depicted in Figure 1 since the first critical value offers a lower refund than would be optimal in $\mathcal{R}(a)$, it must enter the region where consumers prefer to inspect before purchase. For completeness, we analytically verify what the figure depicts to complete the proof.

The regions $[U_A = U_B \ge U_L] = \{(p, \tau) : U_A(p, \tau, a) = U_B(p, a) \ge U_L(a)\}$ and $[U_A = U_L \ge U_B] = \{(p, \tau) : U_A(p, \tau, a) = U_L(a) \ge U_B(p, a)\}$ contain ordered pairs of contracts with the property that if (p, τ) and (p', τ') either both lie in the first or both lie in the second set and $\tau < \tau'$, then p < p'. Moreover, in the region $[U_A = U_B \ge U_L]$, pairs with higher prices deliver less utility as U_B is decreasing. As a result, refunds in $[U_A = U_L \ge U_B]$ must exceed those in $[U_A = U_B \ge U_L]$. It follows that the critical value $\tau = \eta$ lies in the region $[U_A = U_L < U_B]$ and so consumers will opt to inspect the product before purchase if a firm makes this deviation.

B.1 ACTIVE MARKET

To ensure equilibria can support an active market, we must limit the magnitude of the search and production costs. The following three lemmas give the bounds that are needed. First, we characterize the points for which a firm would prefer to play its part in a Wolinsky equilibrium (or equivalently a refund equilibrium when $c = \eta$ and $\beta = 1$) rather than cut its price to incentivize consumers to buy without inspection. In terms of notation, denote the elements in two generic points by $\omega = (c, \beta, \eta, s, \epsilon)$ and $\omega' = (c', \beta', \eta', s', \epsilon')$. **Lemma B.2.** Let $\Omega_N \subset \Omega$ be the points for which the profit when all firms charge the Wolinsky price and consumers inspect before purchase exceeds the profit from cutting the price to induce no inspection. Then, (a) Ω_N is nonempty, and (b) If $\omega \in \Omega_N$ and $\omega' \in \Omega$ is another point with $s' \leq s$ and $\epsilon' \leq \epsilon$, then $\omega' \in \Omega_N$.

Proof. Offering the Wolinsky price yields profit $\pi_B = p - c$ while cutting to the optimal price inducing no inspection yields profit $\pi_N = \frac{b(s)-a-c}{1-G(a+p)}$. Making use of the Envelope Theorem, we differentiate $(1 - G(a+p))(\pi_B - \pi_N)$ in *s* to obtain

$$\frac{d}{ds}(1 - G(a + p))(\pi_B - \pi_N) = -(p - c)g(a + p)\frac{da}{ds} - \left(\frac{db}{ds} - \frac{da}{ds}\right)$$
$$= G(a + p)\frac{da}{ds} - \frac{db}{ds}$$

which is necessarily negative because $\frac{da}{ds} < 0 < \frac{db}{ds}$. Similarly, differentiating in ϵ

$$\frac{d}{d\epsilon}(1-G(a+p))(\pi_B-\pi_N)=G(a+p)\frac{da}{d\epsilon}$$

is negative because $\frac{da}{d\epsilon} < 0$. Substituting $a + p = r(s + \epsilon)$ and $p = \frac{1 - G(a+p)}{g(a+p)} + c$ we see that both expressions for profit are constant in all parameters except for *s* and ϵ . Finally, to prove that Ω_N is nonempty, from

$$(1 - G(a + p))(\pi_B - \pi_N) = \frac{(1 - G(r(s + \epsilon)))^2}{g(r(s + \epsilon))} - b(s) + r(s + \epsilon) - \frac{1 - G(r(s + \epsilon))}{g(r(s + \epsilon))}$$
$$= -b(s) + r(s + \epsilon) - \frac{(1 - G(r(s + \epsilon)))G(r(s + \epsilon))}{g(r(s + \epsilon))}$$

the difference goes to $\bar{v} - \underline{v} > 0$ as $\epsilon + s \to 0$ and if the difference is positive at $(\epsilon, s) = (0, s_0)$ then it must also be positive at $(\epsilon, s) = (s_0, 0)$.

Next, consumers must receive nonnegative utility to be willing to participate in a Wolinsky equilibrium.

Lemma B.3. Let $\Omega_W \subset \Omega$ be the points for which consumers yield positive utility from inspecting before purchase when all firms charge the Wolinsky price. Then, (a) Ω_W is nonempty, and (b) If $\omega \in \Omega_W$ and $\omega' \in \Omega$ is another point with $c' \leq c$ and $s' + \epsilon' \leq s + \epsilon$, then $\omega' \in \Omega_W$. *Proof.* Consumer surplus is $r(s + \epsilon) - \frac{1 - G(r(s + \epsilon))}{g(r(s + \epsilon))} - c$ when all firms charge the Wolinsky price. Surplus is positive when $(c, s, \epsilon) \rightarrow (0, 0, 0)$ and by the logconcavity of 1 - G, it is strictly decreasing in these three arguments.

While the parameters (β, η) continue to have no effect when consumers inspect before purchase, consumers fully absorb the production cost *c*. Therefore, for consumer surplus to be positive in a candidate Wolinsky equilibrium, the search costs along with the production cost cannot be too large.

Finally, consumers must receive nonnegative utility to be willing to participate in a refund equilibrium. Recall that a candidate refund equilibrium corresponds to the unique triple (p, τ, a) in which all firms choose the same contract in $\mathcal{R}(a)$ satisfying the first order conditions for a maximum and a is the utility from inspecting products after purchase when these are the prices $a = U_A(p, \tau, a) - \epsilon$.

Lemma B.4. Let $\Omega_R \subset \Omega$ be the points for which consumers yield positive utility from inspecting after purchase when all firms play their part in a candidate refund equilibrium. Then, (a) Ω_R is nonempty, and (b) There is a nonempty subset $\Omega'_R \subset \Omega_R$ such that, if $\omega \in \Omega'_R$ and $\omega' \in \Omega$ is another point with $\eta' \leq \eta$, $s' \leq s$ and $\epsilon' \leq \epsilon$, then $\omega' \in \Omega'_R$.

Proof. With $\hat{r} = r(\delta + \beta s + \epsilon)$, a symmetric refund equilibrium offers consumers surplus

$$a = \hat{r} - \tau = \hat{r} - \frac{1 - G(\hat{r} + \delta)}{G(\hat{r} + \delta)} \frac{G(\hat{r})}{g(\hat{r})} - \eta \ge \hat{r} - \frac{1 - G(\hat{r})}{g(\hat{r})} - \eta.$$
(12)

Define Ω'_R to be the set of points for which, if we replace the value for β with 0, the right side of (12) is positive. That the set Ω'_R is nonempty follows from noting that $\hat{r} \to \bar{v}$ as $s + \epsilon \to 0$. Observe that the right side of (12) is increasing in β and decreasing in η , ϵ , and s as a result of the logconcavity of 1 - G and Propositions 1-2 (the explicit derivatives of δ and \hat{r} with respect to s and ϵ are contained in Online Appendix D.2). From this, if the right side of (12) is positive at any point with $\beta = 0$, then it is also positive at any other point with a (weakly) smaller salvage value, search cost, and inspection cost.

Our equilibrium characterization will begin at a point in the subset satisfying the conditions of the three preceding lemmas $\Omega^* = \Omega_N \cap \Omega_W \cap \Omega_R$. That Ω^* is nonempty follows simply from taking any $\omega \in \Omega_W$, possibly lowering (η, s, ϵ) to obtain a point in $\Omega'_R \cap \Omega_W$, and then possibly lowering (s, ϵ) again to obtain a point in $\Omega_N \cap \Omega'_R \cap \Omega_W$.

B.2 Equilibria

For the remainder of this section, we fix a point $\omega \in \Omega^*$ and characterize equilibria as we vary the production cost *c* and the post-purchase inspection parameter β . That is, given ω , we characterize equilibria for the set of points { $\omega' \in \Omega : (\eta', s', \epsilon') = (\eta, s, \epsilon)$ }. We start by demonstrating the nonexistence of a symmetric equilibrium in which consumers make a purchase without ever inspecting the good.

Lemma B.5. *There does not exist an equilibrium with an active market in which, at each firm, consumers prefer to buy without inspection.*

Proof. Consider an active market where consumers prefer to buy without inspection. It is clear that there cannot be such an equilibrium with $U_N > U_B$, U_L as each firm would have an incentive to raise its price. Thus, in such an equilibrium, it must be that $U_N = U_j$ for a $j \in \{B, L\}$. However, from inspecting the consumer's pay-off of the different options it follows that upon visiting a firm the consumer prefers to buy without inspection if $a \le b(s) - p$, to inspect before purchase if $b(s) - p < a \le r(s) - p$, and to leave if r(s) - p < a.²⁰ Thus, as the consumer cannot be indifferent between not inspecting at all and leaving, an equilibrium without inspection must equate $U_N = U_B$ so that the price is p = b(s) - a.

Assuming all other firms charge an average price p' and all induce consumers to not inspect the product, a consumer's outside option is simply to incur the search cost to visit another firm and buy without inspection, hence, $a = E(v) - p' - \epsilon$. But this implies that a firm must charge a smaller price than the competition $p = b(s) - E(v) + \epsilon + p' < p'^{21}$ to ensure that consumers do not inspect the product. As each firm's best reply is to charge strictly less than the average market price, such an equilibrium cannot exist.

Lemma B.6. $U_A(p_R(a), \tau_R(a), a) - \epsilon - a$ and $U_B(p_W(a), a) - \epsilon - a$ have unique roots at a_R and a_W , are positive when $a < a_i$ and negative when $a > a_i$ for i = R, W respectively.

Proof. Given that there is a unique candidate Wolinsky equilibrium and candidate refund equilibrium, the functions $U_B(p_W(a), a) - \epsilon - a$ and $U_A(p_R(a), \tau_R(a), a) - \epsilon - a$ have unique roots at a_W and a_R . The Berge Maximum Theorem²² guarantees the Wolinsky price and refund contract to be continuous in a and thus $U_B(p_W(a), a) - \epsilon - a$ and

²⁰Proposition 0 in Doval (2018) identifies this to be the optimal decision rule.

²¹Lemma D.3 verifies $b(s) - E(v) + \epsilon < 0$ when $s + \epsilon < s^*$.

²²Theorem 17.31 in Aliprantis and Border (2006).

 $U_A(p_R(a), \tau_R(a), a) - \epsilon - a$ are also continuous in a. Finally to show that the functions cross zero from above, for $a \ge r(s)$, $p_W(a) = p_R(a) = 0$ and thus $U_A(p_R(a), \tau_R(a), a) - \epsilon - a = U_B(p_R(a), a) - \epsilon - a = U_B(p_W(a), a) - \epsilon - a = -\epsilon < 0$.

Lemma B.7. Consider an equilibrium with $\beta < 1$, $\rho_R + \rho_W = 1$ and consumer surplus a^* . If $\Pi_R(a^*) \le \Pi_W(a^*)$, then (a) $p_R(a^*) < p_W(a^*)$, (b) $a^* \in [a_W, a_R]$ with $a_W < a_R$, and (c) If $\rho_R \in (0, 1)$, then $a_W < a^* < a_R$.

Proof. An equilibrium must feature $a^* < r(s)$ by virtue of the fact that firms will not charge a negative prices. We first argue that $p_R(a^*) < p_W(a^*)$. To the contrary, suppose $p_R \ge p_W$. If the Wolinsky price lies on the boundary $p_W = r(s) - a^*$, then by assumption $p_R = r(s) - a^*$, implying a contradiction as consumer surplus can then be computed to be $a^* - \epsilon$. Now suppose that the Wolinsky price lies in the interior. Then, because τ_R is optimally chosen, we have $\tau_R - \eta$ is weakly less than

$$\frac{1 - G(a^* + p_R)}{g(a^* + p_R)} \cdot \frac{g(a^* + p_R)}{G(a^* + p_R)} \cdot \frac{G(a^* + \tau_R)}{g(a^* + \tau_R)} < \frac{1 - G(a^* + p_R)}{g(a^* + p_R)} \le \frac{1 - G(a^* + p_W)}{g(a^* + p_W)}$$

which is equal to $p_W - c$. The first inequality follows from the logconcavity of *G* and $\tau_R < p_R$ and the second inequality follows from the logconcavity of 1-G and the assumption $p_R \ge p_W$. Thus we have $\tau_R - \eta < p_W - c$, further implying $\tau_R < p_W$. Equilibrium profit therefore satisfies

$$\Pi_R(a^*) = \frac{p_R - c - (\tau_R - \eta)G(a^* + \tau_R)}{1 - q} > \frac{(p_W - c)(1 - G(a^* + p_W))}{1 - q} = \Pi_W(a^*)$$

contradicting $\Pi_R(a^*) \leq \Pi_W(a^*)$. Hence, we must have $p_W(a^*) > p_R(a^*)$, proving part (a). For (b), the implication of this is that visiting the firms charging the Wolinsky price yields less utility than visiting firms offering the refund contract.

$$U_B(p_W(a^*), a^*) < U_B(p_R(a^*), a^*) = U_A(p_R(a^*), \tau_R(a^*), a^*)$$

The conclusion for (c) follows from the equilibrium condition $a^* = \rho_R (U_B(p_R(a^*), a^*) - \epsilon) + \rho_W (U_B(p_W(a^*), a^*) - \epsilon)$ and Lemma B.6.

Lemma B.8. Differentiating profit in the consumer surplus yields $\frac{d}{da}(1-q) \cdot \Pi_R(a) = -1 + G(a + \tau_R), \frac{d}{da}(1-q) \cdot \Pi_W(a) = -1 + G(a + p_W), and \frac{d}{da}(1-q) \cdot \Pi_N(a) = -1.$

The computations supporting this lemma are standard and can be found in Section D.2 of the Online Appendix. Next we simplify the problem so that we need only examine a restricted version of the game. Define the *restricted game* to be the same as the original game, except that firms can only offer either the refund contract or charge the Wolinsky price. In other words, the restricted game rules out equilibria and deviations to no inspection.

Lemma B.9. A strategy profile is an equilibrium if and only if it is an equilibrium of the restricted game.

Proof. We begin by showing that any equilibrium must involve $\rho_N = 0$ and thus constitutes an equilibrium of the restricted game. Consider an equilibrium in which a fraction, ρ_R offer the refund contract, ρ_W charge the Wolinsky price, and ρ_N induce no returns. Supposing a nonzero fraction of firms offer each contract, $\rho_i > 0$ for $i \in \{W, R, N\}$, and equilibrium utility is a^* we have $p_N(a^*) \le p_R(a^*) \le p_W(a^*)$. The relationship $p_R(a^*) \le p_W(a^*)$ follows from Lemma B.7 and the inequality $p_N(a^*) \le p_R(a^*)$ results from consumers preferring to inspect before purchase over not inspecting at all when the price is $p_R(a^*)$ and $U_N(p, a^*) - U_B(p, a^*)$ is strictly decreasing in the price. Moreover, the inequality $p_N(a^*) < p_W(a^*)$ must be strict or else firms charging $p_N(a^*)$ yield strictly higher profit than those charging $p_W(a^*)$.

Due to the order of prices, the utility from visiting a firm charging the Wolinsky price must lie strictly below a^* . Thus, from Lemma B.6, we know $a_W < a^*$. From Lemma D.5 it follows that the difference $(1 - q) (\Pi_W(a) - \Pi_N(a))$ is increasing in a. Given that the parameter is at a point in Ω_N , we have $\Pi_W(a_W) \ge \Pi_N(a_W)$, implying $\Pi_W(a^*) > \Pi_N(a^*)$, contradicting $\Pi_W(a^*) = \Pi_N(a^*)$. In other words, firms charging the Wolinsky price must yield strictly higher profit than those inducing no inspection. By an analogous argument, there can be no equilibrium with $\rho_N > 0$ where $\rho_R = 0$ and $\rho_W > 0$ or $\rho_R > 0$ and $\rho_W = 0$. Finally, Lemma B.5 proves that there can be no such equilibrium when $\rho_R = \rho_W = 0$, i.e., no equilibrium in which all firms induce no inspection.

Next, we show that any equilibrium of the restricted game constitutes an equilibrium of the original game. To do so, it is sufficient to show that there is no incentive for firms to deviate from such an equilibrium by cutting the price to induce no inspection.

By an analogous argument to that above, if the equilibrium involves $\rho_W > 0$, then profit from the Wolinsky price is demonstrably higher than deviating and cutting the price to induce no inspection. Consider $\rho_W = 0$, so that firms are playing a symmetric refund equilibrium. Theorem 1 provides that the contract lies in the interior of $\mathcal{R}(a_R)$. By Lemma B.2, if $\beta = 1$ and $c = \eta$, there is no incentive to deviate given that the parameter belongs to Ω_N . The difference $(1 - q) \cdot (\Pi_R(a_R) - \Pi_N(a_R))$ is demonstrably decreasing in β and constant in *c*; hence, there is no incentive to deviate for any other value $\beta < 1$ or $c > \eta$. \Box

This lemma allows us to draw two conclusions. First, firms only offer either the refund contract or the Wolinsky price in equilibrium. Second, when considering whether a strategy profile in which one or both of these contracts are offered is an equilibrium, the only possible deviations are indeed to one of these two contracts. We now characterize equilibria.

Proof of Theorem 2. Fixing $\omega \in \Omega^*$, the proof proceeds by characterizing the equilibria that arise as we vary β and c. Throughout, we use Lemma B.9 and characterize equilibria of the restricted game. Finally, we also consider the variable a to satisfy $0 \le a < r(s)$ as this must hold in any equilibrium. Let $c^* = c^*(s, \epsilon)$ denote the production cost at which $a_W = 0$.

The difference in profit between the two strategies is expressed by

$$\Pi_R(a) - \Pi_W(a) = \frac{p_R - c - (\tau_R - \eta)G(a + \tau_R)}{1 - q} - \frac{(p_W - c)(1 - G(a + p_W))}{1 - q}.$$

As the denominator does not influence which contract is more profitable, we need only compare the numerators to identify the best reply. Differentiating their difference, Lemma B.8 yields $\frac{d}{da}(1-q) \cdot (\Pi_R(a) - \Pi_W(a)) = G(a + \tau_R) - G(a + p_W)$.

When $\beta < 1$, Lemma B.7 guarantees $p_W > \tau_R$ when $\Pi_R(a) - \Pi_W(a)$ is nonpositive and thus strictly decreasing in this region. Hence, there exists at most one root $\Pi_R(a^*) - \Pi_W(a^*) = 0$ with the difference positive for $a < a^*$ and negative for $a^* < a$. We refer to this fact as *single-crossing*. Using single-crossing and drawing repeatedly from Lemma B.7, we now prove that if there exists an equilibrium at a point with $\beta < 1$, then it is unique.

- If there is a Wolinsky equilibrium, then Π_R(a_W) Π_W(a_W) ≤ 0 and a_W < a_R. There cannot be a refund equilibrium as a_W < a_R implies Π_R(a_R) – Π_W(a_R) < 0. Similarly, an asymmetric equilibrium requires surplus to satisfy a^{*} > a_W, implying Π_R(a^{*}) – Π_W(a^{*}) < 0.
- Suppose there is an asymmetric equilibrium with surplus a^* . Because $\Pi_R(a) \Pi_W(a)$ has a unique root, there can only be one asymmetric equilibrium. Also, because

 $a_W < a^* < a_R$, $\Pi_R(a_W) - \Pi_W(a_W) > 0 > \Pi_R(a_R) - \Pi_W(a_R)$ and so neither a Wolinsky nor a refund equilibrium exist.

• If there is a refund equilibrium, then from the last two points it is the only equilibrium.

In text, we prove the claims of the Theorem for the regions where either $\beta = 1$ or $c = \eta$. To complete the proof, we treat the remaining regions of the parameter space.

Claim B.1. When $\beta < 1$, there is a production cost $\underline{c}(\beta) > \eta$ for which only a refund equilibrium exists when $c \in [\eta, \underline{c}(\beta)]$.

Recall that $a_R > 0$ throughout since varying c has no effect on consumer surplus in a refund equilibrium. At $c = \eta$, we know $\prod_R(a_R; c) > \prod_W(a_R; c)$. Differentiating the difference yields $\frac{d}{dc} (\prod_R(a_R; c) - \prod_W(a_R; c)) = -\frac{G(a_R+p_W)}{1-q}$. As $\lim_{c\to\infty} \prod_R(a_R; c) = -\infty$, there is some cost $\underline{c}(\beta) > \eta$ for which firms prefer to stick to the refund equilibrium when $c \in [\eta, \underline{c}(\beta)]$ and deviate to the Wolinsky price when $c > \underline{c}(\beta)$. The single-crossing property provides that the refund equilibrium is the unique equilibrium in this region.

Claim B.2. When $\beta < 1$, there exists $\bar{c}(\beta) > \underline{c}(\beta)$ for which only a unique asymmetric equilibrium exists when $c \in (\underline{c}(\beta), \overline{c}(\beta))$.

First, we show that the difference $\Pi_R(a_W; c) - \Pi_W(a_W; c)$ is decreasing in *c*. Because profit in the candidate of Wolinsky equilibrium does not depend on the production cost, we have $\frac{d}{dc}\Pi_W(a_W; c) = 0$. Also $G(a_W + p_W) = G(r(s + \epsilon))$ is independent of *c*, thus the probability that a consumer continues search after visiting a firm charging the Wolinsky price is independent of the production cost. Using $\frac{da_W}{dc} = -1$ and differentiating the deviation profit

$$\frac{d}{dc}\Pi_R(a_W;c) = \frac{\partial \Pi_R}{\partial a_W}\frac{da_W}{dc} + \frac{\partial \Pi_R}{\partial c} = -\frac{1-G(a_W+\tau_R)}{1-q}(-1) - \frac{1}{1-q} < 0.$$

Let \hat{c} satisfy $\Pi_W(a_W, \hat{c}) = \Pi_R(a_W, \hat{c})$. As the previous claim guaranteed no Wolinsky equilibrium at \underline{c} , we have $\Pi_W(a_W, \underline{c}) < \Pi_R(a_W, \underline{c})$ and thus $\underline{c} < \hat{c}$. From this and the previous claim, no Wolinsky equilibrium nor refund equilibrium exists in this region.

Now we show that there is an asymmetric equilibrium. In the region $c \in (\underline{c}, \hat{c})$, the absence of a Wolinsky equilibrium or refund equilibrium implies $\Pi_R(a_R) - \Pi_W(a_R) < 0 < 1$

 $\Pi_R(a_W) - \Pi_W(a_W)$ and so the unique root must belong to $a^* \in (a_W, a_R)$. As Lemma B.7 guarantees $p_W(a^*) > p_R(a^*)$, we have

$$U_B(p_W(a^*), a^*) - \epsilon < a^* < U_A(p_R(a^*), \tau_R(a^*), a^*) - \epsilon$$

and thus there exists a unique $\rho_R \in (0, 1)$ for which $a^* = \rho_R(U_B(p_W(a^*), a^*) - \epsilon) + (1 - \rho_R)(U_A(p_R(a^*), \tau_R(a^*), a^*) - \epsilon).$

Finally, we need to ensure that consumers are willing to participate in the market. For a given *a*, increasing the production cost has the effect

$$\frac{d}{dc}(1-q)\cdot(\Pi_R(a;c)-\Pi_W(a;c))=-G(a+p_W)<0.$$

As a consequence of single-crossing, if $a^*(c) \in (a_W(c), a_R)$ is the unique root of $\prod_R(a; c) - \prod_W(a; c)$, then $a^*(c)$ is continuous and strictly decreasing in a neighborhood of c. Let \hat{c}' be the production cost at which $a^*(\hat{c}') = 0$. As $a^*(\underline{c}) > 0$, it must be that $\underline{c} < \hat{c}'$. Define \overline{c} to be the cost at which either the asymmetric equilibrium becomes Wolinsky or consumer surplus in the asymmetric equilibrium drops to zero, i.e. $\overline{c} = \min\{\hat{c}, \hat{c}'\}$. As previously argued, $\overline{c} > \underline{c}$.

Claim B.3. When $\bar{c}(\beta) < c \leq c^*$, only a Wolinsky equilibrium exists.

As the previous claim demonstrated, $\Pi_R(a_W; c) - \Pi_W(a_W; c)$ is strictly decreasing in c and thus $\Pi_R(a_W) - \Pi_W(a_W) < 0$ in this region; hence, a Wolinsky equilibrium exists as long as it delivers nonnegative consumer surplus.

C Appendix: Efficiency

Proof of Proposition 5. First, let us show that for θ close enough to τ_R/p_R , there is a constrained equilibrium in which all firms charge the boundary price. Consider a firm's problem when all other firms charge the boundary price $p(\theta) = \frac{\delta}{1-\theta}$, consumer surplus is set according to these contracts $a(\theta) = r(s + \epsilon) - p(\theta)$, and it must offer a contract in the set $X(\theta)$.

Suppose the planner requires refund rate $\theta_R = \tau_R/p_R$. Given that a refund equilibrium exists at this point, charging p_R is the firm's unique best reply. Moreover, computing the

change in profit in the price along the along ray $\{(p, \tau) \in \mathbb{R}^2_+ : \tau = \theta_R \cdot p\}$ at this point and recalling that $\tau_R - \eta = \frac{1 - G(a_R + p_R)}{G(a_R + p_R)} \frac{G(a_R + \tau_R)}{g(a_R + \tau_R)}$

$$\begin{aligned} \frac{\partial}{\partial p} \pi_A(p, \theta_R \cdot p, a_R)|_{p=p_R} &\propto \frac{\partial}{\partial p} \left(p - c - (\theta_R p - \eta) G(a_R + \theta_R p) \right)|_{p=p_R} \\ &= 1 - \theta_R G(a_R + \theta_R p_R) - (\theta_R p_R - \eta) \theta_R g(a_R + \theta_R p_R) \\ &= 1 - \theta_R \frac{G(a_R + \tau_R)}{G(a_R + p_R)} > 0. \end{aligned}$$

Let *P* be a compact neighborhood of p_R for which profit is increasing in the price $\frac{\partial}{\partial p}\pi_A(p,\theta_R \cdot p,a_R)|_{p=\tilde{p}} > 0$ for all $\tilde{p} \in P$. Let Θ be a neighborhood of θ_R for which $\min_{\tilde{p} \in P} \frac{\partial}{\partial p}\pi_A(p,\theta \cdot p,a(\theta))|_{p=\tilde{p}} > 0$ for all $\theta \in \Theta$. The Berge Maximum Theorem provides that there is a neighborhood $\Theta' \subset \Theta$ of θ_R for which $\theta \in \Theta'$ implies that the firm's best replies are contained in *P*. Thus, when $\theta \in \Theta'$, the firm's unique best reply is on the boundary.

Finally, it is immediate that consumers are made worse off since, on the boundary, the sum $a(\theta) + p(\theta) = r(s + \epsilon)$ is constant and $p(\theta)$ is increasing in θ . In this region, because $(1 - \theta)p(\theta) = \delta$ is constant, social welfare

$$\begin{split} \mathbf{S}(\theta) &= p(\theta) - c + \frac{\delta - c + \eta}{1 - G(r(\delta + \beta s + \epsilon))} G(r(\delta + \beta s + \epsilon)) + a(\theta) \\ &= r(s + \epsilon) - c + \frac{\delta - c + \eta}{1 - G(r(\delta + \beta s + \epsilon))} G(r(\delta + \beta s + \epsilon)) \end{split}$$

is likewise constant, implying that profit is increasing in θ .

Proof of Proposition 6. The proof proceeds by constructing a symmetric constrained equilibrium in which contracts lie interior to the region for which consumers prefer to inspect after purchase. For a symmetric constrained equilibrium to achieve the social optimum, it is necessary that the price p, threshold θ , and consumer surplus a satisfies the firm's first order conditions for an interior optimum $\theta p - \eta = \frac{1-\theta G(a+\theta p)}{\theta g(a+\theta p)}$, the equilibrium condition $a = r((1-\theta)p + \beta s + \epsilon) - \theta p$, and the optimality condition $(1-\theta)p = c - \eta$. Letting $r^* = (c - \eta + \beta s + \epsilon)$, substitute consumer surplus $a = r^* - \theta p$ and the price $p = \frac{c-\eta}{1-\theta}$ into the firm's first order condition

$$\frac{\theta}{1-\theta}(c-\eta) - \eta = \frac{1-\theta G(r^*)}{\theta g(r^*)}$$

When $c > \eta$, the left side is strictly increasing in θ and explodes to infinity, while the right side is strictly decreasing and arbitrarily large when θ is small, implying that there exists a unique $\hat{\theta}$ solving the equation. Then $\hat{p} = \frac{1}{1-\hat{\theta}}(c-\eta)$ is the price and $a^* = r^* - \frac{1-\hat{\theta}G(r^*)}{\hat{\theta}g(r^*)}$ is the surplus. The price must be below the curve $\mathcal{R}(a^*)$ as consumers strictly prefer to inspect after purchase. To verify that consumers are not tempted to forgo inspection altogether, the price and consumer surplus satisfy $a^* + \hat{p} > r(s+\epsilon) > b(s)$; hence, we have $U_N(\hat{p}) < U_B(\hat{p}, a^*) < U_A(\hat{p}, \hat{\theta}\hat{p}, a^*)$.

We now show that firms have no incentive to deviate from this contract as long as the production cost is not too large. As $c \rightarrow \eta$, the optimal threshold $\hat{\theta}$ converges continuously to one from below. At $c = \eta$ and $\hat{\theta} = 1$, profit is precisely equal to the Wolinsky profit. For one, this means that profit is logconcave in the price, implying that \hat{p} is the unique price satisfying firm's first order conditions for an optimum within the region inducing inspection after purchase. Given the parameter belongs to a point in $\{\omega' \in \Omega : (\eta', s', \epsilon') = (\eta, s, \epsilon)\}$ for some $\omega \in \Omega^*$, profit strictly exceeds that from any contract inducing no inspection (see Lemma B.2). Thus for *c* close to η , continuity in the profit functions and the variables $(a^*, \hat{\theta}, \hat{p})$ provides that there is no temptation to deviate to induce no inspection and that \hat{p} is the unique maximizer of $\pi_A(p, \hat{\theta}p, a^*)$.

Because $\beta < 1$, when $\hat{\theta} = 1$, consumers never prefer to inspect before purchase for any price. This is to say that X(1) never intersects $R(a^*)$. As the variables $(a^*, \hat{\theta}, \hat{p})$ are continuous in the production cost, for *c* in a neighborhood of η , $X(\hat{\theta})$ remains bounded away from $\mathcal{R}(a^*)$; hence, there are no contracts the firm could offer to induce inspection before purchase.

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D Online Appendix

D.1 THE RESERVATION AND BACKUP PRICE

The following provides a few basic properties of the reservation and backup price defined by (2) and (3). Integrating by parts, these functions are equivalently implicitly defined by

$$x = \int_{r(x)}^{\bar{v}} (1 - G(v)) \, \mathrm{d}v, \text{ for } x \in [0, S)$$
(13)

$$x = \int_{\underline{v}}^{b(x)} G(v) dv, \text{ for } x \in [0, S')$$
(14)

where $S = E(v - \underline{v})$ and $S' = E(\overline{v} - v)$, as defined in the text.

Lemma D.1. Let r(x) and b(x) be as defined in (2) and (3).

- (a) r'(x) < 0 and b'(x) > 0.
- (b) There exists a unique x^* such that $r(x^*) = b(x^*)$.

(c)
$$r(x^*) = b(x^*) = E(v)$$
.

(d)
$$r(x) > b(x)$$
 for $x < x^*$ and $r(x) < b(x)$ for $x > x^*$.

Proof. (a) follows from differentiating (13) and (14) $\frac{dr}{dx} = -(1 - G(r))^{-1}$ and $\frac{db}{dx} = G(b)^{-1}$. To prove (b), the existence of a unique x^* equating $r(x^*) = b(x^*)$ follows from (a), the continuity of r and b, observing the limiting values of r - b for large and small inspection costs, and the intermediate value theorem. For (c),

$$E(v) - b = \int_{\underline{v}}^{\bar{v}} (v - b) \mathrm{d}G(v) = \int_{b}^{\bar{v}} (v - b) \mathrm{d}G(v) + \int_{\underline{v}}^{b} (v - b) \mathrm{d}G(v)$$
(15)

$$= \int_{b}^{v} (1 - G(v)) dv - x.$$
 (16)

From this, E(v) = b if and only if

$$\int_{b}^{\bar{v}} (1 - G(v)) \mathrm{d}v = x \tag{17}$$

which holds if and only if b = r. Finally, (a) and (b) imply (d).

Lemma D.2. For a sequence $(x_n)_{n \in \mathbb{N}}$ in [0, S), the following are equivalent.

(a) $x_n \to S$.

(b)
$$r(x_n) \to \underline{v}$$
.

(c)
$$x_n + r(x_n) \rightarrow E(v)$$
.

Proof. (a) \implies (b) is immediate. For (b) \implies (c), let $(x_n)_{n \in \mathbb{N}}$ be a sequence of values in [0, S) converging to S so that $r(x_n) \rightarrow \underline{v}$. Using the fact that $v - r(x_n) - \max\{v - r(x_n), 0\}$ almost surely converges to a unit mass at zero and the definition of the reservation price $E(\max\{v - r(x_n), 0\}) = x_n$, we obtain

$$0 = \lim_{n \to \infty} E(v - r(x_n) - \max\{v - r(x_n), 0\})$$
(18)

$$= \lim_{n \to \infty} E(v - r(x_n) - x_n) = E(v) - \lim_{n \to \infty} (r(x_n) + x_n)$$
(19)

from which it follows that $x_n + r(x_n) \to E(v)$. (c) \implies (a) follows from the fact that x + r(x) is strictly decreasing for all $x \in [0, S)$.

Lemma D.3. For $(s, \epsilon) \in \mathbb{R}^2_+$ satisfying $0 < s + \epsilon < s^*$, we have $\epsilon < E(v) - b(s)$.

Proof. The conclusion follows from noting that $E(v) - b(s) - s^* + s$ is decreasing in *s* and is equal to zero at $s = s^*$, implying $s^* - s < E(v) - b(s)$ for $0 < s < s^*$.

D.2 Additional Calculations

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Let us first verify that a refund equilibrium exists for the parameter value in example with uniformly distributed match value in Section 3 of the text. Building on the example, we can compute that $(p - c) + (\eta - \tau)G(a + \tau)$ is given by

$$\frac{2-\sqrt{8s}}{2} - \frac{2\left(1-\sqrt{8s}\right)}{2-\sqrt{8s}}\sqrt{1-\sqrt{8s}+2\beta s} + \eta - c - \frac{\sqrt{8s}}{2-\sqrt{8s}}\left(1-\sqrt{8s}+2\beta s\right)$$

and it follows that firms' profits are proportional to this. The equilibrium is defined for $0 \le s \le 1/8$ and as indicated in Proposition 3 at $s^* = 1/8$ and $\beta = \eta = 0$ we have that a = 0, while $\tau = 0$ and $p = E(v) = \frac{1}{2}$.

To confirm that the refund equilibrium indeed exists for the constellation that is considered in Proposition 3, we have to consider whether a firm is better off deviating to offering a contract p' where the consumer buys without inspection. At price p' a consumer buys without inspection if buying without inspection gives consumers a higher pay-off than inspecting before purchase, *i.e.*, if $Ev - p' \ge \max\{a, \int_{a+p'}^{1} (1-v)dv + a - s\}$.²³ Thus, the best possible deviation is where $p' = \min\{\frac{1}{2} - a, \sqrt{2s} - a\}$ and the best possible deviation is where $p' = \min\{\frac{1}{2} - a, \sqrt{2s} - a\}$ and the best possible deviation is never profitable for $\beta = 0$.

Lemma D.4 (Comparative statics in *s* and ϵ). Let (p, τ, a) be the solution found in Claim *A.1 of Theorem 1 and denote* $\delta = p - \tau$ and $\hat{r} = r(\delta + \beta s + \epsilon)$.

(a)
$$\frac{d\delta}{ds} = \frac{1 - G(a + \tau) - \beta(1 - G(a + p))}{G(a + \tau)(1 - G(a + p))} > 0$$

(b)
$$\frac{d\hat{r}}{ds} = -\frac{1-\beta(1-G(a+p))}{G(a+\tau)(1-G(a+p))} < 0$$

(c)
$$\frac{d\delta}{d\epsilon} = \frac{G(a+p) - G(a+\tau)}{G(a+\tau)(1 - G(a+p))} > 0$$

$$(d) \ \frac{d\hat{r}}{d\epsilon} = -\frac{G(a+p)}{G(a+\tau)(1-G(a+p))} < 0$$

Proof. Differentiating (8) in s and ϵ reveals (a) and (c); differentiating \hat{r} in the same arguments gives (b) and (d).

Lemma D.5. Differentiating profit in the consumer surplus yields the following.

$$\frac{d}{da}(1-q)\cdot\Pi_R(a) = -1 + G(a+\tau_R) \tag{20}$$

$$\frac{d}{da}(1-q) \cdot \Pi_{W}(a) = -1 + G(a+p_{W})$$
(21)

$$\frac{d}{da}(1-q)\cdot\Pi_N(a) = -1\tag{22}$$

Proof. Implicitly defining $\tau(p, a)$ by $\int_{a+\tau(p,a)}^{a+p} G(v) dv = (1 - \beta)s$, the refund contract is

²³It is clear that firms do not want to offer a price such that consumers inspect before purchase as in that case consumers will continue to search if their match value is smaller than the price, which for *s* close to s^* is true for approximately half of the consumers.

found by maximizing

$$\max_{p \in \mathbb{R}_+} p - c - (\tau(p, a) - \eta)G(a + \tau(p, a))$$

s.t. $(b(s) - a)_+ \le p \le (r(s) - a)_+.$

When $p_R(a)$ is interior, one can verify $\frac{d}{da}(p_R(a) + a)$ to be strictly increasing. Thus aside from at most two values of *a*, those equating $p_R(a) + a = b(s)$ and $p_R(a) + a = r(s)$, the function $(1-q) \cdot \prod_R(a)$ is evidently differentiable. We shall now show that the function continues to be differentiable at these two points. Suppose p_R is interior. Noting that $\frac{\partial}{\partial a}\tau(p,a) = \frac{G(a+p)}{G(a+\tau)} - 1$ and employing the Envelope Theorem

$$\begin{aligned} \frac{d}{da}(1-q) \cdot \Pi_R(a) &= \frac{\partial}{\partial a}(1-q) \cdot \pi_A(p_R, \tau(p_R, a), a) = \frac{\partial}{\partial a}\left(p_R - c - (\tau(p_R, a) - \eta)G(a + \tau(p_R, a))\right) \\ &= -G(a+p_R) + G(a + \tau(p_R, a)) - (\tau(p_R, a) - \eta)g(a + \tau(p_R, a))\frac{G(a+p_R)}{G(a + \tau(p_R, a))} \\ &= -1 + G(a + \tau_R). \end{aligned}$$

If the price is at the upper bound $p_R = r(s) - a$, then the refund is set to equate $\int_{a+\tau_R}^{r(s)} G(v) dv = (1 - \beta)s$ and thus $\frac{d\tau_R}{da} = -1$. Differentiating $(1 - q) \cdot \Pi_R(a)$ with respect to *a*

$$\frac{d}{da}\left(r(s)-a-c-(\tau_R-\eta)G(a+\tau_R)\right)=-1+G(a+\tau_R).$$

The same outcome occurs if the price at at the lower bound $p_R = b(s) - a$. Thus whether interior or at the boundary, $\frac{d}{da}(1-q) \cdot \Pi_R(a) = -1 + G(a + \tau_R)$. Since $\tau_R(a)$ is continuous, at all points $\frac{d}{da}(1-q) \cdot \Pi_R(a+) = \frac{d}{da}(1-q) \cdot \Pi_R(a-)$ and thus it is differentiable everywhere, with derivative $-1 + G(a + \tau_R)$.

Turning to Wolinsky, when p_W is interior $p_W(a) + a$ is strictly increasing. It follows immediately that aside from the points equating $p_W + a = b(s)$ and $p_W + a = r(s)$, that $(1 - q) \cdot \Pi_W(a)$ is differentiable. Supposing p_W to be interior, we can apply the Envelope Theorem

$$\frac{d}{da}(1-q) \cdot \Pi_W(a) = \frac{\partial}{\partial a}(1-q) \cdot \pi_B(p_W,a) = \frac{\partial}{\partial a}(p_W-c)(1-G(a+p_W))$$
$$= -(p_W-c)g(a+p_W) = -1 + G(a+p_W).$$

At the upper bound $p_W = r(s) - a$, $\frac{d}{da}(1-q) \cdot \Pi_W(a) = \frac{d}{da}(r(s) - a - c)(1 - G(r(s))) = -1 + G(a + p_W)$. The same outcome occurs if the price is at the lower bound $p_W = b(s) - a$. Thus, the derivative is always $\frac{d}{da}(1-q) \cdot \Pi_W(a) = -1 + G(a + p_W)$. The desired conclusion follows.

Finally, the optimal price inducing no inspection is $p_N(a) = b(s) - a$ which profit is $(1-q) \cdot \prod_N(a) = p_N(a) - c = b(s) - a - c$ and thus differentiating in *a* yields the desired conclusion.