Search Disclosure *

Carl-Christian Groh[†] Marcel Preuss[‡]

February 22, 2022

Abstract

We study a duopoly model in which sellers can disclose to their rival when they encounter a specific buyer, and buyers engage in costly sequential search to find out prices and valuations. The competitive effects of search disclosure crucially depend on whether sellers are able to revise their prices. We show that sellers almost never disclose in equilibrium if revising prices is feasible but always do if it is not. The feasibility of price revisions thus weakly improves buyer surplus and total welfare. Moreover, exogenous search disclosure at all times leads to higher buyer surplus than is attainable when search disclosure is voluntary.

Keywords: Search, Information Exchange, Antitrust, Price Discrimination

JEL Codes: D18, D83, L13, L86

*We would like to thank Volker Nocke, Larry Samuelson, and Ernst-Ludwig von Thadden for valuable discussions and comments. All remaining errors are our own.

[†]University of Mannheim cgroh@mail.uni-mannheim.de

[‡]Cornell University preuss@cornell.edu

1 Introduction

When a buyer visits a seller, the seller can notify rivals about the purchase intention and identity of the buyer. Think of some company B (buyer) that wants to purchase new office furniture. After talking to one of the vendors in town, call this vendor V, this vendor could let its rival, seller S, know that B talked to them. To ensure the credibility of this information, V could add details about B's specific request or identity. Then, S may verify if it has met B before or can recognize the buyer if she visits S in the future. We refer to informing the rival about the visit of a buyer as *search disclosure.*

Search disclosure is particularly applicable to B2B environments with few infrequent buyers. There, letting competitors know of a specific buyer's visit or inquiry is a viable option.[1] Indeed, it is common for competing sellers to occasionally communicate with each other. And while antitrust law clearly forbids price arrangements, it does not prohibit to simply mention a buyer's inquiry. In addition, search disclosure applies to online consumer goods markets where websites can exchange similar information about consumers. In general, the functioning of the most common tracking methods implies that the availability of this information to some firm requires voluntary disclosure by its rival. Like the B2B vendor that can share details about a buyer with a rival to allow them to identify the buyer if they meet, a website obtains a unique identifier for each buyer which it can choose to share with other websites.[2] Search disclosure is thus crucial to facilitate cross-website tracking.

[1]Sellers could, of course, lie and also tell rivals about buyers who have already purchased. Yet those buyers will not visit another seller. Thus, this form of lying to other sellers has no consequences. However, it is important that a seller cannot misinform rivals about buyers that have not actually been there. This can be achieved through adding buyer-specific information to search disclosure that only a previously visited seller could know.

[2]The general concept is not unique to any particular tracking technology. The most basic identifier is obtained when buyers sign in using their email-address, which can be shared easily. Other common forms are cookie-tracking or finger printing. When using cookies, websites install a cookie on the user's device during the user's visit. The website owner controls access to that tracking cookie, i.e. it controls who else may be able to identify that buyer later. Fingerprinting refers to the recording of as detailed information as possible about the user's browser and computer so that it can serve as an identifier. Again, this identifier can be shared with other parties.

In this paper, we analyze the effects of search disclosure, whether search disclosure is anti-competitive, and when search disclosure occurs in equilibrium. We address these questions by integrating the possibility of search disclosure into a duopoly version of the classic Wolinsky (1986) model. We identify the ability to revise prices as a determining factor for whether sellers use search disclosure. Our work thus also represents an investigation into the issue of revisable prices in search markets. In addition, we investigate the surplus effects of search disclosure to assess the need for potential regulation.

When a seller receives search disclosure for a buyer before meeting the buyer, the seller knows that the buyer must have visited the other seller before. In general, we find that this induces sellers to set higher prices than had they not observed search disclosure – a benefit to the disclosing seller. The reasoning essentially follows from Armstrong et al. (2009): a seller visited first faces a higher fraction of fresh demand, which is more price-sensitive.

If revising prices is feasible, however, disclosure can have an additional effect that kicks in if the disclosing firm was visited second. Then, search disclosure informs a rival who was sampled first that a buyer has continued to search. Since search is costly, this signals to the initially visited firm that the buyer's valuation for its product must lie below some cutoff. This creates incentives to revise the originally posted price downward – a disadvantage to the disclosing seller. Notably, a buyer that just waits before buying does not send the same signal as a buyer who samples another firm. In fact, the buyer might just take a break from search, a phenomenon empirically documented by Ursu et al. (2021) in the case of consumer search.

We find that the equilibrium predictions thus depend on whether firms can revise prices along the search path. When price revisions are not possible, both firms disclose to their competitors in the unique pure-strategy equilibrium. Intuitively, if a seller deviated by not disclosing, it could only reduce the other seller's price because this seller would believe to be the first and not the second seller. When price revisions are feasible, however, sellers face a trade-off. To see this, consider an equilibrium where sellers never disclose so that sellers are uncertain whether they are visited first or second. Then, search disclosure leads a lower rival price if the rival knows the buyer already (i.e., the rival was visited first) and to a higher price otherwise. We find that the potential cost of triggering the rival to revise its price downward dominates the gains of increasing the price the rival would offer to the consumer when being visited second. Thus, no disclosure is always an equilibrium when price revisions are feasible.

The reason why the cost of a downward revision dominates is twofold. First, the price reduction when the price is revised downward is significantly larger in magnitude than the price increase when the rival learns it is visited second. In short, this is because receiving search disclosure for a known buyer constitutes a signal about the buyer's valuation for the own firm's product, while receiving search disclosure for an unknown buyer entails a signal about the buyer's valuation for the rival's product. The former affects a buyer's propensity to buy at the own firm more substantially, thus entailing a stronger price change. Second, lower revised prices for buyers who start searching at the rival firm generally harm the disclosing firm more than it benefits from higher prices for buyers who sample the rival firm second, even when the magnitude of the price change is the same. Intuitively, this is because an increase in the price of the second firm in the buyer's search queue does not affect the buyer's consumption decision if she chooses not to search.

While no disclosure is always an equilibrium when price revisions are feasible, partial disclosure can also be supported as an equilibrium when search costs are sufficiently small. In an equilibrium with partial disclosure, firms disclose to their rival if and only if they have not observed search disclosure regarding a buyer before. The reason why partial disclosure is not an equilibrium unless search costs are small is as follows: By not disclosing, a firm can falsely make the other firm believe that it was the first firm a buyer sampled. As a result, the deviating firm may observe search disclosure and, subsequently, receive the chance to revise its initial price downward. Being able to revise prices to price discriminate is more valuable to a firm when search costs are higher because fresh and return demand differ more in that case.

We also derive necessary conditions for an equilibrium in which firms disclose to their competitors both when they have received disclosure before and when they have not. Numerical simulations imply that such a full disclosure equilibrium does not exist. Intuitively, this is because search disclosure after having previously received disclosure triggers an undesirable downward revision of the rival's original price.

An implication of our analysis is that the possibility of price revisions weakly improves welfare and buyer welfare. This is not because firms actually revise prices for returning buyers downward in equilibrium. Instead, the possibility of a price revision discourages search disclosure, which prevails and leads to higher prices if prices revisions are not feasible. Note that this insight holds regardless of which equilibrium we select when both the no disclosure and the partial disclosure equilibrium exist in the model with price revisions. This is because firms do not revise prices in a partial disclosure equilibrium, making it outcome-equivalent to the disclosure equilibrium when price revisions are not feasible.

Finally, we consider the possibility that search disclosure is exogenously guaranteed at all times. This could be achieved by a third party that requires search disclosure at all times in exchange for other services. Alternatively, regulators could force firms to make all inquiries by buyers public. We numerically show that buyer surplus is raised through the introduction of this information source. The intuition is that firms will revise prices downward for any buyer that searches - both on and off the equilibrium path. This encourages search, which creates downward pressure on prices to the benefit of the buyer.

The rest of the paper proceeds as follows. We lay out the related literature in Section 2 and introduce the framework in Section 3. Sections 4 and 5 are devoted to the equilibrium analysis. In section 6, we study the equilibrium outcomes when firms exogenously have access to search history information. Section 7 concludes.

2 Literature Review

Our paper is related to several strands of literature. First, it contributes to the growing literature on consumer search. In the workhorse model by Wolinsky (1986) and the literature that builds upon it, consumers sequentially search sellers to learn prices and match values, which are random draws from the same distribution. Our paper extends that classic model in two novel directions. On the one hand, sellers can share information about a buyer's visit with rivals. Moreover, we provide an analysis of an extended framework in which sellers can revise prices for returning consumers and can condition prices on whether they have encountered a buyer before or not.

Information sharing can inform rival sellers about an arriving buyer's search path, which relates our analysis without price revisions to Armstrong et al. (2009) and Zhou (2011) who study prominence and ordered search, respectively. We contribute to this literature by showing that ordered search can emerge endogenously as a result of sellers' information sharing choices if prices cannot be revised.[3] The idea of revising prices for returning buyers is reminiscent of Armstrong and Zhou (2016), who explore the phenomenon of search deterrence, i.e. when sellers commit to higher prices for returning consumers. The key differences to that paper are that we 1) allow for discrimination not only against returning consumers but also against consumers who visit the rival first, 2) study discrimination that is based on endogenously provided information, and 3) consider a case where firms cannot commit to future prices.

Search disclosure allows individual firms to price discriminate based on the inferred search history of the buyer. A handful of recent papers study price discrimination in search markets. Fabra and Reguant (2020) study a simultaneous search model in which firms price discriminate based on perfect information about the quantity that consumers demand. Preuss (2021) studies price discrimination based on the search behavior of consumers, like this paper. Mauring (2021) considers firms which can discriminate against consumers using information about whether a given consumer is a shopper or a non-shopper. In Bergemann et al. (2021), competing firms receive noisy signals about the size of the consumers choice sets and/or consumers search costs. In Groh (2021), firms receive noisy information about consumers' valu-

[3]Others have studied when ordered search or prominence can result from consumer beliefs or preferences, see for instance Armstrong (2017), Moraga-González and Petrikaitė (2013), and Haan and Moraga-González (2011).

ations who search sequentially.[4] None of these papers consider the incentives of firms to endogenously share information about consumers' search histories.

This paper also contributes to the issue of information exchange between competitors and the social welfare effects thereof. The question when oligopolists gain by sharing their information with one another was first addressed by Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984), and Gal-Or (1985), who studied the effects of agreements to exchange private information about demand conditions, as well as by Shapiro (1986) and Gal-Or (1986), who consider firms sharing information about private costs. Focusing on information about individual consumers, Chen et al. (2001) study settings where firms receive imperfect information about buyer's consideration sets that they can share.[5]

None of these papers, however, consider the exchange of endogenously collected information as it is the case in the present paper. The first such investigation can be found in Taylor (2004), who studies a multi-period model in which sellers can sell their customer lists to one another.[6] Relatedly, Liu and Serfes (2006) study a two-period Hotelling model where firms can share preference information they have acquired for all buyers that initially purchase at their firm.[7] In an online advertising context, Johnson et al. (2021) study when online sellers agree to share unique identifiers of their websites' visitors with ad exchanges. This exchange facilitates re-targeting, but the ad exchanges may also share the data with the firm's rivals. All the listed papers consider the incentives for information sharing, but not when buyers face search costs. Thus, the information that firms can share in these models is different than the information we consider.

Finally, our treatment of price revisions relates this paper to the literature on

[4]While not directly addressing price discrimination, De Corniere (2016) studies a model in which consumers differ based on their search query, providing sellers information they use when setting prices. Similarly, consumers in Yang (2013) differ ex ante and thus search within different pools of firms, again giving firms information relevant to their pricing decision.

[5]Others work that considers the sharing of individual consumer information includes Kim and Choi (2010) , Zhao and Xue (2012) and Zhao (2012).

[6]De Nijs (2017) considers a related model of a three-firm oligopoly.

[7]Extensions are studied by Choe et al. (2020) and Lin et al. (2021), among others.

bargaining versus posted price selling (Riley and Zeckhauser, 1983). In a durable goods monopoly, low valuation buyers benefit more from waiting than high valuation buyers, leading to adverse selection that leads a seller to reduce its price over time (Coase, 1972). Consequently, the seller's inability to commit leads to negative selection and reduces profits (Gul et al., 1986). Board and Pycia (2014) show that the seller does not need commitment power in a search environment because negative selection is prevented due to the fact that low valuation buyers exit immediately. By contrast, we find that sellers lower their prices for returning consumers under competition.[8] More generally, our work contributes to this literature by showing that the inability to set a fixed price affects whether rival sellers exchange information about buyers, which reduces profits even without causing negative selection.

3 Setting

In this section, we introduce the theoretical model we study, which is based on Wolinsky (1986). Two firms indexed $j \in \{1, 2\}$ each produce a horizontally differentiated and indivisible good at constant marginal cost, which are normalized to zero. There is a representative buyer, who wants to buy at most unit of the aforementioned good. When she buys the good from firm j at price p, she attains the following utility:

$$U(u_j, p) = u_j - p \tag{1}$$

The match values u_j are stochastic and drawn uniformly from the unit interval. This distribution, which we denote by F, is common knowledge. The buyer is uncertain about the realizations $\left\{u_j\right\}_{j=1,2}$ at the beginning of the game and has to discover these match values as well as prices via sequential search. The buyer pays the fixed search cost $s > 0$ per firm that she visits, but can costlessly return to a firm that she has previously visited. The order of search is random.[9]

[8] Our findings thus complement insights from Armstrong and Zhou (2016), who observe positive selection when sellers have commitment power or recall is not free.

[9] This is without loss of generality when studying symmetric equilibria in this framework with

Firm j sets its price p_j after the buyer has paid the search cost to sample firm j. At the same time, firm j can disclose to its rival whenever the buyer visits it, which is immediately observed by its rival.[10] We restrict search disclosure to be a binary choice so that each firm j chooses $d_j \in \{D, ND\}$, where $d_j = D$ refers to the choice to disclose. We assume that search disclosure must be truthful so that firms cannot misreport a buyer's visit. This assumption is justified when buyers have unique characteristics or unique identifiers a seller can only know if the firm has indeed met the buyer or received an inquiry from her. In online consumer goods markets, such unique form of identification is enabled, for instance, by the placement of third-party cookies. Buyers do not observe disclosure decisions. Firms neither observe the buyer's search history nor the prices set by the rival firm. Moreover, firms do not observe any match values. Below is an overview of the timing of the game:

1. The buyer randomly visits firm j first and firm j chooses p_j and d_j. If it chooses $d_j = D$, firm $-j$ immediately observes this.

2. The buyer observes p_j and u_j and either buys, exits the market, or decides to sample firm $-j$. The game ends in the first two cases.

3. If the buyer samples firm $-j$, firm $-j$ sets a price p_{-j} and chooses d_{-j}. If $d_{-j} = D$, firm j immediately observes this and can potentially revise its price.

4. The buyer observes p_{-j} and u_{-j} as well as the potentially revised price p_j through free recall.

5. The buyer buys from firm $-j$, returns to and buys from firm j, or does not make a purchase and exits.

The timeline implies that there are up to three information sets as formalized in Remark 1 below.

ex-ante identical firms.

[10] We only use this notion to simplify the exposition. In fact, the exact timing of search disclosure does not matter. The rival can only use the information if the buyer continues to search. Since a buyer does not observe search disclosure, the effect of search disclosure is the same as long as firm $-j$ receives it before quoting a price to the buyer.

Remark 1 *After any history of the game, firm j finds itself in one of three categories of information sets $\mathcal{H}(j) \in \{N, R, KR \times p_j\}$ when a buyer arrives. Specifically,*

$\mathcal{H}(j) = N$ if j has not received disclosure.

$\mathcal{H}(j) = R$ if j has received disclosure and has not met the buyer before.

$\mathcal{H}(j) = KR \times p_j$ if j has received disclosure and has met the buyer before, at which point firm j offered the price p_j .

The restriction that search disclosure by firm $-j$ is necessary for firm j to revise its price is without loss of generality. Without search disclosure by firm $-j$, firm j remains in the same information set. Thus, holding on to the originally posted price is optimal.[11]

As a solution concept, we use perfect Bayesian equilibrium (PBE), in which all players' actions must also be optimal off the equilibrium path. A firm strategy profile in the game without price revisions is thus given by $(d_j, p_j) : \{N, R\} \mapsto \big(\mathbb{R} \times \{D, ND\}\big)$. In addition, each firm j's strategy must also specify $(p_j) : \{KR\} \times \mathbb{R} \mapsto \mathbb{R}$ in the game with price revisions. The buyer's strategy specifies when to buy from the first seller, when to continue searching, and when to exit without a purchase. We define the relevant search-cutoff in the equilibrium analysis. If the buyer samples both sellers, she buys from the seller that maximizes her net utility given by (1) or consumes her outside option which she values at zero.

In a PBE, the buyer's and firms' beliefs must be consistent with Bayes' rule. In information sets that are off the equilibrium path, however, Bayes' rule does not apply. To discipline the results, we impose the following standard assumptions on off-equilibrium beliefs. Firstly, the buyer's beliefs are passive - whenever the buyer is offered an off-equilibrium price, her beliefs and expectations about future prices

[11] Recall that our main applications are B2B and online consumer goods markets. In the former, it is common to receive an offer from firms after sending inquiry. In the latter, consumers simply open a website that quotes them a price. In either case, there is no cost associated with not buying immediately since the buyer does not have to physically "leave" a store. In contrast, sampling another firm is costly and, thus, the only thing that matters.

remain unchanged. Secondly, we also assume that firms hold passive beliefs about their rivals' prices. That is, a firm that unexpectedly receives disclosure continues to believe that the other firm follows the equilibrium pricing strategy.

Thirdly, we need to make assumptions about the beliefs that a firm forms about the possible match values of the buyer it faces in the information set $\mathcal{H}(j) = KR \times p_j$, if this information set is off path. Then, the standard assumption of passive beliefs is not sufficient to pin down beliefs. We specify that these beliefs must be consistent. Consistency requires that (1) the firm believes that the buyer it faces has searched according to her equilibrium search strategy and (2) that the firm takes into account what it believes (or knows) about the prices the consumer has received along the search path.[12]

4 Equilibrium Analysis without Price Revisions

In this section, we solve the model when firms cannot revise prices. There are two possible candidates for a symmetric pure-strategy PBE, namely (1) an equilibrium in which firms do not disclose to their rival and (2) an equilibrium in which firms disclose to their competitor. Without the ability to revise prices, we can ignore the choice of d_j for $\mathcal{H}(j) = R$ since it can only induce $\mathcal{H}(-j) = KR \times p_{-j}$, an information set where firm $-j$ can neither set a different price nor make a relevant disclosure decision itself.

4.1 No Disclosure Equilibrium

Consider an equilibrium candidate in which no firm discloses to its competitor, which we call the *no disclosure equilibrium*. There is just one information set that is on-path for any firm, namely $\mathcal{H}(j) = N$. In a no disclosure equilibrium, firm j knows nothing about an arriving buyer beyond the buyer's equilibrium search strategy if

[12] In the model with no price revisions, the consistency requirement is analogous to the specification of passive beliefs.

$\mathcal{H}(j) = N$. Thus, in equilibrium, the model collapses to the standard Wolinsky-model with two firms. Following Wolinsky (1986), the buyer's search behavior in a no disclosure equilibrium is thus described by the function $w(p_j)$, which is given by:

$$w(p_j) = w^* - p^* + p_j, \tag{2}$$

where p^* denotes the symmetric equilibrium price and where w^* solves:

$$\int_{w^*}^{1} (u - w^*)du = s \tag{3}$$

When the buyer is offered p_j by firm j, she will continue to search if and only if her match value at this firm is strictly below $w(p_j)$. Since each firm expects its rival to set p^* in equilibrium, the buyer is believed to search if and only if her initial match value is strictly below w^*. Having noted this, firm j's profit function can be written as follows:

$$\Pi^*(p_j) = p_j \underbrace{0.5\left[\big[1 - F(w(p_j))\big] + \int_{p_j}^{w(p_j)} F(p^* + u_j - p_j)du_j\right]}_{\text{first arriver demand}} +$$
$$p_j \underbrace{0.5\left[[F(w^*)]\big(1 - F(w(p_j))\big) + \int_{p_j}^{w(p_j)} F(p^* + u_j - p_j)du_j\right]}_{\text{searcher demand}} \tag{4}$$

A buyer who arrives at firm j first and has $u_j > w(p_j)$ will not search and will directly buy, because $w(p_j) > p_j$. A buyer who arrives at firm j after searching must have had $u_{-j} < w^*$. If the buyer visits both firms, she will purchase at firm j if and only if $u_j > p_j$ and her net utility at firm j is greater, for which $u_j > w(p_j)$ is a sufficient condition. In equilibrium, each firm j sets p_j to maximize (4) for a given w^* and equilibrium candidate price p^*. If firms do not use search disclosure, this price follows directly from Wolinsky (1986) and is given by $p^* = \left(1 - (p^*)^2\right)/(1 + w^*)$.

Now consider a possible deviation from this equilibrium in which firm j discloses to firm $-j$. To understand the effects of a deviation to $d_j = D$ on the profits of firm j, we must consider what firm $-j$ will do subsequently. This, in turn, depends on $\mathcal{H}(-j)$. If the buyer has visited firm $-j$ before, j's deviation leads to the off-path information set $\mathcal{H}(-j) = KR \times p_{-j}$. Since revising the already set price p_j is not possible, firm $-j$ cannot take any action in this information set. Thus, the deviation has no effect in this case. By contrast, if the buyer has not visited firm $-j$ before but samples $-j$ later, search disclosure by firm j implies $\mathcal{H}(-j) = R$, where firm $-j$ knows that it is visited second when the buyer arrives.

To pin down the optimal price $p_{-j,2}$ that firm $-j$ sets if $\mathcal{H}(-j) = R$, note that firm $-j$ still believes that firm j has offered the buyer the equilibrium price p^* by the passive beliefs assumption. Consequently, firm $-j$ believes that the buyer would have continued to search if and only if $u_j < w(p^*) = w^*$. With these notions in mind, the perceived profit function of firm $-j$ after observing disclosure by firm j, call this $\Pi_2(p_{-j,2})$, is:

$$p_{-j,2}\left\{0.5F(w^*)\big[1 - F(w(p_{-j,2}))\big] + 0.5\int_{p_{-j,2}}^{w(p_{-j,2})} F(p^* + u_{-j} - p_{-j,2})du_{-j}\right\} \qquad (5)$$

The demand in this information set equals the demand component induced by searchers as defined in equation (4). The price p_2 that maximizes (5) is given by:

$$p_2 = (1/2)\big[1 - \big(w^* - p^*\big)\big] + (1/4)w^* - \frac{(p^*)^2}{4w^*} \qquad (6)$$

Note that we follow the convention of adding the $*$ superscript only to prices for on-path information sets. Lemma 1 establishes a critical relationship between p_2 and the no disclosure equilibrium candidate price.

Lemma 1 *The price p_2 satisfies $p_2 > p^*$.*

This result holds by the following reasoning. Suppose that firm $-j$ unexpectedly received disclosure by firm j for the buyer, who now shows up at firm $-j$ for the first time. Because firm j cannot lie, disclosure by this firm means that the buyer must have visited firm j first. The fact that the buyer has searched implies that she must have had a comparatively low match value u_j at the firm she initially visited. Thus, firm $-j$ understands that the demand entailed by the buyer will be less elastic than that of a randomly drawn buyer, prompting the firm to charge the comparatively high price $p_2 > p^*$.

Thus, the initial deviation by firm j is strictly profitable. Before stating the formal result below, note that we restrict attention to search costs which enable market participation, which is guaranteed if $w^* > p^*$ (see Wolinsky, 1986).

Proposition 1 *Suppose that firms cannot revise prices and $w^* > p^*$ so that the buyer searches in equilibrium. Then, there exists no equilibrium in which firms do not use search disclosure.*

The strength of the disclosure incentives are inverse U-shaped in search costs and converge to zero as search costs go to zero or approach the upper bound of 1/8 (where $w^* = p^*$).[13] This finding is related to the insights by Armstrong et al. (2009), who show that the gains in industry profit (in the respective equilibria) from making a firm prominent are inverse U-shaped in search costs.[14] As a corollary of our result, the introduction of disclosure costs would enable the the existence of equilibria with no disclosure, but only if search costs are either very high or very low.

4.2 Search Disclosure Equilibrium

We now consider an equilibrium candidate in which each firm chooses $d_j = D$ when $\mathcal{H}(j) = N$.[15]. As opposed to the previous analysis, both $\mathcal{H} = N$ and $\mathcal{H} = R$ are on-path histories in an equilibrium with search disclosure.

Because firm j expects firm $-j$ to always disclose when not receiving disclosure beforehand, firm j's beliefs when a buyer arrives and no disclosure was received

[13] We graph this result in appendix B.1. where we also derive the price the deviating firm sets.

[14] This result is obtained by comparing industry profits in the Wolinsky (1986) equilibrium and the ordered search equilibrium of Armstrong et al. (2009). We build on this result by showing that the disclosure incentives within the Wolinsky (1986) equilibrium have a similar form.

[15] As argued in the beginning of Section 4, the choice of d_j does not matter when $\mathcal{H}(j) = R$.

($\mathcal{H}(j) = N$) are different than in the no disclosure equilibrium candidate. Specifically, firm j then believes that the buyer has not visited firm $-j$ before, meaning that j is visited first. We denote the price that j sets in this case by p_1^*. In the alternative information set $\mathcal{H}(j) = R$, firm j has received disclosure before being visited by the buyer. Firm j thus believes that the buyer has visited the other firm first. Let p_2^* be the price firm j sets if $\mathcal{H}(j) = R$.

If both firms play the equilibrium disclosure strategy, the game is identical to the two-firm version of the well-known model of search with prominence put forth by Armstrong et al. (2009). Consequently, we know that $p_2^* > p_1^*$, i.e. the firm that believes to be visited first sets a lower price than the firm that believes to be visited second. This observation immediately leads to the following proposition:

Proposition 2 *Suppose that firms cannot revise prices and $w^* > p^*$ so that the buyer searches in equilibrium. Then, there is an equilibrium with search in which firms use search disclosure and prices are $p_1^* < p_2^*$.*

That is, a strategy profile in which firms always disclose is an equilibrium in this model, independently of the exact value of search costs. To see this, consider a firm j that faces a buyer and has not received disclosure. By deviating and not disclosing to its rival, firm j ensures that firm $-j$ believes the buyer is visiting $-j$ first, if the buyer samples $-j$. This information set is on the equilibrium path, which means that firm $-j$ is sure to set the price p_1^* instead of the higher price p_2^*. Besides intensifying competition for the deviating firm, there is no other effect of such a deviation, which means that it is not profitable.

5 Equilibrium Analysis with Price Revisions

In this section, we solve the model with the possibility of price revisions. That is, the firm that the buyer visits first can potentially revise the first price it offered the buyer. The timing of the game includes all five stages outlined in Section 3.

There are three candidates for a symmetric pure-strategy PBE, namely (1) an equilibrium in which firms never disclose to their rivals, (2) an equilibrium in which firms disclose to their competitors if and only if they do not have received disclosure, and (3) an equilibrium in which firms always disclose to their competitors.[16] We refer to these equilibrium candidates as (1) the *no disclosure equilibria*, (2) the *partial disclosure equilibria*, and (3) the *full disclosure equilibria*, respectively.

5.1 No Disclosure Equilibrium

We consider the *no disclosure equilibrium* candidate first. If this is an equilibrium, the only information set that is on the equilibrium path is $\mathcal{H}(j) = N$. That is, firms know nothing about the buyer's search history and would set the price p^*, i.e. the Wolinksy equilibrium price as shown before in Section 4.1. The existence of a no disclosure equilibrium requires that no firm has a profitable deviation from $d_j = ND$ when $\mathcal{H}(j) = N$.

To see whether this condition holds, consider the reaction by firm $-j$ if firm j deviates to disclosure. This depends on firm $-j$'s information set, which, in turn depends on whether the buyer has sampled firm $-j$ before. If the buyer has not sampled firm $-j$ before, the deviation induces $\mathcal{H}(-j) = R$. Firm $-j$ thus believes that it is visited second by the buyer. In fact, firm $-j$'s problem in this information set is identical to that in Section 4.1, because her beliefs about firm j's prices are passive and because it will optimally never disclose (back) to firm j. Thus, p_{-j} is given by p_2 as defined in (6) if $\mathcal{H}(-j) = R$. Since $p_2 > p^*$ by Lemma 1, the deviation benefits firm j in this case.

Alternatively, the buyer may start her search at firm $-j$ and visit firm j second. At firm $-j$, the buyer initially receives the price p^*. However, disclosure by firm j induces $\mathcal{H}(-j) = KR \times p^*$. That is, firm $-j$ knows that the buyer started at $-j$ and has sampled both firms. In contrast to the the previous analysis, firm $-j$ may now revise its price in this information set. To derive the optimal price p_3 in this case, firm $-j$ takes into account the buyer's optimal search behavior. After

[16] An equilibrium in which firms only disclose after having received disclosure, but not when receiving no previous disclosure would lead to the same equilibrium outcomes as candidate (1).

initially visiting firm $-j$, the buyer used the cut-off rule $w(p_{-j}) = w^* - p^* + p_{-j}$. Since $p_{-j} = p^*$ was initially offered to the buyer by firm $-j$ in equilibrium, firm $-j$ believes that the buyer's match value u_j lies below w^*. Thus, firm $-j$'s expected profit function, which we denote by $\Pi^3(p_{-j})$, is:

$$\Pi^3(p_{-j}) = p_{-j} \int_{p_{-j}}^{w^*} 0.5F(u_{-j} - p_{-j} + p^*)du_{-j} \tag{7}$$

Taking the derivative of (7) with respect to p_{-j}, we see that p_3 is given by:

$$p_3 = (2/3)(w^* + p^*) - (1/3)\sqrt{(w^*)^2 + 2w^*p^* + 4(p^*)^2}. \tag{8}$$

The following lemma establishes an important relationship between p_3 and the no disclosure equilibrium candidate price p^*.

Lemma 2 *The price p_3 satisfies $p_3 < p^*$.*

To understand this result, consider the problem of firm $-j$ after receiving disclosure for a buyer who visited $-j$ before. Given the buyer's optimal search rule, firm $-j$ can infer that the buyer's match value u_{-j} lies below w^*. This induces firm $-j$ to set a price lower than the one it set during the first encounter with the buyer when nothing about u_{-j} was known.

A disclosure deviation by firm j either leads to firm $-j$ setting $p_2 > p^*$ instead of p^* or to firm $-j$ setting $p_3 < p^*$ instead of p^*. To determine which effect dominates, we must analyze the profit function of firm j next. To define this profit function, note that the search behavior of a buyer arriving at the deviating firm j is described by the same cutoff function $w(p_j) = w^* - p^* + p_j$ as in the previous section. This is because the buyer does not anticipate any disclosure and would expect firm $-j$ to offer her the price p^*. Thus, the profit function of firm j after deviating by disclosure, which we call $\Pi^{1,d}(p_j)$, is given by:

$$\Pi^{1,d}(p_j) = \underbrace{p_j 0.5\left[\left[1 - F(w(p_j))\right] + \int_{p_j}^{w(p_j)} F(p_2 + u_j - p_j)du_j\right]}_{\text{modified first arriver profits}} +$$

$$\underbrace{0.5p_j\left[F(w^*)\big(1 - F(w^* - p_3 + p_j)\big) + \int_{p_j}^{w^*-p_3+p_j} F(p_3 + u_j - p_j)du_j\right]}_{\text{modified searcher profits}} \tag{9}$$

To understand the second term in (9), notice that the buyer's match value at firm $-j$, namely u_{-j}, must have been below w^* if the buyer visited firm j second. Thus, such a buyer will surely consume at firm j if $u_j - p_j > w^* - p_3 > 0$ as reflected in the first term of the modified searcher profits. If the buyer's net surplus at firm j is below $w^* - p_3$, she would still consume at firm j if $u_{-j} - p_3$ lies below $u_j - p_j$, which holds with probability $F(p_3 + u_j - p_j)$. If this event holds for a buyer with $u_j < w^* - p_3 + p_j$, the buyer will surely have searched, because:

$$u_{-j} < p_3 + u_j - p_j < p_3 + (w^* - p_3 + p_j) - p_j = w^* \tag{10}$$

Note also that firm $-j$, upon given the chance to revise its price, would not find it optimal to disclose to firm j.[17] With the help of this profit function, we can show that the expected adverse effect of search disclosure always dominates, which makes deviating to disclosure unprofitable. Thus, an equilibrium with no disclosure can be sustained for any search costs, which we formalize in the following proposition.

Proposition 3 *Suppose that revision of prices is possible and $w^* > p^*$ so that the buyer searches in equilibrium. Then, there is an equilibrium in which firms do not use search disclosure and the price is given by p^*.*

There are two reasons that imply Proposition 3. The first is that the rival's price

[17] The details underlying this result may be found in the proof of the following proposition.

reduction ($p^* - p_3$) for buyers that sampled the rival first exceeds the rival's price increase ($p_2 - p^*$) for buyers who sampled the disclosing seller first. This notion is visualized in the following graph, which plots the equilibrium and deviation prices. The second is that even if both price changes were equal in magnitude, the demand reduction from the former group would exceed the demand gains from the latter.

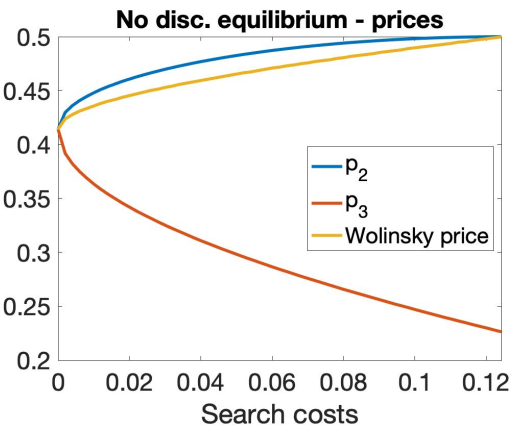


Figure 1: No disclosure equilibrium - prices

The first result obtains because how much sellers update their beliefs about a buyers' propensity to buy depends on the information set. When firm $-j$ receives search disclosure about a buyer it has not seen before, the only inference $-j$ makes when the buyer arrives is that $u_j < w^*$, i.e. that the buyer's match value for the rival's product is low. By contrast, when $-j$ receives search disclosure about a buyer it has seen before, it learns that $u_{-j} < w^*$, i.e. that the buyer's match value for the own product is low. Thus, search disclosure is more informative about the buyer's demand for the own product in the latter case, making the subsequent price reduction greater in magnitude than the price increase in the former case.

To see why the second result holds, suppose that search disclosure induces price changes that are equal in magnitude, i.e. that cause the rival to set a price of $p^* - \delta$ when it receives disclosure about a known buyer and a price of $p^* + \delta$ otherwise. By Lemma 1, we know that $\delta > 0$. Accordingly, rewriting firm j's demand as shown in equation (9) yields j's expected demand if it deviates to search disclosure:

$$\frac{1}{2}\left(w^*(1-w^*+p^*-\delta-p_j)+\int_{p_j}^{w^*-p^*+\delta+p_j}(p^*-\delta+u_j-p_j)\mathrm{d}u_j\right)$$
$$+\frac{1}{2}\left((1-w^*+p^*-p_j)+\int_{p_j}^{w^*-p^*+p_j}(p^*+\delta+u_j-p_j)\mathrm{d}u_j\right).$$

The derivative of this demand function with respect to δ is

$$\frac{1}{2}\int_{p_j}^{w^*-p^*+\delta+p_j}(-1)\mathrm{d}u_j+\frac{1}{2}\int_{p_j}^{w^*-p^*+p_j}(1)\mathrm{d}u_j=-\frac{\delta}{2}<0,$$

which shows that demand falls in δ for any $\delta > 0$. Let us explain the intuition behind the sign of this derivative. The second term on the left-hand side, namely $\frac{1}{2}\int_{p_j}^{w^*-p^*+p_j}(1)\mathrm{d}u_j$, captures the marginal gain from a buyer who samples firm j first.

By contrast, the term on the left-hand side $\frac{1}{2}\int_{p_j}^{w^*-p^*+\delta+p_j}(-1)\mathrm{d}u_j$ captures the marginal reduction in demand due to an increase in δ from a buyer who starts at seller $-j$. It shows that a marginal change in δ changes the likelihood that any buyer who samples both firms buys at firm j by the same amount.

Notably, while the effect on the purchase probability is the same in magnitude in both cases, buyers are more likely (by a difference of $1/2\delta$) to fall into the latter category, i.e. when the purchase probability decreases. To understand why this is the case, observe that if a buyer starts at firm $-j$, a revised price of $p_3 = p^* - \delta$ instead of p^* means that she will be $1/2\delta$ less likely to choose firm j with certainty, and instead buys from j only with probability less than one. This $\delta-$induced increase in the chance of facing a buyer who still considers both firms as viable is possible because a buyer who samples j may still choose firm $-j$.

By contrast, a price of $p_2 = p^* + \delta$ instead of δ has no comparable effect for a buyers who starts at seller j. In particular, seller j cannot increase the likelihood that a buyers purchases immediately because the buyer does not observe search disclosure and expect p^* at firm $-j$. In summary, the rival's potential downward price adjustment weighs more because it affects the buyer with a greater probability.

Consequently, search disclosure harms the deviating seller because overall demand decreases even if price changes are equal in magnitude ($p_2 - p^* = p^* - p_3$).

5.2 Partial Disclosure Equilibrium

We now consider equilibria where firms disclose to their competitors if and only if they have not received disclosure beforehand, i.e. $d_j = D$ if and only if $\mathcal{H}(j) = N$. Two information sets are on path in such an equilibrium, namely $\mathcal{H}(j) = N$ and $\mathcal{H}(j) = R$. If $\mathcal{H}(j) = N$, firm j believes that it is visited first and if $\mathcal{H}(j) = R$, it believes that it is visited second. Consequently, if firms follow the equilibrium disclosure strategy, equilibrium prices are given by $p_j = p_1^*$ if $\mathcal{H}(j) = N$ and $p_j = p_2^*$ if $\mathcal{H}(j) = R$, where (p_1^*, p_2^*) are exactly as in the disclosure equilibrium without price revisions analyzed in Section 4.2. To verify an equilibrium, we must rule out that any of the two deviations below is profitable.

i) $d_j = ND$ must be firm j's best response if $\mathcal{H}(j) = R$

ii) $d_j = D$ must be firm j's best response if $\mathcal{H}(j) = N$

Requirement *i)* is a relatively weak requirement. If $\mathcal{H}(j) = R$, disclosure by firm j gives its rival the chance to revise its price. As argued before, this price revision would lead to a lower price by $-j$, which is not in the interest of firm j. Thus, firms do not have an incentive to deviate after having received disclosure.[18]

To check *ii)*, suppose that firm j deviates and does not disclose to firm $-j$ if $\mathcal{H}(j) = N$. If the buyer continues to search after visiting firm j, then $\mathcal{H}(-j) = N$. Since this information set is on path in the partial disclosure equilibrium under consideration, firm $-j$ will (incorrectly) believe that it is the first firm the buyer visits. If the buyer continues to search, firm $-j$ will thus offer p_1^* instead of p_2^* if j had followed the equilibrium strategy by disclosing. Moreover, firm $-j$ will disclose in the information set $\mathcal{H}(-j) = N$ as part of the equilibrium strategy. This, in turn, induces $\mathcal{H}(j) = KR \times p_j$ so that firm j can revise its price.

[18]This is formally established in Section A.6 of the Appendix.

Deviating and not disclosing thus has two opposing effects. On the one hand, it reduces the rival's price price from p_2^* to p_1^*, which hurts the deviating firm. On the other hand, no disclosure by j enables firm j to revise its price, which constitutes a benefit to the deviating firm. To evaluate which effect dominates, we first derive the price p_3 firm j sets if $\mathcal{H}(j) = KR \times p_j$.

The buyer does not observe when firm j deviates to non-disclosure in the information set $\mathcal{H}(j) = N$ and thus does not expect a price revision. That is, the buyer expects to be able to purchase the good at the price p_1 which firm j initially offered to her (p_1 need not equal p_1^*). Moreover, she also expects the price p_2^* at the second firm she searches. Thus, the buyer continues to search after firm j if and only if $u_j < w^1(p_1)$, where the search cut-off $w^1(p_1)$ is the same as in Section 4.2, given by $w^1(p_1) = w^* - p_2^* + p_1$. Therefore, p_3 is a function of initially set price p_1.[19] Additionally, firm j knows that firm $-j$ sets the equilibrium price p_1^* given $\mathcal{H}(-j) = N$. Thus, the price $p_3(p_1)$ must maximize the following profit function for any p_1:

$$\Pi^{3,d}(p_j; p_1) = p_j \int_{p_j}^{w^1(p_1)} 0.5F(u_j - p_j + p_1^*)du_j. \tag{11}$$

We can now define the profit function that firm j maximizes when choosing the optimal initial price (p_1) it offers to the buyer, conditional on deviating to non-disclosure. This profit function, which we call $\Pi^{1,d}(p_j)$, is:

$$\Pi^{1,d}(p_j) = p_j 0.5\big[1 - F(w^1(p_j))\big] + p_3(p_j) \int_{p_3(p_j)}^{w^1(p_j)} 0.5F(p_1^* + u_j - p_3(p_j))du_j \tag{12}$$

We characterize the first-order conditions for these prices in section $A.6$ of the appendix. However, obtaining closed-form solutions to the resulting system of equations is analytically intractable, which is why we solve for p_1 and $p_3(p_1)$ numerically. We plot these prices together with the equilibrium prices p_1^* and p_2^* for different values of search costs in the Figure 2.

[19]This is because the consistency requirement on beliefs implies that the firm believes that the buyer's match value u_j must have been below $w^1(p_1)$.

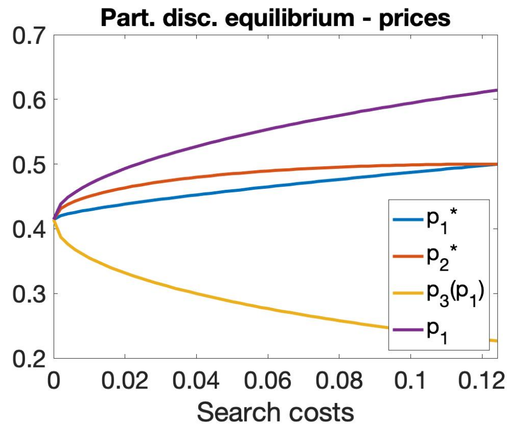


Figure 2: Partial disclosure equilibrium - prices

Our numerical results show that the beneficial effect of deviating dominates for almost all search costs. This is illustrated in the following figure, which plots the gains of deviating over the entire range of search costs for which we know that a no disclosure equilibrium with search exists, i.e. $p^* < w^*$.

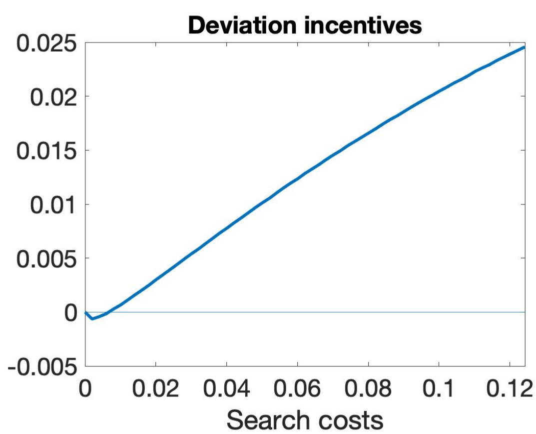


Figure 3: Partial disclosure equilibrium - deviation incentives

To understand why this equilibrium cannot be sustained unless search costs are low, consider the case where search is almost prohibitively costly (i.e. $w^* \approx p^*$). Then, buyers would only search upon receiving an initial price offer that is above or barely below their valuation. Crucially, this implies that any buyer who leaves a firm to search is very unlikely to return to this firm if the firm's price is fixed. Consequently, reducing the rival's price from p_2^* to p_1^* by deviating to no disclosure barely affects firm j's demand By contrast, the possibility to revise its own price is very profitable for firm j when search costs are high because it can significantly increase the probability of selling to the buyer, which is almost zero if firm j does not revise its price. Importantly, the lower revised price does not lead to any self-cannibalization since the buyer does not expect the price to be revised in a partial disclosure equilibrium.

5.3 Full Disclosure Equilibrium

Consider the third possible equilibrium candidate, in which firms always disclose to their competitors, i.e. $d_j = D$ both if $\mathcal{H}(j) = N$ and if when $\mathcal{H}(j) = R$. In such an equilibrium, all three information sets depicted in Remark 1 are on the equilibrium path. If $\mathcal{H}(j) = N$, firm j believes it is visited first. We denote the equilibrium price firm j would set in this information set by p_1^*. If $\mathcal{H}(j) = R$, firm j believes it is visited second and sets the price p_2^* in equilibrium. Lastly, firm j believes that it was visited first but that the buyer has also sampled firm $-j$ if $\mathcal{H}(j) = KR \times p_1$, in which case firm j revised its price according to the equilibrium price function $p_3^*(p_1)$, where $p_1 = p_1^*$ in equilibrium.

When evaluating the consequences of deviating to $d_j = ND$ at $\mathcal{H}(j) = N$ or $\mathcal{H}(j) = R$, notice that such a deviation never changes the rival's search disclosure strategy. This is because no matter whether firm j discloses, firm $-j$ finds itself in an on-path information set and thus discloses as part of the equilibrium strategy profile.

Consider the information set $\mathcal{H}(j) = N$ first. Should the buyer continue to sample firm $-j$, $\mathcal{H}(-j) = R$ if firm j discloses and $\mathcal{H}(-j) = N$ otherwise. Thus, firm j

finds it optimal to disclose if and only if

$$p_2^* \geq p_1^*. \tag{13}$$

Consider the second information set $\mathcal{H}(j) = R$ next, in which firm j knows that the buyer has already sampled firm $-j$. Then, $\mathcal{H}(-j) = N$ if firm j discloses and $\mathcal{H}(-j) = KR \times p_1^*$ otherwise. Consequently, firm j will disclose if and only if

$$p_3^*(p_1^*) \geq p_1^* \tag{14}$$

Intuitively, firm j is better off when it denies its rival the opportunity to revise its price in this case if (14) does not hold.

To calculate p_1^*, p_2^*, and p_3^*, we first derive the sequentially rational buyer search behavior. Note that in a full disclosure equilibrium, the buyer expects the originally visited firm to change its price to p_3^* if she decides to visit the other firm. For any initial price p_1, the buyer's sequentially rational search behavior can thus be characterized by the function $w^1(p_1)$ which, if it is interior for a particular p_1, solves:

$$\int_0^{w^1(p_1)-p_3^*(p_1)+p_2^*} (u - p_3^*(p_1))du + \int_{w^1(p_1)-p_3^*(p_1)+p_2^*}^1 (u - p_2^*)du - s = w^1(p_1) - p_1 \tag{15}$$

The complete derivation of (15) can be found in Appendix A.7. For a given initial price p_1, the buyer searches if and only if her initial match value is below $w^1(p_1)$. Because the buyer expects a price revision when searching, this function does not follow the familiar form of the search cutoff functions from the Wolinsky (1986) model. Given the equilibrium prices p_2^* and p_3^*, let $\hat{u}^1 = w^1(p_1^*)$ denote the equilibrium value of this cutoff. Moreover, let $w^2(p_{-j})$ denote the function that tracks the consumption choices of a buyer who arrives at the second firm $-j$ so that she buys there if and only if $u_{-j} > w^2(p_{-j})$. This function satisfies $w^2(p_{-j}) = \hat{u}^1 - p_3^* + p_{-j}$.

The equilibrium prices are then defined by the following objects:

$$p_2^* = arg\max_{p_{-j}} \left\{ p_{-j}\hat{u}^1 \big[1 - w^2(p_{-j})\big] + p_{-j} \int_{p_{-j}}^{w^2(p_{-j})} F(p_3^* + u_{-j} - p_{-j})du_{-j} \right\} \quad (16)$$

$$p_3^*(p_1) = arg\max_{p_j} \left\{ p_j \int_{p_j}^{w^1(p_1)} F(u_j - p_j + p_2^*)du_j \right\} \quad (17)$$

$$p_1^* = arg\max_{p_j} \left\{ \big[1 - w^1(p_j)\big]p_j + p_3^*(p_j) \int_{p_3^*(p_j)}^{w^1(p_j)} F(p_2^* + u_j - p_3^*(p_j))du_j \right\} \quad (18)$$

The on-path components of this equilibrium are a vector $(p_1^*, p_2^*, p_3^*, \hat{u}^1)$, which jointly have to solve the equations (15), (16), (17), and (18). While an analytical characterization of these objects is impossible to obtain due to the presence of higher-degree polynomials, we can show numerically that a solution to this system of equations has the property $p_1^* > p_3^*$, violating the necessary condition 14. Thus, we conclude that a full disclosure equilibrium never exists.[20]

Note also that the critical finding that $p_3^* < p_1^*$ mirrors the analytically obtained results in Section 5.1. There, the underlying intuition is that the seller learns that the buyer's match value falls short of a certain threshold and, thus, revises the price downward. While the cut-off value is different here, the firm clearly becomes more pessimistic about a known buyer when receiving disclosure, suggesting that $p_1^* > p_3^*$.

6 Exogenous Search Disclosure

In this section, we suppose that a third party guarantees that the firms have comprehensive search history information – as if firms use search disclosure at all times. Thus, tracking no longer requires voluntary search disclosure; each firm is informed

[20]We visualize this notion in appendix $B.3$

about the buyer's search history whenever the buyer samples the firm.

Under this specification, the pricing equilibrium when price revisions are impossible can be fully characterized by existing results from Section 4.2: A firm that is visited first sets the price p_1^* and a firm visited second the price p_2^*. When price revisions are impossible, the exogenous provision of search history information thus has no effects on outcomes.

6.1 Equilibrium Prices

The case with price revisions, however, requires additional analysis. As in Section 5.3, the symmetric pure-strategy equilibrium is characterized by the two prices p_1^* and p_2^* that are offered to a buyer who arrives at a firm first and second, respectively, and the function $p_3^*(p_1)$ that defines the revision price any firm would set as a function of its initial price.

Based on the previous results, we guess (and verify later) that any such equilibrium must satisfy the ordering $p_3^*(p_1^*) < p_1^*$. Based on this conjecture, we subsequently derive the buyer's sequentially rational search behavior. Note that if the gains from sampling the second firm are positive as such, i.e. if

$$\int_{p_2^*}^{1} (u_{-j} - p_2^*) du_{-j} - s > 0 \tag{19}$$

holds, there exists a cutoff function $w^1(p_1)$ such that the buyer searches if and only if her initial match value is below $w^1(p_1)$. A sufficient condition for inequality (19) to hold is that $p_2^* < w^*$, which is a property we document to hold true after solving for the equilibrium. Now consider an arbitrary initial price p_1. Then, any buyer with a match value of $u_j \leq p_1$ will sample the second firm because (19) holds. Moreover, the search cutoff $w^1(p_1)$ will be interior (below 1) if and only if it solves the following equation that is analogous to the one in Section 5.3, i.e.:

$$\int_0^{w^1(p_1)-p_3^*(p_1)+p_2^*} (w^1(p_1) - p_3^*(p_1))du + \int_{w^1(p_1)-p_3^*(p_1)+p_2^*}^{1} (u - p_2^*)du - s = w^1(p_1) - p_1$$

The derivations that lead to this result are presented in Appendix A.8. Note that this cutoff must not be interior. For example, when $s = 0$, the search cutoff $w^1(p^1)$ is equal to 1 (i.e. the buyer will always search after visiting the first firm) whenever $p_3^*(p_1) < p_1$.

These considerations imply that the the buyer's sequentially rational search behavior and thus each firm's demand are characterized by the same functions as in Section 5.3. Consequently, the optimization calculus that determines the firms' prices must be the same. Thus, the equilibrium pricing choices p_1^*, p_2^*, and $p_3(p_1)$ must satisfy equations (16), (17), and (18). As before, we define the equilibrium search cutoff as $\hat{u}^1 := w^1(p_1^*)$. We have previously computed joint solutions for these objects, which we visualize in Figure 4.

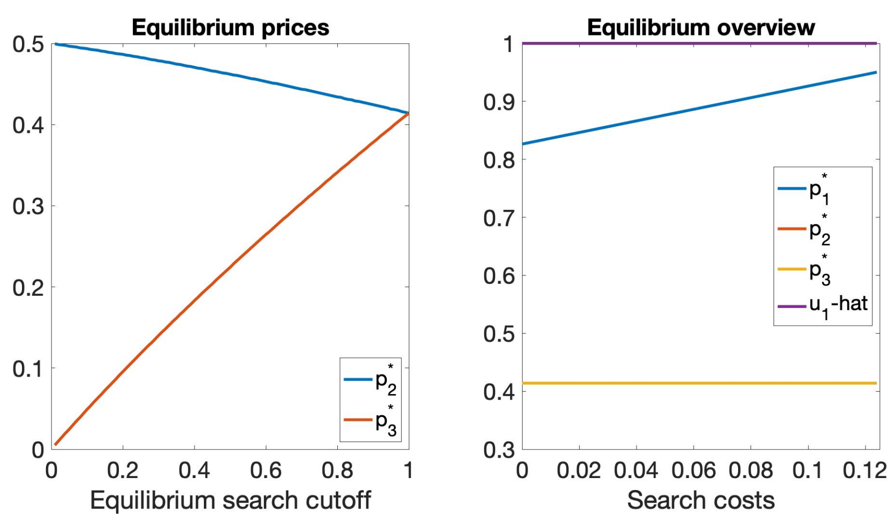


Figure 4: Exogenous information provision - outcomes

In the graph on the left-hand side, we plot the prices p_2^* and p_3^* for different values of the equilibrium search cutoff $\hat{u}^1$. We see that the prices converge to p^* when $\hat{u}^1$ approaches one. In the graph on the right-hand side, we plot all equilibrium prices and the search cutoff over $s \in [0, 1/8]$, the range of search costs for which there would be active search in the standard Wolinsky equilibrium without tracking.

We see that $\hat{u}^1 = 1$ holds always. Thus, $p_2^* = p_3^*$ will always hold, as indicated by the yellow line overlapping with the red one everywhere. The optimal price p_1^* is not uniquely determined; any price p_1 that surely induces the buyer to search constitutes an equilibrium. The blue curve depicts the lowest such p_1.

Given these equilibrium prices, the buyer always samples both firms before making a purchase. To see why this holds, recall that any buyer with $u_j < p_3^* < p_1^*$ samples a second firm because (19) holds. For $u_j \in [p_3^*, p_1^*]$, the gains of search are strictly increasing in u_j, which means that any such buyer will search both firms as well. For $u_j > p_1^*$, the gains from search are also strictly positive because the buyer reaps the full benefits from the price reduction $p_1^* - p_3^*$, which always exceeds the level of search costs. Thus, the buyer will always find it optimal to sample both firms.

Since the buyer surely searches in equilibrium, the problem faced by a firm visited second and the problem faced by a firm that can revise its price are equivalent. In either case, the buyer compares both firms' prices and firms have identical beliefs about the buyer's realized match values, because the decision to search conveys no information at all. Thus, p_2^* and p_3^* are exactly the same.

It remains to explain why the high price p_1^* as depicted in Figure 4 is optimal. The firm is indifferent between all prices p_1 that induce $w^1(p_1) = 1$, of which the blue curve depicts the lower bound. There are of course prices $p_1 < p_1^*$ such that $w^1(p_1) < 1$, at which some buyers would purchase immediately and not continue to search. The reason why deviating to such a price is not profitable is that the buyer correctly anticipates how firms choose $p_3(p_1)$. That is, they know that price revisions tend to be downward because of how the firm's posterior is affected by the buyer's rational decision to continue to search. Thus, any decrease of p_1, which has a direct effect of reducing the buyer's gains from search, will also lead to a decrease of $p_3(p_1)$, which increases the buyer's incentives to search. Thus, firms can only prevent the buyer from searching by setting a very low initial price p_1. Because such a low initial p_1 would induce a low $p_3(p_1)$ as well, the screening attained by achieving $w^1(p_1) < 1$ will not be profitable.[21]

[21] For details, especially regarding the function $w^1(p_1)$, see appendix A.8.

6.2 The Impact of Exogenous Search Disclosure

With all these results in hand, we now discuss how buyer surplus and profits are affected by the exogenous provision of search history information. Specifically, we are interested in how the outcome with exogenous search history information compares with the outcome of the game with endogenous search disclosure.

Again, predicting the effect of exogenously available search history information when price revisions are not feasible is straight forward. As argued in Section 4, disclosure by both firms is the unique equilibrium in this game. Thus, providing exogenous information has no effect on the surplus of firms or buyers.

To compare surplus in the case with price revisions, we mainly focus on the no disclosure equilibrium analyzed in Section 5.1 of the search disclosure game, which we is the unique equilibrium of this game except when search costs are very small. We briefly comment on the comparison with the other partial disclosure equilibrium at the end of this section.

Regarding profits, recall that each buyer obtains two price quotes before making a purchase. Thus, the equilibrium prices p_2^* and p_3^* are identical to the equilibrium price in the Perloff-Salop model where the buyer is perfectly informed (Perloff and Salop, 1985). This is of course the same outcome that obtains in the limit when search costs go to zero in the Wolinsky model (see Anderson and Renault, 1999). This is a notable result, which implies that if firms have full information about buyer search histories, the outcome is as competitive as if consumers have full information about match values, regardless of the level of search costs. Thus, the exogenous provision of search history information reduces profits.

Next, we compare the buyer surplus in the Wolinsky (1986) equilibrium, which is often the unique equilibrium of the game with endogenous disclosure and price revisions, to buyer surplus that is attained when firms receive search history information exogenously. This is visualized in Figure 5.

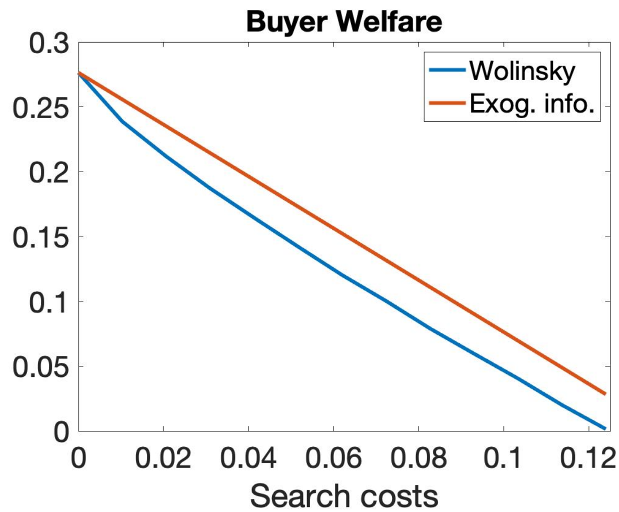


Figure 5: Exog. info. provision - buyer welfare

We see that buyers benefit from exogenous information provision. We also see that surplus decreases linearly in search costs in the exogenous information equilibrium. This is because there is no indirect effect of search costs through the equilibrium prices p_2^* and p_3^*, which are given by $\sqrt{2}-1$ for all search costs. To conclude, we note that buyer surplus is higher in the Wolinsky equilibrium than in the partial partial disclosure equilibrium, which exists if search costs are sufficiently low. Thus, exogenous information provision benefits buyers regardless of the equilibrium we select when search disclosure is endogenous.

7 Conclusion

We have introduced a communication device for firms into a duopoly model in which learning about a seller's offer is costly for buyers due to search frictions. When being visited by the buyer, a firm can notify its rival about this - we refer to this as *search disclosure*. Search disclosure benefits the disclosing firm if it is visited first by the buyer, but detrimental if the buyer visits the disclosing firm second and firms are not bound to one fixed price.

The possibility of price revisions is thus of central importance for the incidence of search disclosure. If revising prices is not feasible, firms will always disclose to their competitors in equilibrium. This prediction is reversed if revising prices is feasible. Then, we show that an equilibrium without disclosure is the unique pure-strategy equilibrium for a large range of search costs.

Our results imply that the possibility of price revisions raises both buyer surplus and welfare. This is because an outcome equivalent to the fully ordered search equilibrium emerges if price revisions are impossible. If price revisions are possible, the equilibrium outcomes will either be exactly the same or the market will revert to the Wolinsky (1986) - equilibrium, in which buyer welfare is higher than in said ordered search equilibrium (see Armstrong et al., 2009).

Thus, an important implication of our work is that policymakers should codify an explicit right for price revisions in the markets we study instead of preventing price discrimination. Arguably, ensuring such a right may be easier than prohibiting communication between firms.

Finally, we note that third parties in the search models we study could ensure access to detailed search history information for the participating firms, rendering voluntary search disclosure by firms replaceable. Our analysis shows that consumers benefit from the exogenous availability of this information if prices are revisable, and are indifferent otherwise. We note, however, that this result is based on a model in which consumers discipline firms with a great deal of foresight and sophistication.

A Mathematical Appendix

A.1 Proof of Lemma 1:

Part 1: Calculating p_2.

The perceived profit function of firm $-j$ is:

$$\Pi^{2,d}(p_{-j}) = p_j \underbrace{\left\{0.5F(w^*)\big[1 - F(w(p_{-j}))\big] + 0.5\int_{p_{-j}}^{w(p_{-j})} F(u_{-j} + p^* - p_{-j})du_{-j}\right\}}_{D^2(p_{-j})}$$

The derivative of demand w.r.t price reads:

$$\frac{\partial D^{2,d}(p_{-j})}{\partial p_{-j}} = -0.5F(w^*)f(w(p_{-j}))+0.5\left[F(w^*)-F(p^*)-\int_{p_{-j}}^{w(p_{-j})} f(u_{-j}+p^*-p_{-j})du_{-j}\right] = -0.5w^*$$

Similarly, demand can be expressed as:

$$D^{2,d}(p_{-j}) = 0.5w^*\big[1 - \big(w^* - p^* + p_{-j}\big)\big] + 0.5\Big[0.5(w^*)^2 - 0.5(p^*)^2\Big]$$

Plugging both these expressions into the first-order condition yields:

$$p_{-j}\frac{\partial D^{2,d}(p_{-j})}{\partial p_{-j}} + D^{2,d}(p_{-j}) = 0$$

$$\Longleftrightarrow$$

$$p_2 = 0.5\big[1 - \big(w^* - p^*\big)\big] + 0.5\big[0.5(w^*) - 0.5(p^*)^2/w^*\big]$$

Part 2: Establishing the ordering $p_2 - p^* > 0$.

Using the previous result, $p_2 - p^* > 0$ if and only if

$$2 > w^* + 2p^* + \frac{p^{*2}}{w^*} \Leftrightarrow \sqrt{2w^*} > w^* + p^* \tag{20}$$

Substituting the equilibrium expression for p^* given by

$$p^* = -\frac{1}{2}(1+w^*)\left(1-\sqrt{1+\frac{4}{(1+w^*)^2}}\right)$$

allows us to rewrite (20) as

$$\sqrt{2w^*} > -\frac{1}{2}(1-w^*) + \frac{1}{2}(1+w^*)\sqrt{1+\frac{4}{(1+w^*)^2}} \tag{21}$$

$$\Leftrightarrow \sqrt{2w^*} + \frac{1}{2}(1-w^*) > \sqrt{\frac{1}{4}(1+w^*)^2+1} \tag{22}$$

$$\Leftrightarrow \frac{1}{4}(1-w^*)^2 + (1-w^*)\sqrt{2w^*} + 2w^* > \frac{1}{4}(1+w^*)^2+1 \tag{23}$$

$$(1-w^*)\sqrt{2w^*} > 1-w^* \Leftrightarrow w^* > 1/2 \tag{24}$$

The last inequality holds true if and only if $p^* < 1/2 \leq w^*$ which proves the claim.

A.2 Proof of Proposition 1

Lemma 1 shows that p_2 is always strictly larger than p^* under the stated assumptions. The profits $\Pi^{1,d}(p_j)$ that the deviating firm faces (by inducing the rival price p_2 instead of p^*) are given by the following function:

$$\Pi^{1,d}(p_j) = p_j\left[0.5\big(1+w^*\big)\big(1-w(p_j)\big)+0.5\int_{p_j}^{w(p_j)} F(p^*+u_j-p_j)du_j+0.5\int_{p_j}^{w(p_j)} F(p_2+u_j-p_j)du\right]$$

For $p_j = p^*$, we have:

$$\Pi^{1,d}(p^*) = p_j\left[0.5\big(1-(w^*)^2\big)+0.5\int_{p^*}^{w^*} F(p^*+u_j-p^*)du_j+0.5\int_{p^*}^{w^*} F(p_2+u_j-p^*)du\right]$$

By contrast, equilibrium profits at p^* are:

$$\Pi^*(p_j) = p_j\left[0.5\big(1-(w^*)^2\big) + \int_{p^*}^{w^*} F(p^* + u_j - p^*)du_j\right]$$

Because $p^* < w^* \in (0,1)$, it must hold that:

$$\Pi^{1,d}(p^*) > \Pi^*(p^*)$$

This implies that there must be strict incentives to disclose - the firm could always set the price p^* after disclosure and obtain strictly higher profits than before.

A.3 Proof of Proposition 2

Part 1: Calculating equilibrium prices

The key part of this result is showing that the equilibrium prices p_1^* and p_2^* satisfy the ordering $p_1^* < p_2^*$. Thus, we first calculate these prices. To do so, we define consumer search behavior using a cutoff function $w^1(p_1)$ that is defined as follows:

$$w^1(p_1) - p_1 = w^* - p_2^*$$

Thus, a consumer would search upon receiving the initial price p^1 if and only if her valuation is below $w^1(p^1)$. In an equilibrium with $p^{2,*} \neq p^{1,*}$, the equilibrium search cutoff will not be equal to w^*. Instead, the equilibrium cutoff, call this $w^{1,*}$, is given by the following:

$$w^{1,*} = w^* - \big(p_2^* - p_1^*\big)$$

We now set up the optimization calculus of a firm who has not yet received disclosure. In equilibrium, any consumer that arrives at this information set must have visited this firm first. Thus, demand is:

$$D^{1,*}(p_1) = 0.5\left[\big[1 - F(w^1(p_1))\big] + \int_{p_1}^{w^1(p_1)} F(p_2^* + u_j - p_1)du_j\right] =$$

$$0.5\big(1 - F(w^1(p_1))\big) + 0.5\bigg[0.5(w^*)^2 - 0.5(p_2^*)^2\bigg]$$

The equilibrium first-period price must satisfy a first-order condition like previous ones. To compute this, note that:

$$\frac{\partial D^{1,*}(p^1)}{\partial p^1} = -0.5$$

Thus, the equilibrium price is characterized by the following first-order condition:

$$p_1^* = \big(1 - (w^* - p_2^* + p_1^*)\big) + \big[0.5(w^*)^2 - 0.5(p_2^*)^2\big]$$

$$\Longleftrightarrow$$

$$p^{1,*} = 0.5\big(1 - (w^* - p_2^*)\big) + 0.25(w^*)^2 - 0.25(p_2^*)^2$$

Since the first-period firm plays the equilibrium price p_1^*, consumers will leave firm j to search if and only if $u_j < w^{1,*}$. Define the function $w^2(p_2)$ as follows:

$$w^2(p_2) - p_2 = w^{1,*} - p_1^*$$

This function tracks the consumption decisions of consumers arriving at firm $-j$ second. Noting this, the demand that this firm faces is:

$$D^{2,*}(p_2) = 0.5F(w^{1,*})\big[1 - F(w^2(p_2))\big] + \int_{p_2}^{w^2(p_2)} 0.5F(p_1^* + u_{-j} - p_2)du_{-j} =$$

$$0.5w^{1,*}\big[1 - (w^{1,*} - p_1^* + p_2)\big] + 0.25(w^{1,*})^2 - 0.25(p_1^*)^2$$

Note firstly that all consumers with $u_{-j} > w^2(p_2)$ would directly buy under the condition that $w^{1,*} - p_1^* > 0$, which we can verify since $p_1^* < p^*$. Note that all consumers in the second integral must have searched, because:

$$u_j \leq p_1^* + u_{-j} - p_2 \leq p^{1,*} + w^2(p_2) - p_2 = w^{1,*}$$

Moreover, note that

$$\frac{\partial D^{2,*}(p_2)}{\partial p_2} = -0.5w^{1,*}$$

Constructing the first-order condition yields:

$$-0.5w^{1,*}p_2^* + \left[0.5w^{1,*}\big[1-(w^{1,*}-p_1^*+p_2^*)\big] + 0.25(w^{1,*})^2 - 0.25(p_1^*)^2\right] = 0$$

$$\iff$$

$$p_2^* = 0.5\big[1-(w^{1,*}-p_1^*)\big] + 0.25w^{1,*} - 0.25\big((p_1^*)^2\big)/w^{1,*}$$

Part 2: Showing that $p_1^* < p_2^*$

This follows from the results of Armstrong et al. (2009). We visually document this result in Appendix B.2.

A.4 Proof of Lemma 2

Part 1: Computation of p_3.

Consider a firm j that knows a consumer has initially visited the firm (and has surely received p^* at firm j) and moved on to search ($u_j < w^*$) - firm j now has a chance to revise the price, believing that this consumer has received the price p^* at the deviating firm (passive beliefs). The implied profit function is:

$$\begin{aligned}\Pi^3(p_j) =& p_j \int_0^{w^*}\int_0^1 \mathbb{1}[u_{-j}-p^* < u_j - p_j]\mathbb{1}[u_j - p_j > 0](0.5)du_{-j}du_j \\ =& p_j \int_{p_j}^{w^*} 0.5F(u_j - p_j + p^*)du_j\end{aligned}$$

This is the correct profit function when restricting attention to prices p_j that satisfy: $w^* - p_j + p^* < 1 \iff p_j > w^* + p^* - 1$. Let's suppose that the optimal price is in this interval (which we verify later). Then, the optimal revision price p_3 is a solution to the following first-order condition:

$$\int_{p_j}^{w^*} 0.5(u_j - p_j + p^*)du_j + p_j\Big[- 0.5F(p^*) + \int_{p_j}^{w^*} 0.5(-1)du_j\Big] = 0$$

$$\Longleftrightarrow$$

$$1.5(p_j)^2 + p_j\big[- w^* - 2p^* - w^*\big] + \big[0.5(w^*)^2 + w^*p^*\big] = 0$$

$$\Longleftrightarrow$$

$$3(p_j)^2 - p_j\big[(4)(w^* + p^*)\big] + \big[(w^*)(w^* + 2p^*)\big]$$

The solutions to this equation are given by the following:

$$p_3 = \frac{4(w^* + p^*) + / - \sqrt{4^2(w^* + p^*)^2 - 4(3)\big((w^*)^2 + 2w^*p^*\big)}}{6}$$

$$=$$

$$p_3 = (2/3)(w^* + p^*) + / - (1/3)\sqrt{(w^*)^2 + 2w^*p^* + 4(p^*)^2}$$

The numerical results indicate that the negative version of this solution is the appropriate one. Moreover, we have numerically verified that this price will always be in the region that we have restricted our attention to, namely $p_3 > w^* + p^* - 1$.

Part 2: Verifying the ordering $p_3 < p^*$

Using the previous results, $p^* - p_3 > 0$ holds if and only if

$$\frac{1}{3}\sqrt{w^{*2} + 2w^*p^* + 4p^{*2}} > \frac{2}{3}w^* - \frac{1}{3}p^* \tag{25}$$

$$\Leftrightarrow \sqrt{w^{*2} + 2w^*p^* + 4p^{*2}} > 2w^* - p^* \tag{26}$$

$$\Leftrightarrow w^{*2} + 2w^*p^* + 4p^{*2} > 4w^{*2} - 4w^*p^* + p^{*2} \tag{27}$$

$$\Leftrightarrow p^{*2} + 2w^*p^* > w^* \tag{28}$$

The necessary equilibrium condition pinning down p^* tells us that

$$p^* = \frac{1-p^{*2}}{1+w^*} \Leftrightarrow w^* = \frac{1-p^*-p^{*2}}{p^*}$$

Substituting the expression for w^* into inequality (28) above yields

$$2-2p^*-p^{*2} > \left(\frac{1-p^*-p^{*2}}{p^*}\right)^2 \Leftrightarrow 2p^*-2p^{*4}+3p^{*2}-4p^{*3} > 1 \tag{29}$$

We know that $p^* \in (\sqrt{2}-1, 1/2]$ in any equilibrium with active search. This follows from the necessary condition that $w^* \geq p^*$ and $w^* = 1-\sqrt{2s}$. It can easily be verified that $2p^*-2p^{*4}+3p^{*2}-4p^{*3} = 1$ for $p = \sqrt{2}-1$. Thus, inequality (29) holds if

$$\partial_{p^*}\left(2p^*-2p^{*4}+3p^{*2}-4p^{*3}\right) > 0 \text{ for all } p^* \geq \sqrt{2}-1 \tag{30}$$

$$\Leftrightarrow (2-8p^{*3})+(6p^*-12p^{*2}) > 0 \text{ for all } p^* \in [\sqrt{2}-1, 1/2] \tag{31}$$

$$\Leftrightarrow (1-4p^{*3})+3p^*(1-2p^*) > 0 \text{ for all } p^* \in [\sqrt{2}-1, 1/2] \tag{32}$$

Since $p^* \leq 1/2$, it is easy to verify that $1-4p^{*3} > 0$ and $3p^*(1-2p^*) \geq 0$, implying that the above inequality always holds. This completes the proof.

A.5 Proof of Proposition 3

Let $D(p_j, p^*)$ represent firm j's demand if it does not disclose and charges a price p_j. Also, let $D^d(p_j, p^*, p_2, p_3)$ denote firm j's demand after disclosure (deviation), where p_2 is the price firm $-j$ sets if $\mathcal{H} = R$ and p_3 the revised price if $\mathcal{H} = KR \times p^*$. In the no disclosure equilibrium profits are $\max_{p_j} p_j D(p_j, p^*)$ while they are $\max_{p_j} p_j D^d(p_j, p^*, p_2, p_3)$ if firm j deviates. Thus, a deviation cannot be profitable if

$$D(p_j, p^*) < D^d(p_j, p^*, p_2, p_3) \text{ for all } p_j. \tag{33}$$

Let $p_2 = p^* + \delta$ and $p_3 = p^* - \delta - \epsilon$. By Lemma 1 and Lemma 2, $\delta > 0$ but the sign of ϵ is unknown. Then, $D^d(p_j, p^*, p_2, p_3) = D^d(p_j, p^*, p^*+\delta, p^*-\delta-\epsilon)$. It is easy to verify that $\partial_\epsilon D^d < 0$. Thus, (33) holds if (i) $\epsilon \geq 0$ and (ii) $\partial_\delta D^d < 0$. Begin with (i) and notice that:

$$\begin{aligned}\epsilon =&(p^* - p_3) - \delta = (p^* - p_3) - (p_2 - p^*) && (34)\\ =&\frac{10}{12}p^* - \frac{5}{12}w^* - \frac{6}{12} + \frac{3}{12}\frac{p^{*2}}{w^*} + \frac{4}{12}\sqrt{w^{*2} + 2w^*p^* + 4p^{*2}} && (35)\end{aligned}$$

Substituting $w^* = (1 - p^* - p^*)/p^*$, which follows from the equilibrium condition pinning down p^*, into the expression above yields that $\epsilon \geq 0$ if and only if

$$\frac{1}{12}\left(-1 - \frac{5}{p^*} + 15p^* - \frac{3(p^*)^3}{-1 + p^* + (p^*)^2} + 4\sqrt{1 + \frac{1}{(p^*)^2} - \frac{2}{p^*} + 3(p^*)^2}\right) \geq 0 \quad (36)$$

$$\Leftrightarrow \left(4\sqrt{1 + \frac{1}{(p^*)^2} - \frac{2}{p^*} + 3(p^*)^2}\right)^2 \geq \left(1 + \frac{5}{p^*} - 15p^* + \frac{3(p^*)^3}{-1 + p^* + (p^*)^2}\right)^2 \quad (37)$$

$$\Leftrightarrow 16\left(1 + \frac{1}{(p^*)^2} - \frac{2}{p^*} + 3(p^*)^2\right) \geq \frac{(5 - 4p^* - 21(p^*)^2 + 14(p^*)^3 + 12(p^*)^4)^2}{(p^*)^2(-1 + p^* + (p^*)^2)^2} \quad (38)$$

$$\begin{aligned}&\Leftrightarrow 16 - 64p^* + 64(p^*)^2 + 32(p^*)^3 - 16(p^*)^4 - 96(p^*)^5 - 32(p^*)^6 + 96(p^*)^7 + 48(p^*)^8 \geq\\ &\quad 25 - 40p^* - 194(p^*)^2 + 308(p^*)^3 + 449(p^*)^4 - 684(p^*)^5 - 308(p^*)^6 + 336(p^*)^7 + 144(p^*)^8\\ &\Leftrightarrow -3(3 + 8p^* - 86(p^*)^2 + 92(p^*)^3 + 155(p^*)^4 - 196(p^*)^5 - 92(p^*)^6 + 80(p^*)^7 + 32(p^*)^8) \geq 0\end{aligned}$$

One can easily verify that the left-hand side equals 0 if $p^* = \sqrt{2} - 1$ and 3/8 if $p^* = 1/2$. Thus, the inequality follows from the concavity of the left-hand side over $[\sqrt{2} - 1, 1/2]$. The second derivative is given by

$$-12(-43 + 138p^* + 465(p^*)^2 - 980(p^*)^3 - 690(p^*)^4 + 840(p^*)^5 + 448(p^*)^6) \quad (39)$$

To show that (39) is negative, we show separately that $465(p^*)^2 - 980(p^*)^3 + 448(p^*)^6$ and $-43 + 138p^* - 690(p^*)^4 + 840(p^*)^5$ are both non-negative for all $p^* \in [\sqrt{2} - 1, 1/2]$.

Consider the first term:

$$465(p^*)^2 - 980(p^*)^3 + 448(p^*)^6 \geq 0 \Leftrightarrow 465 \geq 980p^* - 448(p^*)^4 \tag{40}$$

It can be verified that inequality (40) holds at $p^* = 1/2$ $(465 > 462)$. Moreover, $980p^* - 448(p^*)^4$ increases in p^* for all $p^* \leq (35/64)^{1/3}$ (note $(35/64)^{1/3} > 1/2$) so that (40) holds for all $p^* \in [\sqrt{2}-1, 1/2]$. Consider the second term next:

$$-43 + 138p^* - 690(p^*)^4 + 840(p^*)^5 \geq 0 \tag{41}$$

Again, it can be verified that inequality (41) holds (strictly) at $p^* = \sqrt{2} - 1$. Thus, it is sufficient to show that the left-hand-side of (41) increases in p^*. By taking the derivative, we see that this conditions holds if and only if

$$6(23 - 460(p^*)^3 + 700(p^*)^4) \geq 0 \Leftrightarrow 23 \geq 460(p^*)^3 - 700(p^*)^4 \tag{42}$$

One can check that the function $460(p^*)^3 - 700(p^*)^4$ obtains its maximum at $p^* = 69/140$. Since $23 > 460(69/140)^3 - 700(69/140)^4$, we know that (42) holds for all $p^* \in [\sqrt{2}-1, 1/2]$. This completes the proof of part (i).

Consider part (ii) next. $D^d(p_j, p^*, p^* + \delta, p^* - \delta - \epsilon)$ is given by

$$\frac{1}{2}\left(w^*(1 - w^* + p^* - \delta - \epsilon - p_j) + \int_{p_j}^{w^* - p^* + \delta + \epsilon + p_j} (p^* - \delta - \epsilon + u_j - p_j)\mathrm{d}u_j\right)$$
$$+\frac{1}{2}\left((1 - w^* + p^* - p_j) + \int_{p_j}^{w^* - p^* + p_j} (p^* + \delta + u_j - p_j)\mathrm{d}u_j\right).$$

Thus,

$$\partial_\delta D^d = \frac{1}{2}\left((-1)w^* + (1)w^* + \int_{p_j}^{w^* - p^* + \delta + \epsilon + p_j} (-1)\mathrm{d}u_j\right) + \frac{1}{2}\int_{p_j}^{w^* - p^* + p_j} (1)\mathrm{d}u_j \tag{43}$$

$$= -\frac{\delta + \epsilon}{2} < 0. \tag{44}$$

Non-disclosure by firm j upon being visited second

Suppose that firm j deviates and discloses to firm $-j$, who now receives disclosure for the unknown buyer. This firm $-j$ thus offers the consumer the price p_2, as described above. The previous arguments have assumed that this firm $-j$ will not disclose to firm j even when observing disclosure.

Recall that this firm believes that firm j offered the consumer the price p^*. When disclosing to firm j, firm $-j$ expects firm j to be in the revision information set where it initially offered p^* and consumers left to search if and only if $u_j < w^*$. Thus, firm $-j$ believes firm j to choose the price $p_3(p^*)$ which maximizes the following profit function:

$$\Pi^{3,dd}(p_j) = p_j \int_{p_j}^{w^*} 0.5F(u_j - p_j + p_2)du_j$$

The optimal price in this information set is given by:

$$p_3^{dd} = (2/3)(w^* + p_2) + / - (1/3)\sqrt{(w^*)^2 + 2w^*p_2 + 4(p_2)^2}$$

We verify numerically that this price is always below p^*, which would make disclosure not worthwhile for firm $-j$ in this situation.

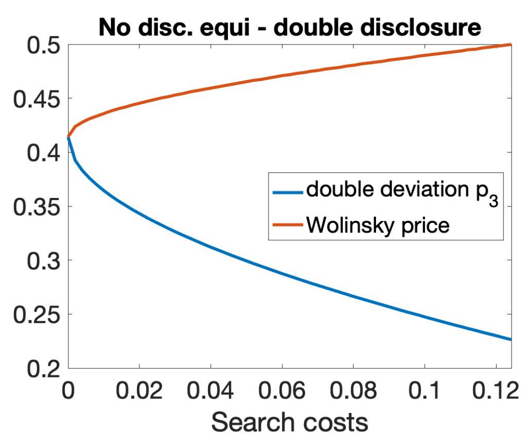


Figure 6: No disclosure equilibria - double disclosure

A.6 Characterization of the partial disclosure equilibrium under price revisions

The equilibrium prices p_1^*, p_2^* have been determined previously. It remains to determine the objects $p_3(p_1)$ and p_1^{nd}, which represent the optimal revision prices (as a function of the initially set price) and the optimal price set after deviating by non-disclosure.

Optimal deviation pricing - general:

Suppose that a firm deviates and does not disclose to it's competitor that a given buyer, for which no disclosure was received previously, has arrived at this firm.

By this deviation, the consumer will now receive the price p_1^* instead of the price p_2^* at the other firm. Moreover, the other firm, believing this consumer to have arrived first, will disclose to the initially deviating firm, allowing for a price revision.

The consumer, however, is not aware of the deviation and will thus expect to receive the price p_2^* at the second firm, which makes the cutoff function $w^1(p_1)$ the correct one to characterize her search behavior.

The price $p_3(p_1)$ that is set conditions on p_1 - this is because the information sets at the time of the revision depend on the price p_1 that was initially set by the deviating firm. Firm beliefs about consumer valuations when revising it's price have to be consistent. Thus, the firm knows that, given an arbitrary initial price p_1, the buyer will have search if and only if $u_j < w^1(p_1)$.

Optimal revision pricing after deviation - $p_3(p_1)$

Now we pin down the function $p_3(p_1)$. In this information set, the profit function of the deviating firm is:

$$\Pi^{3,d}(p_j) = p_j \int_{p_j}^{w^1(p_1)} 0.5F(u_j - p_j + p_1^*)du_j = p_j \int_{p_j}^{w^1(p_1)} 0.5(u_j - p_j + p_1^*)du_j$$

We restrict attention to third period prices which satisfy $w^1(p_1) - p_j + p_1^* < 1$ (and verify later that the optimal revision price will satisfy this). This allows us to rewrite the above profit function with the second equality. Thus, the optimal deviation price p_3 needs to solve the following FOC:

$$\int_{p_j}^{w^1(p_1)} 0.5(u_j - p_j + p_1^*)du_j - p_j 0.5\big(p_1^*\big) + p_j \int_{p_j}^{w^1(p_1)} 0.5(-1)du_j = 0$$

$$\iff$$

$$0.5(w^1(p_1) - p_3 + p_1^*)^2 - 0.5(p_1^*)^2 - p_3 p_1^* - p^3\big(w^1(p_1) - p_3\big) = 0$$

By the implicit function theorem and noting that $w^{1,\prime}(p_1) = 1$, we can write the following:

$$\frac{dp_3(p_1)}{dp_1} = -\frac{(w^1(p_1) - p_3 + p_1^*) - p_3}{-(w^1(p_1) - p_3 + p_1^*) - p_1^* - (w^1(p_1) - 2p_3)} = \frac{w^1(p_1) - 2p_3 + p_1^*}{2w^1(p_1) - 3p_3 + 2p_1^*}$$

Optimal initial pricing after non-disclosure - p_1^{nd}

Note that $p_3(p_1) < w^1(p_1)$ must hold if any profits are to be made by setting $p_3(p_1)$. Thus, the profit function of a firm at the information set just after deviating (by not disclosing) is the following:

$$\Pi^{1,nd}(p_1) = p_1 0.5\big[1 - F(w^1(p_1))\big] + p_3(p^1) \int_{p_3(p_1)}^{w^1(p_1)} 0.5F(p_1^* + u_j - p_3(p_1))du_j$$

Taking the derivative of this w.r.t. the price p^1 is:

$$\frac{\partial \Pi^{1,nd}(p_1)}{\partial p_1} = p_1 0.5[-1] + 0.5\big[1 - F(w^1(p_1))\big] + p_3(p_1)\Big[0.5(p_1^* + w^1(p_1) - p_3(p_1))$$

$$-0.5(p_1^*)\frac{\partial p_3(p_1)}{\partial p_1}-\int_{p_3(p_1)}^{w^1(p_1)}0.5\frac{\partial p_3(p_1)}{\partial p_1}du_j\Bigg]+\frac{\partial p_3(p_1)}{\partial p_1}\left[\int_{p_3(p_1)}^{w^1(p_1)}0.5(p_1^*+u_j-p_3(p_1))du_j\right]$$

Thus, the optimal deviation price p_1^{nd} must solve the following first-order condition:

$$\big[1-p_1-w^1(p_1)\big]+p_3(p_1)(p_1^*+w^1(p_1 1)-p_3(p_1))+$$

$$\frac{\partial p_3(p_1)}{\partial p_1}\left[-p_3(p_1)p_1^*-p_3(p_1)\big(w^1(p_1)-p_3(p_1)\big)+\int_{p_3(p_1)}^{w^1(p_1)}(p_1^*+u_j-p_3(p_1))du_j\right]=0$$

Verifying that no-disclosure is optimal when being visited second

This information set is on-path. Consider a firm $-j$ that is at this information set. The firm is thus sure that firm j set the price p_1^* and consumers searched if and only if $u_j \leq w^{1,*} = w^* - p_2^* + p_1^*$. By deviating and disclosing, the firm thus expects the other firm to set its revision price corresponding to the initial price p_1^*. The objective function which this revision price must maximize is given by:

$$\Pi^{3,dd}(p_j) = p_j \int_{p_j}^{w^{1,*}} 0.5F(u_j - p_j + p_2^*)du_j$$

The optimal revision price is thus

$$p_3^{dd} = (2/3)(w^{1,*} + p_2^*) + / - (1/3)\sqrt{(w^{1,*})^2 + 2w^{1,*}p_2^* + 4(p_2^*)^2}$$

We verify that this revision price p_3^{dd} lies below p_1^* in the following graph:

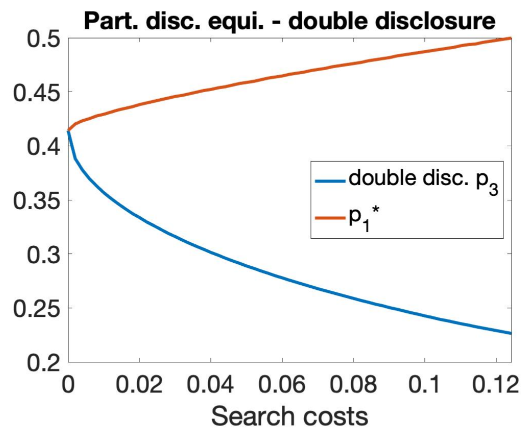


Figure 7: Partial disclosure equilibria - double disclosure

A.7 Characterization of the full disclosure equilibrium under price revisions

Consider first an equilibrium candidate where all firms always disclose to their competitors. Then, the following information sets are on-path: (i) a seller meets the buyer for the first time and has not received disclosure, (ii) a seller meets the buyer for the first time and has received disclosure, and (iii), a seller receives disclosure for a known buyer. For ease of exposition, define the prices that are offered after each of these information sets as p_1^*, p_2^*, and p_3^*, respectively.

As part of the equilibrium pricing strategy, there is also a revision price function for information sets where the initially visited firm gets to revise its price, namely $p_3^*(p_1)$. Note that $p_3^* := p_3(p_1^*)$.

There are two subcases, namely (i) $p_1^* > p_3^*$ and (ii) $p_1^* \leq p_3^*$. The first subcase cannot be an equilibrium - then, no firm would have incentives to disclose when being visited second (since this reduces the competitor's price and intensifies competition), breaking the equilibrium. Thus, we focus on the subcase $p_1^* \leq p_3^*$ in the following.

On-path search:

When receiving the initial price p_1^*, the consumer with match value u_j will thus search if and only if:

$$\int_0^1 \max\{u_j - p_3^*, u_{-j} - p_2^*, 0\} du_{-j} - s > \max\{u_j - p_1^*\} \iff$$

$$\int_0^{u_j - p_3^* + p_2^*} \max\{u_j - p_3^*, 0\} du_{-j} + \int_{u_j - p_3^* + p_2^*}^1 \max\{u_{-j} - p_2^*, 0\} du_{-j} - s > \max\{u_j - p_1^*\}$$

There are three relevant intervals of valuations, namely (i) $[0, p_1^*]$, (ii) $(p_1^*, p_3^*]$, and (iii) and $(p_3^*, 1]$.

We can show that the gains of search are strictly decreasing for $u_j \in (p_1^*, 1]$ and constant before that. Thus, in for there to be search-on-path, the gains of search for consumers with $\nu \leq p_1^*$ must be strictly positive, i.e.:

$$\int_{p_2^*}^1 (u_{-j} - p_2^*) du_{-j} - s > 0$$

Note that we will make use of this condition again later. The actual equilibrium search cutoff $w^1(p_1^*)$ must be strictly above the equilibrium $p_3(p_1^*)$ in order for the latter to be set optimally.

Sequentially rational search strategy:

Consider the search decision of a consumer who has visited firm j first, drawn the match value u_j, and now decides whether or not to search when being offered the (potentially off-equilibrium) price p_1.

Consumers have passive beliefs - thus, they believe that the initially visited firm still discloses to it's competitor. Thus, the price this consumer will expect at firm $-j$ after searching is p_2^*. Moreover, firm j will revise its price after receiving disclosure, ultimately offering the consumer the price $p_3(p_1)$ - note that this function is the one the consumer will expect by the passive beliefs assumption.

When receiving the initial price p_1, the consumer with match value u_j will thus search if and only if:

$$\int_0^{u_j - p_3(p_1) + p_2^*} \max\{u_j - p_3(p_1), 0\} du_{-j} + \int_{u_j - p_3(p_1) + p_2^*}^1 \max\{u_{-j} - p_2^*, 0\} du_{-j} - s > \max\{u_j - p_1, 0\}$$

We are examining perfect Bayesian equilibria where the price $p_3(p_1)$ must be chosen optimally, given $w^1(p_1)$. In order for this price to be optimal, it must induce a $w^1(p_1) > p_3(p_1)$.

Thus, this cutoff must satisfy $w^1(p_1) \in (p_3(p_1), 1]$. For such u_j, the LHS of the search equation thus becomes:

$$\int_0^{u_j - p_3(p_1) + p_2^*} (u_j - p_3(p_1)) du_{-j} + \int_{u_j - p_3(p_1) + p_2^*}^1 (u_{-j} - p_2^*) du_{-j} - s$$

Note that this expression is strictly increasing in u_j. Note also that this expression needs to be strictly positive at $u_j = p_3(p_1)$ in order for there to be search on-path, because it collapses to the following in that case:

$$\int_{p_2^*}^1 (u_{-j} - p_2^*) du_{-j} - s > 0$$

Now suppose $p_1 > p_3(p_1)$. Then, the gains of search in the interval $[p_3(p_1), p_1]$ are also strictly increasing and must be strictly positive. Thus, the search cutoff must be above p_1. It must solve the following expression:

$$\int_0^{u_j-p_3(p_1)+p_2^*}(u_j-p_3(p_1))du_{-j}+\int_{u_j-p_3(p_1)+p_2^*}^1(u_{-j}-p_2^*)du_{-j}-s=(u_j-p_1)$$

All consumers with a match value $u_j < w^1(p_1)$ would have strictly positive incentives to search.

Suppose instead that $p_1 < p_3(p_1)$. In that case, the search cutoff must also solve the above expression. Because the gains of search must be strictly decreasing for $\nu \in [p_1, p_3(p_1)]$ and above, all consumers with a match value below this cutoff will search.

One can thus pin down generalized cutoffs with the function $w^1(p_1)$ that sets the following equation equal to 0.

$$T(u_j,p_j):=F(u_j-p_3^d(p_1)+p_2^*)(u_j-p_3^d(p_1))+\int_{u_j-p_3^d(p_1)+p_2^*}^1(u_{-j}-p_2^*)du_{-j}-s-(u_j-p_1)=0$$

Optimal second-period pricing:

Subcase 2: $p_1^* \leq p_3^*$.

Consider a firm $-j$ that has received disclosure from firm j, which the consumer has visited previously. Suppose the firm $-j$ follows its equilibrium strategy and discloses, which induces firm j to offer any consumer that arrives at this information set to receive the price p_3^* from them.

Note also that any such consumer that arrives after searching must have a match value below $\hat{u}^1$. Define the following function:

$$w^2(p_{-j})=\hat{u}^1-p_3^*+p_{-j}$$

Any consumer with $u_{-j} \geq w^2(p_{-j})$ would thus surely buy at firm $-j$. Any buyer with $u_{-j} \in \big(p_{-j}, w^2(p_{-j})\big)$ prefers firm $-j$ if $u_j < p_3^* + u_{-j} - p_{-j}$, i.e. with probability $F(p_3^* + u_{-j} - p_{-j})$. Note that these conditions are jointly sufficient for search to occur, since:

$$u_j < p_3^* + u_{-j} - p_{-j} \leq p_3^* + w^2(p_{-j}) - p_{-j} = \hat{u}^1$$

Thus, the demand function of a firm in this information set is:

$$D^2(p_{-j}) = 0.5F(\hat{u}^1)\big[1 - F(w^2(p_{-j}))\big] + \int_{p_{-j}}^{w^2(p_{-j})} 0.5F(p_3^* + u_{-j} - p_{-j})du_{-j}$$

The derivative of this w.r.t p_j is:

$$\frac{\partial D^2(p_{-j})}{\partial p_{-j}} = -0.5\hat{u}^1$$

Thus, the equilibrium price must satisfy the following:

$$D^2(p^2)+p^2\frac{\partial D^2(p^2)}{\partial p_j} = 0 \iff F(\hat{u}^1)\big[1-F(w^2(p^2))\big]+\int_{p^2}^{w^2(p^2)} F(p^3+u_{-j}-p^2)du_{-j}+p^2\Big[-\hat{u}^1\Big] = 0$$

$$\iff$$

$$2p_2^*\hat{u}^1 - \hat{u}^1\big[1 - \hat{u}^1 + p_3^*\big] - 0.5(\hat{u}^1)^2 + 0.5(p_3^*)^2 = 0$$

Optimal third-period pricing:

Subcase 2: $p_1^* \leq p_3^*$.

Third-period pricing is made challenging by the fact that the price a given firm has set initially (p_1) is part of this firm's information set when deciding the price it would set upon given the chance to revise.

Note that only the information set where the price p_1^* was set initially is on-path. All other information sets are off-path, which means beliefs at these information sets are determined by our consistency requirement, together with the passive beliefs assumption. Thus, the firm believes that consumers have searched according to the rule $w^1(p_1)$.

Thus, the profits a firm makes in this information set are:

$$\Pi^3(p_3) = p_3 \int_{p_3}^{w^1(p_1)} 0.5F(u_j - p_3 + p_2^*)du_j$$

The derivative of this w.r.t. p_3 for an interior $p_3 \leq w^1(p_1)$ is:

$$\int_{p_3}^{w^1(p_1)} 0.5F(u_j - p_3 + p_2^*)du_j - p_3 0.5F(p_2^*) + p_3 \int_{p_3}^{w^1(p_1)} 0.5(-1)du_j =$$

$$\int_{p_j}^{w^1(p_1)} 0.5(u_j - p_3 + p_2^*)du_j - 0.5p_3\big[w^1(p_1) - p_3 + p_2^*\big]$$

The equilibrium price $p_3(p_1) < w^1(p_1)$ must satisfy the following first-order condition:

$$\int_{p_3}^{w^1(p_1)} 0.5(u_j - p_3 + p_2^*)du_j - 0.5p_3\big[w^1(p_1) - p_3 + p_2^*\big] = 0$$

$$\Longleftrightarrow$$

$$1.5(p_3)^2 + p_3\big(-2w^1(p_1) - 2p_2^*\big) + \big[0.5w^1(p_1)^2 + p_2^* w^1(p_1)\big] = 0$$

The solution to this polynomial is given by the following:

$$p_3(p_1) = (2/3)(w^1(p_1) + p_2^*) + / - (1/3)\sqrt{(w^1(p_1))^2 + 2w^1(p_1)p_2^* + 4(p_2^*)^2}$$

Optimal first-period pricing:

Subcase 2: $p_1^* \leq p_3^*$.

Now we pin down the demand function for consumers that arrive at a firm first. To define this, we have defined a function $w^1(p_1)$ (together with its derivative) that helps characterize which consumers search. Note also that any consumer with $u_j > w^1(p_j)$ will surely consume because the utility of searching must always be strictly positive. Thus, the profit function is:

$$\Pi^1(p_1) = 0.5\big[1 - F(w^1(p_1))\big]p_1 + p_3(p_1)\int_{p_3(p_1)}^{w^1(p_1)} 0.5F(p_2^* + u_j - p_3(p_1))du_j$$

Numerical solution procedure:

Step 1: For any $\hat{u}^1$, find the corresponding p_2^* and p_3^* (i.e. treat $\hat{u}^1$ as a parameter and solve the system of the following two equations):

$$p_2^*\hat{u}^1 - \hat{u}^1\big[1 - \hat{u}^1 + p_3^*\big] - 0.5(\hat{u}^1)^2 + 0.5(p_3^*)^2 = 0$$

$$\Longleftrightarrow$$

$$1.5(p_3^*)^2 + p_3^*\big(-2\hat{u}^1 - 2p_2^*\big) + \big[0.5(\hat{u}^1)^2 + p_2^*\hat{u}^1\big] = 0$$

As a result, we get functions $p_2^*(\hat{u}^1)$ and $p_3^*(\hat{u}^1)$.

Step 2: For any $\hat{u}^1$, find the respective $p_1^*(\hat{u}^1)$ that is optimal. Calculate the resulting search cutoff from the optimal consumer search behavior. If these search cutoffs are equal, we have an equilibrium.

A.8 Exogenous provision of search history information

Now suppose that firms exogenously receive information about the search path of consumers. Thus, firms can set prices to first arrivers, searchers, and return buyers.

We conjecture that $p_1^* > p_3^*$ would hold in such an equilibrium based on the previous findings.

Equilibrium search:
It is instructive to first consider the search decisions of the buyer when she only receives the equilibrium prices p_1^*, p_2^*, and p_3^*.

We characterize this search behavior for three different intervals of match values, (i) $u_j \in [0, p_3^*]$, (ii) $u_j \in (p_3^*, p_1^*]$, (iii) $u_j \in (p_1^*, 1]$.

In general, the buyer will search at p_1^* if and only if:

$$\int_0^{u_j - p_3^* + p_2^*} \max\{u_j - p_3^*, 0\} du_{-j} + \int_{u_j - p_3^* + p_2^*}^1 \max\{u_{-j} - p_2^*, 0\} du_{-j} - s > \max\{u_j - p_1^*, 0\}$$

Consider first $u_j \in [0, p_3^*]$, where the gains of search at $p_1^* > p_3^*$ become:

$$0 + \int_{p_2^*}^1 (u_{-j} - p_2^*) du_{-j} - s - 0$$

Note that these gains of search are independent of the initial match value.

Second, consider $u_j \in (p_3^*, p_1^*]$, where the gains of search become:

$$\int_0^{u_j - p_3^* + p_2^*} (u_j - p_3^*) du_{-j} + \int_{u_j - p_3^* + p_2^*}^1 (u_{-j} - p_2^*) du_{-j} - s - 0$$

Note that these gains of search are strictly increasing in u_j in this interval $u_j \in (p_3^*, p_1^*]$.

Thirdly, consider the gains of search for consumers with $u_j > p_1^*$:

$$\int_0^{u_j - p_3^* + p_2^*} (u_j - p_3^*) du_{-j} + \int_{u_j - p_3^* + p_2^*}^1 (u_{-j} - p_2^*) du_{-j} - s - (u_j - p_1^*) =$$

$$\int_{0}^{u_j-p_3^*+p_2^*}(p_1^*-p_3^*)du_{-j}+\int_{u_j-p_3^*+p_2^*}^{1}\big((u_{-j}-p_2^*)-(u_j-p_1^*)\big)du_{-j}-s$$

The derivative of these gains of search w.r.t. u_j is:

$$\frac{\partial}{\partial u_j}=(p_1^*-p_3^*)-\big((u_j-p_3^*+p_2^*-p_2^*)-(u_j-p_1^*)\big)+\int_{u_j-p_3^*+p_2^*}^{1}\big(-1\big)du_{-j}<0$$

Thus, the maximum of the gains of search will be reached at $u_j=p_1^*$.

Be advised - there are two possible scenarios. Firstly, it could be that:

$$\int_{p_2^*}^{1}(u_{-j}-p_2^*)du_{-j}-s>0$$

Then, there will be an search cutoff $\hat{u}^1>p_1^*$, since the gains of search must be strictly positive for all $u_j\leq p_1^*$.

Secondly, it could hold that:

$$\int_{p_2^*}^{1}(u_{-j}-p_2^*)du_{-j}-s\leq 0$$

Note that $p_2^*<w^*$ is a sufficient condition to ensure that there is an interior search cutoff, since this implies that the latter inequality (first case) is satisfied. In the following, we thus work with the specification that there is a unique search cutoff for on-path search - later, we verify that the necessary property $p_2^*<w^*$ holds true.

Sequentially rational search:
Now consider an arbitrary off-equilibrium initial price p_1. Our earlier specification guarantees that there must be a cutoff above 0, since low- u_j consumer would surely have strictly positive gains of search.

Given this cutoff formulation, we can be sure that $p_3(p_1)<w^1(p_1)$ must hold -

else the firm would make no profits in this information set.

Thus, the cutoff (if it is interior), must set the following gains of search equal to zero.

$$gos(u_j) = \int_0^{\min\{\max\{u_j - p_3(p_1), 0\} + p_2^*, 1\}} \max\{u_j - p_3(p_1), 0\} du_{-j} +$$

$$\int_{\min\{\max\{u_j - p_3(p_1), 0\} + p_2^*, 1\}}^1 (u_{-j} - p_2^*) du_{-j} - s - \max\{u_j - p_1, 0\}$$

If these gains of search are strictly positive for any u_j, the cutoff is 1, i.e. $w^1(p_1) = 1$.

Given that the cutoff formulation is the same, the optimal pricing decisions will remain exactly the same. Thus, we can use the previous calculations to pin down the prices p_1^*, p_2^*, p_3^*.

Buyer welfare

We define buyer welfare as the ex-ante expected utility of the buyer. To achieve this, define $u^s(u_j)$ and $u^{ns}(u_j)$ as the expected utilities of searching and not searching, respectively, for a buyer that draw an initial match value u_j.

Consider the following general formulation where we define p_1^* as the price the buyer would receive at the initial firm she visits and p_2^* and p_3^* as the other prices she could receive second (or after a revision) on the search path. Note that this formulation nests all our equilibria.

Noting this, the ex-ante utility CS of the buyer is:

$$CS = \int_0^{\hat{u}^1} u^s(u_j) du_j + \int_{\hat{u}^1}^1 u^{ns}(u_j) du_j$$

Note that that:

$$u^{ns}(u_j) = \max\{u_j - p_1^*, 0\} - s$$

Note further that:

$$u^s(u_j) = \int_0^{u_j - p_3^* + p_2^*} \max\{u_j - p_3^*, 0\} du_{-j} + \int_{u_j - p_3^* + p_2^*}^1 \max\{u_{-j} - p_2^*, 0\} du_{-j} - 2s$$

$$=$$

$$\int_0^{\max\{u_j - p_3^*, 0\} + p_2^*} \max\{u_j - p_3^*, 0\} du_{-j} + \int_{\max\{u_j - p_3^*, 0\} + p_2^*}^1 (u_{-j} - p_2^*) du_{-j} - 2s$$

Visualization of search cutoff function

In the following graph, we plot the function $w^1(p_1)$ for different levels of search costs, where the initial prices are plotted on the x-axis.

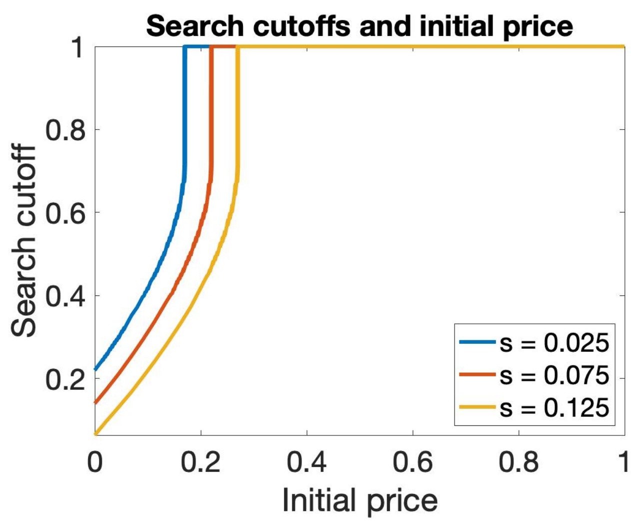


Figure 8: Exogenous information provision - search cutoffs

This graph underscores why deviations to low initial prices where $w^1(p_1)$ is interior are not optimal. In order to achieve an interior $w^1(p_1)$, p_1 must be sufficiently low. Moreover, this figure shows that any such interior $w^1(p_1)$ must be significantly below 1. This follows from the functional form of the gains of search for high u_j-buyers.

For u_j's that are sufficiently close to 1, the gains of search are independent of u_j. This notion implies that the jump of the cutoff function $w_1(p_1)$ towards 1 at the initial price that induces all these consumers to search. Thus, reducing p_1 to attain an interior $w^1(p_1)$ will directly imply a downward jump in the revision price $p_3^*(p_1)$, because the set of consumers who search changes discontinuously. Thus, attaining an interior $w^1(p_1)$ goes along with a low p_1 and a low p_3, which makes this form of screening suboptimal.

B Further material

B.1 No Price Revision - No Disclosure Equilibrium

In this graph, we plot the difference between disclosure profits for an optimally chosen price and equilibrium profits.

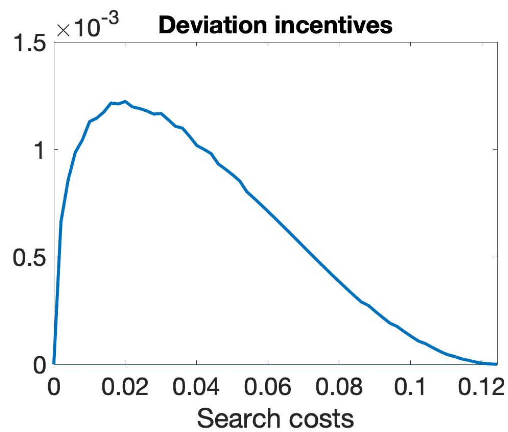


Figure 9: No disclosure equilibria - deviation incentives

B.2 No Price Revision - Disclosure Equilibrium

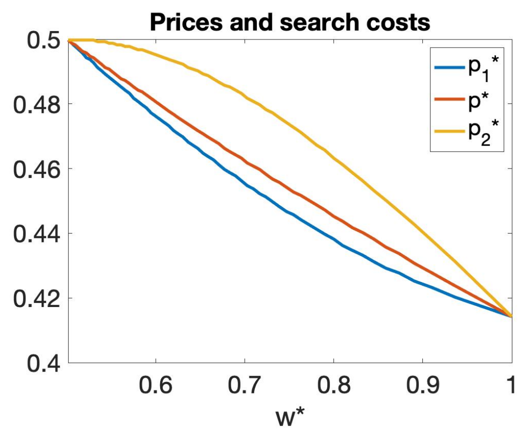
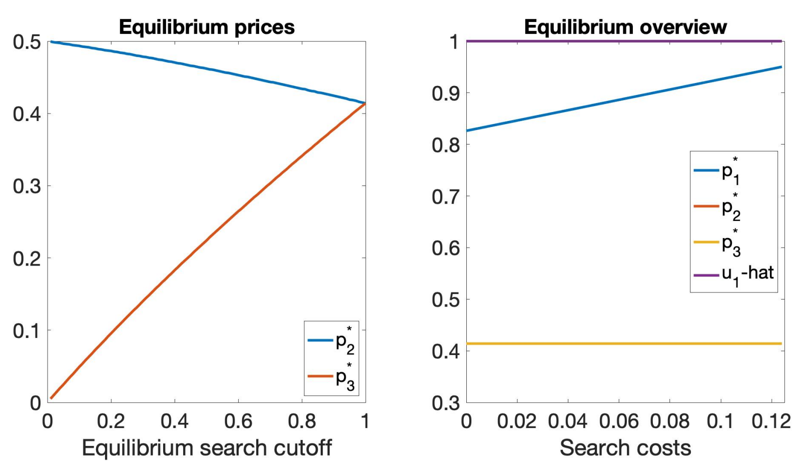


Figure 10: Partial disclosure equilibrium prices

B.3 Price Revisions - Full Disclosure Equilibrium

In the left graph, we plot the equilibrium values of p_2^* and p_3^* that are obtained for a given equilibrium $\hat{u}^1$. In the right graph, we plot the prices and search cutoff that would solve the system of simultaneous equations outlined in the text:



References

S. P. Anderson and R. Renault. Pricing, product diversity, and search costs: A bertrand-chamberlin-diamond model. The RAND Journal of Economics, pages 719–735, 1999.

M. Armstrong. Ordered consumer search. Journal of the European Economic Association, 15(5):989–1024, 2017.

M. Armstrong and J. Zhou. Search deterrence. The Review of Economic Studies, 83 (1):26–57, 2016.

M. Armstrong, J. Vickers, and J. Zhou. Prominence and consumer search. The RAND Journal of Economics, 40(2):209–233, 2009.

D. Bergemann, B. Brooks, and S. Morris. Search, information, and prices. Journal of Political Economy, 129(8):2275–2319, 2021.

S. Board and M. Pycia. Outside options and the failure of the coase conjecture. American Economic Review, 104(2):656–71, 2014.

Y. Chen, C. Narasimhan, and Z. J. Zhang. Individual marketing with imperfect targetability. Marketing Science, 20(1):23–41, 2001.

C. Choe, N. Matsushima, and M. J. Tremblay. Behavior-based personalized pricing: When firms can share customer information. ISER DP, (1083), 2020.

R. N. Clarke. Collusion and the incentives for information sharing. The Bell Journal of Economics, pages 383–394, 1983.

R. H. Coase. Durability and monopoly. The Journal of Law and Economics, 15(1): 143–149, 1972.

A. De Corniere. Search advertising. American Economic Journal: Microeconomics, 8(3):156–88, 2016.

R. De Nijs. Behavior-based price discrimination and customer information sharing. International Journal of Industrial Organization, 50:319–334, 2017.

N. Fabra and M. Reguant. A model of search with price discrimination. European Economic Review, 129:103571, 2020.

E. Gal-Or. Information sharing in oligopoly. Econometrica: Journal of the Econometric Society, pages 329–343, 1985.

E. Gal-Or. Information transmission—cournot and bertrand equilibria. The Review of Economic Studies, 53(1):85–92, 1986.

C.-C. Groh. Competitive price discrimination, imperfect information, and consumer search. 2021.

F. Gul, H. Sonnenschein, and R. Wilson. Foundations of dynamic monopoly and the coase conjecture. Journal of Economic Theory, 39(1):155–190, 1986.

M. A. Haan and J. L. Moraga-González. Advertising for attention in a consumer search model. The Economic Journal, 121(552):552–579, 2011.

J. Johnson, T. Jungbauer, and M. Preuss. Online advertising, data sharing, and consumer control. Data Sharing, and Consumer Control (August 2, 2021), 2021.

B.-C. Kim and J. P. Choi. Customer information sharing: Strategic incentives and new implications. Journal of Economics & Management Strategy, 19(2):403–433, 2010.

X. Lin, X. Huang, S. Liu, Y. Li, H. Luo, and S. Yu. Social welfare analysis under different levels of consumers' privacy regulation. Journal of Theoretical and Applied Electronic Commerce Research, 16(7):2943–2964, 2021.

Q. Liu and K. Serfes. Customer information sharing among rival firms. European Economic Review, 50(6):1571–1600, 2006.

E. Mauring. Search and price discrimination online. 2021.

J. L. Moraga-González and V. Petrikaitė. Search costs, demand-side economies, and the incentives to merge under bertrand competition. The RAND Journal of Economics, 44(3):391–424, 2013.

W. Novshek and H. Sonnenschein. Fulfilled expectations cournot duopoly with information acquisition and release. The Bell Journal of Economics, pages 214–218, 1982.

J. M. Perloff and S. C. Salop. Equilibrium with product differentiation. The Review of Economic Studies, 52(1):107–120, 1985.

M. Preuss. Search, learning and tracking. Working Paper, 2021.

J. Riley and R. Zeckhauser. Optimal selling strategies: When to haggle, when to hold firm. The Quarterly Journal of Economics, 98(2):267–289, 1983.

C. Shapiro. Exchange of cost information in oligopoly. The review of economic studies, 53(3):433–446, 1986.

C. R. Taylor. Consumer privacy and the market for customer information. RAND Journal of Economics, pages 631–650, 2004.

R. Ursu, Q. P. Zhang, and E. Honka. Search gaps. Available at SSRN 3757724, 2021.

X. Vives. Duopoly information equilibrium: Cournot and bertrand. Journal of economic theory, 34(1):71–94, 1984.

A. Wolinsky. True monopolistic competition as a result of imperfect information. The Quarterly Journal of Economics, 101(3):493–511, 1986.

H. Yang. Targeted search and the long tail effect. The RAND Journal of Economics, 44(4):733–756, 2013.

X. Zhao. Service design of consumer data intermediary for competitive individual targeting. Decision support systems, 54(1):699–718, 2012.

X. Zhao and L. Xue. Competitive target advertising and consumer data sharing. Journal of Management Information Systems, 29(3):189–222, 2012.

J. Zhou. Ordered search in differentiated markets. International Journal of Industrial Organization, 29(2):253–262, 2011.