Locally optimal transfer free mechanisms for border dispute settlement

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25th March 2022

Abstract

Individuals living in a contested region are privately informed about their preference for citizenship in two rivalling countries. Not all frontiers are technically feasible which is why not everybody can live in his preferred country. Monetary transfers are not possible. When citizens only care about their own citizenship and types are drawn independently, a simple mechanism with simultaneous binary messages implements a social choice function that maximizes the expected sum of local residents' payoffs. This mechanism picks a feasible allocation that maximizes the number of individuals who live in what they say is their preferred country. An approval voting mechanism reaches the same outcome but does not require knowledge about voters' location. Sequential voting and electoral competition may instead lead to sub-optimal outcomes.

Keywords: mechanism design without transfers, border dispute settlement, voting, approval voting.

JEL Codes: D71, D72, D74, F51.

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1 Introduction

Decisions about how to draw the border between two neighboring countries and, correspondingly, which citizenship the residents of the contested region should have are often at the origin of international conflict and war. While the conflicting parties often claim the entire contested region for themselves as an integral part of their national territory, splitting up a such a region peacefully is a theoretical option[1]. A historic example of a peaceful determination of a border line is the determination of the Danish-German border in 1920 which followed a procedure that was laid out in the treaty of Versailles (for details see Schlürmann, 2019). The new border split up the region of Schleswig which previously was part of Germany but inhabited by many citizens who preferred Danish citizenship.

From a welfare perspective, a method for the settlement of a border dispute should (among possibly other considerations) take into account the preferences of the local residents concerned. This is not a trivial task because individual preferences about citizenship are not directly observable to others. Similarly, the intensity of citizenship preferences is only known privately. No national or international institution can claim to know for sure how important it is for a specific person to be citizen of one country rather than another.

What additionally complicates the process of determining a frontier is that not all frontier lines make equal sense from an economic or purely logistic perspective. A country should ideally be geographically connected to facilitate production, the provision of public goods, travel and transportation. Other geographical factors such as natural obstacles may add further restrictions. Actually, these constraints are a main reason why it makes little sense to let everybody simply be a citizen of his preferred country at the place where he lives.

This paper asks whether there exists a way to efficiently settle border disputes when there are geographical constraints regarding the way in which borders can be drawn. The paper uses mechanism design theory to address the problem of selecting one border from a given feasible set. Since I rule out

[1] This is an exercise in mathematical allocation theory that does not shed any light on the legality of rivalling claims to any specific territory. The paper studies solutions to a given allocation problem but it cannot shed light on whether the underlying claims are legally right or wrong.

mobility, the location of the border determines the allocation of citizenship.[2] Thus, the problem is one of assigning each citizen in the contested region either to one country A or another country B. For the sake of realism, I only consider mechanisms which do not make use of contingent financial transfers. Transfers that are conditioned on individual messages are a powerful tool in theory, but they play little practical role in real world border dispute settlements[3].

Even before summarizing the main results, I would like to clearly spell out some limits of this analysis. Several aspects that may play an important role when countries determine their borders are not considered here. The list includes in particular (i) externalities that the choice of border may have on those who live in- and outside the disputed territory, e.g. because of tax base effects or security concerns, (ii) costs that may be associated with uncertainty generated by some collective choice mechanisms including in particular potential adverse effects on private and public investment and (iii) severe adverse incentive effects that may arise when an international order puts existing frontiers into question. Related to the last point, important normative aspects of existing international law are also not addressed here[4]. The present paper is an exercise in mechanism design addressing only one aspect, asymmetric information about preferences and preference intensity of local residents, that is part of a much larger problem. Thus, it would be far too early to draw any direct and practical normative conclusions from this analysis.

The first main finding of this paper is that a simple decision mechanism implements a constrained optimal social choice in dominant strategies. This mechanism asks all individuals in the contested region to report their

[2] Actually, moving people is considered illegal in international law.

[3] When financial transfers are possible, a Vickrey Clarke Groves (VCG) mechanism can implement an ex post efficient decision in dominant strategies (provided that the revenue is allocated to an unconcerned third party). Similarly, with transfers, an ex post efficienct decision can be reached in a Bayesian Nash equilibrium with a d'Aspremont Gerard-Varet mechanism. An interesting question is what can be implemented with a strategically simple mechanism that uses transfers (Börgers and Li, 2019).

[4] While international law protects the integrity of existing states against attempts of a secession of part of the country, secessions have sometimes been recognized by other states after they occurred. This looks somewhat inconsistent. With a view to the mentioned incentive effects, it may make sense to make the hurdle for such an ex-post recognition particularly or prohibitively high if the seceded part is integrated into another country. These aspects are not addressed in the present analysis.

preferred citizenship and then selects one border from the feasible set that maximizes the number of individuals who live in their preferred country. When type distributions differ across individuals, the mechanism has to be adjusted accordingly, giving more weight to citizens with stronger conditional expected valuations. While this class of mechanisms requires detailed knowledge about the locations of voters in combination with their individual reports, a simple approval voting mechanism implements the same social choice function as the unweighted mechanism without requiring this detailed information.

The second main result is that not all indirect simple majority voting mechanisms are suited to replace the optimal direct mechanism. Although the welfare maximizing solution is a Condorcet winner, a sequence of votes can lead to suboptimal outcomes when voters act strategically. Accordingly, the sometimes used practice of voting on frontiers or secessions can be put into question. I also address the case where local residents not only care about where they live themselves, but where they also care about the location of the border as such. In this case, voting outcomes in indirect democracies turn out to be potentially inefficient while the simple direct mechanism studied in this paper still performs optimally.

The paper is related to the work of Alesina and Spolaore (1997) who analyze the optimal partitioning of a set of locations in a local public good setup and show that politically stable borders can be inefficient. What distinguishes the present work is (i) that it focuses on an information aggregation problem and (ii) that it considers all possible one-stage decision mechanisms. More generally, the paper contributes to the literature on optimal mechanisms without transfers (examples include Börgers 2004, Schmitz and Tröger, 2012, Gershkov, Moldovanu, and Shi 2017, Grüner and Tröger, 2019). From a theoretical perspective, the paper fills a gap in the mechanisms design literature addressing a general class of allocation problems: collective decisions regarding a vector of individual specific binary outcomes with a given set of feasible outcome vectors. Binary voting is one special case of this setup which obtains when the feasibility restriction is that the individual outcome has to be the same for everyone. The mechanism put forward here then turns into the simple majority rule which can therefore also be seen as a special case.

2 The model

Consider two countries A and B and a contested region R that is populated by I citizens. A collective choice has to be made about how to allocate the citizens of R to the countries A and B. Call an individual outcome x_i, where the outcome $x_i = 0$ means that i becomes citizen of country A, and $x_i = 1$ means that i becomes a citizen of country B. An overall outcome is a vector $x \in \{0,1\}^I =: \mathbf{X}$. There are feasibility constraints, captured by the feasible set of allocations $\mathbf{F} \subseteq \mathbf{X}$. An example for a simple meaningful feasibility set is the one that contains all $x \in \mathbf{X}$ with $x_1 \leq x_2 \leq ... \leq x_n$. This restriction obtains when the agents are ordered on a line from 1 to n, and the two countries must be connected. More complex feasibility sets can arise in the two-dimensional case.

Agents' preferences for citizenship are fully represented by a privately known type $\theta_i \in \Theta_i$ that represents the value of living in country B instead of living in country A. Thus, a negative valuation indicates a preference for country A, and a positive valuation a preference for country B. Valuations are drawn independently from a joint distribution $\hat{\phi}(\theta_1, ..., \theta_n) = \phi_1(\theta_1) \times ... \times \phi_n(\theta_n)$.

A social choice function maps all relevant information into an outcome: $x = f(\theta)$. I restrict the analysis of direct mechanisms to deterministic, transfer-free mechanisms. I show that there is an welfare maximizing mechanism that implements a corresponding social choice function in ex-post equilibrium. I define *realized social welfare* as

$$W(x,\theta) = \sum_{i=1}^{I} \theta_i x_i, \tag{1}$$

and *expected social welfare* as $E_\theta W(f(\theta), \theta)$.

We require that a social choice function is implemented in dominant strategies. Thus, an ex-ante welfare maximizing planner solves:

$$\max_{f(\theta)} E_\theta W(f(\theta), \theta) \tag{2}$$

$$s.t. \;\; f(\Theta_1 \times ... \times \Theta_I) \subseteq \mathbf{F}, \tag{3}$$

$$f_i(\theta_i, \theta_{-i}) \geq f_i(\theta_i', \theta_{-i}) \text{ if } \theta_i > 1 \text{ for all } \theta_i, \theta_i', \theta_{-i}, \tag{4}$$

$$f_i(\theta_i, \theta_{-i}) \leq f_i(\theta_i', \theta_{-i}) \text{ if } \theta_i < 1 \text{ for all } \theta_i, \theta_i', \theta_{-i}. \tag{5}$$

3 The mechanism

Consider first the case where individual types are drawn from identical symmetric and independent distributions. A straightforward way to address the design problem in this case is to maximize the number of individuals who live in what they claim to be their strictly preferred country, hereafter called the *number of fits*. Calling the vector of announced types $\hat{\theta}$, the number of fits can be defined as

$$S\left(x,\hat{\theta}\right):=\sum_{j=1}^{n} sgn\left(\hat{\theta}_j\left(x_j-\frac{1}{2}\right)\right).$$

There may be more than one outcome that maximizes the number of fits. To select one of them, I define the *left-minimal outcome* in some (finite) set $\tilde{\mathbf{F}}\subseteq\mathbf{F}$ as the (unique) first ranked outcome in $\tilde{\mathbf{F}}$ according to the lexicographic ordering of the components. According to this criterion, one discards outcomes that put the first individual country in country B if there are other outcomes in $\tilde{\mathbf{F}}$ that put the first individual in country A, and so on.

Definition 1 *The S-mechanism asks all individuals to report their types. It selects the left-minimal outcome in the set* $\tilde{\mathbf{F}}\subseteq\mathbf{F}$ *of alloations that maximize* $S\left(x,\hat{\theta}\right)$.

In the general case where individual valuations are drawn from different distributions, the S-mechanisms can be weighted. I define individual probabilities of positive and negative valuations as follows:

$$\beta_i^{+}\left(\theta_i\right):=\int_{x>0}\phi_i\left(\check{\theta}\right)\cdot d\check{\theta},$$

$$\beta_i^{-}\left(\theta_i\right):=\int_{x<0}\phi_i\left(\check{\theta}\right)\cdot d\check{\theta},$$

Whenever $\pi_i^{+}\left(\theta_i\right)$ or respectively $\pi_i^{-}\left(\theta_i\right)$ are nonzero, the conditional valuations are

$$G_i^{+}\left(\theta_i\right):=\frac{\int_{\check{\theta}>0}\phi_i\left(\check{\theta}\right)\check{\theta}\cdot d\check{\theta}}{\beta_i^{+}\left(\theta_i\right)},$$

$$G_i^{-}\left(\theta_i\right):=\frac{\int_{\check{\theta}>0}\phi_i\left(\check{\theta}\right)\check{\theta}\cdot d\check{\theta}}{\beta_i^{-}\left(\theta_i\right)},$$

and the absolute valuation is

$$G_i(\theta_i) := \begin{cases} G_i^+(\theta_i) & \theta_i > 0 \\ 0 & \theta_i = 0 \\ -G_i^-(\theta_i) & \theta_i < 0 \end{cases}.$$

Definition 2 *The weighted S-mechanism asks all individuals to report their types. It selects the left-minimal outcome in the set $\tilde{\mathbf{F}} \subseteq \mathbf{F}$ that maximize*

$$\tilde{S}\left(x, \hat{\theta}\right) := \sum_{i=1}^{n} G_i\left(\hat{\theta}_i\right) sgn\left(\hat{\theta}_i\right)(2x_j - 1).$$

To study optimality, it is important to note that incentive compatibility of a social choice function $f(\theta)$ requires that for all players the interim probability to live in country B is a step function of θ_i. This is why this mechanism can be replaced by a mechanism that just asks for the sign of the valuation.

Lemma 1 *Consider a revelation mechanism Γ implementing the social choice function $f(\theta)$. Let $\pi_i(\theta_i, f(\cdot)) = E_{\theta_{-i}}[f_i(\theta_i, \theta_{-i})]$. The social choice function $f(\theta)$ is Bayesian incentive compatible if and only if $\pi_i(\theta_i)$ satisfies*

$$\begin{aligned} \pi_i(\theta_i) &= \begin{cases} \pi_i^- & \text{if } \theta_i < 0 \\ \pi_i^+ & \text{if } \theta_i > 0 \end{cases}, \\ \pi_i^- &\leq \pi_i^+, \\ \pi_i^0 &:= \pi_i(0) \in \left[\pi_i^-, \pi_i^+\right]. \end{aligned} \tag{6}$$

The social choice function is dominant strategy implementable only if this condition holds.

Proof: The "if" part is obvious. "Only if" part: Otherwise at least one citizen with nonzero valuation can increase his expected payoff by misreporting his type. *QED*

Note that the interim probabilities π_i^-, π_i^0 and π_i^+ need not be the same for different individuals.

I introduce some more notation:

- Call $\sigma(\theta) := (sgn(\theta_1), ..., sgn(\theta_I))$ the *realized sign profile.* The set of possible sign profiles is $\Sigma := \{1, 0, -1\}^I$ with elements s.

- Call $\Delta(\mathbf{F})$ the set of probability distributions over $\mathbf{F}$.
- Call $\rho : \Sigma \to \Delta(\mathbf{F})$ a *step-assignment rule*.
- Call a step-assignement rule deterministic if the outcomes are deterministic for all $s \in \Sigma$.
- Call the set of deterministic step assignment rules Φ.
- Call $\sigma^{-1}(s)$ the set of type profiles with sign profile s.
- For any given sign profile s and any $\theta \in \sigma^{-1}(s)$ I define the following distribution function

$$\varsigma(\theta, s) = \frac{\phi(\theta)}{\int_{\sigma^{-1}(s)} \phi(x)\, dx}.$$

From any mechanism implementing some social choice function $f(\theta)$, one can construct an associated step assignment rule in the following way.

Definition 3 *Call $\rho_{f(\cdot)}(s) \in \Delta(\mathbf{F})$ the distribution that assigns the density $\varsigma\left(\tilde{\theta}, s\right)$ to the outcome $f\left(\tilde{\theta}\right)$. We say that the step assignment rule that maps profile s into $\rho_{f(\cdot)}(s)$ corresponds to $f(\theta)$.*

Now the following holds:

Lemma 2 *If some direct revelation mechanism implements $f(\theta)$ in Bayesian Nash equilibrium (dominant strategy equilibrium), then the corresponding step assignment rule $\rho_{f(\cdot)}(s)$ implements a social choice function $g(\theta)$ in Bayesian Nash equilibrium (dominant strategy equilibrium) with identical interim welfare for all types.*

Proof: As in Grüner and Tröger (2019), Lemma 1: Consider some direct mechanism $f(\theta)$ with a Bayesian truthtelling equilibrium. Consider the corresponding step assignment rule $\rho_{f(\cdot)}(s)$. A voter who chooses to report a positive valuation and who expects the other voters to report the sign of the valuation truthfully, realizes the same interim probability for living in country B as any other voter i type in the original equilibrium. Deviations yield the same interim probability as any other deviating voter i. A false report does not yield a higher payoff. Thus, there is a truthtelling Bayesian equilibrium with identical interim payoffs and welfare.

The same type of argument applies to dominant strategy equilibria. *QED*

Based on this Lemma, I restrict the further welfare analysis to the comparison of step assignment rules.

Proposition 3 *(i) All weighted S-mechanisms have an ex-post equilibrium in which agents report their types truthfully.*

(ii) All weighted S-mechanisms implement an ex-post efficient social choice.

(iii) The weighted S-mechanism is a solution to 2.

Note that (iii) implies that a detail-free version of the S mechanism is a solution to Problem 2 when absolute conditional valuations on both sides are the same.

Proof:

(i) Equilibrium: Consider w.l.o.g. individual $i = 1$ with $\theta_1 > 0$, i.e. an individual preferring to live in country B. Consider some vector of reports $\hat{\theta} = \left(\theta_i, \hat{\theta}_{-i}\right)$ and a result that maximizes $S\left(x, \theta_i, \hat{\theta}_{-i}\right)$. An alternative individual report $\hat{\theta}_i \neq \theta_i$ with $\hat{\theta}_i > 0$ does not change $S\left(x, \hat{\theta}_i, \hat{\theta}_{-i}\right)$ for any $\hat{\theta}_{-i}$. Thus, the individual does not gain from misreporting.

Consider a false report $\theta_i \leq 0$. By reporting $\theta_i < 0$ instead of $\theta_i > 0$ the values $S\left((1, x_{-i}), \left(\theta_i, \hat{\theta}_{-i}\right)\right)$ weakly decrease for all $\hat{\theta}_{-i}$ and those of all $S\left((0, x_{-i}), \left(\theta_i, \hat{\theta}_{-i}\right)\right)$ weakly increase. Thus, the probability that individual i is allocated to country A weakly increases and misreporting in this direction is suboptimal.

(ii) Ex-post efficiency: Consider any realization of θ and the corresponding outcome of the mechanism $f(\theta)$. A Pareto-improvement implies that all individuals that live in their preferred country continue to live there and that others that did not live in their preferred country now do. This is not possible because the social choice already maximizes $S(x,\theta)$ on $\mathbf{F}$. Thus, there is no Pareto superior outcome relative to the outcome of the S-mechanism.

(iii) Welfare: I have to show that no other incentive compatible mechanism than the weighted S-mechanism yields a higher expected payoff. I can focus on step assignment rules with truthful reports of signs. No other step-assignment rule can yield higher expected welfare than the S-mechanism. The reason is that in equilibrium the S mechanism transmits the entire sign profile to the planner. Not knowing the size of valuations but only their sign, the planner cannot do better than by maximizing $\tilde{S}(x, \theta)$ for all θ. *QED*

Conditional on the true sign profile $\sigma(\theta)$ the optimal choice in $\mathbf{F}$ is the one that maximizes the weighted number of players who live in their preferred country. In the unweighted case, the best use the planner can make of the *realized sign profile* $\sigma(\theta)$ is to maximize the number of players who live in their preferred country. Moreover, any incentive compatible mechanism does not transmit more information to the planner than the sign profile. So if the planner uses a different revelation mechanism to elicit the sign profile then this other mechanism cannot deal in a better way with that profile.

4 Approval voting

Consider the following indirect mechanism. All voters can approve some subset $F_i \subseteq F$. Then, the mechanism implements the left minimal outcome in the set of allocations that are approved by a maximum number of voters. Thus, e.g. if no border is approved by any voter or if all borders are approved by all voters then all citizens are allocated to country A if that is feasible. This mechanism has a dominant strategy equilibrium in which all players approve all allocations that put them in their preferred country. Disapproving one of these borders may be suboptimal in situations where the own statement is pivotal. For the same reason, if there are some allocations that put a voter in country A and some that put him in county B, dominance requires that the latter proposals are not approved by that voter.

The approval voting mechanisms replicates the social choice function of the S-mechanism. In contrast to the S-mechanism, the approval voting mechanism does not require that the designer posesses any knowledge about the identity or location of a citizen. However, it has some signals F_i that are superfluous, namely the ones that involve a contradiction regarding the preferences of individual i.

5 Sequential naive binary voting

An important question is whether the S-mechanism can be replaced by some voting scheme. There are many ways in which one can vote on the set $\mathbf{F}$ which may have more than two elements. I distinguish four setups: (i) direct democracy with two alternatives, (ii) direct democracy with sequential binary votes on the full list of options, (iii) political competition with perfect information, and (iv) political competition with imperfect information. In all four setups, I stick to simple majority rule as the rule for each vote that takes place. Since this rule cannot account for preference intensities and in order to give voting a fair chance, I assume that $G_i^+(\theta_i) = G_i^-(\theta_i)$. Still, I assume that $\pi_i^+(\theta_i)$ and $\pi_i^+(\theta_i)$ need not be the same, i.e. I permit asymmetric probability distributions.

I first consider the case where voters vote naively in the sense that they act as if all their votes were decisive. Consider the case of a binary vote in a direct democracy in which voters may abstain. It is easy to see that any unique welfare maximizing solution wins against all other alternatives if all voters vote for their preferred alternative if they have one and abstain otherwise. Under the same assumption on voting behavior, any sequence of binary votes which covers the entire set of feasible allocations $\mathbf{F}$ leads to unique welfare maximizing solution.

An important insight is that it is necessary to vote or on the entire set of options. Considering only a subclass of feasible partitions can exclude the optimum. In particular, there are examples where one single vote leads "away" from the optimal frontier location. To see why, consider a linear world with one border, seven citizens and valuations $(1, 1, 1, -1, -1, -1, -1)$. Consider a local vote amongst the first five individuals about moving the frontier form the right of position 5 (status quo) the to left of position 1. This referendum moves the first five individuals to country B although all seven individuals should be in country A. Still, the move improves welfare. But welfare could be higher. This implies that referenda on secessions should not be held locally in a subset of the contested region. Similarly, the S-mechanism must be played on the entire feasible set to make sure that welfare is maximized.

6 Strategic binary voting

I now deal with the case where voters act strategically when they vote sequentially. Consider a sequential voting game with a known finite sequence of binary votes. The winning alternative in each voting round enters the next round. The alternative selected in the last round is implemented and counts for payoffs.

With more than two voters, any full-information or Bayesian voting game under simple majority rule has trivial equilibria where all voters vote for the same alternative. The same holds for any sequential voting game. Identical voting behavior on all stages constitutes a Nash equilibrium. In order to rule out these implausible equilibria, I restrict the following analysis to trembling hand perfect equilibria. In this section, I show that a sequential voting game may have trembling hand perfect equilibria, in which the implemented social choice function does not yield a constrained welfare maximum. Thus, the S-mechanism has the advantage that its unique trembling hand perfect equilibrium always selects a welfare maximum.

A sequential Bayesian voting game is a signaling game with potentially many equilibria. I simplify the analysis and consider the limit case where uncertainty disappears. In that case, voter preferences are common knowledge.

The proof of the above claim is by example. I consider a full information benchmark case with four homogenous groups of citizens $g = 1, ..., 4$. Group 1 has $2\hat{n}$ members, where $\hat{n} \geq 1$, group 2 has $2\hat{n} + z$ members, where $z \in \{1, ..., \hat{n}\}$, Group 3 has $\hat{n}$ members, group 4 has z members.[5] Members of groups 1 and 4 prefer living in country A, i.e. for them $\pi_i^+ (\theta_i) = 0$. All other prefer living in country B, i.e. for them $\pi_i^- (\theta_i) = 0$. The set of feasible allocations is $\{x_1, x_2, x_3\}$:

1. x_1: Everybody lives in country B.
2. x_2: Only members of groups 1 and 3 live in country A.
3. x_3: Everybody lives in country A.

Note that the unique welfare maximizing alternative is x_2. It allocates $4\hat{n} + z$ citizens in line with their preferences, while alternative x_1 allocates $3\hat{n} + z$ citizens in line with what they prefer, and x_3 only $2\hat{n} + z$ citizens.

Consider a sequential voting game in which first there is a vote on the two alternatives x_1 and x_2, and second, the winning alternative is entering a vote against x_3.

The following is an equilibrium in undominated strategies: In the second round, everybody votes for his preferred alternative if there is one. In the

[5] This is a simple special case of what will be needed below. Let group i have n_i members. The equilibrium below requires $n_2 + n_3 > n_1 + n_4$ (first and second stage requirement) and $n_1 + n_3 + n_4 > n_2$ (the other second stage vote) and $n_1 + n_2 > n_2 + n_3 \Leftrightarrow n_1 > n_3$ and $n_1 + n_2 > n_1 + n_4 \Leftrightarrow n_2 > n_4$ (optimality of allocation x_2).

second round vote amongst the alternatives $\{x_2, x_3\}$ indifferent voters (the ones in groups 1 and 3) vote for x_3. Note that nobody is indifferent in the second round vote amongst the alternatives $\{x_2, x_3\}$. In the first round, all members of groups 1 and 4 vote for alternative x_2. All others vote for alternative x_1.

Consider the votes that can possibly take place in the second round. In the second round vote on $\{x_1, x_3\}$, $3\hat{n}+z$ voters vote in favor of x_1 and $2\hat{n}+z$ vote in favor of x_3. In the second round vote on $\{x_2, x_3\}$, $2\hat{n}+z$ voters vote in favor of x_2 and $3\hat{n}+z$ vote in favor of x_3.

Note that no one is ever pivotal along the equilibrium path. None of the strategies is dominated. If players were pivotal in the second round, they would behave optimally. Thus, no dominance argument can be made here. Actually, everybody voting for his preferred alternative in the second round must be part of any undominated strategy. If a player was pivotal in the first round, then a dominant strategy would imply that the player is better off no matter what happens in the second stage. This obviously is not true.

We now show that this strategy profile is a trembling hand perfect equilibrium. To see why, consider a sequence of totally mixed strategies where each player must play his seven non-equilibrium strategies with probability $\varepsilon > 0$ and his equilibrium strategy with probability $1-7\varepsilon$. In the two second round votes this yields a small probability for pivotality for all players, thus requiring that the second round vote is in line with voter preferences. This is the case.

The first period vote may also be pivotal. Not that, for any given probability $\varepsilon > 0$, and from the perspective of any player, there is a positive but small probability that the outcome of the second round vote on $\{x_1, x_3\}$ is x_3 and that the outcome of the second round vote on $\{x_2, x_3\}$ is x_2. Taking the almost certain second round outcomes x_1 and x_3 into account, all voters who prefer x_1 to x_3 must vote for x_1 in the first voting round and all voters who prefer x_3 to x_1 must vote for x_2. Again, this is the case. To summarize:

Proposition 4 *Consider a sequence of binary votes that includes all feasible alternatives. There exists a full information environment in which the voting game has a trembling hand perfect equilbirum that does not maximize social welfare.*

The proposition implies that the S-mechanism is more reliable as a tool to implement the welfare maximum than sequential voting procedures.

Note that in the example that underlies the proposition, there is no need to distinguish between different individuals in one group. This only becomes necessary when there is uncertainty about individuals' country preference.

7 Two party competition

Consider the competition of two parties with perfect information. One party wants to maximize the number of people living in country A, the other one wants to maximize the number of people living in country B.

Two cases can be distinguished. In the complete information case, both parties have access to the private information of the citizens. Then both parties offering the same solution that maximizes the number of fits is a Nash equilibrium. In equilibrium both receive one half of the expected votes. Any alternative platform only gains more votes if it puts more voters who previously did not fit into their preferred country than it puts voters who previously were allocated to their preferred country into the other country. Therefore the alternative platform would increase the number of fits which yields a contradiction. Moreover, there are no asymmetric equilibria.

Next, consider the case where two parties A and B try to maximize the size of countries A and B respectively. The parties simultaneously commit to their platform in $\mathbf{F}$. The party that wins the majority of votes implements its platform. The political equilibrium depends on what one assumes about the behavior of indifferent voters. Assume that indifferent voters vote for the party that shares their own country preference (alternatively, one can put an ε weight on the party winning in the utility function). I refer to such voters as partisan voters. Also assume that both complete options (everybody lives in country A or everybody in country B) are in the feasible set. Then an equilibrium is that party proposes to A put everyone in country A and party B proposes to put everyone in country B. The reason is that all A voters vote for party A anyway - no matter what party B proposes. Thus for party B, the best chance to win is to put everybody in country B to maximize the number of votes it receives from B voters. This platform both maximizes the chance of wining and the number of citizens living in country B, conditional on winning. Thus, it is a unique best reply.

Proposition 5 *Two party competition with partisan voters has a unique Nash equilibrium in which party A proposes to put every individual in country*

A and party B proposes to put every individual in country B. The equilibrium is not ex-post welfare maximizing.

Proof Regarding a possible equilibrium where both parties offer non-extreme platforms the same argument as above can be made.

To prove that welfare is not maximized, it suffices to consider the case where the realized sign profile corresponds exactly to a feasible allocation in the sense that putting everybody in his preferred country is feasible. Instead, the majority decides where individuals have to live. *Q.E.D.*

8 Conclusion

The present paper points out that a border choice mechanism that is based on binary voting decisions may result in (local) welfare losses relative to a system that is based on individual reports or approval voting. Similarly, representative democracy can lead to considerable welfare losses when voters have partisan preferences. The simple direct mechanism put forward here yields a superior result. In 1920, the Danish-German border was determined with a similar mechanisms and it is still intact today, indicating that the type of mechanism may be practically robust. However, several relevant issues have not been addressed in the present paper. The list includes (i) possible adverse incentive effects that were mentioned in the introduction, (ii) the possibility of locally correlated types, (iii) the role of voluntary ex-post mobility and (iv) the existence of sequential voting procedures with an optimal trembling hand perfect equilibrium. These issues deserve further research.

References

Alberto A. and E. Spolaore (1997) "On the Number and Size of Nations", *Quarterly Journal of Economics*, 112, 1027-1056.

Börgers, T. (2004), "Costly Voting", *American Economic Review*, 94, 57-66.

Börgers, T. and J. Li (2019) "Strategically Simple Mechanisms" *Econometrica*, 87, 2003-2035.

Grüner, H. P. and T. Tröger (2019) "Linear Voting Rules", *Econometrica*, 87, 2037-2077.

Gershkov, A., B. Moldovanu, and X. Shi (2017), "Optimal voting rules", *The Review of Economic Studies*, 84, 688-717.

Schlürmann, J. (2019) *Eine Grenze für den Frieden*, Wachholtz Verlag, Kiel, Hamburg.

Schmitz, P. W., and T. Tröger (2012), "The (sub-) optimality of the majority rule", *Games and Economic Behavior*, 74, 651-665.