# The Short-Run Employment Effects of Public Infrastructure Investment

Alexander Matusche*
*University of Mannheim*

February 7, 2022
Click here for the latest version.

**Abstract**

I study the stimulus effects of a permanent expansion in public investment that raises productivity in the long run. In anticipation of higher future productivity, firms rush to hire workers while they are still easy to find. Through this *anticipation effect on labor demand*, the policy change causes an immediate increase in employment. I study this mechanism theoretically and quantitatively in a model with search and matching frictions in the labor market. I characterize the *employment multiplier of public investment* analytically and show that it is larger when public investment increases during a recession compared to a boom. When labor demand is inefficiently low, the expansion in public investment improves labor market efficiency. Calibrated to the US economy, the model yields an increase in employment by 0.4 percentage points one year after a permanent expansion of public investment by 1% of GDP. The *anticipation effect* accounts for up to 65% of the additional employment. When the economy is in a recession, the employment gain is 40% larger than in a boom. If the expansion is financed with distortionary taxes or if public investment exhibits implementation delays, the increase in employment is smaller but remains positive and substantial in both cases.

*I am very grateful to Tom Krebs for advice and support and thank Antoine Camous, Antonio Ciccone, Paul Gorny, Andreas Gulyas, Lukas Hack, Anne Hannusch, Matthias Meier, Oliver Pfäuti, Konrad Stahl, Michèle Tertilt, Johannes Wacks, Frederick Zadow and seminar participants at the Mannheim Macro Lunch for helpful comments and discussions. *Address:* Department of Economics, University of Mannheim, L7 3-5, 68161 Mannheim, Germany. *E-Mail*: alexander.matusche@gess.uni-mannheim.de. *Website:* amatusche.github.io.

# 1 Introduction

Faced with aging infrastructure and a need to transition to a green and digital economy, many countries are expanding public infrastructure investment. Most notably, on November 5, 2021, US Congress passed the "Infrastructure Investment and Jobs Act" which appropriates 550 billion US-Dollars in additional federal infrastructure investment that are expected to be spent over the next five years. This would raise federal non-defense infrastructure investment from 0.7% of GDP in 2019 to about 1.3%, a level last seen in the 1970s. Also in Europe, there are plans to expand infrastructure investment. For example, the EU Recovery Fund allocates at least 383 billion Euros to public investment supporting green and digital transformation.[1] These expansions of public investment were put forward during a recession with high unemployment.[2] Since the ability of central banks to stimulate the economy was limited by the zero lower bound, there was a case for fiscal stimulus. Whether a public investment program can provide such short-term stimulus is a debated question. Summers (2009) has argued in favor of expanding public investment during a recession. Ramey (2020) sees little expansionary effects, if any at all, of expanding public investment during a recession.

This paper contributes to this debate. I ask whether a lasting expansion of public infrastructure investment of the kind currently implemented in the US can raise employment in the short run. Is a structural change of fiscal policy towards more public investment conducive to a swift recovery? Should such a change be initiated during a recession?

A vast literature has found large positive effects of public investment on productivity and output *in the long run*.[3] In this paper, I take these positive long-run effects as given and show that they can lead to a substantial increase in employment already in the short run. These short-run employment effects are caused by an *anticipation effect on labor demand*. When public investment increases, firms anticipate higher productivity and tighter labor markets in the future. They expand hiring already in the present when it is still relatively cheap since labor market tightness is low and workers can be found quickly. In a recession, when unemployment is temporarily high, additional labor demand does not impair the ability of other firms to find workers much. Thus, the employment effect is particularly large. To the best of my knowledge, this paper is the first to investigate the anticipation effect of public investment on labor demand and its business cycle dependence.

Labor market frictions are essential for the increase in labor demand in response to

1. The "Recovery and Resilience Facility" contains a total of 672.5 billion Euros of which 57% have to be allocated by the member states to investments and reforms supporting green and digital transformation.

2. In the US, the unemployment rate in June 2021 was 6.9% compared to 4.3% in January 2020 before the beginning of the last recession.

3. See for example Aschauer (1989), Bom and Ligthart (2014), Bouakez et al. (2017), Cubas (2020), Munnell (1990), and Pereira and Frutos (1999). I discuss this literature in detail in Section 4.

higher future productivity. I model them in the standard fashion following Mortensen and Pissarides (Mortensen and Pissarides 1994; Pissarides 1985). Firms enter the labor market by posting vacancies. After a vacancy is filled and a worker is hired, output is produced until the worker is separated from the firm. Firm-worker matches are expected to last more than one period. This allows firms to engage in labor hoarding. When an expansion of public investment is announced, firms anticipate higher future labor productivity and labor market tightness. The increase in future labor market tightness makes it more difficult for firms to find workers in the future which raises the costs of hiring in the future. Firms' optimal response is to substitute hiring intertemporarily, hire earlier and hoard labor. As a result, an expansion in public investment has large positive employment effects in the short run. In contrast, when firms hire labor period by period, as in much of the literature on public investment, there is no anticipation effect on labor demand.

Public investment may also stimulate labor demand directly as the public sector and its contractors hire workers to implement infrastructure projects.[4] In this paper, I abstract from these demand channels and focus on the effect on employment that is due to rising long-run productivity and, therefore, specific to *productive* government spending.

In the first part of this paper, I analyze the employment effects of public investment theoretically. The analysis operates under the assumption of constant labor supply to focus on the anticipation effect on labor demand. In the second part, I quantify the employment effect of public infrastructure investment, also taking into account the labor supply response.

I begin the theoretical analysis by defining the *employment multiplier of public investment*, the change in employment following the announcement of a permanent expansion in public investment at some point in the future. I show that the employment multiplier is strictly positive in the short run even if it is zero in the long run. Thus, the positive employment multiplier is a transitional phenomenon: A higher level of public investment does not imply higher employment in the steady state. Instead, it is the *increase* in public investment that raises employment in the short run.

For the case where the economy is in the steady state, I derive an analytic expression for the employment multiplier. The formula highlights the role of future productivity for the evolution of employment and makes transparent how parameter assumptions and implementation delays shape the multiplier. It is larger if an increase in public investment has larger effects on productivity in the long run. Wage stickiness amplifies the employment effect. When wages rise more slowly following the rise in future productivity, labor hoarding is cheaper and firms expand vacancy creation in the short run more strongly.

4. For example, Michaillat (2014) finds large aggregate employment effects of public sector hiring especially during recessions. Rendahl (2016) studies the effects of public spending on employment through aggregate demand in a matching model and finds large effects during a recession.

When implementation delays are longer, productivity only increases in the more distant future. Therefore, a worker hired today is less likely to be employed at the firm when the productivity effect of investment materializes and the effect on the present value of match output is smaller. Hence, firms expand vacancy creation less strongly in the short run. Implementation delays reduce the employment multiplier and this reduction is larger if separation rates are higher.

The role of discount factors and separation rates for the employment multiplier depends on the degree of wage inertia. In the extreme cases, where wages are fully flexible or completely rigid, higher discount factors and lower separation rates increase the employment effect. A lower separation rate makes it more likely that workers who are hired today will still work at the firm in the future when they benefit from the productivity effects of public investment. Thus, it is easier for firms to substitute hiring over time and they expand hiring more strongly in the short-run. When the discount factor is higher, the future output gain is valued more relative to the short-term costs of hiring so that firms expand short-run labor demand more.

I investigate how the employment multiplier differs between recessions and normal times. I consider two features of recessions, high unemployment and weak labor demand. I show that the employment multiplier is larger when unemployment is high. In this case, an additional vacancy only leads to a small increase in labor market tightness. Therefore, the rate at which other firms can fill their vacancies is not affected much, the congestion externality is small. When labor demand is weaker because of high wages and small short-run profits, the employment multiplier of public investment can be larger or smaller. It depends on the elasticity of the matching function. On the one hand, since short-run profits are small, long-run profits account for a larger share of the total value of a match and an increase in long-run profits has a relatively stronger effect on the value of a match. This leads to a larger effect of public investment on vacancy creation during a recession. On the other hand, when labor demand is weaker and few vacancies are posted, every additional vacancy lowers the filling probability for all other vacancies by more. This means that firms post fewer additional vacancies for the same change in the value of a match. Which of these effects dominates depends on the elasticity of the matching function with respect to vacancies. If it is small, the employment effect is larger when labor demand is weak.

In general, the search and matching equilibrium is not constrained-efficient. Firms neither internalize the positive effect of vacancy creation on the job-finding probability of workers nor the negative effect on the vacancy-filling probability of other firms. Hence, there may be too much or too little vacancy creation in equilibrium. I show that the expansion in hiring brought about by public investment improves labor market efficiency if labor demand in equilibrium is below its constrained efficient level.

These theoretical results rely on the assumption that labor supply is fixed. It allows me to focus on the new mechanism in my model, the anticipation effect of public investment on *labor demand*. However, public investment could also affect *labor supply* in the short run. For example, Leeper et al. (2010) find that an increase in future productivity lowers labor supply through a wealth effect such that public investment has smaller output effects than unproductive spending. When I also incorporate a labor supply margin in my model by allowing unemployed workers to choose search effort, the sign of the employment multiplier of public investment is theoretically ambiguous. In response to an expansion in public investment, workers could increase or decrease search effort. Higher future wages induce workers to search more intensely, whereas better job-finding prospects in the long run lead to lower search effort in the present. The labor supply response also depends on the financing of public investment. If the government levies distortionary labor taxes to finance the expansion in investment, incentives to exert search effort are reduced. How employment responds to an expansion in public investment is thus a quantitative question to which I turn in the second part of this paper.

I calibrate the model to match transition rates between unemployment and employment which I estimate from CPS microdata. The model matches standard moments of the US business cycle such as the volatility and persistence of unemployment, output, and investment. In general it is difficult for the standard search and matching model to generate the volatility of unemployment over the business cycle observed in the data (Shimer 2005). My calibration matches the observed volatility for two reasons. First, since I model private capital explicitly, the profit share is relatively small despite a realistic labor share. Second, I assume that wages are determined by Nash bargaining but exhibit substantial inertia. In the words of Ljungqvist and Sargent (2017), the fundamental surplus is small which leads to a high volatility of unemployment.

I consider a permanent expansion of public infrastructure investment by 1% of GDP. First, I assume that the public investment program is implemented as soon as it is announced and financed with lump-sum taxes. The permanent expansion of public investment leads to a long-run increase in productivity by 3%. It takes about 25 years until the new long-run level of productivity is reached. After one year, the expansion of public investment has increased productivity by only 0.35%, but unemployment is already 0.4 percentage points lower than before. I quantify the contribution of the anticipation effect and find that it accounts for up to 65% of the employment gain after one year. Second, I consider implementation lags. They reduce the employment response upon announcement of the expansion in public investment but the response remains large. For example, when one year passes between the announcement and the implementation of the investment program, unemployment still declines by 0.25 percentage points within the first year after the program was announced. When the government levies distortionary labor taxes to finance the additional public investment, the reduction in unemployment one year after the beginning of the investment program is still close to 0.25 percentage points. Finally, the employment effect is more than 40% larger in a recession than in a boom. Wage inertia are important for the large employment gains in the short run. Under my calibration, wages increase almost in proportion to productivity. If wage inertia is smaller, expectations about higher future productivity lead to a stronger wage increase in the short run as workers demand higher wages. This makes labor hoarding more costly for firms and the employment effect is smaller.

My results imply that recessions are good times to initiate a change in fiscal policy towards more public infrastructure investment. Even if there are substantial implementation delays, the policy change can stimulate employment in the short run. In addition, the employment reduction and the associated output gains are particularly large in a recession.

**Related Literature** A large literature in macroeconomics studies fiscal multipliers. Two strands of this literature are related particularly closely to the present paper. The first is the literature on the short-run effects of public investment (Baxter and King 1993; Boehm 2020; Leeper et al. 2010; Ramey 2020). These studies find smaller short-run effects of public investment than of government consumption because the long-run productivity gains associated with public investment push down labor supply in the short-run through a positive wealth effect. With the exception of Ramey (2020), these papers consider frictionless labor markets. Firms' labor demand decision is static, independent of future productivity and only the labor supply decision of workers is directly affected by changes in expected future productivity due to public investment. The model in Ramey (2020) features labor market frictions in the form of sticky wages but labor demand is still a static decision. Instead, I study a model with search frictions in the labor market in which labor demand depends on future productivity.

I share the emphasis on frictional labor markets with a second strand of literature. Mitman and Rabinovich (2015) focus on unemployment benefit extensions, Rendahl (2016) investigates government consumption, and Michaillat and Saez (2018) and Michaillat (2014) study public sector employment. I add an analysis of a different type of government spending, public investment, which has not been studied in the context of frictional labor markets.

This paper is also related to the literature on news-driven business cycles following Beaudry and Portier (2006) who find that anticipated future TFP growth is an important source of business cycle fluctuations (Beaudry and Portier 2007; Schmitt-Grohé and Uribe 2012). In my model, public investment alters expectations about private future productivity and as such constitutes a news shock causing an expectations-driven boom.[5] Thus, it is closely related to Den Haan and Kaltenbrunner (2009) who also study a model with matching frictions and find that news about higher future productivity can generate a boom in investment, hours worked, consumption and output before pro-

5. I show in Appendix D that the employment effect of public investment when financed with lump-sum taxes, is proportional to the employment effect of a permanent increase in productivity.

ductivity actually increases. My paper differs in the following ways. First, I analyze the employment effect theoretically. Second, I consider anticipated changes in productivity caused by public investment. Since public investment is costly, the government has to raise revenues to finance it and I study the effects of public investment under different assumptions about its financing, e.g. lump-sum taxes and distortionary labor taxes (see Section 5.2). Third, I show that the employment effects can be substantially larger in recessions.

**Outline** The remainder of the paper is structured as follows. In Section 2 I present the model. In Section 3, I define the employment effect of public investment and analyze it theoretically. I calibrate the model in Section 4 and quantify the employment and output effects of public investment in Section 5. Section 6 concludes.

# 2 Model

The model features random search and matching in labor market following Diamond, Mortensen and Pissarides (DMP model). There are two types of physical capital: private and public capital. Firms rent private capital to produce. The government owns the public capital stock which is used in production by all firms simultaneously—it is a public good. Time $t = 0, \ldots, \infty$ is discrete and runs forever.

## 2.1 Households

There are two types of households, a unit mass of workers and a mass $\mu$ of homogeneous firm owners.[6] Workers participate in the labor market and receive labor income when employed. Firm owners do not participate in the labor market. Their income consists of firms' profits and capital returns. I assume that workers consume their income in every period. While this is a strong assumption, it should be noted that a large fraction of US households are hand-to-mouth, especially among the unemployed (Kaplan et al. 2014). For the quantitative analysis, I set the wage replacement rate of unemployment benefits to yield a realistic consumption drop upon job loss. Hand-to-mouth status of workers can also be justified as the equilibrium outcome of an extended model in which all households are allowed to borrow and save in a risk-free bond and in which intermediation costs drive a wedge between saving and borrowing interest rate or in which the bond is in zero net supply (Ravn and Sterk 2020; McKay and Reis 2021).

**Workers** Workers differ regarding their labor market status $s_t$. They can be employed $s_t = e$ or unemployed $s_t = u$ and this labor market status is risky as workers find and

6. This is a common assumption, that is also made in Broer et al. (2019) and Ravn and Sterk (2020).

lose jobs stochastically. Unemployed workers exert search effort $\ell_t \geq 0$ to find a job. The more effort an unemployed worker puts into searching, the higher the probability of finding a job. In addition to effort, the job-finding probability depends on labor market conditions summarized by labor market tightness $\theta_t$, which is determined endogenously in the labor market equilibrium described below. I denote the probability that an unemployed worker finds a job by $\pi^{e|u}(\ell_t, \theta_t)$.The probability of losing a job is exogenous and denoted $\pi^{u|e}$. Unemployed workers receive unemployment benefits $b_t$, whereas employed workers earn labor income $w_t$. Hence, workers face income risk. When they lose their job, their income falls from $w_t$ to $b_t$.

Since workers are hand-to-mouth, consumption equals after tax income. Thus,

$$c_t(s_t) = \begin{cases} (1-\tau_t)w_t, \text{ if } s_t = e \\ b_t, \text{ if } s_t = u, \end{cases} \tag{1}$$

where $\tau_t$ is the tax rate on labor income. Workers value consumption and dislike effort according to the per-period utility function

$$u(c, \ell, s) = \log(c) - d(\ell, s).$$

Note that the disutility from effort depends on labor market status. Employed workers do not exert search effort but experience fixed disutility from working. Workers choose effort to maximize expected lifetime utility,

$$\begin{aligned} \max_{\{\ell_t(s_t), c_t(s^t)\}} \mathbb{E}_0 & \left[ \sum_{t=0}^{\infty} \beta^t \left(\log(c_t(s_t)) - d(\ell_t(s_t), s_t)\right) \Big| s_0, \{\ell_t(s_t)\} \right] \\ \text{s.t. } & (1), \ell_t(s_t) \geq 0 \text{ and given } s_0. \end{aligned} \tag{2}$$

Here, the expectation is taken with respect to labor market status $s_t$. The expected labor market state $s_t$ in period $t$ depends on the initial state $s_0$ and past effort choices $\{\ell_t(s_t)\}$. Denote the expected lifetime utility of a worker in labor market state $s$ in period $t$ by $J_t(s)$. The worker's problem can be formulated recursively as

$$\begin{aligned} J_t(s) = & \max_{\ell, c} \log(c) - d(\ell, s) + \beta \sum_{s' \in \{e,u\}} J_{t+1}(s') \pi_{t+1}^{s'|s}(\ell, \theta_t) \\ & \text{s.t. } c = (1-\tau_t) w_t \mathbb{1}_{s=e} + b_t \mathbb{1}_{s=u}. \end{aligned} \tag{3}$$

The wage and the transition probabilities between labor market states for a given effort choice are determined in the labor market that I describe in the next subsection. The first-order condition for the optimal effort choice is

$$\frac{\partial d(\ell, u)}{\partial \ell} = \beta \left[ J_{t+1}(e) - J_{t+1}(u) \right] \frac{\partial \pi_{t+1}^{e|u}(\ell)}{\partial \ell}. \tag{4}$$

The left-hand side is the utility cost of marginally increasing effort. The right-hand side is the gain in expected lifetime utility from higher effort. More search effort increases the probability of finding a job and thereby expected future income. Note that since all unemployed workers are identical, equilibrium search effort will be the same for all unemployed workers.

**Firm owners** A measure $\mu$ of homogeneous firm owners owns the aggregate capital stock $K_t$ and firm equity in the economy. Due to the equity ownership, each firm owner receives a dividend $\pi_t^F \equiv \frac{\Pi_t}{\mu}$, where $\Pi_t$ denotes aggregate profits. Note that the number of firm owners is constant but the number of firms varies over time. Hence, it is not the case that every firm is owned by one particular firm owner. Instead, every firm owner household owns a share of total firm equity.

Besides equity, firm owners own the private capital stock of the economy. Let $k_t^F$ denote the amount of capital owned by an individual firm owner. It follows the law of motion

$$k_{t+1}^F = (1-\delta_k)k_t^F + i_t^F. \tag{5}$$

Here, $\delta_k$ is the depreciation rate of physical capital and $i_t^F$ denotes investment in the productive capital stock. Firm owners rent out the capital stock to firms at the rental rate $r_t^k$. For the quantitative analysis, I assume that firm owners face adjustment costs $\phi(i_t^F, k_t^F)$ when investing in productive capital. Adjustment costs are needed to obtain quantitatively realistic fluctuations of investment over the business cycle, but as I show in Appendix F, they do not substantially affect the main results on employment. The adjustment cost function is of the quadratic form

$$\phi(i_t^F, k_t^F) = \frac{\phi}{2}\left(\frac{i_t^F}{k_t^F} - \delta_k\right)^2 k_t^F.$$

Thus, the budget constraint of an individual firm owner is

$$i_t^F + c_t^F = r_t^k k_t^F + \pi_t^F - T_t^F - \frac{\phi}{2}\left(\frac{i_t^F}{k_t^F} - \delta_k\right)^2 k_t^F, \tag{6}$$

where $T_t^F$ denotes lump-sum taxes (or transfers if $T_t^F$ is negative). I assume that firm owners are risk neutral. They maximize lifetime utility given by

$$U^F = \sum_{t=0}^{\infty} \beta^t u^F(c_t^F) = \sum_{t=0}^{\infty} \beta^t c_t^F$$

subject to the budget constraint (6) and the law of motion for capital (5). The resulting first-order condition for capital is

$$1+\phi\left(\frac{k_{t+1}^F}{k_t^F}-1\right)=\beta\left(1+r_{t+1}^k-\delta_k+\frac{\phi}{2}\left(\left(\frac{k_{t+2}^F}{k_{t+1}^F}\right)^2-1\right)\right). \tag{7}$$

The left-hand side is the marginal cost of investing one unit of capital, which includes the marginal capital adjustment costs. The right-hand side is the marginal benefit from investing one unit of capital. It consists of the received interest payments net of depreciation and takes into account that a large capital stock in the next period affects the capital adjustment costs that have to be incurred in the next period. If the firm owner plans to grow the capital stock further in the next period, investing in the current period has the additional benefit of lower adjustment costs in the next period. The aggregate capital stock is $K_t = \frac{k_t^F}{\mu}$ such that the first-order condition (7) implies that the aggregate capital stock follows the difference equation

$$1+\phi\left(\frac{K_{t+1}}{K_t}-1\right)=\beta\left(1+r_{t+1}^k-\delta_k+\frac{\phi}{2}\left(\left(\frac{K_{t+2}}{K_{t+1}}\right)^2-1\right)\right). \tag{8}$$

## 2.2 Firms and labor market

Firms enter the economy by posting a vacancy at cost $\kappa_t$. The posting costs can be a constant, $\kappa_t = \bar{\kappa}$, or dependent on the state of the economy. In particular, I will consider the case in which posting costs are proportional to labor productivity $y_t$, i.e.

$$\kappa_t = \bar{\kappa} \cdot y_t.$$

This is a natural assumption, if vacancy posting costs mainly consist of foregone production of the workers who are involved in the hiring process. When posting costs are proportional to labor productivity, unemployment is constant in the long run even if productivity grows over time. In this sense, the assumption that posting costs are proportional to labor productivity, ensures that the model is consistent with balanced growth. In contrast, if posting costs were constant, productivity growth would lead to a decline in unemployment as the costs of posting a vacancy would fall relative to output from a match.

A Cobb-Douglas matching function

$$M(L^u, v) = \zeta\,(L^u)^{\eta}\,(v)^{1-\eta},\ \eta \in (0,1) \tag{9}$$

determines the number of vacancy-worker matches as a function of the measure of vacancies $v$ and of aggregate search effort $L^u$. Aggregate search effort is search effort per unemployed worker $\ell$ times the measure of unemployed workers $U$, $L^u = U\ell$. We can think of individual search effort as the number of applications sent by an unemployed worker. Then, aggregate search effort would be the total number of all applications sent in the economy. The matching function (9) implies that more matches are formed if more vacancies are created, there are more unemployed workers looking for a job, or if unemployed workers search with greater intensity, e.g. send more applications.

With $M(L_t^u, v_t)$ matches being created in period $t$, a total number of $\frac{M(L_t^u, v_t)}{v_t}$ matches are formed for every vacancy in $t$. Since matching is random, this means that every vacancy in period $t$ results in a match with the same probability

$$q_t^v(\theta_t) = \frac{M(L_t^u, v_t)}{v_t} = \zeta\theta_t^{-\eta},$$

where $\theta_t \equiv \frac{v_t}{L_t^u}$ denotes labor market tightness. The job-finding probability of an unemployed worker is determined similarly: The number of matches formed per unit of aggregate search effort in period $t$ is $\frac{M(L_t^u, v_t)}{L_t^u}$. An unemployed worker who exerts search effort $\ell_t$ finds a job with probability

$$\pi_t^{e|u}(\theta_t, \ell_t) = \frac{M(L_t^u, v_t)}{L_t^u}\ell_t = q_t^v(\theta_t)\theta_t\ell_t.$$

We can think of $\frac{M(L_t^u, v_t)}{L_t^u}$ as the number of matches per application sent. Since every application results in a match with the same probability (random matching), a worker who sent out $\ell_t$ applications, finds a job with probability $\pi_t^{e|u}(\theta_t, \ell_t) = \frac{M(L_t^u, v_t)}{L_t^u}\ell_t$. In general, $q_t^v(\theta_t)$ and $\pi_t^{e|u}(\theta_t, \ell_t)$ could exceed one. In order to interpret them as probabilities, I assume and verify that in equilibrium $M(L^U, v) < v_t$ and $M(L_t^U, v_t) < U_t$.

A match continues to exist in the next period with probability $\pi_t^{e|e} = (1-\rho)$ and is dissolved with the complementary probability $\pi_t^{u|e} = \rho$. Hence, at least some matches exist for more than one period if $\rho < 1$. This feature of the model is crucial for the anticipation effect of public investment on labor demand. Since matches formed today persist into the future, expectations about future productivity affect firms' incentives to post vacancies and thereby labor demand. I assume here that the probability of loosing a job is independent of the worker's effort implying that the worker does not exert effort on the job. The model could easily be extended to incorporate work effort.[7] The transition probabilities between employment and unemployment imply that aggregate employment $N_t$ evolves according to

$$N_{t+1} = (1-\rho)N_t + \pi_t^{e|u}U_t. \tag{10}$$

Recall that the total mass of workers is one and every worker is either employed or unemployed. Thus employment $N_t$ and unemployment $U_t$ sum to one and $U_t$ can be

7. There can still be a disutility of working if $d(0, s) > 0$. Such that I could equivalently assume that workers exert a fixed amount of work effort whenever they are employed.

interpreted as the unemployment rate.

When a firm has filled a vacancy, it produces output $y_t$ with the technology

$$y_t = z_t k_t^{\alpha}.$$

Here, $k_t$ is the stock of capital employed in the match, which is rented from firm owners at rate $r_t^k$. The variable $z_t$ is productivity. It depends on the stock of public capital which is provided by the government as described in detail in the next subsection. A firm makes wage payments $w_t$ to the worker. It follows that the value of a filled vacancy is

$$J_t^F = \max_k \; z_t k^{\alpha} - w_t - r_t^k k + \beta \Big\{ \rho V_{t+1} + (1-\rho) J_{t+1}^F \Big\}, \tag{11}$$

where $V_{t+1}$ is the value of an open vacancy in the next period. It is defined according to

$$V_t = -\kappa_t + \beta \left\{ q_t^v J_{t+1}^F + (1 - q_t^v) V_{t+1} \right\}. \tag{12}$$

The first-order condition for the optimal choice of capital in (11) is that the rental rate for capital equals the marginal product of capital,

$$r_t^k = \alpha z_t k_t^{\alpha - 1}. \tag{13}$$

Using the first-order condition, the value of a filled vacancy can be written as

$$J_t^F = (1-\alpha) z_t k_t^{\alpha} - w_t + \beta \Big\{ \rho V_{t+1} + (1-\rho) J_{t+1}^F \Big\}.$$

Many wages are consistent with an equilibrium in the search and matching labor market described so far. Therefore, I must take a stance on how the wage is determined. I assume that the wage in period $t$ is a liner combination of the wage in the previous period and a target wage $w_t*$,

$$w_t = \gamma w_{t-1} + (1-\gamma) w_t^*. \tag{14}$$

For the theoretical results in Section 3, I assume that the target wage is simply a fixed fraction $\omega$ of match output,

$$w_t^* = \omega z_t k_t^{\alpha}. \tag{15}$$

I assume that $\omega$ is such that the wage lies in the bargaining set such that both workers and firms are wiling to sustain the match under the resulting wage. Instead, for the quantitative analysis, I assume that the target wage is determined by Nash bargaining,

$$w_t^* = \arg\max_w \left( J_t(e, w) - J_t(u) \right)^{\psi} \left( J_t^F(w) \right)^{1-\psi}, \tag{16}$$

where $\psi$ is the bargaining power of workers.[8] The parameter $\gamma$ in (14) governs the strength of wage inertia. Wages are completely fixed if $\gamma = 1$ and the wage always equals the target wage if $\gamma = 0$. The assumption of sticky wages has become common in the literature at least since it was put forward by Hall (2005) as an empirically plausible way to resolve the observation by Shimer (2005) that the standard DMP model with Nash bargaining cannot easily generate the degree of counter-cyclicality of unemployment that is observed in the data. Hall (2003, 2005) also proposes the specific functional form (14) and provides a micro foundation for it. Pissarides (2009) challenges the view that wage stickiness can resolve the Shimer puzzle showing that only wages of new hires matter for the volatility of unemployment and that these exhibit much less inertia than wages at large. However, recent evidence by Gertler et al. (2020) indicates that wages in new matches are not as flexible as previously thought once composition effects are taken into account. This lends support to the assumption of wage stickiness.

Much of the literature that employs a DMP framework to study business cycles uses a slight simplification of the wage rule, in which the previous period's wage in equation (14) is replaced by the steady state wage (Blanchard and Galí 2010; Challe 2020). Such a rule removes the wage from the state space which would facilitate solving the model. However, it would not appropriate for the purpose of this study because I am interested in the short-run effect of a *permanent* change of government investment and want to allow the wage to adjust to the new environment in the long run. This is possible under my rule but if would not be possible if the wage in the initial steady state was the wage norm.

A summary of the timing of events in the labor market is as follows: At the beginning of the period, employed workers negotiate wages with their firms. The negotiation will always lead to a wage in the bargaining set such that output is produced and wages are paid. Firms then rent physical capital from firm owners and produce output. At the end of the period, some firms are separated from their workers and new firms enter the economy by posting a vacancy. Some of the new entrants match with workers so they can start production in the next period.

## 2.3 Government

The government collects lump-sum taxes on firm owners $T_t^F$ and taxes labor income at rate $\tau_t$. It pays unemployment benefits $b_t$ and makes investments $I_t^G$ in public infrastructure. The government's per-period budget constraint reads

$$I_t^G + U_t b_t = \mu T_t^F + \tau_t w_t N_t, \tag{17}$$

8. I have made explicit the dependence of the employed worker's value as well as the firm's value on the current wage $w$. In the definition of these value functions (3) and (11) the dependence on $w$ was subsumed in the aggregate state of the economy indicated by the time subscript $t$.

where $U_t$ is the number of unemployed workers and $N_t = 1 - U_t$ is the number of employed workers in period $t$. The left-hand side of the government's budget constraint are government expenditures for public investment and unemployment benefits. The right-hand side captures total tax revenues.

Public investment raises the public capital stock

$$K_{t+1}^G = (1 - \delta_G) K_t^G + I_t^G \tag{18}$$

which determines productivity

$$z_t = A_t \left(K_t^G\right)^{\vartheta}. \tag{19}$$

Here, $A_t$ is an exogenous part of productivity independent of public capital and $\vartheta$ is the output elasticity of public capital. Equation (19) is the standard way of formalizing the idea that public capital enters production as a third factor of production in addition to private physical capital and labor following Baxter and King (1993).[9] Public capital is a public good, all firms can use it and there is no rivalry in use. In particular, I assume that there is no congestion externality when it comes to the use of public capital.

## 2.4 Equilibrium

An equilibrium of this economy is defined as follows.

**Definition 1** (Equilibrium)**.** *An equilibrium is a collection of individual sequences of workers' effort and consumption* $\{\ell_t(s^t), c_t(s^t))\}_{t=0}^{\infty}$*, of labor market tightness, capital rental rates and wages,* $\{\theta_t, r_t^k, w_t\}_{t=0}^{\infty}$*, aggregate employment, aggregate capital, and capital per match,* $\{N_t, K_t, k_t\}_{t=0}^{\infty}$*, and of policies* $\{T_t, \tau_t, K_t^G, I_t^G\}_{t=0}^{\infty}$*, such that*

1. *the sequences of effort and consumption* $\{(\ell_t(s^t), c_t(s^t)\}_{t=0}^{\infty}$ *solve the worker problem* (2),
2. *firms choose capital optimally according to* (13),
3. *the sequence of labor market tightness* $\{\theta_t\}_{t=0}^{\infty}$ *ensures that the value of an open vacancy is zero,* $V_t(\theta_t) = 0$,
4. *wages are determined according to* (14) *together with* (16) *or* (15),
5. *firm owners choose capital optimally according to* (8),
6. *the capital market clears* $K_t = k_t N_t$,
7. *employment follows the law of motion* (10),

9. Baxter and King (1993) explicitly write down an aggregate production function of the form $F(K_t^G, K_t, L_t) = A_t \left(K_t^G\right)^{\vartheta} K_t^{\alpha} L_t^{1-\alpha}$. This is equivalent to my formulation in terms of productivity $z_t$ (see equation (22)).

8. *the government budget constraint* (17) *holds, public capital follows the law of motion* (18) *and determines productivity according to* (19).

In equilibrium, the free entry condition implies that labor market tightness solves the job creation equation

$$\frac{\kappa_t}{q_t^v(\theta_t)} = \beta \left\{ (1-\alpha) z_{t+1} k_{t+1}^{\alpha} - w_{t+1} + (1-\rho) \frac{\kappa_{t+1}}{q_{t+1}^v(\theta_{t+1})} \right\}. \tag{20}$$

Since $\frac{1}{q_t^v}$ is the average duration until a vacancy is filled, the left-hand side of (20) is the expected cost to fill a vacancy. In equilibrium, it has to be equal to the benefit of a filled vacancy on the right-hand side. It consists of the discounted output net of wages and capital costs in the next period plus the expected future value of a match taking into account that the match survives with probability $1-\rho$. Equation (20) emphasizes the dynamic nature of firms' vacancy posting decision and already contains the main intuition for the labor demand effects of public investment. Public investment raises productivity $z$ in the future, which leads to an increase in labor market tightness such that the average time to fill vacancies increases. This raises the expected costs for filling a vacancy in the future. As equation (20) shows, firms respond to higher expected costs of filling a vacancy in the future by expanding hiring in the present, thereby raising labor market tightness. The job creation equation is an affine difference equation of $x_t = \frac{\kappa_t}{q_t^v(\theta_t)}$. As long as the growth rate of productivity does not exceed $\frac{1}{\beta(1-\rho)}$ in the long run, it has a unique solution for $x_t$, and thus for labor market tightness, which satisfies the terminal condition that labor market tightness remains bounded.

Combining the first-order conditions for firms' optimal capital demand (13) and firm owners' optimal capital supply (8) gives

$$1 + \phi \left( \frac{K_{t+1}}{K_t} - 1 \right) = \beta \left( 1 + \alpha z_{t+1} k_{t+1}^{\alpha-1} - \delta_k + \frac{\phi}{2} \left( \left( \frac{K_{t+2}}{K_{t+1}} \right)^2 - 1 \right) \right). \tag{21}$$

Note that, if capital adjustment costs are zero, (21) reduces to the condition that the inverse of the discount factor equals the net return on capital.

Since every firm employs $k_t$ units of capital, the aggregate capital stock is $K_t = k_t N_t$ and aggregate output is

$$Y_t = z_t k_t^{\alpha} N_t = z_t K_t^{\alpha} N_t^{1-\alpha} = A_t \left( K_t^G \right)^{\vartheta} K_t^{\alpha} N_t^{1-\alpha}. \tag{22}$$

This is the same aggregate production function as in Baxter and King (1993).

# 3 Theoretical Analysis: Anticipation Effects on Labor Demand

In this section, I analyze the employment multiplier of government investment theoretically focusing on labor demand. I am interested in the change in employment in some period $t \geq 0$ that is brought about by a public investment program that is announced in period 0 and that permanently raises public investment starting in period $T \geq 0$. Hence, $T$ denotes the implementation lag of public investment. Formally, I define this employment multiplier of public investment as follows.

**Definition 2** (Employment multiplier of public investment)**.** *Let $N_t(\mathcal{X}_0, I_0^G, I_1^G, \dots)$ denote employment in period t in an equilibrium with initial conditions $\mathcal{X}_0 = (N_0, w_0, K_0^G, K_0)$ and public investment sequence $\mathcal{I}^G = \left(I_s^G\right)_{s=0}^{\infty}$. Consider a permanent expansion in public investment starting in period T. The employment multiplier of public investment in t is defined as*

$$M_t^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G) = \frac{\partial N_T(\mathcal{X}_0, I_0^G, \dots, I_{T-1}^G, I_T^G + x, I_{T+1}^G + x, \dots)}{\partial x}|_{x=0}.$$

The employment multiplier tells us how much employment changes in period $t$ when it is unexpectedly announced in period 0 that public investment will rise by 1 dollar in all periods after $T$. I make three assumptions.

**Assumption 1.** *Search effort is fixed at $\ell_t(u) = 1$.*

Hence, I focus on the role of labor demand for the employment multiplier of public investment. I allow for elastic search effort in the quantitative analysis in Section 5 and find that search effort contributes little to the employment effect.

**Assumption 2.** *The target wage is a fixed fraction of output and taken as given by firms, i.e. the target wage is given by* (15)*.*

Assumption 2 implies that there is no feedback from vacancy posting to wages through Nash bargaining which simplifies the analysis. Given the optimal capital choice, the job creation equation (20), together with the law of motion for employment (10) and the accumulation equation of public capital (18), is then sufficient to characterize the employment multiplier.

**Assumption 3.** *Capital adjustment costs are zero, $\phi = 0$.*

This assumption simplifies the law of motion for capital as it eliminates the dependence of the optimal capital choice $k_{t+1}$ on the current capital stock as well as on planned future capital. The choice for $k_{t+1}$ then only depends on expected productivity in $t + 1$ but not on past and future capital choices.

What happens to employment when the government announces a permanent expansion in public investment? Firms anticipate higher productivity in the future which increases the value of a filled vacancy but does not increase the cost of posting a vacancy. Thus firms post more vacancies and employment rises.

Importantly, the short-run employment effect is a dynamic phenomenon. Suppose that, in the long run, wages and posting costs are proportional to labor productivity. These are reasonable assumptions since vacancy posting costs are largely labor costs of the workers involved in the hiring process and since wages follow productivity in the long run. Then, public investment does not affect employment in the long run. Yet, employment still increases along the transition to the new steady state wither high public investment. The reason is that hiring costs are fixed in the short-run whereas the return from a filled vacancy increases with future productivity. Relative to the benefits, the costs of posting a vacancy decline such that firms expand hiring. Proposition 1 formalizes these points.

**Proposition 1** (Positive short-run employment multiplier of public investment)**.** *Suppose that $I_t^G = \delta_G K_0^G$ for all $I_t^G \in \mathcal{I}^G$ and that the initial wage is at least at the steady state level $w_0 \geq \omega \left(\frac{\alpha\beta}{1-\beta(1-\delta_k)}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}}$. Then, under assumptions 1–3, the employment multiplier of public investment is*

*(i) positive, $M_t^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G) > 0$,*

*(ii) zero in the long-run, $\lim_{t\to\infty} M_t^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G) = 0$, if wages are not completely rigid, $\gamma < 1$, and vacancy posting costs are proportional to labor productivity, $\kappa_t = \bar{\kappa} z_t k_t^{\alpha}$,*

*Proof.* See Appendix A.1. □

The employment multiplier can be characterized succinctly when the economy is in the steady state.

**Proposition 2.** *Suppose that the economy is in a steady state with $I_t^G = \delta_G K^G$ for all $I_t^G \in \mathcal{I}^G$ and assumptions 1–3 hold. Then, for $t+1 \leq T$, the employment multiplier of public investment is*

$$\begin{aligned} M_t^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G) = & \frac{(\beta(1-\rho))^{T+1-t}\vartheta}{1-\beta(1-\delta_G)(1-\rho)} \frac{1}{K^G} \frac{1}{1-\alpha} \left[1 + \frac{\gamma\omega(1-\beta(1-\rho))}{(1-\beta\gamma(1-\rho))(1-\alpha-\omega)}\right] \\ & \times \frac{1-\eta}{\eta} \frac{U\pi^{e|u}}{I^G} \frac{1-((1-\rho-\pi^{e|u})\beta(1-\rho))^t}{1-((1-\rho-\pi^{e|u})\beta(1-\rho))} > 0 \end{aligned} \tag{23}$$

*with $\pi^{e|u} = \zeta^{\frac{1}{\eta}} \left(\frac{\kappa(1-\beta(1-\rho))}{\beta(1-\alpha-\omega)}\right)^{\frac{\eta-1}{\eta}}$.*

*Proof.* See Appendix A.2. □

The proposition helps to understand the mechanism through which public investment affects employment in the short run and how the size of the employment multiplier depends on the fundamentals of the economy. To ease the exposition, let us consider the employment multiplier in the first period, i.e. for $t = 1$. In this case, equation (23) reads

$$
\begin{aligned}
M_1^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G) = \; & \underbrace{\frac{\beta^T (1-\rho)^T \vartheta}{1-(1-\delta_G)\beta(1-\rho)} \frac{1}{K^G}}_{\text{Effect of inv. on future productivity}} \\
& \times \underbrace{\underbrace{\frac{1}{1-\alpha}}_{\text{Ela. of profits w.r.t. productivity}} \left[ 1 + \underbrace{\frac{\gamma\omega(1-\beta(1-\rho))}{(1-\gamma\beta(1-\rho))(1-\alpha-\omega)}}_{\text{Wage stickiness}} \right]}_{\text{Effect on expected profits}} \\
& \times \underbrace{\frac{1-\eta}{\eta} U \frac{\pi(e|u)}{I^G}}_{\text{Effect on emp.}} .
\end{aligned}
\tag{24}
$$

The first factor is the change in the present value of expected match output following the increase in public investment,

$$
\frac{\beta^T (1-\rho)^T \vartheta}{1-(1-\delta_G)\beta(1-\rho)} \frac{1}{K_G} = \frac{\sum_{s=T+1}^{\infty} (\beta(1-\rho))^s z k_s^{\alpha}}{\partial x}.
$$

This is what is driving the employment effect—public investment raises productivity which leads to vacancy creation by forward-looking firms. The productivity effect and thereby the employment multiplier are larger if the output elasticity of public capital, $\vartheta$, is higher. This may not be surprising. If the output elasticity of public capital is larger, the future marginal product of labor and therefore future labor demand, labor market tightness and search costs increase more in response to an expansion in public investment. As a result, firms expand hiring by more already in the short run. Note that in a standard RBC model where labor demand is a static decision and public investment affects employment by shifting labor supply, a larger output elasticity of public capital can have the opposite effect (see Ramey 2020). The reason is that public investment raises household wealth relatively more if the output elasticity of public capital is higher. In the short run, this leads to a reduction in labor supply, hours worked and output compared to a case with a low output elasticity of public capital.

The extent to which higher productivity leads to additional employment is captured by the second and third line in (24). The second line determines how match output translates into firm profits. This depends on $\frac{1}{1-\alpha}$, the elasticity of per-period firm profits with respect to instantaneous productivity under flexible wages and on the degree of wage stickiness captured by the term $\frac{\gamma\omega(1-\beta(1-\rho))}{(1-\gamma\beta(1-\rho))(1-\alpha-\omega)}$. The investment program has a stronger effect on employment if firm profits respond more strongly to changes in productivity, which is the case if $\alpha$ is larger and wages are more rigid. Note that the wage stickiness term is zero if wages are fully flexible ($\gamma = 0$) but the overall effect is still strictly positive. Even if wages adjust to higher labor productivity immediately, an increase in expected future productivity raises the expected present value of output net of wages and capital costs of a match but it leaves the costs of posting a vacancy unchanged. This makes it more profitable for firms to post vacancies. However, the employment multiplier is larger if wages are more sticky. The reason is that if wages are more rigid, an expected increase in future productivity does not translate into a proportional increase in wages immediately so that per-period profits from a filled vacancy are expected to increase temporarily.

Finally, the employment multiplier depends on how strongly employment responds to additional vacancy creation, which is determined by the term in the third line of equation (24). It depends on the elasticity of the matching function with respect to vacancies, $1-\eta$ and on initial unemployment $U$. If the matching function elasticity is high, additional vacancies translate into relatively more matches and employment increases more strongly.

The discount factor and the separation rate affect the employment multiplier through two channels. First, they enter the first term in (24), the elasticity of match output with respect to public investment. Second, the discount factor and separation rate matter for the employment multiplier because they determine the importance of wage stickiness. Suppose first, that there is no wage stickiness, $\gamma = 0$. In this case, a higher discount factor and a lower separation rate unambiguously increase the employment effect as both facilitate labor hoarding. When the discount factor is higher, the increase in productivity in the (distant) future is valued more relative to additional costs of hiring and hoarding labor that are incurred in the near future. Hence, the employment effect of public investment is larger. If the separation rate is low, it is more likely that workers hired today will remain with the firm in the future. This makes it easier to substitute hiring inter-temporarily when future costs of filling a vacancy increase as a result of tighter labor markets. When wages are sticky, there is an opposing channel through which the discount factor and the separation rate affect the employment multiplier. A higher discount factor as well as a lower separation rate reduce the term labeled "Wage stickiness" in (24) which would lead to a smaller employment multiplier. When the separation rate is low or the discount factor is high, profits in the distant future are relatively more important for the vacancy posting decision of firms. Since wages adjust to higher levels of productivity over time, wage stickiness does not affect profits in the distant future. Hence, wage stickiness is less important for the employment effect when the discount factor is higher or the separation rate is lower. When wages are completely fixed ($\gamma = 1$) the discount factor and separation rate cancel out in the wage stickiness term. In this case, wages never adjust to higher productivity and the relative importance of wage payments in the distant future does not affect the present value of expected profits and vacancy creation. For the intermediate case with some wage stickiness, the overall effect of discount factor and separation rate on the employment effect of public investment is not clear when wages are sticky. Quantitative analyses suggest that the employment effect increases with the discount factor and declines with the separation rate.

The change in employment at a given point in time declines with the implementation lag $T$. If the implementations lag is long, productivity is expected to increase only in the very distant future and the program has a relatively small effect on employment in the near future. For a given steady state job finding probability $\pi^{e|u}$, the degree to which the implementation lag matters depends on the discount factor $\beta$ and the separation rate $\rho$.

Proposition 2 also shows that the employment multiplier increases in $t$, the time since the investment program has become known. The reason is twofold: first, as $t$ increases the increase in productivity comes closer which raises the value of a filled vacancy and leads to more hiring. Second, if $t$ is larger, more time has passed since news about higher future productivity became known such that firms' expansion in hiring has had more time to reduce unemployment.

## 3.1 Business cycle dependence of employment effects

Are the employment effects of public investment different if the expansion in public investment is announced during a recession? To shed light on this question, I investigate how the employment multiplier depends on two characteristic features of recessions, high unemployment and temporarily weak labor demand.

The law of motion for employment (10) helps to understand how unemployment influences the employment effect of public investment. I restate it here for convenience

$$N_{t+1} = (1-\rho)N_t + \pi_t^{e|u}(\theta_t)U_t.$$

Public investment induces an increase in labor market tightness and in the individual job finding probability of unemployed workers, $\pi_t^{e|u}(\theta_t)$. As can be seen from the law of motion for employment, if the number of unemployed workers is large, a given increase in the job finding probability benefits many workers and aggregate employment increases more strongly.

One can also think about the role of unemployment for the employment multiplier in terms of firms' vacancy creation. When unemployment is high, an additional vacancy has only a small effect on the vacancy filling probability of other firms. Suppose for example, that labor market tightness is one, i.e. there is one vacancy for every unemployment worker. If there is only one unemployed worker, an additional vacancy doubles labor market tightness. In contrast, if there are ten unemployed workers, an additional vacancy increases labor market tightness only by 10%. In the second case, the additional vacancy will have a much smaller effect on the expected costs of all other firms to fill a vacancy than in the first case. The congestion externality is small when unemployment is high. Hence, vacancy creation expands more in response to an increase in public investment that raises future productivity. The employment effect of public infrastructure investment is larger when unemployment is high.

A second feature of recessions that is important for the short-run employment effect of public investment is weak labor demand, i.e. a low labor market tightness and a small job finding probability for unemployed workers. In the model, labor demand is low if the wage is high relative to productivity. Thus, I study how the short-run employment effect of public investment depends on the wage. A high wage corresponds to a situation of weak labor demand—a recession. To build some intuition, consider the job creation equation (20) in period 0. It can be written as

$$\frac{\kappa_0}{q^v(\theta_0)} = \beta(y_1 - w_1 + (1-\rho)J_2^F), \tag{25}$$

where $y_1$ is labor productivity in period 1 and $w_1$ is the wage in period 1. The variable $J_2^F$ is the value of a filled vacancy in period 2. For now, I interpret period 2 as the long run and I suppose that public investment raises the value of a match in the long run $\mathrm{d}J_2^F > 0$. The job creation equation yields

$$\mathrm{d}q^v(\theta_0) = q^{v\prime}(\theta_0)\mathrm{d}\theta_0 = -\frac{\beta(1-\rho)}{\kappa_0}q^v(\theta_0)^2\mathrm{d}J_2^F = -\frac{(1-\rho)\kappa_0}{\beta(y_1 - w_1 + (1-\rho)J_2^F)^2}\mathrm{d}J_2^F < 0.$$

When public investment increases, the vacancy filling probability declines relatively more if labor market tightness is low and the vacancy filling probability is high, i.e. if labor demand is weak. This is the case if the wage $w_1$ is high relative to labor productivity $y_1$ such that the value of a match is relatively small. For this reason, the same increase in the long-run value of a match leads to a relatively larger effect on the value of a match and thereby on the vacancy filling probability. The change in the job finding probability in response to an increase in the long run value of a match is

$$\mathrm{d}\pi_0^{e|u}(\theta_0) = \mathrm{d}q^v(\theta_0)\theta_0 = q^{v\prime}(\theta_0)\theta_0\mathrm{d}\theta_0 + q^v(\theta_0)\mathrm{d}\theta_0 = \frac{\eta-1}{\eta}\theta_0\mathrm{d}q^v(\theta_0) > 0.$$

We know from above that the change in the job finding probability $\mathrm{d}q^v(\theta_0)$ is larger when labor demand is weak. But then, labor market tightness is lower such that the effect of weaker labor demand on the job finding probability is ambiguous. Intuitively, when labor demand is weak, the value of a match is small and the increase in long run productivity brought about by public investment raises it relatively more. This is why labor market tightness increases more. However, since labor demand is weak, the same increase in labor market tightness corresponds to relatively few additional vacancies. Which of these two effects dominates, depends on the elasticity of the matching function with respect to vacancies.

The next proposition shows that these intuitions carry over to the full model.

**Proposition 3** (Business cycle dependence of employment effect). *Suppose that* $I_t^G = \delta_G K_0^G$ *for all* $I_t^G \in \mathcal{I}^G$ *and the wage is at the steady state level* $w_0 = \omega \left(\frac{\alpha\beta}{1-\beta(1-\delta_k)}\right)^{\frac{\alpha}{1-\alpha}} z^{\frac{1}{1-\alpha}}$. *If assumptions 1–3 hold, then, for* $t+1 \leq T$, *the employment multiplier of public investment is*

*(i) increasing in initial unemployment,* $\frac{\partial M_t^{Inv}(T,\mathcal{X}_0,\mathcal{I}^G)}{\partial U_0} > 0$

*(ii) increasing in the initial wage* $\frac{\partial M_t^{Inv}(T,\mathcal{X}_0,\mathcal{I}^G)}{\partial w_0} \geq 0$ *if* $\eta > 0.5$.

*Proof.* See Appendix A.3. □

## 3.2 Welfare effects of public investment

The permanent expansion in public investment raises employment as firms expand hiring in anticipation of higher future productivity. I now show that this increase in employment can constitute a welfare improvement. To that end, I define social welfare in the following way

$$W(\{c_t^F, c_t(s^t), \ell_t(s^t)\}) = \bar{\mu}^F \sum_{t=0}^{\infty} \beta^t u^F(c_t^F) + \sum_{t=0}^{\infty} \beta^t \sum_{s^t} u\left(c_t(s^t), \ell_t(s^t), s_t\right) \pi_t(s^t|s_0)\bar{\mu}(s_0),$$

where $\bar{\mu}^F$, $\bar{\mu}(e)$ and $\bar{\mu}(u)$ are the welfare weights of firm owners, initially employed and initially unemployed workers and $\pi_t(s^t|s_0)$ denotes the share of workers with history $s^t = (s_0, s_1, \ldots, s_t)$ in period $t$. Let $C_t$ denote aggregate consumption in period $t$ and define consumption of individual firm owners and consumption of workers relative to total consumption respectively as $v_t^F \equiv \frac{c_t^F}{C_t}$ and $v_t(s^t) \equiv \frac{c_t(s^t)}{C_t}$. Under Assumption 1 (fixed search effort), the effect of the investment program on welfare is

$$\begin{aligned}\frac{\partial W}{\partial x} = & \underbrace{\sum_{t=0}^{\infty} \beta^t C_t \left(\bar{\mu}^F u_c^F(c_t^F)\frac{\partial v_t^F}{\partial x} + \sum_{s^t} \bar{\mu}(s_0)\pi_t(s^t|s_0)u_c(c_t(s^t))\frac{\partial v_t(s^t)}{\partial x}\right)}_{\text{redistribution (intensive margin)}} \\ & + \underbrace{\sum_{t=0}^{\infty} \beta^t u(c_t(s^t), \ell_t(s^t), s_t)\bar{\mu}(s_0)\frac{\partial \pi_t(s^t|s_0)}{\partial x}}_{\text{redistribution (extensive margin)}} + \underbrace{\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x}}_{\text{aggregate consumption}},\end{aligned} \tag{26}$$

where

$$\mathrm{m}_t \equiv \bar{\mu}^F v_t^F u_c^F(c_t^F) + \sum_{s^t} \bar{\mu}(s_0)\pi_t(s^t|s_0)v_t(s^t)u_c(c_t(s^t)) \tag{27}$$

is the marginal utility of aggregate consumption in period $t$, a weighted average of individual marginal utilities of consumption, where the weight of each agent corresponds to its welfare weight multiplied by its consumption share. As can be seen from equation (26), the effect of the expansion in public investment on welfare can be decomposed into three parts. The first captures the effect of public investment on the distribution of consumption along the intensive margin. Depending on how the increase in public investment is financed, consumption of employed workers, unemployed workers or firm owners increases or falls relative to aggregate consumption and this redistribution changes welfare, even if aggregate consumption remains unchanged. This distributive effect is captured by the first line in equation (26). Note that under Assumptions 2 wages are independent of taxes such that the government can use labor taxes and lump-sum taxes on firm owners to finance investment in a way that leaves the consumption shares of all households unchanged. In this case there is no redistribution of consumption along the intensive margin and the first line in (26) is zero.

The second effect on welfare emerges because the increase in public investment redistributes consumption (and effort) along the extensive margin as it alters the share of workers who are employed. Proposition 1 showed that employment increases in all periods in response to a permanent expansion in public investment if the wage and public investment are in steady state initially. Hence, the extensive margin redistribution raises welfare for sensible parameter choices under which the after-tax wage exceeds unemployment benefits and compensates for potential utility losses from working.

The last summand in equation (26) captures the welfare effect of changes in aggregate consumption due to a permanent increase in public investment. The change in aggregate consumption is

$$\sum_{t=0}^{\infty} \mathrm{m}_t \frac{\partial C_t}{\partial x} = \underbrace{\sum_{t=0}^{\infty} \mathrm{m}_t K_t^{\alpha} N_t^{1-\alpha} \frac{\partial z_t}{\partial x}}_{\text{direct gross return}} - \underbrace{\sum_{t=0}^{\infty} \mathrm{m}_t \frac{\partial I_t^G}{\partial x}}_{\text{costs}} + \underbrace{\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t EG_t}_{\text{efficiency gain}} \tag{28}$$

Equation (28) shows that there are three channels through which the permanent increase in public investment affects aggregate consumption. The first two are standard. On the one hand, public investment raises productivity, which leads to an increase in output and consumption. On the other hand, there is a resource cost of public investment that reduces consumption. In the frictional labor market considered here, there is a third channel through which public investment affects output. I label it $EG_t$ for "Efficiency Gain" in equation (28).

If the economy is in the steady state, the efficiency gain is given by

$$\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t EG_t = \frac{1}{1-\eta} \left[ w - \eta \left( (1-\alpha) z k^{\alpha} + \theta \kappa \right) \right] \sum_{t=0}^{\infty} \beta^t M_{t+1}^{Inv}$$

and comes from the fact that the equilibrium in the matching labor market is not necessarily efficient such that the employment effect of public investment by itself can improve welfare.[10] When a firm posts a vacancy, it imposes a negative externality on other firms,

10. For simplicity, I assume that vacancy posting costs are constant, $\kappa_t = \kappa$. For the quantitative model in Section 5, I instead assume that vacancy posting costs are proportional to labor productivity and therefore

since the additional vacancy makes it more difficult for other firms to fill theirs. However, there is also a positive externality because every additional vacancy makes it easier for workers to find a job. As shown by Hosios (1990), there exists a wage that internalizes both effects and leads to the optimal level of vacancy creation. This wage is such that workers' share of the total match surplus equals the elasticity of the matching function. Here, this is the case if

$$w^* = \eta\left((1-\alpha)zk^\alpha + \theta\kappa\right).$$

As expected, when $w = w^*$, the efficiency gain is zero. If, in contrast, the wage exceeds the efficient wage, $w > w^*$, equilibrium vacancy creation is too low and the expansion in labor demand brought about by the investment program can raise the amount of resources available for consumption.

**Proposition 4** (Efficiency gains from public investment)**.** *Suppose the economy is in a steady state with inefficiently low labor demand, $w > w^*$. Then, the public investment program improves labor market efficiency, $\sum_{t=0}^{\infty} \beta^t \mathrm{m}_t EG_t > 0$.*

*Proof.* See Appendix A.4. □

# 4 Calibration

To quantify the employment effect, I calibrate the model to the US economy and set the period length to one month. The calibration is targeted at the steady state of the model with the exception of two parameters, $\gamma$, the degree of wage stickiness, and $\phi$, which governs the capital adjustment costs. Since the steady state is unaffected by these two parameters, I pick values previously used in the literature and validate this choice by comparing the business cycle moments generated by the model to those in the data.

**Technology** Regarding the production technology, I set $\alpha = 0.33$ and assume the monthly depreciation rate of physical capital is $\delta_k = 0.00874$, which corresponds to 10% annually. Following Baxter and King (1993), the depreciation rate of public capital is also set to $\delta_G = 0.00874$. Regarding the elasticity of productivity with respect to public capital, $\vartheta$, the meta study in Bom and Ligthart (2014) points to an elasticity of 0.12 in the long-run, Bouakez et al. (2017) find 0.065 and Cubas (2020) finds 0.09. I decide on an intermediate value of 0.1 which is also considered in Leeper et al. (2010) and Leduc and Wilson (2013).[11] This is a conservative choice, other empirical studies have found substantially larger values than those above. For example, Aschauer (1989) finds 0.39 and Pereira and Frutos (1999) report 0.63 as a general equilibrium elasticity which according to Ramey (2020) corresponds to a value for $\vartheta$ of 0.39. Of course, the size of the program

depend on public investment. In the Appendix, I characterize the effect of public investment on aggregate consumption also for the case where posting costs depend on public investment.

11. In addition to $\vartheta = 0.1$, Leeper et al. (2010) also consider $\vartheta = 0.05$.

determines the government's financing needs but when lump-sum taxes on firm owners are available it has no additional effects on the economy. Finally, I set $A$ such that labor productivity is normalized to one, $(1-\alpha)zk^{\alpha} = 1$, at a public investment rate of 2.9% which was the average rate for the US between 1990 and 2019.

**Labor market** The calibration approach for the parameters related to the labor market is standard in the literature and follows Shimer (2005). In particular, I match the transition probabilities between employment and unemployment estimated from CPS microdata.

In order to estimate these transition probabilities, I need a definition of unemployment in the data that most closely corresponds to unemployment in the model. Most of the literature uses the unemployment concept U-3 of the Bureau of Labor Statistics (BLS) which defines unemployed workers as those who are not employed, but available to work, and made an effort to find work during the last four weeks or were temporarily laid off and waiting to be recalled. In my model, all workers who are not employed are considered unemployed irrespective of how intensely they search. Therefore, for my baseline calibration, I use a broader definition of unemployment that also encompasses marginally attached workers. These are workers who are not employed but available to work, state that they want a job and have searched for a job during the last twelve months. Hence, I use the unemployment concept U-5 of the BLS. However, I obtain very similar results when using U-3 unemployment instead.

I match individuals over time in CPS micro data from January 1994 to December 2020 to estimate monthly job finding probabilities and separation probabilities from gross flows between labor market states. The estimation approach follows Shimer (2012) and is described in greater detail in Appendix B.1. Table 3 gives an overview of the estimation results.

I find a monthly separation probability of 1.9% which directly informs the choice of the separation parameter $\rho$. It implies that jobs last about 52 months on average. Hagedorn and Manovskii (2008) use a slightly higher but comparable number of 2.6% that leads to an average job duration of 38 months. The parameter $\rho$ is crucial for the size of the employment effect as it determines how long firms can expect a match to last (see Proposition 2). In my model, the rate at which matches are dissolved equals the rate at which workers become unemployed or leave the labor force but this need not be the case if there are job-to-job transitions. This might be a concern since the rate at which matches are dissolved determines the size of the labor demand response to public investment but the rate at which workers exit employment is used to inform my calibration of $\rho$. However, Hyatt and Spletzer (2016) document that average tenure has risen since the 1980s and median job tenure of employed workers was around 4.5 years in 2012, even longer than the median tenure of about three years implied by my choice for $\rho$.

For the monthly job finding probability I estimate a value of 26.9%. In contrast to the separation probability, the job finding probability $\pi_t^{e|u}$ is determined endogenously in the model and I match the estimated value by choosing the remaining labor market parameters as follows.

I set the elasticity of the matching function with respect to unemployment to $\eta = 0.3$. This is on the lower end of the range of empirical estimates surveyed in Petrongolo and Pissarides (2001) but still larger than 0.245 chosen in Hall (2005). A value of $\eta = 0.7$ appears to be more common in the literature (Krusell et al. 2010). Landais et al. (2018) use $\eta = 0.6$ as a benchmark and also consider $\eta = 0.5$ and $\eta = 0.7$. On the other hand, Hall (2005) calibrates $\eta = 0.245$, even lower than my choice.

I assume that unemployment benefits are proportional to the wage, $b_t = \bar{b}(1 - \tau_t)w_t$, and set the replacement rate $\bar{b}$ to 70%. This is higher than the average replacement rate in the US, usually found to be close to 40%. However, it implies a decline in consumption expenditures upon becoming unemployed close to the estimates of Chodorow-Reich and Karabarbounis (2016) from the Consumer Expenditure Survey which lie between 28% for food, clothing, recreation and vacation and 21% for food. I set the workers' bargaining weight to $\psi = 0.4169$ in order to match a labor share of 64%. In Appendix F, I alternatively calibrate the bargaining power such that the steady state wage is efficient.

Vacancy creation costs are assumed to be proportional to labor productivity $\kappa_t = \bar{\kappa} z_t k_t^{\alpha}$. Den Haan et al. (2000) find a vacancy filling probability of $q^v = 71\%$. According to the job creation equation (20), this requires $\bar{\kappa} = 0.7636$. It remains to calibrate the matching efficiency $\zeta$ and the disutility from effort. Regarding the latter, I assume that $d(\ell, s) = d_1 \frac{\ell^{1+\chi}}{1+\chi} + d_{0,s}$ as in Krebs and Scheffel (2017). I set $d_{0,u} = 0$ as a normalization and choose $d_{0,e}$ such that in the steady state there is no difference between the disutility from working and searching. In other words, search effort and other non-pecuniary costs of unemployment such as lower social status offset the utility gain from more leisure, an assumption also made in McKay and Reis (2021). The matching efficiency $\zeta$ and the disutility parameter $d_1$ are not separately identified which is why I normalize $d_1 = 1$. I then choose $\chi = 4.7013$ to obtain a micro elasticity of the job finding probability with respect to unemployment benefits of –0.5.[12] This elasticity is in line with direct empirical evidence in Chetty (2008) who obtains an estimate of –0.53. It is also in the range from –0.6 to –0.2 considered in Landais et al. (2018). I set $\zeta = 0.5631$ to match my estimate for the monthly job finding probability of 26.9%. More specifically, the target for the job finding probability $\pi^{e|u} = 0.269$ together with a vacancy filling probability of $q^v = 0.71$ implies that $\theta\ell(u) = \frac{0.269}{0.71} = 0.379$. Under the parameters calibrated so far, search effort is $\ell(u) = 0.86$ such that $\theta = 0.44$. Since $q^v = \zeta\theta^{-\eta}$, I get $\zeta = 0.5631$.

12. See Appendix B.2 for a derivation of $\chi$ in terms of the micro elasticity of the job finding probability with respect to unemployment benefits.

**Discount factor** Typically, discount factors are chosen to match the observed interest rate. Since I have ruled out saving in the model, there is no interest rate that could inform the choice of the discount factor. However, I show in the appendix that the model can be extended to allow for saving and borrowing and that the extended model features the same equilibrium allocation if the interest rate on savings is at most

$$1 + r_{t+1} = \frac{1}{\beta}\left(\left[\pi_t^{e|e}\frac{(1-\tau_t)w_t}{(1-\tau_{t+1})w_{t+1}}\varphi_t + \pi_t^{u|e}\frac{(1-\tau_t)w_t}{b_{t+1}}\right]\right)^{-1} \tag{29}$$

with

$$\varphi_t = 1 - (1-\gamma)(1-\psi)\frac{1 - \frac{w_{t+1}^N}{b_{t+1}} + J_t(e) - J_t(u)}{1 + (1-\psi)\left(J_t(e) - J_t(u)\right)} \tag{30}$$

and the borrowing rate exceeds the savings rate by a constant borrowing wedge $\xi$. I set the monthly discount factor to $\beta = 0.9911$ to obtain an annual interest rate of 1% according to equation (29).[13] Note that with $\varphi_t = 1$ the right-hand side of (29) is the standard formula for the intertemporal marginal rate of substitution between consumption today and tomorrow. The term $\vartheta_t$ captures an additional savings motive which arises because asset holdings affect the bargaining position of workers. Inspection of (30) shows that this motive is absent if wages are completely rigid ($\gamma = 1$) or if workers have the entire bargaining weight so that they receive the total surplus regardless of their asset holdings ($\psi = 1$).[14] Due to the precautionary savings motive and the effect of savings on the bargaining position, the discount factor is lower than under complete markets which leads to a relatively smaller employment effect as shown in the previous section.

Table 1 provides an overview of the calibrated parameters. In the steady state, the unemployment rate is 6.58%. For comparison, the average U-5 unemployment rate from 1994 to 2020 was 6.86%. The (private) physical investment rate is 18.7%, close to the average of 17.3% observed in the data since 1990.[15]

Finally I set the parameter $\gamma$, which governs the extend of wage stickiness, to 0.993. This choice is also considered in Shimer (2010) who argues that it leads to a reasonable volatility of unemployment over the business cycle. I pick $\phi = 15$ for the capital adjustment cost parameter. As shown in the next subsection, for these choices, the model is able to replicate the volatility of unemployment and investment observed in the data for a realistic process of productivity. In the Appendix, I investigate the role of wage stickiness $\gamma$ and capital adjustment costs $\phi$ for the results.

13. Note that since workers face unemployment risk and firm owners do not, workers always have a higher willingness to save for a given discount factor.

14. See Krusell et al. (2010) for a detailed investigation of this effect on savings and the labor market.

15. The investment rate has not changed much over time, averaging at 17.1% for the time since 1947.

Table 1: Baseline calibration.

| Parameter | | Value | Descripton | Target or source |
|---|---|---|---|---|
| Technology (private) | $\alpha$ | 0.33 | output ela. cap. | standard |
| | $\delta_k$ | 0.0087 | phys. cap. deprec. | ann. depreciation 10.0% |
| | $\phi$ | 15 | cap. adj. costs | see text |
| Technology (public) | $\vartheta$ | 0.10 | output ela. pub. cap. | see text |
| | $\delta_G$ | 0.0087 | pub. cap. deprec. | ann. depreciation 10.0% |
| | $B$ | 0.3571 | productivity | public inv. rate 2.9% |
| Labor market | $\bar{b}$ | 0.7 | wage replacement rate | see text |
| | $\eta$ | 0.3 | match. fct. ela. | Petrongolo and Pissarides (2001) |
| | $\psi$ | 0.4169 | worker barg. weight | labor share 64.0% |
| | $\rho$ | 0.0189 | separation prob. | 1.9% (own est., see text) |
| | $\zeta$ | 0.5631 | match. efficiency | vac. filling prob. 71.0% (HRW00) |
| | $\bar{\kappa}$ | 0.7636 | post. costs (labor) | job finding. prob. 26.9% (own est., see text) |
| | $\gamma$ | 0.9930 | wage stickiness | see text |
| Preferences | $\beta$ | 0.9911 | discount factor | ann. interest rate 1.0% |
| | $\chi$ | 4.7013 | search ela. | $\mathrm{d}\log q^f / \mathrm{d}\log b = -0.5$ |
| | $d_{0,e}$ | 0.0567 | work disutility | $d(\ell(u), u) = d(0, e)$ |

*Notes:* HRW00 stands for Den Haan et al. (2000).

## 4.1 Business cycle properties

I compute standard deviation and quarterly autocorrelation of unemployment, output, investment and labor productivity in the data to evaluate the model's ability to match the volatility and persistence of these variables. To be precise, I compute these moments for the relative deviations from a long-run trend obtained using an HP filter with smoothing parameter 1,600. I use data from the first quarter of 1951 to the fourth quarter of 2019. All moments shown in the first two rows of Table 2 are close to those found in the literature. In particular, the estimates of the standard deviation and autocorrelation of the U-3 unemployment rate are very close to those in Hagedorn and Manovskii (2008) who report 0.125 and 0.870, respectively. Since the calibration focuses on the broader measure of U-5 unemployment, I also report the respective moments for this variable. Relative to U-3 unemployment, it exhibits a slightly lower standard deviation of 0.101 and a higher autocorrelation of 0.944. Standard deviation and autocorrelation of labor productivity are also very close to the estimates in Hagedorn and Manovskii (2008) who find 0.013 and 0.765.

In order to asses the model's ability to replicate these moments, I assume that the public capital stock is constant and $A_t$ follows an AR(1) process in logs

$$\log A_t = \rho \log A_{t-1} + \nu_t, \tag{31}$$

where $\nu_t$ is normally distributed with mean zero and standard deviation $\sigma_\nu$. For the baseline calibration, I set $\rho = 0.9870$ and $\sigma_\nu = 0.0054$. This way, standard deviation and autocorrelation of quarterly TFP in the model match those in the data. I assume that

Table 2: Overview of business cycle moments

| | | u (U-5) | u (U-3) | Y | inv | wages | lab. prod. | z |
|---|---|---|---|---|---|---|---|---|
| Data | Std. dev. | 0.101 | 0.128 | 0.015 | 0.065 | 0.010 | 0.012 | 0.012 |
| | Autocorr | 0.944 | 0.886 | 0.845 | 0.821 | 0.742 | 0.759 | 0.797 |
| Model | Std. dev. | 0.083 | – | 0.017 | 0.094 | 0.008 | 0.011 | 0.012 |
| | Autocorr. | 0.847 | – | 0.848 | 0.255 | 0.947 | 0.790 | 0.791 |

*Notes:* For comparability with the data, all model moments are computed for the relative deviations from the HP trend of the series aggregated to quarterly frequency. I use quarterly data from 1951:I to 2019:IV.

unemployment benefits are fixed at the steady state level. In reality, benefits depend on the individual labor market history. Thus, benefits grow with wages in the long run which is why I assume that benefits are proportional to wages in the next section, when I investigate the employment effects of a permanent expansion in public investment Here, I only consider short-run fluctuations, so that a constant level of benefits is a good approximation to observed benefit schemes. However, the results are very similar when I assume that benefits are proportional to wages.

The last two rows of Table 2 show the model moments corresponding to those in the data. Importantly, the volatility of unemployment and output are close to the data, even though the volatility of unemployment is still slightly lower than observed in the data. As pointed out by Shimer (2005), it is generally difficult for the DMP model to match the volatility of unemployment. My model is able to generate a volatility similar to the data mainly because of the relatively high degree of wage inertia. Nevertheless, the volatility of wages in the model is only slightly lower than in the data. Despite the capital adjustment costs, the volatility of private investment is still larger in the model than in the data but the order of magnitude is the same.

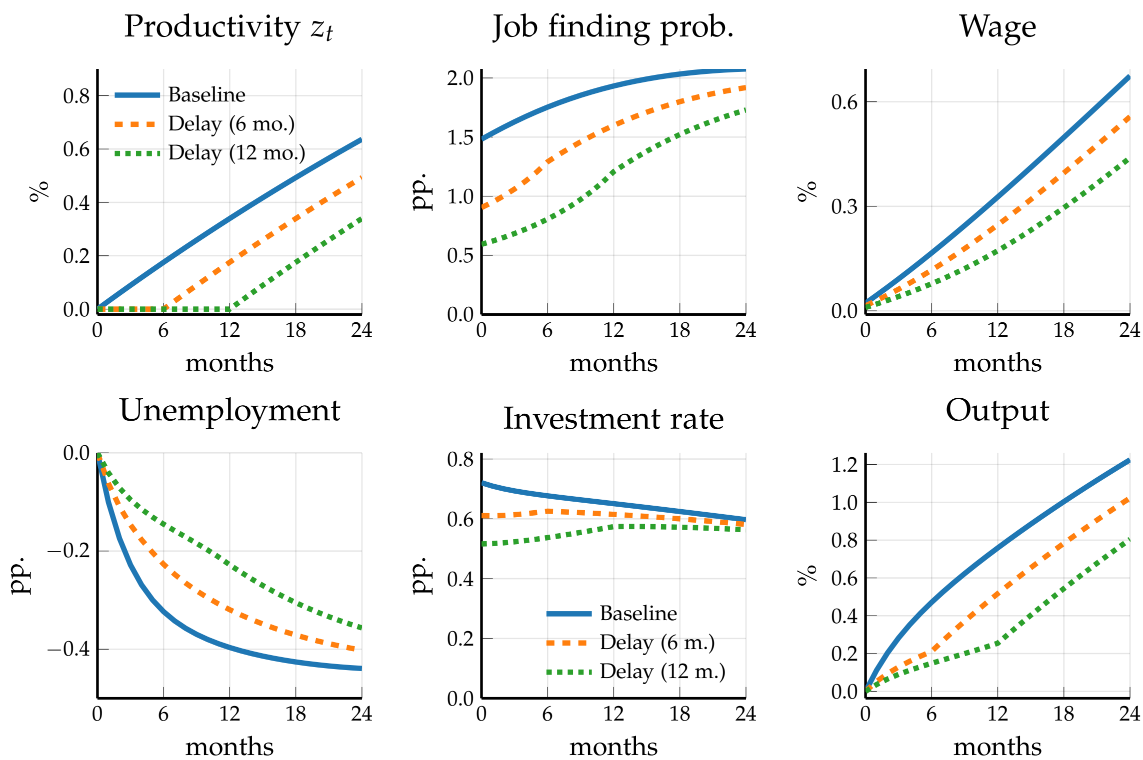
# 5 Quantitative Analysis of the Employment Effect

I assume that the government announces a permanent expansion in government investment by 1% of GDP in period zero in line with the public investment program discussed in the US. The program is financed with lump-sum taxes on firm owners. In the long run, the program increases the public capital stock and thereby raises productivity by 3%. Figure 1 shows the responses of key variables to the announcement of the government investment program. I assume that the economy is in its steady state initially and consider three different scenarios focusing on the response over the first two years.[16] The solid blue line depicts the baseline scenario in which public investment is increased at the same time the program is announced such that productivity starts to rise in the first period. The dashed red line shows the response when it takes six months after

16. See Figures 13 and 12 in the appendix for the long-run responses and the corresponding fiscal policy.

the announcement of the investment program before it is implemented and starts to have an effect on productivity. The dotted green line corresponds to the case where the delay amounts to twelve months. The scenarios with implementation delays are of interest for two reasons. First, the existing literature has emphasized delays as an important characteristic of government investment, which set it apart from consumptive government spending and which can impair its effectiveness as a means of short term stimulus (Leeper et al. 2010). Second, comparing how the economy responds to the investment program under different implementation delays allows us to better understand the mechanism through which it affects the economy in the short-run. In particular, it helps to disentangle the expectations effect from the consequences of the contemporaneous increase in productivity which is zero at first in the case of delay.

Figure 1: Short-run responses to public investment program.



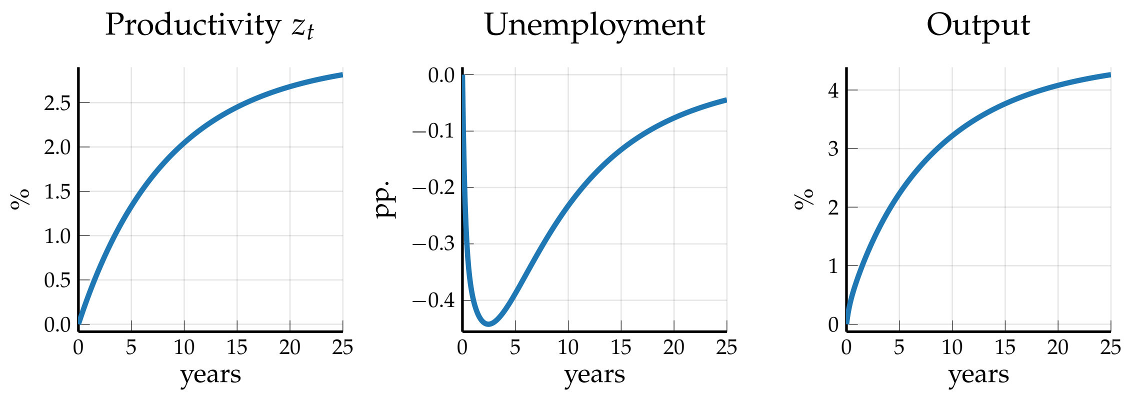

*Notes:* The figure shows the responses to a permanent expansion of public investment by 1% of initial GDP for different implementation delays as deviations from the initial steady state.

Consider the baseline scenario first. Productivity increases almost linearly over the first two years of the program after which it is 0.63 percent higher than before. The increase in productivity brought about by public investment has a substantial effect on unemployment and output. With the start of the program, firms expand vacancy creation such that the job finding probability increases by 1.5 percentage points on impact. The increase in the job finding probability lowers the unemployment rate by 0.4 percentage points after twelve months. The private investment rate increases by about 0.7 percentage points on impact and then quickly returns to a permanently elevated level 0.6

percentage points above the one that would be seen without the investment program. As a consequence of increased hiring and capital investment, output is about 0.8% higher after one year. Wages also increase substantially and are 0.32% percent higher after one year than they would have been without the additional public investment. This might be surprising at first given the seemingly high degree of wage inertia with $\gamma = 0.993$. The reason that wages still respond relatively strongly is that the higher job finding probability improves the bargaining position of workers such that the Nash bargaining wage increases substantially (see Figure 3 and the discussion below). During the first years after the start of the program, the Nash wage substantially exceeds the new long-run wage which leads to a much faster increase of the wage than would be obtained if I naively substituted the new long-run Nash wage into the wage rule and iterated forward.[17]

It is instructive to compare these short-run responses to the long-run effect of the increase in public investment shown in Figure 2. The investment program does not af-

Figure 2: Long-run responses to the expansion of government investment

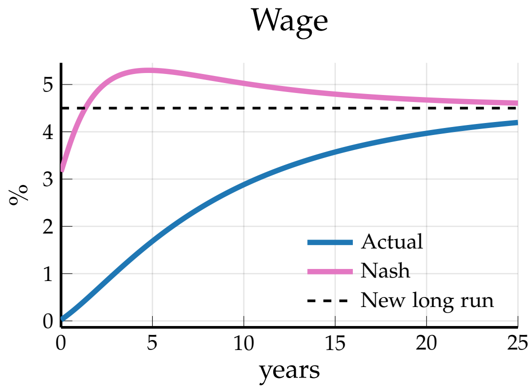



*Notes:* Shown are deviations from the initial steady state.

fect unemployment in the long run. This is because, in the long run, vacancy posting costs, wages and unemployment benefits are all proportional to labor productivity. As a result, firms' incentives to post vacancies are not affected by higher productivity in the long run. Since workers have logarithmic utility, the constant wage replacement rate of unemployment benefits implies that workers' search effort is unaltered in the long-run. Importantly, unemployment falls below its new long-run level temporarily. Its trough is already reached after 2.5 years. The reason for this is twofold. First, wage inertia implies that wages take time to catch up to increased productivity. This raises the share of the match surplus received by firms temporarily who respond by expanding vacancy creation. Second, vacancy posting costs only depend on the current level of labor productivity whereas the value of a filled vacancy to a firm also depends on future productivity. Therefore, when productivity grows, the surplus is large relative to the costs of creating a vacancy which leads to an expansion in vacancy creation and

17. Compare the discussion in Hall (2003, Section V.C).

Figure 3: Actual and Nash bargaining wage




*Notes:* Shown are percentage deviations from the initial steady state.

low unemployment. As growth in labor productivity returns to its long-run trend, the difference between match surplus and vacancy posting costs declines, less vacancies are posted and unemployment increases.

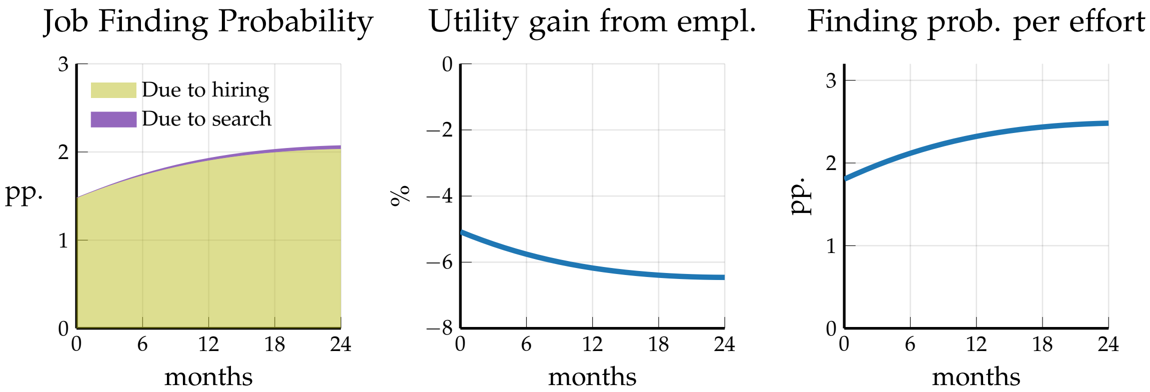
Turning to the short-run responses for the scenarios with implementation delay, we see that even when it takes six or twelve months for the investment program to have an effect on productivity, output and unemployment respond already upon announcement of the investment program. With an implementation delay of six months, unemployment is almost 0.35 percentage points lower twelve months after the announcement. This is more than three quarters of the decline without the delay. Similarly, output after one year is close to 0.6% higher than without the expansion in public investment.

If the delay amounts to twelve months, the investment program still reduces unemployment after one year by about 0.22 percentage points, more than half the reduction without any delay. Output after twelve months is still close to 0.3% higher. Importantly, the increase in output and decline in unemployment take place before the investment program has had any effect on productivity (see the top left panel of Figure 1). The observed effect is entirely due to agents anticipating higher productivity in the future as a result of more government investment.

Higher future productivity due to the announcement of the public investment program not only affects labor demand of firms but also the behavior of workers. Two effects are important. First, workers demand a higher wage. Since higher future productivity increases the expected total surplus from the match, the Nash wage increases already today which raises wages. The increase in wages depends on workers' bargaining weight and the degree of wage inertia. Figure 3 shows that the news about higher future productivity raise the Nash bargaining wage substantially, by 3% on impact. Due to wage inertia, the increase in the Nash bargained wages only gradually translates into actual wage gains and the actual wage increases almost linearly during the first years after the start of the investment program.

Second, workers also respond to the anticipated increase in productivity by adjusting

Figure 4: The response of search effort.



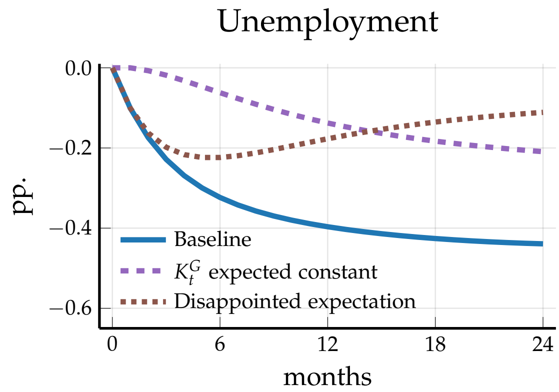

*Notes:* Shown are deviations from the initial steady state.

their search effort. To assess the importance of workers' search effort, I decompose the change in the job finding probability in every period according to

$$\pi_0^{e|u} - \bar{\pi}^{e|u} = \underbrace{\frac{\pi_0^{e|u}}{\ell_0(u)}\bar{\ell}(u) - \frac{\bar{\pi}^{e|u}}{\bar{\ell}(u)}\bar{\ell}(u)}_{\text{vacancy posting}} + \underbrace{\frac{\pi_0^{e|u}}{\ell_0(u)}\ell_0(u) - \frac{\pi_0^{e|u}}{\ell_0(u)}\bar{\ell}(u)}_{\text{search effort}}, \tag{32}$$

where a bar denotes the variable in the initial steady state. The left panel of Figure 4 shows this decomposition graphically. The blue line is the total change in the job finding probability, the left-hand side of (32). The dashed green line is the part due to changes in search effort, the terms labeled "search effort" in (32) and the grey line is the part due to changes in labor demand, the terms labeled "vacancy posting" in (32). The increase in the job finding probability is almost entirely due to changes in firms' labor demand. Search effort also contributes to the increase but its effect is negligible. In the first period, the job finding probability increases by 1.4819 percentage points. Only 0.0005 percentage points are due to the expansion in search effort. Two forces drive the response of effort: the expected gain in lifetime utility from finding a job and the marginal effect of higher effort on the job finding probability (see equation (4)). The center panel in Figure 4 shows that the expected gain in lifetime utility from finding a job declines in response to the investment program. The reason is that job finding rates increase such that unemployed workers can expect to stay unemployed for a shorter period of time. This effect dominates the increase in wages which would lead to an increase in the difference between expected lifetime utility of employed and unemployed workers. The drop in the expected gain in lifetime utility from finding a job would lead workers to lower their search effort. However, for the baseline scenario, this effect is outweighed by the increase in the job finding probability per unit of effort which equals the marginal increase in the job finding probability due to the linearity of the job finding probability in effort. This is depicted in the right panel of Figure 4. In other words, it

Figure 5: The anticipation effect.




*Notes:* The dashed purple line shows the response of unemployment in a scenario where agents only find out about increases in productivity as they materialize and expect constant productivity at every point in time. The dotted brown line is the case where agents always expect the increase in productivity to start in the next period although it never happens.

is the immediate increase in labor demand due to the anticipation effect that prevents workers from reducing search effort. The anticipation effect on short-run labor demand has two effects on employment. It increases the job finding probability directly and thereby raises employment. But it also has an indirect effect on employment as it causes workers to expand search effort.

## 5.1 The anticipation effect

To quantify the contribution of the *anticipated* increase in future productivity to the reduction in unemployment, I consider the following hypothetical scenario. I assume that private agents do not learn about the permanent expansion in public investment when in period zero. Instead, they expect productivity to stay constant at every point in time. In period zero, they expect productivity to stay at its steady state level forever. In period one, they are surprised that productivity has increased but expect it to stay at the new level such that in period two they are surprised again by the additional increase. Hence, agents only learn about increases in productivity as they occur. The dashed purple line in Figure 5 shows the evolution of unemployment in this case. It still declines but more slowly than when the anticipation effect is present. After one year, unemployment has fallen by 0.13 percentage points, more than 65% less than in the baseline scenario. I interpret this difference as the contribution of the anticipation effect to the unemployment reduction.

An alternative way to quantify the anticipation effect is to consider the case where private agents expect the permanent expansion in public investment to begin at every point in time even though this is never the case. In other words, agents anticipate a permanent expansion of public investment in period zero and act accordingly. They are surprised in period one that productivity has not increased but believe that the increase is going to start in the next period when they are disappointed again. I could then interpret the change in unemployment under this scenario as the contribution of the anticipation effect to the overall reduction in unemployment. It is depicted by the dotted brown line in Figure 5. Initially, the response is identical to the one in the baseline scenario. The two then diverge since wages continue to rise as workers keep bargaining for higher wages in anticipation of increasing productivity even though this increase never materializes. After one year, unemployment has declined by 0.18 percentage points under this scenario. This amounts to 45% of the reduction in the baseline scenario. Accordingly, I would attribute 45% of the unemployment reduction to the anticipation effect.

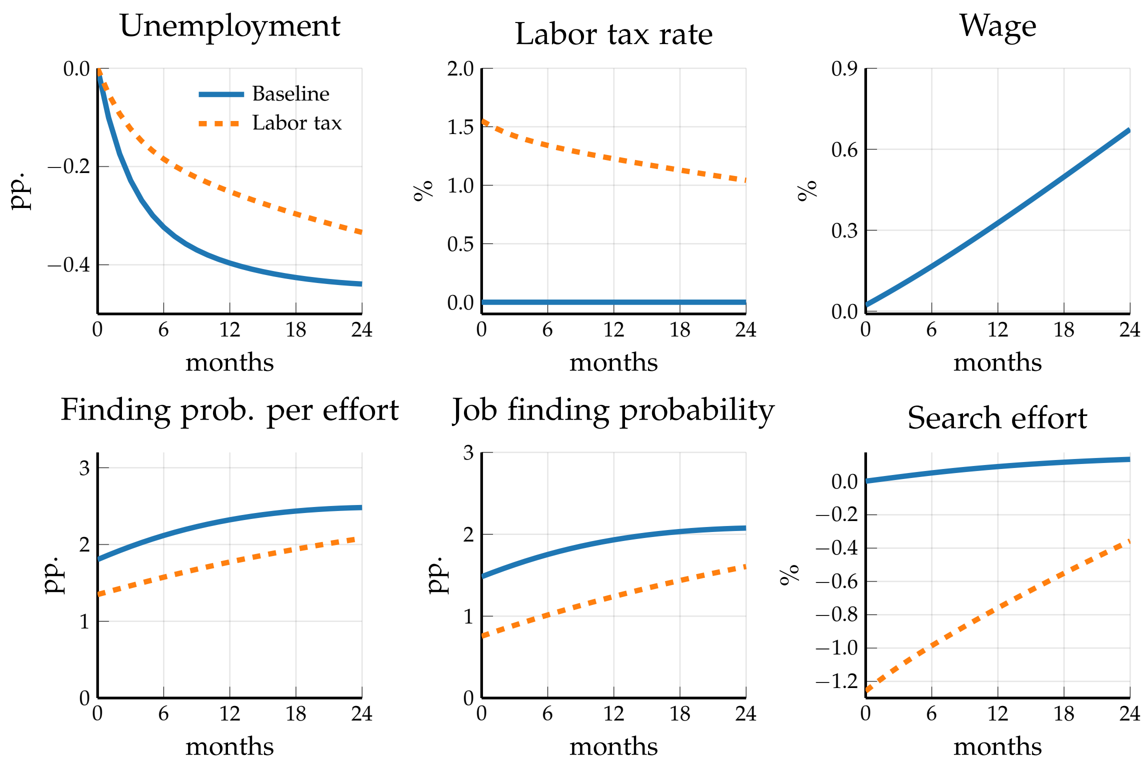
For both definitions, the anticipation effect accounts for a large part of the reduction in unemployment in response to the expansion in public investment.

## 5.2 Financing with distortionary labor taxes

So far, I have assumed that the investment program is financed with non-distortionary lump-sum taxes on firm owners. An alternative policy would be to raise the proportional labor tax to finance public investment. I assume that the government cannot shift the tax burden over time by issuing debt but that it has to raise labor taxes at the same time that expenditures increase. The responses of key variables in this case are shown in Figure 6. As can be seen from the top left panel, unemployment falls less in response to the program in this case but it still declines substantially. After one year it is 0.25 percentage points lower than without the program.

There are two forces that dampen the reduction in unemployment compared to the baseline scenario. First, the increase in the labor tax rate leads to a faster increase in wages as Nash bargaining implies that workers and firms share the tax burden depending on their bargaining weights. Since wages rise faster, firms do not expand vacancy creation as much as in the baseline. This can be seen from the bottom left panel in Figure 6 which shows that the job finding probability per unit of search effort increases less if the program is financed with labor taxes. The second force through which labor taxes reduce the employment effect is workers' search effort. It is lower than in the case of lump-sum taxes and actually declines relative to the steady state. There are two reasons for this. First, firms expand vacancy creation less such that the marginal effect of effort on the job finding probability is lower. Second, the increase in the labor tax reduces the income difference between unemployed and employed workers such that unemployed workers exert less effort.

Figure 6: Responses to the investment program when financed with proportional labor taxes $\tau_t$.



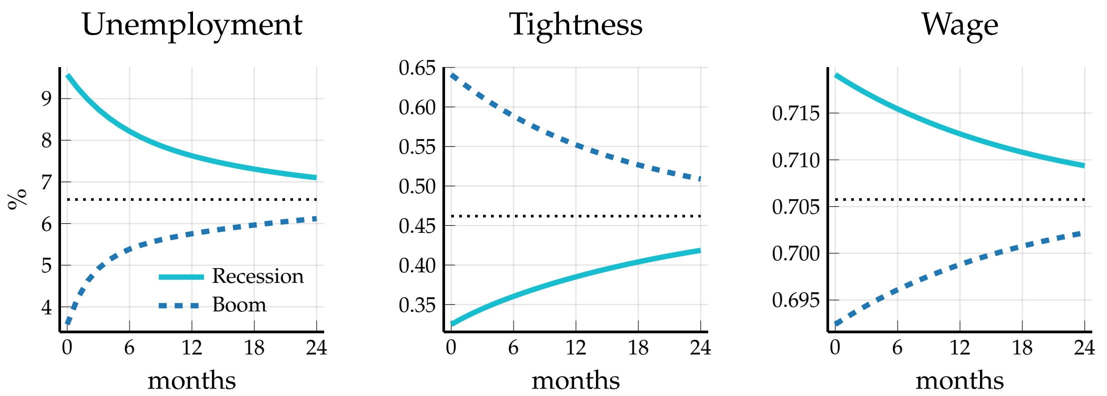

*Notes:* Shown are deviations from the initial steady state.

### 5.3 Business cycle dependence

I established in Proposition 2, that the size of the employment effect depends on the initial level of unemployment. How large are the differences between boom and recession?

To address this question, I follow the same approach as in Section 3 and define a recession as an equilibrium with high unemployment and weak labor demand. More specifically, I assume that unemployment is 3 percentage points higher than in the steady state and the wage is 2% higher. A boom is defined symmetrically as an equilibrium in which initial unemployment is 3 percentage points lower and the wage 2% higher. The unemployment rate in the recession is thus 9.5 percent, similar to the levels in 2009 to 2010 during the Great Recession. The unemployment rate in the boom is 3.5 percent, close to the rates observed in 2019. I further assume that unemployment benefits are constant at the steady state level. Moreover, the capital to labor ratio is also at the steady state level initially, i.e. the private capital stock is smaller in a recession and higher in a boom. I then study the perfect foresight equilibrium under these differential initial conditions and compare the case with an expansion in public investment to the one without. Figure 7 shows the evolution of unemployment, labor market tightness and wages for the two cases. In a recession, labor market tightness is about twice as large as in a boom. A factor of two roughly corresponds to the difference between the trough in tightness at around 0.35 in August 2003 and the peak at 0.73 in March 2007. Comparing the Great Recession to the following expansion, even larger differences are observed in the data. Labor market tightness in 2019 was about 7 times higher than in 2010, 1.2 compared to 0.17.

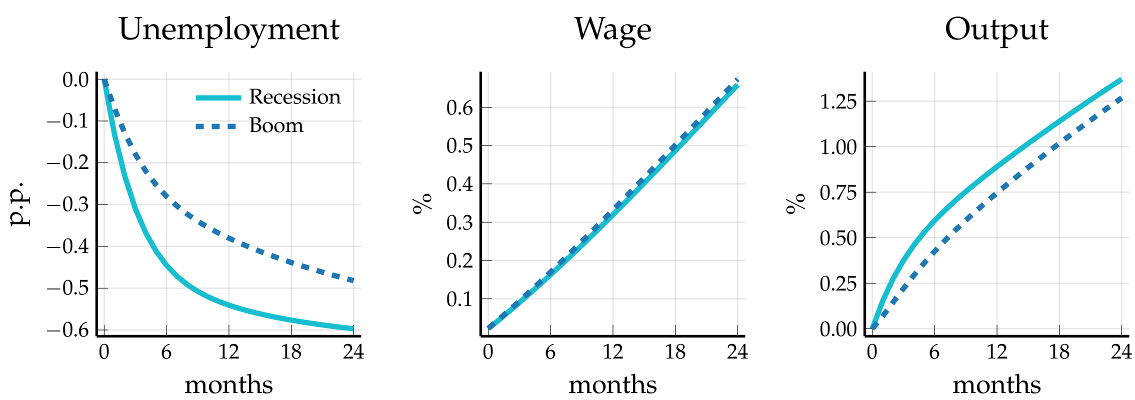
Figure 7: Unemployment, labor market tightness and wages in recession and boom.




*Notes:* The black dotted line denotes the steady state. Unemployment in percent, wages in units of the consumption good.

Figure 8 shows how unemployment and output respond to the expansion in public investment for the case where the economy is in a recession initially and for the case where it is in a boom. Shown are the deviations from the path that would be observed without the investment program, i.e. those shown in Figure 7. When the economy is in a recession initially, the short-run response of both unemployment and output is much larger than when the economy is in a boom. One year after the expansion in public investment, unemployment has fallen by 0.57 percentage points in the case of a recession whereas it has only fallen by 0.4 percentage points in case of a boom. This is a difference of more than 40%.

Figure 8: Response of unemployment and output.




*Notes:* Shown are the deviations from the respective paths that would be observed without the expansion in public investment (see Figure 7).

I also compare recessions and booms that are the result of shocks to productivity. I consider a recession due to a negative shock to productivity of one standard deviation,

$$\log A_0 = -0.0056$$

and accordingly, for a boom

$$\log A_0 = +0.0056.$$

after which productivity $A_t$ evolves according to (31). In this case, the employment effect after one year is about 25% larger when the expansion of public investment is initiated in a recession compared to a boom. The corresponding impulse response functions are shown in Appendix F.1.

# 6 Conclusion

Recently, policymakers in several countries have proposed plans to expand public infrastructure investment. The hope is that public investment not only fosters long-run economic growth but also provides some of the stimulus needed as the economy recovers from the Covid-19 recession. To address the question of whether public investment is suited to provide this stimulus, the existing literature has relied on variants of the neoclassical growth model with frictionless labor markets. In this paper, I revisited this question in another widely used macroeconomic model, the Diamond-Mortensen-Pissarides search and matching model. My theoretical analysis highlighted the role of firms' expectations about future productivity for their hiring decision and the short-run employment effect of public investment. This mechanism is absent in models without labor market frictions. For a realistic calibration of the model, the anticipation effect is large. It accounts for 65% of the reduction in unemployment by 0.4 percentage points within one year after a permanent expansion of public investment by 1% of GDP. The size of the employment effect depends on the state of the business cycle. It is about 40% larger in a recession than in a boom.

These findings are relevant for policymakers. They suggest that a permanent change in fiscal policy towards more public investment can provide a substantial short-run stimulus by raising labor demand. These short-run employment effects are especially large in a recession when labor demand is weak. Thus, a recession might be a good time to initiate such a change in fiscal policy. Because much of the short-run employment effects of the change in fiscal policy are due to the anticipation effect, the announcement of the policy change already leads to significant employment effects. The exact timing of the implementation is of lesser importance, and credibly announcing the change during a recession is enough to stabilize employment.

In this paper, I analyzed and quantified the employment effect of public investment in a standard search and matching model with private and public capital. I focused on the anticipation effect on labor demand and abstracted from some forces, which could further amplify the employment effect of public investment. First, in addition to the anticipation effect, there is likely a direct effect on labor demand as the public sector and its private contractors, who build the additional infrastructure, have to hire more workers. Second, as public investment raises the job-finding probability in the short run, workers will fear unemployment less, reduce precautionary savings, and consume more. This could stimulate the economy further if output is partially demand determined (see for example Den Haan et al. 2017; Ravn and Sterk 2020). Third, separations are exogenous in my model but partly depend on firms' choices in reality. I expect that accounting for endogenous separations will lead to larger employment effects as firms will lay off fewer workers if they anticipate productivity to rise in the future. In addition, if firms can endogenously lower the separation rate, labor hoarding is facilitated, further amplifying the effect of public investment on hiring.

It would also be interesting to investigate heterogeneity in employment effects across industries and occupations. In light of the results presented in this paper, I would expect larger increases in employment in industries and occupations whose productivity benefits most from infrastructure investment. For example, effects may be especially large in the transportation and logistics industry, benefiting from better roads and ports. Employment in repair and maintenance occupations might also respond strongly as these occupations benefit from improvements in telecommunication infrastructure. The employment effects should also be more prominent in industries and occupations in which expected job tenure is longer, making it easier for firms to hoard labor.

The theoretical relationship between public investment and employment studied in this paper crucially depends on the effect of public investment on firms' expectations about future productivity. Firms will only expand hiring in the short run if they believe that productivity will rise. This requires a credible commitment by the government to raise public investment persistently. A short-lived expansion in public investment that only brings public investment forward in time but does not change productivity in the long run will not have significant employment effects. Some of the past expansions in public investment may have satisfied this requirement, whereas others may not. It is an interesting empirical question whether this difference can account for the disparate estimates of the short-run output and employment effects of public investment.

# References

Aschauer, David Alan. 1989. "Does public capital crowd out private capital?" *Journal of Monetary Economics* 24 (2).

Baxter, Marianne, and Robert G King. 1993. "Fiscal Policy in General Equilibrium." *American Economic Review* 83 (3): 315–334.

Beaudry, Paul, and Franck Portier. 2006. "Stock Prices, News, and Economic Fluctuations." *American Economic Review* 96 (4): 1293–1307.

———. 2007. "When can changes in expectations cause business cycle fluctuations in neo-classical settings?" *Journal of Economic Theory* 135 (1): 458–477.

Blanchard, Olivier, and Jordi Galí. 2010. "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment." *American Economic Journal: Macroeconomics* 2 (2): 1–30.

Boehm, Christoph E. 2020. "Government consumption and investment: Does the composition of purchases affect the multiplier?" *Journal of Monetary Economics* 115:80–93.

Bom, Pedro R.D., and Jenny E. Ligthart. 2014. "What have we learned from three decades of research on the productivity of public capital?" *Journal of Economic Surveys* 28 (5): 889–916.

Bouakez, Hafedh, Michel Guillard, and Jordan Roulleau-Pasdeloup. 2017. "Public investment, time to build, and the zero lower bound." *Review of Economic Dynamics* 23:60–79.

Broer, Tobias, Niels-Jakob Harbo Hansen, Per Krusell, and Erik Öberg. 2019. "The New Keynesian Transmission Mechanism: A Heterogeneous-Agent Perspective." *The Review of Economic Studies* 87 (1): 77–101.

Challe, Edouard. 2020. "Uninsured Unemployment Risk and Optimal Monetary Policy in a Zero-Liquidity Economy." *American Economic Journal: Macroeconomics* 12 (2): 241–283.

Chetty, Raj. 2008. "Moral Hazard versus Liquidity and Optimal Unemployment Insurance." *Journal of Political Economy* 116 (2): 173–234.

Chodorow-Reich, Gabriel, and Loukas Karabarbounis. 2016. "The cyclicality of the opportunity cost of employment." *Journal of Political Economy* 124 (6): 1563–1618.

Cubas, German. 2020. "Public Capital and Economic Development." *The Economic Journal* 130 (632): 2354–2381.

Den Haan, Wouter J, Garey Ramey, and Joel Watson. 2000. "Job Destruction and Propagation of Shocks." *American Economic Review* 90 (3): 482–498.

Den Haan, Wouter J, Pontus Rendahl, and Markus Riegler. 2017. "Unemployment (Fears) and Deflationary Spirals." *Journal of the European Economic Association* 16 (5): 1281–1349.

Den Haan, Wouter J., and Georg Kaltenbrunner. 2009. "Anticipated growth and business cycles in matching models." *Journal of Monetary Economics* 56 (3): 309–327.

Gertler, Mark, Christopher Huckfeldt, and Antonella Trigari. 2020. "Unemployment Fluctuations, Match Quality, and the Wage Cyclicality of New Hires." *The Review of Economic Studies* 87 (4): 1876–1914.

Hagedorn, Marcus, and Iourii Manovskii. 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *American Economic Review* 98 (4): 1692–1706.

Hall, Robert E. 2003. *Wage determination and unemployment fluctutations.* NBER Working Paper 9967. National Bureau of Economic Research.

———. 2005. "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review* 95 (1): 50–65.

Hosios, Arthur J. 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment." *The Review of Economic Studies* 57 (2): 279–298.

Hyatt, Henry R., and James R. Spletzer. 2016. "The shifting job tenure distribution." SOLE/EALE conference issue 2015, *Labour Economics* 41:363–377.

Kaplan, Greg, Giovanni L. Violante, and Justin Weidner. 2014. "The Wealthy Hand-to-Mouth." *Brookings papers on Economic Activity,* no. Spring, 77–153.

Krebs, Tom, and Martin Scheffel. 2017. "Labor Market Institutions and the Cost of Recessions."

Krusell, Per, Toshihiko Mukoyama, and Ayegül ahin. 2010. "Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations." *The Review of Economic Studies* 77 (4): 1477–1507.

Landais, Camille, Pascal Michaillat, and Emmanuel Saez. 2018. "A Macroeconomic Approach to Optimal Unemployment Insurance: Applications." *American Economic Journal: Economic Policy* 10 (2): 182–216.

Leduc, Sylvain, and Daniel Wilson. 2013. "Roads to Prosperity or Bridges to Nowhere? Theory and Evidence on the Impact of Public Infrastructure Investment." *NBER Macroeconomics Annual* 27 (1): 89–142.

Leeper, Eric M., Todd B. Walker, and Shu-Chun S. Yang. 2010. "Government investment and fiscal stimulus." *Journal of Monetary Economics* 57 (8): 1000–1012.

Ljungqvist, Lars, and Thomas J. Sargent. 2017. "The Fundamental Surplus." *American Economic Review* 107 (9): 2630–65.

McKay, Alisdair, and Ricardo Reis. 2021. "Optimal Automatic Stabilizers." *The Review of Economic Studies.*

Michaillat, Pascal. 2014. "A Theory of Countercyclical Government Multiplier." *American Economic Journal: Macroeconomics* 6 (1): 190–217.

Michaillat, Pascal, and Emmanuel Saez. 2018. "Optimal Public Expenditure with Inefficient Unemployment." *The Review of Economic Studies* 86 (3): 1301–1331.

Mitman, Kurt, and Stanislav Rabinovich. 2015. "Optimal unemployment insurance in an equilibrium business-cycle model." *Journal of Monetary Economics* 71:99–118.

Mortensen, Dale T., and Christopher A. Pissarides. 1994. "Job Creation and Job Destruction in the Theory of Unemployment." *The Review of Economic Studies* 61 (3): 397–415.

Munnell, Alicia. 1990. "Why has productivity growth declined? Productivity and public investment." *New England Economic Review,* 3–22.

Pereira, Alfredo M., and Rafael Flores de Frutos. 1999. "Public Capital Accumulation and Private Sector Performance." *Journal of Urban Economics* 46 (2): 300–322.

Petrongolo, Barbara, and Christopher A Pissarides. 2001. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature* 39 (2): 390–431.

Pissarides, Christopher A. 1985. "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages." *The American Economic Review* 75 (4): 676–690.

———. 2009. "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?" *Econometrica* 77 (5): 1339–1369.

Ramey, Valerie A. 2020. *The Macroeconomic Consequences of Infrastructure Investment.* NBER Working Paper 27625. National Bureau of Economic Research.

Ravn, Morten O, and Vincent Sterk. 2020. "Macroeconomic Fluctuations with HANK & SAM: an Analytical Approach." *Journal of the European Economic Association* 19 (2): 1162–1202.

Rendahl, Pontus. 2016. "Fiscal Policy in an Unemployment Crisis." *The Review of Economic Studies* 83 (3): 1189–1224.

Schmitt-Grohé, Stephanie, and Martín Uribe. 2012. "What's News in Business Cycles." *Econometrica* 80 (6): 2733–2764.

Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review* 95 (1): 25–49.

———. 2010. *Labor Markets and Business Cycles.* Princeton University Press.

———. 2012. "Reassessing the ins and outs of unemployment." *Review of Economic Dynamics* 15 (2): 127–148.

Summers, Lawrence. 2009. *A Weakened Economy: How to Respond?, 110th Congress (Testimony of Lawrence Summers).*

# A Proofs and Derivations

## A.1 Proof of Proposition 1

Employment evolves as

$$N_{t+1} = (1-\rho)N_t + \pi_t^{e|u}(1-N_t),$$

hence

$$N_t = \sum_{k=0}^{t-1} \pi_k^{e|u} \prod_{j=k+1}^{t-1} (1-\rho-\pi_j^{e|u}) + N_0 \prod_{j=0}^{t-1}(1-\rho-\pi_j^{e|u}).$$

Without adjustment costs, capital in every match is

$$k_t = \left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}} z_t^{\frac{1}{1-\alpha}}.$$

I have from the job creation equation

$$q_t^v = \frac{\kappa a^\alpha z_t^{\frac{1}{1-\alpha}}}{\sum_{s=1}^{\infty} \beta^s (1-\rho)^{s-1} \left\{(1-\alpha)a^\alpha z_{t+s}^{\frac{1}{1-\alpha}} - w_{t+s}\right\}}$$

with $a \equiv \left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}}$. The wage is given by

$$w_t = \gamma^t w_0 + \sum_{n=1}^{t} (1-\gamma)\omega a^\alpha z_n^{\frac{1}{1-\alpha}} \gamma^{t-n}$$

The job finding probability is

$$\pi_t^{e|u} = \zeta^{\frac{1}{\eta}} q_t^{v\,\frac{\eta-1}{\eta}} \ell(u).$$

and using the expressions for $q_t^v$ from above,

$$\begin{aligned}\pi_k^{e|u} =& \zeta^{\frac{1}{\eta}} (1-\rho)^{\frac{1-\eta}{\eta}} \left(z_k^{\frac{1}{1-\alpha}} \kappa a^\alpha\right)^{\frac{\eta-1}{\eta}} \\ &\times \left[\sum_{s=k}^{\infty} (\beta(1-\rho))^{s-k} \left[(1-\alpha)a^\alpha z_{s+1}^{\frac{1}{1-\alpha}} - w_{s+1}\right]\right]^{\frac{1-\eta}{\eta}} \ell(u)\end{aligned}$$

Productivity is

$$z_n = B K_n^{G\,\vartheta}$$

and so

$$\frac{\mathrm{d}z_n}{\mathrm{d}x} = B\vartheta K_n^{G\,\vartheta-1} \frac{\mathrm{d}K_n^G}{\mathrm{d}x} = z \frac{\vartheta}{K_n^G} \frac{\mathrm{d}K_n^G}{\mathrm{d}x}.$$

Furthermore,

$$K_n^G = (1-\delta_G)^n K_0^G + \sum_{j=0}^{n-1}(1-\delta_G)^{n-1-j} I_j^G + \sum_{j=T}^{n-1}(1-\delta_G)^{n-1-j} x$$

such that

$$\frac{\mathrm{d}K_n^G}{\mathrm{d}x} = \sum_{j=T}^{n-1}(1-\delta_G)^{n-1-j}$$

and

$$\frac{\mathrm{d}z_n}{\mathrm{d}x} = \vartheta \frac{z_n}{\delta_G K_n^G} \sum_{j=T}^{n-1}(1-\delta_G)^{n-1-j}.$$

Regarding the wage, it holds

$$\frac{\mathrm{d}w_n}{\mathrm{d}x} = \sum_{s=1}^{n} \gamma^{n-s}(1-\gamma)\omega a^{\alpha} \frac{1}{1-\alpha} z_s^{\frac{\alpha}{1-\alpha}} \frac{\mathrm{d}z_s}{\mathrm{d}x}.$$

By assumption $I^G = \delta_G K_n^G$ for all $n$, such that

$$\frac{\mathrm{d}z_s}{\mathrm{d}x} = \begin{cases} \frac{\vartheta z_s}{I^G}(1-(1-\delta_G)^{s-t}), & \text{if } s > T \\ 0 \quad \text{if } s \leq T \end{cases}$$

I distinguish two cases, $k \leq T$ and $k > T$.

**Case 1:** $k < T$ The semi-elasticity of the job finding probability in period $k < T$ with respect to public investment in the periods after $T$ is

$$\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x}|_{x=0} = \pi_k^{e|u}\left[\frac{(1-\alpha-\omega)a^{\alpha}z^{\frac{1}{1-\alpha}}}{1-\beta(1-\rho)} - \frac{\gamma^{k+1}w_0}{1-\beta\gamma(1-\rho)} + \frac{\omega a^{\alpha}z^{\frac{1}{1-\alpha}}\gamma^{k+1}}{1-\gamma\beta(1-\rho)}\right]^{-1}\frac{1-\eta}{\eta}\frac{1}{1-\alpha}$$

$$z^{\frac{\alpha}{1-\alpha}}\left[\sum_{s=T}^{\infty}(\beta(1-\rho))^{s-k}\left[(1-\alpha)a^{\alpha}\frac{\partial z_{s+1}}{\partial x} - (1-\gamma)\omega a^{\alpha}\sum_{n=T+1}^{s+1}\gamma^{s+1-n}\frac{\partial z_n}{\partial x}\right]\right]$$

$$= \pi_k^{e|u}\left[\frac{(1-\alpha-\omega)a^{\alpha}}{1-\beta(1-\rho)} - \frac{\gamma^{k+1}w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)} + \frac{\omega a^{\alpha}\gamma^{k+1}}{1-\gamma\beta(1-\rho)}\right]^{-1}\frac{1-\eta}{\eta}\frac{1}{1-\alpha}$$

$$\frac{\vartheta}{\delta_G K_G}(\beta(1-\rho))^{T-k}\left[(1-\alpha)a^{\alpha}\left(\frac{1}{1-\beta(1-\rho)} - \frac{(1-\delta_G)}{1-\beta(1-\delta_G)(1-\rho))}\right)\right.$$

$$\left. - (1-\gamma)\omega a^{\alpha}\sum_{s=T}^{\infty}(\beta(1-\rho))^{s-T}\sum_{n=T+1}^{s+1}\gamma^{s+1-n}\left(1-(1-\delta_G)^{n-T}\right)\right]$$

$$= \pi_k^{e|u}\left[\frac{(1-\alpha-\omega)a^{\alpha}}{1-\beta(1-\rho)} - \frac{\gamma^{k+1}w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)} + \frac{\omega a^{\alpha}\gamma^{k+1}}{1-\gamma\beta(1-\rho)}\right]^{-1}\frac{1-\eta}{\eta}\frac{1}{1-\alpha}$$

$$\vartheta(\beta(1-\rho))^{T-k}\left[(1-\alpha)a^{\alpha}\frac{\delta_G}{(1-\beta(1-\rho))(1-\beta(1-\delta_G)(1-\rho)))}\right.$$

$$\left. - (1-\gamma)\omega a^{\alpha}\sum_{s=T}^{\infty}(\beta(1-\rho))^{s-T}\sum_{n=T+1}^{s+1}\gamma^{s+1-n}\left(1-(1-\delta_G)^{n-T}\right)\right]\frac{1}{I^G}$$

Focusing on the last summand, it holds that

$$\sum_{n=T+1}^{s+1}\gamma^{s+1-n}\left(1-(1-\delta_G)^{n-T}\right) = \left(\frac{\gamma^{s-T+1}-1}{\gamma-1} - (1-\delta_G)\frac{\gamma^{s-T+1}-(1-\delta_G)^{s-T+1}}{\gamma-1+\delta}\right)$$

such that

$$\sum_{s=T}^{\infty}(\beta(1-\rho))^{s-t}\sum_{n=T+1}^{s+1}\gamma^{s+1-n}\left(1-(1-\delta_G)^{n-t}\right)$$

$$= \sum_{s=T}^{\infty}(\beta(1-\rho))^{s-t}\left(\frac{\gamma^{s-T+1}-1}{\gamma-1} - (1-\delta_G)\frac{\gamma^{s-T+1}-(1-\delta_G)^{s-T+1}}{\gamma-1+\delta}\right)$$

$$= \left(\frac{1}{1-\beta\gamma(1-\rho)}\left(\frac{\gamma}{\gamma-1} - \frac{(1-\delta_G)\gamma}{\gamma-1+\delta_G}\right) - \frac{1}{\gamma-1}\frac{1}{1-\beta(1-\rho)}\right.$$

$$\left. + (1-\delta_G)^2\frac{1}{\gamma-1+\delta_G}\frac{1}{1-\beta(1-\rho)(1-\delta_G)}\right)$$

$$= \frac{1}{(1-\gamma)(1-\gamma-\delta_G)}\left(\frac{\delta_G\gamma^2}{1-\beta\gamma(1-\rho)} + \frac{1-\gamma-\delta_G}{1-\beta(1-\rho)} - \frac{(1-\gamma)(1-\delta_G)^2}{1-\beta(1-\rho)(1-\delta_G)}\right)$$

such that

$$\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x}|_{x=1} = \pi_k^{e|u}\left[\frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)} - \frac{\gamma^{k+1}w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)} + \frac{\omega a^\alpha \gamma^{k+1}}{1-\gamma\beta(1-\rho)}\right]^{-1}\frac{1-\eta}{\eta}\frac{1}{1-\alpha}$$
$$\vartheta(\beta(1-\rho))^{T-k}\Bigg[a^\alpha\frac{\delta_G(1-\alpha-\omega)}{(1-\beta(1-\rho))(1-\beta(1-\delta_G)(1-\rho)))}$$
$$+\frac{\gamma\omega a^\alpha\delta_G}{(1-\beta\gamma(1-\rho))(1-\beta(1-\rho)(1-\delta_G))}\Bigg] \tag{33}$$

**Case 2:** $k > T$

$$\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x}|_{x=0} = \pi_k^{e|u}\left[z^{\frac{1}{1-\alpha}}\frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)} - \frac{\gamma^{k+1}w_0}{1-\beta\gamma(1-\rho)} + \frac{\omega a^\alpha z^{\frac{1}{1-\alpha}}\gamma^{k+1}}{1-\gamma\beta(1-\rho)}\right]^{-1}$$
$$\frac{1-\eta}{\eta}\frac{1}{1-\alpha}z^{\frac{\alpha}{1-\alpha}}\left[\sum_{s=k}^{\infty}(\beta(1-\rho))^{s-k}\Bigg[(1-\alpha)a^\alpha\frac{\partial z_{s+1}}{\partial x}\right.$$
$$\left.-(1-\gamma)\omega a^\alpha\sum_{n=T+1}^{s+1}\gamma^{s+1-n}\frac{\partial z_n}{\partial x}\Bigg]\right]$$
$$+\pi_k^{e|u}\vartheta\frac{\eta-1}{\eta}\frac{1}{1-\alpha}\left(1-(1-\delta_G)^{k-T}\right)$$
$$=\frac{(1-\eta)\pi_k^{e|u}}{(1-\alpha)K^G\eta}\frac{1}{\delta_G}\vartheta\left[\frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)} - \frac{\gamma^{k+1}w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)} + \frac{\omega a^\alpha\gamma^{k+1}}{1-\gamma\beta(1-\rho)}\right]^{-1}$$
$$\Bigg[(1-\alpha)a^\alpha\left(\frac{1}{1-\beta(1-\rho)} - (1-\delta_G)^{k-T+1}\frac{1}{1-\beta(1-\rho)(1-\delta_G)}\right)$$
$$-\omega a^\alpha\Bigg(\frac{1}{1-\beta(1-\rho)} + \frac{\delta_G\gamma}{1-\gamma-\delta_G}\frac{\gamma^{k-T+1}}{1-\beta\gamma(1-\rho)} \tag{34}$$
$$-\frac{(1-\gamma)(1-\delta_G)}{1-\gamma-\delta_G}\frac{(1-\delta_G)^{k-T+1}}{1-\beta(1-\delta_G)(1-\rho)}\Bigg)\Bigg]$$
$$+\pi_k^{e|u}\vartheta\frac{\eta-1}{\eta}\frac{1}{1-\alpha}\left(1-(1-\delta_G)^{k-T}\right)\frac{1}{I^G}$$
$$=\frac{(1-\eta)\pi_k^{e|u}}{(1-\alpha)\eta}\frac{\vartheta}{I^G}\Bigg\{\left[\frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)} - \frac{\gamma^{k+1}w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)} + \frac{\omega a^\alpha\gamma^{k+1}}{1-\gamma\beta(1-\rho)}\right]^{-1}$$
$$\Bigg[(1-\alpha-\omega)a^\alpha\left(\frac{1}{1-\beta(1-\rho)} - (1-\delta_G)^{k-T+1}\frac{1}{1-\beta(1-\rho)(1-\delta_G)}\right)$$
$$+\frac{\gamma\omega\delta_G a^\alpha}{1-\gamma-\delta_G}\left(\frac{(1-\delta_G)^{k-T+1}}{1-\beta(1-\delta_G)(1-\rho)} - \frac{\gamma^{k-T+1}}{1-\beta\gamma(1-\rho)}\right)\Bigg]$$
$$-(1-(1-\delta_G)^{k-T})\Bigg\}$$

With $w_0 = \omega a^\alpha z^{\frac{1}{1-\alpha}}$, I get

$$\begin{aligned}
\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x}|_{x=0} =& \frac{(1-\eta)\pi_k^{e|u}}{(1-\alpha)\eta}\frac{\vartheta}{I^G}\Bigg\{\left(1-(1-\delta_G)^{k-T+1}\frac{1-\beta(1-\rho)}{1-\beta(1-\rho)(1-\delta_G)}\right)-(1-(1-\delta_G)^{k-T})\\
&+\frac{(1-\beta(1-\rho))\gamma\omega\delta_G}{(1-\gamma-\delta_G)(1-\alpha-\omega)}\left(\frac{(1-\delta_G)^{k-T+1}}{1-\beta(1-\delta_G)(1-\rho)}-\frac{\gamma^{k-T+1}}{1-\beta\gamma(1-\rho)}\right)\Bigg\}\\
=&\frac{(1-\eta)\pi_k^{e|u}}{(1-\alpha)\eta}\frac{\vartheta}{I^G}\Bigg\{\frac{(1-\delta_G)^{k-T+1}\delta_G}{1-\beta(1-\rho)(1-\delta_G)}\\
&+\frac{(1-\beta(1-\rho))\gamma\omega\delta_G}{(1-\gamma-\delta_G)(1-\alpha-\omega)}\left(\frac{(1-\delta_G)^{k-T+1}}{1-\beta(1-\delta_G)(1-\rho)}-\frac{\gamma^{k-T+1}}{1-\beta\gamma(1-\rho)}\right)\Bigg\}
\end{aligned}$$

For the proof of part i), note that in both cases $\frac{\partial\pi_k^{e|u}}{\partial x}|_{x=0} > 0$ (assuming that the job finding probability in the initial equilibrium is strictly positive). The result follows directly for the case $k \le T$ (observe that $\pi_k^{e|u} > 0$ only if $\left[\frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)} - \frac{\gamma^{k+1}w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)} + \frac{\omega a^\alpha\gamma^{k+1}}{1-\gamma\beta(1-\rho)}\right]^{-1} > 0$). For the case $k > T$, the crucial step is to note that the wage stickiness term is always positive,

$$\frac{\gamma\omega\delta_G a^\alpha}{1-\gamma-\delta_G}\left(\frac{(1-\delta_G)^{k-T+1}}{1-\beta(1-\delta_G)(1-\rho)}-\frac{\gamma^{k-T+1}}{1-\beta\gamma(1-\rho)}\right) > 0,$$

this is the case since

$$\left(\frac{(1-\delta_G)^{k-T+1}}{1-\beta(1-\delta_G)(1-\rho)}-\frac{\gamma^{k-T+1}}{1-\beta\gamma(1-\rho)}\right) > 1-\delta_G-\gamma$$

if and only if $\gamma < 1-\delta_G$, but then $1-\delta_G-\gamma > 0$ such that the ratio is positive. If in contrast $\gamma > 1-\delta_G$, then $1-\delta_G-\gamma < 0$ and the ratio is also positive.

I have shown that $\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x}|_{x=0} > 0$ for all $k$ if the initial wage is $w_0 = \omega a^\alpha z^{\frac{1}{1-\alpha}}$. By induction, it then follows from the law of motion for employment, that

$$\frac{\mathrm{d}N_t}{\mathrm{d}x} > 0$$

which proves part (i) of proposition 1 for the case where the initial wage is $w_0 = \omega a^\alpha z^{\frac{1}{1-\alpha}}$.

It can be seen from (33) and (34) that for given $\pi_k^{e|u}$, $\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x}|_{x=0}$ is weakly increasing in $w_0$. Hence, as long as $\pi_k^{e|u} > 0$, the statement in (i) also holds if $w_0 > \omega a^\alpha z^{\frac{1}{1-\alpha}}$.

To prove part (ii), observe that for $t \to \infty$ we are in the case $k > T$ and since $\lim_{k\to\infty}\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x}|_{x=0} = 0$, it follows from the law of motion for employment that

$$\lim_{t\to\infty}\frac{\mathrm{d}N_t}{\mathrm{d}x} = 0.$$

If posting costs are not proportional to labor productivity but constant, the case $k \leq T$ is unchanged but for the case $k > T$, we have that

$$\begin{aligned}
\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x}|_{x=0} =& \frac{(1-\eta)\pi_k^{e|u}}{(1-\alpha)\eta} \frac{\vartheta}{I^G} \Bigg\{ \left[ \frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)} - \frac{\gamma^{k+1} w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)} + \frac{\omega a^\alpha \gamma^{k+1}}{1-\gamma\beta(1-\rho)} \right]^{-1} \\
& \left[ (1-\alpha-\omega)a^\alpha \left( \frac{1}{1-\beta(1-\rho)} - (1-\delta_G)^{k-T+1} \frac{1}{1-\beta(1-\rho)(1-\delta_G)} \right) \right. \\
& \left. + \frac{\gamma\omega\delta_G a^\alpha}{1-\gamma-\delta_G} \left( \frac{(1-\delta_G)^{k-T+1}}{1-\beta(1-\delta_G)(1-\rho)} - \frac{\gamma^{k-T+1}}{1-\beta\gamma(1-\rho)} \right) \right] \Bigg\}.
\end{aligned}$$

By the same argument as above, $\frac{(1-\delta_G)^{k-T+1}}{1-\beta(1-\delta_G)(1-\rho)} - \frac{\gamma^{k-T+1}}{1-\beta\gamma(1-\rho)} > 0$ such that $\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x}|_{x=0} > 0$.

## A.2 Proof of Proposition 2

For the case $k \leq T$, we get from above with $w_0 = \omega a^\alpha z^{\frac{1}{1-\alpha}}$,

$$\begin{aligned}
\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x}|_{x=0} =& \frac{(1-\eta)\pi_k^{e|u}}{(1-\alpha)\eta} \frac{\vartheta}{I^G} (\beta(1-\rho))^{T-k} \left[ \frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)} \right]^{-1} \\
& \left[ (1-\alpha)a^\alpha \left( \frac{1}{1-\beta(1-\rho)} - \frac{1-\delta_G}{1-\beta(1-\delta_G)(1-\rho))} \right) \right. \\
& \left. - \frac{\omega a^\alpha}{(1-\gamma-\delta_G)} \left( \frac{\delta_G \gamma^2}{1-\beta\gamma(1-\rho)} + \frac{1-\gamma-\delta_G}{1-\beta(1-\rho)} - \frac{(1-\gamma)(1-\delta_G)^2}{1-\beta(1-\rho)(1-\delta_G)} \right) \right] \\
=& \frac{(1-\eta)\pi_k^{e|u}}{(1-\alpha)\eta} \frac{\vartheta}{I^G} (\beta(1-\rho))^{T-k} \left[ \frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)} \right]^{-1} \\
& \left[ (1-\alpha-\omega)a^\alpha \left( \frac{1}{1-\beta(1-\rho)} - \frac{1-\delta_G}{1-\beta(1-\delta_G)(1-\rho))} \right) \right. \\
& \left. + \frac{\gamma\omega a^\alpha \delta_G}{(1-\beta\gamma(1-\rho))(1-\beta(1-\rho)(1-\delta_G))} \right] I^G \\
=& \frac{(1-\eta)\pi_k^{e|u}}{(1-\alpha)\eta} \frac{\delta_G \vartheta (\beta(1-\rho))^{T-k}}{1-\beta(1-\delta_G)(1-\rho)} \left[ 1 + \frac{\gamma\omega(1-\beta(1-\rho))}{(1-\beta\gamma(1-\rho))(1-\alpha-\omega)} \right].
\end{aligned}$$

I have used the definition of $a$ for the second equality. If the economy is at the steady state initially, then the employment multiplier is

$$
\begin{aligned}
M_t^{Inv}(T,\mathcal{X}_0,\mathcal{I}^G) &= \sum_{k=0}^{t-1}(1-\rho-\pi^{e|u})^{t-k-1}(1-N)\frac{\partial \pi_k^{e|u}}{\partial x} \\
&= \frac{(\beta(1-\rho))^T(1-N)\vartheta}{1-\beta(1-\delta_G)(1-\rho)}\frac{\pi^{e|u}}{K^G}\frac{1}{1-\alpha}\frac{1-\eta}{\eta}(1-\rho-\pi^{e|u})^{t-1} \\
&\quad \times \frac{1-((1-\rho-\pi^{e|u})\beta(1-\rho))^{-t}}{1-((1-\rho-\pi^{e|u})\beta(1-\rho))^{-1}}\left[1+\frac{\gamma\omega(1-\beta(1-\rho))}{(1-\beta\gamma(1-\rho))(1-\alpha-\omega)}\right] \\
&= \frac{(\beta(1-\rho))^{T+1-t}(1-N)\vartheta}{1-\beta(1-\delta_G)(1-\rho)}\frac{\pi^{e|u}}{K^G}\frac{1}{1-\alpha}\frac{1-\eta}{\eta}\frac{1-((1-\rho-\pi^{e|u})\beta(1-\rho))^t}{1-((1-\rho-\pi^{e|u})\beta(1-\rho))} \\
&\quad \times \left[1+\frac{\gamma\omega(1-\beta(1-\rho))}{(1-\beta\gamma(1-\rho))(1-\alpha-\omega)}\right].
\end{aligned}
$$

## A.3 Proof of Proposition 3

Part i) follows by induction, from the fact that $\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x}|_{x=0} > 0$ for all $k$ and the law of motion for employment,

$$
N_{t+1} = (1-\rho)N_t + \pi_t^{e|u}U_t = (1-\rho-\pi_t^{e|u})N_t + \pi_t^{e|u} = (1-\rho-\pi_t^{e|u})(1-U_t) + \pi_t^{e|u}.
$$

Take two initial levels of unemployment, $\tilde{U}_0$ and $U_0$. Suppose $\tilde{U}_0 > U_0$, then, since $1-\rho > \pi_t^{e|u}$ for all $t$, $\tilde{U}_1 > U_1$. Moreover if $\tilde{U}_t > U_t$, then $\tilde{U}_{t+1} > U_{t+1}$. Hence, $\tilde{U}_t > U_t$ for all $t$. Taking the derivative of the law of motion yields

$$
\frac{\partial N_{t+1}}{\partial x} = (1-\rho-\pi_t^{e|u})\frac{\partial N_t}{\partial x} + \frac{\partial \pi_t^{e|u}}{\partial x}U_t.
$$

Hence, $\frac{\partial \tilde{N}_1}{\partial x} > \frac{\partial N_1}{\partial x}$. In addition, $\frac{\partial \tilde{N}_{t+1}}{\partial x} > \frac{\partial N_{t+1}}{\partial x}$ if $\frac{\partial \tilde{N}_t}{\partial x} > \frac{\partial N_t}{\partial x}$. It follows that that $\frac{\partial \tilde{N}_t}{\partial x} > \frac{\partial N_t}{\partial x}$ for all $t$.

To prove part ii), I show that the change in the job finding probability in every period is increasing in $w_0$ if $\eta > 0.5$. We have from above, that

$$
\begin{aligned}
\frac{\partial^2 \pi_k^{e|u}}{\partial x \partial w_0} =& \frac{\partial \pi_k^{e|u}}{\partial w_0}\frac{1}{\pi_k^{e|u}}\frac{\partial \pi_k^{e|u}}{\partial x} \\
&+ \frac{\partial \pi_k^{e|u}}{\partial x}\left[\frac{(1-\alpha-\omega)a^\alpha}{1-\beta(1-\rho)} - \frac{\gamma^{k+1}w_0 z^{\frac{1}{\alpha-1}}}{1-\beta\gamma(1-\rho)} + \frac{\omega a^\alpha \gamma^{k+1}}{1-\gamma\beta(1-\rho)}\right]^{-1}\frac{\gamma^{k+1}z^{\frac{1}{1-\alpha}}}{1-\beta\gamma(1-\rho)}.
\end{aligned}
$$

We have that

$$\left.\frac{\partial \pi_k^{e|u}}{\partial w_0}\frac{1}{\pi_k^{e|u}}\right|_{x=1,w_0=\omega a^\alpha z^{\frac{1}{1-\alpha}}} = \frac{\eta-1}{\eta}\left[\frac{(1-\alpha-\omega)a^\alpha z^{\frac{1}{1-\alpha}}}{1-\beta\gamma(1-\rho)}\right]^{-1}\frac{\gamma^{k+1}z^{\frac{1}{1-\alpha}}}{1-\beta\gamma(1-\rho)}$$

such that

$$\left.\frac{\partial^2 \pi_k^{e|u}}{\partial x \partial w_0}\right|_{x=1,w_0=\omega a^\alpha z^{\frac{1}{1-\alpha}}} = \frac{2\eta-1}{\eta}\left[\frac{(1-\alpha-\omega)a^\alpha z^{\frac{1}{1-\alpha}}}{1-\beta\gamma(1-\rho)}\right]^{-1}\frac{\gamma^{k+1}z^{\frac{1}{1-\alpha}}}{1-\beta\gamma(1-\rho)}\frac{\partial \pi_k^{e|u}}{\partial x},$$

which is positive if $\eta > 0.5$.

## A.4 Proof of Proposition 4

$$W(\{c_t^F, c_t(s^t), \ell_t(s^t)\}) = \bar{\mu}^F\sum_{t=0}^{\infty}\beta^t u^F(c_t^F) + \sum_{t=0}^{\infty}\beta^t\sum_{s^t} u\left(c_t(s^t), \ell_t(s^t), s_t\right)\pi_t(s^t|s_0)\bar{\mu}(s_0),$$

$$\tilde{W}(\{v_t^F, v_t(s^t), \ell_t(s^t), C_t\}) = \bar{\mu}^F\sum_{t=0}^{\infty}\beta^t u^F(v_t^F C_t) + \sum_{t=0}^{\infty}\beta^t\sum_{s^t} u\left(v_t(s^t)C_t, \ell_t(s^t), s_t\right)\pi_t(s^t|s_0)\bar{\mu}(s_0),$$

such that

$$\begin{aligned}\frac{\partial W}{\partial x} = \frac{\partial \tilde{W}}{\partial x} = &\sum_{t=0}^{\infty}\beta^t\bar{\mu}^F u_c^F(c_t^F)C_t\frac{\partial v_t^F}{\partial x} + \sum_{t=0}^{\infty}\beta^t\sum_{s^t}u_c(c_t(s^t))C_t\frac{\partial v_t(s^t)}{\partial x}\pi_t(s^t|s_0)\bar{\mu}_0(s_0)\\ &+\sum_{t=0}^{\infty}\beta^t\sum_{s^t}u(c_t(s^t),\ell_t(s^t),s_t)\frac{\pi_t(s^t|s_0)}{\partial x}\bar{\mu}_0(s_0)\\ &+\sum_{t=0}^{\infty}\beta^t v_t^F u_c^F(c_t^F)\bar{\mu}^F\frac{\partial C_t}{\partial x} + \sum_{t=0}^{\infty}\sum_{s^t}\beta^t v_t(s^t)u_c(c_t(s^t))\pi_t(s^t|s_0)\bar{\mu}(s_0)\frac{\partial C_t}{\partial x},\end{aligned}$$

which yields (26) with $\mathrm{m}_t$ defined as in the main text. Furthermore,

$$C_t = z_t N_t^{1-\alpha}K_t^{1-\alpha} - \kappa_t\theta_t(1-N_t) - K_{t+1} + (1-\delta_k) - I_t^G,$$

such that

$$\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x} = \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \left( N_t^{1-\alpha}K_t^{\alpha}\frac{\partial z_t}{\partial x} + (1-\alpha)z_t N_t^{-\alpha}K_t^{\alpha}\frac{\partial N_t}{\partial x} + \alpha z_t N_t^{1-\alpha}K_t^{\alpha-1}\frac{\partial K_t}{\partial x} + \kappa_t\theta_t\frac{\partial N_t}{\partial x} \right.$$
$$\left. - \frac{\partial \kappa_t}{\partial x}\theta_t(1-N_t) - \kappa_t\frac{\partial\theta_t}{\partial x}(1-N_t) - \frac{\partial K_{t+1}}{\partial x} + (1-\delta_k)\frac{\partial K_t}{\partial x} - \frac{\partial I_t^G}{\partial x}\right)$$
$$= \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \left( N_t^{1-\alpha}K_t^{\alpha}\frac{\partial z_t}{\partial x} - \frac{\partial I_t^G}{\partial .x}\right)$$
$$+ \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \left( \left(\alpha z_t k_t^{\alpha-1} + 1 - \delta_k\right)\frac{\partial K_t}{\partial x} - \frac{\partial K_{t+1}}{\partial x} + \theta_t(1-N_t)\frac{\partial\kappa_t}{\partial x}\right)$$
$$+ \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \left( \left[(1-\alpha)z_t k_t^{\alpha-1} + \kappa_t\theta_t\right]\frac{\partial N_t}{\partial x} + \kappa_t(1-N_t)\frac{\partial\theta_t}{\partial x}\right)$$

From the law of motion for employment, we get

$$\kappa_t(1-N_t)\frac{\partial\theta_t}{\partial x} = \left[\frac{\partial N_{t+1}}{\partial x} - (1-\rho-q_t^v(\theta_t)\theta_t)\frac{\partial N_t}{\partial x}\right]\frac{\kappa_t}{(1-\eta)q_t^v(\theta_t)}$$

Using this, we have

$$\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x} = \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \left( N_t^{1-\alpha}K_t^{\alpha}\frac{\partial z_t}{\partial x} - \frac{\partial I_t^G}{\partial .x}\right)$$
$$+ \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \left( \left(\alpha z_t k_t^{\alpha-1} + 1 - \delta_k\right)\frac{\partial K_t}{\partial x} - \frac{\partial K_{t+1}}{\partial x} + \theta_t(1-N_t)\frac{\partial\kappa_t}{\partial x}\right)$$
$$+ \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \left( \left[(1-\alpha)z_t k_t^{\alpha-1} + \kappa_t\theta_t - \frac{1-\rho-q_t^v(\theta_t)\theta_t}{(1-\eta)q_t^v(\theta_t)}\right]\frac{\partial N_t}{\partial x} + \frac{\kappa_t}{(1-\eta)q_t^v(\theta_t)}\frac{\partial N_{t+1}}{\partial x}\right)$$

and with the equilibrium condition

$$\frac{\kappa_t}{q_t^v(\theta_t)} = \beta\left\{(1-\alpha)z_{t+1}k_{t+1}^{\alpha} - w_{t+1} + (1-\rho)\frac{\kappa_{t+1}}{m(\theta_{t+1})}\right\}$$

we get

$$\begin{aligned}
\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x} =& \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \left( N_t^{1-\alpha} K_t^{\alpha} \frac{\partial z_t}{\partial x} - \frac{\partial I_t^G}{\partial .x} \right) \\
&+ \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \left( \left( \alpha z_t k_t^{\alpha-1} + 1 - \delta_k \right) \frac{\partial K_t}{\partial x} - \frac{\partial K_{t+1}}{\partial x} + \theta_t (1 - N_t) \frac{\partial \kappa_t}{\partial x} \right) \\
&+ \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \left( [(1-\alpha) k_t^{\alpha} z_t + \kappa_t \theta_t] \frac{\partial N_t}{\partial x} \right) + \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \frac{\kappa_t \theta_t}{1-\eta} \frac{\partial N_t}{\partial x} \\
&+ \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \beta \left( \frac{(1-\alpha) k_{t+1}^{\alpha} z_{t+1} - w_{t+1}}{1-\eta} + \frac{(1-\rho)\kappa_{t+1}}{(1-\eta) m(\theta_{t+1})} \right) \frac{\partial N_{t+1}}{\partial x} \\
&- \sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \frac{(1-\rho)\kappa_t}{(1-\eta) q_t^v(\theta_t)} \frac{\partial N_t}{\partial x}
\end{aligned}$$

Suppose the economy is in a steady state, then the average marginal utility of consumption $\mathrm{m}_t = v^F \bar{\mu}_0 + \frac{1}{C} \sum_{s_0} \bar{\mu}(s_0)$ is constant and so

$$\sum_{t=0}^{\infty}\beta^t \mathrm{m}_t \frac{\partial C_t}{\partial x} = \left( v^F \bar{\mu}_0 + \frac{1}{C} \sum_{s_0} \bar{\mu}(s_0) \right) \sum_{t=0}^{\infty} \beta^t \frac{\partial C_t}{\partial x}.$$

Finally, since $\frac{\partial K_0}{\partial x} = \frac{\partial N_0}{\partial x} = 0$,

$$\begin{aligned}
\sum_{t=0}^{\infty}\beta^t \frac{\partial C_t}{\partial x} =& \sum_{t=0}^{\infty}\beta^t \left( N^{1-\alpha} K^{\alpha} \frac{\partial z_t}{\partial x} - \frac{\partial I_t^G}{\partial x} \right) + \sum_{t=0}^{\infty}\beta^t \theta (1-N) \frac{\partial \kappa_t}{\partial x} \\
&+ \sum_{t=0}^{\infty}\beta^t \frac{1}{1-\eta} \left[ w - \eta \left( (1-\alpha) z k^{\alpha} + \theta \kappa \right) \right] M_{t+1}^{Inv}.
\end{aligned}$$

The second term in the first line are the costs (or benefits) of changing vacancy posting costs. If posting costs are constant, the term drops out.

## A.5 Employment Effect with Search Effort

We have

$$\pi_k^{e|u} = \zeta^{\frac{1}{\eta}} q_k^{v\,\frac{\eta-1}{\eta}} \ell_k$$

such that

$$\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x} = \zeta^{\frac{1}{\eta}} \frac{\eta-1}{\eta} q_k^{v-\frac{1}{\eta}} \ell_k \frac{\mathrm{d}q_k^v}{\mathrm{d}x} + \zeta^{\frac{1}{\eta}} q_k^{v\,\frac{\eta-1}{\eta}} \frac{\mathrm{d}\ell_k}{\mathrm{d}x}. \tag{35}$$

We have to determine $\frac{\mathrm{d}q_k^v}{\mathrm{d}x}$ and $\frac{\mathrm{d}\ell_k}{\mathrm{d}x}$. Starting with $\frac{\mathrm{d}q_k^v}{\mathrm{d}x}$, note that

$$q_k^v = \frac{\kappa a^\alpha z_k^{\frac{1}{1-\alpha}}}{\sum_{s=1}^\infty \beta^s (1-\rho)^{s-1} \left\{(1-\alpha) a^\alpha z_{k+s}^{\frac{1}{1-\alpha}} - w_{k+s}\right\}},$$

hence

$$\frac{\mathrm{d}q_k^v}{\mathrm{d}x} = - \frac{\kappa a^\alpha z_k^{\frac{1}{1-\alpha}}}{\left[\sum_{s=1}^\infty \beta^s (1-\rho)^{s-1} \{(1-\alpha) a^\alpha z_{k+s}^{\frac{1}{1-\alpha}} - w_{k+s}\}\right]^2} \sum_{s=1}^{\infty} \beta^s (1-\rho)^{s-1} \left\{ a^\alpha z_{k+s}^{\frac{\alpha}{1-\alpha}} \frac{\mathrm{d}z_{k+s}}{\mathrm{d}x} - \frac{\mathrm{d}w_{k+s}}{\mathrm{d}x} \right\}$$
$$+ \frac{1}{1-\alpha} \frac{\kappa a^\alpha z_k^{\frac{\alpha}{1-\alpha}}}{\sum_{s=1}^\infty \beta^s (1-\rho)^{s-1} \left\{(1-\alpha) a^\alpha z_{k+s}^{\frac{1}{1-\alpha}} - w_{k+s}\right\}} \frac{\mathrm{d}z_k}{\mathrm{d}x}.$$

Private productivity is

$$z_n = B {K_n^G}^\vartheta$$

and so

$$\frac{\mathrm{d}z_n}{\mathrm{d}x} = B\vartheta {K_n^G}^{\vartheta - 1} \frac{\mathrm{d}K_n^G}{\mathrm{d}x} = z \frac{\vartheta}{K_n^G} \frac{\mathrm{d}K_n^G}{\mathrm{d}x}.$$

Furthermore,

$$K_n^G = (1-\delta_G)^n K_0^G + \sum_{j=0}^{T-1} (1-\delta_G)^{n-1-j} I_j + \sum_{j=T}^{n-1} (1-\delta_G)^{n-1-j} x I_j$$

such that

$$\frac{\mathrm{d}K_n^G}{\mathrm{d}x} = \sum_{j=T}^{n-1} (1-\delta_G)^{n-1-j} I_j^G$$

and

$$\frac{\mathrm{d}z_n}{\mathrm{d}x} = \vartheta \frac{z_n}{K_n^G} \sum_{j=T}^{n-1} (1-\delta_G)^{n-1-j} I_j^G.$$

Regarding the wage, it holds that

$$w_n = \gamma^n w_0 + \sum_{s=1}^{n} \gamma^{n-s} (1-\gamma) \omega a^\alpha z_s^{\frac{1}{1-\alpha}}$$

such that

$$\frac{\mathrm{d}w_n}{\mathrm{d}x} = \sum_{s=1}^{n} \gamma^{n-s} (1-\gamma) \omega a^\alpha \frac{1}{1-\alpha} z_s^{\frac{\alpha}{1-\alpha}} \frac{\mathrm{d}z_s}{\mathrm{d}x}.$$

Next, I turn to the effect of investment on search effort, $\frac{\mathrm{d}\ell_k}{\mathrm{d}x}$. Search effort satisfies the first-order condition

$$\ell_k^\chi = -\beta \Delta_{k+1}^{ue} \zeta^{\frac{1}{\eta}} {q_k^v}^{\frac{\eta-1}{\eta}},$$

with
$$\frac{\mathrm{d}\ell_k}{\mathrm{d}x} = -\beta\frac{1}{\chi}\ell_k^{1-\chi}\zeta^{\frac{1}{\eta}}\left[{q_k^v}^{\frac{\eta-1}{\eta}}\frac{\mathrm{d}\Delta_{k+1}^{ue}}{\mathrm{d}x} + \frac{\eta-1}{\eta}q_k^{v-\frac{1}{\eta}}\Delta_{k+1}^{ue}\frac{\mathrm{d}q_k^v}{\mathrm{d}x}\right].$$

Here,

$$\Delta_{k+1}^{ue} = J_{k+1}(u) - J_{k+1}(e) = \max_{\ell_{k+1}}\log\left(\frac{b_{k+1}}{(1-\tau_{k+1})w_{k+1}}\right) + d_{1,e} - \frac{\ell_{k+1}^{1+\chi}}{1+\chi}$$
$$+\beta(1-\pi_{k+1}^{e|u}-\rho)\Delta_{k+2}^{ue}.$$

is the difference in lifetime utility between unemployed and employed workers. It follows using (35) that

$$\frac{\mathrm{d}\pi_k^{e|u}}{\mathrm{d}x} = \frac{1+\chi}{\chi}\zeta^{\frac{1}{\eta}}\frac{\eta-1}{\eta}\left(q_k^v\right)^{-\frac{1}{\eta}}\ell_k\frac{\mathrm{d}q_k^v}{\mathrm{d}x} + \frac{1}{\chi}\pi_k^{e|u}\frac{1}{\Delta_{k+1}^{ue}}\frac{\mathrm{d}\Delta_{k+1}^{ue}}{\mathrm{d}x}$$

It holds that

$$\frac{\mathrm{d}\Delta_{k+1}^{ue}}{\mathrm{d}x} = -\sum_{s=k+1}^{\infty}\left(\frac{1}{(1-\tau_s)w_s}\frac{\mathrm{d}w_s}{\mathrm{d}x} + \beta\ell_s A^{\frac{1}{\eta}}\frac{\eta-1}{\eta}\Delta_{s+1}^{ue}q_s^{v-\frac{1}{\eta}}\frac{\mathrm{d}q_s^v}{\mathrm{d}x}\right)\prod_{j=k+1}^{s-1}\left(\beta(1-\pi_j^{e|u}-\rho\right).$$

Suppose now that the economy is in the steady state initially. I have that

$$\frac{\mathrm{d}z_n}{\mathrm{d}x} = \begin{cases}\vartheta z(1-(1-\delta_G)^{n-T}) & \text{if } n > T\\ 0 \quad \text{otherwise.}\end{cases}$$

and so for $n > T$

$$\begin{aligned}\frac{\mathrm{d}w_n}{\mathrm{d}x} =& (1-\gamma)\omega a^\alpha\frac{1}{1-\alpha}z^{\frac{\alpha}{1-\alpha}}\vartheta z\sum_{s=1}^{n}\gamma^{n-s}(1-(1-\delta_G)^{s-T})\\ =& (1-\gamma)\omega a^\alpha\frac{1}{1-\alpha}z^{\frac{1}{1-\alpha}}\vartheta\left(\frac{\gamma^{n-T}-1}{\gamma-1} - (1-\delta_G)\frac{\gamma^{n-T}-(1-\delta_G)^{n-T}}{\gamma-1+\delta_G}\right)\end{aligned}$$

and $\frac{\mathrm{d}w_n}{\mathrm{d}x} = 0$ for $n \leq T$. Furthermore

$$q_k^v = \frac{\kappa}{\beta(1-\alpha-\omega)}(1-\beta(1-\rho)).$$

For $k \leq T$, we get

$$\begin{aligned}
\frac{\mathrm{d}q_k^v}{\mathrm{d}x} &= -\frac{\kappa}{\left[\frac{\beta}{1-\beta(1-\rho)}(1-\alpha-\omega)\right]^2 a^\alpha z^{\frac{1}{1-\alpha}}} \sum_{s=1}^{\infty} \beta^s (1-\rho)^{s-1} \left\{ a^\alpha z^{\frac{\alpha}{1-\alpha}} \frac{\mathrm{d}z_{k+s}}{\mathrm{d}x} - \frac{\mathrm{d}w_{k+s}}{\mathrm{d}x} \right\} \\
&= -\frac{\kappa}{\left[\frac{\beta}{1-\beta(1-\rho)}(1-\alpha-\omega)\right]^2 a^\alpha z^{\frac{1}{1-\alpha}}} \beta \sum_{s=0}^{\infty} \beta^s (1-\rho)^{s} \left\{ a^\alpha z^{\frac{\alpha}{1-\alpha}} \frac{\mathrm{d}z_{k+s+1}}{\mathrm{d}x} - \frac{\mathrm{d}w_{k+s+1}}{\mathrm{d}x} \right\} \\
&= -\frac{\kappa(1-\beta(1-\rho))^2}{\beta(1-\alpha-\omega)^2 a^\alpha z^{\frac{1}{1-\alpha}}} \sum_{s=0}^{\infty} \beta^s (1-\rho)^{s} \left\{ a^\alpha z^{\frac{\alpha}{1-\alpha}} \frac{\mathrm{d}z_{k+s+1}}{\mathrm{d}x} - \frac{\mathrm{d}w_{k+s+1}}{\mathrm{d}x} \right\} \\
&= -\frac{\kappa(1-\beta(1-\rho))^2}{\beta(1-\alpha-\omega)^2 a^\alpha z^{\frac{1}{1-\alpha}}} \sum_{s=T-k}^{\infty} \beta^s (1-\rho)^{s} \Bigg\{ a^\alpha z^{\frac{\alpha}{1-\alpha}} \vartheta z (1-(1-\delta_G)^{k+s+1-T}) \\
&- (1-\gamma)\omega a^\alpha \frac{1}{1-\alpha} z^{\frac{1}{1-\alpha}} \vartheta \left( \frac{\gamma^{k+s+1-T}-1}{\gamma-1} - (1-\delta_G) \frac{\gamma^{k+s+1-T}-(1-\delta_G)^{k+s+1-T}}{\gamma-1+\delta_G} \right) \Bigg\} \\
&= -\frac{\kappa(1-\beta(1-\rho))^2}{\beta(1-\alpha-\omega)^2} \vartheta \frac{1}{1-\alpha} (\beta(1-\rho))^{T-k} \sum_{s=0}^{\infty} \beta^s (1-\rho)^{s} \Bigg\{ (1-\alpha)(1-(1-\delta_G)^{s+1}) \\
&- (1-\gamma)\omega \left( \frac{\gamma^{s+1}-1}{\gamma-1} - (1-\delta_G) \frac{\gamma^{s+1}-(1-\delta_G)^{s+1}}{\gamma-1+\delta_G} \right) \Bigg\}.
\end{aligned}$$

Solving the geometric series yields

$$
\begin{aligned}
\frac{\mathrm{d}q_k^v}{\mathrm{d}x} = &-\frac{\kappa(1-\beta(1-\rho))^2}{\beta(1-\alpha-\omega)^2}\vartheta\frac{1}{1-\alpha}(\beta(1-\rho))^{T-k}\Bigg[\frac{\delta_G(1-\alpha)}{(1-\beta(1-\rho))(1-\beta(1-\rho)(1-\delta_G))} \\
&+\omega\left(\frac{\gamma}{1-\gamma\beta(1-\rho)}-\frac{1}{1-\beta(1-\rho)}\right) \\
&+\frac{\omega(1-\gamma)(1-\delta_G)}{\gamma-1+\delta_G}\left(\frac{\gamma}{1-\gamma\beta(1-\rho)}-\frac{1-\delta_G}{1-\beta(1-\delta_G)(1-\rho)}\right)\Bigg] \\
= &-\frac{\kappa(1-\beta(1-\rho))^2}{\beta(1-\alpha-\omega)^2}\vartheta\frac{1}{1-\alpha}(\beta(1-\rho))^{T-k}\Bigg[\frac{\delta_G(1-\alpha)}{(1-\beta(1-\rho))(1-\beta(1-\rho)(1-\delta_G))} \\
&-\frac{\omega}{1-\beta(1-\rho)}+\frac{\omega\gamma^2\delta_G}{\gamma-1+\delta_G}\frac{1}{1-\gamma\beta(1-\rho)}+\frac{(1-\delta_G)\omega}{1-\beta(1-\rho)(1-\delta_G)} \\
&-\frac{\omega(\gamma-\delta_G+\gamma\delta_G)}{\gamma-1+\delta_G}\frac{1}{1-\beta(1-\rho)(1-\delta_G)}\Bigg] \\
= &-\frac{\kappa(1-\beta(1-\rho))^2}{\beta(1-\alpha-\omega)^2}\vartheta\frac{1}{1-\alpha}(\beta(1-\rho))^{T-k}\Bigg[\frac{\delta_G(1-\alpha-\omega)}{(1-\beta(1-\rho))(1-\beta(1-\rho)(1-\delta_G))} \\
&+\frac{\omega\gamma^2\delta_G}{\gamma-1+\delta_G}\frac{1}{1-\gamma\beta(1-\rho)}-\frac{\omega\gamma\delta_G(1-\delta_G)}{\gamma-1+\delta_G}\frac{1}{1-\beta(1-\rho)(1-\delta_G)}\Bigg] \\
= &-\frac{\kappa(1-\beta(1-\rho))^2}{\beta(1-\alpha-\omega)^2}\vartheta\frac{1}{1-\alpha}(\beta(1-\rho))^{T-k}\Bigg[\frac{\delta_G(1-\alpha-\omega)}{(1-\beta(1-\rho))(1-\beta(1-\rho)(1-\delta_G))} \\
&+\frac{\omega\delta_G\gamma}{(1-\gamma\beta(1-\rho))(1-\beta(1-\rho)(1-\delta_G))}\Bigg] \\
= &-\frac{\kappa(1-\beta(1-\rho))\delta_G\vartheta}{\beta(1-\alpha-\omega)(1-\beta(1-\rho)(1-\delta))}\frac{(\beta(1-\rho))^{T-k}}{1-\alpha}\left[1+\frac{\omega\gamma(1-\beta(1-\rho))}{(1-\alpha-\omega)(1-\gamma\beta(1-\rho))}\right]
\end{aligned}
$$

And similarly, for $k > T$,

$$\begin{aligned}
\frac{\mathrm{d}q_k^v}{\mathrm{d}x} &= -\frac{\kappa(1-\beta(1-\rho)^2)}{\beta(1-\alpha-\omega)^2 a^\alpha z^{\frac{1}{1-\alpha}}}\vartheta\sum_{s=0}^{\infty}\beta^s(1-\rho)^s\Bigg\{a^\alpha z^{\frac{1}{1-\alpha}}(1-(1-\delta_G)^{k+s+1-T}) \\
&-(1-\gamma)\omega a^\alpha\frac{1}{1-\alpha}z^{\frac{1}{1-\alpha}}\left(\frac{\gamma^{k+s+1-T}-1}{\gamma-1}-(1-\delta_G)\frac{\gamma^{k+s+1-T}-(1-\delta_G)^{k+s+1-T}}{\gamma-1+\delta_G})\right)\Bigg\} \\
&+\frac{1}{1-\alpha}\frac{\kappa\vartheta}{\beta(1-\alpha-\omega)}(1-\beta(1-\rho))(1-(1-\delta_G)^{k-T}) \\
&= -\frac{\kappa(1-\beta(1-\rho))^2\vartheta}{\beta(1-\alpha-\omega)^2}\Bigg\{\sum_{s=0}^{\infty}(\beta(1-\rho))^s-(1-\delta_G)^{k-T+1}\sum_{s=0}^{\infty}(\beta(1-\rho)(1-\delta_G))^s \\
&-\frac{(1-\gamma)\omega}{1-\alpha}\Bigg[\frac{\gamma^{k+1-T}}{\gamma-1}\sum_{s=0}^{\infty}(\beta(1-\rho)\gamma)^s-\frac{1}{\gamma-1}\sum_{s=0}^{\infty}(\beta(1-\rho))^s \\
&+\frac{(1-\delta_G)\gamma^{k+1-T}}{\gamma-1+\delta_G}\sum_{s=0}^{\infty}(\beta(1-\rho)\gamma)^s-\frac{(1-\delta_G)^{k+2-T}}{\gamma-1+\delta_G}\sum_{s=0}^{\infty}(\beta(1-\rho)(1-\delta_G))^s\Bigg]\Bigg\} \\
&+\frac{1}{1-\alpha}\frac{\kappa\vartheta}{\beta(1-\alpha-\omega)}(1-\beta(1-\rho))(1-(1-\delta_G)^{k-T}) \\
&= -\frac{\kappa(1-\beta(1-\rho))^2}{\beta(1-\alpha-\omega)}\frac{\vartheta}{1-\alpha}\Bigg[\frac{1}{1-\beta(1-\rho)}-\frac{(1-\delta_G)^{k-T+1}}{1-\beta(1-\rho)(1-\delta_G)} \\
&+\frac{\omega\gamma\delta_G}{(1-\gamma-\delta_G)(1-\alpha-\omega)}\left(\frac{\gamma^{k-T+1}}{1-\gamma\beta(1-\rho)}+\frac{(1-\delta_G)^{k-T+1}}{1-\beta(1-\rho)(1-\delta_G)}\right)\Bigg] \\
&+\frac{1}{1-\alpha}\frac{\kappa\vartheta}{\beta(1-\alpha-\omega)}(1-\beta(1-\rho))(1-(1-\delta_G)^{k-T}) \\
&= -\frac{\kappa(1-\beta(1-\rho))}{\beta(1-\alpha-\omega)}\frac{\vartheta}{1-\alpha}\Bigg[1-\frac{(1-\delta_G)^{k-T+1}(1-\beta(1-\rho))}{1-\beta(1-\rho)(1-\delta_G)}-(1-(1-\delta_G)^{k-T}) \\
&+\frac{\omega\gamma\delta_G(1-\beta(1-\rho))}{(1-\gamma-\delta_G)(1-\alpha-\omega)}\left(\frac{\gamma^{k-T+1}}{1-\gamma\beta(1-\rho)}+\frac{(1-\delta_G)^{k-T+1}}{1-\beta(1-\rho)(1-\delta_G)}\right)\Bigg] \\
&= -\frac{\kappa(1-\beta(1-\rho))}{\beta(1-\alpha-\omega)}\frac{\vartheta}{1-\alpha}\Bigg[\frac{(1-\delta_G)^{k-T}\delta_G}{1-\beta(1-\rho)(1-\delta_G)} \\
&+\frac{\omega\gamma\delta_G(1-\beta(1-\rho))}{(1-\gamma-\delta_G)(1-\alpha-\omega)}\left(\frac{\gamma^{k-T+1}}{1-\gamma\beta(1-\rho)}+\frac{(1-\delta_G)^{k-T+1}}{1-\beta(1-\rho)(1-\delta_G)}\right)\Bigg]
\end{aligned}$$

This gives

$$
\begin{aligned}
\frac{\mathrm{d}\Delta^{ue}_{k+1}}{\mathrm{d}x} =& -\frac{1}{(1-\tau)w}\sum_{s=k+1}^{\infty}\Big(\beta(1-\pi^{e|u}-\rho)\Big)^{s-k-2}\frac{\mathrm{d}w_s}{\mathrm{d}x} \\
&-\beta\ell\zeta^{\frac{1}{\eta}}\frac{\eta-1}{\eta}\Delta^{ue}q^{v-\frac{1}{\eta}}\Bigg[\sum_{s=k+1}^{T}\Big(\beta(1-\pi^{e|u}-\rho)\Big)^{s-k-2}\frac{\mathrm{d}q_s}{\mathrm{d}x} \\
&+\sum_{s=T+1}^{\infty}\Big(\beta(1-\pi^{e|u}-\rho)\Big)^{s-k+2}\frac{\mathrm{d}q_s}{\mathrm{d}x}\Bigg].
\end{aligned}
$$

Considering each sum in turn,

$$
\begin{aligned}
&\frac{1}{w}\sum_{s=k+1}^{\infty}\Big(\beta(1-\pi^{e|u}-\rho)\Big)^{s-k-2}\frac{\mathrm{d}w_s}{\mathrm{d}x} \\
&=\sum_{s=T+1}^{\infty}\Big(\beta(1-\pi^{e|u}-\rho)\Big)^{s-k-2}(1-\gamma)\frac{1}{1-\alpha}\vartheta\left(\frac{\gamma^{s-T}-1}{\gamma-1}-(1-\delta_G)\frac{\gamma^{s-T}-(1-\delta_G)^{s-T}}{\gamma-1+\delta_G}\right) \\
&=\frac{\vartheta}{1-\alpha}\Big((\beta(1-\pi^{e|u}-\rho)\Big)^{T-k-1}\frac{1}{1-\gamma-\delta_G} \\
&\times\left(\frac{1-\gamma-\delta_G}{1-\beta(1-\rho-\pi^{e|u})}+\frac{\gamma^2\delta_G}{1-\beta\gamma(1-\rho-\pi^{e|u})}-\frac{(1-\gamma)(1-\delta_G)^2}{1-\beta(1-\delta_G)(1-\rho-\pi^{e|u})}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
&\sum_{s=k+1}^{T}\Big(\beta(1-\pi^{e|u}-\rho)\Big)^{s-k-2}\frac{\mathrm{d}q_s}{\mathrm{d}x} \\
&=-\sum_{s=k+1}^{T}\Big(\beta(1-\pi^{e|u}-\rho)\Big)^{s-k-2}\frac{\kappa(1-\beta(1-\rho))\delta_G\vartheta}{\beta(1-\alpha-\omega)(1-\beta(1-\rho)(1-\delta))}\frac{(\beta(1-\rho))^{T-s}}{1-\alpha} \\
&\times\left[1+\frac{\omega\gamma(1-\beta(1-\rho))}{(1-\alpha-\omega)(1-\gamma\beta(1-\rho))}\right] \\
&=-\frac{\kappa(1-\beta(1-\rho))\delta_G\vartheta}{\beta(1-\alpha-\omega)(1-\beta(1-\rho)(1-\delta))}\frac{1}{1-\alpha}\left[1+\frac{\omega\gamma(1-\beta(1-\rho))}{(1-\alpha-\omega)(1-\gamma\beta(1-\rho))}\right] \\
&\times\beta^{T-k-1}\frac{(1-\rho)^{T-k}-(1-\pi^{e|u}-\rho)^{T-k}}{\pi(1-\pi-\rho)} \\
&=-\frac{\kappa(1-\beta(1-\rho))\delta_G\vartheta}{\beta(1-\alpha-\omega)(1-\beta(1-\rho)(1-\delta))}\frac{1}{1-\alpha}(\beta(1-\rho-\pi)^{T-k-1}\left[1+\frac{\omega\gamma(1-\beta(1-\rho))}{(1-\alpha-\omega)(1-\gamma\beta(1-\rho))}\right] \\
&\times\frac{(1-\rho)^{T-k}(1-\pi^{e|u}-\rho)^{-T+k}-1}{\pi}
\end{aligned}
$$

and

$$\sum_{s=T+1}^{\infty}\Big(\beta(1-\pi^{e|u}-\rho)\Big)^{s-k-2}\frac{\mathrm{d}q_s}{\mathrm{d}x}$$

$$=-\sum_{s=T+1}^{\infty}\Big(\beta(1-\pi^{e|u}-\rho)\Big)^{s-k-2}\frac{\kappa(1-\beta(1-\rho))}{\beta(1-\alpha-\omega)}\frac{\vartheta}{1-\alpha}\Bigg[\frac{(1-\delta_G)^{s-T}\delta_G}{1-\beta(1-\rho)(1-\delta_G)}$$

$$+\frac{\omega\gamma\delta_G(1-\beta(1-\rho))}{(1-\gamma-\delta_G)(1-\alpha-\omega)}\left(\frac{\gamma^{s-T+1}}{1-\gamma\beta(1-\rho)}+\frac{(1-\delta_G)^{s-T+1}}{1-\beta(1-\rho)(1-\delta_G)}\right)\Bigg]$$

$$=-\frac{\kappa(1-\beta(1-\rho))}{\beta(1-\alpha-\omega)}\frac{\vartheta}{1-\alpha}\Bigg[\frac{\delta_G}{1-\beta(1-\rho)(1-\delta_G)}\frac{(1-\delta_G)(\beta(1-\pi^{e|u}-\rho))^{T-k-1}}{1-\beta(1-\pi^{e|u}-\rho)(1-\delta_G)}$$

$$+\frac{\omega\gamma\delta_G(1-\beta(1-\rho))}{(1-\gamma-\delta_G)(1-\alpha-\omega)}\Bigg(\frac{\gamma^2}{1-\gamma\beta(1-\rho)}\frac{(\beta(1-\pi-\rho))^{T-k-1}}{1-\beta\gamma(1-\pi^{e|u}-\rho)}$$

$$+\frac{(1-\delta_G)^2}{1-(1-\delta_G)\beta(1-\rho)}\frac{(\beta(1-\pi-\rho))^{T-k-1}}{1-\beta(1-\delta_G)(1-\pi^{e|u}-\rho)}\Bigg)\Bigg]$$

$$=-\frac{\kappa(1-\beta(1-\rho))\vartheta\delta_G}{\beta(1-\alpha-\omega)(1-\beta(1-\rho)(1-\delta_G))}\frac{1}{1-\alpha}(\beta(1-\pi-\rho))^{T-k-1}\Bigg[\frac{(1-\delta_G)}{1-\beta(1-\pi^{e|u}-\rho)(1-\delta_G)}$$

$$+\frac{\omega\gamma(1-\beta(1-\rho))}{(1-\gamma-\delta_G)(1-\alpha-\omega)}\Bigg(\frac{\gamma^2}{1-\gamma\beta(1-\rho)}\frac{1-\beta(1-\rho)(1-\delta_G)}{1-\beta\gamma(1-\pi^{e|u}-\rho)}$$

$$+\frac{(1-\delta_G)^2}{1-\beta(1-\delta_G)(1-\pi^{e|u}-\rho)}\Bigg)\Bigg]$$

# B Calibration Details

## B.1 Estimation of Job Finding and Separation Probabilities

Crucial targets for calibrating the labor market parameters in the model are the transition probabilities between labor market states, in particular, the the job finding probability and the separation probability. The data source most commonly used to estimate these rates is the Current Population Survey (CPS). There are two main method to estimating the job finding rate from CPS data. Here, I use the one based on gross flows, that is, I use the panel dimension of the monthly CPS microdata to estimate the number of workers who transition from unemployment to employment in a given month. The alternative approach is based solely on the aggregate time series of unemployment as described in Shimer (2012). It requires stronger assumptions than the gross flows method used here, in particular, it assumes a constant labor force. In contrast, the gross flows approach can be extended to incorporate more than two labor market states and arbitrary transitions between them. A discussion and comparison of the two methods can be found in Shimer (2012).

I consider two different definitions of unemployed workers, denoted U-3 and U-5 by the BLS. The most widely used concept is U-3. According to this definition a worker is unemployed if i) he or she does not work but has been actively looking for a job during the last four weeks and would be available to work or if ii) he or she is temporarily laid off and waiting to be recalled. The alternative definition, U-5, also encompasses workers who want a job, searched for a job at some point during the last twelve months, and could have taken a job in the last week if they had been offered one. Hence, this measure includes discouraged and marginally attached workers according to the BLS classification. Figure 9 shows the number of unemployed workers according to the definitions U-3 and U-5 over time.

I estimate the job finding probability from gross flows (method 2) as follows (see also Shimer 2012):

- I match individuals across monthly CPS waves to obtain a panel data set
- For every month I compute the number of workers who transition between each of the three labor market states employed, unemployed, inactive
  - I do this for both concepts of unemployment, U-3 and U-5
  - The series are seasonally adjusted using X13-ARIMA-SEATS
- From these flows I obtain a Markov matrix for the monthly transition between the three states for every month in the sample
- I adjust for time aggregation using the method described in Shimer (2012)
  - Therefore, I compute the continuous time Markov matrix (instantaneous transition probabilities) from the discrete time matrix and obtain the monthly transition probabilities from the instantaneous transition rates. The monthly probabilities obtained in this way capture the probability of experiencing a transition between state A and B over the course of one month. This is different than the probability of being in state B in the next month conditional on being in state A in the current month. The latter is what I observe in the data, the former is what I need to inform the calibration of the model.
- I also use the same procedure but with four states (employed, U-3 unemployed, marginally attached, inactive) to also obtain separate transition probabilities for U-3 unemployed and marginally attached workers.

The CPS did not include the questions that are used to identify discouraged and marginally attached workers prior to 1994. This is why I can only compute job finding probabilities of unemployed workers according to the broader definition U-5 for the time period from 1994 to 2020. For comparison, I also compute the transition probabilities according to the unemployment concept U-3 for the whole time period for which CPS microdata is available, 1976 to 2020. Table 3 shows the average monthly job finding probability for U-3 unemployed, U-5 unemployed, and marginally attached workers for different time periods. For the time period from 1994 to 2020, the average job finding probability for unemployed workers according to the concept U-3 was 29.4%. It was 2.5 percentage points lower for the group of U-5 unemployed workers. Marginally attached workers are much less likely to find a job in a given month, on average their job finding probability is only 10.9%.

Table 3: Monthly transition probabilities.

| | 1976–2020 | 1994–2020 | 2003–2020 |
|---|---|---|---|
| Find. Prob. U-3 | 29.8 | 29.4 | 27.1 |
| Find. Prob. U-5 | - | 26.9 | 24.9 |
| Find. Prob. Marginally attached | - | 10.9 | 11.3 |
| Sep. Rate | 1.9 | 1.8 | 1.8 |

The reason for the small difference in job finding probabilities between U-3 and U-5 can be found in Figure 9 which shows the total numbers of unemployed workers according to definitions U-3 and U-5 and the number of marginally attached workers over time. On average, the number of marginally attached workers is only about one fifth of the number of U-3 unemployed workers. For the group of unemployed workers according to the definition U-5, marginally attached workers play a small role. This is why the substantially lower job finding probability of marginally attached workers does not matter much for the overall job finding probability in the group of U-5 unemployed workers.

Figure 9: Unemployment over time.

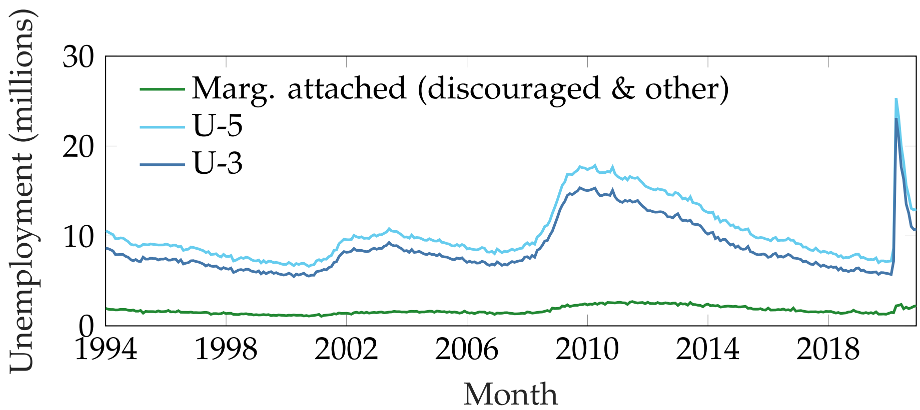
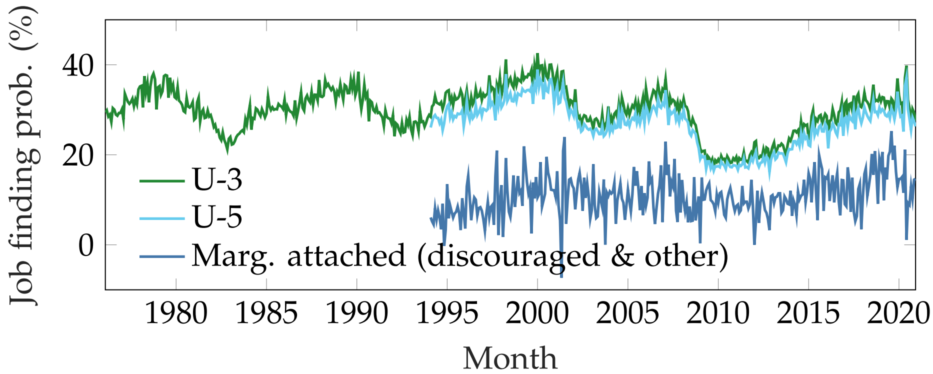


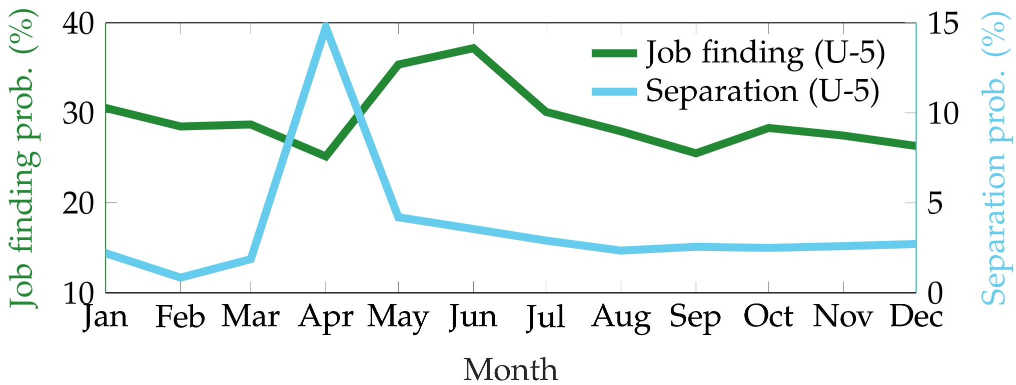

Figure 10 shows the estimated monthly job finding probability over time. The dark blue line shows the estimated monthly job finding probability of unemployed workers, when unemployed according to the concept U-3 are considered. For the time period from 1976Q1 to 2007Q2, I can compare the quarterly averages of this series to the series in Shimer (2012). The two are very similar, the standard deviation of the difference is less than 1.5 percentage points. This difference is likely coming from the different seasonal adjustment procedures used. The light blue line represents the job finding probability for unemployed according to the definition U-5. Finally, the green line shows the job finding rate for marginally employed workers, when I distinguish between four labor market states, employed, U-3 unemployed, marginally attached, and inactive.

Figure 10: Monthly job finding probabilities.




The outbreak of the Covid-19 pandemic and its economic repercussions led to an increase of unemployment to more than 23 million in April 2020. Figure 11 shows the monthly job finding and separation probabilities between labor market states that led to this spike in unemployment. It can be seen that the rise in unemployment is mostly due to a sharp increase of the separation probability in April 2020.

Figure 11: Monthly transition probabilities in 2020.




## B.2 Calibration of Disutility from Effort

I calibrate the parameter $\chi$ to match the elasticity of the job finding probability with respect to unemployment benefits $\epsilon_{q,b} = \frac{\mathrm{d}q^f}{\mathrm{d}b}\frac{b}{q^f}$. From the first-order condition for search effort, I have that

$$\ell^{\chi} = \beta \left( J_t(e) - J_t(u) \right) \frac{q^f}{\ell}. \tag{36}$$

In the steady state and under the assumption that disutility from search and work are equalized, I have that the difference between lifetime utility of employed and unemployed workers is

$$J_t(e) - J_t(u) = \frac{\log(\frac{w}{b})}{1 - \beta + \beta(\rho + q^f)}$$

Hence

$$\left(1 - \beta + \beta(\rho + q^f)\right) \ell^\chi = \beta \left(\log\left(\frac{w}{b}\right) + \frac{\beta}{1-\beta}\right) x$$

where $x = \frac{q^f}{\ell}$ is a constant (partial equilibrium) and

$$\left(1 - \beta + \beta(\rho + q^f)\right) \chi \ell^{\chi-1} \frac{\mathrm{d}\ell}{\mathrm{d}b} + \beta \frac{\mathrm{d}q^f}{\mathrm{d}b} \ell^\chi = -\beta \frac{1}{b} x$$

Therefore

$$\left(1 - \beta + \beta(\rho + q^f)\right) \chi \ell^{\chi-1} \frac{1}{x} \frac{\mathrm{d}q^f}{\mathrm{d}b} + \beta \frac{\mathrm{d}q^f}{\mathrm{d}b} \ell^\chi = -\beta \frac{1}{b} x \tag{37}$$

$$\Leftrightarrow \left(1 - \beta + \beta(\rho + q^f)\right) \chi \ell^{\chi-1} \frac{1}{x^2} \frac{\mathrm{d}q^f}{\mathrm{d}b} \frac{b}{q^f} + \beta \frac{\mathrm{d}q^f}{\mathrm{d}b} \ell^\chi \frac{1}{x} \frac{b}{q^f} = -\beta \frac{1}{q^f} \tag{38}$$

$$\Leftrightarrow \left(1 - \beta + \beta(\rho + q^f)\right) \chi \ell^{\chi-1} \frac{\ell^2}{(q^f)^2} \frac{\mathrm{d}q^f}{\mathrm{d}b} \frac{b}{q^f} + \beta \frac{\mathrm{d}q^f}{\mathrm{d}b} \ell^\chi \frac{\ell}{q^f} \frac{b}{q^f} = -\beta \frac{1}{q^f} \tag{39}$$

$$\Leftrightarrow \left(1 - \beta + \beta(\rho + q^f)\right) \chi \ell^{\chi+1} \frac{1}{q^f} \epsilon_{q,b} + \beta \epsilon_{q,b} \ell^{\chi+1} = -\beta \tag{40}$$

Substituting (36) and rearranging yields

$$\chi = -\frac{1 + \beta \epsilon_{q,b} q^f \left(J_t(e) - J_t(u)\right)}{(1 - \beta + \beta(\rho + q^f))\epsilon_{q,b} \left(J_t(e) - J_t(u)\right)}$$

All terms on the right-hand side follow directly from the calibration targets.

# C Optimal Allocation

In general, the equilibrium in the search and matching labor market described above is inefficient due to two congestion externalities. When posting a vacancy, a firm does not take into account the negative effect this has on the likelihood of other firms to fill their vacancies. Similarly, firms fail to internalize that every additional vacancy makes it easier for workers to find a job. The private benefits of posting a vacancy may exceed or fall below the social benefit. To better understand how these inefficiencies shape the effects of government investment, I analyze the constrained efficient allocation which I define as the one that would be chosen by a utilitarian social planner who is constrained by the matching friction and faces the same capital adjustment costs as firm owners. To that end, I define social welfare as

$$W(\{c_t^F, c_t(s^t)\}) = \bar{\mu}^F \sum_{t=0}^{\infty} \beta^t c_t^F + \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \log\left(c_t(s^t)\right) - d(\ell_t(s^t))\pi_t(s^t|s_0)\bar{\mu}(s_0),$$

where $\bar{\mu}^F$, $\bar{\mu}(e)$ and $\bar{\mu}(u)$ are the welfare weights of firm owners, initially employed and initially unemployed workers and $\pi_t(s^t|s_0)$ denotes the share of workers with history $s^t = (s_0, s_1, \dots, s_t)$ in period $t$.

**Definition 3** (Optimal allocation)**.** *An optimal allocation for a given sequence of productivity is a collection of sequences of aggregate consumption, capital, employment, search effort and labor market tightness and of individual consumption and search effort which solves the planner problem*

$$\begin{aligned}
\max_{\{C_t, N_{t+1}, K_{t+1}, L_t^u, \theta_t, c_t^F, c_t(s^t), \ell_t(s^t)\}} \; & W(\{c_t^F, c_t(s^t)\}) \\
s.t.\; & C_t + K_{t+1} + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t + \kappa_t \theta_t L_t^u \\
& = z_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta_k)K_t \\
& N_{t+1} = (1-\rho)N_t + q_t^v(\theta_t)\theta_t L_t^u \\
& C_t = \mu c_t^F + \sum_{s^t} c_t(s^t)\pi_t(s^t) \\
& L_t^u = \sum_{s^t|s_t=u} \ell_t(s^t)\pi_t(s^t) \\
\textit{given } & K_0,\; N_0.
\end{aligned} \tag{41}$$

Note first that the planner I consider here takes the sequence of productivity as given. In other words, the sequence of public investment and thereby productivity has already been decided and the planner now faces the problem of allocating the remaining resources.[18] The first constraint in the planner problem is the aggregate resource constraint. The right-hand side are total available resources consisting of output and capital after depreciation which can be spend on consumption, investment in next period's capital, and vacancy creation. The second constraint is the law of motion for employment. The planner can increase employment in the next period in two ways. First, the planner can raise tightness $\theta_t$ which comes at a resource cost according to the term $\kappa_t \theta_t L_t^u$ in the resource constraint since more vacancies have to be created for a constant level of aggregate search effort. Second, employment can be increased by raising aggregate search effort $L_t^u$ with comes at a utility cost since effort enters the utility function but there are also resource costs since more vacancies have to be created if tightness is to be held constant. The last two constraints of the planner problem state that individual

18. The costs of public investment could be added to the resource constraint without changing the results that follow. This is because firm owners have linear utility.

consumption must add up to aggregate consumption and individual search effort $\ell_t(s^t)$ has to be consistent with aggregate search effort $L_t^u$.

The next propositions characterize the optimal allocation more closely.

**Proposition 5** (Optimal allocation of capital)**.** *The optimal allocation of capital satisfies*

$$1 + \phi\left(\frac{K_{t+1}}{K_t} - 1\right) = \beta\left(1 + \alpha z_{t+1} k_{t+1}^{\alpha-1} - \delta_k + \frac{\phi}{2}\left(\left(\frac{K_{t+2}}{K_{t+1}}\right)^2 - 1\right) - \frac{\partial \kappa_{t+1}}{\partial K_{t+1}} \theta_{t+1} L_{t+1}^u\right).$$

*Proof.* The result follows immediately from the first-order conditions for consumption and capital associated with (41). □

If vacancy posting costs do not depend on capital, it holds that $\frac{\partial \kappa_{t+1}}{\partial K_{t+1}} = 0$ and the optimal path for the aggregate capital stock coincides with the equilibrium allocation given by equation (21). However, if vacancy posting costs depend on the aggregate capital stock, for example because they are proportional to labor productivity as would be needed for balanced growth, then the aggregate capital stock is too high in equilibrium because existing firms who rent capital do not take into account that more capital per match makes it more expensive for new firms to post a vacancy.

Next, I characterize the sequence of optimal tightness. It will depend on the elasticity of the vacancy filling probability with respect to tightness which I denote as $\eta \equiv -\frac{m'(\theta_t)\theta_t}{q_t^v(\theta_t)}$.

**Proposition 6** (Optimal tightness with fixed search effort)**.** *Suppose individual search effort is fixed at $\ell_t(s^t) = 1$ and $d(1,u) = d(1,e)$, then optimal tightness satisfies*

$$\frac{\kappa_t}{q_t^v(\theta_t)} = \beta\left\{(1-\alpha) z_{t+1} k_{t+1}^{\alpha} - \eta\left[(1-\alpha) z_{t+1} k_{t+1}^{\alpha} + \kappa_{t+1}\theta_{t+1}\right] + (1-\rho)\frac{\kappa_{t+1}}{m(\theta_{t+1})}\right\}.$$

Comparison with the equilibrium condition (20) shows that without search effort, the equilibrium is efficient if the wage is

$$w_t = \eta\left[(1-\alpha) z_t k_t^{\alpha} + \kappa_t \theta_t\right] \tag{42}$$

This is the standard condition for efficiency in the DMP model.

**Proposition 7** (Optimal tightness)**.** *Suppose that the welfare weights of initially unemployed and employed workers are equal to their population shares, $\bar{\mu}(s) = \pi_0(s)$, then optimal tightness satisfies*

$$\begin{aligned}\frac{\kappa_t}{q_t^v(\theta_t)} =& \beta\Bigg\{(1-\alpha) z_{t+1} k_{t+1}^{\alpha} - \eta\left[(1-\alpha) z_{t+1} k_{t+1}^{\alpha} + \kappa_{t+1}\theta_{t+1}\ell_{t+1}(u)\right] \\ &+ (1-\eta)\frac{\mu}{\bar{\mu}^F}\left(d(\ell_{t+1}(u),u) - d(0,e)\right) + (1-\rho)\frac{\kappa_{t+1}}{m(\theta_{t+1})}\Bigg\}.\end{aligned}$$

*where the optimal level of individual search effort solves*

$$d'(\ell_t(u), u) = \frac{\bar{\mu}^F}{\mu} \kappa_t \theta_t \frac{1}{1+\eta}.$$

In this case, the efficient allocation is implemented if the wage amounts to

$$w_t = \eta \left[(1-\alpha) z_t k_t^\alpha + \kappa_t \theta_t \ell_t(u)\right] - (1-\eta) \frac{\mu}{\bar{\mu}^F} \left(d(\ell_t(u), u) - d(0, e)\right). \tag{43}$$

The differences to the optimal wage in the case without effort given by equation (42) are intuitive. First, the term $\kappa_t \theta_t$ is multiplied by individual search effort $\ell_t(u)$. To see why, suppose optimal search effort increases. Then, firms find it easier to fill a vacancy and expand vacancy creation. To prevent an inefficiently high vacancy creation, the wage has to be higher to discourage vacancy creation. Second, the additional summand in (43) takes into account the difference in disutility of effort between employed and unemployed. If the disutility is higher for unemployed, a lower level of unemployment is desirable which is implemented through a lower wage leading to a higher level of labor market tightness.

# D News Shock

The preceding discussion has highlighted the role of expectation about future productivity for the employment effect of public investment. The importance of expected future productivity can also be seen when comparing the public investment employment effect to the change in employment that would result from a permanent change in productivity, defined as follows.

**Definition 4** (Employment effect of (future) productivity). *Let $N_t(\mathcal{X}_0, z_0, z_1, \dots)$ denote employment in period $t$ in an equilibrium with initial conditions $\mathcal{Y}_0 = (N_0, w_0, K_0)$ and productivity sequence $\mathcal{Z} = (z_t)_{t=0}^\infty$. Consider a permanent increase in productivity in period $T$. The employment effect in $t$ is defined as*

$$M_t^z(T, \mathcal{Y}_0, \mathcal{Z}) = \frac{\partial N_t(\mathcal{Y}_0, \dots, z_{T-1}, x z_T, x z_{T+1}, \dots)}{\partial x}|_{x=1}.$$

I get the following result

**Proposition 8.** *If the economy is in its steady state initially, then*

$$M_t^{Inv}(T, \mathcal{X}_0, \mathcal{I}^G) = \frac{\vartheta}{1 - \beta(1-\delta_G)(1-\rho)} \frac{1}{K^G} M_t^z(T, \mathcal{Y}_0, \mathcal{Z}).$$

The public investment employment effect is proportional to the employment change in response to a permanent change in future productivity where the factor of proportionality depends on the elasticity of productivity with respect to public investment. For private agents, the announcement of the public investment expansion constitutes a news shock about productivity and, up to a constant factor, induces the same employment response.

*Proof.* Consider the productivity sequence $(z_k)_{k=0}^{\infty}$ with $z_k = z$ for $k < t$ and $z = xz$ for $k \geq T$. The wage in period $s$ is

$$w_s = \begin{cases} \gamma^s w_0 + (1-\gamma)\omega a^{\alpha} z^{\frac{1}{1-\alpha}} \frac{\gamma^s - 1}{\gamma - 1}, & \text{if } s < T \\ \gamma^s w_0 + (1-\gamma)\omega a^{\alpha} z^{\frac{1}{1-\alpha}} \left( \frac{\gamma^s - \gamma^{s-T+1}}{\gamma - 1} + x^{\frac{1}{1-\alpha}} \frac{\gamma^{s-T+1} - 1}{\gamma - 1} \right), & \text{if } s \geq T \end{cases}$$

and for $k < T$

$$\begin{aligned} \pi_k^{e|u} =& \zeta^{\frac{1}{\eta}} (1-\rho)^{\frac{1-\eta}{\eta}} \left( z^{\frac{1}{1-\alpha}} \kappa a^{\alpha} \right)^{\frac{\eta-1}{\eta}} \\ & \times \Bigg[ \sum_{s=k}^{T-1} (\beta(1-\rho))^{s-k} (1-\alpha) a^{\alpha} z^{\frac{1}{1-\alpha}} + \sum_{s=T}^{\infty} x^{\frac{1}{1-\alpha}} (\beta(1-\rho))^{s-k} (1-\alpha) a^{\alpha} z^{\frac{1}{1-\alpha}} \\ & \quad - \sum_{s=k}^{\infty} (\beta(1-\rho))^{s-k} \gamma^s w_0 + \sum_{s=k}^{T-1} (\beta(1-\rho))^{s-k} \omega a^{\alpha} z^{\frac{1}{1-\alpha}} (\gamma^s - 1) \\ & \quad + \sum_{s=T}^{\infty} (\beta(1-\rho))^{s-k} \omega a^{\alpha} z^{\frac{1}{1-\alpha}} (\gamma^s - \gamma^{s-T+1}) \\ & \quad + \sum_{s=T}^{\infty} (\beta(1-\rho))^{s-k} \omega a^{\alpha} z^{\frac{1}{1-\alpha}} x^{\frac{1}{1-\alpha}} (\gamma^{s-T+1} - 1) \Bigg]^{\frac{1-\eta}{\eta}} \end{aligned}$$

which can be simplified to

$$\begin{aligned} \pi_k^{e|u} =& \zeta^{\frac{1}{\eta}} (1-\rho)^{\frac{1-\eta}{\eta}} \left( z^{\frac{1}{1-\alpha}} \kappa a^{\alpha} \right)^{\frac{\eta-1}{\eta}} \\ & \Bigg\{ \frac{(1-\alpha-\omega) a^{\alpha} z^{\frac{1}{1-\alpha}}}{1-\beta(1-\rho)} \left[ 1 + (\beta(1-\rho))^{T-k} (x^{\frac{1}{1-\alpha}} - 1) \right] \\ & + \gamma \frac{\omega a^{\alpha} z^{\frac{1}{1-\alpha}}}{1-\gamma\beta(1-\rho)} (\beta(1-\rho))^{T-k} (x^{\frac{1}{1-\alpha}} - 1) - \frac{\gamma^k}{1-\gamma\beta(1-\rho)} (w_0 - \omega a^{\alpha} z^{\frac{1}{1-\alpha}}) \Bigg\}^{\frac{1-\eta}{\eta}} \end{aligned}$$

I have that for $k < T$

$$\begin{aligned}\frac{\partial \pi_k^{e|u}}{\partial x}|_{x=1} =&\pi_k^{e|u}\frac{1-\eta}{\eta}\frac{1}{1-\alpha}\Bigg\{\frac{(1-\alpha-\omega)a^\alpha z^{\frac{1}{1-\alpha}}}{1-\beta(1-\rho)} \\ &-\frac{\gamma^k}{1-\gamma\beta(1-\rho)}(w_0-\omega a^\alpha z^{\frac{1}{1-\alpha}})\Bigg\}^{-1}(\beta(1-\rho))^{T-k} \\ &\left(\frac{(1-\alpha-\omega)a^\alpha z^{\frac{1}{1-\alpha}}}{1-\beta(1-\rho)}+\gamma\frac{\omega a^\alpha z^{\frac{1}{1-\alpha}}}{1-\gamma\beta(1-\rho)}\right),\end{aligned}$$

If the wage in period 0 is at its steady state value $w_0 = \omega a^\alpha z^{\frac{1}{1-\alpha}}$, I have for $k < T$

$$\frac{\partial \pi_k^{e|u}}{\partial x}|_{x=1} =(\beta(1-\rho))^{T-k}\pi_k^{e|u}\frac{1-\eta}{\eta}\frac{1}{1-\alpha}\left(1+\frac{\omega\gamma}{1-\omega-\alpha}\frac{1-\beta(1-\rho)}{1-\gamma\beta(1-\rho)}\right)>0.$$

Note that $1-\omega-\alpha>0$ if $\pi_k^{e|u}>0$. The short-run employment effect is

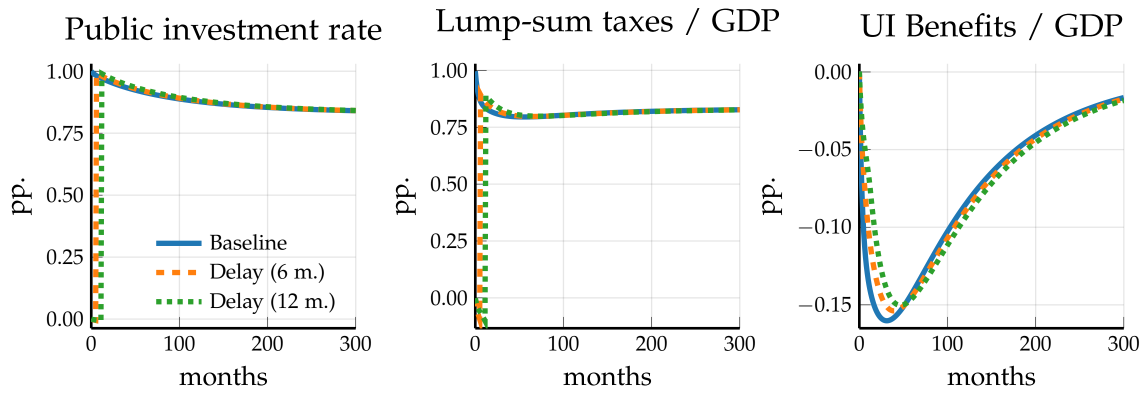
$$\begin{aligned}M_t^z(T,\mathcal{Y}_0,\mathcal{Z}) =&(1-N_0)\frac{\partial \pi_0^{e|u}}{\partial x} = (1-N_0)(\beta(1-\rho))^T\pi_0^{e|u}\frac{1-\eta}{\eta}\frac{1}{1-\alpha} \\ &\times\left(1+\frac{\omega\gamma}{1-\gamma\beta(1-\rho)}\frac{1-\beta(1-\rho)}{1-\omega-\alpha}\right)\end{aligned}$$

If the economy is at the steady state initially, then the employment effect is

$$\begin{aligned}M_t^z(T,\mathcal{Y}_0,\mathcal{Z}) &= \sum_{k=0}^{t-1}(1-\rho-\pi^{e|u})^{t-k-1}(1-N)\frac{\partial \pi_k^{e|u}}{\partial x} \\ &= (\beta(1-\rho))^T(1-N)\pi^{e|u}\frac{1}{1-\alpha}\frac{1-\eta}{\eta}(1-\rho-\pi^{e|u})^{t-1}\frac{1-((1-\rho-\pi^{e|u})\beta(1-\rho))^{-T}}{1-((1-\rho-\pi^{e|u})\beta(1-\rho))^{-1}} \\ &\times\left(1+\frac{\omega\gamma}{1-\gamma\beta(1-\rho)}\frac{1-\beta(1-\rho)}{1-\omega-\alpha}\right) \\ &= (\beta(1-\rho))^{T+1-t}(1-N)\pi^{e|u}\frac{1}{1-\alpha}\frac{1-\eta}{\eta}\frac{1-((1-\rho-\pi^{e|u})\beta(1-\rho))^t}{1-((1-\rho-\pi^{e|u})\beta(1-\rho))} \\ &\times\left(1+\frac{\omega\gamma}{1-\gamma\beta(1-\rho)}\frac{1-\beta(1-\rho)}{1-\omega-\alpha}\right).\end{aligned}$$

□

Figure 12: The fiscal response to the public investment expansion.

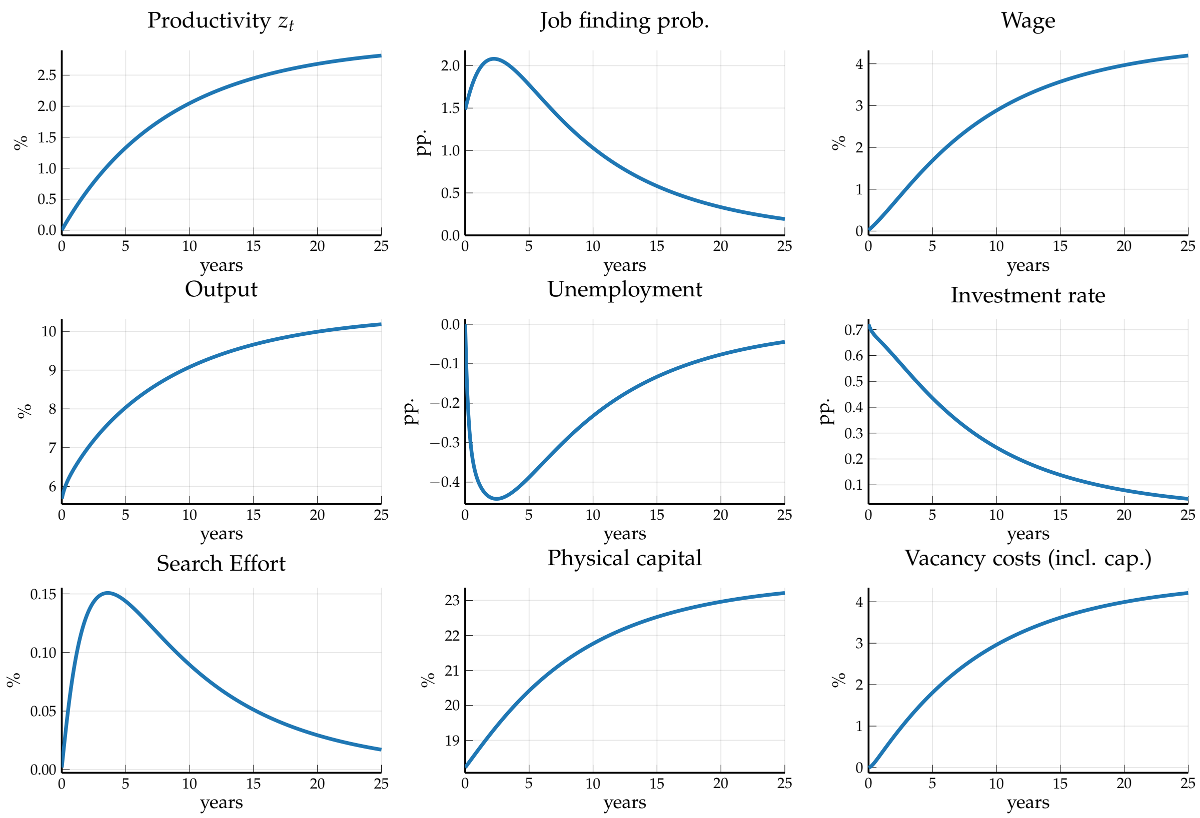



# E Additional Results

## E.1 Fiscal variables

## E.2 Long-run responses

Figure 13: Long-run responses to a government investment program.

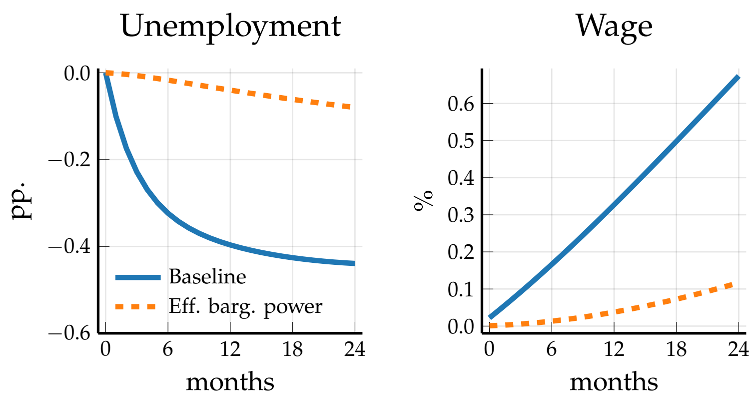



*Notes:* The solid blue line is the baseline calibration with constant benefits $b_t = b$ for all $t$. The dashed red line is for the case where benefits grow with labor productivity, $b_t = \tilde{b} z_t \tilde{k}_t^{\alpha}$.

# F Alternative Calibrations

For the baseline calibration I have chosen the bargaining power of workers $\psi$ such that the labor share is 64% as in the data. Alternatively, I could require that the bargaining power is such that vacancy creation is efficient in the steady state, i.e. the wage is given by (43). Note that the right term in (43) is zero in the steady state given our calibration strategy. The employment and wage response for a re-calibration of the model that requires workers bargaining power to implement efficient vacancy creation in steady state is shown by the dashed red line in Figure 14.

Figure 14: Response if bargaining power implements efficient vacancy creation in steady state.

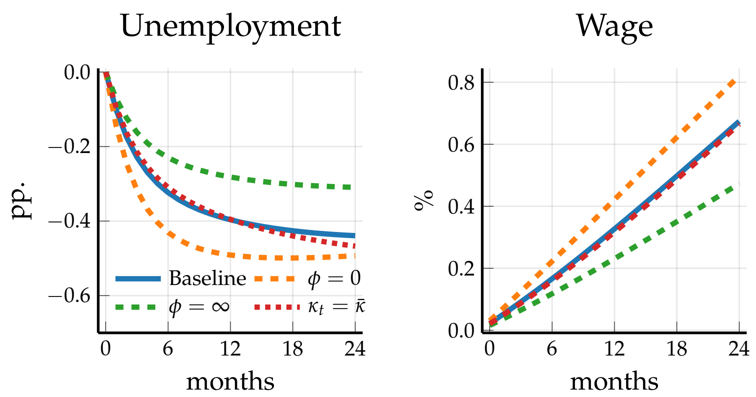



*Notes:* Dashed red line: response of unemployment for calibration where workers' bargaining power is chosen to implement efficient level of vacancy creation in steady state.

Our baseline specification assumes that posting costs are proportional to labor productivity. The dotted red line in Figure 15 shows the short-run response of unemployment and wages if posting costs are constant instead. The dashed orange line shows the responses when capital adjustment costs are zero. The dashed green line shows the responses when capital adjustment costs are infinite, i.e. the private capital stock is constant.

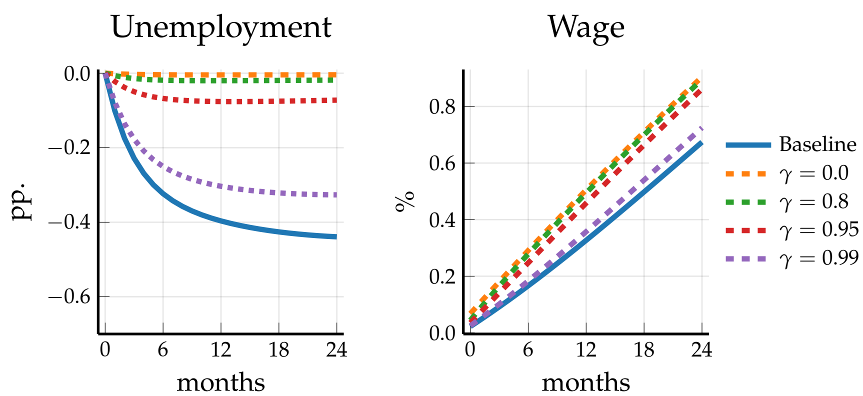
Figure 16 varies the degree of wage stickiness.

Figure 15: Responses without capital adjustment costs and with constant vacancy posting costs.




*Notes:* Dashed orange line: no capital adjustment costs. Dashed green line: infinite capital adjustment costs. Dotted red line: constant posting costs.

Figure 16: Responses for varying degrees of wage stickiness.




## F.1 State dependence for TFP induced recession and boom

In the main text, I study the state dependence of the employment effect of public investment considering a recession that results from a joint positive shock to the separation rate and the wage level (and vice-versa for a boom). Here, I alternatively consider a recession due to a negative shock to productivity of one standard deviation,

$$\log A_0 = -0.0056$$

and accordingly, for a boom

$$\log A_0 = +0.0056.$$

after which productivity $A_t$ then evolves according to (31).

Figure 17 show the response of TFP, unemployment, labor market tightness and wages.

Qualitatively, I obtain the same result as in the main text—the employment effect of public investment is larger in a recession.

Figure 17: Responses of variables to productivity shocks.

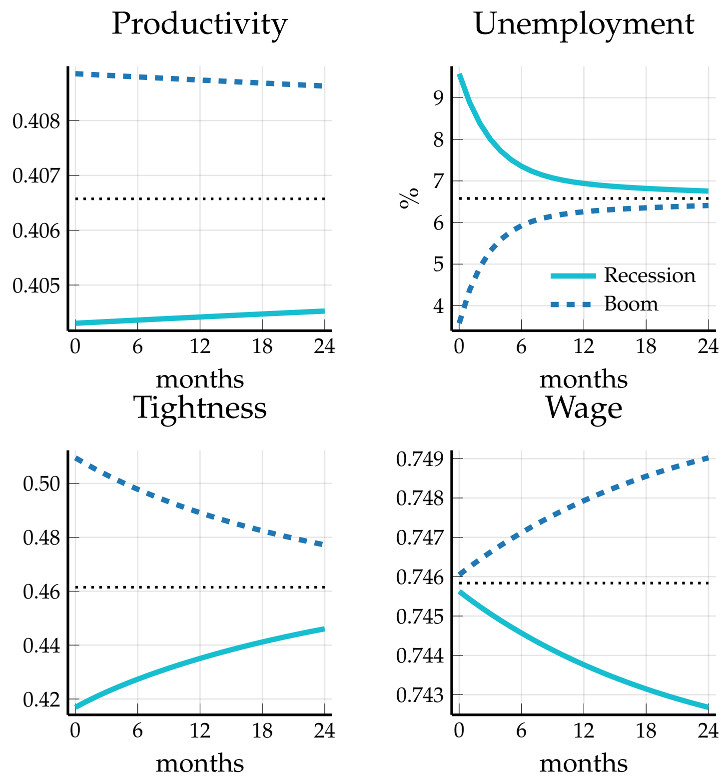
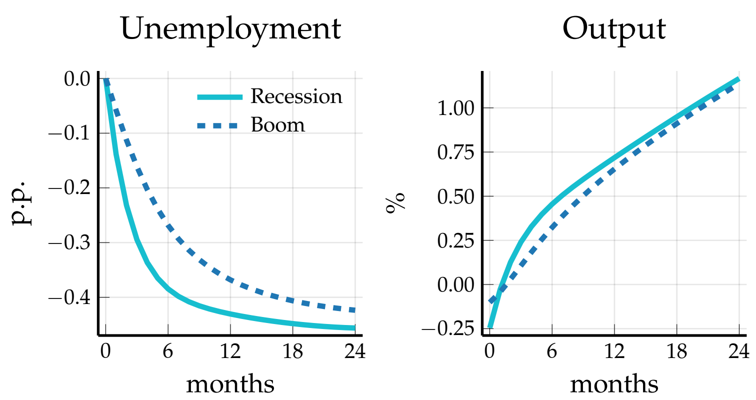



Figure 18: Responses unemployment and output to permanent expansion in public investment in recession and boom.




*Notes:* Shown are the deviations from the paths without an expansion in public investment (see Figure 17)