The Tipping Point: Low Rates and Financial Stability[a]

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Abstract

This paper develops a theory to study the effect of low interest rates on bank stability. While low rates reduce bank profits by compressing interest margins, they also boost the value of banks' long-term assets. The paper's main result is the characterization of a critical interest-rate level. Below this tipping point, the compression of interest margins dominates the revaluation of long-term assets, leading to insolvency and a banking crisis. The tipping point depends on observable bank characteristics. In a quantitative analysis, I find a value of 0.32% for the US economy in the decade before the Global Financial Crisis.

Keywords: Financial crisis, deposit franchise, duration gap, effective lower bound.

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1 Introduction

Interest rates that are low by historical standard have prevailed in advanced economies since the Global Financial Crisis (Holston et al., 2017; Del Negro et al., 2019). Moreover, low interest rates are predicted to define the macroeconomic environment of the future (Kiley and Roberts, 2017; Bernanke et al., 2019). Against this backdrop, policy-makers have expressed concerns about financial stability (Committee on the Global Financial System, 2018; European Central Bank, 2021). In particular, reduced profitability of financial institutions, which may heighten insolvency risk, is regularly attributed to the low-rate environment.

A recent empirical literature brings evidence for an adverse effect of low rates on bank stability. For example, Borio et al. (2017) and Claessens et al. (2018) look at banks' interest margins and find significant reductions associated with low interest rates. However, this evidence stands in contrast to a traditional view in Finance, which focuses on the duration gap between bank assets and liabilities as the key measure of banks' exposure to interest-rate risk (Kaufman, 1984; Basel Committee, 2016). According to this view, low rates are a boon for banks. Since bank assets typically have a long duration, low interest rates push up their value, benefiting banks. With a focus on this mechanism, Akinci et al. (2021) show theoretically that high rates, not low rates, endanger bank stability.

This paper's contribution is to study bank stability in a theoretical framework that features both mechanisms. Low rates lead to two countervailing effects: a compression in banks' interest margins and an increase in the value of banks' long-term assets.[1] The paper finds that low rates are a threat to bank stability. In this sense, concerns about financial stability in the current macroeconomic environment are warranted. However, the risk posed by low rates is mitigated by banks' positive duration gap. This finding is crystallized in a critical interest-rate level, which is decreasing in banks' duration gap. Below this tipping point, low rates lead to bank instability. A quantitative exercise, which uses data from the US economy in the decade before the Global Financial Crisis, finds a tipping point of 0.32%.

I base my study of bank stability on the canonical model developed in Diamond and Dybvig (1983). Banks are crisis-prone because they fund themselves with demandable deposits. If too many depositors withdraw at once, banks fail. In this setting, banking crises can be self-fulfilling or driven by bad fundamentals. Following Allen

[1] Drechsler et al. (2021) find empirical evidence for the co-existence of these two countervailing effects of interest-rate shocks on banks' net worth.

and Gale (1998), I assume that a lender of last resort forestalls panics and thus focus exclusively on fundamental-driven crises. In other words, in this paper banking crises only take place when banks are insolvent.

This canonical model is well-suited to study the effects of low rates because of two elements that naturally emerge in it. First, the model has a lower bound on the interest rate paid on bank deposits. Banks do not set deposit rates below such lower bound, lest they trigger large withdrawals. Second, banks earn an interest margin between the interest rate on their assets and the deposit rate in equilibrium. These are the two key ingredients of the standard mechanism of low rates: low rates reduce the profitability of banks because interest margins are compressed once pass-through to deposit rates is impaired. This mechanism is standard, in the sense that the literature agrees on it and starts from it to investigate the wider economic consequences of low rates on, for example, credit supply (Brunnermeier and Koby, 2019) and risk taking (Dell'Ariccia et al., 2014). In this paper's model, low rates jeopardize bank solvency via the standard mechanism. By compressing banks' interest margins, low rates erode banks' deposit franchise, i.e. the present discounted value of interest margins over the lifetime of deposits. Since the deposit franchise is a component of banks' net worth, net worth falls too.

Along with their deposit franchise, banks have another key characteristic: they hold long-term assets, which go up in value when the interest rate in the economy falls. Through this revaluation effect on long-term assets, low rates increase banks' net worth, bolstering solvency. To account for the latter effect, I generalize the canonical model. I extend the model's time horizon to infinity, so that banking crises do not only take place in the economy's last period, when the remaining duration of bank assets is at most one. In this fully dynamic model, the duration of bank assets can be generalized to any value.

A fall in the interest rate has two competing effects on bank solvency: it erodes the deposit franchise, but also represents a positive windfall for banks holding long-term assets. The paper's main result is the characterization of a critical interest-rate level. If the interest rate falls past it, then the former effect dominates the latter to the extent that banks are tipped into insolvency and a crisis ensues. This tipping point depends on characteristics of the banking system and has a simple analytical formula. It is increasing in banks' interest margins, since a high interest margin implies that deposit rates are near the lower bound. Thus, a relatively small reduction in interest rates is enough for pass-through to deposit rates to be impaired. It is decreasing in the duration of bank assets, since greater duration implies a stronger revaluation effect. Notably, both positive and negative values for the tipping point are possible.

The bank characteristics that determine the tipping point are either directly observable or can be estimated empirically. Therefore, I can use bank data and results from the empirical banking literature to quantify the tipping point. Using US data from the decade before the Global Financial Crisis, I find a value of 0.32%. A shock that reduces the nominal short-term risk-free interest rate permanently below 0.32% makes the banking system insolvent. This quantification has many limitations: banks are highly stylised in the model, real-world shocks do not hit the interest rate in a vacuum and are not permanent. Nevertheless, it illustrates the ease with which bank data and the empirical literature can be mapped into the parameters of the model. Future research can enrich the model to improve the credibility of the quantification.

Related literature. A large empirical literature has studied the adverse effect of low rates on bank profits. Using a comprehensive dataset of banks, Borio et al. (2017) find that the effect of interest rates on bank profitability is positive and stronger at lower levels of the interest rate. Claessens et al. (2018) find that the positive effect on banks' net interest margin of an interest-rate increase is twice as large when rates are low. Ampudia and van den Heuvel (2019) and Bats et al. (2020) document that in low-rate environments interest-rate increases lead to smaller reductions, and even increases, in bank valuations, particularly for banks more heavily funded with deposits.

The wider implications of low rates have attracted a flurry of papers. Many find that financial institutions react to low interest rates by lending to riskier counterparties (Maddaloni and Peydró, 2011; Dell'Ariccia et al., 2014; Jiménez et al., 2014; Di Maggio and Kacperczyk, 2017; Martinez-Miera and Repullo, 2017; Heider et al., 2019; Basten and Mariathasan, 2020). Others focus on the quantity of bank lending and investment in the economy (Amzallag et al., 2019; Brunnermeier and Koby, 2019; Eggertsson et al., 2019; Ulate, 2021; Altavilla et al., forthcoming). This paper abstracts from risk-taking and credit-supply considerations and contributes to the literature by focusing on bank stability.

In the footsteps of the seminal paper by Diamond and Dybvig (1983), I focus on bank instability that is inherent to the supply of liquid demandable deposits. I wish to study the effect of a real shock, an interest-rate shock, on the probability of a crisis. However, this exercise is complicated by the fact that the model features multiple equilibria.[2] Allen and Gale (1998) solves the issue by focusing exclusively on fundamental-driven crises, i.e. crises that cannot be solved merely with better

[2] A strand of the literature studies the effect of financial panics on economic outcomes, embracing equilibrium multiplicity (Gertler et al., 2016; Quadrini, 2017; Gertler et al., 2020).

coordination of depositors. An alternative option to pin down uniquely the conditions for a banking crisis is given by global-games techniques, as in Goldstein and Pauzner (2005). I adopt the former approach and justify it with the presence of a lender of last resort that rules out panics in equilibrium. Baron et al. (2021) show empirically that bank panics are not necessary for banking crises to have severe economic consequences.

Other papers focus on the relationship between interest rates and bank stability. Recently, Akinci et al. (2021) studied this question in a model that features rich macroeconomic dynamics in response to interest-rate shocks. In contrast to my results, they find that low rates are good for bank net worth, while high rates may bring about instability. Since banks do not have a deposit franchise, in their model low rates only have a revaluation effect on bank assets. Hellwig (1994) finds that issuance of demandable deposits exposes banks to excessive interest-rate risk relative to the socially optimal level. However, the paper does not model the long duration of banks' asset holdings. Hence, it does not feature the revaluation effect, which hedges the exposure of banks' deposit franchise to interest-rate risk.

Recent work has argued theoretically (Di Tella and Kurlat, 2021) and shown empirically (Drechsler et al., 2021) that banks are actually not as exposed to interest-rate risk as previously believed. Banks' net exposure is low because deposit franchise and long-term assets are exposed to interest-rate risk in opposite directions. This mechanism emerges clearly in my paper's model and plays an important role in the results about the resilience of banks to interest-rate shocks. In this sense, my paper contributes to the literature on banking crises by linking it to this recent view of banking. Importantly, this view implies that, as in my model, the deposit franchise is an important component of banks' net worth (Egan et al., 2021).

Mine is not the first paper in the literature to model banking crises in a fully dynamic economy. Some adopt an overlapping-generations framework (Bencivenga and Smith, 1991; Ennis and Keister, 2003). Others model infinitely-lived agents (Gertler and Kiyotaki, 2015; He and Manela, 2016; Segura and Suárez, 2017). The latter approach, which I also adopt, improves the match between model outcomes and data, and therefore allows for better calibration of the model.

Paper outline. In the next section, I describe preferences and technologies. In section 3, I illustrate the model's frictions and the agents' optimization problems. In section 4, I define equilibrium and solve for it. Section 5 studies the effects of an interest-rate shock. In section 6, I compare the model's outcomes to data and conduct a quantitative exercise. Proofs of propositions and corollaries are in appendix A. Appendix B contains an extension of the model.

2 Preferences and Technology

A unit mass of households inhabits the economy. They are born at date zero and are infinitely-lived. They enjoy consumption according to felicity function u, which features standard properties $u' > 0$ and $u'' < 0$ and a constant coefficient of relative risk aversion $1/\alpha > 1$. Each household enjoys consumption only at one point in time from date 1 on. The timing of a household's desire to consume is random. In particular, a household, which has not consumed already in the past, has a probability $\phi \in (0,1)$ of turning impatient and enjoying consumption at a given date t. Since the timing of households' desire to consume is idiosyncratic, by the law of large numbers the share of impatient households at any given date is deterministic. In particular, a share ϕ of those households that have not consumed yet turns impatient at a given date. The household's once-in-a-lifetime desire to consume can be interpreted as the random occurrence of an unusually large expense, which the household finances by withdrawing from its bank account. This type of utility was introduced by Diamond and Dybvig (1983), albeit with a finite horizon, and has become standard in the theoretical banking literature. The household's expected utility is given by

$$\sum_{t=1}^{+\infty} (1-\phi)^{t-1} \cdot \phi \cdot u(C_t). \tag{1}$$

The economy features a single good that can be consumed or invested. There are two investment technologies: the productive technology and the storage technology. For any good invested in it, the productive technology produces $1+\rho$ goods at the following date, with $\rho \in (-\phi, \phi/(1-\phi))$. Throughout the paper, I refer to ρ as the economy's interest rate, since by arbitrage it will also be the short-term return on bonds. The bounds on ρ ensure that, on the one hand, the interest rate is high enough for the bank not to be always insolvent and, on the other hand, that household utility is finite. It is noteworthy that the model can accommodate a negative as well as a positive interest rate. As for the storage technology, a stored good today gives one unit of the good tomorrow.

3 Economy

The economy features four frictions. First, households cannot insure each other directly against liquidity risk, i.e. the risk of becoming impatient relatively early and therefore having little time to accumulate wealth before consuming. This is the key reason for the existence of banks in this literature. The micro-foundation is that the individual liquidity shock is not publicly observable and therefore not contractible. Banks emerge as a way to insure liquidity risk by means of an incentive-compatible contract. Second, banks must offer a deposit contract as we observe them in reality. That is, deposits are not contingent on the state of the world and households have the right to withdraw their deposit balance unless the bank is bankrupt.[3] Third, households can only invest in their bank's deposits or in storage. Direct finance is ruled out. This can be justified, for instance, with households' inability to monitor firms. Moreover, the deposit contract is exclusive. That is, once a household has opened an account at one bank, it cannot switch to another one. This can be rationalized with a sufficiently high switching cost. Jacklin (1987) emphasizes that restrictions to households' investment opportunities are essential for banks to provide liquidity-risk insurance in this class of models. If households could costlessly invest directly or defect to another bank, then the insurance mechanism would unravel and with it the role of banks in the economy.[4] Fourth, banks do not invest directly in investment technologies. It is the role of firms to invest in the productive technology. Banks lend to firms in the form of long-term bonds with a fixed duration. This relationship between banks and firms is empirically relevant and has been theoretically justified with the presence of information asymmetries (Gale and Hellwig, 1985). It is worth stressing that there is no cost in this model associated with early liquidation of long-term bonds by banks. Appendix B shows that the introduction of a liquidation cost for long-term bonds does not change the paper's results.

The rest of this section describes the problems solved by the agents of the economy: a representative firm, a unit mass of households and a representative bank.

3.1 Representative firm

Taking the price of bonds q_t as given, the firm chooses how much to borrow B^f_{t+1} and how much to invest in the productive technology K_{t+1} in order to maximise the discounted value of profits

$$\sum_{t=0}^{+\infty} \left(\frac{1}{1+\rho} \right)^t \cdot \Pi_t \tag{2}$$

[3]As shown by Peck and Shell (2003), it is possible to relax these assumptions and still study banking crises. Nonetheless, I adopt the most standard set up.

[4]There is scope to relax this assumption with, for example, a cost for households to directly participate in financial markets (Diamond, 1997).

subject to budget constraints

$$\Pi_t + K_{t+1} + (1 + \delta \cdot q_t) \cdot B^f_t = q_t \cdot B^f_{t+1} + (1 + \rho) \cdot K_t \quad \text{for all } t \geq 0, \tag{3}$$

initial conditions

$$B^f_0 = K_0 = 0, \tag{4}$$

and a boundary condition[5]

$$\lim_{T \to +\infty} \left(\frac{1}{1 + \rho} \right)^T \cdot \left[K_{T+1} - q_{T+1} \cdot B^f_{T+1} \right] = 0. \tag{5}$$

For simplicity, bonds issued by the firm are perpetuities that pay a decaying coupon, as in Woodford (2001). A perpetuity sold at time t pays a coupon δ^{j-1} at every date $t+j$ for $j \geq 1$, with $\delta \leq 1$. This assumption allows us to study long-term assets while keeping the model tractable. Notice that B^f_t, a perpetuity issued at time $t-1$, has the same time-t value as δ perpetuities issued at time t. The duration of the firm's liabilities at the beginning of time t, defined as the average maturity of the firm's outstanding bonds B^f_t weighted by the present discounted value of the coupon paid at each maturity, is $\delta/(1 + \rho - \delta)$. I use this formula in the quantitative exercise in section 6.

Arbitrage by the firm between bonds and capital implies that $1+\rho = (1+\delta \cdot q^*_{t+1})/q^*_t$ in equilibrium. Together with a condition that rules out equilibrium bubbles

$$\lim_{T \to \infty} q^*_T \neq \pm\infty, \tag{6}$$

this pins down the price of new bonds as

$$q^*_t = \frac{1}{1 + \rho - \delta} \quad \text{for all } t \geq 0. \tag{7}$$

When the interest rate falls, the price of a new bond goes up.

3.2 Households and the representative bank's problem

At time 0, households give their endowment of goods to a bank in exchange for a deposit contract. The deposit contract specifies a contingent stream of deposit rates $\{r_t\}^{+\infty}_{t=0}$. Moreover, it allows households to withdraw at any point in time any amount up

[5]The boundary condition is given by the combination of a Ponzi condition and a transversality condition.

to their deposit balance D_t.[6] Patient households do not withdraw at time t, as long as the return on deposits r_t is larger than the zero return on storage. Households that turn impatient withdraw all their deposits immediately.

By perfect competition in the supply of deposits at date zero, the prevailing contract maximises households' expected utility

$$\sum_{t=1}^{+\infty}(1-\phi)^{t-1}\cdot\phi\cdot u(D_t). \tag{8}$$

In other words, the bank acts in the households' interest. At time zero, the household deposits its unit endowment, which implies

$$D_0 = 1. \tag{9}$$

Since the bank has no own resources, namely

$$B_0 = 0, \tag{10}$$

the bank buys bonds at time zero with the households' endowments. That is, the bank's time-zero budget constraint is

$$q_0 \cdot B_1 = D_0. \tag{11}$$

As long as the incentive-compatibility constraints

$$r_t \geq 0 \quad \text{for all } t \geq 1 \tag{12}$$

hold, households only withdraw once they turn impatient. Hence, the law of motion of patient households' deposit balances is given by

$$D_{t+1} = (1+r_t)\cdot D_t \quad \text{for all } t \geq 0, \tag{13}$$

the bank's budget constraints are given by

$$q_t \cdot B_{t+1} + \phi\cdot(1-\phi)^{t-1}\cdot D_t = (1+\delta\cdot q_t)\cdot B_t \quad \text{for all } t \geq 1, \tag{14}$$

[6]In principle, negative withdrawals are allowed. In equilibrium, they do not occur.

and the boundary condition is

$$\lim_{T \to +\infty} \left(\frac{1}{1+\rho} \right)^T \cdot q_T \cdot B_{T+1} = 0. \tag{15}$$

The bank's problem can be broken down in infinitely many subgames indexed by a starting time $j \geq 0$. Subgame 0 corresponds to the bank's problem as described above. In subgame $j > 0$, the bank maximises $\sum_{t=j}^{+\infty} (1-\phi)^{t-1} \cdot \phi \cdot u(D_t)$ subject to incentive-compatibility constraints (12), budget constraints (13) and (14), and the boundary condition (15). The initial conditions B_j and D_j are given. Subgames are useful because they clarify how state-contingency of the deposit contract works. If at a given time t the economy is hit by a shock, the bank's response to the shock solves subgame t. Importantly, the bank cannot immediately change the amount of outstanding deposits or the quantity of bonds it holds, because they are pinned down by initial conditions. It is free to respond to the shock by changing the deposit rate. A focus on subgames of the bank's problem is valid, because a solution to the bank's problem is also a solution to every subgame of the bank's problem, as we will see in the next section.

Bank failure. If there is no incentive-compatible deposit contract that is also feasible (i.e., it satisfies the bank's intertemporal budget constraint), then the bank fails. Failure is brought about by a mass deposit withdrawal by households.[7]

On the other hand, if the bank can offer a feasible and incentive-compatible deposit contract, then the bank is solvent and, I assume, it does not fail. With this assumption, I restrict the analysis to fundamental-driven failure, as in Allen and Gale (1998). A broader analysis, which encompassed panic-driven failure, would have a more demanding notion of feasibility. In the presence of panics, a bank is run-proof only if it can offer an incentive-compatible deposit contract that is feasible even in a scenario in which all households withdraw immediately. A justification for ignoring panics, while focusing on insolvency, is the ease with which public intervention can coordinate households away from such equilibria in this class of models. A lender of last resort committed to rescuing solvent banks in case of a panic eliminates equilibria with panics. Hence, solvent banks can be made run-proof costlessly. To the contrary, the rescue of an insolvent bank involves an equilibrium disbursement of resources. This disbursement is costly as it reduces resources available for alternative uses, such as the provision of public goods by the government (Allen et al., 2018).

[7] In case of failure the bank's assets are distributed to the withdrawing deposit-holders on a pro-rata basis, as specified in definition 1. This modelling choice is irrelevant for the paper's results.

Since this model features no participation constraint, as of time zero there is always a feasible and incentive-compatible deposit contract that the bank can offer.[8] For instance, it can offer a zero deposit rate at any time $t \geq 1$ and set r_0 such that the deposit contract is feasible. Deposit rate r_0 can be negative if necessary. This implies that in an equilibrium with perfect foresight, the bank never fails.

At the heart of the paper are the consequences of a shock hitting the economy at a later date $t \geq 1$. To study these, we must analyse subgame t of the bank's problem.

Proposition 1. *At time $t \geq 1$, the bank does not fail in equilibrium if and only if condition*

$$\frac{(1+\delta \cdot q_t) \cdot B_t}{(1-\phi)^{t-1} \cdot D_t} \geq \frac{\phi \cdot (1+\rho)}{\phi + \rho} \tag{16}$$

holds. We call condition (16) the solvency condition.

The solvency condition tells us whether the bank can offer a feasible and incentive-compatible deposit contract in subgame t, as a function of initial conditions B_t and D_t, price q_t and parameters. The relevant interest rate ρ is the one prevailing from time t on. Intuitively, the bank must hold enough bonds relative to its outstanding deposits. In particular, bond holdings must be enough for the bank to honour withdrawals until the infinite future while paying a zero deposit rate.

4 Equilibrium

In equilibrium, the representative firm and bank solve their optimization problems. As soon as they turn impatient, households withdraw their deposits. As long as they are patient, households do not withdraw unless storage offers a better return than deposits. The market for bonds clears.

Definition 1. *Equilibrium is a sequence $\{B_t^f, B_t, D_t, K_t, q_t, r_t, \Pi_t\}_{t=0}^{+\infty}$ such that:*

1. *Given $\{q_t\}_{t=0}^{+\infty}$, the representative firm chooses $\{B_t^f, K_t, \Pi_t\}_{t=0}^{+\infty}$ to maximise its value (2) subject to budget constraints (3), initial conditions (4) and boundary condition (5).*

2. *Given $\{q_t\}_{t=0}^{+\infty}$, the representative bank chooses $\{B_t, D_t, r_t\}_{t=0}^{+\infty}$ to solve every subgame of its problem of maximising household expected utility (8) subject to initial conditions (9) and (10), incentive-compatibility constraints (12), budget constraints (11), (13) and (14), and a boundary condition (15), if such solution exists.*

[8] Adding a participation constraint at time zero does not make a difference for the response of the model to a shock in the following periods, which is the focus of this paper.

3. *If a subgame starting at time t of the representative bank's problem does not admit a solution, then the bank sets* $B_{s+1} = 0$ *for all* $s \geq t$ *and immediately pays out* $(1+\delta \cdot q_t) \cdot B_t$ *to households in direct proportion to their deposit holdings.*

4. *Prices* $\{q_t\}_{t=0}^{+\infty}$ *ensure that* $B_{t+1}^f = B_{t+1}$ *for all* $t \geq 0$ *and are subject to no-bubble condition* (6).

I denote equilibrium values with stars.

I assume that the bank's resources are distributed to withdrawing households in proportion to their deposit holdings in case of bank failure. This assumption is immaterial for the paper's results. Hence, I choose it for its simplicity and its widespread use in the literature on banking crises, where it is known as pro-rata sharing (Allen and Gale, 1998).

Equilibrium outcomes are characterised by the following proposition.

Proposition 2. *Equilibrium implies that*

$$1 + r_t^* = \max\{1,\ (1+\rho)^{\alpha}\} \quad \textit{for all } t \geq 1, \tag{17}$$

$$\frac{(1+\delta \cdot q_t^*) \cdot B_t^*}{(1-\phi)^{t-1} \cdot D_t^*} = \max\left\{\frac{\phi \cdot (1+\rho)}{\phi + \rho},\ \frac{\phi \cdot (1+\rho)^{1-\alpha}}{(1+\rho)^{1-\alpha} - (1-\phi)}\right\} \quad \textit{for all } t \geq 1 \tag{18}$$

and q_t^* *is given by* (7).

On the equilibrium path, the incentive-compatibility constraints are not binding, unless the interest rate ρ is negative.[9] In case of negative ρ, the bank pays a zero deposit rate. Since incentive compatibility is never violated, bank failures do not occur in equilibrium. Indeed, looking at equation (18), we can confirm that the solvency condition (16) is always satisfied.

At time 1, the economy effectively reaches a steady state, in which important endogenous variables are stable over time. Both the deposit rate and the value of bank-held bonds per unit of outstanding deposits do not change. This is a surprising finding, since elements of the model, such as the number of impatient households in a given period, have a trend. This characteristic of the model helps in terms of analytical tractability. Moreover, it is easier to find empirical counterparts to objects that are stable over time.

[9] Equation (17) corresponds to the equation that determines the ratio between late types' consumption and early types' consumption in the three-date Diamond-Dybvig model.

4.1 Interpretation

In equilibrium, the bank's outstanding deposits $(1-\phi)^{t-1}\cdot D_t^*$ and the value of the bonds it holds $(1+\delta\cdot q_t^*)\cdot B_t^*$ generally do not match, except at time zero. However, the bank never pays any dividends. So, its equilibrium net worth, by definition the value of the bank to the residual claimant, is trivially zero at all times. This is puzzling if we adopt a naïve definition of net worth, given by the difference between holdings of securities and outstanding liabilities.[10]

Solving this puzzle requires us, as a first step, to define the bank's interest margin

$$1+m_t=\frac{1+\rho}{1+r_t}, \tag{19}$$

and to notice that in equilibrium at any date $t\geq 1$ the bank earns an interest margin $1+m_t^*=\min\left\{(1+\rho)^{1-\alpha},\ 1+\rho\right\}$. If the return ρ is positive, it earns a positive margin. If the return ρ is negative, then the incentive-compatibility constraints are binding and the bank earns a negative interest margin. In any case, the interest margin is generally not zero.

The presence of an interest margin implies a discrepancy between the face value of deposits, which is 1, and the economic cost for the bank of owing a unit of deposits. The latter is given by the expected discounted value of outflows of goods from the bank associated with a unit of deposits. Formally, we can define this discrepancy for time $t\geq 1$ as

$$f_t=1-\phi\cdot\sum_{s=t}^{+\infty}\frac{(1-\phi)^{s-t}}{\prod_{j=t}^{s-1}(1+m_j)}. \tag{20}$$

This value is the bank's per-unit deposit franchise. If there was a market for deposits between banks, a bank would not be willing to pay one good to another bank to get rid of a deposit liability with a face value of one. It would pay at most $1-f_t$ units. In equilibrium, the deposit franchise at time $t\geq 1$ is given by

$$f_t^*=\frac{1-\phi}{\phi+m_t^*}\cdot m_t^*, \tag{21}$$

since the interest margin is constant over time. Interestingly, this is approximately the expected time to withdrawal of a deposit multiplied times the interest margin.

The bank's net worth, which is the value of the bank for the residual claimant,

[10] This is the standard definition of bank net worth in the literature that focuses on the financial accelerator (Gertler and Kiyotaki, 2010).

takes into account the existence of the bank's deposit franchise. In fact, it is given by

$$N_t = (1 + \delta \cdot q_t) \cdot B_t - (1 - f_t) \cdot (1 - \phi)^{t-1} \cdot D_t. \tag{22}$$

A bank with a positive deposit franchise effectively has an additional asset, due to the profits associated with its deposit base. This definition of net worth is consistent with the puzzling equilibrium outcomes regarding the bank's balance sheet. The value of bank-held bonds is different from the bank's outstanding deposits. Yet, the bank's value to a residual claimant, that is its net worth, is zero. This is possible because the deposit franchise is a component of bank net worth. Indeed, if we use formula (21) to substitute the deposit franchise's equilibrium value in the definition of net worth (22) along with the equilibrium value of bank-held bonds per unit of outstanding deposits (18), we confirm that in equilibrium $N_t^* = 0$.[11]

The deposit-franchise interpretation of the model's equilibrium outcomes is confirmed by the fact that negative net worth is a necessary and sufficient condition for bank failure.

Proposition 3. *At time $t \geq 1$, the bank does not fail in equilibrium if and only if there exists at least one incentive-compatible deposit contract such that $N_t \geq 0$.*

Indeed, if there is no course of action that can make the bank's net worth positive other than paying a negative deposit rate, then the bank fails.

5 The Tipping Point

Consider an economy on the equilibrium path. Suppose the economy's short-term interest rate changes unexpectedly and permanently from ρ to $\hat{\rho}$ at a given time $t \geq 1$. Does the bank fail?

As we learnt from proposition 1, the solvency condition is necessary and sufficient for the survival of the bank. The bank survives the shock if and only if

$$\frac{(1 + \delta \cdot \hat{q}_t) \cdot B_t^*}{(1 - \phi)^{t-1} \cdot D_t^*} \geq \frac{\phi \cdot (1 + \hat{\rho})}{\phi + \hat{\rho}} \tag{23}$$

holds. The quantity of bonds and of outstanding deposits do not change in response to the shock, as these are predetermined variables. The shock has two effects. For

[11] That all of the bank's resources are paid out to depositing households in equilibrium is a common result in the literature on banking crises. It entails that the bank is worthless to a residual claimant. In other words, that the bank has zero net worth.

illustration, let us consider a fall in the interest rate. A revaluation effect increases the value of the bank's bonds by pushing up the price of new bonds from q_t^* to $\hat{q}_t = 1/(1+\hat{\rho}-\delta)$. This makes the condition likelier to hold and hence the bank likelier to survive the shock. On the other hand, the right-hand side of the inequality goes up. Since bonds have a lower return, the bank needs to hold more bonds per unit of outstanding deposits to make sure it can pay at least a zero deposit rate forever. This makes condition (23) less likely to hold and hence the bank likelier to fail as a result of the shock. Interestingly, the two effects are competing.

Proposition 3 offers another perspective on the effect of the shock on bank stability. We can look at the bank's net worth. The bank survives the shock as long as an incentive-compatible deposit contract $\{\hat{r}_j\}_{j=t}^{+\infty}$ exists such that

$$\hat{N}_t = (1+\delta\cdot\hat{q}_t)\cdot B_t^* - (1-\hat{f}_t)\cdot(1-\phi)^{t-1}\cdot D_t^* \geq 0, \tag{24}$$

where $\hat{f}_t = 1-\phi\cdot\sum_{j=t}^{+\infty}\left[(1-\phi)^{j-t}/\prod_{s=t}^{j-1}(1+\hat{m}_s)\right]$ and $1+\hat{m}_s = (1+\hat{\rho})/(1+\hat{r}_s)$. The first effect on $\hat{N}_t$ of a fall in the interest rate is the same revaluation of the price of bonds $\hat{q}_t$ as above. It directly increases net worth and therefore makes it likelier that an incentive-compatible deposit contract exists such that bank net worth is non-negative. The other effect runs through the deposit franchise $\hat{f}_t$. The deposit franchise is directly pushed down by the fall in the interest rate ρ, since it implies a lower interest margin ceteris paribus. The reduction is undone if the bank passes through the lower interest rate to households in the form of lower deposit rates. However, incentive compatibility sets a limit to this pass-through. For a sufficiently large fall in the interest rate, binding incentive-compatibility constraints make the deposit franchise fall. Hence, net worth falls too. The relative strength of the bond revaluation and of the deposit-franchise effect on net worth determines whether the bank survives.

The key finding of the paper is that a fall in the interest rate past a critical value tips the bank into failure. For this tipping point, there is an analytical solution.

Proposition 4. *Consider an economy on the equilibrium path with $\delta < 1-\phi$. Suppose the economy is hit at time $t \geq 1$ by a shock that changes the interest rate to $\hat{\rho} > -\phi$ permanently. The bank fails if and only if $\hat{\rho} < \underline{\rho}$, with*

$$\underline{\rho} = m_t^* - \delta\cdot\frac{(\rho - m_t^*)\cdot(\phi + m_t^*)}{(1-\phi)\cdot(1+\rho)-\delta\cdot(1+m_t^*)}. \tag{25}$$

I call the value $\underline{\rho}$ the tipping point.

If the bank holds bonds with zero duration (i.e., $\delta = 0$), then there is no revaluation effect. The tipping point is equal to the equilibrium interest margin. If the new interest rate $\hat{\rho}$ is smaller than the equilibrium interest margin m_t^*, then the interest margin falls since incentive-compatibility constraints do not allow the bank to fully pass through the lower interest rate to households. This implies a reduction in the deposit franchise. Since the bank's net worth along the equilibrium path before the shock is zero, any fall in the deposit franchise brings net worth into negative territory. A mass withdrawal of deposits ensues and the bank fails.

If the bank holds long-term bonds, a reduction in the interest rate results in a positive windfall for the bank through the revaluation effect. Thanks to this windfall, the bank can withstand a reduction in the deposit franchise without failing. A stronger revaluation effect (i.e., a larger δ) implies a lower tipping point. In other words, long-term bonds hedge the bank against the adverse effect on the deposit franchise of unanticipated reductions in the interest rate.

The tipping point exists, as described in proposition 4, in a region of the parameter space. The duration of bank-held bonds that is consistent with the existence of the tipping point has an upper bound, $\delta < 1 - \phi$. If $\delta \geq 1 - \phi$, then the revaluation effect is very strong. In this parametric case, I find that there is no risk of bank failure from a low interest rate. To the contrary, it is high rates of interest that may make banks insolvent. In the following two corollaries, I formally analyze the consequences of an interest-rate shock in this region of the parameter space. First, in an economy with a non-negative interest rate before the shock.

Corollary 1. *Consider an economy on the equilibrium path with $\rho \geq 0$. Suppose the economy is hit at time $t \geq 1$ by a shock that changes the interest rate to $\hat{\rho} > -\phi$ permanently. If $1 - \phi \leq \delta \leq (1 - \phi) \cdot (1 + \rho)^{\alpha}$, then for any $\hat{\rho}$ the bank does not fail. If $(1 - \phi) \cdot (1 + \rho)^{\alpha} \leq \delta < 1$, then the bank fails if and only if $\hat{\rho} > \underline{\rho}$, with $\underline{\rho}$ given by equation (25).*

Second, in an economy starting from a negative interest rate.

Corollary 2. *Consider an economy on the equilibrium path with $\rho < 0$. Suppose the economy is hit at time $t \geq 1$ by a shock that changes the interest rate to $\hat{\rho} > -\phi$ permanently. If $\delta = 1 - \phi$, then for any $\hat{\rho}$ the bank does not fail. If $1 - \phi < \delta < 1$, then the bank fails if and only if $\hat{\rho} > \underline{\rho}$, with $\underline{\rho}$ given by equation (25).*

I emphasize these theoretical results less than the result on the tipping point contained in proposition 4 for two reasons. First, the parametric condition $\delta \geq 1 - \phi$ implies maturities of bank-held assets that are one order of magnitude greater than what we observe in data, as shown in the next section. Second, the mechanism whereby high rates lead to bank failure in this parameter space is implausible. An unexpected increase in the interest rate reduces the value of bank assets so much that the bank reacts by cutting its deposit rate. It does so in order to boost its deposit franchise and remain solvent. This implies an unrealistic negative correlation of the interest rate and the deposit rate. The increase in the interest rate leads to insolvency if, due to incentive-compatibility constraints, the bank is unable to respond with a sufficiently large cut in the deposit rate.

6 Quantitative Exercise

In this section, I quantify the tipping point. I use bank data from the US economy in the decade before the Global Financial Crisis, as summarised in table 1. Subject to caveats about the stylized nature of the model, the result of this analysis answers the question: how low can the interest rate fall as a consequence of an unanticipated and permanent shock before banks fail?

The first of the model's variables that I match to an empirical counterpart is interest rate ρ. I take the fed funds rate to be the short-term and safe rate in the US economy. Its average value in the period from 1997Q3 to 2007Q2 was 3.81%.[12]

Second, the deposit rate is an important endogenous object of the model and it can be observed. According to the US Call Reports, the average interest rate paid by the commercial banking sector on core deposits, the sum of checking, savings, and small time deposits, in the period 1997Q3-2007Q2 is 2.39%.

Third, the parameter δ, which regulates the speed at which the perpetuity's coupon decays, is linked to the duration of bank assets. In fact, the average maturity of bank assets weighted by the present value of coupons, the definition of duration, is given by $\delta/(1+\rho-\delta)$, as discussed in section 3. According to English et al. (2018), who use data from the US Call Reports, the average repricing time of bank-held assets in the period 1997Q3-2007Q2 is 4.46 years. I use this as a proxy for bank-asset duration and carry out a robustness analysis later in the text.

The fourth and least immediate of the model's variables with an empirical counterpart is the bank's deposit franchise. Sheehan (2013) focuses directly on estimating the

[12] The interest rate ρ is real and the fed funds rate is nominal. In principle, the empirical counterpart is the real return on fed funds. However, the storage technology, which stands for currency in the model, should then offer a real return given by the negative of the rate of inflation. With both changes in place, the resulting tipping point is the same.

Table 1: Data.

Model	Empirical counterpart	Value	Source
ρ	Average fed funds rate	3.81%	FRED
r^*	Average interest rate on core deposits	2.39%	US Call Reports
$\delta/(1+\rho-\delta)$	Average bank-asset repricing time (years)	4.46	English et al. (2018)
f^*	Average per-unit deposit franchise	20.2%	Sheehan (2013)

Table 2: Model parameters.

Parameter	Description	Value
ρ	Short-term interest rate	3.81%
$1/\alpha$	Coefficient of relative risk aversion	1.58
δ	Common ratio of coupons' progression	84.8%
ϕ	Household's probability of turning impatient	5.13%

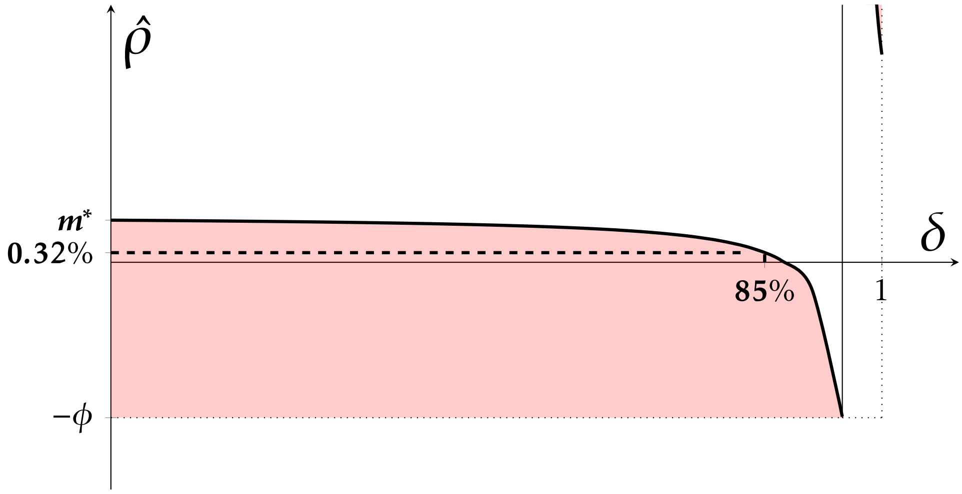
deposit franchise. Exploiting data from large US commercial banks on deposit balances and deposit rates starting in December 1996 and ending in December 2003, the paper finds that deposit franchises are large. The average value across deposit types and banks is 20.2%. This means that a bank would pay at most 79.8 cents to free itself of liabilities in the form of core deposits with a face value of one dollar.

The model contains four parameters and in this exercise I match four variables from the data. Hence, I identify all of the parameters uniquely and report the results in table 2. Noticeably, the resulting coefficient of relative risk aversion is well within the range commonly used in calibrated models and supported by micro-econometric evidence (Mehra and Prescott, 1985). As for the households' probability of turning impatient, the only other attempt in the literature to calibrate it within a similar model, with the exception of purely illustrative exercises, is in Mattana and Panetti (2021). They set the parameter to match the quantity of liquid assets held by banks and find a value of 2%, which is within the same order of magnitude as my finding.

With all parameter values in place, I can check the parametric condition for the existence of a tipping point $\delta < 1 - \phi$, as specified in proposition 4. It is amply satisfied. In particular, the absence of a tipping point would imply either a duration of bank assets of 18.5 years or a deposit franchise of 7.09% of deposits' face value. Both are one order of magnitude away from the empirical evidence.

The quantified tipping point is 0.32%. Taken at face value, this means that the banking sector can withstand a permanent fall in the interest rate down to this level. Past this level, the windfall from the revaluation effect is insufficient to compensate for

Figure 1: Robustness with respect to δ.



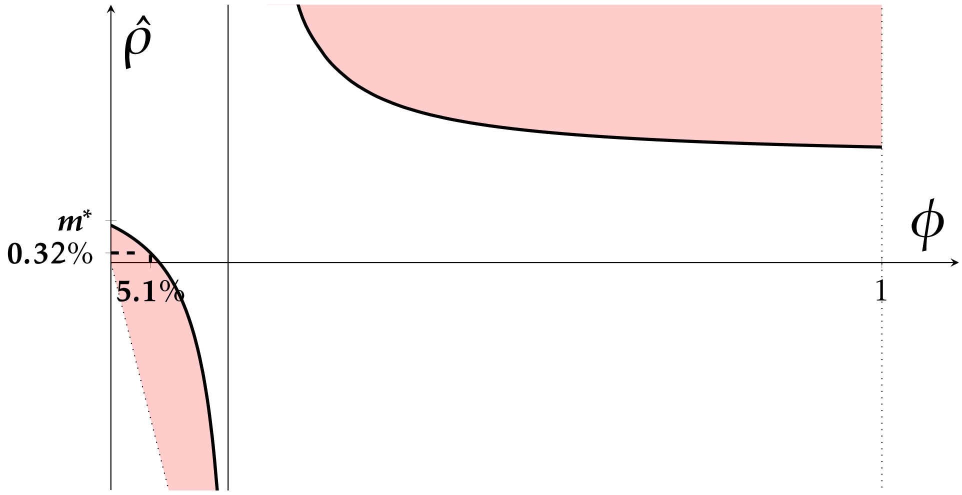
Note: In the red-shaded region $(\delta, \hat{\rho})$, the bank fails. The vertical line represents parametric condition $\delta = 1 - \phi$. To the left of the vertical line, the solid line represents the tipping point.

the reduction in the interest margin and the banking sector fails.

Robustness. A robustness analysis is warranted with regard to values for δ, since the average bank-asset repricing time is only a proxy for bank-asset duration. In fact, it is likely to overestimate the true average duration of bank assets, since prepayment is common for mortgages. Figure 1 shows how the tipping point, tracked by the solid line, varies with different values for δ. In the region shaded in red, the bank fails. A δ of zero implies no revaluation effect and therefore a tipping point equal to the equilibrium interest margin. As δ increases, the tipping point goes down, since the revaluation effect becomes stronger. A tipping point of 1% implies average bank-asset maturity of 2.26 years. A tipping point of zero implies average bank-asset maturity of 6.90 years. For very large values of δ, the tipping point is inverted on account of the very strong revaluation effect, as predicted by corollary 1. In this region, the average bank-asset maturity is at least 18.5 years, which is far from the value in the data.

As already discussed, there is little guidance in the literature on a value for ϕ and the empirical literature that quantifies the deposit franchise is very small. Hence, a robustness analysis is in order with respect to the value that we use. Figure 2 shows how the tipping point, tracked by the solid line, varies with different values for ϕ. In the region shaded in red, the bank fails. The tipping point goes down as ϕ increases. This is because a large ϕ, which means deposits are short-lived, implies a small deposit franchise. In turn, a small deposit franchise implies that a fall in the interest rate, which

Figure 2: Robustness with respect to ϕ.



Note: In the red-shaded region $(\phi, \hat{\rho})$, the bank fails. The vertical line represents parametric condition $\delta = 1 - \phi$. To the left of the vertical line, the solid line represents the tipping point.

compresses interest margins, has a small adverse impact on net worth. Hence, only large negative interest-rate shocks, which strongly compress interest margins, can make the bank insolvent. A tipping point of 1% implies ϕ equal to 2.26% and a corresponding deposit franchise of 37.2% of deposits' face value. A tipping point of zero requires ϕ equal to 7.71% and a corresponding deposit franchise of 14.1% of deposits' face value. If ϕ is large enough, then the deposit-franchise effect is so weak relative to the revaluation effect that there is no tipping point, as predicted by corollary 1. For this to be the case, ϕ must be at least 15.2%, implying a deposit franchise of 7.09% of deposits' face value.

7 Conclusion

Unusually low rates of interest have persisted in advanced economies since the Global Financial Crisis (Holston et al., 2017; Del Negro et al., 2019), drawing attention to the effects of interest-rate levels on economic outcomes. This paper studies the effect of the level of interest rates on bank stability. It marries a tradition in finance, which focuses on the duration gap between bank assets and bank liabilities and thus sees high interest rates as dangerous for banks (Kaufman, 1984), with a more recent view that sees low rates as a threat to bank profitability (Borio et al., 2017). Both effects are at play in this paper's model: a fall in the interest rate increases banks' net worth by increasing the value of bank assets, but also compresses banks' interest margins once deposit rates hit a lower bound. The main finding is that the relative strength of these countervailing effects determines a critical interest-rate level below which banks are tipped into insolvency. This implies that the more recent view is right: low rates are the key menace to the stability of the banking sector. Nonetheless, the duration gap plays an important role in softening the effects of low rates on banks. In fact, a larger duration gap pushes the critical interest-rate level down.

The insurance role of banks' duration gap speaks to the long-standing question in Finance of why deposit taking and long-term lending are conducted under one roof (Kashyap et al., 2002). It is because long-term assets hedge the risk that low interest rates pose to bank profitability, as also found empirically in Drechsler et al. (2021). In fact, this paper's model turns the question on its head: why is the duration gap of banks not large enough to fully hedge interest-rate risk? This question is left to future research. According to the quantitative analysis in this paper, full insulation from interest-rate risk would require an increase in bank-asset duration by a factor of four.

The analysis is conducted through the lens of the canonical model of bank stability developed in Diamond and Dybvig (1983) with time horizon extended to infinity. The extension allows for the introduction of long-term assets into the model in a tractable way. Moreover, the extension has the merit of bringing out in the equations the deposit franchise as the measure of bank profitability that matters for bank stability. The deposit franchise is the value to a bank of its deposit base. It is the product of the bank's average interest margin over time, the expected lifetime of a deposit and total deposits. On its own terms, this insight represents a contribution to the literature. Also, it is a useful connection of model parameters to a bank characteristic, which is not directly observable but is estimated in the empirical banking literature. I use estimates of banks' deposit franchise to discipline the paper's quantitative exercise.

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A Proofs

Proof of Proposition 1. Consider a subgame of the bank's problem starting at time $t \geq 1$. Using budget constraints (13) and (14), boundary condition (15) and anticipating equilibrium condition (7), I can write the intertemporal budget constraint

$$\phi \cdot \sum_{s=0}^{+\infty} \left(\frac{1}{1+\rho} \right)^s \cdot (1-\phi)^{t-1+s} \cdot \prod_{j=0}^{s-1} (1 + r_{t+j}) \cdot D_t = (1 + \delta \cdot q_t) \cdot B_t. \tag{26}$$

Rearranging, I obtain equation

$$\phi \cdot \sum_{s=0}^{+\infty} \left(\frac{1-\phi}{1+\rho} \right)^s \cdot \prod_{j=0}^{s-1} (1 + r_{t+j}) = \frac{(1 + \delta \cdot q_t) \cdot B_t}{(1-\phi)^{t-1} \cdot D_t} \tag{27}$$

that determines feasible deposit contracts $\left\{ r_{t+j} \right\}_{j=0}^{+\infty}$ as a function of initial conditions B_t and D_t, of price q_t and of parameters.

First, I prove the "if" part of the proposition. For this, we verify that a deposit contract that satisfies incentive-compatibility constraints (12) and solves equation (27), where we impose initial conditions satisfying condition (16), exists. Notice that for a deposit contract that pays $r_{t+j} = 0$ for all $j \geq 0$, the left-hand side of equation (27) is equal to $\phi \cdot (1+\rho)/(\phi + \rho)$. Moreover, the left-hand side of the equation is increasing in r_{t+j} and there is no upper bound on r_{t+j}.

Second, I prove the "only if" part of the proposition. This is equivalent to proving that the bank fails if condition (16) does not hold. The left-hand side of equation (27) is equal to $\phi \cdot (1+\rho)/(\phi+\rho)$ when the incentive-compatibility constraint is binding at every date with $r_{t+j} = 0$ for all $j \geq 0$. Since the left-hand side of equation (27) is increasing in r_{t+j}, there is no deposit contract that is both incentive-compatible and feasible if condition (16) does not hold. Hence, the bank fails. □

Proof of Proposition 2. Before I turn to the bank's problem, I look at the firm's problem to pin down equilibrium q_t^*. Arbitrage by the firm implies $1+\rho = (1+\delta \cdot q_{t+1}^*)/q_t^*$. The only sequence that satisfies this condition and no-bubble condition (6) is given by equation (7).

Consider the case with $\rho \geq 0$. First, I solve the bank's full problem starting at time zero. Then, I verify that the solution is also a solution in every subgame of the problem. Using initial conditions (9) and (10), budget constraints (11) and (14), boundary condition

(15), and the equilibrium condition (7), I can write the intertemporal budget constraint

$$\phi \cdot \sum_{t=1}^{+\infty} \left(\frac{1}{1+\rho} \right)^{t} \cdot (1-\phi)^{t-1} \cdot D_t = 1. \tag{28}$$

Maximizing the objective function (8) subject to constraint (28) with respect to choice variables $\{B_{t+1}, D_{t+1}\}_{t=0}^{+\infty}$, I obtain a set of optimality conditions. Once combined with (13), they can be written as

$$1 + r_0^* = (1+\rho)^{\alpha} \cdot \frac{(1+\rho)^{1-\alpha} - (1-\phi)}{\phi} \tag{29}$$

and $1 + r_t^* = (1+\rho)^{\alpha}$ for all $t \geq 1$. Notice that along the optimal path the incentive-compatibility constraints (12) are always slack for $\rho \geq 0$. Hence, I can safely ignore them in this case. Re-arranging and combining initial conditions (9) and (10), budget constraints (11), (13) and (14), and the equilibrium condition (7), I can write a law of motion for the variable of interest given by

$$\frac{(1+\delta \cdot q_1) \cdot B_1}{(1-\phi)^0 \cdot D_1} = \frac{1+\rho}{1+r_0}, \tag{30}$$

$$\frac{(1+\delta \cdot q_{t+1}) \cdot B_{t+1}}{(1-\phi)^t \cdot D_{t+1}} = \frac{1+\rho}{(1-\phi) \cdot (1+r_t)} \cdot \left[\frac{(1+\delta \cdot q_t) \cdot B_t}{(1-\phi)^{t-1} \cdot D_t} - \phi \right] \quad \text{for all } t \geq 1. \tag{31}$$

Substituting in the optimal path $\{r_t^*\}_{t=0}^{+\infty}$, I confirm

$$\frac{(1+\delta \cdot q_t^*) \cdot B_t^*}{(1-\phi)^{t-1} \cdot D_t^*} = \frac{\phi \cdot (1+\rho)^{1-\alpha}}{(1+\rho)^{1-\alpha} - (1-\phi)} \quad \text{for all } t \geq 1. \tag{32}$$

Finally, to verify that the solution above is also a solution to every subgame of the problem, take a subgame starting at time $t \geq 1$ with initial conditions B_t and D_t that satisfy condition (32). The bank's optimality conditions imply that $1 + r_j^* = (1+\rho)^{\alpha}$ for all $j \geq t$. This proves the proposition in the case with $\rho \geq 0$.
Consider $\rho < 0$. The optimal path for the deposit rate described above is not incentive-compatible in this case. The bank sets $r_t^* = 0$ for all $t \geq 0$ and

$$1 + r_0^* = \frac{\phi + \rho}{\phi}. \tag{33}$$

Substituting $\{r_t^*\}_{t=0}^{+\infty}$ in equations (30) and (31), I confirm that

$$\frac{(1+\delta \cdot q_t^*) \cdot B_t^*}{(1-\phi)^{t-1} \cdot D_t^*} = \frac{\phi \cdot (1+\rho)}{\phi + \rho} \quad \text{for all } t \geq 1. \tag{34}$$

Finally, to verify that the solution above is also a solution to every subgame of the problem, take a subgame starting at time $t \geq 1$ with initial conditions B_t and D_t that satisfy condition (34). The bank's optimality conditions imply that $r_j^* = 0$ for all $j \geq t$. □

Proof of Proposition 3. Using equations (19) and (20), it can be shown that, under an incentive-compatible deposit contract with $r_t \geq 0$ for all $t \geq 1$, there is a maximum value to the deposit franchise

$$f_t^0 = 1 - \frac{\phi \cdot (1+\rho)}{\phi + \rho}. \tag{35}$$

Re-arranging equation (22) and using the result above, we can characterize the maximum value of the bank's net worth

$$N_t^0 = (1-\phi)^{t-1} \cdot D_t \cdot \left[\frac{(1+\delta \cdot q_t) \cdot B_t}{(1-\phi)^{t-1} \cdot D_t} - \frac{\phi \cdot (1+\rho)}{\phi + \rho} \right], \tag{36}$$

given initial conditions B_t, D_t and the bond price q_t.

Thanks to proposition 1, we can focus on proving that there exists at least one incentive-compatible deposit contract such that $N_t \geq 0$ if and only if solvency condition (16) holds. First, if condition (16) holds, then $N_t^0 \geq 0$. This proves the "if" part of the proposition. Second, if condition (16) does not hold, then $N_t^0 < 0$. This means that there exists no incentive-compatible deposit contract such that $N_t \geq 0$ and proves the "only if" part of the proposition. □

Proof of Proposition 4. Consider a subgame of the bank's problem starting at time $t \geq 1$. The interest rate is $\hat{\rho} > -\phi$. Using proposition 1, we can conclude that the bank does not fail as long as

$$\frac{(1+\delta \cdot \hat{q}_t) B_t}{(1-\phi)^{t-1} \cdot D_t} \geq \frac{\phi \cdot (1+\hat{\rho})}{\phi + \hat{\rho}} \tag{37}$$

with $\hat{q}_t = 1/(1+\hat{\rho}-\delta)$. Since the economy was running along its equilibrium path before the time-t shock that changed the interest rate to $\hat{\rho}$, the initial conditions B_t and D_t satisfy

$$\frac{B_t}{(1-\phi)^{t-1} \cdot D_t} = (1-f_t^*) \cdot \frac{1+\rho-\delta}{1+\rho}, \tag{38}$$

as per proposition 2 combined with equations (19) and (20). Subbing in these initial conditions, we can write the necessary and sufficient condition under which the bank does not fail as

$$(1 - f_t^*) \cdot \frac{1+\rho-\delta}{1+\rho} \geq \frac{\phi \cdot (1+\hat{\rho}-\delta)}{\phi+\hat{\rho}}. \tag{39}$$

The left-hand side of equation (39) is not a function of $\hat{\rho}$ and is strictly larger than ϕ for parameters within the valid parameter space $\rho \in (-\phi, \phi/(1-\phi))$ and $\delta < 1-\phi$. The right-hand side is continuous, tends to infinity for $\hat{\rho} \to \phi^+$ and tends to ϕ for $\hat{\rho} \to +\infty$. By the intermediate value theorem, there is at least one point $\underline{\rho}$ at which left-hand side and right-hand side are equal. Since the right-hand side is strictly decreasing in $\hat{\rho}$, $\underline{\rho}$ is unique. To the left of $\underline{\rho}$ the right-hand side is larger than the left-hand side of the equation. Hence, the bank fails. To the right of $\underline{\rho}$, the bank does not fail. Solving for $\hat{\rho} = \underline{\rho}$ such that the left-hand side and the right-hand side are equal and substituting in equation (20) gives

$$\underline{\rho} = m_t^* - \delta \cdot \frac{(\rho - m_t^*) \cdot (\phi + m_t^*)}{(1-\phi)\cdot(1+\rho) - \delta \cdot (1+m_t^*)}. \tag{25}$$

□

Proof of Corollary 1. Regardless of the different parametric restriction on δ relative to proposition 4, the necessary and sufficient condition for the bank not to fail is still (39). Restricting our focus on $\rho \geq 0$, it is easy to substitute in f_t^* and obtain

$$\frac{\phi \cdot (1+\rho)^{1-\alpha}}{(1+\rho)^{1-\alpha} - (1-\phi)} \cdot \frac{1+\rho-\delta}{1+\rho} \geq \frac{\phi \cdot (1+\hat{\rho}-\delta)}{\phi+\hat{\rho}}. \tag{40}$$

If I study the left-hand side of the inequality, I notice that it is larger than ϕ for $1-\phi \leq \delta \leq (1-\phi)\cdot(1+\rho)^\alpha$. It is strictly smaller than ϕ for $(1-\phi)\cdot(1+\rho)^\alpha < \delta < 1$. If I study the right-hand side for $\delta \geq 1-\phi$, I find that it is continuous, it tends to minus infinity for $\hat{\rho} \to \phi^+$ and tends to ϕ^- for $\hat{\rho} \to +\infty$. Moreover, it is strictly monotonically increasing. This implies that the right-hand side is smaller than the left-hand side for any $\hat{\rho}$, whenever $1-\phi \leq \delta \leq (1-\phi)\cdot(1+\rho)^\alpha$. Hence, there is no shock such that the bank fails in this case. For $(1-\phi)\cdot(1+\rho)^\alpha < \delta < 1$, there is a unique $\underline{\rho}$ at which right-hand side and left-hand side are equal. For $\hat{\rho} \leq \underline{\rho}$, the left-hand side is larger than the right-hand side. Hence, the bank does not fail. For $\hat{\rho} > \underline{\rho}$, the bank fails. □

Proof of Corollary 2. Regardless of the different parametric restriction on δ relative to proposition 4, the necessary and sufficient condition for the bank not to fail is still (39).

Restricting our focus on $\rho < 0$, it is easy to substitute in f_t^* and obtain

$$\frac{\phi \cdot (1+\rho)}{\phi+\rho} \cdot \frac{1+\rho-\delta}{1+\rho} \geq \frac{\phi \cdot (1+\hat{\rho}-\delta)}{\phi+\hat{\rho}}. \tag{41}$$

If I study the left-hand side of the inequality, I notice that it is equal to ϕ for $\delta = 1-\phi$. For $1-\phi < \delta < 1$, the left-hand side is strictly smaller than ϕ.If I study the right-hand side for $\delta \geq 1-\phi$, I find that it is continuous, it tends to minus infinity for $\hat{\rho} \to \phi^+$ and tends to ϕ^- for $\hat{\rho} \to +\infty$. Moreover, it is strictly monotonically increasing. This implies that the right-hand side is smaller than the left-hand side for any $\hat{\rho}$, whenever $\delta = 1-\phi$. Hence, there is no shock such that the bank fails in this case. For $(1-\phi) < \delta < 1$, there is a unique $\underline{\rho}$ at which right-hand side and left-hand side are equal. For $\hat{\rho} \leq \underline{\rho}$, the left-hand side is larger than the right-hand side. Hence, the bank does not fail. For $\hat{\rho} > \underline{\rho}$, the bank fails. □

B Asset liquidation

The assumption that the bank can sell any amount of bonds frictionlessly is innocuous for the paper's results. In this section, I confirm this.

At any given time t, the bank holds the equivalent of B_t new bonds.[13] Hence, it receives coupons amounting to B_t. Households withdraw in total $\phi \cdot (1-\phi)^{t-1} \cdot D_t$. The bank sells bonds if the coupon is insufficient to cover the withdrawals.

Definition 2. *If at time t*

$$\frac{B_t}{(1-\phi)^{t-1} \cdot D_t} < \phi, \tag{42}$$

a bank sells bonds.

Whether a bank sells bonds is per se irrelevant for equilibrium outcomes in this paper's model, since bond selling is frictionless. Nonetheless, it is in principle interesting to introduce a liquidation cost in the bond market and study its effect on economic outcomes. This friction is theoretically compelling as a result of information asymmetries (Eisfeldt, 2004) and is emphasized in the literature on financial crises. For example, Diamond and Dybvig (1983) posit that assets have a higher per-period return if held for two periods rather than liquidated after one period, reflecting a liquidation cost.

[13] The bank may not actually hold only new bonds but also older vintages of bonds. What matters is the new-bond-equivalent quantity of bonds it holds. For example, it may hold a bond issued at time $t-2$. This pays a coupon of δ at time t and is equivalent to δ new bonds issued at time $t-1$, as explained in section 3.

I find that the introduction of a liquidation cost changes none of the results of this paper. Asset liquidation by the bank never happens, unless the bank is insolvent. In other words, the bank sells its bonds exclusively as a consequence of failure. It cannot become insolvent because of the poor terms at which it must sell bonds. Hence, the size of the shock that makes the bank insolvent does not depend on the liquidation cost.

Proposition 5. *Consider an economy with $\delta \leq 1 - \phi$. If in equilibrium the bank does not fail at time $t \geq 1$, then the bank does not sell bonds at any time $s \geq t$.*

Proof. By definition 1, if a bank does not fail at time $t \geq 1$, there exists a sequence $\left\{B_{j+1}, D_{j+1}, r_j\right\}_{j=t}^{+\infty}$ that, given initial conditions B_t and D_t, satisfies incentive-compatibility constraints (12), budget constraints (13) and (14), and the boundary condition (15). The subsequence $\left\{B_{j+1}, D_{j+1}, r_j\right\}_{j=s}^{+\infty}$ for $s \geq t$, given initial conditions B_s and D_s belonging to the above sequence, also satisfies incentive-compatibility constraints (12), budget constraints (13) and (14), and the boundary condition (15). Hence, there exists a solution to the subgame of the bank's problem starting at any time $s \geq t$. Again by definition 1, this implies the bank does not fail at any time $s \geq t$. Hence, by proposition 1 we have that the solvency condition

$$\frac{(1 + \delta \cdot q_s) \cdot B_s}{(1 - \phi)^{s-1} \cdot D_s} \geq \frac{\phi \cdot (1 + \rho)}{\phi + \rho} \tag{43}$$

holds for any $s \geq t$. Using equation (7) to substitute out q_s, we can verify that, under parametric condition $\delta \leq 1 - \phi$, the solvency condition contradicts inequality (42) in every period. □

Take the perfect-foresight equilibrium path described in proposition 2. This is the path the economy takes in equilibrium if it is never perturbed by a shock from time 0 on. Along this path the bank never fails. As long as the duration of bonds is not too long as captured by parametric restriction $\delta \leq 1 - \phi$, on this path the bank never sells any bonds. It meets withdrawals entirely with the coupons at every point in time. It follows that a liquidation cost would play no role in the economy's equilibrium outcomes. The above proposition is more general: even if an economy is hit by shocks, the bank never sells any bonds along the equilibrium path as long as it survives the shocks. In other words, a solvent bank never sells its bonds.

The parametric condition required for asset liquidation by solvent banks not to occur is implied by the parametric condition necessary for the existence of the tipping point, as specified in proposition 4. The duration of bonds cannot be too long. I discuss theoretical and quantitative reasons supporting this parametric condition in section 5.