Foreign Demand of U.S. Treasury Securities

and the Zero-Lower Bound on Monetary Policy

An Empirical Investigation of Structural Breaks on the U.S. Long-term Interest Rate[1]

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Abstract

Following the literature on the long-term interest rate, we investigate the potential structural breaks in the impact of changes in foreign demand of U.S. Treasury notes and bonds on the U.S. long-term rates. Our sample covers the most recent experience with monetary policy at the zero lower bound (ZLB) and the deployment of unconventional policies (like quantitative easing, QE). Using a battery of stability tests, we find that the end of 2008 is a robust breakpoint. Based on a threshold single equation error correction model with federal funds rate as the threshold variable, the endogenously determined threshold value approximately splits the sample into pre-ZLB and the ZLB regimes. The estimated marginal effect of foreign holdings ratio on the long-term interest rate is larger in absolute value in the ZLB regime than in the pre-ZLB regime, especially for the long-run effect. Therefore, we argue that the impact of foreign holdings ratio on the long-term interest rate shifted when short-term interest rates became stuck at near zero in the US even when we take into account the concurrent impact of the Federal Reserve's purchases of Treasury securities. In addition, through a counterfactual analysis assuming no implementation of the QE by the Fed, the results indicate that the three rounds of QE may have lowered the long-term interest rate by 38 to 55 basis points on average. We also find that changes in China's holdings of U.S. Treasury notes and bonds played an important role in explaining the 2004-2006 interest rate conundrum and kept the long-term interest rate from going ever lower in the recent ZLB period based on counterfactual analyses.

JEL Codes: C24, E43, E58, F21

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1 Introduction

The U.S. long-term interest rate (i.e. 10-year Treasury yield) has trended lower over the past three decades. A large empirical literature exists studying the determinants of the long-term rate. An important part of this literature has focused on the period during 2004-2006 where, despite the fact that the short-term policy rate was raised from 1% to 5.25%, the long-term rate nevertheless kept flat or even declined. The former Federal Reserve chairman, Alan Greenspan, labelled this puzzling behavior the 'interest rate conundrum'.[2] Researchers examining this conundrum have noted the role of foreign purchases or holdings of U.S. long-term Treasury securities as an important force pulling long-term interest rates lower during the conundrum period. The large amount of purchases of Treasuries by foreign investors and institutions mainly come from East Asian countries and the Middle-East oil-exporting countries with excess savings, which has given credence to the so-called global savings glut hypothesis articulated at the time, among others, by Bernanke (2005), Greenspan's replacement at the helm of the Federal Reserve.

A series of insightful empirical studies followed documenting a significant negative relationship between the long-term interest rate and the foreign purchases or holdings of Treasury notes and bonds (Warnock and Warnock 2009, Bandholz et al. 2009, Beltran et al. 2013 and etc.). This paper partly revisits this literature, but putting the spotlight on how the relationship shifted when conventional monetary policy was constrained by the zero-lower bound and through unconventional policy actions (such as quantitative easing, QE).

We argue that the impact of the change in foreign demand of Treasury notes and bonds on the long-term interest rate is subject to structural breaks when we cover a sample period that includes the 2008-2009 financial crisis and the subsequent zero-lower bound period. Our main measure of the foreign demand of Treasury securities is the ratio of foreign holdings of U.S. long-term Treasury securities as a

[2] The interest rate conundrum was pointed out in the testimony of former Fed chairman Alan Greenspan before the Committee on Banking, Housing, and Urban Affairs, U.S. Senate on February 16, 2005. The testimony is available from http://www.federalreserve.gov/boarddocs/hh/2005/february/testimony.htm.

share of marketable Treasury notes and bonds (total outstanding Treasuries excluding the Fed's holdings). This foreign holdings' ratio increased from around 11% in early 1994 to 56% at the end of 2008. Since then, this ratio has declined in spite of the fact that marketable Treasuries have grown less than total outstanding Treasuries due to the beginning of the Fed's policies of QE in the aftermath of the 2008-09 financial recession.

Using a well-known reduced form linear specification that features as a trademark of the existing literature over an extended sample period that includes the zero-lower bound episode, we find robust evidence of parameter instability leading to a breakdown in the relationship. Furthermore, we hypothesize the zero-lower bound combined with the deployment of unconventional monetary policy measures (QE) has led to a shift in the impact that the foreign holdings ratio has on the long-term interest rates—which has behaved differently than during the 'interest rate conundrum' period.

Based on the conventional expectations hypothesis of the term structure of interest rates, we derive a long-run cointegrating relationship between the long-term interest rate, the short-term rate, long- and short-run inflation expectations, and the foreign holdings ratio. Using Hansen (1992) and Gregory and Hansen (1996) tests, we endogenously find that the end of 2008 marks a breakpoint for this long-run relationship. Then, we use a threshold single equation error correction model with the policy rate as the threshold variable to estimate both short-run and long-run effects of the foreign holdings ratio on the long-term interest rate. The endogenously determined threshold value splits the sample into two regimes, which roughly correspond to the period before the zero-lower bound and the period of zero-lower bound. So the results support our hypothesis that the impact of changes in the foreign holdings ratio on the long-term rate shifted when interest rates became stuck at near zero in the U.S. even when we take into account explicitly the concurrent impact of changes in Fed's holdings of outstanding U.S. Treasuries through its QE programs.

From the estimation results and the dynamic multiplier functions obtained in this exercise, we find that the marginal effect of changes in the foreign holdings ratio on the long-term interest rate is amplified in the zero lower bound period relative to what it was in the period before the zero lower

bound, especially for the long-run effects. A one percentage point increase (decrease) in the foreign holdings ratio has an overall impact of reducing (raising) the long-term interest rate by around 6 basis points at the zero lower bound period compared to 4 basis points in the pre-zero-lower bound period.

We also conduct a counterfactual analysis assuming, everything else equal, that without the implementation of policies of QE on the part of the Fed, the marketable securities would have grown as much as outstanding Treasuries did since the onset of the 2008-09 financial recession. Our estimates indicate that the long-term interest rate would have been on average 38 to 55 basis points higher if the Fed had not conducted QE during this period while Fed Funds rates were stuck at the zero-lower bound.

Finally, we examine the role of China's holdings of U.S. Treasuries in affecting the U.S. long-term interest rate through counterfactual analyses. By assuming China's holdings during 2001 – 2006 maintain the same slower growth rate as in 1994 – 2000, we find that the accelerated increase in China's holdings started from 2001 may have lowered the U.S. long-term interest rate by 24 basis points on average in the 2004-2006 conundrum period. In addition, based on a counterfactual analysis assuming China had not discontinued its large amount of purchases of U.S. long-term Treasury securities since 2011m07, we find that the decline of China's holdings may have raised the long-term interest rate by 25 basis points on average during 2011m07 to 2014m12.

The rest of the paper is organized as follows. Section 2 provides an overview of the pertaining literature on the determinants of the long-term interest rate, and points at evidence of parameter instability in the relationship with foreign purchases or holdings of U.S. Treasuries. In Section 3, we derive a baseline theoretical long-run cointegrating relationship from the expectations hypothesis of the term structure of interest rates that motivates our subsequent empirical strategy. In Section 4, we discuss the data used in our model. Section 5 shows our empirical study, the estimation results and the key robustness checks. We also report some counterfactuals to illustrate the importance of the shifting role of foreign holdings at the zero-lower bound and, particularly, the role of China's holdings. And Section 6 concludes.

2 Literature Review

2.1 Overview on the Determinants of the Long-Term Interest Rate

In seventeen Federal Open Market Committee (FOMC) meetings spanning June 2004 to June 2006, the Fed increased the federal funds rate from 1% to 5.25% but the long-term interest rate (i.e. 10-year Treasury yield) remained flat or trended slightly lower during the same period. Former Fed chairman Alan Greenspan called this apparent anomaly, the interest rate conundrum (or the bond yield conundrum).

There has been a growing literature studying possible explanations for the conundrum. The evidence so far suggests that standard macroeconomic fundamentals—such as inflation expectations, the short-term interest rate, fiscal conditions, etc.—are not sufficient to fully account for the observed behavior of the long-term interest rate in particular during the conundrum period.[3] Therefore, researchers have looked intensely for other determinant factors that could help resolve this empirical puzzle.

Among the new factors which can possibly explain the dynamics of the long-term interest rate, the foreign net purchases (or holdings) of U.S. Treasury securities are thought to be an important one—especially the large purchases of U.S. long-term Treasury securities by foreign central banks. Theoretically, higher foreign demand of U.S. long-term Treasury securities pushes the price of Treasury notes and bonds up and, therefore, lowers their yields.

A number of researchers have conducted empirical analyses to quantify the effects of foreign net purchases (or holdings) of U.S. Treasuries on the Treasury yields (e.g., Warnock and Warnock 2009, Craine and Martin 2009, Bertaut et al. 2012, Beltran et al. 2013, etc.). Most of the empirical literature on the subject finds that the effects are negative and statistically significant. However, the range of the estimated values for the effects varies across different econometric methods and across the different

[3] See, e.g., Correia-Nunes and Stemitsiotis (1995), Caporale and Williams (2002), Dewachter and Lyrio (2006), and Diebold et al. (2006) for a discussion about macroeconomic fundamentals as determinants of long-term interest rates.

datasets and sample periods used in each study. In any event, the existing body of work summarized in Table 1 suggests that the estimated marginal effect of foreign purchases of 100 billion dollars of U.S. Treasury securities on the Treasury yield ranges from no effect to 50 basis points in the existing literature.

Other researchers find that a statistically-significant impact can be detected only during certain sample periods and argue that those effects may be shifting over time as the U.S. Treasury market evolves. For example, Wu (2005) finds that the relationship between the 10-year Treasury yield and foreign official net purchases of U.S. Treasuries (as a percentage of U.S. GDP) is unstable and not persistent—appearing only relevant since the early 2000s. Briere et al. (2008) also point out the instability of the impact and, similarly, find that the impact is only statistically significant after 2002. There are also some studies that do not find much of an effect at all. For example, Rudebusch et al. (2006) find that foreign official purchases of U.S. Treasuries had played little to no role in explaining the bond yield conundrum.

This literature is not limited to studies of the relationship between foreign purchases of government bonds and long-term bond yields for the U.S. exclusively. Some scholars have also investigated the same type of relationship for other countries. For example, Carvalho and Fidora (2015) investigate the effect of foreign purchases of government bonds issued by the euro area countries on the long-term interest rates of the euro area countries. Andritzky (2012), Arslanalp and Poghosyan (2014), and Hauner and Kumar (2006) use panel datasets to investigate the effect of foreign demand of sovereign bonds issued by a group of advanced economy countries on their corresponding long-term sovereign bond yields. Peiris (2010), Pradhan et al. (2011), and Ebeke and Lu (2014) study the same relationship for a group of emerging countries. In general, these studies uncovered some effect on sovereign bond long-term yields by the foreign demand of sovereign bonds—although they all assume the marginal effect remains constant over time.

Table 1. Estimated Impact of US$ 100 Billion Foreign Purchases of U.S. Treasury/Agency Securities on the U.S. Long-Term Treasury Yield: An Overview of Previous Studies

Previous Studies	Impact (in basis points)	Measurement of Foreign Variables	Sample Period
Rudebusch et al. (2006)	No significant impact	12-month foreign official flows into Treasury securities (scaled by total outstanding)	1990m05-2005m12
Warnock and Warnock (2009)	-34	12-month foreign official flows into Treasury and Agency securities (scaled by GDP)	1984m01-2005m05
	-16	12-month foreign total flows into Treasury and Agency securities (scaled by GDP)	
Bandholz et al. (2009)	-12	Foreign total holdings of Treasury securities (scaled by total outstanding)	1986m01-2006m06
Bertaut et al. (2012)	-13	Foreign official holdings of Treasury and Agency securities (scaled by total outstanding)	1980q1-2007q2
Beltran et al. (2013)	-46	1-month foreign official flows into Treasury notes and bonds (scaled by total outstanding)	1994m01-2007m06
	-50	1-month foreign official flows into Treasury notes and bonds (scaled by GDP)	
	-16 or -21	Foreign official holdings of Treasury notes and bonds (scaled by total outstanding)	

Note: The study in Beltran et al. (2013) is for the 5-year term premium, while all other studies are for the 10-year Treasury yield. We choose the end of sample in each study as the baseline date to scale the corresponding measure of the foreign variables when computing the reported impacts. The interested reader is also referred to the comparisons made in the previous literature: particularly, Table 4 in Bertaut et al. (2012) and Table 6 in Beltran et al. (2013).

2.2 A Discussion on the Evidence of Parameter Instability with Reference to the Existing Literature

In the existing literature studying the impact of the foreign demand of U.S. Treasury securities on the U.S. long-term Treasury yield, a widely used model specification is the single equation linear specification utilized, among others, in the influential work of Warnock and Warnock (2009). This is a reduced-form specification modelling the long-term Treasury yield motivated by standard assumptions on

the supply and demand functions of Treasury securities. The dependent variable is the long-term Treasury yield, while the explanatory variables are the long-run inflation expectation, the short-term interest rate, the difference between short-run and long-run inflation expectations, the interest risk premium (measured as the rolling 36-month standard deviation of changes in the long-term interest rates), the expected real GDP growth rate over the next year, the budget deficit, and the foreign net purchases of U.S. long-term Treasury securities scaled by U.S. GDP.[4]

Using the full sample period from 1986m01 to 2014m12, there is compelling evidence of parameter instability in the single-equation specification of Warnock and Warnock (2009), as illustrated in Figure A.1 in the Appendix.[5] The recursive residuals go outside the two standard error bands (Panel A in Figure A.1), the cumulative sum of recursive residuals (CUSUM) falls outside the 5% critical line (Panel B of Figure A.1), and the CUSUM of squares test statistic also appears beyond the 5% critical lines (Panel C of Figure A.1), all point toward parameter instability in the coefficients of equation (11).

Panel D of Figure A.1 plots the recursive coefficient estimates on the coefficient of the foreign flows variable to trace their evolution as more sample observations are added into the estimation. The estimated coefficients on the foreign flow variable are not very significant using sample period prior to 1994. Then, the sign of the estimated coefficient becomes statistically significant changing from positive to negative around 2004m07.

We do not take these findings as invalidating the empirical single-equation model specification *per se*, but instead as evidence suggesting that we need to explicitly account for the possibility of structural change in the model specification when modelling the relationship between long-term interest

[4] The specification in Bertaut et al. (2012) builds on the work of Warnock and Warnock (2009). One notable difference is that Bertaut et al. (2012) use a stock variable for foreign holdings instead of a foreign flows variable as their explanatory variable. Another recent contribution that builds on Warnock and Warnock (2009) is Pradhan et al. (2011) who expand the set of model predictors adding an additional variable to the specification, the exchange rate.
[5] Using the same sample period of 1984m01 to 2005m05 and the same dataset as in Warnock and Warnock (2009), only the plot of CUSUM shows clear evidence of parameter instability.

rates and the foreign net purchases (or holdings) of Treasury notes and bonds. This is, in fact, one of the major contributions that our paper makes to the existing literature.

2.3 A Discussion of Structural Breaks with Reference to the Existing Literature

In the previous literature, some studies mention the possibility of structural breaks in the investigation of the impact of foreign net purchases or holdings of U.S. Treasury securities on the long-term Treasury yield. However, often researchers end up selecting certain sample periods to avoid those potential structural breaks. For example, Sierra (2010) chooses the sample period from 1994m05 to 2007m12. For Sierra (2010), the selection of 1994m05 is to avoid the potential break due to the CNY/USD exchange rate change in the early 1994 and the selection of 2007m12 was based on data availability at the time. Beltran et al. (2013) choose the sample period from 1994m01 to 2007m06—they choose 1994m01 as the starting point based on data availability for certain variables, but select 2007m06 as the end of sample period to avoid the 2008 financial crisis and subsequent QE period. Goda et al. (2013) emphasized the importance of structural breaks in their analyses. They conducted statistical tests in order to find these break dates. However, they only focused on the 2004-2006 bond yield conundrum period and conducted Quant-Andrew single break point test only for the sample period 1994m02 to 2007m06. In this paper, in turn, we consider possible structural breaks for the entire available sample period (especially including the 2008 financial crisis and the following QE period) to unearth the impact of foreign demand of Treasury securities on the long-term Treasury yield.

Some studies emphasize the different impacts of foreign demand of U.S Treasury securities on the Treasury yield. But their impacts are calculated as the constant marginal effect multiplied by the change in foreign demand for certain periods.[6] For example, Kaminska and Zinna (2014) calculate the

[6] This way of calculating the overall impacts (also called the cumulative impacts) has potential limitations. By using the constant marginal effect, the different overall impacts of foreign demand of Treasury securities on the long-term interest rate over certain periods only reflect the different magnitudes of the changes in foreign demand over the corresponding periods.

impacts across different periods which show a significantly larger impact before the QE than during the QE episode. Their estimated (constant) marginal effect is that one percent increase of foreign demand has an impact of 4.9 bps on the 10-year real Treasury yield. Their calculated impacts of the change in foreign demand on the 10-year real Treasury yield are 80.8 bps for the period from 2001 to 2008 and 2.4 bps for the periods from March 2009 to November 2009 and from November 2010 to June 2011.

Other studies use regression results for different sub-sample periods to compare the impacts over time. For example, Mann and Klachkin (2012) find that the negative relationship between foreign demand and long-term yields disappeared after QE and they even suggest that the Fed's purchases altered this relationship. In addition, Beltran et al. (2013) show the results for both the sample period before the 2008-09 financial crisis as well as the extended sample period until June 2011. The estimated effect of foreign purchases in their work is a little bit smaller using the extended sample (January 1994 to June 2011) than the shorter sample period (January 1994 to June 2007), which may indicate that QE lowered the impact of foreign demand on the long-term rate.

Our paper goes a step further in this direction, assuming the impact of foreign holdings (or purchases) is time-varying from the beginning while modelling the underlying economic forces behind such time-varying effects. We argue that even controlling for the direct effect of QE policies on marketable securities, the environment of near-zero short-term interest rates that followed the 2008-09 recession has resulted in statistically different impacts of the foreign demand of U.S. Treasuries on U.S. long-term interest rates.

3 A Benchmark Model Based on the Term Structure of Interest Rates

Let $R_{n,t}$ be the nominal yield to maturity of an n-period pure discount bond that is bought at time t and matures after *n* periods. The weakest form of the expectations hypothesis of the term structure of interest rates can be expressed as follows (see, e.g., Hall et al. (1992)):

$$R_{n,t} = \frac{1}{n}\left[\sum_{i=1}^{n} E_t(R_{1,t+i-1})\right] + \theta_{n,t} \tag{1}$$

where $\theta_{n,t} \equiv \frac{1}{n}\sum_{i=1}^{n} \varphi_{i,t}$ denotes a term capturing the effects of the premia along the yield curve. According to the pure expectations hypothesis, the term premia is zero for any given maturity—i.e. $\theta_{n,t} = 0$. A milder version of the expectations hypothesis allows the term premia to be constant over time—i.e. $\theta_{n,t} = \theta_n$. We consider in (1) the weakest form of the expectations hypothesis where the term premia is allowed to vary over time and across maturities.

The weak form of the expectations hypothesis has important statistical implications. To see this, we can rewrite equation (1) by subtracting $R_{1,t}$ on both sides as follows:

$$R_{n,t} - R_{1,t} = \frac{1}{n}\left[\sum_{i=1}^{n} E_t(R_{1,t+i-1})\right] - R_{1,t} + \theta_{n,t} \tag{2}$$

Rearranging terms, we get

$$R_{n,t} - R_{1,t} = \frac{1}{n}\left[\sum_{m=1}^{n-1}\sum_{i=1}^{m} E_t(\Delta R_{1,t+i})\right] + \theta_{n,t} \tag{3}$$

or, alternatively,

$$R_{n,t} - R_{1,t} = \sum_{i=1}^{n-1}\left(1 - \frac{i}{n}\right) E_t\left(\Delta R_{1,t+i}\right) + \theta_{n,t} \tag{4}$$

where Δ is the first-difference operator—i.e., $\Delta R_{1,t+i} = R_{1,t+i} - R_{1,t+i-1}$.

The weak form of the expectations hypothesis shows that yields at different maturities should be linked together, but also gives great importance to the term premia. Here, we consider a weaker version of the expectations hypothesis whereby the term premia is time-varying and can be partly driven by endogenous variables—i.e., we assume that $\theta_{n,t} \equiv \theta_n(w_t, \varepsilon_{n,t})$. The vector containing the known macroeconomic factors influencing the term premia is denoted w_t, but our specification also incorporates a stationary stochastic process $\varepsilon_{n,t}$ to capture the exogenous component of the term premia.

In the empirical literature studying the term premia, different macro variables have been considered to be part of w_t. Breedon et al. (1999) and Caporale and Williams (2002) use fiscal indicators

(the debt-to-GDP ratio and the deficit-to-GDP ratio), real GDP growth, and even the long-term interest rates of other countries. Sierra (2010) and Beltran et al. (2013) are amongst the recent studies to empirically investigate the effect of foreign purchases (and holdings) of Treasury securities on the term premia focusing in particular on the 5-year Treasury notes.[7]

Similar to Beltran et al. (2013), we recognize that the term premia can be influenced by foreign holdings and model it accordingly as $\theta_{n,t} \equiv \theta_n(fh_t, \varepsilon_{n,t})$, where fh_t are the foreign holdings of U.S. Treasury notes and bonds as a percentage of outstanding marketable Treasury notes and bonds (net of Federal Reserve holdings). We specify a linear form for the term premia on both fh_t and $\varepsilon_{n,t}$ as follows,

$$\theta_{n,t} = \theta_n^0 + \theta_n^1 fh_t + \theta_n^2 \varepsilon_{n,t} \tag{5}$$

Combining equations (4) and (5), we derive the following expression

$$R_{n,t} - R_{1,t} - \theta_n^1 fh_t - \theta_n^0 = \sum_{i=1}^{n-1} \left(1 - \frac{i}{n}\right) E_t\left(\Delta R_{1,t+i}\right) + \theta_n^2 \varepsilon_{n,t} \tag{6}$$

which establishes a link between the nominal yields at different maturities and measured foreign holdings.

Campbell and Shiller (1987, 1991) and Campbell (1995) note that an important implication of the expectations hypothesis of the term structure is that the spread between the long and short ends of the yield curve should be related to a sequence of expectations of future movements on the short-term interest rate—and, in the weak form of the hypothesis, to the exogenous component of the term premia as well as the foreign holdings. Equation (6) simply shows the relationship between expected future changes in short-term rates over the lifespan of a long-term bond till maturity $E_t\left(\Delta R_{1,t+i}\right)$, the exogenous

[7] As indicated earlier, there is a theoretical case for placing the spotlight on the effect of foreign purchases (or holdings) of U.S. Treasuries too. The 'global savings glut theory' championed, among others, by former-Federal Reserve chairman Bernanke (2005) suggests that large purchases of U.S. Treasury securities by foreign investors (notably by foreign central banks) can be an important force behind the movements in the term premia—particularly during 2004 to 2006. We also recognize that foreign purchases may play a role making the market and influencing the liquidity risk component of the spreads.

component of the term premium $\varepsilon_{n,t}$, and the yield spread adjusted to account for the endogenous component of the term premia ($R_{n,t} - R_{1,t} - \theta_n^1 fh_t - \theta_n^0$).

The well-known Fisher equation links nominal rates to real rates and inflation expectations, i.e.,

$$R_{1,t} = r_{1,t} + E_t(\pi_{t+1}) \tag{7}$$

where $r_{1,t}$ is the real per-period net yield to maturity of a one-period zero coupon bond at time t and π_{t+1} defines the inflation rate between time t and t+1. Using the Fisher equation in (7) to replace the short-term nominal rates on the right-hand side of (2), we get that

$$\left(R_{n,t} - \pi_{t,n}^e\right) - \left(R_{1,t} - \pi_{t,1}^e\right) = \frac{1}{n}\left[\sum_{i=1}^{n} E_t\left(r_{1,t+i-1}\right)\right] - r_{1,t} + \theta_{n,t} \tag{8}$$

where the short-term inflation expectations are defined as $\pi_{t,1}^e = E_t(\pi_{t+1})$ and long-term inflation expectations over the lifespan of the bond are given by $\pi_{t,n}^e = \frac{1}{n}[\sum_{i=1}^{n} E_t(\pi_{t+i})]$.[8]

Incorporating our preferred specification of the term premia in (5) and after some straightforward algebra, it follows that

$$R_{n,t} - R_{1,t} + \pi_{t,1}^e - \pi_{t,n}^e - \theta_n^1 fh_t - \theta_n^0 = \sum_{i=1}^{n-1}\left(1 - \frac{i}{n}\right) E_t\left(\Delta r_{1,t+i}\right) + \theta_n^2 \varepsilon_{n,t} \tag{9}$$

Generally, we find that $R_{n,t} \sim I(1)$, $R_{1,t} \sim I(1)$, $\pi_{t,n}^e \sim I(1)$, $\pi_{t,1}^e \sim I(1)$, and $fh_t \sim I(1)$. Under the assumption that the short-term real interest rate is $r_{1,t} \sim I(1)$—and hence $\Delta r_{1,t} \sim I(0)$—, if the exogenous component of the term premia $\varepsilon_{n,t}$ is stationary, then all the terms on the right-hand side of equation (9) must be stationary. As a result, the term on the left-hand side of (9) must be stationary also. Hence, the model predicts that the long yield is cointegrated up to a constant with the short yield, with short- and long-term inflation expectations, and with foreign holdings.

[8] Mehra (1998) is among a number of papers advocating that the nominal long-term bond yield is cointegrated with inflation—in particular, with the one-period current inflation rate. The theory of the Fisher equations provides the natural theoretical framework to establish such a connection between inflation expectations and nominal yields.

Let us define $X_t = (R_{n,t}, R_{1,t}, \pi^e_{t,n}, \pi^e_{t,1}, fh_t)'$. Given the theoretical cointegration relationship (up to a constant) implied by equation (9), according to the Granger representation theorem (Engle and Granger (1987)), there exists a statistical representation for the vector X_t in the form of a vector error correction model (VECM) as follows:

$$\Delta X_t = \mu + \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \epsilon_t \tag{10}$$

This provides a rationale for modeling the dynamic interrelationship between interest rates using the implications of a VECM specification.

This framework constitutes the benchmark specification for our empirical analysis. When the long-run relationship among $R_{n,t}, R_{1,t}, \pi^e_{t,n}, \pi^e_{t,1}, fh_t$ is out of equilibrium, either the long-term rate ($R_{n,t}$), the short-term rate ($R_{1,t}$), short- and long-term inflation expectations ($\pi^e_{t,1}$ and $\pi^e_{t,n}$) and/or the foreign holdings (fh_t) must adjust accordingly. Then, the error correction term $R_{n,t} - \gamma_0 - \gamma_1 R_{1,t} - \gamma_2 \pi^e_{t,n} - \gamma_3 \pi^e_{t,1} - \gamma_4 fh_t$ has predictive power for future changes in the long-term interest rate (as shown in equation (9)). Moreover, a single-equation specification in error correction form can be derived for the long-term interest rate from the first row in (10) akin to that used in much of the literature reviewed here.

In this paper we investigate potential instability in the cointegrating relationship—but also in the short-run dynamics—focusing on the shifting impact of foreign holdings (fh_t). In particular, we highlight the changes at the zero-lower bound whenever monetary policy shifted from price-based tools (targeting the Fed Funds rate) to relying more on unconventional monetary policy (including forward guidance but also quantity-based actions of credit and quantitative easing) as short-term interest rates became constrained near zero.

From Equation (6), we know that $R_{n,t} - R_{1,t} - \theta^1_n fh_t - \theta^0_n$ has predictive power for the future expected nominal short-term interest rates. However, when the short-term nominal interest rates are constrained at the zero lower bound (ZLB), this implies that $R_{1,t} = 0$ and $\Delta R_{1,t+i} = 0$ approximately over a number of quarters into the future. In such a situation where short-term rates become effectively unresponsive over a number of periods, the changes in the long-term nominal rate are solely due to

changes in the term premia. Therefore, the ZLB potentially affects how the long-term nominal rate responds to change in the term premia. Given that the foreign holdings ratio is an important contributor to the movements in the term premia, the ZLB may lead to a structural break in the response of the long-term nominal rate from changes in the foreign holdings ratio which we explore empirically in greater detail in the reminder of the paper.

While we conjecture that the ZLB may lead to a structural break in the relationship, we have no strong priors on its direction (weakening or strengthening the effect) and magnitude. We view that as an empirical question, which we are the first to address in this paper, and leave the exploration of the theoretical justification of the empirical evidence presented in the remainder of the paper for future research.

4 Data

Our dataset covers the sample period after the Volcker disinflation of the early 1980s at the onset of the Great Moderation era from 1986m01 till 2014m12—for this period we have complete monthly time series of 348 observations on all variables pertinent to our empirical study.[9] The explanatory variables in our dataset include the short-term and long-term nominal rates, $R_{1,t}$ and $R_{n,t}$, the short-term and long-term inflation expectations, $\pi^e_{t,1}$ and $\pi^e_{t,n}$, and the foreign holdings fh_t. In Figure 1, we plot all these five key variables used in our following empirical analysis.

[9] Our reference sample period begins in the early years of the Great Moderation after the Volcker disinflation with the development and consolidation of the Greenspan-Bernanke regime of price-based monetary policy which came to be characterized by the Taylor (1993) rule targeting the Fed Funds rate. Our reference sample period also includes the major policy shift towards unconventional monetary policies (and the return of quantity-based policies in the form of Quantitative Easing, QE) that followed the 2008-09 financial recession during the latter part of Ben Bernanke's tenure and under Janet Yellen's watch at the helm of the Federal Reserve. In Bandholz et al. (2009), their sample runs from 1986m01 to 2006m06, which also starts from 1986m01. As summarized in Table 1, other well-known studies in the literature similarly study a sample period beginning in the mid-1980s or early-1990s—the most significant differences in time coverage arising from the fact that our dataset includes now the most current data available allowing us to explore the stability of the empirical relationships of interest at the zero-lower bound (ZLB).

For the short-term nominal rate $R_{1,t}$, we use the federal funds rate (monthly average) retrieved from the Federal Reserve Bank of St. Louis' FRED database. For the long-term nominal rate $R_{n,t}$, we use the 10-year Treasury yield (monthly average) also retrieved from the FRED database.[10] Both nominal interest rate series are illustrated in Panel A of Figure 1. The short-term inflation expectations data, $\pi_{t,1}^{e}$, is the monthly one-year-ahead forecast of the (year-over-year) percent change of the quarterly GNP/GDP price deflator from the forecasts reported in the Blue Chip Economic Indicators survey.[11] The long-term inflation expectations data, $\pi_{t,n}^{e}$, is from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters (SPF), extended with Blue Chip Economic Indicators survey data prior to 1991Q4, and linearly interpolated from quarterly to monthly frequency.[12] These short-term and long-term inflation expectations are plotted in Panel B of Figure 1.

The main data source for the key explanatory variable in our empirical model—foreign net purchases or holdings of U.S. Treasury notes and bonds, fh_t—is the Treasury International Capital (TIC) system. The TIC S-Form provides the monthly data of foreign official and private investors' net purchases

[10] Data sources from the FRED database for the 10-year Treasury yield and the policy rate (the Fed Funds rate): https://research.stlouisfed.org/fred2/series/DGS10, https://research.stlouisfed.org/fred2/series/FEDFUNDS.

[11] The Blue Chip Economic Indicators survey reports forecasts of the percent change (from the prior quarter, annualized) of the quarterly GNP/GDP deflator at monthly frequency. We denote the reported monthly inflation forecast at period t for inflation q-quarters-ahead as $g_{t,q}^{qoq}$, where $q = 0$ indicates the current quarter, and collect those forecasts over the next four quarters—i.e., $\left(g_{t,1}^{qoq}, g_{t,2}^{qoq}, g_{t,3}^{qoq}, g_{t,4}^{qoq}\right)$. Our measure of the short-term inflation expectations at time t is the corresponding one-year-ahead forecast of the percent change in the GNP/GDP price deflator expressed in year-over-year rates, which we denote as g_{t+4}^{yoy}. We compute the one-year-ahead inflation forecast at time t with the following formula: $g_t^{yoy} = 100\left(\left(\prod_{q=0}^{3}\left(1+\frac{g_{t,1+q}^{qoq}}{100}\right)\right)^{\frac{1}{4}} - 1\right)$.

[12] The main data source for long-term inflation expectations is the Survey of Professional Forecasters (SPF) whose data can be accessed here: https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/inflation.xls. The SPF forecasts that we use are the expectations for the annual average rate of CPI inflation over the next 10 years ("INFCPI10YR") which are only available from 1991Q4 onwards and at quarterly frequency. The SPF also makes available additional 10-year-ahead inflation forecasts from other sources going further back in time that can be downloaded here: https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/additional-cpie10.xls. We follow the SPF's own recommendation and extend the INFCPI10YR series with the additional forecasts obtained by SPF from the Blue Chip Economic Indicators survey (Additional-CPIE10.xls). The variable forecasted by Blue Chip Economic Indicators since 1983 is the CPI (with the exception of 1983Q4 where it still is the GNP deflator) and those forecasts are taken twice a year (March and October). The biannual Blue Chip Economic Indicators series and the quarterly SPF series are then linearly interpolated to monthly frequency and combined together back to the beginning of our sample period (1986m01).

(gross purchases minus gross sales) of U.S. Treasury notes and bonds starting from 1979m01.[13] The TIC Form SLT reports the monthly data of foreign official and private investors' holdings of U.S. Treasury notes and bonds at market value, but has a limited coverage starting from 2011m09. The TIC annual surveys provide the most accurate data on foreign official and private investors' holdings of U.S. long-term Treasury securities (the amounts are those for the end of June of each year).[14]

It is well-known that TIC data has limitations, though.[15] First, the monthly data of net purchases from TIC S-Form suffers from transaction bias. It only records the direct buyer or seller of the Treasury securities, not the ultimate buyer or seller. Second, the estimated holdings computed from accumulated monthly TIC net flows are not fully consistent with the holdings data in the annual survey (which is regarded as the benchmark data) because of the valuation changes that occur over the course of the year and, potentially, because of transaction bias as well. Third, there are also no official reports for the monthly foreign holdings data before September 2011 when TIC Form SLT got started. Only data on net purchases from the TIC S-Form is available prior to September 2011. Fourth, all data for foreign net purchases and holdings available is subject to custodial bias. For example, a foreign official investor can use a custodian in another country to purchase or sell U.S. Treasury securities. Hence, the geographical allocation of foreign net purchases or holdings may not be accurate and the foreign official holdings or net purchases may be underestimated.

While we cannot overcome the limitations of the TIC data entirely, we rely on a novel dataset of developed by Bertaut and Tryon (2007) and expanded by Bertaut and Judson (2014) which adjust the available TIC data (the TIC S-Form data, the annual survey data, and the more recent release of TIC Form SLT data).[16] This dataset provides benchmark-consistent monthly estimates of foreign official and private holdings. In turn, changes in the benchmark-consistent holdings estimates are decomposed into three

[13] The foreign official institutions mainly include foreign central banks. A partial list of foreign official institutions used by TIC can be found here: https://www.treasury.gov/resource-center/data-chart-center/tic/Pages/foihome.aspx.
[14] The TIC annual surveys have been conducted each year since 2002. Before 2002, the TIC surveys were also conducted in 1974, 1978, 1984, 1989, 1994, and 2000.
[15] See Bertaut and Tryon (2007), Warnock and Warnock (2009), and Bertaut and Judson (2014) for further details.
[16] The data can be downloaded from http://www.federalreserve.gov/pubs/ifdp/2014/1113/ifdp1113_data.zip.

components: adjusted net flows, valuation changes, and residuals. Since the adjusted net flows are also benchmark-consistent, we argue that they offer a more precise measurement of foreign net flows than the unadjusted TIC net flows data after removing mere valuation effects and noise. Hence, we use these benchmark-consistent estimates of adjusted net flows to construct the relative foreign holdings measure fh_t that we use in our empirical model specification.

In the literature, the foreign net purchases or holdings variables are generally scaled by either nominal GDP or total outstanding/marketable Treasury notes and bonds. Our variable fh_t is the ratio of foreign (official, private or total) holdings of Treasury notes and bonds as a percentage of the outstanding marketable Treasury notes and bonds—i.e., outstanding Treasury notes and bonds excluding the Fed's holdings of Treasury notes and bonds.[17] The data on the Fed's holdings of U.S. Treasury notes and bonds is available from the Federal Reserve statistical release H.4.1 measured at the face value of the securities.[18] We define the Federal Reserve's net purchases of Treasury notes and bonds as the first difference of the Fed's holdings. The data on total outstanding Treasury notes and bonds is available in the Monthly Statement of the Public Debt (MSPD) from the Department of the Treasury which reports the face value of those securities going back to 1952m07.[19]

The foreign official holdings ratio, fh_t, used in our model specification is plotted in Panel C of Figure 1.[20] Using the foreign holdings ratio over outstanding marketable Treasury securities, we take account of the foreign demand of Treasury securities relative to the supply of marketable Treasury securities controlling for the impact that the Federal Reserve's holdings of Treasury securities has on the

[17] Bertaut et al. (2012) and Beltran et al. (2013) also use the same marketable Treasury notes and bonds as the scaling factor for foreign holdings or net foreign purchases.
[18] Data source: http://www.federalreserve.gov/releases/h41/.
[19] Data source: https://www.treasurydirect.gov/govt/reports/pd/mspd/mspd.htm. Historical monthly data from the Monthly Statement of the Public Debt (MSPD) is also available for download for the years 1869-1952 from this website, but such data goes beyond the scope of our paper.
[20] Unlike foreign private holdings, the foreign official holdings are generally treated as exogenous in the previous studies because foreign official institutions frequently do not optimize their investment strategy on Treasury securities in response to the prices of Treasuries themselves or U.S. monetary policy (see, e.g. Warnock and Warnock (2009) and Bertaut et al. (2012)). We also follow the literature in this regard and use foreign official holdings as the variable of interest in our analysis. In addition, we explore foreign total holdings (official plus private) as a robustness check.

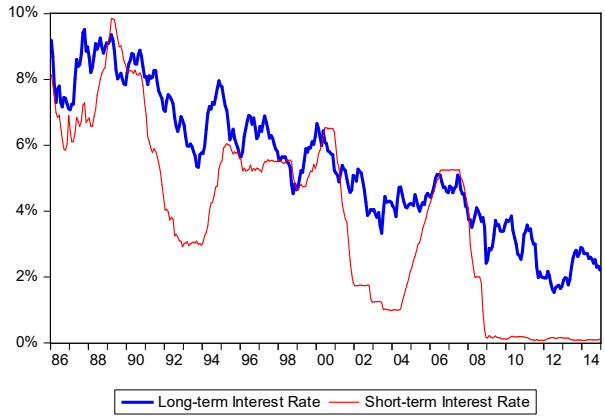
supply that ends up available for the market. A disadvantage of using the ratio of foreign holdings over the outstanding marketable Treasuries, fh_t, is that this ratio has a market value in the numerator but a face value in the denominator. As is noted in the annual survey of foreign portfolio holdings of U.S. Treasury securities, it is not possible to obtain the market value of total outstanding Treasuries on the same basis as the data on foreign holdings or net purchases.[21]

A plausible alternative to construct fh_t that avoids the mismatch in valuation terms between the numerator and the denominator is to use the foreign holdings data at face value (instead of at market value). Table 1A of the FRB H.4.1 release ("Factors Affecting Reserve Balances of Depository Institutions and Condition Statement of Federal Reserve Banks") provides the face value of U.S. Treasury securities held in custody at the Federal Reserve Bank of New York by foreign official institutions. This can be used as an alternative data source for the foreign official holdings of U.S. Treasury securities (at face value) in order to construct another measure of fh_t. However, the FRB H.4.1. release data only partially accounts all foreign official holdings of U.S. Treasury securities. Therefore, we argue that the foreign holdings ratio constructed still offers the most sensible way to scale foreign holdings given the limitations with the data available.

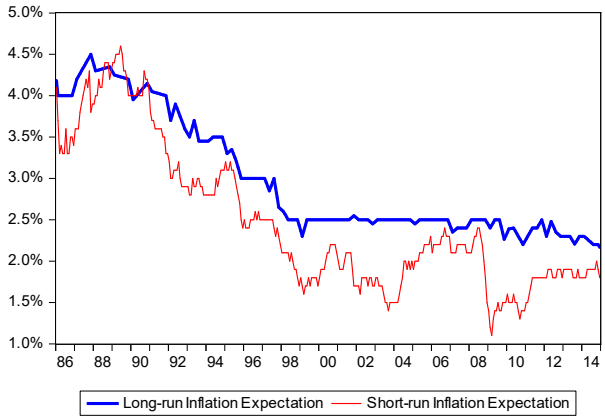
[21] See page 4 on the Foreign Portfolio Holdings of U.S. Securities as of June 30, 2014 released by the U.S. Department of the Treasury.

Figure 1. Data Plots of Key Variables

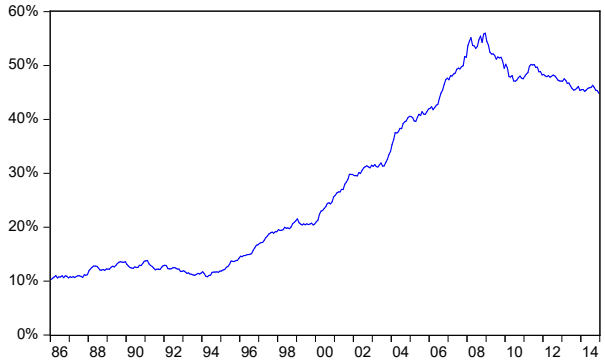
A. Long-term and Short-term Interest Rates



B. Long-run and Short-run Inflation Expectation



C. Foreign Official Holdings as a Share of Outstanding Marketable Treasury Notes and Bonds



Sources: FRED Database, Survey of Professional Forecasters, Blue Chip Economic Indicators, Bertaut and Tryon (2007), Bertaut and Judson (2014), FRB H.4.1., and Department of the Treasury.

Also, the TIC datasets report foreign total (including both official and private) net purchases or holdings by country.[22] Hence, it is possible to analyze foreign net flows or holdings data either by type of investor (official or private) or across countries. As seen in Panel A of Figure 2, large foreign inflows into U.S. Treasury securities took off in the mid-1990s, mainly from major economies—particularly China—with large savings exceeding their domestic investment opportunities, sizeable current account surpluses, and large foreign exchange reserves.

Policy changes coupled with the rapid pace of integration of China into the global economy are often cited among the reasons for the dramatic change in the foreign holdings of U.S. Treasury securities occurring around 1994 (e.g., Sierra (2010), and Goda et al. (2013)).[23] China's accession to the World Trade Organization (WTO) in 2001 further accelerated those trends. Japan, other smaller, but fast-growing economies of East Asia (South Korea, Singapore, Hong Kong, and Taiwan—the so-called Four Asian Tigers), and the oil-exporting countries contributed to a lesser extent to the rise after having been the major foreign players in the U.S. Treasury securities market during the 1980s and the better part of the 1990s.[24]

As shown in Panel B of Figure 2, the ratio of foreign total (the sum of official and private) holdings of U.S. Treasury notes and bonds as a percentage of U.S. total outstanding marketable Treasury notes and bonds is largely accounted for by foreign official holdings. From 1994 to 2008, the foreign official holdings ratio dramatically increased from 10% to 55%. China's large accumulation of foreign exchange reserves alone explains a sizeable part of this shift in foreign ownership. After 2008, the increase in the foreign private holdings ratio has mostly made up for the decline in the foreign official holdings ratio.[25]

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[22] However, the data for decomposing foreign official (or private) flows and holdings into different individual foreign countries are not publicly available (is confidential). Only foreign total flows and holdings can be decomposed into different countries based on publicly available data. Hence, we only have country data on total (including both official and private) flows and holdings of Treasury securities but not on official and private holdings separately.

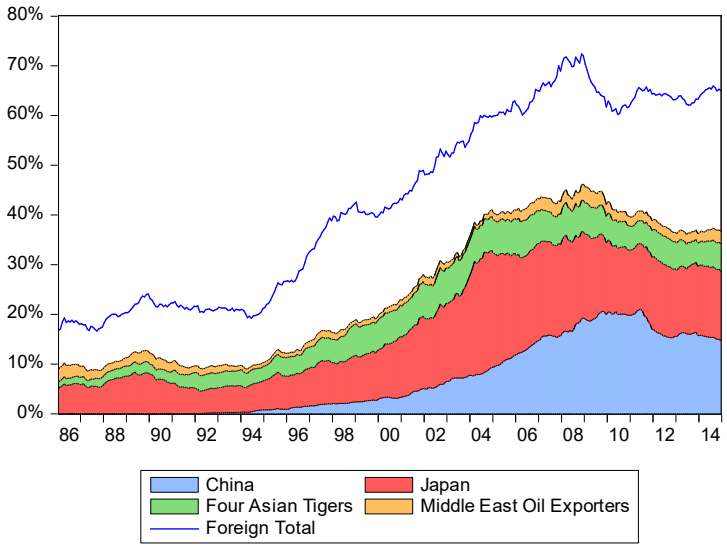
[23] Among the cited policy changes, the sudden change in the exchange rate of the Chinese yuan against U.S. dollars from 5.8 CNY/USD to 8.7 CNY/USD on February 1994 is generally regarded as signaling the major outward transformation of the Chinese economy that would unfold since then.

[24] In Panel A of Figure 2, Four Asian Tigers refers to Singapore, South Korea, Hong Kong, and Taiwan. Middle-East Oil Exporters refers to Bahrain, Iran, Iraq, Kuwait, Oman, Qatar, Saudi Arabia, and the United Arab Emirates.

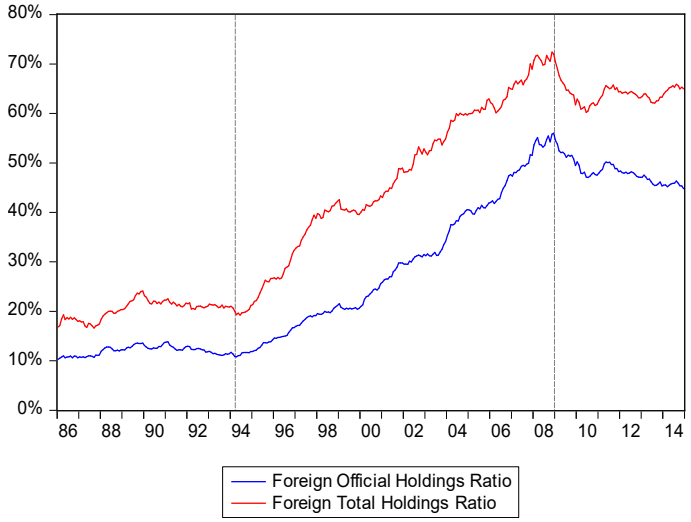
[25] Using Bai-Perron multiple break points test for the regression of the foreign official holdings ratio on a constant and a time trend, two of the breakpoints that we can formally identify occur in 1994m03 and 2009m03. We can informally identify both periods through visual inspection of the time series as well. The Bai-Perron points also to break points on 1999m03 and 2004m02.

Figure 2. Foreign Total Holdings of U.S. Long-term Treasury Securities as a Share of Outstanding Marketable Treasury Notes and Bonds

A. By Different Foreign Holders



B. By Ownership Type: Total Holdings vs. Official-Only Holdings



Source: Bertaut and Tryon (2007), Bertaut and Judson (2014), FRB H.4.1., and U.S. Department of the Treasury.

The aftermath of the 2008-2009 financial recession brought about changes in the monetary policy framework in the U.S. when the Fed Funds rate became constrained at the zero-lower bound, switching policy towards a framework based on unconventional monetary tools. The large scale asset purchases by the Federal Reserve under consecutive rounds of QE had an impact on the share of foreign official holdings over total marketable Treasuries accordingly. However, the policy shift required by the zero-lower bound constraint on conventional monetary policy through the Fed Funds rate may also contribute to a structural break in the empirical relationship between foreign holdings and the long-term interest. Our paper is the first—to the best of our knowledge—to investigate formally the possibility of such a structural break to better quantify and estimate the effects of foreign holdings on long-term yields.

5 Empirical Findings

5.1 Stability of the Long-run Cointegrating Relationship

In Section 3, we argue that in theory the variables $R_{n,t}$, $R_{1,t}$, $\pi^e_{t,n}$, $\pi^e_{t,1}$ and fh_t ought to be cointegrated under the weak form of the expectations hypothesis of the yield curve augmented with the Fisherian equation for the real interest rate. We verify the non-stationary properties of these variables in our data to be consistent with the theory of cointegration. As implied by theory, we find that all our explanatory variables are I(1) over the full sample period between 1986m01 and 2014m12 (see Table A.1 in the Appendix).[26] In addition, the real short-term rate $r_{1,t}$ is also an I(1) variable and accordingly $\Delta r_{1,t}$ is an I(0) variable, which is consistent with the assumptions underlying the theoretical model underpinning our empirical specification.[27] With all this data, we estimate and make inferences on the long-run cointegrating relationship implied by the benchmark theoretical model, i.e.,

[26] The concern about mixing I(0) and I(1) variables in our empirical model is that doing so may lead to spurious regression results. The unit root test results for our data can be found in Table A.1 in the Appendix. That evidence gives us confidence about our empirical estimation results given that we have a balanced dataset of I(1) variables.
[27] We define the real short-term interest rate from the Fisherian equation, $r_{1,t} = R_{1,t} - E_t(\pi_{t+1})$, by subtracting the short-term inflation expectation from the nominal short-term rate.

$$R_{n,t} = \gamma_0 + \gamma_1 R_{1,t} + \gamma_2 \pi_{t,n}^e + \gamma_3 \pi_{t,1}^e + \gamma_4 fh_t + \varepsilon_t \quad (11)$$

with the fully modified OLS (FMOLS) technique developed by Phillips and Hansen (1990).

Parameter Instability in the Long-Run Cointegrating Relationship.

To investigate the parameter stability in the long-run cointegrating relationship posited by equation (11), we apply Hansen (1992)'s instability tests (Tables 2). The three associated statistics—L_c, MeanF, and SupF—can be used to test the null hypothesis of parameter stability against the alternative of parameter instability in the FMOLS cointegrating regression equation. First part of Table 2 presents the results for the Hansen (1992) test using different kernels, bandwidth selection methods and pre-whitening options for the full sample (1986m01-2014m12). Although we find some insignificant test statistics for the full sample, overall the evidence from Hansen (1992) tests tends to reject the null hypothesis of parameter stability in the long-run relationship of equation (11).

In addition, Figure A.2 in the Appendix displays the corresponding sequence of F statistics—which reaches its maximum exactly at 2008m11 in Panel B and Panel C, exceeding the 5% critical line for the SupF statistic. This evidence supports our hypothesis that at the end of 2008 there was a significant breakpoint in the long-run cointegrating relationship given by equation (11) which coincides with the 2008-09 financial recession and the period when short-term rates become constrained at the zero-lower bound.

The Engle and Granger (1987) and Phillips and Ouliaris (1990) cointegration tests (tau-statistic and z-statistic) both indicate that these variables, $R_{n,t}$, $R_{1,t}$, $\pi_{t,n}^e$, $\pi_{t,1}^e$ and fh_t, appear cointegrated for the full sample period (see Panel A of Table A.2 in the Appendix). To further examine possible changes in the long-run cointegrating relation of equation (11), we also conduct FMOLS and cointegration tests using a

smaller subsample, 1986m01-2008m11. Panel B of Table A.2 in the Appendix reports the cointegration test results for the subsample supporting evidence of cointegration as well.[28]

In addition, we also use a battery of stability test proposed by Gregory and Hansen (1996), which introduce three test statistics—i.e., ADF*, Z_{α}^{*}, and Z_{t}^{*}—to test the null hypothesis of no cointegration against the alternative hypothesis of cointegration while allowing for the cointegrating vector to change at a single unknown break date during the sample period. The tests can be applied to three types of structural break models: level shift, level shift with trend, and both level shift and slope shift (called regime shift). These test statistics are helpful in detecting a break in the cointegrating relationships especially when the conventional cointegrating test (e.g., Engle-Granger or Phillips-Ouliaris) cannot reject the null hypothesis of no cointegration. A byproduct of these tests is the estimated breakpoint date, although these tests only allow for one breakpoint.

We applied the Gregory and Hansen (1996) tests using the full sample period (1986m01-2014m12) and results are reported in Panel A of Table A.3 in the Appendix.[29] Since the Engle-Granger and Phillips-Ouliaris cointegration tests reject the null of no cointegration for the full sample, not surprisingly the three test statistics of Gregory and Hansen (1996) reject the null hypothesis of no cointegration as well—for all three types of (structural break) models at the 5% level, except with the Z_{α}^{*} test statistic in the regime shift model (this test statistic is only a little bit less negative than its corresponding 5% critical value). Although both conventional and Gregory and Hansen (1996) tests reject the null of no cointegration, the latter are especially helpful because they provide some meaningful break dates and corroborating evidence of a break around 2009. We focus on the results for the regime shift type model, where the estimated break point is around early 2009 (for the Z_{t}^{*} test statistic, the estimated break date is 2009m02, which is close to the first round of QE after the Fed Funds rate became stuck at zero).

[28] For the subsample of 2008m12-2014m12, the series are more likely to not be cointegrated based on the evidence. One possible reason for the weak evidence here is that the short-term rate $R_{1,t}$ is close to the zero lower bound. Another reason may be the small subsample size (73 observations).
[29] The tests were conducted using the Matlab code downloaded from http://www.ssc.wisc.edu/~bhansen/progs/joe_96.html.

Based on the evidence obtained so far, we argue that the long-run relationship in equation (11) appears to have broken down during the full sample period, 1986m01-2014m12, and we find a likely break date around the end of 2008 or early 2009. As we discussed earlier when documenting the dramatic changes observed in the foreign ratio variable, 1994m02 is another potential breakpoint for the long-run cointegrating relationship. It also signals the time when the foreign holdings ratio started to increase noticeably in no small part due to China's purchases of U.S. Treasuries to build up its war chest of foreign reserves. Therefore, we conduct the Hansen (1992) and Gregory and Hansen (1996) tests using the subsample period 1986m01-2008m11 to examine whether or not the period 1994m02 should be treated as a statistically significant breakpoint too.

Table 2. Hansen (1992) Parameter Stability Test Results for Equation (11)

Test Statistics	Full Sample (1986m01 – 2014m12)		Subsample (1986m01 – 2008m11)	
	Test Statistic Value	P-Value	Test Statistic Value	P-Value
Panel A: Non-prewhitened Bartlett kernel and Newey-West fixed bandwidth				
L_c	0.844	0.075	0.355	$\geq$ 0.20
MeanF	7.156	0.147	5.292	$\geq$ 0.20
SupF	17.175	0.097	12.717	$\geq$ 0.20
Panel B: Prewhitened Quadratic Spectral kernel and Andrew automatic selection bandwidth				
L_c	0.324	$\geq$ 0.20	0.508	$\geq$ 0.20
MeanF	32.902	0.01	9.119	0.049
SupF	330.744	0.01	15.620	0.161
Panel C: Prewhitened Bartlett kernel and Andrew automatic selection bandwidth				
L_c	0.310	$\geq$ 0.20	0.466	$\geq$ 0.20
MeanF	28.473	0.01	8.405	0.074
SupF	279.197	0.01	15.227	0.182

Note: The results in this table were obtained using the Matlab code downloaded from http://www.ssc.wisc.edu/~bhansen/progs/jbes_92.html. The null hypothesis is parameters are stable in the long-run relationship. The SupF and MeanF statistics were calculated using the trimming range [0.15, 0.85].

From the second part of Table 2, all three test statistics in Hansen (1992) cannot reject the null hypothesis of parameter stability at the 5% level for the subsample period 1986m01-2008m11 except the MeanF statistic in Panel B (which is marginal significant at the 5% level). Figure A.3 in the Appendix shows the corresponding plots of F statistic sequence, which achieve their maximal value exactly in 1994m02 for the plots in all three panels. But they are not significant at the 5% level. Furthermore, in Panel B of Table A.3 in the Appendix, the Z_t^* and Z_α^* of Gregory and Hansen (1996) test statistics for the regime shift model detect 1994m02 as a possible breakpoint. However, they are also insignificant—even at the 10% level of significance.

Therefore, the 1994m02 seems to be a plausible break date in the long-run relationship, but the evidence from formal statistical testing indicates that it is not statistically significant after all. Hence, in our following econometric analysis of the long-run cointegrating relationship, we consider only a single break date likely occurring around the end of 2008 or the beginning of 2009 as short-term interest rates became constrained near the zero-lower bound.

FMOLS Estimates of the Long-Run Cointegrating Relationship.

The FMOLS estimation results for equation (11) over the full sample period, 1986m01 to 2014m12, and over a sub-sample excluding the 2008-09 financial recession and its aftermath of monetary policy stuck at zero, 1986m01 to 2008m11, are presented in columns (1) and (2) of Table 3. All the estimated coefficients have their expected signs as suggested by theory and, in most cases, we find them to be statistically significant at least at the 5% level.

Table 3. FMOLS Estimation Results for Equation (11)

	(1) 1986m01-2014m12	(2) 1986m01-2008m11
$R_{1,t}$	0.404***	0.262***
$\pi_{t,n}^{e}$	2.218***	1.259***
$\pi_{t,1}^{e}$	-0.986**	-0.051
fh_t	-0.021***	-0.027***

Note: ***, ** and * represent the level of significance at 1%, 5% and 10%, respectively. The FMOLS regressions were conducted using Bartlett kernel and Newey-West fixed bandwidth. A constant is included but not reported here.

5.2 Linear SEECM

Based on our discussion of the existing literature in Section 2.2 pointing that structural breaks in the relationship have not been fully explored and the theoretical cointegrating relationship derived in Section 3, we propose a single-equation error correction model (SEECM) to address the concern of non-stationarity and incorporate the theoretical long-run relationship into a model for the long-term interest rate. This specification corresponds to the first row in the benchmark model posited in (10), so a natural extension would be to explore later the complete VECM model. The single-equation specification, however, permits us to examine both the long-run and short-run effects of the foreign holdings ratio of Treasury notes and bonds on the long term interest rates and easily testing and estimating for structural breaks.

Our linear SEECM specification takes the following form:

$$\Delta R_{n,t} = \beta_0 + \alpha\left(R_{n,t-1} - \gamma_1 R_{1,t-1} - \gamma_2 \pi_{t-1,n}^{e} - \gamma_3 \pi_{t-1,1}^{e} - \gamma_4 fh_{t-1}\right) + \beta_1 \Delta R_{n,t-1} + \cdots$$

$$\beta_2 \Delta R_{1,t-1} + \beta_3 \Delta \pi_{t-1,n}^{e} + \beta_4 \Delta \pi_{t-1,1}^{e} + \beta_5 \Delta fh_{t-1} + \varepsilon_t \qquad (12)$$

Based on equations (8) and (9) in Section 3, we define the long-run relationship in (12) in relation to the real rates. As customary in the literature we, retain some additional flexibility in the empirical specification for the estimation of the long-run relationship allowing the coefficient on the short-term real

rate to be inferred from the data instead of imposing the restrictive coefficient value implied by theory. Therefore, we impose the following two restrictions on the long-run relationship in equation (12). First, we assume the long-run inflation expectation has a one-to-one relationship with the long-term rate.[30] Second, we assume the coefficients on the short-term interest rate and the short-run inflation expectations are equal in absolute value but with opposite signs. In summary, we impose the following parametric restrictions based on the implications drawn from the theoretical benchmark in Section 3:

$$\gamma_2 = 1 \text{ and } \gamma_1 + \gamma_3 = 0 \tag{13}$$

We use a nonlinear least squares method to simultaneously estimate parameters in both short-run dynamics and the long-run relationship in equation (12) with the restrictions in (13). We present the estimation results for both the sample without the zero lower bound period (1986m01-2008m11) and the full sample including the zero lower bound period (1986m01-2014m12) in Table 4. We also consider different choices of foreign holdings ratio variables, fh_t. In specification (1) of Table 4, fh_t is the foreign official holdings ratio. In specification (2), fh_t is the foreign total holdings ratio. In specification (3), we consider both the foreign official and foreign private holdings ratio as the corresponding variable for fh_t.

The estimated coefficients on the error correction terms are all significant at the 5% level (not reported here but available upon request), which provides additional validation for the use of the error correction model. For the subsample prior to the zero-lower bound episode, the estimated short-run effects of foreign holdings are not significant. But they become significant using the full sample. The estimated long-run effects of foreign holdings are all highly significant (except the long-run effect of foreign private holdings) for the full sample. In addition, the long-run effects are all larger (in absolute value) for the full sample than for the subsample that excludes the period at the zero-lower bound.

[30] Warnock and Warnock (2009) also impose restriction on the long-run inflation expectation. They argue that the impact of long-run inflation expectation would become inconceivably large without any restriction.

Table 4. Linear SEECM Estimation Results

Specifications	Variables of interest	Subsample 1986m01-2008m11	Full sample 1986m01-2014m12
(1)	foreign official holdings (short-run)	-0.040	-0.064**
	foreign official holdings (long-run)	-0.046***	-0.050***
(2)	foreign total holdings (short-run)	-0.030	-0.052**
	foreign total holdings (long-run)	-0.037***	-0.041***
(3)	foreign official holdings (short-run)	-0.037	-0.061**
	foreign official holdings (long-run)	-0.032***	-0.047***
	foreign private holdings (short-run)	-0.024	-0.051*
	foreign private holdings (long-run)	-0.049**	-0.019

Note: ***, ** and * represent the level of significance at 1%, 5% and 10%, respectively.

So the very distinct results between the subsample and full sample estimates in Table 4 further motivates the necessity of explicitly considering structural breaks in the specification. In the existing literature, Bandholz et al. (2009) also use the SEECM specification. Goda et al. (2013) use the autoregressive distributed lag (ADL) specification. Their studies do not cover the extended sample period including the 2008-09 financial crisis and the subsequent period of monetary policy at the zero-lower bound, though.[31] Table 5 summarizes their estimation results for comparison. Similar to our results, Bandholz et al. (2009) finds an insignificant short-run effect for foreign total holdings in the pre-crisis sample. Our estimated long-run effects range from 3 to 5 basis points which is only slightly lower compared to the 7 to 9 basis points in their studies.

[31] They also use contemporaneous (instead of lagged) explanatory variables in their specification, which is different from our specification.

Table 5. SEECM and ADL Estimation Results in the Existing Literature

Paper	Sample Period	Variables of Interest	Estimated Coefficients
Bandholz et al. (2009)	1986m01-2006m06	foreign total holdings (short-run)	Insignificant and not reported
		foreign total holdings (long-run)	-0.07
Goda et al. (2013)	1994m02-2007m06	foreign official holdings (short-run)	-0.127
		foreign official holdings (long-run)	-0.079
		foreign private holdings (short-run)	Insignificant and not reported
		foreign private holdings (long-run)	-0.089

5.3 Threshold SEECM

As discussed before, we are interested in the potential changes in the impact of foreign holdings of Treasury notes and bonds (net of Federal Reserve holdings) on the long-term interest rate. The onset of the period of monetary policy at the zero lower bound around the end of 2008 or the beginning of 2009 is, in our view, a primary candidate to explain this structural change as indicated while discussing the results of the stability tests reported back in Section 5.1. In the following section, we use the threshold SEECM with the Federal Funds rate (specifically, the average of the first and second lag, i.e. $(R_{1,t-1} + R_{1,t-2})/2$) as the threshold variable to investigate the empirical plausibility of this hypothesis to explain the apparent structural break that we detect in the data. We assume the number of thresholds is one so there are two policy rate regimes in this case.

First, we consider the exogenously determined threshold value ($\tau = 0.7$). This is equivalent to imposing an exogenously determined break date of November 2008 to split the sample because the Federal Funds rate (the average of the first and second lag) only fell below 0.7 in our sample after November 2008.

Define the following dummy variable:

$$d = 0 \; if \; (R_{1,t-1} + R_{1,t-2})/2 \geq \tau \; and \; d = 1 \; if \; (R_{1,t-1} + R_{1,t-2})/2 < \tau.$$

Then, we estimate the corresponding threshold SEECM specification (equation (14) below) with the parameter restrictions in (13). In this specification, the coefficients on the foreign holdings ratio in both the short-run and in the long-run are allowed to vary when the federal funds rate is above or below the threshold value:

$$\Delta R_{n,t} = \beta_0 + \alpha \left(R_{n,t-1} - \gamma_1 R_{1,t-1} - \gamma_2 \pi^e_{t-1,n} - \gamma_3 \pi^e_{t-1,1} - \gamma_4 fh_{t-1} * d - \gamma_5 fh_{t-1} * (1-d) \right) + \cdots$$

$$\beta_1 \Delta R_{n,t-1} + \beta_2 \Delta R_{1,t-1} + \beta_3 \Delta \pi^e_{t-1,n} + \beta_4 \Delta \pi^e_{t-1,1} + \beta_5 \Delta fh_{t-1} * d + \beta_6 \Delta fh_{t-1} * (1-d) + \varepsilon_t \quad (14)$$

Second, we consider the endogenously determined threshold value which minimizes the sum of squared residuals of the corresponding specification. For equation (14), the optimal threshold value for the federal funds rate (the average of the first and second lag) under the searching range of [0, 5] is 0.98. Therefore, one regime is when $(R_{1,t-1} + R_{1,t-2})/2 \geq 0.98$ (corresponding to the period of 1986m01-2008m11). The other regime is when $(R_{1,t-1} + R_{1,t-2})/2 < 0.98$ (corresponding to the period of 2008m12- 2014m12, the zero lower bound period). The endogenously determined threshold value separates the model into two regimes exactly the same as the split in the sample that occurs at the zero lower bound. Therefore, as presented in Table 6, we obtain same estimation results for both exogenously and endogenously determined threshold values.

Table 6. Threshold SEECM Estimation Results

Threshold Value	Variable of Interest	when $(R_{1,t-1} + R_{1,t-2})/2 \geq \tau$	when $(R_{1,t-1} + R_{1,t-2})/2 < \tau$
0.7 (exogenously determined)	foreign official holdings (short-run)	-0.048	-0.138***
	foreign official holdings (long-run)	-0.044***	-0.061***
0.98 endogenously determined over range [0,5]	foreign official holdings (short-run)	-0.048	-0.138***
	foreign official holdings (long-run)	-0.044***	-0.061***

Note: *** represents significance level of 1%. The estimated coefficients on the error correction terms are significant at the 1% level (not reported here).

From Table 6, for the long-run effect, in the first regime (the pre-ZLB regime), a one percentage point increase (decrease) in the foreign official holdings ratio is associated with around a 4 basis point decrease (increase) in the long-term rate. In contrast, this marginal effect increases from 4 basis points to around 6 basis points in the second regime (the ZLB regime). For the short-run effect, it is not significant in the pre-ZLB regime. But it becomes larger (in absolute value) and significant in the ZLB regime.[32]

Because the threshold SEECM is a dynamic model (i.e., the change in foreign holdings ratio at time t has impacts on future long-term nominal interest rates over time), in the following, we compute the dynamic multiplier function and the cumulative dynamic multiplier function to illustrate the estimation results.

The dynamic multiplier function and the cumulative dynamic multiplier function are defined in equations (15) and (16), respectively. Using the estimation results of threshold SEECM, we plot the dynamic multiplier function and the cumulative dynamic multiplier function for each regime in Figure 3:

[32] In the existing empirical literature on quantifying the effect of changes in the foreign holdings (or foreign purchases) of U.S. Treasury securities on the U.S. long-term interest rates, the estimated effects can vary over a wide range due in part to the different measures of the foreign holdings (or purchases) used in the specification. For example, some studies consider both Treasury and Agency securities. While some studies consider foreign total holdings or foreign official holdings. Hence, a comparison across all previous empirical studies and our results is not straightforward due to the that a consistent measurement of the foreign holdings variable across model specifications is lacking.

$$DM_i = \frac{dR_{n,t+i}}{dfh_t}, \textit{for all } i = 0, 1, 2, \ldots \quad (15)$$

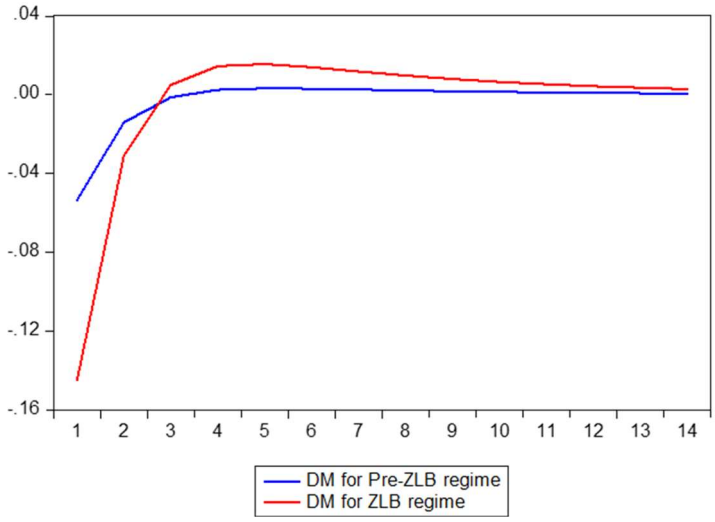
$$Cumulative\_DM_i = \sum_{j=0}^{i} DM_j, \textit{for all } i = 0, 1, 2, \ldots \quad (16)$$

From Figure 3, top panel, the dynamic multiplier functions in both regimes converge to zero very quickly. For the initial period, a one percentage point increase in the foreign holdings ratio will reduce the long-term rate by around 5 basis points in the pre-ZLB regime. But in the ZLB regime, a one percentage point increase in the foreign holdings ratio has relatively larger marginal impact on lowering the long-term rate, which is around 14 basis points, for the initial period.

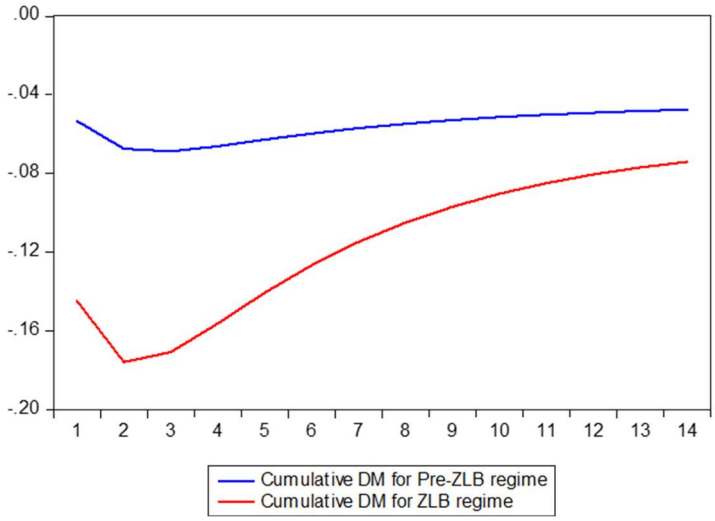
Figure 3, bottom panel, describes the cumulative impacts on the long-term rate over certain period when there is a one percentage point change in the foreign holdings ratio. The value of the cumulative dynamic multiplier function at infinity is called the long-run multiplier (i.e., $\sum_{j=0}^{\infty} DM_j$), which is equal to the long-run effect in the cointegrating vector shown in Table 6. That means, a one percentage point increase in the foreign holdings ratio has an overall impact reducing the long-term interest rate by 6.1 basis points in the ZLB regime compared to 4.4 basis points in the pre-ZLB regime.

Figure 3. Dynamic and Cumulative Dynamic Multiplier Functions

A. Dynamic Multiplier Functions



B. Cumulative Dynamic Multiplier Functions



Note: By transforming the threshold SEECM specification in equation (14) into the autoregressive distributed lag (ADL) form, we can compute the dynamic and cumulative dynamic multiplier functions in equations (15) and (16) straightforwardly.

5.4 Robustness Checks

Threshold SEECM using the Foreign Total Holdings Ratio

As a robustness check, we use the foreign total (sum of official and private) holdings ratio instead of foreign official holdings ratio in our Threshold SEECM specification (equation (14)). The results are presented in Table 7. From Table 7, the endogenously determined threshold value separate the sample into the pre-ZLB and ZLB regimes exactly the same as the case with exogenously determined threshold value. In addition, the effects of foreign total holdings ratio (both short-run and long-run) become larger (more negative) in the ZLB regime than in the pre-ZLB regime. Overall, we obtain similar results as those using the foreign official holdings ratio in Table 6.

Table 7. Threshold SEECM Estimation Results Using Foreign Total Holdings Ratio

Threshold Value	Variable of Interest	when $(R_{1,t-1} + R_{1,t-2})/2 \geq \tau$	when $(R_{1,t-1} + R_{1,t-2})/2 < \tau$
0.7 (exogenously determined)	foreign total holdings (short-run)	-0.036	-0.121***
	foreign total holdings (long-run)	-0.037***	-0.051***
0.98 endogenously determined over range [0,5]	foreign total holdings (short-run)	-0.036	-0.121***
	foreign total holdings (long-run)	-0.037***	-0.051***

Note: *** represents significance level of 1%. The estimated coefficients on the error correction terms are significant at the 1% level (not reported here).

Linear Vector Error Correction Model

To alleviate the concern of endogeneity of the explanatory variables, we consider the following linear Vector Error Correction Model (VECM) specification straight from the theoretical benchmark discussed in Section 3 (equation (10)):

$$\Delta X_t = \mu + \Pi X_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta X_{t-i} + \varepsilon_t \tag{17}$$

where $X_t = (R_{n,t}, R_{1,t}, \pi^e_{t,n}, \pi^e_{t,1}, fh_t)'$.

Using an unrestricted VAR model in levels for the full sample (1986m01-2014m12), the Schwarz information criterion indicates a lag length of 2 (See Table A.4. in the Appendix).[33] Therefore, we choose a lag length of 1 for the VECM specification. In addition, both the Johansen Trace test and Maximum Eigenvalue test indicate one cointegrating relation in the VECM (See Table 8).

By imposing the same restrictions as in (13) on the cointegrating vector of the VECM, the estimation results for the equation of $R_{n,t}$ in the linear VECM are reported in Table 9. Similar to the linear SEECM results in Table 4, the estimated coefficient on the foreign official holdings ratio in the short-run for the subsample excluding the ZLB period is not significant. And the long-run effects are larger (in absolute value) in the full sample than in the subsample.

[33] As an additional robustness check, we expand the sample as far back in time as the dataset would permit starting on 1984m12 but keeping the end period on 2014m12. We find that the result still holds true. These findings are not reported here due to space constraints, but are available upon request from the authors.

Table 8. Johansen's Cointegration Test Results

Trace Test

Hypothesized No. of Cointegrating Equations	Trace Statistic	5% Critical Values	P-values
None	96.05536	69.81889	0.0001
At most 1	43.43587	47.85613	0.1223
At most 2	17.78255	29.79707	0.5822
At most 3	3.990986	15.49471	0.9043
At most 4	0.422666	3.841466	0.5156

Note: The tests are conducted using the MacKinnon-Haug-Michelis (1999) p-values.

Maximum Eigenvalue Test

Hypothesized No. of Cointegrating Equations	Max-Eigenvalue Statistic	5% Critical Values	P-values
None	52.61949	33.87687	0.0001
At most 1	25.65332	27.58434	0.0865
At most 2	13.79156	21.13162	0.3824
At most 3	3.568320	14.26460	0.9019
At most 4	0.422666	3.841466	0.5156

Note: The tests are conducted using the MacKinnon-Haug-Michelis (1999) p-values.

Table 9. Linear VECM Estimation Results

Variable of Interest	Subsample 1986m01-2008m11	Full sample 1986m01-2014m12
foreign official holdings ratio (short-run)	-0.066 [-1.77563]	-0.067 [-2.37879]
foreign official holdings ratio (long-run)	-0.024 [2.81768]	-0.046 [5.67665]

Note: t-statistics are in brackets.

Table 10 reports the estimated coefficients on the error correction terms for each equation in the linear VECM specification. They are significant only for the equation of the long-term interest rate

which is the basis for our SEECM specification. The evidence indicates that the disequilibrium from the long-run cointegrating relationship can only be adjusted through the long-term interest rates. Therefore, the variables $R_{1,t}$, $\pi_{t,n}^{e}$, $\pi_{t,1}^{e}$ and fh_t are weakly exogenous, which provides additional support for the use of the SEECM specification we made in our analysis.

Table 10. Estimated Coefficients on the Error Correction Term in the Linear VECM

$\Delta R_{n,t}$	$\Delta R_{1,t}$	$\Delta\pi_{t,n}^{e}$	$\Delta\pi_{t,1}^{e}$	Δfh_t
For the subsample 1986m01-2008m11				
-0.107	0.027	-0.001	0.004	-0.029
[-4.014]	[1.230]	[-0.232]	[0.401]	[-0.599]
For the full sample 1986m01-2014m12				
-0.113	0.020	0.002	0.003	0.068
[-4.800]	[1.121]	[0.678]	[0.327]	[1.484]

Note: t-statistics are in brackets.

5.5 Counterfactual Analysis

Counterfactual Analysis 1

Based on the estimation results reported for the threshold SEECM specification in Table 6, we consider the following counterfactual analysis to check what would happen if there had been—other things equal—no expansion of the balance sheet of the Federal Reserve through the implementation of various rounds of QE since 2008.

We assume that, for the period of 2009m03 to 2014m12, the Fed's holdings of Treasury notes and bonds could have been kept at the level before the first round of QE. Then, based on our construction of the foreign holdings ratio, we can compute a counterfactual foreign holdings ratio and the one implied by the actual holdings by the Federal Reserve for illustration as shown in Figure 4.

Next, using the estimated coefficients in the threshold SEECM model, we recursively compute the fitted values of the long-term rate based on the actual historical foreign holdings ratio and the counterfactual foreign holdings ratio. As shown in Figure 4, the counterfactual fitted values of the long-term rate are higher than the corresponding historical fitted values of the long-term rate. It implies that the long-term interest rate could have been higher if the Fed had not conducted the successive rounds of QE that it did.

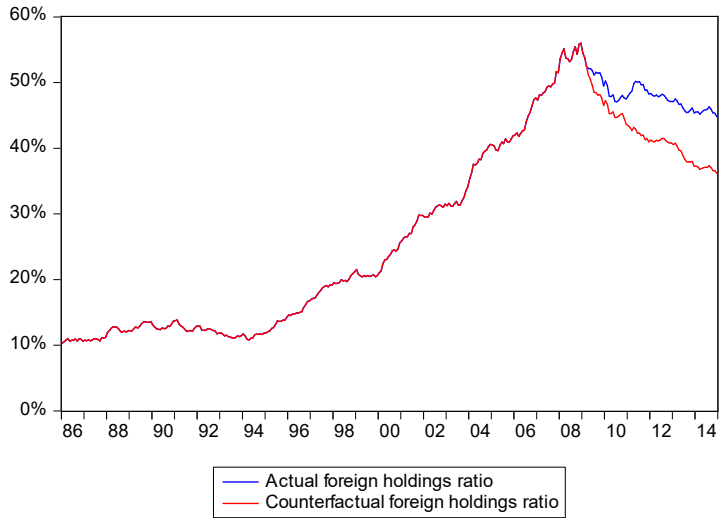
In addition, based on our counterfactual analysis, we can assess the impact of QE on lowering the long-term rate for each round of QE.[34] In Figure 4, the shaded areas represent the three rounds of QE. The differences between the historical and the counterfactual fitted values are summarized in Table 11. By this metric, the QE2 and QE3 rounds may have had relatively larger impacts on lowering the long-term interest rate than QE1. And on average the three rounds of QE pushed down the long-term rate by 38 to 55 basis points.

Comparing with the previous literature studying the effects of QE on the Treasury yield, D'Amico and King (2010) find that from March 2009 to October 2009, the large scale asset purchases by the Fed reduced yields by about 30 basis points on average across the yield curve. They estimate that the yields for 10 to 15 years Treasury securities would have been as much as 50 basis points higher in the absence of the first round of QE. Those findings are, in fact, close to our results in Table 11 for QE1—that appear in line also with our estimates of the impact of QE2 and QE3.

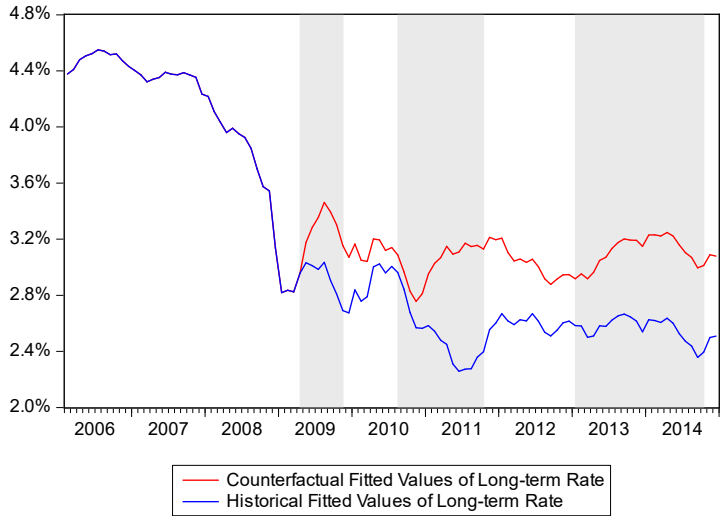
[34] The QE implemented by the Federal Reserve since the end of 2008 included large scale purchases of both long-term Treasury securities and Mortgage-backed securities (MBS). In this case, the timeline of the three rounds of QE is as follows: 2008m11 – 2010m06 for QE1, 2010m11 – 2011m06 for QE2, and 2012m09 – 2014m10 for QE3. However, in this paper, we focus on the foreign demand of U.S. long-term Treasury securities by simultaneously considering the change in the Fed's holdings of Treasury notes and bonds. Therefore, here we define the three rounds of QE as the expansion of the Fed's holdings of Treasury notes and bonds only (not including change in Fed's holdings of MBS). So our defined timeline of the QE are as follows: 2009m04 – 2009m11 for QE1, 2010m08 – 2011m10 for QE2, and 2013m01 – 2014m10 for QE3.

Figure 4. Actual and Counterfactual Foreign Holdings Ratio Effect on Long-Term U.S. Interest Rates

A. Actual and Counterfactual Foreign Official Holdings Ratio



B. Enlarged View of the Fitted Values of the Long-Term Interest Rate



Sources: Bertaut and Tryon (2007), Bertaut and Judson (2014), FRB H.4.1., Department of the Treasury, and authors' calculations based on our benchmark empirical model.

Note: We compute the foreign holdings ratio as foreign holdings of U.S. Treasury notes and bonds divided by the outstanding Treasury notes and bonds excluding Fed's holdings. We construct the counterfactual foreign holdings ratio by assuming that Fed's holdings kept at the level of 2009m03 for the remaining period of our sample.

Table 11. Difference between the Actual and Counterfactual Fitted Values of Long-term Rate

Period		Average	Range
2009m04-2009m11	QE1	38bps	14 – 49 bps
2010m08-2011m10	QE2	53bps	12 – 90 bps
2013m01-2014m10	QE3	55bps	33 – 64 bps

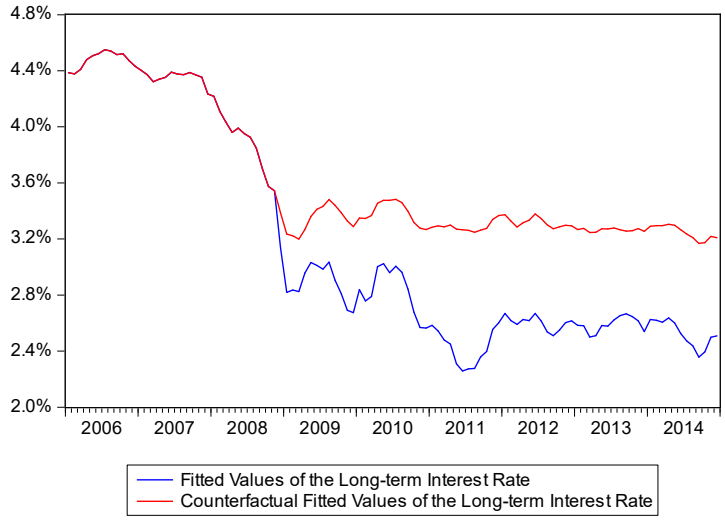
Note: These three periods are determined according to the three rounds of expansion in the holdings of Treasury notes and bonds by the Federal Reserve.

Counterfactual Analysis 2

In order to compare the different impacts on the U.S. long-term interest rate due to the different marginal effects of the foreign holdings ratio in the pre-ZLB and the ZLB regimes, we assume that, in the ZLB regime, the estimated coefficients on the foreign official holdings ratio (both short-run and long-run) stays the same as those in the pre-ZLB regime based on the estimation results from the threshold SEECM in Table 6.

Then, we recursively compute the original and counterfactual fitted values of the long-term interest rate as shown in Figure 5, which illustrates the magnitude of the change in the effect of foreign official holdings ratio on the long-term interest rate in the ZLB regime resulted from the structural break. From Figure 5, the average difference between the original and counterfactual fitted values of the long-term interest rate is 66 basis points.

Figure 5. Fitted Values of the Long-term Interest Rate (Counterfactual Analysis 2)



Source: Authors' calculation based on our benchmark empirical model.

Note: In the counterfactual case, we assume the estimated coefficients on the foreign official holdings ratio in the ZLB regime of the threshold SEECM keep the same as those in the pre-ZLB regime. The original and counterfactual fitted values of the long-term interest rate are computed based on the estimation results in Table 6 and the counterfactual case.

Counterfactual Analysis 3

From Panel A of Figure 2, China's holdings of U.S. Treasury notes and bonds (as a share of outstanding marketable Treasury notes and bonds) had dramatically increased since 1994 and in particular from 2001 to 2011 the speed of increasing was accelerated. As of June 30, 2014, China is the largest foreign holder of U.S. Treasury notes and bonds. So in this part, we focus on China's holdings and investigate the role of changes in China's holdings ratio in explaining the interest rate conundrum around 2004 – 2006.

We are interested in what would happen to the long-term interest rate during the conundrum period if the increasing in China's holdings had not accelerated since 2001. Specifically, we assume that the growth of China's total holdings (sum of official and private holdings) of U.S. Treasury notes and bonds

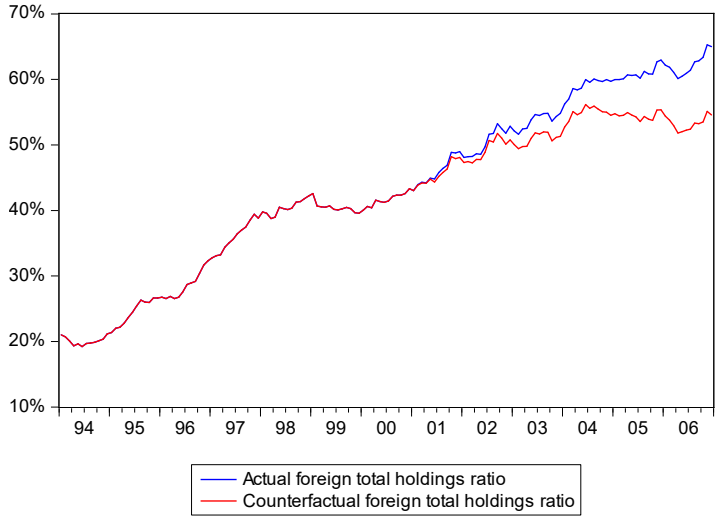
from 2001 to 2006 had been kept at the same pace as during the 1994-2001 period.[35] Based on our counterfactual China's holdings, we can obtain the counterfactual foreign total holdings and accordingly the counterfactual foreign total holdings ratio as shown in Panel A of Figure 6.

Using the estimation results in the threshold SEECM in Table 7, we recursively compute the fitted values of the long-term interest rate (Panel B of Figure 6) based on the actual and counterfactual foreign total holdings ratio. The average differences between the original and counterfactual fitted values of the long-term interest rate is 24 basis points during the conundrum period (2004m06 – 2006m06). That means, if China's holdings after 2001 would had kept at the same pace as in the period of 1994 -2000, the long-term interest rate during the conundrum period would be on average 24 basis points higher. This partially explains the interest rate conundrum during 2004 to 2006.

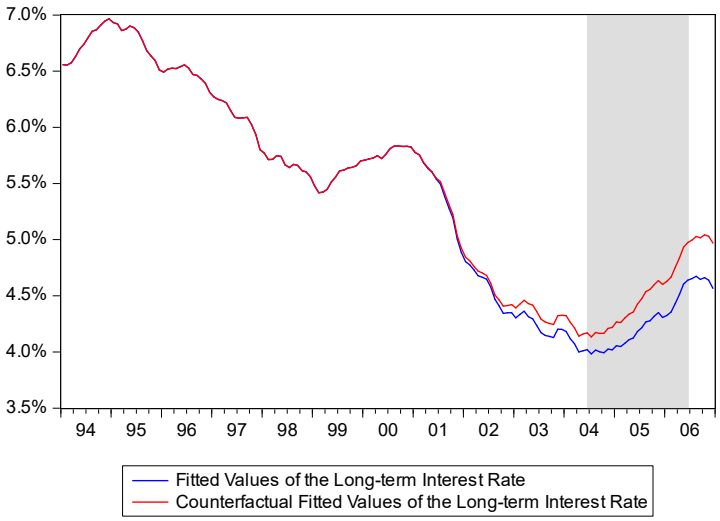
[35] For the construction of the counterfactual of China's holdings, we linearly extrapolate the China's holdings from 2001m01 to 2006m12 using the same slope for the period from 1994m01 to 2000m12. The slope was obtained by running an OLS regression of the China's holdings on a constant and a time trend for the sample period 1994m01 to 2000m12. And the estimated coefficient on the time trend is the slope.

Figure 6. Counterfactual Analysis 3

A. Actual and Counterfactual Foreign Total Holdings Ratio



B. Fitted Values of the Long-Term Interest Rate



Counterfactual Analysis 4

In this part, we continue investigating the impact on long-term interest rate by the change in China's holdings of U.S. Treasuries, but focusing on the recent ZLB period. As shown in Panel A of Figure 2,

China had reduced their holdings of U.S. Treasury notes and bonds since 2011m07. An interesting question is what would happen on the long-term interest rate if China had kept increasing their holdings of U.S. Treasuries during 2011 to 2014.

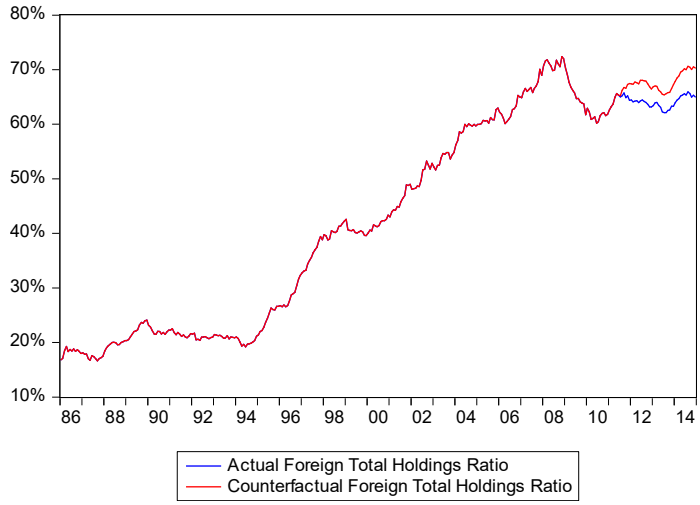
In this counterfactual analysis, we assume that, from 2011m07 to 2014m12, China's holdings had stayed at the same growth rate as in the period of 2001m01 to 2011m06.[36] Similar as in Counterfactual Analysis 3, we can construct the counterfactual foreign total holdings ratio (Panel A of Figure 7) based on the counterfactual China's holdings and compute the original and counterfactual fitted values of the long-term interest rate (Panel B of Figure 7) using the Threshold SEECM estimation results in Table 7.

From Panel B of Figure 7, the average difference between the original and counterfactual fitted values of the long-term interest rate during 2011m07 to 2014m12 is 25 basis points. It implies that, if China had not reduced their holdings since 2011m07 and continues its large amount of purchases of U.S. long-term Treasury securities at the same pace as before, the U.S. long-term interest rate would be on average 25 basis points lower during 2011m07 to 2014m12.

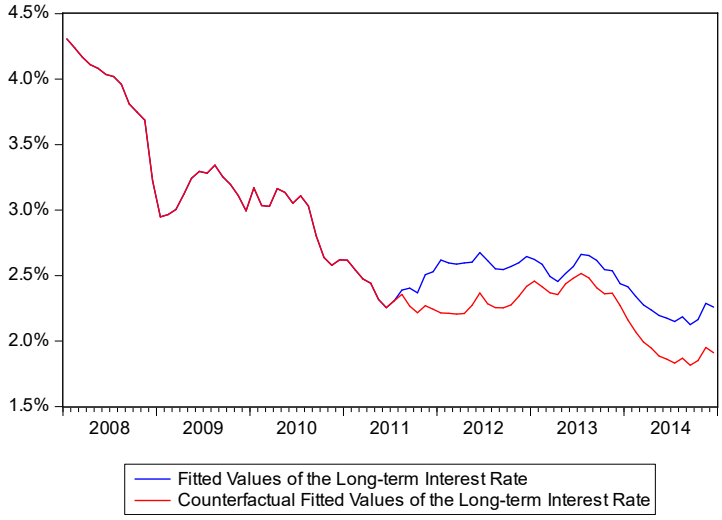
[36] For the construction of the counterfactual China's holdings, we linearly extrapolate the China's holdings from 2011m07 to 2014m12 using the same slope for the period from 2001m01 to 2011m06. The slope was obtained by running an OLS regression of the China's holdings on a constant and a time trend for the sample period 2001m01 to 2011m06. And the estimated coefficient on the time trend is the slope.

Figure 7. Counterfactual Analysis 4

A. Actual and Counterfactual Foreign Total Holdings Ratio



B. Fitted Values of the Long-Term Interest Rate



6 Conclusion

In this paper, we expand the literature on the factors that determine the long-term interest rate behavior by considering an extended sample covering the recent zero lower bound period putting the spotlight on the shifting role of the foreign demand of U.S. Treasuries. We investigate the possible

structural breaks in the impact of foreign demand of U.S Treasury notes and bonds on the U.S. long-term rate. Through a batter of stability tests in the long-run cointegrating relationship, we endogenously find robust empirical evidence supporting the view that the end of 2008 is a significant breakpoint date.

Based on a threshold single-equation error correction model, the endogenously determined threshold value approximately splits the sample into a pre-ZLB and the ZLB regimes. The estimated marginal effect of the foreign holdings ratio on the long-term interest rate become larger (more negative) in the ZLB regime than in the pre-ZLB regime, especially for the long-run effects. So the impact of the foreign holdings ratio on the long-term interest rate shifted when short-term interest rates became stuck at near-zero in the U.S. even when taking into account the concurrent impact of the Fed purchases (QE actions). Therefore, the change in foreign demand of Treasury notes and bonds is still an important contributor to the U.S. long-term rate although its role appears to have shifted. Foreign holdings may have an impact on the effectiveness of monetary policy at the long end of the yield curve not only in the 2004-2006 conundrum period but also in the unconventional monetary policy period that began in the aftermath of the 2008-09 financial recession.

In addition, our results provide a quantitative assessment of the impact of the different rounds of QE on lowering the long-term interest rate. Using a counterfactual analysis assuming no implementation of QE, we find that the three rounds of QE pursued by the Federal Reserve may have lowered the long-term interest rate by 38 to 55 basis points on average.

We also evaluate the effects of China's holdings on the U.S. long-term interest rate during the 2004-2006 conundrum period and the recent ZLB period, respectively. Based on a counterfactual analysis assuming slower growth of China's holdings during 2001 to 2006, we find that change in China's holdings ratio can partially explain the interest rate conundrum by 24 basis points on average. Using a counterfactual analysis assuming continuing increasing of China's holdings until the end of our sample period (2014m12), we find that the recent reduction in China's holdings since 2011m07 had kept the U.S. long-term interest rate from going even lower.

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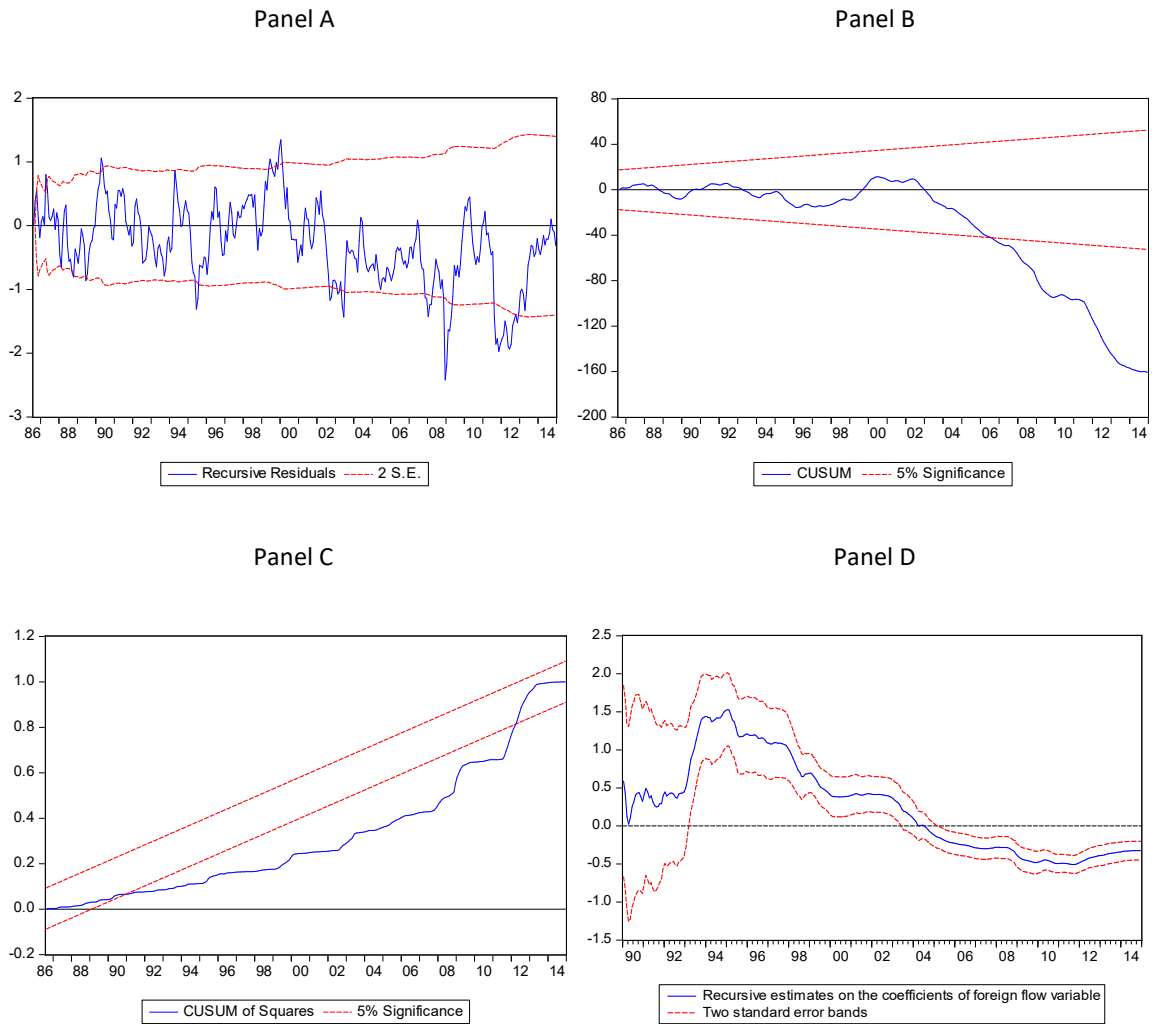
Appendix A. Supplementary Figures and Tables

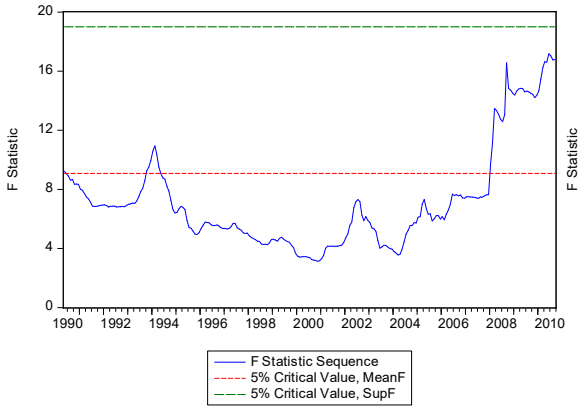
Figure A.1. Evidence of Parameter Instability in the Single-Equation Linear Specification Using an Extended Sample Period of 1986m01-2014m12



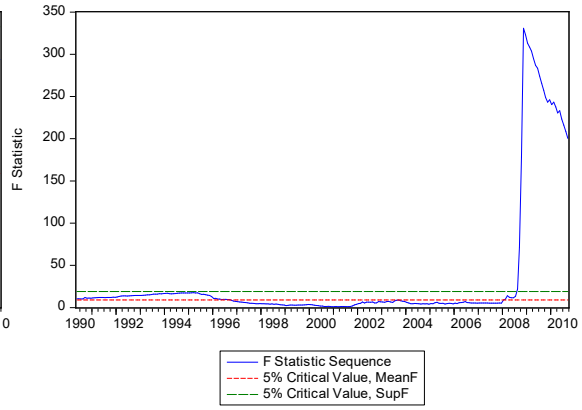
Note: The results in Figure A.1 were obtained using the same specification as in Warnock and Warnock (2009), where the coefficients of the long-run inflation expectation and the short-term interest rate are restricted to sum to one.

Figure A.2. F Statistic Sequence in the Hansen (1992) Tests for the Full Sample

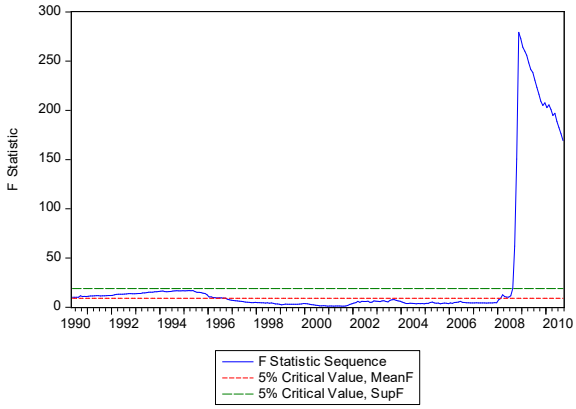
Panel A: Using non-prewhitened Bartlett kernel and Newey-West fixed bandwidth



Panel B: Using prewhitened Quadratic Spectral kernel & Andrew automatic selection bandwidth



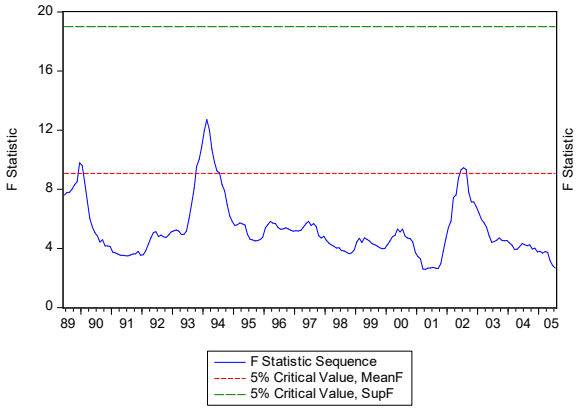
Panel C: Using Prewhitened Bartlett kernel and Andrew automatic selection bandwidth



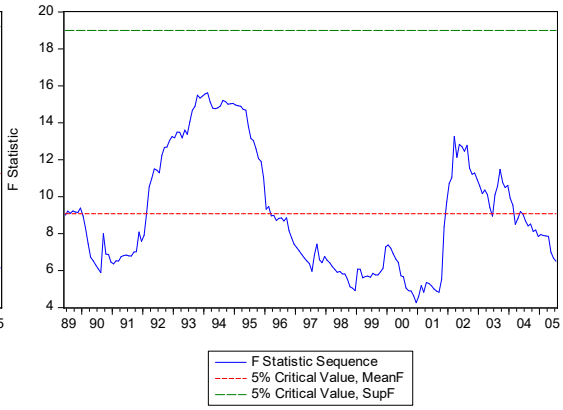
Note: The critical values for SupF and MeanF are from Hansen (1992).

Figure A.3. F Statistic Sequence in Hansen (1992) Tests for the subsample 1986m01-2008m11

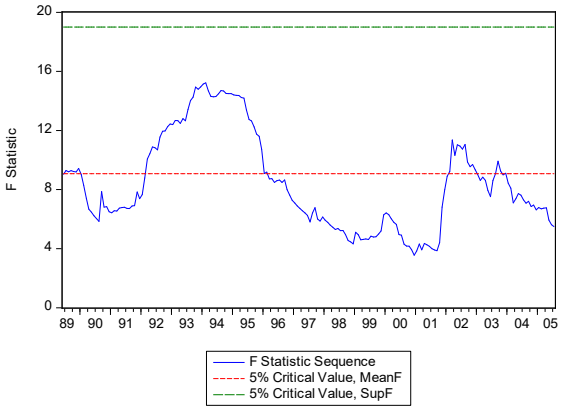
Panel A: Using Non-prewhitened Bartlett kernel and Newey-West fixed bandwidth



Panel B: Using prewhitened Quadratic Spectral kernel & Andrew automatic selection bandwidth



Panel C: Using prewhitened Bartlett kernel and Andrew automatic selection bandwidth



Note: The critical values for SupF and MeanF are from Hansen (1992).

Table A.1. Unit Root Test Results

	Level of Variable		First Difference of Variable		Order of Integration
Variables	t-statistics	p-value	t-statistic	p-value	
$R_{n,t}$	-1.431	0.567	-13.389	0.000	I(1)
$R_{1,t}$	-1.809	0.376	-10.356	0.000	I(1)
$\pi^e_{t,n}$	-1.052	0.735	-5.151	0.000	I(1)
$\pi^e_{t,1}$	-1.738	0.411	-12.993	0.000	I(1)
fh_t	-0.620	0.863	-7.527	0.000	I(1)
$r_{1,t}$	-1.458	0.554	-11.702	0.000	I(1)

Note: This table reports the Augmented Dickey-Fuller unit root test results. The null hypothesis is the series has a unit root. An intercept term is included in the test equation. The sample period is from 1986m01 to 2014m12.

Table A.2. Cointegration Tests for the Long-run Relationship

Test	Test Statistic Value	P-value	Reject the Null
Panel A: 1986m01-2014m12			
Engle-Granger tau-statistic	-6.166	0.000	Yes
Engle-Granger z-statistic	-76.768	0.000	Yes
Phillips-Ouliaris tau-statistic	-5.400	0.003	Yes
Phillips-Ouliaris z-statistic	-55.216	0.002	Yes
Panel B: 1986m01-2008m11			
Engle-Granger tau-statistic	-6.180	0.000	Yes
Engle-Granger z-statistic	-76.408	0.000	Yes
Phillips-Ouliaris tau-statistic	-5.303	0.004	Yes
Phillips-Ouliaris z-statistic	-52.246	0.003	Yes

Note: This table reports the cointegration test results for equation (11) using the full sample period of 1986m01-2014m12 and the subsample period of 1986m01-2008m11 with MacKinnon (1996) P-values. The null hypothesis is that the series are not cointegrated. The tau-statistic is based on the t-statistic and the z-statistic is based on the normalized autocorrelation coefficient.

Table A.3. Gregory and Hansen (1996) Parameter Stability Test Results for Equation (11):

Full Sample vs. 1986m01-2008m11

Model	Test Statistic	5% Critical Value	Breakpoint	Reject the Null
Panel A: Full Sample (1986m01 – 2014m12)				
Level Shift	ADF* = -7.26	-5.56	2010m07	Yes
	Z_t^* = -5.92	-5.56	2010m08	Yes
	Z_α^* = -65.69	-59.40	2010m08	Yes
Level Shift with Trend	ADF* = -7.14	-5.83	2010m07	Yes
	Z_t^* = -6.16	-5.83	2010m08	Yes
	Z_α^* = -70.44	-65.44	2010m08	Yes
Regime Shift	ADF* = -7.71	-6.41	2010m07	Yes
	Z_t^* = -6.44	-6.41	**2009m02**	Yes
	Z_α^* = -77.19	-78.52	2009m08	No
Panel B: Subsample (1986m01 – 2008m11)				
Level Shift	ADF* = -7.58	-5.56	1999m12	Yes
	Z_t^* = -5.72	-5.56	1994m05	Yes
	Z_α^* = -60.21	-59.40	1994m05	Yes
Level Shift with Trend	ADF* = -7.26	-5.83	1991m11	Yes
	Z_t^*= -6.12	-5.83	2002m10	Yes
	Z_α^* = -66.59	-65.44	1996m07	Yes
Regime Shift	ADF* = -7.59	-6.41	1989m11	Yes
	Z_t^* = -6.12	-6.41	**1994m02**	No
	Z_α^* = -67.45	-78.52	**1994m02**	No

Note: This table reports the results of testing the null of no cointegration against the alternative of cointegration with allowance of a possible change in the cointegrating vector at a single unknown break point using the three test statistics for each of the three types of models in Gregory and Hansen (1996) using the full sample period of 1986m01-2014m12 and the subsample period of 1986m01-2008m11. The maximum lag length for the ADF* test is 12 and the lag length was selected using the downward t-statistic method. The 5% critical values are from Gregory and Hansen (1996).

Table A.4. Lag Length Selection

Lag	Schwarz Information Criterion
1	-5.248431
2	**-5.789316***
3	-5.589163
4	-5.549583
5	-5.393838
6	-5.108075
7	-4.852684
8	-4.635849
9	-4.339793
10	-4.042114
11	-3.874639
12	-3.537577

Appendix B. Supplementary Materials

Extended Notes for Table 1 on the Calculations to Back Out the Estimated Impact of US$ 100 Billion Foreign Purchases of U.S. Treasury/Agency Securities on the U.S. Long-Term Treasury Yield in Previous Studies:

Warnock and Warnock (2009): Impact of 100 billion U.S. dollars increase in 12-month foreign total and official purchases of U.S. Treasury and Agency securities on the 10-year U.S. Treasury yield. The date for the scaling factor based on U.S. nominal GDP is 2005m05.

estimated coefficient * (100 / GDP at t-12) * 100 = -0.399 * (100 / 11658) * 100 = -0.34

estimated coefficient * (100 / GDP at t-12) * 100 = -0.188 * (100 / 11658) * 100 = -0.16

Bandholz et al. (2009): Impact of 100 billion U.S. dollars increase in foreign total holdings of U.S. Treasury securities on the 10-year U.S. Treasury yield. The date for the scaling factor based on the total outstanding marketable U.S. Treasury is 2006m06.

estimated coefficient * (100 / total outstanding) * 100 = -7 * (100 / 5714) * 100 = -12.25

Bertaut et al. (2012): Impact of 100 billion U.S. dollars increase in foreign official holdings of U.S. Treasury and Agency securities on the 10-year U.S. Treasury yield. The date for the scaling factor based on the total outstanding U.S. Treasury and Agency securities is 2007.

estimated coefficient * (100 / total outstanding) * 100 =-12.63 * (100 / 10,000) * 100 = -12.63

Beltran et al. (2013): Impact of 100 billion U.S. dollars increase in 1-month foreign official purchases (or 100 billion U.S. dollars increase in foreign official holdings) of U.S. Treasury notes and bonds on the term premium of the 5-year Treasury yield. The date for the scaling factor based on the total outstanding U.S. Treasury notes and bonds (or U.S. nominal GDP) is 2007m06.

For 1-month foreign official flows:

estimated coefficient * (100 / total outstanding) * 100 = -13.5 * (100 / 2915) * 100 = -46

estimated coefficient * (100 / GDP) * 100 = -69.6 * (100 / 14000) * 100 = -50

For foreign official holdings:

estimated coefficient * (100 / total outstanding) * 100 = -(4.6 or 6.2) * (100 / 2915) * 100) = -15.78 or -21.27.