Competitive Price Discrimination, Imperfect Information, and Consumer Search *

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Abstract

I study a homogenous goods model where consumers discover prices via sequential search and firms receive private signals about consumer valuations. The presence of a sufficiently informative signal enables the existence of equilibria with on-path search at intermediate levels of search costs. At low search costs, a structurally different equilibrium without on-path search is played, in which consumers use the threat of searching to ensure low prices. Expansions of the set of consumers that search on-path lead to increased prices. Firm entry is generally only pro-competitive when it eliminates a monopoly and search costs are small.

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1 Introduction

The issue of online price discrimination has gained increased attention by legal authorities around the world in the last years, reflecting the growing body of empirical evidence for its prevalence.[1] In 2016, the competition committee of the OECD recognized that "there are particular reasons to worry that price discrimination in digital markets will be harmful".[2] The European Union (EU) and the state of California have already taken steps to address online price discrimination in the EU GDPR and the CCPA, respectively. In the EU, additional compliance rules for firms engaging in online price discrimination will take effect in 2022.[3] Similarly, price transparency is a core area of competition policy. It is addressed in several directives of the EU, such as the 1998 Unit Prices Directive and the 2005 Unfair Commercial Practices Directive, and in recent legislation in the United States that targets the healthcare market.

The prominence of the aforementioned topics in economic policy poses the question of how they are connected. For instance, is fostering price transparency (in the form of reducing the costs of acquiring price quotes) even desirable in markets where firms price discriminate based on information about consumer valuations? If greater price transparency is beneficial, is this because it induces larger volumes of equilibrium search, and hence more price comparisons? Does the degree of price transparency impact the optimal regulation of price discrimination in said markets? These questions will become even more relevant over time as the prevalence of online price discrimination expands further.

I aim to generate insights pertaining to these questions in the following framework: There is a unit mass of consumers who each want to buy at most one unit of an indivisible and homogeneous good, which is produced by a finite number of firms at zero marginal cost. Consumers have heterogeneous valuations for the good. These valuations are private information to each consumer and are drawn independently and uniformly from the unit interval. Consumers acquire consumption opportunities via sequential search. While the first search is costless, searching any additional firm after that incurs weakly positive search costs. Consumers search randomly - this is without loss of generality when restricting attention to symmetric equilibria in this homogenous goods model.

[1] Hannak et al. (2014) show that e-commerce platforms differentiate prices by whether a consumer uses an app or a website. Larson et al. (2015) demonstrate that prices for Princeton's SAT packages depend on the demographic characteristics of a consumer's ZIP code. Escobari et al. (2019) document that airline ticket prices are higher during business hours, when business travelers are more likely to buy.

[2] The full report may be viewed in CCOECD (2016).

[3] This is outlined in Directive 2019/2161 of the European Parliament, namely EUD2019/2161 (2019).

When a firm is visited by a consumer, the firm receives a noisy private signal about the valuation of this consumer. There are a finite number $K \geq 2$ of possible signal realizations. The probability distribution of the signal depends only on the consumer's valuation and nothing else. Consumers have perfect recall and firms cannot revise a price that they have offered to a given consumer. All firms know nothing about any consumer's search history. Thus, the number of possible information sets that firms have is equal to the number of possible signal realizations.

There are several reasons why different firms could observe varying signals for the same consumer. Most importantly, online retailers may have different information about a given consumer. In principle, web services only have access to first-party data, i.e. information about a consumer's behavior on their own website, aside from basic data such as IP addresses. Because a consumer's behaviour on any particular website is just a snapshot of her underlying preferences, the behavior that different firms observe may be quite heterogeneous and induce different inferences. Similarly, different firms may receive diverse pieces of information from external sources such as data intermediaries. Moreover, even if firms had exactly the same information about a consumer, the way they algorithmically interpret this data may vary.

I initially document that the analysis of noisy price discrimination is intricately linked to the understanding of consumer search choices. In the aforementioned model, no equilibrium with price dispersion exists when search is costless and any consumer has a strictly positive probability of generating any signal. Loosely speaking, there cannot be an equilibrium with price discrimination if any consumer always has a chance of obtaining a low price via search and search is costless. This is because the ability to costlessly search enables consumers to exert significant downward pressure on prices, which precludes price discrimination.

After having documented this result for general signal structures, I restrict attention to binary signal distributions in the remaining analysis. I define the signal realization which becomes more likely to occur when a consumer's valuation rises as the *high signal* and the other signal realization as the *low signal*. A firm's pure strategy is thus a price tuple (p^L, p^H) that consists of a low signal price p^L that is offered to all consumers who generate the low signal and a high signal price p^H that is quoted to the rest. To enable the analysis of equilibria with price discrimination, I also assume that search costs are strictly positive from now on.

Initially, I consider a simplified version of this framework, where the probability to generate the high signal is $1-\alpha$ for any agent with a valuation below 0.5 and α for any agent with a valuation above 0.5.[4] In the following, I refer to consumers with valuation above and below 0.5 as *high-valuation* and *low-valuation consumers*, respectively. In this game, there are three candidates for a symmetric pure-strategy perfect Bayesian equilibrium. In any such equilibrium, $p^L < p^H$ must hold. Two of these equilibrium candidates do not feature search on the equilibrium path, namely the *monopoly equilibrium* and what I refer to as the *search deterrence equilibrium*. In the monopoly equilibrium, any firm sets the prices it would offer to consumers if it were the only active firm or if search was prohibitively costly. In the search deterrence equilibrium, the high signal price is set in such a way that the consumers with the highest incentives to search are just indifferent between searching and not searching.

I refer to the unique pure-strategy equilibrium candidate with on-path search as the *search equilibrium*. Moreover, there potentially also exists a *mixed-strategy equilibrium* with a very particular form. This equilibrium must also feature search on the equilibrium path and firms must offer a deterministic price p^L to all consumers who generate the low signal in this equilibrium.[5] Both these equilibria are characterized by the following results: Firstly, no high-valuation consumer can search on the equilibrium path. If this condition were to be violated, there would either exist undercutting motives or the highest equilibrium price would generate zero profits. Secondly, no consumer would move on to search when being offered a price weakly below or just above the equilibrium low signal price p^L. In addition, any consumer that arrives after searching would directly buy when being offered a price in this interval.[6] These two notions induce a key property of any such equilibrium, namely that the demand created by searchers is fully inelastic around the equilibrium low signal price p^L.

In the following, I describe the parameter regions for which the aforementioned equilibria exist. The monopoly equilibrium exists when search costs are comparatively high. Perhaps surprisingly, the search equilibrium only exists for intermediate search costs. Existence of this equilibrium requires that some low-valuation consumers search on the equilibrium path, while high-valuation consumers do not. High-valuation consumers have comparatively low incentives to search at equilibrium prices, because they have a lower probability of generating the low signal and receiving p^L. Thus, intermediate search costs are necessary and sufficient

[4] I assume that the signal is informative, i.e. $\alpha > 0.5$.

[5] Incidentally, the low signal prices in the mixed-strategy equilibrium and the search equilibrium will be exactly equal, ceteris paribus.

[6] Any consumer that arrives after searching must have received the price p^H at all previous firms and must have a valuation strictly above p^L - otherwise, search would not have been optimal.

to generate this separating behaviour.[7] The interval of search costs for which the search equilibrium can be supported as a perfect Bayesian equilibrium expands as information precision increases.

The search deterrence equilibrium will be played when search costs are low. Existence of this equilibrium requires that high-valuation consumers can credibly promise to search at the monopoly high signal price and at prices slightly below this, which in turn necessitates low search costs. The mixed-strategy equilibrium characterized above exists when search costs are in between the regions of search costs where the search and the search deterrence equilibrium exist, respectively. Loosely speaking, this holds because the structure of any such mixed-strategy equilibrium is a hybrid of the search and the search deterrence equilibrium. Note that these results imply a non-monotonic relationship between search costs and the amount of equilibrium search. At low search costs, no consumer will search on-path. Intuitively, this is because low search costs enable consumers to effectively constrain prices (and their difference) with the threat of searching, which makes the actual act of searching not worthwhile.

Consumer welfare, which I measure by ex-ante consumer utility, is maximal when search costs are zero. However, the effects of changes in search costs are non-monotonic. Reductions of search costs affect prices through two possible channels, namely (i) expanding the set of consumers who search on-path and (ii) changing the search incentives of consumers that do not search on-path. At low levels of search costs for which the search deterrence equilibrium is played, an increase of search costs only affects outcomes through the second channel, because there is no on-path search in this equilibrium. More precisely, an increase of search costs within this equilibrium will lead to higher prices, because the ability of low-valuation consumers to restrict firm pricing by threatening to search is reduced.

There is a threshold level of search costs at which a marginal rise of search costs eliminates the search deterrence equilibrium and the search equilibrium or the mixed-strategy equilibrium will be played. When this happens, both equilibrium prices (or their averages) will jump up discontinuously and consumer welfare is substantially reduced. This upward jump in prices is accompanied by a discontinuous increase in the measure of consumers that search on-path. At this point of discontinuity, both aforementioned channels are active.

[7] Under the weak assumption that income positively correlates with willingness-to-pay, the separating behaviour in the search equilibrium matches the empirical pattern documented in Byrne & Martin (2021), namely that there is an inverse U-shaped relationship between search intensity and income.

High-valuation consumers loose their ability to sustain the search deterrence prices with the credible threat of searching, which induces an upward jump in the high-signal price(s). This, in turn, triggers search by a strictly positive measure of low-valuation consumers. Because these consumers generate locally price inelastic demand around the low signal price, the latter jumps up discontinuously.

In both equilibria with on-path search, an increase of search costs leads to a reduction of the equilibrium low signal price. This result is driven by the first working channel: less consumers arrive after searching, which reduces upward pressure on the low signal price. In the mixed-strategy equilibrium, an increase of search costs leads to an increase in the average high signal price. This reflects the reduced search incentives of high-valuation consumers, who do not search on-path but constrain prices with the threat of searching.

Afterwards, I study the effects of increases in the number of active firms. Changes in the latter do not substantially impact the existence regions of the aforementioned equilibria. The transition from a monopoly to a duopoly can only lead to reduced prices when search costs are low and the search deterrence equilibrium or the mixed-strategy equilibrium would be played under competition. When the market transitions from a monopoly into a duopoly in which the search equilibrium is played, the low signal price increases while the high signal price remains unchanged. Further increases in the number of active firms lead to increased prices in the search equilibrium and the mixed-strategy equilibrium and have no effect in the other equilibria. Thus, increases in the number of active firms only lead to reduced prices when search costs are low and the market transitions from a monopoly to a duopoly. Moreover, the effects of an increase in the number of active firms are not generally equivalent to the impacts of a transition from monopoly to duopoly.

Summing up, the baseline analysis generates the following insights on the interplay of price discrimination and search: Firstly, equilibrium search is not necessarily pro-competitive nor an indicator of low search costs when firms price discriminate. When only consumers with intermediate valuations search on-path, a feature matching the empirical pattern documented by Byrne & Martin (2021), equilibrium search is an imperfect screening device that drives up prices. Secondly, consumer welfare is highest when search costs are zero and price discrimination is infeasible as a result. However, the effects of search cost reductions are non-monotonic, because increases in the measure of consumers that search on-path have detrimental effects on prices. Thirdly, fostering firm entry into such markets is only generally pro-competitive in monopolies and when search costs are low.[8]

To underscore these results, I study generalized binary signal distributions in section 5. Ignoring equilibria that exist only for parameter regions with zero measure, the set of pure-strategy equilibrium candidates is made up of the aforementioned three equilibria and one other candidate when (i) the signal distribution is continuously differentiable and weakly increasing, (ii) the profit functions of a monopolist are strictly concave, and (iii) the set of valuations that satisfy a necessary condition for on-path search is always convex. In the numerical simulations I conduct, the fourth equilibrium candidate almost never actually constitutes an equilibrium. Moreover, the properties of the equilibrium candidates from the baseline model carry over to these generalized settings under weak assumptions.

The rest of the paper proceeds as follows. I lay out the related literature in section 2. In section 3, I set up the general framework and provide initial results. In section 4, I solve the baseline model described above. Section 5 is devoted to the analysis of generalized versions of this model. Section 6 concludes.

2 Related literature

My work represents a theoretical investigation of the interplay between price discrimination and consumer search. Thus, it is related to the developing strand of theoretical research which connects price discrimination to endogenous consumer search choices. Armstrong & Zhou (2016) solve a search model where firms can discriminate against returning consumers. Armstrong & Vickers (2019) analyse a setting where firms face exogenously captive and non-captive consumers and receive information about this status. Fabra & Reguant (2020) study a simultaneous search setting where consumers are heterogeneous in their search costs and the quantity of the good they desire. Firms perfectly observe the consumer's desired quantity and price discriminate based on this information. Braghieri (2019) studies a model where firms condition prices on how a consumer reaches a firm - through an intermediary or via a sequential search process. Preuss (2021) studies a search model where consumers learn about their preferences through search and firms price discriminate based on the search history of a consumer. In contrast to all these papers, I study a model where firms receive information that is (i) noisy and (ii) informative about valuations and not about about search

[8] In appendix E, I solve a version of the above model where firms perfectly know the search history of any arriving consumer in addition to observing the aforementioned signals about consumer valuations. In this model, no symmetric equilibrium with on-path search exists and both prices are below monopoly prices.

costs/history.

Thus, my analysis is more closely related to the papers that model search settings where firms receive noisy information about consumers, which I list in the following. Mauring (2021) and Atayev (2021) study a setting with shoppers and non-shoppers as defined in Burdett & Judd (1983) and Stahl (1989). Mauring (2021) and Atayev (2021) assume that firms receive imperfect information about the affiliation of a particular consumer to the groups of shoppers and non-shoppers. Bergemann et al. (2021) study a homogeneous goods setting where competing firms receive imperfect information about a consumer's search technology and the number of price offers a consumer obtains or has previously obtained. In all these contributions, firms receive information about the search costs of consumers or the size of the choice sets consumers are endowed with. By contrast, I study a setting where firms receive noisy signals about consumer valuations and all consumers are endowed with the same search technology.

Marshall (2020) is the only other paper I am aware of which considers a model of price discrimination based on valuations together with search. In contrast to my work, Marshall (2020) assumes that sellers have perfect information about consumer preferences except for search costs and considers a different search setup: In Marshall (2020), every consumer only interacts with one seller in any period. No recall is possible - when a consumer decides to search in a given period, she foregoes consumption in this period and cannot return to purchase at the firm she interacted with. In addition, Marshall (2020) provides no analytical equilibrium characterization, but empirically calibrates the specified model.

As a model of price discrimination, my work also connects to the extensive literature that exists on price discrimination in itself, such as Villas-Boas (1999), Fudenberg & Tirole (2000), and Acquisti & Varian (2005). More specifically, my work has ties to the theoretical contributions which study price discrimination based on imperfect information in monopoly settings, such as Aron et al. (2006), Belleflamme & Vergote (2016), de Cornière & Montes (2017), Koh et al. (2017), and Valletti & Wu (2020). In contrast to all these papers, I study a setting with competition.

My work is most closely related to the following papers that study price discrimination based on imperfect or partial information about preferences in competitive settings. Esteves (2014) studies a Hotelling-style framework where firms receive noisy information about the horizontal preference parameters of consumers. Peiseler et al. (2021) consider an infinite repetition of a stage game that is very similar to the one analysed in Esteves (2014). Within this framework, the authors analyse under what conditions collusion can be sustained over time. Clavorà Braulin (2021) studies a horizontal differentiation setting where consumer preferences vary in two dimensions. In Clavorà Braulin (2021), firms have perfect information about the realizations of one dimension of consumer preferences, but not both. My work differs from all these papers in the sense that I study a model with endogenous search decisions, which the preceeding papers do not. In addition, I consider a homogenous goods setting with an arbitrary number of firms, while the aforementioned authors examine Hotelling-style duopolies. Moroever, all these papers assume that the market is fully covered, i.e. that all consumers purchase the good in equilibrium, which I do not.[9]

Finally, Belleflamme et al. (2020) study a homogeneous goods model where two competing firms have access to an imperfect profiling technology. The technology they consider differs fundamentally from the one I consider. In Belleflamme et al. (2020), a firm probabilistically either knows a consumer's valuation perfectly or knows nothing about the consumer beyond the ex-ante distribution of preferences. In my setup, a firm receives information about all consumers that visit the firm, but this information is always noisy. Moreover, Belleflamme et al. (2020) do not model endogenous search decisions.

My results are also related to the well-known Diamond paradox established in Diamond (1971). I show that the presence of a sufficiently informative signal about consumer valuations is sufficient to generate equilibria with on-path search when search costs are at intermediate levels. Moreover, the equilibria in my model converge to the Diamond equilibrium whenever the signal becomes uninformative.

3 General framework and initial results

3.1 General framework

I study the following model: There is a unit mass of consumers indexed i, who each want to buy at most one unit of an indivisible and homogenous good. There are N active firms indexed $j \in \{1, 2, ..., N\}$ who all produce this good at zero marginal cost. A firm is free to offer a different price to any consumer that visits the firm. Consumers are heterogeneous in their valuations for the good, namely v_i. These valuations are private information to each

[9] Rhodes & Zhou (2021) show that this assumption is not without loss of generality when studying the welfare effects of price discrimination based on perfect information.

consumer and are known by the consumer at the beginning of the game. The distribution of these valuations, namely the uniform distribution on $[0, 1]$, is common knowledge. Consumers maximize expected utility. When a consumer i buys the good at price p, the utility of the consumer is:

$$u(v_i, p) = v_i - p \tag{1}$$

Consumers visit firms through sequential search. After realizing their valuations, consumers randomly visit some firm first. This incurs no costs to the consumers. The firm that is visited first offers a price to the consumer. After having received this price, a consumer decides whether or not she wants to visit an additional firm, i.e. to search. Searching another firm implies that the consumer incurs a fixed cost equal to $s \geq 0$. The firm that is visited second then also offers the consumer a price. Searching any additional firm after the first always incurs the fixed cost s to the consumer. The game ends when all firms have been sampled or the consumer stops the search process. Then, the consumer decides from which firm to buy the product, or not to buy the good at all. When a consumer does not buy the good, she obtains zero utility. Consumers always choose which firm to visit next randomly - this is without loss of generality when considering symmetric equilibria in this setting.

When a firm j is visited by a consumer i, this firm receives a signal $\tilde{v}_{i,j}$ about the valuation of this consumer. There are a finite number $K \geq 2$ of possible signal realizations, namely $\left\{\tilde{v}^1, ..., \tilde{v}^K\right\}$. The signal probability distribution is expressed by $Pr(\tilde{v}^k|v_i)$. Note that this distribution only depends on v_i and nothing else. In particular, the signals $\tilde{v}_{i,j}$ and $\tilde{v}_{i,-j}$ that two firms j and $-j$ receive for a given consumer i are independent, conditional on v_i.

A firm knows nothing about any consumer's search history. In particular, a firm does not know its position in the search queue of any consumer. Consumers are fully aware of all the prices they have received from any firm they have visited. Moreover, when a consumer visits a firm and decides to search, the consumer can still purchase the good from the initially visited firm at the price that was previously offered to her without further cost. Firms cannot revise a price that they have offered to a consumer.

I use perfect Bayesian equilibrium as a solution concept and focus on symmetric equilibria. A consumer's pure strategy consists of a search strategy and a purchasing strategy. Sequential rationality implies that the consumer's choices must also be optimal off-path, i.e. for prices p_j that the consumer would not be offered if firms play their equilibrium strategies. A firm's pure strategy is a mapping $p : \{\tilde{v}^L, \tilde{v}^H\} \to \mathbb{R}$. There are no relevant off-path decisions for firms. Note that firms form beliefs over three consumer characteristics: (i) the true valuation of the consumer, (ii) the consumer's search queue, and (iii) the prices the consumer has received from the other firms (if any). In the following, I omit the index i when describing a given consumer. I say that there is search on the equilibrium path when there is a strictly positive measure of consumers who search with positive probability in equilibrium.

3.2 General results

To break ties, I assume that consumers will search if and only if it is strictly optimal. Define a cutoff function $\hat{p}(v, n)$ such that a consumer who has already sampled n firms will search another firm if and only if the lowest price she has found sofar is above $\hat{p}(v, n)$. In a symmetric equilibrium, the optimal search rule of any consumer will be myopic.

Lemma 1 *In a symmetric equilibrium, the optimal search rule of any consumer will be myopic, i.e.* $\hat{p}(v, n) = \hat{p}(v)$ *holds for all* $n \in \{1, ...N-1\}$.

The proof of this lemma follows standard arguments and is based on the fact that both search costs and the distribution of prices stay unchanged along the search path. Moreover, any symmetric pure-strategy equilibrium in the above game is characterized by the following:

Proposition 1 *Consider a symmetric pure-strategy equilibrium and define the lowest equilibrium price as* $p^{min} = \min_k p^k$.

- *If* $s > 0$*, no consumer would search after being offered the price* p^{min}*. Moreover, any consumer that arrives at a firm after searching buys immediately when offered* p^{min}.
- *Consider any price* p^k *that is offered in equilibrium. The set of consumers with* $v \geq p^k$ *that (i) have a strictly positive probability of being offered* p^k *in equilibrium and (ii) would search when receiving* p^k *must have measure zero.*

Note firstly that p^{min} is the best possible price offer a consumer can obtain through searching in a symmetric pure-strategy equilibrium. After receiving this best possible outcome, there is no more reason to search. This notion also implies that any consumer that arrives after searching must have received a price strictly above p^{min}. Because $s > 0$, any such consumer must also have $v > p^{min}$. Taken together, these arguments imply the first result.

Now consider any equilibrium price p^k and suppose, for a contradiction, that there is a strictly positive measure of consumers with $v > p^k$ who would search upon receiving this price and receive this price with strictly positive probability. Conditional on the valuation

v, the probability to receive this price will be the same at all firms in a symmetric equilibrium. Morever, any consumer who searches upon receiving p^k at the initial firm will continue searching until obtaining a lower price or there are no more firms to sample. Thus, there will be a strictly positive measure of consumers with $v > p^k$ who receive this price at all N firms. However, this would induce some firm to slightly undercut p^k, because this ensures that the sale is made to all these consumers, which represents a contradiction.

Having established this, the following lemma describes the potential equilibria of the aforementioned game when search is costless. To that end, I define $\Pi^M(p_j|\tilde{v}^k)$ as the profit functions of a monopolist in the above framework, conditional on the signal $\tilde{v}^k$, with global maximizers $\{p^{k,M}\}_{k\in\{1,\dots,K\}}$. Define the minimum of these as $p^{min,M} = \min_k p^{k,M}$. I further say that an equilibrium features price dispersion unless there exists a price that all firms play with probability 1 after all signals.

Proposition 2 *Suppose that $Pr(\tilde{v}^k|v) \in (0,1)$ holds for any $k \in \{1,..,K\}$ and for any $v \in [0,1]$.*

- *If $s = 0$, no equilibrium with price dispersion exists.*
- *Suppose that $s = 0$. If $\Pi^M(p_j|\tilde{v}^k)$ are all strictly concave in p_j, the uniform price that firms offer to all consumers in an equilibrium must satisfy $p^0 \in [0, p^{min,M}]$.*

To see why there cannot be price dispersion under the specified signal structure when search is costless, consider a symmetric pure-strategy equilibrium candidate $(p^1, ...p^K)$ with price dispersion. Define $p^{max} = \max_k p^k$ as the maximal price offered in this equilibrium and consider consumers with $v \geq p^{max}$. Any consumer with $v \geq p^{max}$ will have a strictly positive probability of receiving this price because $Pr(\tilde{v}^k|v) \in (0,1)$ holds for any valuation v and any signal. However, this assumption also implies that all these consumers have a strictly positive chance of receiving a better price by searching. Because search is costless, all these consumers will thus search after receiving p^{max}. This breaks the equilibrium by proposition 1.

Now consider symmetric mixed-strategy equilibrium candidates. Once again, define p^{max} as the highest price a consumer can receive. Any consumer with $v \geq p^{max}$ will search upon receiving this price. If this price is offered with strictly positive probability by the firms, consumers' search decisions create undercutting motives that break the equilibrium. If this price is offered with zero probability, any consumer that sees multiple price offers will buy with probability 0 at a firm that offers her the price p^{max}. Because all consumers with $v \geq p^{max}$ search upon being offered this price, profits at p^{max} will thus be zero. This implies the existence of a profitable deviation at the information sets where firms offer p^{max}. Similar arguments can be made to rule out the existence of asymmetric equilibria with price dispersion.

Thus, proposition 2 establishes that equilibria with price dispersion cannot exist when search is costless and firms receive noisy signals with the specified properties.[10] To analyse price discrimination that is based on these types of signals, I assume $s > 0$ from now on. Equilibria with price dispersion can exist when the signals are deterministic, even if search is costless. To achieve tractability, I restrict attention to binary signals in the following analysis.

4 Baseline model

4.1 Setup and monopoly solution

Consider the following special case of the model outlined above: The valuations of the consumers are uniformly distributed on the interval $[0, 1]$. Search costs are strictly positive and there are just two possible realizations of the signal, namely $\left\{\tilde{v}^L, \tilde{v}^H\right\}$. The signal follows a step-function distribution with precision parameter α, where the following holds:

$$Pr(\tilde{v}^H|v_i) = \begin{cases} \alpha & v_i \geq 0.5 \\ 1-\alpha & v_i < 0.5 \end{cases} \tag{2}$$

I assume that $\alpha > 0.5$, i.e. that the signal is informative. There are $N \geq 2$ firms in the market. In the following, I refer to the setting I have just described as the *baseline model*. I believe that this step-function signal distribution constitutes a useful starting point for the following reasoning: A binary signal about $v_i \sim U[0, 1]$ can be viewed as an information structure which indicates whether a consumer's valuation is in the upper or lower half of its support. To the extent that these groups differ systematically in some manifest way, the signal structure likely varies the most around the cutoff which separates the two groups.

Defining the optimal prices of firms with monopoly power is instructive to understand the competitive equilibria. In the baseline model, the monopoly profit functions are strictly concave piecewise and the implied optimal monopoly prices $p^{L,M}$ and $p^{H,M}$ satisfy the following:

Lemma 2 *Consider the baseline setting and suppose that $s \rightarrow \infty$. The resulting optimal monopoly low and high signal prices are given by $p^{L,M} = 1/4\alpha$ and $p^{H,M} = 0.5$.*

[10] An analogue of this result would also hold in a model of simultaneous search where consumers decide how many firms they want to visit before making the first (random) search. For a discussion of this, please see appendix F.

The monopoly low signal price $p^{L,M}$ is falling in α because a more precise signal implies that the average valuation of consumers that generate the low signal decreases.

4.2 Competitive pure-strategy equilibria

In this section, I characterize the structure of symmetric pure-strategy equilibria, beginning with the following proposition:

Proposition 3 *Consider the baseline setting. A symmetric pure-strategy equilibrium with search on the equilibrium path must satisfy:*

$$p^L < p^H = p^{H,M} \tag{3}$$

There are exactly two candidates for a symmetric pure-strategy equilibrium without search on the equilibrium path, namely $(p^L, p^H) = (p^{L,M}, p^{H,M})$ *and* $(p^L, p^H) = (p^{L,M}, s/\alpha + p^{L,M})$*.*

The above result shows that there are three types of possible symmetric pure-strategy equilibria in the baseline model. Firstly, there potentially exist equilibria with search on the equilibrium path. I show later that there exists a unique candidate for such an equilibrium, which I then refer to as the *search equilibrium*. In addition, there are exactly two candidates for a pure-strategy equilibrium without on-path search. I label these equilibria as follows: The price vector $(p^{L,M}, p^{H,M})$ is labelled the *monopoly equilibrium*, and the price vector $(p^{L,D}, p^{H,D}) = (p^{L,M}, s/\alpha + p^{L,M})$ is labelled the *search deterrence equilibrium*. Note also that the low signal price must always be strictly below the high signal price in any symmetric pure-strategy equilibrium.

In the monopoly equilibrium, firms set the same prices that a monopolistic firm would set. In the search deterrence equilibrium, the high signal price is set in such a way that consumers with valuations $v \in (s/\alpha + p^{L,D}, 0.5)$ are exactly indifferent between searching and not searching.[11] These consumers have the highest probability of generating the low signal and receiving the lower of the two equilibrium prices. Thus, these consumers have greater incentives to search at equilibrium prices than consumers with $v > 0.5$, which means that there will be no on-path search in this equilibrium. Note further that consumers with $v \in (s/\alpha + p^{L,D}, 0.5)$ would search and never return if offered a price above $p^{H,D}$ in this equilibrium.

[11]Consumers with $v < s/\alpha + p^{L,D}$ do not fulfil the necessary conditions for search.

I now establish when the aforementioned equilibria without on-path search exist. To that end, I define $\Pi^C(p_j;\tilde{v})$ as the expected profits a firm obtains (in competitive settings) from consumers that generate the signal $\tilde{v} \in \left\{\tilde{v}^L, \tilde{v}^H\right\}$ when offering a price p_j.

Proposition 4 *Consider the baseline setting. The price tuple* $(p^{L,M}, p^{H,M})$ *can be supported as a perfect Bayesian equilibrium if and only if:*

$$0.5 \leq s/\alpha + p^{L,M} = s/\alpha + 1/(4\alpha) \tag{4}$$

By contrast, the price tuple $(p^{L,D}, p^{H,D}) = (p^{L,M}, s/\alpha + p^{L,M})$ *can be supported as a perfect Bayesian equilibrium if and only if the following two conditions jointly hold:*

$$0.5 > (1-\alpha)p^{L,D} + \alpha p^{H,D} + s \tag{5}$$

$$\Pi^C((1-\alpha)p^{L,D} + \alpha p^{H,D} + s; \tilde{v}^H) \leq \Pi^M(p^{H,D}; \tilde{v}^H) \tag{6}$$

The condition laid out in equation (4) is both necessary and sufficient for existence of the monopoly equilibrium. This inequality is equivalent to the property that the cutoff price $\hat{p}(v)$ of any consumer is above $p^{H,M} = 0.5$. If this holds true, no consumer will find it optimal to search on the equilibrium path. Then, the competitive profit function corresponding to either signal is bounded from above by the respective monopoly profit function. Since both prices maximize monopoly profits after the respective signal, there can be no profitable deviations. Condition (4) is also necessary for existence of the monopoly equilibrium. If this condition is violated, some consumers would search on the equilibrium path, which would create incentives for an upward deviation from the equilibrium low signal price.

As discussed previously, there will not be on-path search in the search deterrence equilibrium. This equilibrium can only be supported when all agents with $v > 0.5$ would search upon being offered the out-of-equilibrium price $p^{H,M} = 0.5$. Condition (5) implies that this holds true. If consumers with $v > 0.5$ would not search when being offered $p^{H,M}$, a deviation to $p^{H,M}$ when observing the high signal would grant the firm profits equal to $\Pi^M(p^{H,M}; \tilde{v}^H)$. Because there is no on-path search in this equilibrium and the monopoly high signal profit function has a strict optimum at $p^{H,M}$, this deviation would be profitable.

It remains to check other deviations from the search deterrence equilibrium prices. There exist no profitable deviations from the low signal price, since there is no on-path search and $p^{L,D}$ maximizes the low signal monopoly profit function. By an analogous logic, there exist no profitable downward deviations from $p^{H,D}$ since $p^{H,D} < p^{H,M}$. Now consider deviation prices

$p_j > p^{H,D}$. Setting such a price triggers search by all consumers with $v \in (s/\alpha + p^{L,D}, 0.5)$. Moreover, consumers with $v > 0.5$ would move on to search at such a price p_j if and only if:

$$p_j > (1-\alpha)p^{L,D} + \alpha p^{H,D} + s \tag{7}$$

Note that condition (5) implies that the cutoff price on the right-hand side of this equation is strictly below 0.5. Note further that any consumer who leaves firm j to search after receiving a price $p_j > p^{H,D}$ will never return. Thus, setting a price above $(1-\alpha)p^{L,D} + \alpha p^{H,D} + s$ will yield zero profits. In addition, any price in the interval $(p^{H,D}, (1-\alpha)p^{L,D} + \alpha p^{H,D} + s)$ is dominated by setting the price $p_j = (1-\alpha)p^{L,D} + \alpha p^{H,D} + s$. At any such price, consumers with $v \leq 0.5$ will surely not buy at the firm while the sale is surely made to all consumers with $v > 0.5$ who arrive at firm j first. Thus, the most profitable deviation from the equilibrium high signal price would be to the price $p_j = (1-\alpha)p^{L,D} + \alpha p^{H,D} + s$, which is not profitable if and only if condition (6) holds true.

This completes the definition of the conditions which guarantee existence of the equilibria without on-path search. I thus turn my attention to the pure-strategy equilibrium candidate with search on the equilibrium path. In this equilibrium, the mass of agents that search on the equilibrium path and the equilibrium prices are determined jointly. To pin down the equilibrium prices, I thus firstly characterize the equilibrium search behavior.

Lemma 3 *Consider the baseline setting. A symmetric pure-strategy equilibrium (p^L, p^H) with search on the equilibrium path must satisfy:*

$$\frac{s}{\alpha} + p^L < 0.5 \leq \frac{s}{1-\alpha} + p^L \tag{8}$$

In such an equilibrium, the search cutoff $\hat{p}(v)$ of consumers with $v \in (s/\alpha + p^L, 0.5]$ equals $\hat{p}(v) = s/\alpha + p^L$. All other consumers have a search cutoff strictly above $p^H = 0.5$.

By proposition 1, agents with $v > 0.5 = p^H$ cannot search on-path in an equilibrium of the above form. The inequality $0.5 \leq s/(1-\alpha) + p^L$ is equivalent to this property. Moreover, an equilibrium with on-path search must have a property which guarantees that on-path search is optimal for some agents. The inequality $s/\alpha + p^L < 0.5$ guarantees that there exists an interval of consumers with $v \in (s/\alpha + p^L, 0.5]$ who optimally search when offered $p^H = 0.5$.

Equipped with the characterization of the equilibrium search strategies, I now move on to define the competitive profit functions of a firm. To that end, I define the density of consumers with valuation v that arrive at a firm in position $k \in \{1, ..., N\}$ as $f^k(v)$. Because the first search is random and valuations are drawn from the uniform distribution, $f^k(v) = 1/N$ holds true for all k.

Firstly, suppose that a firm j sets a price $p_j \in (0, s/\alpha + p^L)$. By lemma 3, no consumer that arrives at firm j first would move on to search when receiving such a price. Thus, a consumer that arrives at firm j first buys at firm j if and only if $v \geq p_j$. By lemma 3, consumers that arrive at firm j after searching must have generated the high signal (and thus received the high price $p^H = 0.5$) at all previously visited firms and must have $v \in (s/\alpha + p^L, 0.5]$. Because $s/\alpha + p^L < 0.5 = p^H$ must hold, consumers arriving after search will thus surely buy at firm j when offered a price $p_j \in (0, s/\alpha + p^L)$. For prices $p_j \in (0, s/\alpha + p^L)$, the competitive profit functions $\Pi^C(p_j; \tilde{v}^k)$ are hence given by:

$$\Pi^C(p_j; \tilde{v}^k) = \underbrace{p_j \int_{p_j}^{1} f^1(v) Pr(\tilde{v}^k|v) dv}_{\Pi^M(p_j;\tilde{v}^k)} + \underbrace{p_j \sum_{j=2}^{N} \left[\int_{s/\alpha+p^L}^{0.5} f^j(v) Pr(\tilde{v}^k|v) Pr(\tilde{v}^H|v)^{j-1} dv \right]}_{M^k(\alpha,s;p^L)} \quad (9)$$

Note that the first term on the right-hand side is equal to the monopoly profit function, evaluated at this price and for the given signal, and that $M^k(\alpha, s; p^L)$ is the measure of consumers that arrive after searching and generate the signal $\tilde{v}^k$.

Profits in the price interval $p_j \in (s/\alpha + p^L, 0.5)$ are only relevant when checking for potential deviations. Thus, consider prices in the interval $p_j \in [0.5, \alpha p^H + (1-\alpha)p^L + s]$, which contains p^H. Consumers that arrive at firm j after searching could not buy the good at these prices. Moreover, no consumer that arrives at firm j first and has $v > 0.5$ would move on to search at these prices. Thus, profits in this price interval are equal to $\Pi^M(p_j; \tilde{v})$. A natural candidate for an equilibrium high signal price would thus be the global maximizer of $\Pi^M(p_j; \tilde{v})$, namely $p^{H,M}$. By proposition 3, we know that this price is the unique high signal price that can be supported in a perfect Bayesian equilibrium with on-path search. In the price interval $p_j \in [\alpha p^H + (1-\alpha)p^L + s, 1]$, profits will be zero. [12]

Given that the equilibrium low signal price must satisfy $p^L \in [0, s/\alpha + p^L]$ and competitive profits are continuously differentiable in this interval, the low signal price p^L must be a

[12]The model can be equivalently formulated with a representative agent, where the competitive profit functions are equivalent to the ones presented, but scaled by constants which represent the probabilities of reaching the high signal and low signal information sets, respectively - please see appendix B.3. for details.

fixed point of the following first-order condition:

$$\left.\frac{\partial \Pi^M(p_j;\tilde{v}^L)}{\partial p_j}\right|_{p_j=p^L} + M^L(\alpha, s; p^L) = 0 \tag{10}$$

Strict concavity of the monopoly low signal profit function $\Pi^M(p_j;\tilde{v}^L)$ in the interval $[0, 0.5]$ implies that there is a unique solution to this equation (if a solution that satisfies the properties laid out in lemma 3 exists). Moreover, stricty concavity of $\Pi^M(p_j;\tilde{v}^L)$ will imply the following important result: The low signal price $p^{L,S}$ will be *higher* in this equilibrium than the monopoly low signal price $p^{L,M}$. The following proposition defines when the search equilibrium can be supported as a perfect Bayesian equilibrium:

Proposition 5 *There is a unique pure-strategy equilibrium candidate $(p^{L,S}, p^{H,S})$ with search on the equilibrium path, where $p^{L,S}$ solves equation (10) and $p^{H,S} = p^{H,M} = 0.5$.*

This price tuple constitutes a perfect Bayesian equilibrium (with the sequentially rational search behaviour given in lemma 3) if and only if the following conditions jointly hold:

$$\frac{s}{\alpha} + p^{L,S} < 0.5 \leq \frac{s}{1-\alpha} + p^{L,S} \tag{11}$$

$$\Pi^C(s/\alpha + p^L; \tilde{v}^H) \leq \Pi^M(0.5; \tilde{v}^H) \tag{12}$$

Moreover, firm profits are always strictly higher in the search equilibrium than in the monopoly equilibrium and in the search deterrence equilibrium.

Uniqueness of the equilibrium follows from previous arguments. Equation (11) is equivalent to the condition defined in lemma 3. In addition, it needs to be ensured that there are no profitable deviations from equilibrium prices. In this equilibrium, both competitive profit functions are bounded from above by the corresponding monopoly profit functions in the price interval $p_j \in (s/\alpha + p^L, 1]$. This is because any consumers (first arrivers and searchers) with $v \in (s/\alpha + p^L, 0.5)$ could only buy at such prices when generating the high signal at all firms. However, this is quite unlikely. Thus, a firm loses more first arrivers at these prices (in comparison to the monopoly outcome) than it gains consumers who arrive after search.

Having noted this, consider possible deviations from the low signal price $p^{L,S}$. Recall that the equilibrium low signal price in the search equilibrium is higher than the monopoly low signal price $p^{L,M}$. Note further that low signal profits in the price interval $p_j \in (0, p^{L,S} + s/\alpha]$

were given by the following expression:

$$\Pi^C(p_j; \tilde{v}^L) = \Pi^M(p_j; \tilde{v}^L) + p_j M^L(\alpha, s; p^{L,S}) \tag{13}$$

Because $p^{L,M} < p^{L,S}$, equation (13) implies that competitive profits when setting $p^{L,M}$ are higher than the monopoly profits when setting this price. Moreover, recall that $p^{L,S}$ must maximize $\Pi^C(p_j; \tilde{v}^L)$ in the interval $p_j \in [0, s/\alpha + p^{L,S}]$, which includes $p^{L,M}$. Together, these two notions imply that the profits a firm obtains from consumers that generate the low signal are higher in the search equilibrium than in the monopoly outcome. This result, together with the fact that the competitive profit functions are bounded from above by the monopoly profit functions for $p_j \in (s/\alpha + p^L, 1]$, implies there are no profitable deviations from $p^{L,S}$. By a similar logic, the most profitable deviation from $p^{H,S}$ would be to $p_j = s/\alpha + p^L$. This deviation is not profitable if and only if equation 12 is satisfied.

4.3 Competitive mixed-strategy equilibria

Now, I move on to characterize the set of symmetric mixed-strategy equilibria (MSE) that can exist. First, note the results of the following lemma:

Lemma 4 *Consider a symmetric mixed-strategy equilibrium in the baseline setting. Define p^{min} and p^{max} as the lowest and highest prices that are offered in this equilibrium. It must hold that:*

- *The firm offers a deterministic price when observing the low signal, namely p^{min}.*
- *The probability that a firm offers a price above 0.5 after either signal is 0. There is on-path search, but no consumer with $v > 0.5$ will search on-path.*

In the following, I provide some intuition for these results when restricting attention to equilibria where actions are drawn from atomless, gapless distributions. The formal arguments that deal with price distributions that have gaps and atoms are relegated to the appendix. No consumer would search when receiving a price around p^{min} and all consumers that arrive after searching directly buy. This implies the first result, since the corresponding competitive objective function has the structure laid out in equation (9) and must thus be strictly concave. The price p^{max}, which must be offered after $\tilde{v}^H$, has to be weakly below 0.5. If $p^{max} > 0.5$, all consumers with $v \geq p^{max}$ have identical search incentives and there are just two possible outcomes: Either all of the consumers with $v \geq p^{max}$ search upon receiving p^{max}, or none of them search. If all of them search, there are either undercutting incentives from p^{max} (if p^{max} is played with positive probability) or p^{max} yields zero profits (if p^{max} is played with zero probability). Both of these notions break any such equilibrium. If none of them search at this price p^{max}, the corresponding competitive profit function would be strictly increasing on $[0.5, p^{max}]$, a contradiction.

One can further narrow down the structure of possible mixed-strategy equilibria. Define p^L as the price that is set after $\tilde{v}^L$. Suppose, for simplicity, that the distribution of prices that are offered after the high signal is atomless and gapless, with support $[\underline{p}^H, \bar{p}^H]$.

Lemma 5 *Consider a symmetric mixed-strategy equilibrium in the baseline setting where the high signal action is drawn from an atomless, gapless distribution.*

- *It must hold that $p^L + s/\alpha \leq \underline{p}^H$. Any consumer with $v \in (s/\alpha + p^L, 0.5]$ will search with probability 1 if she generates the high signal. No other consumer will search on-path.*
- *For given parameters s, α, and N, the price p^L must equal the low signal price in the search equilibrium, i.e. $p^{L,S}$.*
- *Firm profits in such a MSE are weakly below those made in the search equilibrium, with a strict inequality unless $\bar{p}^H = 0.5$. Whenever the MSE with $\bar{p}^H = 0.5$ exists, so does the search equilibrium.*

The first result holds because $p^L + s/\alpha$ is the cutoff price of any consumer with $v < 0.5$. If this cutoff price were above $\underline{p}^H$, no such consumer would search when receiving a price in an open ball around $\underline{p}^H$. Then, competitive profits around this price would be strictly increasing, a contradiction to mixing indifference. This result implies that the structure of any such mixed-strategy equilibrium is very similar to that of the search equilibrium. Only consumers with $v \in (p^L + s/\alpha, 0.5]$ will search on path and they will do so if and only if they generate the high signal. This property, together with the notions that (i) no consumer would leave to search when receiving a price around p^L and (ii) any consumer that arrives after searching would immediately buy upon receiving such a price, implies that the measure of consumers that search-on-path is the same in the search equilibrium and in this mixed-strategy equilibrium, ceteris paribus. This feature generates the second result.

The third result of this lemma indicates that a subset of the MSE under consideration is of particular interest, namely the MSE where $\bar{p}^H < 0.5$. By contrast, firms have no incentives to play an MSE with $\bar{p}^H = 0.5$, because the simpler search equilibrium would also be available to play. Thus, I focus on the computation of the MSE with $\bar{p}^H < 0.5$ in the following. To that end, three equilibrium objects need to be pinned down, namely: $\underline{p}^H, \bar{p}^H$ (the bounds of the distribution), and $F^H(p)$, the distribution over prices itself.

Proposition 6 *In a symmetric mixed-strategy equilibrium in the baseline setting where* $\bar{p}^H < 0.5$ *and* $F^H(p)$ *is atomless and gapless, the equilibrium objects are characterized by:*

$$\bar{p}^H = (1-\alpha)p^L + \alpha \int_{\underline{p}^H}^{\bar{p}^H} p dF^H(p) + s \tag{14}$$

$$\bar{p}^H = \underline{p}^H + \underline{p}^H(0.5 - \underline{p}^H)\left(\frac{2N(1-\alpha)^N}{\alpha}\right) \tag{15}$$

$$F^H(p_j) = 1 - \left(\frac{\alpha}{2N(1-\alpha)^N}\frac{\bar{p}^H - p_j}{p_j(0.5 - p_j)}\right)^{1/(N-1)} \tag{16}$$

The price $\bar{p}^H$ must be equal to the cutoff price on the left side of equation (14), because consumers with $v > 0.5$ must be indifferent between searching and not searching when offered $\bar{p}^H$. To see this, note that this cutoff price must be weakly above $\bar{p}^H$, because consumers with $\nu > 0.5$ cannot search on path. If this cutoff price would be strictly above $\bar{p}^H$, there would exist a profitable upward deviation. This is because $\bar{p}^H < 0.5$ and the demand for prices in an open interval above $\bar{p}^H$ would only consist of consumers with $v > 0.5$ that arrive at a firm first, who would all directly buy at these prices. The two other equilibrium objects are computed using the mixing indifference conditions. Conditions for the existence of said equilibrium are given in the following proposition:

Proposition 7 *The mixed-strategy equilibrium characterized by equations (14), (15), and (16) exists if and only if the following two conditions hold jointly:*

$$p^{L,S} + s/\alpha \leq \underline{p}^H < \bar{p}^H < 0.5 \tag{17}$$

$$\Pi^C(p^{L,S} + s/\alpha, \tilde{v}^H) \leq \Pi^M(\bar{p}^H; \tilde{v}^H) \tag{18}$$

The first condition follows from what was previously established. Note that the second condition is very similar to the no-deviation condition in the search equilibrium. There will be no profitable deviations from the low signal price, and the most profitable deviation when observing the high signal will be to $p^{L,S} + s/\alpha$, the cutoff price of consumers with $v < 0.5$.

4.4 Equilibrium predictions and comparative statics

A corollary of the previous results is that the equilibria of the aforementioned model converge to the Diamond equilibrium when the signal becomes uninformative.

Corollary 1 *Consider any* $s > 0$*. As* $\alpha \to 0.5$*, the only aforementioned equilibrium that exists is the monopoly equilibrium and we have* $\lim_{\alpha \to 0.5} |p^{H,M} - p^{L,M}| = 0$*.*

Having established this, I visualize the existence results pinned down in propositions 4, 5, and 7 in figure 1, where I plot realizations of signal precision (α) on the x-axis and realizations of search costs (s) on the y-axis. To facilitate interpretability in the face of equilibrium multiplicity for some parameter regions, the different colors indicate the most profitable equilibrium that exists for a given parameter combination. Yellow dots indicate existence of the search equilibrium. Pink dots indicate existence of the mixed-strategy equilibrium described in proposition 6, which yields profits strictly between those attained in the search and the search deterrence equilibrium, ceteris paribus. Green dots indicate that the search deterrence equilibrium is the only equilibrium that exists. Blue dots indicate that only the monopoly equilibrium exists. Black dots indicate that no equilibrium exists. I visualize the aforementioned content when there are two ($N = 2$), three ($N = 3$) and four ($N = 4$) active firms, respectively.

The monopoly equilibrium exists if and only if consumers with $v \leq 0.5$, who have the highest probability to generate the low signal, have no incentives to search on-path. The incentives to search for these agents are falling in search costs and rising in information precision. Thus, this equilibrium exists when search costs are relatively high or information precision is relatively low.

The search equilibrium requires that some consumers with $v < 0.5$ would search when offered the price $p^{H,M} = 0.5$, but consumers with $v \geq 0.5$ would not search at this price. Such an outcome is possible because the incentives to search at equilibrium prices are lower for consumers with $v \geq 0.5$ than for consumers with $v \in (s/\alpha + p^L, 0.5)$. In order for said outcome to be an equilibrium, search costs must be in an intermediate range. Then, search costs are low enough such that consumers with $v \in (s/\alpha + p^L, 0.5)$ would optimally search at the price $p^{H,M}$, but also high enough to ensure that consumers with $v \geq 0.5$ do not search on path. The possible range of search costs that can support this equilibrium shrinks as information precision falls. This is because the difference in between the search incentives of consumers with $v < 0.5$ and those with $v \geq 0.5$ becomes smaller as signal precision falls.

Now consider the search deterrence equilibrium. Recall that the equilibrium high signal price was set to make consumers with $v \in (s/\alpha + p^{L,D}, 0.5)$ exactly indifferent between searching and not searching. By contrast, existence of this equilibrium is determined by the search incentives of agents with $v > 0.5$. The search deterrence equilibrium exists if and only if the cutoff price of consumers with $v < 0.5$ is sufficiently far below $p^{H,M} = 0.5$. This is necessary to ensure that a deviation from the high signal price to this cutoff price is not profitable. This property, in turn, requires low search costs.

Finally, consider the mixed-strategy equilibrium outlined in proposition (6). This equilibrium exists even for low search costs where the search equilibrium does not exist - this is because the decreased high signal prices reduce the search incentives of consumers with $v > 0.5$, thus enabling these consumers to refrain from searching on-path. Like the search equilibrium, the MSE does not exist when search costs and information precision are simultaneously low. This follows from the fact that consumers must separate w.r.t. their search behaviour in the MSE, which requires appropriately high information precision, accompanied by search costs that are not prohibitively high.

Finally, there also exists a small parameter region around $\alpha \approx 0.58$ and $s \approx 0.03$ where no equilibrium exists. Problems with equilibrium non-existence may thus be ruled out by restricting attention to appropriately precise signal structures with $\alpha \geq 0.6$. Moreover, note that increases in the number of firms (N) do not substantially alter the existence regions.

Now I investigate the comparative statics of prices and profits in any equilibrium. The above considerations already indicate that the effects of changes in the parameters are non-monotonic and fundamentally depend on the equilibrium that is being played. I begin by outlining the comparative statics in the search equilibrium:

Corollary 2 *In the search equilibrium, $p^{H,S}$ is independent of s and N. Moreover, $p^{L,S}(s, N)$ is falling in s and rising in N.*

Standard intuition regarding the effect of search costs on prices suggests the following: An increase of search costs should reduce the number of firms an average consumer has in her choice set, thus reducing competitive pressure and raising prices. In the search equilibrium, both the sign of the comparative statics result and the working channel behind it are diametrically opposed to the standard intuition.

The low signal price will fall in response to an increase of search costs. Recall that any consumer that arrives at a firm after searching generates fully inelastic demand at prices $p_j \in (0, s/\alpha + p^{L,S})$. By contrast, the demand created by consumers that arrive at firm j first is sensitive to the price p_j offered by this firm. When search costs rise, less consumers search on the equilibrium path. Thus, the weight of consumers that arrive after searching and generate locally inelastic demand in the low signal profit function falls. As a consequence, the optimal low signal price falls. Similar logic underlies the result that $p^{L,S}$ is increasing in the number of active firms. As N increases, consumers that arrive after searching receive higher weight, which creates additional upward pressure on the low signal price.

Now consider the search deterrence equilibrium:

Corollary 3 *In the search deterrence equilibrium, the price $p^{L,D}(N, s)$ is independent of s and N. The price $p^{H,D}(N, s)$ is rising in s and independent of N.*

The low signal equilibrium price is equal to the monopoly low signal price, which is independent of s and N by design. By contrast, the high price $p^{H,D}$ is rising in search costs. Recall that the high price is set in a way that makes consumers with $v \in (s/\alpha + p^{L,D}, 0.5)$ exactly indifferent between searching and not searching. As s rises, their gains of search at any given price offer fall. This implies that the price achieving indifference rises. This comparative statics result can also be explained by market forces. When s rises from s' to s'', competitive profits are now equal to monopoly profits in the interval $(s'/\alpha + p^{L,D}, s''/\alpha + p^{L,D})$. Note that the old high signal price $p^{L,D}(\alpha, s')$ has to be strictly below 0.5 and monopoly profits must thus be strictly increasing in prices at $p^{L,D}(\alpha, s')$. Thus, firms will raise their high signal price in response to the rise of s.

This concludes the study of the effects of parameter changes on equilibrium prices within a given equilibrium. Next, I study how the equilibrium prices respond to parameter changes when these changes may switch the equilibrium that is being played. When parameter changes induce shifts in the nature of the equilibrium that is played, this has a considerable effect on prices. Given the research questions I have raised in the introduction, I will focus on the effect of search costs (s) and competition (N) on equilibrium outcomes.

In visualizing these, I fix three levels of α at 0.65, 0.75, and 0.85. First, I study the equilibrium prices under different search costs by fixing the number of firms at $N = 2$. The corresponding effects are visualized in figure 2. Equilibrium prices are plotted on the y-axis while search costs are plotted on the x-axis.

Changes in search costs affect prices through two channels in this model. Firstly, increases of search costs reduce the search incentives of consumers who don't search on-path. The reductions of their search incentives imply increased high signal prices in the search deterrence and the mixed-strategy equilibrium. Secondly, increases of search costs reduce the measure of consumers that search on-path. This channel, while underlying the negative effect of increases in s on $p^{L,S}$, has no impact on the high signal prices. This is because any consumer that is detained from on-path search would not be able to consume at any of the high signal prices in the search and the mixed-strategy equilibrium.[13] In the appendix, I show (for the 2-firm case) that increased search costs lead to improved consumer welfare in the search equilibrium for a substantial range of parameters.[14]

Thus, any increase in the measure of consumers that search never has a beneficial effect on prices as such. When the market reverts from the search deterrence equilibrium to any equilibrium with on-path search, there is an upward jump of prices that is accompanied by a discontinuous increase in the measure of consumers that search. Moreover, while increases of s reduce this measure in the mixed-strategy equilibrium and are accompanied by increases of the expected high signal price, this relationship is not causal. Increases of the high signal prices within this equilibrium are entirely driven by the reduced search incentives of high-valuation consumers. Holding the search incentives of these consumers (and thus $\bar{p}^H$) fixed, the high signal prices would not respond to changes of s within the MSE.

Now, I turn my attention to the comparative statics effects of N. Within the search deterrence equilibrium and the monopoly equilibrium, changes of N do not affect prices, since there is no on-path search. In figure 3, I fix $s = 0.05$ and study the effects of increases in N on equilibrium prices in the search equilibrium.

In figure 4, I fix $s = 0.025$ and study the effects of increases in N on equilibrium prices in the MSE for slightly different levels of α than before, namely $\alpha \in \{0.75, 0.8, 0.85\}$. I consider these α to ensure that the MSE always exists for the parameters under consideration.

When the market transitions from a monopoly to a duopoly in which the MSE is played, the high signal price falls. Afterwards, the presence of more firms in the market is not pro-competitive. The low signal price $p^{L,S}$ is increasing in N. Moreover, increases in N also lead to increased high signal prices in the MSE by the following logic: At the high signal prices, two groups of consumers are relevant: (i) consumers with $v > 0.5$, who all buy at $p_j \in (\underline{p}^H, \bar{p}^H)$ and thus entail locally price inelastic demand, and (ii) consumers with $v < 0.5$, who only buy at a firm when generating the high signal at all N firms and who entail price elastic demand. When N rises, the latter have less chance of consuming at firm j for any

[13] This follows from the result that $p^{L,S} + s/\alpha < p^{H,S}$ and $p^{L,S} + s/\alpha < \underline{p}^H$, respectively.

[14] Details may be found in appendix B.14.

given price $p_j \in (\underline{p}^H, \bar{p}^H)$. Thus, these consumers receive less weight in the firm's profit function, making the high signal demand less elastic overall. This leads to higher prices.

5 Generalized signal distributions

5.1 Setup and initial remarks

This section establishes that the key insights of the previous model also hold for more general signal distributions. Firstly, the presence of equilibrium search is neither an indicator for high levels of competitive pressure nor low search costs in the markets I study. Secondly, consumer welfare is maximized when search costs are neglibly small, but the effect of search cost reductions on prices and consumer welfare is non-monotonic. Thirdly, an increase in the number of active firms can only reduce prices when search costs are low. In the following, I merely retain the specification that there are just two possible signals, i.e. $K = 2$.

5.2 Equilibrium structure

In this subsection, I show that the structure of potential pure-strategy equilibria in general settings is the same as in the baseline model when the signal distribution is continuous, weakly increasing, and satisfies $Pr(\tilde{v}^H|v) \in (0,1)$ for all $v \in [0,1]$. Further, I provide conditions on $Pr(\tilde{v}^H|v)$ which ensure that the key properties of the mixed-strategy equilibria from the previous framework are retained as well.

Proposition 1 still holds in any such settings. In a pure-strategy equilibrium $p := (p^L, p^H)$ with $p^L < p^H$, all consumers that arrive at a firm after searching can not buy at the high price p^H. In this section, it is useful to devote closer attention to the optimal search rule. To do so, I define the set $\hat{V}(p^L)$, which defines what consumers *can* search on-path:

$$\hat{V}(p^L) = \left\{ v \in [0,1] : Pr(\tilde{v}^L|v)(v - p^L) - s > 0 \right\} \tag{19}$$

A consumer can search on the equilibrium path if and only if her valuation v is in the set $\hat{V}(p^L)$. Define further that $S(p^L, p^H)$ is the set of consumer valuations that actually search on the equilibrium path, with $\underline{v} := \inf S(p^L, p^H)$, and $\bar{v} = \sup S(p^L, p^H)$. The properties of these sets are pinned down by the following lemma:

Lemma 6 *Suppose $Pr(\tilde{v}^H|v)$ is weakly increasing and consider a pure-strategy equilibrium (p^L, p^H) with $p^L < p^H$, in which $\hat{V}(p^L)$ is non-empty. The following must hold:*

- *$Pr(\tilde{v}^L|v)(v-p^L)-s=0$ must hold at $v=\inf \hat{V}(p^L)$, which implies that $\inf \hat{V}(p^L) > p^L$. Moreover, the function $Pr(\tilde{v}^L|v)(v-p^L)$ must be continuous at $v=\inf \hat{V}(p^L)$.*

- *The set of consumer valuations that search on the equilibrium path (ignoring measure zero sets) is $\hat{V}(p^L) \cap [p^L, p^H]$. Thus, $\underline{v} = \inf \hat{V}(p^L)$.*

The first result has mostly technical relevance. Intuitively, it holds because any jumps in $Pr(\tilde{v}^L|v)$ at points of discontinuities must always be downwards by the assumption that $Pr(\tilde{v}^H|v)$ is weakly increasing in v. The second result pins down the measure of consumers that search on the equilibrium path. The following lemma establishes useful connections between the optimal search and consumption choices:

Lemma 7 *Suppose that $Pr(\tilde{v}^H|v)$ is weakly increasing. Consider a symmetric pure-strategy equilibrium (p^L, p^H) with $p^L < p^H$ and search on the equilibrium path. Ignoring sets of valuations with measure zero, the following must hold:*

- *All consumers that arrive at firm j after searching would buy at this firm when being offered a price $p_j \leq \underline{v}$.*

- *No consumer would search after receiving a price $p_j \leq \underline{v}$.*

By lemma 6, all consumers that arrive after searching must have a valuation above $\underline{v}$ and must have received p^H at all previously visited firms. The assumption that there is search on the equilibrium path implies that the set $S(p^L, p^H)$ is not empty, which in turn requires that $\underline{v} < p^H$. Together, these notions imply the first result. To understand the second result, recall that a consumer with valuation v will search if and only if she receives an initial price offer $p_j > \hat{p}(v)$. Any consumer with $v \in S(p^L, p^H)$ has a cutoff price $\hat{p}(v) > v$. Together with the fact that $\inf S(p^L, p^H) = \underline{v}$, this notion implies the second result.

These two lemmas imply that the structure of profits around the equilibrium low signal price in general settings mirrors the structure of this function in the baseline model. Consider an equilibrium with on-path search, where $p^L < \underline{v}$ must hold. When receiving a price close enough to p^L, no consumer searches. Moreover, the demand that is implied by searchers is also fully inelastic around the equilibrium low signal price p^L in these generalized settings. In other words, there must exist an interval of prices $p_j \in [0, \underline{v}]$ with p^L in its interior where the profit functions for either signal $\tilde{v}^k$ satisfy the following stucture:

$$\Pi^C(p_j; \tilde{v}^k) = p_j \int_{p_j}^{1} (1/N) Pr(\tilde{v}^k|v) dv + p_j M^k(p^L, p^H) \tag{20}$$

Note that $M^k(p^L, p^H)$, i.e. the measure of consumers that arrive after searching and generate the signal $\tilde{v}^k$, is defined as follows:

$$M^k(p^L, p^H) = \sum_{j=2}^{N} \int_{v \in S(p^L, p^H)} \left[Pr(\tilde{v}^H|v)\right]^{j-1} Pr(\tilde{v}^k|v)(1/N)dv \tag{21}$$

The equilibrium low signal price in generalized settings is thus determined by an optimization calculus that is analogous to its counterpart in the baseline setting. Similar notions hold true for the equilibrium high signal price. Recall that the set of valuations that could search on the equilibrium path in the baseline model was $[s/\alpha + p^L, 0.5]$ and $\inf \hat{V}(p^L) = s/\alpha + p^L$. Thus, the high signal price in the search deterrence equilibrium, namely $p^{H,D} = \inf \hat{V}(p^L)$, satisfied $Pr(\tilde{v}^L|p^{H,D})(p^{H,D} - p^{L,D}) - s = 0$. In all other equilibria, the high signal price was a maximizer of $\Pi^M(p_j; \tilde{v}^H)$, for which there was only one candidate, namely $p^{H,M}$. The following proposition formalizes that this dichotomy is retained under weak assumptions on $Pr(\tilde{v}^H|v)$:

Proposition 8 *Suppose that $Pr(\tilde{v}^H|v)$ is continuous and weakly increasing. In a symmetric pure-strategy equilibrium (p^L, p^H) with $p^H > p^L$, the low signal price p^L must satisfy:*

$$\left.\frac{\partial \Pi^M(p_j; \tilde{v}^L)}{\partial p_j}\right|_{p_j = p^L} + M^L(p^L, p^H) = 0 \tag{22}$$

Suppose further that $Pr(\tilde{v}^H|v) \in (0,1)$ holds for all $v \in [0,1]$. Then, the equilibrium high price must satisfy one of the following expressions:

$$\left.\frac{\partial \Pi^M(p_j; \tilde{v}^H)}{\partial p_j}\right|_{p_j = p^H} = 0 \tag{23}$$

$$Pr(\tilde{v}^L|p^H)(p^H - p^L) - s = 0 \tag{24}$$

The equilibrium low price must satisfy expression (22). This is because the competitive low signal profit function is differentiable when $Pr(\tilde{v}^H|v)$ is continuous and satisfies the structure laid out in equation (20) for prices $p_j \in [0, \underline{v}]$, which includes p^L in its interior.

Given that competitive profits are equal to monopoly profits at p^H, a natural candidate for the equilibrium high signal price is a maximizer of the monopoly high signal profit function. Now consider an equilibrium candidate that does not satisfy the corresponding first-order condition. Suppose that $Pr(\tilde{v}^L|p^H)(p^H - p^L) - s > 0$. By continuity of $Pr(\tilde{v}^L|v)$, there would exist an open interval of valuations above p^H for which a consumer would search at p^H, which yields a contradiction to proposition 1. Suppose alternatively that $Pr(\tilde{v}^L|p^H)(p^H - p^L) - s < 0$. Then, the competitive high signal profit function is equal to the monopoly high signal profit function in an open ball around p^H. Because $\Pi^M(p_j, \tilde{v}^H)$ is differentiable, p^H must thus satisfy the first-order condition given in (23). Hence, a high signal price that does not satisfy this first-order condition must satisfy equation (24). Having established this, note the following:

Lemma 8 *Suppose that $Pr(\tilde{v}^H|v)$ is continuously differentiable, weakly increasing, and satisfies $Pr(\tilde{v}^H|v) \in (0,1)$ for all v.*

- *If $NPr(\tilde{v}^H|p^{H,M})^{N-1} \geq 1$ holds, then $p^H \leq p^{H,M}$ must hold true.*
- *The regularity condition $NPr(\tilde{v}^H|p^{H,M})^{N-1} \geq 1$ holds true, for example, if $N = 2$ and $Pr(\tilde{v}^H|0.5) = 0.5$.*

The above lemma provides regularity conditions that rule out equilibria where $p^H > p^{H,M}$. Such outcomes would be quite unintuitive - this is because competition only puts downward pressure on p^H, since no consumer that arrives after searching can buy at p^H. In the appendix, I show that the regularity condition $NPr(\tilde{v}^H|p^{H,M})^{N-1} \geq 1$, which is always satisfied when $N = 2$ and $Pr(\tilde{v}^H|0.5) = 0.5$, is also satisfied for a wide range of parameters when $N > 2$. Loosely speaking, the validity of this condition requires a sufficiently precise signal.[15]

It remains to establish how many price tuples can satisfy the structure of equilibria that was established in proposition 8. Strict concavity of the monopoly high signal profit functions is sufficient to ensure that there are unique solutions to the equations (22) and (23). Visual inspection of the function $Pr(\tilde{v}^L|p^H)(p^H - p^L) - s$ for given prices p^L makes it clear that this function has, in most situations, at most two zeros. Strict quasiconcavity of this function formally narrows down its number of zeros.

Corollary 4 *Suppose that (i) $Pr(\tilde{v}^H|v)$ is once continuously differentiable and $Pr(\tilde{v}^H|v) \in (0,1)$ $\forall v \in [0,1]$, (ii) $N\big[Pr(\tilde{v}^H|p^{H,M})\big]^{N-1} \geq 1$, (iii) $\Pi^M(p_j, \tilde{v}^L)$ is strictly concave in p_j, and (iv) $g(v) := Pr(\tilde{v}^L|v)(v - p^L) - s$ is a strictly quasiconcave function on $v \in [0, p^{H,M}]$ for any $p^L \in [0,1]$.*[16]

[15] For details, please see appendix D.5.

[16] In appendix D.6., I document that the function $g(v)$ has at most two zeros on $[0, p^{H,M}]$ for an overwhelming share of possible scenarios, which is the sole role that assumption (iv) plays in generating the listed implications.

Then, there are exactly four candidates for a symmetric pure-strategy equilibrium (ignoring a candidate that only exists for an interval of search costs with zero measure), namely:

- *The search equilibrium* $(p^{L,S}, p^{H,S})$, *defined by* $\frac{\partial \Pi^M(p_j;\tilde{v}^L)}{\partial p_j}\Big|_{p^{L,S}} + M^L(p^{L,S}, p^{H,S}) = 0$ *with* $M^L(p^{L,S}, p^{H,S}) > 0$ *and* $\frac{\partial \Pi^M(p_j;\tilde{v}^H)}{\partial p_j}\Big|_{p^{H,S}} = 0$.
- *The monopoly equilibrium* $(p^{L,M}, p^{H,M})$ *, where* $\frac{\partial \Pi^M(p_j;\tilde{v}^L)}{\partial p_j}\Big|_{p^{L,M}} = \frac{\partial \Pi^M(p_j;\tilde{v}^H)}{\partial p_j}\Big|_{p^{H,M}} = 0$.
- *The search deterrence equilibrium* $(p^{L,D}, p^{H,D})$, *where* $p^{L,D} = p^{L,M}$ *and* $p^{H,D} = \inf \hat{V}(p^{L,D})$.
- *An equilibrium candidate* $(p^{L,C}, p^{H,C})$, *defined by* $\frac{\partial \Pi^M(p_j;\tilde{v}^L)}{\partial p_j}\Big|_{p^{L,C}} + M^L(p^{L,C}, p^{H,C}) = 0$ *with* $M^L(p^{L,C}, p^{H,C}) > 0$ *and* $p^{H,C} = \sup\left[\hat{V}(p^{L,C}) \cap [0, p^{H,M}]\right]$.

The only new equilibrium candidate is the fourth one, which I call the *constrained search equilibrium*. The numerical simulations that I conduct, in which I restrict attention to the intuitive case where $p^{H,C} \leq p^{H,M}$, highlight that this equilibrium candidate is extremely unlikely to actually exist. Essentially, there are two conflicting requirements which are necessary to support this price tuple as an equilibrium. As before, no consumer with a valuation above p^H can search on the equilibrium path. To ensure that an upward deviation from p^H towards $p^{H,M}$ is not profitable, enough agents with a valuation in the interval $v \in (p^H, 1]$ need to search for prices above p^H. Given that these consumers cannot search at p^H, they are relatively unlikely to search at prices $p_j \in (p^H, p^{H,M})$ under continuity of the signal probability distribution. In my numerical analysis, I have found exactly one parameter combination (out of 600) for which this equilibrium exists. Moreover, the more profitable search equilibrium would also exist at this particular parameter combination.[17]

Now, I move on to characterize the MSE that exist in these general settings. Providing closed-form expressions for these without exact specifications of $Pr(\tilde{v}^H|v)$ is quite challenging. Thus, I focus on showing that key characteristics of the mixed-strategy equilibria which were established for the baseline setting go through to these more general settings.

Proposition 9 *Suppose that* $Pr(\tilde{v}^H|v)$ *is continuous, weakly increasing, and satisfies* $Pr(\tilde{v}^H|v) \in (0,1)$ *for all* $v \in [0,1]$. *Suppose further that the monopoly profit functions are both strictly concave and that the function* $g(v)$ *is strictly quasiconcave for all* $p^L \in [0,1]$.

Consider a symmetric mixed-strategy equilibrium and define p^{min} *and* p^{max} *as the lowest and highest prices that are offered in this equilibrium.*

[17] Details may be found in appendix D.3.

- *There must be on-path search in this equilibrium.*
- *The firm offers a deterministic price* p^L *when observing the low signal and* $p^L = p^{min}$*.*
- *Suppose that the high signal prices are drawn from an atomless, gapless distribution with support* $[\underline{p}^H, \bar{p}^H]$*. It must hold that* $\inf \hat{V}(p^L) \leq \underline{p}^H$*.*

I conjecture that the relevant regions where these equilibria exist feature high information precision and low search costs, as before. Low search costs and high information precision are necessary and sufficient to facilitate the separating search behavior that is necessary to sustain such an equilibrium. Given that there must be on-path search, the low-signal price in such an MSE must be higher than the monopoly low signal price. Thus, prices in this equilibrium must be strictly above the search deterrence equilibrium prices, because $\inf \hat{V}(p^L) < \underline{p}^H$ also holds.

In appendix D, I numerically show that the properties of the aforementioned pure-strategy equilibria, i.e. the existence regions and comparative statics, carry over to generalized settings when the signal distribution is given by the following parametric form:

$$Pr(\tilde{v}^L|v) = \alpha\left(1 - \frac{1}{1 + e^{k(0.5-v)}}\right) + (1-\alpha)\left(\frac{1}{1 + e^{k(0.5-v)}}\right) \tag{25}$$

Lower values of k amount to making the signal distribution more linear, while the parameter α governs the upper and lower bounds of the probability distribution. In figure 6, I plot this distribution for different values of α and k. Under the aforementioned interpretation of signal precision, the latter is rising both in k and α.

6 Conclusion

I have studied price discrimination based on imperfect information in homogeneous goods markets where consumers engage in sequential search to obtain price offers. Whenever a consumer visits a firm, this firm receives a binary and informative signal about the consumer's valuation. In the baseline framework, a firm observes nothing else for any consumer. I have highlighted that different search costs give rise to fundamentally different equilibria.

My results refute the notion that a high volume of equilibrium search generally reflects high levels of competitive pressure or low search costs. It crucially matters which consumers choose to search in equilibrium. When the only consumers that search on-path have intermediate valuations, a feature matching the empirical phenomenon documented by Byrne

& Martin (2021), equilibrium search is an imperfect screening device that is indicative of intermediate search costs and allows firms to sustain high prices.

My work sheds light on the effects of three potential regulatory interventions in markets where firms can price discriminate, namely: (i) fostering price transparency (in the form of reducing search costs), (ii) promoting firm entry, and (iii) prohibiting firms from accessing information about consumers' search histories.[18] Pushing search costs down to negligible levels is a very effective way of regulating markets where firms can price discriminate. This maximizes consumer welfare and renders firms unable to price discriminate, which may be desirable in itself. However, the effects of search cost reductions on prices and consumer welfare are non-monotonic. Expansions of the set of consumers that search on the equilibrium path lead to higher prices. Search cost reductions unfold pro-competitive effects only via strengthening the search incentives of consumers that do not search on-path. This is beneficial whenever it enables them to constrain prices more effectively with the threat of search.

Analogously, fostering firm entry is not generally pro-competitive in the markets I study. It leads to reduced prices only when it eliminates a monopoly and search costs are small. In non-monopolistic markets with equilibrium search, firm entry leads to higher prices. Similarly, banning firms from accessing search history information is only sensible when search costs are low, because firms with access to search history information will find it impossible to sustain equilibria with search.

[18] The associated model analysing the equilibrium outcomes when firms have access to search history information in addition to receiving the aforementioned signals is found in appendix E.

A Proofs of section 3

A.1 Proof of lemma 1

This proof works by induction and follows existing proofs in the literature. Consider any valuation $v \in [0, 1]$ and suppose that firms play a symmetric equibrium. Given the equilibrium distribution of prices, one can define the differences $\max\{v - p, 0\}$ as the "prizes", which are drawn from the distribution $F(x)$ that is the same at all firms.

Define the cutoff prize of a consumer that has visited $N-1$ firms previously as r_{N-1}. When the best prize in hand $y \in [0, \infty)$ satisfies $y < r_{N-1}$, it is strictly optimal to search. When the best prize in hand satisfies $y \geq r_{N-1}$, the consumer finds it (weakly) optimal not to search.

Such a unique prize must exist since the gains of search after having visited $N-1$ firms, call these $g(y, r_{N-1})$, must be weakly decreasing in y. Formally, we have that $g(y, N-1) > 0$ if $y < r_{N-1}$ and $g(y, N-1) \leq 0$ if $y \geq r_{N-1}$. Search will occur iff $y < r_{N-1}$.

Now I conduct the induction step. Suppose n firms have been visited sofar. Suppose that the cutoff prize (defined as above) is equal to r_{N-1} whenever the amount of previously visited firms is weakly above $n+1$. I show that the consumer will find it strictly optimal to search when her best prize in hand (after having visited n firms) satisfies $y < r_{N-1}$ and weakly optimal not to search otherwise.

Firstly, suppose the best prize that the consumer has in hand after visiting n firms satisfies $y \geq r_{N-1}$. This specification implies that the consumer will never search again after having visited $n+1$ or more firms. The gains of searching are thus equal to $g(y, N-1)$ defined above, which are weakly negative because $y \geq r_{N-1}$.

Secondly, suppose the best prize the consumer has in hand after visiting n firms satisfies $y < r_{N-1}$. The expected utility gain this consumer obtains through search is weakly larger than the expected utility gain this consumer obtains if she searches just one more time, which is equal to $g(y, N-1)$. This follows because stopping the search process is just one of several viable choices for any decision maker after having visited $n+1$ firms.

Because $y < r_{N-1}$, $g(y, N-1) > 0$ holds and it is strictly optimal to search.

This completes the proof, since the induction step was written down for a general n.

A.2 Proof of proposition 1

Part 1:

Suppose that search costs are strictly positive. No consumer would search after receiving the price p^{min}, since the gains of search would be strictly negative.

Moreover, no consumer with $v \leq p^{min}$ would ever search. Thus, any consumer that arrives after searching must have received a price strictly above p^{min} at all firms previously visited and must have a valuation strictly above this price. Since they will not search at p^{min}, such consumers will all immediately buy at a firm offering them the price p^{min}.

Part 2:

Consider a symmetric pure strategy equilibrium $(p^1, ..., p^K)$. If the price p^k is offered in equilibrium, the set $\left\{v \geq p^k : \hat{p}(v) > p^k, Pr(p^k|v > 0)\right\}$ must have zero measure.

Suppose, for a contradiction, that this set has strictly positive measure. Note that the probability distribution over the prices a consumer (fixing v) can receive is identical for all firms because $Pr(\tilde{v}^H|v)$ only depends on v and we study symmetric equilibria.

By our assumptions, there is a strictly positive measure of consumers who will receive the price p^k from all firms in this equilibrium (since receiving p^k always triggers search because search is myopic), which is given by:

$$\int_{\left\{v \geq p^k : \hat{p}(v) < p^K, Pr(p^k|v) > 0\right\}} \left[Pr(p^k|v)\right]^N dv > 0$$

When setting the price p^k, some firm will only make the sale to these consumers with probability below 1. When marginally undercutting this price, the sale will be made to all these consumers, representing an upward jump in this component of demand.

All other components of demand can only weakly increase after a decrease in price - any such decrease in price will (i) reduce the search incentives of consumers and (ii) allow more consumers to buy. Thus, marginally undercutting p^k will imply a discontinuous upward jump of total demand. This will represent a profitable deviation and we have a contradiction.

A.3 Proof of proposition 2

Part 1:

Symmetric pure-strategy equilibrium candidates

Define $p^{max} = \max_k p^k$. Because there is price dispersion, $p^{max} > p^{min}$ must hold.

Any consumer with $v \geq p^{max}$ will receive p^{max} with strictly positive probability under our assumptions. Given that $s = 0$ and the probability to obtain a lower price is strictly positive (because there is price dispersion and any signal is generated with strictly positive probability), receiving the price p^{max} induces search.

Note that $p^{max} < 1$ must hold - otherwise, this price would warrant zero profits. Thus, we have a strictly positive measure of consumers with $v \geq p^{max}$ that receive p^{max} with strictly positive probability and move on to search when receiving p^{max}. This contradicts proposition 1. Thus, no such equilibrium can exist.

Symmetric mixed-strategy equilibrium candidates

Consider a candidate for a symmetric mixed-strategy equilibrium in which there is price dispersion. Define p^{max} as the highest possible price that can be offered in this equilibrium. At this price p^{max}, all consumers will surely search because there is price dispersion and any consumer has strictly positive probability of receiving a lower price.

Suppose firstly that the highest price is played with zero probability after all signals. Then, the probability that a consumer finds a price below this through search is 1. Thus, all consumers with $v \geq p^{max}$ leave to search and never return. The measure of consumers who arrive after searching and buy at p^{max} is also 0. Thus, profits at this highest price are zero.

This implies a contradiction. If the firm makes strictly positive profits for another price that is played at an information set where p^{max} is offered, mixing indifference fails. If no such price exists, there exists a profitable deviation in this information set.

Suppose, instead, that the highest price is played with strictly positive probability after some signal and call this signal $\tilde{v}^{max}$. Any consumer with $v \geq p^{max}$ has a strictly positive probability of generating the signal $\tilde{v}^{max}$ and receiving the price p^{max}. Any such consumer will search at this price since there is price dispersion, $s = 0$, and $Pr(\tilde{v}^k|v) \in (0,1)$ holds true for any consumer and any signal.

As a result, there is a strictly positive measure of consumers with $v \geq p^{max}$ who receive the price p^{max} at all firms, no matter at which firm they start the search process. This set creates undercutting incentives that break the equilibrium. Once again, such an equilibrium can thus not exist.

<u>Asymmetric equilibria:</u>

Now consider asymmetric equilibria and find the highest price that is offered by any firm. Define this price as p^{max}. Call a firm that offers this price j.

Suppose there exists a firm $-j$ that does not offer this price. Because search is costless, and consumer with $v \geq p^{max}$ that receives p^{max} from firm j is guaranteed to visit the firm $-j$ and retrieve a strictly lower price with certainty. Thus, firm j would obtain zero profits when offering p^{max}, a contradiction.

Thus, the price p^{max} must be in the price support of any firm. Suppose firstly that there exists a firm $-j$ that offers this price with probability 0 after all signals. Once again, a consumer who visits firm j and receives p^{max} is guaranteed to visit firm $-j$ and retrieve $p_{-j} < p^{max}$, implying that firm j makes zero profits by setting p^{max}, a contradiction.

Thus, suppose that all firms offer this price with strictly positive probability after some signal $\tilde{v}^{max}$. Then, consumers with $v \geq p^{max}$ have a strictly positive probability of receiving p^{max} at all firms.

There will exist some firm $-j$ that offers a price strictly below p^{max} with positive probability by the assumption that there is price dispersion. All consumers with $v \geq p^{max}$ that start at the firm j would search when receiving p^{max} at this firm - these consumers would receive p^{max} at all firms with positive probability. Once again, this strictly positive measure of consumers would create undercutting motives.

Part 2: Uniform price equilibrium candidates

Thus, any equilibrium must be of the following form: Independently of the signal, all firms must offer a uniform price p^0. In such a symmetric PSE, the measure of consumers with $v \geq p^0$ that search must be zero - see proposition 1. Define $p^{min,M}$ as the lowest possible monopoly price.

Suppose that $p^0 > p^{min,M}$. By the above results, the equilibrium profits after any signal equal monopoly profits at p^0 (for the respective signal). When deviating downwards to $p^{min,M}$, any consumer that arrives at your firm first with $v \geq p^{min,M}$ will surely buy, no matter whether they search or not. Thus, the firm will at least make monopoly profits at $p^{min,M}$ when deviating. By strict concavity of all monopoly profit functions, this deviation is strictly profitable when observing the signal $\tilde{v}^{min,M}$.

Now, I have to show that any price $p^0 \in [0, p^{min,M}]$ constitutes an equilibrium. In equilibrium, it is optimal for any consumer not to search. Thus, no consumer arrives after searching.

There are no profitable upward deviations, since these invoke zero profits - this holds because any consumer with $v > p^0$ would move on to search.

No consumer arrives after searching. Thus, the competitive profit function is equal to the monopoly profit function when deviating downwards. Note that all monopoly profit functions (for the respective signals) are strictly concave. Thus, for prices $p_j < p^0$, all monopoly profit functions must be strictly increasing. Thus, such a deviation can never be profitable.

B Proofs of section 4

B.1 Proof of lemma 2

Suppose that $s \to \infty$, which makes any firm into a monopolist. Since the first search is random, the profit functions are:

$$\Pi^M(p_j; \tilde{v}^k) = p_j \int_{p_j}^{1} f^1(v_i = v, \tilde{v}_{i,j} = \tilde{v}^k) dv = p_j \int_{p_j}^{1} (1/N) f(v_i = v, \tilde{v}_{i,j} = \tilde{v}^k) dv$$

Note that:

$$f(v_i = x, \tilde{v}_{i,j} = \tilde{v}^L) = \begin{cases} (1-\alpha)(1) & \text{, if } x \geq 0.5 \\ (\alpha)(1) & \text{, if } x < 0.5 \end{cases}$$

$$f(v_i = x, \tilde{v}_{i,j} = \tilde{v}^H) = \begin{cases} (\alpha)(1) & \text{, if } x \geq 0.5 \\ (1-\alpha)(1) & \text{, if } x < 0.5 \end{cases}$$

We can show that:

$$\left.\frac{\partial \Pi^M(p_j; \tilde{v}^L)}{\partial p_j}\right|_{p_j} = 0 \iff 0.5 - 2\alpha p_j = 0 \iff p^{L,M} = \frac{1}{4\alpha}$$

Similarly, one can derive $p^{H,M}$ by the following logic:

$$\left.\frac{\partial \Pi^M(p_j; \tilde{v}^L)}{\partial p_j}\right|_{p_j} > 0 \iff p_j < 0.5 \quad , \quad \left.\frac{\partial \Pi^M(p_j; \tilde{v}^L)}{\partial p_j}\right|_{p_j} < 0 \iff p_j > 0.5$$

This implies that $p^{H,M} = 0.5$. Note also that both these functions are strictly concave when $\alpha \in (0.5, 1)$.

B.2 Proof of proposition 3

Recall that we assume $s > 0$ throughout this section.

Part 1: $\alpha < 1$.

Crucially, this implies that both monopoly profit functions are strictly concave. Recall also that the optimal search rule will be myopic.

<u>Possible equilibrium category 1:</u> $p = p^L = p^H$.

In such an equilibrium, there is no search on path because there are zero incentives to search but positive costs of doing so. Thus, no consumer would arrive at any firm j after search.

Since search costs are strictly positive, there exists an interval of prices $p_j \in [p, p+s]$ where consumers would not search. Thus, no equilibrium with $p < 0.5$ can be supported, since an upward deviation would be optimal when observing the high signal.

Consider instead a price $p \geq 0.5$. Since $p^{L,M} < 0.5$ holds true, there would be a profitable downward deviation, since no consumer would move on to search after this deviation. Thus, no equilibrium of this type can be supported.

Possible equilibrium category 2: $p^H < p^L$.

(i) Suppose $p^L > 0.5$.

Note that $p^L < 1$ must hold - otherwise zero profits will be obtained after $\tilde{v}^L$. Consider the search calculus of an agent with $v \in [0.5, p^L]$ with a best price $p_j \leq p^L$.

$$\alpha(v-p^H)+(1-\alpha)\max\{v-p_j,0\}-s > \max\{v-p_j,0\} \iff \alpha(v-p^H)-s > \alpha\max\{v-p_j,0\}$$

An agent with $v > p^L$ that receives a price p^L will search iff:

$$\alpha(v-p^H)+(1-\alpha)(v-p^L)-s > (v-p^L) \iff \alpha(v-p^H)-s > \alpha(v-p^L) \iff \alpha(p^L-p^H) > s$$

Suppose:

$$\alpha(p^L - p^H) - s > 0$$

This is a direct contradiction, since agents with $v \in (p^L, 1]$ will search upon being offered the price p^L. Since these agents are offered this price with strictly positive probability, the equilibrium is broken by proposition 1.

Thus, it must hold that: $\alpha(p^L - p^H) \leq s$. Thus, for all agents with $v \in [0.5, p^L]$, we have that:

$$\alpha(v - p^H) \leq \alpha(p^L - p^H) \leq s$$

This means that all these consumers cannot search at prices $p_j \in [0.5, p^L]$. Moreover, consumers with $v \geq p^L$ would also not search when being offered the price p^L ever. Thus, no consumer with $v > 0.5$ searches on path.

In the price interval, $p_j \in [0.5, p^L]$ profits would thus equal monopoly profits. No consumer with $v > 0.5$ can arrive after searching. No consumer with $v > 0.5$ that arrives first would search. Thus, there is a profitable downward deviation since monopoly profits are strictly concave on $[0.5, p^L]$, a contradiction.

(ii) $p^H < p^L \leq 0.5$

Assume that the following condition holds:

$$(1-\alpha)(p^L - p^H) - s > 0 \implies \alpha(p^L - p^H) - s > 0$$

Consider agents with $v > 0.5 > p^L$, who will search at price $p_j = p^L$ if and only if:

$$(\alpha)(v - p^H) + (1-\alpha)(v - p^L) - s > (v - p^L) \iff \alpha(p^L - p^H) > s$$

Thus, these consumers will search at $p_j = p^L$, which breaks the equilibrium, since all these consumers are offered p^L with positive probability.

Thus, it must hold that:

$$(1-\alpha)(p^L - p^H) - s \leq 0 \implies p^L \leq p^H + \frac{s}{1-\alpha}$$

Now consider agents with $v \in [p^H, 0.5]$, who search for prices $p_j \leq p^L$ if and only if:

$$(1-\alpha)(v - p^H) - s > (1-\alpha)\max\{v - p_j, 0\}$$

Agents with $v \in [p^H, p^L]$ have a negative LHS, i.e. won't search for any price $p_j \leq p^L$. For the consumers with $v \in (p^L, 0.5]$, the cutoff price is interior and above p^L, (this means search won't occur at this price), i.e.:

$$\hat{p}(v) = p^H + \frac{s}{1-\alpha} \geq p^L$$

Moreover, no consumer with $v > 0.5$ can search when being offered a price $p_j = p^L$ (since all these consumers have a strictly positive probability of receiving p^L). Thus, no consumers search when being offered the price p^L or any price below this. This means that there will not be search on the equilibrium path and profits for prices $p_j \leq p^L$ equal monopoly profits.

A deviation to the price p^L is profitable after the high signal since $p^L \leq 0.5$ and monopoly profits after the high signal are rising in this price interval. Thus, we have a contradiction once more.

<u>Possible equilibria category 3:</u> $p^L < p^H$

The fact that $\alpha > 0.5$ implies that:

$$\frac{s}{\alpha} + p^L < \frac{s}{1-\alpha} + p^L$$

<u>Subcase 1:</u> Suppose that $s/\alpha + p^L < 0.5$.

There are two possible equilibrium high signal prices, namely $p^H = p^L + s/\alpha$ and $p^H = 0.5$.

(i) $p^H < s/\alpha + p^L$.

Consider consumers with $v \in [p^H, 0.5]$. At best price $p_j \geq p^H$ in hand, search occurs iff:

$$Pr(\tilde{v}^L|v)(v-p^L)+Pr(\tilde{v}^H|v)(v-p^H)-s > \max\{v-p_j, 0\} \iff v-\alpha p^L-(1-\alpha)p^H-s > \max\{v-p_j, 0\}$$

Solving for the interior cutoff price in this interval of valuations yields that:

$$\hat{p}(v) = \alpha p^L + (1-\alpha)p^H + s$$

Note that this is above p^H, which makes it the correct cutoff price:

$$\alpha p^L + (1-\alpha)p^H + s > p^H \iff \alpha(p^L - p^H) + s > 0 \iff p^L + \frac{s}{\alpha} > p^H$$

In the price interval $p_j \in [p^H, \alpha p^L + (1-\alpha)p^H + s]$, no consumer would search. For consumers with $v > 0.5 > p^H$, the price cutoff will also surely be above p^H. To see this, consider the search calculus of these agents for prices $p_j \in [p^L, p^H]$:

$$Pr(\tilde{v}^L|v)(v-p^L)+Pr(\tilde{v}^H|v)\max\{v-p_j, 0\}-s > \max\{v-p_j, 0\} \iff (1-\alpha)(v-p^L)-s > (1-\alpha)(v-p_j)$$

Their cutoff price cannot be in this interval since:

$$\hat{p}(v) = p^L + \frac{s}{1-\alpha} > p^L + \frac{s}{\alpha} > p^H$$

Thus, their cutoff price would be equal to:

$$\hat{p}(v) = (1-\alpha)p^L + \alpha p^H + s > (\alpha)p^L + (1-\alpha)p^H + s > p^H$$

No agent would arrive after search. Thus, profits are equal to monopoly profits in the price interval $p_j \in [p^H, \alpha p^L + (1-\alpha)p^H + s]$. Because $p^H < s/\alpha + p^L < 0.5$, monopoly profits are increasing in this price interval. Thus, an upward deviation from p^H is profitable.

(ii) $p^H = s/\alpha + p^L$: This could be an equilibrium and will be verified later.

(iii) $p^H \in (s/\alpha + p^L, 0.5)$.

Consider a consumer with $v \in (p^H, 0.5)$. Such a consumer would search at price p^H if and only if:

$$Pr(\tilde{v}^L|v)(v - p^L) + Pr(\tilde{v}^H|v)(v - p^H) - s > (v - p^H)$$

$$\Longleftrightarrow$$

$$\alpha(-p^L) - s > \alpha(-p^H) \iff p^H > p^L + \frac{s}{\alpha}$$

This holds true for all such consumers. Thus, all these consumers would search at p^H, which breaks the equilibrium because they have a strictly positive probability of generating p^L.

(iv) $p^H = 0.5$. This could also be an equilibrium.

(v) $p^H > 0.5$:

Suppose that $p^H \leq p^L + \frac{s}{1-\alpha}$. Consumers with $v > p^H$ will not search on-path or for prices $p_j \leq p^H$, since:

$$(1-\alpha)(v - p^L) + \alpha(v - p^H) - s \leq (v - p^H) \iff p^H \leq p^L + \frac{s}{1-\alpha}$$

Similarly, consumers with $v \leq p^H$ will also not search on path or for prices $p_j \leq p^H$.

Then, a downward deviation would be optimal since consumers with $v \in (0.5, 1)$ would not move on to search for prices weakly below p^H. Thus, downward deviations will imply profits weakly above monopoly profits, making these deviations be profitable.

Thus, suppose that $p^H > p^L + \frac{s}{1-\alpha}$. Then, consumers with $v \in (p^H, 1)$ will search on-path, breaking the equilibrium.

<u>Case 2:</u> Suppose that $s/\alpha + p^L > 0.5$. No such equilibrium can exist.

The premise implies: $\alpha(0.5 - p^L) < s \implies (1-\alpha)(0.5 - p^L) < s$

(i) $p^H < 0.5$.

Note firstly that this implies that $p^H < s/\alpha + p^L$

Consider any consumer with $v \in (p^H, 0.5)$. If the cutoff price of these consumers is below p^H, it equals: $s/\alpha + p^L$. We know this cannot be true by assumption. The resulting cutoff price for these consumers must hence be:

$$\hat{p}(v) = \alpha p^L + (1-\alpha)p^H + s > p^H \iff \alpha p^L + s > \alpha p^H \iff p^L + \frac{s}{\alpha} > p^H$$

Moreover, the cutoff price for $v > 0.5$, which has to be above p^H as well, will be even higher. Thus, there is no search on path and no consumer would ever leave to search for prices just above p^H.

Thus, you would have a profitable marginal upward deviation after the high signal since profits are monopoly profits in this interval.

(ii) $p^H \in (0.5, 1]$

For these prices, only the search behaviour of agents with $v > 0.5$ is relevant, since all other agents cannot buy.

Suppose:

$$0.5 < p^H \leq p^L + \frac{s}{1-\alpha}$$

Now consider consumers with $v \in [0.5, 1]$. These consumers would all not search on path.

For the price $p_j = 0.5$ which is a better deal than p^H, search occurs iff:

$$(1-\alpha)(v - p^L) - s > (1-\alpha)(v - 0.5) \iff 0.5 > p^L + \frac{s}{1-\alpha}$$

Thus, no consumer will ever search with best price $p_j = 0.5$. A downward deviation to $p_j = 0.5$ is profitable, because monopoly profits are guaranteed.

Suppose instead that:

$$p^H > p^L + \frac{s}{1-\alpha}$$

As before, consumers with $v > p^H$ would search, breaking the equilibrium.

Part 2: Structure of equilibria when $\alpha < 1$

Sofar, we have obtained that an equilibrium must either satisfy $p^H = p^{H,M}$ or $p^H = p^L + s/\alpha < 0.5$. In the second equilibrium, there will be no on-path search. To see this consider some p^L and $p^H = s/\alpha + p^L$. It must hold that $p^L + s/\alpha < 0.5$

Since $p^H = p^L + \frac{s}{\alpha}$, consumers with $v < p^H$ will strictly prefer to not search at $p^H = s/\alpha + p^L$. Consumers with $v \in [s/\alpha + p^L, 0.5)$ will strictly prefer to search for prices above p^H, but will be indifferent between searching and not searching at best price p^H. However, the measure of these consumers that would search at p^H must be zero.

Consumers with $v > 0.5$ will strictly prefer not to search at best prize p^H since:

$$(1-\alpha)(v - p^L) - s < (1-\alpha)(v - p^H) \iff p^H < \frac{s}{1-\alpha} + p^L$$

If there is no on-path search, the only optimal solution for the low price is $p^L = p^{L,M}$. The latter is unique. Setting $p^L > p^{L,M}$ is not optimal, since a downward deviation to $p^{L,M}$ is optimal.

Setting $p^L < p^{L,M}$ is also not optimal since sequentially rational search implies that marginally raising the price above p^L will not trigger search. Strict concavity of the low signal monopoly profit function on $p_j \in [0, 0.5]$ then implies the result.

Thus, there are two possible equilibria without search on path, where p^L is always equal to $p^{L,M}$. An equilibrium with search must satisfy $p^H = p^{H,M}$ and $p^L < p^H$ by initial arguments.

Part 3: Possible equilibria when $\alpha = 1$.

Then, no people with $v \leq 0.5$ arrive at firm j and generate the high signal and vice versa. The monopoly prices are:

$$p^{L,M} = 0.25 \quad ; \quad p^{H,M} = 0.5$$

There will be no search on the equilibrium path, since the price offer that any agent receives is non-stochastic in a symmetric equilibrium and $s > 0$. Thus, the equilibrium profits equal the corresponding monopoly profits.

Possible equilibrium candidate 1: $p = p^L = p^H$

Setting $p \geq 0.5$ cannot be an equilibrium, since there exists a profitable downward deviation from p^L, which would yield zero profits.

Setting $p < 0.5$ will imply that there is a profitable deviation from p^H. Consumers with $v > 0.5$ have a cutoff price above or equal to: $\hat{p}(v) = p^H + s$. Setting a price just above $p^H < 0.5$ will thus not trigger search by consumers with $v > 0.5$. All these consumers would hence still consume under this deviation. Thus, this deviation is profitable since only consumers with $v \geq 0.5$ generate the high signal.

Possible equilibrium candidate 2: $p^H < p^L$.

Because $\alpha = 1$, no consumers arrive at any given firm j after searching.

Suppose $p^L > 0.5$. Only consumers with $v \leq 0.5$ will generate the low signal. This means that low signal profits will be zero, a contradiction.

Suppose instead that $p^H < p^L \leq 0.5$. For prices just above p^H no agent with $v > 0.5$ will move on to search, which implies that there is a profitable upward deviation from the high signal price, a contradiction.

Possible equilibrium candidate 3: $p^L < p^H$

Examine the search incentives of a consumer with $v > 0.5$. Only such consumers can arrive at a firm and generate the high signal.

These consumers would never search when receiving the high price. Their cutoff price is: $\hat{p}(v) = p^H + s$.

Suppose $p^H > 0.5$. No consumer with $v > 0.5$ would search at p^H by the above arguments, which means there cannot be search on the equilibrium path by agents with $v > 0.5$.

Equilibrium profits are thus monopoly profits - and profits at a deviation price $p_j = 0.5$ would also equal monopoly profits, which makes this deviation profitable.

Suppose $p^H < 0.5$. There is no search on-path by agents with $v > 0.5$, and there is an interval of prices $p_j \in (p^H, p^H + s)$ for which no such consumer would move on to search. Since high signal profits are only obtained from such consumers, there is a profitable upward deviation from p^H.

Thus, $p^H = 0.5 = p^{H,M}$ must hold when $\alpha = 1$. Since there is no search on-path, $p^L = p^{L,M}$ must hold.

B.3 Derivation of competitive objective functions

Consider the setting with a representative agent. The firm can be called to act at two information sets - (i) the consumer shows up and generates $\tilde{v}^L$, and (ii) the consumer shows up and generates $\tilde{v}^H$.

Call the information set where a consumer shows up and generates the signal $\tilde{v}^k$ I^k. The expected profit a firm makes at this information set is then equal to the price that is set, multiplied by the probability that the sale is made to the consumer that arrives at firm j and generates the signal $\tilde{v}^k$. I call this $Pr(sale|I^k)$.

Thus, profits are:

$$\Pi^C(p_j, I^k) = p_j Pr(sale|I^k) = p_j \left(\frac{Pr(sale \wedge I^k)}{Pr(I^k)} \right)$$

By the law of total probability for continuous random variables, it holds that:

$$Pr(sale|I^k) = \frac{Pr(sale \wedge I^k)}{Pr(I^k)} = \frac{1}{Pr(I^k)} \int_0^1 Pr(sale \wedge I^k|v) f(v) d(v)$$

I define $r^* \in \{1, 2, ..., N\}$ as the exogenous position in which a consumer would visit a firm. By a modified law of total probability, it holds that:

$$Pr(sale \wedge I^k|v) = \sum_{f=1}^{N} Pr(sale \wedge I^k|v \wedge r^* = f) Pr(r_i^* = j|v) = (1/N) \sum_{f=1}^{N} Pr(sale \wedge I^k|v \wedge r^* = f)$$

Equilibria without on-path search

Consider an equilibrium without search on the equilibrium path.

In an equilibrium without search, it is impossible for an agent with $r^* \geq 2$ to arrive at a firm. This implies that:

$$Pr(sale \wedge I^k | v \wedge r^* = f) = 0 \quad \forall f \geq 2$$

If $r^* = 1$ holds for a consumer with valuation v, then the event I^K is realized with probability $Pr(\tilde{v}^k|v)$. Conditional on v and $r^* = 1$ and for a given p_j, the fact that a consumer generates the signal $\tilde{v}^k$ is not informative about his consumption decision at a given price p_j. Thus, we can write:

$$Pr(sale \wedge I^k | v \wedge r^* = 1) = Pr(sale | v \wedge r^* = 1) Pr(I^k | v \wedge r^* = 1)$$

Thus, in an equilibrium without search, we have the following at prices $p_j \leq p^H$ (for which nobody searches):

$$Pr(sale \wedge I^k | v \wedge r^* = 1) = \mathbb{1}[p_j \leq v] Pr(\tilde{v}^k | v)$$

For prices $p_j > p^H$, where search is hypothetically possible, we have:

$$Pr(sale \wedge I^k | v \wedge r^* = 1) = \mathbb{1}[p_j \leq v] \mathbb{1}[p_j \leq \hat{p}(v)] Pr(\tilde{v}^k | v)$$

Thus, we have the following in an equilibrium without search:

$$Pr(sale \wedge I^k | v) = \begin{cases} (1/N) \mathbb{1}[p_j \leq v] Pr(\tilde{v}^k | v) & p_j \leq p^H \\ (1/N) \mathbb{1}[p_j \leq v] \mathbb{1}[p_j \leq \hat{p}(v)] Pr(\tilde{v}^k | v) & p_j > p^H \end{cases}$$

Based on this, we can calculate the profit functions. For $p_j \leq p^H$, these are:

$$\Pi(p_j; I^k) = \frac{p_j}{Pr(I^k)} \int_0^1 Pr(sale \wedge I^k | v) f(v) d(v) = \frac{p_j}{Pr(I^k)} \int_{p_j}^1 (1/N) Pr(\tilde{v}^k | v) f(v) d(v)$$

For prices $p_j > p^H$, these are:

$$\Pi(p_j; I^k) = \frac{p_j}{Pr(I^k)} \int_{p_j}^1 (1/N) \mathbb{1}[p_j \leq \hat{p}(v)] Pr(\tilde{v}^k | v) f(v) d(v)$$

Note that $Pr(I^k)$ depends only on the equilibrium strategies of the other players and not on the choice of the price p_j. Thus, it can be safely ignored in the maximization problem.

Equilibria with search on path

Consider an equilibrium price tuple (p^L, p^H) with search on the equilibrium path. Here, it is possible for an agent with $r^* \geq 2$ to arrive at a given firm.

For prices $p_j < p^H$ and $f \geq 2$, it holds that:

$$Pr(sale \wedge I^k | v \wedge r^* = f) =$$

$$\mathbb{1}[p^H > \hat{p}(v)] Pr(\tilde{v}^H | v)^{f-1} Pr(\tilde{v}^k | v) \mathbb{1}[v \geq p_j] \left[\mathbb{1}[p_j \leq \hat{p}(v)] + \mathbb{1}[p_j > \hat{p}(v)] Pr(\tilde{v}^H | v)^{N-f} \right]$$

For the price $p_j = p^H$, this becomes:

$$Pr(sale \wedge I^k | v \wedge r^* = f) = \mathbb{1}[p^H > \hat{p}(v)] Pr(\tilde{v}^H | v)^{f-1} Pr(\tilde{v}^k | v) \mathbb{1}[v \geq p_j] \left[\rho_N Pr(\tilde{v}^H | v)^{N-f} \right]$$

The constant $\rho_N < 1$ denotes the probability with which consumption occurs at a given firm in case of a price tie.

For prices $p_j > p^H$ and $f \geq 2$, it holds that $Pr(sale \wedge I^k | v \wedge r^* = f) = 0$.

Now consider consumers that arrive at firm j first. For prices $p_j < p^H$, it holds that:

$$Pr(sale \wedge I^k | v \wedge r^* = 1) = \mathbb{1}[p_j \leq v] \left[\mathbb{1}[p_j > \hat{p}(v)] Pr(\tilde{v}^H | v)^{N-1} + \mathbb{1}[p_j \leq \hat{p}(v)] \right] Pr(\tilde{v}^k | v)$$

Note that $\hat{p}(v) > p^L$ must hold for any consumer in any symmetric pure-strategy equilibrium. Thus, when $p_j > \hat{p}(v) > p^L$, a consumer that generates the low signal at the other firm will not buy at the firm that is visited first. Moreover, we have considered a $p_j < p^H$, so $\hat{p}(v) < p_j < p^H$, which implies that a consumer would either search all firms or none.

For the price $p_j = p^H$, it holds that:

$$Pr(sale \wedge I^k | v \wedge r^* = 1) = \mathbb{1}[p_j \leq v] \left[\mathbb{1}[p_j > \hat{p}(v)] \rho_N Pr(\tilde{v}^H | v)^{N-1} + \mathbb{1}[p_j \leq \hat{p}(v)] \right] Pr(\tilde{v}^k | v)$$

For prices $p_j > p^H$, it holds that:

$$Pr(sale \wedge I^k|v \wedge r^* = 1) = \mathbb{1}[p_j \leq v]\Big[0 + \mathbb{1}[p_j \leq \hat{p}(v)]\Big]Pr(\tilde{v}^k|v)$$

For prices $p_j < p^H$, we have the following:

$$Pr(sale \wedge I^k|v) = (1/N)\sum_{f=1}^{N} Pr(sale \wedge I^k|v \wedge r^* = f) =$$

$$(1/N)\mathbb{1}[p_j \leq v]\Big[\mathbb{1}[p_j > \hat{p}(v)]Pr(\tilde{v}^H|v)^{N-1} + \mathbb{1}[p_j \leq \hat{p}(v)]\Big]Pr(\tilde{v}^k|v)+$$

$$(1/N)\sum_{f=2}^{N}\mathbb{1}[p^H > \hat{p}(v)]Pr(\tilde{v}^H|v)^{f-1}Pr(\tilde{v}^k|v)\mathbb{1}[v \geq p_j]\Big[\mathbb{1}[p_j \leq \hat{p}(v)]+\mathbb{1}[p_j > \hat{p}(v)]Pr(\tilde{v}^H|v)^{N-f}\Big]$$

Profits are then:

$$\Pi(p_j; I^k) = \frac{p_j}{Pr(I^k)}\int_0^1 Pr(sale \wedge I^k|v)f(v)dv =$$

$$\frac{p_j}{NPr(I^k)}\int_0^1 \mathbb{1}[p_j \leq v]\Big[\mathbb{1}[p_j > \hat{p}(v)]Pr(\tilde{v}^H|v)^{N-1} + \mathbb{1}[p_j \leq \hat{p}(v)]\Big]Pr(\tilde{v}^k|v)f(v)dv+$$

$$\frac{p_j}{NPr(I^k)}\int_0^1 \sum_{f=2}^{N}\mathbb{1}[p^H > \hat{p}(v)]Pr(\tilde{v}^H|v)^{f-1}Pr(\tilde{v}^k|v)\mathbb{1}[v \geq p_j]\Big[\mathbb{1}[p_j \leq \hat{p}(v)]+\mathbb{1}[p_j > \hat{p}(v)]Pr(\tilde{v}^H|v)^{N-f}\Big]f(v)dv$$

$$=$$

$$\frac{p_j}{NPr(I^k)}\int_{p_j}^1 \Big[\mathbb{1}[p_j > \hat{p}(v)]Pr(\tilde{v}^H|v)^{N-1} + \mathbb{1}[p_j \leq \hat{p}(v)]\Big]Pr(\tilde{v}^k|v)f(v)dv+$$

$$\frac{p_j}{NPr(I^k)}\int_{p_j}^1 \mathbb{1}[p^H > \hat{p}(v)]Pr(\tilde{v}^k|v)\sum_{f=2}^{N}\Big[\mathbb{1}[p_j \leq \hat{p}(v)]Pr(\tilde{v}^H|v)^{f-1}+\mathbb{1}[p_j > \hat{p}(v)]Pr(\tilde{v}^H|v)^{N-1}\Big]f(v)dv$$

For prices $p_j > p^H$, we thus have the following:

$$Pr(sale \wedge I^k|v) = (1/N)\sum_{f=1}^{N} Pr(sale \wedge I^k|v \wedge r^* = f) = (1/N)\mathbb{1}[p_j \leq v]\mathbb{1}[p_j \leq \hat{p}(v)]Pr(\tilde{v}^k|v)$$

Profits are then:

$$\Pi(p_j; I^k) = \frac{p_j}{Pr(I^k)}\int_0^1 Pr(sale \wedge I^k|v)f(v)dv = \frac{p_j}{NPr(I^k)}\int_{p_j}^1 \mathbb{1}[p_j \leq \hat{p}(v)]Pr(\tilde{v}^k|v)f(v)dv$$

B.4 Proof of proposition 4

Part 1: Monopoly equilibrium

In the monopoly equilibrium, $p^L = 1/4\alpha$ and $p^H = 0.5$. At p^H, no agent may search, i.e. all agents must have a cutoff price above 0.5. This requires:

$$0.5 \leq \frac{s}{\alpha} + \frac{1}{4\alpha} < \frac{s}{1-\alpha} + \frac{1}{4\alpha}$$

Consider any consumer with $v \leq 0.5$. This consumer will search at $p^H = 0.5$ if and only if:

$$\alpha(v - p^L) - s > 0 \iff v > \frac{s}{\alpha} + p^L$$

This cannot be true. Note that if the above condition is violated, there exists a positive measure agents with $v \in (p^L + s/\alpha, 0.5)$ that will search, which makes $p^{L,M}$ not optimal. Now examine consumers with $v > 0.5$. These agents will not search at p^H iff:

$$(1-\alpha)(v - p^L) - s \leq (1-\alpha)\max\{v - 0.5, 0\} \iff 0.5 \leq \frac{s}{1-\alpha} + p^L$$

This holds by assumption. Thus, there will be no search on the equilibrium path.

There are no deviations in terms of prices, as competitive profits are below monopoly profits everywhere (this holds because there is no search on-path).

Part 2: Search deterrence equilibrium:

There are no deviations from the low signal price. There is no search on-path, which means that the monopoly profit function is an upper envelope for competitive profits. Since p^L maximizes the former, there will be no deviations from p^L.

Consider deviations from p^H. There will not be any profitable deviations to $p_j < p^H$, since profits are equal to monopoly profits here. Monopoly profits are rising in this price interval since $p^H = s/\alpha + p^L < 0.5$ holds true.

First consider a deviation to $p_j = 0.5$. To ensure that this is not profitable, it must hold that some consumers with $v \geq 0.5$ will move on to search at this price (otherwise there is a profitable deviation to $p_j = 0.5$ since monopoly profits are attained there).

A consumer with $v > 0.5$ will move on to search at price $p_j = 0.5$ in this equilibrium if and only if:

$$(1-\alpha)(v-p^L) + \alpha(v-p^H) - s > (v-0.5) \iff 0.5 > (1-\alpha)p^L + \alpha p^H + s$$

Note that $v \geq 0.5 > p^H > p^L$, which implies the structure of the gains of search in the above equation. Note further that this condition is independent of v, so long as $v > 0.5$. Thus, suppose for a contradiction, that:

$$0.5 \leq (1-\alpha)p^L + \alpha p^H + s$$

Then, no consumer with $v \geq 0.5$ will move on to search when receiving the price $p_j = 0.5$ in said equilibrium. Then, a deviation to $p_j = 0.5$ would be strictly profitable. Thus, the following must hold:

$$0.5 > (1-\alpha)p^L + \alpha p^H + s$$

Moreover, the above condition implies that the initial condition that $p^H < 0.5$ holds true. To see this, recall that $p^H = p^L + s/\alpha$, which yields that:

$$0.5 > (1-\alpha)p^L + \alpha p^H + s = (1-\alpha)p^L + \alpha p^L + s + s \iff 0.5 > p^L + 2s > p^L + \frac{s}{\alpha}$$

Note also that the condition $0.5 > (1-\alpha)p^L + \alpha p^H + s$ implies that the necessary condition for search by all consumers with $v > 0.5$ is satisfied.

We can thus repeat the above arguments for generic prices $p_j > p^H$ to pin down the (interior) cutoff price of consumers with $v \in [0.5, 1]$ as:

$$\hat{p}(v) = (1-\alpha)p^L + \alpha p^H + s \in (p^H, 0.5)$$

No deviations to prices $p_j \in ((1-\alpha)p^L + \alpha p^H + s, 1)$ will be profitable, since profits will be zero as all consumers move on to search and never return.

Now consider price deviations in the interval $p_j \in (p^H, (1-\alpha)p^L + \alpha p^H + s)$. This interval is non-degenerate.

In this interval of prices, all consumers with $v \leq 0.5$ that can search (i.e. $v \in (p^L + s/\alpha, 0.5)$) will search and will not return. All consumers with $v \geq 0.5$ do not search at these prices and buy directly because this cutoff price is below 0.5 by assumption. Thus, profits in the price interval $p_j \in (p^H, (1-\alpha)p^L + \alpha p^H + s)$ are:

$$\Pi(p_j; \tilde{v}^H) = p_j \int_{0.5}^{1} f^1(v) Pr(\tilde{v}^H|v) dv$$

This holds because all consumers with $v < 0.5$ either left to search ($v > p^H = p^L + s/\alpha$) and won't return or cannot buy ($v \leq p^H = p^L + s/\alpha$). Moreover, no consumer with $v > 0.5$ will search for these prices.

Thus, the best deviation price is $p_j = (1-\alpha)p^L + \alpha p^H + s < 0.5$, since profits are strictly increasing for prices in the interval just discussed. Note that no consumers arrive after searching in this equilibrium.

When setting the price $p_j = (1-\alpha)p^L + \alpha p^H + s < 0.5$, the firm will only make the sale to consumers with $v \geq 0.5$ that arrive at this firm first - but the sale will be made to all these consumers. Thus, profits from the deviation are:

$$\Pi^C((1-\alpha)p^L + \alpha p^H + s; \tilde{v}^H) = \big((1-\alpha)p^L + \alpha p^H + s\big)(0.5/N)\alpha$$

By contrast, equilibrium profits are:

$$\Pi(p_j = s/\alpha + p^L; \tilde{v}^H) = \left(\frac{s}{\alpha} + p^L\right)\left[\int_{\frac{s}{\alpha}+p^L}^{0.5} Pr(\tilde{v}^H|v)(1)(1/N)dv + \int_{0.5}^{1} Pr(\tilde{v}^H|v)(1)(1/N)dv\right] =$$

$$\left(\frac{s}{\alpha} + p^L\right)\left[(1/N)(1-\alpha)\left(0.5 - \frac{s}{\alpha} - p^L\right) + (0.5/N)\alpha\right]$$

Thus, a necessary condition for equilibrium existence (which is then also sufficient given that said cutoff price is below 0.5) is:

$$\left(\frac{s}{\alpha} + p^L\right)\left[(1-\alpha)\left(0.5 - \frac{s}{\alpha} - p^L\right) + 0.5\alpha\right] \geq 0.5\alpha\big((1-\alpha)p^L + \alpha p^H + s\big)$$

B.5 Proof of lemma 3

We are studying an equilibrium with search, where it must hold that $p^L < p^H = 0.5$.

Part 1: Ordering

(i) Suppose:

$$p^H = 0.5 \leq \frac{s}{\alpha} + p^L < \frac{s}{1-\alpha} + p^L$$

Then, no consumer will search on-path. Consider consumers with $v \leq 0.5$, who search at $p_j = p^H$ iff:

$$\alpha(v - p^L) + (1-\alpha)(0) - s > (0) \iff v > \frac{s}{\alpha} + p^L$$

Such consumers don't exist under the above assumption.

Consider consumers with $v > 0.5$, who will not search at $p_j = p^H$ iff:

$$(1-\alpha)(v - p^L) + (\alpha)(v - 0.5) - s \leq (v - 0.5) \iff 0.5 \leq \frac{s}{1-\alpha} + p^L$$

Thus, these consumers will also not search on path.

(ii) Suppose instead that:

$$0.5 > \frac{s}{1-\alpha} + p^L$$

Then, consumers with $v > 0.5$ will search on path by the above logic, which breaks the equilibrium. This pins down the ordering such a PSE needs to satisfy.

Part 2: Calculating the seq. rational search strategy

Note that the optimal search rule is myopic, so we can consider the last search decision without loss. I will pin down the optimal search process for the relevant intervals of valuations separately.

<u>(i)</u> $v \leq p^L$

There is no price after which such a consumer would search. This implies that $\hat{p}(v) = \infty$ for these consumers.

<u>(ii)</u>: $v \in (p^L, 0.5)$:

Search is strictly optimal for prices $p_j \in [p^L, p^H]$ if and only if:

$$\alpha(v-p^L)+(1-\alpha)\max\{v-p_j, 0\}-s > \max\{v-p_j, 0\} \iff \alpha(v-p^L)-s > \underbrace{\alpha\max\{v-p_j, 0\}}_{\geq 0}$$

Since the RHS is weakly positive, a necessary condition for search to occur at these prices is $\alpha[v - p^L] - s > 0$. If this is true, the indifference price is pinned down by:

$$\alpha(v - p^L) - s = \alpha(v - p_j) \iff \hat{p}(v) = p^L + s/\alpha$$

Note that our assumption implies that there exist consumers with $v \in [p^L + s/\alpha, 0.5]$ for which the necessary condition for search is fulfilled and this is the cutoff price.

For all consumers with $v \leq p^L + s/\alpha$, search is never optimal. For prices $p_j \leq p^H$, the above reasoning prooves it. For prices $p_j \geq p^H$, $v < 0.5 = p^H \leq p_j$ holds and the search calculus is the same as above.

<u>(iii)</u> $v > p^H = 0.5$: Such a consumer will find it strictly optimal to search for prices $p_j \in (p^L, p^H)$ if and only if:

$$(1 - \alpha)(v - p^L) + \alpha(v - p_j) - s > (v - p_j)$$

Supposing that the price cutoff if below p^H, it will be: $\hat{p}(v) = s/(1-\alpha)+p^L$. Our assumption was that $0.5 = p^H < \frac{s}{1-\alpha} + p^L$, which means that this cannot be the correct search cutoff. In other words, these consumers will never search at these prices.

If $p_j > p^H$, the search inequality becomes:

$$\alpha(v - p^H) + (1 - \alpha)(v - p^L) - s > \max\{v - p_j, 0\}$$

The relevant necessary condition for search is now $\alpha(v - p^H) + (1 - \alpha)(v - p^L) - s > 0 \iff v > s + \alpha p^H + (1 - \alpha)p^L$. If this is weakly negative, a consumer won't search. If this is strictly positive, you will have an interior solution for your search cutoff and:

$$\hat{p}(v) = \alpha p^H + (1 - \alpha)p^L + s \geq p^H \iff p^H \leq p^L + \frac{s}{1 - \alpha}$$

This pins down search behaviour.

B.6 Proof of proposition 5

Part 1: Closed-form solution for p^L:

I first focus on pinning down the optimal low signal price p^L. Until a price $p_j \in (0, s/\alpha + p^L]$, the objective function after the low signal is:

$$\Pi^C(p_j;\tilde{v}^L) = p_j \int_{p_j}^{1} f^1(v)Pr(\tilde{v}^L|v)dv + p_j \underbrace{\sum_{j=2}^{N}\left[\int_{s/\alpha+p^L}^{0.5} f^j(v)Pr(\tilde{v}^L|v)Pr(\tilde{v}^H|v)^{j-1}dv\right]}_{M^L(\alpha,s;p^L)}$$

To evaluate this, note the following:

$$\int_{s/\alpha+p^L}^{0.5} f^j(v)Pr(\tilde{v}^L|v)Pr(\tilde{v}^H|v)^{j-1}dv = (1/N)(1-\alpha)^{j-1}\Big[(0.5-p^L)\alpha - s\Big]$$

It follows that:

$$M^L(\alpha,s;p^L) = \sum_{j=2}^{N}\left[\int_{s/\alpha+p^L}^{0.5} f^j(v)Pr(\tilde{v}^L|v)Pr(\tilde{v}^H|v)^{j-1}dv\right] = \sum_{j=2}^{N}(1/N)(1-\alpha)^{j-1}\Big[(0.5-p^L)\alpha-s\Big]$$

$$=$$

$$\left(\sum_{j=1}^{N-1}(1-\alpha)^j\right)(1/N)\Big[(0.5-p^L)\alpha-s\Big] = \left(\frac{(1-\alpha)\big(1-(1-\alpha)^{N-1}\big)}{1-(1-\alpha)}\right)(1/N)\Big[(0.5-p^L)\alpha-s\Big]$$

Now, let us plug this function $M(.)$ into the objective for the prices $p_j \in (0, s/\alpha+p^L]$:

$$\Pi^C(p_j;\tilde{v}^L) = p_j \int_{p_j}^{0.5} \alpha(1/N)dv + p_j \int_{0.5}^{1}(1-\alpha)(1/N)dv + p_j M(\alpha,s;p^L)$$

Integrating up, the objective function becomes:

$$\Pi^C(p_j;\tilde{v}^L) = p_j\alpha(1/N)[0.5-p_j]+p_j(1-\alpha)(1/N)[0.5]+p_jM(\alpha,p^L) = 0.5(1/N)p_j-\alpha(1/N)(p_j)^2+p_jM(\alpha,p^L)$$

Setting the derivative equal to zero and assuming that such a solution exists in the price interval we are examining yields:

$$\frac{\partial\Pi^C(p_j;\tilde{v}^L)}{\partial p_j} = 0 \iff 0.5(1/N) - 2\alpha(1/N)p_j + M(\alpha,p^L) = 0 \iff p_j = \frac{1}{4\alpha} + \frac{M(\alpha,p^L)}{(2/N)\alpha}$$

Note strict concavity of the objective function for $p_j \in [0, s/\alpha+p^L]$. Now let's obtain a closed-form expression for p^L in symmetric equilibrium:

$$p^L = \frac{\alpha + 2(1-\alpha)\big(1-(1-\alpha)^{N-1}\big)(0.5\alpha-s)}{4\alpha^2+2\alpha(1-\alpha)\big(1-(1-\alpha)^{N-1}\big)}$$

$$\implies \frac{\partial p^L}{\partial s} = \frac{-2(1-\alpha)\big(1-(1-\alpha)^{N-1}\big)}{4\alpha^2 + 2\alpha(1-\alpha)\big(1-(1-\alpha)^{N-1}\big)} > -1 \iff$$

$$-2(1-\alpha)\big(1-(1-\alpha)^{N-1}\big) > -4\alpha^2 - 2\alpha(1-\alpha)\big(1-(1-\alpha)^{N-1}\big) \iff$$

$$4\alpha^2 > 2(1-\alpha)^2\big(1-(1-\alpha)^{N-1}\big)$$

This holds because $\alpha > 1-\alpha$ and $\big(1-(1-\alpha)^{N-1}\big) < 1$.

Part 2: Checking deviations from p^L:

Consider first prices $p_j \in [0, s/\alpha + p^L]$. Note that the objective function is strictly concave in this price range. Thus, there will be no deviations in this interval.

Secondly, consider prices $p_j \in (s/\alpha + p^L, 1]$. At these prices, the monopoly profit functions are an upper envelope for the competitive profits function for both signals:

$$\Pi^C(p_j; \tilde{v}^k) < \Pi^M(p_j; \tilde{v}^k)$$

$$\iff$$

$$p_j \int_{0.5}^{1} (1/N)Pr(\tilde{v}^k|v)dv + p_j \int_{p_j}^{0.5} (1/N)Pr(\tilde{v}^H|v)^{N-1}Pr(\tilde{v}^k|v)dv + p_j \sum_{f=2}^{N} \int_{p_j}^{0.5} (1/N)Pr(\tilde{v}^k|v)Pr(\tilde{v}^H|v)^{N-1}dv$$

$$<$$

$$p_j \int_{p_j}^{0.5} (1/N)Pr(\tilde{v}^k|v)dv + p_j \int_{0.5}^{1} (1/N)Pr(\tilde{v}^k|v)dv$$

$$\iff$$

$$p_j \int_{p_j}^{0.5} Pr(\tilde{v}^H|v)^{N-1}Pr(\tilde{v}^k|v)dv + p_j \sum_{f=2}^{N} \int_{p_j}^{0.5} Pr(\tilde{v}^k|v)Pr(\tilde{v}^H|v)^{N-1}dv < p_j \int_{p_j}^{0.5} Pr(\tilde{v}^k|v)dv$$

$$\iff$$

$$p_j \int_{p_j}^{0.5} NPr(\tilde{v}^H|v)^{N-1}Pr(\tilde{v}^k|v)dv + < p_j \int_{p_j}^{0.5} Pr(\tilde{v}^k|v)dv \iff N\big(1-\alpha\big)^{N-1} < 1$$

This equality holds for all relevant N since $1-\alpha < 0.5$.

Now consider prices $p_j \geq 0.5$. At these prices, the sale will not be made to any searchers. Thus, competitive profits are below monopoly profits for any prices $p_j \in (s/\alpha + p^L, 1)$, i.e.:

$$\Pi^C(p_j; \tilde{v}^L) \leq \Pi^M(p_j; \tilde{v}^L)$$

By contrast, note that $\Pi^C(p_j;\tilde{v}^L) > \Pi^M(p_j;\tilde{v}^L)$ holds for all prices $p_j \leq s/\alpha + p^L$.

Moreover, one can show the following regarding the relationship between the search equilibrium low signal price and the monopoly low signal price.

$$p^L = \frac{1}{4\alpha} + \frac{M(\alpha, p^L)}{(2/N)\alpha} > \frac{1}{4\alpha} = p^{L,M}$$

Thus, $p^{L,M} < p^L < p^L + s/\alpha$ holds. Taking note of this and the fact that p^L maximizes $\Pi^C(p_j;\tilde{v}^L)$ on $p_j \in (0, s/\alpha + p^L]$ then yields:

$$\Pi^C(p^L;\tilde{v}^L) \geq \Pi^C(p^{L,M};\tilde{v}^L) > \Pi^M(p^{L,M};\tilde{v}^L)$$

Since $p^{L,M}$ maximizes $\Pi^M(p_j;\tilde{v}^L)$ over the entire domain, $\Pi^M(p^{L,M};\tilde{v}^L) \geq \Pi^M(p_j;\tilde{v}^L)$ holds for $p_j \in (s/\alpha + p^L, 1]$: These arguments imply the following for the prices $p_j \in (s/\alpha + p^L, 1)$:

$$\Pi^C(p^L;\tilde{v}^L) > \Pi^M(p^{L,M};\tilde{v}^L) \geq \Pi^M(p_j;\tilde{v}^L) \geq \Pi^C(p_j;\tilde{v}^L)$$

This shows there are no deviations from p^L.

Part 3: Checking deviations from $p^H = 0.5$

As argued before, the firm's profits are bounded from above by $\Pi^M(p_j)$ in the price interval $p_j \in (s/\alpha + p^L, 1)$. Since $p^{H,S}$ maximizes monopoly profits, there won't be any deviations into this region.

Finally, I need to show when there is no deviation to a price in the interval $p_j \in (0, s/\alpha + p^L]$. Competitive high signal profits in this region are:

$$\Pi^C(p_j;\tilde{v}^H) = \Pi^M(p_j;\tilde{v}^H) + p_j M^H(s, \alpha; p^L)$$

To evaluate this, recall that $\Pi^M(p_j;\tilde{v}^H)$ is strictly rising for all prices $p_j \leq 0.5$. Thus, the maximum of the above will be at $p_j = s/\alpha + p^L$. Existence of the search equilibrium requires that this deviation is not profitable. Equilibrium profits at $p^{H,M} = 0.5$ are:

$$\Pi^C(0.5;\tilde{v}^H) = 0.5 \int_{0.5}^{1} (1/N)\alpha dv = (1/4N)\alpha$$

By contrast, deviation profits are given by the following function at prices $p_j \in (0, p^L + s/\alpha]$:

$$\Pi^C(p_j; \tilde{v}^H) = \Pi^M(p_j; \tilde{v}^H) + p_j \sum_{j=2}^{N} \left[\int_{s/\alpha + p^L}^{0.5} f^j(v) Pr(\tilde{v}^H|v) Pr(\tilde{v}^H|v)^{j-1} dv \right] =$$

$$0.5p_j(1/N) - (1-\alpha)(1/N)p_j^2 + p_j\big[0.5 - p^L - s/\alpha\big](1/N)\left(\frac{(1-\alpha)^2}{\alpha}\big(1-(1-\alpha)^{N-1}\big)\right)$$

Part 4: Ordering of profits in the different equilibria and premise of the search equilibrium.

The equilibrium prices also need to satisfy the ordering established in the previous lemma regarding the optimal search process.

In the search deterrence and the monopoly equilibrium, there is no search on-path and the low signal price is the same in the monopoly equilibrium and in the search deterrence equilibrium. Thus, low signal profits are the same in the monopoly and the search deterrence equilibrium.

In the search deterrence equilibrium, the firm makes the high signal profits it would make in the monopoly setting when offering the price $p^H = s/\alpha + p^L < 0.5$. We know this must be below the monopoly price and that high signal monopoly profits have a strict global maximum at $p^{H,M}$. Thus, high signal profits are higher in monopoly than in the search deterrence equilibrium.

In the search equilibrium and the monopoly equilibrium, the high signal price is the same. This is because no consumer that arrives after searching in the search equilibrium can buy at the high signal price.

Thus, high signal profits are the same in the monopoly and the search equilibrium. This means that high signal profits in the search equilibrium are strictly higher than in the search deterrence equilibrium.

By previous arguments, low signal profits in the search equilibrium are higher than in the monopoly equilibrium, which equal low signal profits in the search deterrence equilibrium.

B.7 Proof of lemma 4

Part 1: The probability that a firm plays a price strictly above 0.5 is 0.

Find the highest price p^{max} that is in the support of the possible prices that can be played. If this price is weakly below 0.5, we are done.

Suppose instead that this price p^{max} is strictly above 0.5, which means that only consumers with $v > 0.5$ can consume at this price. Define the signal which warrants the offer of this price as $\tilde{v}^{max}$. Further define the support of prices after the signals (with possible gaps) as $[\underline{p}^{min}, \bar{p}^{min}]$ and $[\underline{p}^{max}, \bar{p}^{max}]$, respectively, where $p^{max} = \bar{p}^{max}$.

At p^{max}, all consumers with $v \geq p^{max}$ will optimally search iff:

$$Pr(\tilde{v}^{min}|v) \int_{\underline{p}^{min}}^{\bar{p}^{min}} (v-p)dF^{min}(v) + Pr(\tilde{v}^{max}|v) \int_{\underline{p}^{max}}^{\bar{p}^{max}} (v-p)dF^{max}(v) - s > (v - p^{max})$$

$$\Longleftrightarrow$$

$$p^{max} > Pr(\tilde{v}^{min}|v) \int_{\underline{p}^{min}}^{\bar{p}^{min}} pdF^{min}(v) + Pr(\tilde{v}^{max}|v) \int_{\underline{p}^{max}}^{\bar{p}^{max}} pdF^{max}(v) + s$$

Since the signal probabilities are constant for these consumers, their search incentives are identical.

Suppose that they all have strict incentives to search - then, they all search. If p^{max} is played with zero probability after both signals, they never return, thus implying that profits from setting this price are zero, a contradiction. If p^{max} is played with positive probability after some signal, there are undercutting motives from this price, breaking the equilibrium.

Thus, consumers with $v \geq p^{max}$ must not have strict incentives to search - then, they also would not search for any price $p_j \leq \bar{p}^{max}$. One can show that consumers with $v \in [0.5, \bar{p}^{max})$ have lower search incentives and would not search at p^{max} because:

$$Pr(\tilde{v}^{min}|v) \int_{\underline{p}^{min}}^{\min\{v,\bar{p}^{min}\}} (v-p)dF^{min}(v) + Pr(\tilde{v}^{max}|v) \int_{\underline{p}^{max}}^{\min\{v,\bar{p}^{max}\}} (v-p)dF^{max}(v) - s < 0$$

This holds because:

$$Pr(\tilde{v}^{min}|v)\int_{\underline{p}^{min}}^{\min\{v,\bar{p}^{min}\}}(v-p)dF^{min}(v)+Pr(\tilde{v}^{max}|v)\int_{\underline{p}^{max}}^{\min\{v,\bar{p}^{max}\}}(v-p)dF^{max}(v)-s<$$

$$Pr(\tilde{v}^{min}|v)\int_{\underline{p}^{min}}^{\bar{p}^{min}}(\bar{p}^{max}-p)dF^{min}(v)+Pr(\tilde{v}^{max}|v)\int_{\underline{p}^{max}}^{\bar{p}^{max}}(\bar{p}^{max}-p)dF^{max}(v)-s\leq 0$$

This expression must be negative by the case we are currently in because the signal probabilities are constant for consumers with $v > 0.5$.

Thus, consumers with $v \in [0.5, \bar{p}^{max}]$ would also not search for any prices $p_j \leq p^{max}$. As a result, no such consumers would arrive at the firm after searching and no such consumers would leave the firm to search at the prices $p_j \in [0.5, \bar{p}^{max}]$.

Thus, the profits a firm would make when setting any price $p_j \in [0.5, p^{max}]$ would be equal to monopoly profits. By strict concavity of the monopoly profit functions (for either signal), monopoly profits would be strictly decreasing in this region.

Suppose that there are two or more different prices $p_j \in [0.5, p^{max}]$ in the support of F^{max}. By the above results, this would violate the mixing indifference condition, since monopoly profits are strictly decreasing in this interval of prices.

Thus, suppose that it is just p^{max} that is in the support of F^{max}. If this is played with positive probability, there is a profitable deviation towards 0.5.

Thus, if there is a $p^{max} > 0.5$ in the support of F^{max}, it must be played with zero probability, and there are no other prices in $[0.5, p^{max}]$ that are in the support of F^{max}.

Now consider the distribution of prices F^{min}. We are still in the case $p^{max} > 0.5$, where we know that no consumer with $v > 0.5$ can search for prices $p_j \leq p^{max}$. Thus, all prices $p_j > 0.5$ that are played after $\tilde{v}^{min}$ will yield monopoly profits at the respective price, which are strictly decreasing in the interval $[0.5, 1]$.

If there are two or more such prices, we have a contradiction to mixing indifference. If there is exactly one, it needs to satisfy a FOC whenever it is played with positive probability, which cannot be fulfilled for prices $p_j > 0.5$. Thus, no price $p_j > 0.5$ can be set after $\tilde{v}^L$

with positive probability either.

Part 2: The lowest price that is played in such an MSE, call this p^{min}, cannot be part of a stochastic action.

Suppose p^{min} is part of a stochastic action. For prices in the open ball $[p^{min}, p^{min} + s)$, no consumer will search. To see this, define the distribution of prices a consumer with v can expect as $F(p; v)$. Note that a consumer will not search at a price p_j if:

$$\int_{p^{min}}^{p^{max}} \max\{v - p, v - p_j, 0\} dF(p; v) - s \leq \max\{v - p_j, 0\}$$

The corresponding gains of search satisfy:

$$\int_{p^{min}}^{p_j} \big[\max\{v - p, 0\} - \max\{v - p_j, 0\}\big] dF(p; v) - s < \big[(v - p^{min}) - \max\{v - p_j, 0\}\big] - s$$

Now consider prices p_j in the open ball $p_j \in [p^{min}, p^{min} + s)$. If $v \leq p_j$, the gains of search are bounded from above by:

$$\big[(v - p^{min}) - \max\{v - p_j, 0\}\big] - s < (v - p^{min}) - s < 0$$

If $v > p_j$, the gains of search are bounded from above by:

$$\big[(v - p^{min}) - \max\{v - p_j, 0\}\big] - s < (p_j - p^{min}) - s < 0$$

Thus, no consumer would move on to search for any of the prices in the open ball $p_j \in [p^{min}, p^{min} + s)$

Similarly, no consumer with $v \in [p^{min}, p^{min} + s)$ would ever search on the equilibrium path after any price. If $v \leq p_j$, the gains of search take the aforementioned form, which are negative.

Suppose instead that $v > p_j$. Then, it must hold that $p_j < v < p^{min} + s$, which implies the result given the gains of search highlighted above.

Thus, all consumers that arrive after search must have a valuation $v \geq p^{min} + s$ and must have initially received a price $p_j \geq p^{min} + s$. Note that these are just lower bounds, but not necessarily infima.

Thus, when setting a price in this open ball above p^{min}, the sale will be made to all searchers and no consumer leaves to search.

Suppose that the set of prices $p_j \in [p^{min}, p^{min} + s]$ are played with probability/measure zero. Then, these prices are irrelevant for the consumer's search decision. Define $p^{min,*} = p^{\min} + s$ and repeat the above steps until you find a $p^{min,*}$ such that the prices in the open ball around this are played with positive probability when observing the low signal. Such an interval must exist after some repetitions, since $s > 0$ and the interval of possible prices can be partitioned into a finite number of subsets. Moreover, $p^{min,*} < 0.5$ must hold.

If the first such interval satisfies $p^{min,*} + s > 0.5$, we know that the prices $p_j > 0.5$ in this interval can only be played with zero probability by previous arguments. Then, replace $p^{min,*} + s$ with 0.5 in the following arguments - the interval between $[p^{min,*}, 0.5]$ must be played with positive probability.

No consumer with $v < p^{min,*} + s$ would arrive after search and all consumers that searched must have received prices strictly above $p^{min,*} + s$ previously. Moreover, no first arriver would search after these prices. Thus, for all these prices, competitive profits will be monopoly profits + price, multiplied by the measure of searches. Thus, profits in this interval will be strictly concave, since we are considering prices below 0.5.

Thus, consider such a set $[p^{min,*}, p^{min,*} + s]$. Suppose there are three or more prices in this interval that are played in F^{min}. This implies an immediate contradiction to the mixing indifference condition by strict concavity of the competitive profit function in this interval.

Suppose that there are exactly two prices in this interval that are in the support of F^{min}. Since prices in the interval must be played with positive probability, at least one of these prices must be played with positive probability. Then, this price must be a local maximizer of the objective function, otherwise there would be a deviation. By strict concavity, this must also be a (unique) maximizer on $[0, 0.5]$, a contradiction to mixing indifference.

Thus, there must be exactly one price in this interval, which must be played with positive probability. Then, there can be no other prices in the support of F^{min}. By a similar logic as above, $p^{min,*}$ must be a local maximizer of the competitive profit function monopoly profits + price*searchers.

By strict concavity of this function on $[0, 0.5]$, $p^{min,*}$ must uniquely maximize this function on this price interval. Since this function is an upper bound for the true competitive profits for all prices, the firm cannot attain equal profits for any other price $p_j \leq 0.5$.

Now consider prices $p_j > 0.5$. Then, we are in the case where $p^{max} > 0.5$. It was previously established that no consumer with $v > 0.5$ can search at p^{max} or lower prices. Thus, any price $p_j \geq 0.5$ would only yield monopoly profits.

Since $p^{min,*}$ must satisfy a FOC for the corresponding competitive profits $p^{min,*} > p^{L,M}$ must hold. Thus, profits at $p^{min,*}$ would be strictly above the optimal monopoly profits. Thus, any prices $p_j > 0.5$, which can only achieve monopoly profits at the respective price, cannot satisfy mixing indifference.

Part 3: $p^{min} < 0.5$ and this price must be offered after the low signal.

Suppose, for a contradiction, that $p^{min} \geq 0.5$. As a result, $p^{max} > 0.5$ must hold, but any no prices above 0.5 can be played with positive probability. Thus, we do not have a MSE in the classical sense, a contradiction. Also, there would be a downward deviation after $\tilde{v}^L$ since there would not be search on path.

Thus, $p^{min} < 0.5$. Also note that it is played with strictly positive probability. Suppose, for a contradiction, that it is played after the high signal $\tilde{v}^H$. There is a contradiction, since profits in an open ball above it are monopoly profits + price*searchers, which are strictly increasing after the high signal in the interval $[0, 0.5]$.

Part 4: There must be search in such an equilibrium.

Define $[\underline{p}^H, \bar{p}^H]$ as the interval of prices offered after the high signal. Suppose, for a contradiction, that there is no search on the equilibrium path in a MSE.

Restrict attention to the prices in F^H that satisfy $p_j \leq 0.5$. We know that such prices need to be played with positive probability. Because no consumer arrives after searching, profits for all the prices set after any signal equal the monopoly profits at these prices, which are strictly concave on $[0, 0.5]$.

If there are more than 2 such prices, strict concavity implies a direct contradiction. If there are two prices or one such price, one of them must be played with positive probability. This price must then be a local maximizer - this cannot exist in the price interval, a contradiction.

Part 5: Consumers with $v > 0.5$ cannot search on path.

If $p^{max} > 0.5$, we know that consumers with $v > 0.5$ cannot search at p^{max}, which means that they cannot search on-path.

Suppose $\bar{p}^H = p^{max} \leq 0.5$. Given what was previously proven, these consumers have lower incentives to search than consumers with $v \in [\bar{p}^H, 0.5]$.

Suppose that consumers with $v > 0.5$ search on path - then they must search at $\bar{p}^H$. Since they have a lower probability of generating the favorable low signal, they will have lower search incentives than consumers with $v \in [\bar{p}^H, 0.5]$, who would thus also search at $\bar{p}^H$.

This means that all consumers with $v > \bar{p}^H$ would search at $\bar{p}^H$. If $\bar{p}^H$ is played with zero probability, all consumers that leave to search never return - thus, profits are zero, a contradiction. If $\bar{p}^H$ is played with positive probability, there will be undercutting motives that break the equilibrium.

B.8 Proof of lemma 5

Consider first the structure of such an equilibrium. A price p^L is offered to all consumers that generate the low signal. Any consumer that generates the high signal is offered a price that is drawn from the distribution $[\underline{p}^H, \bar{p}^H]$, which is atomless and gapless by assumption.

We know $\bar{p}^H \leq 0.5$ must hold in such an equilibrium - by the atomless and gapless specification and the previous result that prices above 0.5 are only played with 0 probability.

Part 1: $p^L + s/\alpha \leq \underline{p}^H$ must hold in a MSE.

The previous results imply that p^L must be the smallest price that is offered, i.e. $p^L \leq \underline{p}^H$. Suppose, for a contradiction, that $p^L + s/\alpha > \underline{p}^H$ and consider the search decision of a consumer with $v \in (p^L, 0.5]$ for prices $p_j \in [\underline{p}^H, \underline{p}^H + \epsilon]$, with a small ϵ.

When receiving such a price, a consumer with $v \in [p_j, 0.5]$ will choose not to search if:

$$\alpha(v-p^L)+(1-\alpha)\left[\int_{\underline{p}^H}^{p_j} \max\{v-p,0\}dF^H(p)+\int_{p_j}^{\bar{p}^H} \max\{v-p_j,0\}dF^H(p)\right]-s \leq \max\{v-p_j,0\}$$

$$\Longleftrightarrow$$

$$\alpha(p_j-p^L)+(1-\alpha)\left[\int_{\underline{p}^H}^{p_j} \big[(p_j-p)\big]dF^H(p)\right]-s \leq 0$$

Note that the gains of search are continuous in p_j, which follows from the fact that we are studying atomless and gapless distributions F^H.

As $p_j \rightarrow \underline{p}^H$, such a consumer will surely not search. This is because the gains of search converge to $\alpha(\underline{p}^H - p^L) - s$, which must be strictly negative by our assumption.

Now consider consumers with $v < p_j$. Such a consumer will not search at p_j if and only if:

$$\alpha(v-p^L)+(1-\alpha)\left[\int_{\underline{p}^H}^{v} \big[(v-p)\big]dF^H(p)\right]-s \leq 0$$

As $p_j \rightarrow \underline{p}^H$, the gains of search for such a consumer must also be strictly negative, since they are bounded from above by the expression introduced previously.

Under our assumption, there would thus be an interval of prices $p_j \in [\underline{p}^H, \underline{p}^H + \epsilon]$ for which no consumer with $v \leq 0.5$ would move on to search. We also know that no consumer with a valuation $v > 0.5$ can search for any such prices, since they cannot search on-path. At these prices, the sale would thus be made to any first arriver with $v \geq p_j$.

Now examine the purchasing choices of any consumer that arrives after searching. Any such consumer must have received a price weakly above $\underline{p}^H + \epsilon$ by the above logic.

Any consumer that arrives after searching must have a valuation $v \geq \underline{p}^H + \delta$. Consumers with $v \leq \underline{p}^H < p^L + s/\alpha$ could never search on-path. Moreover, consider a consumer with $v > \underline{p}^H$. Such a consumer would not search if and only if:

$$\alpha(v-p^L)+(1-\alpha)\left[\int_{\underline{p}^H}^{p_j} \max\{v-p,0\}dF^H(p)+\int_{p_j}^{\bar{p}^H} \max\{v-p_j,0\}dF^H(p)\right]-s \leq \max\{v-p_j\}$$

Suppose this consumer has received an initial price $p_j > v$. Then, they will not search iff:

$$\alpha(v - p^L) + (1-\alpha)\left[\int_{\underline{p}^H}^{v} \max\{v - p, 0\} dF^H(p)\right] - s \leq 0$$

This is continuous in v and converges to $\alpha(\underline{p}^H - p^L) - s$ as $v \to \underline{p}^H$, which is strictly negative.

Suppose this consumer has $p_j \leq v$. Then, they will not search if:

$$\alpha(p_j - p^L) + (1-\alpha)\left[\int_{\underline{p}^H}^{p_j} \max\{p_j - p, 0\} dF^H(p)\right] - s \leq 0$$

Once again, this is bounded from above by the aforementioned expression, which are negative when v is in an open ball around $\underline{p}^H$.

Thus: When setting a price $p_j \in [\underline{p}^H, \underline{p}^H + \psi]$, with $\psi = \min\{\epsilon, \delta\}$, no consumer would move on to search and any consumer arriving after search would surely buy. Thus, competitive profits are equal to:

$$\Pi^C(p_j, \tilde{v}^H) = \Pi^M(p_j, \tilde{v}^H) + p_j M^H(p^L, F^H(p))$$

Note that $M^H(p^L, F^H(p))$ is the mass of consumers that arrive after searching and generate the signal $\tilde{v}^H$. Monopoly profits are strictly concave for $p_j \in [\underline{p}^H, \bar{p}^H]$, since $\bar{p}^H \leq 0.5$, and the second component is linear in p_j. Given that competitive profits are differentiable in this interval, the derivative of competitive profits is strictly positive here, which violates the mixing indifference condition, a contradiction.

Part 2: Any consumer with $v \in (s/\alpha + p^L, 0.5]$ will search when receiving any price in $(\underline{p}^H, \bar{p}^H]$. No other consumers will search on path.

First, consider $v \in (s/\alpha + p^L, \underline{p}^H]$, where $v < 0.5$. Recall that $s/\alpha + p^L \leq \underline{p}^H$ was established. These consumers will search at any price $p_j \in (\underline{p}^H, \bar{p}^H]$ if and only if:

$$\alpha(v - p^L) + (1-\alpha)[0] - s > 0$$

This equality is satisfied - thus, all these consumers will search.

Second, consider consumers with $v \in (\underline{p}^H, 0.5]$. These consumers will search at any price

$p_j \in (\underline{p}^H, \bar{p}^H]$ if and only if:

$$\alpha(v-p^L)+(1-\alpha)\left[\int_{\underline{p}^H}^{p_j} \max\{v-p,0\}dF^H(p)+\int_{p_j}^{\bar{p}^H} \max\{v-p_j,0\}dF^H(p)\right]-s > \max\{v-p_j,0\}$$

If $v \leq p_j$, this inequality will be satisfied, because $v > \underline{p}^H \geq s/\alpha + p^L$.

If $v > p_j$, the above inequality becomes:

$$\alpha(p_j - p^L) + (1-\alpha)\left[\int_{\underline{p}^H}^{p_j} (p_j - p)dF^H(p) + 0\right] - s > 0$$

Since $p_j > \underline{p}^H \geq s/\alpha + p^L$, this inequality will also be satisfied. Thus, these consumers will thus search for any such price.

Thirdly, note that consumers with $v \geq 0.5$ cannot search on-path - this would break the equilibrium.

Fourth, consumers with $v \leq p^L + s/\alpha \leq \underline{p}^H$ won't search after $p_j \in [\underline{p}^H, \bar{p}^H]$, since:

$$\alpha(v - p^L) + (1-\alpha)(0) - s \leq 0$$

This completes the characterization, since no consumer would ever search at p^L.

Part 3: The equilibrium low signal price is identical to the low signal price in the search equilibrium.

In an open ball above p^L, no consumer would move on to search. Any consumer that arrives after search must have $v \geq s/\alpha + p^L$ and must have received a price strictly above p^L. Thus, competitive profits are the following in an open ball around p^L

$$\Pi^C(p_j; \tilde{v}^L) = \Pi^M(p_j; \tilde{v}^L) + p_j M^L(p^L, p^H)$$

Moreover, the mass of consumers who arrive after searching is also exactly the same as in the search equilibrium. All consumers with $v \in (s/\alpha + p^L, 0.5]$ will search with probability 1 if they generate the high signal but never if they generate the low signal.

Thus, p^L is a fixed point of the following FOC:

$$\frac{\partial \Pi^M(p_j, \tilde{v}^L)}{\partial p_j}\bigg|_{p_j=p^L} + \sum_{j=2}^{N} \int_{s/\alpha+p^L}^{0.5} \alpha(1-\alpha)^{j-1}(1/N)dv = 0$$

By strict concavity of $\Pi^M(p_j, \tilde{v}^L)$, the solution to this equation is unique and equals $p^{L,S}$.

Part 4: Comparison of profits to those made in the search equilibrium.

Profits made for all consumers that generate the low signal are exactly identical as in the search equilibrium - this holds because the structure of profits is identical and the low signal price is the same.

Now consider the equilibrium high signal profits. By mixing indifference, the firm must make the same profits for any price in the support of F^H. Thus, consider $\bar{p}^H$. At this price, all consumers with $v \in [\bar{p}^H, 0.5]$ leave the firm to search. They never return, since F^H is atomless.

By construction, no consumer with $v > 0.5$ can leave the firm to search at these prices (or arrive after searching). Hence, competitive profits at this price (and thus at all prices played after the high signal) are:

$$\Pi^C(\bar{p}^H; \tilde{v}^H) = p_j \int_{0.5}^{1} (1/N)\alpha dv$$

If $\bar{p}^H < 0.5$, these profits are strictly smaller than those attained in the search equilibrium. If $\bar{p}^H = 0.5$, profits will be exactly equal. We are done.

Part 5: Whenever the MSE with $\bar{p}^H$ exists, so must the search equilibrium.

Existence of the MSE with $\bar{p}^H = 0.5$ requires that the following conditions hold (amongst others): (1) $p^{L,S} + s/\alpha < 0.5$, (2) No consumer with $v > 0.5$ would search at $\bar{p}^H = 0.5$ in the MSE, and (3) $\Pi^M(0.5; \tilde{v}^H) \geq \Pi^C(p^{L,S} + s/\alpha; \tilde{v}^H)$

The search equilibrium exists if and only if the following three conditions are met: (1) $p^{L,S} + s/\alpha < 0.5$, (2) no consumer with $v > 0.5$ would search at $p^H = 0.5$ in the search equilibrium, and (3) $\Pi^M(0.5; \tilde{v}^H) \geq \Pi^C(p^{L,S} + s/\alpha; \tilde{v}^H)$

Conditions 1 are the same in the two equilibria. Condition 3 is also the same in both equilibria, for the following reasoning:

Both in the search equilibrium and in the MSE with $\bar{p}^H = 0.5$, setting the highest price will grant you monopoly profits. At the price $p^L + s/\alpha$, nobody searches in the search equilibrium - this also holds true in the MSE with $\bar{p}^H = 0.5$.

Consumers with $v \leq p^L + s/\alpha$ never search. All consumers that arrive after searching must have $v \geq p^L + s/\alpha$ and must have generated the high signal at all previous firms. Thus, the structure of profits at the deviation price is the same in the two equilibria.

It remains to show that when condition 2 of the MSE requirements holds, so will condition 2 of the search equilibrium.

Suppose that consumers with $v > 0.5$ do not search at $\bar{p}^H = 0.5$ in the MSE. This is equivalent to saying that:

$$(1-\alpha)(v-p^{L,S})+\alpha \int_{\underline{p}^H}^{0.5} (v-p)dF^H(p)-s \leq (v-0.5) \iff 0.5 \leq (1-\alpha)p^{L,S}+\alpha \int_{\underline{p}^H}^{0.5} pdF^H(p)+s$$

Consumers with $v > 0.5$ do not search in the search equilibrium at $p^{H,S} = 0.5$ if and only if:

$$(1-\alpha)(v-p^{L,S}) + \alpha(v-0.5) - s \leq (v-0.5) \iff 0.5 \leq (1-\alpha)p^{L,S} + \alpha(0.5) + s$$

The RHS in the MSE condition is below the RHS in the search equilibrium condition, which means we are done.

B.9 Proof of proposition 6

Part 1: Upper bound $\bar{p}^H$

Consider the equilibrium with $\bar{p}^H < 0.5$, the only relevant MSE. Note that this equilibrium is characterized by:

$$p^{L,S} + s/\alpha < \underline{p}^H < \bar{p}^H < 0.5$$

The price $\bar{p}^H$ must be set to make consumers with $v > 0.5$ exactly indifferent between searching and not searching. Thus, this price needs to satisfy:

$$\bar{p}^H = (1-\alpha)p^L + \alpha\mathbb{E}[p|\tilde{v}^H] + s$$

If they strictly prefer to search, we have a contradiction, since the price $\bar{p}^H$ will yield zero profits. If they strictly prefer not to search, there is a profitable deviation to the cutoff price listed above after the high signal, since profits there will be strictly higher.

Part 2: Distribution F^H:

The profits for any price $p_j \in (\underline{p}^H, \bar{p}^H)$ are made up of profits from first arrivers and searchers.

We know that any consumer with $v \in (p^L + s/\alpha, 0.5)$ will move on to search when receiving a price in the interval $p_j \in (\underline{p}^H, \bar{p}^H)$. No other consumer will search on path.

All initial arrivers with $v \in (p^L + s/\alpha, 0.5)$ will not return from searching when generating the low signal at any other firm. They will return if and only if the high signal is generated at all other firms (and thus the best price in hand is always above their search cutoff and they keep searching), and then only when p_j is lower than any other price they have received at any other firm, which occurs with conditional probability $[1 - F^H(p_j)]^{N-1}$.

Any consumer with $v \in (\underline{p}^H, 0.5)$ that arrives after searching (no other consumer with $v > p_j$ arrives after searching) will move on to search again upon receiving a price $p_j \in (\underline{p}^H, \bar{p}^H)$. Note that these consumers arrive after searching if and only if they generate the high signal at all previous firms.

Such a consumer will then buy at firm j only when generating the high signal at all other firms and receiving a higher price at all these firms, which occurs with probability $[1-F^H(p_j)]^{N-1}$, conditional on generating the appropriate signals.

Thus, competitive high signal profits from a price $p_j \in (\underline{p}^H, \bar{p}^H)$ are:

$$\Pi^C(p_j;\tilde{v}^H) = p_j \int_{0.5}^{1} (1/N)\alpha dv + p_j \left[\sum_{j=1}^{N} \int_{p_j}^{0.5} (1/N)(1-\alpha)^N \left[1 - F^H(p_j)\right]^{N-1} dv \right] =$$

$$p_j(0.5)(1/N)\alpha + Np_j\big[0.5 - p_j\big](1/N)(1-\alpha)^N\big[1 - F^H(p_j)\big]^{N-1}$$

By contrast, total profits from the price $\bar{p}^H$ are:

$$\Pi^C(\bar{p}^H;\tilde{v}^H) = \bar{p}^H \int_{0.5}^{1}(1/N)\alpha dv = 0.5(1/N)\alpha\bar{p}^H$$

By the mixing indifference condition, these profits have to be equal for any such price p_j:

$$0.5(1/N)\alpha\bar{p}^H = p_j(0.5)(1/N)\alpha + p_j\big[0.5 - p_j\big](1-\alpha)^N\big[1 - F^H(p_j)\big]^{N-1}$$

$$\iff$$

$$\bar{p}^H = p_j + \left(\frac{2N(1-\alpha)^N}{\alpha}\right)\left[p_j[0.5-p_j]\big[1-F^H(p_j)\big]^{N-1}\right] \iff \frac{\alpha}{2N(1-\alpha)^N}\frac{\bar{p}^H - p_j}{p_j(0.5-p_j)} = \big[1-F^H(p_j)\big]^{N-1}$$

$$\iff$$

$$F^H(p_j) = 1 - \left(\frac{\alpha}{2N(1-\alpha)^N}\frac{\bar{p}^H - p_j}{p_j(0.5-p_j)}\right)^{1/(N-1)} \tag{26}$$

Part 3: Closed-form expression for $\underline{p}^H$

Profits at $\underline{p}^H$ are:

$$\Pi^C(\underline{p}^H;\tilde{v}^H) = (0.5)(1/N)\alpha\underline{p}^H + \underline{p}^H\big[0.5 - \underline{p}^H\big](1-\alpha)^N$$

The derivative of these profits w.r.t $\underline{p}^H$ is the following:

$$(0.5)(1/N)\alpha + \big[0.5 - 2\underline{p}^H\big](1-\alpha)^N$$

Note that:

$$(0.5)(1/N)\alpha + \big[0.5 - 2\underline{p}^H\big](1-\alpha)^N > (0.5)(1/N)\alpha - 0.5(1-\alpha)^N > 0.5\alpha\Big[(1/N) - (1-\alpha)^{N-1}\Big]$$

Because $1 > N(1-\alpha)^{N-1}$, this derivative is strictly positive. This implies that there will be a unique solution for $\underline{p}^H$. The profits made at $\underline{p}^H$ must be equal to the profits made at $\bar{p}^H$:

$$\Pi^C(\bar{p}^H;\tilde{v}^H) = \Pi^C(\underline{p}^H;\tilde{v}^H)$$

$$\iff$$

$$(0.5)(1/N)\alpha\bar{p}^H = (0.5)(1/N)\alpha\underline{p}^H + \underline{p}^H\big[0.5 - \underline{p}^H\big](1-\alpha)^N \iff \bar{p}^H = \underline{p}^H + \underline{p}^H\big[0.5 - \underline{p}^H\big]\left(\frac{2N(1-\alpha)^N}{\alpha}\right)$$

Part 4: Numerical solution procedure:

Thus, a solution procedure for $\bar{p}^H$ would be the following.

1. Start with an initial guess for $\bar{p}^H \rightarrow$ calculate $\underline{p}^H$ and $\mathbb{E}[p_j|\tilde{v}^H]$.
2. Check whether these satisfy the indifference condition highlighted above.

When calculating $\mathbb{E}[p_j|\tilde{v}^H]$, note that:

$$\mathbb{E}[p_j|\tilde{v}^H] = \int_{\underline{p}^H}^{\bar{p}^H} pf^H(p)dp = \Big[pF^H(p)\Big]_{\underline{p}^H}^{\bar{p}^H} - \int_{\underline{p}^H}^{\bar{p}^H} F^H(p)dp = \bar{p}^H - \int_{\underline{p}^H}^{\bar{p}^H} F^H(p)dp$$

B.10 Proof of proposition 7

Part 1:

Previous results imply that the following ordering of prices must hold in this equilibrium:

$$p^L + s/\alpha \leq \underline{p}^H < \bar{p}^H < 0.5$$

Part 2: Deviations from the high signal price.

The following regions of deviations need to be checked: (i) $[0, p^L + s/\alpha]$, (ii) $[p^L + s/\alpha, \underline{p}^H]$, (iii) $[\bar{p}^H, 1]$. I will go through them in the following. By construction of the equilibrium, the profits a firm makes for all prices $p_j \in [\underline{p}^H, \bar{p}^H]$ will be equal, so there can be no deviations into this region.

(i) $p_j \in [0, p^L + s/\alpha]$

Recall that no consumer will leave to search for these prices. Moreover, all consumers that arrive after search must have a valuation $v \geq p^L + s/\alpha$. Also, the price beats any price received at the other firm with probability 1, because search only occurs after the high signal. Thus, the sale is made to all consumers that arrive after search.

Competitive profits are thus monopoly profits + searcher profits. Thus, competitive profits are strictly increasing in this price interval. The most profitable deviation in this price interval would be to the price $p_j = p^L + s/\alpha$. This is because $p^L + s/\alpha < 0.5$ by assumption.

Profits at this deviation would be:

$$\Pi^C(p^L + s/\alpha; \tilde{v}^H) = \Pi^M(p^L + s/\alpha; \tilde{v}^H) + (p^L + s/\alpha) \sum_{j=2}^{N} \left[\int_{p^L+s/\alpha}^{0.5} (1/N)(1-\alpha)^j dv \right] =$$

$$0.5(1/N)(p^L+s/\alpha)-(1/N)(1-\alpha)(p^L+s/\alpha)^2+(p^L+s/\alpha)\big(0.5-(p^L+s/\alpha)\big)(1/N)\left(\frac{(1-\alpha)^2\big(1-(1-\alpha)^{N-1}\big)}{1-(1-\alpha)}\right)$$

We need to ensure that these are below the equilibrium profits, namely $0.5(1/N)\alpha\bar{p}^H$.

(ii) $p_j \in [p^L + s/\alpha, \underline{p}^H]$

For this set of prices, all consumers with $v \in (p^L + s/\alpha, 0.5]$ that arrive at firm j first will move on to search. If they generate the low signal at any other firm, they won't return. They will return iff and only if they generate the high signal at all other firms - since the resulting price will always be strictly higher than p_j.

For the prices $p_j \leq \underline{p}^H$, no consumer with $v > 0.5$ can search.

For these prices, all consumers that arrive after searching will have received a strictly higher price $p_j \in (\underline{p}^H, \bar{p}^H)$ at all other firms with probability 1 . Recall that these consumers have $v \in (p^L + s/\alpha, 0.5]$. These consumers will continue searching and will buy at firm j if and only if they generate the high signal at all subsequent firms.

Thus, competitive profits for these deviation prices $p_j \in [p^L + s/\alpha, \underline{p}^H]$ are:

$$\Pi^C(p_j; \tilde{v}^H) = p_j \int_{0.5}^{1} (1/N)\alpha dv + p_j \left[\sum_{j=1}^{N} \int_{p_j}^{0.5} (1/N)(1-\alpha)^N dv \right] = 0.5(1/N)\alpha p_j + [0.5p_j - (p_j)^2](1-\alpha)^N$$

The derivative of this w.r.t. p_j is:

$$\frac{\partial \Pi^C(p_j; \tilde{v}^H)}{\partial p_j} = 0.5(1/N)\alpha + [0.5 - 2p_j](1-\alpha)^N > 0.5\alpha\big[(1/N) - (1-\alpha)^{N-1}\big]$$

As indicated before, this derivative will be strictly positive for any N. Thus, profits in this interval are strictly lower than profits at $\underline{p}^H$ - which are the equilibrium high signal profits. Thus, there will be no profitable deviations.

(iii) $p_j \in (\bar{p}^H, 1]$.

We know all consumers with $v \in (p^L + s/\alpha, 0.5)$ search at $\bar{p}^H$ and higher prices. We know all consumers with $v > 0.5$ are exactly indifferent between searching and not searching at $\bar{p}^H$ - thus, a deviation $p_j > \bar{p}^H$ will trigger search by all these consumers.

Any consumer that arrives after search will not buy - these consumers must have received a strictly lower price. Any consumer that leaves to search will not return.

By this logic, deviation profits are zero. Thus, as previously: The most profitable deviation is to the price $p_j = p^L + s/\alpha$ - the condition that this is not profitable was listed above.

Part 3: Possible deviations from the equilibrium low signal price.

The following regions of deviations need to be checked: (i) $[0, p^L + s/\alpha]$, (ii) $[p^L + s/\alpha, \bar{p}^H]$, (iii) $[\bar{p}^H, 1]$. I will go through them in the following:

(i) $p_j \in [0, p^L + s/\alpha]$

We know that competitive profits in the price interval $p_j \in [0, p^L + s/\alpha]$ take the same structure as in the search equilibrium - the low signal price is a maximizer of profits in this interval by construction.

Moreover, strict concavity of the competitive profit function guarantees it is the unique maximizer within this price interval.

(ii) $[p^L + s/\alpha, \bar{p}^H]$

Competitive low signal profits for these deviation prices $p_j \in [p^L + s/\alpha, \bar{p}^H]$ are:

$$\Pi^C(p_j; \tilde{v}^L) = p_j \int_{0.5}^{1} (1/N)(1-\alpha)dv + \sum_{j=1}^{N} p_j \int_{p_j}^{0.5} (1/N)\alpha(1-\alpha)^{N-1}[1 - F^H(p_j)]^{N-1} dv$$

$$=$$

$$p_j \int_{0.5}^{1} (1/N)(1-\alpha)dv + p_j \int_{p_j}^{0.5} \alpha(1-\alpha)^{N-1}[1-F^H(p_j)]^{N-1}dv$$

We can use the same trick as we did in the baseline setting. We know that, in equilibrium, $p^{L,S}$ will grant low signal profits strictly above the monopoly low signal profits.

I will show that competitive profits for prices $p_j \in [p^L + s/\alpha, \bar{p}^H]$ will be strictly below monopoly profits. To see this, recall that:

$$\Pi^M(p_j; \tilde{v}^L) = p_j \int_{0.5}^{1} (1/N)(1-\alpha)dv + p_j \int_{p_j}^{0.5} (1/N)\alpha dv$$

Thus, $\Pi^M(p_j; \tilde{v}^L) > \Pi^C(p_j; \tilde{v}^L)$ holds for any such price because:

$$p_j \int_{0.5}^{1} (1/N)(1-\alpha)dv + p_j \int_{p_j}^{0.5} (1/N)\alpha dv > p_j \int_{0.5}^{1} (1/N)(1-\alpha)dv + p_j \int_{p_j}^{0.5} \alpha(1-\alpha)^{N-1}[1-F^H(p_j)]^{N-1}dv$$

$$\iff$$

$$p_j \int_{p_j}^{0.5} (1/N)\alpha dv > p_j \int_{p_j}^{0.5} \alpha(1-\alpha)^{N-1}[1-F^H(p_j)]^{N-1}dv \iff 1 > N(1-\alpha)^{N-1}[1-F^H(p_j)]^{N-1}$$

This holds because the following inequality holds: $N(1-\alpha)^{N-1} < 1$. Thus, there will not be any profitable deviations in this region.

(iii) $p_j \in (\bar{p}^H, 1]$:

Profits will be zero - there will be no profitable deviations.

B.11 Proof of corollary 1

Consider any $s > 0$ and suppose that $\alpha \to 0.5$.

Then, $p^{L,M} \to 0.5$, which means that $\lim_{\alpha \to 0.5}[p^{L,M} + s/\alpha] = 0.5 + 2s > 0.5$

This implies that the search deterrence equilibrium cannot exist, because existence of this equilibrium requires that $p^{L,M} + s/\alpha < 0.5$.

In the search equilibrium, we have $p^{L,S} > p^{L,M}$, which means that $\lim_{\alpha \to 0.5}[p^{L,S} + s/\alpha] \geq 0.5 + 2s > 0.5$ must also hold true, which rules out existence of the search equilibrium.

Thus, both the search and the search deterrence equilibrium cannot exist.

Similarly, recall that any MSE where the high signal price is drawn from an atomless, gapless distribution must satisfy the following properties: The highest price must be weakly below 0.5 and $p^{L,S} + s/\alpha \leq \underline{p}^H$.

When $p^{L,S} + s/\alpha > 0.5$, these two conditions cannot be jointly satisfied, implying that this equilibrium cannot exist.

By constrast, the fact that $\lim_{\alpha \to 0.5}[p^{L,M} + s/\alpha] > 0.5$ implies that the monopoly equilibrium exists. The last result follows because $p^{L,M} \to 0.5$.

B.12 Proof of corollary 2

Part 1: High price comparative statics:

The equilibrium high price is unaffected by search costs, signal precision, and the number of active firms N. It is always equal to 0.5.

Part 2: Low price comparative statics:

Note that the equilibrium low price has to satisfy the following FOC:

$$\left.\frac{\partial \Pi^M(p_j; \tilde{v}^L)}{\partial p_j}\right|_{p_j = p^L} + M^L(s, N; p^L) = 0$$

Note that:

$$M^L(s, N; p^L) = \left(\frac{\alpha(1-\alpha)\big(1-(1-\alpha)^{N-1}\big)}{1-(1-\alpha)}\right)(1/N)\Big[(0.5 - p^L) - s/\alpha\Big]$$

Thus, it holds that:

$$\frac{\partial M^L(.)}{\partial s} < 0$$

The implicit function theorem and weak concavity of the monopoly profit function implies that a rise of s will lead to a fall in $p^{L,S}$.

Now consider the effect of a rise in the number of active firms. Recall that the equilibrium low price must solve:

$$T(p^L,\alpha) = p^L - \frac{1}{4\alpha} - \frac{M^L(s,N;p^L)}{(2/N)\alpha} = 0 \iff$$

$$T(p^L,\alpha) = p^L - \frac{1}{4\alpha} - \frac{1}{(2/N)\alpha}\left(\frac{(1-\alpha)\big(1-(1-\alpha)^{N-1}\big)}{1-(1-\alpha)}\right)(1/N)\Big[(0.5-p^L)\alpha - s\Big] \iff$$

$$T(p^L,\alpha) = p^L - \frac{1}{4\alpha} - \left(\frac{(1-\alpha)\big(1-(1-\alpha)^{N-1}\big)}{2\alpha}\right)\Big[(0.5-p^L) - s/\alpha\Big]$$

Note that:

$$\frac{\partial T}{\partial p^L} = 1 - \left(\frac{(1-\alpha)\big(1-(1-\alpha)^{N-1}\big)}{2\alpha}\right)(-1) > 0$$

$$\frac{\partial T}{\partial N} = -\frac{(1-\alpha)}{2\alpha}\left(-\big(log(1-\alpha)\big)(1-\alpha)^{N-1}\right)\Big[(0.5-p^L) - s/\alpha\Big] < 0$$

Since $1-\alpha < 1$, $log(1-\alpha) < 0$, which implies that the second expression is strictly negative. The result then follows from application of the implicit function theorem - p^L is rising in N.

B.13 Proof of corollary 3

In the search deterrence equilibrium, it holds that $p^{L,D} = \frac{1}{4\alpha}$, $p^{H,D} = s/\alpha + p^{L,D}$. The comparative statics are immediate.

B.14 Comparative statics - consumer welfare (N=2)

Consider a pure-strategy equilibrium without search. In such an equilibrium, the expected utility of a consumer with valuation v is given by the following:

$$EU(v) = Pr(\tilde{v}^H|v)\max\{v-p^H,0\} + Pr(\tilde{v}^L|v)\max\{v-p^L,0\}$$

Consider instead a pure-strategy equilibrium with search. In such an equilibrium, the expected utility of a consumer that would not search at p^H is given by the above. By constrast, the expected utility of a consumer who searches at p^H is given by:

$$EU(v) = Pr(\tilde{v}^H|v)\Big[Pr(\tilde{v}^H|v)\max\{v-p^H,0\}+Pr(\tilde{v}^L|v)\max\{v-p^L,0\}-s\Big]+Pr(\tilde{v}^L|v)\max\{v-p^L,0\}$$

Defining welfare as the ex-ante expected utility of consumers, $U = \int_0^1 EU(v)dv$, figure 5 plots the (numerically calculated) relationship between search costs and consumer welfare (when restricting attention to pure-strategy equilibria).

C Proofs of section 5

C.1 Proof of lemma 6

Part 1:

I have previously defined the function $g(v) := Pr(\tilde{v}^L|v)(v - p^L) - s$. The first part requires that $lim_{v\uparrow x} g(v) \geq g(x) \geq lim_{v\downarrow x} g(v)$. By the limit rule for products, we have:

$$\lim_{v\uparrow x} Pr(\tilde{v}^L|v)(v - p^L) = \lim_{v\uparrow x} Pr(\tilde{v}^L|v) \lim_{v\uparrow x}(v - p^L) = \lim_{v\uparrow x} Pr(\tilde{v}^L|v)(x - p^L) \geq$$

$$Pr(\tilde{v}^L|x)(x - p^L) \geq \lim_{v\downarrow x} Pr(\tilde{v}^L|v)(x - p^L) = \lim_{v\downarrow x} \left[Pr(\tilde{v}^L|v)(v - p^L)\right]$$

Note that these limits exist because we are dealing with monotonic functions.

Now I need to show that $Pr(\tilde{v}^L|v^1)(v^1 - p^L) - s = 0$, when v^1 is defined as $v^1 = \inf \hat{V}(p^L)$, and it holds that:

$$\hat{V}(p^L) = \left\{ Pr(\tilde{v}^L|v)(v - p^L) - s > 0 \right\}$$

Suppose that $g(v^1) = Pr(\tilde{v}^L|v^1)(v^1 - p^L) - s > 0$. We know that $lim_{v\uparrow v^1} g(v) \geq g(v^1)$. By definition, v^1 cannot be the infimum of $\hat{V}(p^L)$ then, since it would not constitute a lower bound.

Now suppose that $Pr(\tilde{v}^L|v^1)(v^1 - p^L) - s < 0$. We know that $lim_{v\downarrow v^1} g(v) \leq g(v^1) < 0$. By definition of limits, any point just above v^1 would also be a lower bound, which means we cannot have an infimum at v^1.

Next, I need to show continuity. Assume, for a contradiction, that $lim_{v\uparrow v^1} g(v) > g(v^1) = 0$. This is a contradiction to the fact that v^1 is an infimum, because it cannot be a lower bound.

Assume, for a contradiction, that $0 = g(v^1) > lim_{v\downarrow v^1} g(v)$. We can show that this would be a contradiction to the infimum definition as well, since a point just above v^1 would also be an infimum. This prooves continuity.

Part 2:

Search on the equilibrium path is only possible when receving the price offer $p_j = p^H$.

We know consumers with $v \notin \hat{V}(p^L)$ cannot search on path. Similarly, consumers with

$v < p^L$ cannot search on path. Moreover, only sets of consumers with $v > p^H$ that have zero measure can search on-path.

The set of consumers that searches on path (ignoring measure zero sets) is thus a subset of $\hat{V}(p^L) \cap [p^L, p^H]$. Now I show that any consumer in this set will search when being offered the price p^H. Such a consumer will search then if and only if:

$$Pr(\tilde{v}^L|v)(v-p^L)+Pr(\tilde{v}^H|v)\max\{v-p^H,0\}-s > \max\{v-p^H,0\} \iff Pr(\tilde{v}^L|v)(v-p^L)-s > 0$$

This holds true by construction.

C.2 Proof of lemma 7

In an equilibrium with search, we must have $\underline{v} = \inf \hat{V}(p^L) < p^H$.

Part 1: Consider any consumer that has arrived after search. This requires that this consumer has received p^H at all other firms that were previously visited, which means the firm beats the price of all other firms when offering a price $p_j \leq \underline{v} < p^H$.

A consumer that arrives after searching must have $v \in \hat{V}(p^L)$. We know that the infimum of this set is $\underline{v}$, so consumption is possible for all these agents. This implies the result.

Part 2: I need to show that no consumer would move on to search when receiving a price $p_j \leq \underline{v}$.

We know that no consumer with $v \notin \hat{V}(p^L)$ or $v \leq p^L$ can search at p^H, which implies that such a consumer would also not search at any price $p_j \leq \underline{v} < p^H$.

Now consider a $v \in \hat{V}(p^L)$. All these consumers have a price cutoff $\hat{p}(v)$ equal to:

$$\hat{p}(v) = p^L + \frac{s}{Pr(\tilde{v}^L|v)}$$

It was shown previously that: $Pr(\tilde{v}^L|\underline{v})(\underline{v} - p^L) - s = 0$. This implies that $\hat{p}(\underline{v}) = \underline{v}$. Note that $\hat{p}'(v) > 0$, which means that for all $v \in \hat{V}(p^L) \cap [p^L, p^H]$:

$$\hat{p}(v) \geq \hat{p}(\underline{v}) = \underline{v}$$

This means that none of these consumers will search when offered the price $p_j \leq \underline{v}$. Finally, note that the set of consumers with $v \geq p^H$ that search on path must have measure zero, which means that all these consumer cannot search for prices $p_j \leq p^H$ either.

C.3 Proof of proposition 8

Part 1:

Given that the signal probability functions are continuous, the monopoly low signal profit function is continuously differentiable. Since competitive low signal profits take the given form in the price interval $p_j \in [0, \underline{v}]$, we know that this FOC must hold.

Part 2:

(i) $Pr(\tilde{v}^L|p^H)(p^H - p^L) - s > 0$ cannot hold true in a symmetric equilibrium.

Suppose, for a contradiction, that $Pr(\tilde{v}^L|p^H)(p^H - p^L) - s > 0$ holds true in a symmetric PSE. If this holds true, $p^H \in \hat{V}(p^L)$. Consider the search decision of an agent with $v > p^H$. This consumer searches at p^H if and only if:

$$Pr(\tilde{v}^L|v)(v - p^L) + Pr(\tilde{v}^H|v)(v - p^H) - s > (v - p^H) \iff Pr(\tilde{v}^L|v)(p^H - p^L) - s > 0$$

These gains of search continuous in v and we know that this inequality is satisfied at $v = p^H$. This shows that there exists an interval of valuations $v \in (p^H, p^H + \delta)$ who would search when receiving the equilibrium high price.

Given the assumption that $Pr(\tilde{v}^H|v) \in (0, 1)$ holds for all agents, there is a strictly positive measure of consumers with $v > p^H$ that search after p^H and receive this price with positive probability in equilibrium, a contradiction to proposition 1.

(ii) If $Pr(\tilde{v}^L|p^H)(p^H - p^L) - s < 0$, the equilibrium must satisfy:

$$\frac{\partial \Pi^M(p_j; \tilde{v}^H)}{\partial p_j} = 0$$

Consider any consumer with $v \in [p^H, 1]$. This consumer would search when offered a price $p_j \geq p^H$ if and only if:

$$Pr(\tilde{v}^L|v)(v - p^L) + Pr(\tilde{v}^H|v)(v - p^H) - s > \max\{v - p_j, 0\}$$

In order for a consumer with $v \geq p^H$ to search at a price $p_j \geq p^H$, the LHS of this expression needs to be strictly positive. Define the following set:

$$\hat{V}^H(p) = \left\{v - p^H + Pr(\tilde{v}^L|v)(p^H - p^L) - s > 0\right\}$$

Further recall that the cutoff price (if it is weakly above p^H) of agents with a valuation in the above set is:

$$\hat{p}^H(v) = Pr(\tilde{v}^L|v)\big(p^L - p^H\big) + p^H + s$$

By continuity of $Pr(\tilde{v}^L|v)$, it holds that $\lim_{v \to p^H} Pr(\tilde{v}^L|v) = Pr(\tilde{v}^L|p^H)$. This implies that:

$$\lim_{v \to p^H} \left[v - p^H + Pr(\tilde{v}^L|v)(p^H - p^L) - s\right] = 0 + Pr(\tilde{v}^L|p^H)(p^H - p^L) - s < 0$$

Continuity implies that you can find an open interval $[p^H - \delta, p^H + \delta]$ such that any $v' \in [p^H - \delta, p^H + \delta]$ will satisfy $v' \notin \hat{V}^H(p)$ and $v' \notin \hat{V}^L(p)$. Thus, for this price interval, all consumers will have $\hat{p}(v') = \infty$ by construction.

Find the first valuation $v' \leq 1$ above p^H that solves:

$$v' - p^H + Pr(\tilde{v}^L|v')(p^H - p^L) - s = 0$$

If this does not exist, no consumer with $v > p^H$ will satisfy the necessary condition for search at prices $p_j > p^H$. Thus, $\hat{p}(v) = \infty$ holds for all consumers with $v > p^H$ in this case.

Suppose that this exists. By continuity arguments, $\hat{p}(v) = \infty$ must hold for all consumers with $v \in [p^H, v')$. They cannot search at prices $p_j < p^H$. Moreover, they do not fulfil the necessary condition for search at $p_j > p^H$ since $v' \notin \hat{V}^H(p)$.

Now consider consumers with $v \geq v'$. Note that $\hat{p}(v') = v' > p^H$. All consumers with $v > v'$ will have a cutoff price above this, since $\hat{p}'(v) \geq 0$.

Summing up, consumers with $v \in [p^H, v')$ will have $\hat{p}(v) = \infty$ and consumers with $v \in [v', 1]$

will have $\hat{p}(v) \geq v' > p^H$. Thus, all consumers with $v > p^H$ will not search for prices $p_j \in [p^H, v']$ and hence not for prices below this either.

This means that profits are equal to monopoly profits in the interval $[p^H, v']$. No consumer with a valuation above p^H can arrive at firm j after searching. No consumer with $v > p^H$ that arrives at firm j first will search for these prices.

Consider prices just below p^H. We know that consumers with $v \in [p^H - \delta, p^H]$ must have $v \notin \hat{V}(p^L)$, which is the necessary condition for search at prices $p_j \leq p^H$, i.e. for equilibrium search. Furthermore, we know that no consumer with $v \geq p^H$ can search at p^H, which implies that these consumers would also not search at $p_j \leq p^H$.

Thus, no consumers with $v > p^H - \delta$ will arrive at firm j after search. All consumers that arrive first and have $v > p^H - \delta$ won't move on to search at prices $p_j \leq p^H$. In the price interval $[p^H - \delta, p^H]$, profits will thus also be monopoly profits.

Thus, competitive profits equal monopoly profits in an open ball around p^H. Differentiability of this function implies that the high signal price must satisfy said FOC.

C.4 Proof of lemma 8

Part 1: If $Pr(\tilde{v}^H|v)$ is weakly increasing and once continuously differentiable, $\Pi^M(p_j; \tilde{v}^H)$ is strictly concave.

Note that monopoly high signal profits are given by the following:

$$\Pi^M(p_j; \tilde{v}^H) = p_j \int_{p_j}^{1} (1/N) Pr(\tilde{v}^H|v) dv$$

Taking the derivative of this w.r.t prices yields:

$$\frac{\partial \Pi^M(p_j; \tilde{v}^H)}{\partial p_j} = \int_{p_j}^{1} (1/N) Pr(\tilde{v}^H|v) dv - (1/N) p_j Pr(\tilde{v}^H|p_j)$$

Note that this derivative is continuous, since the signal probability function is continuous.

Now let's evaluate the second derivative of profits, which is:

$$\frac{\partial^2 \Pi^M(p_j;\tilde{v}^H)}{\partial p_j^2} = -(1/N)Pr(\tilde{v}^H|p_j) - (1/N)Pr(\tilde{v}^H|p_j) - (1/N)p_j\frac{\partial Pr(\tilde{v}^H|p_j)}{\partial p_j} < 0$$

This is strictly negative because $\frac{\partial Pr(\tilde{v}^H|p_j)}{\partial p_j} \geq 0$.

Part 2: If $NPr(\tilde{v}^H|p^{H,M})^{N-1} \geq 1$ holds true, $p^H \leq p^{H,M}$ must hold.

Consider a symmetric pure-strategy equilibrium candidate where $p^H > p^{H,M}$ holds. Note that there must be search on the equilibrium path - otherwise, there would be an immediate downward deviation.

For ease of exposition, define $Pr(\tilde{v}^H|v) := Pr^H(v)$.

Consider demand for a price in an open ball below p^H, such that any such p_j must satisfy $p_j > p^{H,M}$. The demand a firm receives at such a deviation price is given by:

$$D^C(p_j;\tilde{v}^H) = \int_{p^H}^1 (1/N)Pr^H(v)dv + \int_{p_j}^{p^H}(1/N)Pr^H(v)\Big[\mathbb{1}[\hat{p}(v) \geq p_j] + \mathbb{1}[\hat{p}(v) < p_j][Pr^H(v)]^{N-1}\Big]dv$$

$$+\sum_{i=2}^{N}\int_{p_j}^{p^H}(1/N)Pr^H(v)\mathbb{1}[\hat{p}(v) < p^H]\Big[\mathbb{1}[\hat{p}(v) \geq p_j][Pr^H(v)]^{i-1} + \mathbb{1}[\hat{p}(v) < p_j][Pr^H(v)]^{N-1}\Big]dv$$

$$\Longrightarrow$$

$$D^C(p_j;\tilde{v}^H) = \int_{p^H}^1 (1/N)Pr(\tilde{v}^H|v)dv +$$

$$\int_{p_j}^{p^H}\frac{1}{N}Pr^H(v)\Bigg[\Bigg(\mathbb{1}[\hat{p}(v) \geq p_j] + \mathbb{1}[\hat{p}(v) < p_j][Pr^H(v)]^{N-1}\Bigg) +$$

$$\Bigg(\sum_{i=2}^{N}\mathbb{1}[\hat{p}(v) < p^H]\big[\mathbb{1}[\hat{p}(v) \geq p_j][Pr^H(v)]^{i-1} + \mathbb{1}[\hat{p}(v) < p_j][Pr^H(v)]^{N-1}\big]\Bigg)\Bigg]dv$$

I will show the following: For any $v \in [p_j, p^H]$, (which must satisfy $v \geq p_j > p^{H,M}$) the argument of the second integral will be weakly greater than $(1/N)Pr^H(v)$, which implies that said function lies above the monopoly demand function in this interval.

For any v such that $\hat{p}(v) \geq p_j$, the argument is weakly greater than $(1/N)Pr^H(v)$, because it becomes:

$$\frac{Pr^H(v)}{N}\left[\left[(1)\right] + \sum_{i=2}^{N} \mathbb{1}[\hat{p}(v) < p^H]\left[[Pr^H(v)]^{i-1}\right]\right]$$

For any v such that $\hat{p}(v) < p_j$, the argument is also weakly greater than $(1/N)Pr^H(v)$, because $\hat{p}(v) < p_j \leq p^H$ and this argument thus becomes:

$$\frac{Pr^H(v)}{N}\left[\left[[Pr^H(v)]^{N-1}\right] + \sum_{i=2}^{N}\left[[Pr^H(v)]^{N-1}\right]\right] = \frac{Pr^H(v)}{N}\left[N[Pr^H(v)]^{N-1}\right]$$

The fact that $N[Pr^H(p^{H,M})]^{N-1} \geq 1$, together with our assumption that $Pr^H(v)$ is weakly increasing, implies that $N[Pr^H(v)]^{N-1} \geq 1$ holds true for any such $v > p^{H,M}$. Thus, this term is weakly greater than $\frac{Pr^H(v)}{N}$.

Under the assumption that $N[Pr^H(p^{H,M})]^{N-1} \geq 1$, we have thus shown that:

$$D^C(p_j;\tilde{v}^H) \geq \int_{p^H}^{1} (1/N)Pr(\tilde{v}^H|v)dv + \int_{p_j}^{p^H} \frac{1}{N}Pr^H(v)dv$$

Thus, for prices p_j just below p^H, we have $D^C(p_j;\tilde{v}^H) \geq D^M(p_j;\tilde{v}^H)$, which implies the following for any p_j just below p^H:

$$\Pi^C(p_j;\tilde{v}^H) \geq \Pi^M(p_j;\tilde{v}^H) \implies \Pi^C(p_j;\tilde{v}^H) \geq \Pi^M(p_j;\tilde{v}^H) > \Pi^M(p^H;\tilde{v}^H) = \Pi^C(p^H;\tilde{v}^H)$$

Note that $\Pi^M(p_j;\tilde{v}^H) > \Pi^M(p^H;\tilde{v}^H)$ holds by strict concavity, since $p^H > p^{H,M}$. Thus, the profit function must be strictly decreasing at the prices we study. This prooves the existence of a profitable downward deviation.

Part 3: The regularity condition holds when $N = 2$ and $Pr(\tilde{v}^H|0.5) = 0.5$.

I firstly show that $p^{H,M} \geq 0.5$ generally holds true. Suppose, for a contradiction, that $p^{H,M} < 0.5$. Recall that the monopoly high signal price must solve:

$$\int_{p_j}^{1} (1/N)Pr(\tilde{v}^H|v)dv - (1/N)p_jPr(\tilde{v}^H|p_j) = 0 \iff p^{H,M}Pr(\tilde{v}^H|p^{H,M}) = \int_{p^{H,M}}^{1} Pr(\tilde{v}^H|v)dv$$

Because $Pr(\tilde{v}^H|v)$ is weakly increasing, we have:

$$\int_{p^{H,M}}^{1} Pr(\tilde{v}^H|v)dv \geq \int_{p^{H,M}}^{1} Pr(\tilde{v}^H|p^{H,M})dv = [1 - p^{H,M}]Pr(\tilde{v}^H|p^{H,M})$$

Because $p^{H,M} < 0.5$, we have:

$$[1 - p^{H,M}] > p^{H,M}$$

This implies a contradiction to the first-order condition.

Thus, $p^{H,M} \geq 0.5$. Because $Pr(\tilde{v}^H|v)$ is weakly increasing and $Pr(\tilde{v}^H|0.5) = 0.5$, our regularity condition will be satisfied, because:

$$NPr(\tilde{v}^H|p^{H,M})^{N-1} = 2Pr(\tilde{v}^H|p^{H,M}) \geq 2(0.5) = 1$$

C.5 Proof of corollary 4

Part 1: If $Pr(\tilde{v}^H|v)$ is once continuously differentiable, there is a unique maximizer of the monopoly profit function, which I call $p^{H,M}$. This holds because the former implies that monopoly high signal profits are strictly concave, which implies uniqueness.

Part 2: Suppose that $g(v) := Pr(\tilde{v}^L|v)(v - p^L) - s$ is a strictly quasiconcave function on $[0, p^{H,M}]$ .Then, it has at most two zeros on the interval $[0, p^{H,M}]$.

If $g(v) := Pr(\tilde{v}^L|v)(v - p^L) - s$ is a strictly quasiconcave function on $v \in [0, p^{H,M}]$ for any p^L, it will have a unique maximum on $[0, p^{H,M}]$, call this v^*, for any given p^L.

Suppose, for a contradiction, that there are three (or more) distinct solutions to $g(v) = 0$ on $v \in [0, p^{H,M}]$. Label these solutions v^1, v^2, v^3 and assume, without loss, that $v^1 < v^2 < v^3$.

This is a direct violation to strict quasiconcavity. Since $v^2 \in (v^1, v^3)$, strict quasiconcavity implies that $g(v^2) > \min\{g(v^1), g(v^3)\} = 0$.

Part 3: If $\hat{V}(p^L) \cap [0, p^{H,M}]$ is non-empty, the only possible solutions to $g(v) = 0$ in the interval $[0, p^{H,M}]$ are $\inf \hat{V}(p^L)$ and $\sup \left[\hat{V}(p^L) \cap [0, p^{H,M}]\right]$.

Note that our assumptions guarantee continuity of $g(v)$. Because $g(v)$ is strictly quasiconcave, $g(v^*) > 0$ must hold - otherwise the set $\hat{V}(p^L) \cap [0, p^{H,M}]$ is empty.

Thus, we can surely find an infimum of the set $\hat{V}(p^L)$ that lies below $p^{H,M}$. It must hold that $\inf \hat{V}(p^L)$ satisfies $g(\inf \hat{V}(p^L)) = 0$.

Suppose, for a contradiction, that $g(\inf \hat{V}(p^L)) > 0$. If this holds true, it must be true that $\inf \hat{V}(p^L) > 0$.

By continuity of g, there exists an open ball below $\inf \hat{V}(p^L)$ for which $g(.) > 0$ also holds true. Hence, $\inf \hat{V}(p^L)$ cannot be a lower bound of $\hat{V}(p^L)$, a contradiction.

Suppose, for a contradiction, that $g(\inf \hat{V}(p^L)) < 0$. We know there cannot exist valuations $v \leq \inf \hat{V}(p^L)$ for which $g(v) > 0$. By continuity of $g(.)$, there exists an open ball above $\inf \hat{V}(p^L)$ for which $g(.) < 0$ holds true. This implies that $\inf \hat{V}(p^L)$ is not the smallest lower bound of $\hat{V}(p^L)$, a contradiction.

The supremum of this set could be $p^{H,M}$ or something below $p^{H,M}$. If the supremum of $\hat{V}(p^L) \cap [0, p^{H,M}]$ is strictly below $p^{H,M}$, then $g(\sup \hat{V}(p^L) \cap [0, p^{H,M}]) = 0$ must hold by analogous arguments.

Suppose $\sup \hat{V}(p^L) \cap [0, p^{H,M}] < p^{H,M}$. Any solution to $g(v) = 0$ must be $\sup \hat{V}(p^L) \cap [0, p^{H,M}]$ or the inf of $\hat{V}(p^L)$. Otherwise, we would have a contradiction to strict quasiconcavity, since $g\big(\inf \hat{V}(p^L)\big) = g\big(\sup \hat{V}(p^L) \cap [0, p^{H,M}]\big) = 0$ must hold and there can be no other zeros of this function on $[0, p^{H,M}]$.

Suppose $\sup \hat{V}(p^L) \cap [0, p^{H,M}] = p^{H,M}$. Then, it must hold that $g(\sup \hat{V}(p^L)) \geq 0$. If $g(\sup \hat{V}(p^L)) < 0$, there would be a contradiction, since the true supremum would be below $p^{H,M}$.

Once again, any solution to $g(v) = 0$ on $v \in [0, p^{H,M}]$ must satisfy $v = \sup \hat{V}(p^L) \cap [0, p^{H,M}]$ or $v = \inf \hat{V}(p^L)$. Suppose there exists another solution to this, which must then be strictly below $\sup \hat{V}(p^L) \cap [0, p^{H,M}] = p^{H,M}$. This would violate strict quasiconcavity, since $g(\inf \hat{V}(p^L)) = 0$ and $g(\sup \hat{V}(p^L) \cap [0, p^{H,M}]) \geq 0$.

Part 4: If $\hat{V}(p^L) \cap [0, p^{H,M}]$ is empty, there is at most one solution to $g(v) = 0$, namely v^*.

Note that $g(v^*) \leq 0$ is equivalent to saying that the set $\hat{V}(p^L) \cap [0, p^{H,M}]$ is empty under strict quasiconcavity and continuity of $g(v)$.

By strict quasiconcavity of g, it has a unique maximum on $[0, p^{H,M}]$. If this maximum is below 0, there exists no solution.

If this maximum is 0, v^* is a solution - but it is the only possible solution.

Part 5: If p^L satisfies $\frac{\partial \Pi^M(p_j;\tilde{v}^L)}{\partial p_j}\Big|_{p^L} = 0$, then p^H must be equal to $v^* = arg\max_{v\in[0,p^{H,M}]} g(v)$, $\inf \hat{V}(p^L)$ or $p^{H,M}$.

Our regularity conditions imply that only one other candidate for an equilibrium high signal price can exist, namely $p^H = \sup \hat{V}(p^L) \cap [0, p^{H,M}]$.

Consider an equilibrium candidate where p^L satisfies the above FOC and $p^H = \sup \hat{V}(p^L) \cap [0, p^{H,M}]$. This construction requires that $\hat{V}(p^L) \cap [0, p^{H,M}]$ is non-empty, which in turn implies that $\inf \hat{V}(p^L) < \sup \hat{V}(p^L) \cap [0, p^{H,M}]$.

We know $p^L < \inf \hat{V}(p^L) < \sup \hat{V}(p^L) \cap [0, p^{H,M}] = p^H$, which then directly implies that a strictly positive measure of consumers will search on path, i.e. $M^L(p^L, p^H) > 0$. Then, there is a profitable upward deviation from p^L.

Part 6: If p^L satisfies $\frac{\partial \Pi^M(p_j;\tilde{v}^L)}{\partial p_j}\Big|_{p^L} + M^L(p^L, p^H) = 0$, with $M^L(.) > 0$, then $p^H = \sup \hat{V}(p^L) \cap [0, p^{H,M}]$ or $p^H = p^{H,M}$ must hold.

Note first that $\hat{V}(p^L) \cap [0, p^{H,M}]$ cannot be empty, since $v \in \hat{V}(p^L)$ is a necessary condition for equilibrium search and all consumers with $v \leq p^H \leq p^{H,M}$ cannot satisfy this condition. This rules out the equilibrium candidate $p^H = arg\max_{v\geq p^{H,M}} g(v)$.

Now suppose that $\hat{V}(p^L)$ is non-empty, but that $p^H = \inf \hat{V}(p^L)$. All consumers with $v \geq p^H$ cannot search on-path. All consumers with $v < \inf \hat{V}(p^L)$ must have $v \notin \hat{V}(p^L)$ by the definition of the infimum. Thus, they cannot search at p^H either, which implies that there will not be search-on-path and $M^L(p) = 0$, a contradiction.

Part 7: An equilibrium candidate (p^L, p^H) with $\frac{\partial \Pi^M(p_j;\tilde{v}^L)}{\partial p_j}\Big|_{p^L} = 0$ and corresponding empty $\hat{V}(p^L)$ and $g(p^H) = 0$ only exists for exactly one value of search costs.

By strict quasiconcavity of $g(v)$, it has a unique maximum on $[0, p^{H,M}]$ for any given p^L. Under strict concavity of the low signal monopoly profit function, there is only one p^L, namely $p^{L,M}$, that can constitute a solution to the low-signal first-order condition. Thus, there is a unique candidate v^* for the equilibrium high signal price.

This must satisfy $g(v^*) = 0 \iff Pr(\tilde{v}^L|v^*)(v^* - p^{L,M}) - s = 0$. This equation has only one solution for s. If $s \neq Pr(\tilde{v}^L|v^*)(v^* - p^{L,M})$, this equilibrium cannot exist.

Thus, the interval of search costs for which this equilibrium exists has zero measure.

Summary of previous steps:

The previous results show that there are four equilibrium candidates we can have (ignoring candidates that only exist for a zero measure of search costs).

For the monopoly equilibrium, we know there is a unique candidate that can satisfy the construction.

In the search deterrence equilibrium, $p^L = p^{L,M}$. The infimum of $\hat{V}(p^{L,M})$, if it exists, is uniquely determined, so there is just one candidate for this.

In the following, I proove that there exists exactly one candidate that satisfies the definition of the search equilibrium and one candidate that satisfies the definition of the constrained search equilibrium.

Part 8: Uniqueness of the search equilibrium.

I now show that there is at most one candidate for a low signal price that satisfies the definition of the search equilibrium.

Because $p^H = p^{H,M}$, the set of consumers that search on-path is given by $\hat{V}(p^L) \cap [0, p^{H,M}]$ and $\inf \hat{V}(p^L) = \inf \hat{V}(p^L) \cap [0, p^{H,M}]$. To see this, note that $\inf \hat{V}(p^L) < p^{H,M}$ must hold, otherwise there would not be search on-path.

Here, $p^{H,S} = p^{H,M}$ and $p^{L,S}$ must satisfy the following equation:

$$\left.\frac{\partial \Pi^M(p_j, \tilde{v}^L)}{\partial p_j}\right|_{p^L} + \sum_{j=2}^{N} \int_{\inf \hat{V}(p^L)}^{\sup \hat{V}(p^L) \cap [0, p^{H,M}]} (1/N) Pr(\tilde{v}^L|v) Pr(\tilde{v}^H|v)^{j-1} dv = 0$$

I have to show that the LHS is strictly falling in p^L for the relevant interval of possible low signal prices. An equilibrium low signal price p^L must induce a non-empty $\hat{V}(p^L)$. Thus, for all possible equilibrium candidates of p^L, $\inf \hat{V}(p^L) \in (0, 1)$ must hold.

The first component is already strictly falling in p^L by strict concavity of the monopoly low signal profit function.

Showing that $\inf \hat{V}(p^L)$ is weakly increasing in p^L and $\sup \hat{V}(p^L) \cap [0, p^{H,M}]$ is weakly decreasing in p^L will be sufficient to show that the second component will also be weakly decreasing in p^L.

First, I show that $\inf \hat{V}(p^L)$ must be weakly increasing in p^L.

Suppose, for a contradiction, that $\inf \hat{V}(p^L)$ is decreasing in p^L at some point. Pick two $p^{L,1}, p^{L,2}$ with $p^{L,1} < p^{L,2}$, for which:

$$\inf \hat{V}(p^{L,1}) > \inf \hat{V}(p^{L,2})$$

It was already established that such an infimum must set $g(v; p^L) = 0$. Thus, we have:

$$g(\inf \hat{V}(p^{L,1}), p^{L,1}) = 0$$

Strict quasiconcavity, together with $\inf \hat{V}(p^{L,2}) < \inf \hat{V}(p^{L,1})$, implies that:

$$g(\inf \hat{V}(p^{L,2}), p^{L,1}) < 0 \implies Pr(\tilde{v}^L | \inf \hat{V}(p^{L,2})) \bigg(\inf \hat{V}(p^{L,2}) - p^{L,1} \bigg) - s < 0$$

However, since $p^{L,1} < p^{L,2}$, we have:

$$Pr(\tilde{v}^L | \inf \hat{V}(p^{L,2})) \bigg(\inf \hat{V}(p^{L,2}) - p^{L,2} \bigg) - s < Pr(\tilde{v}^L | \inf \hat{V}(p^{L,2})) \bigg(\inf \hat{V}(p^{L,2}) - p^{L,1} \bigg) - s < 0$$

Hence, $\inf \hat{V}(p^{L,2}) \in (0, 1)$ does not satisfy the definition of an infimum, a contradiction.

Now I need to show the analogue of this for the supremum. I will show that the supremum $\sup \hat{V}(p^L) \cap [0, p^{H,M}]$ is weakly decreasing in p^L. Recall that this supremum must be weakly greater than 0 in order for $\hat{V}(p^L)$ to be non-empty. For notational clarity, I abbreviate this by sup in what follows.

Pick two $p^{L,1}, p^{L,2}$ with $p^{L,1} < p^{L,2}$ for which sup is increasing, i.e.:

$$\sup(p^{L,1}) < \sup(p^{L,2}) \leq p^{H,M}$$

This specification implies that $\sup(p^{L,1}) < p^{H,M}$, which means that $g(\sup(p^{L,1}); p^{L,1}) = 0$ must hold. By strict quasiconcavity of g, it must hold that:

$$g(\sup(p^{L,2}); p^{L,1}) < 0$$

However, since $p^{L,1} < p^{L,2}$, we have that:

$$Pr(\tilde{v}^L | \sup(p^{L,2}))\bigg(\sup(p^{L,2}) - p^{L,2}\bigg) - s < Pr(\tilde{v}^L | \sup(p^{L,2}))\bigg(\sup(p^{L,2}) - p^{L,1}\bigg) - s < 0$$

Once again, this represents a contradiction.

Thus, I have shown that $\sup \hat{V}(p^L) \cap [0, p^{H,M}]$ is weakly decreasing in p^L and $\inf \hat{V}(p^L)$ is weakly increasing in p^L. Thus, the entire integral of searchers will be weakly decreasing in p^L, which completes the proof.

Part 9: Uniqueness of constrained search equilibrium.

Note that $\inf \hat{V}(p^L)$ is weakly increasing in p^L and $\sup \hat{V}(p^L) \cap [0, p^{H,M}]$ is weakly decreasing.

In the constrained search equilibrium (and by our strict quasiconcavity assumption), the interval of consumers that search on-path is given by $\big[\inf \hat{V}(p^L), p^H\big]$.

Uniqueness of the low signal price in the constrained search equilibrium follows if the following equation only has one solution:

$$\frac{\partial \Pi^M(p_j, \tilde{v}^L)}{\partial p_j}\bigg|_{p^L} + \int_{\inf \hat{V}(p^L)}^{\sup \hat{V}(p^L) \cap [0, p^{H,M}]} (1/N) Pr(\tilde{v}^L | v) Pr(\tilde{v}^H | v) dv = 0$$

The LHS is strictly decreasing in p^L by previous arguments, implying that there is only one solution for p^L.

Given that $p^{L,C}$ is unique, $p^{H,C} = \sup \hat{V}(p^{L,C}) \cap [0, p^{H,M}]$ will also be unique by the definition of a supremum.

C.6 Proof of proposition 9

Part 1: There must be search in such a mixed-strategy equilibrium.

Suppose that no consumer searches on-path. Then, each firm attains monopoly profits (for the respective prices) at all prices that are played on the equilibrium path. In a mixed-strategy equilibrium, there must be signal $\tilde{v}^k$ after which at least two prices are offered by a firm. Strict concavity of profits implies a contradiction to the mixing indifference condition.

Part 2:

By the arguments in the analogous proposition of the previous section, the price p^{min} will be deterministically played after it's corresponding signal $\tilde{v}^{min}$.

Now suppose, for a contradiction, that this price is played after $\tilde{v}^H$. We know that no other price can be played after this signal by previous arguments. Thus, it must hold that $p^{min} > p^{H,M}$ - otherwise, the first-order condition that p^{min} must satisfy would surely be violated by strict concavity of the monopoly profit functions.

Consider the gains of search of an agent with $v > p^{min}$ at $\bar{p}^L$, the highest possible price that an agent can receive.

$$Pr(\tilde{v}^H|v)(v - p^{min}) + Pr(\tilde{v}^L|v) \int_{\underline{p}^L}^{\bar{p}^L} \max\{v - p, 0\} dF^L(p) - s > \max\{v - \bar{p}^L, 0\}$$

For $v \leq \bar{p}^L$, the gains of search are:

$$Pr(\tilde{v}^H|v)(v - p^{min}) + Pr(\tilde{v}^L|v) \int_{\underline{p}^L}^{v} (v - p) dF^L(p) - s - 0$$

The derivative of these w.r.t. v is:

$$Pr(\tilde{v}^H|v) + Pr(\tilde{v}^L|v) \int_{\underline{p}^L}^{v} dF^L(p) + \frac{\partial Pr(\tilde{v}^H|v)}{\partial v}(v - p^{min}) - \frac{\partial Pr(\tilde{v}^H|v)}{\partial v} \int_{\underline{p}^L}^{v} (v - p) dF^L(p)$$

This will be strictly positive.

Now consider consumers with $v > \bar{p}^L$. For them, the gains of search are:

$$Pr(\tilde{v}^H|v)(\bar{p}^L - p^{min}) + Pr(\tilde{v}^L|v) \int_{\underline{p}^L}^{\bar{p}^L} (\bar{p}^L - p)dF^L(p) - s$$

These are also increasing in v because p^{min} is smaller than any other price a consumer could receive. Thus, if any consumer with valuation v' searches at $\bar{p}^L$, so will all consumers with a valuation above this.

If $v' < \bar{p}^L$, all consumers with $v \geq \bar{p}^L$ will search, which will imply a contradiction. If $\bar{p}^L$ is played with zero probability, it will invoke zero profits. If it is played with positive probability, there are undercutting motives.

Thus, $v' \geq \bar{p}^L$ must hold. Firstly, suppose that $\bar{p}^L$ is played with strictly positive probability. We know $v' < 1$ must hold because there must be search in such an equilibrium. All consumers with $v \in [v', 1]$ receice $\bar{p}^L$ with positive probability and would search thereafter - this creates undercutting motives.

Thus, $\bar{p}^L$ must be played with zero probability. Any consumer who arrives after search would buy with probability zero when being offered this price. Thus, profits at this price must be weakly below monopoly profits.

Thus, low signal profits must be below $\Pi^M(\bar{p}^L, \tilde{v}^L)$. We know $\bar{p}^L > p^{min} > p^{H,M} > p^{L,M}$ and thus that monopoly profits are rising in this region. When offering p^{min}, no consumer would search - thus granting monopoly profits at the very least. Thus, this deviation would be profitable and we have a contradiction.

Part 3: The lowest price offered after $\tilde{v}^H$, namely $\underline{p}^H$, must be weakly above $\inf \hat{V}(p^L)$.

Note that the set $\hat{V}(p^L)$ now captures necessary conditions for search at $\underline{p}^H$. Any consumer would search upon receiving this price if and only if:

$$Pr(\tilde{v}^L|v)(v - p^L) + Pr(\tilde{v}^H|v)(0) - s > \max\{v - \underline{p}^H, 0\}$$

Now suppose, for a contradiction, that $\inf \hat{V}(p^L) > \underline{p}^H$. Then, no consumer would search at $\underline{p}^H$. To see this, note that membership in the set $\hat{V}(p^L)$ is clearly a necessary condition for search at this price.

Thus, the only consumers that could search at $\bar{p}^H$ must have $v \in [\inf \hat{V}(p^L), \sup \hat{V}(p^L)]$. No such consumer would search at $\underline{p}^H$, because $v \geq \inf \hat{V}(p^L) > \underline{p}^H$ and hence:

$$Pr(\tilde{v}^L|v)(v - p^L) + Pr(\tilde{v}^H|v)(v - \underline{p}^H) - s \leq (v - \underline{p}^H) \iff Pr(\tilde{v}^L|v)(\underline{p}^H - p^L) - s < 0$$

We know this must hold true because $\underline{p}^H < \inf \hat{V}(p^L)$ and $Pr(\tilde{v}^L|v) \leq Pr(\tilde{v}^L|\underline{p}^H)$ (because $v > \underline{p}^H$), which guarantees:

$$Pr(\tilde{v}^L|v)(\underline{p}^H - p^L) - s \leq Pr(\tilde{v}^L|\underline{p}^H)(\underline{p}^H - p^L) - s < 0$$

The second inequality holds by strict quasiconcavity of $g(.)$.

Similarly, no consumer will search after receiving a price just above $\underline{p}^H$. By the assumption that F^H is atomless and gapless, there will be a violation of the mixing indifference condition, because profits in this interval equal monopoly profits + searchers*price, which must be strictly concave since monopoly profits are strictly concave.

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D Numerical results for general signal distributions

D.1 Overview

In this subsection, I numerically show that the properties of the aforementioned equilibria carry over to generalized settings. I will consider signal distributions that take the following form:

$$Pr(\tilde{v}^L|v) = \alpha\left(1 - \frac{1}{1 + e^{k(0.5-v)}}\right) + (1 - \alpha)\left(\frac{1}{1 + e^{k(0.5-v)}}\right) \tag{27}$$

For $k \to \infty$, this distribution is for all intents and purposes equal to the one used in the baseline setting. Lower values of k amount to making the signal distribution more linear, while the parameter α governs the upper and lower bounds of the probability distribution. In figure 6, I plot this distribution for different values of α and k.

Under the aforementioned interpretation of signal precision, the latter is rising both in k and α. In figure 7, I study different parameter combinations and show when the aforementioned equilibria exist. A given graph in this figure always corresponds to a fixed level of

α and a fixed number of firms, namely $\alpha \in \{0.75, 0.85\}$ and $N \in \{2, 4\}$, respectively. On the x-axis, different levels of k are being plotted. On the y-axis, different levels of s are plotted. As before, the different colors indicate existence of a given equilibrium. The search deterrence equilibrium exists at green points, the search equilibrium exists at yellow points, and the monopoly equilibrium exists at blue points.

The general trends outlined previously are being confirmed. The search deterrence equilibrium exists for low search costs, the search equilibrium exists for intermediate search costs, and the monopoly equilibrium exists for high search costs. Both the search equilibrium and the search deterrence equilibrium exist for substantial parameter ranges even when k is in the region $[25, 40]$, at which the signal distribution is relatively linear.

High levels of information precision facilitate existence of the search equilibrium. Shifting α from 0.85 to 0.75 and decreasing k shrinks the interval of search costs for which the search equilibrium exists. There is a potential issue of equilibrium non-existence when search costs are too high to support the search deterrence equilibrium but not high enough to support the search equilibrium. I conjecture that the mixed-strategy equilibria discussed previously would occupy these spaces.

In figure 8, I plot the comparative statics effects of an increase of search costs, fixing levels of k and α. These comparative statics effects mirror those in the baseline model.

D.2 Equilibrium search in general settings

The measure of consumers who arrive after search and generate the signal $\tilde{v}^k$ is given by:

$$M^k(.) = \sum_{j=2}^{N} \int_{\inf \hat{V}(p^L)}^{\sup \hat{V}(p^L)} \left[Pr(\tilde{v}^H|v)\right]^{j-1} Pr(\tilde{v}^k|v)(1/N)dv = \int_{\inf \hat{V}(p^L)}^{\sup \hat{V}(p^L)} \left(\sum_{j=2}^{N} \left[Pr(\tilde{v}^H|v)\right]^{j-1} \right) Pr(\tilde{v}^k|v)(1/N)dv$$

Note that:

$$\sum_{j=2}^{N} \left[Pr(\tilde{v}^H|v)\right]^{j-1} = \sum_{j=1}^{N-1} \left[Pr(\tilde{v}^H|v)\right]^{j} = \frac{Pr(\tilde{v}^H|v)\left(1 - Pr(\tilde{v}^H|v)^{N-1}\right)}{1 - Pr(\tilde{v}^H|v)}$$

Plugging this back in yields:

$$M^k(p^L, p^H) = \int_{\inf \hat{V}(p^L)}^{\sup \hat{V}(p^L)} \left(\frac{Pr(\tilde{v}^H|v)\left(1 - Pr(\tilde{v}^H|v)^{N-1}\right)}{1 - Pr(\tilde{v}^H|v)} \right) Pr(\tilde{v}^k|v)(1/N)dv$$

Now I have to calculate the profits for off-equilibrium prices, namely $p_j \in [\inf \hat{V}(p^L), \sup \hat{V}(p^L)]$ for which search is possible. Note that $\sup \hat{V}(p^L) \leq p^H$. These are:

$$\Pi^C(p_j; \tilde{v}^k) = p_j \int_{p_j}^{1} (1/N) Pr(\tilde{v}^k|v) \Big[\mathbb{1}[p_j \leq \hat{p}(v)] + \mathbb{1}[p_j > \hat{p}(v)] Pr(\tilde{v}^H|v)^{N-1} \mathbb{1}[p_j \leq p^H] \Big] dv +$$

$$p_j \sum_{j=2}^{N} \int_{p_j}^{\sup \hat{V}(p^L)} (1/N) Pr(\tilde{v}^k|v) Pr(\tilde{v}^H|v)^{j-1} \Big[\mathbb{1}[p_j \leq \hat{p}(v)] + \mathbb{1}[p_j > \hat{p}(v)] Pr(\tilde{v}^H|v)^{N-j} \mathbb{1}[p_j \leq p^H] \Big] dv$$

D.3 Existence results - constrained search equilibrium

This equilibrium exists if and only if (i) there is search on the equilibrium path but no consumer with $v > p^{H,C}$ searches on-path **and** (ii) there are no profitable deviations from the equilibrium high signal price $p^{H,C}$. In figure 9, I visualize when these requirements are met - if one of these requirements fails, then the equilibrium does not exist. This is visualized by a red dot. At green dots, the equilibrium exists.

D.4 Search costs and prices - general settings

In figure 8, I visualize the relationship between search costs and prices in general settings (when restricting attention to pure-strategy equilibria).

D.5 Documenting when $NPr(\tilde{v}^H|p^{H,M})^{N-1} \geq 1$ holds true

I consider signal distributions with the following form:

$$Pr(\tilde{v}^L|v) = \alpha \left(1 - \frac{1}{1 + e^{k(0.5-v)}} \right) + (1-\alpha) \left(\frac{1}{1 + e^{k(0.5-v)}} \right)$$

Fixing a given level of N, I calculate the value of $NPr(\tilde{v}^H|p^{H,M})^{N-1}$ for different combinations of k and α, which govern the signal distribution. In figure 10, the following data is plotted: For a given combination of k and α, green dots indicate that the regularity condition holds - red dots indicate that it is violated. Every graph corresponds to a given N.

D.6 Quasiconcavity of $g(v)$

In the following, I document how many zeros the function $g(v)$ has. The following data is visualized in figure 11. Every graph corresponds to a given combination of p^L and s in the sets $p^L \in \{0.25, 0.3, 0.35, 0.4\}$ and $s \in \{0.01, 0.05, 0.09, 0.13\}$. The first row corresponds to the first value of s and documents the results for different values of p^L. The second row corresponds to the second value of s, and so on. In a given column, the p^L under consideration is always the same.

Within each graph, I check whether the function $g(v)$ has at most two zeros in the interval $[0, p^{H,M}]$ for a given k and α, which governs $Pr(\tilde{v}^H|v)$. Green dots indicate that this is fulfilled. Red dots indicate that there are more than two zeros.

E Extension - information about search history

E.1 Overview

In the previous analysis, it was assumed that firms have no information about the search history of any consumer that visits the firm. In real-world settings, firms could design and store third-party cookies in a way that allows them to determine whether a consumer has previously acquired other price offers.

To study the effects of such a technology when firms already receive signals about consumer valuations, I consider the following model: I assume, for simplicity, that there are just two firms. For any consumer i that visits a firm j, the firm j receives an informative binary signal $\tilde{v}_{i,j}$ about the consumer's valuation and realizes whether this consumer i visits firm j first or second. Thus, a firm has four information sets. The prices $(p^{1,L}, p^{1,H})$, which I call *first arriver prices*, are offered to consumers that arrive at a firm first and generate the low and high signal, respectively. The prices $(p^{2,L}, p^{2,H})$, which I call *searcher prices*, are offered to consumers that arrive at a firm second and generate the low and high signal, respectively.

As in the baseline model, I assume that the signal distribution only depends on a consumer's valuation and is given by the following distribution with information parameter $\alpha \in (0.5, 1)$:

$$Pr(\tilde{v}^H|v) = \begin{cases} \alpha & v \geq 0.5 \\ 1-\alpha & v < 0.5 \end{cases} \tag{28}$$

I further assume that $s > 0$. I label the resulting framework the *framework with search history information*. Crucially, no symmetric equilibrium with on-path search can exist in this setting, as demonstrated by the following propostion.

Proposition 10 *There exists no symmetric equilibrium with search on the equilibrium path in the framework with search history information.*

Any such equilibrium is ruled out by a fundamental inconsistency between the optimal search behaviour of consumers and optimal firm pricing. To fix ideas, consider a potential pure-strategy equilibrium with search. Define the infimum of the set of consumer valuations that search on the equilibrium path as $\underline{v}$ and the price $p^{2,min} = \min\{p^{2,L}, p^{2,H}\}$ as the lowest possible price a consumer can obtain via search. Optimal firm pricing requires that $p^{2,min} \geq \underline{v}$ must hold. However, sequentially rational search behaviour implies that $p^{2,min} < \underline{v}$ must be true. Since these properties cannot be fulfilled simultaneously, there exists no symmetric pure-strategy equilibrium with search on the equilibrium path.

To gain intuition, suppose that $p^{2,min} < \underline{v}$ holds. Then, all consumers that arrive at some firm j after searching will buy at firm j if offered the price $p^{2,min}$ or a price just above it - this holds because $p^{2,min} < \underline{v}$ and since $p^{2,min}$ surely beats the price that such a consumer has received at the firm that was initially visited. The latter is a necessary condition for search to have been optimal. If $p^{2,min} < \underline{v}$, firms would thus find it optimal to deviate to a slightly higher price, a contradiction. This means that $p^{2,min} \geq \underline{v}$ must hold in such an equilibrium. However, this result is not consistent with sequentially rational search behaviour. Because $s > 0$, consumers with a valuation below $p^{2,min}$ or just above $p^{2,min}$ will find it strictly optimal not to search, no matter the price they have initially received. This implies that $\underline{v}$, the infimum of the valuations that search on-path, must satisfy $\underline{v} > p^{2,min}$.

Similar arguments rule out the existence of symmetric mixed-strategy equilibria with on-path search. Having established this, I characterize equilibria without on-path search.

Proposition 11 *In a pure-strategy equilibrium without on-path search in the framework with search history information, $p^{1,L} \leq p^{L,M}$ and $p^{1,H} \leq p^{H,M}$ must hold. There always exists an equilibrium where first arriver prices equal monopoly prices and firms attain monopoly profits.*

Because there is no on-path search, any price $p^{1,k}$ (for a signal $\tilde{v}^k$) above the corresponding monopoly price $p^{M,k}$ cannot be sustained as an equilibrium, because there would always be a profitable downward deviation. To understand the second result, note that firm beliefs at an information set where a consumer arrives after searching are arbitrary, because such information sets must be off-path when there is no on-path search. If a firm believes that only consumers with $v = 0$ would ever search, setting the prices $p^{2,L} = p^{2,H} = 1$ is optimal.

Then, offering the monopoly prices $(p^{1,L}, p^{1,H})$ to first arrivers is optimal, as consumers cannot constrain prices with the threat of searching.

It remains to study when consumers benefit from regulation that prohibits firms from using information about their search histories. When this is not forbidden and certain browsers enable the availability of search history information, propositions (10) and (11) establish that the equilibrium prices in these situations will be weakly lower than their monopoly counterparts. By contrast, the equilibria of the baseline setting will be played if firms cannot access search history information. Thus, this policy measure would lead to increased prices at high or intermediate search costs, when the monopoly equilibrium or the search equilibrium would be played in the baseline setting. By implication, prohibiting firms from accessing search history information can only be pro-competitive when search costs are low and the search deterrence equilibrium or the MSE would be played in the baseline setting.

E.2 Proof of proposition 10

Part 1: Symmetric pure-strategy equilibria

Define $p^{1,max} = \max\{p^{1,L}, p^{1,H}\}$ and $p^{2,min} = \min\{p^{2,L}, p^{2,H}\}$. Further, define $\underline{v}$ as the infimum of the set of valuations that would search on the equilibrium path.

(i) Equilibrium candidates where search only occurs after $p^{1,max}$.

Note firstly that $p^{2,min} < p^{1,max}$ must hold - otherwise, search would never be optimal after receiving $p^{1,max}$.

Secondly, it must hold that $p^{2,min} < \underline{v}$. Suppose, for a contradiction, that $p^{2,min} \geq \underline{v}$. Any consumer will search upon receiving the initial price p_j if and only if:

$$Pr(\tilde{v}^H|v)\max\{v - p^{H,2}, v - p_j, 0\} + Pr(\tilde{v}^L|v)\max\{v - p^{L,2}, v - p_j, 0\} - s - \max\{v - p_j, 0\} > 0$$

These gains from search are bounded from above by:

$$gos(v, p_j) = Pr(\tilde{v}^H|v)\max\{v - p^{2.H}, v - p_j, 0\} + Pr(\tilde{v}^L|v)\max\{v - p^{2,L}, v - p_j, 0\} - s - \max\{v - p_j, 0\}$$

$$\leq$$

$$\max\{v - p^{2,min}, v - p_j, 0\} - s - \max\{v - p_j, 0\}$$

Evaluating this at $p_j = p^{1,max} > p^{2,min}$ yields:

$$gos(v, p^{1,max}) \leq \max\{v - p^{2,min}, 0\} - s - \max\{v - p^{1,max}, 0\}$$

When $v \in [0, p^{2,min} + s)$, these gains of search will be strictly negative. Thus, no consumer with $v \in [0, p^{2,min} + s)$ can ever search, which implies that $\underline{v} > p^{2,min}$ must hold.

But this leads to a contradiction. Since $\alpha \in (0, 1)$, any consumer that searches has a positive probability of generating either signal. Thus, there is a strictly positive measure of consumers that generate either signal after searching because there is a strictly positive measure of consumers that search. We have established that all consumers who arrive after searching must have $v \geq \underline{v} > p^{2,min}$ and must have received the price $p^{1,max} > p^{2,min}$.

Thus, searcher demand is fully inelastic in an open ball around $p^{2,min}$, which implies a the existence of a profitable upward deviation from $p^{2,min}$ and rules out any such equilibrium.

<u>(ii)</u> Equilibrium candidates where search occurs after both prices.

In order for search to be optimal after both first arriver prices, it must hold that $p^{2,min} < p^{1,min} \leq p^{1,max}$.

Secondly, it must once more hold that $p^{2,min} < \underline{v}$. Once more, we can bound the gains of search at any price $p^1 \in \{p^{1,min}, p^{1,max}\}$, which must be below $p^{2,min}$, from above by the following:

$$gos(v, p^1) \leq \max\{v - p^{2,min}, 0\} - s - \max\{v - p^1, 0\}$$

For any consumer with $v < p^{2,min} + s$, these will be strictly negative.

All consumers who search must have $v \geq \underline{v} > p^{2,min}$ and must have received a price strictly above $p^{2,min}$, which induces the existence of a profitable deviation from this price.

Part 2: Symmetric mixed-strategy equilibrium candidates

Consider a symmetric mixed-strategy equilibrium candidate with $p^{2,min}$ being the lowest possible price that firms offer to second arrivers. Define $\underline{p}^1$ as the infimum of the set of prices P^1 for which there exists a v that finds it strictly optimal to search after such a price. Thus, any consumer who arrives after searching must have received an initial price $p^1 \geq \underline{p}^1$.

Once more, any price $p^1 \in P^1$ must satisfy $p^{2,min} < p^1$. For any such prices, only consumers with $v > p^{2,min} + s$ would ever search. Thus, once again: Any consumer that arrives after searching must have received a price $p^1 > p^{2,min}$ and must have $v \geq \underline{v} \geq p^{2,min} + s$.

Moreover, search by any consumer requires that the probability of receiving a price $p^2 \in [p^{2,min}, \underline{p}^1)$ after searching must be strictly positive. Suppose that this probability is 0. Then, search would never be optimal for prices $p_j \in [\underline{p}^1, \underline{p}^1 + s)$, making $\underline{p}^1$ an incorrect infimum. To see this, note that the gains of search at prices $p_j \in [\underline{p}^1, \underline{p}^1 + s)$ bounded from above by:

$$gos(v, p^1) \leq \max\{v - \underline{p}^1, 0\} - s - \max\{v - p^1, 0\}$$

If $v < \underline{p}^1 + s$, these are negative. If $v \geq \underline{p}^1 + s$, then they become:

$$gos(v, p^1) \leq (v - \underline{p}^1) - s - (v - p^1) = p^1 - \underline{p}^1 - s < 0$$

Because $\alpha \in (0.5, 1)$, said probability is strictly positive for any consumer valuation v iff it is positive for all consumer valuations.

Thus, there must exist a signal $\tilde{v}^k$ after which the probability of receiving these prices must be strictly positive, i.e. $Pr(p^{2,k} \in [p^{2,min}, \underline{p}^1)|\tilde{v}^k) > 0$.

Suppose that $\underline{p}^1 \leq \underline{v}$. Then, for any price in the range $p_j \in [p^{2,min}, \underline{p}^1)$, the sale will be made to all consumers who arrive after searching.

Suppose there is just one price $p_j \in [p^{2,min}, \underline{p}^1)$ in the support of these prices offered after $\tilde{v}^k$. Then, there is a profitable upward deviation from this price since it has to be played with strictly positive probability. Suppose there are at least two prices $p_j \in [p^{2,min}, \underline{p}^1)$ in said support - this is a contradiction, as demand is entirely inelastic in this price interval and profits are thus strictly increasing. Thus, we cannot have an equilibrium.

Thus, suppose that $\underline{v} < \underline{p}^1$. Then, demand will only be fully inelastic in the price interval $p_j \in [p^{2,min}, \underline{v}]$.

Prices in this interval need to be offered with strictly positive probability to any consumer that arrives after searching. Suppose, for a contradiction, that that prices in the interval $p_j \in [p^{2,min}, \underline{v}]$ are offered with zero probability. Now consider the search decision of any consumer with $v \in (\underline{v}, \underline{p}^1]$ for any price $p_j \in P^1$ (at which the consumer cannot consume). Then, this consumer would search iff:

$$\int_{p^{2,min}}^{1} \max\{v - p, 0\} dF^2(p) - s - 0 > 0$$

Because $p_j \in [p^{2,min}, \underline{v}]$ are offered with zero probability, these gains of search are bounded from above by:

$$\max\{v - \underline{v}, 0\} - s$$

For $v \in B_\epsilon(\underline{v})$, these gains of search will thus be strictly negative again - implying that $\underline{v}$ could not be the appropriate infimum. Thus, prices $p_j \in [p^{2,min}, \underline{v}]$ must be offered with strictly positive probability to all searchers.

Because the demand is fully inelastic in this interval (which must be offered with positive probability to searchers), we obtain a contradiction once more.

E.3 Proof of proposition 11

Part 1: First arriver prices must be lower than monopoly prices.

Suppose that $p^{1,k} > p^{k,M}$ for some signal $\tilde{v}^k$. No consumer would move on to search for prices $p_j \le p^{1,k}$ - otherwise, there would be on-path search. Thus, competitive profits for $p_j \le p^{1,k}$ are equal to monopoly profits, and there is a profitable downward deviation.

Part 2: Independently of α and s, monopoly profits can always be attained.

The following vector of prices can always be supported as an equilibrium:

$$(p^{1,L}, p^{1,H}, p^{2,L}, p^{2,H}) = (p^{L,M}, p^{H,M}, 1, 1)$$

Given these prices, no consumer will ever search for any price under sequential rationality. Thus, first arriver profit functions are exactly equal to the monopoly profit functions. Thus, first arriver profits are maximized at $(p^{L,M}, p^{H,M})$.

The second arriver information sets are never reached on path - thus, beliefs are arbitrary. Firms could believe that only consumers with $v = 0$ search. Then, the aforementioned prices are optimal. No matter which prices are offered to second arrivers, the perceived profits are always 0.

F Simultaneous search

Consider a model with N firms where consumers, upon learning their type v, decide how many firms they want to sample. This choice is made before visiting any firm. Search is random. While the first search is free, visiting any firm after that incurs search costs equal to s. Firms receive signals about consumer valuations as in the main text. There are K signals $\left\{\tilde{v}^1, ..., \tilde{v}^K\right\}$ and $Pr(\tilde{v}^k) \in (0,1)$ holds for any valuation v.

When $s = 0$, there cannot be an equilibrium with price dispersion in this model. Consider, for a contradiction, an equilibrium candidate with price dispersion.

Consider a symmetric equilibrium with the highest equilibrium price p^{max}. Because there is price dispersion, there must exist a price $p_j < p^{max}$ that is played with positive probability after some signal. Any consumer has a strictly positive probability of obtaining this price via search.

Suppose that p^{max} is played with positive probability after some signal. To attain the best possible consumption opportunities, any consumer will optimally search all N firms. This optimality is strict, because there is always a chance of receiving the worst price p^{max} at any firm. Thus, all consumers will visit all firms in any such equilibrium candidate.

Because every consumer has a strictly positive chance of receiving p^{max}, there is a strictly positive measure of these consumers who receive this price at all N firms - this creates undercutting incentives that break the equilibrium.

Suppose instead that p^{max} is played with zero probability after all signals. Then, this price will yield zero profits, because every consumer is guaranteed to visit all firms (since there is price dispersion) and receive a price below this with probability 1. This represents a contradiction.

Finally, consider possible asymmetric equilibria with price dispersion and define the highest price that is played by any firm as p^{max}. Because the equilibrium is asymmetric and search is random, consumers will optimally commit to searching all firms.

Suppose that there exists some firm that plays p^{max} with zero probability. Then, any firm receives zero profits by offering p^{max}, a contradiction - because all consumers that visit this firm will also visit all other firms and receive a better price with probability 1.

If all firms play p^{max} with positive probability after some signal, there will be a strictly positive measure of consumers who receive this price with positive probability at all firms, which creates undercutting motives, a contradiction.

G Figures

G.1 Figure 1:

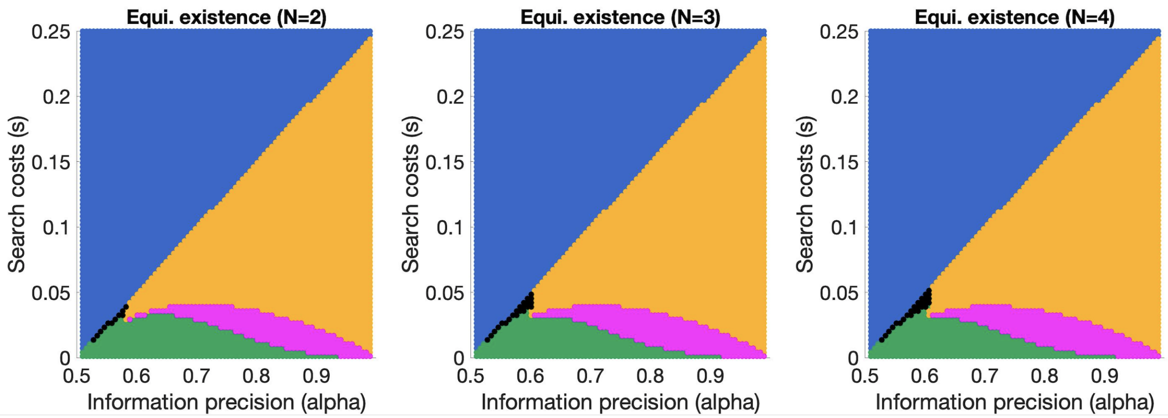

Figure 1: Equilibrium existence in the baseline model

G.2 Figure 2

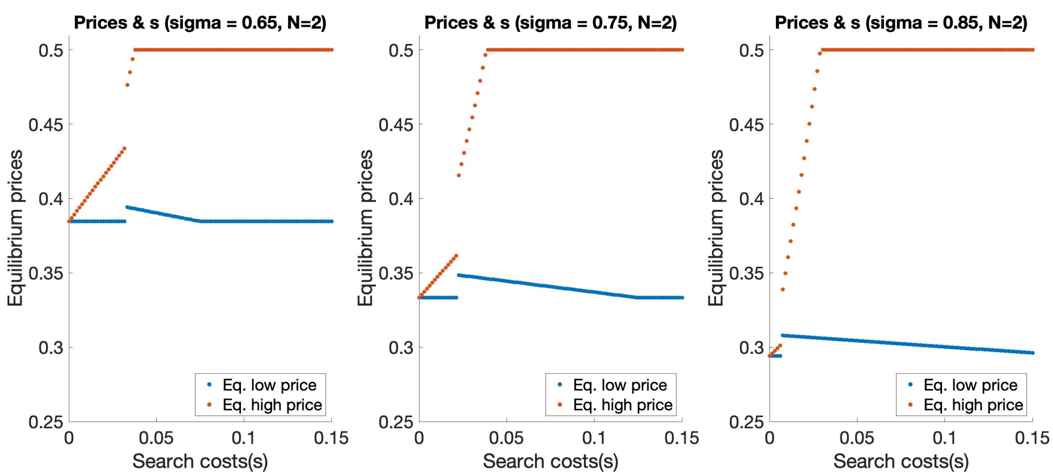

Figure 2: Search costs and prices in the baseline model

G.3 Figure 3

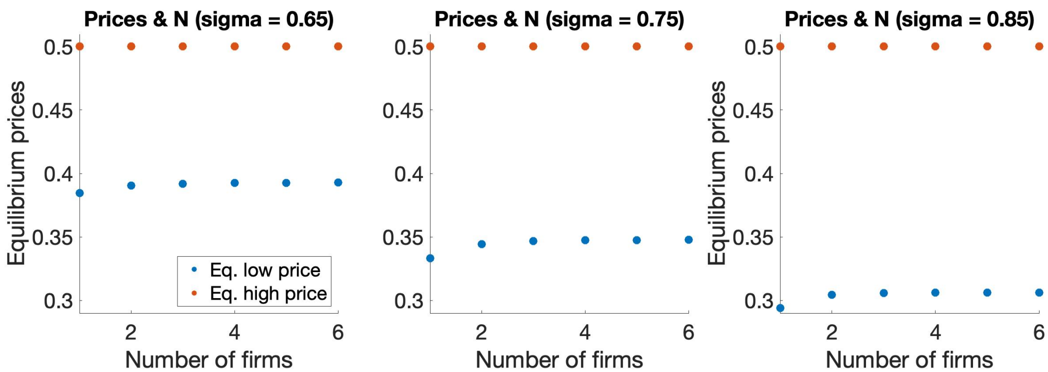

Figure 3: Competition and prices in the search equilibrium (baseline model)

G.4 Figure 4

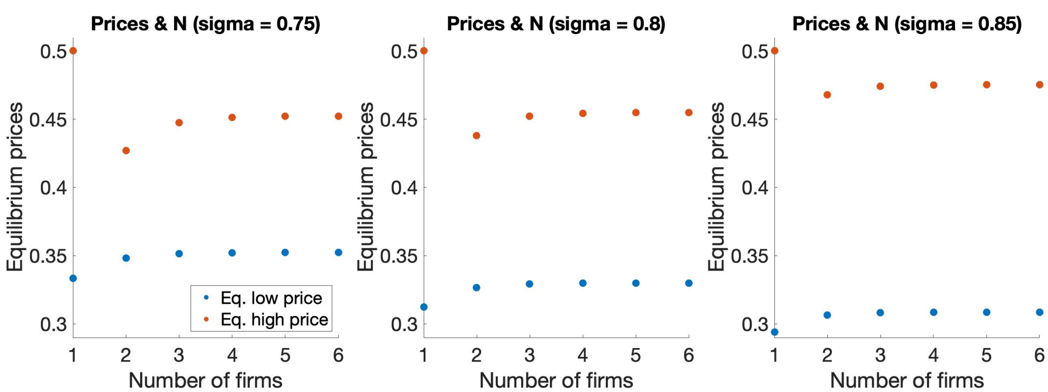

Figure 4: Competition and prices in the MSE (baseline model)

G.5 Figure 5

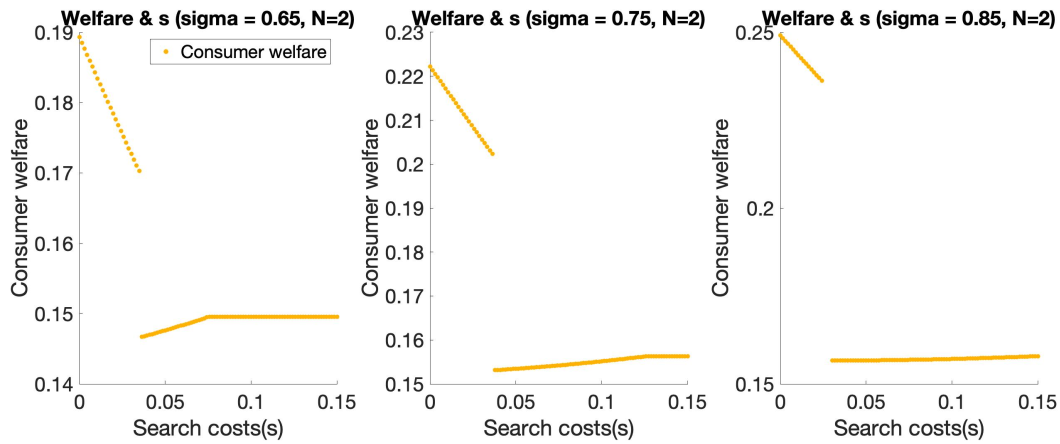

Figure 5: Search costs and consumer welfare

G.6 Figure 6

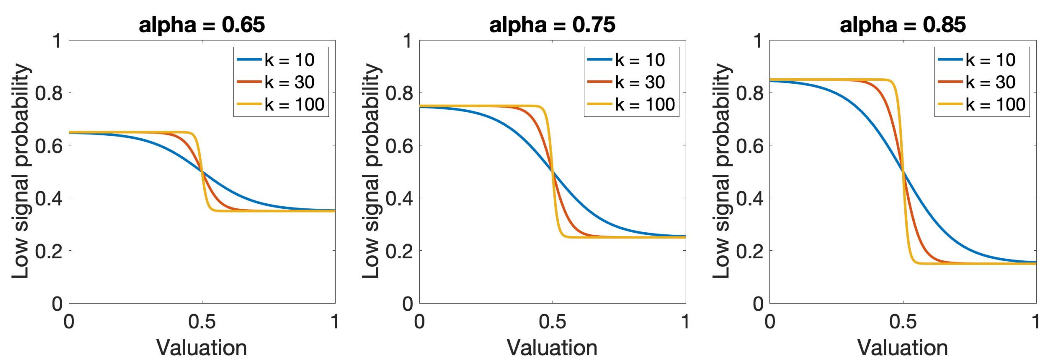

Figure 6: General signal distributions

G.7 Figure 7

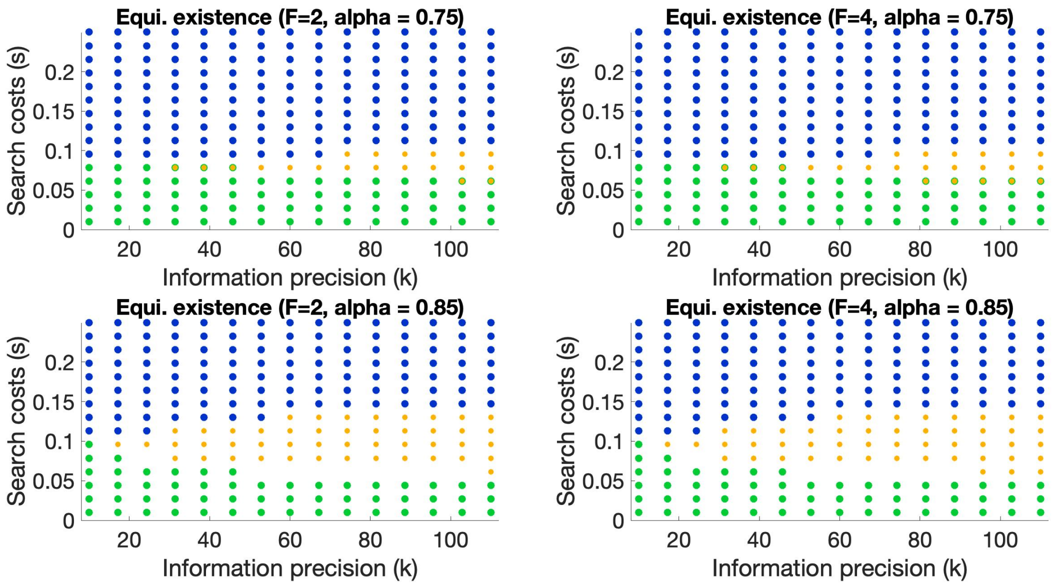

Figure 7: Existence results for general signal distributions

G.8 Figure 8

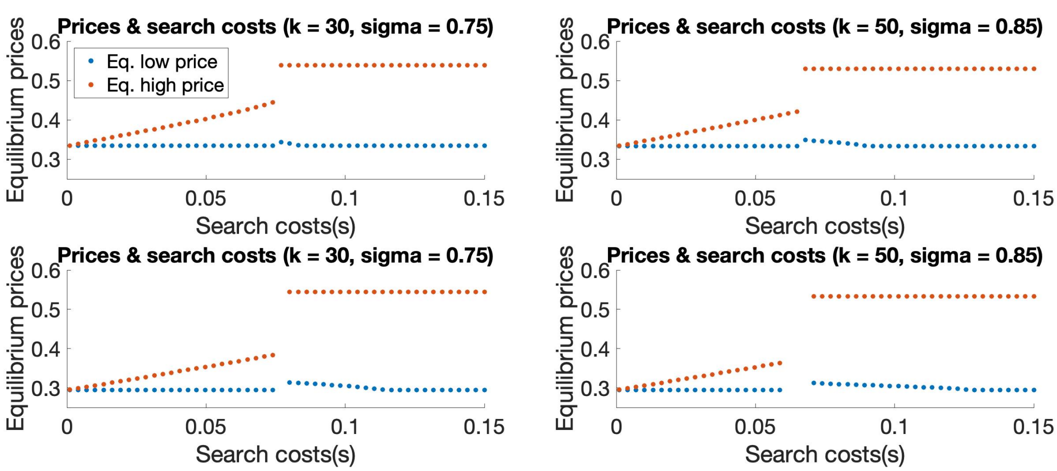

Figure 8: Comparative statics - search costs

G.9 Figure 9

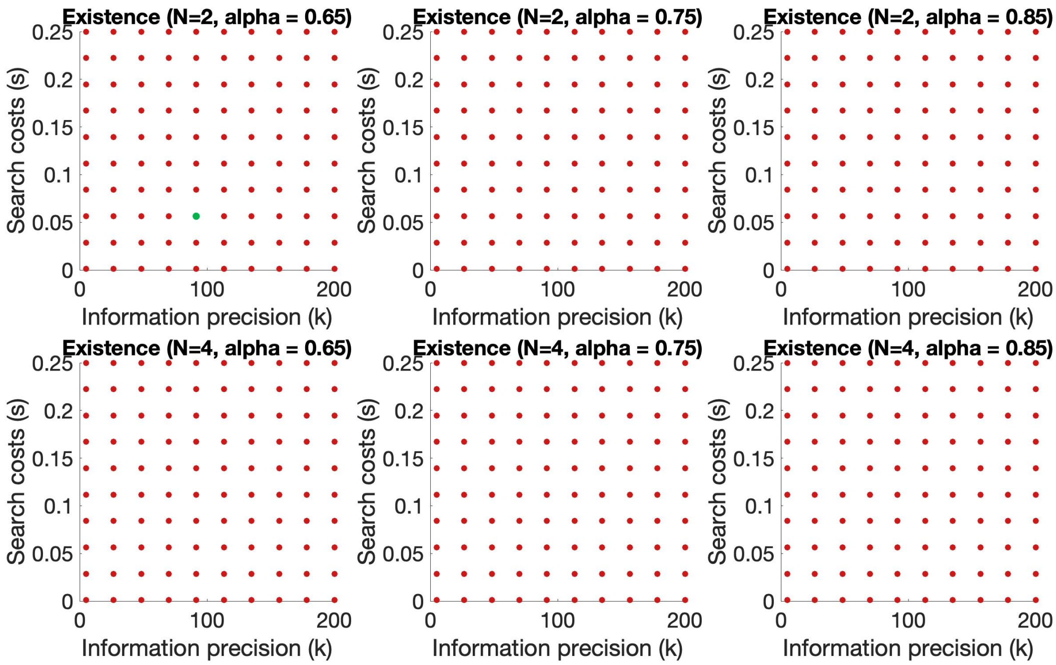

Figure 9: Existence of the constrained search equilibrium

G.10 Figure 10

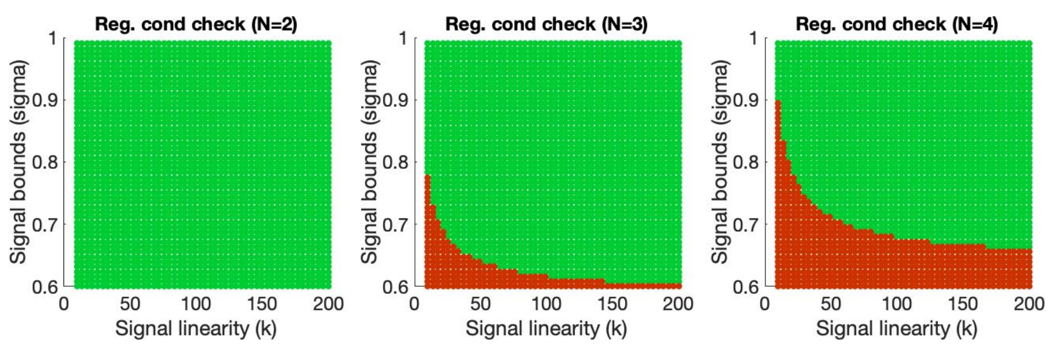

Figure 10: Regularity condition checks

G.11 Figure 11

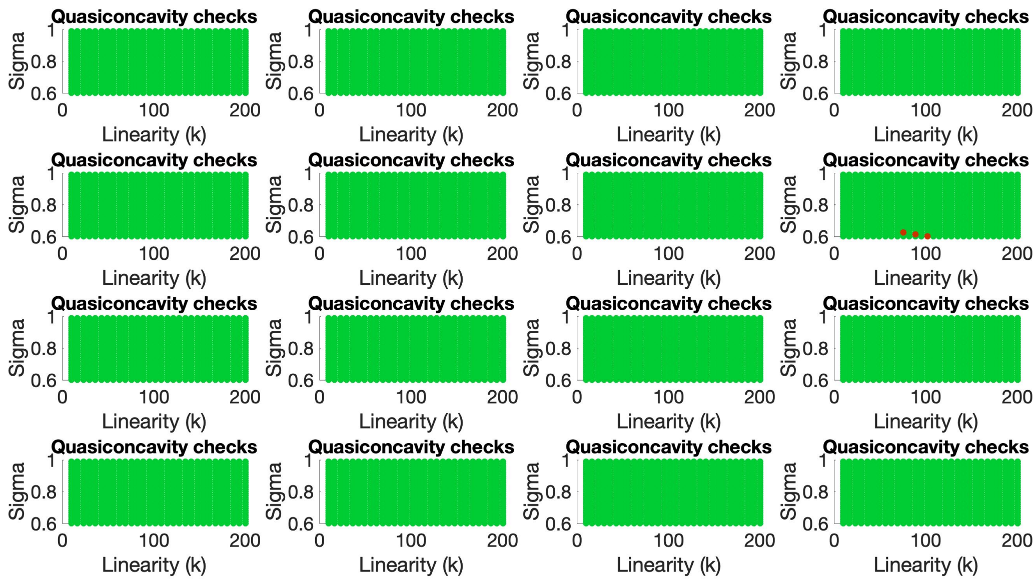

Figure 11: Quasiconcavity condition checks

References

Acquisti, A., & Varian, H. R. (2005). Conditioning prices on purchase history. *Marketing Science*, *24*(3), 367–381.

Armstrong, M., & Vickers, J. (2019). Discriminating against captive customers. *American Economic Review: Insights*, *1*(3), 257–72.

Armstrong, M., & Zhou, J. (2016). Search deterrence. *The Review of Economic Studies*, *83*(1), 26–57.

Aron, R., Sundararajan, A., & Viswanathan, S. (2006). Intelligent agents in electronic markets for information goods: customization, preference revelation and pricing. *Decision Support Systems*, *41*(4), 764–786.

Atayev, A. (2021). Search cost based price discrimination. *Working paper*.

Belleflamme, P., Lam, W. M. W., & Vergote, W. (2020). Competitive imperfect price discrimination and market power. *Marketing Science*, *39*(5), 996–1015.

Belleflamme, P., & Vergote, W. (2016). Monopoly price discrimination and privacy: The hidden cost of hiding. *Economics Letters*, *149*, 141–144.

Bergemann, D., Brooks, B., & Morris, S. (2021). Search, information, and prices. *Journal of Political Economy*, *129*(8), 000–000.

Braghieri, L. (2019). Targeted advertising and price discrimination in intermediated online markets. *working paper*.

Burdett, K., & Judd, K. L. (1983). Equilibrium price dispersion. *Econometrica: Journal of the Econometric Society*, 955–969.

Byrne, D. P., & Martin, L. A. (2021). Consumer search and income inequality. *International Journal of Industrial Organization*, 102716.

CCOECD. (2016). Price discrimination and competition. *OECD roundtable*.

Clavorà Braulin, F. (2021). The effects of personal information on competition: Consumer privacy and partial price discrimination. *ZEW-Centre for European Economic Research Discussion Paper*(21-007).

de Cornière, A., & Montes, R. (2017). Consumer privacy and the incentives to price-discriminate in online markets. *Review of Network Economics*, *16*(3), 291–305.

Diamond, P. A. (1971). A model of price adjustment. *Journal of economic theory*, *3*(2), 156–168.

Escobari, D., Rupp, N. G., & Meskey, J. (2019). An analysis of dynamic price discrimination in airlines. *Southern Economic Journal*, *85*(3), 639–662.

Esteves, R.-B. (2014). Price discrimination with private and imperfect information. *The Scandinavian Journal of Economics*, *116*(3), 766–796.

EUD2019/2161. (2019). Directive (eu) 2019/2161 of the european parliament and of the council. *Official Journal of the European Union*.

Fabra, N., & Reguant, M. (2020). A model of search with price discrimination. *European Economic Review*, *129*, 103571.

Fudenberg, D., & Tirole, J. (2000). Customer poaching and brand switching. *RAND Journal of Economics*, 634–657.

Hannak, A., Soeller, G., Lazer, D., Mislove, A., & Wilson, C. (2014). Measuring price discrimination and steering on e-commerce web sites. In *Proceedings of the 2014 conference on internet measurement conference* (pp. 305–318).

Koh, B., Raghunathan, S., & Nault, B. R. (2017). Is voluntary profiling welfare enhancing? *Management Information Systems Quarterly*, *41*(1), 23–41.

Larson, J., Mattu, S., & Angwin, J. (2015). Unintended consequences of geographic targeting. *Technology Science*.

Marshall, G. (2020). Search and wholesale price discrimination. *The RAND Journal of Economics*, *51*(2), 346–374.

Mauring, E. (2021). Search and price discrimination online. *Working paper*.

Peiseler, F., Rasch, A., & Shekhar, S. (2021). Imperfect information, algorithmic price discrimination, and collusion. *The Scandinavian Journal of Economics*.

Preuss, M. (2021). Search, learning and tracking. *working paper*.

Rhodes, A., & Zhou, J. (2021). Personalized pricing and privacy choice. *working paper*.

Stahl, D. O. (1989). Oligopolistic pricing with sequential consumer search. *The American Economic Review*, 700–712.

Valletti, T., & Wu, J. (2020). Consumer profiling with data requirements: Structure and policy implications. *Production and Operations Management*, *29*(2), 309–329.

Villas-Boas, J. M. (1999). Dynamic competition with customer recognition. *The Rand Journal of Economics*, 604–631.