Over-workers and Drop-Outs in Competitions: Contests with Expectation-Based Loss-Averse Agents*

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Abstract

Competition is often presumed to enhance performance, however, empirical evidence points out bi-modal effort provision in competitive environments: over-workers and drop-outs. In this paper, I analyze a multiple-prize contest with expectations-based loss-averse contestants. The predictions of the model are able to align the observed behavior on effort provision, which is hard to reconcile under classical preferences. In particular, I show that high-ability players, holding high expectations, exert effort aggressively while low-ability players, holding low expectations, exert little or no effort in comparison to the standard theoretical predictions. Furthermore, the optimal prize allocation differs markedly in the presence of loss-averse contestants. I show that awarding multiple prizes can become optimal in the cases where a single prize is predicted to be optimal under standard preferences. Intuitively, awarding an additional prize motivates drop-outs to exert effort, while it demotivates over-workers due to lowered competition. Which of these two countervailing forces dominates, and thus the optimal allocation of prizes, hinges on the interplay between the number of competitors, the ability heterogeneity of competitors, and the degree of loss-aversion. These results have significant implications for the optimal contest design: muting competition by awarding more prizes can increase total output.

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I did not come to Tokyo for a silver medal. It will not give me satisfaction. I could not achieve what I wanted to.

Ravi Dahiya, Silver Medal Winner, Tokyo Olympics 2021

1. Introduction

Contests are commonly-used mechanisms for encouraging agents to achieve a goal. The principal, aiming to attain a goal, announces a prize structure, and contestants compete with each other by means of expending resources to win prizes. Many economic, political and social environments can be described as contests. In firms, workers spend effort to receive bonuses, in sales contests salespeople compete for rewards, at universities researches engage in scientific work to receive grants or tenure positions, in sports athletes perform for winning medals, in R&D races research teams work on projects to win prizes and in board elections firms compete for seats. The prevalence of such environments resulted in contests and their design being studied extensively in the economic literature both theoretically and experimentally. A proven design instrument in contests is prize-allocation schemes. Galton (1902) is one of the first to pose the question of how to "most suitably" allocate prizes in a competition.[1] Galton's problem is addressed in various settings, including rent seeking (Tullock, 1980), models of worker incentives (Glazer and Hassin, 1988; Barut and Kovenock, 1998; Moldovanu and Sela, 2001, 2006), labor contracts (Lazear and Rosen, 1981; Krishna and Morgan, 1998), elimination tournaments (Rosen, 1986), innovation contests (Halac et al., 2017) and status contests (Moldovanu et al., 2007).

A well-documented and puzzling evidence reported in the literature is the discrepancy between the theoretical predictions and observed behavior on effort provision. Competition among participants is often presumed to enhance performance, however, empirical evidence points out bi-modal behavior: over-workers and dropouts. More specifically, it is observed that contestants with high abilities over-exert effort while contestants with low abilities withhold or withdraw effort in comparison to the predictions of standard theory (see, for example Müller and Schotter, 2010;

[1] Sisak (2009) presents a survey of theoretical papers studying the optimal prize allocations in contests, and Dechenaux et al. (2015) documents an extensive review of the experimental literature on contests.

Schotter and Weigelt, 1992; Fershtman and Gneezy, 2011; Barut and Noussair, 2002; Noussair and Silver, 2006; Ernst and Thöni, 2013; Sbriglia and Hey, 1994). Besides the observed behavior in laboratory and field experiments, causal empiricism also suggests that in many organizations, a group of workers is observed to be workaholics, while another group drops out by not exerting any effort (Kemeny, 2002; Stinebrickner and Stinebrickner, 2012, see) [2] The observation raises the following two key questions: What is the deriving force behind it? What are the implications of it for the optimal prize-allocation schemes in contests?

To address these questions, I analyze a canonical all-pay contest under the assumption that participants are expectation-based loss-averse. It is established that competing agents not only evaluate outcomes in absolute terms but also relative to their expectations and therefore expectations play a key role in effort provision in competitive environments (Abeler et al., 2011; Gill and Prowse, 2012; Delgado et al., 2008; Allen et al., 2017; Bartling et al., 2015). My model is able to align the empirical evidence on bi-modal effort provision which is hard to reconcile with a model under standard economic assumptions. Expectation-based loss aversion serves as a key driver of this bimodal behavior and has important implications for the optimal prize allocation. Muting competition by awarding multiple prizes becomes optimal in the cases where a single prize is predicted to be optimal under the assumption of standard preferences. These results appear to be consistent with the prevalence of multiple prize contests in the real world. For example, employees expend effort to be promoted to positions in organizational hierarchies, athletes perform to win gold, silver and bronze medals and students study for grades and so on.

My model builds on the workhorse contest model of Moldovanu and Sela (2001). In my setup, contestants covertly exert efforts simultaneously to win prizes. Effort is always costly and the cost-of-effort depends on effort levels as well as abilities. Contestant with higher abilities bears lower costs and higher effort levels leads to higher costs. Contestants know their private abilities and the distribution of abilities, while they do not know the abilities of their competitors. The principal chooses a prize-allocation scheme, specifying how a fixed budget sum is be allocated into prizes to maximize the total expected effort. Prizes are allocated according to the rankings of

[2] Kemeny (2002) reports that around 30% of workers in the US and Canada self-identify themselves as workaholics. Stinebrickner and Stinebrickner (2012) reports that students' learning that their academic ability is low plays a prominent role in college drop-outs.

players with respect to their effort levels. The first prize is allocated to the contestant with the highest effort, the second highest prize is allocated to the contestant with the highest second effort and so on until all prizes are allocated. In my setup, contestants not only derive utility from evaluating outcomes in absolute terms (for example the absolute value of prizes), but also by evaluating them in comparison to their expectations. A contestant derive a gain-loss utility next to the standard consumption utility, by comparing the actual outcome with his expectations about outcomes. I posit that contestants are loss averse around their rational expectations à la Kőszegi and Rabin (2006, 2007), where expectations are determined endogenously within the economic environment.

My first main result is that expectation-based loss aversion induces a bifurcating force among the efforts of high- and low-ability players: high-ability players choose higher effort levels while low-ability players choose lower effort levels than predicted by standard theory. Intuitively, a high-ability player, who has *ex-ante* high chances of winning a prize, holds high expectations of winning a prize. To avoid the loss sensation he faces in case of not wining any prize, he increases his effort level to further increase his chances of winning. A low-ability player, on the other hand, having *ex-ante* low chances of winning a prize, holds low expectations of winning a prize. He reduces his effort level to further decrease his expectation since lowering his expectations of winning a prize makes an outcome of not winning less disappointing. Moreover, for sufficiently high degrees of loss aversion, players with low abilities drop out of the contest by exerting zero effort since their gain-loss utility dominates their standard consumption utility. Consequently, they end up with negative expected utility if they exert positive effort levels. In order to secure themselves a non-negative expected utility, these players reduce their effort level to the minimum possible level and exert zero effort.

The principal, anticipating the contestants' behavior, aims to maximize the total expected effort. Thus, any change in the contestants' effort provision has important implications on the principal's prize allocation decision. My second main result is that awarding multiple prizes may become optimal in the cases where standard preferences predict the optimality of a single grand prize. Intuitively, the marginal effect of introducing another smaller prize has two countervailing effects on the revenue of the principal: a beneficial effect on the low- and middle-ability players and a detrimental effect on high-ability players. While an additional prize motivates drop-outs to exert effort, it de-motivates over-workers due to decreased value of larger prizes. Which of these two forces dominates, and thus the optimal allocation of prizes, depends on the interplay between the number of competitors, the ability heterogeneity, and the degree of loss-aversion.

An important implication of these results is that the principal can obtain higher levels of total effort by muting competition when contestants have reference dependent preferences.[3] The principal mutes competition by decreasing prize inequality, namely either by transferring an amount from higher- to lower-ranked prizes or by introducing additional prizes. The principal is better off by awarding additional prizes when there is contest entry, when ability range of contestants is less dispersed, and when contestants are more loss averse. Contest entry means adding external contestants to the competition without adding any prizes, for example accepting applications from outsiders for a promotion at a firm. A less dispersed ability range means having more homogeneous contestants with lower abilities, for example athletes in lower leagues. A higher loss aversion means contestants are putting more weight on avoiding disappointment which leads to more bifurcated effort. In all of these cases, the principal gets better off by lowering the prize inequality and potentially introducing additional prizes.

Before reviewing the literature explaining micro and macro evidence by incorporating expectation-based reference-dependent preferences, I briefly discuss the existing evidence documenting the role of expectations and loss-aversion on effort provision. Abeler et al. (2011) manipulate the rational expectations of subjects in a real-effort experiment and find that effort provision is in line with the theory of expectation-based reference-dependent models: subjects work more when expectations are high, and many subjects stop otherwise. Sbriglia and Hey (1994) conduct a series of real-effort experiments on R&D competition and report that players who are ahead of others in the contest raise their investment to guarantee a win, while players who are lagging behind drop out of the contest. Medvec and Gilovich (1995) analyze the emotional reactions of bronze and silver medalists at the 1992 Summer Olympics and find that the reactions are mainly influenced by medalists' thoughts

[3] Fang et al. (2020) finds a similar result in a complete information all-pay contest model with risk-neutral homogeneous contestants and convex cost-of-effort functions.

about "what might have happened", which is a key property of Kőszegi and Rabin (2006). Gill and Prowse (2012) experimentally study a real effort contest and find that subjects competing with each other are loss averse around reference points given by endogenous expectations, which affect subjects' effort levels. Delgado et al. (2008) combines neuroeconomic and behavioral economic techniques using functional magnetic resonance imaging (fMRI) to investigate the overbidding in auctions, similar to workaholics in organizations. Their findings highlight a role for the contemplation of loss in explaining the tendency to over-bid.

This paper fits well into the recent and growing literature utilizing expectation-based loss aversion in different settings to give a rationale for a variety of empirical findings. Lange and Ratan (2010) study first- and second-price sealed-bid auctions for a single item with expectation-based loss-averse bidders. Their model predicts overbidding in first-prize auctions, in line with evidence from recent laboratory experiments. Balzer et al. (2020) study first-price and Dutch auctions with expectations-based loss-averse bidders and show that the strategic equivalence between the two formats breaks down. von Wangenheim (2021) analyze expectation-based loss-aversion in the context of dynamic and static auctions and show that the Vickrey auction yields strictly higher revenue than the English auction, violating the well-known revenue equivalence. Crawford and Meng (2011) analyze field data on cab drivers' working hours and propose a model of labor supply for cab drivers. Their estimates suggest that their reference-dependent model of labor supply rationalizes the cab drivers' behavior observed in the field data. Herweg et al. (2010) study the principal-agent model with moral hazard in the presence of expectation-based loss-averse agents. They show that the optimal contract is a binary payment scheme, consistent with the observed prevalence of simple contracts. Rosato (2017) studies sequential negotiations with one-sided incomplete information and breakdown risk. It is shown that in comparison to the standard benchmark case, loss aversion on the buyer's side softens the rent efficiency trade-off for the seller. Pagel (2017) demonstrates that applications of expectations-based reference-dependent preferences are not limited to explaining micro evidence, but also rationalize widely analyzed macro consumption puzzles: excess smoothness and sensitivity in consumption.

The remainder of this paper is organized as follows. In Section 2 I present the contest model, introduce the preferences and the equilibrium concept. Section

3 discusses participation in the contest. In Section 4 I focus on linear cost-of-effort functions and derive the equilibrium effort function of the players. I show that the model's predictions reconcile the empirical evidence on effort provision with the theoretical predictions. Then, I formulate the principal's problem and provide a sufficient condition for the optimality of multiple prizes. In Section 5 I generalize Section 4 to the case of convex and concave cost-of-effort functions. Section 6 concludes. Proofs are relegated to the Appendix.

2. The Model

The Contest Setup

I first present the contest model, build on Moldovanu and Sela (2001). Consider a contest with P prizes $V_1 \geq V_2 \geq ... \geq V_P \geq 0$, where V_i denotes the value of the i-th prize. The values of the prizes are announced by the principal and are common knowledge. The prize sum is fixed and assumed to be normalized to 1, $\sum_{i=1}^{P} V_i = 1$.

There is a principal who chooses the number and levels of prizes in order to maximize the total expected effort of players. There are $k \geq P$ players, i.e. there are at least as many players as there are prizes. Each players has an ability (cost) parameter c_i, which is private information. Ability parameters are drawn independently from a continuous distribution function F on the interval $[m, 1]$. The distribution function F is assumed to have a strictly positive and continuous density $F' > 0$. It is assumed that F is common knowledge.

Each player i exerts effort x_i, which is always costly regardless of winning any prize. The cost-of-effort is denoted by $c_i\gamma(x_i)$. The cost-of-effort function $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is assumed to be a strictly increasing function with $\gamma(0) = 0$ and is assumed to be either linear, or concave or convex. Note that a player with a high c_i is of low ability since it is more costly for him to put effort. In the remainder of the text, the players having higher c_i parameters will be referred to as low-ability players, and those with low c_i parameters will be referred to as high-ability players. In order to avoid infinite efforts caused by zero costs, the highest possible ability m is assumed to be strictly positive.

Preferences

Having presented the contest model, I now introduce the contestants' preferences. Following Kőszegi and Rabin (2006), I posit that players have reference-dependent preferences. The utility is assumed to have two components: a consumption (intrinsic) utility and a gain-loss utility. The consumption utility is the standard outcome-based utility and solely depends on consumption. The gain-loss utility depends on how consumption of the individual compares to his reference point and is evaluated according to the following universal gain-loss utility function μ:

$$\mu(w) = \begin{cases} \eta w, & \text{if } w \geq 0 \\ \eta \lambda w, & \text{if } w < 0. \end{cases}$$

The gain-loss utility function μ is assumed to satisfy the assumptions Kahneman and Tversky (1979) put on their value function, where w is the distance of individual's consumption to his reference point, $\lambda \geq 1$ is the loss-aversion index weighting losses relative to gains and $\eta > 0$ weighs gain-loss utility component of the utility relative to consumption utility component.

In my setting, the consumption space of the players comprises two dimensions: the prize dimension and the effort dimension. The contestant's consumption utility for prizes is assumed to be the value of the prize. For example, if a player wins the i-th prize, his consumption utility is equal to V_i. The consumption disutility from exerting effort x for a player with ability c is equal to $-c\gamma(x)$, namely to his cost-of-effort.

The reference point of the players is determined by his rational expectations about outcomes. The gain-loss utility for a given outcome is evaluated by comparing it with all other possible outcomes and weighting each comparison with the *ex-ante* probability of the alternative outcome. I apply the concept of choice-acclimating personal equilibrium (CPE) as defined in Kőszegi and Rabin (2007). CPE assumes that an individual correctly predicts his choice set, the set of possible outcomes, and how the distribution of these outcomes depends on his decision. An essential feature of CPE is that the reference point of the player is affected by his own action. In a contest environment, a player not only affects his probability of winning a prize but also his reference point by changing his effort level. As laid out in Kőszegi and

Rabin (2007), CPE refers to the analysis of risk preferences regarding outcomes that are resolved long after all decisions are committed to. This environment seems well suited for a contest environment, where the prizes are allocated long after players exert effort.

The Environment

In the following I formulate the expected utility of a players with ability c and an effort level x. Suppose that there are P prizes to be awarded, $V_1 \geq V_2 \geq ... \geq V_P \geq 0$, and $k > P$ players. In this case there are $P+1$ possible outcomes for the players: Winning the first prize V_1, winning the second prize V_2, $\cdots$, winning the P-th prize V_P and not winning any prize. Denote the probabilities with which these $P+1$ outcomes occur respectively by p_1, p_2,..., p_P and $1 - \sum_{i=1}^{p} p_i$. Namely, p_s is the probability that $(k-s)$ competitors of a players exert less effort than him while $s-1$ competitor exerts more effort. It is important to note that, a players can affect these probabilities by putting more or less effort. For the convenience of presentation, denote the probability of not winning any prize, by

$$p_{P+1} \equiv 1 - \sum_{i=1}^{p} p_i. \tag{1}$$

To be consistent, let $V_{P+1} = 0$ denote the consumption in the prize dimension when the players does not win any prize. The utility from winning the first prize V_1 is given as follows:

$$V_1 - c\gamma(x) + \eta \sum_{i=1}^{P+1} p_i(V_1 - V_i) \tag{2}$$

In this formulation, the term $V_1 - c\gamma(x)$ is the consumption utility from winning the first prize plus the consumption disutility from exerting effort. The next term, $\sum_{i=1}^{P+1} p_i(V_1 - V_i)$, is the gain-loss utility, where η is its weight relative to the standard consumption utility. The player forms his gain-loss utility in the prize dimension as follows: he first compares the actual outcome, namely winning V_1, with all the other possible outcomes, namely winning any other prize and not winning any prize. Then he weights these comparisons with the ex-ante probabilities of alternative outcomes. Note that winning the first prize V_1 is the best outcome in the outcome space as $V_1 - V_i \geq 0$ for $i \in 2, \cdots, P+1$. Thus, in each of these comparisons the player experiences a gain of $V_1 - V_i$ for any $i \in 2, \cdots, P+1$, as he could have ended up winning the prize V_i with p_i probability. Note that, the ex-ante expected gain-loss utility is zero in the effort dimension as effort is a deterministic outcome (contestant's expected and actual effort choices coincide).

The utility of a player when he does not win any prize –the worst outcome– is given as follows:

$$V_{P+1} - c\gamma(x) + \eta \left(\sum_{i=1}^{P+1} p_i \lambda(0 - V_i) \right) \tag{3}$$

Here the consumption utility is $0 - c\gamma(x)$, where the players has only the disutility of exerting effort, while the gain-loss utility from not winning any prize is $\sum_{i=1}^{P+1} p_i \lambda(0 - V_i)$. Since not winning any prize is the worst outcome in the outcome space, the player will be always in the loss domain in his comparisons, namely $\mu(0 - V_i) = -\eta\lambda V_i$. The experienced loss of $0 - V_i$ is weighted with p_i, as he could have ended up winning the prize V_i with this probability, for any $i \geq 1$.

The utility of winning the s-th prize V_s with $s > 1$ is evaluated exactly in the same way and given as follows:

$$V_s - c\gamma(x) + \eta \left(\sum_{i=s+1}^{P+1} p_i (V_s - V_i) + \sum_{i=1}^{s} p_i \lambda(V_s - V_i) \right) \tag{4}$$

The players derives the consumption utility of V_s and the consumption disutility of effort $-c\gamma(x)$. The gain-loss utility $\sum_{i=s+1}^{P+1} p_i(V_s - V_i) + \sum_{i=1}^{s} p_i \lambda(V_s - V_i)$ has two parts: the first part $\sum_{i=s+1}^{P+1} p_i(V_s - V_i)$ captures the gain sensation from winning V_s, while the players could have ended up with winning a worse prize V_i with $s < i$ and the second part $\sum_{i=1}^{s} p_i \lambda(V_s - V_i)$ captures the loss sensation from from winning V_s, while the players could have ended up with winning a better prize V_i where $s > i$. Note that $V_s - V_i \leq 0$ for $i < s$ since $V_s \leq V_{s-1} \leq \cdots \leq V_1$.

Being the actual outcome uncertain, the expected utility of the players is obtained by averaging over all possible outcomes, namely the weighted average of the terms in equations (2), (3) and (4), where the weights are the probabilities with which these outcomes realize. The expected utility of a player with type c is given as follows:

$$EU = \sum_{s=1}^{P} p_s V_s - c\gamma(x) + \eta \left[\sum_{s=1}^{P+1} p_s \left(\sum_{i=s+1}^{P+1} p_i (V_s - V_i) + \sum_{i=1}^{s} p_i \lambda (V_s - V_i) \right) \right] \quad (5)$$

The expected utility becomes a sum of the standard consumption utility and the gain-loss utility derived from outcome comparisons. With $\lambda = 0$, the expected utility in equation (5) reduces to the standard consumption utility.

The timing of the contest game is as follows: The principal chooses and announces the number and the levels of the prizes. Each player gets privately informed about his ability. Each player chooses his effort level and forms rational expectations about the prize outcomes. All players simultaneously exert effort. The player with the highest effort wins first prize V_1, the player with the second-highest effort wins second prize V_2 and so on until all the P prizes are allocated, while each player bears the cost-of-effort regardless of winning any prize.

3. Participation in the contest

In my analysis, as standard in the contest and auction literature, I concentrate on symmetric equilibria, where the equilibrium effort is assumed to be decreasing in type parameter c. That is, a player with an ability (cost) parameter c always puts more effort than a player with a ability (cost) parameter larger than c. Before launching out into the discussion on participation in the contest, I introduce the following notation. For any $s \in \{1, 2, ..., P\}$ let $F_{s,k}$ denote the distribution of the s-th order statistic out of the k independent variables distributed according to F, and $F_s(c) = F_{s-1,k-1}(c) - F_{s,k-1}(c)$. To facilitate the exposition, define $\Lambda = \eta(\lambda - 1)$, where η is the weight placed on the gain-loss utility relative to the consumption utility and λ is the classical loss aversion index. For $\lambda = 1$, there is no loss aversion and the contestant's expected utility equals expected net consumption utility. With this notation, the expected utility of the players with type c becomes:

$$EU = \sum_{s=1}^{P} p_s V_s - c\gamma(x) - \Lambda \sum_{s=1}^{P} \left(\sum_{i=s+1}^{P+1} p_s p_i (V_s - V_i) \right). \quad (6)$$

From this formulation of the expected utility of the players, it becomes clear that Λ captures not only the weight put on losses relative to gains, but also captures the weight of gain-loss utility relative to consumption utility. So, Λ can be interpreted as the overall measure of a contestant's degree of loss aversion. The expected utility in equation (6) has two parts: the standard consumption utility part $\sum_{s=1}^{P} p_s V_s - c\gamma(x)$ and the gain-loss utility part $-\Lambda \sum_{s=1}^{P} \left(\sum_{i=s+1}^{P+1} p_s p_i (V_s - V_i) \right)$. For $\Lambda = \eta(\lambda - 1) = 1$, the expected utility of the players reduces to the case without loss aversion. Since $V_s \geq V_i$ for any $i > s$, the gain-loss utility part is always negative. This is because for non-deterministic outcomes (i.e. the outcomes in the prize dimension) since losses from comparing an outcome to a counterfactual loom larger than the symmetric gains. Therefore, when choosing his effort level, the players has to balance off two possibly conflicting targets: maximizing the expected net consumption utility and minimizing the gain-loss utility.

According to Kőszegi and Rabin (2007), under CPE a player might choose a stochastically dominated option when the degree of loss-aversion is sufficiently pronounced with $\eta(\lambda - 1) = \Lambda > 1$. Although choosing a stochastically dominated option of putting zero effort seems counterintuitive, the player gets worse off if he exerts any positive effort level. The reason is that, when loss aversion is sufficiently high, the standard consumption utility of a player with low probabilities of winning a prize is dominated by his gain-loss utility, resulting in him ending up with a net loss. *Ex-ante* expecting to experience a net loss, contestant's main concern becomes reducing the scope of possible losses. He does so by spending zero effort and secures himself a zero net loss. The following proposition formalizes this intuition of withdrawing effort and provides the comparative statistics on how the critical type $\tilde{c}$ changes as contestants' degree of loss aversion increases.

Proposition 1 *Assume that there are P prizes with $V_1 \geq V_2 \geq \cdots \geq V_P$ to be awarded and there are $k > P$ loss-averse players. When the loss-aversion index $\Lambda \leq 1$, there is full participation in the contest, that is each players with type $c < 1$ exerts positive effort. When players are sufficiently loss averse, namely when $\Lambda > 1$, there exists a critical type $\tilde{c}$ such that every players with $c > \tilde{c}$ exerts zero effort and* *drops out the contest, where the critical type $\tilde{c}$ satisfies:*

$$\frac{\sum_{s=1}^{P}(F_s(\tilde{c}))^2 V_s + \sum_{s=2}^{P}\sum_{i=1}^{s-1} 2V_s F_s(\tilde{c}) F_i(\tilde{c})}{\sum_{s=1}^{P} F_s(\tilde{c}) V_s} = 1 - \frac{1}{\Lambda}. \tag{7}$$

The value of $\tilde{c}$ decreases as the contestants' degree of loss aversion increases.

Proof. See appendix A.

Proposition 1 gives two main messages. First, there is full participation in the contest if loss aversion is mild. In other words, all players except the lowest-ability one put positive effort when $\Lambda \leq 1$. Second, if the contestants' loss-aversion degree is sufficiently high but still plausible, $\Lambda > 1$, then there is a group of players who put zero effort and drop out of the contest. Moreover, as contestants' degree of loss aversion increases, more and more players drop out of the contest.

Drop-out behavior has been observed in a number of experimental and field studies. Müller and Schotter (2010) experimentally tests the predictions of Moldovanu and Sela (2001) and reports that observed effort provision is bifurcated: Low-ability subjects drop put, while high-ability subjects try too hard. Schotter and Weigelt (1992) reports that subjects with higher marginal cost-of-efforts drop out of the contest even when they are not expected to lose money. Fershtman and Gneezy (2011) reports the findings from running races among elementary school students: some students simply stop running and drop out when it is clear they have no chance of winning. Finally, dropping out is also observed in studies of multiple unit all-pay auctions (see Barut and Noussair (2002), Noussair and Silver (2006) and Ernst and Thöni (2013)).

Drop-out behavior is closely related to the cognitive strategy called "internal self-handicapping": an individual's withdrawal of effort when he expects a low probability of success, as defined by Arkin and Baumgardner (2011).[4] One cause of self-handicapping is uncertainty about decision makers' own performance –in my

[4]Self-handicapping, as a strategy to create obstacles by the decision-maker in anticipation of a failing performance, is first introduced by Jones and Berglas (1978). Psychological literature demonstrating a wide range of self-handicapping strategies (see in Rhodewalt (1990)), Arkin and Baumgardner (2011) classified self-handicapping into two main categories: internal (e.g., withdrawal of effort) or external (e.g., choice of non-diagnostic performance settings). The dropping-out behavior discussed here is related to internal self-handicapping.

context uncertainty about winning a prize or not– especially when others have high expectations of success. By withdrawing effort, the decision-maker makes an outcome of failure –not winning any prize – less painful for himself. Examples of internal self-handicapping include students not studying on exam papers and workers' underachievement at organizations.

4. Linear Cost Functions

This section contains the main results about the optimal effort provision and the optimal prize structure in a multiple prize contest when the players have linear cost-of-effort functions. I will first solve the subgame and derive the equilibrium effort levels of contestants for a given prize structure. I will then solve the problem of the principal and elaborate the optimal allocation of prizes.

4.1. Equilibrium Effort Provision

Assume that there are P prizes with $V_1 \geq V_2 \geq \cdots \geq V_P$ to be awarded and there are $k > P$ loss-averse players with linear cost-of-effort function $\gamma(x) = x$. A players with type c chooses his effort level x in order to maximize his expected utility. Then the contestant's problem is the following optimization problem:

$$\max_{x} \quad \sum_{s=1}^{P} p_s V_s - c\gamma(x) - \Lambda \sum_{s=1}^{P} \left(\sum_{i=s+1}^{P+1} p_s p_i (V_s - V_i) \right). \tag{8}$$

The following proposition displays the solution to this optimization problem, namely the equilibrium effort function.

Proposition 2 *Assume that there are P prizes $V_1 \geq V_2 \geq ... \geq V_P \geq 0$ to be awarded and $k > P$ players. If $\Lambda > 1$, then there exists a critical type $\tilde{c}$ satisfying (7) with equality, such that in equilibrium players with $c \geq \tilde{c}$ exert zero effort and players with $c < \tilde{c}$ exert effort according to:*

$$b(c) = \sum_{s=1}^{P} A_s(c) V_s \tag{9}$$

where the coefficients of s-th prize is given by:

$$A_s(c) = (1-\Lambda)\int_c^{\tilde{c}} -\frac{1}{a}F_s(a)'da + \Lambda\int_c^{\tilde{c}} -\frac{1}{a}\left(\left(F_s(a)^2\right)' + \sum_{i=1}^{s-1}\left(2F_i(a)F_s(a)\right)'\right)da. \tag{10}$$

If $\Lambda \leq 1$, then there is full participation and each players exerts effort according to equations (9) and (10) with $\tilde{c} = 1$.

Proof. See appendix A.

Put verbally, in equilibrium each player exerts an effort equal to a weighted sum of the P prizes. The weights of the prizes differ for each player depending on his chances of winning these prizes. If players are sufficiently loss-averse, then there is a mass of players who puts zero effort as discussed in Section 3. The following example illustrates the equilibrium effort function of players when there are $P = 2$ prizes to be awarded with $V_1 \geq V_2 \geq 0$.

Example 1 *Assume that there are two prizes to be awarded $V_1 \geq V_2 \geq 0$ and $k = 3$ players whose abilities are drawn from the uniform distribution $F(c) = 2c - 1$ on the interval $[1/2, 1]$. Using Proposition 2, the optimal effort function in this case is the following weighted sum of the first and the second prizes:*

$$b(c) = A_1(c)V_1 + A_2(c)V_2,$$

where the coefficients of the first and the second prize are given by:

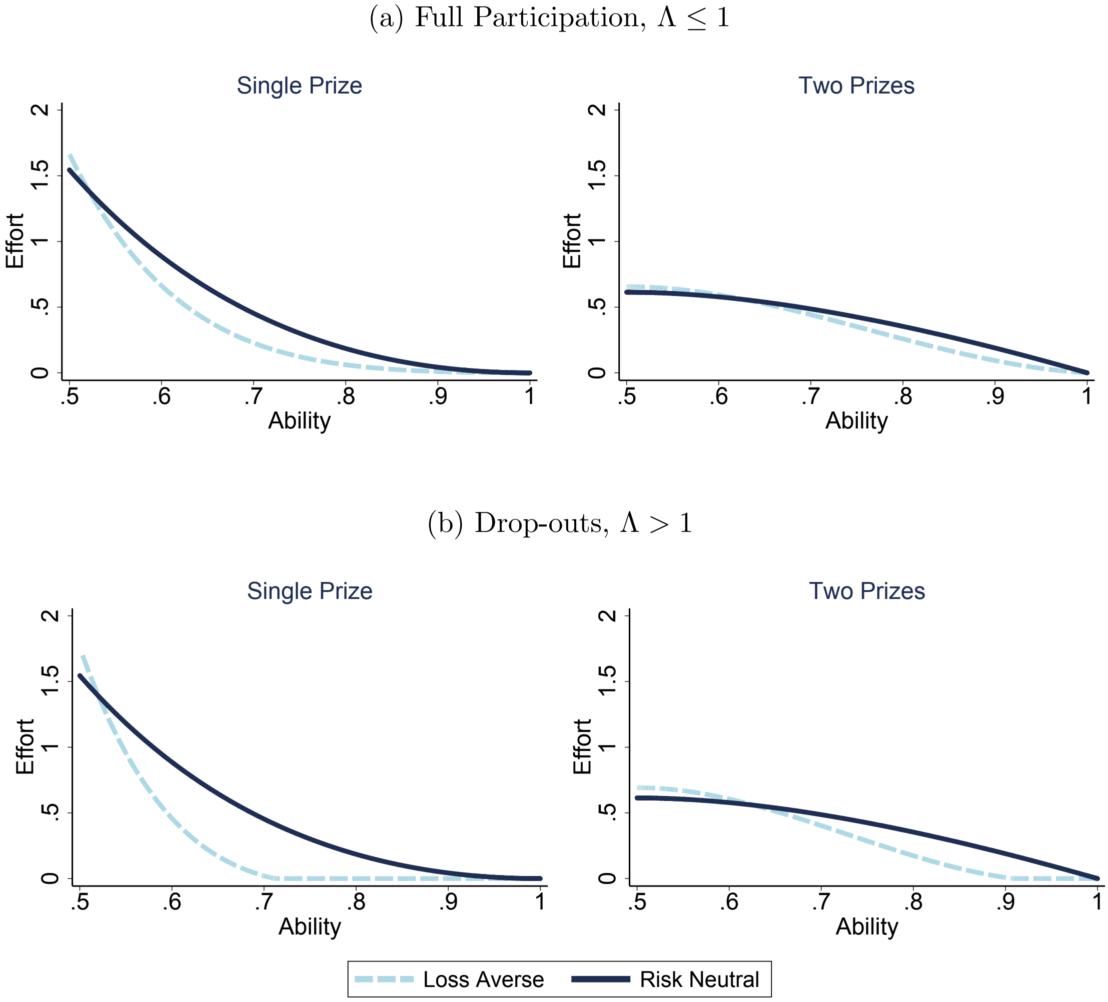
$$\begin{aligned} A_1(c) &= (1-\Lambda)\int_c^{\tilde{c}} -\frac{1}{a}F_1'(a)da + \Lambda\int_c^{\tilde{c}} -\frac{1}{a}(F_1^2(a))'da \\ &= (1-\Lambda)\int_c^1 4\frac{1}{a}(2-2a)^3 da + \Lambda\int_c^1 8\frac{1}{a}(2-2a)^3 da \end{aligned} \tag{11}$$

and

$$\begin{aligned} A_2(c) &= (1-\Lambda)\int_c^{\tilde{c}} -\frac{1}{a}F_2'(a)da + \Lambda\int_c^{\tilde{c}} -\frac{1}{a}\left((F_2^2(a))' + (2F_1(a)F_2(a))'\right)da \\ &= (1-\Lambda)\int_c^1 4\frac{1}{a}\left(-1+2(1-2a)\right)da + \Lambda\int_c^1 8\frac{1}{a}(2-2a)\left(-1+3(1-2a)\right)da. \end{aligned} \tag{12}$$

Figure 1 depicts the equilibrium effort function for both standard preferences (solid line) and expectation-based reference-dependent preferences (dashed line). The

Figure 1: Equilibrium Effort Functions



Notes: The upper and lower panels depict the equilibrium effort functions when players have linear cost-of-effort functions and have the loss-aversion index $\Lambda = 0.8$ and $\Lambda = 1.5$ respectively, on the left-hand side for a single prize and on the right-hand side for two prizes.

upper panel shows the equilibrium effort provision when $\Lambda = 0.8 \leq 1$ *and the lower panel when* $\Lambda = 1.5 > 1$.

The upper panel of Figure 1 shows that when $\Lambda = 0.8 \leq 1$, *every players but the one with the lowest ability* $c = 1$ *exert positive effort, as proven in Proposition 1. In the lower panel of the figure, however, there is a critical type* $\tilde{c}$, *all players having types* c *above which put zero effort and drop-out of the contest. The value of the critical type depends on the distribution of order statistics and the ratio of prizes as laid out in Proposition 1. Using the equation (7), the critical type is* $\tilde{c} = 0.71$ *when the principal awards a single prize, while it increases to* $\tilde{c} = 0.9$ *when the principal awards two equal prizes. The reason is that putting a second prize motivates low-*

ability players who otherwise give up the slim hope of winning any prize. In the presence of a second prize, players with abilities between $0.71 < c < 0.9$ *come back to the competition to win the second prize and start exerting positive efforts. While introducing a second prize motivates low-ability players, it leads to a decrease in the value of the first prize and results in an effort drop on the side of high-ability players, as seen in Figure 1.*

The effort provision discussed above is not particular to the numerical values in the example. More precisely, a high-ability player always overexerts effort, while a low-ability player always under-exerts effort. When the gain-loss utility is sufficiently important, low-ability players with $c > \tilde{c}$ do not exert any effort at all and drop out of the contest. The following theorem formalizes the bimodal effort provision illustrated in example 1 to the general case of P prizes, $k > P$ players with abilities drawn from a distribution F on $[m, 1]$.

Theorem 1 *Assume that there are* P *prizes with* $V_1 \geq V_2 \geq ... \geq V_P \geq 0$ *to be awarded and* $k > P$ *players with linear cost-of-effort functions. Denote the equilibrium effort function of a players with reference dependent preferences by* b^{LA} *and of players with standard preferences by* b^S*. For any players with ability* c *in an* ϵ *neighbourhood of* m*,* $b^{LA}(c) > b^S(c)$*. For any players with ability* c *in an* ϵ *neighbourhood of* $\tilde{c}$*,* $b^{LA}(c) < b^S(c)$*, where* $\tilde{c} = 1$ *whenever* $\Lambda \leq 1$*.*

Proof. See appendix A.

The presence of expectation-based reference-dependent preferences introduces a bifurcating force among the high- and low-ability players. A high-ability player has *ex-ante* high probabilities of winning a prize. He, therefore, holds high expectations regarding winning a prize. Losses looming larger than gains of equal size, he tries to avoid the loss of not winning a prize. To do so, he increases his effort level to further increase his chances to win a prize. On the other hand, low-ability players have *ex-ante* low probabilities of winning a prize. Thus, he holds low expectations regarding winning a prize. Intuitively, lowering his expectations of winning a prize makes the outcome of not winning less painful. Therefore, he reduces his effort level to avoid such disappointments.

Furthermore, as discussed in Section 3, when the gain-loss utility is sufficiently important, decision-makers central concern is reducing exposure to sensations of loss according to Kőszegi and Rabin (2007). For a low-ability player who is sufficiently loss-averse, the gain-loss utility becomes more dominant than the standard consumption utility. In this case, the players might end up with a negative expected utility. To avoid the net loss arising from the strong disappointment he exerts zero effort and secures himself a zero expected utility.

It is important to note that there are two different channels through which loss-averse players try to avoid losses, creating the bimodal effort provision. A player chooses either one of these two channels depending on his expectations. A high-ability player, holding high expectations, chooses to increase his effort level to avoid the disappointment of not winning. A low-ability player, on the other hand, holding low expectations, chooses to lower his effort level to lower his expectations further, which makes not winning a prize less painful. Therefore, high-ability players exert effort aggressively while low-ability ones withhold effort.

4.2. The Optimal Prize Allocation

The contestants' problem being discussed in the previous section, I now turn into the principal's problem. Assume that there are P prizes $V_1 \geq V_2 \geq \cdots \geq V_P$ to be awarded with $\sum_{s=1}^{P} V_s = 1$ and $k > P$ players with linear cost-of-effort functions $\gamma(x) = x$. Given the optimal effort provision of players for any prize allocation, the principal chooses the number and the level of the prizes. The goal of the principal is to maximize his expected revenue, namely the total expected effort exerted by players. players exert effort according to

$$b(c) = \sum_{s=1}^{P} A_s V_s$$

where the weights $A_s(c)$ are as in equation (10). Noting that $V_1 = 1 - \left(\sum_{i=2}^{P} V_i\right)$, the average effort of each players is given by:

$$\int_m^{\tilde{c}} \left(\sum_{i=1}^{P} V_i A_i(c) \right) F'(c)dc = \int_m^{\tilde{c}} \left(A_1 + \sum_{i=1}^{P} V_i (A_i(c) - A_1(c)) \right) F'(c)dc$$

The expected revenue of the principal becomes:

$$R(V_2, V_3, ..., V_P) = k \int_m^{\tilde{c}} \left(A_1 + \sum_{i=1}^{P} V_i (A_i(c) - A_1(c)) \right) F'(c) dc. \tag{13}$$

The principal's problem is choosing the number and the levels of the prizes to maximize his revenue, namely:

$$\max_{\{V_i\}_{i=2}^P} k \int_m^{\tilde{c}} \left(A_1 + \sum_{i=1}^{P} V_i (A_i(c) - A_1(c)) \right) F'(c) dc.$$

The optimal number of prizes depends on the shape of the revenue function. If the revenue function with respect to the i-th prize V_i at $V_i = 0$ is increasing, then it is optimal to award $V_i > 0$. Otherwise, it is optimal to set $V_i = 0$. Note that when $\Lambda \leq 1$, the critical type becomes $\tilde{c} = 1$ and the only place V_is appears in the principal's revenue is the function inside the integral. When $\Lambda > 1$, however, the critical type $\tilde{c} < 1$ depends on the values of V_is as well as the degree of loss aversion. In the following, I will discuss the principal's problem respectively for these two cases: $\Lambda \leq 1$ and $\Lambda > 1$.

First, consider the case when $\Lambda \leq 1$. Note that, in this case, there is full participation in the contest with $\tilde{c} = 1$ and so the maximization problem of the principal becomes a linear programming problem, where the maximum value of the objective function is achieved at the corners of the feasibility set $0 \leq V_i \leq \frac{1}{i+1}$. Therefore, it is always optimal to either award a grand single prize or multiple prizes of equal sizes whenever $\Lambda \leq 1$. The following proposition provides a sufficient condition for the optimality of the i-th prize and characterizes the optimal values of the prizes for the case of $\Lambda \leq 1$.

Proposition 3 *Assume that there are P prizes $V_1 \geq V_2 \geq ... \geq V_P \geq 0$ to be awarded with $\sum_{i=1}^{P} V_i = 1$ and $k > P$ players with linear cost-of-effort functions. Moreover, assume that $\Lambda \leq 1$ as such there is full participation in the contest. It is optimal to allocate the s-th prize if and only if*

$$\int_m^1 (A_s - A_1) F'(c) dc > 0. \tag{14}$$

Moreover, the optimal value of the prizes are given by $V_1 = \cdots = V_s = \frac{1}{s}$ whenever *it is optimal to put s prizes. In other words, it is always optimal to put equal prizes.*

Proof. See Appendix B.

Now consider the case when $\Lambda > 1$. In this case, V_is appear in the principal's revenue not only in the function inside the integral but also in the upper bound of the integral in equation (43). This is because the critical type $\tilde{c} < 1$ depends on the value of the prizes $V_2, V_3, \cdots, V_P$ as defined in equation (7). In this case, the derivative of the principal's revenue with respect to the i-th prize at $V_i = 0$ becomes:

$$\int_m^{\tilde{c}} (A_s - A_1)\, F'(c)dc + \int_m^{\tilde{c}} \left[\frac{\partial}{\partial \tilde{c}} \left(A_1(c) + \sum_{i=1}^{P} V_i (A_i - A_1) \right) \frac{\partial \tilde{c}}{\partial V_s}\bigg|_{V_s=0} \right] F'(c)dc > 0, \tag{15}$$

where

$$\frac{\partial \tilde{c}}{\partial V_s}\bigg|_{V_s=0} = -\frac{F_s(\tilde{c})^2 - F_1(\tilde{c})^2 + \sum_{i=1}^{s-1} \left(2F_s(\tilde{c})^2 F_i(\tilde{c})^2\right) - (1-\frac{1}{\Lambda})(F_s(\tilde{c}) - F_1(\tilde{c}))}{\left(F_1(\tilde{c})^2\right)' + \sum_{\substack{i=1\\ i\neq s}}^{P} V_i \left(F_i(\tilde{c})^2 F_1(\tilde{c})^2\right)' - (1-\frac{1}{\Lambda}) \left[F_1(\tilde{c})' + \sum_{\substack{i=1\\ i\neq s}}^{P} V_i (F_i(\tilde{c})' - F_1(\tilde{c})') \right]}.$$

To evaluate the expression in (15) for the i-th prize, one should first calculate the value of $\tilde{c}$, which in turn depends on the optimal values of $V_2, V_3, \cdots, V_{i-1}$. Therefore, when $\Lambda > 1$, the expression analogous to the sufficient condition in Proposition 3 does not provide a practical way to check the optimality of the i-th prize. Indeed, it is more straightforward to maximize the revenue function and directly calculate the optimal values of the prizes $V_2, V_3, \cdots, V_P$ and $V_1 = 1 - \sum_{s=1}^{P} V_s$.

The following example illustrates the optimal prize allocation when there are $P = 2$ prizes to be awarded $V_1 \geq V_2 \geq 0$ in the setting of Example 1, comparing the cases of standard- and reference-dependent preferences.

Example 2 *Assume that there are 3 players, whose abilities are drawn from a uniform distribution $F(c) = 2c - 1$ on the interval $[0.5, 1]$. Assume, moreover, that the cost-of-effort function is $\gamma(x) = x$. The optimal effort function in this case becomes $b = A_1 V_1 + A_2 V_2$, where A_1 and A_2 are as in equation (10). For the ease of expression, rewrite A_1 and A_2 as follows:*

$$A_1 = (1-\Lambda)A_1^1 + \Lambda A_1^2 \qquad \textit{and} \qquad A_2 = (1-\Lambda)A_2^1 + \Lambda A_2^2.$$

First, consider the standard case with $\Lambda = 0$. *Letting* $V_2 = \alpha$ *and* $V_1 = 1 - \alpha$, *where* $0 \leq \alpha \leq 1/2$, *the revenue of the principal reads:*

$$R(\alpha) = k \int_m^1 (A_1^1(c) + \alpha(A_2^1(c) - A_1^1(c)))F'(c)dc. \tag{16}$$

Awarding a single prize is optimal if $R(\alpha)$ *is strictly decreasing, that is if the revenue function has its maximum at* $\alpha = 0$. *Otherwise, the revenue function* $R(\alpha)$ *has its maximum at* $\alpha > 0$, *leading to the optimality of two prizes. A sufficient and necessary condition for the optimality of two prizes reduces (see Moldovanu and Sela (2001)):*

$$R'(0) = \int_m^1 \left(A_2^1(c) - A_1^1(c)\right) F'(c)dc > 0.$$

Substituting the numerical values, $R'(0) = -0.137 < 0$, *violating the sufficient condition. Therefore, it is optimal to award a grand single prize in this case. Now suppose that players are loss-averse and contestants' degree of loss aversion is* $\Lambda = 0.8$. *The sufficient condition for the optimality of two prizes becomes:*

$$R'(0) = \int_m^{\tilde{c}} (A_2(c) - A_1(c))\, F'(c)dc > 0.$$

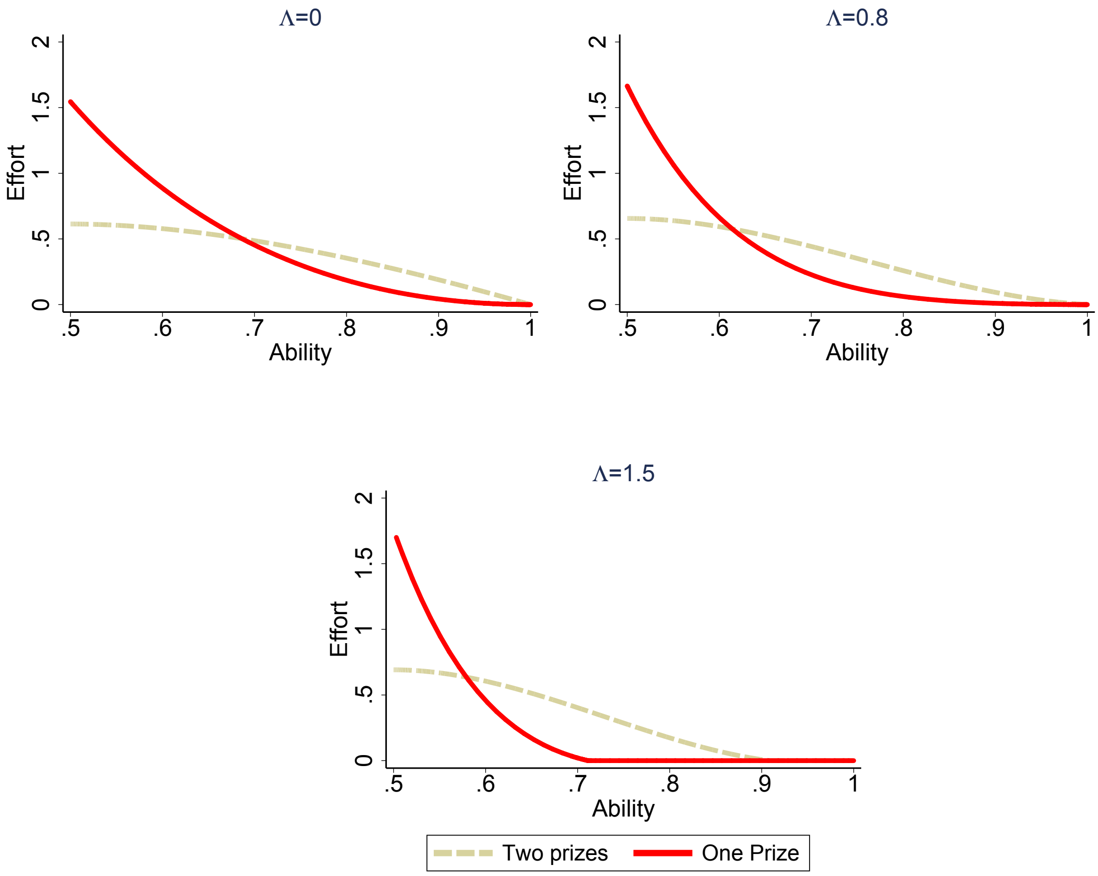
Substituting the numerical values, $R'(0) = 0.152 > 0$ *and the sufficient condition is satisfied. Therefore, awarding a second prize becomes optimal with* $V_2 = 0.5$ *when the degree of loss aversion is* 0.8.

For the specific values taken in the above example, it is optimal to award a single prize in the standard case without loss aversion, while awarding a second prize becomes optimal in the presence of loss aversion. To grasp this finding intuitively, recall the intuition underlying Theorem 1. Award of a second prize creates two opposite effects: a beneficial effect on low- and middle-ability players and a detrimental effect on high-ability players.

On the one hand, when there is only a single prize, a loss-averse player with low ability loses his hope of winning the prize and gives up the competition by exerting either very little or no effort, as shown in the previous section. Award of a second prize will motivate low- and middle-ability players, who otherwise give up the competition, resulting in an increase in the total expected effort.

On the other hand, the award of a second prize lowers the value of the first one,

Figure 2: The Beneficial Effect of a Second Prize



Notes: The figure illustrates the beneficial effect of a second prize when players have linear cost-of-effort functions for different levels of degree of loss aversion.

since the principal has a fixed budget sum. A smaller first prize will not motivate the high-ability players as much as a single grand prize does. Therefore, loss-averse players with high ability will lower their effort level relative to the case of a single prize, leading to a decrease in the total expected effort.

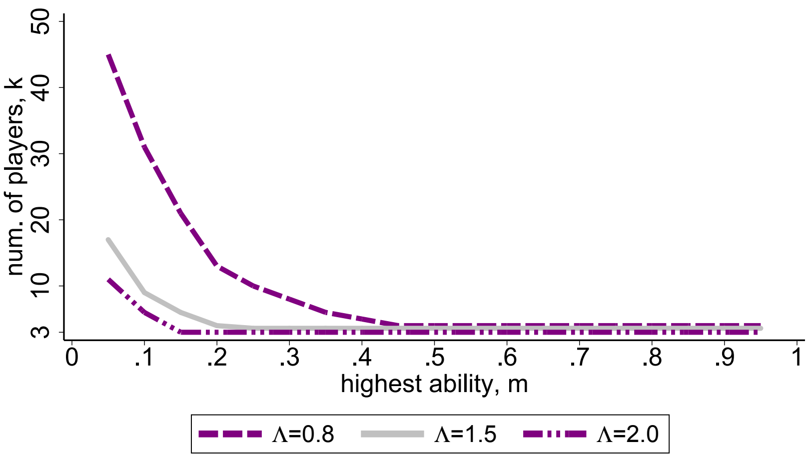
The optimality of the award of a second prize depends on which of these two effects is the dominant one. In example 2, the beneficial effect of the second prize on the effort level of low- and middle-ability types dominates the detrimental effect on high-ability types and so allocating a second prize becomes optimal. Figure 2 illustrates both the resulting effort increase for low- and middle-ability types and the effort decrease of the high-ability ones, relative to the award of a grand single prize, respectively for the standard case, moderate degrees of loss aversion ($\Lambda = 0.8 \leq 1$) and higher degrees of loss aversion ($\Lambda = 1.5 > 1$).

More generally, when players are loss-averse, the optimality of awarding a second prize (or a third or more when $P > 2$) depends on the specific properties of the ability distribution function, the number of players, and the degree of loss-aversion. In the standard case without loss aversion, Moldovanu and Sela (2001) show that the result to contestant's problem is independent of any of those variables and it is always optimal to award a grand single prize even if, a priori, the principal is allowed to award P prizes. It is important to note that a successful mechanism must elicit behavior identical to the one assumed by the principal. Therefore, optimal prize structure predicted in the presence of loss aversion could possibly yield better performance than the standard predictions in applications to real life, e.g., the number of promotions in a firm.

Figure 3 illustrates the optimal number of prizes when $P = 2$ depending on the number of players k, the minimum effort cost m under a uniform distribution of abilities, respectively for three different levels of the loss-aversion degree, $\Lambda = 0.8$, $\Lambda = 1.5$ and $\Lambda = 2.0$. The plotted curves separate the k-m values for which the award of a single prize is optimal from those for which the award of a second prize becomes optimal. For example, when the degree of loss aversion is $\Lambda = 0.8$, for any k-m combination below the large-dashed curve award of a grand single prize is optimal, while introducing a second prize becomes optimal for k-m values above the large-dashed curve. Inspecting Figure 3, it becomes clear that as contestants' degree of loss aversion increases, the award of a second prize is more likely to become optimal. Intuitively, as the degree of loss aversion increases, losses looming larger than gains, even middle-ability players start to give up the race. The aggressive effort provision of high-ability players can not compensate for a large number of drop-outs. The primary target of the principal becomes motivating the drop-outs by introducing a second (or possibly more when $P > 2$) prize(s).

As the number of players k increases, keeping everything else constant, the beneficial effect of the second prize on middle- and low-ability players is amplified. Intuitively, it becomes less likely to win a prize for a low-or a middle-ability player when there are more competitors. So, there will be more players who put little or no effort, decreasing the total expected effort further in comparison to the case of a smaller k. In this case, the principal is better off when he motivates the low- and middle-ability players – rather than the high-ability ones– by introducing a second prize.

Figure 3: Optimal Prize Allocation with $P = 2$



Notes: The figure illustrates the optimal number of prizes for different degrees of loss aversion when there are at most two prizes $P = 2$ and abilities are drawn from a uniform distribution on $[m, 1]$. For (k,m) values below the graphs award of a grand single prize is optimal, while for (k,m) values above the graphs award of a second prize becomes optimal.

The higher the minimum effort cost m is, the more likely a second prize to become optimal. If m is small, players have lower cost-of-efforts on average in comparison to the case of a larger m. In this case, the main contribution to the revenue of the principal is from the abler players. Therefore, the principal is better off by motivating those players with a grand single prize. In contrast, when m is larger, the reasoning goes in the opposite direction: not only the number of players who overexert effort is much less but also more low- and middle-ability players put very little or no effort. In this case, the principal is better off by awarding two prizes to motivate low- and middle-ability players. In other words, when m is larger the beneficial effect of a second prize on the low- and middle-ability players dominates the detrimental effect on high-ability players.

5. Concave and Convex Cost Functions

In this section, I generalize the results from the previous section to the cases of convex or concave cost-of-effort functions.

5.1. Equilibrium Effort Provision

Assume that there are P prizes with $V_1 \geq V_2 \geq \cdots \geq V_P$ to be awarded and there are $k > P$ loss-averse players with convex or concave cost-of-effort functions.

The optimization problem of a player with type c is given by equation (8). The equilibrium in the cases of convex or concave costs is obtained by a simple transformation of the equilibrium strategies found in the previous section. The following proposition displays the equilibrium effort function of a player.

Proposition 4 *Assume that there are P prizes $V_1 \geq V_2 \geq ... \geq V_P \geq 0$ to be awarded and $k > P$ players with either convex or concave cost-of-effort function $\gamma(x)$. If $\Lambda > 1$, then there exists a critical type $\tilde{c}$ satisfying (7) with equality, such that in equilibrium players with $c \geq \tilde{c}$ exert zero effort and players with $c < \tilde{c}$ exert effort according to:*

$$b(c) = \gamma^{-1}\left(\sum_{s=1}^{P} A_s(c)V_s\right) \tag{17}$$

where the coefficients of s-th prize are given by equation (10). If $\Lambda \leq 1$, then there is full participation and each players exerts effort according to equations (17) and (10) with $\tilde{c} = 1$.

Proof. See appendix A.

As in the case of linear cost case, when players are sufficiently loss-averse, there is a critical type $\tilde{c}$ satisfying equation (7) such that players with higher cost parameters than the critical type put zero effort. Full participation in the contest is guaranteed when contestants' degree of loss aversion is $\Lambda \leq 1$. The following example illustrates the equilibrium effort function of players when there are $P = 2$ prizes to be awarded $V_1 \geq V_2 \geq 0$ and the cost-of-effort function is convex.

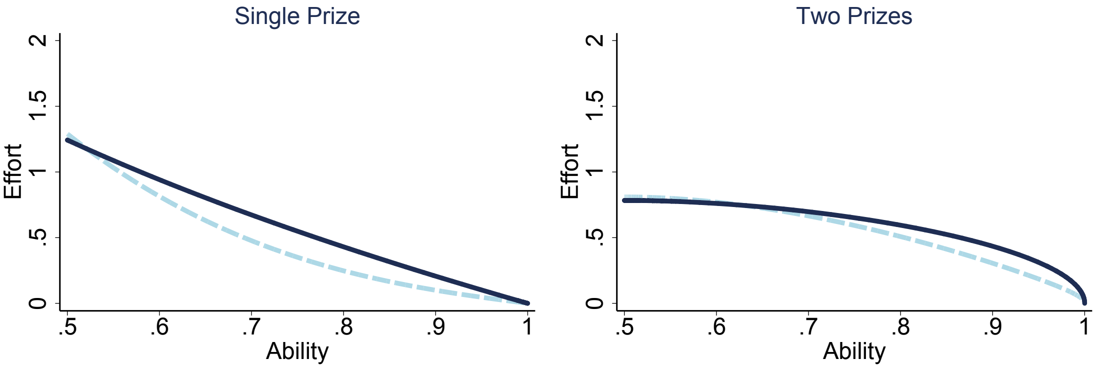
Example 3 *Assume that there are $k = 3$ players, whose abilities are drawn independently from the uniform distribution $F(c) = 2c - 1$ on the interval $[1/2, 1]$, as in example 1. Assume that the cost-of-effort function of players is given by $\gamma(x) = x^2$. Using Proposition 4, the optimal effort function is given by:*

$$b(c) = \gamma^{-1}A_1(c)V_1 + A_2(c)V_2,$$

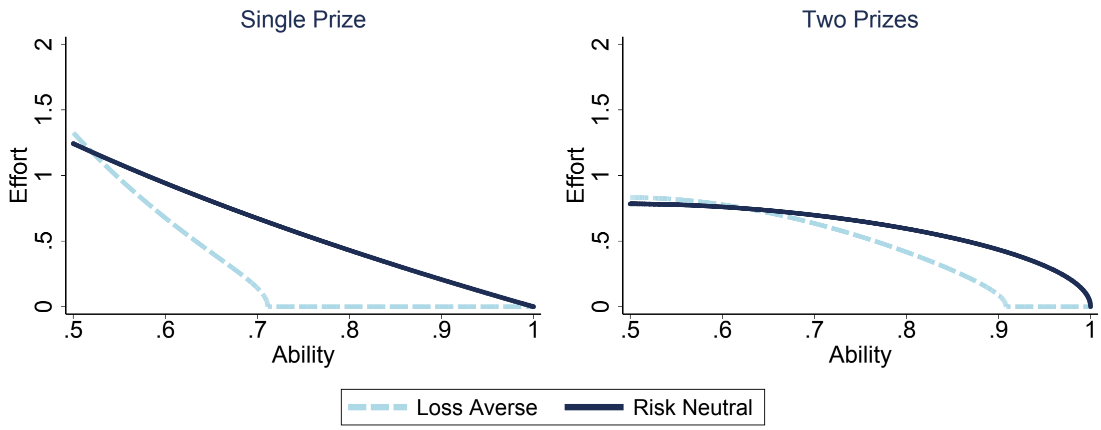
where the coefficients of the first and the second prize are given by equations (11) and (12). Figure 4 depicts the equilibrium effort function for both standard preferences (solid line) and expectation-based reference-dependent preferences (dashed line), in the upper panel for $\Lambda = 0.8 \leq 1$ and in the lower panel for $\Lambda = 1.5 > 1$.

Figure 4: Equilibrium Effort Functions

(a) Full Participation, $\Lambda \leq 1$



(b) Drop-outs, $\Lambda > 1$



Notes: The figure illustrats the equilibrium effort functions when players have convex cost-of-effort function $\gamma(x) = x^2$, for the degree of loss-aversion $\Lambda = 0.8$ in the upper panel and for $\Lambda = 1.5$ in the lower panel.

The upper panel of Figure 4 shows that there is full participation when $\Lambda = 0.8 \leq 1$*, while in the lower panel of the figure there is a mass of players with* $c \geq \tilde{c}$ *putting zero effort. It is important to note that the critical type, as in the case of linear cost-of-efforts, depends on the distribution of order statistics and the ratio of prizes is calculated using the equation (7). As in example 1, the critical type is much smaller in the case of the award of a single prize than the case of the award of a second prize. Intuitively a second prize motivates low- and middle-ability players who would otherwise give up the fragile hope of winning any prize and making gains. Inspecting the left- and right-hand sides of the figure reveal that introducing a second prize motivates low- and high-ability players while it discourages the high-ability* *players from putting high effort levels.*

The equilibrium effort functions in the cases of convex or concave cost-of-effort functions are obtained by a simple transformation of the equilibrium strategies in the linear cost-of-effort case. Therefore, the intuition presented in Section 4.1 for explaining the over- and under exertion of effort of high- and middle-ability players and drop-outs of low-ability players in comparison to the standard predictions applies here exactly in the same way.

The following theorem generalizes the bimodal effort provision illustrated in example 3 to the general case with any number of prizes P, any distribution of abilities F, and any number of players k.

Theorem 2 *Assume that there are P prizes with $V_1 \geq V_2 \geq ... \geq V_P \geq 0$ to be awarded and $k > P$ players with convex or concave cost-of-effort functions. Denote the equilibrium effort function of a players with reference dependent preferences by b^{LA} and of players with standard preferences by b^S. For any players with ability c in an ϵ neighbourhood of m, $b^{LA}(c) > b^S(c)$. For any players with ability c in an ϵ neighbourhood of $\tilde{c}$, $b^{LA}(c) < b^S(c)$, where $\tilde{c} = 1$ whenever $\Lambda \leq 1$.*

Proof. See appendix A.

5.2. The Optimal Prize Allocation

Assume that there are P prizes $V_1 \geq V_2 \geq \cdots \geq V_P \geq 0$ to be awarded with $\sum_{s=1}^{P} V_s = 1$ and $k > P$ players with convex or concave cost-of-effort functions $\gamma(x)$. Given the optimal effort functions for any prize allocation, the principal chooses the number and the level of the prizes to maximize the total expected effort. players exert effort according to equation (17), where A_s for any $s \in \{1, \cdots P\}$ is as in equation (10). The average effort of a players with type c becomes:

$$\int_m^{\tilde{c}} \gamma^{-1} \left(\sum_{i=1}^{P} V_i A_i(c) \right) F'(c)dc = \int_m^{\tilde{c}} \gamma^{-1} \left(A_1 + \sum_{i=1}^{P} V_i (A_i(c) - A_1(c)) \right) F'(c)dc$$

The revenue of the principal, the expected total effort of all players, becomes:

$$R(V_2, V_3, \cdots, V_P) = k \int_m^{\tilde{c}} \gamma^{-1} \left(A_1 + \sum_{i=1}^{P} V_i (A_i(c) - A_1(c)) \right) F'(c)dc. \tag{18}$$

The principal's problem becomes choosing $V_{i i=1}^{P}$ to maximize the total expected effort:

$$\max_{\{V_i\}_{i=2}^{P}} k \int_m^{\tilde{c}} \gamma^{-1} \left(A_1 + \sum_{i=1}^{P} V_i (A_i(c) - A_1(c)) \right) F'(c) dc.$$

Similar to the case of linear cost-of-effort functions, the optimal number of prizes depends on the shape of the revenue function. If the derivative of the revenue function is positive with respect to the i-th prize V_i at $V_i = 0$, then it is optimal to award a positive i-th prize and it is optimal to set $V_i = 0$ otherwise. As in Section 4,the principal's problem will be discussed separately for the cases of $\Lambda \leq 1$ and $\Lambda > 1$. The following proposition provides a sufficient condition for the optimality of the i-th prize for the case of $\Lambda \leq 1$ and characterises the levels of prizes for a concave cost-of-effort case.

Proposition 5 *Assume that there are P prizes $V_1 \geq V_2 \geq ... \geq V_p \geq 0$ to be awarded with $\sum_{i=1}^{P} V_i = 1$ and $k > P$ loss-averse players with convex or concave cost-of-effort functions. Moreover assume that $\Lambda \leq 1$ so that there is full participation in the contest. It is optimal to allocate the s-th prize if*

$$\int_m^{\tilde{c}} \gamma^{-1\prime} \left(A_1 + \sum_{s=1}^{P} V_s (A_s - A_1) \right) (A_s - A_1) F'(c) dc > 0. \tag{19}$$

Moreover, if the cost-of-effort function is concave, it is always optimal to award either a single grand prize or multiple equal prizes.

Proof. See Appendix B.

Now suppose that contestants' degree of loss aversion is $\Lambda > 1$. Similar to the case of linear costs, the upper bound of the integral in equation (18) $\tilde{c}$ depends on the values of the prizes $V_2, V_3, \cdots, V_P$. Therefore, $\tilde{c}$ appears in the derivative of the principal's revenue function $R'(V_2, V_3, \cdots, V_P)$ in the bounds of the integrals, which in turn depends on the optimal values of prizes. For this reason, calculating the numerical values for $R'(V_2, V_3, \cdots, V_P)$ does not becomes computationally less demanding than finding the optimal values of the prizes $V_2, V_3, \cdots, V_P$ and $V_1 = 1 - \sum_{s=1}^{P} V_s$ by maximizing the revenue function R.

The following example illustrates the optimal prize structure when there are $P = 2$ prizes to be awarded $V_1 \geq V_2 \geq 0$ and the cost-of-effort function is concave, comparing the cases of standard- and reference-dependent preferences.

Example 4 *Assume that there are* 5 *players, whose abilities are drawn from a uniform distribution* $F(c) = 2c - 1$ *on the interval* $[0.5, 1]$. *Assume, moreover, that the cost-of-effort function is* $\gamma(x) = \sqrt{x}$. *Suppose first that* $\Lambda = 0$, *so that we are in the standard case without loss aversion. Following the results of Moldovanu and Sela (2001), it is optimal to award a grand single prize in this case. Suppose that* $\Lambda = 0.8$, *so that there is full participation in the contest. In this case, the revenue of the principal is maximized at* $V_2 = 0$ *and thus awarding a single grand prize is optimal. Now, suppose that players are more loss averse with* $\Lambda = 3$, *so that there are some drop-outs. In this case, the revenue of the principal is maximized at* $V_2 = 0.5$ *with the critical type being* $\tilde{c} = 0.65$. *Thus the award of a second prize becomes optimal when* $\Lambda = 3$.

The optimal prize structure illustrated in example 4 is qualitatively very similar to the one presented in example 2. The equilibrium effort strategies with convex or concave costs being simple transformations of the equilibrium strategies with linear costs, as shown in proposition 4, the intuition presented in Section 4 provides the rationale behind these results.

6. Conclusion

Competition among participants is often presumed to enhance performance, however, empirical evidence points out bi-modal behavior: over-workers and drop-outs. More specifically, it is observed that contestants with high abilities over-exert effort while contestants with low abilities withhold or withdraw effort in comparison to the predictions of standard theory. This observation is puzzling from the point of view of classical economic theory, since contestants with positive probability of winning a prize (i.e. any contestants except for the lowest type) are exerting zero effort and staying out of competition. This observation raises to important questions: (1) What is the deriving force behind this bifurcated effort provision? and (2) What is its implication for the optimal allocation of prizes?

To address these questions, I analyze a canonical incomplete information all-pay contest with heterogeneous contestants under the assumption that participants are expectation-based loss-averse. It is established that competing agents not only evaluate outcomes in absolute terms but also relative to their expectations and therefore expectations play a key role in effort provision in competitive environments. Therefore, expectations of contestants are critical in deciding how much effort to put. My model provides a backbone theory for the observed bimodal effort provision. I show that high-ability players exert effort aggressively while low-ability players withhold effort, in comparison to the predictions with standard preferences. Players being loss averse around an endogenous reference point determined by rational expectations creates the following bifurcating force on effort provision. On the one hand, a high-ability player who has *ex-ante* high expectations regarding winning a prize, increases his effort level further to avoid disappointment in case of not winning a prize. On the other hand, a low-ability player who has *ex-ante* low expectations of winning a prize, lowers his expectations of winning a prize to make the outcome of not winning any prize less painful. If the degree of loss aversion is sufficiently high, $\Lambda > 1$, a low-ability player chooses a stochastically dominated option: withdraws effort. The reason is that loss aversion being sufficiently pronounced, the gain-loss utility of a player with *ex-ante* low chances of winning a prize dominates his consumption utility. In this case, *ex-ante* expecting a net loss, he withdraws effort to secure himself the minimum possible net loss of zero.

Expectation-based loss aversion serves as a key driver of this bifurcated behaviour and has important implications for the optimal prize allocation. Muting competition by awarding multiple prizes becomes optimal in the cases where a single prize is predicted to be optimal under the assumption of standard preferences. Intuitively, the marginal effect of introducing another smaller prize has two countervailing effects on the revenue of the principal: a beneficial effect on the low- and middle-ability players and a detrimental effect on high-ability players. While an additional prize motivates drop-outs to exert effort, it de-motivates over-workers due to decreased value of larger prizes. The beneficial effect of a second (or more) prize(s) dominates its detrimental effect when the relative increase in principal's revenue due to aggressive effort provision of high-ability players is outsized by the relative effort decrease of low-ability players. The balance of these two opposing forces and thus the optimal allocation of prizes, hinges on the interplay between the number of competitors, the ability heterogeneity, and the degree of loss-aversion.

The above conclusion has policy-relevant implications. A principal can obtain higher levels of total effort by muting competition when contestants expectation-based loss averse. The principal can mute competition by decreasing prize inequality and by potentially introducing additional prizes. The discouraging effect of competition is more pronounced when there is contest entry, when ability range of contestants is less dispersed, and when contestants are more loss averse.

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Appendices

A. Derivation of Equilibria

Proof of Proposition 1. The expected utility of a players with ability c putting effort x is given by equation (5). Using the notation introduced in Section 3, the it reduces to:

$$EU = \sum_{s=1}^{P} p_s V_s - c\gamma(x) - \Lambda \sum_{s=1}^{P} \left(\sum_{i=s+1}^{P+1} p_s p_i (V_s - V_i) \right),$$

as expressed in equation (6). The players derives positive expected utility only if

$$(1-\Lambda)\left(\sum_{s=1}^{P} F_s(c)V_s\right) + \Lambda\left(\sum_{s=1}^{P}(F_s(c))^2 V_s + \sum_{s=2}^{P}\sum_{i=1}^{s-1} 2V_s F_s(c) F_i(c)\right) \geq 0. \quad (20)$$

Note that, here I already use the monotonicity of effort function, so that order statistics are used in the above expression. It is straightforward to see that the inequality (20) is automatically satisfied for all $c \in [m, 1]$ if $\Lambda \leq 1$. Therefore, there is full participation in the contest. If $\Lambda > 1$, $\tilde{c}$ is the highest type for which the inequality (20) holds, where $\tilde{c}$ satisfies the the condition in (20) with equality:

$$\frac{\sum_{s=1}^{P}(F_s(\tilde{c}))^2 V_s + \sum_{s=2}^{P}\sum_{i=1}^{s-1} 2V_s F_s(\tilde{c}) F_i(\tilde{c})}{\sum_{s=1}^{P} F_s(\tilde{c}) V_s} = 1 - \frac{1}{\Lambda}.$$

Therefore, any player with type $c > \tilde{c}$ ends up with negative expected utility if he puts positive effort. Instead, these players put zero effort and secure themselves a zero expected utility, a better outcome than a negative expected utility. Now it remains to show that $\frac{\delta \tilde{c}}{\delta \Lambda} < 0$. By definition $\tilde{c}$ is a function of Λ (see equation (7)). Using Implicit Function Theorem:

$$\frac{\delta \tilde{c}}{\delta \Lambda} = -\frac{\frac{\delta EU}{\delta \Lambda}}{\frac{\delta EU}{\delta c}},$$

where EU is the expected utility given in equation (6). It is straightforward to see that $\frac{\delta EU}{\delta \Lambda} < 0$ using the equation (6) as the coefficient of Λ is always negative. Now I will show that $\frac{\delta EU}{\delta c}$ is also negative, which together with $\frac{\delta EU}{\delta \Lambda} < 0$ implies that $\frac{\delta \tilde{c}}{\delta \Lambda} < 0$. Denote the sum of expected utility of a players with type, c $EU(c)$ and his disutility of cost-of-effort $c\gamma(x)$ by $\bar{EU}(c)$. Namely $EU(c) = \bar{EU}(c) - c\gamma(x)$. Take any two players with abilities $c_1 < c_2$. I will show that $EU(c_1) > EU(c_2)$. Suppose that $\bar{EU}(c_1) < \bar{EU}(c_2)$. The players with type c_1, being a better type, can imitate the players with type c_2 by putting exactly the same effort level as him (since the bidding function is monotonically decreasing), implying $\bar{EU}(c_1) > \bar{EU}(c_2)$. Since $c_1 < c_2$, the cost-of-effort for the players with type c_1 is lower than the cost-of-effort for the players with type c_2. Combining the two we get the following:

$$EU(c_1) = \bar{EU}(c_1) - c_1\gamma(x) > \bar{EU}(c_2) - c_2\gamma(x) = EU(c_2).$$

Thus $\frac{\delta EU}{\delta c} < 0$, completing the proof.

Proof of Proposition 2. To ease the exposition, I first concentrate on the case of $P = 2$ prizes with $V_1 \geq V_2 \geq 0$. I discuss the cases when there is full participation in the contests (i.e. $\Lambda \leq 1$) and when some players drop out of the contest (i.e. $\Lambda > 1$) separately. I elaborate the case of P prizes afterwards. Assume that there are two prizes to be awarded with $V_1 + V_2 = 1$. First suppose that $\Lambda \leq 1$. Assume that all players except i exert effort according to the function b and assume that b is strictly monotonic and differentiable. The maximization problem of the players i is:

$$\begin{aligned} \max_x \quad & \Big[p_1\Big(V_1 + \eta p_2(V_1 - V_2) + \eta(1 - p_1 - p_2)V_1 - cx\Big) \\ & + p_2\Big(V_2 - \eta p_1\lambda(V_1 - V_2) + \eta(1 - p_1 - p_2)V_2 - cx\Big). \\ & + (1 - p_1 - p_2)\Big(- \eta p_1\lambda V_1 - \eta p_2\lambda V_2 - cx\Big)\Big]. \end{aligned} \tag{21}$$

where the probabilities of winning the first and the second prize are

$$p_1 = (1 - F(b^{-1}(x)))^{k-1} \quad \text{and} \quad p_2 = (k-1)(1 - F(b^{-1}(x)))^{k-2}F(b^{-1}(x)). \tag{22}$$

Denote the inverse of b by y. Substituting p_1, p_2, $\Lambda = \eta(\lambda - 1)$ and y, the maximization problem becomes:

$$\max_{x} \quad \Big[(1-\Lambda)(1-F(y))^{k-1}V_1 + (1-\Lambda)(k-1)(1-F(y))^{k-2}F(y)V_2$$
$$+ \Lambda(1-F(y))^{2k-2}V_1 + (\Lambda)(k-1)^2(1-F(y))^{2k-4}F^2(y)V_2$$
$$+ 2\Lambda(k-1)(1-F(y))^{2k-3}F(y)V_2 - cx\Big].$$

Using the strict monotonicity of b and symmetry, the first-order condition (FOC) is given by:

$$\begin{aligned}
1 =& V_1\frac{1}{y}\Big(- (1-\Lambda)(k-1)(1-F(y))^{k-2}F'(y)y' \\
& - \Lambda(2k-2)(1-F(y))^{2k-3}F'(y)y'\Big) \\
& + V_2\frac{1}{y}\Big(- (1-\Lambda)(k-1)(1-F(y))^{k-3}F'(y)y'(1-(k-1))F(y) \\
& + 2\Lambda(k-1)(1-F(y))^{2k-5}F'(y)y'\Big((1-kF(y)-((k-1)^2-1\Big)F(y)^2\Big)
\end{aligned} \tag{23}$$

A player with the highest possible type $c = 1$ never wins a prize under the assumption $k > 2$. Thus the optimal effort of this player is always 0, providing $y(0) = 1$ as a boundary condition. Note that the FOC is a differential equation with separated variables, since the left hand side of the equation (23) is a function of y only. Define the function $H(y)$:

$$\begin{aligned}
H(y) =& V_1\left((1-\Lambda)(k-1)\int_y^1 \frac{1}{t}(1-F(t))^{k-2}F'(t)dt\right. \\
& \left.+ \Lambda(2k-2)\int_y^1 \frac{1}{t}(1-F(t))^{2k-3}F'(t)dt\right) \\
& + V_2\left((1-\Lambda)(k-1)\int_y^1 \frac{1}{t}(1-F(t))^{k-3}(1-(k-1))F(t)F'(t)dt\right. \\
& \left.+2\Lambda(k-1)\int_y^1 \frac{1}{t}(1-F(t))^{2k-5}F'(t)(1-kF(t)-((k-1)^2-1)F(t)^2)dt\right).
\end{aligned} \tag{24}$$

The solution to the differential equation (23) with the boundary condition $y(0) = 1$ becomes:

$$\int_x^0 dt = -H(y). \tag{25}$$

Integrating both sides of the equation (25) gives $x = H(y) = H(b^{-1}(x))$ and thus

$b \equiv H$. Therefore, the effort function of each player is given by:

$$b(c) = A_1(c)V_1 + A_2(c)V_2, \tag{26}$$

where

$$\begin{aligned} A_1(c) =& (1-\Lambda)\int_c^1 \frac{1}{a}(k-1)(1-F(a))^{k-2}F^{'}(a)da \\ &+ \Lambda \int_c^1 \frac{1}{a}(2k-2)(1-F(a))^{2k-3}F^{'}(a)da \end{aligned} \tag{27}$$

and

$$\begin{aligned} A_2(c) =& (1-\Lambda)\int_c^1 \frac{1}{a}(k-1)(1-F(a))^{k-3}\left(-1+(k-1)F(a)\right)F^{'}(a)da \\ &+ \Lambda \int_c^1 \frac{1}{a}(2k-2)(1-F(a))^{2k-5}\left(-1+kF(a)+((k-2)^2-1)F(a)^2\right)F^{'}(a)da. \end{aligned} \tag{28}$$

Replacing the terms in equations 27 and 28 with the order statistics one gets:

$$A_1(c) = (1-\Lambda)\int_c^1 -\frac{1}{a}F_1^{\prime}(a)da - \Lambda \int_c^1 \frac{1}{a}(F_1^2(a))^{\prime}da \tag{29}$$

and

$$A_2(c) = (1-\Lambda)\int_c^1 -\frac{1}{a}F_2^{\prime}(a)da - \Lambda \int_c^1 \frac{1}{a}\left((F_2^2(a))^{\prime} + (2F_1(a)F_2(a))^{\prime}\right)da. \tag{30}$$

It remains to show that the equilibrium effort function $b(c)$ is differentiable and strictly decreasing. The former one is obvious. To show that the effort function is strictly decreasing, consider the derivatives of $A_1(c)$ and $A_2(c)$:

$$\begin{aligned} A_1^{\prime}(c) =& (1-\Lambda) - \frac{1}{c}(k-1)(1-F(c))^{k-2}F^{'}(c) \\ &- \Lambda\frac{1}{c}(2k-2)(1-F(c))^{2k-3}F^{'}(c). \end{aligned}$$

and

$$\begin{aligned} A_2^{\prime}(c) =& (1-\Lambda) - \frac{1}{c}(k-1)(1-F(c))^{k-3}\left(-1+(k-1)F(c)\right)F^{'}(c) \\ &- \Lambda\frac{1}{c}(2k-2)(1-F(c))^{2k-5}\left(-1+kF(c)+((k-2)^2-1)F(c)^2\right)F^{'}(c). \end{aligned}$$

The derivative of the effort function $b(c)$ becomes:

$$\begin{aligned} b'(c) &= A_1'(c)V_1 + A_2'(c)V_2 \\ &\leq V_2\Big(A_1'(c) + A_2'(c)\Big) \end{aligned} \tag{31}$$

where the last inequality follows from $V_2 \leq V_1$. Now it remains to show that $A_1'(c) + A_2'(c) < 0$

$$\begin{aligned} A_1'(c) + A_2'(c) =& -(1-\Lambda)(k-1)(k-2)\frac{1}{c}(1-F(c))^{k-3}F'(c)F(c) \\ & - \Lambda(2k-2)\frac{1}{c}(1-F(c))^{2k-5}F'(c)\Big((k-2)F(c) + (k-2)^2F(c)^2\Big) \\ & < 0. \end{aligned} \tag{32}$$

Now suppose that $\Lambda > 1$. By Proposition 1 there exists a critical type $\tilde{c}$ such that all types with $c > \tilde{c}$ puts zero effort, i.e. drops out to avoid negative expected utility. Thus the boundary condition becomes $y(0) = \tilde{c}$, while the maximization problem of the players expressed in equation (21) remains the same, leading to the FOC in equation (23). Note that the boundary condition $y(0) = 1$ in the case of $\Lambda \leq 1$ can be viewed as a special case of the boundary condition $y(0) = \tilde{c}$ with $\tilde{c} = 1$. Thus the solution to the differential equation (23) with the boundary condition $y(0) = \tilde{c}$ becomes:

$$\begin{aligned} \int_x^0 dt =& V_1\left((1-\Lambda)(k-1)\int_y^{\tilde{c}} \frac{1}{t}(1-F(t))^{k-2}F'(t)dt \right. \\ & \left. + \Lambda(2k-2)\int_y^{\tilde{c}} \frac{1}{t}(1-F(t))^{2k-3}F'(t)dt\right) \\ & + V_2\left((1-\Lambda)(k-1)\int_y^{\tilde{c}} \frac{1}{t}(1-F(t))^{k-3}(1-(k-1))F(t)F'(t)dt \right. \\ & \left. + 2\Lambda(k-1)\int_y^{\tilde{c}} \frac{1}{t}(1-F(t))^{2k-5}F'(t)(1-kF(t)-((k-1)^2-1)F(t)^2)dt\right). \end{aligned}$$

Integrating both sides, a players with $c \leq \tilde{c}$ puts effort according to:

$$b(c) = A_1(c)V_1 + A_2(c)V_2,$$

where

$$\begin{aligned} A_1(c) =&(1-\Lambda)\int_c^{\tilde{c}} \frac{1}{a}(k-1)(1-F(a))^{k-2}F^{'}(a)da \\ &+\Lambda\int_c^{\tilde{c}} \frac{1}{a}(2k-2)(1-F(a))^{2k-3}F^{'}(a)da \end{aligned} \tag{33}$$

and

$$\begin{aligned} A_2(c) =&(1-\Lambda)\int_c^{\tilde{c}} \frac{1}{a}(k-1)(1-F(a))^{k-3}\left(-1+(k-1)F(a)\right)F^{'}(a)da \\ &+\Lambda\int_c^{\tilde{c}} \frac{1}{a}(2k-2)(1-F(a))^{2k-5}\left(-1+kF(a)+((k-2)^2-1)F(a)^2\right)F^{'}(a)da. \end{aligned} \tag{34}$$

Again, replacing the terms in equations (27) and (28) with the order statistics gives the following coefficients for the first and the second prizes:

$$A_1(c) =(1-\Lambda)\int_c^{\tilde{c}} -\frac{1}{a}F_1'(a)da - \Lambda\int_c^{\tilde{c}} \frac{1}{a}(F_1^2(a))'da$$

and

$$A_2(c) =(1-\Lambda)\int_c^{\tilde{c}} -\frac{1}{a}F_2'(a)da - \Lambda\int_c^{\tilde{c}} \frac{1}{a}\left((F_2^2(a))'+(2F_1(a)F_2(a))'\right)da.$$

The equilibrium effort function $b(c)$ being differentiable and strictly decreasing can be shown exactly in the same way as above. The last thing to show is for any type c, the effort $b(c)$ indeed maximizes the expected utility of that type. The necessary FOC is clearly satisfied, now it remains to show that a sufficient second-order condition is satisfied. In other words, I will show that the derivative of the expected utility of players with type c given by equation (21) with respect to x is nonnegative if $x < b(c)$ and nonpositive if $x > b(c)$. Since the expected payoff in equation (21) is continuous in x, this implies that $b(c)$ indeed maximizes the expected utility. The derivative of the expected utility in equation (21) with respect to x is given by:

$$\begin{aligned}\frac{dEU(x,c)}{dx} &= V_1\frac{db^{-1}(x)}{dx}\Big(-(1-\Lambda)(k-1)(1-F(y))^{k-2}F'(y)y' \\ &\quad -\Lambda(2k-2)(1-F(y))^{2k-3}F'(y)y'\Big) \\ &\quad + V_2\frac{db^{-1}(x)}{dx}\Big(-(1-\Lambda)(k-1)(1-F(y))^{k-3}F'(y)y'(1-(k-1))F(y) \\ &\quad + 2\Lambda(k-1)(1-F(y))^{2k-5}F'(y)y'\Big((1-kF(y)-((k-1)^2-1\Big)F(y)^2\Big) - c\end{aligned}$$

I will show that $\frac{dEU(x,c)}{dx} \geq 0$ for every $x < b(c)$. Let $x < b(c)$ and $\bar{c}$ with $b(\bar{c}) = x$. Since b is strictly decreasing, $\bar{c} > c$. Note that the derivative of $\frac{dEU(x,c)}{dx}$ with respect to c is $-1 < 0$, so that $\frac{dEU}{dx}$ is decreasing in c. This implies, $\frac{dEU(x,c)}{dx} \geq \frac{dEU(x,\bar{c})}{dx} = 0$, where the last inequality follows from $x = b(\bar{c})$. Thus, $\frac{dEU(x,c)}{dx} \geq 0$ for every $x > b(c)$. A similar argument shows that for every $x > b(c)$, $\frac{dEU(x,c)}{dx} \leq 0$.

Following the same steps as in the case of 2 prizes, I will derive the optimal effort functions when there are P prizes as follows. Assume that there are $P \leq k$ prizes to be awarded with $V_1 \geq V_2 \geq \cdots \geq V_P$ and $k > P$ players. Assume that the cost-of-effort of players is given by $c\gamma(x)$, where $\gamma(x) = x$. The expected utility of players with ability c and effort level x is given by, as discussed in Section 2.

$$EU = \sum_{s=1}^{P} p_s V_s - cx + \eta\left[\sum_{s=1}^{P+1} p_s\left(\sum_{i=s+1}^{P+1} p_i(V_s - V_i) + \sum_{i=1}^{s} p_i\lambda(V_s - V_i)\right)\right]$$

Substituting $\Lambda = \eta(\lambda - 1)$ and rearranging the terms, the expected utility of the players becomes:

$$EU = \sum_{s=1}^{P} p_s V_s - cx - \Lambda\sum_{s}^{P}\left(\sum_{i=s+1}^{P+1} p_s p_i(V_s - V_i)\right).$$

Rearranging the terms, one gets:

$$EU = (1-\Lambda)\sum_{s=1}^{P} p_s V_s - cx + \Lambda\left\{\sum_{s=1}^{P} p_s^2 V_s + \sum_{s=2}^{P} 2V_s F_s \sum_{i=1}^{s-1} F_1\right\}.$$

The player chooses how much effort to put by maximizes his expected utility as follows:

$$\max_x (1-\Lambda)\sum_{s=1}^{P} p_s V_s - cx + \Lambda\left\{\sum_{s=1}^{P} p_s^2 V_s + \sum_{s=2}^{P} 2V_s F_s \sum_{i=1}^{s-1} F_1\right\}.$$

Taking the derivative of the objective function with respect to the choice variable x, first order condition becomes:

$$\sum_{s=1}^{P} V_s \left\{(1-\Lambda)p_s^{'} + \Lambda\left((p_s^2)' + \sum_{i=1}^{s-1}(2p_s p_i)'\right)\right\} = c,$$

the probability of winning the s-th prize is

$$p_s = \frac{(k-1)!}{(s-1)!(k-s)!}(1-F(a))^{k-s}F(a)^{s-1} \quad \text{with} \quad F(a) = (b^{-1}(x)). \tag{35}$$

Solving the differential equation with the boundary condition $b(1) = 0$, the equilibrium effort function becomes:

$$b(c) = \sum_{s}^{P} V_s A_s \tag{36}$$

where $A_s(c)$ is

$$A_s(c) = (1-\Lambda)\int_c^1 -\frac{1}{a}F_s(a)'da + \Lambda\int_c^1 -\frac{1}{a}\left(\left(F_s(a)^2\right)' + \sum_{i=1}^{s-1}\left(2F_i(a)F_s(a)\right)'\right)da.$$

Whenever $\Lambda > 1$, there exist a critical type $\tilde{c}$ satisfying the equation (7),

$$\frac{\sum_{s=1}^{P}(F_s(\tilde{c}))^2 V_s + \sum_{s=2,i<s}^{P} 2V_s F_s(\tilde{c})F_i(\tilde{c})}{\sum_{s=1}^{P} F_s(\tilde{c})V_s} = 1 - \frac{1}{\Lambda}.$$

such that any players with $c \geq \tilde{c}$ exerts zero effort in equilibrium, while players with $c < \tilde{c}$ exert effort in equilibrium according to equation (36). Showing that the equilibrium effort function is decreasing, $b'(c) < 0$, and that the second order condition is also satisfied is completely analogous to the case of $P = 2$.

Proof of Theorem 1. First I will show that for each players with ability c in a neighbourhood of m, $b^{LA}(c) > b^S(c)$. Proof of this part will be done by induction on the number of prizes P. First assume that there are only two prizes V_1 and V_2 with $V_1 + V_2 = 1$, $P = 2$. The equilibrium effort function of a loss-averse player is given by

$$b^{LA}(c) = A_1(c)V_1 + A_2(c)V_2$$

where the coefficients of the first and second prize are given by:

$$A_1(c) = (1 - \Lambda)\int_c^{\tilde{c}} -\frac{1}{a}F_1'(a)da + \Lambda\int_c^{\tilde{c}} -\frac{1}{a}(F_1^2(a))'da$$

and

$$A_2(c) = (1 - \Lambda)\int_c^{\tilde{c}} -\frac{1}{a}F_2'(a)da + \Lambda\int_c^{\tilde{c}} -\frac{1}{a}\left((F_2^2(a))' + (2F_1(a)F_2(a))'\right)da.$$

For the easiness of representation I introduce the following notation: denote $A_1(c) = (1 - \Lambda)A_1^1(c) + \Lambda A_1^2(c)$ and $A_2(c) = (1 - \Lambda)A_2^1(c) + \Lambda A_2^2(c)$. The equilibrium effort function of a player with standard preferences is obtained by substituting $\Lambda = 0$:

$$b^S(c) = A_1^1(c)V_1 + A_{"}^1(c)V_2.$$

First I will show that $b^{LA}(m) - b^S(m) > 0$ for $p = 2$, concluding that there is over-exertion of effort at $c = m$. Using this and the continuity of function b, for any c in the ε neighbourhood of m: $b^{LA}(c) - b^S(c) > 0$ is true.

Claim: $\quad b^{LA}(m) - b^S(m) = V_1\Lambda\Big(A_1^2(m) - A_1^1(m)\Big) + V_2\Lambda\Big(A_2^2(m) - A_2^1(m)\Big) > 0$

I will first look at the difference $A_1^2(m) - A_1^1(m)$ and show that it is always positive.

$$\begin{aligned}
A_1^2(m) - A_1^1(m) &= \int_m^{\tilde{c}} \frac{-1}{a} F_1'(a)\Big[2F_1(a) - 1\Big] da \\
&= \int_{v(m)}^{v(\tilde{c})} \frac{1}{a(v)} \frac{1}{2} - v dv \\
&= \int_{v(\tilde{c})}^{1} \frac{1}{2} \frac{1}{a(v)} v dv \\
&> \int_{v(\tilde{c})}^{1} \frac{1}{2} v dv \int_{v(\tilde{c})}^{1} \frac{1}{a(v)} dv \\
&\geq \int_{-1}^{1} \frac{1}{2} v dv \int_{v(\tilde{c})}^{1} \frac{1}{a(v)} dv \\
&= 0,
\end{aligned}$$

where the following substitution is made in the second equality: $v(a) = 2F_1(a) - 1$, with $v(m) = 1$, $v(1) = -1$ and $v(a)$ is decreasing. Notice that $a(v)$ is the inverse of $v(a)$. Now I will look at the difference $A_2^2(m) - A_2^1(m)$ and show that it is equal to $A_1^2(m) - A_1^1(m)$ plus some strictly positive term.

$$\begin{aligned}
A_2^2(m) - A_2^1(m) &= \int_m^{\tilde{c}} \frac{-1}{a} F_2'(a)\Big[2F_1(a) + 2F_2(a) - 1\Big] da + \int_m^{\tilde{c}} \frac{-1}{a} 2F_1'(a) F_2(a) da \\
&= \int_m^{\tilde{c}} \frac{-1}{a} \Big[F_2'(a) + F_1'(a)\Big]\Big[2F_1(a) + 2F_2(a) - 1\Big] da - \int_m^{\tilde{c}} \frac{-1}{a} F_1'(a)\Big[2F_1(a) - 1\Big] da \\
&= \int_m^{\tilde{c}} \frac{-1}{a} \Big[F_1'(a) + F_2'(a)\Big]\Big[2(F_1(a) + F_2(a)) - 1\Big] da - (A_1^2(m) - A_1^1(m)) \\
&= \int_{v(m)}^{v(\tilde{c})} \frac{-1}{a(v)} \frac{1}{2} v dv - \Big(A_1^2(m) - A_1^1(m)\Big) \\
&= \int_{v(\tilde{c})}^{1} \frac{1}{a(v)} \frac{1}{2} v dv - \Big(A_1^2(m) - A_1^1(m)\Big) \\
&> \int_{v(\tilde{c})}^{1} \frac{1}{2} v dv \int_{v(\tilde{c})}^{1} \frac{1}{a(v)} dv - \Big(A_1^2(m) - A_1^1(m)\Big) \\
&\geq \int_{-1}^{1} \frac{1}{2} v dv \int_{v(\tilde{c})}^{1} \frac{1}{a(v)} dv - \Big(A_1^2(m) - A_1^1(m)\Big) \\
&= 0 - \Big((A_1^2(m) - A_1^1(m)\Big).
\end{aligned}$$

In the fourth equality the the following substitution is made: $v(a) = 2(F_1(a) + F_2(a)) - 1$, with $v(m) = 1$, $v(1) = -1$ and $v(a)$ is decreasing. Again, $a(v)$ is the inverse of $v(a)$. Going back to the original claim:

$$\begin{aligned}
b^{LA}(m) - b^{S}(m) &= V_1\Lambda\Big(A_1^2(m) - A_1^1(m)\Big) + V_2\Lambda\Big(A_2^2(m) + \Lambda A_2^1(m)\Big) \\
&= \Big(V_1 - V_2\Big)\Big(A_1^2(m) - A_1^1(m)\Big) + V_2\Big(\int_m^{\tilde{c}} \frac{-1}{a} F_{12}'(a)(2F_{12}(a) - 1)da\Big) \\
&> \int_m^{\tilde{c}} \frac{-1}{a} F_{12}'(a)(2F_{12}(a) - 1)da + 0 \\
&> 0
\end{aligned},$$

where the last inequality follows from $A_1^2(m) - A_1^1(m)$ being positive, $\Lambda > 0$ for a player to be loss-averse and the fact that the second prize is at most as large as the first one. Thus over-exertion of effort of type $c = m$ is proven. By the continuity of the effort function b, there is an over-exertion of effort for all types in a neighborhood of m.

Now suppose that the claim $b^{LA}(m) - b^{S}(m)$ is true for $p = l$ prizes and now I will show that it holds for $p = l + 1$ prizes. Namely, for $p = l + 1$ prizes the following needs to be shown:

$$\text{Claim:} \qquad b^{LA}(m) - b^{S}(m) = \sum_{i=1}^{l+1} V_i\Lambda\Big(A_i^2(m) - A_i^1(m)\Big) > 0.$$

First lets look at the difference $A^2_{l+1}(m) - A^1_{l+1}(m)$.

$$\begin{aligned}
A^2_{l+1}(m) - A^1_{l+1}(m) &= \int_m^{\tilde{c}} \frac{-1}{a} F'_{l+1}(a)[2\sum_{i=1}^{l+1} F_i(a) - 1]da + \int_m^{\tilde{c}} \frac{-1}{a} \sum_{i=1}^{l} 2F_{l+1}(a)F'_i(a)da \\
&= \int_m^{\tilde{c}} \frac{-1}{a}[\sum_{i=1}^{l+1} F'_i(a)][2\sum_{i=1}^{l+1} F_i(a) - 1]da - \int_m^{\tilde{c}} \frac{-1}{a}[\sum_{i=1}^{l} F'_i(a)][2\sum_{i=1}^{l} F_i(a) - 1]da \\
&= \int_m^{\tilde{c}} \frac{-1}{a}[\sum_{i=1}^{l+1} F'_i(a)][2\sum_{i=1}^{l+1} F_i(a) - 1]da - \sum_{i=1}^{l} A^2_i(m) - A^1_i(m) \\
&= \int_{v(m)}^{v(\tilde{c})} \frac{-1}{a(v)} \frac{1}{2} v dv - \sum_{i=1}^{l} A^2_i(m) - A^1_i(m) \\
&= \int_{v(\tilde{c})}^{1} \frac{1}{a(v)} \frac{1}{2} v dv - \sum_{i=1}^{l} A^2_i(m) - A^1_i(m) \\
&> \int_{v(\tilde{c})}^{1} \frac{1}{2} v dv \int_{v(\tilde{c})}^{1} \frac{1}{a(v)} dv \sum_{i=1}^{l} A^2_i(m) - A^1_i(m) \\
&\geq \int_{-1}^{1} \frac{1}{2} v dv \int_{v(\tilde{c})}^{1} \frac{1}{a(v)} dv - \sum_{i=1}^{l} A^2_i(m) - A^1_i(m) \\
&= 0 - \sum_{i=1}^{l} A^2_i(m) - A^1_i(m).
\end{aligned}$$

The third equality follows from the supposition for $p = l$. The fourth equality follows by making the substitution is $v(a) = 2\sum_{i=1}^{l+1} F_i(a) - 1$, where v is strictly increasing with $v(m) = 1$ and $v(1) = -1$. Going back to the claim:

$$\begin{aligned}
b^{LA}(m) - b^S(m) &= \sum_{i=1}^{l+1} V_i \Lambda (A^2_i(m) - A^1_i(m)) \\
&> \sum_{i=1}^{l} (V_i - V_{l+1}) \Lambda (A^2_i(m) - A^1_i(m)) \\
&> \sum_{i=1}^{l-1} (V_i - V_l) \Lambda (A^2_i(m) - A^1_i(m)) \\
&\vdots \\
&> \sum_{i=1}^{l-2} (V_i - V_3) \Lambda (A^2_i(m) - A^1_i(m)) \\
&> (V_1 - V_2) \Lambda (A^2_1 - A^1_1) \\
&> 0,
\end{aligned}$$

where the last inequality follows from $A_1^2(m) - A_1^1(m)$ being positive, $\Lambda > 0$ for a player to be loss-averse and the fact that the second prize is at most as large as the first one. Thus the highest-ability type with $c = m$ over-exerts effort. By the continuity of the effort function b, all types in a neighborhood of m overexert effort. Second, I will show that for each players with ability c in a neighbourhood of m, $b^{LA}(c) < b^S(c)$. If $\tilde{c} < 1$, then for any c in a neighbourhood of 1, $b^{LA}(c) = 0$as shown in Proposition 2, while $b^S(c) > 0$ except for $c = 1$. Thus $b^{LA}(c) < b^S(c)$ is true whenever $\tilde{c} < 1$. Now consider the case when $\tilde{c} = 1$. It has been already shown that:

$$A_2^2(c) - A_2^1(c) = \int_c^{\tilde{c}} \frac{-1}{a} \Big[\sum_{i=1}^{l+1} F_i'(a) \Big] \Big[2 \sum_{i=1}^{l+1} F_i(a) - 1 \Big] da - \sum_{i=1}^{l} A_i^2(c) - A_i^1(c).$$

Putting the second part of RHS to the LHS one gets:

$$\sum_{i=1}^{l+1} A_i^2(c) - A_i^1(c) = \int_c^{\tilde{c}} \frac{-1}{a} \Big[\sum_{i=1}^{l+1} F_i'(a) \Big] \Big[2 \sum_{i=1}^{l+1} F_i(a) - 1 \Big] da. \tag{37}$$

Recalling

$$b^{LA}(c) - b^S(c) = \sum_{i=1}^{l+1} V_i \Lambda \Big(A_i^2(c) - A_i^1(c) \Big),$$

it only remains to show that RHS of equality 37 is negative for any c in a neighbourhood of $\tilde{c} = 1$. Note that $\sum_{i=1}^{l+1} F_i'(a)$ will be negative since this is change in the probability of winning any prize for players with type a as c increases, where $a \in [c, 1]$ with c being close to 1. $2 \sum_{i=1}^{l+1} F_i(a) - 1$ is also negative since $\sum_{i=1}^{l+1} F_i(a)$ is the probability that a players with ability $a \in [c, 1]$ wins any prize, which is almost zero. Combining these signs, one gets the desired inequality. Thus,

$$b^{LA}(c) - b^S(c) = \sum_{i=1}^{l+1} V_i \Lambda \Big(A_i^2(c) - A_i^1(c) \Big) < 0,$$

for any c in a neighbourhood of 1, as both $V_i > 0$ and $\Lambda > 0$.

Proof of Proposition 4. The equilibrium effort function in the case of convex or concave cost-of-effort is derived following the same steps in the proof of Proposition 2. Assume that all players except i exert effort according to the function b and assume that b is strictly monotonic and differentiable. The maximization problem of the players i is:

$$\begin{aligned} \max_{x} \quad & \Big[p_1\Big(V_1 + \eta p_2(V_1 - V_2) + \eta(1 - p_1 - p_2)V_1 - cx\Big) \\ & + p_2\Big(V_2 - \eta p_1\lambda(V_1 - V_2) + \eta(1 - p_1 - p_2)V_2 - cx\Big). \\ & + (1 - p_1 - p_2)\Big(- \eta p_1\lambda V_1 - \eta p_2\lambda V_2 - c\gamma(x)\Big)\Big]. \end{aligned} \tag{38}$$

where the probabilities of winning the first and the second prize are given by equation (35). Denote the inverse of b by y. Substituting p_1, p_2, $\Lambda = \eta(\lambda - 1)$ and y, the maximization problem becomes:

$$\begin{aligned} \max_{x} \quad & \Big[(1 - \Lambda)(1 - F(y))^{k-1}V_1 + (1 - \Lambda)(k - 1)(1 - F(y))^{k-2}F(y)V_2 \\ & + \Lambda(1 - F(y))^{2k-2}V_1 + (\Lambda)(k - 1)^2(1 - F(y))^{2k-4}F^2(y)V_2 \\ & + 2\Lambda(k - 1)(1 - F(y))^{2k-3}F(y)V_2 - c\gamma(x)\Big]. \end{aligned}$$

Using the strict monotonicity of b and symmetry, the first-order condition (FOC) is given by:

$$\begin{aligned} \gamma^{'}(x) =& V_1\frac{1}{y}\Big(- (1 - \Lambda)(k - 1)(1 - F(y))^{k-2}F'(y)y' \\ & - \Lambda(2k - 2)(1 - F(y))^{2k-3}F'(y)y'\Big) \\ & + V_2\frac{1}{y}\Big(- (1 - \Lambda)(k - 1)(1 - F(y))^{k-3}F'(y)y'(1 - (k - 1))F(y) \\ & + 2\Lambda(k - 1)(1 - F(y))^{2k-5}F'(y)y'\Big((1 - kF(y) - ((k - 1)^2 - 1\Big)F(y)^2\Big) \end{aligned} \tag{39}$$

Assume that $\Lambda > 1$, so that there is a critical type $\tilde{c}$ such that all types with $c \geq \tilde{c}$ puts zero effort, i.e. drops out to avoid negative expected utility as discussed in Proposition 1. Thus the optimal effort of the players with type $\tilde{c}$ is zero yielding the boundary condition $y(0) = \tilde{c}$. Using this boundary condition, the solution to the differential equation in (39) is given by $\gamma(x) = H(y)$, where $H(y)$ is given by equation (24). Thus $x = \gamma^{-1}(H(b^{-1}(x)))$ implying that $b = \gamma^{-1}(H)$. The effort function of each players with type $c < \tilde{c}$ is given by

$$b(c) = \gamma^{-1}\left(A_1(c)V_1 + A_2(c)V_2\right), \tag{40}$$

where $A_1(c)$ and $A_2(c)$ are given by equation (33) and (34). Now assume that $\Lambda \leq 1$. The maximization problems of the players remains the same, while the boundary condition becomes $y(0) = 1$. Thus, the equilibrium effort of a players with type c is given by equation (40), where $\tilde{c} = 1$. It remains to show that the equilibrium effort function $b(c)$ is differentiable and strictly decreasing. The former one is obvious. To show that the effort function is strictly decreasing, consider the derivative of the effort function, $b'(c)$:

$$b'(c) = \gamma^{-1}(A(c)V_1 + B(c)V_2)\,(A'(c)V_1 + B'(c)V_2) < 0,$$

where the last inequality follows from the facts that $\gamma^{-1} > 0$ and equation (32). Thus, $b(c)$ is strictly decreasing. The last thing to show is for any type c, the effort $b(c)$ indeed maximizes the expected utility of that type. Using the same arguments as above, the derivative of the expected utility in equation (38) with respect to x is given by:

$$\begin{aligned}\frac{dEU(x,c)}{dx} &= V_1\frac{db^{-1}(x)}{dx}\Big(- (1-\Lambda)(k-1)(1-F(y))^{k-2}F'(y)y' \\ &- \Lambda(2k-2)(1-F(y))^{2k-3}F'(y)y'\Big) \\ &+ V_2\frac{db^{-1}(x)}{dx}\Big(- (1-\Lambda)(k-1)(1-F(y))^{k-3}F'(y)y'(1-(k-1))F(y) \\ &+ 2\Lambda(k-1)(1-F(y))^{2k-5}F'(y)y'\Big((1-kF(y)-((k-1)^2-1\Big)F(y)^2\Big) - c\gamma(x)\end{aligned}$$

I will show that $\frac{dEU(x,c)}{dx} \geq 0$ for every $x < b(c)$. Let $x < b(c)$ and $\bar{c}$ with $b(\bar{c}) = x$. Since b is strictly decreasing, $\bar{c} > c$. Note that the derivative of $\frac{dEU(x,c)}{dx}$ with respect to c is $-\gamma'(x) < 0$, since *gamma* is strictly increasing. Thus $\frac{dEU}{dx}$ is decreasing in c implying $\frac{dEU(x,c)}{dx} \geq \frac{dEU(x,\bar{c})}{dx} = 0$. Therefore, $\frac{dEU(x,c)}{dx} \geq 0$ for every $x > b(c)$. A similar argument shows that for every $x > b(c)$, $\frac{dEU(x,c)}{dx} \leq 0$.

Proof of Theorem 2. First I will show that for each players with ability c in a neighbourhood of m, $b^{LA}(c) > b^S(c)$, where $b^{LA}(c)$ and $b^S(c)$ are given by:

$$b^{LA}(c) = \gamma^{-1}\Big(\sum_{i=1}^{P} V_i\Big((1-\Lambda)A_i(c)^1 + \Lambda A_i^2(c)\Big)\Big) \qquad \text{and} \qquad b^S(c) = \Big(\sum_{i=1}^{P} V_i A_i^1(c)\Big),$$

where $A_i^1(c)$ and $A_i^2(c)$ are given by

$$A_i^1(c) = \int_c^1 -\frac{1}{a}F_s(a)'da \qquad \text{and} \qquad A_i^2(c) = \int_c^1 -\frac{1}{a}\left(\left(F_s(a)^2\right)' + \sum_{i=1}^{s-1}(2F_i(a)F_s(a))'\right)da.$$

Now, the following claim needs to be shown:

$$\text{Claim:} \quad b^{LA}(m) - b^S(m) = \gamma^{-1}\left(\sum_{i=1}^{P} V_i\Big((1-\Lambda)A_i^1(m) + \Lambda A_i^2(m)\Big)\right) - \gamma^{-1}\left(\sum_{i=1}^{P} V_i A_i^1(m)\right) > 0 \quad .$$

In the proof of Proposition 1 the following is shown:

$$\left(\sum_{i=1}^{P} V_i\Big((1-\Lambda)A_i^1(m) + \Lambda A_i^2(m)\Big)\right) > \left(\sum_{i=1}^{P} V_i A_i^1(m)\right).$$

Since both sides of the above inequality is positive and γ^{-1} is strictly increasing, we have:

$$\gamma^{-1}\left(\sum_{i=1}^{P} V_i\Big((1-\Lambda)A_i^1(m) + \Lambda A_i^2(m)\Big)\right) > \gamma^{-1}\left(\sum_{i=1}^{P} V_i A_i^1(m)\right),$$

proving the claim.

B. Optimal Allocation of Prizes

Proof of Proposition 3. Assume that there are P prizes $V_1 \geq V_2 \geq \cdots \geq V_P \geq 0$ to be awarded and $k > P$ players. Assume that players have linear cost-of-effort functions. A players with type c exerts effort according to:

$$b(c) = \sum_{s=1}^{P} A_s V_s,$$

where where the weights $A_s(c)$ are as in equation (10):

$$A_s = \Bigg\{(1-\Lambda)\int_c^{\tilde{c}} -\frac{1}{a}F_s(a)'da$$
$$+\Lambda \int_c^{\tilde{c}} -\frac{1}{a}\left(\left(F_s(a)^2\right)' + \sum_{i=1}^{s-1}(2F_i(a)F_s(a))'\right)da\Bigg\}.$$

Noting that $V_1 = 1 - \left(\sum_{i=2}^{P} V_i\right)$, one can re-write the bidding function as:

$$\begin{aligned} b(c) &= \sum_{s=1}^{p} V_s A_s(c) \\ &= \left(1 - \sum_{i=1}^{p-1} V_{i+1}\right) A_1(c) + \sum_{i=2}^{p} V_i A_i \\ &= A_1 + \sum_{i=2}^{p} V_i \left(A_i(c) - A_1(c)\right) \end{aligned}$$

Given this, the average effort of each players is given by:

$$\int_m^1 \left(\sum_{i=1}^{P} V_i A_i(c)\right) F'(c)dc = \int_m^1 \left(A_1 + \sum_{i=1}^{P} V_i(A_i(c) - A_1(c))\right) F'(c)dc$$

The revenue of the principal becomes:

$$R(V_2, V_3, ..., V_P) = k \int_m^1 \left(A_1 + \sum_{i=1}^{P} V_i(A_i(c) - A_1(c))\right) F'(c)dc. \tag{41}$$

The principal's problem is choosing the number and the levels of the prizes to maximize his revenue, namely:

$$\max_{0 \leq \{V_i\}_{i=2}^{P} \leq \frac{1}{i}} k \int_m^1 \left\{A_1(c) + \sum_{i=2}^{p} V_i \left(A_i(c) - A_1(c)\right)\right\} F'(c)dc$$

subject to the following $P-1$ conditions:

$$\begin{aligned} 1 - \sum_{i=1}^{p} V_i &\geqslant V_2 \\ V_2 &\geqslant V_3 \\ &\vdots \\ V_{P-1} &\geqslant V_P \end{aligned} \tag{42}$$

Since A_1 does not have a coefficient of type V_i, one can remove the term A_1 from the objective of the maximization problem. Since the summation is finite, one can interchange the integral and the summation signs. Then the maximization problem reads:

$$\max_{0\leq\{V_i\}_{i=2}^{P}\leq\frac{1}{i}} \sum_{i=2}^{P}\left\{V_i\int_m^1 \left(A_i(c)-A_1(c)\right)F'(c)dc\right\}$$

subject to equation (42). It is optimal to award a single first prize if and only if each summand in the maximization problem is zero, that is

$$\int_m^1 \left(A_i(c)-A_1(c)\right)F'(c)dc<0.$$

for each $i\in\{2,\ldots,p\}$. Otherwise, it is optimal to award equal prizes only, due to the linearity of the program. That is the constraints in equation (42) will all bind. To see this, suppose to the contrary that there is an interior solution. Without loss of generality assume that the interior solution is $(\sigma,\varsigma,\tau,0,\ldots,0)\in[0,1]^p$, where $\sigma>\varsigma>\tau$ and $\sigma+\varsigma+\tau=1$. For the sake of easiness denote $G_i := \int_m^{\tilde{c}}\left(A_i(c)-A_1(c)\right)F'(c)dc>0$. Since this allocation is optimal, and σ, ς, τ are all positive it means that G_4 is positive(otherwise it would be optimal to transfer the weight τ to σ and ς). Since $\tau>0$, G_2 should be greater than both G_3 and G_4 (otherwise it would be optimal to transfer the weight τ to σ and ς). But then τ should take the biggest value it could take, which is in this case $\frac{1}{3}$ (otherwise$(\sigma,\varsigma,\tau,0,\ldots,0)$ would not be optimal). Applying the same reasoning to both σ and ς, we conclude that $\sigma=\varsigma=\tau=\frac{1}{3}$. In order to obtain the optimal prize allocation one needs to evaluate the objective function only on the boundary values, namely on the set $\{(1,0,\ldots,0),(\frac{1}{2},\frac{1}{2},0,\ldots,0),\ldots,(\frac{1}{p},\frac{1}{p},\ldots,\frac{1}{p})\}$ and take the allocation which gives the maximum value. It is optimal to award $2\leq r\leq p$ equal prices if and only if

$$r=\arg\max_{j\in 2,\ldots,P}\frac{1}{j}\sum_{i=2}^{j}\left\{\int_m^1\left(A_i(c)-A_1(c)\right)F'(c)dc\right\}.$$

Proof of Proposition 5. Assume that there are P prizes $V_1\geq V_2\geq\cdots\geq V_P\geq 0$ to be awarded and $k>P$ players. Assume that players have linear cost-of-effort functions. A players with type c exerts effort according to:

$$b(c)=\sum_{s=1}^{P}\gamma^{-1}\left(A_sV_s\right),$$

where where the weights $A_s(c)$ are as in equation (10). Similar to the proof of proposition 3, the average effort of each contestant is given by:

$$\int_m^1 \gamma^{-1}\left(\sum_{i=1}^{P} V_i A_i(c)\right) F'(c)dc = \int_m^1 \gamma^{-1}\left(A_1 + \sum_{i=1}^{P} V_i(A_i(c) - A_1(c))\right) F'(c)dc$$

The revenue of the principal becomes:

$$R(V_2, V_3, ..., V_P) = k\int_m^1 \gamma^{-1}\left(A_1 + \sum_{i=1}^{P} V_i(A_i(c) - A_1(c))\right) F'(c)dc. \tag{43}$$

The principal's problem is choosing the number and the levels of the prizes to maximize his revenue, namely:

$$\max_{0\leq\{V_i\}_{i=2}^{P}\leq\frac{1}{i}} k\int_m^1 \gamma^{-1}\left(A_1(c) + \sum_{i=2}^{p} V_i\left(A_i(c) - A_1(c)\right)\right) F'(c)dc$$

subject to the $P-1$ conditions in equations 42. If the condition in (19) is satisfied for some i, then

$$\frac{\partial R}{\partial V_i} = \int_m^{\tilde{c}} \gamma^{-1\prime}\left(A_1 + \sum_{s=1}^{P} V_s(A_s - A_1)\right)(A_s - A_1)F'(c)dc > 0,$$

which means the revenue function cannot be maximized at $V_i = 0$ given that it is increasing in the i-th dimension. Thus $V_i > 0$ should be true. Now it remains to show that the solutions to this maximization problem are at the corners when the cost-of-effort function is concave. Assume that γ is concave, thus γ^{-1} is convex. I will show that $\frac{\partial^2 R}{\partial V_i^2} > 0$, which implies that the revenue function is convex. Thus one can only have corner solutions. First, suppose that there are $P = 2$ prizes to be awarded. In this case,

$$R(V_2) = k\int_m^1 \gamma^{-1}\left(A_1 + V_2\left(A_2 - A_1\right)\right) F'(a)da.$$

The second derivative of the revenue function is:

$$R''(V_2) = k\int_m^1 \gamma^{-1\prime\prime}\left(A_1 + V_2\left(A_2 - A_1\right)\right)\left(A_2 - A_1\right)^2 F'(a)da > 0,$$

since $\gamma^{-1\prime\prime} > 0$ by the concavity if γ. Now it remains to show it for $P > 2$ prizes.

This should be done using the Hessian matrix as follows:

$$\begin{bmatrix} x_2 & x_3 & \dots & x_P \end{bmatrix} \begin{bmatrix} \frac{\delta^2 R}{\delta V_2^2} & \frac{\delta^2 R}{\delta V_2 \delta V_3} & \cdots & \frac{\delta^2 R}{\delta V_2 \delta V_P} & \\ \frac{\delta^2 R}{\delta V_3 \delta V_2} & \frac{\delta^2 R}{\delta V_3^2} & \cdots & \frac{\delta^3 R}{\delta V_2 \delta V_P} & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\delta^2 R}{\delta V_P \delta V_2} & \frac{\delta^2 R}{\delta V_P \delta V_3} & \cdots & \frac{\delta^2 R}{\delta V_P^2} & \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ x_P \end{bmatrix} > 0$$

Making the multiplication and substituting the cross partial derivatives, the above expression reduces to:

$$\left(\sum_{i=2}^{P} x_i (A_i - A_1) \right)^2 > 0.$$

Since this expression is a square of a sum, it is always positive, implying that the revenue function R is convex. Therefore, the solutions to the principal's problem is always boundary solutions.