Combining microsimulation and optimization to identify optimal universalistic tax-transfer rules

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Abstract

As a response to a changing labour market scenario and to the concerns for increasing costs and bad incentives of traditional income support policies, the last decades have witnessed, in many countries, reforms introducing more sophisticated designs of means-testing, eligibility and tagging. In this paper, we consider an alternative direction of reform that points towards universality, unconditionality and simplicity. Our main research question is whether tax-transfer rules designed according to these alternative criteria might be superior to the current one and could therefore be proposed as a policy reform. We adopt a computational approach to the design of optimal tax-transfer rules, within a flexible class. The exercise is applied to France, Germany, Italy, Luxembourg, Spain and the United Kingdom. The results suggest some common features in all the countries. The optimal tax-transfer rules feature a universal unconditional basic income or, equivalently, a negative income tax with a guaranteed minimum income. The tax profiles are much flatter than the current ones. For most social welfare criteria, and most countries, the simulated tax-transfer rules are superior to the current ones. These results confirm that policy reforms inspired by the principle of Universal Basic Income and Flat Tax might have good chances to dominate the current tax-transfer rules.

JEL: H2, H3, C6

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1. Introduction

During the last decades, a variety of social, economic and technological processes has been leading to a new labour market scenario: fewer available jobs, more intermittent careers; more mobility across jobs with different locations, contents and required skills; more heterogeneity of income sources. At the same time, it emerged a concern for increasing costs of traditional income support policies, due to both a higher demand for support and to welfare "traps" or dependence. The new scenario and its implications represented a challenge to the traditional welfare policies, which – as a response – tried to evolve toward a more sophisticated design of means-testing, eligibility and tagging, aimed at sustaining work incentives (e.g., through in-work benefits or tax credits) and at matching specific needs and attitudes of different sociodemographic groups with categorical policies.[1] In spite of some appreciable results on labour participation incentives, these policies have been criticised for introducing new inequalities, distortions and high monitoring costs.[2] The policy debate, however, considers also an alternative direction of reform that aims at over-coming (or complementing) means-testing and categorical policies, pointing towards universality, unconditionality and simplicity.[3] In this paper, we adopt this latter alternative view. We consider a simple though flexible class of TTRs, where net available household income is defined as 4-th degree polynomial in taxable income plus a constant term that is scaled according to the household's size. The rule is universally applied to all the households. Within this class of TTRs, we look for the social-welfare-optimal member according to the criterion proposed by Kolm (1976), analysing the data of six European countries: France, Germany, Italy, Luxembourg, Spain and the United Kingdom. Our research question is whether the social-welfare-optimal members of the considered class are social-welfare superior to the current TTRs. We formulate the problem as an instance of empirical optimal taxation. However, we do not adopt the familiar analytical approach, not in the original version (e.g., Mirrlees 1971) nor in the "sufficient statistics" version (e.g. Saez 2001, 2002). We develop a computational approach that combines microeconometric modelling, microsimulation, numerical optimization and social welfare evaluation. We identify optimal TTRs for different degrees of social inequality aversion and compare them to the current rules. With respect to the literature on empirical optimal taxation, this paper provides three main contributions that are relevant both from the methodological and the policy point-of-view.

First, it considers a flexible – even though parametric – class of TTRs. This is important both for guaranteeing the generality of our results and for allowing the analysis of issues that are crucial in tax-transfer reforms, such as

[1] Analyses of these reforms are provided by, among others, Eissa and Liebman (1996), Shoemi and Blabk (2000), Blank (2002), Moffit (2003), Francesconi et al. (2009).

[2] See for example Bryan (2005), Handler and Babcock (2006), Standing (2011).

[3] See for example: Atkinson (1996), Colombino (2019), Colombino and Islam (2018), Colombino and Narazani (2013), Gentilini et al. (2020), Ghatak and Jaravel (2020), Ghatak and Maniquet (2019), Grimalda et al. (2020), Islam and Colombino (2018), Magnani and Piccoli (2020), Moene and Raj (2016), Standing (2012), Van Parijs and Vanderborght (2017).

the size of a guaranteed minimum income, the degree of progressivity and the trade-off between efficiency and equality. The heterogeneity accounted for in the data and in the microeconometric model in principle might allow us to consider TTRs based on some categorical articulation of tax rates and subsidies, which might be welfare-superior to our optimal polynomial TTRs. However, categorical and means-tested designs of the TTR bear administrative and political costs that instead are smaller or even non-existent in simple and universalistic designs. In view of policy reforms, our research motivation consists precisely of testing the performance of a very simple, transparent and universalistic TTR against the current (typically categorical and means-tested) TTR.

Second, most of the quantitative analysis of tax-transfer reforms in the last three decades are dedicated to mean-tested and categorical policies, which are also the prevailing policies in the considered countries. However, more recently, the policy debate has witnessed an increasing interest in unconditional and universal policies.[4] Our analysis permits an evaluation of the latter approach using a polynomial tax-transfer rules as compared to the prevailing current one.

Third, we identify some significant effects of "primitives" (i.e., basic characteristic of the economy) on the features of the optimal polynomial Tax Transfer Rules (TTRs). Despite the common features, the results show also large differences between the optimal polynomial TTRs in the different countries. They depend indeed on various characteristics of the population and the economic environment. An explanation of these differences requires to identify a general relationship between the basic ("primitive") characteristics of the economy and the features of the optimal TTRs. Actually, this is the direct result of analytical optimal taxation. We can come close to a similar result by identifying a "mapping" from the set of country-specific "primitives" to the set of country-specific optimal TTRs.

For most countries and most values of the inequality aversion parameter, the optimal polynomial rules provide a higher social welfare than the current ones. The results suggest some common features in all the countries. The optimal polynomial TTRs envisage a universal unconditional basic income with a (almost) flat tax or, depending on the interpretations, a (almost) flat negative income tax with guaranteed minimum. The profiles of the MTRs are pretty similar in different countries and definitely flatter than under the current TTRs. Section 2 presents the computational approach. Section 3 describes the microeconometric model and the procedure adopted to identify the optimal country-specific TTRs and a general rule for mapping from the "primitives" to some features of the

[4] Besides the references of footnote 3, there is abundant evidence upon experiments in developing countries focussing on universal basic income and many pilots or projects also in developed countries: e.g. https://www.visualcapitalist.com/map-basic-income-experiments-world/.

optimal TTRs (Appendix A provides a more detailed description of the econometric model and reports the estimates for the couples and the singles in the six countries and the simulated elasticities of labour supply). Section 4 illustrates the results and Section 5 concludes.

2. The computational approach to empirical optimal taxation

In the basic framework of optimal taxation theory, the Government chooses the taxes to be applied to households' personal incomes with the aim of maximizing some social welfare criterion that accounts for both total welfare and its distribution among the households. While doing so, the Government takes into account a public budget constraint - i.e., the taxes must collect a given amount (to be used in public expenditures) – and an incentive constraint, i.e. household incomes and paid taxes are determined by household utility maximizing (labour supply) choices subject to budget constraints that are also defined by taxes. Using appropriate analytical optimization techniques one can obtain, under convenient assumptions, closed-form expressions for the optimal tax rate for a given level of taxable income. The analytical optimal taxation, pioneered by Mirrlees (1971), is a fundamental contribution since it sets the basic problem to be solved. Its empirical applications (e.g. Mirrlees 1971, Tuomala 1990, 2010, Saez 2001) can also indicate promising directions of policy reform. However, the relevance of those results and policy prescriptions critically depends on the flexibility and generality of the assumptions and on the ability to account for the heterogeneity and realistic features of the economies. In view of the above perspective, we will propose and apply a computational approach. A combination of micro-econometric modelling, behavioural microsimulation and numerical optimization provides an alternative or a complement with respect to the analytical approach.[5] Modern micro-econometric models of labour supply can be specified according to general and flexible assumptions. They can account for many realistic features such as heterogeneous preferences and opportunity sets, simultaneous decisions of couples, complicated budget constraints, multidimensional heterogeneity of both households and jobs, quantitative constraints, etc. It might not be feasible or practical to obtain analytical solutions for the optimal taxation problem in such economic environments. Yet those features are likely to be relevant and important for evaluating or designing reforms. The ability to adopt more general assumptions and to account for realistic features of the population and of the opportunities might provide more robust policy prescriptions The implementation of the computational approach is articulated in four steps.

[5] The background of the computational approach is exemplified by a series of papers: Islam and Colombino (2018) identify optimal TTRs within the Negative Income Tax matched with a Flat Tax (NIT+FT) class in eight European countries. Aaberge and Colombino (2006, 2013) identify optimal taxes for Norway within the class of 9-parameter piece-wise linear TTRs. Aaberge and Colombino (2012) perform a similar exercise for Italy. Blundell and Shephard (2012) design an optimal TTR for lone mother in the UK. Closely related contributions are Fortin et al. (1993), Sefton and Van de Ven (2009), Creedy and Hérault (2012), Ericson and Flood (2012) and Colombino (2015).

First, we estimate a microeconometric model of household labour supply. The model accounts for both singles and couples, wage employed, self-employed and non-participants, extensive and intensive labour supply responses, multidimensional sources of welfare, heterogeneous preferences and quantity constraints.

Second, for any member of a given class of TTRs, we can simulate the new household choices based on the estimated household preferences and compute the attained value of households' utility.

Third, we apply a maximization algorithm and iterate step two in order to identify the TTR that maximizes a Social Welfare function. The Social Welfare function takes as arguments (an appropriate transformation of) the previously computed household utility level.[6]

Fourth, obtaining a general rule. The first three steps explained above will identify a specific optimal polynomial TTR for each country. Given the country-specific optimal TTRs and a set of country-specific "primitives" (i.e. basic characteristics of the economy) we can identify the mapping from the "primitives" to the optimal TTRs, i.e. a general rule analogous to the one identified by the analytical approach. As a matter of fact, the path of the computational approach is opposite to path of the analytical approach. The latter solves for a general rule and then can obtain country-specific rules by assigning country-specific values to the "primitives". The former identifies country-specific rules and from those rules a general rule can be inferred. The general rule can be used for many purposes, e.g. providing indications for tax reforms in countries were reliable micro data are not available; making "out-of-sample" predictions in order to test the whole optimal taxation procedure; forecasting the need for fiscal reforms based on predictions about trends or future changes of the "primitives".[7]

[6] Of course one might adopt different evaluation criteria, such as the effects on employment, poverty etc. We adopt a social welfare criterion but we will also report and discuss results on other indices that might be policy-relevant.
[7] In principle, the analytical approach is more general regarding the representation of the optimal TTR, which is non-parametric, while our approach adopts a parametric specification. However, the non-parametric representation comes at the price of restrictive assumptions upon other elements of the problem. Moreover, the non-parametric solutions produced by the analytical approach typically produce shapes of the optimal TTR that can be easily approximated by parametric polynomial functions (e.g. Heathcote and Tsujiyama 2015).

3. Model, Evaluation criterion and Optimization

The microeconometric model. A detailed description of the microeconometric model is given in Appendix A. In this Section we present the main elements. The household chooses one alternative from an opportunity set contains jobs that belong to different types indexed by *j*. Type are defined by hours of work (**h**) and sector of employment **s** (wage employment or self-employment). Non-market activities ("leisure") are "jobs" that will be indexed as $j = 0$. There are M types, including j = 0. The utility level attained by household *i* when holding a job of type *j* given wage rate w_i and TTR τ (a vector of parameters that define the tax-transfer rule) is written as follows,

$$U_i(j; w_i, \tau, \varepsilon_{ij}) = V_i(j; w_i, \tau) + \varepsilon_{ij} \tag{1}$$

where $V_i(j; w_i, \tau)$ is the "systematic" part (a parametric function of observed variables: income, hours of work and socio-economic characteristics of the household) and ε_{ij} is a random variable that accounts for unobserved features of the match (*i,j*). w_i is a scalar for singles and a vector for couples. Aaberge and Colombino (2013) presents a "Random Utility Random Opportunity (RURO)" approach where the probability that household i holds a job is -

$$P_i(j; w_i, \tau) = \frac{\exp\{V_i(j; w_i, \tau) + \delta_i D_i\}}{\sum_{x=0}^{M} \exp\{V_i(x; w_i, \tau) + \delta_x D_x\}} \tag{2}$$

where the D are alternative-specific dummies (see Appendix A for more details).

The estimates for couples (32 parameters), singles females (17 parameters) and single males (17 parameters) are available on request. Labour supply elasticities in all the five countries are reported in Tables A1 of Appendix A.

The class of polynomial TTRs. We look for optimal TTRs within the class of rules defined as a polynomial functions of total household taxable income $y_i = \mathbf{w}_i^{'}\mathbf{h}_i + I_i - \text{SSC}_i$ where SSC_i denotes social security contributions. Net available income C_i is specified as follows:

$$C_i = \tau_0\sqrt{N_i} + \tau_1 y_i + \tau_2 y_i^2 + \tau_3 y_i^3 + \tau_4 y_i^4 \qquad (3)$$

where y_i (= total taxable household income) and N_i = household size. The choice of this simple specification is due to three main motivations. First, since we compare six different countries, our results are made more easily interpretable by abstracting from details and keeping the optimal TTRs as simple as possible. Second, even though the 4[th] degree polynomial specification is a parametric, it is flexible enough to be judged close to a non-parametric rule. Third, our specific research interest is investigating whether a very simple and universalistic TTR can outperform the (typically categorical and complex) current TTRs.

The corresponding TTR is:

$$T(y_i;\boldsymbol{\tau}) = y_i - \tau_0\sqrt{N} - \tau_1 y_i - \tau_2 y_i^2 - \tau_3 y_i^3 - \tau_4 y_i^4 \qquad (4)$$

The marginal tax rate and the average tax rate are respectively:

$$MT(y;\boldsymbol{\tau}) = \frac{\partial T(y;\boldsymbol{\tau})}{\partial y} = 1 - \tau_1 - 2\tau_2 y - 3\tau_3 y^2 - 4\tau_4 y^3 \qquad (5)$$

$$AT(y;\boldsymbol{\tau}) = \frac{T(y;\boldsymbol{\tau})}{y} = 1 - \frac{\tau_0\sqrt{N}}{y} - \tau_1 - \tau_2 y - \tau_3 y^2 - \tau_4 y^3$$

The rule is sufficiently flexible to represent many alternative versions of TTRs. Provided $\tau_0 > 0$, the rule can be interpreted as a negative income tax or a basic income matched with a generic tax rule.[8] In the former case $\tau_0\sqrt{N_i}$

[8] In all the optimal TTRs we obtain $\tau_0 > 0$. The equivalence of a universal basic income and a universal negative income tax with guaranteed minimum income can be easily seen in the flat tax case, although it carries over to non-flat taxes. Consider first the universal basic income. The household receives $\tau_0\sqrt{N_i}$ and pays taxes on its own taxable income y_i according to a constant marginal tax rate $1-\tau_1$, so that its net disposable income will be $C_i = \tau_0\sqrt{N_i} + \tau_1 y_i$. With the negative income tax, it is useful to define the exemption level $E_i = \frac{\tau_0\sqrt{N_i}}{1-\tau_1}$. If taxable income y_i is smaller than E_i the household receives a transfer $(E_i - y_i)(1-\tau_1)$ so that its net disposable income will be $C_i = y_i + (E_i - y_i)(1-\tau_1) = y_i + \left(\frac{\tau_0\sqrt{N_i}}{1-\tau_1} - y_i\right)(1-\tau_1) = \tau_0\sqrt{N_i} + \tau_1 y_i$. If taxable income is larger than E_i the household only pays a tax $(y_i - E_i)(1-\tau_1)$ so that its net available income will be

is the universal guaranteed minimum income when $y_i = 0$, in the latter case it is a universal basic income. The case $C_i = \tau_0\sqrt{N_i} + \tau_1 y_i$ corresponds to a NIT+FT, i.e. the TTR class considered in Islam and Colombino (2018). The term $\sqrt{N_i}$ rescales the guaranteed minimum income or the basic income according to the household size (square root rule). A pure flat tax rule is the special case $C_i = \tau_1 y_i$. Also rules with negative marginal taxes (such as In-work Benefits or Tax Credits) are accounted for, depending on the values of the paramers τ.

When identifying the optimal TTR, the rule of expression (4) completely replaces the current TTR. A correct interpretation of the comparison of the optimal TTR to the current one must take into account important differences between the two. First, the rule of expression (4) applies to the sum of all household personal taxable incomes, whatever the source. The current rules might use different rules depending on the source and might apply to individual or household incomes. Second, the current income support mechanisms are typically a combination of (mostly) means-tested and categorical transfers. The rule of expression (4), instead, envisages a universal mechanism that can be interpreted as a guaranteed minimum income or as a basic income, provided $\tau_0 > 0$. In the evaluation of the relative performance of the optimal polynomial TTRs as compared to the current TTRs, we can only conclude that a certain TTR is better or worse (according to a given criterion) to another one. We cannot identify the specific contribution of, say, income support mechanisms, or the treatment of different income sources, to the relative performance of optimal TTRs as compared to current TTRs. However, in order to get a summary comparison of the current TTR and the optimized TTRs, we compute an approximation to the current TTR, which will be used in commenting the results. The approximation is the 4th degree polynomial that satisfies the public budget constraint and minimizes the sum of squared differences between the household observed disposable income and the household disposable income computed according to expression (3).

Welfare evaluation. We define the Comparable Money-metric Utility (CMU). This concept is based on the approach proposed by King (1983), where different preferences are due to different characteristics within a common parametric utility function. The characteristics account for a different productivity in obtaining utility from the opportunities available in the budget set. The utility levels attained by households with different preferences are made comparable by using a common "reference" household. The CMU of a given household *i*

$C_i = y_i - (y_i - E_i)(1-\tau_1) = y_i - \left(y_i - \frac{\tau_0\sqrt{N_i}}{1-\tau_1} \right)(1-\tau_1) = \tau_0\sqrt{N_i} + \tau_1 y_i$. Despite the budget-wise equivalence, when it comes to policy implementation, universal basic income and universal negative income tax might differ in their implications for their administrative costs, the timing of transfers and the perception by the households. However, these aspects are beyond the scope of our analysis.

is the level of income that the "reference" household would need to attaine the same utility level attaine by household i. The procedure is analogous to using a reference price vector in order to compare utility levels attained under different price vectors. Empirical examples of this approach are provided by King (1983), Aaberge et al. (2004) and Islam and Colombino (2018). Our CMU transforms the household utility level into an inter-household comparable monetary measure that will enter as argument of the Social Welfare function. First, we calculate the expected maximum utility attained by household i under tax-transfer regime τ (McFadden 1978): $\ln\left(\sum_j \exp\{V_i(j;w_i,\tau)\}\right)$. Analogously, we define $\ln\left(\sum_j \exp\{V_R(j;w_R,\tau_R)\}\right)$ as the expected maximum utility attained by the "reference" household R under the "reference" tax-transfer regime $\boldsymbol{\tau}_R$. The reference household is the couple household at the median value of the distribution of the expected maximum utility. The reference TTR $\boldsymbol{\tau}_R$ is a pure flat tax that satisfy the public budget constraint. The CMU of household i under tax regime $\boldsymbol{\tau}$, $\mu_i(\boldsymbol{\tau})$, is defined as the gross income that a reference household under a reference tax-transfer regime $\boldsymbol{\tau}_R$ would need in order to attain an expected maximum utility attained undert tax-transfer rule to $\boldsymbol{\tau}$. Although the choice of the reference household is essentially arbitrary, some choices make more sense than others. Our choice of the median household as the reference household can be justified in terms of representativeness or centrality of its preferences. Aaberge and Colombino (2006, 2013) adopt a related, although not identical, procedure that consists of using a common utility function as argument of the social welfare function (Deaton and Muelbauer 1980).

In order to aggregate the household-specific welfare levels, we choose the Social Welfare index proposed by Kolm (1976), which can be defined as:

$$W = \bar{\mu} - \frac{1}{k}\ln\left[\sum_{\mathrm{i}} \frac{\exp\{-k(\mu_{\mathrm{i}} - \bar{\mu})\}}{\mathrm{N}}\right] \tag{6}$$

W has limit $\bar{\mu}$ as $k \to 0$ and $\min\{\mu_1,...,\mu_N\}$ as $k \to \infty$.

$\bar{\mu} = \frac{1}{N}\sum_i \mu_i$ is an index of efficiency

$$\frac{1}{k}\ln\left[\sum_{\mathrm{i}} \frac{\exp\{-k(\mu_{\mathrm{i}} - \bar{\mu})\}}{\mathrm{N}}\right] = \text{Kolm's Inequality Index} \tag{7}$$

$k =$ inequality aversion parameter[9]

μ_i = comparable money-metric utility of household i (defined in Section 3.3).

Therefore, Social Welfare = Efficiency – Inequality. Social Welfare and its components are monetary measures. The Inequality Index can be interpreted as the cost of inequality.[10]

Identification of optimal TTRs. The problem to be solved can be written as follows:

$$\begin{aligned} &\max_{\boldsymbol{\tau}} W\left(\sum_i \mu_i(\boldsymbol{\tau})\right) \\ &\text{s.t.} \\ &\sum_i \sum_j P_{ij}(\boldsymbol{\tau}) T_{ij}(\boldsymbol{\tau}) \geq R \end{aligned} \tag{8}$$

where $P_{ij}(\boldsymbol{\tau})$ is the probability that household i chooses alternative j under TTR $\boldsymbol{\tau}$ (according to expression (2) and $T_{ij}(\boldsymbol{\tau})$ is the net tax paid by household i when choosing alternative under TTR $\boldsymbol{\tau}$. The constraint requires that the total expected net tax revenue be greater than (or equal to) a given amount R. Note that problem (8) assumes that the households are maximizing their utility functions, since the arguments of W are the (comparable money-metric) maximized utilities. The problem is solved with a numerical procedure. Given a vector of parameters $\boldsymbol{\tau}$, the microeconometric model simulates $\mu_i(\boldsymbol{\tau})$, $P_{ij}(\boldsymbol{\tau})$ and $T_{ij}(\boldsymbol{\tau})$ for I = 1,…,H and j = 1,…,M. An optimization algorithm iterates the above simulation updating the value of $\boldsymbol{\tau}$ until W cannot be further improved.[11]

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[9] In this paper we identify optimal TTRs for six value of k: 0.0, 0.05, 0.075, 0.10, 0.125, and 0.15. It can be shown (Islam and Colombino 2018) that the correspong values of the popular Atkinson's parameter of inequality aversion are 0.000, 0.114, 0.180, 0.252, 0.333 and 0.424.

[10] Kolm's Inequality Index is an absolute index, meaning that it is invariant with respect to translations (i.e. adding a constant to every μi). Absolute indices are less popular than relative indices (e.g. Gini's or Atkinson's), although there is no strict logical or economic motivation for preferring one to the other. Atkinson and Brandolini (2010) provide a discussion of relative indexes, absolute indexes and intermediate cases. Depending on the specific applcation, there might be a motivation of computational convenience for choosing one or another type of index. Blundell and Shephard (2012) adopt a social welfare index which turns out to be very close to Kolm's index. Their main motivation for their index seems to be computational, since it handles negative numbers (random utility levels). Our motivation is analogous.

[11] In orde to locate a global maximum, we partition the parameter space and try different starting values $\boldsymbol{\tau}^0$.

From the "primitives" to the optimal polynomial TTRs. The analytical optimal taxation identifies general TTR as a function of generic exogenous parameters $\boldsymbol{\pi}$ called "primitives", i.e. fundamental exogenous characteristics of the economy: $TTR = f(\boldsymbol{\pi})$. In order to specify the optimal TTR for a specific country c , the analytical approach imputes to the primitives the values π_c in order to get $TTR_c = f(\boldsymbol{\pi}_c)$. With the computational approach, we can follow the inverse path. First, we identify $\boldsymbol{\tau}_c = \varphi(z_c)$, c = 1, 2, …, T, for T countries. where typically the optimal $\boldsymbol{\tau}_c$ depends on characteristics z_c and φ is a computational procedure instead of a closed form function *f*. As a further step we can retrieve a mapping $(\boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \ldots, \boldsymbol{\pi}_T) \rightarrow (\boldsymbol{\tau}_1, \boldsymbol{\tau}_2, \ldots, \boldsymbol{\tau}_T)$. We will present a simple example that uses regression analysis. We consider the following "primitives".

1) **Kolm's *k***. The inequality aversion parameter *k*, multiplied by 100. As a matter of fact, we have six different values of *k* for each one of the six countries, which makes 36 observations.
2) **Productivity**. The current average monthly taxable household income, as a measure of productivity.
3) **Extensive Elasticity**. The current own wage elasticity of labour supply (Table 7).
4) **Intensive Elasticity**.
5) **Budget.** The current monthly net tax revenue to be attained in order to satisfy the public budget constraint.[12]

We characterize the optimal TTRs with:

1) τ_0. It is the universal basic income
2) 100(1-τ_1). This is the percentage Leading Tax Rate. The definition is motivated by the fact that the other tax parameters τ_2, τ_3 and τ_4 are very small and have a sensible effect only on high levels of taxable income.
3) Local progressivity (MTR/ATR) at taxable income = 10000.
4) Local progressivity (MTR/ATR) at taxable income = 100000.

These are by far the most important features of the polynomial optimal TTRs. Moreover, notice that the higher τ_0 and the lower 1-τ_1, the larger is the range of taxable income with negative net taxes, therefore the ratio τ_0 /(1-τ_1) can be interpreted as an index of global progressivity. We estimate the regressions of the two characteristics of the optimal TTRs against the four "primitives". The results are shown in Table 1 and commented in the next Section 4. The identification of the mapping ("primitives") → (features of the optimal TTR) an be useful in many different circumstances. For example, it can be used to suggest direction for reforms in countries where

[12] It might be argued that "primitives" 2 – 5, are not really primitives, since they are also determined by the current TTRs. This is true, but it is not really relevant. We interpret our analysis as conditional upon the current economy.

microdata are absent or not sufficiently rich, or to forecast the future need for reforms in view of expectations upon future changes in some of the “primitives”.

4. Results

The results of our exercise are documented by Table 1-2 and by Graphs 1a – 6b and 7 – 14. As a general synthesis: with the exception of Luxembourg, the optimal polynomial TTRs are welfare-superior to the current TTR for most values of Kolm'*k*; optimal basic income is larger than the current expenditure on income maintenance, however it can also be smaller in some cases; optimal MTRs are almost constant for a large range of values of taxable income; the optimal TTRs lead to lower poverty; the distribution of welfare gains or losses across demographic groups is very heterogeneous across the different countries. Hereafter we provide more detailed comments and illustrations.

The optimal TTRs. Table 1 reports the parameters of the polynomial optimal TTRs, the polynomial approximation to the current TTRs and the welfare gain of the optimal TTRs with respect to the (real) current TTR. In all the countries, the shape of the optimal TTRs is dominated by τ_1, while the other parameters are very small and might except some influence only at very large taxable incomes (e.g. above 2000 euro a year). As a consequence, the optimal TTRs are very close to a flat tax, with (almost) constant MTR equal to 1-τ_1. In contrast, the polynomial approximation to the current TTRs features important non-linearities, as it is revealed in particular by the values of parameters τ_1 and as compared to the corresponding parameters of their polynomial optimal TTRs. Parameter τ_0 is the monthly universal basic income (or guaranteed minimum income according to the NIT interpretation) for a one-person household. For a N-person household it must be multiplied by $N^{1/2}$. We also report the value of τ_0 in the polynomial approximation of the current TTRs. It is not strictly comparable to the optimized values of τ_0, since the last ones are attained by universal and unconditional transfers to be received with certainty, while the former is an expected value across the population of various - mostly means-tested, contingent and categorical - transfers. It makes sense, however, to interpret τ_0 as a measure of the basic current expenditure in income support policies from view-point of the public budget constraint. In this perspective, one can judge that the current policies are more or less cost-effective than those dictated by the optimized rules. In France and Luxembourg, the current income support policies appear to be "too costly": a less expensive universal and unconditional basic income would attain a higher Social Welfare (for $k < 0.125$ in France and for $k < 0.05$ in Luxembourg). The opposite holds in Germany and Italy (for $k > 0.05$), Spain (for all considered value of k) and the United Kingdom (for $k \geq 0.05$). The main features of the optimal polynomial TTRs are also illustrated in the Graphs 1a – 6b. The effects of the other parameters $\tau_1,\ldots,\tau_4$ on the shape of the TTRs are illustrated by the Graphs 1a – 6a and 1b – 6b, which respectively represent MTR and ATR as functions the household total taxable income. It appears that in all the countries, the optimal MTR is almost constant over a large range of values of taxable income. This happens because, as we have seen in Table 1, the parameters $\tau_2,\ldots,\tau_4$ are very small, so that the MTR, except for very large taxable incomes, reduces to $1-\tau_1$. The conclusion on this point is that, within the (flexible) TTR class considered, a certain degree of progressivity is more efficiently attained by an unconditional and universal Basic Income with non-distortive MTRs rather than by (increasing and distortive MTRs. We also represent the MTR of the polynomial approximation to the current TTR, which show striking differences both between the countries and with respect to the optimal polynomial TTRs. The optimal polynomial TTRs essentially feature a universal basic income with a (almost) flat tax or a negative income tax with a transfer reduction rate equal to the (almost) flat tax above the exemption level. It must be stressed that the "current" MTRs represented in the Graphs in general do not correspond to the formal current MTRs. They represent the effective marginal change in paid taxes for a household with average size. The current systems in France and Luxembourg appear to envisage relatively generous income support policies at low or zero income followed by very high implicit marginal benefit reduction rates. The optimal rules suggest less expensive (although universal and unconditional) income support and a longer and smoother phase-out. Germany envisages an expensive current income support policies and yet a slowly increasing MTR on low incomes. In Italy and Spain, the current MTRs are first steeply increasing up to taxable incomes around 100000 and then decreasing. The recent decades have witnessed a big interest in policies like In-work benefits or Tax Credits, which essentially imply negative effective MTRs on some range of (typically low) incomes. As a matter of fact, In-Work Benefits and Tax Credits are two pillars of the evolution of traditional income support policies to face the new scenario mentioned in the Introduction. Our results do not confirm a better performance of this type of policies: negative MTRs on low taxable incomes never emerge as elements of the optimal polynomial TTRs (although there are negative average net taxes on low incomes. However, it must be remembered that these policies have received a favourable evaluation mostly due to their effects on labour supply incentives, while our evaluation criterion is a Social Welfare index. Recent contributions adopting the analytical approach (e.g. Saez 2002) have argued that a high participation elasticity of labour supply tends to favour policies like In-Work Benefits or Tax-Credits. In our results – as we will see later on when commenting the "mapping" results - a high participation elasticity favours instead a large Basic Income. The implication seems to be that unconditional and universal basic income reduces distortions more effectively than lower initial tax rates in low incomes.

Given that the optimal MTRs are very close to a constant, the ATRs (Graphs 1b – 6b) are useful to show the level and type of progressivity implied by the various TTRs in the different countries.[13] If the Social Welfare criterion ignores inequality effects (i.e. $k = 0.00$), in all the countries the optimal TTR – as compared to the current TTR – is more progressive on low levels of taxable income and more progressive on middle or high taxable incomes.

[13] A simple index of progressivity is MTR/ATR.

The opposite happens with k=0.15. This holds in general, although in Luxembourg and Germany the ATRs are very close for different values of k. , where the ATR behaves approximately in the same way whatever the value of k. For k = 0.075, the optimal ATR is closer to the Current one, but less progressive on middle and high incomes in France and Italy. Judging from the point-of view of the optimal TTRs, the main flaws of the current TTRs – and the corresponding implicit reform recommendations – can be summarized as follows. France: current income support too expensive, current MTR two high on low and high income. Germany: current MTR too low on low incomes and too high on high incomes. Italy: current income support not cost-effective, current MTR too low on low and high incomes and too high on middle incomes. Luxemburg: nothing to complain about, except maybe current income support not cost-effective if k = 0.00. Spain: current income support too low, current MTR on low incomes too low. The United Kingdom: current MTR too low on low incomes and too high on middle incomes.

Welfare effects. The Social Welfare gains (SWG) and the Equality Gains (EG) due to the optimal polynomial TTRs by country and Kolm's k are reported in Table 1, in the last two rows of each country-specific section.[14] Note that the current Social Welfare values are those attained the actual current TTRs, not by the polynomial approximations to them. For most countries and most values of k, the optimal polynomial TTR is social welfare superior to the current TTR. This result holds in France and Italy for k < 0.15, in Luxembourg for k < 0.05, in Germany and Spain for $k \geq$ 0.075 and in the United Kingdom for $k \geq$ 0.05. What happens is that the polynomial optimal TTRs are mainly disequalizing in France, Italy and Luxembourg but equalizing (for a majority of k values) in Spain and in the United Kingdom. As consequence, higher values of k – i.e. higher costs of inequality – tend overcome the efficiency effect in the former group of countries and strengthen it in the latter one. These results are also due to the efficiency effect (i.e. SWG – EG), which decreases with k in the first group of countries while it increases in the second one.

Besides the overall Social Welfare effects, we can identify the winners and the losers of a new TTR. For different demographic groups (couples, single male and single females) we have computed the CMU (Section 3) under the current TTR and under the optimal TTRs for k = .075.[15] Graphs 7 – 12 show the average CMU gains in different demographic groups, by decile (1-3, 4-7, 8-10) of current CMU[16]. The graphs show an extreme heterogeneity across countries, demographic groups and deciles. Depending on the country, some groups and/or some deciles are penalized by the optimal polynomial TTRs. System like NIT+FT are typically expected to penalize middle

[14] The Equality Gain is computed as -Inequality Gain, where Inequality is defined by expression (7). Note that the Efficiency Gain can also be retrieved as SWG – EG.
[15] The shape of results is similar for different values of k.
[16] We prefer CMU instead of the ordinal criterion based only on the proportion of winners, since CMU also accounts for the size of the gains or losses.

deciles. In our results this is indeed the case except for France and Germany. There are however important differences among countries and demographic groups. In view of implementing the optimal rules, most of these problems might probably be moderated by a country-specific design of the equivalence scale applied to the basic income and/or to other parameters of the Optimal TTRs.

Economic effects. Graphs 13 and 14 represent the % change in disposable income and the Poverty Gap Index respectively, by country and Kolm's *k*. The two graphs illustrate a manifestation of the efficiency-equality trade-off. Disposable income increases as long as $k \leq 0.05$, with the exception of Germany. With $k > 0.05$ it keeps increasing in France and Luxembourg, while it decreases in Germany, Italy and the United Kingdom. The Poverty Gap Index in Graph 14 shows a similar pattern. The aggregate effects on labour supply (not reported) incomes are small, although we might observe more significant changes for different demographic groups or income levels (not reported in this paper).

The "primitives" and the optimal TTRs. Table 1 shows the results of inferring a general rule that links the "primitives" to the optimal TTRs, i.e. it presents the results of the analysis explained in Section 3. We have well-defined results on Basic Income (τ_0): all the coefficients are significant at standard levels with the expected sign. Kolm's *k*, Productivity and Elasticity (both extensive and intensive) elasticities favour a higher Basic Income. A stricter Budget require a lower Basic Income. Among the above results, the surprising one is the effect of elasticities. A possible explanation is that Basic Income, as compared to means-tested policies does not suffer from poverty-traps, therefore its relative advantage is greater the more elastic is household behaviour. Kolm's *k* and Extensive elasticity respectively favour a lower and a higher Leading Tax rate Productivity: the former result, taken together with k's effect on Basic Income, seems to mean that more egalitarian social preferences favour a higher Basic Income rather than higher taxes; the latter result might mean that less distortions are better achieved with a universal basic income than with lower taxes. The regressions on local indices of progressivity are less informative. Kolm'*k* asks for more progressivity (measured as MTR/ATR) both at low and high levels of taxable income. There is a positive effect of (intensive) Elasticity upon Local progressivity at taxable income = 10000. At this value, we are just out the negative tax area, so it probably makes sense that a higher Elasticity (which favours a higher Basic Income) starts asking for more progressivity in order to raise tax revenue. Just as an illustrative example, let's us imagine that we want to propose a common TTR to all the countries, based on the averages of the "primitives". Let us also suppose that social preferences are such that $k = 0.075$. Then the basic income and the leading tax rate of the common polynomial optimal TTR would be 456 monthly euros (for one-person household) and 29.8% respectively. It is close to the optimal TTR in Germany for $k = 0.05$.

5. Concluding remarks

As a response to a new labour market scenario emerged during the last decades, featuring a larger heterogeneity in household behaviour, preferences and needs and implying adverse labour incentives and raising costs for the welfare system, many countries have introduced reforms with more sophisticated design of mean-tested and categorical tax-transfer polices. In this exercise we have followed a different policy direction, inspired by simplicity and universality. We have adopted the perspective of optimal taxation. Two main approaches to empirical optimal income taxation have been used so far in the literature: the analytical and the computational approach. In this paper we develop a version of the computational approach that combines microeconometric modelling, microsimulation, numerical optimization and social welfare evaluation in a consistent way. We consider the class of 4th polynomial TTRs, i.e. a generic rule that represents total household disposable income as a 4th degree polynomial function plus a constant. We adopt the Kolm's social welfare function. A specific TTR is defined by the parameter vector containing the four coefficients of the polynomial and the constant. We identify optimal TTRs for different degrees of social inequality aversion and compare them to the current rules in six European countries. For most countries and most values of the inequality aversion parameter, the optimal polynomial rules provide a higher social welfare than the current ones.

The class of TTRs considered as candidates for welfare optimality, although flexible, is extremely simple. It is applied to total taxable household income, irrespective of the source of income. It does not depend household's socio-economic characteristics, with the exception on the number of household member that affects the basic income transfer. It is of course quite possible that we might do better by taking households' heterogeneity into account when designing the optimal TTR. However, finely categorized and means-tested TTR bear administration and political costs (complex forms to file out, monitoring, political manipulation, lack of transparency, conflict resolution etc.) that are certainly less important or even not existent in simple and universalistic TTRs. Our research goal consists of investigating whether a very simple and universalistic design is able to outperform, social welfare-wise, the current system: in most countries it does.

The results suggest some common features in all the countries. The optimal polynomial TTRs envisage a universal unconditional basic income with a (almost) flat tax or, depending on the interpretations, a (almost) flat negative income tax with guaranteed minimum. The profiles of the MTRs are pretty similar in different countries and definitely flatter than under the current TTRs. This result holds for all the considered countries, despite the fact that the polynomial TTR class is very flexible and the heterogeneous responses allowed by the microeconometric model might induce very different shapes of the optimal TTRs. These results confirm those of Islam and Colombino 2018). This is remarkable, since Islam and Colombino (2018) only compare the

NIT+FT rule to the current TTR, while the polynomial class considered in this paper is very flexible. The social welfare gains due to the optimal polynomial TTRs are admittedly small. However, there are other benefits – not accounted for in this exercise – that presumably can be provided by that type of TTR, such as simplicity, lower administration costs and reduced opportunities for corruption and political manipulation.

Despite the above common features, we can see large differences between the levels of the basic income and the values of the MTRs under the optimal polynomial TTRs in the different countries. They depend indeed on various characteristics of the population and the economic environment. The differences of optimal TTRs in different countries, therefore, call for a further step. An explanation of these differences among countries requires to identify a general relationship between the basic ("primitive") characteristics of the economy and the features of the optimal TTRs. Actually, this is the direct result of analytical optimal taxation. We can come close to a similar result by replacing the analytical solution with microsimulation and numerical optimization. Even with a limited number of countries, we are able to identify some significant effects of "primitives" (Kolm's k, Productivity, Extensive and Intensive Elasticities, Public budget constraint) upon four characteristics of the optimal TTRs (Basic Income, Leading tax rate, Local Progressivity at low and at high taxable income). Overall, the results are not at odds with the typical results obtained by the analytical approach. However we also have specific results that seem to stem from the resulting social preference for universal basic income and the (almost) flat tax. This supports a complementary role of the computational approach. While the analytical solutions might indicate promising reform directions, the computational approach in principle is able to account for more realistic features of the economies.

Tables

Table 1. "Current" and optimal TTRs								
		"Current"	**k = .00**	**k = .05**	**k= .075**	**k = .10**	**k =.125**	**k = .15**
France	τ_0	603.27	61.43	181.72	265.57	367.97	453.77	592.02
	τ_1	0.52	0.93	0.88	0.84	0.80	0.76	0.70
	τ_2	3.01×10^{-6}	-0.0000×10^{-6}	0.0106×10^{-6}	0.013×10^{-6}	0.011×10^{-6}	0.0098×10^{-6}	0.0097×10^{-6}
	τ_3	-1.51×10^{-11}	0.0004×10^{-11}	0.0062×10^{-11}	0.015×10^{-11}	0.005×10^{-11}	0.0064×10^{-11}	0.0053×10^{-11}
	τ_4	0.20×10^{-16}	0.0009×10^{-16}	0.008×10^{-16}	0.014×10^{-16}	0.017×10^{-16}	0.0098×10^{-16}	0.0051×10^{-16}
	SWG	-	159.23	89.63	53.17	16.37	-8.6	-45.47
	EG	-	0	-20.31	-20.64	-12.4	-1.86	20.3
Germany	τ_0	427.9	427.9	454.23	554.48	600.91	600.9075	701.7375
	τ_1	0.71	0.71	0.69	0.66	0.64	0.64	0.61
	τ_2	0.0016×10^{-6}	0.003×10^{-6}	0.001×10^{-6}	0.0001×10^{-6}	0.001×10^{-6}	0.002×10^{-6}	0.002×10^{-6}
	τ_3	0.0009×10^{-11}	0.005×10^{-11}	-0.002×10^{-11}	0.001×10^{-11}	0.001×10^{-11}	0.001×10^{-11}	0.001×10^{-11}
	τ_4	0.0022×10^{-16}	0.010×10^{-16}	0.004×10^{-16}	0.001×10^{-16}	0.001×10^{-16}	0.004×10^{-16}	0.002×10^{-16}
	SWG	-	-7.48	-5.00	41.05	71.39	87.65	155.27
	EG	-	0	-2.02	8.51	23.68	39.94	78.61
Italy	τ_0	217.24	98.99	177.28	236.69	270.67	350.15	417.72
	τ_1	0.745	0.752	0.698	0.66	0.63	0.58	0.53
	τ_2	-1.98×10^{-6}	-0.02×10^{-6}	-0.01×10^{-6}	0.002×10^{-6}	0.005×10^{-6}	0.0002×10^{-6}	0.0008×10^{-6}
	τ_3	0.69×10^{-11}	0.04×10^{-11}	0.01×10^{-11}	0.004×10^{-11}	0.004×10^{-11}	0.0000×10^{-11}	-0.0004×10^{-11}
	τ_4	-0.07×10^{-16}	-0.02×10^{-16}	-0.01×10^{-16}	-0.0000×10^{-16}	0.003×10^{-16}	0.0005×10^{-16}	0.0025×10^{-16}
	SWG	-	72.9	45.93	28.4	18.64	-1.35	-18.54
	EG	-	0	-3	-3.06	-2.97	0.13	3.16
Luxembourg	τ_0	1469.68	615.73	680.78	746.64	809.23	858.74	926.17
	τ_1	0.316	0.761	0.75	0.717	0.706	0.692	0.676
	τ_2	4.12×10^{-6}	0.228×10^{-6}	0.24×10^{-6}	0.23×10^{-6}	0.245×10^{-6}	0.227×10^{-6}	0.223×10^{-6}
	τ_3	-1.869×10^{-11}	0.019×10^{-11}	0.022×10^{-11}	0.014×10^{-11}	0.051×10^{-11}	0.041×10^{-11}	0.013×10^{-11}
	τ_4	0.25×10^{-16}	0.07×10^{-16}	0.017×10^{-16}	0.075×10^{-16}	0.008×10^{-16}	0.009×10^{-16}	-0.004×10^{-16}
	SWG	-	2.06	-35.63	-68.35	-62.63	-67.54	-66.94

	EG	-	0	-41.82	-43.04	-46.45	-40.26	-27.94
Spain	τ_0	196.42	227.43	225.51	436.88	481.18	493.54	498.23
	τ_1	0.97	0.81	0.81	0.65	0.61	0.60	0.60
	τ_2	-4.24×10^{-6}	0.001×10^{-6}	0.0005×10^{-6}	-0.0012×10^{-6}	0.001×10^{-6}	0.0005×10^{-6}	-0.0001×10^{-6}
	τ_3	2.05×10^{-11}	0.001×10^{-11}	$.0008\times10^{-11}$	$-.0008\times10^{-11}$	0.0003×10^{-11}	$-.0001\times10^{-11}$	0.0000×10^{-11}
	τ_4	-0.36×10^{-16}	0.001×10^{-16}	0.0008×10^{-16}	-0.003×10^{-16}	0.001×10^{-16}	0.0002×10^{-16}	0.0000×10^{-16}
	SWG	-	-16.99	-13.87	10.65	13.83	21.35	27.17
	EG		0	2.17	13.88	20.25	25.47	30.29
United Kingdom	τ_0	455.51	289.08	564.61	564.82	628.61	749.57	864.62
	τ_1	0.66	0.80	0.61	0.61	0.57	0.48	0.40
	τ_2	1.717×10^{-6}	-0.0002×10^{-6}	0.0003×10^{-6}	0.0003×10^{-6}	0.001×10^{-6}	-0.005×10^{-6}	-0.001×10^{-6}
	τ_3	-2.008×10^{-11}	0.0002×10^{-11}	0.0001×10^{-11}	0.001×10^{-11}	-0.0006×10^{-11}	0.002×10^{-11}	-0.002×10^{-11}
	τ_4	0.505×10^{-16}	0.0001×10^{-16}	0.0001×10^{-16}	0.0004×10^{-16}	-0.0004×10^{-16}	-0.003×10^{-16}	-0.001×10^{-16}
	SWG	-	-18.37	37.42	37.81	48.34	65.19	77.19
	EG		0	0.81	1.14	1.24	0.64	-0.17

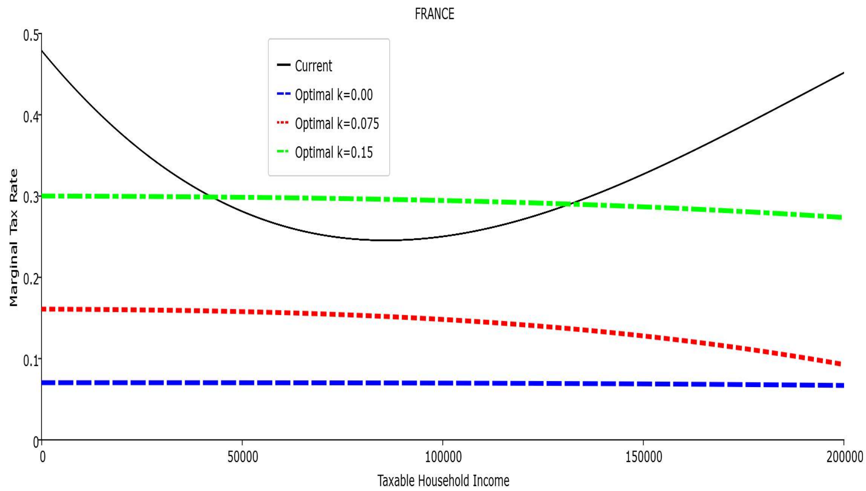
Note to Table 1: **SWG** = social Welfare Gain, **EG** = Equality Gain

Table 2. "Primitives" of the economy and characteristics of the optimal polynomial TTRs

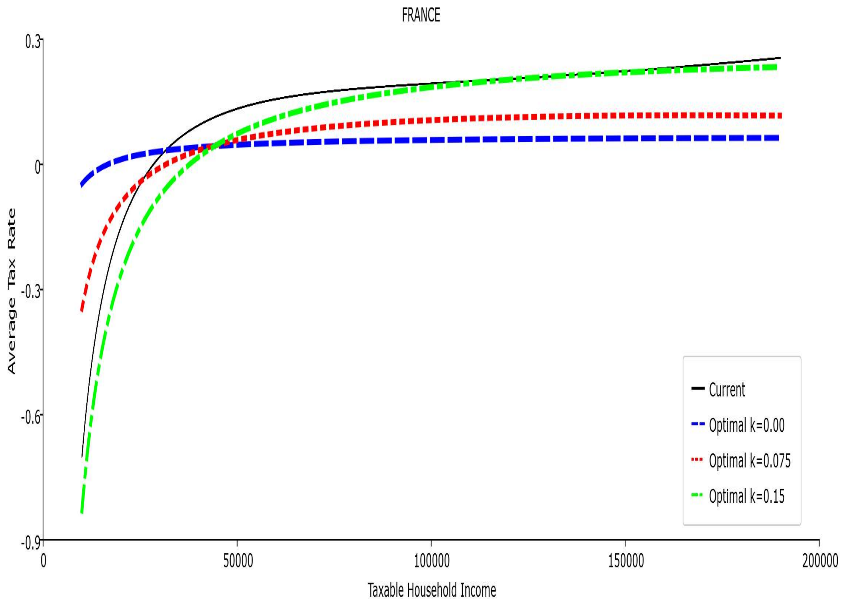
	Basic Income	**Leading Tax Rate**	**Local Progressivity at taxable = 10000**	**Local Progressivity at taxable = 100000**
Constant	**-357.28** *-2.14*	-5.04 *-0.3*	-36.97 *-0.48*	**1.37** *3.57*
Kolm's *k*	**25.44** *14.01*	**1.40** *9.95*	**1.51** *1.79*	**0.01** *3.38*
Productivity	**0.22** *11.29*	0.002 *1.48*	0.001 *0.135*	-0.00 *-0.71*
Extensive Elasticity	**117.35** *3.05*	**9.90** *3.31*	8.41 *0.47*	-0.06 *-0.67*
Intensive Elasticity	**253.55** *8.44*	-1.46 *-0.62*	***27.28*** *1.96*	0.032 *0.39*
Budget	**-0.25** *-8.08*	-0.002 *-0.85*	0.001 *0.08*	-0.00 *-1.10*
R^2	0.83	0.88	0.25	0.31
Standard error of the estimate	53.69	0.04	24.91	0.12
t-values in italics below the estimates Bold estimates are significant at standard levels				

Graphs

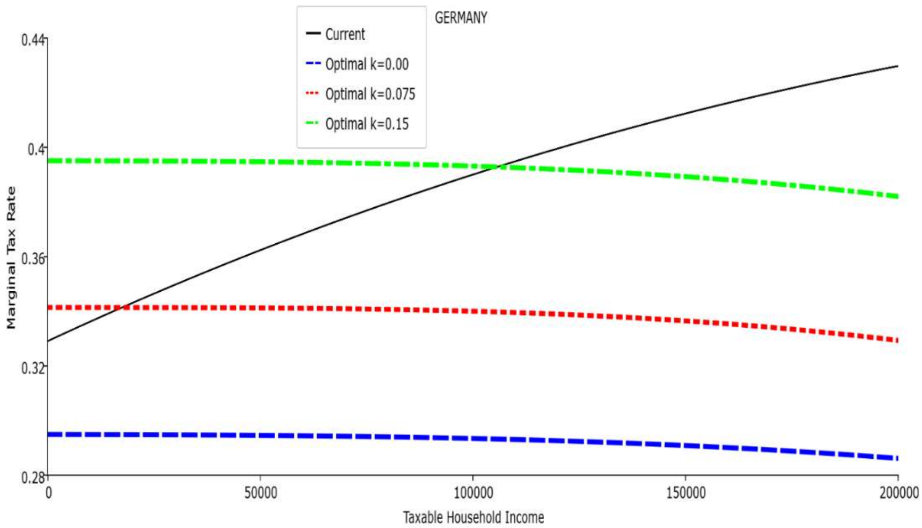
Graph 1a. Marginal Tax Rate vs. Taxable Income. France



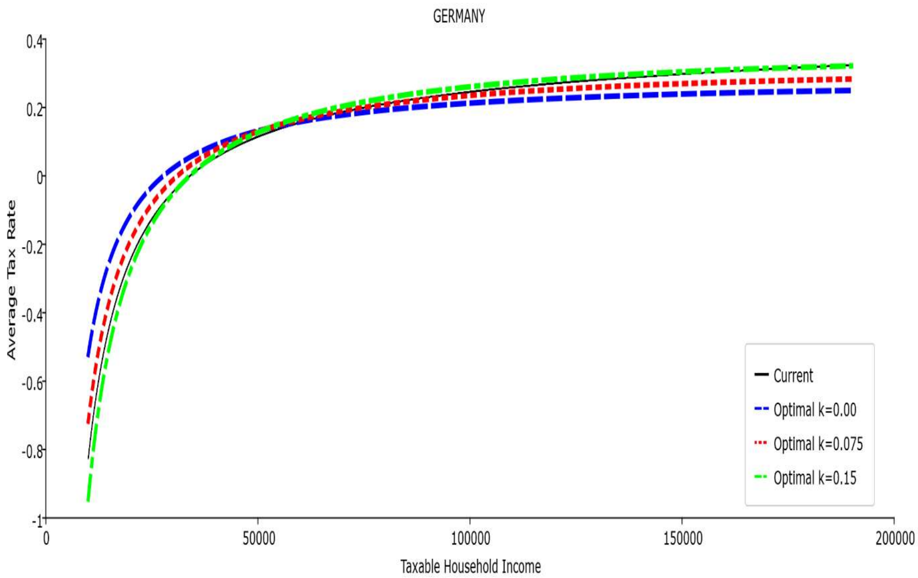
Graph 1b. Average Tax Rate vs. Taxable Income. France



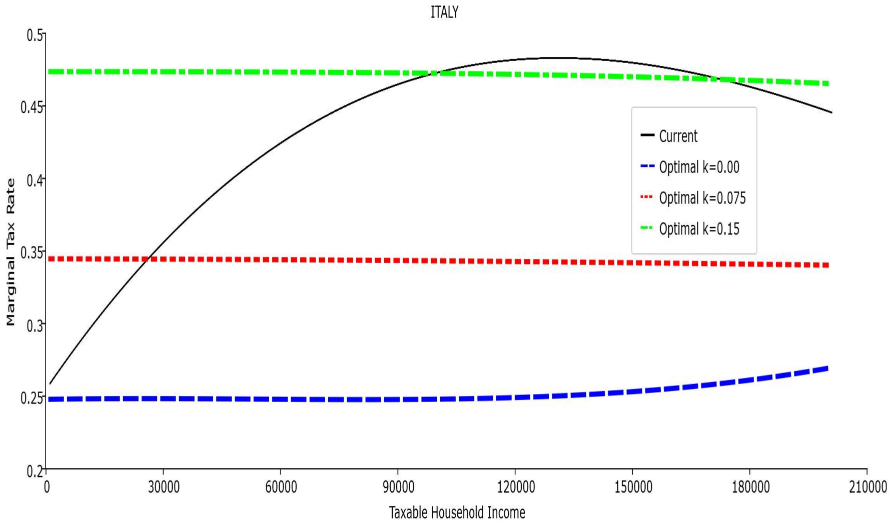
Graph 2a. Marginal Tax Rate vs. Taxable Income. Germany



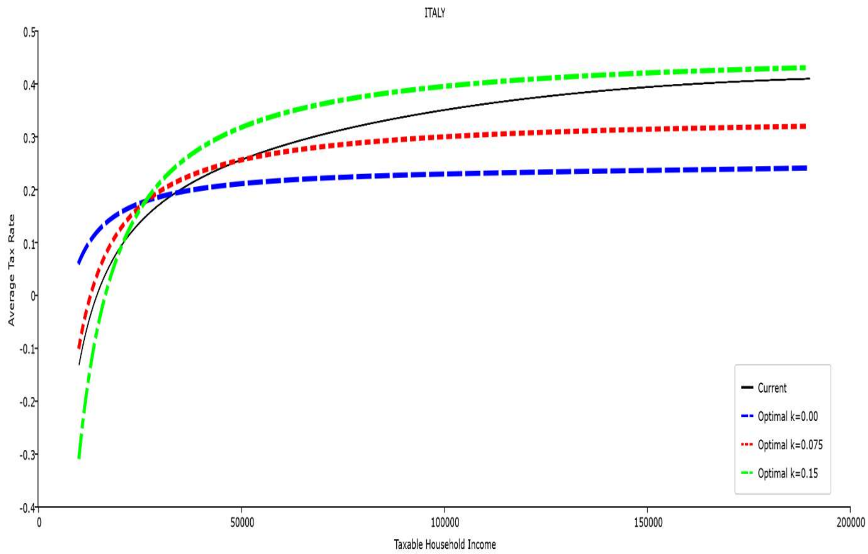
Graph 2b. Average Tax Rate vs. Taxable Income. Germany



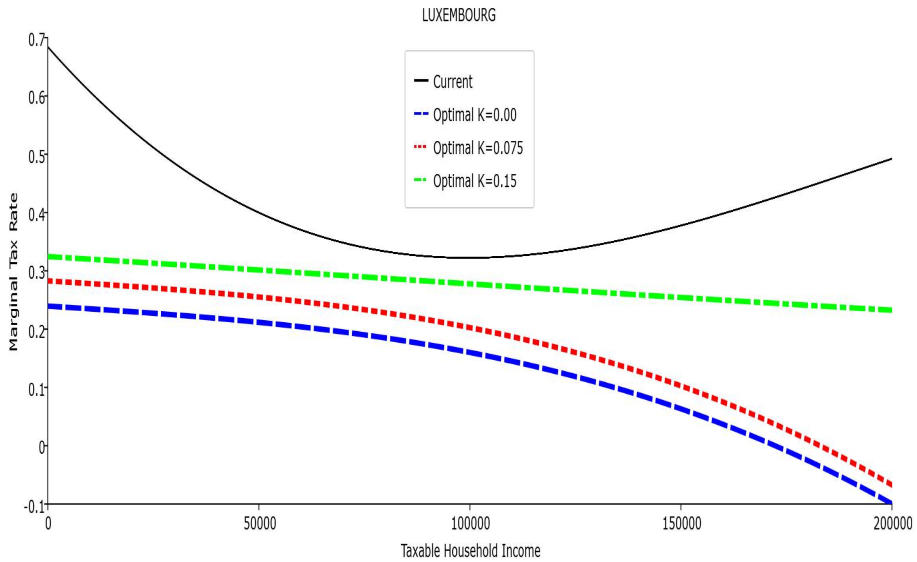
Graph 3a. Marginal Tax Rate vs. Taxable Income. Italy



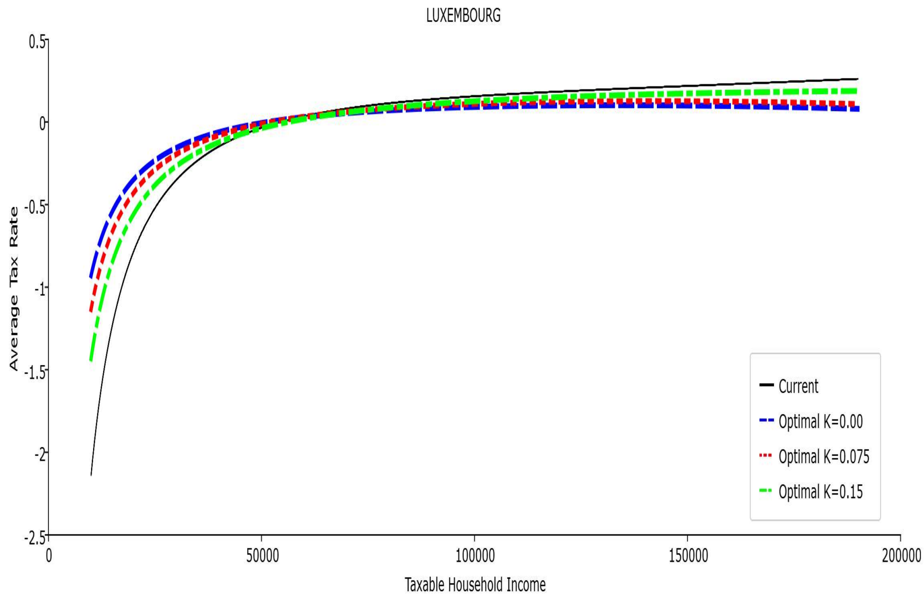
Graph 3b. Average Tax Rate vs. Taxable Income. Italy



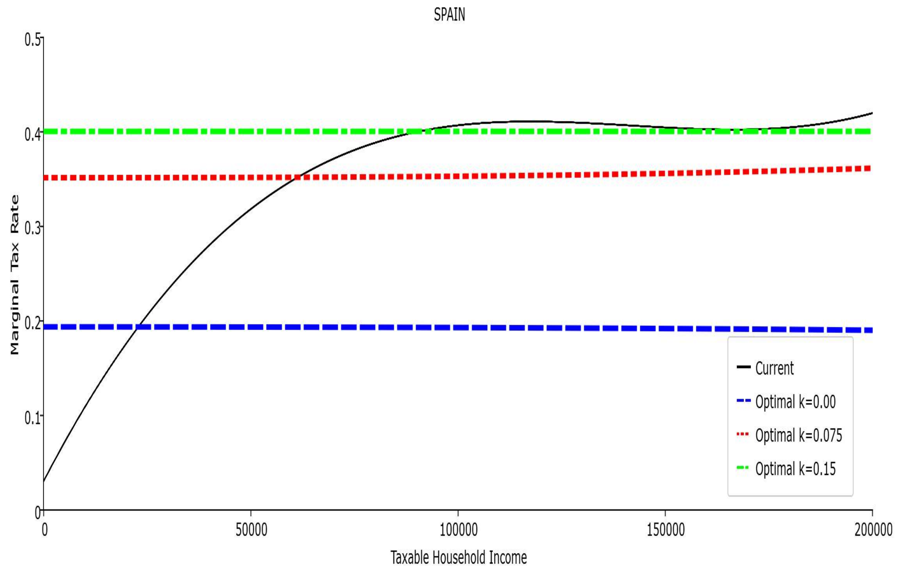
Graph 4a. Marginal Tax Rate vs. Taxable Income. Luxembourg



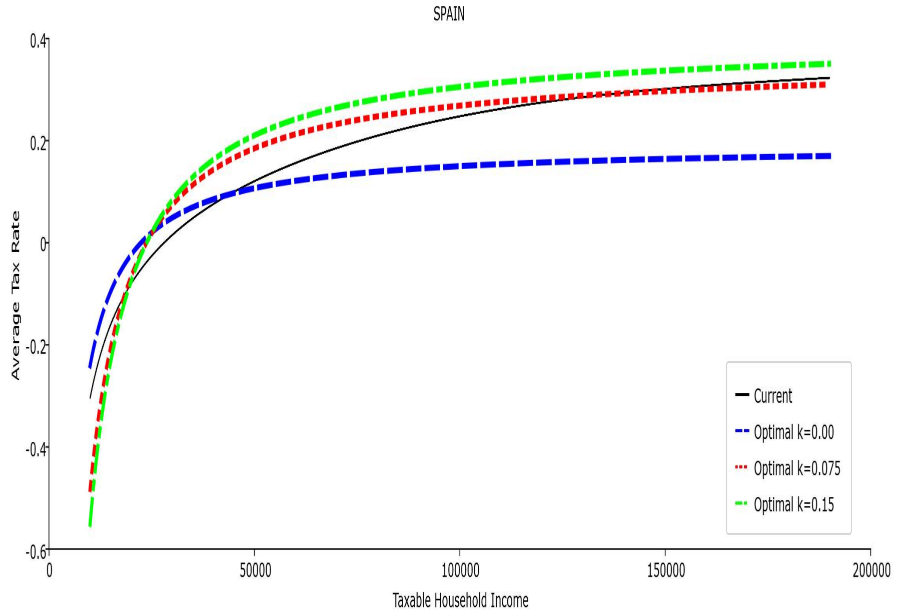
Graph 4b. Average Tax Rate vs. Taxable Income. Luxembourg



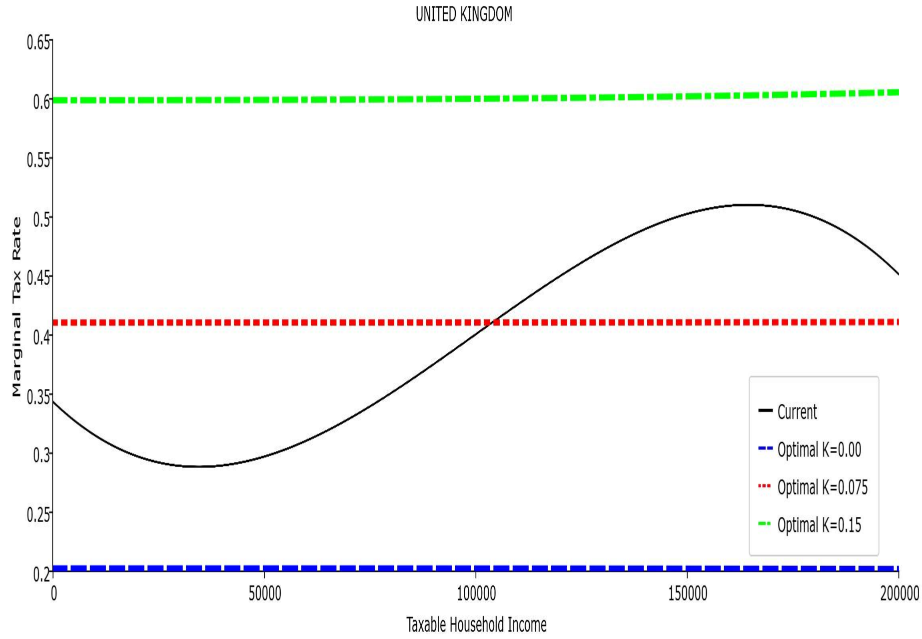
Graph 5a. Marginal Tax Rate vs. Taxable Income. Spain



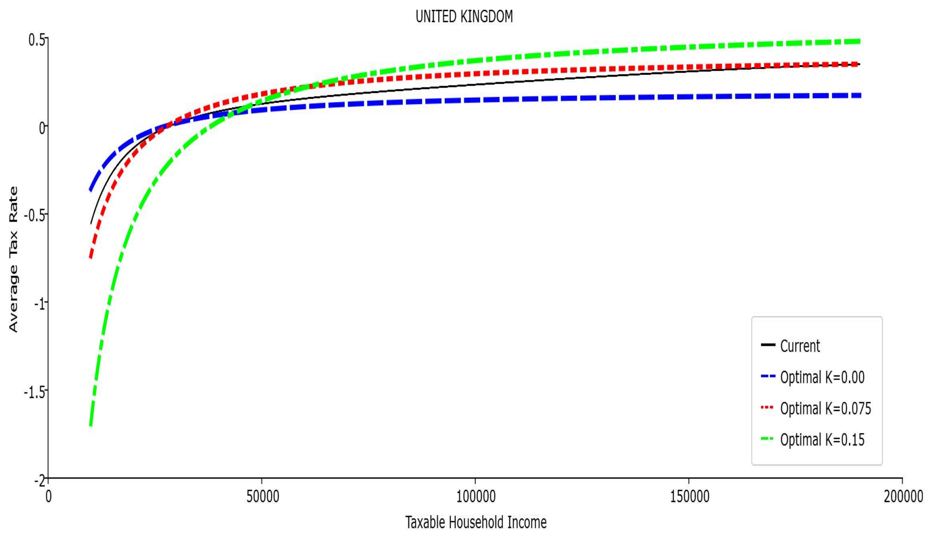
Graph 5b. Average Tax Rate vs. Taxable Income. Spain

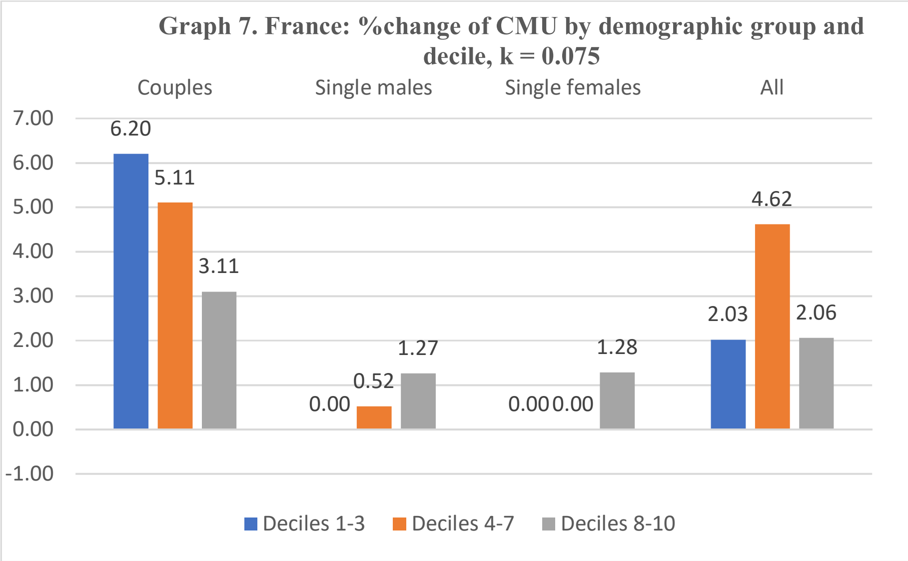


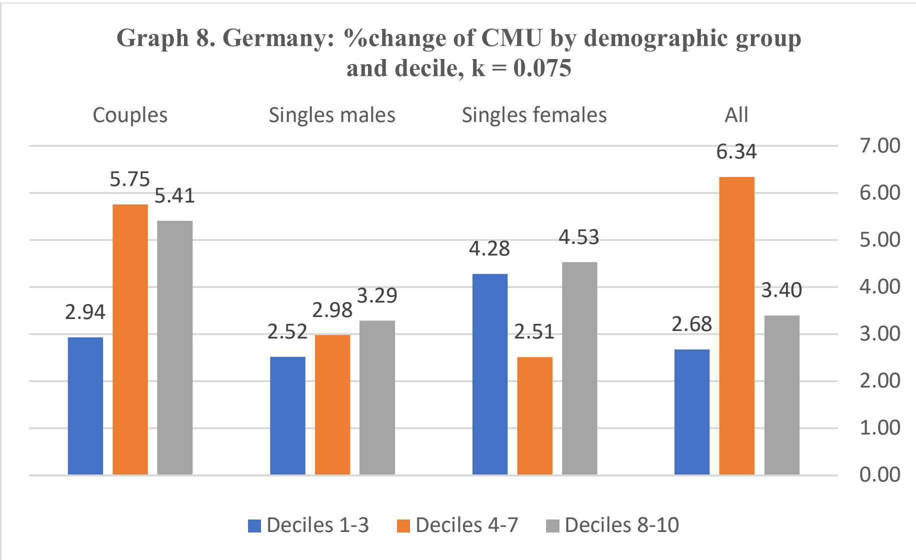
Graph 6a. Marginal Tax Rate vs. Taxable Income. United Kingdom

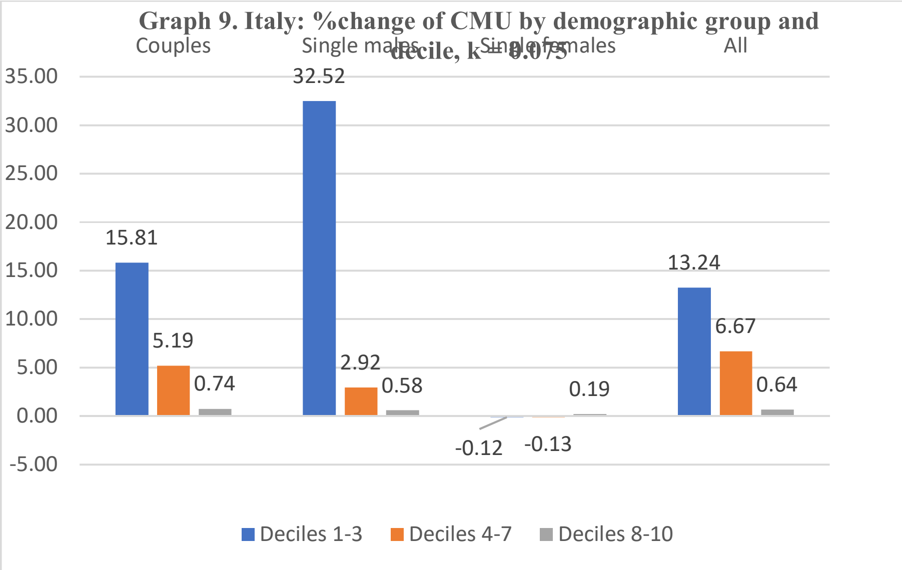


Graph 6b. Average Tax Rate vs. Taxable Income. United Kingdom

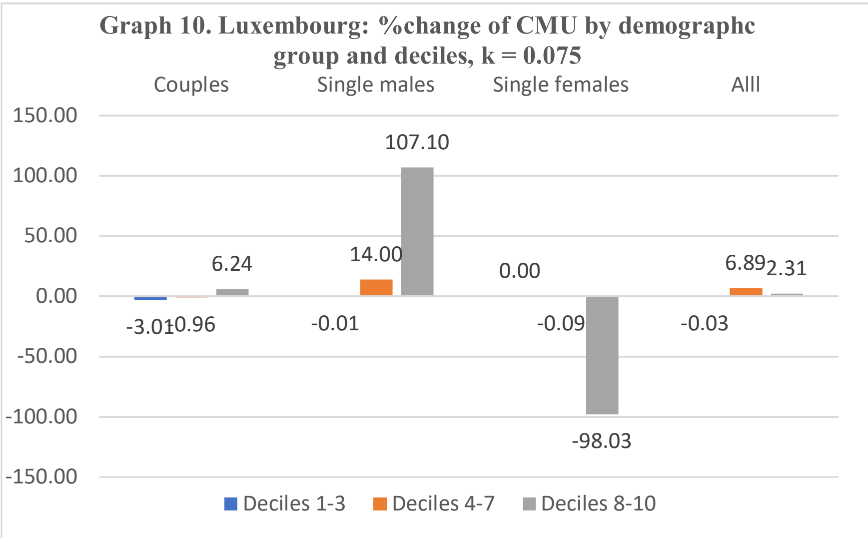



Graph 7. France: %change of CMU by demographic group and decile, k = 0.075
Couples
Single males
Single females
All
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
-1.00
6.20
5.11
3.11
0.00
0.52
1.27
0.00
0.00
1.28
2.03
4.62
2.06
Deciles 1-3
Deciles 4-7
Deciles 8-10


Graph 8. Germany: %change of CMU by demographic group and decile, k = 0.075
Couples
Singles males
Singles females
All
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
2.94
5.75
5.41
2.52
2.98
3.29
4.28
2.51
4.53
2.68
6.34
3.40
Deciles 1-3
Deciles 4-7
Deciles 8-10

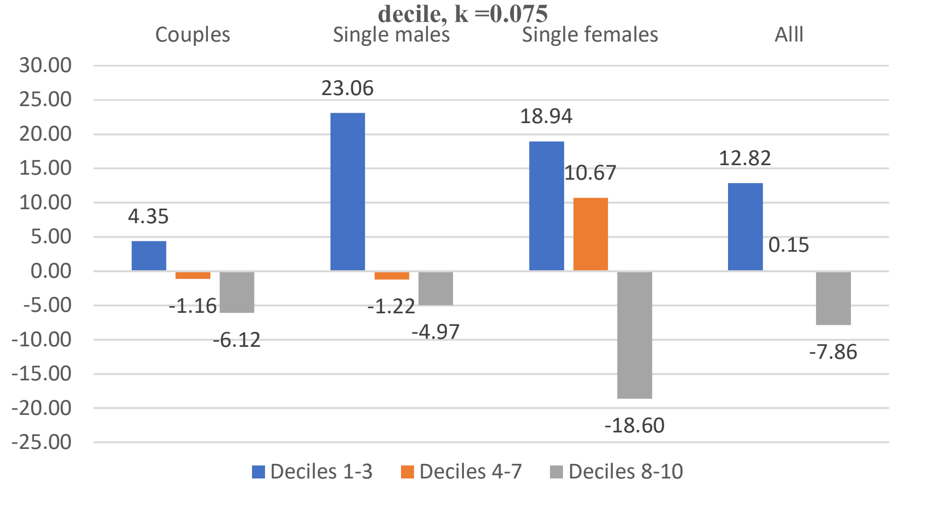


Graph 9. Italy: %change of CMU by demographic group and decile, k = 0.075

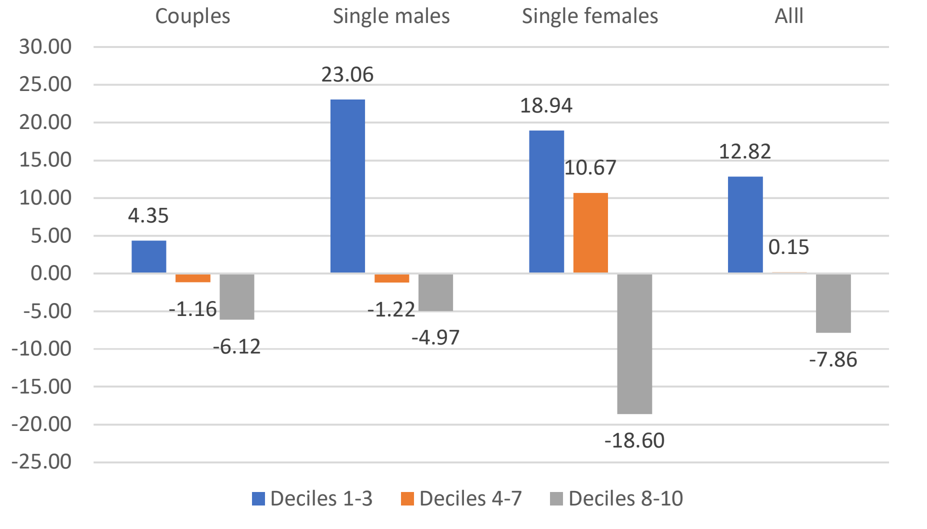


Graph 10. Luxembourg: %change of CMU by demographc group and deciles, k = 0.075

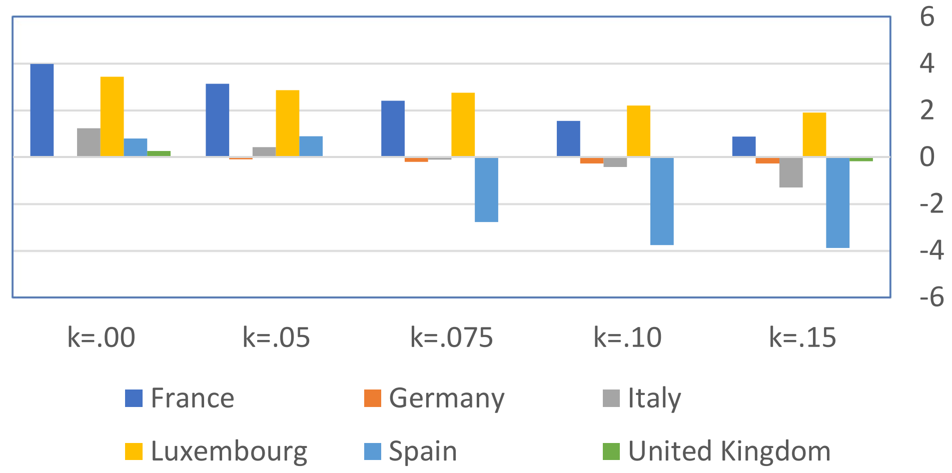
Graph 11. Spain: %change of CMU by demographic group and decile, k =0.075



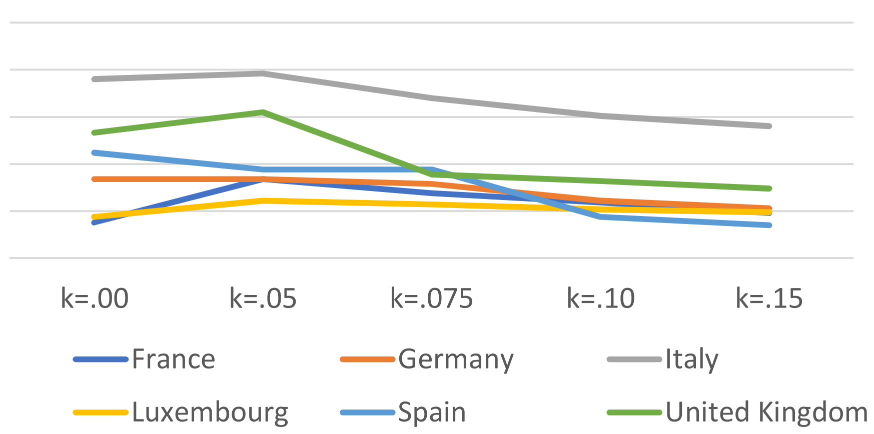
Graph 12. The UK: %change of CMU by demographic group and decile, k = 0.075



Graph 13. %change of disposable income w.r.t. current values by country and Kolm's k



Graph 14. Povery Gap Index by country and Kolm's k



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Appendices

Appendix A

The empirical model of household labour supply

The microeconometric model has the same specification as in Islam and Colombino (2018), although it is estimated on more recent datasets and on a partially different set of countries. We model the households as agents who can choose within an opportunity set Ω containing jobs or activities characterized by hours of work *h*, wage rate *w* and sector of market job *s* (wage employment or self-employment) and other characteristics (observed by the household but not by us). We define **h** and **w** as vectors with one element for the singles and two elements for the couples, $\mathbf{h} = \begin{pmatrix} h_F \\ h_M \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_F \\ w_M \end{pmatrix}$, where the subscripts F and M refer to the female and the male partner respectively. Analogously, in the case of couples, $\mathbf{s} = \begin{pmatrix} s_F \\ s_M \end{pmatrix}$. The above notation assumes that each household member can work only in one sector. We write the utility function of the i-th household at a (**h, s, ε**) under TTR **τ** job as follows (Coda Moscarola et al. 2020):

$$U_i(\mathbf{h},\mathbf{s},\varepsilon;\boldsymbol{\tau}) = \mathbf{Y}_i(\mathbf{h},\mathbf{s};\boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda} + \varepsilon \qquad (9)$$

where:

$\boldsymbol{\gamma}$ and $\boldsymbol{\lambda}$ are parameters to be estimated;

$\mathbf{Y}_i(\mathbf{h},\mathbf{s};\boldsymbol{\tau})$ is a vector including

- $C_i(\mathbf{w}_i^{'}\mathbf{h}, I_i, \mathbf{s};\boldsymbol{\tau})$ = household disposable income on a (**h**,**s**) job given the tax-benefit parameters $\boldsymbol{\tau}$;
- the square of the household disposable income $C_i(\mathbf{w}_i^{'}\mathbf{h}, I_i, \mathbf{s};\boldsymbol{\tau})$ defined above;
- the product of disposable income $C_i(\mathbf{w}_i^{'}\mathbf{h}, I_i, \mathbf{s};\boldsymbol{\tau})$ and household size *N* (interaction term);

$L_i(\mathbf{h})$ is a row vector including

- the leisure time (defined as the total number of available weekly hours (80) minus the hours of work *h*) of the two partners (for a couple) or of the individual (for a single): $L_{ig} = 80 - h_{ig}$, where $g = F, M$.
- the square of leisure time(s), $(L_{ig})^2$;
- the interaction(s) of leisure time(s) with household disposable income ($L_{ig} \times C_i$), with age of the couple's partners of the single, age square and three dummy variables indicating presence of children of different age range (any age, 0-6, 7-10);

ε is a random variable that measures the effect of unobserved (by the analyst) characteristics of the job-household match.

The opportunity set each individual can choose among is $\Omega = \{(0,0),(h_1,s),(h_2,s),(h_3,s)\}$, where (0,0) denotes a non-market "job" or activity (non-participation), h_1, h_2, h_3 are values drawn from the observed distribution of hours in each hour interval 1-26 (part time), 27-52 (full time), 52-80 (extra time) and sector indicator *s* is equal to 1 (wage employment) or 2 (self-employment).

A **(h,s)** job is "available" to household *i* with p.d.f. $f_i(\mathbf{h,s})$, which we call "opportunity density".

We estimate the labour supply models of couples and singles separately. In the case of singles, we have 7 alternatives, while in the case of couples, who make joint labour-supply decision, we combine the choice alternatives of two partners, thus getting 49 alternatives.

When computing the earnings of any particular job **(h, s)** we face the problem that the wage rates of sector *s* are observed only for those who work in sector *s*. Moreover, for individuals who are not working we do not observe any wage rate. To deal with this issue, we follow a two-stage procedure presented in Dagsvik and Strøm (2006) and also adopted in Coda-Moscarola et al. (2020). The procedure is analogous to the well-known Heckman correction for selectivity but is specifically appropriate for distribution assumed for ε. The random component of the impute wage is taken into account when simulating the expected likelihood

By assuming the ε is i.i.d. Type I extreme value we obtain the following expression for the probability that household *i* holds a $(\mathbf{h}_i, \mathbf{s}_i)$ job (e.g. Aaberge and Colombino 2013)

$$P_i(\mathbf{h}_i, \mathbf{s}_i; \boldsymbol{\tau}) = \frac{\exp\{\mathbf{Y}_i(\mathbf{h}_i, \mathbf{s}_i; \boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h}_i)'\boldsymbol{\lambda} + \ln f_i(\mathbf{h}_i, \mathbf{s}_i)\}}{\sum_{\mathbf{s}} \sum_{\mathbf{h} \in \Omega} \exp\{\mathbf{Y}_i(\mathbf{h,s}; \boldsymbol{\tau})'\boldsymbol{\gamma} + \mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda} + \ln f_i(\mathbf{h,s})\}} \tag{10}$$

By choosing a convenient (uniform with peaks") specification for the opportunity density *f(.,.)* it turns out that expression (10) can be rewritten as follows (e.g. Aaberge and Colombino 2013, Colombino 2013),

$$P_i(\mathbf{h}_i,\mathbf{s}_i;\boldsymbol{\tau})=\frac{\exp\left\{\mathbf{Y}_i(\mathbf{h}_i,\mathbf{s}_i;\boldsymbol{\tau})'\boldsymbol{\gamma}+\mathbf{L}_i(\mathbf{h}_i)'\boldsymbol{\lambda}+\mathbf{D}_i(\mathbf{h}_i,\mathbf{s}_i)'\boldsymbol{\delta}\right\}}{\sum_{\mathbf{s}}\sum_{\mathbf{h}\in\Omega}\exp\left\{\mathbf{Y}_i(\mathbf{h},\mathbf{s};\boldsymbol{\tau})'\boldsymbol{\gamma}+\mathbf{L}_i(\mathbf{h})'\boldsymbol{\lambda}+\mathbf{D}_i(\mathbf{h},\mathbf{s})'\boldsymbol{\delta}\right\}} \tag{11}$$

where, for a single household, $\mathbf{D}_i$ is the vector (with 1[.] denoting the indicator function)

$$\begin{aligned}
&D_{1,0}=1\left[s=1,h>0\right],\\
&D_{1,1}=1\left[s=1,1\le h\le 26\right],\\
&D_{1,2}=1\left[s=1,27\le h\le 52\right],\\
&D_{2,0}=1\left[s=2,h>0\right],\\
&D_{2,1}=1\left[s=2,1\le h\le 26\right],\\
&D_{2,2}=1\left[s=2,27\le h\le 52\right].
\end{aligned} \tag{12}$$

and $\boldsymbol{\delta}$ is vector of parameters to be estimated. For couples, $\mathbf{D}_i$ contains two analogous sets of variables, one for each partner (F = wife, M = husband):

$$\begin{aligned}
&D_{F,1,0}=1\left[s_F=1,h_F>0\right],\\
&D_{F,1,1}=1\left[s_F=1,1\le h_F\le 26\right],\\
&D_{F,1,2}=1\left[s_F=1,27\le h_F\le 52\right],\\
&D_{F,2,0}=1\left[s_F=2,h_F>0\right],\\
&D_{F,2,1}=1\left[s_F=2,1\le h_F\le 26\right],\\
&D_{F,2,2}=1\left[s_F=2,27\le h_F\le 52\right]\\
&D_{M,1,0}=1\left[s_M=1,h_M>0\right],\\
&D_{M,1,1}=1\left[s_M=1,1\le h_M\le 26\right],\\
&D_{M,1,2}=1\left[s_M=1,27\le h_M\le 52\right],\\
&D_{M,2,0}=1\left[s_M=2,h_M>0\right],\\
&D_{M,2,1}=1\left[s_M=2,1\le h_M\le 26\right],\\
&D_{M,2,2}=1\left[s_M=2,27\le h_M\le 52\right].
\end{aligned} \tag{13}$$

It can be shown (Colombino 2013) that the coefficients δ of the dummies **D** depend on the number and types of available jobs and therefore reflect quantity constraints in the opportunity sets.

The model is a simplified version of the so-called RURO model (Aaberge and Colombino 2013, 2014). The main simplification concerns the wage rates. In the most general versions of the RURO model the wage rates densities are estimated simultaneously with the preference parameters and the hours' opportunity density. In this paper we use instead pre-estimated wage densities.

Expression (11) and its extensions to couples are the contribution to the likelihood function to be maximized in order to estimate the parameters γ, λ and δ.

The datasets used in the analysis are the EUROMOD input data based on the European Union Statistics on Income and Living Conditions (EU-SILC) for the year 2015 in France, Italy, Germany, Luxembourg, Spain and the United Kingdom. The input data provide all required information on demographic characteristics and human capital, employment and wages of household members, as well as information about various sources of non-labour income. We apply common sample selection criteria for all countries under study by selecting individuals in the age range 18-55 who are not retired or disabled. EUROMOD[17] is used for two different operations. First, for every household in the sample computes the net available income under the current TTR at each of the 49 (7) alternatives available to the couples (singles). The net available incomes are used in the estimation of the labour supply model. Second, for each household, it computes the gross income at each alternative. Gross incomes are used in the simulation and optimization steps, where EUROMOD is not used anymore and new values of net available incomes are generated by applying the new TTRs to the gross incomes. The estimates of the model are available on request.

The analytical approach typically uses point estimates of the wage elasticities of labour supply. As explained in the illustration of the computational approach, we do not use elasticity estimates in order to identify optimal TTRs, instead we directly use the microeconometric model to iteratively simulate decisions in order to identify optimal TTRs. We do use extensive and intensive elasticities (as reflecting "primitive" characteristics of the economy) in the exercise that "maps" from the primitives to the optimal TTRs (explained in Section 3 and commented in Section 4). Moreover, elasticities estimates are useful to illustrate how the model works. We report them in Table A1.

[17] EUROMOD is a large-scale pan-European tax-benefit static micro-simulation engine (e.g. Sutherland and Figari, 2013). It covers the tax-benefit schemes of the majority of European countries and allows computation of predicted household disposable income, on the basis of gross earnings, employment and other household characteristics.

Table A1. Wage elasticity

		FR		DE		IT		LU		ES		UK	
		E	I	E	I	E	I	E	I	E	I	E	I
Direct	Wife	0.87	0.65	2.3	-0.67	4.71	0.44	1.76	-0.08	3.89	0.01	3.33	0.01
	Husband	0.21	-0.02	1.11	-0.53	0.53	0.14	0.20	0.12	1.11	0.14	1.49	0.24
	Single female	1.497	0.43	2.05	-0.64	2.66	2.35	2.20	-0.14	2.20	0.01	4.9	-0.38
	Single male	1.907	-1.85	2.66	-0.66	3.70	-2.63	2.20	0.07	2.05	0.03	4.71	0.02
	All	0.87	0.36	1.90	-0.90	2.66	-0.23	1.36	0.16	2.20	0.02	3.16	-0.17
Indirect	Wife	0.87	-0.11	4.71	-3.41	4.71	-0.22	1.76	-0.44	3.88	-0.10	3.33	-0.02
	Husband	0.20	0.57	0.53	0.40	0.53	-0.04	0.20	-0.06	1.11	0.001	1.49	0.19

E = extensive elasticity
I = intensive elasticity