# Optimal Disclosure of Private Information to Competitors

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**Abstract**

I consider a duopoly model with differentiated substitutes, price competition, and uncertain demand, in which one firm has an information advantage over a competitor. I study the incentives of the informed firm to share its private information with its competitor and the incentives of a regulator to constrain or enforce disclosure in order to benefit consumers. I show that full disclosure of information is optimal for the informed firm, because it increases price correlation and surplus extraction from consumers. A regulator can increase expected consumer surplus and welfare by restricting disclosure, but consumers can benefit from the regulator privately disclosing some information to the competitor. Disclosure increases the ability of firms to extract surplus from consumers, but private disclosure creates a coordination failure in firm pricing. Private partial disclosure is optimal for consumers when firms offer sufficiently close substitutes. My findings highlight the effect of an uneven distribution of consumer data between firms on welfare allocation. They also inform an ongoing policy debate about how to control the dissemination of information between firms to protect consumers.

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Some firms can gather more information than their competitors about market features like demand, given their size or incumbency status. For instance, online platforms like Amazon engage in massive data analysis and demand estimation by gathering information generated through trade and consumer searches that other sellers on the platform can't replicate. As sellers themselves, they can use this information to guide their own pricing, control the information observed by other sellers, and potentially make price recommendations as in Amazon's Seller's coaching program. In settings of information asymmetry, information disclosure between firms affects firm behavior and therefore also impacts consumers and welfare. The use of private information as a competitive advantage by online platforms and the role of price recommendations as a collusive device have attracted the attention of regulatory entities in the US and Europe.[1] This is because, when there is an uneven distribution of consumer data between firms, regulatory interventions to control information disclosure can redistribute surplus between firms and consumers, as well as impact welfare.

In this paper, I study the role of information disclosure as a pricing persuasion device through which a firm with an information advantage or a regulator can influence the pricing of a competing firm. I examine the informed firm's incentives to commit to share its private information with its competitor and the role of a regulator who commits to control information disclosure between firms to benefit consumers. Specifically, this paper analyzes a stylized duopoly model with information asymmetry about demand and a binary state, with an informed firm which privately learns the level of the demand and an uninformed firm that has no private information. Demand is linear and firms face uncertainty about its level, which can be either low or high. Firms offer differentiated goods, such that consumer willingness to pay for a good depends on its substitutability with the competitor's. Firms compete by simultaneously and non-cooperatively setting prices to maximize their expected profits. In this context, I address the following questions: What is the informed firm's optimal disclosure policy as a competitor in the market? How can a regulator constrain or enforce information disclosure to benefit consumers?

I characterize the optimal disclosure for firms and consumers. The welfare implications of disclosure are determined by the degree of differentiation between goods, because it determines the extent to which disclosure affects firm pricing and relative demand across markets. Regarding optimal disclosure for firms, with substitutes, firm choices are strategic complements and the informed firm thus benefits from sharing its private information with the uninformed firm through increased price correlation. As a result, full disclosure is optimal

[1] See for example media coverage in Fung (2020), Green (2018) and Lardieri (2019).

for the informed firm. Full disclosure also maximizes producer surplus, because the uninformed firm also benefits from price correlation as well as from learning about the state. This result highlights that an informed firm may have incentives to share information even when it has no information to gain in return, because it can influence the pricing of its competitor. Furthermore, I generalize this result by showing that the informed firm's optimal disclosure doesn't rely on the linearity of demand: when the informed firm's expected equilibrium profit is supermodular in the state and the choice of the uninformed firm, firms' choices are strategic complements and, accordingly, full disclosure is optimal. Also, no disclosure is optimal when the informed firm's expected equilibrium profit is submodular.[2]

Regarding optimal disclosure for consumers, a regulator should restrict information disclosure, at least partially.[3] However, some information disclosure is not necessarily detrimental to consumers. First, I show that the optimal disclosure is private, such that the informed firm doesn't observe the signal realization of the uninformed firm. Second, Proposition 2 shows that optimal disclosure is determined by the degree of differentiation between goods. Partial disclosure is optimal if firms offer sufficiently close substitutes and no disclosure is optimal otherwise. Information disclosure creates a trade-off for consumers. On the one hand, it reduces the uninformed firm's uncertainty about the state, improving the ability of firms to extract surplus from consumers by increasing price correlation. On the other hand, private partial disclosure introduces uncertainty about the information observed by a firm's competitor. This expands the range of prices in each state, since firms price according to the expected price of its competitor and its own expected level of demand. Namely, the uninformed firm may observe a signal which conflicts with the realized state, but the informed firm doesn't observe the signal and therefore cannot adjust, creating a coordination failure. Consumers can benefit from this price heterogeneity by choosing from which firm to buy after observing prices. Overall, the regulator trades-off the opportunity to create this coordination failure in prices with allowing firms to better extract surplus from consumers. The net effect depends on the differentiation between goods, because it determines consumers' willingness to substitute between goods and therefore the extent to which disclosure affects relative demand across firms.

[2]When the informed firm's profits are supermodular in the state and the choice of the uninformed firm, an increase in the uninformed firm's price has a increasing effect on the informed firm's profits as the state increases. Focusing instead on decision problems, Kolotilin and Wolitzky (2020) shows that supermodularity of a sender's objective function with respect to the state and the receiver's action is a sufficient condition for the optimality of full disclosure.

[3]Luco (2019) presents empirical evidence that full disclosure can be detrimental for consumers in the gasoline market in Chile.

To maximize expected welfare, the regulator trades off the effect of disclosure on consumers and firms. When firms offer sufficiently differentiated goods, no disclosure is optimal, since the expected loss from disclosure for consumers exceeds the expected gain for firms. Conversely, when firms offer sufficiently close substitutes, full disclosure is optimal. For intermediate levels of differentiation, partial disclosure maximizes expected welfare.

When partial disclosure is optimal, I also fully characterize the consumer and welfare optimal disclosure policies. Signals act as equilibrium price recommendations, recommending a price to each firm conditional on the state subject to obedience constraints. Proposition 3 shows that the regulator recommends at most two prices. One of the prices is only recommended when the state is low, revealing the state to the uninformed firm. The other price is recommended in both states, obfuscating the level of demand. The optimality of partial disclosure contrasts with previous work focusing on firm incentives, highlighting that optimal disclosure is more nuanced when considering implications for consumers and welfare.

My analysis emphasizes the wide scope for intervention by a regulator, based on product differentiation and their objective function. My results are of particular interest given current policy debates on the use of private information by firms who act as both a trading platform and a competitor in the market, as well as the debate about whether retail price recommendations act as a collusive device. As I show, it can be optimal for a regulator to intervene by completely preventing or forcing information disclosure, or by designing disclosure policies to partially inform the uninformed firm. Since disclosure policies can be interpreted as price recommendations, these recommendations can help consumers, and abstaining from regulation minimizes consumer surplus. Lastly, my results highlight that it is crucial to consider the strategic environment to understand the welfare consequences of information sharing.

**Related literature.** This paper contributes to the literature on strategic information sharing in oligopolies with commitment and the literature on information design in games.[4] Incentives for information sharing about demand among competing firms with symmetric private information and normally distributed linear demand were first studied in Novshek and Sonnenschein (1982), Clarke (1983) as well as Vives (1984), and later generalized in Raith (1996).[5] In these papers, firms commit to share their private information with an

[4]Papers like Benoit and Dubra (2006) show that agents' ex-ante and ex-post incentives for information sharing can be disaligned, such that commitment is key.

[5]Other papers in this literature include Gal-Or (1985), Li (1985), Kirby (1988) and Vives (1990). Information sharing about costs are studied in papers like Fried (1984), Gal-Or (1986), Sakai (1986) and Shapiro (1986), in which incentives to share information are reversed for firms. Information sharing about costs with Bertrand competition is strategically equivalent to sharing about values in first price auctions

intermediary, which then discloses a common signal to all firms to maximize industry-wide profits. These papers focus on the producer surplus optimal public disclosure and on the regulation of industry-wide information sharing by trading organizations. They show the optimality of full disclosure for firms when they compete by choosing prices and offer imperfect substitutes. Instead, I study the incentives of an individual firm to share information to influence its competitor's behavior in a setting of informational advantage, in which the distribution of the uninformed firm's signal is unrestricted.[6] My results show that it can be optimal for a firm to unilaterally disclose information about demand to a competitor even without receiving information in return, because disclosure influences competitor behavior and acts as a pricing persuasion device.[7] Further, full disclosure is not only optimal for the informed firm, but also for producer surplus. The intuition for this result relates to Angeletos and Pavan (2007), who study the social value of information with normally distributed signals and find that producer surplus increases with the precision of both public and private signals.

In contrast with this literature, I also analyze the effects of information disclosure on consumers. Vives (1984) and Calzolari and Pavan (2006) show that information disclosure is not necessarily harmful to consumers. Vives (1984) illustrates this by comparing the utility of a representative consumer across full and no disclosure when firms share symmetric normally distributed private information. Calzolari and Pavan (2006) study a sequential setting in which the Stackelberg leader must provide incentives to consumers to reveal their private information to be able to share it with its follower. They focus on the leader's optimal disclosure policy, whereas I focus on the optimal disclosure for consumers. Regarding welfare, Vives (1984) also shows that full disclosure dominates no disclosure if and only if firms offer sufficiently close substitutes, yet I show that restricting to full and no disclosure is with loss of generality since partial disclosure can be consumer and welfare optimal. My results regarding welfare relate to Ui and Yoshizawa (2015), who study the social value of information restricted to symmetric normally distributed signals and symmetric equilibria. They show that welfare decreases in the precision of private information and increases in the precision of public information if goods are close substitutes, intuitively related to the optimality of

(Engelbrecht-Wiggans et al. (1983), Fang and Morris (2006), and Bergemann et al. (2017)).

[6]Bergemann and Morris (2013), Bergemann et al. (2015b) and Eliaz and Forges (2015) analyze producer optimal disclosure in Cournot settings with perfect substitutes and information symmetry. They show that it is with loss of generality to restrict attention to a common and, hence, perfectly correlated disclosure.

[7]In sequential settings, the role of current choices as a costly persuasion device to influence the precision of future information has been studied in Mailath (1989), Mirman et al. (1993), Mirman et al. (1994), Harrington (1995), Keller and Rady (2003), Taylor (2004), Bernhardt and Taub (2015), Bonatti et al. (2017).

either full or private partial disclosure as I fully characterize in this paper.

More broadly, the paper contributes to the literature on information design in games as studied in papers like Taneva (2019) and Mathevet et al. (2020). I characterize the optimal recommendation mechanism in a Bertrand setting with product differentiation and information asymmetry.[8] It is most closely related to the literature on consumer optimal information design, which analyzes the effect of information about buyers' valuation on pricing and welfare allocation. This literature has focused on buyer optimal learning, consumer optimal market segmentation and on the incentives of consumers to disclose their preferences to firms. Within the buyer optimal learning literature, Roesler and Szentes (2017) analyzes the effect of a buyer's information on monopoly pricing and characterizes optimal buyer learning. In a duopoly setting, Armstrong and Zhou (2019) studies competition between firms when consumers observe a private signal about their valuation and characterizes consumer optimal learning. Within the consumer optimal segmentation literature, Bergemann et al. (2015a) analyzes the welfare consequences of a monopolist having access to additional information about consumer preferences and characterize the feasible welfare outcomes achieved by segmentation. Li (2020) extends the insights from Bergemann et al. (2015a) to an oligopoly setting and characterizes the consumer-optimal market segmentation in competitive markets. Elliott et al. (2020) studies how information about consumer preferences should be distributed across firms which compete by offering personalized discounts to consumers and provides necessary and sufficient conditions under which perfect segmentation can be achieved. Lastly, Ichihashi (2020) studies the welfare effects of consumers disclosing information about their valuation with a monopolist, whereas Ali et al. (2020) analyzes the consumer optimal disclosure of information about their preferences in monopolistic and competitive markets. In contrast, I focus on the welfare consequences of an unequal distribution of consumer data across firms and the effect of information disclosure between firms. In particular, I characterize the consumer optimal disclosure policy between firms, which affects consumers indirectly by affecting prices.

The remainder of the paper is organized as follows: Section 1 presents the model, Section 2 derives the informed firm optimal disclosure, Section 3 derives the consumer optimal disclosure, Section 4 derives the producer and welfare optimal disclosures, Section 5 discusses extensions and robustness of results, and Section 6 concludes.

[8] In contrast, Bergemann et al. (2021) study a setting in which identical firms offer an homogeneous good, compete by setting prices and are uncertain about the number of price quotes a consumer receives. They identify how the equilibrium price dispersion depends on the distribution of the price count and the information firms have.

# 1 The model

Two symmetric firms offer horizontally differentiated substitutes and compete by simultaneously setting prices. Firm profits depend on the realization of a binary payoff-relevant state, $\theta \in \Theta = \{\theta_L, \theta_H\}$ with $\theta_H > \theta_L > 0$. Firms share a common prior about the state where the probability of $\theta \in \Theta$ is denoted by $\mu_\theta \in (0,1)$. Firm $i$'s demand, $q_i((p_i, p_{-i});\theta)$, is given by

$$q_i((p_i, p_{-i});\theta) = \max\{0, \theta - ap_i + bp_{-i}\} \tag{1}$$

where $a$ and $b$ are known parameters with $a > b > 0$.[9] As can be seen from (1), the state represents the level of demand and, since firms offer substitutes, both the state and the price of the competitor are positive demand shifters which increase quantity demanded at every price. Define $\delta$ as the ratio of $b$ and $a$, which measures the degree of differentiation. As $\delta$ converges to 0, goods are more differentiated and as $\delta$ converges to 1, goods are closer substitutes. I restrict attention to distributions of the payoff-relevant state that satisfy Assumption 1.

**Assumption 1** *The support of the payoff-relevant state's distribution satisfies* $\theta_H < \frac{4a^2-b^2}{2a^2-b^2}\theta_L$.

Assumption 1 imposes an upper bound on the high state, ensuring that equilibrium prices and quantities are strictly positive for both firms, for any information they may have about the state. This assumption restricts attention to the effect of information disclosure on prices, isolating it from the potential effect of inducing a firm to be "priced out" of the market when it selects prices that are not competitive.

Further, assume that firms' costs are zero.[10] Hence, firm $i$'s ex-post profits, $\Pi_i : \mathbb{R}^2_+ \times \Theta \to \mathbb{R}$, correspond to

$$\Pi_i((p_i, p_{-i});\theta) = p_i \cdot q_i((p_i, p_{-i});\theta).$$

Given the state and prices $(p_i, p_{-i})$, ex-post consumer surplus in the market of firm $i$ is the difference between consumers' ex-post willingness to pay for the good and the equilibrium price. The ex-post willingness to pay of consumers in market $i$ is characterized by the ex-post inverse demand, $p_i(q_i; p_{-i}, \theta)$, given by

$$p_i(q_i; p_{-i}, \theta) = \max\left\{0, \frac{\theta + bp_{-i} - q_i}{a}\right\}$$

[9] The relationship between $a$ and $b$ implies that demand is more sensitive to a firm's own price than the price of its competitor, ensuring that equilibrium prices are finite.

[10] Including linear or quadratic costs has no impact on the results.

where demand is generated by a continuum of heterogeneous consumers making discrete choices (Armstrong and Vickers, 2015).[11] Then, ex-post consumer surplus in market $i$, $CS_i : \mathbb{R}_+^2 \times \Theta \to \mathbb{R}$, corresponds to

$$CS_i((p_i, p_{-i}); \theta) = \frac{1}{2}\left[\frac{\theta + bp_{-i}}{a} - \frac{\theta + bp_{-i} - q_i((p_i, p_{-i}); \theta)}{a}\right] q_i((p_i, p_{-i}); \theta) = \frac{1}{2a} q_i((p_i, p_{-i}); \theta)^2, \tag{2}$$

where the term in square brackets corresponds to the difference between the demand intercept and the equilibrium price.

**Information environment.** Firm 1 (the informed firm) learns the state, whereas firm 2 (the uninformed firm) initially has no information beyond the common prior. Assume that a designer can restrict (or require) information sharing between firms by choosing the information observed by the uninformed firm. The designer selects and commits to an information structure before the realization of the state which discloses none, some, or all of the informed firm's private information to the uninformed firm. Let $S_2$ be the set of signal realizations observed by firm 2. An information structure consists of a set of signal realizations $S_2$ and a family of conditional distributions $\psi_2 : \Theta \to \Delta(S_2)$. The information structure is observed by both firms but signal realizations are private.

Specifically, the timing is as follows: (i) the designer selects and commits to an information structure $(S_2, \psi_2)$ observed by both firms; (ii) the state $\theta$ is realized and privately observed by the informed firm; (iii) the signal realization is realized and privately observed by the uninformed firm according to $(S_2, \psi_2)$; (iv) firms update their beliefs according to Bayes' rule and simultaneously choose prices; (v) payoffs are realized.

Signals play two roles. First, they act as a coordination device by providing information about the actions of others, even when there is no uncertainty about the state. Second, they inform the uninformed firm about the state which influences firms' pricing choices. All correlation between firms' choices is generated through the state.

[11] Assume a continuum of consumers with heterogeneous preferences. Consumer $\ell$ has valuation $v_{\ell,i}$ for one unit of the good offered by firm $i$, where $v = (v_{\ell,1}, v_{\ell,2})$ is drawn from a joint cumulative distribution $G(v)$. Consumer $\ell$ attaches no value to more than one unit of either good and wishes to buy either a single unit of one good or to not buy any of them. Then, consumer $\ell$ buys from firm $i$ if $v_{\ell,i} - p_i \geq \max_{j \neq i}\{0, v_{\ell,j} - p_j\}$, where the outside option is normalized to zero. The demand for product $i$, $q_i(p)$, is then the measure of consumers $\ell$ who satisfy $v_{\ell,i} - p_i \geq \max_{j \neq i}\{0, v_{\ell,j} - p_j\}$. Armstrong and Vickers (2015) show that the linear demand model defined by (1) can be micro-founded by this discrete choice model. In this context, consumer $\ell$ who buys from firm $i$ receives surplus $v_{\ell,i} - p_i$ and the consumer surplus in market $i$ is simply the sum of the surpluses of consumers $\ell$ who purchase good $i$, which coincides with (2).

Two assumptions on the information structure warrant further discussion. First, the commitment assumption is standard in the literature and can be interpreted as a reputation concern.[12] In practice, Amazon shares data with other firms through algorithmic price recommendations based on consumer searches and purchases. We should expect that Amazon commits to these automated recommendations, rather than designing a new algorithm each time demand for a given product is realized. Second, Lemma 16 shows that, for any disclosure policy, consumers are better off when signals are private, while it has no effect on optimal disclosure for firms.[13] One interpretation of private disclosure is that the informed firm observes the uninformed firm's signal realization, but doesn't condition its pricing on it. For example, Amazon may observe the recommendations made to sellers, but any given recommendation typically does not feed back into its pricing.

**Pricing game.** *Fixing the information structure* $(S_2, \psi_2)$, firms play a pricing game in which they condition their pricing on their information by selecting mappings

$$\hat{\beta}_1 : \Theta \to \Delta(\mathbb{R}_+) \text{ and } \hat{\beta}_2 : S_2 \to \Delta(\mathbb{R}_+)$$

to maximize their expected profits.[14] The solution concept is Bayes Nash equilibrium (BNE). A strategy profile $(\hat{\beta}_1, \hat{\beta}_2)$ is a BNE if, for all $p_i \in \text{supp}\ \hat{\beta}_i$,

$$\int_{S_2} \int_{\mathbb{R}_+} \Pi_1((p_1, p_2); \theta) \mathrm{d}\hat{\beta}_2(p_2|s_2) \mathrm{d}\psi_2(s_2|\theta) \geq \int_{S_2} \int_{\mathbb{R}_+} \Pi_1((p_1', p_2); \theta) \mathrm{d}\hat{\beta}_2(p_2|s_2) \mathrm{d}\psi_2(s_2|\theta) \tag{3}$$

for all $p_1' \in \mathbb{R}_+$ and $\theta \in \Theta$ and

$$\sum_{\theta \in \Theta} \mu_\theta \int_{\mathbb{R}_+} \Pi_2((p_2, p_1); \theta) \mathrm{d}\hat{\beta}_1(p_1|\theta) \geq \sum_{\theta \in \Theta} \mu_\theta \int_{\mathbb{R}_+} \Pi_2((p_2', p_1); \theta) \mathrm{d}\hat{\beta}_1(p_1|\theta) \tag{4}$$

for all $p_2' \in \mathbb{R}_+$ and $s_2 \in S_2$. Denote by $\hat{\mathcal{E}}(S_2, \psi_2)$ the set of BNE in the pricing game.

For any information structure $(S_2, \psi_2)$, the existence and uniqueness of the BNE is guaranteed by Ui (2016), which provides sufficient conditions for the existence and uniqueness of the BNE in Bayesian games with concave and continuously differentiable payoff functions. This result is formalized in Lemma 1. The proofs of this result and all subsequent others are in Appendix A.3.

**Lemma 1** *For all information structures* $(S_2, \psi_2)$*, the set of BNE in the pricing game* $\hat{\mathcal{E}}(S_2, \psi_2)$ *is a singleton.*

[12] See for example Vives (1984), Novshek and Sonnenschein (1982) or Bergemann et al. (2015b).

[13] Similarly, Bergemann et al. (2015b) shows in a Cournot setting that it is with loss of generality to restrict attention to public information disclosure, since it comes at the cost of ex-ante welfare.

[14] Note that different information structures $(S_2, \psi_2)$ induce different optimal strategies for both firms.

**Information disclosure.** The choice of information structure $(S_2, \psi_2)$ determines the equilibrium in the pricing game. The designer chooses an information structure to maximize its ex-ante expected payoff such that $(\hat{\beta}_1^*(p_1|\theta), \hat{\beta}_2^*(p_2|s_2))$ is the BNE of the pricing game. I consider four objective functions for the designer:

i) Informed firm expected profits:

$$\mathbb{E}\left[\Pi_1((p_1,p_2);\theta)\right] = \sum_{\theta\in\Theta} \mu_\theta \int_{S_2}\int_{\mathbb{R}_+}\int_{\mathbb{R}_+} \Pi_1((p_1,p_2);\theta)\mathrm{d}\hat{\beta}_2^*(p_2|s_2)\mathrm{d}\hat{\beta}_1^*(p_1|\theta)\mathrm{d}\psi_2(s_2|\theta)$$

ii) Expected consumer surplus:

$$\mathbb{E}\left[CS((p_1,p_2);\theta)\right] = \frac{1}{2a}\sum_{i\in\{1,2\}}\sum_{\theta\in\Theta} \mu_\theta \int_{S_2}\int_{\mathbb{R}_+}\int_{\mathbb{R}_+} q_i((p_i,p_{-i});\theta)^2\mathrm{d}\hat{\beta}_1^*(p_1|\theta)\mathrm{d}\hat{\beta}_2^*(p_2|s_2)\mathrm{d}\psi_2(s_2|\theta)$$

iii) Expected producer surplus:

$$\sum_{i=1,2}\mathbb{E}\left[\Pi_i((p_1,p_2);\theta)\right] = \sum_{i\in\{1,2\}}\sum_{\theta\in\Theta} \mu_\theta \int_{S_2}\int_{\mathbb{R}_+}\int_{\mathbb{R}_+} \Pi_i((p_i,p_{-i});\theta)\mathrm{d}\hat{\beta}_1^*(p_1|\theta)\mathrm{d}\hat{\beta}_2^*(p_2|s_2)\mathrm{d}\psi_2(s_2|\theta)$$

iv) Expected welfare:

$$\mathbb{E}\left[W((p_1,p_2);\theta)\right] := \mathbb{E}\left[CS((p_1,p_2);\theta)\right] + \sum_{i=1,2}\mathbb{E}\left[\Pi_i((p_1,p_2);\theta)\right].$$

The interpretation of the role of the designer varies depending on their objective function. If the designer's objective is to maximize the informed firm's expected profits, then it is as if the informed firm is choosing how much information to disclose to its competitor. If the designer's objective is to maximize expected producer surplus, it is as if there is a collusive agreement between firms to determine optimal disclosure of information among them. If the designer's objective is to maximize expected consumer surplus or welfare, the interpretation of the designer is as a regulator.

The main effects of information disclosure are captured by the trade offs arising from the informed firm and the consumer optimal disclosures. The insights obtained by analyzing them extend to the producer surplus and welfare optimal disclosures.

**Equivalence to recommendation mechanisms.** The revelation principle of games of communication simplifies the information design problem by constraining the set of information structures. Taneva (2019) shows that it is without loss of generality to restrict attention to information structures where signals are equilibrium recommendations conditional on the state. I present an extension to compact action spaces and bounded, continuous real-valued payoff functions, restricting attention to $p_i \in \left[0, \frac{\theta_H}{a-b}\right]$ for all $i \in \{1,2\}$.[15] In a recommendation mechanism, the pricing rule $\sigma : \Theta \to \Delta\left(\left[0, \frac{\theta_H}{a-b}\right]^2\right)$ recommends a price for each firm such that the obedience constraints are satisfied, ensuring that firms are willing to follow the recommendation. Any pricing rule which satisfies the obedience constraints is a Bayes Correlated Equilibrium (BCE) as introduced by Bergemann and Morris (2013).[16] That is, a pricing rule $\sigma : \Theta \to \Delta\left(\left[0, \frac{\theta_H}{a-b}\right]^2\right)$ is a BCE if

$$\sum_{\theta\in\Theta} \mu_\theta \int_{p_{-i}\in\left[0,\frac{\theta_H}{a-b}\right]} \Pi_i((p_i, p_{-i}), \theta)\mathrm{d}\sigma((p_i, p_{-i})|\theta) \geq \sum_{\theta\in\Theta} \mu_\theta \int_{p_{-i}\in\left[0,\frac{\theta_H}{a-b}\right]} \Pi_i((p_i', p_{-i}), \theta)\mathrm{d}\sigma((p_i, p_{-i})|\theta)$$

for all $p_i \in \operatorname{supp} \sigma$, $p_i' \in \left[0, \frac{\theta_H}{a-b}\right]$, and $i \in \{1,2\}$, such that the distribution of the informed firm's price given the state is degenerated.[17] Every possible BCE distribution can be replicated as a BNE by appropriately choosing the information structure. A detailed discussion of this equivalence is presented in Appendix A.2.

# 2 Informed firm optimal disclosure

In this section, the informed firm directly determines its optimal information disclosure. That is, assume that the designer's objective is to maximize the informed firm's expected profits,

$$\mathbb{E}_{(\mu,\sigma)}[\Pi_1((p_1, p_2); \theta)] = \sum_{\theta\in\Theta} \mu_\theta \int \Pi_1((p_1, p_2); \theta)\mathrm{d}\sigma((p_1, p_2)|\theta).$$

The informed firm chooses a feasible obedient recommendation mechanism $\sigma$ to maximize its expected equilibrium profits in the pricing game. From its point of view, whether disclosure is private or public has no impact on the optimal disclosure policy.[18] Proposition 1 states that it is optimal for the informed firm to share its information.

**Proposition 1 (Informed firm optimal disclosure)** *It is optimal for the informed firm to fully reveal its private information to the uninformed firm.*

The optimal disclosure policy is determined by the fact that pricing choices are strategic complements, which determines the effect of changes in the precision of the uninformed firm's

[15] This is without loss of generality, since any price above $\frac{\theta_H}{a-b}$ induces no trade and zero profits for firm $i$.

[16] In my model, unlike in Bergemann and Morris (2013) in which both players are uninformed about the state, firm 1 learns the state before selecting prices. The definition of BCE is adapted to account for this.

[17] The informed firm observes a perfectly informative signal and its equilibrium prices are $p_1^\sigma(\theta) = \frac{\theta+\mathbb{E}_\sigma[p_2|\theta]}{2a}$.

[18] See the discussion in Section 5 and details in Appendix A.5.

signal on the informed firm's expected profits. In particular, using the uninformed firm's best response function, the informed firm's expected equilibrium profits, $\mathbb{E}_{(\mu,\sigma)}[\Pi_1^*((p_1, p_2); \theta)]$, can be expressed as

$$\mathbb{E}_{(\mu,\sigma)}[\Pi_1^*((p_1, p_2); \theta)] = a\mathbb{E}_\mu \left[ \left( \frac{\theta + b\mathbb{E}_\sigma[p_2|\theta]}{2a} \right)^2 \right],$$

where the expectation is taken with respect to the prior $\mu$ and the recommendation mechanism $\sigma$ since firm 1 commits to a recommendation mechanism before learning the state. Note that the informed firm's expected equilibrium profits are convex with respect to the conditional expectation of the uninformed firm's price. Then, maximizing the informed firm's expected equilibrium profits is equivalent to maximizing the distance between the expected equilibrium prices set by the uninformed firm across states, $\mathbb{E}_\sigma[p_2|\theta_L]$ and $\mathbb{E}_\sigma[p_2|\theta_H]$. Increasing the precision of the signal observed by the uninformed firm increases the correlation between its expected price and the state and, therefore, variation in its expected price. As a result, full disclosure maximizes the informed firm's expected profits.[19]

Intuitively, increasing the precision of the signal observed by the uninformed firm increases its certainty about the state, increasing (decreasing) expected demand when its posterior beliefs suggest that the high (low) state is more likely. Accordingly, the uninformed firm increases its expected equilibrium price in the high state and decreases it in the low state. As a result, with substitutes, more precise information disclosure increases (decreases) the informed firm's expected demand in the high (low) state. In the high state, a higher expected demand allows it to increase its price. The informed firm then increases its profits by raising the price on inframarginal consumers who were already buying its product and by gaining marginal consumers from the uninformed firm's market. The opposite is true in the low state since it charges a lower price and faces lower demand, but the expected profit gain in the high state exceeds the expected loss in the low state given the larger size of the market. Hence, the informed firm benefits from price correlation and its expected equilibrium profits increase in the precision of the uninformed firm's signal. Since this precision is maximized by full disclosure, it is optimal for the informed firm to fully disclose its private information.

This result is intuitively related to Kamenica and Gentzkow (2011), which finds that full disclosure is optimal when a sender's expected payoff is strictly convex. However, this

[19]This result relies on Assumption 1. Otherwise, if the high state is sufficiently high and the uninformed firm observes a sufficiently uninformative signal about demand, its prices can be only competitive when the state is high. Then, the informed firm can be effectively a monopolist when demand is low if it shares sufficiently imprecise information about the state. Therefore, from the informed firm's perspective, sharing their private information could harm it, because it can induce more competition when demand is low.

result doesn't directly apply to my setting, since the informed firm (the sender) and the uninformed firm (the receiver) play a game after the uninformed firm privately observes its signal realization. As such, payoffs not only depend on the state and the action of the uninformed firm, but also on the action of the informed firm. My results highlight that their intuition holds more broadly, not only in decision problems, but also in games in which payoffs are supermodular in the state and the actions of others.

Indeed, I show that the optimality of full disclosure doesn't rely on the convexity of payoffs or, equivalently, on the linearity of demand. As formalized in Proposition 6 in Appendix A.4, full disclosure is optimal if the informed firm's expected equilibrium profits are supermodular in the state and the uninformed firm's price. I also show that no disclosure is optimal if the informed firm's expected equilibrium profits are submodular in the state and the uninformed firm's price. This implies that it is optimal for the informed firm to reveal no information to its competitor when they compete by setting prices and offer differentiated complement goods. Kolotilin and Wolitzky (2020) obtain a related result in a setting in which the sender and the receiver do not interact. They show that supermodularity of the sender's objective function with respect to the state and the receiver's action is a sufficient condition for the optimality of full disclosure in decision problems. My results strengthen findings from previous work (Vives (1984), Vives (1990) and Raith (1996)), by showing the optimality of either full or no information disclosure in a setting of information asymmetry where the distribution of the uninformed firm's signal and the correlation with the informed firm's signal are unrestricted.[20] One takeaway is that it can be optimal for a firm to disclose information to a competitor even when it has no information to gain in return, because the firm can use disclosure to influence competitor prices.

# 3 Consumer optimal disclosure

In this section, I interpret the designer as a regulator whose objective is to maximize expected consumer surplus. We can interpret the regulator as a consumer protection agency who requires the informed firm to make its private information available to them. It can then privately share all or a subset of this information with the uninformed firm.[21]

In particular, assume that the designer's objective is to choose an obedient price recom-

[20] They also strengthen results from Novshek and Sonnenschein (1982), Clarke (1983) and Gal-Or (1985), given that Cournot with substitutes (complements) is equivalent to Bertrand with complements (substitutes) from the point of view of firms, as discussed in Raith (1996).

[21] Alternatively, the regulator can require the informed firm to directly share a specific subset of its information with the uninformed firm, as long as the informed firm's pricing cannot be conditioned on it.

mendation mechanism $\sigma$ that maximizes expected consumer surplus, given by

$$\mathbb{E}_{(\mu,\sigma)}[CS((p_1,p_2);\theta)] = \frac{1}{2a}\sum_{i\in\{1,2\}}\sum_{\theta\in\Theta}\mu_\theta\int q_i((p_i,p_{-i});\theta)^2\mathrm{d}\sigma((p_i,p_{-i})|\theta).$$

The optimal disclosure is formalized in Proposition 2. Partial disclosure is optimal for consumers when firms offer sufficiently close substitutes. Otherwise, no disclosure is optimal. Disclosure allows the uninformed firm to better tailor its price to the state, which in turns increases the ability of firms to extract surplus from consumers. But, private disclosure increases the benefit for consumers of reallocating across markets by creating a potential coordination failure in firm pricing.

**Proposition 2 (Consumer optimal disclosure)** *If the designer's objective is to maximize expected consumer surplus, there exists $\hat{\alpha}\in(0,1)$ such that partial disclosure is optimal if $\delta\in(\hat{\alpha},1)$ and no disclosure is optimal otherwise.*

Intuitively, the impact of disclosure on consumer surplus is determined through two channels. On the one hand, disclosure provides the uninformed firm with information about the state, which increases the correlation between its pricing and the state. Indirectly, this also increases pricing correlation across firms. Accordingly, in expectation, firms more accurately tailor their prices to the demand they face, allowing them to better extract surplus from consumers. On the other hand, it creates uncertainty in firms' pricing decisions, because both firms now have private information. Even if disclosure increases expected price correlation between firms, uncertainty about the signal realization observed by their competitor generates a pricing coordination failure with positive probability. That is, the uninformed firm may observe a signal realization that mismatches with the state, setting a price tailored to the incorrect state. In contrast, the informed firm sets a price tailored to the realized state. When the mismatch occurs and firms set different prices, consumers benefit by selecting from which firm to purchase after observing prices.

The relative impact of these effects is determined by the degree of differentiation between goods. When goods are close substitutes, a price differential between firms caused by partial private disclosure induces a large segment of the market to buy from the firm with a comparatively low price, creating large gains in consumer surplus with positive probability. In contrast, when goods are not close substitutes, the pricing coordination failure has little impact on the demand that firms face, yielding negligible benefits. Accordingly, when goods are sufficiently close substitutes, private partial disclosure creates a large enough expected benefit from a potential price coordination failure for the regulator to impose partial disclosure. Otherwise, no disclosure is optimal.

The sketch of the proof is as follows. First, I verify that no disclosure is always better for consumers than full disclosure. Second, I show that the difference between the expected consumer surplus with partial and no disclosure is a continuous and strictly increasing function of the degree of substitution. Third, I show that there exists a cutoff in the degree of differentiation above which partial disclosure is optimal.

More specifically, the difference between expected consumer surplus with partial and no disclosure is a linear combination of three moments: the variance of the uninformed firm's price, $\mathbb{V}_{(\mu,\sigma)}[p_2]$, the covariance between the uninformed firm's price and the state, $\mathrm{Cov}_{(\mu,\sigma)}(\theta, p_2)$, and the variance of the conditional expectation of the uninformed firm's price conditional on the state, $\mathbb{V}_{\mu}[\mathbb{E}_{\sigma}[p_2|\theta]]$. The expected gain in consumer surplus:

i) increases in $\mathbb{V}_{(\mu,\sigma)}[p_2]$, because it increases the opportunity for consumers to benefit from price differences between firms and substitute between them.
ii) decreases in $\mathrm{Cov}_{(\mu,\sigma)}[p_2, \theta]$, because it captures surplus extraction from consumers through the uninformed firm's better pricing decision.
iii) decreases in $\mathbb{V}_{\mu}[\mathbb{E}_{\sigma}[p_2|\theta]]$, because it reduces the informed firm's uncertainty about the uninformed firm's pricing and increases price correlation between firms.

Increasing information disclosure increases all three moments, but their relative magnitude is determined by the degree of differentiation, which pins down optimal pricing and consumers' willingness to substitute between goods. In particular, the benefit for consumers increases as firms offer closer substitutes, whereas the ability of firms to extract surplus decreases due to decreased market power. As a result, partial disclosure is optimal for consumers when firms offer sufficiently close substitutes.

The information environment I consider represents a lower bound on the potential benefits for consumers, because the benefits of private disclosure are minimized when there is complete information asymmetry between firms. That is, when the informed firm learns the state and the uninformed firm has no private information. This is because, when the informed firm is only partially informed, the regulator can induce a coordination failure in firm pricing with higher probability.

Furthermore, consumers can also benefit from disclosure when the uninformed firm's signal realization is noisily observed by the informed firm. As long as the informed firm is sufficiently uncertain about the information observed by the uninformed firm, it is possible for the regulator to create the coordination failure in prices with sufficiently high probability. Therefore, the optimality of partial disclosure doesn't rely on the fact that information disclosure is private, even though private disclosure maximizes the probability of inducing the coordination failure and is, as a result, optimal for consumers.

Next, when partial disclosure is optimal, I characterize the optimal partially informative recommendation mechanism in Proposition 3. The optimal price recommendation mechanism recommends at most two prices: a low price only recommended in the low state and a high price recommended in both states. The recommended prices maximize the uninformed firm's expected profits given its beliefs about the state. Then, the optimal price recommendation mechanism is characterized by the probability of recommending the low price in the low state, denoted by $\lambda^*$, where $\lambda^*$ determines the recommended prices $\hat{p}_L$ and $\hat{p}_H$ and is chosen to maximize expected consumer surplus subject to firm optimal pricing.[22]

**Proposition 3 (Consumer optimal recommendation mechanism)** *Any consumer optimal recommendation mechanism recommends at most two prices. If an optimal mechanism discloses information, then there exists an optimal mechanism that recommends one price $\hat{p}_L$ only when the state is low and another price $\hat{p}_H$ in both states, where*

$$\hat{p}_L = \frac{4a^2[1-\mu_L\lambda^*]\theta_L + b^2\mu_H[(1-\lambda^*)\theta_H - \theta_L]}{(2a-b)\left[4a^2(1-\mu_L\lambda^*) - b^2\mu_H\lambda^*\right]},\ \hat{p}_H = \frac{4a^2\left[\mu_H\theta_H + \mu_L(1-\lambda^*)\theta_L\right] - b^2\mu_H\lambda^*\theta_H}{(2a-b)\left[4a^2(1-\mu_L\lambda^*) - b^2\mu_H\lambda^*\right]},$$

*and $\lambda^* := \sigma(\hat{p}_L|\theta_L) \in (0,1)$ maximizes expected consumer surplus.*

Intuitively, consumers gain from disclosure when there are differences in firm pricing. When the state is high and the informed firm sets a corresponding high price, recommending an intermediate rather than a low price to the uninformed firm would provide less benefit to consumers, implying that it is best for consumers for at most a low and a high price to be recommended. Given that no intermediate price would be recommended in the high state, an intermediate price recommendation would reveal to the uninformed firm that the state is low, but the uninformed firm would only be willing to set the low price in that case. Hence, an optimal price recommendation mechanism recommends at most two prices.

With linear demand, recommending a unique price in the low or the high state is equivalent. Both options yield the same expected consumer surplus given that consumers benefit from price differentials induced by uncertainty among firms. Without loss of generality, I

[22] In this context, the price coordination failure occurs when the low state is realized and the uninformed firm is recommended to price high.

focus on the case in which two prices are recommended in the low state, which minimizes the level of prices set by firms.[23]

The sketch of the proof of Proposition 3 is as follows. First, I show that at most two prices are recommended in a state if only one price is recommended in the other state. If a unique price $\hat{p}$ is recommended in one state, observing any other recommendation $p_2 \neq \hat{p}$ reveals the state to the uninformed firm. When the uninformed firm knows the level of demand, the obedience constraint implies that there is a unique price that it is willing to set. As a result, it is not possible to recommend more than two obedient prices across states. Second, I show that it is optimal for the regulator to recommend a unique price in one state. These results imply that the optimal information structure sends at most two price recommendations.

Some of the intuition behind the characterization of the optimal disclosure relates to Kamenica and Gentzkow (2011). They show that there exists an optimal mechanism which induces a distribution of posteriors whose support has no more than $|\Theta|$ elements, here corresponding to at most two prices. However, as they discuss themselves, their results do not apply to settings with multiple receivers whose payoffs depend on each others' actions and in which the designer (the regulator) can send private signals to each receiver. This is because, for a given set of beliefs that firms hold after observing their signals, their actions may vary as a function of the disclosure policy that produced those beliefs. Accordingly, I extend the intuition behind their results to a setting in which firms privately observe a signal about demand before engaging in Bertrand competition with differentiated goods.

# 4 Producer surplus and welfare optimal disclosure

Information disclosure impacts surplus allocation between firms and between firms and consumers, with implications for total welfare. In this section, I first characterize the disclosure policy that maximizes expected producer surplus and, combining this result with the consumer optimal disclosure, derive the expected welfare maximizing disclosure policy.

[23] The first order conditions of the regulator's maximization problem are collinear. As a result, it is possible to set either the probability of recommending a low price in the low state or a high price in the high state to 1, since the optimality conditions define a relationship between them. In contrast, with a quadratic demand given by $q_i(p_i, p_{-i}; \theta) = \max\{0, \theta + bp_{-i} - ap_i - cp_i^2\}$ where $c$ is positive and sufficiently small, it is optimal to recommend two prices in the low state and one price in the high state.

## 4.1 Producer Surplus optimal disclosure

In this section, I interpret the designer as a collusive agreement between firms whose objective is to choose a disclosure policy to maximize expected producer surplus given by

$$\sum_{i=1,2} \mathbb{E}_{(\mu,\sigma)}[\Pi_i((p_i,p_{-i});\theta)] = \sum_{i\in\{1,2\}} \sum_{\theta\in\Theta} \mu_\theta \int \Pi_i((p_i,p_{-i});\theta)\mathrm{d}\sigma((p_i,p_{-i})|\theta).$$

First, it is optimal for the uninformed firm to learn the state, as stated in Lemma 2, because it increases the correlation between its pricing decisions and the state.

**Lemma 2** *The expected profits of the uninformed firm are maximized by full disclosure.*

Proposition 1 and Lemma 2 indicate that full disclosure is optimal for both firms. Thus, full disclosure maximizes expected producer surplus.

## 4.2 Welfare optimal disclosure

Assume that the designer, interpreted as a regulator, wants to maximize expected welfare, defined as the sum of expected consumer and producer surplus. The regulator trades off the effect of information disclosure on firms and consumers, given their conflicting preferences over disclosure policies. In particular, firms' expected profits are maximized by full disclosure, whereas expected consumer surplus is maximized by no or partial disclosure. However, the benefits from disclosure for both firms and consumers increase as firms offer closer substitutes. As a result, the optimal disclosure is again determined by the degree of differentiation, as stated in Proposition 4.

**Proposition 4 (Welfare optimal disclosure)** *If the designer's objective is to maximize expected welfare, there exists $\tilde{\alpha}_1 \in (0,1)$ and $\tilde{\alpha}_2 \in (0,1)$ such that $\tilde{\alpha}_1 \leq \tilde{\alpha}_2$ and*

*i) no disclosure is optimal when $\delta \in (0, \tilde{\alpha}_1]$.*

*ii) partial disclosure is optimal when $\delta \in (\tilde{\alpha}_1, \tilde{\alpha}_2)$.*

*iii) full disclosure is optimal when $\delta \in [\tilde{\alpha}_2, 1)$.*

When firms offer sufficiently close substitutes, full disclosure is optimal since it is optimal for firms and their expected gains exceed expected losses for consumers. When firms offer sufficiently differentiated substitutes, no disclosure maximizes expected welfare since it is optimal for consumers and the expected gains for firms from disclosure are small. For intermediate levels of differentiation, partial disclosure is optimal.[24]

Ui and Yoshizawa (2015) reach a similar conclusion, restricting attention to symmetric normally distributed private and public signals. When firms offer substitutes, they show that welfare decreases in the precision of private information and increases in the precision of public information, related to the optimality of either partial or full disclosure.

Proposition 5 characterizes the welfare optimal partially informative disclosure policy. Incentives for partial disclosure are driven by the effect of disclosure on consumer surplus. The qualitative features of the policy are shared with the consumer optimal one stated in Proposition 3. That is, any optimal partially informative recommendation mechanism has binary support, recommends one price only when the state is low, and another price in both states.

**Proposition 5 (Welfare optimal recommendation mechanism)** *Any welfare optimal recommendation mechanism recommends at most two prices. If the optimal mechanism discloses information, then it recommends one price $\hat{p}_L$ only when the state is low and another price $\hat{p}_H$ in both states, where*

$$\hat{p}_L = \frac{4a^2[1-\mu_L\lambda]\theta_L + b^2\mu_H[(1-\lambda)\theta_H - \theta_L]}{(2a-b)\left[4a^2(1-\mu_L\lambda) - b^2\mu_H\lambda\right]}, \; \hat{p}_H = \frac{4a^2\left[\mu_H\theta_H + \mu_L(1-\lambda)\theta_L\right] - b^2\mu_H\lambda\theta_H}{(2a-b)\left[4a^2(1-\mu_L\lambda) - b^2\mu_H\lambda\right]},$$

*and $\lambda^* := \sigma(\hat{p}_L|\theta_L) \in (0,1)$ maximizes expected welfare.*

These results suggest that a regulator whose objective is to maximize welfare faces a trade off between consumer and producer surplus, and must take into account the relationship between markets. They highlight that the task of a regulator can be more nuanced than simply banning or releasing information: the exact design of information matters.

# 5 Robustness of results

In this section, I discuss the robustness of the results and how they change as I enrich the model. A detailed discussion and proofs are provided in Appendix A.5.

First, consumers are better off when signal realizations are private instead of public since they benefit from the induced uncertainty between firms, whereas firms' optimal disclosure

[24] Suppose instead that the regulator maximizes the weighted sum of producer and consumer surplus, where $\omega \in [0,1]$ represents the weight assigned to consumers. The cutoffs $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ increase with $\omega$: for sufficiently high $\omega$, only no or partial disclosure can be optimal; for sufficiently low values of $\omega$, full disclosure is always optimal.

remains the same. When signals are public, the gain from partial disclosure disappears and no disclosure is optimal for consumers. However, consumers can also benefit from disclosure when the uninformed firm's signal realization is noisily observed by the informed firm. As long as the informed firm is sufficiently uncertain about the information observed by the uninformed firm, it is possible for the regulator to create the coordination failure in prices with sufficiently high probability. Therefore, the optimality of partial disclosure doesn't rely on information disclosure being private.

Second, consumers benefit more from partial disclosure as the asymmetry in market size between firms increases, implying that their benefits from partial disclosure are minimized when firms face markets of the same size. While firm preferences for information remain unaltered, partial disclosure increases consumer surplus in the informed firm's market and reduces it in the uninformed firm's market. Then, when the informed firm faces a bigger market than the uninformed firm, incentives for partial disclosure are larger. Hence, partial disclosure is optimal for a bigger range of degrees of differentiation. In practice, this is specially relevant since firms with an information advantage may often be larger, like Amazon. Lastly, the features of the consumer and welfare optimal recommendation mechanism generalize to this case.

Third, the main intuition of the consumer optimal disclosure extends to the case in which $N$ firms compete à la Bertrand and the designer selects an information structure from a constrained set. I consider the case in which the designer commits to an information structure with private signals to share all of the informed firm's private information with a subset of firms and no information with the rest. The informed firm's expected equilibrium profits are maximized by sharing its private information with all other firms, because it benefits from price correlation. However, if the designer's objective is to maximize expected consumer surplus, information disclosure between firms is at least partially restricted. The optimal information structure is determined by the degree of substitution and the number of firms in the market. It is optimal to share information with more firms as the number of firms increase in the market and as firms offer closer substitutes, but it is optimal to leave at least a fraction of firms uninformed. By leaving some firms uninformed, the designer is able to increase price heterogeneity, benefiting consumers.

Fourth, the informed firm's incentives to share information are amplified if it can charge a fee to the other firm to use its platform, but that consumer optimal and welfare optimal disclosures remain unaltered. Furthermore, the producer surplus optimal disclosure remains unchanged, since this represents a transfer between firms.

Fifth, the informed firm's incentives for information sharing are reversed when firms offer complements and the producer surplus optimal disclosure depends on the degree of complementary between goods. When goods are complements, disclosure increases the uninformed firm's profits, but reduces the informed firm's profits. In particular, if goods are sufficiently complementary, competitor prices have a significant impact on demand. Then, the negative effect of increased pricing correlation on the informed firm's profits exceeds the positive effect of learning about the state on the uninformed firm's profits. As a result, no disclosure is optimal. Otherwise, full disclosure is optimal.

Sixth, the proofs of Proposition 1 and Proposition 2 hold more generally for $[\theta_L, \theta_H]$, whereas the characterization of the optimal disclosure policy, Proposition 3, holds for $\{\theta_L, \theta_H\}$. In particular, considering more states would require increasing the number of price recommendations.

# 6 Conclusion

This paper studies information disclosure in a setting where two competing firms face ex-ante information asymmetry about the level of demand. I examine the incentives of an informed firm to share its private information with a competitor in a market with product differentiation and price competition. I show that the informed firm can have incentives to fully disclose its private information even without receiving information in return, because it allows it to influence competitor pricing. When firms offer substitutes, they benefit from price correlation, which implies that it is optimal for the informed firm to fully reveal its private information to the uninformed firm. When firms offer complements, it is optimal for the informed firm to not share any private information, which reduces the expected profits of its competitor. Accordingly, it can be optimal for a designer with the objective of maximizing producer surplus or maintaining competition to intervene and force information disclosure.

Information disclosure also impacts consumers. Even though complete information disclosure can help firms, it hurts consumers. I find that a regulator with the objective of protecting consumers would either completely restrict information disclosure between firms or only allow private partial disclosure, determined by the degree of differentiation between goods. If goods are sufficiently close substitutes, partial disclosure is optimal, because it increases the benefit for consumers to reallocate across markets. The consumer optimal partial disclosure reveals low levels of demand and obfuscates high levels to the uninformed firm.

Moreover, preferences for information disclosure between firms and consumers are not aligned. When firms offer substitutes, optimal disclosure depends on the degree of substitution, which determines the effect of disclosure on consumers and firms. If firms offer sufficiently differentiated goods, no disclosure maximizes expected welfare. If firms offer sufficiently close substitutes, full disclosure is optimal. For intermediate levels of differentiation, partial disclosure is optimal. Since incentives for partial disclosure arise from consumers, the optimal partial disclosure also reveals low levels and obfuscates high levels to the uninformed firm. My results highlight the wide scope for potential intervention by regulators, depending on their objective function and product differentiation.

This paper speaks in a preliminary form about the competition issues that arise when there is an unequal distribution of consumer data among firms. An important aspect not considered in this paper is the effect of information disclosure on firm entry and exit decisions. In particular, the informed firm could reduce its information disclosure, reducing its current profits to increase its market share and future profits by inducing uninformed firms to exit the market. In this context, a regulator may have incentives to force information disclosure between firms to maintain competition, which could indirectly benefit consumers. Furthermore, if firms could choose their product offering and the state reflected consumer preferences over horizontally differentiated goods, an informed firm may not want to disclose information if it would lead their competitor to offer a similar product and intensify price competition.

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# A Appendix

## A.1 Benchmark: binary signals

In this section, I derive the optimal information structures for the benchmark case in which signals are restricted to be binary and given by $S = \{s_L, s_H\}^2$. The joint distribution over signals is represented in Table 1, where the set of feasible distributions, denoted by $\mathcal{D}$, is $\mathcal{D} := \{(x_L, x_H) \in [0,1]^2 : x_L + x_H \geq 1\}$.

| $\theta = \theta_L$ | $s_2 = s_L$ | $s_2 = s_H$ |
|---|---|---|
| $s_1 = s_L$ | $x_L$ | $1 - x_L$ |
| $s_1 = s_H$ | $0$ | $0$ |

| $\theta = \theta_H$ | $s_2 = s_L$ | $s_2 = s_H$ |
|---|---|---|
| $s_1 = s_L$ | $0$ | $0$ |
| $s_1 = s_H$ | $1 - x_H$ | $x_H$ |

Table 1: Binary distribution of signals

**Lemma 3 (Binary Consumer optimal disclosure)** *If the designer's objective is to maximize expected consumer surplus, partial disclosure is optimal if $\delta \in (\hat{c}, 1)$ and no disclosure is optimal, otherwise.*

**Proof. Lemma 3.** Define $\Delta\mathbb{E}[CS](x_L, x_H)$ as the difference between the expected consumer surplus with no disclosure and with disclosure $(x_L, x_H)$. First, full disclosure is never optimal since

$$\Delta\mathbb{E}[CS](1,1) \geq \frac{\mu_L \mu_H \left(a^4 + b^4\right) \left(\theta_H - \theta_L\right)^2}{8a^3(2a-b)^2} \geq 0,$$

implying that either no or partial disclosure maximizes consumer surplus.

Second, I show that there exists $\hat{c} \in (0,1)$ such that partial disclosure is optimal if $\delta \geq \hat{c}$ and no disclosure is optimal otherwise. Note that the sign of $\Delta\mathbb{E}[CS](x_L, x_H)$ is determined by

$$\Phi(a, b, (x_L, x_H)) := f_1(a,b)\mathbb{V}[s_2] + f_2(a,b)\mu_L\mu_H(x_L + x_H - 1)^2 - f_3(a,b)\mathbb{E}[\mathbb{V}[s_2|\theta]],$$

where $f_k(a,b) > 0$ for all $k \in \{1,2,3\}$ and $\min\{f_1(a,b), f_3(a,b)\} > f_2(a,b)$. Furthermore, there exists $\hat{c} \in (0,1)$ such that $f_1(a,b) > f_3(a,b)$ if and only if $\delta < \hat{c}$. This implies that $f_1(a,b)\mathbb{V}[s_2] > f_3(a,b)\mathbb{E}[\mathbb{V}[s_2|\theta]]$ for $\delta < \hat{c}$ since $f_1(a,b) > f_3(a,b)$ and $\mathbb{V}[s_2] > \mathbb{E}[\mathbb{V}[s_2|\theta]]$. Thus, no disclosure is optimal when $\delta \leq \hat{c}$. Otherwise, partial disclosure is optimal since for all $\delta > \hat{c}$, there exists $(x_L, x_H) \in \mathcal{D}$ such that $\Phi(a, b, (x_L, x_H)) < 0$. ∎

**Lemma 4 (Binary Welfare optimal disclosure)** *Assume that the designer's objective is to maximize expected welfare. Then: i) if $\delta \in (0, \tilde{c}_1]$, no disclosure is optimal; ii) if $\delta \in (\tilde{c}_1, \tilde{c}_2)$, partial disclosure is optimal; and iii) if $\delta \in [\tilde{c}_2, 1)$, full disclosure is optimal.*

**Proof. Lemma 4.** It is straightforward to show that there exists $\overline{c} \in (0,1)$ such that full disclosure yields higher welfare than no disclosure if and only if $\delta \geq \tilde{c}$. Consider first the case in which $\delta \geq \overline{c}$ and define $\Delta\mathbb{E}[TS_1](x_L, x_H)$ as the difference in expected welfare with full and partial disclosure $(x_L, x_H)$. The sign of $\Delta\mathbb{E}[TS_1](x_L, x_H)$ is determined by

$$\rho_1(a,b,\mu,(x_L,x_H)) = f_4(a,b)\mathbb{V}[s_2] + f_5(a,b)\mathbb{E}[\mathbb{V}[s_2|\theta]] + \mu_L\mu_H f_6(a,b)(x_L + x_H - 1)^2 \text{ where}$$
$$f_4(a,b) = 16a^5(3b-a),\ \ f_5(a,b) = 4a^2b^2\left(5a^2 - b^2\right) \text{and } f_6(a,b) = b^2(16a^4 - 12a^3b + a^2b^2 - b^4).$$

Note that there exists $\tilde{c}_2 \in (\overline{c}, 1)$ such that $\rho_1(a,b,\mu,(x_L,x_H)) > 0$ for all $(x_L,x_H) \in \mathcal{D}$ if $\delta \geq \tilde{c}_2$ and there exists $(x_L,x_H) \in \mathcal{D}$ such that $\rho_1(a,b,\mu,(x_L,x_H)) < 0$ if $\delta < \tilde{c}_2$. Thus, full disclosure is optimal if $\delta \geq \tilde{c}_2$ and partial is optimal when $\delta \in (\overline{c}, \tilde{c}_2]$. Similarly, consider the case in which $\delta < \overline{c}$ and define $\Delta\mathbb{E}[TS_2](x_L, x_H)$ as the difference of expected welfare with no and partial disclosure $(x_L, x_H)$. Analogously, it is possible to show that no disclosure is optimal when $\delta \leq \tilde{c}_1$ and that partial disclosure is optimal when $\delta \in (\tilde{c}_1, \overline{c}]$. In summary, no disclosure is optimal if $\delta \leq \tilde{c}_1$, partial disclosure is optimal if $\delta \in (\tilde{c}_1, \tilde{c}_2]$ and full disclosure is optimal if $\delta > \tilde{c}_2$. ■

## A.2 Preliminary results

### A.2.1 Equivalence to recommendation mechanisms

This section simplifies the information design problem by constraining the set of information structures. Taneva (2019) shows that it is without loss of generality to restrict attention to information structures where signals are equilibrium recommendations conditional on the state. I present an extension to compact action spaces and bounded, continuous real-valued payoff functions, restricting attention to $p_i \in \left[0, \frac{\theta_H}{a-b}\right]$ for all $i \in \{1,2\}$.[25] In a recommendation mechanism, the pricing rule $\tilde{\sigma} : \Theta \to \Delta\left(\left[0, \frac{\theta_H}{a-b}\right]^2\right)$ recommends a price for each firm such that the obedience constraints are satisfied, ensuring that firms are willing to follow the price recommendation. Any pricing rule which satisfies the obedience constraints is a Bayes Correlated Equilibrium (BCE), as introduced by Bergemann and Morris (2013).[26]

[25] This is without loss of generality, since any price above $\frac{\theta_H}{a-b}$ induces profits of zero for firm $i$.

[26] Unlike in Bergemann and Morris (2013) in which both players are uninformed about the state, firm 1 learns the state before selecting prices. The definition of BCE is adapted to take this into account.

A pricing rule $\tilde{\sigma} : \Theta \to \Delta\left(\left[0, \frac{\theta_H}{a-b}\right]^2\right)$ is a BCE if

$$\int_{p_2 \in \left[0, \frac{\theta_H}{a-b}\right]} \Pi_1((p_1, p_2), \theta) \mathrm{d}\tilde{\sigma}((p_1, p_2)|\theta) \geq \int_{p_2 \in \left[0, \frac{\theta_H}{a-b}\right]} \Pi_1((p_1', p_2), \theta) \mathrm{d}\tilde{\sigma}((p_1, p_2)|\theta)$$

for all $p_1 \in \text{supp } \tilde{\sigma}$, $p_1' \in \left[0, \frac{\theta_H}{a-b}\right]$ and

$$\sum_{\theta \in \Theta} \mu_\theta \int_{p_1 \in \left[0, \frac{\theta_H}{a-b}\right]} \Pi_2((p_2, p_1), \theta) \mathrm{d}\tilde{\sigma}((p_2, p_1)|\theta) \geq \sum_{\theta \in \Theta} \mu_\theta \int_{p_1 \in \left[0, \frac{\theta_H}{a-b}\right]} \Pi_2((p_2', p_1), \theta) \mathrm{d}\tilde{\sigma}((p_2, p_1)|\theta)$$

for all $p_2 \in \text{supp } \tilde{\sigma}$ and $p_2' \in \left[0, \frac{\theta_H}{a-b}\right]$.

Consider an analogous information environment in which both firms observe a signal about the state with private signal realizations such that the informed firm's signal is perfectly informative. In what follows, I show that it is without loss of generality to interpret signals $(s_1, s_2)$ as equilibrium recommendations in which each signal recommends a price to each firm. Define the information structure as the joint distribution of signals. Let $S_i$ be the set of private signal realizations for firm $i$. An information structure consists of a set of signal realizations and a family of conditional distributions $\psi : \Theta \to \Delta(S)$, where $S = S_1 \times S_2 = \{s_L, s_H\} \times S_2$. Let $\psi_i : \Theta \to \Delta(S_i)$ be the marginal distribution of signal $s_i \in S_i$ given the information structure $(S, \pi)$. The distribution $\psi_1$ is fully informative, which implies that the probability of observing signal $s_k$ conditional on state $\theta_k$ is 1.

Given the information structure $(S, \psi)$, firms play a pricing game in which they condition their pricing choices on their signal realization by selecting a mapping $\beta_i : S_i \to \Delta(\left[0, \frac{\theta_H}{a-b}\right])$ to maximize their expected profits. A strategy profile $(\beta_1, \beta_2)$ is a BNE if, for all $p_i \in \left[0, \frac{\theta_H}{a-b}\right]$ with $\beta_i(p_i|s_i) > 0$ for all $i$, we have

$$\begin{aligned} &\sum_{\theta \in \Theta} \mu_\theta \int_{S_{-i}} \int_0^{\frac{\theta_H}{a-b}} \Pi_i((p_i, p_{-i}); \theta) \mathrm{d}\beta_{-i}(p_{-i}|s_{-i}) \mathrm{d}\psi((s_i, s_{-i})|\theta) \\ \geq &\sum_{\theta \in \Theta} \mu_\theta \int_{S_{-i}} \int_0^{\frac{\theta_H}{a-b}} \Pi_i((p_i', p_{-i}); \theta) \mathrm{d}\beta_{-i}(p_{-i}|s_{-i}) \mathrm{d}\psi((s_i, s_{-i})|\theta) \end{aligned} \tag{5}$$

for all $p_i' \in \left[0, \frac{\theta_H}{a-b}\right]$, $s \in S$ and $i \in \{1, 2\}$. Denote by $\mathcal{E}(S, \psi)$ the set of BNE.

Similarly, we can define a pricing rule $\sigma : \Theta \to \Delta\left(\left[0, \frac{\theta_H}{a-b}\right]^2\right)$ which is a BCE if

$$\sum_{\theta \in \Theta} \mu_\theta \int_{p_{-i} \in \left[0, \frac{\theta_H}{a-b}\right]} \Pi_i((p_i, p_{-i}), \theta) \mathrm{d}\sigma((p_i, p_{-i})|\theta) \geq \sum_{\theta \in \Theta} \mu_\theta \int_{p_{-i} \in \left[0, \frac{\theta_H}{a-b}\right]} \Pi_i((p_i', p_{-i}), \theta) \mathrm{d}\sigma((p_i, p_{-i})|\theta) \tag{6}$$

for all $p_i \in \text{supp } \sigma$, $p_i' \in \left[0, \frac{\theta_H}{a-b}\right]$, and $i \in \{1, 2\}$, in which the distribution of the informed firm's price given the state is degenerated.[27]

First, Lemma 5 is an equivalence result stating that every possible BCE distribution can be replicated as a BNE by appropriately choosing the information structure. Intuitively, any correlation between obedient pricing choices can be generated as a BCE. In a BNE, all the correlation between pricing choices is generated through the information structure $(S, \psi)$.

**Lemma 5** *The set of BCE coincides with* $\cup_{(S,\psi)}\mathcal{E}(S, \psi)$.

Second, Lemma 6 implies that it is without loss of generality to restrict attention to recommendation mechanisms. Formally, an information structure $(S, \psi)$ is a recommendation mechanism if $S = \left[0, \frac{\theta_H}{a-b}\right]^2$. In a recommendation mechanism, signals act as pricing recommendations which firms are willing to follow as long as their competitor does as well.

**Lemma 6** *For every* $\sigma \in \cup_{(S,\psi)}\mathcal{E}(S, \psi)$, *there exists a recommendation mechanism* $\left(\left[0, \frac{\theta_H}{a-b}\right]^2, \sigma\right)$ *such that* $\sigma \in \mathcal{E}\left(\left[0, \frac{\theta_H}{a-b}\right]^2, \sigma\right)$.

### A.2.2 Existence of optimal recommendation mechanism

The existence of the optimal recommendation mechanism stated in Lemma 7 is guaranteed by the Weierstrass extreme value theorem. First, the existence of correlated equilibria for games in which players receive private signals and simultaneously choose actions from compact sets is established in Stinchcombe (2011). Second, the set of BCE is compact in the weak* topology, since it is the set of all probability measures on a compact set.[28] Then, the designer's problem is to maximize a continuous function of $\sigma$ over a non-empty compact set.

**Lemma 7** *The optimal recommendation mechanism exists.*

[27] The informed firm's equilibrium prices are given by $p_1^\sigma(\theta) = \frac{\theta + \mathbb{E}_\sigma[p_2|\theta]}{2a}$.

[28] With full disclosure, equilibrium prices are

$$p^F(\theta) = \frac{\theta}{2a - b}.$$

It follows that firms have no incentives to set prices above $p^F(\theta_H)$ or below $p^F(\theta_L)$, because such prices would never be part of a BNE of the pricing game. Hence, the support of any obedient recommendation mechanism must be a subset of $[p^F(\theta_L), p^F(\theta_H)]^2$. See Appendix A.3 for a formal argument.

## A.3 Proofs

### A.3.1 Preliminary results: proofs

**Proof. Lemma 1.** The pricing game is a smooth concave game since $\Pi_i((\cdot, p_{-i});\theta) : \mathbb{R}_+ \to \mathbb{R}$ is concave and continuously differentiable for each $p_{-i} \in \mathbb{R}_+$ since the demand is linear in $p_{-i}$. Define the payoff gradient as

$$\nabla\Pi(\mathbf{p}, \theta) := \left( \frac{\partial \Pi_i((p_i, p_{-i});\theta)}{\partial p_i} \right)_{i\in\{1,2\}},$$

where firm $i$'s ex-post payoff function is given by $\Pi_i((p_i, p_{-i});\theta) = p_i(\theta - ap_i + bp_{-i})$. Then, the payoff gradient, given by

$$\nabla\Pi(\mathbf{p}, \theta) = (\theta + bp_{-i} - 2ap_i)_{i\in\{1,2\}},$$

is continuously differentible. The Jacobian matrix of the payoff gradient, given by

$$F_{\nabla\Pi}(\mathbf{p}, \theta) := \begin{pmatrix} \frac{\partial^2\Pi_1((p_1,p_2);\theta)}{\partial p_1^2} & \frac{\partial^2\Pi_1((p_1,p_2);\theta)}{\partial p_1 \partial p_2} \\ \frac{\partial^2\Pi_2((p_2,p_1);\theta)}{\partial p_1 \partial p_2} & \frac{\partial^2\Pi_2((p_2,p_1);\theta)}{\partial p_2^2} \end{pmatrix} = \begin{pmatrix} -2a & b \\ b & -2a \end{pmatrix},$$

is negative definite because $-2a < 0$ and $4a^2 - b^2 > 0$ since $a > |b|$. This implies that the payoff gradient $\nabla\Pi(\mathbf{p}, \theta)$ is strictly monotone by Lemma 4 from Ui (2016). Furthermore, since for all $\mathbf{p}$, there exists $c > 0$ such that

$$\mathbf{p}^T F_{\nabla\Pi}(\mathbf{p}, \theta)\mathbf{p} < -c\mathbf{p}^T\mathbf{p},$$

the payoff gradient is also strongly monotone by the same lemma. Then, the uniqueness of the Bayesian Nash equilibrium of the pricing game follows from Proposition 1 from Ui (2016), which states that if the payoff gradient is strictly monotone, the Bayesian game as at most one Bayesian Nash equilibrium. The existence of a unique Bayesian Nash equilibrium follows from Proposition 2 from Ui (2016). ■

**Proof. Lemma 5.** First, I show that the set of BCE is a subset of $\cup_{(S,\psi)}\mathcal{E}(S, \psi)$. Assume $\sigma \in BCE$. Then, $\sigma$ satisfies

$$\sum_{\theta\in\Theta} \mu_\theta \int_{p_{-i}\in\left[0, \frac{\theta_H}{a-b}\right]} \Pi_i((p_i, p_{-i}), \theta) \mathrm{d}\sigma((p_i, p_{-i})|\theta) \geq \sum_{\theta\in\Theta} \mu_\theta \int_{p_{-i}\in\left[0, \frac{\theta_H}{a-b}\right]} \Pi_i((p_i', p_{-i}), \theta) \mathrm{d}\sigma((p_i, p_{-i})|\theta) \quad (7)$$

for all $p_i \in \text{supp } \sigma$, $p_i' \in \left[0, \frac{\theta_H}{a-b}\right]$ and $i \in \{1, 2\}$.

Consider an information structure $\left(\left[0, \frac{\theta_H}{a-b}\right]^2, \psi^*\right)$ where $\left[0, \frac{\theta_H}{a-b}\right]^2$ is the set of signal realizations and $\psi^* : \Theta \to \Delta(\left[0, \frac{\theta_H}{a-b}\right]^2)$ coincides with $\sigma$, i.e. $\sigma = \psi^*$. Let

$$\beta_i^*(p_i|p_i') = \begin{cases} 1 & if\ p_i = p_i' \\ 0 & otherwise \end{cases}$$

be the obedient strategy. Then, the right-hand side of (7) can be written as

$$
\begin{aligned}
&\sum_{\theta\in\Theta}\mu_\theta\int_{p_{-i}\in\left[0,\frac{\theta_H}{a-b}\right]}\Pi_i((p_i',p_{-i}),\theta)\mathrm{d}\sigma((p_i,p_{-i})|\theta)\\
&=\sum_{\theta\in\Theta}\mu_\theta\int_{p_{-i}\in\left[0,\frac{\theta_H}{a-b}\right]}\Pi_i((p_i',p_{-i}),\theta)\mathrm{d}\psi^*((p_i,p_{-i})|\theta)\\
&=\sum_{\theta\in\Theta}\mu_\theta\int_{s_{-i}\in\left[0,\frac{\theta_H}{a-b}\right]}\int_{p_{-i}\in\left[0,\frac{\theta_H}{a-b}\right]}\Pi_i((p_i',p_{-i});\theta)\mathrm{d}\beta^*_{-i}(p_{-i}|s_{-i})\mathrm{d}\psi^*((s_i,s_{-i})|\theta)
\end{aligned}
$$

The first equality holds by definition of $\psi^*$. The second equality holds by definition of the obedient strategy and Fubini's theorem since, fixing $\theta$, $\Pi_i((p_i,p_{-i});\theta)$ is $\sigma$-integrable because $\Pi_i|\theta:\left[0,\frac{\theta_H}{a-b}\right]^2\to\mathbb{R}_+$ is a bounded and continuous real-valued function on a compact set.[29] Hence, the BNE incentive-compatibility constraints are implied by the BCE obedience constraints. This, in turn, implies that if $\sigma\in BCE$, then $\sigma$ is also a BNE of the game. Thus, the set of BCE is a subset of the set of BNE of the game.

Second, I show that $\cup_{(S,\psi)}\mathcal{E}(S,\psi)$ is a subset of BCE. Consider a BNE composed by an information structure $(\hat{S},\hat{\psi})$ with $\hat{\psi}:\Theta\to\Delta(S)$ and measurable behavioral strategies $(\hat{\beta}_i,\hat{\beta}_{-i})$.[30] Given the behavioral strategies $(\hat{\beta}_i,\hat{\beta}_{-i})$, define $\hat{\beta}:S\to\Delta\left(\left[0,\frac{\theta_H}{a-b}\right]^2\right)$ as the joint measure. Let $\hat{\sigma}:\Theta\to\Delta\left(\left[0,\frac{\theta_H}{a-b}\right]^2\right)$ be the composition of $\hat{\psi}$ and $\hat{\beta}$, defined as $\hat{\sigma}=\hat{\beta}\circ\hat{\psi}$. Then, by definition $\hat{\sigma}\in\cup_{(S,\psi)}\mathcal{E}(S,\psi)$. The definition of BNE implies that $(\hat{S},\hat{\psi})$ and $\hat{\beta}$ satisfy:

$$
\begin{aligned}
&\sum_{\theta\in\Theta}\mu_\theta\int_{\hat{S}_{-i}}\int_{p_{-i}\in\left[0,\frac{\theta_H}{a-b}\right]}\Pi_i((p_i,p_{-i});\theta)\mathrm{d}\hat{\beta}_{-i}(p_{-i}|s_{-i})\mathrm{d}\hat{\psi}((s_i,s_{-i})|\theta)\\
\geq&\sum_{\theta\in\Theta}\mu_\theta\int_{\hat{S}_{-i}}\int_{p_{-i}\in\left[0,\frac{\theta_H}{a-b}\right]}\Pi_i((p_i',p_{-i});\theta)\mathrm{d}\hat{\beta}_{-i}(p_{-i}|s_{-i})\mathrm{d}\hat{\psi}((s_i,s_{-i})|\theta)
\end{aligned}\tag{8}
$$

for all $p_i'\in\left[0,\frac{\theta_H}{a-b}\right]$, $s\in S$ and $i\in\{1,2\}$. Integrating both sides of the BNE incentive-compatibility constraint, we have

$$
\begin{aligned}
&\sum_{\theta\in\Theta}\mu_\theta\int_{\hat{S}_i}\int_{\hat{S}_{-i}}\int_{p_{-i}\in\left[0,\frac{\theta_H}{a-b}\right]}\Pi_i((p_i,p_{-i});\theta)\mathrm{d}\hat{\beta}_i(p_i|s_i)\mathrm{d}\hat{\beta}_{-i}(p_{-i}|s_{-i})\mathrm{d}\hat{\psi}(s|\theta)\\
\geq&\sum_{\theta\in\Theta}\mu_\theta\int_{\hat{S}_i}\int_{\hat{S}_{-i}}\int_{p_{-i}\in\left[0,\frac{\theta_H}{a-b}\right]}\Pi_i((p_i',p_{-i});\theta)\mathrm{d}\hat{\beta}_i(p_i|s_i)\mathrm{d}\hat{\beta}_{-i}(p_{-i}|s_{-i})\mathrm{d}\hat{\psi}(s|\theta)
\end{aligned}
$$

[29]See theorem 11.27 from Aliprantis and Border (2013) where the condition of theorem are satisfied by Proposition 3.3 and Theorem 4.4 from from Royden (1968)

[30]Behavioral strategies $\beta_i:S_i\to\Delta\left(\left[0,\frac{\theta_H}{a-b}\right]\right)$ for all $i\in\{1,2\}$ are defined as a regular conditional probabilities as defined in Appendix C from Bass (2011).

Then, (8) implies that

$$\sum_{\theta\in\Theta}\mu_\theta\int_{p_{-i}\in\left[0,\frac{\theta_H}{a-b}\right]}\Pi_i((p_i,p_{-i}),\theta)\mathrm{d}\sigma((p_i,p_{-i})|\theta)\geq\sum_{\theta\in\Theta}\mu_\theta\int_{p_{-i}\in\left[0,\frac{\theta_H}{a-b}\right]}\Pi_i((p_i',p_{-i}),\theta)\mathrm{d}\sigma((p_i,p_{-i})|\theta)$$

■

**Proof. Lemma 6.** Consider a distribution $\sigma\in\cup_{(S,\psi)}\mathcal{E}(S,\psi)$. Lemma 5 implies that $\sigma\in BCE$. Consider the recommendation mechanism $(\left[0,\frac{\theta_H}{a-b}\right]^2,\psi_\sigma)$ where $\psi_\sigma=\sigma$ for all $(p_1,p_2)\in\left[0,\frac{\theta_H}{a-b}\right]^2$ and $\theta\in\Theta$ and the obedient behavioral strategy

$$\beta_i^*(p_i|p_i')=\begin{cases}1 & if\ p_i=p_i'\\ 0 & otherwise\end{cases}.$$

The interim expected payoff of firm $i$ when firm $-i$ follows $\beta_{-i}^*$ is

$$\begin{aligned}\sum_{\theta\in\Theta}\mu_\theta\int_{S_{-i}}\int_0^{\frac{\theta_H}{a-b}}&\Pi_i((p_i',p_{-i});\theta)\mathrm{d}\beta_{-i}^*(p_{-i}|p_{-i}')\mathrm{d}\psi_\sigma((p_i,p_{-i}')|\theta)\\ &=\sum_{\theta\in\Theta}\mu_\theta\int_0^{\frac{\theta_H}{a-b}}\Pi_i((p_i',p_{-i});\theta)\mathrm{d}\psi_\sigma((p_i,p_{-i})|\theta)\\ &=\sum_{\theta\in\Theta}\mu_\theta\int_0^{\frac{\theta_H}{a-b}}\Pi_i((p_i',p_{-i});\theta)\mathrm{d}\sigma((p_i,p_{-i})|\theta)\end{aligned}\tag{9}$$

for all $i$. Hence, the definition of BCE and (9) imply

$$\sum_{\theta\in\Theta}\mu_\theta\int_0^{\frac{\theta_H}{a-b}}\Pi_i((p_i,p_{-i});\theta)\mathrm{d}\psi_\sigma((p_i,p_{-i})|\theta)\geq\sum_{\theta\in\Theta}\mu_\theta\int_0^{\frac{\theta_H}{a-b}}\Pi_i((p_i',p_{-i});\theta)\mathrm{d}\psi_\sigma((p_i,p_{-i})|\theta)$$

for all $p_i'\in\left[0,\frac{\theta_H}{a-b}\right]$ and $i$. The distribution of prices conditional on the state $\theta$ under $\beta^*$ and $(\left[0,\frac{\theta_H}{a-b}\right]^2,\sigma)$ is $\psi_\sigma=\sigma$. Thus, $\sigma\in\mathcal{E}\left(\left[0,\frac{\theta_H}{a-b}\right]^2,\sigma\right)$. ■

**Lemma 8** *The support of the distribution $\sigma((p_1,p_2)|\theta)$ is a subset of $[p^F(\theta_L),p^F(\theta_H)]^2$ for all $\theta\in\Theta$, where $p^F(\theta)$ is the equilibrium price with full disclosure when the state $\theta$ is realized.*

**Proof. Lemma 8.** The minimum and maximum price in any equilibrium is charged when both firms know that the state is low and that the state is high, respectively. That is, the highest and lowest equilibrium prices occur with full disclosure. Under full disclosure

$\sigma^F$, both firms learn the state. Let $p^F(\theta)$ be the equilibrium price under full disclosure when the state is $\theta$, where

$$p^F(\theta_L) = \frac{\theta_L}{(2a-b)} \text{ and } p^F(\theta_H) = \frac{\theta_H}{(2a-b)}$$

Hence, any obedient recommendation mechanism must recommend prices in the set of feasible equilibrium prices denoted by $[p^F(\theta_L), p^F(\theta_H)]^2$. ■

**Proof. Lemma 7.** The set of BCE is the collection of distributions $\sigma : \Theta \to \Delta([p^F(\theta_L), p^F(\theta_H)]^2)$ such that

*i)* $\sigma((p_1, p_2)|\theta) \geq 0$ for all $(p_1, p_2) \in [p^F(\theta_L), p^F(\theta_H)]^2$ and $\theta \in \Theta$,

*ii)* $\displaystyle\int \mathrm{d}\sigma((p_1, p_2)|\theta) = 1$ for all $\theta \in \Theta$ and

*iii)* $\displaystyle\sum_{\theta\in\Theta} \mu_\theta \int_{p_{-i}\in\mathbb{R}_+} \Pi_i((p_i, p_{-i}), \theta)\mathrm{d}\sigma((p_i, p_{-i})|\theta) \geq \sum_{\theta\in\Theta} \mu_\theta \int_{p_{-i}\in\mathbb{R}_+} \Pi_i((p_i', p_{-i}), \theta)\mathrm{d}\sigma((p_i, p_{-i})|\theta)$

for all $p_i \in \text{supp } \sigma,\ p_i' \in \mathbb{R}_+$ and $i \in \{1, 2\}$.

First, Theorem A from Stinchcombe (2011) establishes the existence of Correlated equilibrium in games in which players receive private signals and then simultaneously choose actions from compact sets. Formally, consider a game in which the set of players $I$ is finite and for each $i$, the type $\omega_i$ belongs to the measure space $(\Omega_i, \mathcal{F}_i)$. Each player $i$ simultaneously chooses an action from a compact set $A_i$ and denote by $\Delta_i$ the set of countably additive Borel probabilities in $A_i$, with the weak* topology. Let $\mathbb{B}_i(\mathcal{F}_i)$ be the set of $i$'s behavioral strategies, defined as the $\mathcal{F}_i$-measurable functions from $\Omega_i$ to $\Delta_i$. Given a vector $b \in \mathbb{B} := \times_i \mathbb{B}_i(\mathcal{F}_i)$, player $i$'s expected utility if $b$ is played is defined by

$$u_i^P(b) = \int_\Omega \langle u_i(\omega), \times_i b_i(\omega)\rangle P(\mathrm{d}\omega)$$

where $\langle f, \nu\rangle := \int_A f(a)\nu(\mathrm{d}a)$ for $f : A \to \mathbb{R}$ and Borel probabilities $\nu$, and $\times_i b_i$ is the product probability on $A$ having $b_i$ as the marginal. $(\mathbb{B}_i(\mathcal{F}_i), u_i^P)_{i\in I})$ denotes the normal form game. Then, Theorem A shows that all games $(\mathbb{B}_i(\mathcal{F}_i), u_i^P)_{i\in I})$ have correlated equilibria.

In the pricing game, two firms simultaneously choose a price to maximize their expected equilibrium profits,

$$\sum_{\theta\in\Theta} \mu_\theta \int_{p_{-i}\in\mathbb{R}_+} \Pi_i((p_i, p_{-i}), \theta)\mathrm{d}\sigma((p_i, p_{-i})|\theta).$$

Note that $p_i \in \left[0, \frac{\theta_H}{a-b}\right]$ for all $i \in \{1, 2\}$. Hence, firms simultaneously choose prices from compact sets. Thus, this result implies that the set of BCE is non-empty.

Second, the set of BCE is the collection of distributions

$$\sigma : \Theta \to \Delta([p^F(\theta_L), p^F(\theta_H)]^2),$$

which corresponds to the set of all probability measures on $[p^F(\theta_L), p^F(\theta_H)]^2$ for each $\theta \in \Theta$ where $\Theta$ is finite. Then, the set of BCE is compact since $[p^F(\theta_L), p^F(\theta_H)]^2$ is compact in the weak* topology, by Theorem 15.11 from Aliprantis and Border (2013).

The designer's objectives are

$$
\begin{aligned}
&i) \text{ Informed firm optimal}: \sum_{\theta\in\Theta} \mu_\theta \int \Pi_1((p_1,p_2),\theta)\mathrm{d}\sigma((p_1,p_2)|\theta) \\
&ii) \text{ PS optimal}: \sum_{i\in\{1,2\}} \sum_{\theta\in\Theta} \mu_\theta \int \Pi_i((p_i,p_{-i}),\theta)\mathrm{d}\sigma((p_i,p_{-i})|\theta) \\
&iii) \text{ CS-optimal}: \frac{1}{2a} \sum_{i\in\{1,2\}} \sum_{\theta\in\Theta} \mu_\theta \int q_i((p_i,p_{-i});\theta)^2\mathrm{d}\sigma((p_i,p_{-i})|\theta)
\end{aligned}
$$

and

$$
\begin{aligned}
iv) \text{ Welfare-optimal}: &\sum_{i\in\{1,2\}} \sum_{\theta\in\Theta} \mu_\theta \int \Pi_i((p_i,p_{-i}),\theta)\mathrm{d}\sigma((p_i,p_{-i})|\theta) \\
&+ \frac{1}{2a} \sum_{i\in\{1,2\}} \sum_{\theta\in\Theta} \mu_\theta \int q_i((p_i,p_{-i});\theta)^2\mathrm{d}\sigma((p_i,p_{-i})|\theta)
\end{aligned}
$$

Third, the continuity of all objective functions in the weak* topology follows from Corollary 15.7 from Aliprantis and Border since because both $\Pi_i((p_i,p_{-i}),\theta)$ and $q_i((p_i,p_{-i}),\theta)$ are continuous and bounded functions. Hence, the integral $\int \Pi_i \mathrm{d}\sigma((p_i,p_{-i})|\theta)$ and $\int q_i^2 \mathrm{d}\sigma((p_i,p_{-i})|\theta)$ is continuous in $\sigma$. Thus, the designer's problem is to maximize a continuous objective function in a compact set. The existence of a solution is guaranteed by the Weierstrass extreme value theorem. ■

### A.3.2 Informed firm optimal disclosure

**Proof. Proposition 1.** The fully disclosing information structure recommends prices $(p^F(\theta), p^F(\theta))$ with probability 1 for all $\theta \in \Theta$. Full disclosure is optimal for the informed firm if her expected equilibrium payoffs with full disclosure exceed her expected equilibrium payoffs induced by any other obedient recommendation mechanism. That is,

$$\sum_{\theta\in\Theta} \mu_\theta \Pi_1((p^F(\theta), p^F(\theta));\theta) \geq \sum_{\theta\in\Theta} \mu_\theta \int \Pi_1((p_1,p_2);\theta)\mathrm{d}\sigma((p_1,p_2)|\theta) \tag{10}$$

for all $\sigma : \Theta \to \Delta([p^F(\theta_L), p^F(\theta_H)]^2)$ that satisfy the obedience constraints and $p_1$. The obedience constraints requires that $p_1$ must be a best response for firm 1. Then, for all recommendation mechanism $\sigma$ that satisfy the obedience constraints, the RHS of (10) is given by

$$\sum_{\theta\in\Theta}\mu_\theta\int\Pi_1((p_1,p_2);\theta)\mathrm{d}\sigma((p_1,p_2)|\theta) = \sum_{\theta\in\Theta}\mu_\theta\int\frac{\theta+b\mathbb{E}_\sigma[p_2|\theta]}{2a}\left[\theta+bp_2-\frac{\theta+b\mathbb{E}_\sigma[p_2|\theta]}{2}\right]\mathrm{d}\sigma(p_1,p_2|\theta)$$
$$= a\mathbb{E}_\mu\left[\left(\frac{\theta+b\mathbb{E}_\sigma[p_2|\theta]}{2a}\right)^2\right] = a\mathbb{E}_\mu[p_1^\sigma(\theta)^2]$$

The first equality holds since $\Pi_1((p_1,p_2);\theta) = p_1(\theta + bp_2 - ap_1)$ and since firm 1's best response, denoted by $p_1^\theta(\theta)$, is

$$p_1^\sigma(\theta) = \frac{\theta+b\mathbb{E}_\sigma[p_2|\theta]}{2a}.$$

When firms offer substitutes, firm 1's expected equilibrium profit is an increasing and convex function of the expected equilibrium price $p_2$. Then, Jensen's inequality implies that maximizing expected equilibrium profits is equivalent to maximizing the distance between the expected equilibrium prices set by firm 2, $\mathbb{E}_\sigma[p_2|\theta]$ (or, equivantly, by maximizing the distance between $p_1^\sigma(\theta)$). When firms offer substitutes, Lemma 8 shows that $\sup\sigma((p_1,p_2)|\theta) \in [p^F(\theta_L), p^F(\theta_H)]^2$ for all $\theta\in\Theta$. Hence, recommending $(p^F(\theta_L), p^F(\theta_L)))$ in the low state and $(p^F(\theta_H), p^F(\theta_H)))$ in the high state maximizes expected equilibrium profit which implies that full disclosure is optimal for the informed firm.[31] ■

### A.3.3 Consumer optimal disclosure

**Proof. Proposition 2.** Consider any partial disclosure policy $\sigma$ and define $\sigma(s_2|\theta)$ the distribution of price recommendation $p_2$ conditional on the state $\theta$. Expected consumer surplus, denoted by $\mathbb{E}_{(\mu,\sigma)}[CS((p_1,p_2);\theta)]$, is

$$\mathbb{E}_{(\mu,\sigma)}[CS((p_1,p_2);\theta)] = \frac{1}{2a}\sum_{\theta\in\Theta}\mu_\theta\left[\int(\theta+bp_2-ap_1)^2\mathrm{d}\sigma((p_1,p_2)|\theta)+\int(\theta+bp_1-ap_2)^2\mathrm{d}\sigma((p_1,p_2)|\theta)\right] \tag{11}$$

where, in the unique BNE, $p_1$ satisfies

$$p_1 = \frac{1}{2a}\left[\theta + b\int p_2\mathrm{d}\sigma(p_2|\theta)\right].$$

[31] That is, to maximize the expectation of a quadratic function in an interval, it is necessary to put all mass on the extremes of such interval.

First, Lemma 3 shows that full disclosure is never optimal for consumers. Second, define $\Delta\mathbb{E}[CS](\sigma)$ as the difference in expected consumer surplus with partial and no disclosure. This difference is

$$\Delta\mathbb{E}[CS](\sigma) = \frac{a}{2}\left(\delta^2+1\right)\mathbb{V}_{(\mu,\sigma)}[p_2] - \left[\left(1-\frac{\delta^2}{2}\right)\left(\frac{\delta}{2}+1\right)-\frac{\delta}{4}\right]\mathrm{Cov}_{(\mu,\sigma)}(\theta,p_2) - \frac{b\delta}{8}\left(7-\delta^2\right)\mathbb{V}_{\mu}[\mathbb{E}_{\sigma}[p_2|\theta]],$$

where the equality holds by the law of iterated expectations, the definition of variance, conditional expectation, conditional variance and covariance and the law of total variance. Note that $\Delta\mathbb{E}[CS](\sigma)$ is a continuous in $\delta$. Third, Lemma 3 also implies that $\Delta\mathbb{E}[CS](\sigma)$ converges to a positive number as $\delta \to 1$. This, in turn, implies that

$$b > \frac{2\mathrm{Cov}_{(\mu,\sigma)}[\theta,p_2]}{4\mathbb{V}_{(\mu,\sigma)}[p_2] - 3\mathbb{V}_{\mu}[\mathbb{E}_{\sigma}[p_2|\theta]]},$$

which is a sufficient condition that ensures that $\Delta\mathbb{E}[CS](\sigma)$ is a strictly increasing in $\delta$. Similarly, when $\delta \to 0$, $\Delta\mathbb{E}[CS](\sigma)$ converges to

$$\Delta\mathbb{E}[CS](\sigma) \underset{\delta\to 0}{\to} \frac{a}{2}V_{(\mu,\sigma)}[p_2] - \mathrm{Cov}_{(\mu,\sigma)}[\theta,p_2] = -\mathrm{Cov}_{(\mu,\sigma)}\left[\theta - \frac{a}{2}p_2, p_2\right].$$

The price $p_2$ is an increasing function of $\theta$ since the state is a positive demand shifter and

$$\frac{\partial p_2}{\partial\theta} \leq \frac{1}{2a-b} \leq \frac{2}{a}$$

since $a > b > 0$. Then, the covariance between $\theta - \frac{a}{2}p_2$ and $p_2$ is the covariance between two increasing functions of $\theta$. Hence, this covariance is positive, which implies that $\Delta\mathbb{E}[CS](\sigma)$ converges to a negative number when $\delta \to 0$.

Lastly, since Lemma 3 shows that $\Delta\mathbb{E}[CS](\sigma) > 0$ for all $\delta \in (\hat{c},1)$, the Intermediate Value theorem implies that there exists $\hat{\alpha} \in (0,\hat{c}]$ such that $\Delta\mathbb{E}[CS](\sigma) = 0$ when $\delta = \hat{\alpha}$. Moreover, since $\Delta\mathbb{E}[CS](\sigma)$ is strictly increasing in $\delta$, partial disclosure is optimal for all $\delta \in (\hat{\alpha},1)$ where $\hat{\alpha} \in (0,\hat{c}]$ and no disclosure is optimal otherwise. ■

**Lemma 9** *Assume that $\sigma$ is partially informative and $\sigma(p_2|\theta)$ is degenerated, placing all mass on $\hat{p} \in [p_L^F, p_H^F]$. For any obedient $\sigma$, supp $\sigma(p_2|\theta') = \{\hat{p}, \hat{p}'\}$ for all $\theta \neq \theta'$.*

**Proof. Lemma 9.** The recommendation mechanism $\sigma$ is not fully informative. First, I show that $\hat{p} \in \text{supp }\sigma(p_2|\theta')$. Suppose not. Then, supp $\sigma|\theta \cap \text{supp }\sigma|\theta' = \emptyset$ which implies that price recommendations fully reveal the state. However, this contradicts the assumption that $\sigma$ is partially informative. Hence, $\hat{p} \in \text{supp }\sigma(p_2|\theta')$.

Second, I show that the support of $\sigma(p_2|\theta')$ is binary. Firm $i$'s obedience constraint is

$$\sum_{\theta\in\Theta}\mu_\theta\int_{p_{-i}\in\mathbb{R}_+}\Pi_i((p_i,p_{-i}),\theta)\mathrm{d}\sigma((p_i,p_{-i})|\theta)\geq\sum_{\theta\in\Theta}\mu_\theta\int_{p_{-i}\in\mathbb{R}_+}\Pi_i((p_i',p_{-i}),\theta)\mathrm{d}\sigma((p_i,p_{-i})|\theta)$$

for all $i$, $p_i\in\text{supp }\sigma$ and $p_i'\in[p_L^F,p_H^F]$. The left-hand side of the uninformed firm obedience constraint can be simplified as follows:

$$\begin{aligned}\sum_{\theta\in\Theta}\mu_\theta\int_{p_1\in[p_L^F,p_H^F]}p_2\left(\theta+bp_1-ap_2\right)\mathrm{d}\sigma((p_1,p_2)|\theta)&=\sum_{\theta\in\Theta}\mu_\theta\int_{p_1\in[p_L^F,p_H^F]}p_2\left[\theta+b\left(\frac{\theta+b\mathbb{E}_\sigma[p_2|\theta]}{2a}\right)-ap_2\right]\mathrm{d}\sigma((p_1,p_2)|\theta)\\&=\sum_{\theta\in\Theta}\mu_\theta p_2\left[\theta+b\left(\frac{\theta+b\mathbb{E}_\sigma[p_2|\theta]}{2a}\right)-ap_2\right]\int_{p_1\in[p_L^F,p_H^F]}\mathrm{d}\sigma((p_1,p_2)|\theta)\\&=\sum_{\theta\in\Theta}\mu_\theta p_2\left[\theta+b\left(\frac{\theta+b\mathbb{E}_\sigma[p_2|\theta]}{2a}\right)-ap_2\right]\sigma(p_2|\theta)\end{aligned}$$

The first equality holds by the best response function of firm 1. The last equality holds since $\int_{p_1\in[p_L^F,p_H^F]}\mathrm{d}\sigma((p_1,p_2)|\theta)=\sigma(p_2|\theta)$. Hence, the uninformed firm obedience constraint is

$$\sum_{\theta\in\Theta}\mu_\theta p_2\left[\theta+b\left(\frac{\theta+b\mathbb{E}_\sigma[p_2|\theta]}{2a}\right)-ap_2\right]\sigma(p_2|\theta)\geq\sum_{\theta\in\Theta}\mu_\theta p_2'\left[\theta+b\left(\frac{\theta+b\mathbb{E}_\sigma[p_2|\theta]}{2a}\right)-ap_2'\right]\sigma(p_2|\theta)$$

for all $p_2\in\text{supp }\sigma(p_2|\theta)$ and $p_2'\in[p_L^F,p_H^F]$. The obedience constraint for $p_2=\hat{p}$ is

$$\begin{aligned}&\mu_{\theta'}\sigma(\hat{p}|\theta')\hat{p}\left[\theta'+b\left(\frac{\theta'+b\mathbb{E}_\sigma[p_2|\theta']}{2a}\right)-a\hat{p}\right]+\mu_\theta\hat{p}\left[\theta+b\left(\frac{\theta+b\hat{p}}{2a}\right)-a\hat{p}\right]\\&\geq\mu_{\theta'}\sigma(\hat{p}|\theta')p_2'\left[\theta'+b\left(\frac{\theta'+b\mathbb{E}_\sigma[p_2|\theta']}{2a}\right)-ap_2'\right]+\mu_\theta p_2'\left[\theta+b\left(\frac{\theta+b\hat{p}}{2a}\right)-ap_2'\right]\end{aligned}$$

for all $p_2'\in[p_L^F,p_H^F]$. Similarly, the obedience constraint of $\sigma$ for $p_2\neq\hat{p}$ is

$$p_2\left[\theta'+b\left(\frac{\theta'+b\mathbb{E}_\sigma[p_2|\theta']}{2a}\right)-ap_2\right]\geq p_2'\left[\theta'+b\left(\frac{\theta'+b\mathbb{E}_\sigma[p_2|\theta']}{2a}\right)-ap_2'\right]\tag{12}$$

for all $p_2'\in[p_L^F,p_H^F]$. The uninformed firm's profits are strictly concave in $p_2$ which implies there exists a unique $\hat{p}'\in[p_L^F,p_H^F]$ that satisfies (12) and $\hat{p}'\neq\hat{p}$. Hence, the support of $\hat{\sigma}|\theta'$ is binary and given by $\{\hat{p},\hat{p}'\}$. ■

**Lemma 10** *Assume that $\sigma$ is partially informative and $\sigma(p_2|\theta_H)$ is degenerated, placing all mass on $\hat{p}_H\in[p_L^F,p_H^F]$. For any obedient $\sigma$, supp $\sigma(p_2|\theta_L)=\{\hat{p}_L,\hat{p}_H\}$ where $\lambda=\sigma(\hat{p}_L|\theta_L)$,*

$$\hat{p}_L=\frac{4a^2[1-\mu_L\lambda]\theta_L+b^2mu_H[(1-\lambda)\theta_H-\theta_L]}{(2a-b)\left[4a^2(1-\mu_L\lambda)-b^2\mu_H\lambda\right]}\text{ and }\hat{p}_H=\frac{4a^2\left[\mu_H\theta_H+\mu_L(1-\lambda)\theta_L\right]-b^2\mu_H\lambda\theta_H}{(2a-b)\left[4a^2(1-\mu_L\lambda)-b^2\mu_H\lambda\right]}.$$

**Proof. Lemma 10.** Lemma 9 implies that the support of $\sigma(p_2|\theta_L)$ is binary and given by $\{\hat{p}_L, \hat{p}_H\}$ if the support of $\sigma(p_2|\theta_H)$ is degenerated and given by $\hat{p}_H$. Define $\sigma(\hat{p}_L|\theta_L) = 1 - \sigma(\hat{p}_H|\theta_L) := \lambda \in (0,1)$. By definition,

$$\mathbb{E}_\sigma[p_2|\theta_L] = \lambda\hat{p}_L + (1-\lambda)\hat{p}_H \text{ and } \mathbb{E}_\sigma[p_2|\theta_H] = \hat{p}_H.$$

Then, taking $\mathbb{E}_\sigma[p_2|\theta_L]$ and $\mathbb{E}_\sigma[p_2|\theta_H]$ as given, $\hat{p}_L$ and $\hat{p}_H$ are characterized by

$$\hat{p}_L = \arg\max_{p_2} p_2 \left[\theta_L + b\left(\frac{\theta_L + b\mathbb{E}_\sigma[p_2|\theta_L]}{2a}\right) - ap_2\right]$$
$$\hat{p}_H = \arg\max_{p_2} \mu_L(1-\lambda)p_2 \left[\theta_L + b\left(\frac{\theta_L + b\mathbb{E}_\sigma[p_2|\theta_L]}{2a}\right) - ap_2\right] + \mu_H p_2 \left[\theta_H + b\left(\frac{\theta_H + b\mathbb{E}_\sigma[p_2|\theta_H]}{2a}\right) - ap_2\right]$$

The first order conditions of the previous maximization problems are

$$\hat{p}_L = \frac{1}{2a}\left[\theta_L + \frac{b}{2a}\left(\theta_L + b\mathbb{E}_\sigma[p_2|\theta_L]\right)\right]$$
$$\hat{p}_H = \frac{\mu_L(1-\lambda)\left[\theta_L + \frac{b}{2a}\left(\theta_L + b\mathbb{E}_\sigma[p_2|\theta_L]\right)\right] + \mu_H\left[\theta_H + \frac{b}{2a}\left(\theta_H + b\mathbb{E}_\sigma[p_2|\theta_H]\right)\right]}{2a\left(\mu_L(1-\lambda) + \mu_H\right)}$$

Using the definition of $\mathbb{E}_\sigma[p_2|\theta_L]$ and $\mathbb{E}_\sigma[p_2|\theta_H]$, we have that $\hat{p}_L$ and $\hat{p}_H$ are given by

$$\hat{p}_L = \frac{4a^2[1-\mu_L\lambda]\theta_L + b^2\mu_H[(1-\lambda)\theta_H - \theta_L]}{(2a-b)\left[4a^2(1-\mu_L\lambda) - b^2\mu_H\lambda\right]} \text{ and } \hat{p}_H = \frac{4a^2\left[\mu_H\theta_H + \mu_L(1-\lambda)\theta_L\right] - b^2\mu_H\lambda\theta_H}{(2a-b)\left[4a^2(1-\mu_L\lambda) - b^2\mu_H\lambda\right]}$$

where $\lambda$ fully characterizes $\sigma$. ■

**Proof. Proposition 3.** This proof applies to a more general result which states that is optimal for the designer to select $\sigma(p_2|\theta)$ to be degenerated for any $\theta$. Here I present the proof for $\sigma(p_2|\theta_H)$ but the proof for the other case is analogous.

Suppose not. Assume that the optimal recommendation mechanism $\sigma^* = \{\sigma^*(p_2|\theta)\}_{\theta\in\Theta}$ is partially informative where both $\sigma^*(p_2|\theta)$ are not degenerated. Consider an alternative partially informative recommendation mechanism $\hat{\sigma}$ in which $\hat{\sigma}(p_2|\theta_H)$ is degenerated and places all its mass on one point $\hat{p}_H \in [p^F(\theta_L), p^F(\theta_H)] = [p^F_L, p^F_H]$ where $\hat{p}_H \in \text{supp } \hat{\sigma}(p_2|\theta_L)$. By Lemma 10, for any obedient $\hat{\sigma}$, the support of $\hat{\sigma}|\theta_L$ is $\{\hat{p}_L, \hat{p}_H\}$ where $\hat{p}_L$ and $\hat{p}_H$ are defined in Lemma 9 and $\lambda = \hat{\sigma}(\hat{p}_L|\theta_L)$ fully characterizes $\hat{\sigma}$. Next, I show that there exists $\lambda \in (0,1)$ such that $\Delta\mathbb{E}[CS](\hat{\sigma}) \geq \Delta\mathbb{E}[CS](\sigma^*)$. Given that $\mathbb{E}_\sigma[p_2] = \mathbb{E}_{\sigma'}[p_2]$ for all feasible $\sigma, \sigma'$,[32] the difference between $\Delta\mathbb{E}[CS](\hat{\sigma})$ and $\Delta\mathbb{E}[CS](\sigma^*)$, denoted as $\Delta\mathbb{E}[CS]_{\hat{\sigma}-\sigma^*}$, is

[32]Note that $\mathbb{E}_\pi[p_2] = \mathbb{E}_{\pi'}[p_2]$ for all feasible $\pi, \pi'$ since

$$\mathbb{E}_\pi[p_2] = \frac{1}{2a}\left[\mathbb{E}[\theta]\left(1 + \frac{b}{2a}\right) + \frac{b^2}{2a}\mathbb{E}_\pi[p_2]\right] \Leftrightarrow \mathbb{E}_\pi[p_2] = \frac{\mathbb{E}_\mu[\theta]}{2a-b}.$$

The equality holds by the uninformed firm's and informed firm's best response functions and by the law of iterated expectations. Then, $\mathbb{E}_\pi[p_2]$ doesn't depend on $\pi$. Given the equivalence between $\pi_2$ and $\sigma$, it also follows that $\mathbb{E}_\sigma[p_2] = \mathbb{E}_{\sigma'}[p_2]$ for all feasible $\sigma, \sigma'$.

$$\Delta\mathbb{E}[CS]_{\hat{\sigma}-\sigma^*} = \frac{a}{2}\left(1+\delta^2\right)\left(\mathbb{E}_{\hat{\sigma}}[p_2^2] - \mathbb{E}_{\sigma^*}[p_2^2]\right) - \left[\left(1-\frac{\delta^2}{2}\right)\left(1+\frac{\delta}{2}\right) - \frac{\delta}{4}\right]\left(\mathbb{E}_{\hat{\sigma}}[\theta\cdot p_2] - \mathbb{E}_{\sigma^*}[\theta\cdot p_2]\right)$$
$$- \frac{b\delta}{8}(7-\delta^2)\left(\mathbb{E}_{\hat{\sigma}}[\mathbb{E}[p_2|\theta]^2] - \mathbb{E}_{\sigma^*}[\mathbb{E}[p_2|\theta]^2]\right)$$

For any feasible $\sigma^*$, the expectation $\mathbb{E}_{\sigma^*}[p_2|\theta_L]$ satisfies

$$\mathbb{E}_{\sigma^*}[p_2|\theta_L] \in \left(\frac{\theta_L}{2a-b}, \frac{\mathbb{E}_\mu[\theta]}{2a-b}\right).$$

Moreover, by definition, $\mathbb{E}_{\hat{\sigma}}[p_2|\theta_L] = \lambda\hat{p}_L + (1-\lambda)\hat{p}_H$, and

$$\mathbb{E}_{\hat{\sigma}}[p_2|\theta_L] = \frac{\theta_L}{2a-b} \text{ if } \lambda = 1 \text{ and } \mathbb{E}_{\hat{\sigma}}[p_2|\theta_L] = \frac{\mathbb{E}[\theta]}{2a-b} \text{ if } \lambda = 0.$$

The intermediate value theorem implies that there exists $\tilde{\lambda} \in (0,1)$ such that $\mathbb{E}_{\hat{\sigma}}[p_2|\theta_L] = \mathbb{E}_{\sigma^*}[p_2|\theta_L]$ since $\mathbb{E}_{\hat{\sigma}}[p_2|\theta_L]$ is a continuous function of $\lambda$. Since $\mathbb{E}_\sigma[p_2] = \mathbb{E}_{\sigma'}[p_2]$ for all feasible $\sigma$ and $\sigma'$, $\tilde{\lambda}$ also satisfies $\mathbb{E}_{\hat{\sigma}}[p_2|\theta_H] = \mathbb{E}_{\sigma^*}[p_2|\theta_H]$. Then, the difference between $\Delta\mathbb{E}[CS](\hat{\sigma})$ and $\Delta\mathbb{E}[CS](\sigma^*)$ for $\hat{\sigma}$ characterized by $\tilde{\lambda}$ is

$$\begin{aligned}\Delta\mathbb{E}[CS]_{\hat{\sigma}-\sigma^*} &= \frac{a}{2}\left(1+\delta^2\right)\left[\mathbb{E}_{\hat{\sigma}}[p_2^2] - \mathbb{E}_{\sigma^*}[p_2^2]\right] \\ &= \frac{a}{2}\left(1+\delta^2\right)\left[\mu_L\left(\tilde{\lambda}\hat{p}_L^2 + (1-\tilde{\lambda})\hat{p}_H^2 - \mathbb{E}_{\sigma^*}[p_2^2|\theta_L]\right) + \mu_H\left(\hat{p}_H^2 - \mathbb{E}_{\sigma^*}[p_2^2|\theta_H]\right)\right]\end{aligned}$$

Hence, $\mathbb{E}_{\hat{\sigma}}[p_2^2] \geq \mathbb{E}_{\sigma^*}[p_2^2]$ by Jensen's inequality. Then, for all demand parameters and $\sigma^*$ such that $\delta \leq \hat{c}$, there exists $\lambda \in (0,1)$ such that $\Delta\mathbb{E}[CS]_{\hat{\sigma}-\sigma^*} \geq 0$. This contradicts the optimality of $\sigma^*$. Thus, the optimal partially informative recommendation mechanism is such that supp $\sigma|\theta_H = \{\hat{p}_H\}$ and supp $\sigma|\theta_L = \{\hat{p}_L, \hat{p}_H\}$.

Lastly, the optimal recommendation mechanism is characterized by $\lambda^* \in \arg\max_{\lambda\in[0,1]} \Delta\mathbb{E}[CS](\lambda)$ where

$$\begin{aligned}\Delta\mathbb{E}[CS](\lambda) = &\frac{a}{2}\left(\delta^2+1\right)\mu_L\lambda\left[\mu_L(1-\lambda)+\mu_H\right](\hat{p}_H - \hat{p}_L)^2 \\ &- \left[\left(1-\frac{\delta^2}{2}\right)\left(\frac{\delta}{2}+1\right) - \frac{\delta}{4}\right]\left[\mu_L\theta_L\left[\lambda\hat{p}_L + (1-\lambda)\hat{p}_H\right] + \mu_H\theta_H\hat{p}_H - \frac{(\mu_L\theta_L+\mu_H\theta_H)^2}{2a-b}\right] \\ &- \frac{b\delta}{8}\left(7-\delta^2\right)\left[\mu_L\left[\lambda\hat{p}_L + (1-\lambda)\hat{p}_H\right]^2 + \mu_H\hat{p}_H^2 - \left[\mu_L\lambda\hat{p}_L + (1-\mu_L\lambda)\hat{p}_H\right]^2\right]\end{aligned}$$

and $\hat{p}_L$ and $\hat{p}_H$ are functions of $\lambda$ defined in Lemma 9. The optimal $\lambda^* \in (0,1)$ is characterized by the first order condition of $\Delta\mathbb{E}[CS](\lambda)$ and it is given by

$$\lambda^* = \frac{4\left[\delta(1-3\delta^2) + 6(1-\delta^2)\right]}{\mu_H\delta^5 + 2\mu_H\delta^4 - (12-\mu_H)\delta^3 - 6(4-\mu_H)\delta^2 + 4(1-\mu_H)\delta + 24(1-\mu_H)}.$$

■

### A.3.4 Producer surplus optimal disclosure

**Proof. Lemma 2.** For the uninformed firm, the difference in expected profits with full disclosure $\sigma^F$ and any disclosure $\sigma$ is

$$\Pi_2(\sigma^F) - \Pi_2(\sigma) = a\mathbb{E}_\mu\left[\left(\frac{\theta}{2a-b}\right)^2 - 2\frac{\theta}{2a-b}\mathbb{E}_\sigma[p_2|\theta] + \mathbb{E}_\sigma[p_2^2|\theta]\right] + \frac{b^2}{2a}\mathbb{E}_\mu\left[\frac{\theta}{2a-b}\mathbb{E}_\sigma[p_2|\theta] - \mathbb{E}_\sigma[p_2|\theta]^2\right]$$

Then, this difference is positive since

$$\begin{aligned}\Pi_2(\sigma^F) - \Pi_2(\sigma) &\geq a\mathbb{E}_\mu\left[\left(\frac{\theta}{2a-b} - \mathbb{E}_\sigma[p_2|\theta]\right)^2\right] + \frac{b^2}{2a}\sum_{\theta\in\theta}\mu_\theta\left[\mathbb{E}_\sigma[p_2|\theta]\left(\frac{\theta}{2a-b} - \mathbb{E}_\sigma[p_2|\theta]\right)\right] \\ &\geq a\mathbb{E}_\mu\left[\left(\frac{\theta}{2a-b} - \mathbb{E}_\sigma[p_2|\theta]\right)^2\right] \geq 0\end{aligned}$$

The first inequality holds by Jensen's inequality. The second since $a > |b| > 0$,

$$\frac{\theta_L}{2a-b} \leq \mathbb{E}_\sigma[p_2|\theta_L] \leq \mathbb{E}_\sigma[p_2|\theta_H] \leq \frac{\theta_H}{2a-b} \text{ and } \sum_{\theta\in\Theta}\mu_\theta\mathbb{E}_\sigma[p_2|\theta] = \frac{\mathbb{E}_\mu[\theta]}{2a-b}$$

for all feasible $\sigma$. Hence, $\Pi_2(\sigma^F) \geq \Pi_2(\sigma)$ which implies that full disclosure is optimal for the uninformed firm. ■

### A.3.5 Welfare optimal disclosure

**Proof. Proposition 4.** First, there exists $\tilde{\alpha} \in (0,1)$ such that full disclosure yields a higher expected welfare than no disclosure if and only if $\delta \geq \tilde{\alpha}$. Hence, if $\delta \geq \tilde{\alpha}$, either full or partial disclosure is optimal whereas if $\delta < \tilde{\alpha}$, either no or partial disclosure is optimal.

Consider $\delta < \tilde{\alpha}$. The difference in welfare with partial and no disclosure is given by

$$\begin{aligned}&\mathbb{E}_{(\mu,\sigma)}\left[W((p_1,p_2);\theta)\right] - \mathbb{E}_{(\mu,\sigma^N)}\left[W((p_1,p_2);\theta)\right] \\ &= \frac{\delta}{2}\left[\delta\left(\frac{\delta}{2}+1\right) + \frac{3}{2}\right]\mathrm{Cov}_{(\mu,\sigma)}[\theta,p_2] - \frac{a}{2}\left(1-\delta^2\right)\mathbb{V}_{(\mu,\sigma)}[p_2] - \frac{b}{8}\delta\left(1-\delta^2\right)\mathbb{V}_\mu[\mathbb{E}_\sigma[p_2|\theta]].\end{aligned}$$

This difference is a continuous and strictly increasing in $\delta$. Moreover, as $\delta$ converges to 0, the difference in expected consumer surplus converges to

$$\mathbb{E}_{(\mu,\sigma)}\left[W((p_1,p_2);\theta)\right] - \mathbb{E}_{(\mu,\sigma^N)}\left[W((p_1,p_2);\theta)\right] \underset{\delta\to 0}{\to} -\frac{a}{2}\mathbb{V}_{(\mu,\sigma)}[p_2] < 0$$

and Lemma 4 shows that $\mathbb{E}_{(\mu,\sigma)}\left[W((p_1,p_2);\theta)\right] > \mathbb{E}_{(\mu,\sigma^N)}\left[W((p_1,p_2);\theta)\right]$ for all $\delta > \tilde{c}_1$. Then, the intermediate value theorem implies that there exists a $\tilde{\alpha}_1 \in (0, \tilde{c}_1]$ such that

$$\mathbb{E}_{(\mu,\sigma)}\left[W((p_1,p_2);\theta)\right] = \mathbb{E}_{(\mu,\sigma^N)}\left[W((p_1,p_2);\theta)\right].$$

Also, since this difference is strictly increasing in $\delta$, this also implies that

$$\mathbb{E}_{(\mu,\sigma)}\left[W((p_1,p_2);\theta)\right] > \mathbb{E}_{(\mu,\sigma^N)}\left[W((p_1,p_2);\theta)\right] \text{ for all } \delta > \tilde{\alpha}_1 \text{ and}$$

$$\mathbb{E}_{(\mu,\sigma)}\left[W((p_1,p_2);\theta)\right] < \mathbb{E}_{(\mu,\sigma^N)}\left[W((p_1,p_2);\theta)\right] \text{ for all } \delta < \tilde{\alpha}_1.$$

That is, partial disclosure is optimal when $\delta \in [\tilde{\alpha}_1, \tilde{\alpha})$ whereas no disclosure is optimal when $\delta < \tilde{\alpha}_1$.

Now, consider $\delta \geq \tilde{\alpha}$ and define the difference in welfare with full and with partial disclosure as follows:

$$\begin{aligned}
&\mathbb{E}_{(\mu,\sigma^F)}\left[W((p_1,p_2);\theta)\right] - \mathbb{E}_{(\mu,\sigma)}\left[W((p_1,p_2);\theta)\right] \\
&= \frac{b\delta}{2}\left(\delta^2 - \frac{3}{2}\right)\mathbb{E}_\mu\left[\left(\frac{\theta}{2a-b}\right)^2 - \mathbb{E}_\sigma[p_2|\theta]^2\right] - \frac{b\delta}{2}\mathbb{E}_\mu\left[\mathbb{E}_\sigma[p_2^2|\theta] - \mathbb{E}_\sigma[p_2|\theta]^2\right] \\
&+ \delta\left(\frac{3}{4} + \frac{\delta}{2} + \frac{\delta^2}{4}\right)\mathbb{E}_\mu\left[\theta\left(\frac{\theta}{2a-b} - \mathbb{E}_\sigma[p_2|\theta]\right)\right] - \frac{a}{2}\mathbb{E}_\mu\left[\left(\frac{\theta}{2a-b}\right)^2 - \mathbb{E}_\sigma[p_2^2|\theta]\right],
\end{aligned}$$

which is a continuous function of $\delta$. Analogously, we can show that full disclosure is optimal when $\delta > \tilde{\alpha}_2$ and partial disclosure is optimal when $\delta \in [\alpha, \tilde{\alpha}_2)$. In summary, full disclosure is optimal if $\delta \geq \tilde{\alpha}_2$, partial disclosure is optimal if $\delta \in (\tilde{\alpha}_1, \tilde{\alpha}_2)$ and no disclosure is optimal if $\delta \in (0, \tilde{\alpha}_1]$. ■

**Proof. Proposition 5.** The proof is analogous to the proof of Proposition 3. ■

## A.4 Non-linear demand

Consider the same environment as before but assume firm $i$'s demand, $q(p_i, p_{-i};\theta)$, is continuous and differentiable and satisfies the following properties:

$$i)\ \frac{\partial q(p_i,p_{-i};\theta)}{\partial p_i} \leq 0,\ \ ii)\quad \frac{\partial q(p_i,p_{-i};\theta)}{\partial \theta} > 0,\ \text{ and } iii)\ \left|\frac{\partial q(p_i,p_{-i};\theta)}{\partial p_i}\right| > \left|\frac{\partial q(p_i,p_{-i};\theta)}{\partial p_{-i}}\right|$$

The first condition ensures that quantity demanded decreases as price increases, the second condition implies that the state is a positive demand shifter and, lastly, the third condition implies that goods are differentiated and that a change of its own price has a bigger effect on the demand than a change of the price of a competitor.[33] Assume that firm's ex-post profits are strictly concave in $p_i$. That is,

$$p_i \frac{\partial^2 q(p_i,p_{-i};\theta)}{\partial p_i^2} < -2\frac{\partial q(p_i,p_{-i};\theta)}{\partial p_i} \text{ for all } p_i.$$

[33]This ensures that equilibrium prices are finite.

Furthermore, assume that

$$\frac{\partial^2 \Pi_i(p_i, p_{-i}; \theta)}{\partial p_i^2} \frac{\partial^2 \Pi_{-i}(p_{-i}, p_i; \theta)}{\partial p_{-i}^2} \geq \left( \frac{\partial^2 \Pi_i(p_i, p_{-i}; \theta)}{\partial p_i \partial p_{-i}} \right)^2 .$$

Firms offer substitutes (complements) if

$$\frac{\partial q(p_i, p_{-i}; \theta)}{\partial p_{-i}} > 0(< 0).$$

When firms offer substitutes, assume that the elasticity of demand of firm $i$ is a non-increasing function of the other firm' price and that the demand is supermodular in the state $\theta$ and the price of the other firm $p_{-i}$, i.e.,

$$\frac{\partial^2 q(p_i, p_{-i}; \theta)}{\partial p_i \partial p_{-i}} \geq 0 \text{ for all } (p_i, p_{-i}) \text{ and } \frac{\partial^2 q(p_i, p_{-i}; \theta)}{\partial \theta \partial p_{-i}} \geq 0.$$

Similarly, when firms offer complements, assume that the elasticity of demand of firm $i$ is a non-decreasing function of the other firm' price and that the demand is submodular in the state $\theta$ and the price of the other firm. That is,

$$\frac{\partial^2 q(p_i, p_{-i}; \theta)}{\partial p_i \partial p_{-i}} \leq 0 \text{ for all } (p_i, p_{-i}) \text{ and } \frac{\partial^2 q(p_i, p_{-i}; \theta)}{\partial \theta \partial p_{-i}} \leq 0.$$

Note that these assumptions imply that firms choices are strategic complements (substitutes) when they offer substitutes (complements).

**Pricing game equilibrium.** For all information structures $(S_2, \pi_2)$, the existence and uniqueness of the BNE is guaranteed by Ui (2016), which provides sufficient conditions for the existence and uniqueness of BNE in Bayesian games with concave and continuously differentiable payoff functions. This is formalized in Lemma 11.

**Lemma 11** *For all information structures $(S_2, \pi_2)$, the set of Bayesian Nash equilibria in the pricing game $\hat{\mathcal{E}}(S_2, \pi_2)$ is a singleton.*

**Simplifications.** The strict concavity of firm's ex-post profits in $p_i$ imply that firms' profits are bounded and continuous functions and that there exists $\overline{p}$ such that it is without loss of generality to restrict attention to the compact action space $p_i \in [0, \overline{p}]$. The equivalence to recommendation mechanism $\sigma$ is established in Lemma 5 and Lemma 6. The existence and uniqueness of BNE imply that it is sufficient to restrict attention to the distribution $\sigma(p_2|\theta)$ since for any obedient recommendation mechanism there exists a function $p_1(\theta, \sigma(p_2|\theta))$

which represents firm 1's best response when the state is $\theta$ and the price recommendations are given by $\sigma$ where

$$p_1(\theta, \sigma(p_2|\theta)) = \arg\max_{p_1} \int_{p_2} v_1(p_1, p_2; \theta)\mathrm{d}\sigma(p_2|\theta).$$

By Leibniz rule, $p_1(\theta, \sigma(p_2|\theta))$ is implicitly characterized by

$$\int_{p_2} q(p_1, p_2; \theta)\mathrm{d}\sigma(p_2|\theta) + p_1 \int_{p_2} \frac{\partial q(p_1, p_2; \theta)}{\partial p_1}\mathrm{d}\sigma(p_2|\theta) = 0.$$

Then, firm 1's expected equilibrium profits given information structure $\sigma$ are

$$\mathbb{E}_(\mu, \sigma)[\Pi_1^*(p_1, p_2; \theta)] = \sum_{\theta \in \Theta} \mu_\theta \mathbb{E}_\sigma[\Pi_1^*(p_1, p_2; \theta)|\theta] = \sum_{\theta \in \Theta} \mu_\theta \int_{p_2} \Pi_1(p_1(\theta, \sigma(p_2|\theta)), p_2; \theta)\mathrm{d}\sigma(p_2|\theta)$$

Furthermore, for any information structure $\sigma$, the set of recommended equilibrium prices for firm 2 in the pricing game is a subset of the interval between the equilibrium prices with full disclosure. This is formalized in Lemma 12.

**Lemma 12** *The support of any obedient distribution $\sigma(p_2|\theta)$ is a subset of $[p^F(\theta_L), p^F(\theta_H)]$ for all $\theta \in \Theta$ where $p^F(\theta)$ is the equilibrium price with full disclosure when the state $\theta$ is realized.*

**Informed firm optimal disclosure.** Assume the designer wants to maximize the informed firm's expected equilibrium payoffs. First, I show that when firms offer substitutes, the informed firm's expected equilibrium payoff conditional on the state is supermodular in the state and the price of the other firm. Similarly, I also show that the informed firm's expected equilibrium payoff conditional on the state is submodular in the state and the price of the other firm when firms offer complements. This is formalized in Lemma 13.

**Lemma 13** *When firms offer substitutes (complements), $\mathbb{E}_\sigma[\Pi_1^*(p_1, p_2; \theta)|\theta]$ is supermodular (submodular) in $\theta$ and $p_2$.*

Second, I show that it is optimal for the informed firm to share all its private information with the uninformed firm when the informed firm expected equilibrium profits are supermodular in the state and the uninformed firm's price. I also show that it is optimal for the informed firm to share none of its private information with the uninformed firm when the informed firm expected equilibrium profits are submodular in the state and the uninformed firm's price. This is formalized in Proposition 6.

**Proposition 6** *If* $\mathbb{E}_\sigma[\Pi_1^*(p_1, p_2; \theta)|\theta]$ *is supermodular in* $p_2$ *and* $\theta$*, full disclosure is optimal for the informed firm. Similarly, if* $\mathbb{E}_\sigma[\Pi_1^*(p_1, p_2; \theta)|\theta]$ *is submodular in* $p_2$ *and* $\theta$*, no disclosure is optimal for the informed firm.*

These two results imply that full disclosure is optimal for the informed firm when firms offer substitutes and no disclosure is optimal when firms offer complements. These results also extend to Cournot competition using same equivalence arguments as before.

### A.4.1 Proofs

**Proof. Lemma 11.** The pricing game is a smooth concave game since $\Pi_i(\cdot, p_{-i}; \theta) : \mathbb{R}_+ \to \mathbb{R}$ is concave and continuously differentiable for each $p_{-i} \in \mathbb{R}_+$ since

$$\frac{\partial^2 \Pi_i(p_i, p_{-i}; \theta)}{\partial p_i^2} < 0 \text{ for all } p_{-i} \in \mathbb{R}_+.$$

Define the payoff gradient as

$$\nabla \Pi(\mathbf{p}, \theta) := \left( \frac{\partial \Pi_i((p_i, p_{-i}); \theta)}{\partial p_i} \right)_{i \in \{1,2\}}.$$

The payoff gradient is continuously differentible. The Jacobian matrix of the payoff gradient, given by

$$F_{\nabla \Pi}(\mathbf{p}, \theta) := \begin{pmatrix} \frac{\partial^2 \Pi_1((p_1, p_2); \theta)}{\partial p_1^2} & \frac{\partial^2 \Pi_1((p_1, p_2); \theta)}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \Pi_2((p_2, p_1); \theta)}{\partial p_1 \partial p_2} & \frac{\partial^2 \Pi_2((p_2, p_1); \theta)}{\partial p_2^2} \end{pmatrix},$$

is negative definite because

$$\frac{\partial^2 \Pi_i(p_i, p_{-i}; \theta)}{\partial p_i^2} < 0 \text{ and } \frac{\partial^2 \Pi_i(p_i, p_{-i}; \theta)}{\partial p_i^2} \frac{\partial^2 \Pi_{-i}(p_{-i}, p_i; \theta)}{\partial p_{-i}^2} \geq \left( \frac{\partial^2 \Pi_i(p_i, p_{-i}; \theta)}{\partial p_i \partial p_{-i}} \right)^2.$$

This implies that the payoff gradient $\nabla \Pi(\mathbf{p}, \theta)$ is strictly monotone by Lemma 4 from Ui (2016). Furthermore, since for all $\mathbf{p} := (p_i, p_{-i})$, there exists $c > 0$ such that

$$\mathbf{p}^T F_{\nabla \Pi}(\mathbf{p}, \theta) \mathbf{p} < -c \mathbf{p}^T \mathbf{p},$$

the payoff gradient is also strongly monotone by the same lemma. Then, the uniqueness of the Bayesian Nash equilibrium of the pricing game follows from Proposition 1 from Ui (2016), which states that if the payoff gradient is strictly monotone, the Bayesian game as at most one Bayesian Nash equilibrium. The existence of a unique Bayesian Nash equilibrium follows from Proposition 2 from Ui (2016). ■

**Proof. Lemma 12.** With full disclosure, there is no uncertainty about the state. Each firm chooses $p_i : \Theta \to \mathbb{R}_+$ to maximize $\Pi_i(p_i, p_{-i}; \theta)$. That is, firm $i$'s best response to $p_{-i}$ is implicitly defined by

$$q(p_i, p_{-i}; \theta) + p_i \frac{\partial q(p_i, p_{-i}; \theta)}{\partial p_i} = 0$$

In equilibrium, both firms choose the same price, denoted by $p^F(\theta)$. Since $q(p_2, p_1; \theta_L) < q(p_2, p_1, \theta_H)$, the highest (lowest) equilibrium price the uninformed firm is willing to price is when both firms are certain that the state is high (low). Hence, the support of any obedient recommendation $\sigma(p_2|\theta)$ is a subset of $[p^F(\theta_L), p^F(\theta_H)]$. ■

**Proof. Lemma 13.** By definition, $\mathbb{E}_\sigma[\Pi_1^*(p_1, p_2; \theta)|\theta]$ is given by

$$\mathbb{E}_\sigma[\Pi_1^*(p_1, p_2; \theta)|\theta] = \int_{p_2} \Pi_1(p_1(\theta, \sigma(p_2|\theta)), p_2; \theta) \mathrm{d}\sigma(p_2|\theta)$$

When firms offer substitutes, for any obedient $\sigma(p_2|\theta)$ we have that

$$\begin{aligned} &\int_{p_2} q(p_1, p_2; \theta) \mathrm{d}\sigma(p_2|\theta_H) \geq \int_{p_2} q(p_1, p_2; \theta) \mathrm{d}\sigma(p_2|\theta_L) \text{ and} \\ &\int_{p_2} \frac{\partial q(p_1, p_2; \theta)}{\partial p_1} \mathrm{d}\sigma(p_2|\theta_H) \geq \int_{p_2} \frac{\partial q(p_1, p_2; \theta)}{\partial p_1} \mathrm{d}\sigma(p_2|\theta_L) \text{ for all } p_1 \text{ and } \theta \end{aligned} \tag{13}$$

since $q(p_1, p_2; \theta)$ is strictly increasing in $p_2$, $\int_0^x \mathrm{d}\sigma(p_2|\theta_L) \geq \int_0^x \mathrm{d}\sigma(p_2|\theta_H)$ for all $x$ and $\frac{\partial^2 q(p_1, p_2; \theta)}{\partial p_1 \partial p_2} > 0$. Then, since $p_1(\theta, \sigma(p_2|\theta))$ is implicitly defined by

$$\int_{p_2} q(p_1, p_2; \theta) \mathrm{d}\sigma(p_2|\theta) + p_1 \int_{p_2} \frac{\partial q(p_1, p_2; \theta)}{\partial p_1} \mathrm{d}\sigma(p_2|\theta) = 0,$$

(13) implies that $p_1(\theta, \sigma(p_2|\theta_H)) \geq p_1(\theta, \sigma(p_2|\theta_L))$ for all $\theta \in \Theta$. Then, $\frac{\partial^2 q(p_1, p_2, \theta)}{\partial p_1 \partial p_2} \geq 0$ also implies that

$$\int_{p_2} q(p_1(\theta, \sigma(p_2|\theta_H)), p_2; \theta) \mathrm{d}\sigma(p_2|\theta_H) \geq \int_{p_2} q(p_1(\theta, \sigma(p_2|\theta_L)), p_2; \theta) \mathrm{d}\sigma(p_2|\theta_L).$$

Then, when firms offer substitutes,

$$\int_{p_2} \Pi_1(p_1(\theta, \sigma(p_2|\theta_H)), p_2; \theta) \mathrm{d}\sigma(p_2|\theta_H) - \int_{p_2} \Pi_1(p_1(\theta, \sigma(p_2|\theta_L)), p_2; \theta) \mathrm{d}\sigma(p_2|\theta_L) \geq 0 \tag{14}$$

for all $\theta \in \Theta$. By Leibnitz rule,

$$\frac{\partial \int_{p_2} \Pi_1(p_1(\theta,\sigma(p_2|\theta_H)), p_2; \theta) \mathrm{d}\sigma(p_2|\theta_H)}{\partial \theta}$$
$$= \int_{p_2} \left.\frac{\partial p_1(t,\sigma(p_2|\theta_H))}{\partial t}\right|_{t=\theta} \left[q(p_1(\theta,\sigma(p_2|\theta_H)), p_2; \theta) + p_1(\theta,\sigma(p_2|\theta_H)) \frac{\partial q(p_1,p_2;t)}{\partial p_1}\right] \mathrm{d}\sigma(p_2|\theta_H)$$
$$+ \int_{p_2} p_1(\theta,\sigma(p_2|\theta_H)) \left.\frac{\partial q(p_1(\theta,\sigma(p_2|\theta_H)), p_2; t)}{\partial t}\right|_{t=\theta} \mathrm{d}\sigma(p_2|\theta_H)$$
$$= \int_{p_2} p_1(\theta,\sigma(p_2|\theta_H)) \left.\frac{\partial q(p_1(\theta,\sigma(p_2|\theta_H)), p_2; t)}{\partial t}\right|_{t=\theta} \mathrm{d}\sigma(p_2|\theta_H)$$

where the last inequality holds by the first order condition of the informed firm's pricing decision. Similarly,

$$\frac{\partial \int_{p_2} \Pi_1(p_1(\theta,\sigma(p_2|\theta_L)), p_2; \theta) \mathrm{d}\sigma(p_2|\theta_L)}{\partial \theta} = \int_{p_2} p_1(\theta,\sigma(p_2|\theta_L)) \left.\frac{\partial q(p_1(\theta,\sigma(p_2|\theta_L)), p_2; t)}{\partial t}\right|_{t=\theta} \mathrm{d}\sigma(p_2|\theta_L)$$

Then, the left-hand side of (14) is non-decreasing in $\theta$ since

$$\int_{p_2} p_1(\theta,\sigma(p_2|\theta_H)) \left.\frac{\partial q(p_1(\theta,\sigma(p_2|\theta_H)), p_2; t)}{\partial t}\right|_{t=\theta} \mathrm{d}\sigma(p_2|\theta_H) \geq \int_{p_2} p_1(\theta,\sigma(p_2|\theta_L)) \left.\frac{\partial q(p_1(\theta,\sigma(p_2|\theta_L)), p_2; t)}{\partial t}\right|_{t=\theta} \mathrm{d}\sigma(p_2|\theta_L)$$

because $p_1(\theta,\sigma(p_2|\theta_H)) > p_1(\theta,\sigma(p_2|\theta_L))$ and $\frac{\partial^2 q(p_1,p_2;\theta)}{\partial\theta\partial p_2} > 0$. Thus, when firms offer substitutes,

$$\int_{p_2} \Pi_1(p_1(\theta_H,\sigma(p_2|\theta_H)), p_2; \theta_H) \mathrm{d}\sigma(p_2|\theta_H) - \int_{p_2} \Pi_1(p_1(\theta_H,\sigma(p_2|\theta_L)), p_2; \theta_H) \mathrm{d}\sigma(p_2|\theta_L)$$
$$\geq \int_{p_2} \Pi_1(p_1(\theta_L,\sigma(p_2|\theta_H)), p_2; \theta_L) \mathrm{d}\sigma(p_2|\theta_H) - \int_{p_2} \Pi_1(p_1(\theta_L,\sigma(p_2|\theta_L)), p_2; \theta_L) \mathrm{d}\sigma(p_2|\theta_L)$$

which implies that $\mathbb{E}_\sigma[\Pi_1^*(p_1,p_2;\theta)|\theta]$ is supermodular in $\theta$ and $p_2$. The proof for the complement case is analogous. ■

**Proof. Proposition 6.** Consider first the case in which $\mathbb{E}_\sigma[\Pi_1^*(p_1,p_2\theta)|\theta]$ is supermodular in $p_2$ and $\theta$. Next, I show that for all $\sigma$ and $p_2 \in [p^F(\theta_L), p^F(\theta_H)]$,

$$\mathbb{E}_{\sigma^F}[\Pi_1^*(p_1,p_2;\theta_L)|\theta_L] \leq \mathbb{E}_\sigma[\Pi_1^*(p_1,p_2;\theta_L)|\theta_L] \leq \mathbb{E}_\sigma[\Pi_1^*(p_1,p_2;\theta_H)|\theta_H] \leq \mathbb{E}_{\sigma^F}[\Pi_1^*(p_1,p_2;\theta_H)|\theta_H].$$

That is,

$$\Pi_1(p_1(\theta_L,\sigma^F(p_2|\theta_L)), p^F(\theta_L); \theta_L) \leq \int_{p_2} \Pi_1(p_1(\theta_L,\sigma(p_2|\theta_L)), p_2; \theta_L) \mathrm{d}\sigma(p_2|\theta_L)$$
$$\leq \int_{p_2} \Pi_1(p_1(\theta_H,\sigma(p_2|\theta_H)), p_2; \theta_H) \mathrm{d}\sigma(p_2|\theta_H) \leq \Pi_1(p_1(\theta_H,\sigma^F(p_2|\theta_H)), p^F(\theta_H); \theta_H)$$

First,

$$\int_{p_2} \Pi_1(p_1(\theta_L, \sigma(p_2|\theta_L)), p_2; \theta_L) \mathrm{d}\sigma(p_2|\theta_L) \geq \int_{p_2} \Pi_1(p_1(\theta_L, \sigma^F(p_2|\theta_L)), p_2; \theta_L) \mathrm{d}\sigma^F(p_2|\theta_L)$$
$$= \Pi_1(p_1(\theta_L, \sigma^F(p_2|\theta_L)), p^F(\theta_L); \theta_L) \tag{15}$$

since $\sigma^F(p_2|\theta_L)$ recommends $p^F(\theta_L)$ with probability 1, the informed firm's demand is increasing in $p_2$ and $p_1(\theta_L, \sigma(p_2|\theta_L)) \geq p_1(\theta_L, \sigma^F(p_2|\theta_L))$.[34] Similarly,

$$\int_{p_2} \Pi_1(p_1(\theta_H, \sigma(p_2|\theta_H)), p_2; \theta_H) \mathrm{d}\sigma(p_2|\theta_H) \leq \int_{p_2} \Pi_1(p_1(\theta_H, \sigma^F(p_2|\theta_H)), p_2; \theta_H) \mathrm{d}\sigma^F(p_2|\theta_H)$$
$$= \Pi_1(p_1(\theta_H, \sigma^F(p_2|\theta_H)), p^F(\theta_H); \theta_H) \tag{16}$$

because $\sigma^F(p_2|\theta_H)$ recommends $p^F(\theta_H)$ with probability 1, the informed firm's demand is increasing in $p_2$ and $p_1(\theta_H, \sigma(p_2|\theta_H)) \leq p_1(\theta_H, \sigma^F(p_2|\theta_H))$.

Second, supermodularity implies that

$$\int_{p_2} \Pi_1(p_1(\theta_L, \sigma(p_2|\theta_L)), p_2; \theta_L) \mathrm{d}\sigma(p_2|\theta_L) \leq \int_{p_2} \Pi_1(p_1(\theta_H, \sigma(p_2|\theta_H)), p_2; \theta_H) \mathrm{d}\sigma(p_2|\theta_H) \tag{17}$$

since $p_1(\theta_L, \sigma(p_2|\theta_L)) \leq p_1(\theta_H, \sigma(p_2|\theta_H))$, $\frac{\partial^2 q(p_1,p_2;\theta)}{\partial\theta\partial p_2} > 0$ and the state is a positive demand shifter, implying that $\sigma(p_2|\theta_H)$ recommends on average higher prices than $\sigma(p_2|\theta_L)$. Thus, (15), (16) and (17) imply that

$$\Pi_1(p_1(\theta_L, \sigma^F(p_2|\theta_L)), p^F(\theta_L); \theta_L) \leq \int_{p_2} \Pi_1(p_1(\theta_L, \sigma(p_2|\theta_L)), p_2; \theta_L) \mathrm{d}\sigma(p_2|\theta_L)$$
$$\leq \int_{p_2} \Pi_1(p_1(\theta_H, \sigma(p_2|\theta_H)), p_2; \theta_H) \mathrm{d}\sigma(p_2|\theta_H) \leq \Pi_1(p_1(\theta_H, \sigma^F(p_2|\theta_H)), p^F(\theta_H); \theta_H)$$

Then,

$$\begin{aligned}
\mathbb{E}_{\sigma^F,\mu}[\Pi_1^*(p_1, p_2; \theta)] &= \sum_{\theta\in\Theta} \mu_\theta \Pi_1(p_1(\theta, \sigma^F(p_2|\theta)), p^F(\theta); \theta) \\
&\geq \sum_{\theta\in\Theta} \mu_\theta \int_{p_2} \Pi_1(p_1(\theta, \sigma(p_2|\theta)), p_2; \theta) \mathrm{d}\sigma(p_2|\theta) \\
&= \mathbb{E}_{\sigma,\mu}[\Pi_1^*(p_1, p_2; \theta)]
\end{aligned}$$

where the inequality holds by Jensen's inequality.

[34] The proof of $p_1(\theta_L, \sigma(p_2|\theta_L)) \geq p_1(\theta_L, \sigma^F(p_2|\theta_L))$ follows an analogous argument as in Lemma 13.

Consider now the case in which $\mathbb{E}_\sigma[\Pi_1^*(p_1,p_2;\theta)|\theta]$ is submodular in $\theta$ and $\sigma(p_2|\theta)$. Analogously as in the supermodular case, it is possible to show that

$$\Pi_1(p_1(\theta_L,\sigma^N(p_2|\theta_L)),p^N;\theta_L) \leq \int_{p_2} \Pi_1(p_1(\theta_L,\sigma(p_2|\theta_L)),p_2;\theta_L)\mathrm{d}\sigma(p_2|\theta_L)$$
$$\leq \int_{p_2} \Pi_1(p_1(\theta_H,\sigma(p_2|\theta_H)),p_2;\theta_H)\mathrm{d}\sigma(p_2|\theta_H) \leq \Pi_1(p_1(\theta_H,\sigma^N(p_2|\theta_H)),p^N;\theta_H)$$

which in turn implies that

$$\begin{aligned}\mathbb{E}_{\sigma^N,\mu}[\Pi_1^*(p_1,p_2;\theta)] &= \sum_{\theta\in\Theta}\mu_\theta\Pi_1(p_1(\theta,p^N),p^N;\theta)\\ &\geq \sum_{\theta\in\Theta}\mu_\theta\int_{p_2}\Pi_1(p_1(\theta,\sigma(p_2|\theta)),p_2;\theta)\mathrm{d}\sigma(p_2|\theta_H)\\ &= \mathbb{E}_{\sigma,\mu}[\Pi_1^*(p_1,p_2;\theta)]\end{aligned}$$

where the inequality holds by Jensen's inequality. ■

## A.5 Extensions

**Firm optimal disclosure with complements.** In this section, I show that the informed firm's incentives for information sharing are reversed when firms offer complements and that the producer surplus optimal disclosure depends on the degree of complementary between goods. Formally, assume $b \in (-a,0)$ and define the degree of complementary between goods as $\delta = |\frac{b}{a}|$. When goods are complements, disclosure increases the uninformed firm's profits, as stated in Lemma 2, but reduces the informed firm's profits, as formalized in Proposition 6 in Appendix A.4. The optimal disclosure policy is determined by comparing the gains of the uninformed firm to the losses of the informed firm, which are determined by the degree of complementarity between goods. In particular, if goods are sufficiently complementary ($\delta \geq \hat{\gamma}$), competitor prices have a significant impact on demand. Then, the negative effect of increased pricing correlation on the informed firm's profits exceeds the positive effect of learning about the state on the uninformed firm's profits. As a result, no disclosure is optimal. Otherwise, the informed firm's expected loss from information disclosure is smaller than the uninformed firm's expected gain, implying that full disclosure is optimal. These results are stated in Lemma 14.

**Lemma 14 (Producer surplus optimal disclosure)** *If the designer's objective is to maximize expected producer surplus and firms offer complements, full disclosure is optimal if $\delta \in (0,\hat{\gamma})$. Otherwise, no disclosure is optimal.*

Eliaz and Forges (2015) study the producer surplus optimal disclosure policy in a Cournot duopoly with no private information and unknown demand. They show that it is optimal to fully inform one of the duopolists and disclose no information to the other when firms offer perfect substitutes. They also show that the producer surplus optimal disclosure consists of fully informing both firms when they offer perfect complements. Given the correspondence between Cournot and Bertrand discussed in Raith (1996), my results for complements and substitutes nest theirs, while allowing for more general patterns of complementarity and substitution between goods. Relatedly, Angeletos and Pavan (2007) show that producer surplus increases with the precision of public and private normally distributed signals when firms offer substitutes, related to the optimality of full disclosure. When firms offer complements, they show that producer surplus increases in the precision of private information, but can decrease in the precision of public information. In my context, this is exemplified by the designer who may have incentives to force information disclosure between firms when they offer complementary goods, decreasing the informational advantage of the informed firm at the benefit of its competitor.

**Public signals.** In this section, I characterize the optimal disclosure with public signals and show that consumers are better off when signal realizations are private instead of public since consumers benefit from the induced uncertainty between firms. Assume that the designer commits to an information structure $(S_2, \pi_2)$ with public signal realizations. Given the information structure, firms play a pricing game in which they condition their choices on their information by selecting a mapping $p_1 : \Theta \times S_2 \to \Delta(\mathbb{R}_+)$ and $p_2 : S_2 \to \Delta(\mathbb{R}_+)$ to maximize their expected profits.

The next two lemmas fix the disclosure policy $\sigma$, which represents the information conveyed to firm 2, and compare the case in which firm 1 observes firm 2's signal realization (public disclosure) to the case in which firm 1 doesn't observe it (private disclosure).

**Lemma 15** *For any disclosure policy $\sigma$, the informed firm's profits are higher with public disclosure than private disclosure.*

Public signals reinforce the informed firm's incentives to disclose information when firms offer substitutes, implying that full disclosure is optimal for the informed firm. When firms offer complements, it is optimal for the informed firm to disclose no information, by the same reasoning as with private signals. In contrast, consumers are better off with private disclosure, since they can benefit from information asymmetry as stated in Lemma 16.

**Lemma 16** *For any disclosure policy $\sigma$, expected consumer surplus is higher with private disclosure than public disclosure.*

When signals are public, the gain from partial disclosure disappears and, as a result, no disclosure is optimal for consumers as stated in Lemma 17.

**Lemma 17** *With public disclosure, no disclosure is optimal for consumers.*

Consumers can also benefit from disclosure when the uninformed firm's signal realization is noisily observed by the informed firm. As long as the informed firm is sufficiently uncertain about the information observed by the uninformed firm, it is possible for the regulator to create the coordination failure in prices with sufficiently high probability. Therefore, the optimality of partial disclosure doesn't rely on information disclosure being private.

**Informed firm as the owner of an online platform.** In this section, I show that the informed firm incentives to share information are amplified if it can charge a fee to the other firm to use its platform. Consider the case in which trade occurs on an online platform run by the informed firm. The informed firm charges the uninformed firm a percentage of its sales for the use of the platform. Given the disclosure policy $\sigma$, the informed firm's expected equilibrium payoff is

$$\mathbb{E}_{(\mu,\sigma)}[\Pi_1^*((p_1,p_2);\theta)] = a\mathbb{E}_\mu\left[\left(\frac{\theta + b\mathbb{E}_\sigma[p_2|\theta]}{2a}\right)^2\right] + \alpha\Pi_2^*(\sigma),$$

where $\alpha \in [0,1]$ is the percentage of sales charged to the uninformed firm.

The informed firm's incentives for information sharing are minimized by setting $\alpha = 0$, since the uninformed firm always benefits from observing information. When firms offer substitutes, the informed firm optimal disclosure doesn't change with $\alpha > 0$. In this case, the informed firm discloses all of its private information for any $\alpha \in [0,1]$. When firms offer complements, the informed firm optimal disclosure shares the same qualitative properties as the producer surplus maximizing disclosure with $\alpha = 0$. Full disclosure is optimal if the degree of complementarity is below a certain cutoff, no disclosure is optimal above the cutoff, and the cutoff is an increasing function of $\alpha$. Furthermore, the producer surplus optimal disclosure remains unchanged, since $\alpha$ represents a transfer between firms. Lastly, the consumer and welfare optimal disclosure also remain unchanged, since they are not affected by transfers between firms.

**Asymmetric firm-size.** In this section, I show that consumers benefit more from partial disclosure as the asymmetry in market size between firms increases, implying that the benefits from partial disclosure for consumers are minimized when firms face markets of the same size. To see this, assume

$$q_1((p_1, p_2); \theta) = \max\{0, \theta - ap_1 + bp_2\} \text{ and } q_2((p_2, p_1); \theta) = \max\{0, \kappa\theta - ap_2 + bp_1\},$$

where $\kappa \in (0, 1]$. Firms' preferences for information disclosure remain unaltered, implying that full disclosure maximizes expected producer surplus. Similarly, the features of the optimal disclosure policy for consumers also remain unaltered. Lemma 18 generalizes Proposition 2 and Proposition 3 to the case in which $\kappa \in (0, 1]$. In particular, for any $\kappa \in (0, 1]$, partial disclosure is optimal for consumers when firms offer sufficiently close substitutes and no disclosure is optimal otherwise. Moreover, the features of the optimal recommendation mechanism also generalize to this case.

**Lemma 18** *If the designer's objective is to maximize expected consumer surplus, for all* $\kappa \in (0, 1)$ *there exists an* $\hat{\alpha}(\kappa) \in (0, \hat{c}(\kappa)]$ *such that partial disclosure is optimal if* $\delta \in (\hat{\alpha}(\kappa), 1)$ *and no disclosure is optimal otherwise, where* $\hat{c}(\kappa)$ *is the cutoff for binary information structures and is decreasing in* $\kappa$*. A partially informative recommendation mechanism recommends two prices: one price when the state is low and another price in both states.*

From the perspective of consumers, partial disclosure increases their surplus in the informed firm's market and reduces it in the uninformed firm's market. Then, when the informed firm faces a bigger market than the uninformed firm ($\kappa \in (0, 1)$), incentives for partial disclosure are larger than when market sizes are symmetric ($\kappa = 1$). This implies that the cutoff in the degree of differentiation $\hat{c}(\kappa)$ must be a decreasing function of $\kappa$.

From the point of view of welfare, Proposition 4 and Proposition 5 also generalize to this setting, with the cutoffs that characterize optimal disclosure also decreasing with $\kappa$. The intuition is analogous to the case in which firms face markets of the same size.

**N firms with constrained disclosure policies.** In this setting, I show that the main intuition of the consumer optimal disclosure extends to the case in which $N$ firms compete a la Bertrand. Consider a setting with $N \geq 3$ firms who compete by choosing prices. The level of demand depends on the state $\theta \in \{\theta_L, \theta_H\}$ with $\theta_H > \theta_L > 0$ such that firms are active in the market in both states. Firms share a common prior about the state, where the probability of $\theta$ is denoted by $\mu_\theta \in (0, 1)$. Firm $i$'s demand is given by

$$q_i(\mathbf{p}) = \theta - ap_i + \frac{b}{N-1} \sum_{j \neq i} p_j$$

where $a$ and $b$ are known parameters with $a > b > 0$. Firms' costs are zero.

The designer commits to an information structure with private signals, denoted by $\hat{\psi}_k$, to share all of the informed firm's private information with $k$ firms and no information with $N - 1 - k$ firms, where $k \in \{0, 1, 2, ..., N - 1\}$. Firms who observe a perfectly informative signal condition their pricing choices on the state and select a mapping $p^F : \Theta \to \mathbb{R}_+$ to maximize their expected profits, whereas firms who observe no information select a price $p^N \in \mathbb{R}_+$ to maximize their expected profits. The optimal disclosure is stated in Lemma 19.

**Lemma 19** *If the designer's objective is to maximize the informed firm's expected equilibrium profits or to maximize expected producer surplus, it is optimal to share the informed firm's private information with all other firms. In contrast, if the designer's objective is to maximize expected consumer surplus, it is optimal to share the informed firm's private information with $k^*(N, \delta)$ firms where $\frac{k^*(N,\delta)}{N} \leq \frac{2}{3}$.*

First, the informed firm's expected equilibrium profits are maximized by sharing its private information with all other firms because it benefits from price correlation. Similarly, when the designer's objective is to maximize expected producer surplus, it is optimal to share information with all firms, eliminating information asymmetry between firms, allowing them to better extract surplus from consumers.

Second, if the designer's objective is to maximize expected consumer surplus, information disclosure between firms is at least partially restricted. The optimal information structure, characterized by $k^*(N, \delta)$, is determined by the degree of substitution and the number of firms in the market. In particular, it is optimal to not disclose information to any other firm when $\delta \leq \frac{3}{4}$. When $\delta > \frac{3}{4}$, optimal disclosure is determined by the number of firms in the market and $\delta$. In particular, the optimal $k^*(N, \delta)$ increases in both $\delta$ and $N$, and $\frac{k^*(N,\delta)}{N} \leq \frac{2}{3}$. This means that it is optimal to share information with more firms as the number of firms increase in the market and as firms offer closer substitutes, but that it is optimal to leave at least a third of firms uninformed. By leaving some firms uninformed, the designer is able to increase the heterogeneity of prices, benefiting consumers.

### A.5.1 Extensions proofs

**Proof. Lemma 14.** Proposition 1 and Lemma 2 imply that full disclosure is optimal if firms offer imperfect substitutes. If firms offer complements, the informed firm prefers to not disclose her private information whereas the uninformed firm prefers to learn the state. First, full disclosure yields higher producer surplus than no disclosure if and only if $\delta < \frac{2}{1+\sqrt{2}}$.

Next, I show that full disclosure is optimal when firms offer complements if $\delta < \frac{2}{1+\sqrt{2}}$ and no disclosure is optimal otherwise.

Consider first the case in which $\delta < \frac{2}{1+\sqrt{2}}$. The difference in expected producer surplus of full disclosure $\sigma^F$ and disclosure policy $\sigma$ is

$$PS(\sigma^F) - PS(\sigma) \geq \left(a + b - \frac{b^2}{4a}\right) \mathbb{E}_\mu \left[ \left( \frac{\theta}{2a-b} - \mathbb{E}_\sigma[p_2|\theta] \right)^2 \right] \geq 0$$

The first inequality holds by Jensen's inequality, $a > |b|$ and $b < 0$, whereas the second one holds for all $\delta < \frac{2}{1+\sqrt{2}}$. Hence, full disclosure is optimal if $\delta < \frac{2}{1+\sqrt{2}}$.

Consider now the case in which $\delta \geq \frac{2}{1+\sqrt{2}}$. The difference in expected producer surplus of no disclosure $\sigma^N$ and disclosure policy $\sigma$ is

$$PS(\sigma^N) - PS(\sigma) = a\mathbb{V}_{(\mu,\sigma)}[p_2] - (1-\delta)\operatorname{Cov}_{(\mu,\sigma)}[p_2,\theta] + \frac{3b}{4} \cdot \delta \mathbb{V}_\mu[\mathbb{E}_\sigma[p_2|\theta]]$$

Note that this difference is a strictly increasing function of $\delta$ because

$$\begin{aligned}
&\frac{\partial PS(\sigma^N) - PS(\sigma)}{\partial \delta} = \operatorname{Cov}_{(\mu,\sigma)}[p_2,\theta] + \frac{3b}{4}\mathbb{V}_\mu[\mathbb{E}_\sigma[p_2|\theta]], \\
&\operatorname{Cov}_{(\mu,\sigma)}[p_2,\theta] > 2a\mathbb{V}_{(\mu,\sigma)}[p_2], \;\; \delta < 1 \text{ and } \mathbb{V}_{(\mu,\sigma)}[p_2] \geq \mathbb{V}_\mu[\mathbb{E}_\sigma[p_2|\theta]] \geq 0.
\end{aligned}$$

This implies that

$$PS(\sigma^N) - PS(\sigma) \geq a\mathbb{V}_{(\mu,\sigma)}[p_2] - \left(1 - \frac{2}{1+\sqrt{2}}\right)\operatorname{Cov}_{(\mu,\sigma)}[p_2,\theta] + \frac{3b}{4} \cdot \frac{2}{1+\sqrt{2}}\mathbb{V}_\mu[\mathbb{E}_\sigma[p_2|\theta]] \geq 0$$

for all $\delta > \frac{2}{1+\sqrt{2}}$ since $\operatorname{Cov}_{(\mu,\sigma)}[p_2,\theta] \in \left[2a\mathbb{V}_{(\mu,\sigma)}[p_2], (2a-b)\mathbb{V}_{(\mu,\sigma)}[p_2]\right]$ and $\mathbb{V}_{(\mu,\sigma)}[p_2] \geq \mathbb{V}_\mu[\mathbb{E}_\sigma[p_2|\theta]]$.[35] Hence, no disclosure is optimal if $\delta > \frac{2}{1+\sqrt{2}}$. ■

**Proof. Lemma 15.** With public signals, the informed firm's expected equilibrium profits, given by

$$\mathbb{E}_{(\mu,\sigma^{Pub})}[\Pi_1^*((p_1,p_2);\theta)] = a\mathbb{E}_\mu \left[ \mathbb{E}_{\sigma^{Pub}} \left[ \left( \frac{\theta + bp_2}{2a} \right)^2 \middle| \theta \right] \right],$$

[35] The highest covariance between prices and the state occurs with full disclosure. In this case, $p_2(\theta) = \frac{\theta}{2a-b}$. Hence,

$$\operatorname{Cov}_{(\mu,\sigma)}[p_2,\theta] \leq \operatorname{Cov}_{(\mu,\sigma)}[p_2,(2a-b)p_2] = (2a-b)\mathbb{V}_{(\mu,\sigma)}[p_2]$$

are higher than its expected equilibrium profits with private signals. This holds because

$$\begin{aligned}\mathbb{E}_{(\mu,\sigma^{Pub})}[\Pi_1^*((p_1,p_2);\theta)] &= a\mathbb{E}_\mu\left[\mathbb{E}_{\sigma^{Pub}}\left[\left(\frac{\theta+bp_2}{2a}\right)^2\middle|\theta\right]\right] \geq a\mathbb{E}_\mu\left[\left(\mathbb{E}_{\sigma^{Pub}}\left[\frac{\theta+bp_2}{2a}\middle|\theta\right]\right)^2\right]\\ &= a\mathbb{E}_\mu\left[\left(\frac{\theta+b\mathbb{E}_{\sigma^{Pub}}\left[p_2|\theta\right]}{2a}\right)^2\right] = a\mathbb{E}_\mu\left[\left(\frac{\theta+b\mathbb{E}_{\sigma^{Priv}}\left[p_2|\theta\right]}{2a}\right)^2\right]\\ &= \mathbb{E}_{(\mu,\sigma^{Priv})}[\Pi_1^*((p_1,p_2);\theta)]\end{aligned}$$

where the inequality holds by Jensen's inequality. ■

**Proof. Lemma 16.** The expected consumer surplus with public disclosure is given by

$$CS(\sigma^{Pub}) = \frac{1}{2a}\mathbb{E}_\mu\left[\mathbb{E}_{\sigma^{Pub}}\left[q_1\left(\frac{\theta+bp_2}{2a},p_2;\theta\right)^2 + q_2\left(p_2,\frac{\theta+bp_2}{2a};\theta\right)^2\middle|\theta\right]\right],$$

whereas expected consumer surplus with private disclosure is

$$CS(\sigma^{Priv}) = \frac{1}{2a}\mathbb{E}_\mu\left[\mathbb{E}_{\sigma^{Priv}}\left[q_1\left(\frac{\theta+b\mathbb{E}_{\sigma^{Priv}}[p_2|\theta]}{2a},p_2;\theta\right)^2 + q_2\left(p_2,\frac{\theta+b\mathbb{E}_{\sigma^{Priv}}[p_2|\theta]}{2a};\theta\right)^2\middle|\theta\right]\right].$$

The difference between expected consumer surplus with private and public disclosure is

$$CS(\sigma^{Priv}) - CS(\sigma^{Pub}) = \frac{b^2}{8a}\left(7-\frac{b^2}{a^2}\right)\left(\mathbb{E}_{(\mu,\sigma)}[p_2^2] - \mathbb{E}_\mu\left[\mathbb{E}_\sigma[p_2|\theta]^2\right]\right)$$

Then, $CS(\sigma^{Priv}) \geq CS(\sigma^{Pub})$ because $a > |b|$ and

$$\mathbb{E}_{(\mu,\sigma)}[p_2^2] - \mathbb{E}_\mu\left[\mathbb{E}_\sigma[p_2|\theta]^2\right] = \mathbb{E}_\mu[\mathbb{E}_\sigma[p_2^2|\theta]] - \mathbb{E}_\mu\left[\mathbb{E}_\sigma[p_2|\theta]^2\right] \geq 0$$

where the equality holds by the law of iterated expectations and the inequality by Jensen's inequality. ■

**Proof. Lemma 17.** First, no disclosure is optimal when firms offer complements since $CS(\sigma^{Pub}) \leq CS(\sigma^{Priv})$. Similarly, no disclosure is optimal when firms offer substitutes and firms offer sufficiently far substitutes ($\delta < \hat{c}$). Consider then the case in which firms offer substitutes ($b > 0$) and $\delta \geq \hat{c}$. The expected gain of consumer surplus with public disclosure with respect to no disclosure is given by:

$$CS(\sigma^{Pub}) - CS(\sigma^N) \leq \frac{1}{2a}\left(\frac{a}{2}V_{(\mu,\sigma^{Pub})}[p_2] - \text{Cov}_{(\mu,\sigma^{Pub})}(\theta,p_2)\right) < 0$$

where the first inequality holds by definition of variance, covariance and $\delta$. The second inequality holds because $CS(\sigma^{Pub}) < CS(\sigma^N)$ for $\delta = 0$. ■

**Proof. Lemma 18.** The proof of this lemma is analogous to Proposition 2 and Proposition 3. Using an analogous argument as in Lemma 3, it is possible to show that full disclosure of information is never optimal for consumers. The expected difference in expected consumer surplus with partial $\sigma$ and no disclosure, $\Delta\mathbb{E}[CS](\sigma)$, now is

$$\Delta\mathbb{E}[CS](\sigma) = \frac{a}{2}\left(1+\delta^2\right)\mathbb{V}_{(\mu,\sigma)}[p_2] - \left[\left(1-\frac{\delta^2}{2}\right)\left(\kappa+\frac{\delta}{2}\right)-\frac{\delta}{4}\right] - \frac{b\delta}{8}\left(7-\delta^2\right)\mathbb{V}_{\mu}[\mathbb{E}_{\sigma}[p_2|\theta]]$$

For all $\kappa \in (0,1]$, using analogous arguments as in Proposition 2, I show that $\Delta\mathbb{E}[CS](\sigma)$ is continuous and strictly increasing in $\delta$, $\Delta\mathbb{E}[CS](\sigma)$ converges to a negative value when $\delta \to 0$ and to a positive one when $\delta \to 1$. The result follows then from the Intermediate Value Theorem. It is also straightforward to show that $\hat{c}(\kappa)$ is decreasing in $\kappa$. The same argument as in Proposition 3 implies that it is optimal to recommend at most two prices, one price when the state is low and another price in both states. ■

**Proof. Lemma 19.** The designer commits to an information structure with private signals, denoted by $\hat{\psi}_k$, to share all the informed firm's private information with $k$ firms and share no information with $N-1-k$ firms, where $k \in \{0,1,2,...,N-1\}$. Firms who observe a perfectly informative signal condition their pricing choices on the state and select a mapping $p^F : \Theta \to \mathbb{R}_+$ to maximize their expected profits, whereas firms who observe no information select a price $p^N \in \mathbb{R}_+$ to maximize their expected profits. Equilibrium prices are

$$\begin{aligned}
p^F(\theta_L) &= \frac{\theta_L(2a(N-1)-bk) + b\mu_H(N-k-1)\left(\theta_H-\theta_L\right)}{(2a-b)(2a(N-1)-bk)}, \\
p^F(\theta_H) &= \frac{\theta_H(2a(N-1)-bk) - b\mu_L(N-k-1)\left(\theta_H-\theta_L\right)}{(2a-b)(2a(N-1)-bk)}, \text{ and} \\
p^N &= \frac{\mu_L\theta_L+\mu_H\theta_H}{2a-b}.
\end{aligned}$$

Consider first the case in which the designer's objective is to maximize the informed firm's expected equilibrium profits, given by

$$\mathbb{E}[\Pi_1^*(\hat{\psi}_k)] = a\sum_{\theta\in\Theta}\mu_\theta p^F(\theta)^2.$$

The informed firm's expected equilibrium profits are maximized by sharing its private information with all other firms ($k^* = N-1$). Similarly, when the designer's objective is to maximize expected producer surplus, given by

$$PS(\hat{\psi}_k) = (N-k-1)a(p^N)^2 + (k+1)a\sum_{\theta\in\Theta}\mu_\theta p^F(\theta)^2,$$

it is optimal to share information with all firms ($k^* = N-1$), eliminating all information asymmetry between firms.

In contrast, if the designer's objective is to maximize expected consumer surplus, information disclosure between firms is at least partially restricted. Expected consumer surplus, given by,

$$CS(\hat{\psi}_k) = \frac{(k+1)}{2a}\sum_{\theta\in\Theta}\mu_\theta\left[\theta + b\frac{(N-k-1)}{N-1}p^N - \left(a - b\frac{k}{N-1}\right)p^F(\theta)\right]^2$$
$$+\frac{(N-k-1)}{2a}\sum_{\theta\in\Theta}\mu_\theta\left[\theta + b\frac{k+1}{N-1}p^F(\theta) - \left(a - b\frac{N-k-2}{N-1}\right)p^N\right]^2$$

The optimal information structure, characterized by $k^*(N,\delta)$, is determined by the degree of substitution and the number of firms in the market, where

$$k^*(N,\delta) = \begin{cases} 0 & \text{if } \delta \leq \frac{3}{4} \text{ for all } N \geq 3 \\ 0 & \text{if } \delta \in \left(\frac{3}{4}, 0.76\right) \text{ and } N \in \left[3, 1 + \frac{1}{2}\sqrt{\frac{\delta^2}{4\delta-3}} - \frac{\delta}{2}\right] \\ f(N,\delta) & \text{otherwise} \end{cases}$$

and

$$f(N,\delta) = \lceil\frac{2(N-1)\left(\delta^3 + \delta^2(4N-5) + \delta(4N-7)(N-1) - 3(N-1)^2\right)}{\delta\left(\delta + (N-1)\right)\left(\delta + 3(N-1)\right)}\rceil$$

if

$$CS\left[\hat{\pi}_{\lceil\frac{2(N-1)\left(\delta^3+\delta^2(4N-5)+\delta(4N-7)(N-1)-3(N-1)^2\right)}{\delta(\delta+(N-1))(\delta+3(N-1))}\rceil}\right] \geq CS\left[\hat{\pi}_{\lfloor\frac{2(N-1)\left(\delta^3+\delta^2(4N-5)+\delta(4N-7)(N-1)-3(N-1)^2\right)}{\delta(\delta+(N-1))(\delta+3(N-1))}\rfloor}\right]$$

and

$$f(N,\delta) = \lfloor\frac{2(N-1)\left(\delta^3 + \delta^2(4N-5) + \delta(4N-7)(N-1) - 3(N-1)^2\right)}{\delta\left(\delta + (N-1)\right)\left(\delta + 3(N-1)\right)}\rfloor,$$

otherwise.

■