Obfuscation and Rational Inattention*

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Abstract

We study the behavior of duopolistic firms that can obfuscate their prices before competing on price. Obfuscation affects the rational inattentive consumers' optimal information strategy, which determines the probabilistic demand. Our model advances related models by allowing consumers to update their unrestricted prior beliefs with an informative signal of any form. We show that the game may result in an obfuscation equilibrium with high prices or a transparency equilibrium with low prices and no obfuscation, providing an argument for market regulation. Obfuscation equilibria cease to exist for low information costs and if one firm seems a priori considerably more attractive.

Keywords: Rational Inattention, Obfuscation, Price Competition, Digitalized Markets

JEL Codes: D11, D21, D43

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1 Introduction

Consumers often need to exert costly effort to learn about the price and quality of different products. In many markets, the complexity of price-quality assessment results from obfuscation practices by firms that make it harder for consumers to learn about their offers.[1] Examples of such practices include add-on pricing (e.g., Ellison, 2005; Gabaix and Laibson, 2006), hidden shipping and handling costs (e.g., Hossain and Morgan, 2006; Brown et al., 2010), and complicated or confusing product descriptions (e.g., Ellison and Ellison, 2009; Chioveanu and Zhou, 2013). The optimal degree of obfuscation is a strategic decision by firms subject to consumers' information acquisition process, which largely depends on exogenous factors, such as availability of information, information costs, market structure, or consumers' prior beliefs.[2] In recent years, digital technology has reshaped many of these factors.[3] In particular, it has markedly increased the amount of available and accessible information.

To account for this development, we study equilibrium pricing and obfuscation behavior of profit-maximizing firms in a market with rational inattentive consumers. In contrast to existing models of obfuscation, consumers' information acquisition is endogenous with no restrictions on consumers' prior beliefs or the type and extent of information processing. We consider a duopoly where firms compete on price and decide beforehand whether to obfuscate their prices or not. Obfuscation implies that consumers do not perfectly observe prices. Consumers can, however, exert costly efforts to decrease uncertainty about obfuscated offers. The optimal information strategy, which depends on consumers' information costs and the prior beliefs about the price-quality differentials, determines the endogenous demands for the products. Our model has important implications for effective market regulation as obfuscation makes it more costly for consumers to learn about firms' offers and, therefore, decreases competition.

The structure of our model is motivated by recent developments in the mobile subscription market, where pricing schemes have become less complex in recent years. Mobile subscriptions were previously characterized by a complex, variable add-on pricing structure. Today, most subscription plans are subject to fixed, postpaid pricing schemes. A decrease in prices accompanied this development.[4] In our model, this development can arise endogenously as a result of digitalization, equilibrium pricing, and obfuscation behavior of profit-maximizing firms. In general, our model also applies well to other markets where firms, broadly speaking, can choose between: i) a transparent flat pricing scheme, which makes it easy for consumers to assess products' "final" price per month or year; and ii) a complex variable pricing scheme that depends on many details over different dimensions and, therefore, requiring consumers to exert costly

[1]Other related terms used in the literature include shrouding attributes, strategic price complexity, and price frame dispersion.

[2]Several studies examine a variety of firms' actions that aim at manipulating consumers' information acquisition process in different market settings and with diverging implications for the resulting complexity of price-quality assessment in equilibrium. For example, see Gabaix and Laibson (2006), Carlin (2009), Ellison and Ellison (2009), Ellison and Wolitzky (2012), Chioveanu and Zhou (2013), and Heidhues et al. (2016).

[3]See Goldfarb and Tucker (2019) for an overview.

[4]In Appendix A, we provide a descriptive analysis of the Swedish mobile subscription market to exemplify our motivation. Turnbull et al. (2000) documents the importance of customer confusion in the early times of mobile subscriptions in the UK.

efforts to get a reasonable price estimate.[5] Other examples of such markets include different platform services (e.g. social networking sites and online marketplaces), video and online gaming, commercial news websites, and some services in the financial and health industries.

We show that our model may result in an obfuscation equilibrium with high prices where both firms obfuscate and a transparency equilibrium with low prices and no obfuscation. These results first highlight that rational inattentive consumers could incentivize firms in duopolies to optimally obfuscate their prices. If both firms obfuscate their price, competition is lower, and equilibrium prices and profits increase. Obfuscation may be a mutual best reply. This result extends findings from the standard search cost literature (e.g. Ellison and Wolitzky, 2012).

Prices and obfuscation behavior in equilibrium crucially depend on consumers' information costs to learn about obfuscated prices. An equilibrium with high prices and obfuscation only exists if information costs are large enough. Otherwise, the rents from decreased competition may be too low, giving firms an incentive to choose a transparent pricing scheme in the quest to secure the entire market. Eventually, this makes obfuscation a dominated strategy, leading to zero equilibrium profits as in Bertrand (1883) – a prisoner's dilemma for the firms. Following our theoretical results, the development towards more transparent pricing schemes, such as for mobile services, may result from digitalization. Digital technologies may have decreased individual information costs to the extent that a transparent pricing scheme has become the dominant strategy for firms. The difference to other markets, in which pricing schemes remain complex, such as service contracts in the financial and health industries, could be attributed to lower initial information costs or a comparably larger relative change in information costs due to digitalization.[6]

Additionally, we find that the equilibrium outcome depends on the shape of the consumers' prior beliefs. If, according to prior beliefs, one firm seems more attractive than the other, i.e., has higher prominence, the consumers assign a larger weight to the more attractive firm.[7] The weight arises endogenously from the optimal information strategy of the consumers and will bias the purchase decisions towards more prominent firms.[8] In a scenario where both firms obfuscate, a higher prominence implies higher optimal prices, a higher market share, and higher profits.[9] The less prominent firm will claim less of the rent arising from obfuscation, which increases the incentive to use a transparent pricing scheme in an attempt to claim the entire market. As a consequence, obfuscation in equilibrium ceases to exist if differences in prominence are large.

[5] In our model, we refer to case ii) as price obfuscation, which, in contrast to case i), requires costly effort by the rational inattentive consumers to decrease the uncertainty about prices.

[6] For general evidence of inertia in insurance markets, see, for example, Abaluck and Gruber (2016), Handel (2013), Handel and Kolstad (2015), Marquis and Holmer (1996), or Marzilli Ericson (2014). Woodward and Hall (2010) provide evidence for information costs and confusion in the mortgage market.

[7] Prominence in our model implies that, according to their prior beliefs, consumers expect the price-quality differential of one firm to be higher than that of the competing firm. We are agnostic to the underlying source of diverging prominence levels or how firms become prominent (e.g. Armstrong and Zhou, 2011). However, as "prominence" in our model arises due to unrestricted prior beliefs, it can accommodate a variety of different "prominence" concepts.

[8] This weight can be directly related to a quality parameter in linear random utility models (see Anderson et al., 1992)

[9] In our model, a larger weight on one product is connected to a smaller weight on the other one. This depends on the fact that the exact weights will be a monotonic transformation of the unconditional choice probabilities of all possible actions, which have to sum up to 1. See the discussion in sections 3 and 4 for further details.

Finally, we find that competition in the form of new market entrants alters optimal obfuscation behavior and prices. First, if the new competitor is significantly less prominent than existing firms, obfuscation equilibria cease to exist. Second, if the new competitor is sufficiently prominent, two channels become important. The competition channel increases competition among obfuscating firms. As a result, existing firms' optimal prices and profits decrease, thereby decreasing the likelihood of obfuscation equilibria. The information channel affects the equilibrium outcome by changing the prior belief and thus the consumers' optimal information strategy. This channel may have opposing effects if a new market entry increases the incentive to process at least some information and if the new information strategy favors the less prominent firm, such that resulting profits under obfuscation are more balanced. The effect on obfuscation behavior and prices will largely depend on the consumers' prior belief about the new product and its correlation with existing options.

Our results have important implications for market regulation. In addition to direct attention costs for consumers, obfuscation reduces competition among firms and leads to a decrease in consumer surplus. As a consequence, a policymaker may want to regulate markets to avoid obfuscation. According to our model, two main policy options exist. First, there is the option to decrease the information costs for consumers to learn about prices. This can be achieved either by increasing comparability between prices and facilitating search, for example, by digitalization, or by restricting product features that complicate consumers' assessment of the product's final price, such as add-on pricing.[10] Simply providing additional information about the products and prices does not help, as consumers' attention is the limiting factor. Second, there is the option to increase the difference in the prominence of firms, for example, by decreasing market entry barriers for new and unknown competitors that are likely to be less prominent than existing firms. Both of these policies make obfuscation and high prices less likely in equilibrium and increase consumer welfare.

Our demand-side approach takes advantage of a seminal paper by (e.g., Matějka and McKay, 2015, henceforth MM15) that formalizes the idea of rational inattentive consumers, initiated by Sims (2003), in a discrete choice framework.[11] Importantly, our framework inherits the idea that consumers have access to all information and freely choose – subject to a generally applicable information cost function – what kind of information to process.[12] This makes our approach particularly appealing in situations where relevant information is easily and abundantly available, which holds true for most digitalized markets. Through our demand-side approach, we extend several strands of literature that study optimal obfuscation and pricing in markets with imperfectly informed consumers.

[10] Heidhues et al. (forthcoming) reach a similar conclusion, studying secondary price features of products in a similar setting.

[11] Caplin et al. (2019) extend this framework by providing necessary and sufficient conditions that allow identification of products about which a rational inattentive consumers will process at least some information. For our paper, these conditions are particularly relevant for the cases with opposing obfuscation behavior by firms. Huettner et al. (2019) further extend the framework of MM15 for heterogeneous information costs for different choices.

[12] Our convex cost function is based on Shannon entropy, commonly used in the literature (e.g., Sims, 2003; Caplin et al., 2019).

Many of these studies utilize exogenous information structures where a fraction of consumers are sophisticated and make optimal decisions while another fraction of consumers are naive and choose randomly. In these settings, the firms usually manipulate the relative number of sophisticated consumers by obfuscation. Examples include Gabaix and Laibson (2006), who study add-on pricing; Carlin (2009), who considers the retail financial industry; and Chioveanu and Zhou (2013) and Piccione and Spiegler (2012), who focus on the comparability of prices. Notably, these models do not model the endogenous character of consumers' information search, which has considerable implications for firms' optimal obfuscation and pricing behavior as well as market regulation.

Other related studies analyze optimal obfuscation behavior in a search-theoretic model, along the lines of Salop and Stiglitz (1977), Varian (1980), and Stahl (1989). In these (sequential) search models, the information set is endogenous because it depends on the consumers' search costs.[13] Ellison and Wolitzky (2012) present a sequential search model in which firms affect the time needed to assess their prices. In the model of Wilson (2010), firms can influence the consumers' order of the search by obfuscation. Taylor (2017) adopt a search-theoretic model in which obfuscation helps firms to target the most valuable consumers. The crucial difference of traditional search models, as compared to our approach, is that information about one product is binary. Consumers either do not learn anything about a product or identify the relevant valuation perfectly. Furthermore, the order of search usually plays an important role. While the traditional setup is a good fit for window-shopping or browsing simply priced products, it fails to account for the abundance of relevant information and the complexity of pricing structures in mobile subscription or many other digitalized markets, which depend on many details, often related to expected future consumption. For these pricing structures, perfectly identifying the final price of products seems unrealistic. We present an alternative model in which the consumers are typically not perfectly informed about the obfuscated products' true valuation at the time of purchase. This may have important implications for the equilibrium behavior of firms and optimal regulatory policies, particularly with regards to the role of consumers' prior beliefs.

The role of prior beliefs is directly related to the literature on the "prominence" of firms (e.g., Armstrong et al., 2009; Armstrong and Zhou, 2011; Rhodes, 2011; Chioveanu, 2019) that deploy similar "binary" search models or exogenous information structures on the demand side. In a related study, Gu and Wenzel (2014) find that more prominent firms always choose maximum obfuscation in a market with either sophisticated or naive consumers. Similarly, Chioveanu (2019) study a duopoly where obfuscation influences the share of naive consumers in the market. Naive consumers choose randomly and are more likely to buy from the more prominent firm, thereby, providing an incentive for the more prominent firm to choose a more complex pricing scheme. In contrast to these models, prominence affects the equilibrium outcome in our model through consumers' endogenous information acquisition, which can lead to

[13] Grubb (2015), Spiegler (2016), and Armstrong (2017) provide overviews of recent relevant theoretical work on the intersection of industrial organization and consumer search.

diverging predictions of the model as prominence can have large or small effects depending on consumers' information costs and the exact shape of prior beliefs.

The paper by MM15 triggered several recent studies in industrial organization that focus on different aspects, such as inattentive sellers (Matějka, 2015) or inattention to quality and optimal pricing (Martin, 2017).[14] Two studies that incorporate rational inattentive demand in an industrial organization model are particularly relevant. First, Matějka and McKay (2012) investigate equilibria in an oligopoly when consumers are rational inattentive. Second, Boyacı and Akçay (2017) study the optimal pricing behavior of a monopolistic firm that faces a rational inattentive consumer. We add to this literature by extending the models along the obfuscation dimension. This important dimension has a natural connection to rational inattention as the attractiveness of obfuscation practices depends on how and which type of information consumers process.[15]

2 Model

2.1 Motivation

In Appendix A, we provide a descriptive analysis of developments in the Swedish mobile subscription market to exemplify our motivation. We document a change in the predominant pricing scheme and a decrease in prices in recent years. A development that can be observed in many different countries (e.g. European Commission, 2019). In the past, mobile subscriptions were characterized by a variable add-on pricing structure over different dimensions. Today, most subscription plans are based on fixed, post-paid pricing schemes.[16] These changes were accompanied by a decrease in price.[17] In comparison, service contracts in the financial or health industry are still based on complex pricing schemes.[18] We present a model in which these diverging developments can be explained by equilibrium pricing and obfuscation behavior of profit-maximizing firms when determining factors of endogenous consumers' information acquisition have been reshaped.

[14] See Mackowiak et al. (2020) for a detailed review of the budding rational inattention literature in other fields of economics

[15] Indeed, Grubb (2015) highlights the importance of "research [that] would be to consider firms investing in obfuscation that raises the cost of attention and hence increases the noise with which prices are evaluated." Similarly, Matějka and McKay (2012) note that "there is a natural connection between our model and the literature on obfuscation [...] which aims to understand practices by firms that serve to make the terms of their offers less transparent."

[16] In 2017, 83% of revenues in the Swedish mobile subscription market could be attributed to fixed fees, compared to 54% in 2011.

[17] For instance, the producer prices for telecommunication decreased by around 13% between 2014 and 2020

[18] For the health industry see for example Abaluck and Gruber (2016), Handel (2013), Handel and Kolstad (2015), Marquis and Holmer (1996); or Marzilli Ericson (2014) Woodward and Hall (2010) for evidence in the mortgage market.

2.2 Model Structure

Firms

There are two firms, indexed by $i = 1, 2$, that produce a homogeneous good of quality $q \in \mathbb{R}$. The firms sell their product to a unit mass of consumers and face two sequential strategic decisions. In $t = 1$, they decide simultaneously on the individual obfuscation parameter $\lambda_i \in \{0, \lambda\}$, i.e., between a transparent pricing scheme ($\lambda_i = 0$) and an obfuscated pricing scheme ($\lambda_i = \lambda$). Obfuscation comes at a fixed cost of $\xi > 0$.[19] After observing the obfuscation choice of the other firm, firms simultaneously set their prices ($p_i \geq 0$) in a subsequent second step ($t = 2$). Firms decide about obfuscation as well as prices non-cooperatively and with complete information. The pricing game can be summarized in a game tree, described in Figure 1.

The maximization problem of firm i in the second stage is given by:

$$\max_{p_i \geq 0} \eta_i(p_i, p_{-i}, \lambda_i, \lambda_{-i}, G_0(\cdot)) * p_i - \mathbf{1}_{\lambda_i = \lambda} * \xi,$$

where η_i represents the expected market share or, equivalently, the conditional probability that consumers buy the product of firm i at prices p_i and p_{-i}. The conditional probabilities are a result of the consumers' optimal information strategy (see section 3.2.), which depends on the firms' obfuscation choices λ_i and λ_{-i} as well as the exogenous consumers' prior beliefs about prices, $G_0(\cdot)$.

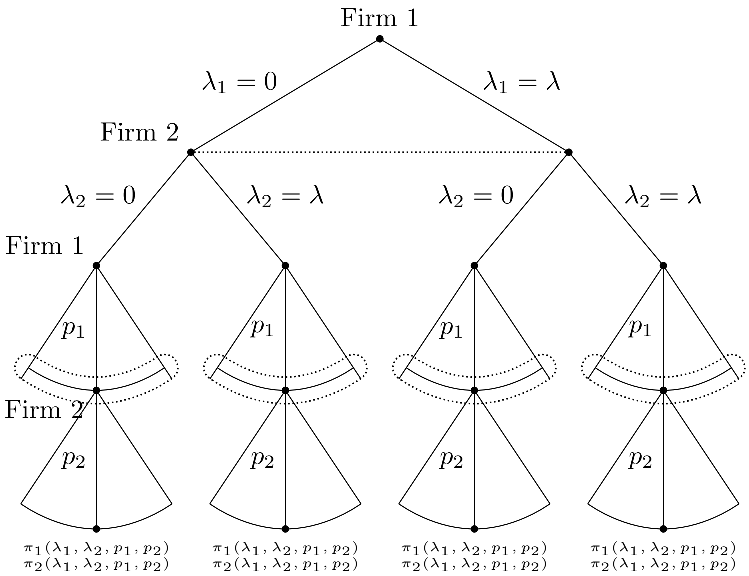
Consumers

Consumers can either buy from firm 1 or firm 2. The payoff of buying product i is given by the respective price-quality differential: $k_i = q - p_i$. We suppose consumers are aware of quality q, but do not observe the prices set by firms. They have an exogenously given prior belief about the joint distribution of prices, which we denote in terms of the quality-price differentials k_i: $G_0(k_1, k_2) = G_0(\mathbf{k})$. The prior belief fulfills two main functions in our model. First, it initiates uncertainty about prices. We do not restrict consumers' prior beliefs to allow for different rationales behind the uncertainty in our model.[20] Accordingly, consumers' prior beliefs can be "wrong" and may refer to (a combination of) many different factors, including uncertain future consumption, unintelligible firm structures and production costs as well as other behavioral biases and misunderstandings. Second, prior beliefs determine how prominent the firms are relative to each other. We consider a firm to be more prominent if consumers expect that

[19] Obfuscation costs ξ may refer to both direct and indirect costs. Examples of direct costs include increased costs for labeling the same product differently or (online) marketing expenses targeted at making firms' offers less transparent or comparable to other products. Indirect cost may, for instance, arise because complex pricing schemes may increase administrative efforts, necessary to keep track of diverging individual consumption and contracts, or lead to higher demand for customer services, dealing with complaints and questions.

[20] We implicitly assume that consumers cannot infer firms' optimal prices from the structure of the model. To put more structure on prior beliefs, one could, for instance, assume exogenous shocks that influence the optimal pricing strategies of firms, whose realizations cannot be perfectly observed by the consumers. If this would be the only source of uncertainty, the prior belief of consumers would be about the realizations of these shocks.

Figure 1: Game Tree



Notes: The figure describes the game tree. Initially, both firms set the obfuscation parameter λ_i simultaneously. The decision of λ_i is binary as firms can choose a transparent pricing scheme ($\lambda_i = 0$) or price obfuscation ($\lambda_i = \lambda$). The dotted line refers to an information set. After observing both choices of obfuscation, the firms choose prices p_i simultaneously. The payoff functions for firm 1 and firm 2 are denoted as π_1 and π_2.

the respective firm sets a lower relative price according to their prior beliefs.[21]

The consumers have the possibility to gather and process signals $\mathbf{z}$ to decrease the uncertainty about prices set by firms. The information strategy is unrestricted and can be denoted as a joint distribution of signals and payoffs, $F(\mathbf{z}, \mathbf{k}) \in \Delta(\mathbb{R}^{2N})$.[22] Receiving signals $\mathbf{z}$, according to the respective information strategy, results in a posterior belief, $F(\mathbf{k}|\mathbf{z})$. Given this posterior belief, the consumers choose product a with the highest expected payoff, i.e. the lowest expected price:

$$a(F(\mathbf{k}|\mathbf{z})) = \arg\max_i E_{F(\mathbf{k}|\mathbf{z})}(q - p_i)$$

When choosing the optimal information strategy, the consumers trade off the cost of information processing with its expected benefits. The benefits arise from the fact that choosing a distribution of more precise signals results in less uncertain posterior beliefs, which in turn translates into a higher expected value of the optimal choice. More precise signals are, however, more costly. We follow the rational inattention literature and assume an information cost function that is linear in the expected Shannon entropy reduction by means of the processed signal: $\hat{c}(F) = \lambda(H(G_0) - E_z[H(F(\mathbf{k}|\mathbf{z}))])$. $H(B)$ denotes the Shannon entropy of distribution B and λ is the unit cost of information, which is equivalent to the firms' obfuscation parameter.[23] Formally, the consumers choose an information strategy to solve the

[21] Similar qualitative results would arise if we assume that different levels of prominence result from diverging prior beliefs regarding the relative qualities of the products.

[22] $\Delta(\mathbb{R}^{2N})$ denotes the set of all probability distributions on $\mathbb{R}^2$. The information strategy has to be consistent with the consumers' prior belief. The condition $\int_z F(dz, \mathbf{k}) = G_0(\mathbf{k})$ assures this.

[23] This notation is only possible since we restrict the individual information costs to zero and λ. If the firms can set

following maximization problem:

$$\max_{F\in\Delta(\mathbb{R}^{2N})} \int_{\mathbf{k}}\int_{\mathbf{z}} k_{a(F(\mathbf{k}|\mathbf{z}))} F(d\mathbf{z}|\mathbf{k}) G_0(d\mathbf{k}) - \hat{c}(F)$$
$$\text{s.t. } \int_z F(d\mathbf{z},\mathbf{k}) = G_0(\mathbf{k})$$

3 Equilibrium

3.1 Characterization

We focus on subgame perfect equilibria in pure strategies. S is the set of strategy profiles, i.e., the set of possible pairs of firms' strategies, $(s_1, s_2) \in S$. Firm i's strategy $(s_i = (\lambda_i, p_i(\lambda_i, \lambda_{-i}))$ consists of the obfuscation decision in the first stage (λ_i) and the pricing decision in the second stage conditional on firms' obfuscation choices $(p_i(\lambda_i, \lambda_{-i}))$, taking into account the optimal information strategy of the rational inattentive consumers. A strategy profile $s \in S$ is subgame perfect if for each subgame the strategy profile constitutes a Nash equilibrium. A strategy profile $s \in S$ is a pure Nash equilibrium if $\forall i$ and $\forall s_i \in S_i$, $U_i(s_i, s_{-i}) \geq U_i(s_i', s_{-i})$.

3.2 Consumer demand

In the case in which both firms choose a transparent pricing scheme $(\lambda_1 = \lambda_2 = 0)$, the information strategy and the derivation of the consumer demands are trivial. The consumers will choose (costless) signals that perfectly identify firms' prices, implying that our model becomes deterministic. Consumers will buy the product with the lowest price. The conditional choice probabilities $\eta_i(\mathbf{k})$, which signify the probability that consumers choose product i at $k_i = q - p_i$, $\forall i \in 2$, will thus equal 1 if $p_i < p_j, \forall j \neq i$ and zero otherwise.[24]

If at least one firm chooses to obfuscate, information processing is costly, implying probabilistic demands and posterior uncertainty about prices set by firms. In these cases, one important and well-known feature of rational inattention is that each action strategy is associated with a particular signal (MM15). This allows us to rewrite the formal maximization problem of the consumers in terms of choice probabilities. Solving the reformulated consumers' maximization problem, we show, in accordance with MM15, that the optimal conditional choice probabilities $\eta_i(\mathbf{k})$ will follow a multinomial logit function (See Appendix B):

individual non-zero information costs different from each other, $0 < \lambda_1 < \lambda_2 < \infty$, the problem gets more pronounced (see Huettner et al., 2019).

[24] If two or more actions inhibit the same state-contingent payout, a tie-breaking rule is applied, resulting in equally distributed conditional choice probabilities among these options.

$$\eta_i(\mathbf{k}) = \begin{cases} 0 & \text{if } \eta_i^0 = 0 \\ \dfrac{\eta_i^0 e^{(q-p_1)/\lambda}}{\eta_1^0 e^{(q-p_1)/\lambda} + \eta_2^0 e^{(q-p_2)/\lambda}} & \text{if } 0 < \eta_i^0 < 1, \\ 1 & \text{if } \eta_i^0 = 1 \end{cases} \tag{1}$$

where η_i^0 are the unconditional choice probabilities, which are the collection of all conditional choice probabilities $\eta_i(\mathbf{k})$ integrated over the respective prior beliefs $G_0(\mathbf{k})$: $\eta_i^0 \equiv \int_{\mathbf{k}} \eta_i(\mathbf{k}) G_0(d\mathbf{k})$. Accordingly, the unconditional choice probabilities are a result of the consumers' maximization problem and will shift the demand towards the more prominent product. η_i^0 is independent of the prices set by firms, but affected by the prior belief and λ. As a result, even large differences in prominence may only marginally affect conditional choice probabilities if λ is small or prior beliefs are very uncertain. Intuitively, prominence has a small effect if prior beliefs are unimportant relative to the informative signal $\mathbf{z}$. Similar to standard Bayesian updating, this can happen either because the received signal will be very informative due to small information costs or because the prior beliefs are not very precise. If consumers do not distinguish between the different products according to their prior beliefs, both unconditional choice probabilities will be equal to $\eta_i^0 = 0.5$. In this case, the model reduces to a symmetric logit model and both products are completely homogeneous.

The consumers' optimal probabilistic choice will follow a multinomial logit form, irrespective of whether only one firm chooses to obfuscate or both firms choose to obfuscate (See Appendix B for derivations). However, the cases differ with respect to the relevant (conditional) prior beliefs, as rational inattention implies that consumers process all costless information, i.e., the transparent price, before choosing the optimal (non-trivial) information strategy. For the case in which only one firm obfuscates, this implies that the prior belief conditional on all costless information $\tilde{k}$, $G_0(\mathbf{k}|\tilde{k})$ is relevant for the consumers' maximization problem.[25] As a result, the corresponding unconditional choice probabilities η_i^0 differ, which has important implications, particularly with respect to border cases where the consumers do not process any information. Unconditional choice probabilities of zero and one signify these border cases. $\eta_i^0 = 1$ implies that the consumer would not process any information and buys product i with a probability of one ($\eta_i(\mathbf{k}) = 1$) irrespective of the prices set by obfuscating firm(s). Accordingly, the consumers never choose or consider a product with $\eta_i^0 = 0$.[26]

The price cutoff level

Caplin et al. (2019) show that rational inattention in a discrete choice setting implies that some options

[25] We assume that, according to the consumers prior belief, the price of one firm is independent of the price of the other firm such that learning the price of the transparent firm does not imply anything about the other price.

[26] Caplin et al. (2019) provide the corresponding necessary and sufficient conditions for the unconditional choice probabilities that we apply in our simulations later. If $0 < \eta_i^0 < 1$, the normalization conditions laid out by MM15 apply:

$$\int_{\mathbf{k}} \left(\frac{e^{k_i/\lambda}}{\sum_{j=0}^{2} \eta_j^0 e^{k_j/\lambda}} \right) \Omega_0(d\mathbf{k}) = 1, \quad \forall i \eta_i^0 > 0$$

will never be considered. These non-considerations of options arise endogenously and will depend on a unique cutoff that depends on the information cost and prior beliefs (Caplin et al., 2019). In our setting, non-considerations of a product exist if information costs are comparably large and one of the products seems a priori much more attractive to the consumer than the other option. While we assume that the prior beliefs are such that consumers always find it optimal to consider both products if both firms choose to obfuscate, the cutoff level becomes important in the case of opposing obfuscation choices by firms.

If the price of the transparent firm, which is costless information to the consumers, is low enough the consumers will not process any information and will buy from the transparent firm with a probability of one. We denote the cutoff price level as $\bar{p}_i$ ($\lambda_i = 0$ and $\lambda_{-i} = \lambda$): $\bar{p}_i = \max p_i$ such that $\eta_i^0 = \eta_i(\mathbf{k}) = 1$. If the transparent price is below that cutoff, the agent will not process any information and will always buy the transparent product. If the transparent price is above the cutoff, the likelihood that the consumer will buy the obfuscated product will be positive. While the exact cutoff level will depend on the shape of the prior belief and the information costs (see simulations below), some properties are important for the outcome of the game.

First, the cutoff level will always be larger than zero for positive information costs and some prior uncertainty about the obfuscated price, which implies there will also be some remaining posterior uncertainty about the obfuscated price after processing information. Consider a transparent price that is set to or close to zero. As we assume positive prices - and accordingly a support of the prior belief of zero - this implies that even in the most favorable (uncertain) state, the transparent option is as good as the obfuscated option. As a result, consumers would never process any information and just buy the transparent product if the transparent price is close enough to zero.

Second, the price cutoff level is increasing in λ. Generally, the obfuscated option is more valuable to the consumers. From the perspective of the consumer, the prices are uncertain and they customize their information strategy such that they purchase the transparent product if the obfuscated price is high and the obfuscated product otherwise. This implies that the conditional choice probability will be larger for the obfuscated product compared to the transparent option, if both expected prices at prior beliefs are equal. In this sense, uncertainty is beneficial in a rational inattention setting because the consumers can customize their choice to the realization of the state, which, in our case, refers to the prices set by the obfuscating firm (see also Caplin et al., 2019). If information costs increase, it gets more costly to customize the choice. The advantage of the uncertain option vanishes and the consumer will choose the transparent option without processing any information for larger prices, implying an increasing cutoff level.

Third, the cutoff level converges to the expected price of the obfuscated product if information costs strive to infinity. If $\lambda = \infty$, the consumer will not process any information and will always buy the obfuscated product if its expected price at prior beliefs is lower than the transparent price and vice versa. Accordingly, the expected price at prior beliefs constitutes an asymptote for the price cutoff level for increasing levels of λ.

3.3 Firms

Second Stage

In the spirit of backward induction, we start by solving for subgame perfect equilibria of the competitive pricing game of the firms conditional on the obfuscation decision made in the first stage.

Proposition 1. *If both firms choose to obfuscate, the optimal prices and profits are uniquely determined by following set of equations:*

$$p_i^* = \frac{\lambda}{(1-\eta_i^*(\boldsymbol{k}))}. \tag{2}$$

for $i \in N$, where $\eta_i^*(\boldsymbol{k})$ denote the equilibrium market share, and the corresponding equilibrium profits of firm i are given by:

$$\pi_i^* = \frac{\lambda}{(1-\eta_i^*(\boldsymbol{k}))}\eta_i^*(\boldsymbol{k}) \tag{3}$$

Proof. Given the multinomial logit demand function derived above, the derivation of equilibrium prices and profits in the case of simultaneous obfuscation follows a similar logic as in Anderson et al. (1992).[27] The first-order conditions are given as follows:

$$\eta_i(\mathbf{k}) - \frac{\eta_i(\mathbf{k})(1-\eta_i(\mathbf{k}))}{\lambda}p_i^* = 0,$$

For an interior solution, the optimal price of firm i is thus given by equation (2). Anderson et al. (1992) show that such a system of equations has a unique solution. Equilibrium profits are given by equation (3). □

As noted above, the unconditional choice probabilities are independent of firms' prices as they do not affect the consumer's prior belief. As a result, firms cannot influence the unconditional choice probabilities in the second stage if $\lambda_i = \lambda$. If only one firm chooses to obfuscate, the consumer will be perfectly informed about the transparent price set in the second stage. Consequently, the prior belief conditional on all costless information ($G_0(\mathbf{k}|\tilde{k})$) changes, which affects the consumer's optimal information strategy. As a result, the transparent firm can influence the unconditional choice probabilities.[28]

Proposition 2. *If only one firm chooses to obfuscate, the optimal price of the transparent firm ($\lambda_i = 0$) is equal to $\bar{p}_i$, where $\bar{p}_i = \max p_i$ such that $\eta_i^0 = \eta_i(\boldsymbol{k}) = 1$.*

Proof. First, note that the transparent firm will not set a price below $\bar{p}_i$, because, if $p_i < \bar{p}_i$, $\eta_i(\mathbf{k}) = 1$ and accordingly $\dfrac{\partial \pi_i}{\partial p_i} = 1 > 0$.

[27] In Anderson et al. (1992) the multinomial logit demand arises from a representative consumer with linear random utility and double exponential distributed error terms. In contrast, under rational inattention, the randomness arises not from noise with respect to preferences, but from the signal the consumer receives.

[28] See Appendix B for further descriptions.

If $p_i \geq \bar{p}_i$, both firms will face a multinomial demand as derived above. The maximized profit function for a given unconditional choice probability is equal to $\pi_i^*(\eta_i^0) = \frac{\lambda}{(1-\eta_i^*(\mathbf{k}))}\eta_i^*(\mathbf{k})$. Plugging in the corresponding demand function, the profit function can be rewritten to:

$$\pi_i^*(\eta_i^0) = \lambda \frac{\eta_i^0}{(1-\eta_i^0)} \frac{e^{(q-p_1^*)/\lambda}}{e^{(q-p_2^*)/\lambda}} = \lambda \frac{\eta_i^0}{(1-\eta_i^0)} e^{(p_2^*-p_1^*)/\lambda}$$

Applying the Envelope theorem, the derivative of the profit function with respect to the unconditional choice probability is given by:

$$\frac{\partial \pi_i^*}{\partial \eta_i^0} = \frac{\lambda}{(1-\eta_i^0)^2} e^{(p_2^*-p_1^*)/\lambda} > 0,$$

which is strictly larger than zero if $\eta_i^0 < 1$.

□

As a consequence of Proposition 2, the transparent firm's profits and optimal price in equilibrium are given by $\pi_i = p_i^* = \bar{p}_i$. The obfuscating firm has a market share of zero and can set any p_2 in equilibrium. The equilibrium profits of the obfuscating firm are equal to $-\xi$. Proposition 2 further implies that the consumer will not process any information. The exact value of the cutoff level $\bar{p}_i$ depends on the prior beliefs and the information costs parameter λ, as we discuss in the sections below.

If both firms choose not to obfuscate, the pricing game is equal to a well-studied Bertrand duopoly. Both obfuscation parameters are equal to zero such that the rational inattentive consumer observes prices perfectly. The unique best reply of both firms is given by $p_1 = p_2 = 0$, the Bertrand paradox. The corresponding firm profits are zero.

First Stage

In the first stage, both firms have the option to obfuscate or choose a transparent pricing structure. The simultaneous and discrete decision to obfuscate leads to four different nodes of the game (see Figure 1) and three different possible equilibria.

Proposition 3. ***Subgame perfect equilibria:***

i.) ***Transparency equilibrium:*** *With positive obfuscation costs ($\xi > 0$), there always exists a subgame perfect equilibrium that is defined by the following strategies:*

$$s_i = (\lambda_i = 0, p_i = 0) \quad \forall i \in N$$

ii.) ***Opposing equilibrium:*** *With positive obfuscation costs ($\xi > 0$), there is no opposing equilibrium in which only one firm obfuscates.*

iii.) ***Obfuscation equilibrium:*** *There exists a subgame perfect equilibrium that is defined by the following strategies and conditions:*

Strategies:

$$s_i = (\lambda_i = \lambda, p_i = c + \frac{\lambda}{(1-\eta_i^*(\boldsymbol{k}))}) \quad \forall i \in N$$

Condition:

$$\frac{\lambda}{(1-\eta_i^*(\boldsymbol{k}))}\eta_i^*(\boldsymbol{k}) - \xi \geq \bar{p}_i(G_0(\cdot)) \quad \forall i \in N \tag{4}$$

Following the results of the second stage, we can represent the simultaneous move game in the first stage in a symmetric 2×2 payoff matrix presented in Table 1.

Table 1: First Stage Payoff matrix

		Firm 2	
		$\lambda_2 = 0$	$\lambda_2 = \lambda$
Firm 1	$\lambda_1 = 0$	$(0, 0)$	$(\bar{p}_1(G_0(\cdot), \lambda),\ -\xi)$
	$\lambda_1 = \lambda$	$(-\xi,\ \bar{p}_2(G_0(\cdot), \lambda))$	$\left(\frac{\lambda}{(1-\eta_1^*(\mathbf{k}))}\eta_1^*(\mathbf{k}) - \xi,\ \frac{\lambda}{(1-\eta_2^*(\mathbf{k}))}\eta_2^*(\mathbf{k}) - \xi\right)$

Notes: The table presents the payoff matrix of both firms in the first stage. Firms 1 and 2 set their obfuscation parameters simultaneously. Each combination of obfuscation parameters relates to a payoff function.

If both firms choose a transparent pricing scheme, prices and profits are zero. In the case that the opponent of a firm does not obfuscate, i.e., $\lambda_{-i} = 0$ where $-i \in N \setminus i$, it is always the best reply for firm i to be transparent ($\lambda_i = 0$) as $0 > -\xi$. Thus, the Transparency equilibrium $s_i = (\lambda_i = 0, p_i = 0) \quad \forall i \in N$ always exists independent of prior beliefs and information costs if $\xi > 0$ (*Proposition 3.i.*).

Following Proposition 2, the transparent firm claims the entire market if $\xi > 0$ and $\lambda_i \neq \lambda_{-i}$. As a result, only the transparent firm would make non-negative profits, while the obfuscating firm suffers a loss due to positive obfuscation costs. Consequently, it is the dominant strategy to be transparent as well, yielding a payoff of zero. An opposing equilibrium does not exist (*Proposition 3.ii.*).

According to *Proposition 3.iii.)*, the game has an obfuscation equilibrium ($s_i = (\lambda_i = \lambda, p_i = \frac{\lambda}{(1-\eta_i^*(\mathbf{k}))})$) if condition (4) holds. Condition (4) states that firm i's profit in the joint obfuscation case is larger than the profit that firm i would gain with a transparent pricing scheme, given the other firm chooses to obfuscate. As shown above, the profit of firm i in the joint obfuscation case is given by: $\frac{\lambda}{(1-\eta_i^*(\mathbf{k}))}\eta_i^*(\mathbf{k}) - \xi$ (Proposition 1); and the profit of the transparent firm in the opposing case is equal to the cutoff price level $\bar{p}_i(G_0(\cdot))$ (Proposition 2), which may be different for each firm (see below). If condition (4) does not hold, firm i would have an incentive to deviate from choosing obfuscation in the first stage and the obfuscation equilibrium cease to exist.

Overall, the condition (4) is crucial for the outcome of the game in the first stage. If the condition holds, the first stage is a pure coordination game with two equilibria. The payoff is strictly higher in the obfuscation equilibrium compared to the transparency equilibrium. If the condition does not hold, the game is equivalent to a prisoner's dilemma. Even though mutual obfuscation would result in a higher payoff, the game has a unique equilibrium in pure strategies, in which both firms set $\lambda_i = 0 \; \forall i \in N$.

Furthermore, condition (4) also illustrates the role of obfuscation costs ξ. While ξ does not affect optimal prices in the second stage, it has important implications for the outcome of the first stage. We assume positive obfuscation costs, referring to both direct and indirect cost (see examples above). In most relevant markets, this seems like a realistic assumption. However, there might be situations, in which obfuscation costs may be zero or even negative. For $\xi \leq 0$, the equilibrium outcome can change. Generally, it makes obfuscation more attractive, implying that obfuscation equilibria exist for a larger range of parameters. If $\xi \leq 0$, "opposing" equilibria may exist as both options in the first stage will at least yield a profit of zero, given the other firm chooses a transparent pricing scheme. For strictly negative obfuscation costs, transparency equilibria cease to exist. Furthermore, if ξ is small enough obfuscation may become the dominant strategy for both firms.

While no closed-form solution for the condition in Proposition 3 exists, we provide comparative static exercises in the next section to show for which parameters of the model an obfuscation equilibrium exists.

4 Comparative statics

4.1 The Effect of Information Cost Parameter λ

Proposition 4. *An obfuscation equilibrium only exists if the information cost parameter λ is large enough.*

To get an idea of how different information costs influence the equilibrium outcomes, we start by considering that consumers have symmetric prior beliefs about the attractiveness of the options. Both firms are equally prominent, implying that consumers do not distinguish between the different products before processing information.[29] The unconditional choice probabilities will be identical across firms, $\eta_i^0 = 0.5$, making them completely homogeneous. The optimal prices are given by $p_i^* = 2\lambda$ with corresponding profits of $\pi_i = \lambda - \xi$ in the case where both firms obfuscate. In the other obfuscation cases, firms' optimal behavior is symmetric as well and follows the optimality conditions derived above.

Figure 2 shows the payoffs for increasing information costs for different decisions in the first stage.[30] If both firms choose to obfuscate, the equilibrium profits and prices are linearly increasing in λ. Both

[29] If not stated otherwise, we assume that prior beliefs follow a multivariate normal distribution $G_0(\mathbf{k}) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $\boldsymbol{\mu} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ and $\boldsymbol{\Sigma} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. Accordingly, the (conditional) prior belief of the price difference is given by $(p_2 - p_1) \sim N(\mu_2 - \mu_1, \sigma_{11}^2 + \sigma_{22}^2)$ in the case where both firms obfuscate, $(p_2 - p_1) \sim N(\mu_2 - p_1, \sigma_{22}^2)$ if only firm 2 obfuscates, and $(p_2 - p_1) \sim N(p_2 - \mu_1, \sigma_{11}^2)$ if only firm 1 obfuscates.

[30] Note that, as the game is symmetric and the consumer does not distinguish between the two products before processing information, the graph is equivalent for both firms.

firms benefit from increasing information costs, as this decreases competition among firms. Symmetric prior beliefs ensure that the market is shared equally.

For the transparent firm in the opposing case ($\lambda_1 = 0, \lambda_2 = \lambda$), profits are also increasing in λ, while marginal profits are decreasing in λ. This is because the profits for the transparent firm mirror the cutoff price level $\bar{p_1}$, for which the consumer finds it optimal to process at least some information.

In general, the obfuscated product is more valuable for the consumers *a priori* as the prices vary in every state (from the perspective of the consumers) and the consumers will choose an information strategy that they will choose the obfuscated products if the obfuscated price turns out to be low and the transparent product otherwise. As a consequence, variance is beneficial under rational inattention, which explains the convex transformation of payoffs (Caplin et al., 2019). This comparative advantage of the obfuscated product increases for low values of lambda as tailoring the purchase decision to the obfuscated prices set becomes less costly.[31] Vice versa, if λ approaches infinity the purchase decision will only depend on the expected value of the obfuscated product at prior beliefs, which explains why $\bar{p_1}$ converges to the mean of the underlying prior belief. Accordingly, if processing information is costly, the transparent firm can set a higher price and still claim the entire market, as the benefit for the consumer of potentially learning about a lower price of the obfuscating firm is decreasing, making the consumer more likely to choose the transparent ("safe") option.

Following from the argument above, the consumer prefers the transparent option for lower values of λ if the variance of the obfuscated price according to the prior belief (σ_{22}^2) is smaller. As a result, the transparent firm's profits in the opposing case are higher for the same values of λ and lower variance of the prior belief, given that the mean of the prior belief does not change. If both firms do not obfuscate, profits are always zero. In a opposing equilibrium, the obfuscating firm ($\lambda_1 = \lambda, \lambda_2 = 0$) has a negative profit of $-\xi$.

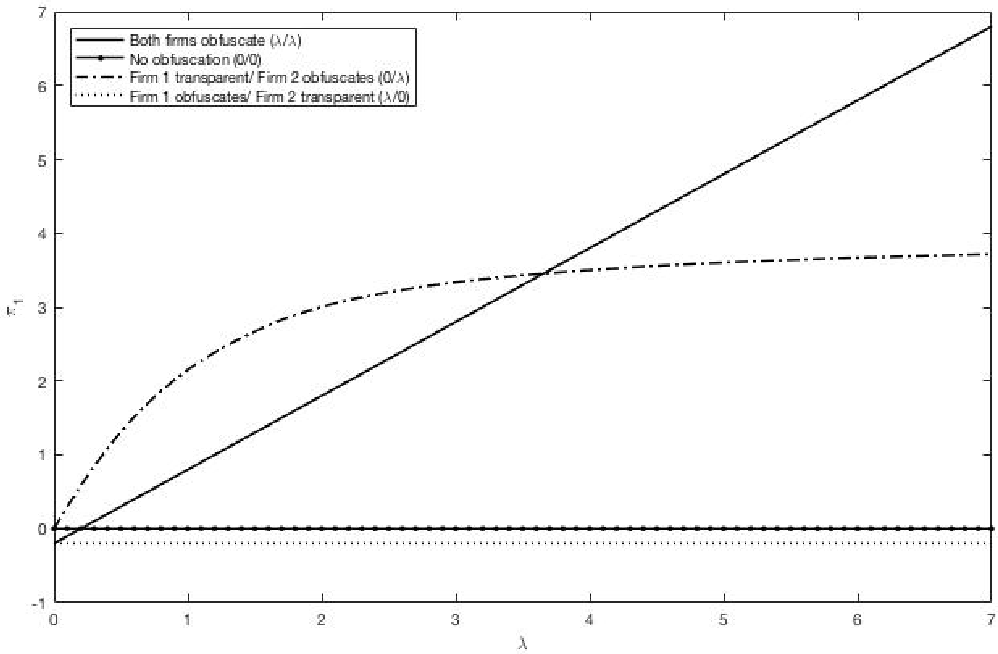
4.2 The Effect of Different Levels of Prominence

Proposition 5. *Obfuscation equilibria cease to exist if the difference in prominence of firms is too large.*

With asymmetric priors, the consumer prefers one firm prior to any information processing. In our simulation, we assume that $G_0(p_1) \sim N(4.4, 2)$ and $G_0(p_2) \sim N(4, 2)$, implying that, before processing information, the consumer expects the product of firm 1 to be more expensive, i.e., firm 2 seems more attractive. The prominence translates into different unconditional choice probabilities that can be interpreted as diverging product qualities in a standard logit-demand model (see Matějka and McKay, 2012). Accordingly, the ex-ante preferred firm (firm 2 in our example) sets a higher price than firm 1 if both firms obfuscate $\lambda_i = \lambda \; \forall i \in N$. As market shares, i.e., conditional choice probabilities, are also larger for more prominent firms, it directly translates into higher equilibrium profits. The difference in

[31] This case is closely related to the independent consumer problem discussed in section 3.2. of Caplin et al. (2019)

Figure 2: Profit Functions for Symmetric Prior Beliefs



Notes: The graph describes the profits of firm 1 for increasing obfuscation (λ) and different obfuscation choices when consumers have symmetric prior beliefs across all prices. The graph for firm 2 is identical.

the unconditional choice probabilities (see Figure 3a), and accordingly profits (see Figure 3b), will be higher for larger information costs (λ). If information processing is more costly, the rational inattentive consumer will optimally choose less precise signals, relying more on prior beliefs. As in the symmetric case, the competition among firms decreases as a result, which benefits both firms as long as the consumer processes some information about both products ($\eta_i^0 > 0$). Furthermore, if there is lower variance of the prior beliefs about both prices (σ_{ii}^2, $i = 1, 2$), the importance of the difference in prominence will play a bigger role, as the consumer's expected value of learning about the actual price difference decreases. Accordingly, the difference between the unconditional choice probabilities (see Figure 3c) and thus profits (Figure 3d) becomes larger for the same values of λ.

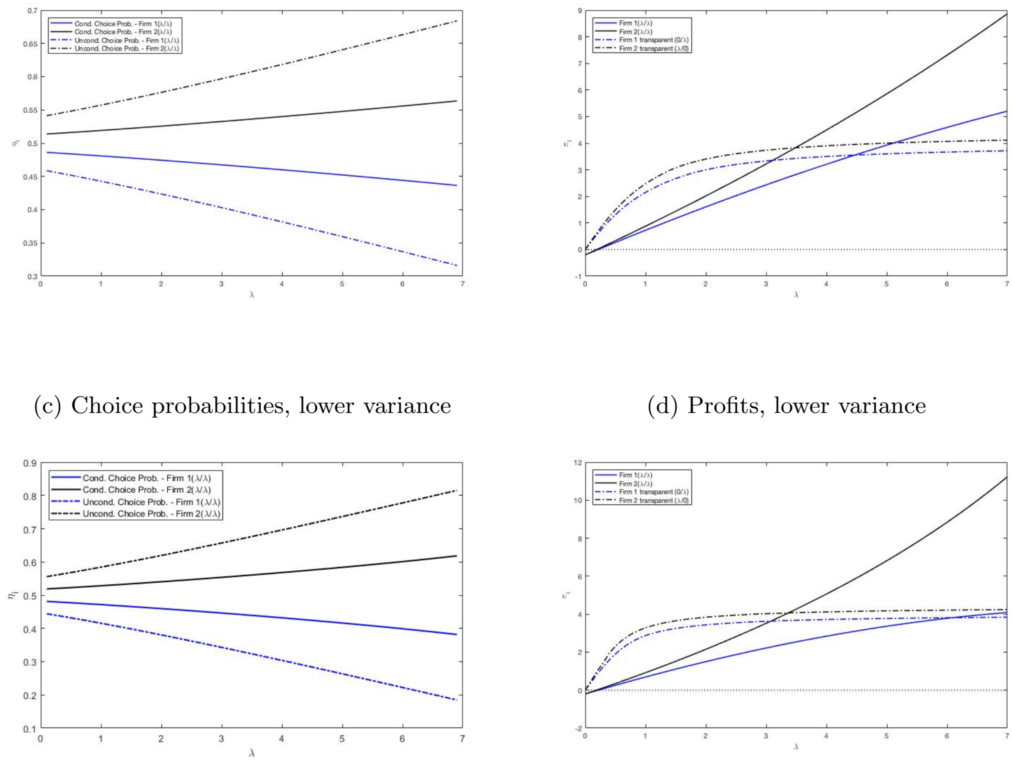
As in the symmetric case, the obfuscating firm ($\lambda_1 = \lambda, \lambda_2 = 0$) makes a negative profit of $-\xi$ if the other firm chooses a transparent pricing scheme. The transparent firm will set $\bar{p}_i$ and has a market share of one. The difference for the asymmetric case is, however, that the relevant expected price, constituting the asymptote for profits of the transparent firm, varies conditional on which firm obfuscates. The consumer finds it less beneficial to learn something about an obfuscated price if the consumer expects the price to be comparably high. This implies that the more prominent firm, as compared to the less prominent firm, can set a higher $\bar{p}_i$, yielding higher profits, for the same values of λ.

These simulation results for the subgame perfect equilibria in the second stage have important implications for the obfuscation choice in the first stage, which are analyzed in more detail in the next

Figure 3: Choice Probabilities and Profit Functions for Asymmetric Prior Beliefs

(a) Choice probabilities (b) Profits

(c) Choice probabilities, lower variance (d) Profits, lower variance



Notes: The four parts of this figure illustrate conditional $\eta_i(\mathbf{k})$ and unconditional choice probabilities η_i^0 of the consumer and profits of firms for increasing obfuscation parameter (λ) and different obfuscation choices. We simulate asymmetric prior beliefs as an independent multinomial normal distribution with a mean of $\mu = (4.4, 4)$. Parts (a) and (b) of this figure present choice probabilities with a variance of $\sigma^2 = (2, 2)$. The blue line refers to the choice unconditional and conditional choice probabilities as well as the profit functions for firm 1 while the black line shows profits for firm 2. Parts (c) and (d) of the figure present results for a lower variance of prior beliefs, i.e., $\sigma^2 = (1.5, 1.5)$.

section.

4.3 The existence of an obfuscation equlibrium

We have shown that two equilibria may exist in the first stage. First, there always exists a transparency equilibrium, in which both firms do not obfuscate. This equilibrium is independent of λ and consumers' prior beliefs. Second, there exists an obfuscation equilibrium in which both firms obfuscate. The condition for the existence of the obfuscation equilibrium is described in Proposition 3. The existence depends on the prior beliefs as well as on λ.

First, we consider symmetric prior beliefs. Figure 2 presents a firm's profit for different obfuscation decisions and different values of λ. For small values of λ, obfuscating the price is a dominated strategy. Independent of the opponent's decision, it is optimal for a firm not to obfuscate. The condition in Proposition 3 does not hold because conditional on the opponent choosing to obfuscate, the best reply is to be transparent. The unique subgame perfect equilibrium is that both firms abstain from obfuscation in the first stage and set prices equal to zero in the second stage. The first-stage game reduces to a prisoner's dilemma as both firms could increase their profits if both obfuscate.

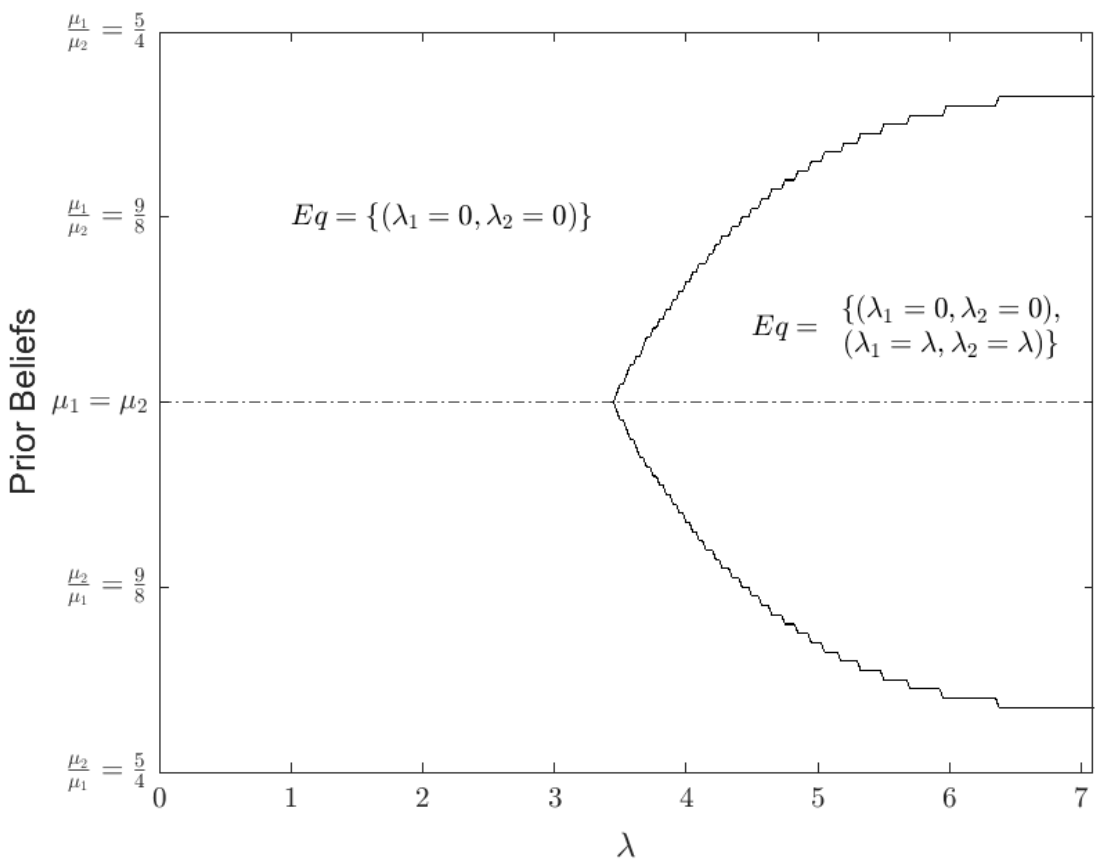
For large values of λ, conditional on an obfuscating opponent, it may become optimal to obfuscate prices as well and the game may result in a different equilibrium. In Figure 2 we observe that, for large values of λ, the profit of mutual obfuscation exceeds the profit of an opposing equilibrium for the transparent firm. Thus, for symmetric priors and high enough values of λ, there exists a subgame perfect equilibrium where both firms obfuscate and set positive prices, as described in Proposition 3. As there exist two equilibria, the game is characterized by a coordination problem. Both firms have an incentive to coordinate to attain the high-payoff equilibrium with positive profits.

Next, we turn to analyze equilibria under asymmetric prior beliefs of consumers, i.e. diverging prominence of firms. Now, the degree of asymmetry in prior beliefs as well as the size of information costs determine if the game has one or two subgame perfect equilibria. Figure 4 shows the relationship between consumers' prior beliefs, information costs, and the existence of an obfuscation equilibrium. In detail, we show the area in which the condition of Proposition 3 holds. We observe that a higher asymmetry of prior beliefs requires a higher λ such that mutual obfuscation in the first stage is an equilibrium, everything else being equal. The intuition can be derived from Figure 3. In this example and before processing information, consumers prefer firm 2 to firm 1. In Figure 3, we show both firms' profit functions for increasing values of information costs λ, differentiating between the case in which both firms obfuscate and the case in which only one firm obfuscates. Proposition 3 requires that the profit from mutual obfuscation exceeds the profit in the opposing case for each firm. Comparing the profit functions for asymmetric priors to those with symmetric priors in Figure 2, we see that the ex-ante less preferred firm requires a higher value of λ , conditional on the other firm chooses to obfuscate, to prefer obfuscation compared to a transparent pricing scheme. Intuitively, while the benefit of decreased competition persists for both firms, the market share and relative profits for the ex-ante less preferred firm in the mutual obfuscation case will be lower. This in turn increases the relative attractiveness of choosing a transparent pricing scheme. Accordingly, for some values of λ where obfuscation would have been a best reply in the symmetric case, the ex-ante less preferred firm now chooses a transparent pricing scheme such that mutual obfuscation is not an equilibrium anymore.

4.4 New Competitor

We now consider the entry of a new competitor. The market entry of a new competitor may influence the obfuscation behavior of firms in equilibrium. We differentiate between two cases. First, the prior belief may be such that the consumer will not process any information about the new product and thus will not consider buying the product. The optimal reply of the new competitor, in this case, is either not to

Figure 4: Equilibria in First Stage



Notes: The graph describes equilibria regions for the first stage of the game given different prior beliefs and increasing values of the obfuscation parameter λ. The solid line represents the border between the equilibria regions.

enter the market or to set a transparent price, which in turn will lead to zero equilibrium profits.[32] Thus, consumers' prior belief creates an additional source of a market entry barrier in a market with rational inattentive consumers. Besides a high expected price, this scenario may also arise if the consumer thinks the new product duplicates an existing one, i.e., yielding the same payoff, in contrast to the standard logit model (Caplin et al., 2019, MM15). In our analysis, we assume a second case in which the consumer's prior belief is such that the consumer finds it optimal to process some information about all products if all firms obfuscate, $\eta_i^0 > 0, \forall\, i$.

If two or more firms choose a transparent pricing scheme, a Bertrand type of situation occurs. In the second stage, transparent firms split the entire market, resulting in zero equilibrium profits. As a result, obfuscating firm(s) would incur a loss of $\pi_i = -\xi$. This implies that $\lambda_i = 0$ is always the best reply for firm i conditional on $\exists \lambda_{-i} = 0$ where $-i \in N \setminus i$. As in the duopoly case (see Proposition 3), there always exists a subgame perfect equilibrium in which all firms do not obfuscate and choose to set a price of zero, $s_i = (\lambda_i = 0, p_i = 0) \quad \forall i \in N$.

For the existence of a pure strategy equilibrium, in which all firms choose to obfuscate, the profits in the joint obfuscation case ($\lambda_i = \lambda, \forall i \in N$) have to be higher than in the opposing cases ($\lambda_i = 0, \lambda_{-i} = \lambda, \quad i = 1, 2, ..., N$) for all firms. Otherwise, at least one of the firms would have an incentive to deviate, implying that no obfuscation equilibrium exists (see Proposition 3 for the equivalent condition in the duopoly case). Therefore, to determine whether we would expect a new market entry to increase or decrease the occurrence of obfuscation in equilibrium, the effect on firms' profits in the second-stage subgame perfect equilibrium is crucial. The derivation of the subgame perfect equilibria follows the same logic as in the duopoly case. In a market in which all firms obfuscate, the optimal pricing of the firm's strategy results in the following equilibrium profit for firm i (see Appendix C):

$$\pi_i^* = \frac{\lambda}{(1 - \eta_i^*(\mathbf{k}))} \eta_i^*(\mathbf{k}), \quad \forall i \in N$$

A new competitor affects the optimal behavior of firms through two channels. First, as $\eta_i(\mathbf{k}) > 0 \,\forall\, i \in N$, a new competitor decreases aggregated demand. This decreases the prices and profits of existing firms through increased market competition. In turn, obfuscation becomes less attractive and the obfuscation equilibrium less likely, compared to the duopoly case.

Second, a new market entry also changes the consumer's prior belief and thus alters the optimal information strategy of the consumer. As a result, the choice among existing alternatives may change, and a single existing firm may benefit from a new market entry under rational inattention, in contrast to any random utility model (e.g. MM15). This may also shift the equilibrium in the first stage, potentially in both directions, depending on whether the new information strategy favors the less prominent or more prominent firm.

[32] The necessary and sufficient condition provided by Caplin et al. (2019) can be used as a simple test to determine whether the consumer would consider the new product or not.

The outcome depends on the exact shape of the consumer's prior belief, particularly on the correlation of the new competitor's price with the price of existing alternatives. Deriving the exact cutoff levels for different beliefs goes beyond this paper's scope, but consider the following stylized example, in which the likelihood of an obfuscation equilibrium may increase. There are two firms in the market with completely independent prices according to the consumer's prior belief. One option has small variance, and the other option has thicker tails, implying a high probability that the firm's price is either very low or very high, a risky option. The option with small variance seems more attractive to the consumer, such that obfuscation equilibria only exist for large values of λ. Now, there is a new competitor, which also appears risky to the consumer. Prices of the new competitor are perceived to be negatively correlated with prices of the less prominent firm but independent from the more prominent firm's prices. This negative perceived correlation increases the incentive for the consumer to investigate whether one of the risky options is very cheap, similar to the red bus–blue bus–train example from MM15. If, in addition, unconditional choice probabilities and resulting profits are more balanced after market entry, the likelihood of an obfuscation equilibrium may increase, even if aggregated demand in the obfuscation case decreases for existing firms.

In addition, a new competitor may increase the incentive to investigate prices at all. Similar to the duopoly case, equilibrium profits are increasing in η_i^0, implying that, in the case of only one transparent firm, the transparent firm will again choose the maximum price that will claim the entire market $\bar{p}$ (see Appendix C).[33] The higher the attractiveness for the consumer to investigate the obfuscated alternatives, the lower $\bar{p}$ is, and thus the incentive for one particular firm to deviate from the obfuscation equilibrium is lower. As a consequence, the likelihood of an obfuscation equilibrium may increase.

5 Conclusion

This paper examines firms' equilibrium behavior in a duopoly with rational inattentive consumers. Before firms compete on the price of a homogeneous product, firms decide whether to obfuscate the prices or disclose all relevant information. In our motivating example of the mobile subscription market, mobile operators may obfuscate prices by using add-on pricing schemes with variable fees for the service's components, such as diverging fees for different operators and call time, non-linear prices for data usage, or varying roaming fees. Compared to a fixed-fee pricing scheme, assessment of the final prices is more complicated and requires costly effort by consumers as it depends on a good estimate of future consumption and understanding all contractual details.

The presence of rational inattentive consumers may make it rational for firms to obfuscate prices in equilibrium. However, mutual obfuscation equilibria with high prices and profits only exist if consumers'

[33]This finding only holds true under certain conditions that are always fulfilled if we assume that, conditional on not choosing the transparent option, the other products are a priori homogeneous, or, less restrictively, if increasing the "prominence," i.e., unconditional choice probabilities, of firm 1 decreases the prominence of both other firms.

information costs of learning about obfuscated prices are large enough. If information costs are low, the rent from obfuscation is small, and firms have an incentive to deviate from obfuscation in equilibrium. Obfuscation is not a mutually best reply anymore, and the unique equilibrium will be one with transparent prices and equilibrium profits of zero. Furthermore, we show that firms' mutual obfuscation in equilibrium cease to exist if, according to the prior beliefs, the consumers perceive one offer to be superior to other offers.

Despite attentional costs for consumers, obfuscation has negative welfare implications, as it implies decreased competition among firms. For these reasons, policymakers may want to limit obfuscation. According to our findings, this can be achieved by decreasing consumers' information costs. Potential policy measures include fostering the creation of neutral product comparison portals, ensuring consumers' access to digital services, and increasing comparability between products, such as forbidding some price dimensions that are difficult to access or standardizing product descriptions. Furthermore, new competitors may decrease obfuscation behavior in equilibrium, not only because of decreased market power of existing firms but also because differences in prominence between existing firms and new competitors alter consumers' information strategy and, thus, the optimal obfuscation behavior by firms. Therefore, lowering market entry barriers may have two positive effects on social welfare.

Our study has several limitations and implications that open several avenues for future research. First, firms' obfuscation decision in the first stage is binary. In addition, consumers' information costs to learn about obfuscated prices are exogenously given, and homogeneous across firms and consumers. While this makes our results tractable, it may be worthwhile to allow for more flexibility in these respects to assess questions about the fragmentation of markets and different scopes of obfuscation. Huettner et al. (2019) provide a suitable framework for the consumer side that can be applied.

Second, prior beliefs in our model are unrestricted and exogenously given, which allows for an unifying and general framework. However, it also implies that our model is agnostic about the mechanisms behind the prominence of firms and the source of uncertainty in our model. Putting more structure on prior beliefs may thus help to learn more about the mechanisms and their implications for optimal obfuscation and pricing behavior of firms. Related to the prior beliefs, one could also discuss obfuscation that not only affect the information costs of consumers, but also the shape of consumers' prior belief.

Third, we consider a static model. Extending our framework to a dynamic setup would allow us to study the effects of memory and learning under rational inattention on market equilibria and optimal obfuscation behavior over time. Steiner et al. (2017) and Maćkowiak et al. (2018) study rational inattention in a dynamic framework. While dynamic applications of rational inattention in industrial organization are rare, this approach seems particularly promising in markets characterized by a dynamic environment.

Fourth, the effect of digitalization on economic activity is an empirical question that has received a lot of attention from scholars in recent years (e.g., Goldfarb and Tucker, 2019). Combining the insights from rational inattention theory with empirical work on these issues can help to better identify the decisive role of information in the far-reaching digital transformation process that reshapes many disciplines, including economics.

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Appendix

A Application: The Swedish Mobile Subscription Market

This section provides a descriptive analysis of the Swedish mobile subscription market to exemplify our motivation. We document stylized facts about the development of mobile subscriptions.

Data

We use yearly market share data for the Swedish mobile subscription market between 2011 and 2018 provided by the Swedish Post and Telecom Authority. In detail, we observe yearly revenues of firms fragmented by type of subscription (i.e., prepaid and postpaid) and fees (i.e., variable and fixed fees). We adjust revenues for inflation using general yearly consumer price indices. Additionally, we use the consumer price index (CPI) for the product group of telephone services and equipment from Statistics Sweden.

Prices and Subscriptions

We start by evaluating prices in the subscription market. Even though we do not observe prices in our data set, we use several measures that allow us to infer average price movements. Figure A.1a shows the development of the CPI for telecommunication and telecommunication equipment between 2011 and 2015. In this time period, the yearly CPI dropped by 10%.[34] In Figure A.2 of Appendix A we document that the producer prices, which do not include equipment expenses, show the same trend. Between 2014 and 2020, producer prices decreased by 13%. The same holds true for average revenues per subscription in Figure A.3, which also decreased over the last two decades. Furthermore, decreasing prices are not a unique phenomenon of the Swedish mobile subscription market. Indeed, average revenues per user in the mobile subscription market decreased in most European countries (Commission, 2019), which leads us to our first stylized fact.

Stylized Fact 1. In recent years, prices in the mobile subscription market decreased.

Figure A.1b illustrates aggregated yearly revenues between 2011 and 2017. Total revenues increase slightly. Looking at yearly revenues divided into fixed and variable fees subscriptions, we see that this increase can be attributed to a strong increase in fixed subscription fees. In comparison, the revenues that are due to variable fees significantly decreased. In 2011 54% of the yearly revenues could be attributed to fixed fees, while in 2017 fixed fees were responsible for 83% of the revenues. We observe a similar heterogeneous development in the number of subscriptions in Figure A.1c. Overall, the market size of the mobile market increased slightly. Between 2011 and 2017, the total number of subscriptions increased from 13.4 to 14.4 million. As we saw above, the increasing number of subscriptions increases the aggregate

[34]Note that Statistics Sweden solely offers data on the CPI for telecommunication expenses until 2011.

revenues, while average revenues per subscriptions decrease in accordance with stylized fact 1 (see Figure A.3). If we divide the number of subscriptions into postpaid and prepaid subscriptions between 2011 and 2017, we see an increase in the share of postpaid subscriptions from 66% to 76%, while the share of prepaid subscriptions decreased. This is due to the fact that fixed fee subscriptions are usually postpaid, while variable fees are indicative for prepaid subscriptions. Looking at the characteristics of the subscriptions, we conclude:

Stylized Fact 2. Fixed-fee and postpaid subscriptions became more common, replacing variable-fee and prepaid subscriptions.

Intuitively, both the increase in fixed fees and the increase in postpaid contracts are in line with the reasoning that consumers purchase easier contracts. The complexity in pricing schemes of mobile subscriptions has decreased. At the same time, the average price decreased, while the number of products increased. We argue that the descriptive example of the Swedish market is representative of mobile subscription markets in general. Historically, we observed markets with variable pricing structures where it has been difficult to assess mobile services' final price per year or month. In recent years, we observe not only reduced prices but also less complex pricing schemes.

Competition

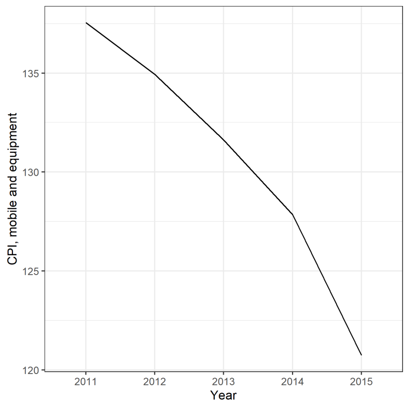
Additionally, we observe an increase in competition in the Swedish mobile subscription market. The source of competition in the market is manifold. On the one hand, the distribution of 3G licenses in 2000 led to the entry of a new competitor that gained market share relatively fast (OECD, 2015). On the other hand, network-sharing agreements that allow easier entry are common and frequently used (OECD, 2015). Figure A.1d shows the market shares of the four biggest operators. After the entry of a fourth competitor in 2010, market shares of all companies remained largely unchanged. However, we observe slight decreases in market shares of the two largest firms (around 5 percentage points each) while smaller firms increased their market shares.

Producer Price Index and Average Revenues

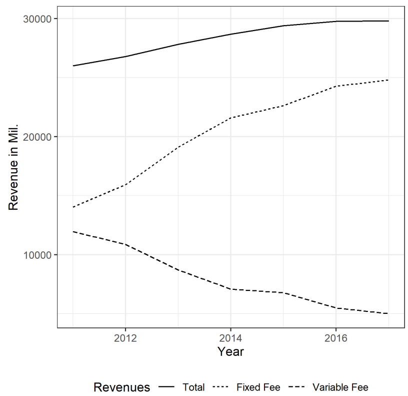
Our main empirical analysis argues that the decrease in the consumer price index in telecommunication services and equipment is a clear indicator of decreasing prices. Within this section, we show that the result is similar when evaluating the producer price index or average revenues. Figure A.2 shows the yearly producer price index for telecommunication services in Sweden between 2014 and 2020. Figure A.3 presents monthly average revenues in the mobile subscription market between 2000 and 2018. Similar to the results for the consumer price index, the producer price index and the average revenues decrease.

Figure A.1: The Swedish Mobile Subscription Market

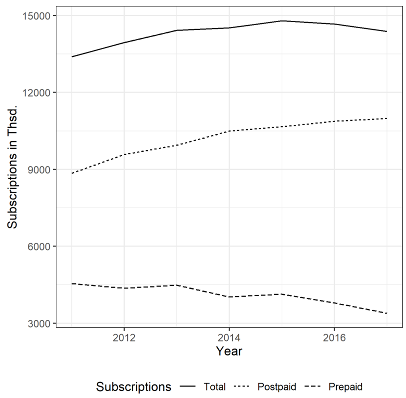
(a) Consumer Price Index, Telecommunication



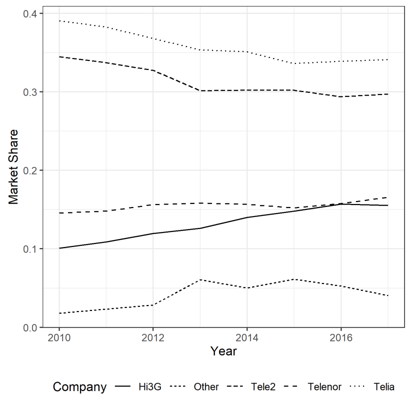
(b) Revenues



(c) Subscriptions



(d) Market Shares



Notes: The four parts of this figure illustrate key stylized facts of the Swedish mobile subscription market: (a) the yearly consumer price index for telecommunication and telecommunication equipment between 2011 and 2015; (b) aggregated revenue, divided into fixed and variable fees and adjusted for inflation, in 2011 prices; (c) the number of mobile subscriptions in the Swedish mobile subscription market between 2011 and 2017, divided into postpaid and prepaid subscriptions; and (d) the four biggest competitors' market shares between 2011 and 2017.

B Solving the Consumer Maximization Problem

Solving the consumer maximization problem, we mainly follow MM15. Formally, in the non-trivial cases the consumer chooses an information strategy to solve the following maximization problem:

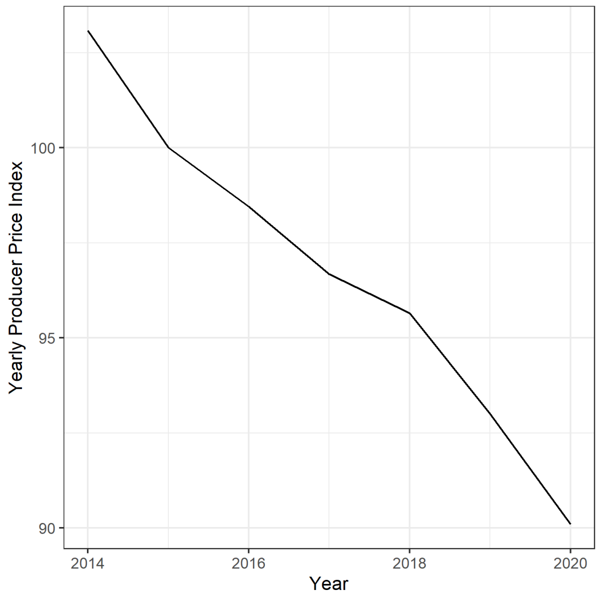
$$\max_{F\in\Delta(\mathbb{R}^{2N})} \int_{\mathbf{k}} \int_{\mathbf{z}} E(k_{a(F(\mathbf{k}|\mathbf{z}))}) F(d\mathbf{z}|\mathbf{k}) G_0(d\mathbf{k}) - \hat{c}(F)$$

$$\text{s.t. } \textstyle\int_z F(d\mathbf{z},\mathbf{k}) = G_0(\mathbf{k})$$

Let Z_i further be the set of signals $\mathbf{z}$ that result in action strategy a:

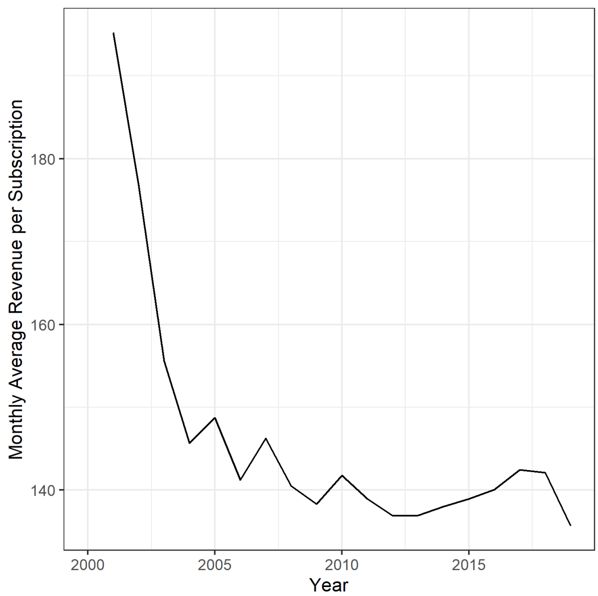
$$Z_i = \{\mathbf{z} \in \mathbb{R}^N : a(F(\mathbf{k}|\mathbf{z})) = i\},$$

Figure A.2: Producer Price Index



Notes: The figure presents the yearly producer price index for telecommunication services in Sweden between 2014 and 2020. The baseline of the index is in the year 2015.

Figure A.3: Monthly Average Revenue



Notes: The figure presents the monthly average revenue per subscriber in the Swedish mobile subscription market between 2000 and 2015. Average revenues are adjusted for inflation and in prices of 2000.

where N is the number of options available to the consumer ($N = 2$ in our base case). Building upon this, we define the conditional probability of selecting action i depending on state $\mathbf{k}$:

$$\eta_i(\mathbf{k}) \equiv \int_{z \in Z_i} F(d\mathbf{z}|\mathbf{k})$$

The unconditional choice probabilities before observing the costly signal but after processing all costless information is given by the following:[35]

$$\eta_i^0 \equiv \int_{\mathbf{k}} \eta_i(\mathbf{k}) G_0(d\mathbf{k}) \tag{5}$$

It can be shown that, due to convex information costs, one action strategy is selected in at most one posterior (MM15). This implies that each action strategy is associated with a particular signal, which in turn allows us to rewrite the maximization problem of the consumer in terms of choice probabilities, which facilitates the derivation of the respective demand. Without loss of generality we restrict the derivation of the conditional demand to two options, i.e., two products the consumer can choose from. Using the definition of Shannon entropy, our maximization problem then reads:

$$\max_{\eta \in \{\eta_i(\mathbf{k})\}_{i=1}^2} \sum_{i=1}^{2} \int_{\mathbf{k}} k_i \eta_i(\mathbf{k}) G_0(d\mathbf{k})) - \lambda(-\sum_{i=1}^{2} \eta_i^0 log \eta_i^0 + \int_{\mathbf{k}} \sum_{i=1}^{2} \eta_i(\mathbf{k}) log \eta_i(\mathbf{k})) G_0(d\mathbf{k}) \tag{6}$$

such that

$$\eta_1(\mathbf{k}) + \eta_2(\mathbf{k}) = 1 \tag{7}$$

$$\forall i: \quad \eta_i(\mathbf{k}) \geq 0, \tag{8}$$

where η is the collection of conditional probabilities $\{\eta_i(\mathbf{k})\}_{i=1}^2$. The maximization problem is applicable to all cases in which at least one firm chooses to obfuscate its prices. However, the cases differ with respect to the relevant (conditional) prior beliefs. We denote the prior belief conditional on all costless information $\tilde{k}$ by: $\Omega_0(\mathbf{k}) = G_0(\mathbf{k}|\tilde{k})$.

Both Firms Obfuscate

In the case in which both firms obfuscate ($\lambda_1 = \lambda_2 = \lambda$), the consumer chooses an information strategy based on $G_0(\mathbf{k})$ and homogeneous information costs across both products. Choosing an informative signal, she will thus consider the joint distribution about both prices.[36]

[35] Again, what is relevant to our problem is not the unconditional choice probability before any information $G(\cdot)$ is processed, but the unconditional choice probability after all costless information is processed.

[36] This makes our problem directly applicable to the framework studied by MM15.

Solving the corresponding Lagrangian, where $\zeta(\mathbf{k})$ signifies the Lagrange multipliers on (4) and $\tau_i(\mathbf{k})$ are Lagrange multipliers on (5), gives us the following first-order conditions (assuming an interior solution $\eta_i^0 > 0$):

$$k_i - \lambda \left(-log(\eta_i^0) - 1 + log(\eta_i(\mathbf{k}) + 1\right) + \tau_i(\mathbf{k}) - \zeta(\mathbf{k}) = 0$$

MM15 show that if $\eta_i^0 > 0$ and $k_i > -\infty$, it has to hold that $\eta_i(\mathbf{k}) > 0$, which in turn implies that the Lagrange multiplier on (6) is zero, $\tau_i(\mathbf{k}) = 0$. Taking the exponential of both sides and rearranging the first-order condition results in:

$$\eta_i(\mathbf{k}) = \eta_i^0 e^{(k_i - \zeta(\mathbf{k}))/\lambda} \tag{9}$$

Plugging (7) into (5) gives us:

$$e^{\zeta(\mathbf{k})/\lambda} = \sum_{i=1}^{2} \eta_i^0 e^{k_i/\lambda},$$

which in turn can be plugged back into (7) to arrive at our demand function (1) for $0 < \eta_i^0 < 1$. Note that η_i^0 is not only determined by the prior beliefs but is rather a result of the maximization problem itself. We plug the conditional choice probabilities (1) into the definition of the unconditional choice probabilities (3) to arrive at the normalization conditions that allow us to numerically solve for the unconditional choice probabilities:

$$\int_{\mathbf{k}} \left(\frac{e^{k_i/\lambda}}{\sum_{j=0}^{2} \eta_j^0 e^{k_j/\lambda}} \right) G_0(d\mathbf{k}) = 1, \quad \forall i \eta_i^0 > 0$$

If $\exists \eta_i^0 = 0$, then $\eta_i(\mathbf{k}) = 0$. This implies that the consumer does not process any information about i and, in the duopoly case, always buys the product of the competitor. Caplin et al. (2019) provide the necessary and sufficient boundary conditions for this case that can be applied to our simple two-product setting.

Only One Firm Obfuscates

The logic of calculating the choice probabilities if only one firm obfuscates is similar. Note that we assume here without loss of generality that $\lambda_1 = 0$ and $\lambda_2 = \lambda$. However, when deciding how much information to process (about firm 2's price), the consumer knows the price of product 1. This has some implications for the relevant conditional prior belief that is given by $\Omega_0(k_2) = G_0(k_2|k_1)$, implying that the uncertainty is one-dimensional, considering only product 2's price.[37]

[37] Note that if the price of product 1 is low enough compared to the expected price of firm 2, the consumer will not process any information about product 2 and will just buy product 1. Similarly, if the expected price of product 2 is lower than p_1, the consumer will pay no attention and will just buy (uncertain) product 2. This case's logic is similar to that of the case in which the consumer can decide to buy a product/enter the market or choose an outside option with a certain outcome. A similar setting is discussed in MM15 or Boyacı and Akçay (2017) for a binary state variable and corresponding prior belief

Plugging constraint (5) into the maximization problem above, noting that k_1 is perfectly known by the consumer, and assuming $0 < \eta_2^0 < 1$, the Lagrangian for the maximization problem of the consumer in the case where only one firm obfuscates can be rewritten to:

$$\max_{\eta_2(\mathbf{k})} \int_{k_2} k_2 \eta_2(\mathbf{k}) \Omega_0(dk_2) + k_1 \int_{k_2} (1-\eta_2(\mathbf{k}))\Omega_0(dk_2)$$
$$-\lambda \left(-\eta_2^0 log \eta_2^0 - (1-\eta_2^0) log(1-\eta_2^0) + \int_{k_2} (\eta_2(\mathbf{k}) log \eta_2(\mathbf{k}) + (1-\eta_2(\mathbf{k})) log(1-\eta_2(\mathbf{k})))\Omega_0(dk_2) \right)$$

Differentiating with respect to $\eta_2(\mathbf{k})$ and noting that $\eta_2(\mathbf{k}) > 0$ almost surely if $\eta_2^0 > 0$ and $k_2 > -\infty$ (see MM15) gives us the following first-order condition:[38]

$$(k_2 - k_1) - \lambda \left(-log(\eta_2^0) - 1 + log(1-\eta_2^0) + 1 + log(\eta_2(\mathbf{k})) + 1 - log(1-\eta_2(\mathbf{k})) - 1 \right) = 0$$

Combining the terms and taking the exponential on both sides gives us:

$$e^{(k_2-k_1)/\lambda} = \frac{1-\eta_2^0}{\eta_2^0} \frac{\eta_2(\mathbf{k})}{1-\eta_2(\mathbf{k})}$$

Rearranging and adding 1 (respectively $\frac{(1-\eta_2^0)}{(1-\eta_2^0)}$ and $\frac{(1-\eta_2^0)e^{k_1/\lambda}}{(1-\eta_2^0)e^{k_1/\lambda}}$) on both sides gives us:

$$\frac{1}{1-\eta_2(\mathbf{k})} = \frac{(1-\eta_2^0)e^{k_1/\lambda} + \eta_2^0 e^{k_2/\lambda}}{(1-\eta_2^0)e^{k_1/\lambda}}$$

Using the definition of k_i gives us our final demand:

$$\eta_2(\mathbf{k}) = \frac{\eta_2^0 e^{k_2/\lambda}}{(1-\eta_2^1)e^{k_1/\lambda} + \eta_2^0 e^{k_2/\lambda}}$$

$$\eta_1(\mathbf{k}) = 1 - \eta_2(\mathbf{k})$$

The normalization condition gives us our unconditional choice probabilities:

$$\int_{k_2} \left(\frac{e^{k_2/\lambda}}{\eta_2^1 e^{k_2/\lambda} + (1-\eta_2^1)e^{k_1/\lambda}} \right) \Omega_1(dk_2) = 1$$

:

that follows a Bernoulli distribution.

[38] Note that $\eta_i^l \equiv \int_{\mathbf{k}} \eta_i(\mathbf{k}) G_l(d\mathbf{k})$.

C Derivation of Subgame Perfect Equilibria with N Firms

In a market in which all firms obfuscate, the optimal information strategy results in a generalized multinomial logit demand function for all three products:[39]

$$\eta_i(\mathbf{k}) = \frac{\eta_i^0 e^{(q-p_i)/\lambda}}{\sum_{j=1}^{3} \eta_j^0 e^{(q-p_j)/\lambda}}, \quad \forall i \in N$$

Following Anderson et al. (1992) and as in the duopoly, the subgame perfect equilibria are uniquely determined by following system of equations:

$$p_i^* = c + \frac{\lambda}{(1-\eta_i^*(\mathbf{k}))}, \quad \forall i \in N$$

with equilibrium profits of

$$\pi_i^* = \frac{\lambda}{(1-\eta_i^*(\mathbf{k}))}\eta_i^*(\mathbf{k}), \quad \forall i \in N$$

This proposition and proof are the generalization of Proposition 2 to a market with N competing firms.

Proposition 6. *With $N-1$ obfuscating firms $(-i)$ and one transparent firm (i), the optimal price of the transparent firm ($\lambda_i = 0$) is equal to $\bar{p}_i$, where $\bar{p}_i = \max p_i$ such that $\eta_i^0 = \eta_i(\boldsymbol{k}) = 1$.*

Proof. As in Proposition 2, note that the transparent firm will not set a price below $\bar{p}_i$, as $\dfrac{\partial \pi_i}{\partial p_i} = 1 > 0$ if $p_i < \bar{p}_i$, $\eta_i(\mathbf{k}) = 1$.

All firms will face a multinomial logit demand if $p_i \geq \bar{p}_i$ (see above). The maximized profit function for a given unconditional choice probability is equal to $\pi_i^*(\eta_i^0) = \dfrac{\lambda}{(1-\eta_i^*(\mathbf{k}))}\eta_i^*(\mathbf{k})$. Without loss of generality, we assume $N = 3$ and that firm 1 is using the transparent pricing scheme.

Plugging in the corresponding demand function, the profit function of firm 1 can be rewritten to:

$$\pi_1^*(\eta_i^0) = \lambda \frac{\eta_1^0 e^{(q-p_1^*)/\lambda}}{\eta_2^0 e^{(q-p_2^*)/\lambda} + \eta_3^0 e^{(q-p_3^*)/\lambda}} = \lambda \frac{\eta_1^0 e^{-p_1^*/\lambda}}{\eta_2^0 e^{-p_2^*/\lambda} + \eta_3^0 e^{-p_3^*/\lambda}}$$

If we assume that, conditional on not choosing the transparent option (option 1), the other products are a priori homogeneous, implying equal unconditional choice probabilities (η_i^0) and optimal prices (see, e.g., Matějka and McKay, 2012). Then, the unconditional choice probabilities can be rewritten as $\eta_2^0 = \eta_3^0 = \frac{(1-\eta_1^0)}{2}$. This simplifies the maximized profit function to:

$$\pi_1^*(\eta_i^0) = \lambda \frac{\eta_1^0 e^{(q-p_1^*)/\lambda}}{\frac{(1-\eta_1^0)}{2}\left(e^{(q-p_2^*)/\lambda} + e^{(q-p_3^*)/\lambda}\right)} = 2\lambda \frac{\eta_i^0}{(1-\eta_i^0)} \frac{e^{(-p_1^*)/\lambda}}{\left(e^{(-p_2^*)/\lambda} + e^{(-p_3^*)/\lambda}\right)}$$

Applying the envelope theorem, differentiating with respect to η_1^0 equals:

[39] See Appendix A for the derivation.

$$\frac{\partial \pi_i^*}{\partial \eta_i^0} = \frac{2\lambda}{(1-\eta_1^0)^2} \frac{e^{(-p_1^*)/\lambda}}{\left(e^{(-p_2^*)/\lambda} + e^{(-p_3^*)/\lambda}\right)} > 0,$$

which is strictly larger than zero if $\eta_1^0 < 1$.

If the options are not a priori homogeneous, the derivative of the maximized profit function with respect to η_1^0 is given by:

$$\frac{\partial \pi_1^*}{\partial \eta_1^0} = \lambda \frac{e^{(-p_1^*)/\lambda}\left((\eta_2^0 e^{(-p_2^*)/\lambda} + \eta_3^0 e^{(-p_3^*)/\lambda}) - \eta_1^0 \left(\frac{\partial \eta_2^0}{\partial \eta_1^0} e^{(-p_2^*)/\lambda} + \frac{\partial \eta_3^0}{\partial \eta_1^0} e^{(-p_3^*)/\lambda} \right) \right)}{(\eta_2^0 e^{(-p_2^*)/\lambda} + \eta_3^0 e^{(-p_3^*)/\lambda})^2}$$

This is always larger than zero if $\left(\frac{\partial \eta_2^0}{\partial \eta_1^0} e^{(-p_2^*)/\lambda} + \frac{\partial \eta_3^0}{\partial \eta_1^0} e^{(-p_3^*)/\lambda} \right) \leq 0$, which always holds if we assume that increasing the "prominence" i.e., unconditional choice probabilities, of firm 1 decreases the prominence of both other firms.[40]

□

[40]This assumption can further be relaxed by considering the entire numerator.