Monitoring Team Members: Information Waste and the Self-Promotion Trap*

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Abstract

We analyze a moral hazard problem where a firm incentivizes a team of complementary workers by designing a robust incentive scheme that relies on individual and team performance measures. While using both measures minimizes information rents, team-performance bonuses expose workers to *strategic uncertainty* about their colleagues' effort. We show that the firm typically sacrifices statistically-relevant information to curb strategic uncertainty, compensating some workers solely based on their individual performance. We provide a complete characterization of the optimal incentive scheme, highlighting how the firm discriminates among (possibly homogeneous) workers in terms of total rents, type of contract offered, and monitoring some workers more closely than others. Finally, we use this characterization to study the workers' incentives to facilitate or hinder the firm's monitoring. We show that competition for better contracts incentivizes workers to be more transparent, triggering an unraveling result that only benefits the firm, delivering the same payoffs as the firm-preferred equilibrium. That is, competition gives rise to a self-promotion trap.

Keywords: Teamwork, Robustness, Bonus design, Information waste, Endogenous monitoring, Self-promotion

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1 Introduction

"Coming together is a beginning. Keeping together is progress. Working together is success." – Henry Ford

At least since the mid-1980s, the classical top-down hierarchical organization model has been giving way to a new organization system based on teams (Devine et al., 1999).[1] The ability to incentivize teamwork has thus become a stepping stone for success in almost every organization. As it emerges from *Payscale 2019 Compensation Best Practices Report*, more than 70% of American firms resort to performance-contingent bonuses to incentivize their employees to work together toward a common goal. Since individual efforts are rarely observable, firms typically design bonuses that rely on a combination of imperfect signals about individual performance (e.g., working hours, individual reports, and some individual outcomes) and team performance (e.g., overall success or failure of a project). Depending on the monitoring accuracy and the team characteristics, firms need to find the optimal balance between individual and team-performance bonuses to incentivize teamwork at the lowest possible cost.

In modern contexts, where teams are often multidisciplinary, and workers' efforts are more complements than substitutes, team members have higher incentives to work when they expect higher effort from their colleagues. As a result, when workers trust their teammates to work hard, firms would be able to incentivize teamwork while keeping compensation low. This mutual trust can be thought of as a strong corporate culture. However, in some circumstances, corporate culture might be weak. Or it might be difficult to engineer in the first place, given its foundations in mutual expectations of hard work. In such cases, firms might provide extra bonuses to some workers to ensure their hard work even when they expect their colleagues to shirk. We refer to these additional payments as strategic insurance or dependability rents. Importantly, dependability rents can lead workers to believe that their colleagues will work hard. That is, they can potentially create a robust foundation of a positive corporate culture.

This paper studies how a firm optimally balances individual and team-performance bonuses to robustly implement teamwork without relying on corporate culture, i.e., without relying on favorable equilibrium selection. In particular, we explore the impact of strategic insurance rents on the firm's best contractual practices, providing a complete characterization of the compensation structure that arises for any given monitoring structure. Finally,

[1]According to Forbes Bersin (2016), in 2016 76% of large companies ($>5{,}000$ employees) claim to be organized in teams and 62% overall; see also Huthwelker, Peltzer and Wemhoner (2011), and Bikfalvi, Jäger and Lay (2014).

we exploit such a characterization to analyze how the monitoring structure arises in the first place. Specifically, we study the firm's incentives to invest in monitoring and the workers' incentives to facilitate or hinder such investment or engage in self-promotion or other activities that increase effort transparency.

The analysis delivers the following predictions: (1) firms should mostly resort to individual bonuses, optimally ignoring statistically-relevant information, and (2) workers optimally engage in a self-promotion race that only benefits the firm, leading to higher effort transparency and the same payoffs as under the firm-preferred equilibrium.

To shed light on the interplay between the main trade-offs, we consider the following stylized model. A firm incentivizes a team of risk-neutral employees to work together toward a common goal. Each team member privately chooses between exerting costly effort and shirking. Consistently with the idea of multidisciplinary teams, we assume that workers' efforts are strategic complements: the impact of a worker's effort on team success positively depends on the effort exerted by her colleagues. The firm's objective is to maximize the probability that the team succeeds at the lowest possible cost. The firm can compensate each worker based on the team's success and/or (noisy) signals about the worker's individual performance.[2]

As a first step of the analysis, we consider the classical benchmark case where the firm can coordinate the workers on its preferred equilibrium for any incentive scheme of its choice. In this context, consistently with Holmström (1979, 1982)'s *sufficiency principle*, we find that the firm optimally exploits all available statistically-relevant information about the workers' effort to limit the moral hazard problem and thus the worker's information rents. In particular, the firm pays each worker a bonus only when the team succeeds, and the signal about the worker's individual performance is positive. However, we show that such a bonus scheme is insufficient to incentivize workers who expect some of their colleagues to shirk, opening the door to a less favorable equilibrium where all workers shirk. In this sense, focusing on the firm-preferred equilibrium requires quite an optimistic view about the firm's corporate culture and its ability to shape the worker's beliefs.

Taking a less optimistic view, we focus on a more wary firm, willing to concede strategic insurance rents to implement teamwork as the *unique equilibrium outcome*. An example of a wary firm would be one that has been formed by the merger of two entities with very different workforces or different cultures. Our first contribution is to show that, in such a setting, the classical sufficiency principle of Holmström (1979, 1982) ceases to hold. To understand the intuition behind such a result, note that a worker is exposed to strategic

[2] In section 2.1 we thoroughly discuss the assumption that a worker's wage cannot directly depend on a coworker's individual signal conditional on the coworker's effort.

uncertainty only insofar as her bonus depends on her coworkers' individual signals or the team's performance. On the other hand, bonuses paid solely based on individual performance insulate the recipients from strategic uncertainty. We show that under the firm's optimal compensation scheme that robustly incentivizes teamwork, some team members only receive individual-performance bonuses, even though this implies wasting the statistically-relevant information about their efforts contained in the team performance outcome. Differently from the classical firm-preferred equilibrium benchmark, our model is thus broadly consistent with the empirical observation that firms mostly resort to individual bonuses and that team bonuses typically cover only a small fraction of the employees (Ledford Jr, Lawler III and Mohrman (1995), Payscale (2019))).[3]

Our second contribution is to provide a sharp characterization of the optimal incentive scheme, showing that the firm optimally discriminates among (possibly homogeneous) team members both in terms of the total rent granted and the type of bonuses offered. First, we show that the firm optimally creates two contractual categories with different exposure to strategic uncertainty: *insulated workers* (IW) rewarded only based on their individual signals, and *non-insulated workers* (NW) rewarded based on both team and individual performances signals. While exploiting the information contained in the team performance allows the firm to reduce information rents, assigning a worker to NW increases the workers' exposure to strategic uncertainty and thus the strategic insurance rents that the firm needs to grant. As a result of this trade-off, both categories are typically non-empty. Furthermore, in line with Winter (2004) and the literature that followed, we show that the firm optimally creates a ranking among the workers in NW, offering each worker the minimum bonus that incentivizes her to work even when expecting lower-ranked colleagues to shirk. In other words, the firm grants higher strategic insurance rents to higher-ranked workers to compensate them for the possibility that lower-ranked coworkers may shirk.

Our third contribution is to characterize how workers' heterogeneity affects their allocation to the different contractual categories and the rents they ultimately receive from the firm. We prove that, even though workers with higher signal precision would be cheaper both in IW and in NW, the firm optimally assigns to IW those workers with the highest signal precisions. Furthermore, we show that when the strategic complementarities are sufficiently strong, the firm grants higher ranks to more monitorable workers in order to limit strategic insurance rents at the top.

[3]Payscale (2019) highlights how 73% of the firms resort to some type of bonuses, 66% use team bonuses, and 22% team bonuses. Beyond these categories, firms also use spot bonuses (45%), employee referral bonuses (45%), retention and hiring bonuses, and other types of bonuses.

Finally, we show that the logic behind the optimal contract structure is robust to the possibility for the firm to invest in monitoring, selecting the precision of the different individual signals. In contrast with the firm-preferred equilibrium benchmark, we show that, when the monitoring cost is increasing and convex, the firm optimally chooses to monitor some workers more closely than others, even when the workers are homogeneous ex-ante. Specifically, the firm monitors the workers that it plans to assign to IW more closely than the others, and, within NW, the firm monitors the higher ranks more closely than the lower ranks.[4]

However, the choice of the monitoring structure is rarely unilateral. While the firm can set up a monitoring system at a cost, the workers can also influence the firm's monitoring effectiveness or cost. For example, a worker can deny consent or even deactivate the computer's camera, use external devices to elude browsing restrictions, affect the transparency, and, ultimately, the accuracy of a report resorting to omissions, creative interpretations, or outright lies. Furthermore, the workers can actively provide signals about their work by engaging in self-promotion and seeking face time with the management. Such practices of highlighting own effort and results obviously increase effort transparency and thus the precision of individual signals.[5] To study the workers' incentives to engage in such practices, we assume that each worker can freely choose the precision of her individual-performance signal (hereafter, *monitorability*) before the firm offers the contract schedule.[6]

To establish a natural benchmark, we start by analyzing the case in which the firm is not concerned about contract robustness, expecting the workers to coordinate on its preferred equilibrium for every given contract scheme. In this case, since workers receive only information rents in the equilibrium of the subgame that follows the monitorability choices, all workers optimally reduce their monitorability to the minimum. As a result, individual signals become uninformative and individual-performance bonuses ineffective. This analysis is arguably the most natural extension of the classical literature (e.g., Holmström (1982)) and, as we show, also delivers to the firm the highest possible equilibrium payoff. For this reason, I refer to this equilibrium as the *firm-preferred equilibrium benchmark*.

In contrast, when the firm implements teamwork robustly (i.e., when workers cannot be assumed to coordinate on the equilibrium the firm most prefers) the workers could, in

[4]In case monitoring costs were intrinsically heterogeneous, we show that the firm would monitor most closely cheaper workers. However, as we show in an extension, the latter would no longer be true if we allow workers to have a heterogeneous impact on team success.

[5]Lack of communication from a self-promoting employee would typically be interpreted as a negative individual signal about her effort.

[6]Qualitatively equivalent results can be obtained when we assume that the workers choose the cost that the firm needs to sustain to increase the precision of her own individual signals.

principle, obtain higher payoffs by choosing the same monitor ability as in the firm-preferred equilibrium but also collecting strategic insurance rents. However, we show that competition for such rents traps the workers into a race toward higher monitorability, which we refer to as a self-promotion race, that ultimately delivers the same payoffs as in the firm-preferred equilibrium benchmark. Intuitively, when all workers select the level of monitorability of the firm-preferred equilibrium benchmark, they are assigned to NW, ranked arbitrarily, and obtain payoffs higher than in the firm-preferred equilibrium benchmark. However, by increasing her own monitorability by an epsilon, a worker can make sure to be ranked above the others, giving up little in terms of information rents but gaining much in terms of strategic insurance rents. This change benefits the firm and the worker at the expense of the other team members, who thus have incentives to do the same and increase their monitorability even further, fuelling the race to higher monitorability levels. However, as the race proceeds and higher monitorability is reached, the sacrifice in terms of information rents eventually exceeds the gain in terms of strategic insurance rents. When this happens, the race comes to an end. As we show, the resulting equilibrium payoffs are the same as under the firm-preferred equilibrium benchmark. In this sense, robustness comes for free in this context: the workers' self-promotion race ultimately annihilates all strategic insurance rents. However, workers' monitorability is significantly higher than in the firm-preferred equilibrium benchmark, and some workers are rewarded solely based on their individual performance.

In terms of predictions, our model thus offers a possible explanation for why workers often want to make their effort more transparent and engage in self-promotion.[7] Self-promotion serves the scope of climbing the team ranking, ensuring higher strategic insurance rents at the colleagues' expenses. Moreover, consistently with the evidence in Ledford Jr, Lawler III and Mohrman (1995), and Payscale (2019), our model predicts that, when workers can impact their effort transparency, individual-performance bonuses continue to be paid and possibly become even more relevant.[8] Note that these predictions and the empirical facts they capture would be at odds with the classical firm-preferred equilibrium benchmark, which simply predicts that all workers would reduce their monitorability as much as possible, to the point at which individual bonuses become essentially useless.

[7]According to *Reward Gateway 2018 report* 43% of workers feel invisible or undervalued and look for a way to signal themselves to their manager. Many articles and books, including *HBR Guide to Office Politics* indicate self-promotion as a way to "make sure people understand and see what you do," increasing chances of recognition and career advancements.

[8]For example, we show that when the firm is constrained to additively separable bonuses, the workers' monitorability race results in the firm optimally offering only individual-performance bonuses to all team members but at most one.

Finally, while workers are indifferent to their colleagues' monitorability in the firm-preferred equilibrium benchmark, they benefit from keeping their colleagues' monitorability low in our setting. This result casts doubt on the firm's ability to delegate monitoring to higher-ranked workers, especially the team manager, who is typically assigned the highest rank, as we show in an extension. In this sense, the paper also suggests that the typical team manager's rhetoric about teamwork and lack of result attributability might speak more to the team manager's incentive to protect her own rent than to her intent of motivating team members.

The rest of the paper is organized as follows. In the next subsection we discuss the contribution to the literature. In Section 2 we setup and analyze the model for fixed signals' precisions. In Section 3 we endogenize the monitoring structure accounting for the firm's and the workers' incentives to invest in monitorability. In Section 4 we study incentives to limit coworkers' monitorability. Finally, in Section 5 we conclude by discussing directions for future work.

1.1 Contribution to the Literature

Since the seminal papers of Alchian and Demsetz (1972) and Holmström (1982), the problem of providing incentives for teamwork has gained a prominent role in organization and contract design. Such literature studies how a firm should combine individual and team signals to limit the workers' moral hazard rents. In an influential contribution, McAfee and McMillan (1991) find that team-performance bonuses are sufficient to implement teamwork when the workers are risk-neutral and face no limited liability. On the other hand, Holmström (1982) argues that if such conditions are violated, the firm should exploit all available statistically-relevant information about workers' effort to limit the workers' rents (sufficiency principle); see also Vander Veen (1995), and Bag and Wang (2019).[9] We contribute to such a debate about *contributions vs. output monitoring* by studying the impact of strategic uncertainty and dependability concerns on the optimal incentive scheme to implement teamwork. In contrast to Holmström (1982) and most of the literature that followed, we assume the firm aims at implementing teamwork robustly. As anticipated above, we find that while the firm always finds it optimal to exploit individual-performance signals, it may strictly benefit from

[9]An influential literature, pioneered by Lazear and Rosen (1981), advocated the use of incentive schemes that induce competition among employees via tournaments or relative performance evaluation. On the other hand, Che and Yoo (2001) and Dai and Toikka (2017) highlight how team-performance bonuses should be expected to outperform relative performance bonuses, respectively, in long-term relations and when the principal has a limited understanding of the game.

wasting some statistically-relevant information.

Within the broader literature on contracting with externalities, pioneered by Segal (1999), a recent series of papers including Segal (2003), Winter (2004), Bernstein and Winter (2012), Halac, Kremer and Winter (2020), Halac, Lipnowski and Rappoport (2021), and Halac, Kremer and Winter (2021) study unique implementation mechanisms, highlighting the optimality for a firm to resort to ranking schemes paying higher bonus to higher-ranked agents.[10][11] This literature however either assumes that agents' choices are directly observable (e.g., Segal (2003), Winter (2004), Bernstein and Winter (2012), Halac, Kremer and Winter (2020)), or that only the team's overall success is observable and contractible (e.g., Winter (2004) and Halac, Lipnowski and Rappoport (2021)). As a result, such literature is essentially silent about the debate of contributions vs. output monitoring and thus the optimal balance between individual and team performance bonuses, which is the main focus of our analysis. Our paper is the first to look at the robust design of compensation schemes when the signal structure is rich enough to permit the firm to combine individual-performance bonuses with team-performance ones. We show that the firm optimally discriminates among (possibly homogeneous) workers both in terms of monitoring and types of contract offered.

A further contribution to this literature is that we endogenize the underlying heterogeneity among the workers. In this respect, the paper which is most closely related to ours is the contemporaneous work by Halac, Kremer and Winter (2021). That paper also studies the firm's incentives to monitor a team of workers when the principal wants to uniquely implement teamwork. However, while Halac, Kremer and Winter (2021) studies how to design teams within a firm, analyzing the optimal size and composition of teams, we focus on the complementary question of how to monitor individual inputs *within* a given team. In Halac, Kremer and Winter (2021) the firm observes, for every agent, only the overall success or failure of the sub-team she is part of, and the firm can assign an agent to a smaller or a larger team depending on whether it prefers to monitor such a worker more or less closely. Instead, in our setting, all workers are part of the same team, and the firm obtains multiple signals about every worker, an individual and a team-performance one. Another difference between

[10]Note that, given the supermodularity of the game, in our setting robust implementation is equivalent to unique implementation of teamwork, i.e., ensuring the effort of all workers in all Nash equilibria

[11]Bernstein and Winter (2012) and Halac, Kremer and Winter (2020) characterize, in different applied contexts, the specific ranking that arise as a function of some underlying agents' heterogeneity. Halac, Lipnowski and Rappoport (2021) shows that the firm might benefit from keeping the workers uncertain about their ranking within the team, e.g., preventing communication among the workers; as the authors highlight, this results in homogeneous workers being assigned approximately the same team-performance bonus.

our paper and Halac, Kremer and Winter (2021) is that we study the workers' incentives to facilitate or hinder the monitoring process.

Finally, our paper relates to the literature on monitoring in teams, including Miller (1997), Strausz (1999), Winter (2010), Rahman (2012), Miller and Rozen (2014), and Gershkov and Winter (2015). The vast majority of this literature focuses on partial rather than robust implementation. We contribute to this literature by showing how robustness impacts firm and peer monitoring.

2 Model

Setting. A firm owns a project that, if successful, yields a fixed surplus Y. The success of the project depends on the effort exerted by a team of n risk-neutral workers. Each worker $i \in N = \{1, ..., n\}$ privately chooses between working, i.e., $e_i = 1$, and shirking, i.e., $e_i = 0$. While shirking is free, the worker incurs in a cost $c > 0$ if she chooses to work. To capture effort complementarities, we assume that the positive impact of one worker's effort on team success increases in the number of colleagues that work. Formally, we assume the probability of team success to be an increasing and convex function of the total effort exerted by the team members:

$$F\left(\sum_{i=1}^{n} e_i\right) \in (0,1).$$

Whether the project succeeds or fails is publicly observable and provides valuable information about the aggregate effort exerted by the team, $\sum_{i=1}^{n} e_i$. Beyond observing the team performance (i.e., the project outcome), the firm obtains signals about the workers' individual contributions. In particular, we assume the principal has access to the following signal structure: a *team-performance signal* indicating whether the team succeeds or fails,

$$S^{team} = \begin{cases} 0 & \text{if team failure} \\ 1 & \text{if team success} \end{cases}$$

and n imperfect *individual-performance signals* about the workers' individual effort

$$S_i^{ind} = \begin{cases} e_i & \text{wp } p_i \\ 1 - e_i & \text{wp } 1 - p_i \end{cases}$$

where $p_i \in \left[\frac{1}{2}, 1\right]$ is the precision of i's individual signal. Note that the precisions of the individual monitoring $(p_i)_{i\in N}$ can be exogenous, as in the first part of the paper, or emerge endogenously from the firm's and workers' decision, as in the second part of the paper.

Independently on how it arises, such monitoring structure allows the firm to incentivize teamwork by publicly offering every worker i a state-contingent wage $W_i\left(S^{team}, S_i^{ind}\right)$ that depends on the team performance S^{team} and on the worker's individual-performance signal S_i^{ind}.[12]

As we assume that the workers have limited liability and thus payments need to be non-negative in every state of the world, it is without loss to focus on the case in which the incentive scheme takes the form of a bonus plan. Since granting a positive bonus when $W_i\left(S^{team}=0, S_i^{ind}=0\right)>0$ could only decrease worker's i incentives to work, we can rewrite the state-contingent wage as

$$W_i\left(S^{team}, S_i^{ind}\right) = b_i^{team} * S^{team} + b_i^{ind} * S_i^{ind} + b_i^{both} * S_i^{ind} * S^{team},$$

where $b_i^{team} \geq 0$ is the team-performance bonus, $b_i^{ind} \geq 0$ is the individual-performance bonus, and $b_i^{both} > -(b_i^{ind}+b_i^{team})$ is the bonus adjustment when both team and individual-performance signals are positive.

Timing. The order of moves is the following:

1. For every given distribution of signal precisions, $(p_i)_{i\in N}$, the firm publicly offers each employee a contract $\boldsymbol{b_i} = \left(b_i^{team}, b_i^{ind}, b_i^{both}\right)$ specifying team and individual-performance bonuses and the eventual adjustment.

2. The workers simultaneously choose whether or not to exert effort.

3. The project succeeds or fails and the individual signals are generated.

4. The firm collects the possible project surplus and pays the employees according to the specified contracts.

In the second part of the paper, we will introduce two ex-ante steps where the firm can choose how much to invest in monitoring the individual contributions and the workers how much to facilitate or hinder such monitoring. We postpone to the dedicated sections the detailed description of those steps.

The workers' problem. For every monitoring structure $(p_i)_{i\in N}$ and incentive scheme $\boldsymbol{b} = \left(b_i^{team}, b_i^{ind}, b_i^{both}\right)_{i\in N}$ the workers needs to choose whether to work or shirk. Formally,

[12]See the next section for a discussion of why we rule out the possibility that a worker's wage depends on the individual signals of some coworkers.

every worker i maximizes

$$\max_{e_i \in \{0,1\}} \sum_{n_{-i}=0}^{n-1} \mu_i(n_{-i}) \mathbb{E}\left[W_i | n_{-i}, e_i\right] - c e_i$$

where $\mu_i(n_{-i})$ is worker i's belief that exactly n_{-i} of her coworkers choose to work.[13] As a result, worker i works if and only if

$$\sum_{n_{-i}=0}^{n-1} \mu_i(n_{-i}) \Big(\mathbb{E}\left[W_i | n_{-i}, e_i = 1\right] - \mathbb{E}\left[W_i | n_{-i}, e_i = 0\right] \Big) \geq c, \qquad (IR_{i,\mu_i})$$

which can be rewritten as

$$(2p_i - 1) b_i^{ind} + \sum_{n_{-i}=0}^{n-1} \mu_i(n_{-i}) \Bigg(\Big(F(1+n_{-i}) - F(n_{-i}) \Big) b_i^{team} + \Big(p_i F(1+n_{-i}) - (1-p_i) F(n_{-i}) \Big) b_i^{both} \Bigg) \geq c,$$

Where $(2p_i - 1)$ is i's impact of working on the probability of generating a positive individual performance $S_i^{ind} = 1$; $F(1+n_{-i}) - F(n_{-i})$ and $p_i F(1+n_{-i}) - (1-p_i) F(n_{-i})$ are the differential impacts of i's work respectively on the probability of team success, $S^{team} = 1$, and on the probability that both team and individual performance are positive, $\left(S_i^{ind} = 1, S^{team} = 1\right)$, when other n_{-i} workers work.

Note that the worker's participation constraint in IR_{i,μ_i} crucially depends on the worker's belief μ about teammates' efforts. Indeed, due to effort complementarities, the expected impact of the worker's effort on team success, $S^{team} = 1$, is higher when more teammates work. Thus, whenever a worker's contingent wage depends on team success, i.e., $b_i^{team} > 0$ or $b_i^{both} > 0$, the worker is more willing to work if she expects more colleagues to work as well.

The Firm's Problem. We consider a cautious firm that wants to ensure the highest possible probability of team success at the lowest possible cost. In particular, the firm's objective is to minimize the cost of robustly implementing teamwork (**RITW**), i.e., inducing all workers to work as the unique rationalizable outcome.[14] In order to overcome the technical issue that the set of incentive schemes that robustly implement work is open, and thus a minimum does not exist, we assume that when indifferent between working and shirking, the workers choose to work.

[13] With a slight abuse of notation we write W_i rather than $W_i\left(S^{team}, S_i^{ind}\right)$.

[14] See next section for the discussion of this assumption.

For given signal precisions $p_1, ..., p_N$ the firm problem is thus

$$\min_{\boldsymbol{b}} \sum_{i \in N} \left(p_i b_i^{ind} + F(n) b_i^{team} + p_i F(n) b_i^{both} \right)$$

subject to: $\quad \boldsymbol{b} = \left(b_i^{team}, b_i^{ind}, b_i^{both} \right)$ **RITW**,

i.e., subject to: IR_{i,μ_i} for all $i \in N$, μ_i consistent with rationalizability.

2.1 Discussion of the Assumptions

Before solving the problem, we provide a more detailed discussion of our assumptions.

The most important assumption of our model is that the firm wants to implement teamwork as the unique rationalizable outcome of the game (robust implementation); see Bergemann and Morris (2009). Such assumption is a significant departure from the classical literature, e.g., Holmström (1982), that typically assumes the principal to be able to coordinate the agents on its preferred equilibrium (partial implementation). On the other hand, our principal does not trust its ability to coordinate the agents and wants to ensure teamwork independently of the workers' beliefs about their colleagues. Given the supermodularity of the game, the firm's objective of implementing effort as the unique rationalizable outcome coincides, in our context, with implementing effort as the unique equilibrium outcome of the game, bringing us close to the unique implementation literature (Segal, 2003; Winter, 2004, 2006; Bernstein and Winter, 2012; Halac, Kremer and Winter, 2020, 2021; Halac, Lipnowski and Rappoport, 2021). Given the supermodularity of the game, robust implementation is also equivalent, in our context, to assume that the firm maximizes profits when expecting the workers to coordinate on the firm's least-preferred equilibrium.[15]

The second assumption of our model is that the firm wants to implement the effort of every worker in N and is willing to pay high bonuses to reach that scope. This assumption, however, is without loss of generality. In particular, we can think of our analysis as the second step of a two-step firm's problem, where the firm chooses the optimal contract scheme that maximizes its least-preferred equilibrium payoff.Indeed, we can decompose the firm's problem in two: first, the firm selects the optimal set of workers to incentivize, and then the firm sets up the optimal incentive scheme to implement their effort in the firm's least-preferred equilibrium or, equivalently, to robustly implement teamwork.[16]

[15]See (Segal, 2003) for a general argument.

[16]Another possible interpretation is that the potential project surplus is so high that the

The third assumption is that a worker's contract cannot depend on the private signals that the firm receives from her colleagues. Although theoretically intriguing, considering such a possibility would significantly complicate the analysis without changing the main contribution of the paper.[17] In an extension of the paper, we show that the firm would never provide a larger bonus to a worker upon observing the positive individual signal of a colleague. Due to strategic uncertainty, doing so could only be counterproductive in our setting. On the other hand, giving a worker an extra bonus if a coworker fails might be interesting but opens to a whole new set of criticisms. Indeed, creating direct competition among workers can easily backfire when cooperation is crucial. Workers might not cooperate and even sabotage each other as a result of such competition (e.g., Dai and Toikka (2017),Chowdhury and Gürtler (2015)). Workers might also suffer from the so-called "discouragement effect" when the workers are pessimistic about their own abilities; see Sheremeta (2016) for a review of pros and cons of relative performance bonuses in the workplace. Finally, in many applications of interest, we might expect the workers to be unable to verify the private signals that the firm observes on their coworkers, questioning their actual contractability.

2.2 A Useful Benchmark: The firm-preferred Equilibrium

As discussed, the most important assumption of our paper is that the firm wants to implement work as the unique rationalizable outcome of the game (or, equivalently, the unique equilibrium outcome of the game). To fully understand the impact of this assumption in our analysis, it is helpful to start by studying the classical benchmark case in which the firm can coordinate the workers on its preferred equilibrium (partial implementation). The main argument behind focusing on the firm-preferred equilibrium in this context would be that the firm might have successfully established a very positive team culture: all workers have no doubt about their teammates exerting efforts. As a result, to secure the effort of all team members in the firm-preferred equilibrium, the firm only needs to provide all workers a wage that makes them indifferent whenever they expect all their teammates to work. Indeed, note that, due to effort complementarities, this would be the lowest wage that still guarantees

cost of incentivizing a worker to exert effort is always relatively small with respect to the expected benefit.

[17] Indeed, strategic uncertainty also significantly affects every contract that depends on such third-party signals. As a result, our main trade off between strategic insurance rents and information rents persists.

teamwork in some equilibrium of the game. Thus, the firm's problem in this case would be

$$\min_{\boldsymbol{b}} \sum_{i \in N} \Big(p_i b_i^{ind} + F(n) b_i^{team} + p_i F(n) b_i^{both} \Big)$$

subject to:

$$(2p_i - 1) b_i^{ind} + \Big(F(n) - F(n-1) \Big) b_i^{team} + \Big(p_i F(n) - (1 - p_i) F(n-1) \Big) b_i^{both} \geq c, \quad \forall i \in N$$

where we plugged $\mu_i(n-1) = 1$ in the worker's participation constraint in IR_{i,μ_i}. Note that in this case, the workers would face no strategic uncertainty. Thus the firm only need to provide the classical information rents to limit the workers' moral hazard problem.

In terms of the wage structure, we can show that the firm would provide a positive bonus to every worker i if and only if both the individual and the team-performance signals are positive, i.e., for all $i \in N$, $b_i^{team} = b_i^{ind} = 0$ and $b_i^{both} > 0$.

Proposition 1 *For every given precision vector* $\big(p_i\big)_{i \in N}$*, in the firm-preferred equilibrium the incentive scheme* $\boldsymbol{b} = \Big(b_i^{team}, b_i^{ind}, b_i^{both}\Big)_{i \in N}$ *is such that for all* $i \in N$*,* $b_i^{team} = b_i^{ind} = 0$ *and*

$$b_i^{both} = \frac{c}{p_i F_N - (1 - p_i) F(n-1)}.$$

Intuitively, in the spirit Holmström (1979) sufficient statistic principle, since S_i^{ind} and S^{team} carry complementary information about i's effort, we could expect the firm to find optimal to use both signal to incentivize the agent. Moreover, given that workers are risk-neutral, the firm does not benefit from switching workers' payments from one state of the world to another. Thus the firm optimally rewards every worker $i \in N$ only conditional the combination of signals' realizations that most strongly indicates that i worked rather than shirking. Not surprisingly, the best combination turns out to be the one that maximizes the likelihood ratio, namely when both team and individual-performance signals are positive.

Note that $b_i^{both} = \frac{c}{p_i F_N - (1-p_i) F(n-1)}$ is the bonus that makes worker i exactly indifferent between working and shirking when she expects all her coworkers to work. Such a bonus would be insufficient to incentivize a *doubtful* worker who assigns positive probability to some of her coworkers shirking. Indeed, another possible equilibrium outcome that arises following such incentive scheme provision involves all workers shirking, expecting that all the coworkers shirk as well. Note that, differently from other contexts, no variation argument justifies the selection of the firm-preferred equilibrium in this case: starting from the firm-preferred equilibrium, no epsilon variation in the offered contracts would ensure the effort of all workers as the unique equilibrium. In this sense, focusing solely on the firm's preferred equilibrium requires a quite optimistic view about the firm's team culture's ability to shape the workers' beliefs about their colleagues and their beliefs.

Finally, to better argue that the firm-preferred equilibrium represents a knife-edge case, in an extension, we show how, when we enrich our model to account for some intermediate level of the firm's ability to shape workers' beliefs, the logic behind our results still holds.

2.3 Optimality of the ranking mechanism

This section returns to analyze the firm's problem of offering an incentive scheme that robustly implements effort at the lowest possible cost, given any fixed precision of the individual signals. In this case the firm problem is

$$\min_{\boldsymbol{b}} \sum_{i\in N} \left(p_i b_i^{ind} + F(n) b_i^{team} + p_i F(n) b_i^{both} \right) \tag{1}$$

subject to:

$$\sum_{n_{-i}=0}^{n-1} \mu_i(n_{-i}) \left(\Delta^{F}\!\left(1+n_{-i}\right) b_i^{team} + \Delta^{p_iF}\!\left(1+n_{-i}\right) b_i^{both} \right) + (2p_i-1)\, b_i^{ind} \geq c, \quad \forall i \in N, \mu_i \in \Gamma_i(\boldsymbol{b})$$

where $\Gamma_i(\boldsymbol{b})$ is the set of all beliefs of i that are consistent with rationalizability when the firms offers the bonus scheme $\boldsymbol{b}$. Moreover, for notation purposes, we denoted by $\Delta^{F}(x)$ the differential impact of e_i on the probability of team success, $\Delta^{F}(x) = F(x) - F(x-1)$, and by $\Delta^{p_iF}(x)$ $F_{p_i}^{\Delta}(x)$ the differential impact of e_i on the probability that both team and individual signals are positive, $\Delta^{p_iF}(x) = p_iF(x) - (1-p_i)F(x-1)$, when x teammates work.

Note that worker i's incentive to work depends on her beliefs about her coworkers' efforts if and only if b_i^{both} or b_i^{ind} are different from zero. In other words, team-performance bonuses b_i^{team}, as well as the wage adjustment b_i^{both}, expose the workers to strategic uncertainty: they are more valuable to the worker when she expects greater effort from her teammates. On the other hand, workers whose wages only consist of individual-performance bonuses b^{ind}, are fully insulated from their coworkers' choices, facing no strategic uncertainty.

As we want to focus on the impact of strategic uncertainty, it is thus useful to note that every incentive scheme $\boldsymbol{b} = \left(b_i^{team}, b_i^{ind}, b_i^{both}\right)_{i\in N}$ divides the workers into two mutually exclusive contractual categories:

- ***Insulated Workers*** **(IW)**, whose wages depend only on their individual-performance signals S_i^{ind}:

$$IW_b = \left\{ i \in N : \left(b_i^{team}, b_i^{ind}, b_i^{both}\right) = \left(0, b_i^{ind}, 0\right) \right\}.$$

- ***Non-insulated Workers* (NW)**, whose wages depend also on the team performance S^{team}.[18]

$$NW_b := N \setminus IW_b.$$

Workers in IW are fully dependable in the eyes of their colleagues as they are insulated from strategic uncertainty and thus would work (or shirk) independently on their beliefs about the others. On the other hand, workers in NW are exposed to strategic uncertainty, and thus, for some proposed $\boldsymbol{b_i}$, they would work only when they have sufficiently high expectations about their teammates working.

Insulated and non-insulated workers categories will prove crucial for a sharp characterization of the contract structure that arises in our context. However, before going into detail on any specific aspect, it is helpful to provide a broad overview of the contract structure.

1. Non-insulated Workers (NW) are rewarded only when both team and individual signals are positive. As in the partial implementation benchmark, $b_i^{ind} = b_i^{team} = 0$, and $b_i^{both} > 0$ for all $i \in NW$.

2. Typically the firm insulates some workers from strategic uncertainty and exposes others. Both NW and IW are typically non-empty.

3. The workers are those with the highest signal precision.

4. The firm optimally creates a ranking within NW, granting higher strategic insurance rents to higher-ranked workers.

5. If complementarities strong, within NW, the firm ranks higher the workers with higher signal precision.

As a result, the overall contract structure of the firm could be represented by the following graph:

2.3.1 Optimality of Rank Mechanism within NW

A first useful step toward providing intuition for how the above contract structure arises in equilibrium is to show the optimality of a ranking mechanism within NW. Winter (2004) showed that, in absence of individual signals about the workers' efforts, i.e. when only team-performance bonuses b_i^{team} are available, the firm finds optimal to deal with the workers'

[18]Note that, the wages within NW can depend, to some extent, also on the workers' individual signals.

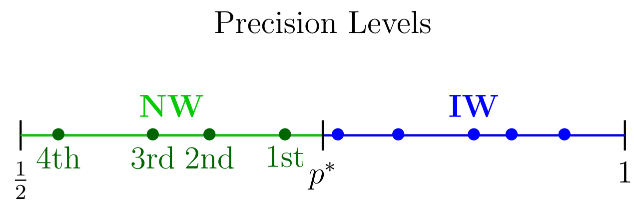

Figure 1: Mapping from individual signal precisions to contracts. Note that the 1st in the ranking is the highest-ranked worker in NW.

strategic uncertainty by creating a ranking among the team-members, offering each worker a wage that makes her indifferent between working and shirking when all (and only) the higher ranked colleagues work. The first contribution of our paper is to show that such logic still applies, at least in part, even when the firm has access to individual signals about the workers' performances and to a larger set of contracts. To this end, we introduce the following definition.

Definition 1 *An incentive scheme $\boldsymbol{b} = \left(b_i^{team}, b_i^{ind}, b_i^{both}\right)_{i\in N}$ creates a ranking within a subset of workers $A \subset N$ if there exist a one to one mapping $O : A \to \{1, .., |A|\}$, the ranking, such that each worker $i \in A$ obtains a payment scheme $\boldsymbol{b_i}$ such that*

$$\Delta^{F}\Big(x+O(i)\Big)\; b_i^{team} + \Delta^{p_i F}\Big(x+O(i)\Big)\; b_i^{both} + (2p_i-1)\, b_i^{ind} = c,$$

for $x = |N| - |A|$.

Interpreting the definition, $\boldsymbol{b}$ creates a ranking within $A \subset N$ if and only if every worker $i \in A$ is indifferent between working and shirking when the only colleagues who shirk are those that are ranked lower, i.e., all $j \in A$ such that $O(j) > O(i)$.

Exploiting this definition, we can establish that the optimal incentive mechanism $\boldsymbol{b}$ creates a ranking among those workers whose wages optimally depend on the team performance S^{team}, i.e., the workers in NW.

Theorem 1 *For every vector of individual signals precisions $(p_i)_{i\in N}$, if $\boldsymbol{b} = \left(b_i^{team}, b_i^{ind}, b_i^{both}\right)_{i\in N}$ is the optimal incentive mechanism that robustly implements effort, then $\boldsymbol{b}$ creates a ranking within NW.*

To gain some intuition, consider two workers that are paid based, at least in part, on team performance: with $b^{team} \neq 0$ or $b_i^{both} \neq 0$. If no worker is incentivized enough to work even when her colleague does not work, then there exists a possible equilibrium in which both workers shirk as they expect the other to shirk as well. On the other hand, the firm could robustly implement teamwork by offering both of them high enough bonuses such that both of them work even when expecting the other to shirk. However, when the firm offers such high incentives to the first worker, her colleague anticipates that the first worker is going to work, and thus she would be willing to accept a lower rent. In particular, the firm can offer to the second worker a lower bonus that only makes her indifferent between working and shirking when the first worker works.[19] In this sense, we refer to the second worker as the lower-ranked. As the first worker (ranked higher) would work independently on her belief, the second worker faces lower strategic uncertainty and is thus cheaper to incentivize. Suppose there is a third worker, but the firm incentivizes her to work through an individual bonus only, i.e., the third worker is in IW. In this case, the other two workers anticipate that the third worker would work independently of her beliefs about her colleagues. As a result, assigning one worker to IW reduces the strategic uncertainty and thus the required rent of all her colleagues. Iterating this argument, we get that all workers in NW need to be ranked and will obtain a bonus that makes them indifferent between working and shirking when they expect only the workers in IW and the higher-ranked coworkers in NW to work. For every worker $i \in IW$ we refer to such subset of coworkers as *fully dependable* from the perspective of i:

$$D_i = IW \cup \{j \in NW : O(j) < O(i)\}.$$

Even though in equilibrium all workers work, the worker expects that if the firm were to offer her a slightly lower bonus, only workers in D_i would continue to work, the other ones would shirk.

2.3.2 Optimal Bonuses

In this section we characterize the bonus that the workers would receive within IW and when assigned to a specific rank within NW. From the participation constraint IR_{i,μ_i}, we conclude that the cheapest way to incentivize a worker in IW is to offer her $b_i^{ind} = \frac{c}{2p_i - 1}$. In light of theorem 1, the firm's problem in (2) can thus be rewritten as

$$\min_{NW \subset I,\ O \in NW^{\{1,\ldots,|NW|\}},\ \boldsymbol{b}} \quad \sum_i \left(b_i^{team} F(n) + b_i^{ind} p_i + p_i F(n) b_i^{both} \right)$$

subject to:

$$(2p_i - 1)\, b_i^{Ind} = c, \qquad \forall i \in IW$$

$$\Delta^{F}\Big(\big|IW\big| + O(i)\Big)\ b_i^{team} + \Delta^{p_i F}\Big(\big|IW\big| + O(i)\Big)\ b_i^{both} + (2p_i - 1)\, b_i^{ind} \geq c \qquad \forall i \in NW$$

[19] Lower rents are required in this case due to effort complementarities.

This reformulation suggests that we can set up a three-steps procedure to find the optimal incentive mechanism:

1. Given precisions $p = \left(p_i\right)_{i \in N}$, the firm chooses which team members to incentivize with their individual signals only and which ones to incentivize also through team performance. Formally, the firm chooses how to partition the workers into the two contractual categories IW and NW as a function of p.

2. The firm chooses how to rank workers within NW. Note that, this is equivalent to select the set of workers that are dependable from the perspective of every $i \in NW$: $D_i = IW \cup \{j \in NW : O(j) < O(i)\}$.

3. Given the IW, NW, and the ranking selected within NW, the firm minimizes the expected wage that guarantees the workers' efforts.

Starting from the last step of the procedure, we show that, similarly to the partial implementation benchmark, the firm finds it optimal to reward workers in NW only when both team and individual signals are positive. As a result, we can write the following proposition.

Proposition 2 *For every vector of individual signals precisions* $(p_i)_{N \in I}$*,* IW*,* NW*, and ranking* $O \in NW^{\{1,\ldots,|NW|\}}$*, the optimal incentive mechanism* $\boldsymbol{b} = \left(b_i^{team}, b_i^{ind}, b_i^{both}\right)_{i \in N}$ *is such that*

- *if* $i \in IW$*, then* $b_i^{team} = b_i^{both} = 0$ *and*

$$b_i = \frac{c}{2p_i - 1}$$

- *if* $i \in NW$*, then* $b_i^{team} = b_i^{ind} = 0$ *and*

$$b_i^{both} = \frac{c}{p_i F\left(|D_i|+1\right) - (1-p_i) F\left(|D_i|\right)}.$$

To understand why workers in NW receive b_i^{both} only, note that, as we showed, the firm's problem could be rewritten as a three-step procedure where the firm chooses the workers' bonus only after selecting their positions ranking. As a result, conditioning on the worker's rank, the firm selects the bonus scheme that minimizes the worker's information rent at that given rank. As in the partial implementation benchmark, the optimal bonus for this purpose is b_i^{both}.

2.3.3 Information Waste and Rent Decomposition

In this section, we show that, unlike the partial implementation benchmark, the firm typically benefits from offering contracts that purposely ignore statistically-relevant information, violating Holmström (1979)'s sufficiency principle. In particular, we show that to curb the negative effects of strategic uncertainty, the firm typically assigns some workers to IW, where they receive only individual-performance bonuses; team performance is informative and yet ignored for workers in IW. In order to understand the crucial trade-off driving the firm's incentives to assign a worker to IW rather than to NW, it is helpful to identify and decompose the rent obtained by the workers in the two different contractual categories.

As in the partial implementation benchmark, workers in NW receive only b_i^{both}. However, incentivizing such workers is more expensive when the firm cannot coordinate them on its preferred equilibrium, and strategic uncertainty becomes thus a concern. Indeed, unlike the partial implementation benchmark, information rents are no longer sufficient in the robust implementation case. In particular, the firm needs to provide also strategic insurance rents to implement teamwork robustly, ensuring that workers work even when they have less optimistic expectations about their teammates' efforts. We can thus decompose the rent that NW workers obtain in our model into two components:

- **information rent**:

$$\underbrace{p_i F(n)\left(\frac{c}{p_i F\left(n\right)-(1-p_i) F\left(n-1\right)}\right)-c}_{\text{Information rent}},$$

  namely the rent obtained in the partial implementation benchmark, when strategic uncertainty was not a concern.

- **strategic insurance rent**:

$$\underbrace{p_i F(n)\left(\left(\frac{c}{p_i F\left(|D_i|+1\right)-(1-p_i) F\left(|D_i|\right)}\right)-\frac{c}{p_i F\left(n\right)-(1-p_i) F\left(n-1\right)}\right)}_{\text{Strategic insurance rent}}.$$

  arising from the firm's robustness concerns.

Note that strategic insurance rents arise precisely because every worker i views as fully dependable only a subset D_i of her colleagues rather than all of them as in the partial implementation case. The only exception is the lowest-ranked worker l from whose perspective all the other workers are made dependable. As $|D_l| = n-1$, the lowest-ranked worker in NW

would receive no information rent and thus the same (lower) payoff as in the firm-preferred equilibrium.

On the other hand, all workers in IW are fully insulated from strategic uncertainty and thus obtain only information rents, as in the partial implementation benchmark. However, such information rents are higher than in the partial implementation benchmark, and thus higher than in NW. Indeed, by definition, the firm does not use all the information available on IW workers' efforts but just the one contained in their own individual signals; team performance is informative and yet ignored for workers in IW. In particular if $i \in IW$ and $p_i < 1$, worker i's expected information rent is

$$\underbrace{p_i\left(\frac{c}{2p_i-1}\right)-c}_{\text{IW:Information rent with } b_i^{ind}} > \underbrace{\frac{c}{p_iF\left(n\right)-(1-p_i)F\left(n-1\right)}-c}_{\text{Information rent with } b_i^{both}}$$

The crucial trade-off between IW and NW is the one between information and strategic insurance rent. Placing a worker in NW allows the firm to use all the information about the worker's effort, keeping the information rents low, but creates the need for strategic insurance rents. On the other hand, placing a worker in IW allows the firm to save on strategic insurance rents but increases the information rent that the firm needs to grant to the worker. As it turns out, whether the firm prefers IW or NW worker does not only depends on the success probability function F and the individual signal precision, but also on how many workers are already in NW. As a result both IW or NW are typically non-empty.

Proposition 3 (Information Waste) *For every vector of individual signals precisions* $(p_i)_{N\in I}$, *the optimal incentive mechanism* $\boldsymbol{b} = \left(b_i^{team}, b_i^{ind}, b_i^{both}\right)_{i\in N}$ *is such that* $NW \neq \emptyset$, *and also* $IW \neq \emptyset$ *if*

$$\frac{p_i}{2p_i-1} < \frac{p_iF_N}{p_iF(1)-(1-p_i)F(0)} \quad \textit{for all } i \in N$$

The result that NW is non-empty is straightforward. Indeed, recall that the lowest rank coworker in NW is payed as much as in the partial implementation benchmark and thus less than in IW. On the other hand to understand why also IW is typically non-empty is a bit trickier. Even though all workers exert effort in equilibrium, the firm needs to provide the workers in NW with strategic insurance rents so high to guarantee their effort even when they expect their lower-ranked coworker to shirk. The only way in which the firm can limit such rents is by limiting the strategic uncertainty that generates them. Intuitively, individual bonuses serve precisely that purpose, as they insulate a worker from her coworkers' choices, reducing (or eliminating) the strategic uncertainty. Moreover, assigning i to IW has the further benefit of making i fully dependable in the eyes of her coworkers, reducing their strategic uncertainty. On the other hand, conditioning a worker's wage on the team signal increases her strategic uncertainty and creates a negative externality on all lower-ranked colleagues, increasing their strategic uncertainty. As a result, every contractual agreement that relies on team success imposes an extra cost on the firm in terms of strategic uncertainty, and such cost increases as the firm adds workers to NW. As the proposition shows, this logic typically results in the firm purposely ignoring the team signal for some workers and placing them in IW.

The comparison with the partial implementation case suggests that, when the firm cannot perfectly coordinate the workers on her preferred equilibrium, the firm typically benefit from sacrificing statistically-relevant information to curb strategic uncertainty, violating Holmström (1979, 1982)'s sufficiency principle.

Finally, differently from the classical firm-preferred equilibrium benchmark, our model is thus broadly consistent with the empirical observation that firms resort to individual bonuses three times more frequently than to team bonuses and that, even in place, team bonuses cover only a small fraction of the employees (Ledford Jr, Lawler III and Mohrman (1995), Payscale (2019))).[20]

2.3.4 Optimal Contract Structure under Homogeneous Precision

So far, we have characterized the incentive structure abstracting away from the role played by the precision of individual signals. If all individual signals have the same precision, i.e., $p_i = p_j = p$, then all workers are homogeneous, and thus the firm would be indifferent on how to rank them within NW. Thus, the relation between p and $F(\cdot)$ solely determines IW and NW's size. In particular, we can prove the following proposition.

Proposition 4 (Homogeneous Precision) *If all workers have the same signal precision, i.e., $p_i = p$ for all $i \in N$, then*

$$|IW| = \max_{s \in \mathbb{N}} \left\{ s \in \mathbb{N} : \frac{F(n)}{pF(s) - (1-p)\,F(s-1)} - \frac{1}{2p-1} \geq 0 \right\}, \text{ or } 0 \text{ if empty.}$$

The ranking within NW as well as the choice about which workers allocate to IW is arbitrary.

Intuitively, whether the highest-ranked worker i in NW is kept in NW or switched to IW would not affect the incentives of all other workers: lower-ranked workers in IW would

[20] Note that teamwork is employed by the vast majority of the firms (Bersin, 2016; Bikfalvi, Jäger and Lay, 2014), so the large disparity in the frequency of two compensation practices could not be simply attributed to teamwork being uncommon.

continue to have no strategic uncertainty on i and workers in IW continue not to care whether i works or not. Since all workers are homogeneous, switching any worker would have the same effect. Thus the firm finds it convenient to increase the size of IW whenever

$$\frac{F(n)}{pF(|IW|+1)-(1-p)F(|IW|)}>\frac{1}{2p-1},$$

i.e., whenever the highest rank in NW gains less in expectation than what she would gain in IW.

2.3.5 Heterogeneous Precision: in IW the Most Monitorable Workers

In this section, we explore how the heterogeneity in signal precision can shape the firm's optimal incentive scheme: how the firm will divide workers into IW and NW? Moreover, within NW, which workers will receive higher ranks? The precision of worker i's individual signal and how such precision relates to the ones of the other workers crucially determine i's optimal contract. Although workers with higher signal precision are cheaper to incentivize both in IW and in NW, we can show that only the workers with the highest signal precisions will be included in IW. The subtlety of this result lies in the fact that, due to strategic uncertainty, the contractual arrangement that the firm has with worker i imposes externalities on the other workers: saving on a worker's wage typically implies spending more on the others. As a result, when choosing whether to assign i to IW or to NW, the firm should consider not only i's expected salary in the two contractual categories but also the effect that this choice would have on the other workers. As we show that, in general, the relation between signal precision and rank within NW is not simple (it can be positive, negative, or even non-monotone), then the effect of assigning i to IW and j to NW, when $p_i>p_j$, to NW rather than vice versa is far from obvious. For example, we can show that, if forced to choose between i,j whom to place in IW and whom to place in NW, the firm might even prefer to place the one with lower precision to IW. This observation appears to contradict the naive intuition that the firm should assign the workers with the highest signal precision to IW. However, we show that this can occur only when the firm finds it optimal to assign both workers to IW. As a result, we can show the following proposition.

Proposition 5 *If worker j is optimally assigned to IW and $p_i>p_j$, then also i is assigned to IW. Moreover, if $|IW|\geq 1$ then*

$$IW\subset\{i\in I:p_i\geq p^*\},$$

where

$$p^* = \frac{F(n) - F(|IW| - 1)}{2F(n) - F(|IW| - 1) - F(|IW|)}$$

increasing in $|IW|$.

To understand the intuition behind such proposition, recall that placing a worker in IW involves an expected wage of $\frac{p_i c}{2-p_i}$ which becomes extremely high when a worker's signal precision is very low (p_i close to $\frac{1}{2}$). Not surprisingly, the firm would benefit from reducing such extreme information rent by exploiting the team's overall signal about the project's success or failure. Placing such low precision workers in NW would thus be optimal for the firm. On the other hand, when the precision of the individual signal is very high (p_i close to 1), the workers' information rents in IW would be very low: while placing the worker in NW would further reduce such rents, such reduction would be almost irrelevant. On the contrary, assigning an additional worker to NW generates a sharp increase in the exposure to strategic uncertainty both for the worker and her higher-ranked coworkers. As a result, the firm is typically better off by placing highly monitorable workers in IW as a way to save on the strategic insurance rents needed to incentivize teamwork.

This simple intuition explains why, in equilibrium, all and only workers with signal precisions above a certain threshold p^* are placed in IW. Finally, note that, if $i \in IW$ in equilibrium, then it must be the case that the firm saves on her wage by assigning i to IW rather than to the highest rank of NW; indeed, the firm could make this change without affecting the incentives needed by all the other workers. Mathematically this translates into

$$\frac{p_i}{2 - p_i} \leq \frac{F(n)}{p_i F(|IW|) - (1 - p_i) F(|IW| - 1)},$$

and thus

$$p_i \geq p^* = \frac{F(n) - F(|IW| - 1)}{2F(n) - F(|IW| - 1) - F(|IW|)}.$$

Finally the proposition shows that, due to the effort complementarities (convexity of F), p^* is increasing in $|IW|$. Thus higher signal precision is necessary to be considered for IW when more workers are placed in IW.

2.3.6 Optimal Ranking

This section analyzes how the firm would optimally set up a ranking within the workers in NW when they have heterogeneous signal precisions. As a first step, note that, without further assumptions, it is unclear whether the firm wants to place the most monitorable workers at the top or the bottom of the ranking. While it is true that a higher signal precision allows higher savings on a worker's expected wage, it is not obvious whether such savings would be more significant at the top or at the bottom of the hierarchy. As a result, the relation between signal precision and rank might be positive, negative, or even non-monotone.

In the following proposition, we show that if the effort complementarities are sufficiently strong, then the workers with higher signal precisions will be placed higher in the ranking. In other words, the firm benefits from making more predictable the effort of those agents who can be monitored more easily. Furthermore, we show that, in this case, we obtain a simple condition that pins down the set of workers optimally assigned to IW.

Proposition 6 *The production function F is such that $\frac{(F(x+1)-F(x))^2}{F(x)}$ is increasing in x if and only if for all precision vector p the following results hold:*

1. *Within NW, workers with higher signal precision are ranked higher, i.e.*

$$\text{if } i,j \in NW \text{ and } p_i > p_j \text{ then } O(i) < O(j).$$

2. *If $p_i \neq p_j$ for all $i,j \in N$, then IW is uniquely determined as a function of $\boldsymbol{p}$ by*

$$IW = \{i \in N : p_i \geq p^*(F,p)\},$$

where $p^*(F,p)$ is the lowest $p \in \left[\frac{1}{2},1\right]$ such that

$$p \geq \frac{F(n) - F(n-1)}{2F(n) - F(n-1) - F(n)},$$

with $n = |\{s \in N : p_s > p\}|$.[21]

Intuitively, when effort complementarities are particularly high, the strategic insurance required from workers in NW steeply decreases along with the ranking. The firm resorts to placing highly monitorable workers at the top of the hierarchy as a way to limit the strategic insurance rents of higher ranks workers. To see how the above condition on F translates

[21] In case $|\{s \in N : p_s = p^*\}| > 1$ then

$$IW = \{s \in N : p_s > p^*\} \cup T$$

where $T \subset \{s \in N : p_s = p^*\}$ and $|T|$ such that $\frac{p}{1-p} \leq \frac{F(n)-F(n+|T|-1)}{F(n)-F(n+|T|)}$ and $\frac{p}{1-p} > \frac{F(n)-F(n+|T|)}{F(n)-F(n+|T|+1)}$.

into the effort complementarities being strong, note that we can rewrite it as

$$(F(x+1)-F(x))\frac{(F(x+1)-F(x))}{F(x)}$$

being increasing in x. Due to effort complementarieties the impact of one extra unit of labor is increasing in how many coworkers exert effort, i.e. $F(x+1)-F(x)$ is increasing in x. Thus our assumption on F requires the percentage increase of such impact $\frac{F(x+1)-F(x)}{F(x)}$ to be increasing (or not too decreasing) in x. Finally, since we have that the highest rank in NW needs to be also the most monitorable worker in NW, we can conclude that p_j being higher than $p^* = \frac{F(n)-F(|IW|-1)}{2F(n)-F(|IW|-1)-F(|IW|)}$ would not only be necessary but also sufficient for j being included in IW: if j were not at the highest rank still the agent i at the highest rank would have $p_i > p_j$ and thus the firm would prefer to place i in IW rather than in the highest rank of NW. Exploiting this observation, we can uniquely determine IW as highlighted in the proposition.

Finally note that the condition in the above proposition is sharp. In particular if $\frac{(F(x+1)-F(x))^2}{F(x)}$ is decreasing for some x, i.e. if

$$\frac{(F(x+1)-F(x))^2}{F(x)} > \frac{(F(x+2)-F(x+1))^2}{F(x+1)}$$

then there exist signal precision vectors under such that the firm might prefer assign higher rank to some less monitorable workers; i.e. such that $i, j \in IW$, $p_i > p_j$ and still $O(i) > O(i)$.

2.4 An Interesting Restriction: Separable Bonuses

So far, we considered the possibility for the firm to set up contracts that depend on both individual and team performances in a rather flexible way. However, in contexts where contract simplicity or behavioral considerations play an essential role, the firm might benefit from restricting the attention to additively separable contracts that simply grant a bonus b^{team} whenever the team performance is positive and a bonus b^{ind} when the individual performance is positive. Formally, relative to the previous case, this section considers the case where the firm is constrained to set $b_i^{both} = 0$ for all workers.

We find that the same logic highlighted so far continues to hold even in this constrained case. Namely, the firm still finds it optimal to waste statistically-relevant information, divide workers into two contractual categories IW and NW, and create a ranking within NW. The main difference in the contract structure, in this constrained case, is that workers within NW will receive bonuses based only on team performance, i.e., b_i^{team}, rather than on both individual and team performance, b_i^{both}. As a result, we will have a cleaner separation between the workers' rents in the two categories:

- workers in IW receives only individual information rents, as before
- workers in NW no longer receive individual information rents but just strategic insurance.

Formally we can show the following proposition

Proposition 7 *For every vector of individual signals precisions $(p_i)_{i\in I}$, the optimal separable incentive mechanism $b = \left(b_i^{team}, b_i^{ind}\right)_{i\in N}$ divides workers into two contractual categories IW and $NW = N \setminus IW$ such that*

- *if $i \in IW$, then $b_i^{team} = 0$ and*
$$b_i^{ind} = \frac{c}{2p_i - 1}$$
- *if $i \in NW$, then $b_i^{ind} = 0$ and*
$$b_i^{team} = \frac{c}{F\left(\left|IW\right|+O(i)\right) - F\left(\left|IW\right|+O(i)-1\right)} \text{ for some ranking } O.$$

Note that the restriction to additively separable contracts results in the wages of workers in NW being optimally independent of p_i. As a result, under such restriction, the ranking within NW is arbitrary, i.e., all ranking permutations within NW would deliver the same expected payoff for the firm. On the other hand, workers within IW are completely insulated from strategic uncertainty as their expected wage only depends on the precision of their individual signals. Finally, similarly to proposition 6, we show that it will still be the case that the workers with the highest precision would be the ones placed in IW.

Proposition 8 *For every vector of individual signals precisions $(p_i)_{i\in I}$, the optimal separable incentive mechanism $b = \left(b_i^{team}, b_i^{ind}\right)_{i\in N}$ is such that, the cardinality of IW, $|IW|$, is uniquely determined. Moreover, there exists a a precision level $p^*(F,p)$ such that all workers with precision higher than $p^*(F,p)$ will be placed in IW, i.e.*

$$IW = \{i \in N : p_i \geq p^*(F,p)\},$$

where $p^*(F,p)$ is the lowest $p \in \left(\frac{1}{2}, 1\right)$ such that

$$\frac{p}{1-p} \geq \frac{F(n)}{F(n) - F(n+1) + F(n)},$$

with $n = |\{s \in N : p_s > p\}|$.

3 Endogenous Monitoring

So far, we have considered the firm problem of incentivizing a team of complementary workers for every fixed signal precision. We showed that for every $\boldsymbol{p}$ there exists an almost unique way for the firm to organize workers between $IW(\mathbf{p})$ and $NW(\mathbf{p})$, with the only exceptions being due to the interchangeability of homogeneous workers. In this section, we want to focus on the firm's incentives to monitor its workers more or less closely and on the workers' incentives to facilitate or hinder such monitoring and, in general, to make their work more or less transparent.

3.1 Firm's monitoring choices

In this section, we consider that the firm can not only offer contracts to the workers but also select the monitoring precision p_i of every worker $i \in N$ at a cost $k(p_i)$ which is increasing and convex in p_i. The timing in this case would thus be as follows.

1. Firm choose p_i at cost $k(p_i)$.

2. The firm publicly sets up the optimal incentive scheme that robustly implements teamwork (i.e., our previous analysis).

3. Workers simultaneously choose whether to work or shirk.

4. Signals are realized and payoffs are determined.

3.1.1 Firm-Preferred Equilibrium Benchmark

As a first step of the analysis we show that in the firm-preferred equilibrium benchmark the firm would select the same monitoring level for all workers. Indeed note that all workers are ex-ante homogeneous and, given the result in proposition 1, their wages are independent of their coworkers. As a result the firm problem of selecting the vector of signal precision can simply be reduced n identical problems, one for every worker.

3.1.2 Robust Implementation Case

In contrast with the firm-preferred equilibrium benchmark, we show that, when concerned about robustness, the firm optimally chooses to monitor some workers more closely than the others, even if the workers were homogeneous ex-ante. Specifically, the firm monitors the workers that it plans to assign to IW more closely than the others, and, within NW, the firm monitors the higher ranks more closely than lower ranks.[22]

Proposition 9 *If effort complementarities are sufficiently strong,*[23] *then firm chooses heterogeneous monitoring precisions*

- *Same (highest)* p_i *for those that will be in IW*
- *Within NW: higher* p_j *for those that will be ranked higher.*

Moreover, both NW *and* IW *are non-empty, if monitoring is sufficiently cheap.*

Intuitively, in the robust implementation case, the firm's incentives to monitor one worker crucially depend on how much the firm is monitoring the others. So, for example, investing in a NW worker signal precision would have a higher impact in reducing her expected salary when the worker expects more coworkers to shirk. Thus if all workers in NW had the same monitorability, the firm would benefit from transferring monitoring investments from lower ranks to higher ranks. Also, as IW workers receive information rents only, we can show that the firm prefers to transfer monitoring resources from NW to IW. Finally, we show that when the monitoring cost function $k(\bullet)$ is sufficiently low, then it is optimal for the firm to invest in some workers' signal precision enough to make it optimal to place theem in IW.

Thus, our results about the optimal contract structure are robust to the possibility for the firm to invest in monitoring the workers.

3.2 Workers' Monitoring Choices

In the previous section, we studied the firm's incentives to invest in monitoring the workers. However, the choice of the monitoring structure is rarely unilateral. While it is true that the firm can set up a monitoring system at a cost, the workers can largely impact the monitoring effectiveness or cost. For example, a worker can oppose or even deactivate the computer's camera, use external devices to elude browsing restrictions, affect the transparency, and, ultimately, the accuracy of a report resorting to omissions, creative interpretations, or outright lies. Also, the workers can actively provide extra signals about their work by engaging in self-promotion and seeking face time with the management. Such practices of highlighting

[22]Here we assume that the cost of monitoring each worker is homogeneous. If monitoring costs were intrinsically different, the firm would monitor most closely cheaper workers.

[23]As before, by sufficiently strong we mean that F is such that $\frac{(F(x+1)-F(x))^2}{F(x)}$ is increasing in x.

own effort and results obviously increase effort transparency and thus the precision of the individual signal: the lack of communication from a self-promoting employee would typically be interpreted as a negative individual signal about her effort. To study the workers' incentives toward monitoring, we assume that each worker can freely choose her individual precision level (hereafter, *monitorability*) before the firm offers the contract schedule. The timing is as follows.

1. Workers simultaneously choose their own individual monitorability p_i at no cost.
2. The firm publicly sets up the optimal incentive scheme that robustly implements teamwork (i.e., our previous analysis).[24]
3. Workers simultaneously choose whether to work or shirk.
4. Signals are realized and payoffs are determined.

3.2.1 Firm-Preferred Equilibrium Benchmark

To establish a natural benchmark, we start by considering the equilibrium in which the firm can coordinate workers on its preferred equilibrium in the second stage of the game. In this equilibrium, the firm's robustness concerns obviously play no role: every worker obtains the same expected wage as a function of p_i. Workers receive information rents only, which are decreasing in the workers' individual signal precisions p_i. Thus, to maximize the information rents, every worker finds it optimal to reduce her monitorability as much as possible. In this case all workers would select $p_i = \frac{1}{2}$, and obtain the expected wage $\mathbb{E}(W_i) = F(n)\frac{c}{F(n)-F(n-1)}$. Note that, since by selecting the lowest monitorability every worker can obtain an equilibrium wage that is at least as high as $F(n)\frac{c}{F(n)-F(n-1)}$ when the firms wants to implement teamwork in the second stage of the game. Thus such an equilibrium is also the firm-preferred equilibrium of the whole game.

Corollary 1 *In the firm-preferred equilibrium every $i \in N$ chooses the lowest precision for her individual signal, i.e., $p_i = \frac{1}{2}$, and obtain the expected wage*

$$\mathbb{E}(W_i) = F(n)\frac{c}{F(n) - F(n-1)}.$$

[24]Qualitatively equivalent results can be obtained when we assume that the workers choose the cost that the firm needs to sustain to increase the precision of her own individual signals (see Appendix).

3.2.2 Robust Implementation Case

In contrast with the firm-preferred equilibrium benchmark, we show that when the workers anticipate that the firm would robustly implement teamwork, they would facilitate monitoring and resort to self-promotion, increasing their efforts' transparency. While an increase in the firm's monitoring precision penalizes workers overall, we can show that competition for better contracts leads to the following unraveling result.

Theorem 2 (Self-Promotion Race) *In equilibrium, all agents work and select precision levels so high that they completely annihilate the strategic insurance rent. Namely, all workers obtain an expected wage equal to*

$$\mathbb{E}(W_i) = F(n)\frac{c}{F\big(n\big) - F\big(n-1\big)}.$$

Note that $\frac{F(n)}{F(n)-F(n-1)}$ is indeed the expected wage that any worker would get in the firm-preferred equilibrium, i.e., in the absence of strategic uncertainty. Interestingly, such a result holds both when we restrict our attention to additively separable bonuses and when we allow for more complex and flexible contracts. This observation suggests that the firm benefits from limiting itself to offer additively separable bonuses whenever contract complexity is a concern. For this reason, we choose to start by focusing on such a simple class of contracts, with the additional benefit of simplifying the analysis and clarifying the forces at stake. Indeed in this setting, NW workers' wages are independent of their individual signal but only on the team's overall outcome, and, thus, the actual ranking within NW is independent of the individual signal precisions.[25]

As a first step, we show that starting from any distribution of signal precisions and, thus, from any optimal contract structure, workers in NW can induce the firm to switch them to IW by increasing their monitorability above a certain threshold. The critical step to characterize this threshold is to note that all the workers in NW would consider a coworker dependable both when included in IW and when assigned to the highest rank in NW.[26] Thus, the firm would choose between these two scenarios solely based on the worker's salary, without worrying about any externality on the other workers who would be indifferent between the two alternatives. Thus, if a worker increases her monitorability above the threshold

[25] The logic of the results broadly extends to the case of more flexible contracts, treated in the appendix.

[26] Indeed, the worker receives enough incentives to work independently of their beliefs not only when assigned to IW, but also when assigned to the highest rank of IW, as she expects workers in IW to work.

where her expected salary is the same in IW and in the highest rank of NW, the firm optimally switches her to IW, and the worker obtains the same payoff as the highest rank. We denote such threshold as $\overset{N\to I}{P}(\boldsymbol{p})$, highlighting its dependency on the initial precision distribution $\boldsymbol{p}$.[27] Since the other workers in NW gain less than the highest rank, they would strictly benefit from increasing their monitorability slightly above such $\overset{N\to I}{P}(\boldsymbol{p})$.

Lemma 1 *Suppose the initial distribution of signal precisions is* $\boldsymbol{p} = (p_i)_{i\in N}$, *generating* $IW(\boldsymbol{p})$ *and* $NW(\boldsymbol{p})$ *as the optimal response for the firm. If* $i \in NW(\boldsymbol{p})$ *and not the highest ranked worker, then* i *benefits from increasing her* p_i *to* $\overset{N\to I}{P}(\boldsymbol{p})^+$.

Finally, we show that when a firm switches a worker from NW to IW, all her higher-ranked coworkers, including the highest rank, receive lower strategic insurance rents as the worker de facto becomes fully dependable in their eyes. Consequently, the expected wage of the highest rank decreases with the number of workers in IW and thus the monitorability threshold $\overset{N\to I}{P}(\boldsymbol{p})$ increases with the number of workers in IW. In other words, higher signal precision is required to switch from NW to IW (or stay in IW) when $\boldsymbol{p}$ is such that more workers are in NW. From the worker's perspective, as strategic insurance rents decrease in $|IW(\boldsymbol{p})|$, workers need to accept lower information rents (thus, higher signal precision), if they want to be included in IW.

Remark 1 $\overset{N\to I}{P}(\boldsymbol{p})$ *increases when* $|IW(\boldsymbol{p})|$ *increases.*

As a result, the workers engage in a race toward higher and higher monitorability levels or, in other words, in a self-promotion race. Intuitively, if a worker expects her colleagues to engage in self-promotion, she has incentives to engage in self-promotion even more. This self-promotion race leads to monitorability levels so high that the firm finds it optimal to offer only individual bonuses in equilibrium. Indeed, as every worker in NW, apart from the highest rank, is happy to increase her signal precision to switch to IW, this race would not stop until all workers select a precision level so high that at most only one worker is placed in NW. As this worker can obtain the same payoff as another by increasing her monitorability above the other's, it must be the case that all workers get the same expected wage. This observation leads to two possible payoff equivalent equilibria, one in which only one worker is in NW and one in which all workers are in IW. In both equilibria, we show that all the workers select the same precision level: the one that delivers the same expected wage in IW

[27] Recall that, when the contracts are additively separable, the expected wages within $NW(\boldsymbol{p})$ only depend on the cardinality $NW(\boldsymbol{p})$. Since $\boldsymbol{p}$ fully determines the cardinality of $NW(\boldsymbol{p})$, it also determines the expected wage of the highest ranked worker in $NW(\boldsymbol{p})$ and thus the threshold.

and NW when the worker expects all her coworkers to work.[28] Thus, in terms of payoff, such a race leads to the complete annihilation of the workers' strategic insurance rents: the firms and the worker get the same payoff as in the firm-preferred equilibrium, albeit with higher monitorability levels.[29]

Proposition 10 (Self-Promotion Race) *Suppose bonuses are constrained to be separable. In every pure action equilibrium, all agents select a precision level $p_i = p^*$ so high that:*

1. *The firm finds it optimal to offer only individual-performance bonuses to robustly implement teamwork.*
2. *The workers' expected payments are the same as in the firm-preferred equilibrium benchmark.*

$$p^* \frac{c}{2p^* - 1} = F(n) \frac{c}{F(n) - F(n-1)}.$$

Finally, we compare our results with those of the classical firm-preferred equilibrium benchmark, finding that our model speaks to empirical facts that would be at odds with the latter. For example, while the classical model predicts that the workers should try to keep their efforts as private as possible, we often observe workers complaining that they feel unseen (more than 40%, according to *Reward Gateway 2018 report*). Many articles and books, including *HBR Guide to Office Politics* indicate self-promotion as a way to "make sure people understand and see what you do," increasing chances of recognition and career advancements, which is in line with our model. Also, in terms of the type of bonuses offered, while the classical model suggests that team bonuses should be at least as common as individual ones and tend to become the only bonuses when workers can affect the transparency of their job, Payscale (2019) highlights that individual bonuses are by far the most common type of bonus. This observation is in line with our model that predicts individual bonuses to be more common than the team ones, even more so when workers can affect the transparency of their jobs. These ideas can be summarized in the following table.

[28] Note that the only worker in NW would, in fact, consider all other workers dependable.

[29] Workers in IW have higher signal precision than in the firm-preferred equilibrium benchmark.

	firm-preferred	RITW	Evidence
Signal Precision	Lowest	Higher	Self-promotion and seek face time with boss.
Bonus Type	Only Team	Mostly Individual	Mostly Individual bonuses

Lastly, as anticipated, when the simplicity of the contracts is not a concern, and the firm can resort to more flexible contracts, the same logic applies. The main differences are that there are no pure action equilibria, and workers might select different mixed strategies in equilibrium. Still, theorem 2 continues to hold, and both the firm and the workers would obtain the same equilibrium payoffs.

3.3 Resistance to Monitoring

In this section, we study how a change in a worker's monitorability affects her coworkers. We show that both in the flexible and in the additively separable cases, workers are better off when their colleagues' efforts are not transparent.[30] In particular, we can prove the following proposition.

Proposition 11 *Workers benefits from limiting the monitorability of their colleagues.*

First consider an increase in p_i. Intuitively, workers in IW are unaffected by an increase in p_i as far as the firm does not switch them to NW as a result, in which case they are damaged. Within NW, lower-ranked workers would be unaffected: i is and continues to be dependable in their eyes. Higher-ranked workers can only be negatively affected as i might overcome them in the ranking, or, if p_i increases enough, i might be switched to NW; In both cases, i would become dependable in their eyes, and thus the firm would decrease their strategic insurance rents and expected bonuses. On the other hand, consider a decrease in p_i. Workers in IW would not be affected. Workers in NW are unaffected if $i \in NW$ and ranked lower, but benefit if $i \in IW$ or $i \in NW$ and ranked-higher and p_i decreases below their own monitorability level. Indeed, in the latter case, worker i would switch to a lower rank and thus increase their strategic insurance rents and thus expected bonuses.

Interpreting proposition 11, we can conclude that, when workers can organize themselves in representative institutions, e.g., unions, such institutions should vehemently oppose any

[30] As before in the flexible case we assume that effort complementarities are strong enough, i.e. $\frac{(F(x+1)-F(x))^2}{F(x)}$ is increasing in x.

increase in monitoring and, possibly, discourage individual workers from self-promoting. Also, proposition 11 casts doubts on the firm's ability to delegate monitoring to other team members, as they benefit from limiting the monitorability of their colleagues. This benefit is particularly high for the team manager who, as we show in an extension, can be interpreted as the highest-ranked worker in NW. Indeed, the team manager would enjoy the highest possible strategic insurance rents (and thus bonuses) if she manages to keep her colleagues' monitorability so low that the firm optimally includes all of them in NW. In this sense, our paper suggests that rhetoric about teamwork and lack of result attributability might speak more to the team manager's incentive to protect her own rent against the firm than to the intent of motivating team members.

In fact, in corporate practices, it is not infrequent that the team manager obstacle any form of direct monitoring of the subordinates, avoiding detailed reports and regularly retaliating against subordinates that try to report straight to the upper management or the property.

4 Conclusions and Directions for Future Work

This paper studies monitoring incentives and the optimal balance between individual and team-performance bonuses in teams of complementary workers. As a first contribution, we show that the firm optimally incentivizes some team members with individual-performance bonuses only, sacrificing statistically-relevant information in order to reduce the strategic insurance rents. Consistent with the available evidence for corporate best practices, our model predicts that most bonuses should be linked to workers' individual performances rather than the team output. As a second contribution, we show that the firm optimally discriminates among (possibly homogeneous) workers regarding total rent granted, type of bonus offered, and how closely they are monitored (targeted monitoring). As a third contribution, we show that, even if the firm's monitoring ability damages workers overall, competition for better contracts incentivizes them to facilitate monitoring even when the firm cannot credibly threaten the workers. In particular, workers engage in a self-promotion race that yields to the annihilation of the strategic insurance rents: firm and workers obtain the same payoffs as in the firm-preferred equilibrium, albeit with a very different contract structure and higher monitoring precisions. Thus, differently from the classical benchmark and in line with the evidence, our model predicts that workers actively engage in practices that increase their effort's transparency, like self-promotion, and bonuses remain primarily individual. As a final contribution, we show that workers' monitorability damages their colleagues, providing a possible explanation to why unions typically oppose increases in monitoring and casting doubts on the possibility that workers could be effective supervisors for their colleagues.

An important assumption of our model is that the firm wants to robustly (rather than partially) implement teamwork. As discussed, robust implementation coincides with unique implementation and with implementation in the firm's least-preferred equilibrium in our setting. One can see our assumption as just as extreme as the classical firm-preferred equilibrium assumption. However, we can show that the dynamics introduced by our model arise as long as workers expect their colleagues not to coordinate on the firm-preferred equilibrium with positive probability when multiple equilibria are available.

A restriction of our model is that a worker's contract cannot depend on the private signals that the firm receives from her colleagues. This restriction is realistic in many contexts and helps simplify the analysis and isolate the main effects of the firm's robustness concerns on the optimal balance between individual and team bonuses. However, considering such a possibility opens up a new exciting set of questions about how the firm could use competition among workers to limit strategic insurance rent. For example, we can show that the firm might benefit from granting extra bonuses when the signal about the workers' colleagues is negative despite the team performance being positive.[31] It would be interesting to study the trade-off between limiting strategic uncertainty and exposing team members to sabotage from their colleagues.

In another extension, we study the role of a team manager whose job is to allocate tasks to the workers. His job is scarcely monitorable but has a crucial impact on team success. We find that the firm will assign the manager to the highest rank in NW. Furthermore, we find that the manager might benefit from limiting the task monitorability even at the expense of productivity. This observation opens up interesting questions about whether managers should receive team-performance bonuses only or also be incentivized based on tasks' transparency.

In our last extension, we start exploring the role of heterogeneous skills in our setting. For homogeneous signal precision, we can show that higher-skilled workers will be ranked higher. It would be interesting to explore how such results might impact investment in human capital and firms hiring policies. We conjecture that workers will invest more in firm-specific skills when they are closer in terms of ability. On the other hand, when a worker perceives his colleague to be much more or much less skilled, she has a lower incentive to invest in human capital. This observation suggests that the firm might benefit from hiring workers with similar abilities to trigger such competition.

In future research, we plan to explore the question of peer monitoring further. While we

[31] We can show that the firm optimally avoids granting extra bonuses in case of positive signals from the colleagues.

provide clean comparative statics about how peer monitoring might be problematic in the presence of strategic insurance rents, our results suggest that monitoring from the bottom might be much less challenging than monitoring from the top. Indeed, suppose the ranking among workers is independent of p_i; this would be the case, for example, if the ranking were fixed ex-ante, or if skills were sufficiently heterogeneous to be the only determinant of the ranking). In this case, we would have that subordinates would have no incentives to limit the effort transparency of superiors because they consider superiors dependable anyway. As a result, the firm might easily incentivize them to do so. The same does not hold for the superiors who, on the contrary, are afraid to see their strategic insurance rents deteriorate. Indeed, if they give away too much information about their subordinates' efforts, the firm might find it optimal to switch the subordinates to IW, reducing the superiors' strategic insurance rents.

Another exciting direction that we still did not explore is team design. In particular, while our framework takes the team as given, it would be interesting to study how the firm optimally creates teams of complementary workers and how robustness concerns and the presence of individual signals impact such design.[32]

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5 Appendix

5.1 Extensions

5.1.1 The Team-Manager

In this section we consider the role of a team-manager m, whose job is fundamental for the team success. in particular we assume that

$$F(\boldsymbol{e}) = \begin{cases} 0 & \text{if } e_m = 0; \\ F\left(\sum_{i \neq m} e_i\right) & \text{if } e_m = 1. \end{cases}$$

We further assume that, given the nature of the job, the team-manager is scarcely monitorable, i.e. we assume that $p_m = \frac{1}{2}$. In this case we can prove the following result.

Proposition 12 *The team manager obtains the highest rank within NW and the highest wage with respect to the entire team.*

Intuitively, note that if m is not dependable for worker $i \in NW$, worker i will anticipate the team's failure, thus assigning no value to any incentives linked to team success. As a result, the firm should make m dependable from the perspective of all NW workers. Moreover, since the manager's individual signal precision is extremely low, it would be too expensive to reward her with individual bonuses only. As a result, m would be granted the highest rank in NW. Applying proposition 11 to this context, we obtain that the team manager would try to limit team members' effort transparency to preserve her own rent. Finally, consider the case in which the manager's job is to assign tasks to the team members, selecting from a pool of tasks that are heterogeneous both in their impact on team success and monitorability. In this case, we will obtain that the manager might even be willing to sacrifice productivity for limiting effort transparency and keeping her strategic insurance rents high.

5.1.2 Heterogeneous Skills

In this extension we consider the impact of introducing heterogeneity in the team-members' skills. Each worker i is thus endowed with an ability type θ and has to choose whether or not to exert effort: $e_i \in \{0, 1\}$. We assume that the effort of higher skilled workers is more productive and that, even if all workers exert effort, there is no guarantee that the project is implemented. In particular we assume that the probability that the project is successful is $F\left(\sum_i \theta_i e_i\right)$, with F strictly increasing and $F\left(\sum_i \theta_i e_i\right) < 1$. For simplicity we assume that all workers have the same signal precisions $p_i = p_j = p$ for all $i, j \in N$. For simplicity, I restrict the attention to the case where contracts are additively separable. In this case we can prove the following characterization of the optimal ranking scheme.

Proposition 13 *In the optimal ranking scheme that robustly implement effort, agents are either assigned to IW or to NW, with* $IW \bigcap NW = \emptyset$*. Within NW, higher skilled workers are ranked higher and paid more.*

Moreover, in terms of the composition of NW and IW we can prove the following.

Proposition 14 *If highest skilled worker* i *is paid through a wage contingent to individual performance, then all workers are.*

Note that, within each category higher skill corresponds to higher wages, however it is still possible that some workers in IW gets a higher expected wage than some workers in NW with higher ability. Intuitively that might occur because workers in IW might be more difficult o monitor than workers in NW.

5.1.3 More Flexible Contracts: include third party performances

The third important assumption is that a worker's contract cannot depend on the private signals that the firm receives from her colleagues. As we anticipated, considering such a possibility would significantly complicate the analysis without changing the main messages of the paper. In particular, since strategic uncertainty significantly affects also every contract that depends on such third-party signals, the trade-off between strategic insurance rents and information rents continues to bite. As a result we can still prove that team performance bonuses would be lower-powered when the firm is concerned about trust than when trust is not a concern (our benchmark case).

Proposition 15 *Trust concerns strictly decrease the weight of team performance bonuses.*

Moreover, we can show that the firm would never provide a larger bonus to a worker upon observing the positive individual signal of a colleague. If the wage does not depend on the team performance, conditioning on coworkers' individual signal would be useless: it would just add variance to the salary without changing the expectation.[33] Indeed, when not combined with team performance, colleagues' individual signals are uninformative for the worker's effort, and thus ineffective in reducing information rents.[34] On the other hand, if

[33]Recall that both the firm and the workers are risk-neutral.

[34]Note that colleagues' individual signals would be ineffective in reducing information rents also for any given belief about coworkers' effort as it would provide no additional information about the worker's effort.

a worker's wage positively depends on the team performance, granting higher bonus when the colleagues' performance is positive would be counterproductive: doing so would just increase strategic uncertainty and thus strategic insurance rents. However, granting higher wages when some of the colleagues' individual signal are negative might help in reducing the negative impact of strategic uncertainty. We are still exploring this possibility. Intuitively, denote by h the highest-ranked worker, who needs to be made indifferent between working and shirking when she expects all other workers in NW to shirk. The firm benefits from introducing for h an extra bonus when the signal of one of his colleague is negative. Indeed such occurrence is unlikely in equilibrium, given that all workers exert effort, but is able to reduce strategic insurance rents that need to be granted to h.[35]

However, creating direct competition among workers can easily backfire when cooperation is crucial. Workers might not cooperate and even sabotage each other as a result of such competition (e.g., Dai and Toikka (2017),Chowdhury and Gürtler (2015)). Workers might also suffer from the so-called "discouragement effect" when the workers are pessimistic about their own abilities; see Sheremeta (2016) for a review of pros and cons of relative performance bonuses in the workplace.

5.1.4 Inequality Aversion (preliminary)

In our analysis, we abstracted away from inequality concerns. This section explores the effects of such concerns over the contract structure that arises.

While a small degree of inequality aversion does not change the optimal contract structure, strong inequality aversion penalizes the adoption of a divide and conquer strategy, potentially providing a challenge for our result. To capture this logic, we consider the case in which all workers have inequality aversion so strong that the firm is forced to pay all workers the same expected bonus. We can show that our results would just become starker in this case. In particular, we can show that if p and F are such that all workers are rewarded with b_i^{both} in absence of inequality aversion, then all workers are rewarded with b_i^{both} also with inequality aversion. On the other hand, if m workers are in IW (rewarded only with b_i^{ind}) in the absence of inequality aversion, then at least m workers are rewarded with b_i^{ind} also with inequality aversion; if there is any aversion or cost in having different bonus compositions then, in this second case, all workers would be rewarded with b_i^{ind} only. Intuitively, if forced to pay all workers the same, the firm would only care to minimize how much it should pay the highest-paid worker to robustly implement teamwork. On the other hand, if it is cheaper to

[35]Given the out of equilibrium beliefs, the impact on the h's participation constraint is bigger than the impact on the firm's objective function.

induce the highest-paid worker to exert effort using b^{ind} then the firm would find it optimal to reward all workers that way.

In a nutshell, we concluded that strong inequality aversion leads to even less frequent use of team performance bonuses (in favor of individual performance bonuses) and result in a larger information waste on the side of the firm.

5.2 Mathematical Appendix

Proof of Proposition 1

Proof. Note that the firm chooses $\boldsymbol{b} = \left(b_i^{ind}, b_i^{team}, b_i^{both}\right)$ so to solve

$$\min_{\boldsymbol{b}} \sum_{i \in N} \left(p_i b_i^{ind} + F(n) b_i^{team} + p_i F(n) b_i^{both}\right)$$

subject to:

$$(2p_i - 1)b_i^{ind} + \Big(F(n) - F(n-1)\Big)b_i^{team} + \Big(p_i F(n) - (1-p_i)F(n-1)\Big)b_i^{both} = c, \quad \forall i \in N.$$

Note that, since $p_i > \frac{1}{2}$, we have

$$\frac{F(n)}{F(n) - F(n-1)} > \frac{p_i F(n)}{p_i F(n) - (1-p_i) F(n-1)}.$$

Thus the firm always prefer to use b^{both} rather than b^{team}. Moreover, since $F(n) - F(n-1) > 0$, we have

$$\frac{p_i}{(2p_i - 1)} > \frac{p_i F(n)}{p_i F(n) - (1-p_i) F(n-1)}$$

thus the firm always prefer to use b^{both} rather than b^{ind}.
As a result we can conclude that

$$\left(b_i^{ind}, b_i^{team}, b_i^{both}\right) = \left(0, 0, \frac{p_i F(n)}{p_i F(n) - (1-p_i) F(n-1)}\right).$$

■

Proof of Theorem 1

Proof. Aiming to implement workers' efforts at the lowest possible cost, the firm offers to every $i \in IW$ an individual bonus of exactly $W_i^{ind} = \frac{c}{2p_i - 1}$. As a result, every $i \in IW$ exerts effort independently of the belief she holds about her coworkers' efforts.

Moreover, note that, by definition, the NW workers' incentives depend on how many coworkers they expect to exert effort. Such workers take for granted the effort of the workers in IW, but are exposed to strategic uncertainty when it comes to the choices of the other workers in NW. The effort choice of their NW colleagues also depend on their expectations on the team effort level as well. This logic clearly generates multiple Nash equilibria in the game. Following Bergemann and Morris (2009) we focus on equilibria that robustly implement work, which in our case would also coincide with both unique implementation of work and cheapest implementation of work in the firm's least preferred equilibrium. In this case the firm problem is

$$\min_{\boldsymbol{b}} \sum_{i \in N} \left(p_i b_i^{ind} + F(n) b_i^{team} + p_i F(n) b_i^{both} \right) \tag{2}$$

subject to:

$$\sum_{n_{-i}=0}^{n-1} \mu_i(n_{-i}) \left(\Delta^F\left(1+n_{-i}\right) b_i^{team} + \Delta^{p_i F}\left(1+n_{-i}\right) b_i^{both} \right) + (2p_i - 1)\, b_i^{ind} \geq c, \quad \forall i \in N, \mu_i \in \Gamma_i(\boldsymbol{b})$$

where $\Gamma_i(\boldsymbol{b})$ is the set of all beliefs of i that are consistent with rationalizability when the firms offers the bonus scheme $\boldsymbol{b}$.

Note that in order for all workers to work even under the most adversarial belief we need that

- there is no single agent $i \in NW$ for whom

$$\Delta^F(n)\, b_i^{team} + \Delta^{p_i F}(n)\, b_i^{both} + (2p_i - 1)\, b_i^{ind} < c$$

- there are no two agents $l, j \in NW$ for whom

$$\Delta^F\left(n-1\right) b_i^{team} + \Delta^{p_i F}\left(n-1\right) b_i^{both} + (2p_i - 1)\, b_i^{ind} < c$$

- there are no $s+1$ agents in NW for whom

$$\Delta^F\left(n-s\right) b_i^{team} + \Delta^{p_i F}\left(n-s\right) b_i^{both} + (2p_i - 1)\, b_i^{ind} < c.$$

Rewriting this condition we get that at least h agents in NW get bonuses such that

$$\Delta^F\left(|IW|+h\right) b_i^{team} + \Delta^{p_i F}\left(|IW|+h\right) b_i^{both} + (2p_i - 1)\, b_i^{ind} \geq c.$$

Furthermore, note that both $\Delta^{F}(x)$ and $\Delta^{p_iF}(x)$ are increasing in x for all s, and $p_i \geq \frac{1}{2}$. Thus, if

$$\Delta^{F}\Big(|IW|+h\Big)\, b_i^{team} + \Delta^{p_iF}\Big(|IW|+h\Big)\, b_i^{both} + (2p_i - 1)\, b_i^{ind} \geq c,$$

then

$$\Delta^{F}\Big(|IW|+h+1\Big)\, b_i^{team} + \Delta^{p_iF}\Big(|IW|+h+1\Big)\, b_i^{both} + (2p_i - 1)\, b_i^{ind} \geq c.$$

Aiming to minimize the expected payments, the firm's optimal incentive sceme that robustly implement teamwork must be such that

- no agent in NW obtains bonuses such that

  $$\Delta^{F}\Big(|IW|+1\Big)\, b_i^{team} + \Delta^{p_iF}\Big(|IW|+1\Big)\, b_i^{both} + (2p_i - 1)\, b_i^{ind} > c:$$

  indeed all workers would continue to work even if the firm lowers $\boldsymbol{b}_i$ until

  $$\Delta^{F}\Big(|IW|+1\Big)\, b_i^{team} + \Delta^{p_iF}\Big(|IW|+1\Big)\, b_i^{both} + (2p_i - 1)\, b_i^{ind} = c.$$

- no two agents in NW obtains bonuses such that

  $$\Delta^{F}\Big(|IW|+2\Big)\, b_i^{team} + \Delta^{p_iF}\Big(|IW|+2\Big)\, b_i^{both} + (2p_i - 1)\, b_i^{ind} > c:$$

  indeed all workers would continue to work even if the firm lowers the bonuses of one of the two, $\boldsymbol{b}_i$, until

  $$\Delta^{F}\Big(|IW|+2\Big)\, b_i^{team} + \Delta^{p_iF}\Big(|IW|+2\Big)\, b_i^{both} + (2p_i - 1)\, b_i^{ind} = c.$$

- no h agents in NW obtains bonuses such that

  $$\Delta^{F}\Big(|IW|+h\Big)\, b_i^{team} + \Delta^{p_iF}\Big(|IW|+h\Big)\, b_i^{both} + (2p_i - 1)\, b_i^{ind} > c:$$

  indeed all workers would continue to work even if the firm lowers the bonuses of one of them, $\boldsymbol{b}_i$, until

  $$\Delta^{F}\Big(|IW|+h\Big)\, b_i^{team} + \Delta^{p_iF}\Big(|IW|+h\Big)\, b_i^{both} + (2p_i - 1)\, b_i^{ind} = c.$$

Combined with the previous result, this argument shows that it is optimal to create a ranking among different agents in NW, $O : NW \to \{1...n - |IW|\}$, so that for all $i \in NW$

$$\Delta^{F}\Big(|IW|+i\Big)\, b_i^{team} + \Delta^{p_iF}\Big(|IW|+i\Big)\, b_i^{both} + (2p_i - 1)\, b_i^{ind} = c.$$

In this way we showed that, in order to induce all agents to work, the firm optimally creates a ranking among the workers and assign each $i \in NW$ a ranking $O(i)$ and thus a wage that makes him indifferent between working or not when she expects $|IW| + O(i) - 1$ of her colleagues working.

■

Proof of Propositions 2 and 3

Proposition 16 *For every vector of individual signals precisions* $(p_i)_{N\in I}$*, the optimal incentive mechanism* $\boldsymbol{b} = \left(b_i^{team}, b_i^{ind}, b_i^{both}\right)_{i\in N}$ *divides workers into two contractual categories* IW *and* $NW = N \setminus IW$ *such that*

- *if* $i \in IW$*, then* $(b_i^{team} = b_i^{both} = 0$ *and*

$$b_i^{ind} = \frac{c}{2p_i - 1}$$

- *if* $i \in NW$*, then* $(b_i^{team} = b_i^{ind} = 0$ *and*

$$b_i^{both} = \frac{c}{\Delta^{p_iF}\Big(|IW|+O(i)\Big)} \text{ for some ranking } O : NW \xrightarrow{1:1} 1, ..., |NW|$$

Moreover $NW \neq \emptyset$*; and also* $IW \neq \emptyset$ *for all* i*,* p_i *is such that*

$$\frac{p_i}{2p_i - 1} < \frac{p_i F_N}{p_i F(1) - (1 - p_i) F(0)}$$

Proof. As we noted, the firm finds optimal to offer all workers in IW

$$b_i^{ind} = \frac{c}{2p_i - 1} \text{ if } i \in IW.$$

As for the workers in NW, the optimal wage depends on the ranking O. For every fixed ranking O, the firm would solve

$$\min_{\boldsymbol{b_i}_{i\in NW}} \sum_i \left(b_i^{team}F(n) + b_i^{ind}p_i + p_iF(n)b_i^{both}\right)$$

$$sub$$

$$\Delta^F\Big(\big|IW\big|+O(i)\Big)\ b_i^{team} + \Delta^{p_iF}\Big(\big|IW\big|+O(i)\Big)\ b_i^{both} + (2p_i-1)\, b_i^{ind} \geq c \qquad \forall i \in NW\ .$$

Since

$$\frac{F(n)}{\Delta^F\Big(\big|IW\big|+O(i)\Big)} > \frac{p_iF(n)}{\Delta^{p_iF}\Big(\big|IW\big|+O(i)\Big)}$$

whenever S_i^{ind} is informative, i.e. $p_i > \frac{1}{2}$, then we conclude that $b_i^{team} = 0$: the firm would always prefer b_i^{both} to b_i^{team}. Moreover, if $i \in NW$ then it must be the case that[36]

$$\frac{p_iF(n)}{\Delta^{p_iF}\Big(\big|IW\big|+O(i)\Big)} \leq \frac{p_i}{2p_i-1}.$$

As for the second part of the proposition: suppose all $j \in N \setminus \{i\}$ are placed in IW then if $p_i < 1$,

$$\frac{p_iF(n)}{\Delta^{p_iF}(n)} = \frac{p_iF(n)}{p_iF_N - (1-p_i)F(n-1)} = \frac{p_i}{p_i - (1-p_i)\frac{F(n-1)}{F_N}} < \frac{p_i}{2p_i-1}$$

because $\frac{F(n-1)}{F_N} < 1$. Thus firm would save to place one worker in NW when the worker expects all hell colleagues to work (as in the firm-preferred equilibrium); thus $NW \neq \emptyset$. On the other hand suppose all workers are in NW and for all i

$$\frac{p_i}{2p_i-1} < \frac{p_iF_N}{p_iF(1)-(1-p_i)F(0)}$$

then surely the firm is better off by switching at least one worker to IW

■

Proof of Proposition 5

Proof. Suppose by contrapposition that $j \in IW$, $i \in NW$, but still $p_i > p_j$, i.e. $Z_i = \frac{1-p_i}{p_i} < \frac{1-p_j}{p_j} = Z_j$. Since $j \in IW$ then the firm must be saving by assigning j to IW

[36] Note that, if $O(i) > 1$ the inequality should hold strictly otherwise firm could save on coworkers by rewarding i only with b_i^{ind}, i.e. placing i in IW.

rather than to the highest rank of NW, i.e.

$$\frac{1}{1-Z_j} < \frac{F(n)}{F(|IW|) - Z_j F(|IW|-1)}$$

and thus

$$Z_j < \frac{F(n) - F(|IW|)}{(F(n) - F(|IW|-1))}. \tag{3}$$

Since $Z_i < Z_j$ then

$$Z_i < \frac{F(n) - F(|IW|)}{(F(n) - F(|IW|-1))}$$

i.e. the firm also prefers to place i in IW rather than in the highest rank of NW. If higher precision were to correspond to higher rank, we could use this observation to conclude that only the workers with the highest precision can be placed in IW. However it might be the case that higher precision worker optimally placed below in the ranking. Suppose that worker i is placed in position $r+1$ of the ranking in NW and denote $x = |IW| + r$. Then it must be the case that placing i there allows the firm to save with respect to placing i in IW. Since all other agents would just be better off (and thus the firm could save on the other workers by making this change), we conclude that

$$\frac{1}{1-Z_i} > \frac{F(n)}{F(x+1) - Z_i F(x)}$$

i.e.

$$Z_i > \frac{F(n) - F(x+1)}{(F(n) - F(x))}$$

and thus

$$Z_j > \frac{F(n) - F(x+1)}{(F(n) - F(x))}$$

Finally note that we work on the assumption that the firm is better off by not switching the workers i and j, (which would keep all the other indifferent). Then it must be the case that

$$\frac{1}{1-Z_j} + \frac{F(n)}{F(x+1) - Z_i F(x)} < \frac{1}{1-Z_i} + \frac{F(n)}{F(x+1) - Z_j F(x)}$$

Using $F_2 = x+1$, $F_1 = x$

$$\frac{1}{1-Z_j} - \frac{1}{1-Z_i} < \frac{F(n)}{F_2 - Z_j F_1} - \frac{F(n)}{F_2 - Z_i F_1}$$

or

$$\frac{1}{1-Z_j} - \frac{F(n)}{F_2 - Z_j F_1} < \frac{1}{1-Z_i} - \frac{F(n)}{F_2 - Z_i F_1}$$

Thus $\frac{1}{1-Z}-\frac{F(n)}{F_2-ZF_1}$ needs to be decreasing in Z for some Z, i.e.

$$\frac{\partial\left(\frac{1}{1-Z}-\frac{F(n)}{F_2-ZF_1}\right)}{\partial Z}=\frac{\left(Z^2F_1^2-F(n)Z^2F_1-2ZF_1F_2+2F(n)ZF_1-F(n)F_1+F_2^2\right)}{\left(ZF_1-F_2\right)^2(1-Z)^2}<0.$$

This is true if and only if

$$\left(\frac{F_2-F_1Z}{1-Z}\right)^2<F(n)F_1.$$

Note that since $\left(\frac{F_2-F_1Z}{1-Z}\right)^2$ is increasing in Z,[37] we conclude that

$$\left(\frac{F_2-F_1Z}{1-Z}\right)^2<F(n)F_1$$

can be true only if it is true at $Z=0$ (i.e. $p=1$), namely if

$$\left(F_2\right)^2<F(n)F_1.$$

So we assume that $\left(F_2\right)^2<F(n)F_1$.
Note that we can rewrite

$$\left(\frac{F_2-F_1Z}{1-Z}\right)^2<F(n)F_1$$

as

$$\frac{F_2-F_1Z}{1-Z}<\sqrt{F(n)F_1}$$

and thus

$$F_2-\sqrt{F(n)F_1}<Z\left(F_1-\sqrt{F(n)F_1}\right)$$

i.e.

$$\sqrt{F(n)F_1}-F_2>Z\left(\sqrt{F(n)F_1}-F_1\right)$$

Since $\left(F_2\right)^2<F(n)F_1$ and thus $\sqrt{F(n)F_1}-F_1>\sqrt{F(n)F_1}-F_2>0$ then i.e.

$$Z<\frac{\sqrt{F(n)F_1}-F_2}{\left(\sqrt{F(n)F_1}-F_1\right)}$$

[37] Indeed note that

$$\frac{\partial\left(\left(\frac{F_2-F_1Z}{1-Z}\right)^2\right)}{\partial Z}=2\frac{F_2-F_1}{(1-Z)^3}\left(F_2-F_1Z\right)>0.$$

which is possible.

To recap:
Suppose $p_i > p_j$ (i.e. $Z_i < Z_j$) and $j \in IW$, $i \in NW$. Then it must be the case that

1. Z_i, Z_j are such that

$$\frac{F(n) - F(x+1)}{(F(n) - F(x))} < Z_i < Z_j < \frac{F(n) - F(|IW|)}{(F(n) - F(|IW| - 1))}$$

2. F is such that

$$\sqrt{F(n)F(x)} - F(x+1) > 0$$

3. there exist $Z \in [Z_i, Z_j]$ such that

$$Z < \frac{\sqrt{F(n)F(x)} - F(x+1)}{\left(\sqrt{F(n)F(x)} - F(x)\right)}$$

However it is impossible that

$$\frac{F(n) - F(x+1)}{(F(n) - F(x))} < \frac{\sqrt{F(n)F(x)} - F(x+1)}{\left(\sqrt{F(n)F(x)} - F(x)\right)}$$

indeed

$$\frac{\sqrt{F(n)F(x)} - F(x+1)}{\left(\sqrt{F(n)F(x)} - F(x)\right)} - \frac{F(n) - F(x+1)}{F(n) - F(x)} = -\frac{(F(x+1) - F(x))}{F(x)\,(F(n) - F(x))}\sqrt{F(n)F(x)} < 0$$

Thus there is no Z such that

$$\frac{F(n) - F(x+1)}{(F(n) - F(x))} < Z < \frac{\sqrt{F(n)F(x)} - F(x+1)}{\left(\sqrt{F(n)F(x)} - F(x)\right)}$$

and we can conclude that it must be the case that if $j \in IW$ and $p_i > p_j$, then $i \in IW$. As a result, we can conclude that

$$IW = \{i \in I : p_i \geq p^{**}\},$$

for some p^{**}. Note that, if $j \in IW$ then inequality (3) must be satisfied. As a result, we obtain that if $|IW| \geq 1$,

$$p^{**} \geq \frac{F(n) - F(|IW| - 1)}{2F(n) - F(|IW| - 1) - F(|IW|)}$$

and thus

$$IW \subseteq \left\{ i \in I : p_i \geq \frac{F(n) - F(|IW| - 1)}{2F(n) - F(|IW| - 1) - F(|IW|)} \right\}.$$

Furthermore note that

$$\frac{\partial \frac{F(n)-F(x-1)}{2F(n)-F(x-1)-F(x)}}{\partial x} > 0$$

if and only if

$$xF(x)(F(n) - F(x-1)) - (x-1)F(x)(F(n) - F(x)) > 0,$$

which is always the case. Then $\frac{F(n)-F(|IW|-1)}{2F(n)-F(|IW|-1)-F(|IW|)}$ is increasing in $|IW|$.

■

Proof of Proposition 6

Proof. We start from proving the first bullet point. Consider two workers $i, j \in NW$, $p_i > p_j$ assigned to two consecutive places in the ranking. We want to show that assigning higher ranking to i would allow the firm to save on the expected wages

$$\frac{p_i F(n)}{F_1 p_i - F_0(1 - p_i)} + \frac{p_j F(n)}{F_2 p_j - F_1(1 - p_j)} < \frac{p_j F(n)}{F_1 p_j - F_0(1 - p_j)} + \frac{p_i F(n)}{F_2 p_i - F_1(1 - p_i)},$$

where $F_0 = F(x)$, $F_1 = F(x+1)$, $F_2 = F(x+2)$. Note that all the other workers would not be impacted from any change in the order between i and j.

Note that

$$\frac{p_i F(n)}{F_1 p_i - F_0(1 - p_i)} + \frac{p_j F(n)}{F_2 p_j - F_1(1 - p_j)} < \frac{p_j F(n)}{F_1 p_j - F_0(1 - p_j)} + \frac{p_i F(n)}{F_2 p_i - F_1(1 - p_i)}$$

for all $p_i > p_j$ if and only if

$$\frac{F(n)}{F_1 - F_0 \frac{(1-p_i)}{p_i}} - \frac{F(n)}{F_2 - F_1 \frac{(1-p_i)}{p_i}} < \frac{F(n)}{F_1 - F_0 \frac{(1-p_j)}{p_j}} - \frac{F(n)}{F_2 - F_1 \frac{(1-p_j)}{p_j}}$$

i.e. if and only if

$$\frac{1}{F_1 - F_0 \frac{(1-p_i)}{p_i}} - \frac{1}{F_2 - F_1 \frac{(1-p_i)}{p_i}}$$

decreasing in p_i. In other words, ranking i on top of j when $p_i > p_j$ is optimal if and only if. $\frac{1}{F_1 - F_0 Y} - \frac{1}{F_2 - F_1 Y}$ is increasing in Y, (indeed $\frac{\partial\left(\frac{1-p}{p}\right)}{\partial p} = -\frac{1}{p^2} < 0$). Note that

$$\frac{\partial\left(\frac{1}{F_1 - F_0 Y} - \frac{1}{F_2 - F_1 Y}\right)}{\partial Y} = \frac{F_0}{(F_1 - F_0 Y)^2} - \frac{F_1}{(F_2 - F_1 Y)^2} \geq 0$$

if and only if

$$F_0 (F_2 - F_1 Y)^2 - F_1 (F_1 - F_0 Y)^2 \geq 0.$$

Since

$$\frac{\partial\left(F_0 (F_2 - F_1 Y)^2 - F_1 (F_1 - F_0 Y)^2\right)}{\partial Y} = 2F_0 F_1 \left((F_1 - F_0) Y - (F_2 - F_1)\right) < 0$$

because F is convex and $Y \in (0, 1)$,[38] then

$$F_0 (F_2 - F_1 Y)^2 - F_1 (F_1 - F_0 Y)^2 \geq F_0 (F_2 - F_1)^2 - F_1 (F_1 - F_0)^2 .$$

Finally note that, since $\frac{(F(x+1) - F(x))^2}{F(x)}$ is assumed to be increasing in x, then

$$\frac{(F(x+1) - F(x))^2}{F(x)} < \frac{(F(x+2) - F(x+1))^2}{F(x+1)}$$

and thus

$$F(x+1) (F(x+1) - F(x))^2 < F(x) (F(x+2) - F(x+1))^2$$

i.e.

$$F_1 (F_1 - F_0)^2 < F_0 (F_2 - F_1)^2$$

Thus we get

$$F_0 (F_2 - F_1 Y)^2 - F_1 (F_1 - F_0 Y)^2 \geq F_0 (F_2 - F_1)^2 - F_1 (F_1 - F_0)^2 \geq 0.$$

which implies

$$\frac{\partial\left(\frac{1}{F_1 - F_0 Y} - \frac{1}{F_2 - F_1 Y}\right)}{\partial Y} > 0,$$

[38]Note indeed $\frac{1-p}{p} \in (0, 1)$ as $p \in \left(\frac{1}{2}, 1\right)$

showing that, in the optimum incentive scheme, if i and j are in consecutive spots of the ranking and $p_i > p_j$ then i is ranked above j. Iterating this argument we prove the first bullet point.
As a result, since the workers with higher precision in NW would be ranked higher, having a signal precision p_i such that

$$\frac{p_i F(n)}{F\left(|IW|\right)p_i - F\left(|IW|-1\right)(1-p_i)} - \frac{p_i}{2p_i-1} \geq 0$$

would not only be necessary but also sufficient for i being included in IW; thus

$$IW = \left\{ i \in I : \frac{p_i F(n)}{p_i F\left(|IW|\right) - (1-p_i)\, F\left(|IW|-1\right)} - \frac{p_i}{2p_i-1} \geq 0 \right\}.$$

Finally, since the above characterization would hold for all possible $|IW|$, we exploit the fact that in equilibrium there cannot be an i such that $\frac{F(n)}{F(|IW|)-F(|IW|-1)\frac{(1-p_i)}{p_i}} - \frac{p_i}{2p_i-1} > 0$ and still $i \in NW$, to conclude that

$$|IW| = \max_{s\in\mathbb{N}^+} \left\{ s \in \mathbb{N}^+ : \left| \left\{ i \in \mathbb{N} : \frac{F(n)}{p_i F\left(s\right) - (1-p_i)\, F\left(s-1\right)} - \frac{1}{2p_i-1} \geq 0 \right\} \right| \geq s \right\}, \text{ or 0 if empty.}$$

Note that

$$\frac{p_i F(n)}{p_i F\left(|IW|\right) - (1-p_i)\, F\left(|IW|-1\right)} - \frac{p_i}{2p_i-1} \geq 0$$

if and only if

$$p_i\left(F(n) - F\left(|IW|\right)\right) - (1-p_i)\left(F(n) - F\left(|IW|-1\right)\right) \geq 0.$$

if and only if

$$\frac{p_i}{1-p_i} \geq \frac{\left(F(n) - F\left(|IW|-1\right)\right)}{\left(F(n) - F\left(|IW|\right)\right)}$$

Since the left hand side is increasing in p_i we conclude that if $i \in IW$ and $p_j > p_i$ then $j \in IW$. Thus

$$IW = \left\{ s \in \mathbb{N}^+ : p_s \geq p^* \right\},$$

where p^* is the lowest $p \in \left(\frac{1}{2}, 1\right)$ such that

$$\frac{p}{1-p} \geq \frac{F(n) - F\left(n\right)}{F(n) - F\left(n+1\right)},$$

with $n_{p^*} = |\{s \in \mathbb{N}^+ : p_s > p^*\}|$. In case $|\{s \in \mathbb{N}^+ : p_s = p^*\}| > 1$ then

$$IW = \left\{s \in \mathbb{N}^+ : p_s > p^*\right\} \cup T$$

where $T \subset \{s \in \mathbb{N}^+ : p_s = p^*\}$ and $|T|$ such that $\frac{p}{1-p} \leq \frac{F(n)-F(n+|T|-1)}{F(n)-F(n+|T|)}$ and $\frac{p}{1-p} > \frac{F(n)-F(n+|T|)}{F(n)-F(n+|T|+1)}$.
Rewriting

$$\frac{p}{1-p} \geq \frac{F(n) - F_n}{F(n) - F_{n+1}}$$

we get that p^* is the minimum p such that

$$p \geq \frac{F(n) - F_n}{(2F(n) - F_{n+1} - F_n)}.$$

Note that $\frac{F(n)-F_n}{(2F(n)-F_{n+1}-F_n)}$ is decreasing in p. Indeed n decreasing in p and

$$\frac{F(n) - F_n}{(2F(n) - F_{n+1} - F_n)} > \frac{F(n) - F_{n'}}{(2F(n) - F_{n'+1} - F_{n'})}$$

with $p' > p$. Indeed

$$\frac{F(n) - F_1}{(2F(n) - F_2 - F_1)} > \frac{F(n) - F_0}{(2F(n) - F_1 - F_0)}$$

because

$$(F(n) - F_1)(2F(n) - F_1 - F_0) - (F(n) - F_0)(2F(n) - F_2 - F_1) > 0$$

$$F_1^2 - F_0F_2 + F_0F(n) + F_2F(n) - 2F(n)F_1 > 0$$

Note that the LHS is increasing in F_N and $F_N > F_2$, then

$$\begin{aligned} F_1^2 - 2F(n)F_1 - F_0F_2 + F_0F(n) + F_2F(n) > \\ F_1^2 - 2F_2F_1 - F_0F_2 + F_0F_2 + F_2F_2 = \\ (F_1 - F_2)^2 > 0 \end{aligned}$$

Uniqueness follows from $\frac{F(n)-F_n}{(2F(n)-F_{n+1}-F_n)}$ being decreasing in p; indeed as we consider lower p_i the threshold $\frac{F(n)-F_n}{(2F(n)-F_{n+1}-F_n)}$ increases; paired with

$$IW = \left\{s \in \mathbb{N}^+ : p_s \geq \frac{F(n) - F(n_{p_s})}{(2F(n) - F(n_{p_s} + 1) - F(n_{p_s}))}\right\}$$

this finding implies uniqueness of IW.

We now move to show the other direction of the proposition.

First we show that if there exist x such that $\frac{(F(x+1)-F(x))^2}{F(x)} > \frac{(F(x+2)-F(x+1))^2}{F(x+1)}$, then there is a precision vector $\mathbf{p}$ such that $i,j \in NW$, $p_i > p_j$ and still j is optimally ranked higher than i, i.e. $O(i) > O(j)$.

Consider two workers $i,j \in NW$, $p_i > p_j$ assigned to two consecutive places in the ranking. We want to show that it is possible that assigning higher ranking to j would allow the firm to save on the expected wages, namely

$$\frac{p_i F(n)}{F_1 p_i - F_0(1-p_i)} + \frac{p_j F(n)}{F_2 p_j - F_1(1-p_j)} > \frac{p_j F(n)}{F_1 p_j - F_0(1-p_j)} + \frac{p_i F(n)}{F_2 p_i - F_1(1-p_i)},$$

where $F_0 = F(x)$, $F_1 = F(x+1)$, $F_2 = F(x+2)$. Note that all the other workers would not be impacted from any change in the order between i and j.

Note that

$$\frac{p_i F(n)}{F_1 p_i - F_0(1-p_i)} + \frac{p_j F(n)}{F_2 p_j - F_1(1-p_j)} > \frac{p_j F(n)}{F_1 p_j - F_0(1-p_j)} + \frac{p_i F(n)}{F_2 p_i - F_1(1-p_i)}$$

for some $p_i > p_j$ if and only if

$$\frac{p_i F(n)}{F_1 p_i - F_0(1-p_i)} - \frac{p_i F(n)}{F_2 p_i - F_1(1-p_i)} > \frac{p_j F(n)}{F_1 p_j - F_0(1-p_j)} - \frac{p_j F(n)}{F_2 p_j - F_1(1-p_j)}$$

for some $p_i > p_j$

i.e. if and only if

$$\frac{1}{F_1 - F_0\frac{(1-p_i)}{p_i}} - \frac{1}{F_2 - F_1\frac{(1-p_i)}{p_i}}$$

increasing in p_i for some p_i, i.e. if and only if $\frac{1}{F_1 - F_0 Y} - \frac{1}{F_2 - F_1 Y}$ decreasing in Y for some Y (indeed $\frac{\partial\left(\frac{1-p}{p}\right)}{\partial p} = -\frac{1}{p^2} < 0$). Note that

$$\frac{\partial\left(\frac{1}{F_1 - F_0 Y} - \frac{1}{F_2 - F_1 Y}\right)}{\partial Y} = \frac{F_0}{(F_1 - F_0 Y)^2} - \frac{F_1}{(F_2 - F_1 Y)^2} < 0$$

if and only if

$$F_0(F_2 - F_1 Y)^2 - F_1(F_1 - F_0 Y)^2 < 0.$$

Consider $p_i = \frac{1}{2}$ and thus $Y = 1$, then

$$F_0(F_2 - F_1 Y)^2 - F_1(F_1 - F_0 Y)^2 = F_0(F_2 - F_1)^2 - F_1(F_1 - F_0)^2$$

which is indeed lower than 0 if

$$\frac{\left(F_2 - F_1\right)^2}{F_1} < \frac{\left(F_1 - F_0\right)^2}{F_0}$$

i.e.

$$\frac{\left(F\left(x+2\right) - F\left(x+1\right)\right)^2}{F\left(x+1\right)} < \frac{\left(F\left(x+1\right) - F\left(x\right)\right)^2}{F\left(x\right)}$$

which is the case for some x by assumption. This implies that at $p_i = \frac{1}{2}$ and thus $Y = 1$,

$$\frac{\partial\left(\frac{1}{F_1 - F_0 Y} - \frac{1}{F_2 - F_1 Y}\right)}{\partial Y} < 0$$

and thus that

$$\frac{p_i F(n)}{F_1 p_i - F_0\left(1 - p_i\right)} + \frac{p_j F(n)}{F_2 p_j - F_1\left(1 - p_j\right)} > \frac{p_j F(n)}{F_1 p_j - F_0\left(1 - p_j\right)} + \frac{p_i F(n)}{F_2 p_i - F_1\left(1 - p_i\right)}$$

for $p_j = \frac{1}{2}$, $p_i = \frac{1}{2} + \varepsilon$ with $\varepsilon > 0$ sufficiently small. So j is placed above i if they optimally placed in $x+1$ and $x+2$ where $\frac{(F(x+1)-F(x))^2}{F(x)}$ decreasing, i.e.

$\frac{(F(x+2)-F(x+1))^2}{F(x+1)} < \frac{(F(x+1)-F(x))^2}{F(x)}$. Furthermore note that if p_i sufficiently close to $\frac{1}{2}$ then i is in NW. To see this note that $\frac{p_i}{2p_i - 1}$ goes to infinity when p_i goes to $\frac{1}{2}$. Thus the firm can surely do better by placing i and j in NW. Finally, since there can be as many p_s close to $\frac{1}{2}$ as we want such that $s \in NW$, this shows that $\frac{(F(x+2)-F(x+1))^2}{F(x+1)} < \frac{(F(x+1)-F(x))^2}{F(x)}$ for some x implies that there exist precision vectors such that some workers with lower precision agent are optimally ranked higher.

■

Proof of Proposition 7

Proof.

The firm optimally minimizes

$$min_{b_i^{team}, b_i^{ind}} \sum_i \left(b_i^{team} F(n) + b_i^{ind} p_i\right)$$

sub

$$b_i^{team}\left(E_{\mu_i}\left(F\left(n+1\right)\right) - E_{\mu_i}\left(F\left(n\right)\right)\right) + b_i^{ind}\left(2p_i - 1\right) \geq c \text{ for all } i,$$

for all μ_i consistent with rationality and common belief in rationality.

To show that it is optimal to set up a ranking system we proceed as follows:
There cannot be a single agent for which

$$b_i^{team}\left(F\left(N\right)-F\left(N-1\right)\right)+b_i^{ind}\left(2p_i-1\right)<c$$

there cannot be a two agents i,j for which

$$b_i^{team}\left(F\left(N-1\right)-F\left(N-2\right)\right)+b_i^{ind}\left(2p_i-1\right)<c$$

there cannot be a s agents for which

$$b_i^{team}\left(F\left(N-s+1\right)-F\left(N-s\right)\right)+b_i^{ind}\left(2p_i-1\right)<c$$

Conversely, there must be 1 agent for which

$$b_i^{team}\left(F\left(1\right)-F\left(0\right)\right)+b_i^{ind}\left(2p_i-1\right)=c$$

there must be 1 agent for which

$$b_i^{team}\left(F\left(2\right)-F\left(1\right)\right)+b_i^{ind}\left(2p_i-1\right)=c$$

for all i,

$$b_i^{team}\left(F\left(i\right)-F\left(i-1\right)\right)+b_i^{ind}\left(2p_i-1\right)=c.$$

This create an order on workers. Finally, assume an order on $N:O:N\xrightarrow{1:1}1,...,n$. Then for all i, given the optimal $O(i)$ the firm solves

$$\begin{aligned}\min_{b_i^{team},b_i^{ind}} b_i^{team}F(n)+b_i^{ind}p_i \\ \text{sub} \\ b_i^{team}\left(F\left(O(i)\right)-F\left(O(i)-1\right)\right)+b_i^{ind}\left(2p_i-1\right) &= c\end{aligned}$$

$b_i^{team}=\frac{c}{(F(O(i))-F(O(i)-1))}$ and $b_i^{ind}=0$ if and only if $\frac{F(n)}{(F(O(i))-F(i-1))}<\frac{p_i}{(2p_i-1)}$ and

$b_i^{ind} = \frac{c}{(2p_i - 1)}, b_i^{team} = 0$ if and only if $\frac{F(n)}{(F(O(i)) - F(O(i)-1))} > \frac{p_i}{(2p_i - 1)}$ (abstracting away from ties). Finally, if i in IW and $p_j > p_i$, then $j \in IW$. Indeed note that ranking among NW is totally arbitrary in this case. ■

Proof of Proposition 8

Proof. We start by considering the case in which $p_i \neq p_j$ for all $i, j \in N$. As a first step, it is useful to prove the following simple lemma.

Lemma 2 *If $i \in IW$ and $p_j > p_i$ then $j \in IW$.*

Proof. *Suppose j is in NW and i in IW. Switching them is strictly beneficial for the firm as*

$$\frac{p_i}{2p_i - 1} + \frac{F_N}{F(x+1) - F(x)} > \frac{p_j}{2p_j - 1} + \frac{F_N}{F(x+1) - F(x)}$$

. ■

As a result, we can reformulate the firm problem as follows:

$$\min_{p^* in (0,1)} \left(\sum_{i: p_i > p^*} p_i \frac{c}{2p_i - 1} + \sum_{i: p_i \leq p^*} (\frac{F(n)}{(F(O(i)) - F(O(i) - 1))} \right),$$

where O is an arbitrary ranking among workers with $p_i > p^*$.

We can then show the following lemma.

Lemma 3 *Worker $i \in IW$ if and only if*

$$p_i \frac{c}{2p_i - 1} < F(n) \frac{c}{F(n_{p_i} + 1) - F(n_{p_i})},$$

i.e.

$$p_i > \frac{1}{\left(2 - \frac{\left(F\left(n_{p_i} + 1\right) - F\left(n_{p_i}\right)\right)}{F(n)}\right)},$$

where $n_{p_i} = |\{j : p_j > p_i\}|$

Proof. *Any worker in IW could be assigned in NW and assigned the highest rank, keeping fixed the wage of the other in NW as well as the one in IW. Thus $i \in IW$ implies*

$$p_i \frac{c}{2p_i - 1} < F(n) \frac{c}{F(n_{p_i} + 1) - F(n_{p_i})}.$$

If $i \in NW$ and $p_i \in \max_{j \in NW} p_j$ then it must be that

$$p_i \frac{c}{2p_i - 1} > F(n) \frac{c}{F(n_{p_i} + 1) - F(n_{p_i})}$$

otherwise assigning i to IW would allow the firm decrease her wage as well as that of the other workers in NW, keeping unchanged the ones in IW. ■

Thus $i \in IW$ if and only if

$$p_i \frac{c}{2p_i - 1} < F(n) \frac{c}{F(n_{p_i} + 1) - F(n_{p_i})}. \tag{4}$$

Note that the LHS is increasing in p_i: indeed $p_i \frac{c}{2p_i - 1} > p_s \frac{c}{2p_s - 1}$ if $p_s > p_i$. On the other hand, the RHS is decreasing with p_i as

$$F(n) \frac{c}{F(n_{p_i} + 1) - F(n_{p_i})} > F(n) \frac{c}{F(n_{p_s} + 1) - F(n_{p_s})}$$

given that F is convex.
Finally note that in case we allow for ties, i.e. if we allow $p_i = p_j$ for some $i, j \in N$. In this case, the same reasoning goes through, with the only caveat that it is possible that despite $p_i = p_j$, i is placed in IW and j in NW. As a result, only the cardinalities (but not necessarily the identities) of IW and NW are uniquely determined. Indeed the right hand side of (4) is increasing in n_{p_i}. ■

Proof of Proposition 9

Proof. As a first step, note that, if p^* is optimal for the firm then, every component p_i^* needs to be optimal for the equilibrium contractual position assigned to i. If $i \in IW$, then the firm minimizes

$$\min_{p_I} \left(\frac{p_i}{2p_i - 1} \right) c + k(p_i).$$

Since $k'\left(\frac{1}{2}\right) = 0, k'(1) = \infty$ and k convex, then the solution is internal and we can take the first order condition:

$$\frac{1}{(2p_i - 1)^2} = k'(p_i)$$

If instead $i \in NW$, then the firm minimizes

$$\min_{p_i} \frac{p_i F(n)}{p_i F_1 - F_0 (1 - p_i)} + k(p_i).$$

From the first order condition we get

$$F_0 \frac{F(n)}{(p_i F_1 - F_0(1-p_i))^2} c = k'(p_i)$$

Note that

$$\frac{\partial \left(F_0 \frac{F(n)}{(p_i F_1 - F_0(1-p_i))^2} \right)}{\partial p_i} = -2F_0 F(n) \frac{F_0 + F_1}{(F_0 p_i - F_0 + F_1 p_i)^3} < 0$$

thus the left hand side is decreasing in p_i and the right hand side is increasing in p_i, thus the solution is unique.
Further note that

$$F_0 \frac{F(n)}{(p_i F_1 - F_0(1-p_i))^2}$$

is increasing in the ranking (racall that rank 1 is higher than rank 2) if

$$F_0 \frac{F(n)}{(p_i F_1 - F_0(1-p_i))^2} - F_1 \frac{F(n)}{(p_i F_2 - (1-p_i) F_1)^2} > 0$$

which holds if and only if

$$F_0 (p_i F_2 - (1-p_i) F_1)^2 - F_1 (p_i F_1 - F_0(1-p_i))^2 > 0$$

i.e.,

$$F_0 \left(F_2 - F_1 + \frac{(2p_i - 1)}{p_i} F_1 \right)^2 - F_1 \left(F_1 - F_0 + \frac{(2p_i - 1)}{p_i} F_0 \right)^2 > 0.$$

Rearranging we get

$$\begin{aligned}
&(F_2 - F_1)^2 F_0 - (F_1 - F_0)^2 F_1 \\
&+ \left(F_1 \frac{(2p_i - 1)}{p_i} \right)^2 F_0 - \left(F_0 \frac{(2p_i - 1)}{p_i} \right)^2 F_1 \\
&+ 2(F_2 - F_1) F_0 F_1 \frac{(2p_i - 1)}{p_i} - 2(F_1 - F_0) F_0 \frac{(2p_i - 1)}{p_i} F_1 \\
&> 0
\end{aligned}$$

which holds when strategic complementarities are high enough. In particular, recall that by assumption $(F_2 - F_1)^2 F_0 > (F_1 - F_0)^2 F_1$ so

$$\begin{aligned}
&(F_2 - F_1)^2 F_0 - (F_1 - F_0)^2 F_1 && (> 0 \text{ by assumption}) \\
&+ \frac{(2p_i - 1)}{p_i} \left((F_1)^2 F_0 - (F_0)^2 F_1\right) && (> 0 \text{ because } F_1 > F_0) \\
&+ 2F_0F_1 \frac{(2p_i - 1)}{p_i} (F_0 - 2F_1 + F_2) && (> 0 \text{ by effort complementarities}) \\
&> 0
\end{aligned}$$

Thus, higher ranked workers would be monitored more closely.

Moreover, we show that the firm optimally monitors more closely the workers in IW than those in NW. Suppose there are two workers $i \in NW$ and $j \in IW$ with $p_i = p_j$. Note that if $i \in NW$ then

$$\frac{p_j}{(2p_j - 1)} = \frac{p_i}{(2p_i - 1)} > \frac{p_i F(n)}{(p_i F(D_i + 1) - (1 - p_i) F(D_i))},$$

otherwhise the firm would optimally switch i to IW. At this level of signal precision, the firm would have higher incentive to invest in p_i if i were in IW rather than NW. Indeed

$$\frac{1}{(2p_j - 1)} > \frac{F(n)}{(p_i F(D_i + 1) - (1 - p_i) F(D_i))}$$

and thus

$$\begin{aligned}
\frac{1}{(2p_j - 1)^2} &> \frac{F(n)^2}{(p_i F(D_i + 1) - (1 - p_i) F(D_i))^2} \\
&> F_0 \frac{F(n)}{(p_i F(D_i + 1) - (1 - p_i) F(D_i))^2}.
\end{aligned}$$

However, the optimum precision for $j \in IW$ we have $\frac{1}{\left(2p_j^* - 1\right)^2} = k'\left(p_j^*\right)$ and for NW we have $F_0 \frac{F(n)}{\left(p_i^* F(D_i+1) - \left(1 - p_i^*\right) F(D_i)\right)^2} = k'(p_i^*)$, so it cannot be the case that $p_i^* = p_j^*$.

More precisely, as $\frac{1}{\left(2p_j^* - 1\right)^2} - k'\left(p_j^*\right)$ is decreasing in p_j, we get $p_j^* > p_i^*$ and thus that the firm acquires more precise signals about workers in IW than in NW.

Finally, we want to show that NW and IW are non-empty. First we show that

$$\frac{p_j}{(2p_j - 1)} > \frac{p_j F(n)}{(p_j F(n) - (1 - p_j) F(n - 1))}$$

and thus NW is non-empty. Indeed note that the previous inequality could can be rewritten as

$$2p_j > \frac{F(n-1)}{F(n)}$$

which is always true because $p_j \geq \frac{1}{2}$.
As for IW, it could be the case that IW is empty. For example, if the monitoring cost is very low, e.g., $k(1) < \varepsilon$ then assigning a worker to IW, while optimally investing in monitoring her
would cost

$$\frac{p_j^*}{\left(2p_j^* - 1\right)} + k\left(p_j^*\right) < 1 + \varepsilon.$$

On the other hand, assigning even the last worker to NW would cost

$$\frac{p_j^{NW} F(n)}{\left(p_j^{NW} F(1) - \left(1 - p_j^{NW}\right) F(0)\right)} + k\left(p_j^{NW}\right) > \frac{F(n)}{F(1)} > 1.$$

So for $\varepsilon < \frac{F(n)}{F(1)} - 1$ the firm certainly benefits by assigning at least one worker to IW. ■

Proof of Theorem 2

Proof. First, note that workers in IW gain more when the precision of their individual signals is lower:

$$\frac{p_i}{2p_i - 1}c, \text{ is decreasing in } p_i.$$

As for workers in NW the argument is not that straightforward. On one side for every given rank , lower precision would benefit the worker, as

$$\frac{F(n)p_i}{F(x)\,p_i - F(x-1)\,(1-p_i)}c = \frac{F(n)}{F(x) - F(x-1)\,\frac{(1-p_i)}{p_i}}c$$

is decreasing in p_i. However, as workers with higher precision would be ranked higher and thus get an higher salary with respect to what they could get at a lower rank. From these observation we can conclude that:

- In equilibrium all workers get the same payoff: suppose i get higher expected payof than j, by selecting the possible degenerate mixed strategy π_i over the possible signal precision $\pi_i \in \Delta\left[\frac{1}{2}, 1\right]$. Indeed if this were the case j would strictly benefit from deviating to π_j' such that $\pi_j'\left(\max_{p_i \in Supp(\pi_i)} p_i + \varepsilon\right) = 1$ with $\varepsilon \to 0^+$: by doing so it would obtain almost the same payoff as i if $\max_{p_i \in Supp(\pi_i)} p_i$ used to lead to IW or the highest rank in NW, and strictly higher payoff otherwise.

- In equilibrium no worker assign positive probability mass to a precision level that lead to a lower rank (not the first) of NW with positive probability. Indeed if $\Pr_{\pi_i}(p_i) > 0$ and $\Pr(i \in NW, O(i) > 1)$ then j could guarantee herself higher payoff by selecting π_j such that $\Pr_{\pi_j}(p_i + \varepsilon) = 1$, for $\varepsilon > 0$. Combined with the previous bullet point, this would contraddict that p is part of an equilibrium to start with.

- There exists i such that $\frac{1}{2} \in Supp(\pi_i)$. Suppose $\min_i (\min_{p_i} Supp(\pi_i)) = x > \frac{1}{2}$ and no other π_j has athom then i would benefit from switching to $p_i = \frac{1}{2}$: i is still made indifferent between working and shirking when everybody else work, so lower precision corresponds to higher salary. Note that one worker needs be assigned to NW. Assume some workers has athom in $\min_i (\min_{p_i} Supp(\pi_i)) = x > \frac{1}{2}$ then i would have either the incentive to switch to $p_i = \min_i (\min_{p_i} Supp(\pi_i)) + \varepsilon$, $\varepsilon > 0$ or to $p_i = \frac{1}{2}$. Indeed if it is not better off with $p_i = \frac{1}{2}$, it would necessary be the case that being ranked higher, which can be obtained with probability 1 in case $p_i = \min_i (\min_{p_i} Supp(\pi_i)) + \varepsilon$ is selected, is strictly better than selecting $p_i = \min_i (\min_{p_i} Supp(\pi_i))$ and being ranked lower.

■

Proof of Lemma 1

Proof. First of all, note that $\overset{N\nrightarrow I}{P}(\boldsymbol{p})$ only depends on $|IW(\boldsymbol{p})|$; so I will use the following notation: $\bar{P}(|IW(\boldsymbol{p})| + 1) := \overset{N\nrightarrow I}{P}(\boldsymbol{p})$.
As a first step, note that the expected wages in NW, $\frac{F_n c}{F(|IW|+O(i)) - F(|IW|+O(i)-1)}$ are decreasing in x and thus increasing in the ranking. As a result, the highest rank in NW (i.e. the one with $O(i) = 1$) gains, in expectation, strictly more than all other workers in NW. In other words, if $i \in NW$ and $O(i) > 1$ then i gains, in expectation, less than

$$F(n)\frac{c}{F(|IW| + 1) - F(|IW|)} = \bar{P}(|IW| + 1)\frac{c}{2\bar{P}(|IW| + 1)}.$$

By increasing p_i slightly above $\bar{P}(|IW| + 1)$, the worker would guarantee herself to be placed in IW and thus obtain a payoff arbitrarily close to the one that the highest rank is gaining under the original **p**. Indeed, from the firm's perspective, the ranks within NW are interchangeable; thus the firm would switch one worker to IW whenever the expected wage needed to incentivize such worker is lower in IW than in the highest rank in NW. ■

Proof of Lemma 10

Proof. As a first step, note that the expected wage of worker $i \in IW$, i.e. $\frac{p_i c}{2p_i+1}$ is decreasing in p_i. As a result, conditioning on staying in IW, every worker $i \in IW$ has incentive to decrease her precision level as much as possible. However, decreasing her wage below $\max\left\{\bar{P}\left(|IW|\right), \max_{j\in NW}\left\{p_j\right\}\right\}$ would induce the firm to switch her to NW. Indeed if p_i decreases below $p_s = \max_{j\in NW}\left\{p_j\right\}$, then the firm would optimally switch i and j. Moreover independently of p_s, if p_i decreases below $\bar{P}\left(|IW|\right)$, then the firm would include i in NW. To see why this is the case note that the firm could incentivize all the other agents with the same bonus schedule and just switch i from IW to the highest rank in NW, that would now obtain a payoff of

$$F(n)\frac{c}{F\left(|IW|\right) - F\left(|IW| - 1\right)} = \bar{P}\left(|IW|\right)\frac{c}{2\bar{P}\left(|IW|\right)}.$$

as the cardinality of IW decreases by one. This change would be beneficial for the firm whenever p_i decreases below $\bar{P}\left(|IW|\right)$. Furthermore note that avoiding such switch would be in the best interest for the worker unless she expects with sufficiently high probability to be placed to the highest rank of NW (with probability one if $\bar{P}\left(|IW|\right) > \max_{j\in NW}\left\{p_j\right\}$). Moreover, by lemma 1 workers that are not sure to be placed in the highest rank within NW will continue to increase their precision levels slightly above $\bar{P}\left(|IW| + 1\right)$. Thus, in any equilibrium with pure action there must be at most a single worker in NW. Thus there would be only two pure equilibria in pure strategies: one in which all workers are in IW and one in which only one worker is in NW. In both equilibria all workers select $p_i = \bar{P}\left(n\right)$ and obtain the same expected equilibrium payoff: all workers exert effort and get expected wage

$$\mathbb{E}(W_i) = F(n)\frac{c}{F\big(n\big) - F\big(n-1\big)}.$$

■

Proof of Proposition 11

Proof. The following proof holds both for the additively separable case and when we allow for more flexible contracts. First, note that any increase in the monitorability of worker $i \in IW$ would not affect any other worker. Second note that even if $i \in NW$ an increase in IW would not affect any lower-ranked worker in NW. All workers in IW are also unaffected by such change, with the only exception of the worker in IW with the lowest signal precision, $l \in \arg\min_{j\in NW} p_j$. In particular, worker l might be switched to NW if p_i increases enough (above p_l) and l gains more in IW than in the highest rank of NW given that i switched to IW. Thus l can only be worse off. Finally, consider the possible increase of the signal precision p_i of a worker $i \in NW$ and denote by r a higher ranked worker within NW. If p_i increases above p_r and contracts are flexible, then i's ranking would decrease. If p_i increases so much that i is switched to IW, then strategic insurance rent of r would decrease. As a result, r would be worse off.[39] ■

Proof of Proposition 15

Proof. It is easy to show that, in the firm preferred equilibrium workers continue to offer a bonus which is positive if and only if both individual and team performance signals are positive.Conditioning also on the coworkers' signals would be irrelevant. In any case, workers receive a bonus only if the team signal is positive. We can show that this is no longer optimal when we look at robust implementation of teamwork. ■

[39]Note that here, in the case of additively separable payoffs, we are considering equilibrium transitions in which the relative ranking among the workers in $NW \setminus i$ are unaffected by changes in p_i. Alternatively, we could prove the same result with r being the highest rank or focusing on the sum of the expected wages of all coworkers in NW.