Optimal minimum wages[*]

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Abstract

We develop a quantitative spatial model with heterogeneous firms and a monopsonistic labour market to derive minimum wages that maximize employment or welfare. Quantifying the model for German micro regions, we find that the German minimum wage, set at 48% of the national mean wage, has increased aggregate worker welfare by about 2.1% at the cost or reducing employment by about 0.3%. The *welfare-maximizing federal* minimum wage, at 60% of the national mean wage, would increase aggregate worker welfare by 4%, but reduce employment by 5.6%. An *employment-maximizing regional* wage, set at 50% of the regional mean wage, would achieve a similar aggregate welfare effect and increase employment by 1.1%.

Key words: General equilibrium, minimum wage, monopsony, employment, Germany, inequality
JEL: J31, J58, R12

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1 Introduction

On few questions do economists disagree so passionately as on the desirability of minimum wages. The controversy is primarily an empirical one since there is arguably a theoretical consensus that a sufficiently high minimum wage will reduce employment. That a sufficiently low minimum wage may increase employment and welfare in a monopsonistic labour market also seems consensual. The open policy question is which minimum wage level maximizes employment or welfare. We develop a quantitative model that offers an answer.

Our approach differs from a vast literature using reduced-form methods to study employment effects of minimum wages summarized by Manning (2021) and Neumark and Shirley (2021). Instead, we develop a quantitative spatial model with heterogeneous firms and a monopsonistic labour market to study minimum wage effects in a spatial general equilibrium. Our model is uniquely equipped to derive optimal minimum wage schedules. For one thing, our model accounts for qualitatively and quantitatively heterogeneous employment responses in regions of distinct productivity (Christl et al., 2018). This allows us to predict both regionally differentiated and aggregate employment effects. For another, our model accounts for a broad range of minimum-wage effects that have recently been documented in the literature, including effects on labour force participation (Lavecchia, 2020), tradable goods prices (Harasztosi and Lindner, 2019), housing rents (Yamagishi, 2021), or commuting costs (Pérez Pérez, 2018) and worker-firm matching (Dustmann et al., 2022). This allows us to derive a worker welfare measure that incorporates all of those general equilibrium channels along with the effects on wages and employment probabilities.

We use our model to derive bounds for optimal minimum wages that are novel to the literature and of immediate policy interest. For Germany, we find an employment-maximizing *federal* minimum wage of 38% of the national mean wage, corresponding to 42% of the median wage. At less than 0.5%, however, the positive employment effect is small, and so is the impact on welfare. The welfare-maximizing federal minimum wage level is 60% of the national mean wage, corresponding to 66% of the median wage. While welfare increases by about 4%, there is a reduction in aggregate employment by 5.5%, driven by low-productivity regions. One important conclusion from our analysis is that within the bounds of the employment-maximizing and welfare-maximizing federal minimum wages, an increase in welfare can only be achieved at a cost of reducing employment. The implication is that ambitious federal minimum wages in the range of 60-70% of the national median wage—which are currently debated in the EU, the UK, and the US—may increase welfare at the cost of sizable job loss. Moderate *regional* minimum wages offer an attractive alternative that can achieve similar welfare gains as ambitious federal minimum wages, plus significant job creation.

While our model is sufficiently tractable to be implemented in arbitrary empirical contexts that satisfy the data requirements, choosing Germany as our case in point comes with three advantages. First, the first-time introduction of a relatively high nationally uniform minimum wage (54% of the national median wage)[1] as of 2015 provides an opportunity to contrast theoretical predictions with evidence. Second, we are able to quantify the model at unprecedented spatial coverage and detail at the level of 4,421 micro regions (*Verbandsgemeinden*), owing to the availability of linked linked-employer-employee data covering the universe of 30M workers from the Institute for Employment Research (IAB) and a micro-geographic property price index recently developed by Ahlfeldt et al. (2021). Third, our ability to observe the spatial economy in the recent past before a minimum wage was introduced greatly simplifies the quantification since we can treat observed labour market outcomes as undistorted market outcomes.

This data set paves the way for our methodological contribution, which is to develop a quantitative spatial model with heterogeneous firms that possess monopsony power. We start from a canonical setup in the spirit of Redding and Rossi-Hansberg (2017). Workers choose where to live, where to work and how much to consume of a composite tradable good and housing, trading expected wages and amenities against commuting cost, goods prices and housing rents. Goods are produced in a monopolistically competitive market and traded at a cost. Housing is supplied inelastically, creating a congestion force that restores the spatial equilibrium. We extend this canonical framework in three important respects. First, we borrow from the trade literature and introduce a Pareto-shaped productivity distribution of firms (Redding, 2011; Gaubert, 2018). This extension is critical to generating a wage distribution within regions and enabling the minimum wage to reallocate workers to more productive establishments. Second, we follow Egger et al. (2021), who build on Card et al. (2018), and generate an upward-sloping labour supply curve to the firm via Gumbel-distributed idiosyncratic preferences for employers, in addition to allowing for idiosyncrasy in preferences for residence and workplace locations (Ahlfeldt et al., 2015).[2] This extension is critical to awarding employers monopsony power. Third, we generate imperfectly elastic aggregate labour supply via a Gumbel-distributed idiosyncratic utility from non-employment. This extension is critical to capturing incentives minimum wages can create for workers to become active on the labour market and search for jobs (Mincer, 1976; Lavecchia, 2020).

To develop the intuition for the regional employment response in our model, it is instructive to consider three firm types. The minimum wage has no effect on the most productive firms, which we term *unconstrained* because they voluntarily pay wages above the minimum wage. These firms still exercise their full monopsony power after the minimum wage is introduced. For all less productive firms, the minimum wage is binding. These firms can no longer lower the wage below the minimum wage, which implies that they lose some of their monopsony power. The more productive among the constrained firms will respond by hiring all workers they can attract at the minimum wage—the new

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[1] This quantity is based on the hourly wages of full-time and part-time workers (see Section 2.2 for further information). Based on the wages of full-time workers, the Minimum Wage Commission reports that the Kaitz Index was 46% in Germany in the year 2018 (Mindestlohnkommission, 2018)

[2] Dustmann et al. (2022) model non-pecuniary aspects of job choice in a similar way.

marginal cost of labour—which is why we refer to them as *supply-constrained*. Consequentially, they will increase employment. Less productive firms will hire until the marginal revenue product falls below the minimum wage level, which is why we term them *demand-constrained*. While some demand-constrained firms will react to the introduction of the minimum wage by increasing employment, any demand-constrained firm that produces at a MRPL below the minimum wage will have to reduce employment to stay in the market once a minimum wage is introduced. The aggregation of the employment response across all firms within a region delivers the prediction that the regional employment effect of a federal minimum wage is a hump-shaped function of regional productivity. We substantiate this predictions in two complementary approaches.

In the first step, we employ a reduced-form methodology that uses high-productivity regions in which firms are mostly unconstrained as a counterfactual in the spirit of a dynamic difference-in-difference model. This approach allows us to test a central prediction of the model without imposing the full structure of the quantitative model. Consistent with model predictions, we find that the employment response is flat in the regional wage level for high-productivity regions, where the 2014 mean hourly wage exceeds €18.6. Compared to this group, regions with a mean hourly wage of more than €13.1 tend to gain employment whereas those with a lower mean wage tend to lose. These estimates of a theory-consistent regional distribution of minimum-wage induced employment effects adds to a literature that has mostly focused on average effects for selected spatial units (e.g. Card and Krueger, 1994; Dustmann et al., 2022) or point estimates of the effect of the minimum wage bite (e.g. Machin et al., 2003; Ahlfeldt et al., 2018). Indirectly, they provide evidence supporting the monopsonistic labour market model that is still scarce (Neumark, 2018). Importantly, we bring to light a sizable negative employment effect in the least productive micro regions that has gone unnoticed in previous studies analyzing larger spatial units (Ahlfeldt et al., 2018; Caliendo et al., 2018; Dustmann et al., 2022).

In the second step, we quantify the full model to take the analysis into the general equilibrium. We exploit our matched worker-establishment micro data to estimate the structural parameters that govern the wage distribution within regions. We then invert the model in 2014—the year before the introduction of the minimum wage. Solving the model under the minimum wage of 48% of the national mean that we observe in our data delivers the comparative statics from which we infer the minimum wage effect. We find almost exactly the same regional wage levels that characterize the hump-shape of the regional employment response as in the reduced-form analysis. The important advantage of the model-based general-equilibrium approach is that we do not have to assume any group of firms, workers, or regions to be unaffected by the minimum wage, which allows us to establish the aggregate employment effect. While the hump-shape in the model resembles our reduced-form estimates, we gain the additional insight that employment increases in regions of intermediate productivity at the expense of the least *and* most productive regions. In the national aggregate, employment decreases by about 0.3% or 100K jobs, which is less than predicted by the competitive labour market model (Knabe et al., 2014).

For our purposes, the ability of our model to speak to welfare effects is, at least, as important as establishing aggregate employment effects. We find that the German minimum wage has increased welfare by 2%. This estimate of the minimum wage welfare effect is unprecedented in the literature in that it accounts for changes in nominal wages, employment probabilities, goods prices, housing rents, the quality of the worker-firm match, the reallocation of workers across firms, commuting destinations, residences, and the growing number of workers who decide to be active on the labour market. In other words, the increase in real wage—adjusted for changes in tradeble goods prices, housing rents, and commuting costs—dominates the reduction in the employment probability. As as a result, about 180K workers become active on the labour market and start searching for jobs. Again, there is significant spatial heterogeneity. The net-winners are low-productivity regions such as in the eastern states, resulting in long-run incentives for workers to relocate to regions that have experienced sustained population loss over the past decades.

Given the absence of a credible counterfactual, we cannot over-identify the aggregate effects of the German minimum wage our model delivers. We show, however, that the model's predictions for minimum wage effect in wages, employment, housing rents and commuting distances are closely correlated with observed before-after changes in the data at the regional level. We also show that our model predicts changes in the Gini coefficient of wage inequality across all workers in all regions that are in line with before-after changes observed in data. This suggests significant out-of-sample predictive power, which is reassuring with respect to our key normative contribution: The derivation of optimal minimum wage schedules.

To this end, we compute aggregate employment and welfare effects for a broad range of federal and regional minimum-wage schedules. We also provide an equity measure based on the Gini coefficient of wage inequality. Hence, we equip our readers with the key ingredients to compute their own optimal minimum wage. Under canonical welfare functions, the optimal *federal* minimum wage will not be lower than the employment-maximizing minimum wage, at 38% of the national mean wage. Up to 60%, the minimum wage can be justified on the grounds of welfare effects. Higher levels require equity (among those in employment) as an objective. Ambitious minimum wages need to be defended against negative employment effects that start building up rapidly beyond 50% of the national wage. Against this background, it is important to note that the employment-maximizing *regional* minimum wage, at 50% of the regional mean wage, would deliver positive welfare effects that are similar to the federal welfare-maximizing minimum wage (4%), plus increase employment by 1.1%, suggesting that regional minimum wages are targeted policy instruments that warrant more attention.

With these results, we contribute to the identification of turning points where the costs of minimum wages start exceeding the benefits, a challenge that allegedly lies ahead of the field (Manning, 2021). In doing so, we complement a large literature using reduced-form approaches that suggest that minimum wages may (Meer and West, 2016; Clemens and

Wither, 2019) or may not have negative employment effects (Dube et al., 2010; Cengiz et al., 2019).[3] This includes a growing literature evaluating the labour market effects of the German minimum wage, which we review in more detail in Appendix A (e.g. Ahlfeldt et al., 2018; Bossler and Gerner, 2019; Caliendo et al., 2018; Dustmann et al., 2022). We also contribute to a smaller normative literature on minimum wages that considers distributional effects of minimum wages (Lee and Saez, 2021; Simon and Wilson, 2021).[4] In current working papers, Berger et al. (2022) and Drechsel-Grau (2021) study aggregate and distributional effects of minimum wages within non-spatial dynamic macroeconomic models.

Our theoretical contribution builds on a literature showing that, in a monopsonistic labour market,[5] a minimum wage can raise the wage without reducing employment (Stigler, 1946; Manning, 2003a).[6] We also draw from a literature on quantitative spatial models, which have recently emerged as general-purpose tools for policy evaluation that can account for mobility of residents across residence and workplace locations (Allen and Arkolakis, 2014; Ahlfeldt et al., 2015).[7] In particular, our model nests Monte et al. (2018) as a special case in which the dispersion of firm productivity approaches infinity, there is no idiosyncrasy in worker tastes for employers, and workers supply labour inelastically.

Most closely, we connect to a small literature that studies the effects of minimum wages in a spatial equilibrium. Our contribution complements Monras (2019) and Simon and Wilson (2021) who consider a competitive labour market. To our knowledge, the only other model that nests a monopsonistic labour market in a spatial general equilibrium is in the current working paper by Bamford (2021), who also provides an evaluation of the German minimum wage. Similar to us, he uses worker-firm-specific idiosyncratic utility to generate an upward-sloping labour-supply curve to the firm. By making the labour supply elasticity dependent on the density of nearby firms within relatively large regions that roughly correspond to local labour markets, he shows that lower monopsony power acts as an important concentration force in the spatial economy (see also Azar et al. (2019)). In contrast, our focus is the development of a comprehensive welfare measure for the normative evaluation of minimum wages. Hence, we develop our model at the micro-regional level to capture minimum wage effects on commuting costs explicitly.[8] We also account for how frictional trade shapes the spatial distribution of minimum wage effects and account for employment and welfare effects that arise when minimum wages

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[3] A new wave of empirical minimum wage research, based on difference-in-differences designs, started with the seminal paper by Card and Krueger (1994) whose findings, subsequently challenged by Neumark and Wascher (2000), cast doubt on the competitive labour market model which predicts that binding minimum wages necessarily lead to job loss.

[4] Minimum wages also interact with the optimal tax system (Allen, 1987; Guesnerie and Roberts, 1987).

[5] Manning (2020) offers a recent review of the literature.

[6] Similarly, search models do not restrict the sign of the employment effect of a minimum wage (Brown et al., 2014; Blömer et al., 2018).

[7] Other recent models that quantitatively account for commuting include Tsivanidis (2019); Heblich et al. (2020); Almagro and Domínguez-Iino (2021).

[8] We generate larger employment elasticities (Monte et al., 2018) and less monopsony power in thicker labour markets as workers can substitute across commuting destinations (Manning, 2003b; Datta, 2021).

incentivize workers to become active on the labour market, which is important to enable aggregate employment gains.

The remainder of the paper is structured as follows. Section 2 introduces the institutional context and our data, and presents stylized evidence that informs our modelling choices. Section 3 introduces a partial equilibrium version of our model and provides transparent reduced-form evidence that is consistent with stylized predictions. Section 4 develops the full quantitative model and takes the analysis to the general equilibrium. Section 5 concludes.

2 Empirical context

In this section, we introduce the German minimum wage policy, the various sources of data we rely on, and some stylized facts that inform our modelling choices.

2.1 The German minimum wage

The first uniformly binding federal minimum wage in Germany was introduced in 2015. Since then, German employers had to pay at least €8.50 euros per hour corresponding to 48% of the mean salary of full-time workers. Because no similar regulation preceded the statutory wage floor, it represented a potentially significant shock to regions in the left tail of the regional wage distribution. Subsequently, the minimum wage has been raised to €8.84 in 2017, €9.19 in 2019 and €9.35 in 2020. In relative terms, it has fluctuated within a close range of 47% to 49% of the national mean wage, suggesting that it is reasonable to treat the introduction of the minimum wage as a singular intervention in 2015. We provide a detailed discussion of the institutional context in Section 2.1.

2.2 Data

We compile a novel data set for German micro regions that is unique in terms of its national coverage of labour and housing market outcomes at sub-city level. We provide a brief summary of the various data sources here and refer to Appendix B.2 for details.

Employment, establishments and wages. We use the Employment Histories (BeH) and the Integrated Employment Biographies (IEB) provided by the Institute for Employment Research (IAB) which contain individual-level panel data containing workplace, residence, establishment, wage, and characteristics such as age, gender, and skill on the universe of about 30M labour market participants in Germany.

Hours worked. We follow Ahlfeldt et al. (2018) and impute average working hours separately for full-time and part-time workers from an auxiliary regression that accounts for the sector of employment, federal state of employment, and various socio-demographic attributes and using the 1% sample from the 2012 census. We find that full-time employees work approximately 40 hours per week while the number is lower for regularly employed

(21 hours) and for marginally employed part-time workers (10 hours). Combining working hours with average daily earnings delivers hourly wages.

Real estate. We use a locally-weighted regression approach proposed by Ahlfeldt et al. (2021) to generate an area-year housing cost index. The raw data comes from Immoscout24, accessed via the FDZ-Ruhr (Boelmann and Schaffner, 2019). It covers nearly 20 million residential observations between 2007 and 2018.

Trade. Trade volumes are taken from the Forecast of Nationwide Transport Relations in Germany which are provided by the Clearing House of Transport Data at the Institute of Transport Research of German Aerospace Center. The data set contains information about bilateral trade volumes between German counties in the year 2010 for different product groups. Following Henkel et al. (2021), we aggregate trade volumes across all modes of transport (road, rail and water). To convert volumes (measured in metric tonnes) into monetary quantities, we use information on national unit prices for the different product groups. Finally, we aggregate the value of trade flows across all product groups.

Spatial unit. The primary spatial unit of analysis are 4,421 municipal associations (*Verbandsgemeinden*) according to the delineation from 31 December 2018 (see Figure 1 for a map). Municipal associations are spatial aggregates of 11,089 municipalities (*Gemeinden*) that ensure a more even distribution of population and geographic size. Henceforth, we refer to municipal associations as municipalities for simplicity. On average, a municipality hosts 541 establishments employing 6,769 workers on less than 80 square kilometers, making it about a tenth of the size of an average county. For each pair of municipalities, we compute the Euclidean distance using the geographic centroids.

2.3 Stylized facts

Figure 1, illustrates a measure of the regionally differentiated "bite" of the national minimum wage, very much in the tradition of Machin et al. (2003). Concretely, we compute a bite exposure measure at the residence by taking the average over the shares of below-minimum-wage workers at the workplace across nearby municipalities, weighted by the bilateral commuting flows in 2014.[9] This way, we capture the bite within the actual commuting zone of a municipality. Evidently, the minimum wage had a greater bite in the east, in line with the generally lower productivity. Changes in low wages, defined as the 10th percentile in the within-area wage distribution, from 2014 to 2016 closely follow the distribution of the bite, suggesting a significant degree of compliance. Together, the two maps suggest that the minimum wage contributed to the reduction of spatial wage disparities in Germany, an impression that we substantiate with further evidence in Appendix B.3.

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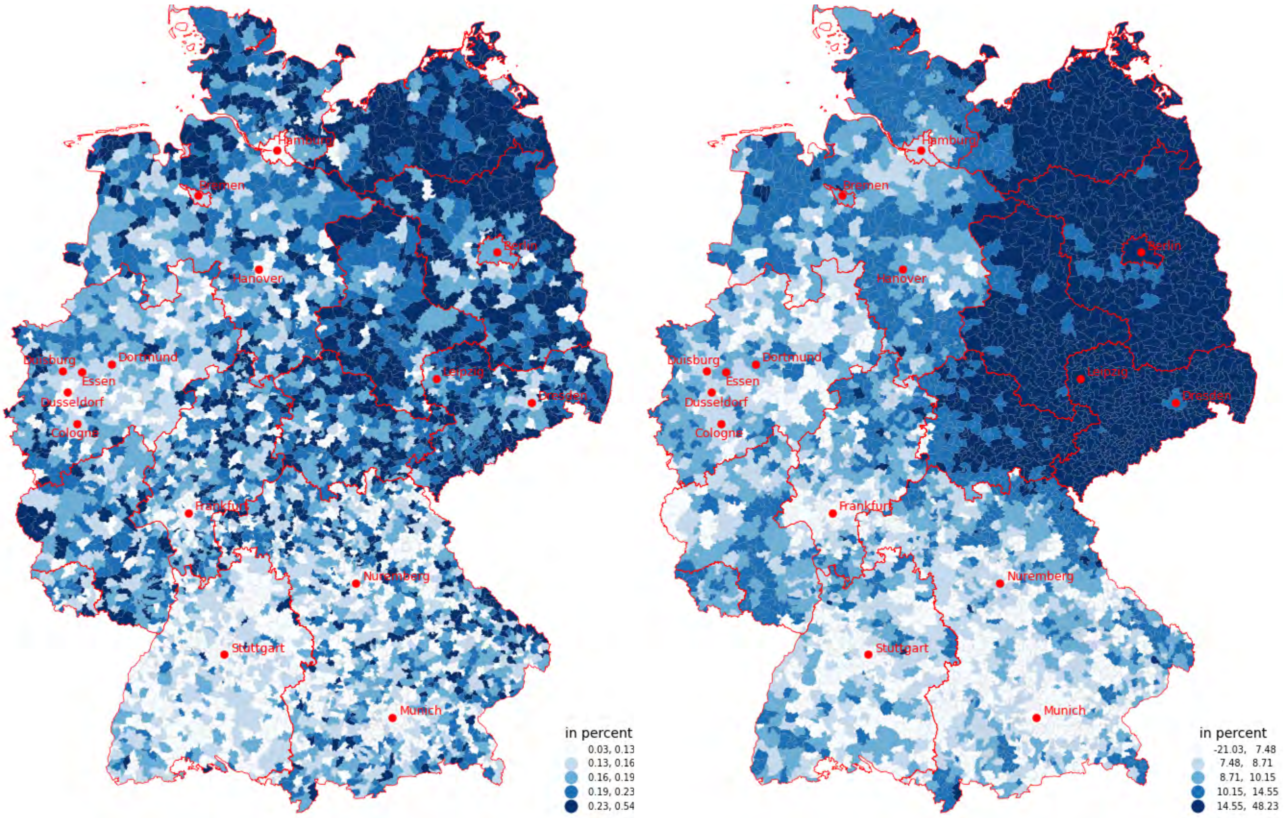
[9] Formally, we define the bite as $\mathcal{B}_i = \sum_j \frac{L_{i,j}}{\sum_j L_{i,j}} S_j^{MW}$, where $L_{i,j}$ is the number of employees who live in municipality i and commute into municipality j for work and S_j^{MW} is the share of workers compensated below the minimum wage in j.

Figure 1: Minimum wage bite and change in 10th pct. regional wages



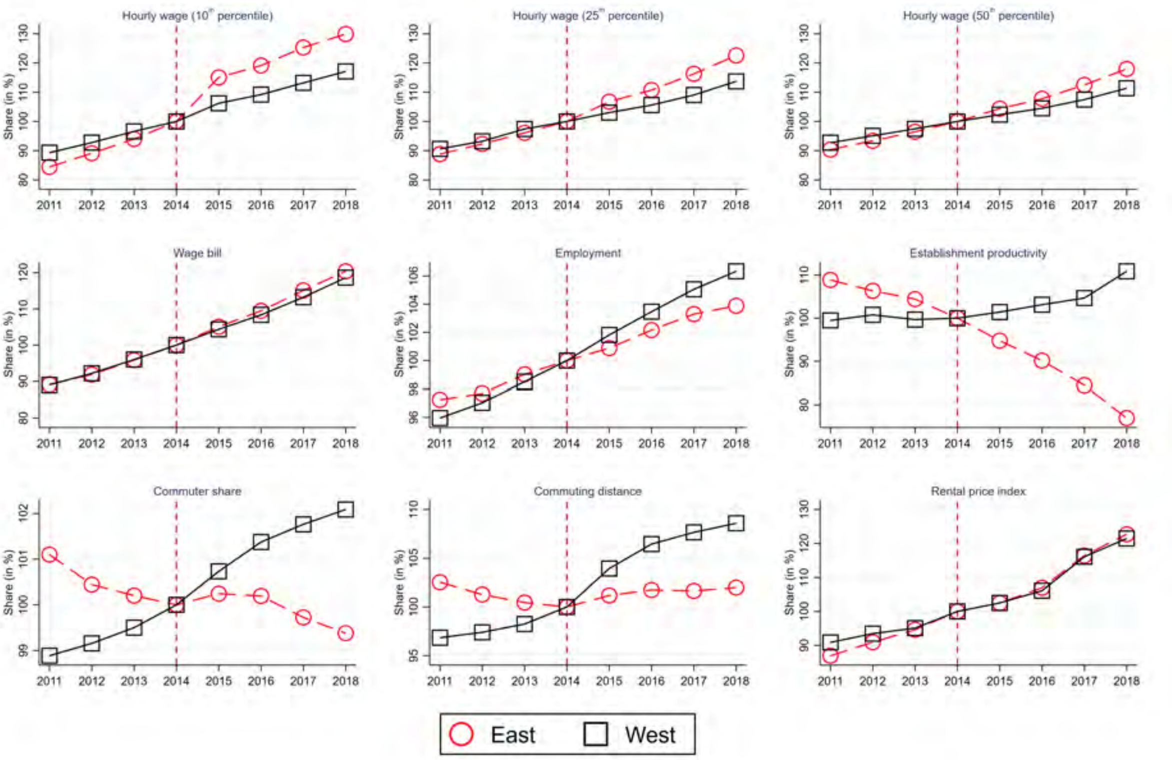
(a) Minimum wage bite in 2014

(b) 2014-2016 wage growth at 10th pct.

Note: Unit of observation is 4,421 municipality groups. The 10^{th} percentile wage refers to the 10^{th} percentile in the distribution of individuals within a workplace municipality, re-weighted to the residence using commuting flows. Wage and employment data based on the universe of full-time workers from the *BeH*.

The striking heterogeneity in the policy-induced wage increase between the eastern and the western states makes it instructive to compare how employment and other outcomes evolved in the respective parts of the country over time. We offer this purely descriptive comparison in Figure 2. Confirming Figure 1, a jump at the 10^{th} percentile of the wage distribution in the east is immediately apparent. A more moderate increase is also visible for the west. For higher percentiles, it is possible to eyeball some increase in the east, but not in the west. A first-order question from a policy-perspective is whether the policy-induced wage increase came at the cost of job loss as predicted by the competitive labour market model. While we argue that—without a general equilibrium model—it is difficult to establish a counterfactual for aggregate employment trends, the absence of an immediately apparent employment effect in these time series is still informative. It is worth noticing that, while employment continues to grow in both parts of the country after the minimum wage introduction, the rate of growth appears to slow down in the east compared to the west. However, even if one is willing to interpret this as suggestive evidence of a negative employment effect, it will be difficult to argue that negative employment effects

Figure 2: Outcome trends in western and eastern states



Note: All time series are normalized to 100% in 2014, the year before the minimum wage introduction. The establishment wage premium is the employment-weighted average across firm-year fixed effects from a decomposition of wages into worker and firm fixed effects following Abowd et al. (1999) (see Appendix B.4 for details).

turned out to be as severe as in some pessimistic scenarios circulated ahead of the implementation (Ragnitz and Thum, 2008). Since, following the minimum-wage introduction, the aggregate wage bill increases in the east, relative to the west, it seems fair to conclude that a positive wage effect has dominated a possibly negative employment effect, pointing to positive welfare effects. Figure 2 also illustrates the reallocation of workers to more productive establishments at greater commuting distance documented by Dustmann et al. (2022). Indeed, it appears that the effect has gained momentum subsequent to 2016, when their analysis ends. Finally, there appears to be a slight increase in the rate of property price appreciation after the minimum wage which could be reflective of increased demand.

These stylized facts echo a growing empirical literature on the effects of the German minimum wage which we summarize in Appendix A. They motivate us to develop a quantitative spatial model that nests a monopsonistic labour market in which a statutory minimum wage triggers different employment responses by firms of distinct productivity and workers decide where to live and work trading higher wages against greater commuting costs.

3 Partial equilibrium analysis

In this section, we develop a model of optimal behaviour of heterogeneous firms in a monopsonistic labour market with a minimum wage. We first use the model to develop the intuition for why the employment response to a uniform minimum wage differs qualitatively across firms, depending on their productivity. We then derive the novel prediction that the regional employment response is a hump-shaped function of regional productivity. Finally, we provide novel area-specific estimates of the employment effect of the minimum-wage that confirm this prediction.

3.1 Model I

For now, we take upward-sloping labour supply to the firm as well as downward-sloping product demand as exogenously given. We nest the firm problem introduced here into a QSM in Section 4. The extended model will provide the micro-foundations for the labour supply and product demand functions and allow us to solve for the spatial general equilibrium of labour, goods, and housing markets.

3.1.1 Optimal firm behaviour

A firm in location $j \in J$ sells its product variety at monopolistically competitive goods markets across all locations $i \in J$. Because one firm produces only one variety, we use ω_j to denote both a firm and its variety. Given a productivity φ_j, firm ω_j hires $l_j(\omega_j)$ units of labour in a monopsonistically competitive labour market which it uses to produce output $y_j(\omega_j) = \varphi_j(\omega_j) l_j(\omega_j)$.

Labour supply. Firm ω_j faces an iso-elastic labour supply function

$$h_j(\omega_j) = S_j^h \left[\psi_j(\omega_j) w_j(\omega_j)\right]^{\varepsilon} \tag{1}$$

of the expected wage $\psi_j(\omega_j) w_j(\omega_j)$ that a worker earns in this firm, with $w_j(\omega_j) > 0$ being the firm's wage rate and $\psi_j(\omega_j) \in (0, 1]$ being the firm's hiring probability. Unless otherwise indicated, we assume $\psi_j(\omega_j) = 1$ to ease notations. We denote the firm's constant labour supply elasticity by $\varepsilon > 0$ and introduce $S_j^h > 0$ as an aggregate shift variable that summarizes all general equilibrium effects operating through location j's labour market (specified in more detail below and solved in general equilibrium in Section 4).

Goods demand. Similarly, there is iso-elastic demand for variety ω_j in location i

$$q_{ij}(\omega_j) = S_i^q p_{ij}(\omega_j)^{-\sigma}, \tag{2}$$

which depends inversely on the variety's consumer price $p_{ij}(\omega_j)$ with a constant price elasticity of demand $\sigma > 1$, and which is directly proportional to an aggregate shift variable $S_i^q > 0$ that summarizes all general equilibrium effects operating through location

i's goods market (specified in more detail below and solved in general equilibrium in Section 4). Under profit maximization and goods market clearing, we can express the revenue function as

$$r_j(\omega_j) = \sum_i p_{ij}(\omega_j) q_{ij}(\omega_j) = \left(S_j^r\right)^{\frac{1}{\sigma}} [y_j(\omega_j)]^{\rho}, \tag{3}$$

where $\rho = \frac{\sigma-1}{\sigma} \in (0,1)$. Intuitively, a greater market access $S_j^r \equiv \sum_i \tau_{ij}^{1-\sigma} S_i^q > 0$ implies that a smaller fraction of output melts away due to iceberg trade costs $\tau_{ij} \geq 1$, leading to relatively larger revenues (see Appendix C.1).

Minimum wage. In deriving the effects of a statutory minimum wage $\underline{w}$ on price, output, and labour input, it is instructive to distinguish between three firm-types: unconstrained firms (indexed by superscript u), for which the minimum wage $\underline{w}$ is non-binding; supply-constrained firms (indexed by superscript s), whose labour demand exceeds labour supply at the binding minimum wage $\underline{w}$; and demand-constrained firms (indexed by superscript d), that attract more workers than they require when the minimum wage $\underline{w}$ is binding. We present the key results for the three firm types below and refer to Appendix C.2 for further derivations. As each firm can be fully characterized by its productivity level and its firm-type, we drop the firm index ω_j in favour of a more parsimonious notation, combining the firm's productivity level φ_j with superscript $z \in \{u, s, d\}$.

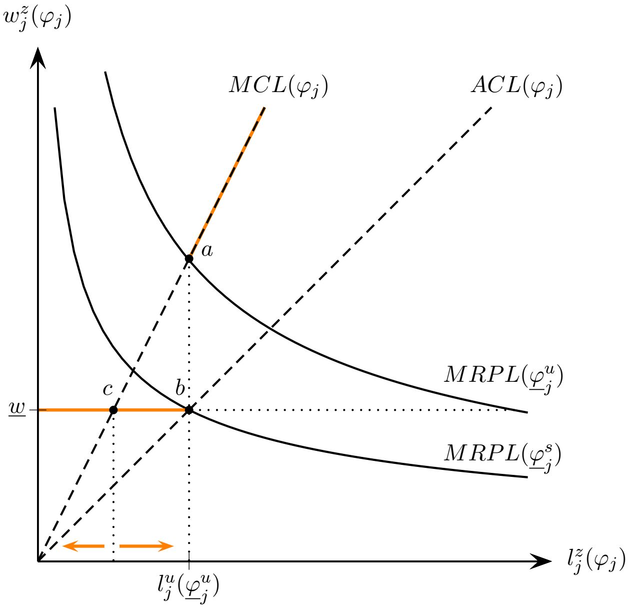
Unconstrained firms choose profit-maximizing wages that are larger or equal to the minimum wage level. Therefore, we can use the labour supply function to the firm in Eq. (1) to derive the relevant cost function

$$c_j^u(\varphi_j) = w_j^u(\varphi_j) l_j^u(\varphi_j) = \left(S_j^h\right)^{-\frac{1}{\varepsilon}} l_j^u(\varphi_j)^{\frac{\varepsilon+1}{\varepsilon}}. \tag{4}$$

Facing an upward-sloping labour supply function, firms can only increase their employment by offering higher wages. Hence, the average cost of labour $ACL(\varphi_j) = c_j^u(\varphi_j)/l_j^u(\varphi_j)$ is upward-sloping as illustrated in Figure 3. The marginal cost of labour $MCL(\varphi_j) = \partial c_j^u(\varphi_j)/\partial l_j^u(\varphi_j) = \frac{\varepsilon+1}{\varepsilon} ACL(\varphi_j)$ is also upward-sloping and strictly greater than $ACL(\varphi_j)$. Since demand for any variety is downward-sloping, an expansion of production and labour input is associated with a lower marginal revenue product of labour $MRPL(\varphi_j) = \partial r_j^u(\varphi_j)/\partial l_j^u(\varphi_j)$. Unconstrained firms find the profit-maximizing employment level by setting $MRPL(\varphi_j) = MCL(\varphi_j)$ which corresponds to point a in Figure 3. Since a higher productivity shifts the $MRPL(\varphi_j)$ function outwards, more productive firms hire more workers at higher wages (Oi and Idson, 1999). Unconstrained firms simultaneously act as monopolists in the goods market and monopsonists in the labour market, setting their prices as a constant mark-up $\sigma/(\sigma-1) > 1$ over marginal revenues and their wages as a constant mark-down $\varepsilon/(\varepsilon+1) < 1$ below marginal costs. The combined mark-up/mark-down factor is $1/\eta \equiv [\sigma/(\sigma-1)][(\varepsilon+1)/\varepsilon] > 1$.

We refer to $\underline{\varphi}_j^u$ as the least-productive unconstrained firm that is identified by setting

Figure 3: Optimal firm employment



$w_j(\underline{\varphi}_j^u) = \underline{w}$, so we obtain

$$\underline{\varphi}_j^u(\underline{w}) = \left(\frac{1}{\eta}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{S_j^h}{S_j^r}\right)^{\frac{1}{\sigma-1}} \underline{w}^{\frac{\sigma+\varepsilon}{\sigma-1}}. \tag{5}$$

All firms with $\varphi_j < \underline{\varphi}_j^u$ are constrained by the minimum wage. Any increase in the minimum wage level will lead to a firm with a greater productivity becoming the marginal unconstrained firm.

Supply-constrained firms face a binding minimum wage, resulting in $MRPL(\varphi_j) = \underline{w}$. At this wage, workers are willing to supply no more than $h_j^s(\varphi_j) = S_j^h \underline{w}^\varepsilon$ units of labour, which corresponds to $l_j^u(\underline{\varphi}_j^u)$ in Figure 3. Employment is constrained by labour supply because supply-constrained firms would be willing to hire more workers as the MRPL function intersects with $\underline{w}$ at an employment level greater than $l_j^u(\underline{\varphi}_j^u)$. In the absence of the minimum wage, supply-constrained firms would set a wage below $\underline{w}$ to equate MRPL and MCL. At this wage, workers would supply less than $l_j^u(\underline{\varphi}_j^u)$ units of labour. By removing the monopsony power, the mandatory wage floor raises employment for all firms with $\underline{\varphi}_j^s \leq \varphi_j < \underline{\varphi}_j^u$, where $\underline{\varphi}_j^s$ defines the least-productive supply-constrained firm given by

$$\underline{\varphi}_j^s(\underline{w}) = \left(\frac{\eta}{\rho}\right)^{\frac{\sigma}{\sigma-1}} \underline{\varphi}_j^u(\underline{w}) < \underline{\varphi}_j^u(\underline{w}) \quad \text{with} \quad \frac{\eta}{\rho} = \frac{\varepsilon}{\varepsilon+1} < 1. \tag{6}$$

Notice that all supply-constrained firms set the same wage (i.e. the minimum wage) and hire the same number of workers $l_j^s(\varphi_j) = h^s(\varphi_j) = \underline{w}^\varepsilon S_j^h = l_j^u(\underline{\varphi}_j^u)$, (determined by b in Figure 3).

Demand-constrained firms also face a binding minimum wage, resulting in $MRPL(\varphi_j) = \underline{w}$. For these firms with productivities $\varphi_j < \underline{\varphi}_j^s(\underline{w})$, however, employment is constrained by labour demand because at a wage of $\underline{w}$ firms demand less units of labour than workers are willing to supply. To see this, consider the MRPL curve for any firm with productivity $\varphi_j < \underline{\varphi}_j^s$ in Figure 3, which will be below $MRPL(\underline{\varphi}_j^s)$. Since $\underline{w}$ intersects with the MRPL before it intersects with ACL, there is job rationing with a hiring probability $\psi_j^d(\varphi_j) = l_j^d(\varphi_j)/h_j^d(\varphi_j) < 1$. Yet, demand-constrained firms do not necessarily reduce employment. As long as a demand-constrained firm is sufficiently productive for its MRPL curve to be above point c, the MRPL in the monopsony market equilibrium exceeds $\underline{w}$. Therefore, the intersection of MRPL and $\underline{w}$ is necessarily to the right of the intersection of MRPL and MCL, implying greater employment under the minimum wage. The opposite is true, however, for any firm whose productivity is sufficiently small for the MRPL curve to be below point c. Because the MRPL in the monopsony market equilibrium is smaller than $\underline{w}$, the firm has to reduce output and labour input to raise the MRPL to the minimum wage level.

3.1.2 Aggregate outcomes

Having characterized the optimal behaviour of the three firm types, we now explore how the introduction of a minimum wage affects aggregate outcomes at the regional level. To this end, we assume that firm productivity follows a Pareto distribution with shape parameter $k > 0$ and lower bound $\underline{\varphi}_j > 0$. For the following discussion, it is instructive to introduce the critical minimum wage levels $\underline{w}_j^z \ \forall \ z \in \{s, u\}$ as a function of $\underline{\varphi}_j$. They are implicitly defined through $\underline{\varphi}_j^z(\underline{w}_j^z) = \underline{\varphi}_j \ \forall \ z \in \{s, u\}$ and have the following interpretation: For a sufficiently small minimum wage, $\underline{w} < \underline{w}_j^u$, location j features only unconstrained firms. For higher minimum wages, $\underline{w} < \underline{w}_j^s$, location j also features supply-constrained, but no demand-constrained firms. Using Eq. (5), we obtain

$$\underline{w}_j^u = w_j^u(\underline{\varphi}_j) = \left(\eta^\sigma \underline{\varphi}_j^{\sigma-1} \frac{S_j^r}{S_j^h}\right)^{\frac{1}{\sigma+\varepsilon}} \tag{7}$$

as an implicit solution to $\underline{\varphi}_j^u(\underline{w}_j^u) = \underline{\varphi}_j$. Using Eq. (5) in Eq. (6) and solving $\underline{\varphi}_j^s(\underline{w}_j^s) = \underline{\varphi}_j$ for $\underline{w}_j^s$ results in

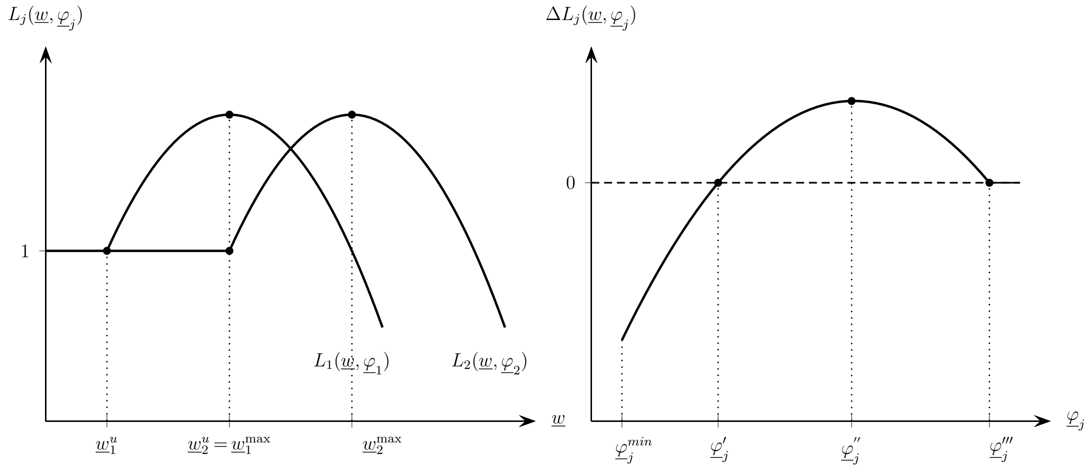
$$\underline{w}_j^s = \left(\rho^\sigma \underline{\varphi}_j^{\sigma-1} \frac{S_j^r}{S_j^h}\right)^{\frac{1}{\sigma+\varepsilon}}, \tag{8}$$

implying $\underline{w}_j^s/\underline{w}_j^u = (\rho/\eta)^{\frac{\sigma}{\sigma+\varepsilon}} > 1$. Using these critical minimum wages we derive the following proposition:

Proposition 1. *Assuming Pareto-distributed firm productivities, aggregate employment L_j, aggregate labour supply H_j and aggregate revenues R_j are hump-shaped in the minimum wage level. Aggregate profits, Π_j, are declining in $\underline{w}$.*

Proof see Appendix C.4.

Figure 4: Regional employment, minimum wages and productivity



(a) Minimum wages (b) Regional productivity

Note: In this partial-equilibrium illustration, we assume constant general equilibrium terms $\{S_j^r, S_j^h\}$ that are invariant across regions and not affected by the minimum wage.

To develop the intuition, let's first consider the region indexed by $j = 1$ in panel a) of Figure 4. Any minimum wage $\underline{w}_1 \leq \underline{w}_1^u$ will have no effect because all firms in the region are unconstrained as they voluntarily set higher wages. A marginal increase in $\underline{w}_1$ turns some unconstrained firms into supply-constrained firms, whose response to the loss of monopsony power is to hire all workers who are willing to supply their labour at wage $\underline{w}_1^u$. Hence, regional employment increases. Once $\underline{w}_1 > \underline{w}_1^s$, some firms become demand-constrained. The marginal effect of an increase in $\underline{w}_1$ remains initially positive even beyond $\underline{w}_1^s$ because demand-constrained firms still increase the labour input as long as their MRPL exceeds $\underline{w}_1$ in the monopsony market equilibrium. At some point, however, $\underline{w}_1$ will exceed the MRPL of the least productive firms in the market equilibrium and these firms will respond by reducing output and labour input. The marginal effect of $\underline{w}_1$ declines and becomes zero at the employment-maximizing minimum wage $\underline{w}_1^{\max}$. Further increases have negative marginal effects and, eventually, the absolute employment effect will turn negative. The generalizable insight is that for given fundamentals $\{S_j^r, S_j^h\}$ and regional productivity summarized by $\underline{\varphi}_j$, aggregate employment $L_j(\underline{w}_j, \underline{\varphi}_j)$ is hump-shaped in the minimum wage level $\underline{w}_j$.

To clear the regional labour market, the hump-shaped pattern must carry through to labour supply. Since the hiring probability is $\psi_j^z = 1 \ \forall \ z \in \{s, u\}$ for unconstrained and supply-constrained firms, labour supply defined in Eq. (1) increases for low but binding minimum wage levels $\underline{w}_j^u \leq \underline{w}_j < \underline{w}_j^s$. At a higher minimum wage level, the expected hiring probability adjusts to the hiring rate to account for the job rationing of demand-constrained firms ($\psi_j^d(\varphi_j) = l_j^d(\varphi_j)/h_j^d(\varphi_j)$). In other words, workers who are unlikely to get a job withdraw from the labour force. In the spatial general equilibrium introduced in Section 4.1, workers will adjust labour supply by choosing *if* to work and *where* to live (migration) and work (commuting). For a discussion of the effect of the minimum wage on firm profits and revenues, we refer to Appendix C.4.

Let us now compare the effect of a uniform minimum wage in region $j = 1$ to a region $j = 2$ in which firms are generally more productive, for example, due to better infrastructure or institutions. To ease the comparison, we normalize initial employment to unity. A low minimum wage $\underline{w}_1^u < \underline{w} \leq \underline{w}_1^{\max}$ leads demand- and supply-constrained firms to hire more workers in region $j = 1$, whereas there is no employment effect in region $j = 2$ since all firms remain unconstrained. At a higher level $\underline{w}_1^{\max} < \underline{w} < \underline{w}_2^{\max}$, an increase in the minimum wage reduces employment in region $j = 1$ because the MRPL of the marginal firm falls below $\underline{w}$, whereas employment increases in region $i = 2$ owing to the loss of monopsony power of formerly unconstrained firms. Hence, the same increase in the minimum wage level can have qualitatively different employment effects in different regions because the employment-maximizing minimum wage depends on regional productivity. This is an important theoretical result that rationalizes why a large empirical literature has failed to reach consensus regarding the employment effects of minimum wage rises (Manning, 2021).

Of course, regional productivity not only affects the marginal effect of a minimum wage increase, but also the aggregate effect relative to the situation without a minimum wage. In panel (a) of Figure 4, the aggregate effect is given by $\Delta L_j(\underline{w}, \underline{\varphi}_j) = L_j(\underline{w}_j) - L_j(\underline{w}_j^u)$. In panel (b) of Figure 4, we plot $\Delta L_j(\underline{w}, \underline{\varphi}_j)$ against $\underline{\varphi}_j$, which directly maps into the average regional productivity given the Pareto-shaped firm productivity distribution. We consider a continuum of regions with heterogeneous productivity, but only one universal national minimum wage $\underline{w}$, which resembles the empirical setting in Germany and many other countries. We refer to Appendix C.5 for a detailed and formal discussion of the comparative statics. Briefly summarized, we can distinguish between three types of regions. The minimum wage has no effect in regions where even the least productive firm is unconstrained ($\underline{\varphi}_j \geq \underline{\varphi}_j^{'''}$). In the least productive regions, there are negative aggregate employment effects driven by demand-constrained firms ($\underline{\varphi}_j < \underline{\varphi}_j^{'}$). In between, there are positive employment effects driven by supply-constrained firms (and some demand-constrained firms) that peak at the regional productivity level $\underline{\varphi}_j^{''}$. Hence, the regional employment effect of a national minimum wage is hump-shaped in regional productivity. This is a novel theoretical prediction which we take to the data using a transparent reduced-form methodology before we return to the model to establish the spatial general equilibrium.

3.2 Reduced-form evidence

To empirically evaluate the central prediction that the regional employment effect of the German national minimum wage is hump-shaped in regional productivity, we require estimates of the minimum wage effect by spatial units that are sufficiently small to exhibit sizable variation in average productivity. The empirical challenge in establishing the regional minimum wage effect is that the counterfactual outcome in the absence of the minimum wage is unlikely to be independent of the regional productivity level $\underline{\varphi}_j$. Consider the following data generating process (DGP):

$$\ln L_{j,t} = \left[\overline{f} + f(\underline{\varphi}_j)\right] I(t \geq \mathcal{J}) + \boldsymbol{a_j} + t\boldsymbol{b_j} + \epsilon_{j,t}, \tag{9}$$

where $\mathcal{J} = 2015$ is the year of the minimum wage introduction, $L_{j,t}$ is employment in area j in year t, $\boldsymbol{a_j}$ is a $1 \times J$ vector of regional fixed effects and $\boldsymbol{b_j}$ is a vector of parameters that moderate regional-specific time trends of the same dimension. $\boldsymbol{a_j}$ is likely positively correlated with employment since more productive regions attract more workers. Conditional on $\boldsymbol{a_j}$, $\boldsymbol{b_j}$ can be positively or negatively correlated with employment depending on whether the economy experiences spatial convergence or divergence. $\epsilon_{j,t}$ is a random error term. Unless we hold $\boldsymbol{a_j}$ and $t\boldsymbol{b_j}$ constant, we will fail to recover the correct conditional expectation $\mathbb{E}[\ln L_{j,t} | \varphi_j), t \geq \mathcal{J}] - \mathbb{E}[\ln L_{j,t} | \varphi_j), t < \mathcal{J}]$. To address this concern, we difference Eq. (9) twice to get

$$[\ln L_{j,t} - \ln L_{j,t-n}] - [\ln L_{j,t-n} - \ln L_{j,t-m}] = \Delta^2 \ln L_j = \overline{f} + f(\underline{\varphi}_j) + \tilde{\epsilon}_{i,t}, \tag{10}$$

where $t - n < \mathcal{J}$, $t - m < \mathcal{J}$ and $\tilde{\epsilon}_{i,t}$ is the twice differenced error term. Guided by the theoretical predictions summarized in Figure 4, we define the relative (up to the constant $\overline{f}$) before-after minimum wage effect as a polynomial spline function

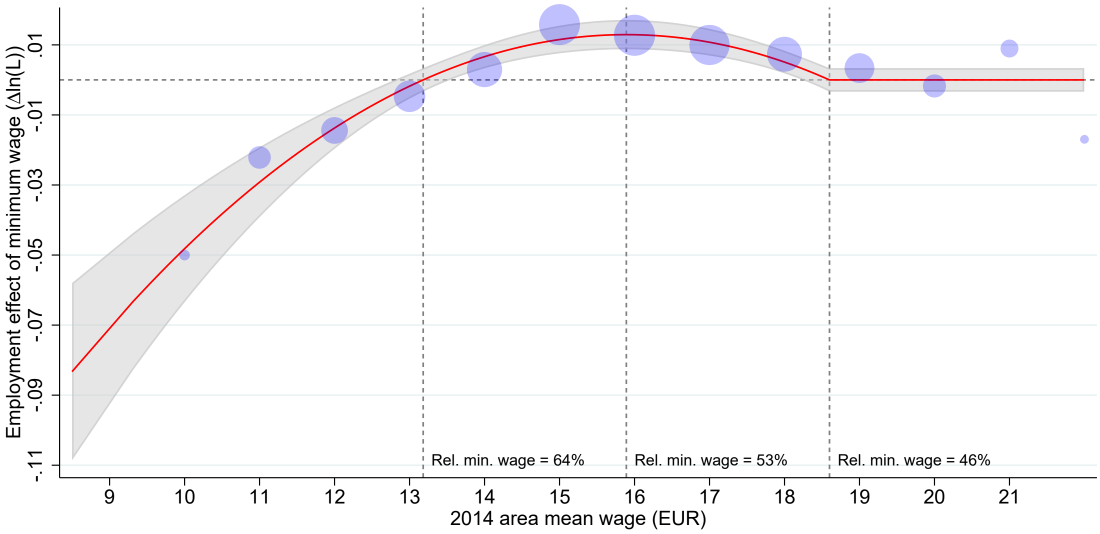
$$\begin{aligned} f(\underline{\varphi}_j) &= \mathbb{E}\left[\Delta^2 \ln L_j | w_j^{\text{mean}}, (w^{\text{mean}_j} \leq \alpha_0)\right] - \mathbb{E}\left[\Delta^2 \ln L_j | w_j^{\text{mean}}, (w^{\text{mean}_j} > \alpha_0)\right] \\ &= \mathbb{1}\left(w_j^{\text{mean}} \leq \alpha_0\right) \times \left[\sum_{g=1}^{2} \alpha_g \left(w_j^{\text{mean}} - \alpha_0\right)^g\right], \end{aligned} \tag{11}$$

with the theory-consistent parameter restrictions $\{\alpha_0 > \frac{\alpha_1}{2\alpha_2}, \alpha_1 < 0, \alpha_2 < 0\}$. Since higher fundamental productivity maps to higher wages in our model, we use the 2014 mean wage w_j^{mean} as a proxy for regional productivity. Notice that the interpretation of $f(\underline{\varphi}_j)$ is akin to the treatment effect in an intensive-margin difference-in-difference setting in which regions populated solely by unconstrained firms form a control group to establish a counterfactual.

Substituting in Eq. (11), we are ready to estimate Eq. (10) for given years $\{t, t-n, t-m\}$. To obtain parameter values $\{\alpha_0, \alpha_1, \alpha_2\}$, we nest an OLS estimation of $\{\alpha_1, \alpha_2\}$ in a grid search over a parameter space $\alpha_0 \in [\underline{\alpha}_o, \bar{\alpha}_o]$ and pick the parameter combination that minimizes the sum of squared residuals. From the identified parameters $\{\alpha_0, \alpha_1, \alpha_2\}$, there is a one-to-one mapping to regional mean wage levels that correspond to regional productivity levels $\{\underline{\varphi}_j^{'}, \underline{\varphi}_j^{''}, \underline{\varphi}_j^{'''}\}$ in Figure 4 (see Appendix C.6 for details). Note that consistent with the partial-equilibrium nature of the analysis, Eq. (11) lends a difference-in-difference interpretation to the predicted employment effect $\hat{f}(\underline{\varphi}_j)$ as regions dominated by unconstrained firms $(\underline{\varphi}_j \geq \underline{\varphi}_j^{'''}\})$ serve as the counterfactual.

In the DGP laid out in Eq. (9), we assume that the linear area-specific trends extend from the $[t-m, t-n]$ to the $[t-n, t]$ period. This assumption is more likely to be true over shorter study periods. Hence, we set $\{t = 2016, m = 4, n = 2\}$ in Figure 5, which restricts the comparison to two years before and after the minimum wage introduction. The results with one- or three-year windows are very similar (see Appendix C.6).

Figure 5: Regional minimum wage effects: Reduced-form evidence



Note: Dependent variable is the second difference in log employment over the 2012-14 and 2014-16 periods. Markers give averages within one-euro bins, with the marker size representing the number of municipalities within a bin. The last bin (22) includes all municipalities with higher wages because observations are sparse. The red solid line illustrates the quadratic fit, weighted by bin size. Two outlier bins are excluded to improve readability, but they are included in the estimation of the quadratic fit. Confidence bands (gray-shaded area) are at the 95% level. The relative minimum wage is the ratio of the 2015 minimum wage level $\underline{w} = 8.50$ over the 2014 mean wage (when there was no minimum wage).

Consistent with theory, we find an employment effect that is hump-shaped in the 2014 mean wage. The greatest positive employment effect is predicted for an area with a 2014 mean wage of about €16, which corresponds to $\underline{\varphi}_j^{''}$. The implication is that the regional employment effect is maximized for the area where the relative minimum wage amounts to €8.5/€16=53% of the mean wage. Municipalities with a lower mean wage, where the relative minimum wage is higher, have smaller predicted employment effects. At a relative minimum wage of 64% the predicted employment effect turns negative, a point that corresponds to $\underline{\varphi}_j^{'}$ in Figure 4. The empirical correspondent to productivity level $\underline{\varphi}_j^{'''}$—beyond which the minimum wage has no bite— is a regional mean wage of €18.6, which corresponds to a relative minimum wage of 46%.

The 46-64% range for the relative minimum wage derived in this section represents a first point of reference for those wishing to ground the minimum-wage setting in transparent reduced-form evidence of employment effects. Yet, the reduced-form approach constrains us to identifying relative employment effects. By assumption, we do not capture any general equilibrium effects that affect the control group (unconstrained regions). Moreover, the reduced-form approach naturally does not allow us to derive the welfare effect, which not only depends on the effects on wages and employment probabilities, but also on changes in commuting costs, tradable goods and housing prices. We, therefore, take the analysis to the spatial general equilibrium in the next section.

4 General equilibrium analysis

We develop the model in Section 4.1 and discuss the quantification in Section 4.2 before lay out how to use the model for quantitative counterfactual analyses in Section 4.3. Then, we proceed to a three-step application. First, we use the model to quantitatively evaluate the general equilibrium effects of the German minimum wage introduced in 2015 in a counterfactual analysis in Section 4.4.1. Second, we treat the model's predictions of changes in endogenous outcomes as forecasts that we subject to over-identification tests by comparing them to observed before-after changes in the data in Section 4.4.2. Third, we find the optimal minimum wage in a series of counterfactuals in which we consider a range of national and regional minimum wages in Section 4.5.

4.1 Model II

Building on the partial equilibrium framework introduced in Section 3.1, we now expand the model to account for the interaction of goods and factor markets, free entry of firms and an endogenous choice of workers to enter the labour market. We refer to $\bar{N}$ as the working-age population and denote the labour force measured at the place of residence i by N_i and the labour force measured at the workplace j by H_j. L_j represents employment (at the workplace) and can generally be smaller than the labour force when minimum wages are binding.

4.1.1 Preferences and endowments

Workers are geographically mobile and have heterogeneous preferences to work for firms in different locations. Given the choices of other firms and workers, each worker maximizes utility by choosing a residence location i and a (potential) employer φ_j – thereby pinning down the (potential) workplace location j. The preferences of a worker ν who lives and consumes in location i and works at firm φ_j in location j are defined over final goods consumption $Q_{i\nu}$, residential land use $T_{i\nu}$, an idiosyncratic amenity shock $\exp[b_{ij\nu}(\varphi_j)]$, and commuting costs $\kappa_{ij} > 1$, according to the Cobb-Douglas form

$$U_{ij\nu}(\varphi_j) = \frac{\exp[b_{ij\nu}(\varphi_j)]}{\kappa_{ij}} \left(\frac{Q_{i\nu}}{\alpha}\right)^{\alpha} \left(\frac{T_{i\nu}}{1-\alpha}\right)^{1-\alpha}. \tag{12}$$

The amenity shock captures the idea that workers can have idiosyncratic reasons for living in different locations and working in different firms (Egger et al., 2021). We assume that $b_{ij\nu}(\varphi_j)$ is drawn from an independent Type I extreme value (Gumbel) distribution

$$F_{ij}(b) = \exp(-B_{ij} \exp\{-[\varepsilon b + \Gamma'(1)]\}), \quad \text{with} \quad B_{ij} > 0 \quad \text{and} \quad \varepsilon > 0, \tag{13}$$

in which B_{ij} is the scale parameter determining the average amenities from living in location i and working in location j, ε is the shape parameter controlling the dispersion of amenities, and $\Gamma'(1)$ is the Euler-Mascheroni constant (Jha and Rodriguez-Lopez, 2021).

The goods consumption index Q_i in location i is a constant elasticity of substitution (CES) function of a continuum of tradable varieties

$$Q_i = \left[\sum_j \int_{\varphi_j} q_{ij}(\varphi_j)^{\frac{\sigma-1}{\sigma}} d\varphi_j \right]^{\frac{\sigma}{\sigma-1}} \tag{14}$$

with $q_{ij}(\varphi_j) > 0$ denoting the quantity of variety φ_j sourced from location j and $\sigma > 1$ as the constant elasticity of substitution. Utility maximization yields $q_{ij}(\varphi_j) = S_i^q p_{ij}(\varphi_j)^{-\sigma}$ with $S_i^q \equiv E_i^Q \left(P_i^Q\right)^{\sigma-1}$ as defined in Eq. (2), in which E_i^Q is aggregate expenditure in location i for tradables, P_i^Q is the price index dual to Q_i in Eq. (14), and $p_{ij}(\varphi_j)$ is the consumer price of variety φ_j in location i.

The economy is further endowed with a fixed housing stock $\bar{T}_i$. Denoting by E_i^T total expenditure for housing in location i, we can equate supply with demand, $T_i^D = E_i^T / P_i^T$, to derive the market-clearing price for housing:

$$P_i^T = \left(\frac{E_i^T}{\bar{T}_i} \right). \tag{15}$$

4.1.2 Free entry and goods trade

Firms learn their productivity φ_j only after paying market entry costs, $f_j^e P_j^T$, which consist of some start-up space f_j^e acquired at housing rent P_j^T. The investment is profitable whenever expected profits exceed these costs and we refer to this relation as the free-entry condition given by

$$\tilde{\pi}_j = \frac{\Pi_j}{M_j} = f_j^e P_j^T. \tag{16}$$

Using the facts that $\Pi_j = (1-\eta)[\Phi_j^\Pi(\underline{w})/\Phi_j^R(\underline{w})]R_j$ and that also the aggregate wage bill is proportional to revenues, $\tilde{w}_j L_j = [1-(1-\eta)\Phi_j^\Pi(\underline{w})/\Phi_j^R(\underline{w})]R_j$, we can reformulate Eq. (16) to get

$$M_j = \frac{\Phi_j^\Pi(\underline{w})(1-\eta)}{\Phi_j^R(\underline{w}) - \Phi_j^\Pi(\underline{w})(1-\eta)} \frac{\tilde{w}_j L_j}{P_j^T f_j^e}, \tag{17}$$

where

$$\tilde{w}_j = \frac{R_j - \Pi_j}{L_j} = \frac{1-(1-\eta)\Phi_j^\Pi(\underline{w})/\Phi_j^R(\underline{w})}{\eta} \frac{\chi_R \Phi_j^R(\underline{w})}{\chi_L \Phi_j^L(\underline{w})} w_j^u(\underline{\varphi}_j) \tag{18}$$

denotes the average wage rate in location j which is proportional to the cut-off wage $w_j^u(\underline{\varphi}_j)$ of an unconstrained firm with productivity $\underline{\varphi}_j$ given that $w_j^u(\varphi_j) l_j^u(\varphi_j)/\eta = r_j^u(\varphi_j) = \pi_j^u(\varphi_j)/(1-\eta)$.

With firm entry costs being paid in terms of housing and assuming that land owners spend their entire income on the tradable good, we can state that total housing expenditure in location i is given by $E_i^T = (1-\alpha)\tilde{v}_i N_i + \Pi_i$ and aggregate expenditure on tradable goods results as

$$E_i^Q = \alpha \tilde{v}_i N_i + E_i^T = \tilde{v}_i N_i + \Pi_i, \tag{19}$$

where $\tilde{v}_i$ is the average labour income of the residential labour force N_i across employment locations.

Building on optimal firm behaviour derived in Section 3.1, our model implies a gravity equation for bilateral trade between locations. Using the CES expenditure function and the measure of firms M_j, the share of location i's expenditure on goods produced in location j is given by

$$\begin{aligned} \theta_{ij} &= \frac{M_j \int_{\varphi_j} p_{ij}(\varphi_j)^{1-\sigma} dG(\varphi_j)}{\sum_{k \in J} M_k \int_{\varphi_k} p_{ik}(\varphi_k)^{1-\sigma} dG(\varphi_k)}, \\ &= \frac{M_j \Phi_j^P(\underline{w}) \left(\left\{ \Phi_j^L(\underline{w}) / [\Phi_j^R(\underline{w}) - (1-\eta)\Phi_j^\Pi(\underline{w})] \right\} \tau_{ij} \tilde{w}_j / \underline{\varphi}_j \right)^{1-\sigma}}{\sum_{k \in J} M_k \Phi_k^P(\underline{w}) \left(\left\{ \Phi_k^L(\underline{w}) / [\Phi_k^R(\underline{w}) - (1-\eta)\Phi_k^\Pi(\underline{w})] \right\} \tau_{ik} \tilde{w}_k / \underline{\varphi}_k \right)^{1-\sigma}}. \end{aligned} \tag{20}$$

To derive Eq. (20) we take advantage of the ideal price index $P_{ij} \equiv [\int_{\varphi_j} p_{ij}(\varphi_j)^{1-\sigma} d\varphi_j]^{1/(1-\sigma)}$ for the subset of commodities that are consumed in location i and produced in location j. As formally shown in Appendix C, it can be computed as

$$P_{ij} = \chi_P^{\frac{1}{1-\sigma}} \Phi_j^P(\underline{w})^{\frac{1}{1-\sigma}} M_j^{\frac{1}{1-\sigma}} p_{ij}^u(\underline{\varphi}_j), \tag{21}$$

with $\chi_P > 1$ as a constant and $\Phi_j^P(\underline{w}) > 0$ as a term that captures the aggregate effect of the minimum wage $\underline{w}$ on the price index P_{ij}. Notice that $\Phi_j^P(\underline{w}) = 1$ if the minimum wage $\underline{w}$ is not binding in location j. If the minimum wage is binding, $\Phi_j^p(\underline{w})$ can be larger or smaller than one, reflecting two opposing forces: Supply-constrained firms and highly productive demand-constrained firms lose their monopsony power and therefore set lower prices, which reduces the average price of firms from location j. At the same time, a binding minimum wage raises the costs – in particular for unproductive demand-constrained firms, which pass through this increase to their consumers in form of higher prices. The expenditure share θ_{ij} declines in bilateral trade costs τ_{ij} in the numerator ("bilateral resistance") relative to the trade costs to all possible sources of supply in the denominator ("multilateral resistance").

Using optimal prices together with Eqs. (18) and (21) to substitute for P_{ij}, $p_{ij}^u(\underline{\varphi}_j)$, and $w_j^u(\underline{\varphi}_j)$, into the price index $(P_i^Q)^{1-\sigma} \equiv \sum_j P_{ij}^{1-\sigma}$ dual to the consumption index in Eq. (14) we obtain

$$\begin{aligned} P_i^Q &= \frac{\chi_L}{\chi_R} \chi_P^{\frac{1}{1-\sigma}} \left\{ \sum_j M_j \Phi_j^P(\underline{w}) \left[\frac{\Phi_j^L(\underline{w})}{\Phi_j^R(\underline{w}) - (1-\eta)\Phi_j^\Pi(\underline{w})} \frac{\tau_{ij}\tilde{w}_j}{\underline{\varphi}_j} \right]^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}, \\ &= \frac{\chi_L}{\chi_R} \chi_P^{\frac{1}{1-\sigma}} \left[\frac{M_i \Phi_i^P(\underline{w})}{\theta_{ii}} \right]^{\frac{1}{1-\sigma}} \frac{\Phi_i^L(\underline{w})}{\Phi_i^R(\underline{w}) - (1-\eta)\Phi_i^\Pi(\underline{w})} \frac{\tau_{ii}\tilde{w}_i}{\underline{\varphi}_i}, \end{aligned} \tag{22}$$

which we can rewrite in terms of location i's own expenditure share θ_{ii}.

Location j's aggregate labour income $\tilde{w}_j L_j$ is proportional to aggregate revenue R_j in location j, which equals total expenditure on goods produced in this location:

$$\tilde{w}_j L_j = \frac{\Phi_j^R(\underline{w}) - (1-\eta)\Phi_j^\Pi(\underline{w})}{\Phi_j^R(\underline{w})} \sum_i \theta_{ij} \left(\tilde{v}_i N_i + \Pi_i\right). \tag{23}$$

4.1.3 Labour mobility, commuting, and labour supply

A worker's decision where to live, whether to enter the labour market and where to work depends on the indirect utility function $V_{ij\nu}(\varphi_j)$ dual to $U_{ij\nu}(\varphi_j)$ in Eq. (12) given by

$$V_{ij\nu}(\varphi_j) = \frac{\exp[b_{ij\nu}(\varphi_j)]}{\kappa_{ij}} \frac{\psi_j(\varphi_j) w_j(\varphi_j)}{\left(P_i^Q\right)^\alpha \left(P_i^T\right)^{1-\alpha}}, \tag{24}$$

in which the expected income of those seeking employment at firm φ_j in location j is the firm's wage rate $w_j(\varphi_j)$ evaluated at the hiring probability $\psi_j(\varphi_j)$. The probability that a worker chooses to live in location i and work in firm φ_j in location j then can be derived as

$$\lambda_{ij}(\varphi_j) = \frac{B_{ij} \left[\kappa_{ij} \left(P_i^Q\right)^\alpha \left(P_i^T\right)^{1-\alpha}\right]^{-\varepsilon} \left[\psi_j(\varphi_j) w_j(\varphi_j)\right]^\varepsilon}{\sum_r \sum_s B_{rs} \left[\kappa_{rs} \left(P_r^Q\right)^\alpha \left(P_r^T\right)^{1-\alpha}\right]^{-\varepsilon} \int_{\varphi_s} \left[\psi_s(\varphi_s) w_s(\varphi_s)\right]^\varepsilon d\varphi_s}. \tag{25}$$

The idiosyncratic shock to preferences $\exp[b_{ij\nu}(\varphi_j)]$ implies that individual workers choose different bilateral commutes and different employers when faced with the same prices and location characteristics. Other things equal, workers are more likely to live in location i and work for firm φ_j in location j, the lower the prices for consumption and housing P_i^Q and P_i^T in i; the higher the expected income $\psi_j(\varphi_j) w_j(\varphi_j)$ from working for firm φ_j in j; the more attractive average amenities B_{ij}; and the lower the commuting costs κ_{ij}. Summing across all residential locations i yields the probability that a worker is seeking employment at firm φ_j, $\lambda_j(\varphi_j) = \sum_i \lambda_{ij}(\varphi_j) = h_j(\varphi_j)/N$ with $N = \sum_i N_i$. The labour supply $h_j(\varphi_j)$ to firm φ_j therefore is given by Eq. (1) with

$$S_j^h \equiv \frac{\sum_i B_{ij} \left[\kappa_{ij} \left(P_i^Q\right)^\alpha \left(P_i^T\right)^{1-\alpha}\right]^{-\varepsilon}}{\sum_r \sum_s B_{rs} \left[\kappa_{rs} \left(P_r^Q\right)^\alpha \left(P_r^T\right)^{1-\alpha}\right]^{-\varepsilon} W_s^\varepsilon} N, \tag{26}$$

in which $W_j \equiv \{\int_{\varphi_j} [\psi_j(\varphi_j) w_j(\varphi_s)]^\varepsilon \, d\varphi_j\}^{\frac{1}{\varepsilon}}$ denotes an index of (expected) wages. In Appendix C.3, we demonstrate that W_j can be rewritten as a function of location j's cut-off wage $w_j^u(\underline{\varphi}_j)$, which according to Eq. (18) is proportional to the average wage $\tilde{w}_j$

in location j

$$\begin{aligned} W_j &= \chi_W^{\frac{1}{\varepsilon}} \Phi_j^W(\underline{w})^{\frac{1}{\varepsilon}} M_j^{\frac{1}{\varepsilon}} w_j^u(\underline{\varphi}_j) \\ &= \Omega_j(\underline{w}) \tilde{w}_j \frac{\chi_L}{\chi_R} \chi_W^{\frac{1}{\varepsilon}} M_j^{\frac{1}{\varepsilon}}, \text{where} \\ \Omega_j(\underline{w}) &\equiv \frac{\eta \Phi_j^W(\underline{w})^{\frac{1}{\varepsilon}} \Phi_j^L(\underline{w})}{\Phi_j^R(\underline{w}) - (1-\eta)\Phi_j^{\Pi}(\underline{w})} \end{aligned} \tag{27}$$

is a composite adjustment factor that captures various channels through which the minimum wage affects the wage index. Henceforth, we refer to $\Omega_j(\underline{w})\tilde{w}_j$ as *expected wage* for convenience. If the minimum wage $\underline{w}$ is not binding in location j, we have $\Phi_j^{X\in\{W,L,R,\Pi\}}(\underline{w}) = 1$ and, hence, $\Omega_j(\overline{w}) = 1$. If the minimum wage is binding, $\Omega_j(\underline{w})$ can be larger or smaller than one, reflecting two opposing forces: On the one hand, there is a direct effect captured by $\Phi_j^W(\underline{w})$. Because a binding minimum wage $\underline{w}$ exceeds the wages that supply- and demand-constrained firms would pay otherwise, the wage index increases. On the other hand, a binding minimum wage $\underline{w}$ causes demand-constrained firms to practice job rationing, such that the employment probability at these firms $\psi_j^d(\varphi_j)$ falls below one. If there are enough demand-constrained firms, the employment response captured by $\Phi_j^L(\underline{w})$ will be negative (dominating the positive response by supply-constrained firms). It is possible that a lower hiring rate more than compensates for rising wages so that the minimum wage causes the expected wage index to fall.

Aggregating $\lambda_{ij}(\varphi_j)$ across all firms φ_j in workplace j, we obtain the overall probability that a worker living in i applies to a firm in j, to which we refer as unconditional commuting probability.

$$\lambda_{ij} = \int_{\varphi_j} \lambda_{ij}(\varphi_j) d\varphi_j = \frac{B_{ij} M_j \left[\frac{\Omega_j(\underline{w})\tilde{w}_j}{\kappa_{ij}\left(P_i^Q\right)^{\alpha}\left(P_i^T\right)^{1-\alpha}}\right]^{\varepsilon}}{\sum_r \sum_s B_{rs} M_j \left[\frac{\Omega_s(\underline{w})\tilde{w}_s}{\kappa_{rs}\left(P_r^Q\right)^{\alpha}\left(P_r^T\right)^{1-\alpha}}\right]^{\varepsilon}}, \tag{28}$$

From Eq. (28), we obtain the residential choice probability λ_i^N and the workplace choice probability λ_j^H as $\lambda_i^N = \frac{N_i}{N} = \sum_j \lambda_{ij}$ and $\lambda_j^H = \frac{H_j}{N} = \sum_i \lambda_{ij}$, with $\sum_i \lambda_i^N = \sum_j \lambda_j^H = 1$. In order to solve for location j's aggregate employment L_j, we have to account for the fact that not all workers H_j, who are willing to work in j, will necessarily find a job. This is a novel feature in the context of quantitative spatial models and results in a labour-market clearing condition that equates the number of workers *working* at j, L_j to the number of workers *working or searching* in j, $\lambda_j^H N$, discounted by the employment probability Φ_j^L/Φ_j^H (which is equal to one in the absence of the minimum wage):

$$L_j = \frac{L_j}{H_j}\lambda_j^H N = \frac{\Phi_j^L(\underline{w})}{\Phi_j^H(\underline{w})}\lambda_j^H N, \tag{29}$$

with the second equality following from results derived in Appendix C.3 and $h_j^u(\underline{\varphi}_j) = l_j^u(\underline{\varphi}_j)$.

The average income of a worker living in location i depends on the expected wages in all employment locations. To construct this average income of residents, note first that the probability that a worker commutes to location j conditional on living in location i is given by:

$$\lambda_{ij|i}^{N} \equiv \frac{\int_{\varphi_j} \lambda_{ij}(\varphi_j) d\varphi_j}{\lambda_i^N} = \frac{B_{ij} M_j \left[\Omega_j(\underline{w}) \frac{\tilde{w}_j}{\kappa_{ij}}\right]^{\varepsilon}}{\sum_s B_{is} M_s \left[\Omega_s(\underline{w}) \frac{\tilde{w}_s}{\kappa_{is}}\right]^{\varepsilon}}, \tag{30}$$

in which ε can be interpreted as the elasticity of commuting flows with respect to commuting costs. Using these conditional commuting probabilities, we obtain the following condition that equates the measure of workers L_j employed in location j with the measure of workers that choose to commute to that location and that are successful in finding a job, namely,

$$L_j = \frac{L_j}{H_j} \sum_i \lambda_{ij|i}^N N_i = \frac{\Phi_j^L(\underline{w})}{\Phi_j^H(\underline{w})} \sum_i \lambda_{ij|i}^N N_i. \tag{31}$$

Expected worker income conditional on living in location i is then equal to the expected income in all workplaces weighted by the probabilities of being employed in those locations conditional on living in i:

$$\tilde{v}_i = \sum_j \lambda_{ij|i}^N \frac{L_j}{H_j} \tilde{w}_j = \sum_j \lambda_{ij|i}^N \frac{\Phi_j^L(\underline{w})}{\Phi_j^H(\underline{w})} \tilde{w}_j. \tag{32}$$

The expected utility, conditional on being active on the labour market, is

$$\overline{V} = \left\{ \sum_i \sum_j B_{ij} M_j \left[\frac{\Omega_j(\underline{\omega}) \tilde{w}_j}{\kappa_{ij} \left(P_i^Q\right)^{\alpha} \left(P_i^T\right)^{1-\alpha}} \right]^{\varepsilon} \right\}^{\frac{1}{\varepsilon}}. \tag{33}$$

4.1.4 Labour market entry

Workers have the discrete choice between entering the labour market and abstaining. Since workers do not observe the idiosyncratic residence-workplace-employer shock $b_{ijv}(\varphi_i)$ when deciding on entering the labour market, they compare the correctly anticipated expected utility from working in Eq. (33) to the expected leisure utility. Following the conventions in the discrete choice literature (McFadden, 1974), we assume that individuals have Gumbel-distributed idiosyncratic preferences for the two alternatives. As we formally derive in Appendix D.2, we can express the labour force participation rate as

$$\mu = \frac{\overline{V}^{\zeta}}{\overline{V}^{\zeta} + A}, \tag{34}$$

where ζ is the Gumbel shape parameter that is a transformation of the Hicksian extensive-margin labour supply elasticity, and A is the shift parameter that captures the leisure amenity. Intuitively, workers are more likely to abstain from the labour market if there are greater leisure amenities and if the utility from entering the labour market is lower.

Naturally, the labour force participation rate plays a key role in the aggregate labour market clearing condition

$$\sum_j H_j = \mu \overline{N}, \tag{35}$$

where the left-hand side represents the national labour force and $\overline{N}$ is the working-age population. Finally, the Gumbel distribution of idiosyncratic taste shocks implies that expected welfare across all workers (working, searching, and abstaining) takes the following form:

$$\overline{\mathcal{V}} = \left(A + \overline{V}^{\zeta}\right)^{\frac{1}{\zeta}} \tag{36}$$

4.1.5 General equilibrium

The general equilibrium of the model can be referenced by the following vector of seven variables $\{\tilde{w}_i, \tilde{v}_i, M_j, P_i^T, L_i, N_i, P_i^Q\}_{i=1}^J$ and the scalars $\{\mu, \overline{V}\}$. Given the equilibrium values of these variables and scalars, all other endogenous objects can be determined conditional on the model's primitives. This equilibrium vector solves the following seven sets of equations: income equals expenditure from Eq. (23); average residential income from Eq. (32); firm entry from Eq. (17); housing market clearing from Eq. (15); aggregate local employment from Eq. (31); $N_i = \lambda_i^N N$ based on Eq. (28) and the price index from Eq. (22). The conditions needed to determine the scalars $\{\mu, \overline{V}\}$ are labour force participation from Eq. (34) and the labour market clearing condition from Eq. (35).

4.2 Quantification

The primitives of the model consist of the structural parameters $\{\underline{w}, k, \alpha, \sigma, \epsilon, \zeta, \mu\}$ and the structural fundamentals $\{\tau_{ij}, \kappa_{ij}, B_{ij}, \underline{\varphi}_j, \bar{T}_i, f_j^e, A\}$. If these primitives are given alongside the endowment $\{\bar{N}\}$, we can solve for the variables $\{\tilde{w}_i, \tilde{v}_i, P_i^T, L_i, N_i, P_i^Q, M_i\}_{i=1}^J$ and the scalars $\{\mu, \overline{V}\}$ that reference the general equilibrium. We quantify the model using data from 2014, the year before the minimum wage introduction. Therefore, we can treat all firms as unconstrained and set $\underline{w} = 0$ in the quantification, which implies that $\Phi_j^{X \in \{L,H,R,P,W,\Pi\}} = 1$. We borrow $\{\alpha, \zeta\}$ from the literature and set σ such that all parameter restrictions of the model are satisfied. We infer all other primitives from the data using observed values of $\{P^T, \lambda_{ij} N_i, M_j, w_j(\omega), \tilde{w}_j, (p_{ij} q_{ij}), \mu\}$. We provide a brief discussion below and refer to Appendix Section D.3 for details.

Expenditure share on housing $(1-\alpha)$. We set the housing expenditure share to $1 - \alpha = 0.33$, which is in line with a literature summarized in Ahlfeldt and Pietrostefani (2019) and official data from Germany (Statistisches Bundesamt, 2020).

Labour force participation rate (μ). We use the 2014 employment rate of $\mu = 73.6\%$ reported by the German Federal Statistical Office.

Working-age population $(\bar{N})$. Based on the labour force participation rate and total employment in in 2014, we get $\bar{N} = N/\mu$.

Reservation utility heterogeneity (ζ). As we show in Appendix D.2, we can express the heterogeneity of idiosyncratic shocks to the utility from non-employment ζ as a function of the Hicksian extensive-margin labour supply elasticity $\tilde{\zeta}$ and the labour force participation rate μ. Setting the former to the canonical value of $\tilde{\zeta} = 0.2$ in the literature (Chetty et al., 2011) and the latter to the value observed in German data, we obtain $\zeta = 0.8$.

Preference heterogeneity (ε). We use a novel estimation strategy that leverages on our firm-level data. We exploit the firm-level wage and firm size scale in firm productivity at elasticities that differ by multiplicative factor ε (see Table A3). This allows us to obtain a theory-consistent estimate of ε from an establishment-level regression of the log of wage against the log of employment, controlling for area fixed effects. Our estimate of $\varepsilon = 5.2$ is in between (Monte et al., 2018), who use larger spatial units, and (Ahlfeldt et al., 2015), who use smaller spatial units. We refer to Appendix D.3.1 for details.

Productivity heterogeneity and elasticity of substitution (k, σ). Intuitively, we identify k by fitting a Pareto cumulative distribution function (CDF) of wages as conventional in the trade literature (Arkolakis, 2010; Egger et al., 2013). We take a structural approach to the estimation of k because $\{k, \sigma, \epsilon\}$ jointly determine the dispersion of wages and the regional lower-bound wage, conditional on observed values of $\tilde{w}_j$. The conventional reduced-form approach in the literature emphasizes the dispersion. However, in the context of the minimum wage evaluation, the left tail of the distribution is of particular relevance. Moreover, the aggregation of firm level outcomes requires several parameter constraints to be met. Therefore, we take our estimate of $\varepsilon = 5.2$ as given and nest the estimation of k using a GMM estimator into a grid search over σ values. We choose $\sigma = 1.5$ as the value that is closest to the conventions in the literature and still satisfies all parameter restrictions of the model. Conditional on these values for $\{\epsilon, \sigma\}$, we obtain an estimate for k of 0.53. These values are smaller than the typical values found in the trade literature (Egger et al., 2013; Simonovska and Waugh, 2014), but they ensure that we obtain a decent fit of the wage distribution in the left tail. We refer to Appendix D.3.2 for details.

Minimum wage ($\underline{\mathbf{w}}$). Since we use the worker-weighted mean wage as the numeraire in our model, it is straightforward to define the minimum wage in relative terms as $\underline{w} = 0.48$, which is the share of the minimum wage at the national minimum wage observed in the data (across full time and part-time workers). Notice that this share remains remarkably constant over time, suggesting that the adjustments to the absolute minimum wage level made in 2017, 2019, and 2020 aimed at keeping the relative level constant.

Trade cost (τ_{ij}). We estimate a gravity equation of bilateral trade volumes ($p_{lk}q_{lk}$) between county pairs lk within Germany allowing for an direction-specific inner-German border effect and origin-specific distance effects. Using the estimated reduced-form parameters and our set value of σ we predict τ_{ij} at the bilateral area level in a theory-consistent way. We refer to Appendix Section D.3.3 for details. With this approach, we account for the legacy of German cold war history and the centrality bias in inter-city trade (Mori and Wrona, 2021).

Fundamental productivity ($\underline{\varphi_j}$). Given observed values of $\{L_j, N_i, \lambda^N_{ij|i}, \tilde{w}_j, M_j\}$, the set or estimated values of $\{\varepsilon, \sigma\}$, the predicted values of τ_{ij}, and exploiting that $\tilde{v}_j = \sum_j^J \lambda^N_{ij|i}\tilde{w}_j$, we can invert $\underline{\varphi}_j$ from Eq. (23) (substituting in Eq. (20)) using a conventional fixed-point solver. We refer to Appendix Section D.3.4 for details.

Ease of commuting ($B_{ij}\kappa_{ij}^{-\varepsilon}$). Following Monte et al. (2018), we refer to the composite term $B_{ij}\kappa_{ij}^{-\varepsilon}$ as ease of commuting since, conditional on a given residence i, it captures the attractiveness of commuting to a destination j holding the number of firms M_j and workplace wages $\tilde{w}_j$ constant. Given values of $\{\alpha, \varepsilon, \sigma, k, \tau_{ij}, \underline{\varphi}\}$ and observed values of $\{\lambda^N_{ij|i}, M_j, \tilde{w}_j, P_i^T\}$, we invert $B_{ij}\kappa^{-\epsilon}$ using the unconditional commuting probabilities λ_{ij} using Eq. (28) and a conventional fixed-point solver.

Start-up space (f_j^e). Given values of $\{\varepsilon, \sigma\}$ and observed values of $\{M_j, P_j^T, \tilde{w}_j, L_j\}$ it is straightforward to invert the start-up space firms need to acquire to enter the market, f_j^e, using the firm-entry condition in Eq. (16).

Housing supply ($\overline{T}_i$). For given values of $\{\lambda^N_{ij|i}, \tilde{w}_i, L_i\}$, we can exploit that Π_i scales at known parameters in $w_i L_i$ along with $\tilde{v}_j = \sum_j^J \lambda^N_{ij|i}\tilde{w}_j$ and $E_i^T = (1-\alpha)\tilde{v}_i + \Pi_i$ to infer housing supply $\overline{T}_i$ using the housing market clearing condition in Eq. (15).

Leisure amenity (A). Using observed values of $\{\mu_i, M_j, \tilde{w}_j, P_i^T\}$, inverted values of $\{\underline{\varphi}_j, \tau_{ij}, B_{ij}\kappa_{ij}^{-\varepsilon}\}$ and the estimated and set parameter values for $\{\alpha, \varepsilon, \sigma, \zeta, k\}$, we invert fundamental utility A using Eqs. (22), (33) and (34).

4.3 Quantitative analysis

Given the fully quantified model, the evaluation of the effects of an exogenous change in the minimum wage $\underline{w}$ on the vector of endogenous outcomes that references the general equilibrium $\mathbf{X} = \{\tilde{w}_i, \tilde{v}_i, P_i^T, L_i, N_i, P_i^Q, M_i, \mu, \overline{V}\}$ can be established by solving the model under different values of $\underline{w}$, holding all other primitives constant. We model the solution as a fixed point for which we solve using a conventional numerical procedure that we discuss in Appendix D.4.

We first solve the model for $\underline{w} = 0$ expressing all endogenous goods and factor prices in terms of the worker-weighted mean, which becomes the numeraire. This delivers equilibrium values of the vector of endogenous outcomes which we denote by $\mathbf{X^0}$. We use these, instead of the observed values in the data, because the lower-bound fundamental productivity $\underline{\phi}_j$ is identified up to a constant. The normalization of nominal wages does not affect the interpretation of real wages, which are relevant for welfare. We then set $\underline{w}$ to the desired value (in units of the numeraire) and solve the model for a vector of counterfactual outcomes $\mathbf{X^C}$. With this approach, we acknowledge that policy makers set minimum wages that are routinely adjusted to maintain purchasing power. Using conventional *exact hat algebra* notations (Dekle et al., 2007), we can express the relative change in endogenous outcomes as $\widehat{\mathbf{X}} = \frac{\mathbf{X^C}}{\mathbf{X^0}}$ and the absolute change as $\Delta\mathbf{X} = \widehat{\mathbf{X}} \cdot \mathbf{X^D}$, where $\mathbf{X^D}$ indicates values observed in data. Whenever we refer to welfare, we use the expected utility $\overline{\mathcal{V}}$ from Eq. (36) which takes into account workers inside (working and searching) and outside the labour market.

We follow the canonical approach in the spatial equilibrium literature and pin down residential location choices by assuming perfect mobility, which results in a spatially invariant welfare $\overline{\mathcal{V}}$ (Roback, 1982). However, the assumption that residents are perfectly mobile across residential locations is obviously more plausible in the long-run than in the short-run. Therefore, we also evaluate a special case that approximates short-run spatial equilibrium adjustments. To this end, we make workers immobile across residences. This restriction is straightforward to implement in our counterfactual by solving the model conditional on holding $\{\bar{N}_i\}$ constant at the values observed in data. For further details on the short-run evaluation, we refer to Appendix Section D.4.2.

4.4 The German minimum wage

We now use the model to quantitatively evaluate the effects of the German minimum wage in general equilibrium. In Section 4.4.1, we use the procedure outlined in Section 4.3 to predict the effects a federal minimum wage of 48% of the national mean wage (the value we observe in data) has on endogenous model outcomes. Because migration costs are high (Koşar et al., 2021), relocations across local labour markets are rare events (Ahlfeldt et al., 2020). Since it is unlikely that workers have fully re-optimized their residential location choices as of now, we provide a short-run evaluation in which residents are immobile across residential areas (but mobile across workplaces) and a *long-run* evaluation in which residents are fully mobile. In Section 4.4.2, we compare the predicted effects to observed before-after changes in our data. Note that our model-based counterfactuals deliver forecasts in the sense that they are based solely on data observed before the introduction of the minimum wage. Hence, the comparison of the model's predictions to changes in data represents an over-identification test that allows us to evaluate the out-of-sample predictive power of our model.

4.4.1 Model-based counterfactuals

In Table 1, we summarize the simulated short-run and long-run effects of the German minimum wage on various endogenous outcomes. We report the worker-weighted average across regions as well as the regional minimum and maximum values. The high-level conclusion is that the minimum wage increases welfare at the cost of reducing employment. Given a workforce of approx. 30M, the 0.3%-reduction in employment translates into about 100k jobs lost, which is less than extant ex-ante predictions based on competitive labour market models (Knabe et al., 2014). Applying the relative welfare effect of about 3%

to the 2018 average annual wage of €34.4K to about 30M workers, we can monetize the aggregate welfare effect as equivalent to an increase in annual worker income of about €30BN.

This increase in welfare is driven by an increase in *expected* real wage, i.e. higher real wages more than compensate for lower employment probabilities. This is why the aggregate labour force increases by about 0.6%. Intuitively, these workers become active on the labour market because they prefer well-paid jobs with a lower hiring probability over badly paid jobs with a higher hiring probability. Thus, although the number of jobs decreases, the number of workers *working* or *searching* increases by about 180K. Notice that the near-zero effect on the mean wage is an artifact of the choice of the numeraire in our model: the worker-weighted average of regional wages. Since, expressed in units of this numeraire, tradable goods prices and real housing rents decrease, real wages actually increase. Ease of commuting is another source of the positive welfare effect, implying that workers find jobs in more convenient reach. Since Figure 2 does not point to a significant reduction in commuting distance, it is likely that the increase in ease of commuting is driven by workers finding jobs at places that are well connected by transport infrastructure. In contrast, the reduction in the number of establishments has negative welfare effects because the chance of a good worker-firm match decreases. The mean welfare effects on all workers, at 2.2%, $(\mathcal{V})$ is smaller that the 2.9%-effect on those working (V) because about a quarter of the working-age population abstains from the labour market and, hence, experiences no welfare effect.

Table 1 also reveals that the national averages mask striking spatial heterogeneity. Some municipalities experience substantial job loss whereas employment increases in others. This mirrors a highly heterogeneous increase in real wages. While the regional spread in short-run and long-run effects is mostly similar, there are two important exceptions. When we fix worker residences in the short run, we essentially switch off an important margin of the spatial arbitrage process. Within each area, the size of the labour force can only change due to workers entering or exiting the labour market. This rules out migration-induced adjustments in wages and rents that would equalize utility. As a result, there is significant spatial heterogeneity in the welfare incidence. When we allow for free residential choices in the long run, migration-induced spatial arbitrage equalizes the welfare incidence, but we observe much greater changes in the spatial distribution of the labour force.

We dig deeper into the spatial heterogeneity in minimum wage effects in Figure 6, where we correlate our simulated relative changes in selected outcomes to the 2014 mean wage observed in our data. As expected, the employment effect follows the hump-shaped pattern that we have derived theoretically and substantiated empirically in partial equilibrium in Section 3. It is straightforward to eyeball the critical points introduced in Figure 4. For regions where the 2014 mean hourly wage exceeds €19, the employment effect is flat in the initial regional wage $(\underline{\phi}_j^{'''})$. We find the most positive employment response for

Table 1: Short-run and long-run effects of the German minimum wage

	Short run			Long run		
	Mean	Min	Max	Mean	Min	Max
Panel a: Employment						
Employment at workplace (L)	-0.250	-21.31	5.350	-0.350	-25.91	5.810
Labour supply at residence (N)	0.590	0.120	1.550	0.590	-6.540	14.23
Employment probability (L/H)	-0.820	-19.99	0	-0.880	-21.15	0
Panel b: Wage and prices						
(Normalized) wage ($\tilde{w}$)	0.320	-1.360	25.72	0.390	-1.310	24.70
Real tradables price index (P^Q)	-3.040	-4.620	-2.200	-2.930	-5.630	-1.600
Real housing rent (P^T)	-1.040	-7.170	1.100	-1.070	-5.390	2.520
Panel c: Welfare components						
Exp. real wage $\tilde{v}\left[(P^Q)^{\alpha}(P^T)^{(1-\alpha)}\right]$	1.620	-0.260	5.510	1.630	0.370	4.350
# establishments (M)	-0.100	-7.290	0.920	-0.120	-16.43	2.770
Ease of commuting ($B\kappa^{-\epsilon}$)	1.160	-4.290	7.090	0.880	-14.04	8.440
Panel d: Welfare						
Worker welfare $\mid$ working (V)	2.910	0.560	7.830	2.860	2.860	2.860
Worker welfare, all ($\mathcal{V}$)	2.150	0.410	5.770	2.110	2.110	2.110

Notes: All outcomes are given in terms of % changes. Mean is the mean outcome across municipalities, weighted by initial workplace or residence employment. Min and max are minimum and maximum values in the distribution across municipalities. Short run gives simulation results when workers are immobile across residences whereas long-run results allow workers to be fully mobile. Outcomes are normalized by the mean wage across all municipalities. Expected real wage effect captures the direct (positive) effect of the minimum wage on wages and the effect on the hiring probability that can be negative in municipalities with sufficient demand-constrained firms.

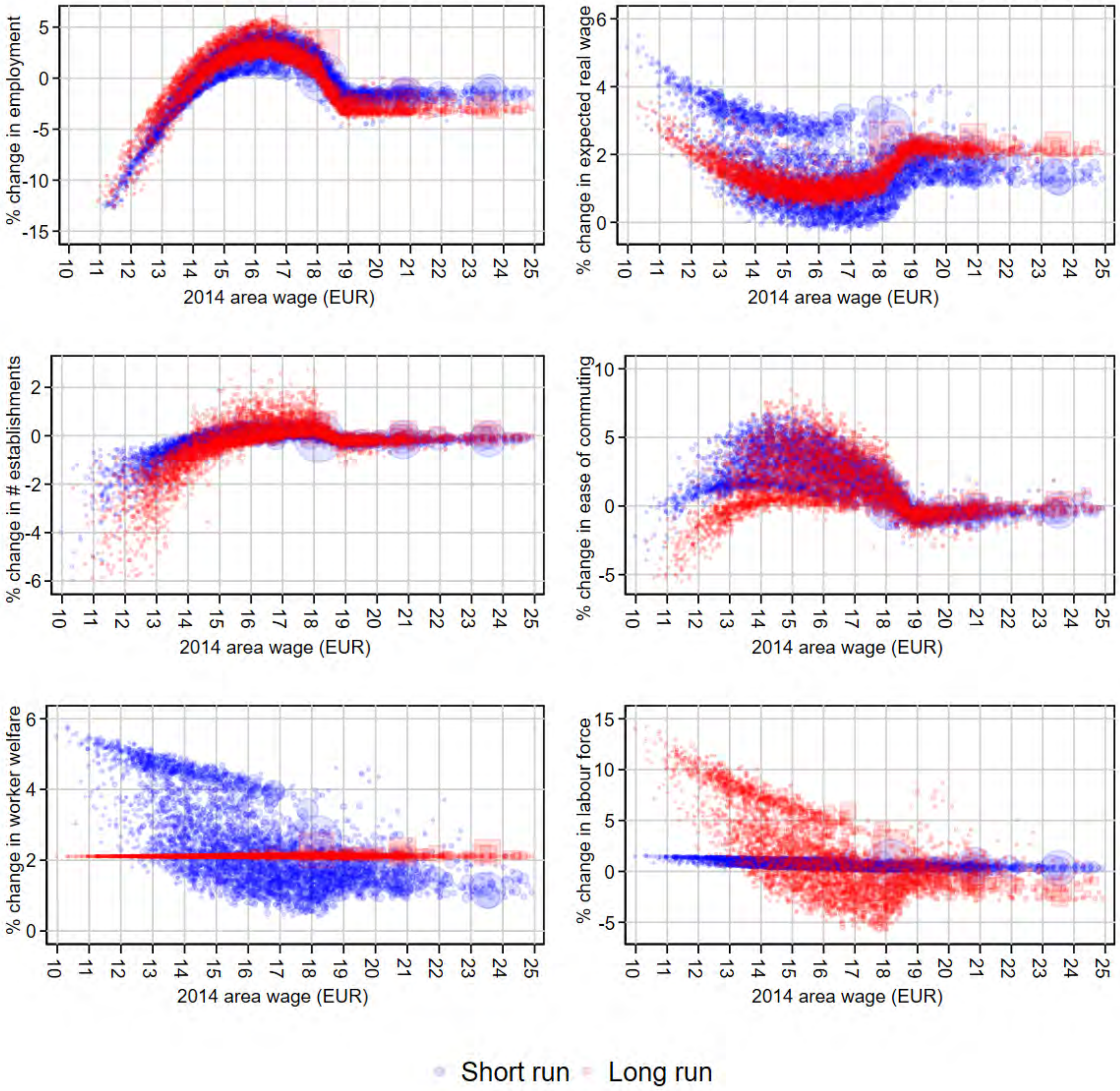
regions where the wage is about €16 ($\underline{\phi}_j''$). For regions where the wage is below €13 ($\underline{\phi}_j'$)), the employment effects tend to be more negative than for the high-productivity regions. Reassuringly these critical points derived from model-based counterfactuals are close to those in Figure 5, which are based on a reduced-form before-after comparison.

Our general equilibrium analysis adds the important insight that there is a negative level effect on unconstrained regions in the right tail of the regional productivity distribution, which is unidentifiable with the reduced-form approach in Figure 5. Intuitively, the expansion of employment and production in municipalities of intermediate productivity comes at the expense of the most productive municipalities since aggregate demand is, albeit endogenous, finite. In keeping with intuition, this displacement effect is reinforced by worker mobility in the long-run.

The effect on the number of establishments follows the employment effect qualitatively. The effect on the ease of commuting also has a hump shape. This is consistent with a reallocation effect of workers towards more productive establishments further away from their residences in low-productivity regions (Dustmann et al., 2022). In contrast, the ease of commuting increases in municipalities of intermediate productivity, revealing that workers find attractive employment opportunities that are more convenient to reach.

While low-productivity regions are those that experience the largest decline in employment, they are also those where the minimum wage has had the greatest effect on wages. Figure 7 shows that municipalities experiencing real wage growth and a reduction in em-

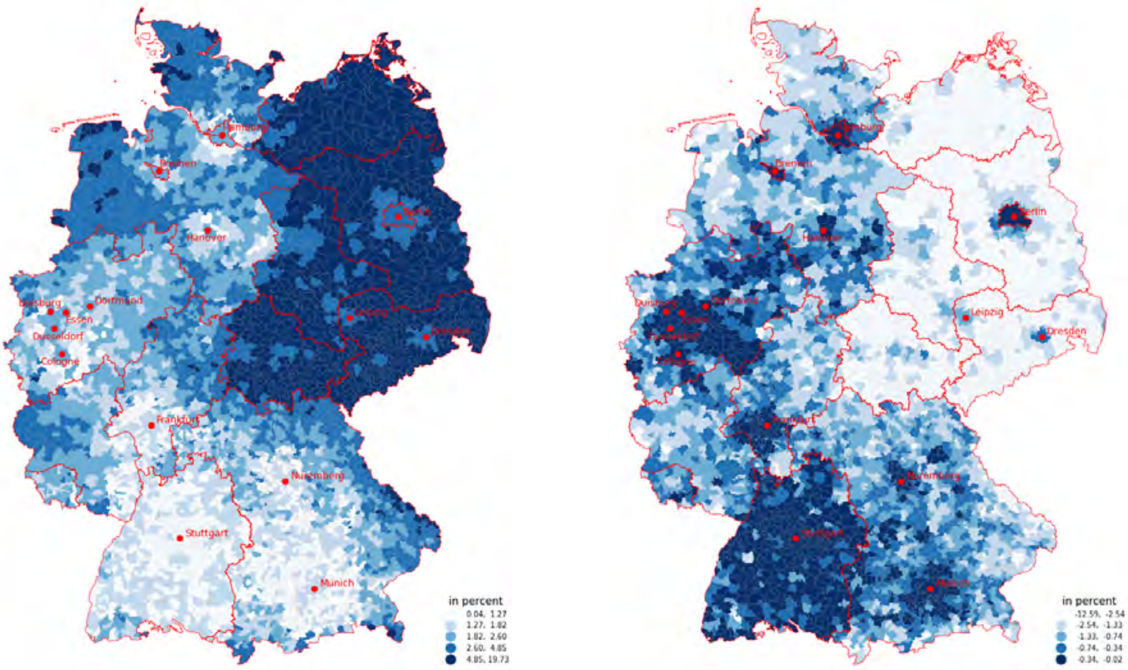
Figure 6: Short-run and long-run effects by regional productivity



Note: Each icon represents one outcome for one area (*Verbandsgemeinde*). Results of model-based counterfactuals comparing the equilibrium under a federal minimum of 48% (the value observed in data) of national mean wage to the equilibrium with a zero minimum wage. *Blue circles* show outcomes when workers are immobile across residences (short run). *Red squares* show outcomes when workers are mobile across residence (long run). Expected real wage is measured at the residence and incorporates changes in (normalized) nominal wages at workplace, employment probabilities at workplace, bilateral commuting probabilities, housing rents at residence, and tradable goods prices at residence. For a more intuitive interpretation, we multiply the normalized regional mean wage on the x-axes by the 2014 national mean wage. To improve the presentation, we crop the right tail of the regional productivity distribution (about one percent).

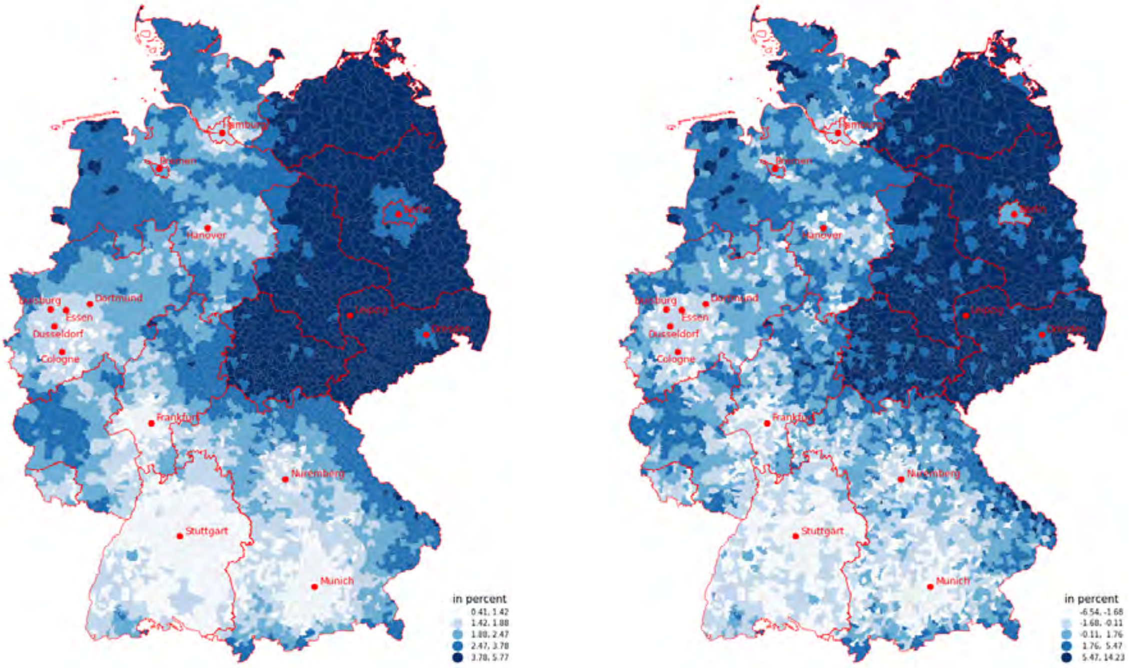
ployment probability are over-represented in the east (resembling the minimum wage bite in Figure 1). Because the former dominates the latter, expected real wages increase (see top-middle panel in Figure 6). The bottom panels of Figure 6 and 7 illustrate how, as a result, welfare increases in the short run and the labour force increases in the long run. The important take-away for policy is that the German minimum wage has disproportionately improved welfare in economically weak municipalities, but the effect will become more uniform in the long run as workers re-optimize their location choices. That said, population growth in economically weaker regions may well represent a policy objective in its own right, especially in Germany where there has been substantive out-migration from former East Germany after the fall of the iron curtain.

Figure 7: Regional effects of the German minimum wage



(a) Real wage, short run

(b) Employment probability, short run



(c) Welfare, short run

(d) Labour force, long run

Note: Unit of observation are 4,421 municipality groups. Results from model-based counterfactuals are expressed as percentage changes. All outcomes are measured at the place of residence. To generate the data displayed in panels a) and b), we break down residential income from Eq. (32) into two components. The first is the residential wage conditional on working $\sum_j \lambda_{ij|i}^N \tilde{w}_j$, which we normalize by the consumer price index (the weighted combination of goods prices and housing rent) to obtain the real wage. The second is the residential employment probability $\sum_j \lambda_{ij|i}^N L_j/H_j$, which captures the probability that a worker finds a job within the area-specific commuting zone.

4.4.2 Comparison to data

To evaluate whether the model successfully predicts observed changes in the data, we use the area-based minimum wage effects discussed in Section 4.4.1 as treatment variables in a conventional dynamic difference-in-differences estimation. Intuitively, this approach compares before-after changes in selected outcomes in the data to respective before-after changes predicted by the model. For further detail on the empirical specification and the interpretation of the estimated treatment effect, we refer to Appendix D.5.

Figure 8 summarizes the results. The first insight is that the before-after changes in regional mean hourly wage and employment observed in data converge towards the predictions of the model over time. One interpretation is that compliance has been imperfect, but increasing over time. Imperfect compliance with minimum wage laws is a well-known phenomenon (Ashenfelter and Smith, 1979) that can mitigate employment effects (Garnero and Lucifora, 2021). While Germany is no exception (Mindestlohnkommission, 2020), evidence from labour force surveys suggests that compliance has increased over time (Weinkopf, 2020).

In contrast, workers do not seem to have started to relocate to regions with positive short-run welfare gains within the first four years of the policy. If anythings, those regions, which are mostly located in the East (see Figure 7), have continued to lose population, suggesting that migration decisions have been dominated by other forces. Notice that the correlations with the short-run predictions need to be taken with a grain of salt since—given fixed residential locations—the model predicts only small changes in the local labour force (see Table 1) owing to some workers starting searching for jobs, for which we do not have a good equivalent in the data.

For commuting distances, both short-run and long-run predictions are positively correlated with before-after changes in the data, but, as time goes by, the long-run predictions start outperforming the short-run predictions. Given that few workers appear to have re-optimized their residential locations with respect to the effects of the minimum wage, it is no surprise that we find positive correlations between predicted and observed changes for house prices based on the short-run predictions, but not for the long-run predictions.[10] Finally, a pooled regression in which we evaluate the predictive power for all outcomes simultaneously suggests that, overall, our short-run predictions provide a better description of the minimum-wage impact. We conclude that, four years after the introduction in Germany, the full effect of the minimum wage law has not yet materialized in the data.

This impression is confirmed by another metric that is of first-order relevance in the context of minimum wage laws: The Gini coefficient of nominal wage inequality (across all workers in all regions), which we can derive within our model as discussed in Appendix D.6.1. Our model predicts a short-run reduction of the Gini coefficient of about two percentage points (from 32.7% to 30.7%). This is qualitatively and quantitatively in line with an empirically observed steady decline in the Gini coefficient from 30.7% to 29.1%

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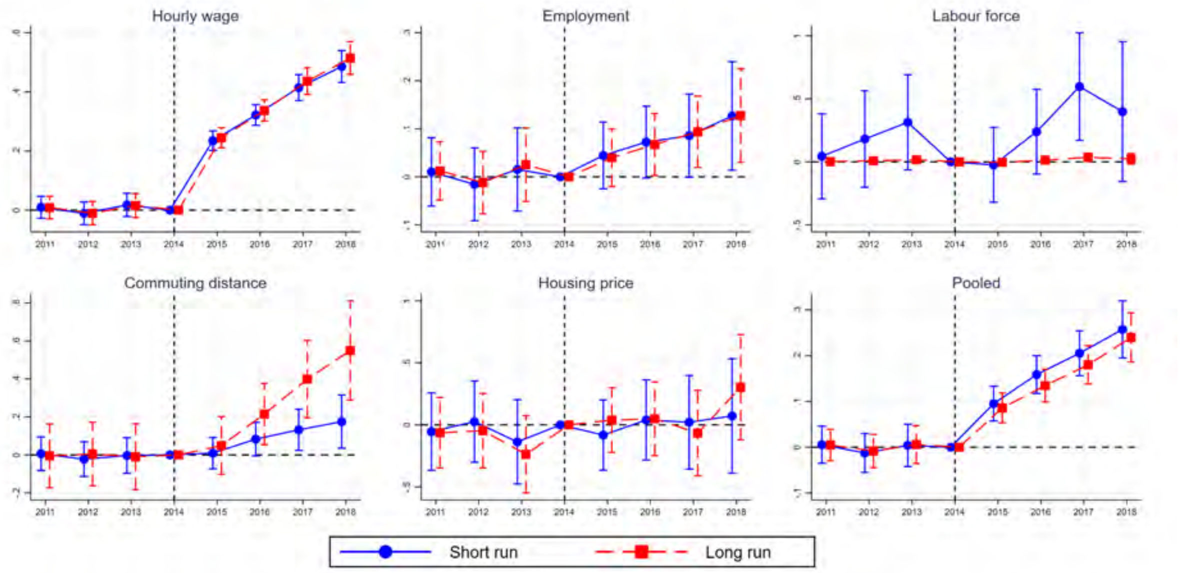
[10] This confirms Yamagishi (2021), who shows that desirable minimum wages increase housing rents.

Figure 8: Regional minimum wage effects: Model vs. data



Note: For each panel, we run a regression of the log of an outcome variable against the log of the relative change (the ratio of the model-predicted outcome with the minimum wage over the baseline) interacted with year dummies (omitting 2014 as the baseline), controlling for area and year-by-zone (former East and West Germany) dummies. Prior to this regression, we adjust all area-level time series for the pre-minimum wage time trend following Monras (2019). Icons denote point estimates. Error bars give 95% confidence bands. In the bottom-left panel, we pool over all outcomes, using area-by-outcome and year-by-zone-by-outcome fixed effects.

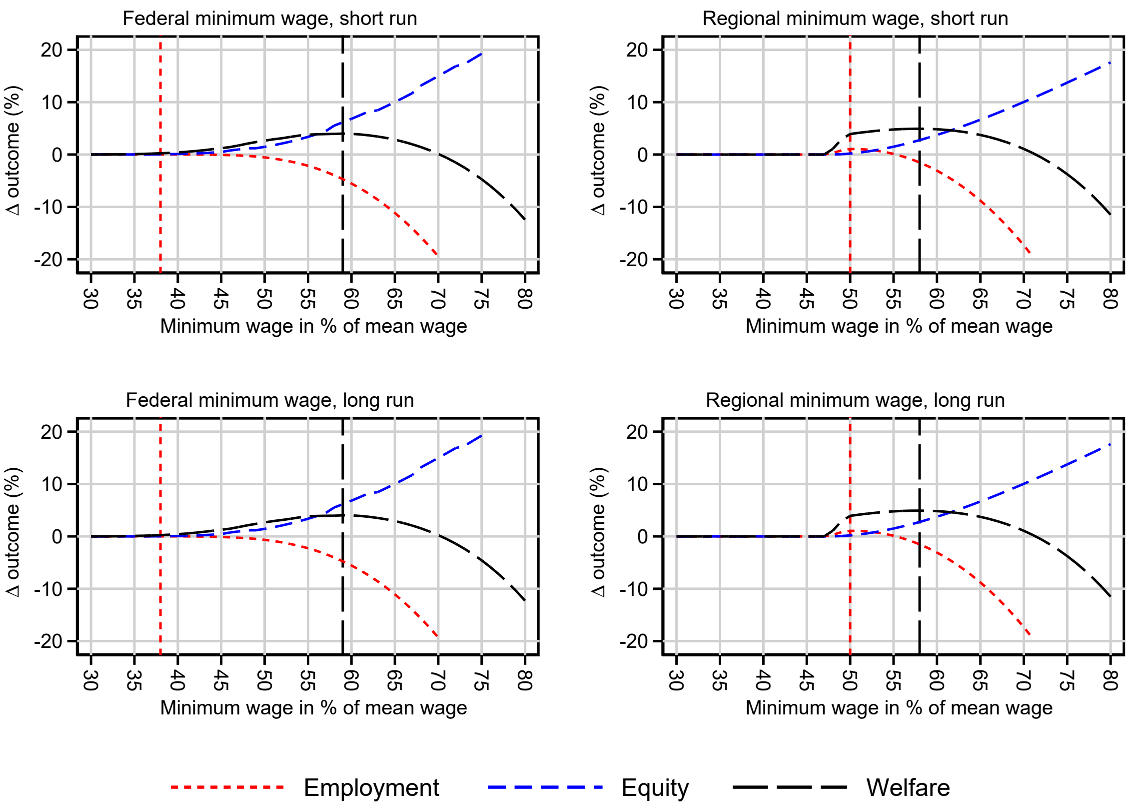
during the first three years of the minimum wage (see Appendix D.6.2 for details).

4.5 Optimal minimum wages

We now turn from the positive evaluation of the effects of the German minimum wage to a normative evaluation of optimal minimum wages. To this end, we conduct a series of counterfactual exercises using the procedure outlined in Section 4.3. We evaluate two alternative minimum wage schedules that are fairly straightforward to implement from a policy perspective. For one thing, we consider a $1 \times \mathcal{N}$ vector of uniform relative *national* minimum wages $\underline{\mathbf{w}}^{\mathbf{n}} \in (0.3, 0.31, ...0.8)$ that correspond to a fraction of the national mean wage, the numeraire in our model. For another, we consider a $J \times \mathcal{N}$ vector of *regional* minimum wages $\underline{\mathbf{w}}^{\mathbf{r}}_{\mathbf{j}} = \mathbf{w}^{\mathbf{m}}_{\mathbf{j}} \cdot \underline{\mathbf{w}}^{\mathbf{n}}$ that represents the regional minimum wage as a fraction of the $J \times 1$ vector of regional mean wages $\mathbf{w}^{\mathbf{m}}_{\mathbf{j}}$. As in Section 4.3, we evaluate the effects of both minimum wage schedules in a short-run scenario in which residential locations are fixed, and in a long-run scenario in which workers are fully mobile.

The most obvious optimality criterion for a successful minimum wage policy in the context of our model is expected worker welfare as defined in Eq. (36). Since the literature on minimum wages is very much concerned with employment effects, we also evaluate the aggregate employment effect. In practice, one of the main policy objectives associated with minimum wages is a reduction in income inequality. Therefore, we also report an equity measure $1 - \mathcal{G}$, where $\mathcal{G}$ is the Gini coefficient of nominal wage inequality across all workers in all regions (see Appendix D.6 for details). Figure 9 summarizes how employment,

Figure 9: Minimum wage effects on employment, equity, and welfare



Note: Results of model-based counterfactuals. Employment is the total number of workers in employment. Equity is measured as 1-$\mathcal{G}$, where $\mathcal{G}$ is the Gini coefficient of real wage inequality across all workers in employment. Welfare is the expected utility of as defined by Eq. (36). It captures individual who are active on and absent from the labour market and accounts for minimum wage effects on employment probabilities, wages, tradable goods prices, housing rents, commuting costs, and worker-firm matching qualities. In the short run, workers are immobile across residence locations whereas workers re-optimize their residential location choice in the long run.

equity, and welfare effects vary in the level of a federal or regional minimum wage. We also compare the employment, equity, and welfare effects of employment-maximizing and welfare-maximizing federal and regional minimum wages to the effects of the actual German minimum wage in Table 2. With these ingredients at hand, the interested reader will be able to infer a social welfare effect according to their preferred social welfare function that trades aggregate welfare, equity and employment effects.

The first insight is that the welfare effect is hump-shaped in the minimum wage level, whether workers are mobile or not and whether the minimum wage is nationally uniform or regionally differentiated. The intuition is that up to the welfare-maximizing minimum wage, the positive effect on real wages dominates the negative effect on employment probabilities, such that expected wages and welfare increase. With a federal minimum wage, this point is reached at a level of 60% of the national mean wage. Beyond this point, the negative effect on employment probabilities dominates at the margin. At 70%, the absolute welfare effect turns negative.

Since minimum wages mechanically compresses the nominal wage distribution, it is no surprise that our measure of equity increases monotonically in the level of the minimum wage. Under conventional social welfare functions that discount aggregate welfare by the Gini coefficient of income inequality (Newbery, 1970), an increase beyond the welfare-maximizing minimum wage can be justified. Yet, policy makers may wish to take into account that beyond a minimum wage of 50% of the national mean wage, negative employment effects start building up as more and more firms must reduce their labour input in order to raise their MRPL to the minimum wage level.

Intuitively, the employment-maximizing minimum wage must be lower than the welfare-maximizing minimum wage since, unlike the latter, the former does not take into account positive welfare effects from higher wages earned by those who remain in employment. Indeed, the long-run employment-maximizing federal minimum wage is as low as 38%. While this moderate minimum wage does increase employment, the effect is very small, and so are the effects on equity and welfare. The employment-maximizing minimum wage is almost identical to the *efficiency maximizing minimum wage* of 37% of the mean wage simulated by Berger et al. (2022) for the US within a dynamic macroeconomic model. Like our employment-maximizing minimum wage, their *efficiency maximizing minimum wage* is only concerned with correcting for the inefficiencies that originate from the employer monopsony. Similar to ours, their model predicts higher optimal minimum wages once worker welfare effects are taken into account which, however, come at the cost of reducing employment.[11] The important takeaway is that, in setting federal minimum wages, policy makers trade positive aggregate welfare effects and progressive distributional effects (within employed workers) against negative employment effects.

This trade-off can be mitigated by setting minimum wages at the regional level. The employment-maximizing regional minimum wage—at 50% of the municipality mean wage—delivers a similar welfare gain as the welfare-maximizing federal minimum wage, plus a positive employment effect of 1.1%. Intuitively, the regional minimum wage is a more targeted policy instrument that avoids the main problem of the federal minimum wage: Reducing the monopsony power of supply-constrained firms in high productivity municipalities comes at the cost of increasing the wage beyond the MRPL of low-productivity firms in low-productivity regions. Instead, the regional minimum wage, by accounting for regional productivity heterogeneity, affects mostly supply-constrained firms in all regions. Notice that we find similar effects if we set the minimum wage at the county level (*Kreise*) whereas a state (Bundesland) minimum wage has effects that resemble the federal minimum wage (see Appendix D.8. This confirms the intuition that a minimum wage needs to be sufficiently localized to account for productivity differentials *between* commuting zones since workers can relatively easily re-optimize to heterogeneous effects caused by productivity differentials *within* commuting zones. In this context, it is worth highlighting

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[11] Their optimal minimum wage of \$15/h under utilitarian welfare weights would correspond to 77% of the national mean wage, resulting in an employment loss of about 3.5%. We thank the authors for converting the absolute relative minimum wages reported in Berger et al. (2022) into the relative minimum wages reported here.

Table 2: Minimum wage schedules

		Level rel. to		Employment		Equity		Welfare	
Objective	Scheme	Mean	p50	SR	LR	SR	LR	SR	LR
Actual	Federal	48.00	52.81	-0.25	-0.35	1.20	1.16	2.08	2.11
Employment	Federal	38.00	41.81	0.02	0.02	0.05	0.03	0.24	0.25
Welfare	Federal	58.00	63.82	-3.94	-4.02	5.51	5.51	3.95	3.95
Employment	Regional	50.00	55.01	1.06	1.06	0.19	0.19	3.92	3.92
Welfare	Regional	58.00	63.82	-1.51	-1.51	2.75	2.75	4.93	4.92

Notes: All values are given in %. *Objective* describes if the minimum wage is employment-maximizing or welfare-maximizing. *Federal* indicates a uniform minimum wage, where the minimum wage *level* is given as a percentage of the national mean wage. *Regional* indicates a minimum wage that is set the respective *level* of the municipality mean. Results are from model-based counterfactuals. Employment is the total number of workers in employment. Equity is measured as 1-$\mathcal{G}$, where $\mathcal{G}$ is the Gini coefficient of real wage inequality across all workers in employment. Welfare is the expected utility of as defined by Eq. (36). It captures individual who are active on and absent from the labour market and accounts for minimum wage effects on employment probabilities, wages, tradable goods prices, housing rents, commuting costs, and worker-firm matching qualities. In the short run, workers are immobile across residence locations whereas workers re-optimize their residential location choice in the long run. We strictly select the long-run maximizing minimum wages.

that the employment effects we simulate for the municipality and county regional minimum wages are closer to Drechsel-Grau (2021) than our simulations for federal minimum wages. This is intuitive since, in relative terms, the regional minimum wage is uniform within our spatial economy, similar to the federal minimum wage in Drechsel-Grau's macroeconomic model with only one region.

Another insight from Figure 9 and Table 2 is that long-run and short-run welfare effects are generally similar in the national aggregate. It is important, however, to recall that there is substantial regional heterogeneity in the welfare effect of federal minimum wages in the short-run, which is equalized through migration in the long run (see Figures 6 and 7). How this regional heterogeneity plays out very much depends on the level of the minimum wage and the regional productivity distribution. While the actual German minimum wage benefits many low-productivity municipalities in the eastern states, the regional fortunes reverse under the 25% higher welfare-maximizing federal minimum wage. Welfare increases more in the more productive west, resulting in a long-run increase in labour force at the expense of the east. In contrast, because the regional minimum wage "bites" similarly in all regions, there is little spatial heterogeneity in the short-run effects on welfare and the long-run effects labour force (see Appendix D.7).

In policy contexts, it is common to express minimum wages in terms of median wages. To convert the relative minimum wages discussed thus far into this metric, we just need to multiply them by the inverse of the ratio of the median wage over the mean wage. In Germany, this ratio was 0.908 in 2015, with remarkably little variation over time. For convenience, we also report the relative minimum wage in per-median terms in Table 2. Accordingly, there is a range between the employment-maximizing and the welfare-maximizing minimum wage from 43-64% of the national median wage within which policy makers trade welfare gains against employment losses. Connecting to the current policy debate in many countries, our simulations suggest that ambitious minimum wages in the range of 60-70%, will likely increase aggregate welfare, but also put a sizable fraction of jobs at risk. In contrast a moderate regional minimum wage, set at about 55% of the municipal median wage, could deliver similar welfare effects *and* generate employment.

5 Conclusion

Minimum wage policies have been popular policy tools to reduce wage inequality. In light of the success of the monopsony model and a growing body of reduced-form evidence, they have also become more popular among economists as the fear of catastrophic employment effects is fading. As a result, more ambitious minimum wages are now being debated in many countries. The European Commission advocates an *adequate minimum wage* of 60% of the median wage. A recent report published by *HM Treasury* recommends a similar level. Some German political parties have recently proposed a minimum wage of €12 that would exceed 70% of the median wage. The *Raise the Wage Act* would increase the U.S. federal minimum wage to $15 per hour by 2025, putting it in a similar ballpark, in relative terms.[12] We inform this debate in a concrete, yet nuanced fashion.

Our simulations within a quantitative model calibrated to German micro-regional data reveal that such ambitious federal minimum wages may achieve a reduction in wage inequality without having a detrimental effect on welfare – compared to the counterfactual of no minimum wage. However, they will likely cause significant job loss. While employment effects remain small up until about 50% of the national mean wage, they build up at an increasing rate at higher levels. Therefore, we caution against extrapolating from encouraging reduced-form evidence on employment effects of moderate minimum wages to the likely effects of more ambitious levels. We recommend that ambitious minimum wages are implemented in small steps, under careful evaluation of short-run employment effects so that potential tipping points can be detected in time.

More generally, our results illustrate how the desirability of any minimum wage will depend on the considered relative level and the social welfare function. In setting minimum wages, policy makers trade employment, equity, and, welfare effects, and, depending on priorities, different minimum wage levels will be optimal. As an example, maximizing employment requires setting a relatively low level—in the case of Germany about 42% of the national median wage. This should generate a small positive employment effect, but also negligible equity and welfare effects. Maximizing welfare requires a more ambitious minimum wage of 64% of the national median wage, which will also lead to greater reduction in nominal wage inequality. Any increase in the minimum wage level within these bounds will trade positive equity and welfare effects against negative employment effects. Of course, even higher minimum wage levels can be advocated on the grounds of an equity objective. In fact, our simulations suggest that the minimum wage could be set as high as

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[12] For background on these initiatives, see European Commission (2020); Dube (2019); Deutscher Bundestag (2020a,b); H. R. 603 (2021).

77% of the national median wage before the welfare effect would turn negative. However, recommending such a high minimum wage level would imply that one strictly cared about the expected real wage—the product of the real wage earned conditional on being in employment and the employment probability—without any aversion to high unemployment rates.

While these trade-offs may appear frustrating from a policy perspective, our analysis also reveals some more encouraging news. Rather than going down the route of ever higher federal minimum wages, policy makers have the alternative of implementing regional minimum wages. We find that regional minimum wages—if set for spatial units no larger than counties—are targeted policy instruments that mitigate the trade-off of negative employment effects and positive welfare effects. To illustrate the potential, the employment-maximizing minimum wage, at 50% of the municipality mean wage, could increase welfare by 4%—as much as the welfare-maximizing federal minimum wage—*and* generate a sizable positive employment effect of 1.1%.

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