Labor Markets, Inequality, and Hiring Selection*

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Abstract

The most qualified applicants often get hired and workers with the poorest fit are fired. Employers usually poach the most talented workers of other firms. Labor markets reward the most talented workers through high wages and beneficial transitions, and the opposite occurs for the most unproductive ones. These casual observations relating worker flows and abilty or match quality find empirical support: the unemployment rate and wage inequality have highly significant, positive and concave relation over time and across states in the US. Job finding rates and Job-to-job transitions also share this common pattern. We rationalize these facts through employer hiring selectivity in a nonsequential search model with on-the-job search. Most qualified applicants, either unemployed or employed, have higher chances to be hired, especially when unemployment is high. Employer selectivity also generates a general equilibrium composition effect affecting inequality. We estimate our model using CPS worker flows and wages data via Generalized Method of Moments. We explain part of the positive and concave relation between unemployment and wage inequality by shifts in the average worker/match productivity.

Keywords: Nonsequential search, Inequality, Unemployment, Worker Flows, Efficiency.

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1 Introduction

The most qualified applicants often get hired and workers with the poorest fit are fired. Employers usually poach the most talented workers of other firms. That ability drives labor market transitions is a commonsensical fact. Labor markets reward the most talented workers through high wages and beneficial transitions, and the opposite occurs for the most unproductive ones. Since "all the good things come together", a common focus on earnings of the employed probably understates the true role of individual ability in shaping (labor) income inequality. In particular, the role of labor market frictions in amplifying or dampening inequality has been scarcely researched empirically or theoretically.

To do so, in this paper we first document a number of empirical facts for the US labor market. There is a large positive cross-sectional and time correlation between wage inequality and the state-time unemployment rate using merged CPS monthly and CPS-ORG data. More detailed analysis shows that high inequality is often related to low job finding probabilities and job-to-job transitions, suggesting that a part of the phenomenon may be explained by delving deeper into the hiring mechanism of firms.

To rationalize this evidence, in a second part of the paper, we construct a general equilibrium model of nonsequential employer search with hiring selectivity and heterogeneous workers, and characterize its equilibrium. The model departs from the standard search model by allowing firms to simultaneously meet several applicants and choose the best. Since the seminal work of Stigler (1961) there have been scattered empirical and theoretical pieces of research that have been pointing out the importance of nonsequential search in modeling behavior (Stern 1989; van Ours and Ridder 1992; Villena-Roldan 2010; van Ommeren and Russo 2013; Wolthoff 2017; Davis and Samaniego de la Parra 2017). However, nonsequential search seems a critical ingredient to rationalize how individual productivity affects jointly wages and transitions in the labor market.

The nonsequential search model provides an endogenous matching process in which the job finding and job-to-job probabilities increase in worker productivity. Under selective hiring, lifetime inequality increases relative to a sequential search benchmark because low productivity workers go through longer unemployment spells and face lower chances of job-to-job transitions. In general equilibrium, more productive workers, more often employed and potentially engaged in on-the-job search, compete for positions with the unemployed, as in Eeckhout and Lindenlaub (2018). In equilibrium, employers receive a pool of both employed and unemployed applicants, learn the true productivity type by exerting some effort, and offer the position to the most productive candidate. In the competitive equilibrium solution, employers do not take into account that they worsen the average composition of applicants when hiring the top candidates and dismissing the less attractive ones.

We take the model to the data by calibrating the average unemployment rate for the US, as well as job-to-job and separation transition frequencies, and the wage distribution during 1994-2019. Since we do not try to match observed correlations between inequality and worker flow measures per se, we investigate –for instance– what type of shocks can replicate a positive comovement between the unemployment rate and different measures of wage inequality. We find that an increase in the mean of productivities replicates the aforementioned empirical fact. The first intuition goes as standard search and matching models: an increase in the average productivity of workers spurs greater vacancy creation, making it easier to find jobs, and reducing the unemployment rate. As the number of applicants per vacancy (average queue length) declines, screening becomes less selective, and thus the composition of the pool of the unemployed *improves* its average productivity. The employed pool, on the other hand, *worsens*. Hence, the pool of applicants, a mixture employed and unemployed workers, become more similar to the population distribution of productivities. Hence, hired applicants tend to be more similar to each other, reducing inequality. However, additional general equilibrium effects also occurs: as the distribution exhibits lower variance, the likelihood of hiring top applicants decrease, offsetting the increased average productivity, and deterring to some extent vacancy posting. The average number of applicants per vacancy increases and makes screening activity more selective. In the end, the general equilibrium adjustment partially undo the negative initial impact of higher productivity on both unemployment and inequality.

Our model is also related to theoretical and empirical approaches explaining raising income or labor income inequality in recent years. While increased education, skill-bias technical change, globalization, and taxation are often regarded as principal drivers of these trends, our theory complements them by explaining how labor markets amplify or attenuate secular trends in the distribution of productivities.[1] In that sense, we do not have a theory of why productivity dispersion has seemingly increased over time. Instead, our purpose is to theoretically understand how the selectivity prevailing in hiring and poaching workers in the labor market can affect pre-market productivity inequality, whatever the underlying cause is. We also try to provide an explanation simultaneously accounting for empirical correlations between inequality and worker flows, and a sensible measurement of the effect of the selective

[1]There is a large literature examining these issues. An incomplete enumeration of papers studying merits and demerits of these hypotheses: Krusell, Ohanian, Ríos-Rull, and Violante (2000), Card and DiNardo (2002), Moore and Ranjan (2005), Goldin and Katz (2008), Helpman (2016), and Ravallion (2018).

activity into inequality.

Literature review Our paper is linked to several strands of the literature. First, considering the non-sequential matching process, there is a long-standing tradition of modelling the matching function as an urn-ball process, starting from the papers by Butters (1977), on which Blanchard and Diamond (1994) and Moen (1999) build their models. Blanchard and Diamond (1994) analyse a framework in which firms can receive more than one application and rank candidates according to the unemployment duration, so that the equilibrium exit rate from unemployment negatively depends on unemployment duration, as well as on the measure of labor market tightness. Moen (1999) builds on Blanchard and Diamond (1994) and extends the analysis by considering that workers' ranking for firms depends on their productivity level, and productivity is linked to education, which can be acquired through a monetary cost. He thus focuses on the optimal level of education, considering that on the one side education improves employers prospects, but on the other it introduces a so-called "rat-race" among workers, who acquire education just to move up in the ranking.

Some authors have considered a ranking mechanism for hiring in a directed search approach, as for example Shimer (2005), Shi (2006) and more recently Fernández-Blanco and Preugschat (2018), who endogeneize the mechanism highlighted by Blanchard and Diamond (1994), Wolthoff (2017), and Cai, Gautier, and Wolthoff (2021).Wolthoff (2017) considers a framework in which *ex ante* homogeneous workers can send multiple applications to firms who decide on the level of wage to post, the hiring standard (a minimum threshold level of productivity) and recruiting intensity. He uses this model to study the effects of aggregate productivity shocks on the decisions of firms regarding screening and hiring standards. Cai, Gautier, and Wolthoff (2021) develop a directed search model with two-sided heterogeneity to study the conditions for sorting in applications as well as matches.

Secondly, there are a number of papers who recently focused on the importance of the composition of the pool of employed and unemployed agents in relation with the presence of on-the-job search. Eeckhout and Lindenlaub (2018) develop a framework with random search, endogenous on-the-job search, endogenous vacancy creation and sorting, and show that the presence of a complementarity between vacancy creation and on the-job search has an impact in the cyclical composition of the searchers and unemployed agents, thus creating self-fulling states of booms and recessions.

Engbom (2020) considers a similar setting in which however firms pay a cost to screen job applicants, who decide to apply based on a noisy signal on the productivity of the match, and shows that, differently from Eeckhout and Lindenlaub (2018), it is not the pro-cyclical search behavior of workers that amplifies shocks to the labor market, but rather the counter-cyclical recruiting costs for firms: when unemployment is high in fact firms receive more applications from unemployed workers who are more likely to be a bad fit. Similarly to Engbom (2020), Bradley (2020) considers a model with on-the-job search where firms screen applicants; the frequency with which unemployed and employed search, as well the number of available vacancies they can see is different, so that unemployed workers search more intensively but are exposed to fewer possibilities, thus as if unemplyed and employed were sampling from different wage distributions.

The last three cited papers consider workers as *ex ante* homogenous, and focus on match-specific shocks. Merkl and van Rens (2019) develop a framework where there are no search frictions but workers are heterogeneous in terms or training costs, and consider a polar case in which this heterogenity is permanent or individual-specific; this case gives rise to what they define selective hiring, where the unemployed pool is essentially composed by "lemons". They then use their framework to study the welfare costs of unemployment and the trade-offs a benevolent social planner faces in providing unemployment insurance.

2 Empirics

In this section, we document significant relations between worker flows and inequality that have been under the radar. We are *not* claiming any causal effects. Our preferred interpretation is that worker flows cause changes in wage inequality and we will provide a model that provides a plausible mechanism that generates this fact. In this section, however, our purpose is to document correlations and partial correlations that may be interpreted later by using a structural model.

Figures 1-4 show the fitted values of a quadratic regression between the standard deviation of real log hourly wages (national CPI adjusted) by state-year on a number of flow/stock variables, one at a time. Confidence intervals of fitted values are depicted, too. The results are also separated in college vs non-college since this is a natural way to segment the labor market in high and low skill jobs.

The standard deviation of log hourly wages peaks at an unemployment rate (U) near 11% for non-college workers. For lower and upper levels of unemployment, wage inequality decreases. The pattern seems quite similar when controlling for state or time fixed effects, separately. The hump shape relationship seems less clear, especially for high unemployment levels when both state and time fixed effects are put together. Since we are interested in accounting for the inequality level, the evidence suggests that this could be generated either by different labor market conditions across states, aggregate labor market cyclicality, or state-specific labor market cyclicality. Since the unemployment rate averages 7% for this group, in most state-year labor markets there is a clear increasing relationship between wage inequality and unemployment rate.

Figures 2 and 3 try to dig deeper into the previous correlation, as both flows in and out unemployment shape the unemployment rate in a standard two-state search and matching model (Pissarides 2000). The job finding frequency (UE) is mostly negatively correlated with wage dispersion, showing that the wage inequality tend to increase in times of long unemployment duration, which remains true after controlling for state fixed effects. The sensitivity of wage inequality is of similar magnitude for both college and non-college workers, although the latter exhibit greater concavity.

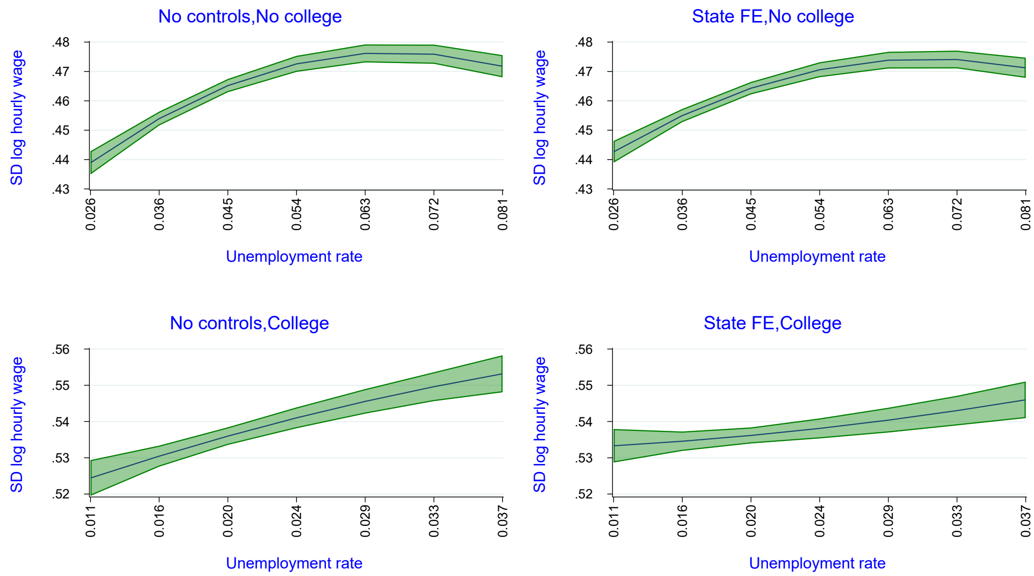
In the case of separation frequency (EU flow) we observe mostly positive correlation. The sensitivity remains if we control for state fixed effects. Both job finding and separation frequencies are associated to wage dispersion in a way that is consistent with the correlation between wage dispersion and unemployment. Taken together, these pieces suggests a linkage between worker flows and inequality.

For job-to-job transitions, only data from 1994 onwards is available at the CPS. As occurs with the job finding frequency (UE), there is a negative relation between job-to-job frequency transitions and inequality for non-college workers, although the slope of the relationship is flatter than the one for the UE flow. The correlation for non-college workers decreases until it hits a rate of 2.4% and essentially remains flat beyond that point. For the college worlers, the slope is negative but quite flat, exhibiting slightly concave patterns.

Our preferred explanation, among many possible, for the relation between wage inequality and unemployment assumes that employers hire selectively, i.e. they compare applicants and offer the job to the most appropriate one. When the unemployment rate increases, there are larger queues of workers applying for jobs. In this situation, employers hire better applicants on average, all else constant. Therefore, jobs are allocated to most productive workers or matches on average, so that the composition of employed workers improves, widening the gap with previously employed workers. However, as the unemployment rate keeps increasing, the composition of the employed becomes better but less heterogeneous, driving down inequality. This would explain why the increase of inequality associated with unemployment becomes smaller in higher unemployment markets, a concave relationship. For a very high unemployment rate, selectivity is so bold that hired workers may be homogeneous enough to force wage inequality to decrease.

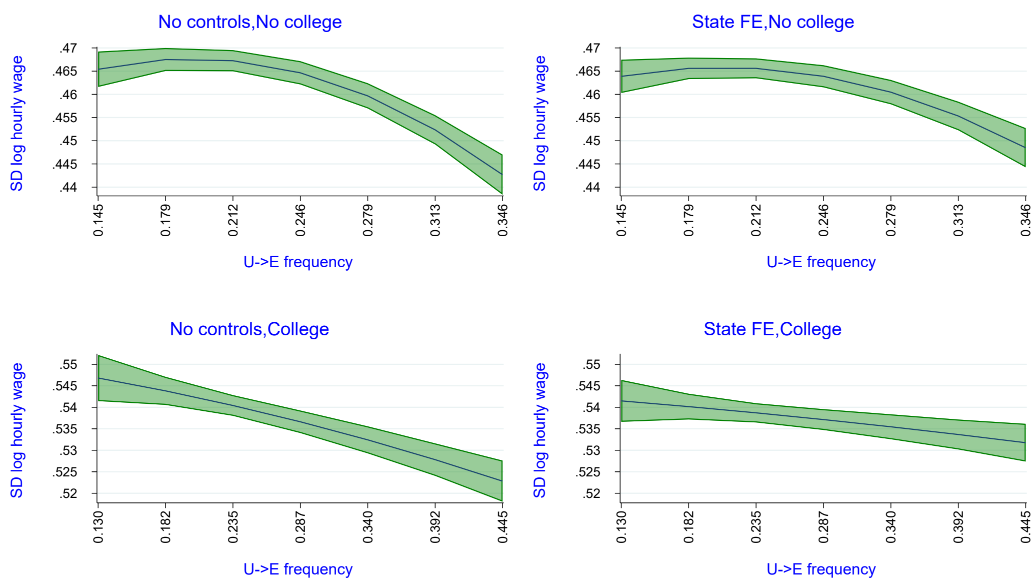
There are a number of robustness checks we perform to be sure these relations meaningful

Figure 1: Standard deviation of wages vs Unemployment Rate by State & Year



Sample 1976-2019. US real log hourly wages.

Figure 2: Standard deviation of wages vs Job finding Freq. (UE) by State & Year

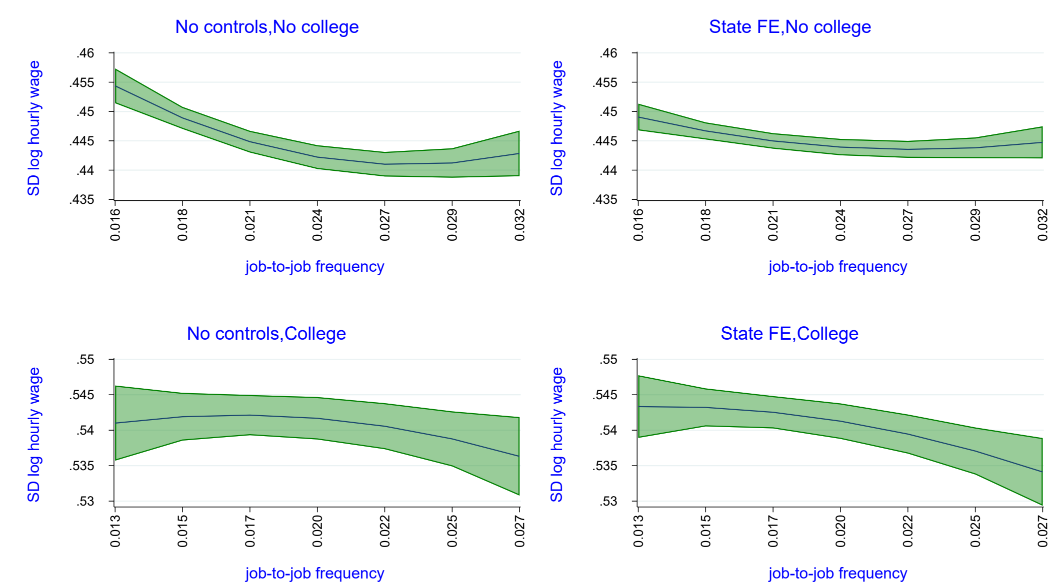


Sample 1976-2019. US real log hourly wages.

Figure 3: Standard deviation of wages vs Job separation Freq. (EU) by State & Year

Sample 1976-2019. US real log hourly wages.

Figure 4: Standard deviation of wages vs Job-to-job Freq. (JJ) by State & Year



Sample 1994-2019. US real log hourly wages.

and unrelated to mechanical biases. We refer the reader the relevant figures to the Appendix. In particular, we checked that

- The 1994 CPS sampling redesign does not substantially change the observed patterns in the relationship between log wage dispersion and the unemployment rate, nor the relation between the wage dispersion and EU and UE flow frequencies. These can be seen in Figures 15, 16, and 17.
- The findings are not sensitive to the CPI deflator used to compute real wages. In particular, we use regional CPI (Northeast, Midwest, South, and West) and obtain roughly the same results in Figures 18, 19, 20, and 21.
- The findings generally remain unaltered for alternative measures of wage dispersion such as the 75-25 and 90-10 percentile wage gaps, as can be seen in Figures 22 - 29, which are less sensitive to capture a meaningful measure of dispersion for skewed distributions.
- We introduce controls for year fixed effects only in Figures 30, 31, 32, and 33, obtaining qualitatively similar results. This shows that the underlying mechanism generating the shapes in the previous evidence is pertinent to explain the cross-sectional diversity of inequality across different states.

3 The Model

In the model time is discrete. There is a continuum of homogeneous risk-neutral firms or employers that can post ex ante identical job vacancies. There is also a fixed mass of size 1 of workers who have a time-invariant productivity θ according to an exogenous distribution with density $g(\theta)$. Workers can either be employed or unemployed. Employed workers can apply to a position with a fixed exogenous probability λ. The mass of applicants $\mathcal{A}$ is thus given by the sum of the unemployed and the proportion of employed who can apply to a job: $\mathcal{A} = \mathcal{U} + \lambda(1 - \mathcal{U})$. Workers cannot borrow nor save.

The general state of the economy is a tuple $\mathcal{X} \equiv (\mathcal{A}, \mathcal{V}, G_A(\theta))$, where $\mathcal{A}$ represents the mass of applicants in the economy, $\mathcal{V}$ is the mass of aggregate vacancies posted in equilibrium, and $G_A(\theta)$ is the endogenous joint distribution of types of the applicants, including unemployed and employed jobseekers. While the setting could be extended into a dynamic framework, we solely focus on the symmetric steady-state equilibrium of this economy in this paper.

Since all jobs are ex ante identical, workers randomly apply to vacancies. Thus, the probability that the application of a particular worker arrives to a given vacancy is $1/\mathcal{V}$ and the number of applications arrived to a vacancy, K, follows a binomial distribution.

$$Prob(K=k) = \binom{\mathcal{A}}{k} (1/\mathcal{V})^k (1 - 1/\mathcal{V})^{\mathcal{U}-k}$$

As both $\mathcal{A}, \mathcal{V} \to \infty$ with its ratio $q = \mathcal{A}/\mathcal{V}$ constant, the number of applicants received per vacancy K converges to a Poisson distribution with mean q, the queue length.

3.1 Workers

All unemployed workers receive an exogenous income $\rho\theta$ with $0 < \rho < 1$. An applicant randomly chooses a vacancy and faces an *equilibrium* job finding probability $p(\theta; q)$ which depends on the applicant's type and the market queue length, as derived later. In case the applicant obtains the job, the worker gets the value of being employed $W(\cdot)$ earning a wage $w(\theta)$ set by the employer. If the applicant comes from unemployment and receives no offers, she remains in this state and applies again the next period.

With an exogenous probability λ an employed worker applies to another job. She actually moves to the new job with overall probability $\lambda p(\theta; q)$; if she does not move, she remains in her current job until the next period.

If the worker does not apply to another job, she can become unemployed with exogenous probability η and with probability $1-\eta$ the match continues. Workers have linear preferences over consumption and have a constant discount factor $\beta \in (0, 1)$. Hence, an unemployed worker's lifetime utility is

$$U(\theta) = \rho\theta + \beta[p(\theta; q)W(\theta) + (1 - p(\theta; q))U(\theta)] \tag{1}$$

while for the employed agent, the value of being employed W depends on the potential value of a new job $\tilde{W}$.

$$W(\theta) = w(\theta) + \beta[\lambda \left(p(\theta, q)\tilde{W}(\theta) + (1 - p(\theta, q))W(\theta) \right) + (1 - \lambda) \left((1 - \eta)W(\theta) + \eta U(\theta) \right)$$

and

$$W(\theta) = (1 - \lambda)W(\theta) + \lambda p(\theta)\tilde{W}(\theta) + [1 - (1 - \lambda) - \lambda p(\theta)]W(\theta) \tag{2}$$

3.2 Firms

A filled job with a worker of productivity θ generates a flow profit $\theta - w(\theta)$. After production, if the worker does not apply to another job, the job can be destroyed with exogenous probability η, in which case the employer obtains the value of posting a vacancy V described below.

The match can also be destroyed if the worker applies to another job and leaves the current match. Hence, the value of a filled job equals

$$J(\theta) = \theta - w(\theta) + \beta\{(1-\lambda)\eta V + \lambda p(\theta, q)V + [1-(1-\lambda)\eta - \lambda p(\theta)]J(\theta)\} \tag{3}$$

Employers observe the aggregate state of the labor market and pay a fixed entrance cost χ when they open a vacancy if it is profitable to do so. This cost is related to securing financial funds, obtaining capital goods, or access to product markets in environments modeling aspects of other markets (Petrosky-Nadeau and Wasmer 2015; Petrosky-Nadeau and Wasmer 2017). After this, employers optimally create vacancies by paying a fixed flow cost κ. Vacancies simultaneously meet K applications drawn from the distribution of applicants $G_A(\theta)$, which will be defined below. This simultaneous hiring is a key departure from most of sequential search and matching models. In addition, after receiving applications, the employer attaches a probability ϕ of interviewing or screening each applicant with marginal cost $\xi \geq 0$.[2]

After each costly interview, the employer perfectly learns the applicant's type θ. Due to this assumption, we focus on selection issues, leaving aside informational effects. The employer offers the position to the most profitable worker, or posts a vacancy again. Thus, the value of posting a vacancy is

$$V = \max\left\{\max_{\phi}\left\{-\kappa + \beta(Prob(K>0)H(k) + Prob(K=0)V)\right\}, V\right\}$$

where $H(k)$ is the maximum profit obtained out from a pool of $k > 0$ effectively screened applicants.[3]

$$H(k) = \mathbb{E}_K\left[-\xi K + \max_j \left\{J(\theta_j)\right\}_{j=1}^{K} | K > 0\right]$$

[INCORPORATE A NOTE INCLUDING THE MIXTURE POISSON-BINOMINAL TO FORMALLY ADDRESS AN EFFECTIVE POISSON DISTRIBUTION WITH MEAN $q = \phi\tilde{q}$]

3.3 Solving the Competitive Equilibrium

The model solution schematically involves these steps:

1. Find out the job finding rate schedule $p(\theta, q)$.

[2] This decision could be contingent on the realized number of arrived applicants k, and an optimal hiring policy would set of cap on the number of interviewed candidates. This would assuage the hiring advantage high productivity applicants have, as does the ex ante hiring probability ϕ at the cost of substantially decreasing tractability. Therefore, for the sake of simplicity, we only consider the case in which ϕ is constant and set before learning the realized number of applicants.

[3] [PENDING] In the appendix, we show the equivalence between the model with an effective queue length q and one with an ex ante probability of screening ϕ and an average number of applicants per vacancy $\tilde{q}$.

2. Using the job finding rate schedule $p(\theta, q)$ to derive the invariant joint distribution $G_A(\theta)$ in terms of the equilibrium queue length q.

3. Having $p(\theta)$ and $G_A(\theta)$ find out the equilibrium queue length q using the free-entry condition $V = 0$

3.3.1 Equilibrium Job Finding Rate

A worker is hired whenever she generates a profit value $J(\theta)$ for the employer which is greater than that of other applicants to the same vacancy. Under our hypothesis for the wage setting mechanism, the highest productivity applicant yields the highest profit in equilibrium. In particular, if an employer screens s applicants, the top candidate gets the offer with probability $(\phi G_A(\theta))^{s-1}$.

The number of screened applicants follows a binomial distribution. By Bayes' law, a worker of type θ has a probability of being hired

$$Prob(\theta \text{ hired}|k \text{ total applicants}, \theta) = \sum_{s=1}^{k} \binom{k}{s} \phi^{s-1}(1-\phi)^{k-s+1} G_A(\theta)^{s-1} = (\phi G_A(\theta) + 1 - \phi)^{k-1}$$

Nevertheless, at the moment a worker applies, she ignores how many applications are competing for the same job she applied to. Considering that the number of applicants follow a Poisson distribution, the probability of being hired $p(G_A(\theta), q)$ is

$$p(G_A(\theta), q) = \sum_{k=1}^{\infty} \frac{e^{-\phi q}(\phi q)^{k-1}}{(k-1)!} (\phi G_A(\theta) + 1 - \phi)^{k-1} = e^{-\phi q(1-G_A(\theta))} \tag{4}$$

The average probability of being hired of an applicant, $\mathbb{E}[p(\theta, q)|q]$ is therefore give by

$$\mathbb{E}[p(G_A(\theta), q)|q] = \overline{p}_A = \int \sum_{k=1}^{\infty} \frac{e^{-\phi q}(\phi q)^{k-1}}{(k-1)!} (\phi G_A(\theta) + 1 - \phi)^{k-1} \, dG_A(\theta) = \frac{1 - e^{-\phi q}}{\phi q} \tag{5}$$

3.3.2 Distributions

So far the analysis has been in partial equilibrium because the distribution of applicants $G_A(\theta)$ has been considered as given. However, the recruiting selection process affects the distribution of unemployed and employed workers. In this section we show how this distribution is endogenously determined in equilibrium.

First, the exogenous density of types is composed by a weighted average of the densities of the unemployed g_U and of the employed g_E as follows

$$g(\theta) = \mathcal{U} g_U(\theta) + (1 - \mathcal{U}) g_E(\theta)$$

In steady state, for a given type θ and queue length q, the flows in and out from unemployment must be equal, i.e

$$p(G_A(\theta), q)g_U(\theta) = \eta^* g_E(\theta)$$

Combining both conditions, we obtain the following intuitive condition: the population density of the type θ is weighted by its steady-state probability of unemployment, and scaled by the mass of the unemployed to ensure the expression integrates out to 1.

$$g_U(\theta) = \frac{g(\theta)}{\mathcal{U}} \frac{\eta^*}{\eta^* + p(G_A(\theta), q)} \tag{6}$$

In a similar fashion, we can obtain the density of employed workers as

$$g_E(\theta) = \frac{g(\theta)}{1-\mathcal{U}} \frac{p(G_A(\theta), q)}{\eta^* + p(G_A(\theta), q)} \tag{7}$$

Since a fraction λ of the employed workers apply for jobs as the unemployed do, the density of the applicants is

$$g_A(\theta) = \frac{g_U(\theta)\mathcal{U} + g_E(\theta)\lambda(1-\mathcal{U})}{\mathcal{U} + \lambda(1-\mathcal{U})}$$

which turns out to be a separable differential equation

$$\frac{dG_A(\theta)}{d\theta} = \frac{\eta + \lambda p(G_A(\theta), q)}{\eta + p(G_A(\theta), q)} \frac{1}{\mathcal{U} + \lambda(1-\mathcal{U})} g(\theta) \tag{8}$$

Let us adopt a change of variable and define the quantile of the distribution of applicants as $G_A(\theta) = x$. Since $G_A(\theta)$ is a cumulative distribution function, the boundary conditions are $G_A(\infty) = G(\infty) = 1$ and $G_A(0) = G(0) = 0$.

Using the boundary conditions we can determine the value of the constant of integration and therefore the expression for the unemployment rate.[4]

$$\mathcal{U} = \frac{\lambda \log(\eta + \lambda e^{-\phi q}) - \log(\eta + \lambda) + \phi q}{(1-\lambda)\left(\log(\eta + \lambda) - \log(\eta + \lambda e^{-\phi q})\right) + \lambda \phi q} \tag{9}$$

Equation (8) shows us that, even if it is not possible, in general, to obtain a close-form solution for $G_A(\theta)$ in terms of the primitives, however, there is a close-form mapping between quantiles of the applicants distribution and the quantiles of the original population, given an equilibrium queue length q.

[4] Using the L'Hôpital rule, one can show that the unemployment rate converges to a well-known formula when there is no on-the-job search, i.e. when $\lambda \to 0$

$$\lim_{\lambda \to 0} \mathcal{U} = \frac{\eta}{\eta + \frac{1-e^{-\phi q}}{\phi q}}$$

where $\frac{1-e^{-\phi q}}{\phi q}$ is the average probability of being hired when there is no on-the-job search.

Using the transformation $x = G_A(\theta) \to G_A^{-1}(x) = \theta$, we therefore obtain

$$G^{-1}(M(x; q, \eta, \lambda)) = G_A^{-1}(x) = \theta \tag{10}$$

with

$$M(x; q, \eta, \lambda) \equiv \frac{\log(\eta + \lambda e^{-\phi q(1-x)})(1-\lambda) + \lambda\phi q x - \log(\eta + \lambda e^{-\phi q})(1-\lambda)}{\log(\eta + \lambda)(1-\lambda) + \lambda\phi q - \log(\eta + \lambda e^{-\phi q})(1-\lambda)} \tag{11}$$

The result in equation (10) is key to express the equilibrium conditions in a way that they do not depend on the unknown distribution $G_A(\theta)$, but rather on the distribution of population's productivity $G(\theta)$, which is a primitive of the model.

We can also define the average job finding probability (the average probability of an unemployed to find a job) as well as the job-to-job average transition probability: it is in fact important to see how the information from the data map into our model.

The probability of being hired of an unemployed, i.e. the transition probability from the state of unemployed to employed (UE) is given by

$$\overline{p}_U = \int e^{-\phi q(1-G_A(\theta))} dG_U(\theta) = \frac{\mathcal{A}\overline{p}_A - \lambda \int_0^1 e^{-\phi q(1-x)} dM(x; q, \eta, \lambda)}{(1-\lambda)\mathcal{U}}$$

The probability of being hired of an employed is instead given by

$$\overline{p}_E = \int e^{-\phi q(1-G_A(\theta))} dG_E(\theta) = \frac{\int_0^1 e^{-\phi q(1-x)} dM(x; q, \eta, \lambda) - \mathcal{A}\overline{p}_A}{(1-\lambda)(1-\mathcal{U})}$$

The total job-to-job transition rate (EE) is therefore given by $\lambda\overline{p}_E$.

3.3.3 Solving the hiring problem

In Appendix B we show that the free entry condition can be written as

$$\kappa + \beta\xi\phi q(1 - e^{-q}) + \chi(1 - \beta e^{-q}) = \beta \int_0^1 J(G^{-1}(M(x; q, \eta, \lambda)))\phi q e^{-\phi q(1-x)} dx \tag{12}$$

In general equilibrium, then, the value of the queue length must satisfy the free-entry condition $V = \varphi$, that is, employer post vacancies up to the point the expected value of doing so exactly compensates the opportunity cost of the entry cost.

In this set-up, quantiles of an unknown distribution $G_A(\theta)$ are mapped to the quantiles of a known distribution. By doing so, we obtain an equation that characterizes the equilibrium value of q without figuring out $G_A(\theta)$.

The mapping $M(x; q, \eta, \lambda)$ always maps quantiles x of the applicant distribution to a lower quantiles of the population distribution $G(\theta) = M(x; q, \eta, \lambda)$. As the queue length q increases, the recruiting selectivity intensifies.

3.3.4 The choice of the probability ϕ

The information we can retrieve from the data is the average number of interviews per vacancy $\tilde{q}$. This can be thought as resulting from a choice of the firm, who decides to interview each applicant with probability ϕ: $\tilde{q} \equiv \phi q$, where q is the number of applications received per vacancy.

When we consider the optimal choice of vacancy opening, we realize that in fact ϕ and q always enter in a multiplicative way in the model, as we can see in Equation 13, where we reported for the reader's convenience the expressions of $p(G_A(\theta))$ and $J(\theta)$.

$$V = \max_{\phi} -\kappa - \beta\xi\phi q(1 - e^{-q}) + \beta \int_0^{\infty} J(\theta)\phi q p(G_A(\theta))d(G_A(\theta)) \tag{13}$$

$$J(\theta) = \frac{\theta(1-\rho) + \beta[\lambda p(G_A(\theta)) + \eta^*]\chi}{1 - \beta[1 - \eta^* - \lambda p(G_A(\theta))]} \tag{14}$$

$$p(G_A(\theta)) = e^{-\phi q(1-G_A(\theta))} \tag{15}$$

Since they always enter as a product in the model, we cannot distinguish between ϕ and $\tilde{q}$; for this reason we do not focus on the choice of ϕ and simply assume in the following that $\tilde{q} = q$, or that $\phi = 1$.

3.4 Wages

Since we consider job-to-job transitions, the traditional Nash bargaining between the worker and the firm would pose some theoretical problems of non-uniqueness of the equilibrium, as discussed by (Shimer 2006). (Gottfries 2017) recently proposed a bargaining protocol, with infrequent renegotiation, to address the problems raised by (Shimer 2006)[5], However, in order to keep the model as simple as possible, we decide to be conservative and adopt the widely used assumption in the literature that the firm has full bargaining power: to maximize profits, wages are set to make the employer indifferent between taking the job and taking her outside option.

Bargained wages are never turned down in equilibrium because: (1) the value of the match is solely determined by θ; (2) all employers are identical and therefore pay the same wages, and (3) Workers only send one application per period. The Nash axiomatic solution solves

[5] Gottfries (2017) shows that when wages can be continously renegotiated then the solution corresponds to the Nash equilibrium as in Mortensen and Pissarides (1994), while the framework described in Shimer (2006) corresponds to the case in which wages cannot be renegotiated at all.

the following problem.

$$\max_w J(w,\theta) - V$$
$$\text{s. t. } W(\theta) > U$$

Substituting equations (2) and (3), and using the free entry condition $V = \chi$ for an interior solution, the problem can be expressed as

$$\max_w \frac{\theta - w(\theta) + \beta\chi[\lambda p(\theta) + \eta^*]}{1 - \beta[1 - \eta^* - \lambda p(\theta)]}$$
$$\text{s. t. } \frac{w - (1-\beta)U}{1 - \beta(1-\eta^*)} \geq 0$$

and therefore by substituting equation (1), the solution is simply

$$w(\theta) = \rho\theta \tag{16}$$

4 Calibration

Once we solved the model, we need to bring it to the data to estimate some of the parameters, and test its empirical performance. Our strategy consists in using information from the data about the unemployment rate, the job-to-job transition probabilities, the separation rate (EU) and the moments of the wage distribution. We posit a specific distributional form for the productivity distribution for the whole population: we assume $\theta \sim$ lognormal (μ, σ^2).

We use a Simulated Method of Moments approach: we look for the parameters' values that minimize the distance between the moments from the data and those implied by the model. It is important to notice that for our steady state calibration we need to estimate also the value of the queue length, even if this is not a parameter but an endogenous variable of the model. Once we obtain the estimated values of the parameters of interest, we perform some counter-factual experiments: in these cases, the value of q changes endogenously through the free entry condition.

The set of values we need to estimate is given by $\Psi = \{\eta, \lambda, \mu, \sigma, \rho, q\}$, where η is the exogenous match destruction rate[6], λ is the probability of applying to a job while working, μ and σ are the parameters characterizing the productivity distribution of the population, ρ is the proportion of the productivity that the worker receives when unemployed[7], and q is the

[6]The overall separation rate is given by $\eta^* = \eta(1-\lambda)$

[7]Since in our setting we do not explicitly model labor market institutions such as unemployment benefits, the coefficient ρ can be seen as an overall measure of the outside options of the workers, including income support as unemployment insurance.

equilibrium queue length. Once we obtained the estimated values, we close the models using the equilibrium entry condition (Equation 12), with data on the screening costs (ξ), in order to recover the vacancy posting cost κ.

4.1 The data

We use monthly data from the CPS for the labor market transitions and from ORG-CPS for wages. The data span a period from 1994 to 2016.[8]. We aggregate the data that are available at state level: our hypothesis is therefore that the parameters are time invariant. We also distinguish between college and non-college, acknowledging the well established fact that the labor market performance for these two groups of agents are quite different.

4.2 Estimation procedure

In this Section we describe the estimation procedure. The moments we consider as informative are: the unemployment rate, the job-to-job transition rates, the separation rates and a continuum of moments characterizing the wage distributions[9]. The set of parameters we want to identify is $\Psi_0 = \{\eta, \lambda, \mu, \sigma, \rho\}$; within the same procedure we also identify q, even if it is an equilibrium object (the queue lenght), and not a parameter. For what it regards the wage distributions, we compare the predictions of the model with the data in terms of the level of the wage for each of the 1000 points of the cumulative distribution function. It has to be noticed that we solved the model in terms of the applicants cumulative distribution function: the predictions of the model are therefore made in terms of applicant's percentiles. We therefore have to apply a transformation to the empirical wage distribution to obtain the percentile of the applicants' distribution that corresponds to the observed wage distribution.

Using the definition of the density of employed, we can write

$$G_E(\theta) = \frac{G(\theta) - \mathcal{A}G_A(\theta)}{(1-\lambda)(1-\mathcal{U})}$$

In Section 3.3.2 we showed that $G(\theta) = M(x; q, \eta, \lambda)$, which implies that $\theta = G^{-1}(M(x; q, \eta, \lambda)) = G_A^{-1}(x)$, so that using this definition in the previous expression we can rewrite it as

$$G_E(G^{-1}(M(x; q, \eta, \lambda))) = \frac{M(x; q, \eta, \lambda) - \mathcal{A}x}{(1-\lambda)(1-\mathcal{U})}$$

[8]Data on job-to-job transitions are available strting from 1994

[9]In particular, we approximate the continuum of moments characterizing the wage distributions by focusing on the 10th of a percentile, i.e. we divide the wage distribution in 1000 points.

Let us call $\tilde{G}_w(.)$ the empirical cumulative distribution function of observed wages. The cumulative distribution function of the employed $G_E(.)$ is unknown, but we can approximate it using the empirical wage cdf: $G_E(G^{-1}(M(x;q,\eta,\lambda))) \approx \hat{G}_w(\hat{w}(x)) = \frac{M(x;q)-\mathcal{A}x}{(1-\lambda)(1-\mathcal{U})}$. Thus if we want to obtain the level of wages implied by the observed wage distribution for a certain percentile x (referring to the applicants' distribution), we just need to invert the expression for $\hat{G}_w$:

$$\hat{w}(x) = \widehat{G}_w^{-1}\left(\frac{M(x;q,\eta,\lambda) - \mathcal{A}x}{(1-\lambda)(1-\mathcal{U})}\right) \tag{17}$$

In our model the wage is simply the reservation productivity of the worker, i.e. $w(\theta) = \rho\theta = \rho G^{-1}(M(x;q,\eta,\lambda))$.

To find the parameters' values, we thus solve the following minimization problem:

$$\min_{\eta,\lambda,\mu,\sigma,\rho,q} Q = \left\{\varphi_1[\mathcal{U}(q,\eta,\lambda) - \hat{\mathcal{U}}]^2 + \varphi_2[\lambda\overline{p}_E(q,\eta,\lambda) - \hat{J}J]^2 + \varphi_3\left[\eta(1-\lambda) - \hat{EU}\right]^2 \right.$$
$$\left. + \varphi_4\left[\int_0^1 \left(\rho G^{-1}(M(x;q,\eta,\lambda)) - \hat{w}(x)\right)^2 dx\right]\right\}$$

4.3 Results

Tables 1 and 2 show the results of our benchmark model estimation. The results in terms of the exogenous separation rate η and the poaching probability λ are in line with standard values in the literature. The parameter ρ is a rough measure that includes the value of home production and unemployment benefits, and it is also quite in line with standard calibration values.

Tables 2 compares the informative moments from the data with the ones obtained by simulating the model: the performance of the model is quite good, however it systematically underestimates the average values for the log wage distribution. Figure 5 shows the actual and simulated cdf for low wages.

In order to assess the goodness of the fit, we report a measure of the Mean Square Errors for the four types of moments we used in the estimation. For each variable x, let us define as x_t the monthly observations, $\bar{x}$ is the simple average across time and $\tilde{x}$ is the simulated value according to the model. For the unemployment rate, separation rates and job-to-job transitions, we compute the MSE in the standard way:

$\bar{Q}_1 = \Sigma_t(\mathcal{U}_t - \bar{\mathcal{U}})^2; \bar{Q}_2 = \Sigma_t(JJ_t - \bar{J}J)^2; \bar{Q}_3 = \Sigma_t(EU_t - \bar{EU})^2$
$\tilde{Q}_1 = \Sigma_t(\mathcal{U}_t - \tilde{\mathcal{U}})^2; \bar{Q}_2 = \Sigma_t(JJ_t - \tilde{J}J)^2; \bar{Q}_3 = \Sigma_t(EU_t - \tilde{EU})^2$

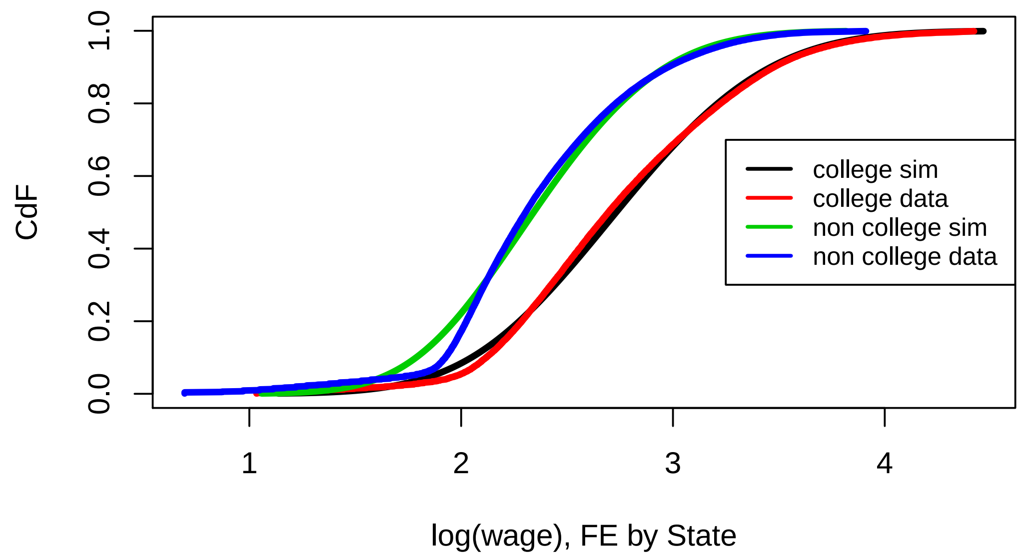
Table 1: Parameters' estimation

	Baseline		No adjust	
Parameter	College	Non College	College	Non College
η	0.007	0.017	0.008	0.017
λ	0.045	0.057	0.044	0.057
q	2.465	2.867	2.379	2.866
μ	3.976	3.447	4.144	3.651
σ	0.533	0.440	0.533	0.439
ρ	0.330	0.397	0.280	0.323
Min fun	0.418	0.893	0.428	1.08

Table 2: Data vs. Model generated moments

	Baseline		No adjust	
Statistic	College	Non college	College	Non college
Unempl. rate data	0.027	0.065	0.027	0.065
Unempl. rate model	0.026	0.064	0.026	0.064
Job-to-job trans. data	0.019	0.023	0.019	0.023
Job-to-job trans. model	0.019	0.024	0.019	0.024
Separation rates data	0.006	0.014	0.006	0.014
Separation rates model	0.007	0.016	0.007	0.016
Median wage in $ data	17.267	12.251	17.291	12.251
Median wage in $ model	15.361	10.403	15.473	10.385

Figure 5: CdF of baseline log wages



For the moments of the wage distribution, it is important to remark that the data are not "model independent", since we used the transformation of Equation 17: that is why we report the values for the fourth moment in Tables 3 separately from the others.

4.4 Closing the model

In the previous section we estimated the values of the parameters of interest and of the queue length, distinguishing between education achievement $j = \{\text{college, non college}\}$, i.e. we obtained the values for the set $\hat{\Psi}_{i,j} = \{\hat{\eta}_{i,j}, \hat{\lambda}_{i,j}, \tilde{\mu}_{i,j}, \hat{\sigma}_{i,j}, \hat{\rho}_{i,j}, \hat{q}_{i,j}\}$. We now proceed to recover the vacancy posting costs κ, by using the general equilibrium condition given by Equation 12. In order to do so, we need additional evidence on the screening costs ξ. We follow (Villena-Roldan 2010) and consider the National Employer Survey 1997 (NES97) to compute the average monetary cost incurred in that specific year for recruiting activities. To do this, we use the following questions of the survey:

Q29: What percent of total labor costs is spent annually on the recruitment and selection of employees?

Q3: What was the total labor cost used in the production of your 1996 sales?

Q30A: How many people have you hired in the past two years?

Table 3: Mean Square Errors

	Baseline		No adjust	
MSE Statistics	College	Non College	College	Non College
$\bar{Q}_1$ (data)	0.145	0.606	0.149	0.792
$\tilde{Q}_1$ (model)	0.146	0.608	0.150	0.793
$\bar{Q}_2$ (data)	0.026	0.038	0.027	0.038
$\tilde{Q}_2$ (model)	0.027	0.039	0.027	0.040
$\bar{Q}_3$ (data)	0.006	0.017	0.006	0.017
$\tilde{Q}_3$ (model)	0.008	0.022	0.009	0.022
$\bar{Q}_4$ (data)	0.211	0.145	0.215	0.145
$\tilde{Q}_4$ (model)	0.237	0.224	0.241	0.224

Q41: How many candidates do you interview for each [JOB TITLE] opening?

Therefore, using these responses the annual recruitment and selection cost (RSC) in 1996-1997 is computed as $RSC = Q29/100 \times Q3$. The total number of interviews over the last two years is $NIN = Q30A/2 \times Q41$, assuming each position is filled after interviewing $Q41$ applicants on average. Hence an estimate for the average recruiting cost is $\xi = RSC/NIN$. Indeed, RSC could be interpreted as the total amount spent including fixed costs. In a model without capital, κ would also reflect some fixed entry cost not accounted here. Vacancy-posting costs are probably negligible compared to the cost of capital required by the firm to produce.

Nevertheless, there is a substantial non-response rate in the survey. Only 48.2% (1486 out of 3081 respondants) answered the four questions we need to compute ξ. We then regress the observed ξ on a set of categorical variables for size, industry and multi-establishment firm, using the weights of the survey. Since these variables are observed for all surveyed employers, we predict the value non-respondents would have declared. The actual distribution of ξ and the one with imputed values are highly skewed. The results are reported in Table 4.

Table 4: Measurement of screening cost ξ (NES 1997)

	Mean (USD 1997)	Median (USD 1997)
Actual ξ	1172	80
ξ with imputations	1337	193

Given the influence that outliers and measurement error may have on the averages, we may consider medians as more reliable measures for ξ in 1997.

How can we compute values for ξ for other years? We adapt the idea of (Landais, Michaillat, and Saez 2018) who use some NES information reported by (Villena-Roldan 2010). Instead of measuring the share of workers, we use the average hourly wage of workers in areas such as

Occupation codes 1992-2002:

- 8: Personnel and labor relations workers.
- 27: Personal, training and labor relation specialists.

Occupation codes 2003-2010:

- 130: Human Resources Managers.
- 620: Human resources, training, and labor relations specialists.
- 5360: Human resources assistants, except payroll and timekeeping.

Occupation codes since 2011:

- 136: Human Resources Managers.
- 630: Human resource workers.
- 5360: Human resources assistants, except payroll and timekeeping.

To compute the recruiting cost ξ, we determine there is a proportionality, given by an adjustment factor ζ, between screening costs and recruiters' wages at any moment in time, such that

$$\xi_{1997} = \zeta \times \text{wage recruiters}_{1997}$$

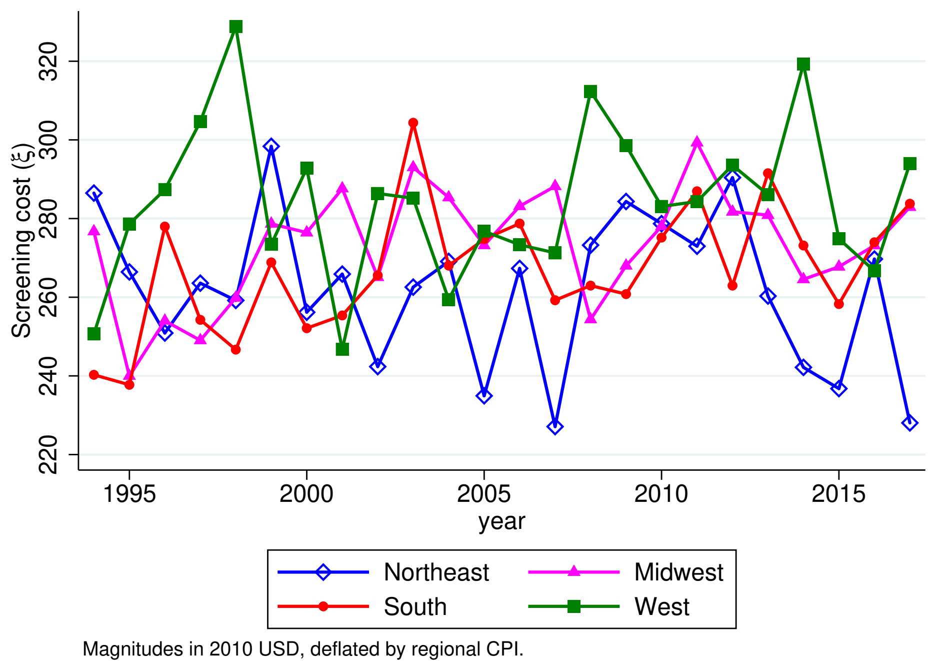
The recruiters wage in CPS-ORG data is 13.57 USD per hour, using the median of the NES97 screening cost adjusted by non-response, we compute an adjustment factor $\zeta = 14.2$. The series of reconstructed recruiting costs is reported in Table 6.

The vacancy posting costs κ can be considered as the cost of keeping an online job post for one month: current costs oscillates between 200 USD and 400 USD, so we consider an average value of 300 USD.

Considering the mean of screening costs ξ over the sample period, and the vacancy posting costs κ, we then use the free entry condition to obtain the value of the fixed entry cost χ:

$$\kappa + \beta\xi\phi\hat{q}(1 - e^{-\hat{q}}) + \chi(1 - e^{-\hat{q}}) = \beta \int_0^1 J(G^{-1}(M(x; \hat{q}, \hat{\eta}, \hat{\lambda}, \chi)))\phi\hat{q}e^{-\phi\hat{q}(1-x)}dx$$

Figure 6: Real recruiting cost wages CPS-ORG since 1994



Estimated recruiting cost is based on the median of the NES97 recruiting cost with imputations due to non-response.

The values for χ we obtain for our benchmark case for college and non college are respectively 97,322 USD and 34,022 USD.

5 Counterfactual experiments

In this section we perform some counterfactual experiments to better understand the functioning of the model, and to rationalize the empirical findings of Section 2.

We proceed as following: we consider an exogenous change in one of the model's parameters, namely *i)* an increase in the mean of the exogenous distribution of produtivity that preserves the spread, and *ii)* a mean preserving spread in the exogenous distribution of productivity.[10]

For each case, we keep all other parameters as fixed and recompute the value of the vacancy posting costs κ that solves the zero profit condition (Equation 12).

[10] In the Appendix we report additional counterfactual experiments, in particular: an increase in the separation rate η, an increase in the exogenous poaching probability λ, and an increase in the screening costs ξ.

5.1 The effects of changes in average productivity

We start by focusing on the effects of shifts in the productivity distribution that leave the variance unchanged. We assumed that productivity follows a log-normal distribution: $\theta \sim$ lognormal (μ, σ^2). Therefore we consider an exogenous change in the parameter μ, and we compute the parameter σ that allows to keep the variance constant.

The increase in average productivity is beneficial for the firm, which therefore opens more vacancies: the queue length and therefore the unemployment rate decrease monotonically as productivity increases, as we can see in the top panels of Figure 7 (the dots represent the benchmark equilibrium values).[11].

The bottom left panel of Figure 7 shows that as productivity increases (and therefore as the queue length decreases), both the job-to-job (EE) and the unemployment-to-employment (UE) transition rates increase. Moreover, the bottom right panel of Figure 7 shows that when the queue length decreases the unemployed agents gain relatively more in terms of transitions (the ratio EE/UE decreases as the lenght of the queue decreases, i.e. with the decrease in unemployment).

In terms of wages, the left Panel of Figure 8 shows that mean productivity is, quite naturally, positively related to the mean (log) wage. Moreover, wage inequality, expressed as the standard deviation of (log) wages also decreases, as mean productivity increases. The overall result in terms of the relation between the unemployment rate and wage inequality is thus an increasing concave function, as it can be seen in the right Panel of Figure 8.[12]

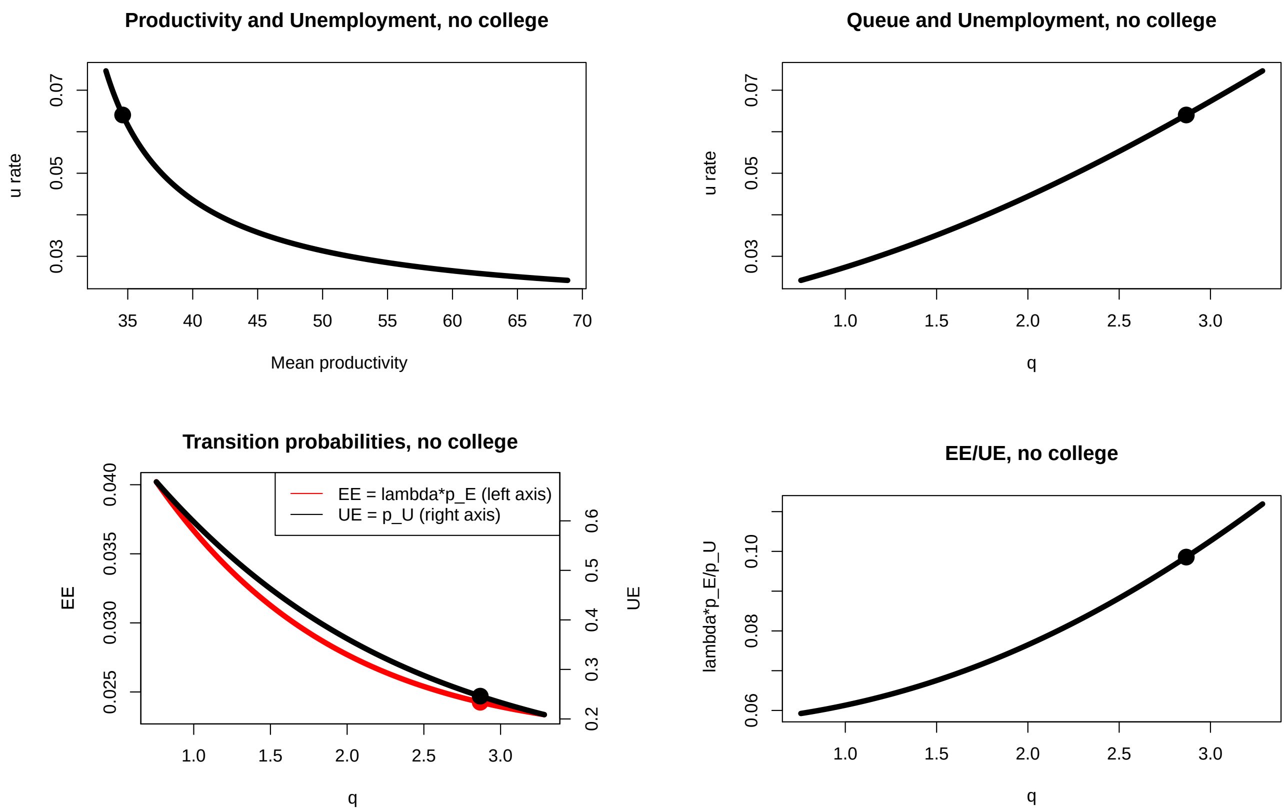
The overall effect of an increase in productivity which implies a reduction of the unemployment rate from 8.1% to 3% on the distributions of applicants is shown in the left Panel of Figure 9. The right Panel shows that when the unemployment rate is high, the distributions of employed and unemployed agents, in terms of their ability level, are relatively different; as mean productivity increases and thus unemployment decreases, the two distributions look more similar.

The bottom left Panel of Figure 9 shows the quantile mapping $M(x; q)$, where x represents the quintile and the equilibrium value of the queue length changes in equilibrium implying three different levels of unemployment rate. The closer to the 45 degrees line, the closer the distribution of population and applicants are. We can thus recover the results that, starting from an high unemployment rate (7.3%, associated to the black solid line), a decrease in unemployment rate to, for example, 6.1% is associated to a little reduction in inequality in terms

[11]Figure 7 shows the simulations for non college workers, but results are very similar for workers with a college degree, as it can be seen in Figure 35, Appendix A

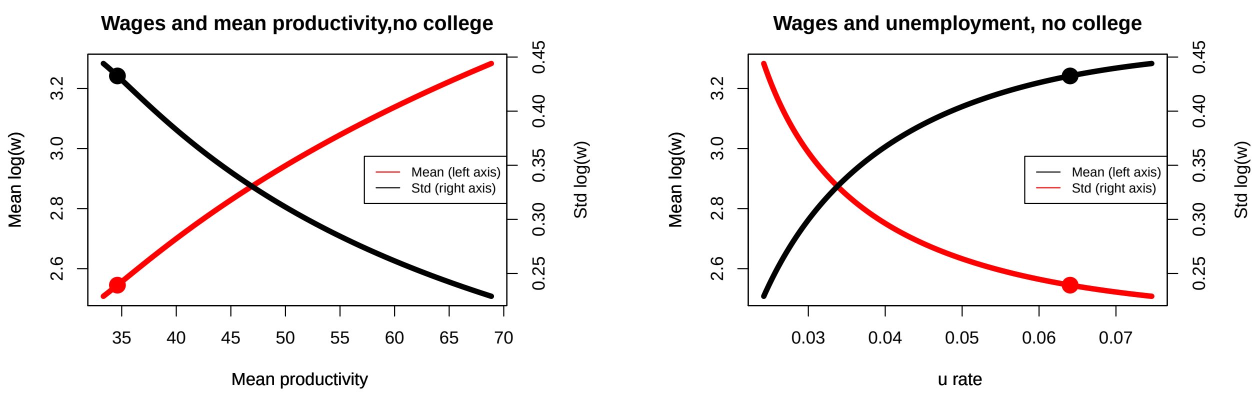
[12]Results are very similar for workers with a college degree, as it can be seen in Figure 35 in Appendix A.

Figure 7: The effects of an increase in average productivity on unemployment and transition probabilities



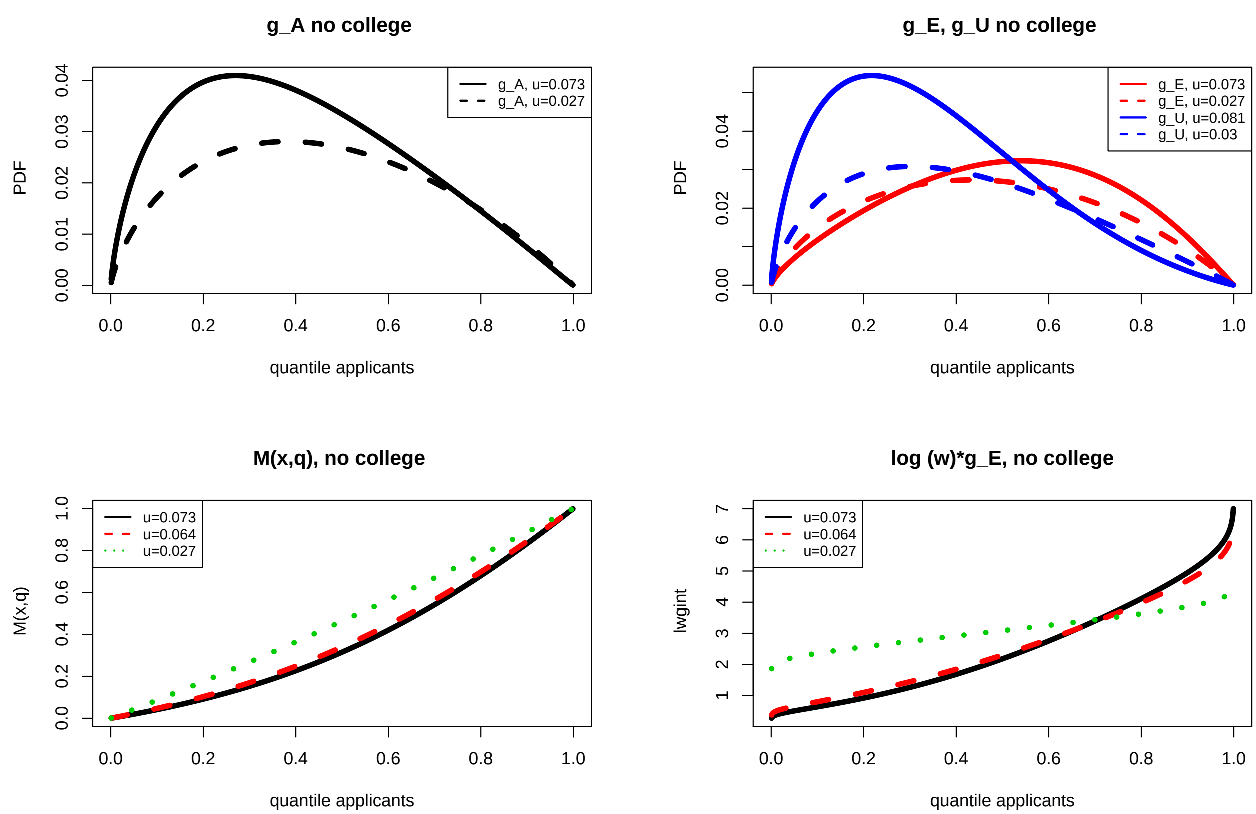
The dots represent the values for the benchmark calibration and estimation exercise.

Figure 8: The effects of an increase in average productivity on wages



The dots represent the values for the benchmark calibration and estimation exercise.

Figure 9: The effects of an increase in average productivity on wages and mass functions

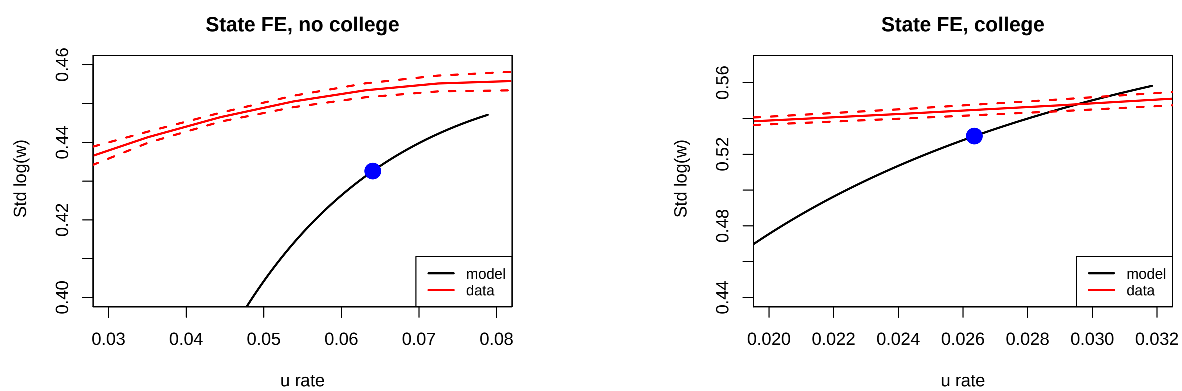


of quality composition of the two pools (the solid black line and the red dashed line are close to each other). A further decrease of unemployment to 2.7% brings instead a more visible change (from the dashed red line to the dotted green line). The right Panel of Figure 9 shows the integrand of the average (log) wage: as mean productivity increases the wage function, which just reflects productivity since $w(\theta) = \rho\theta$, also improves; however, the unemployment decreases, thus affecting the quality composition of the pool of the employed (the newly employed people are worse). The overall effect, coming from the composition of the two forces, is an increase in the average wage.

Figures 10 compares the model results with the data. We can observe that the performace of the main calibration exercise (how far the blue dot is with respect to the corresponding standard deviation of log wage for the targeted unemployment rate) is in general worse for non-college workers; however, the model, without being calibrated to match these effects, does imply a similar evolution to what we observe in the data.

For college workers the model in its benchmark calibration produces a value for the standard deviation of log wages which is closer to the data, but it overpredicts its reaction to changes in mean productivity.

Figure 10: The effects of an increase in average productivity on unemployment and wage inequality: model and data



The dots represent the values for the benchmark calibration and estimation exercise.

5.2 The effects of a mean preserving spread in productivity

In this section we show the effects of a mean preserving spread in the exogenous distribution of productivity. In particular, we consider exogenous changes in parameters μ and σ that imply a change in variance, while keeping constant the mean of the population distribution.

As expected, a higher variance in the ability distribution is reflected, in equilibrium, in a higher variance in terms of wages. Our model also implies that the average log wage decreases, as the left Panel of Figure 11 shows. With an increasing standard deviation of the productivity distribution, the unemployment rate decreases too, but only very slightly, as the Right Panel of Figure 11 shows. The overall relation between the unemployment rate and wage inequality (Right Panel of Figure 12), driven by an exogenous mean preserving spread in the ability distribution, is thus a decreasing one.

The Left Panel of Figure 12 shows the profit function for the firm for two different values of the standard deviation of the productivity distribution.

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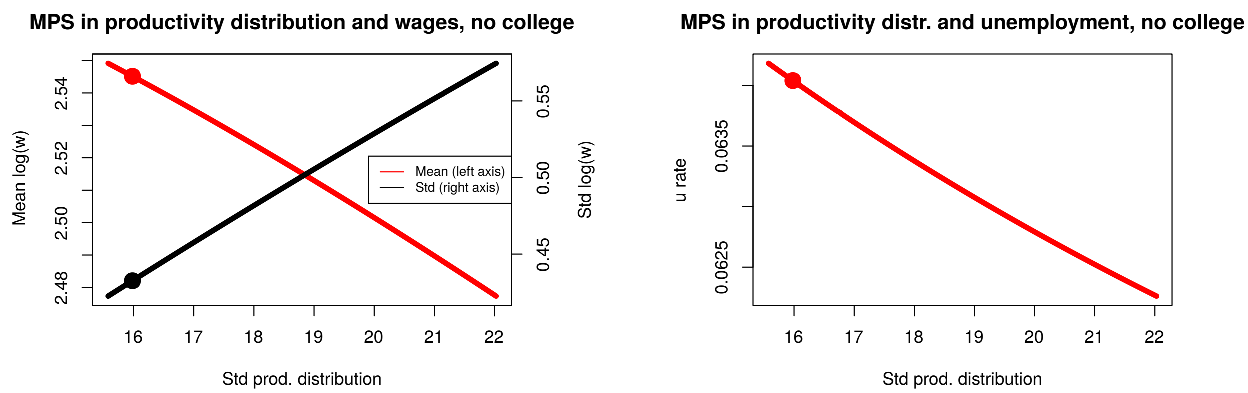
Figure 11: The effects of a mean preserving spread in productivity on wages and unemployment rate



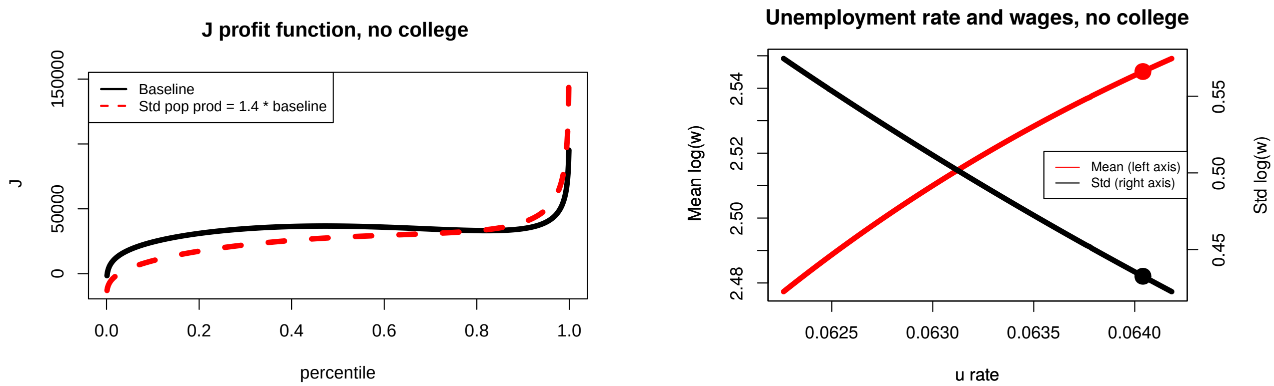
The dots represent the values for the benchmark calibration and estimation exercise.

5.3 The effects of changes in both mean and variance of productivity distribution

The counterfactual exercises on the effects of changes in only the mean or only the standard deviation of the population productivity distribution showed that: *(i)* an increase in the mean implies a reduction in the standard deviation of the wage distribution, as well as an overall increasing relation between unemployment rate and wage inequality; *(ii)* an increase in the standard deviation of the population productivity distribution implies a reduction in the average wage, and an overall decreasing relation between unemployment and wage inequality.

Our model can reproduce the increasing relation between unemployment rate and the standard deviation of wages, but it also overstates it. We do not claim that only changes in the average production distribution drove the relation between unemployment and wage inequality, but are rather interested in proposing potential mechanisms: we therefore also

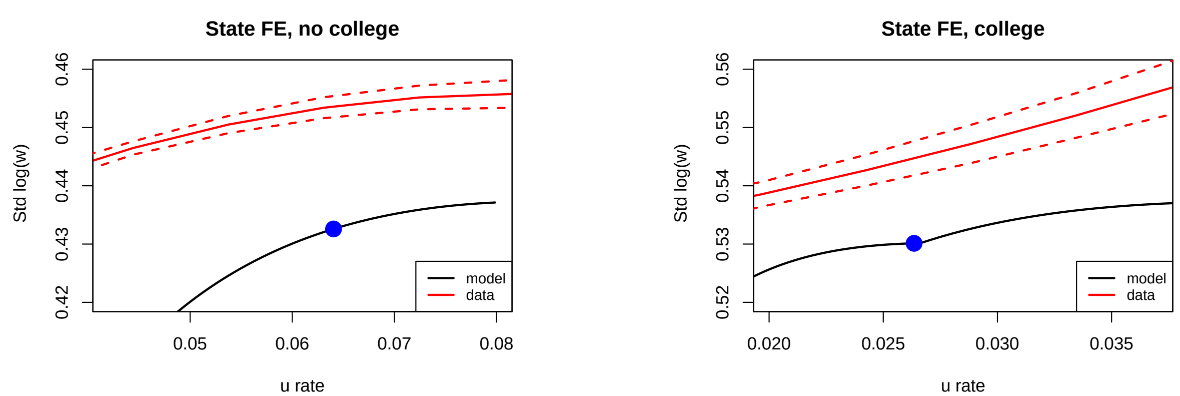
Figure 12: The effects of a mean preserving spread in productivity on unemployment and transition probabilities



The dots represent the values for the benchmark calibration and estimation exercise.

considered at the same time time changes in the mean and in the standard deviation of productivity distribution, considering that they counteract each other. The results are shown in Figure 13: the combination of both shocks is better suited to explain the overall trend observed in the data.

Figure 13: The effects of changes in both average and Std productivity



6 Efficiency Analysis

As it is common in matching models, there are many Pareto allocations. Indeed, it is not possible to switch a given employed worker for an unemployed one without hurting the former. Instead, we focus in allocations that maximize aggregate welfare or production (since workers are risk neutral). The Social Planner faces the same constraints that private agents have, in particular the same recruiting and screening technologies. The Social Planner may instruct firms to post vacancies and workers to apply. The screening effort control is embedded into an optimal q decision because the planner can affect the effective queue length by changing the probability of screening ϕ or by posting vacancies and changing q,i.e $q = \phi\tilde{q}$.

The matching process faced by the firms instructed by the SP still face the risk of not matching a worker at all. Since workers are still sending their applications to firms, the number of applications k received still follows a Poisson distribution with mean $q = \mathcal{A}/\mathcal{V}$. Therefore, the probability of matching at least one applicant is $1 - e^{-q}$. Thus, the Social Planner's objective is therefore

$$Y = \max_{q,\mathcal{V}} \left\{ \beta(1-\mathcal{U}) \int_0^\infty \theta g_E(\theta) d\theta + \beta\mathcal{U} \int_0^\infty \rho\theta g_U(\theta) d\theta - \underbrace{[(1-e^{-q})(\kappa\mathcal{V} + \chi + \beta\xi\mathcal{A}) + e^{-q}(\kappa\mathcal{V} + \chi)]}_{\text{recruiting costs}} \right\}$$

$$Y = \max_{q,\mathcal{V}} \left\{ (1-\mathcal{U}) \int_0^\infty \theta g_E(\theta) d\theta + \mathcal{U} \int_0^\infty \rho\theta g_U(\theta) d\theta - \underbrace{[\kappa\mathcal{V} + \chi + (1-e^{-q})\beta\xi\mathcal{A}]}_{\text{recruiting costs}} \right\}$$

where $g_E(\theta) \equiv \frac{g(\theta)p(\theta)}{(1-\mathcal{U})(\eta+p(\theta))}$ is the density of productivities of employed workers and $g_U(\theta) \equiv \frac{g(\theta)\eta}{\mathcal{U}(\eta+p(\theta))}$.

The recruiting costs faced by the Social Planner are given by the fact that with probability $(1 - e^{-q})$ the vacancy is filled so that, in addition to the vacancy posting costs ($\kappa\mathcal{V}$), in the next period it is necessary to pay the screening costs ($\beta\xi\mathcal{A}$) and to entry again in the market, therefore paying the cost χ. With probability (e^{-q}) the vacancy is not filled, because not even one application is received, so that the costs consist just the vacancy posting costs (it is not becessary to screen or to re-enter the market in the following period, since the vacancy is still available).

Using the definition of the distribution of applicants $g_A(\theta)$ and the fact that the population distribution always equals $g(\theta) = \mathcal{U}g_U(\theta) + (1-\mathcal{U})g_E(\theta)$, we realize that the distribution of the unemployed is

$$g_U(\theta) = \frac{\mathcal{A}g_A(\theta) - \lambda g(\theta)}{\mathcal{U}(1-\lambda)},$$

while the distribution of the employed can be written as

$$g_E(\theta) = \frac{g(\theta) - \mathcal{A}g_A(\theta)}{(1-\mathcal{U})(1-\lambda)}.$$

With some algebra and using quantile mapping, we obtain

$$\max_q Y(q) = \frac{\beta(1-\rho\lambda)}{(1-\lambda)}\mathbb{E}[\theta] - \mathcal{A}(q)\left[\frac{\kappa}{q} + (1-e^{-q})\beta\xi + \frac{(1-\rho)}{1-\lambda}\int_0^1 G^{-1}(M(x;q))dx\right] - \chi \quad (18)$$

considering that

$$\mathcal{A}(q) = \frac{1}{1 + \frac{1-\lambda}{\lambda q}\log\left(\frac{\eta+\lambda}{\eta+\lambda e^{-q}}\right)}.$$

The first term of equation (18) is proportional to the output obtained in a frictionless environment, $\overline{Y} = \mathbb{E}[\theta]$. The proportionality factor $\frac{1-\rho\lambda}{1-\lambda}$ indicates there is an obvious loss because unemployed workers only generate a fraction ρ of their productivity. However, the probability of on-the-job search, λ attenuates this effect to some because a fraction of new hirings do not originate through jobless workers.

The first two terms in the square parehtesis indicate the recruiting and screening costs. The number of posted vacancies $\mathcal{V}$ is managed by the Social Planner to control the extensive margin of hiring at the firm level, whereas q controls the screening activity.

The last term in the square parehtesis refers to the loss due to the negative impact that recruiting activities generate on the quality of the pool of applicants.

The more selective the recruiting process is through a higher q, the lower the quality of the average pool of applicants. Although more frequent on-the-job search offsets this selectivity effect to some extent, the overall impact is a decline in the productivity of the average hiring of firms in the economy.

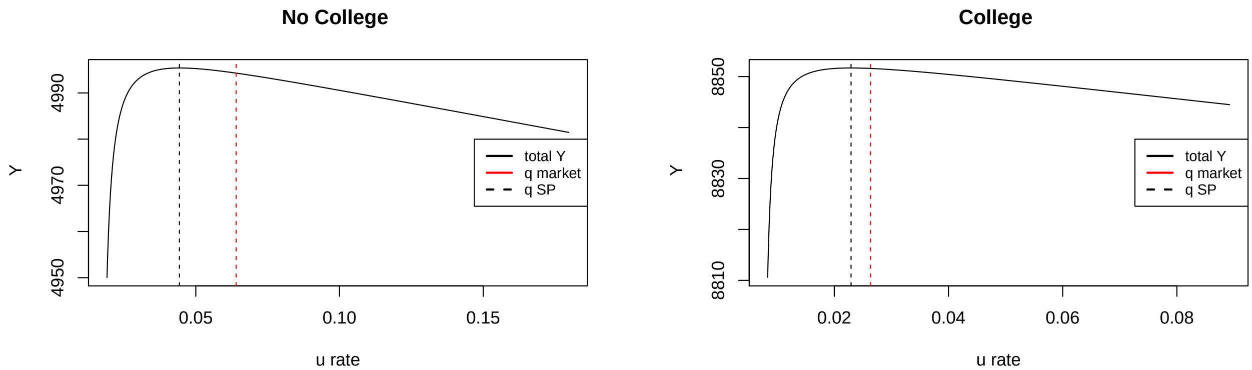
If the Social Planner is allowed to instruct vacancy-posting and screening, she is clearly constrained by the identity $q = \mathcal{A}(q)/\mathcal{V}$.

The average productivity of the workers in the economy is not affected by SP decisions, so that the optimal queue length q^{SP} chosen can be written as

$$q^{SP} = \underset{q}{\operatorname{argmin}}\left\{\log(\mathcal{A}(q)) + \log\left(\frac{\beta(1-\rho)}{1-\lambda}\int_0^1 G^{-1}(M(x,q))dx + \frac{\kappa}{q} + (1-e^{-q})\beta\xi\right)\right\}$$

Taking first-order conditions on the Social Planner program, we obtain the following expression:

Figure 14: **Queue length and unemployment: market versus social planner**



$$\frac{\kappa}{q}[1-\widetilde{\mathcal{A}}(q)]-\widetilde{\mathcal{A}}(q)\beta\xi(1-e^{-q})-q\xi\beta e^{-q}=$$
$$\frac{\beta(1-\rho)}{1-\lambda}\left[\widetilde{\mathcal{A}}(q)\int_0^1 G^{-1}(M(x,q))dx+q\int_0^1\frac{M_q(x;q)}{g(G^{-1}(M(x,q)))}dx\right]$$

where $\widetilde{\mathcal{A}}(q)=\frac{\partial\log\mathcal{A}(q)}{\partial\log q}$ is the elasticity of $\mathcal{A}$ with respect to q.

The denominator of the right-hand side of (**??**) indicates the expected social cost of an additional applicant since the term $\frac{\beta(1-\rho)}{1-\lambda}\int_0^1 G^{-1}(M(x,q))dx$ is the expected loss due to an unmatched applicant given the prevailing equilibrium composition of applicants reflected by $G_A^{-1}(x,q)=G^{-1}(M(x,q))$ and the queue length q. In addition to this loss, the SP also considers the recruiting cost associated per applicant, including the interview cost, ξ, an the average vacancy cost, κ/q. The numerator is just the marginal cost of the average cost of recruiting. Therefore, the optimality condition just says that the increase of applicants that the SP generates when choosing a higher must equate the whole expected cost increase of that decision, including more selectivity through the $M_q(x,q)$ term, larger screening costs (through the ξ term in the denominator, and lower average fixed costs, through the term κ/q.

On the other hand, in a competitive equilibrium, q is determined by the Free-Entry condition

$$\kappa+\beta\xi\phi q(1-e^{-q})+\chi(1-\beta e^{-q})=\beta\int_0^1 J(G^{-1}(M(x;q,\eta,\lambda)))\phi q e^{-\phi q(1-x)}dx \qquad (19)$$

and substituting the expression for the value function of a filled job from eq. (14), it can be rewritten as

$$\kappa+\beta\xi\phi q(1-e^{-q})+\chi(1-\beta e^{-q}) = \beta\int_0^1 \frac{(1-\rho)G^{-1}(M(x;q))+\beta\chi[\eta^*+\lambda e^{-q(1-x)}]}{1-\beta[1-\eta^*-\lambda e^{-q(1-x)}]}e^{-q(1-x)}(1-\tau(x;q))dx$$

A tax $\tau(x;q)$ can be obtained so that the decentralised equilibrium queue lenght is the same the social planner would choose: the first equation of system (20) is the Social Planner FOC, while the second one is the decentralized equilibrium free entry condition obtained when substituting the expression for the value function of a filled job from eq. (14).

$$\begin{cases} \frac{\kappa}{q}+\beta\xi e^{-q}\widetilde{\mathcal{A}}(q) = \widetilde{\mathcal{A}}(q)\left(\frac{\kappa}{q}+\xi\beta\right)+q\xi\beta e^{-q}+\frac{\beta(1-\rho)}{1-\lambda}\left[\widetilde{\mathcal{A}}(q)\int_0^1 G^{-1}(M(x,q))dx+q\int_0^1\frac{M_q(x;q)}{g(G^{-1}(M(x,q)))}dx\right] \\ \kappa+\beta\xi\phi q(1-e^{-q})+\chi(1-\beta e^{-q}) = \beta\int_0^1\frac{(1-\rho)G^{-1}(M(x;q))+\beta\chi[\eta^*+\lambda e^{-q(1-x)}]}{1-\beta[1-\eta^*-\lambda e^{-q(1-x)}]}e^{-q(1-x)}(1-\tau(x;q))dx \end{cases} \tag{20}$$

We notice that the optimal tax is generally dependent in x and q, which implies some progressive and labor-market contingent tax scheme.

To be implementable in a competitive equilibrium, the tax schedule must preserve a coincidence ranking equilibrium so that firms strictly prefer hiring higher types after paying the corresponding tax. Since $\theta = G^{-1}(M(x;q))$, the coincidence ranking implies that

$$\frac{dJ(\theta)}{d\theta}>0 \quad\Rightarrow\quad \frac{dJ(x)}{\frac{M_x(x;q)dx}{g(G^{-1}(M(x;q)))}} = \frac{\partial\left(\frac{[(1-\rho)G^{-1}(M(x,q))+\beta\chi(\eta^*+\lambda e^{-q(1-x)})]}{1-\beta[1-\eta^*-\lambda e^{-q(1-x)}]}(1-\tau(x,q))\right)}{\partial x}\frac{g(G^{-1}(M(x;q)))}{M_x(x;q)}>0$$

$\forall x\in[0,1], q>0$, which can be rewritten as

$$\frac{\partial\log((1-\rho)G^{-1}(M(x,q))+\beta\chi(\eta^*+\lambda e^{-q(1-x)}))}{\partial x}+\frac{\partial\log(1-\tau(x,q))}{\partial x}>\frac{\partial\log(1-\beta[1-\eta^*-\lambda e^{-q(1-x)}])}{\partial x} \tag{21}$$

$\forall x\in[0,1], q>0$. We can therefore find a constant φ_1 such that

$$\frac{\partial\log((1-\rho)G^{-1}(M(x,q))+\beta\chi(\eta^*+\lambda e^{-q(1-x)}))}{\partial x}+\frac{\partial\log(1-\tau(x,q))}{\partial x}=\varphi_1\frac{\partial\log(1-\beta[1-\eta^*-\lambda e^{-q(1-x)}])}{\partial x} \tag{22}$$

Yet a feasibility condition for implementation is a tax balanced budget, i.e. that the net revenue is zero. Assuming that only employed workers are taxed (or equivalently, that only firms are taxed), we obtain that

$$(1-\mathcal{U})\int_0^\infty \tau(G_A(\theta),q)\theta dG_E(\theta)=0 \tag{23}$$

TO BE COMPLETED

7 Conclusions

We developed a theoretical model that can shed some light into the consequences of selection of workers who are heterogenous in productivities. We started from some empirical facts that show a positive correlation between the unemployment rate and a measure of inequality which is the standard deviation of log wages, and a negative correlation between job finding probabilities as well as job-to-job transition rates and our measure of inequality. Then we constructed a non sequential search model in which firm can pay a screening cost and perfectly learn the productivity type of the worker. Workers can apply to jobs while employed, so the pool of applicants is composed by a mixture of employed and unemployed agents. Labor market transitions affect the composition of the employed pool, so that the *ex post* distribution is an endogenous object.

We estimated a set of parameters using a Simulated Method of Moments. We then used the calibrated version of the model to perform some counterfactual experiments. We learnt that a possible driver of the correlations we found in the data is given by shifts in the productivity distribution that affect its mean, but not its standard deviation. However, our counterfactuals also show that some non-linearities can appear in the relationship between our labor market variables and inequalities.

References

Blanchard, O. and P. Diamond (1994). Ranking, unemployment duration, and wages. *Review of Economic Studies 61*(3), 417–434.

Bradley, J. (2020). Worker-Firm Screening and the Business Cycle. Working paper, University of Nottingham.

Butters, G. R. (1977). Equilibrium distributions of sales and advertising prices. *Review of Economic Studies 44*(3), 465–491.

Cai, X., P. Gautier, and R. Wolthoff (2021, June). Search, Screening and Sorting. Tinbergen Institute Discussion Papers 21-058/VI, Tinbergen Institute.

Card, D. and J. E. DiNardo (2002). Skill-biased technological change and rising wage inequality: Some problems and puzzles. *Journal of Labor Economics 20*(4), 733–783.

Davis, S. J. and B. Samaniego de la Parra (2017). Application Flows. Unpublished manuscript.

Eeckhout, J. and I. Lindenlaub (2018). Unemployment cycles. *American Economic Journal: Macroeconomics forthcoming*.

Engbom, N. (2020). Contagious Unemployment. Working paper, New York University.

Fernández-Blanco, J. and E. Preugschat (2018). On the effects of ranking by unemployment duration. *European Economic Review 104*(C), 92–110.

Goldin, C. and L. F. Katz (2008). *The race between education and technology*. Harvard Univiersity Press.

Gottfries, A. (2017). Bargaining with renegotiation in models with on-the-job search.

Helpman, E. (2016, December). Globalization and Wage Inequality. Working Paper 22944, National Bureau of Economic Research.

Krusell, P., L. E. Ohanian, J.-V. Ríos-Rull, and G. L. Violante (2000). Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica 68*(5), pp. 1029–1053.

Landais, C., P. Michaillat, and E. Saez (2018, May). A macroeconomic approach to optimal unemployment insurance: Applications. *American Economic Journal: Economic Policy 10*(2), 182–216.

Merkl, C. and T. van Rens (2019). Selective hiring and welfare analysis in labor market models. *Labour Economics 57*(C), 117–130.

Moen, E. (1999). Education, Ranking, and Competition for Jobs. *Journal of Labor Economics 17*(4), 694.

Moore, M. P. and P. Ranjan (2005). Globalisation vs Skill-Biased Technological Change: Implications for Unemployment and Wage Inequality. *The Economic Journal 115*(503), 391–422.

Mortensen, D. T. and C. A. Pissarides (1994, jul). Job Creation and Job Destruction in the Theory of Unemployment. *The Review of Economic Studies 61*(3), 397–415.

Petrosky-Nadeau, N. and E. Wasmer (2015). Macroeconomic dynamics in a model of goods, labor, and credit market frictions. *Journal of Monetary Economics 72*, 97–113.

Petrosky-Nadeau, N. and E. Wasmer (2017). *Labor, Credit, and Goods Markets: The macroeconomics of search and unemployment*. MIT Press.

Pissarides, C. (2000). *Equilibrium Unemployment Theory* (Second ed.). The MIT Press.

Ravallion, M. (2018, June). Inequality and globalization: A review essay. *Journal of Economic Literature 56*(2), 620–42.

Shi, S. (2006, May). Wage differentials, discrimination and efficiency. *European Economic Review 50*(4), 849–875.

Shimer, R. (2005, Oct). The Assignment of Workers to Jobs in an Economy with Coordination Frictions. *Journal of Political Economy 113*(5), 996–1025.

Shimer, R. (2006). On-the-job search and strategic bargaining. *European Economic Review 50*(4), 811 – 830.

Stern, S. (1989, jul). Estimating a Simultaneous Search Model. *Journal of Labor Economics 7*(3), 348–369.

Stigler, G. (1961). The Economics of Information. *Journal of Political Economy 3*(69), 213–225.

van Ommeren, J. and G. Russo (2013). Firm Recruitment Behaviour: Sequential or Nonsequential Search? *Oxford Bulletin of Economics and Statistics*, 1–24.

van Ours, J. and G. Ridder (1992). Vacancies and the Recruitment of New Employees. *Journal of Labor Economics 10*(2), 138–155.

Villena-Roldan, B. (2010). Aggregate Implications of Employer Search and Recruiting Selection. Working Paper 271, Center for Applied Economics, University of Chile.

Wolthoff, R. (2017, 08). Applications and Interviews: Firms' Recruiting Decisions in a Frictional Labour Market. *The Review of Economic Studies 85*(2), 1314–1351.

Appendix

Appendix A Robustness of empirical facts

A Robustness to 1994 CPS sampling redesign

Figure 15: Standard deviation of log wages vs Unemployment Rate by State & Year

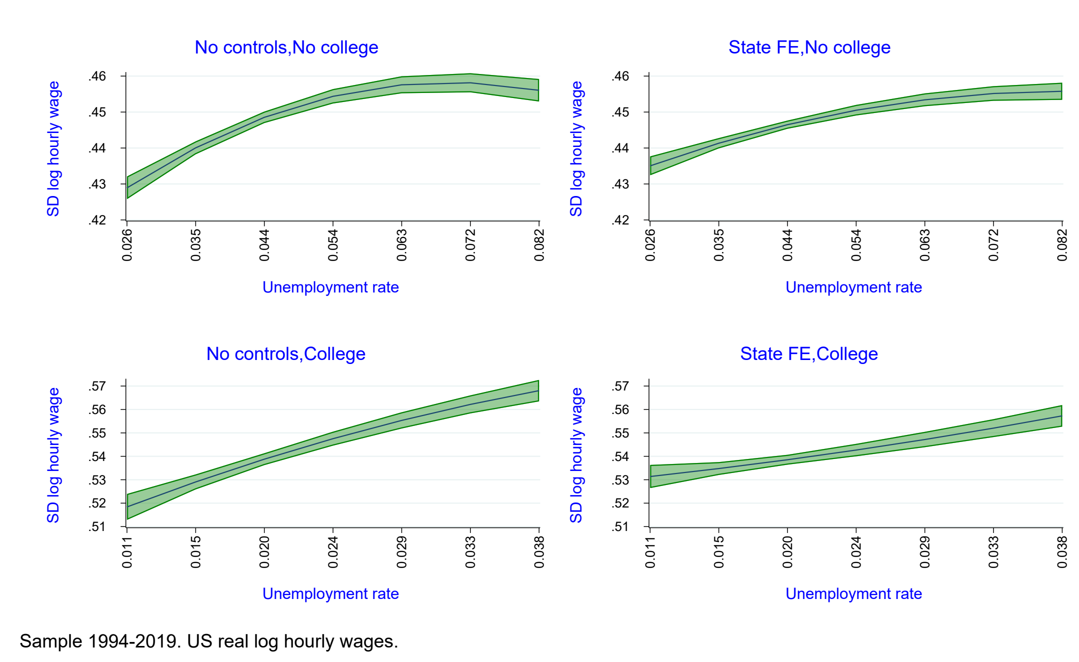
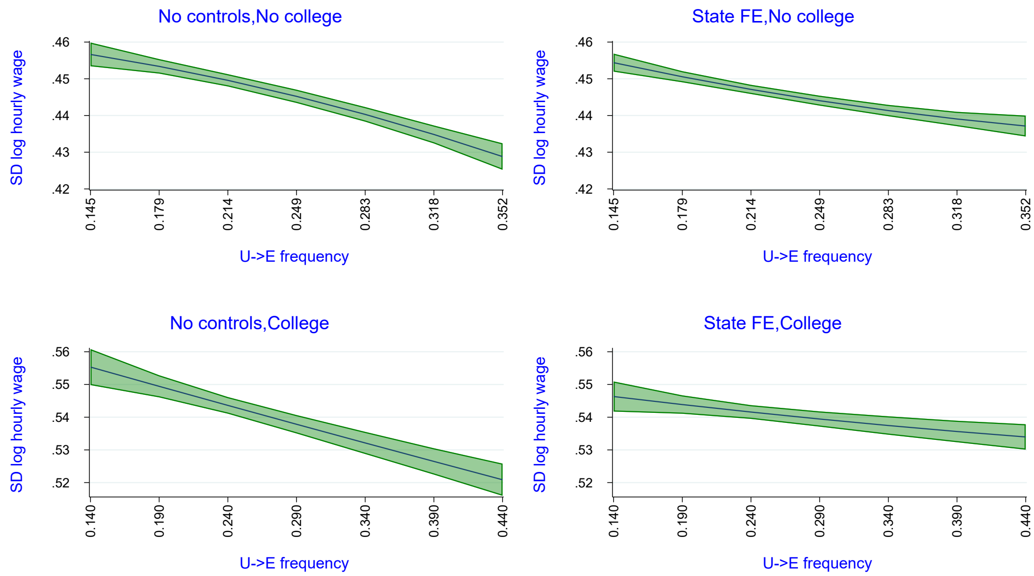


Figure 16: Standard deviation of log wages vs UE frequency by State & Year



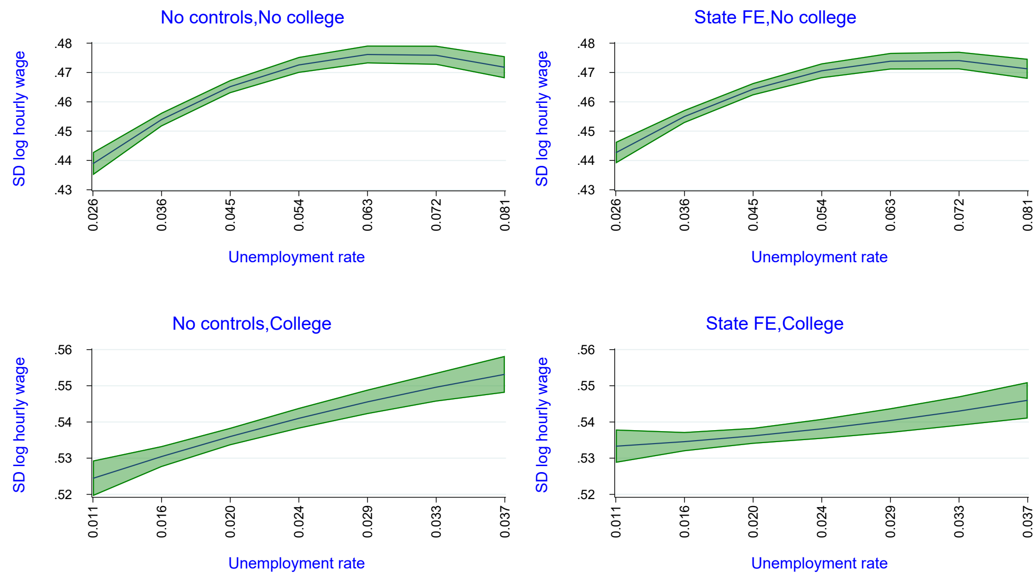
Sample 1994-2019. US real log hourly wages.

Figure 17: Standard deviation of log wages vs EU frequency by State & Year

Sample 1994-2019. US real log hourly wages.

B Robustness to regional adjusted real wages

Figure 18: Standard deviation of log wages vs Unemployment Rate by State & Year

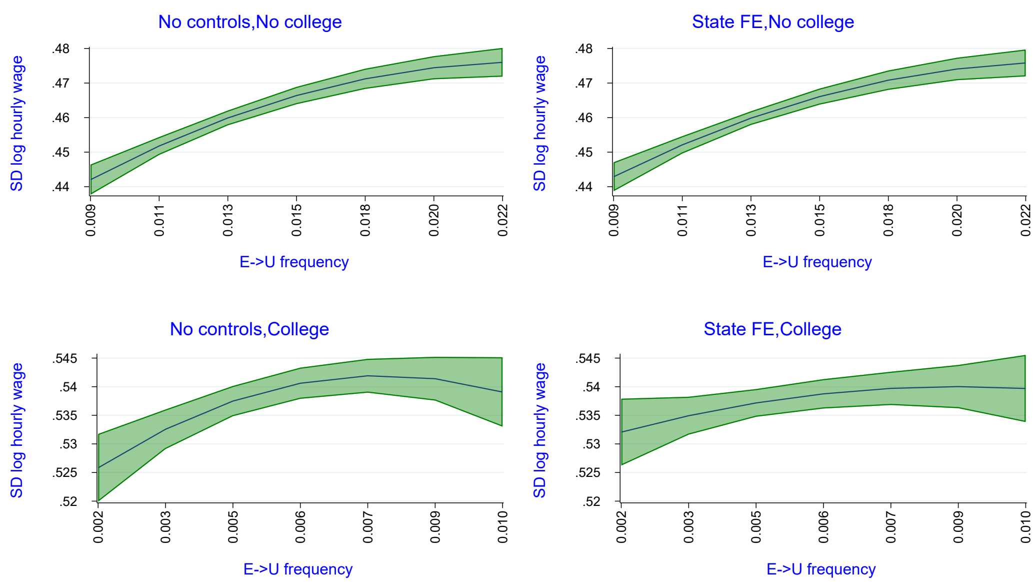


Sample 1976-2019. Region real log hourly wages.

Figure 19: Standard deviation of log wages vs UE frequency by State & Year

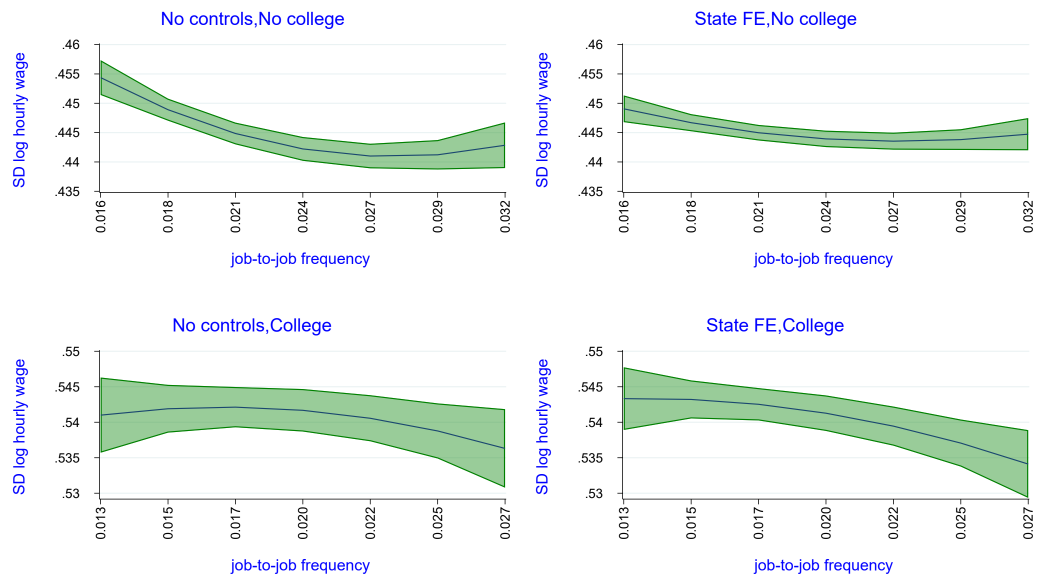
Sample 1976-2019. Region real log hourly wages.

Figure 20: Standard deviation of log wages vs EU frequency by State & Year



Sample 1976-2019. Region real log hourly wages.

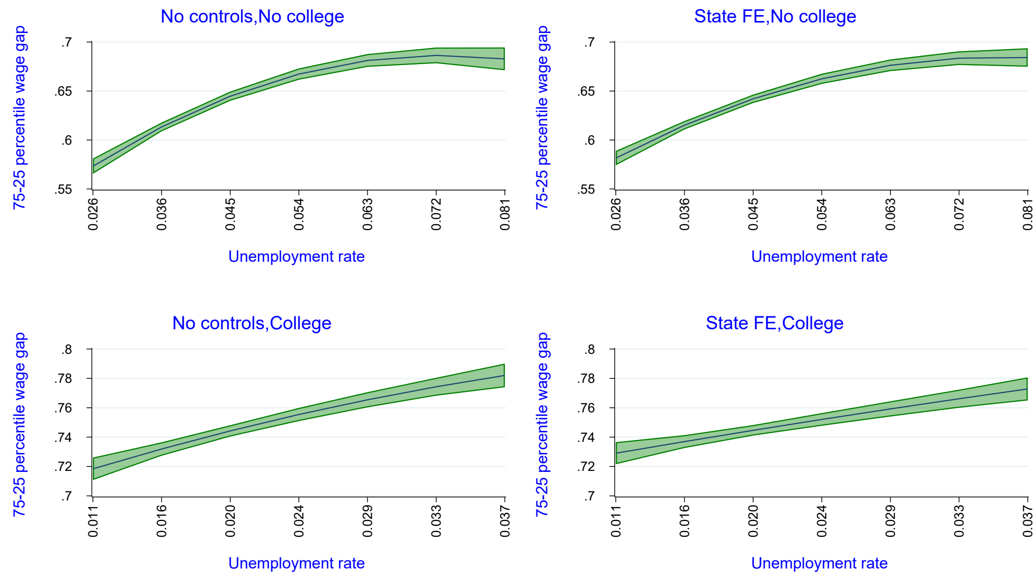
Figure 21: Standard deviation of log wages vs JJ frequency by State & Year



Sample 1994-2019. Region real log hourly wages.

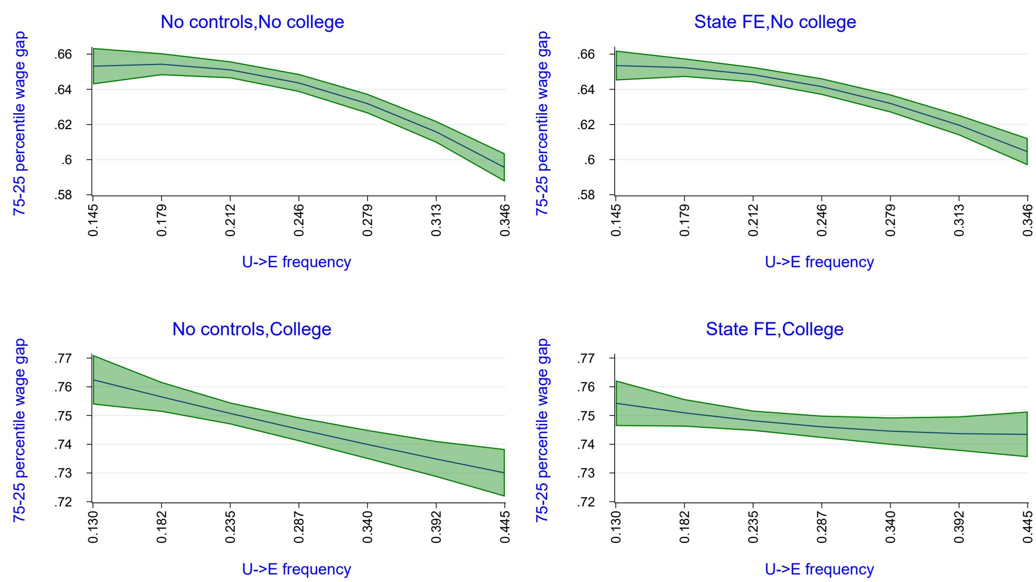
C Alternative measure of dispersion

Figure 22: 75-25 percentile log wage gap vs Unemployment Rate by State & Year



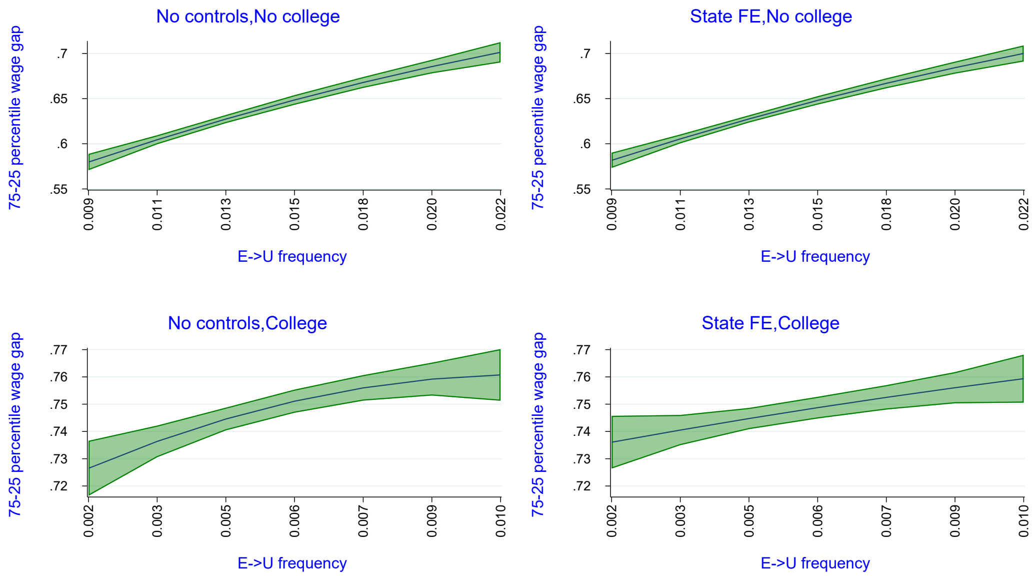
Sample 1976-2019. US real log hourly wages.

Figure 23: 75-25 percentile log wage gap vs UE frequency by State & Year



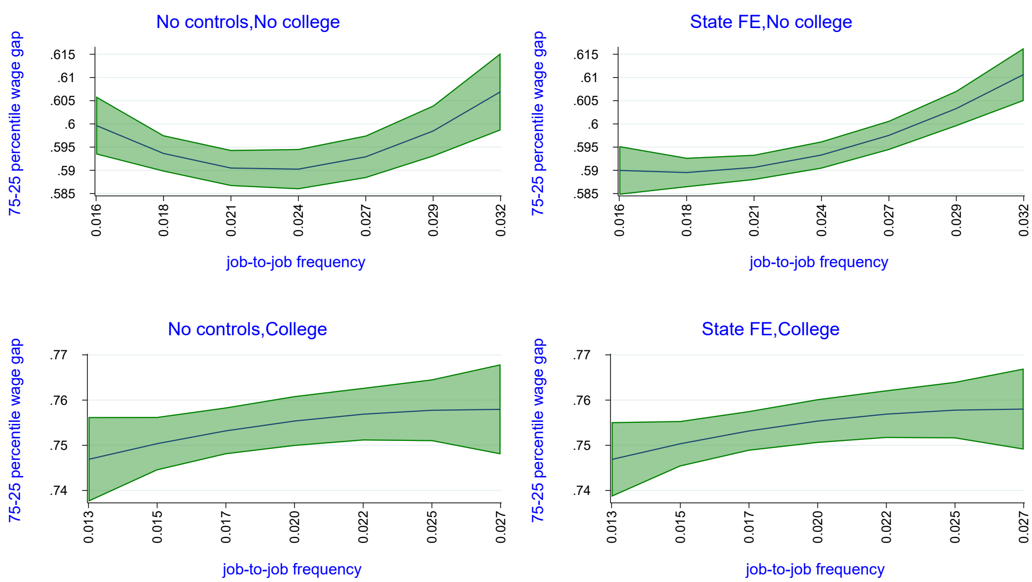
Sample 1976-2019. US real log hourly wages.

Figure 24: 75-25 percentile log wage gap vs EU frequency by State & Year



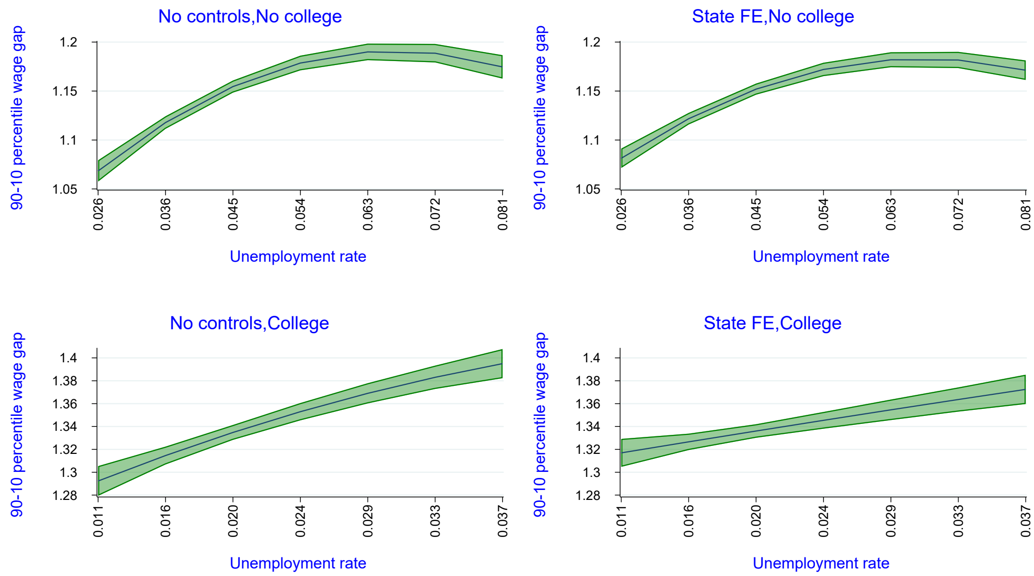
Sample 1976-2019. US real log hourly wages.

Figure 25: 75-25 percentile log wage gap vs JJ frequency by State & Year



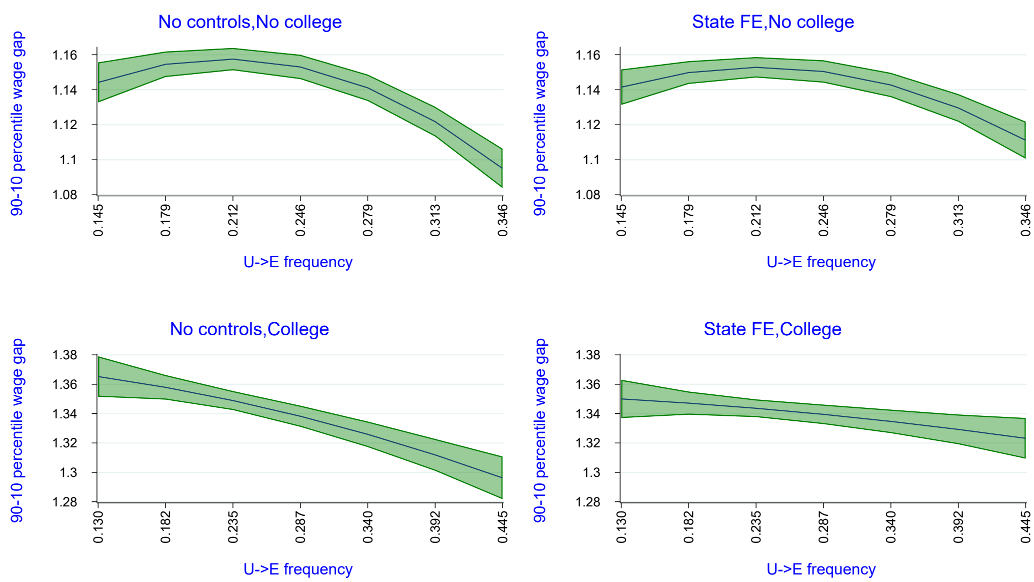
Sample 1994-2019. US real log hourly wages.

Figure 26: 90-10 percentile log wage gap vs Unemployment Rate by State & Year



Sample 1976-2019. US real log hourly wages.

Figure 27: 90-10 percentile log wage gap vs UE frequency by State & Year

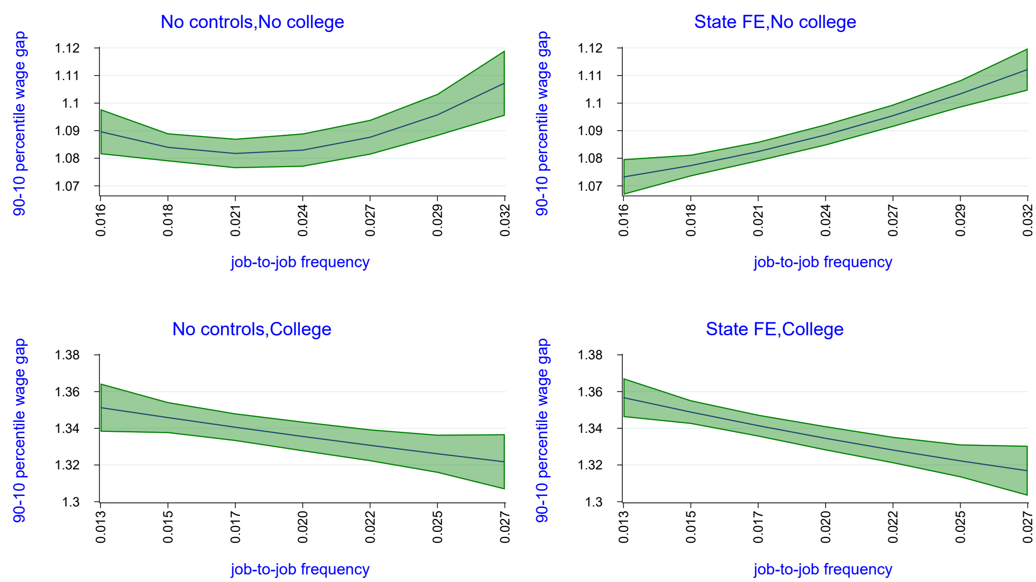


Sample 1976-2019. US real log hourly wages.

Figure 28: 90-10 percentile log wage gap vs EU frequency by State & Year

Sample 1976-2019. US real log hourly wages.

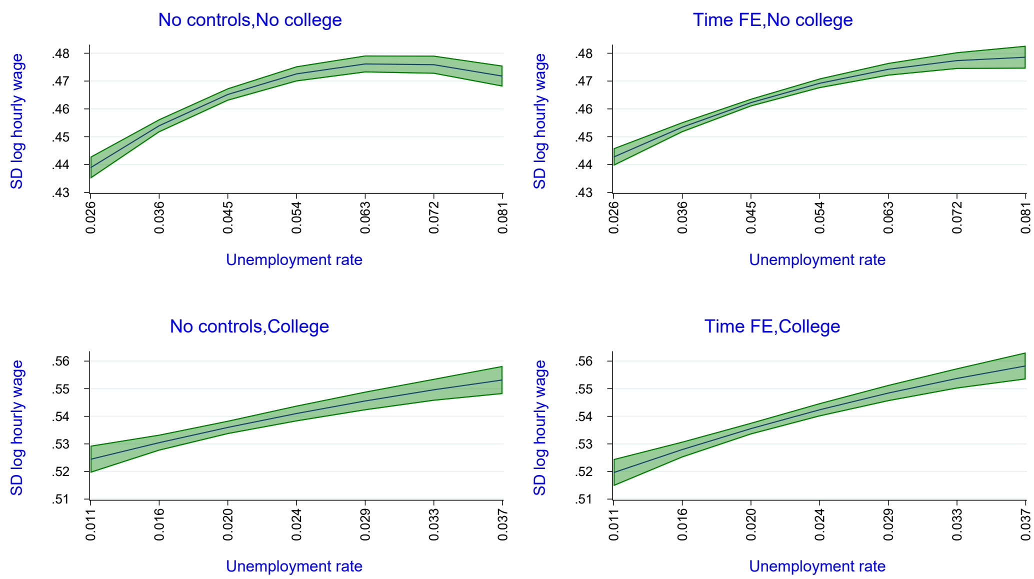
Figure 29: 90-10 percentile wage gap vs JJ frequency by State & Year



Sample 1994-2019. US real log hourly wages.

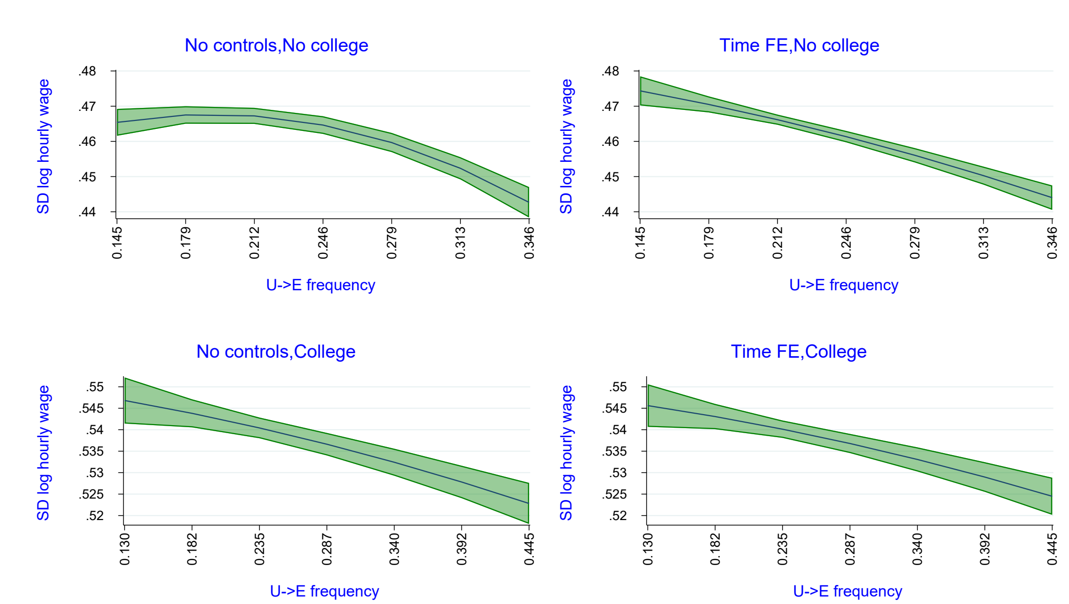
D Time fixed effects

Figure 30: Standard deviation of wages vs Unemployment Rate by State & Year



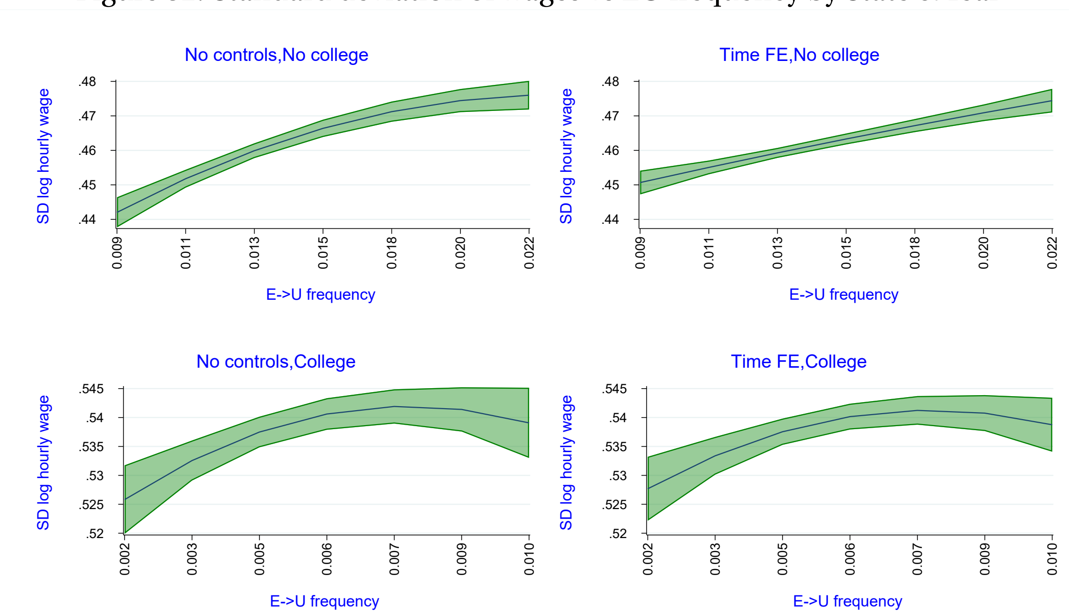
Sample 1976-2019. US real log hourly wages.

Figure 31: Standard deviation of wages vs UE frequency by State & Year



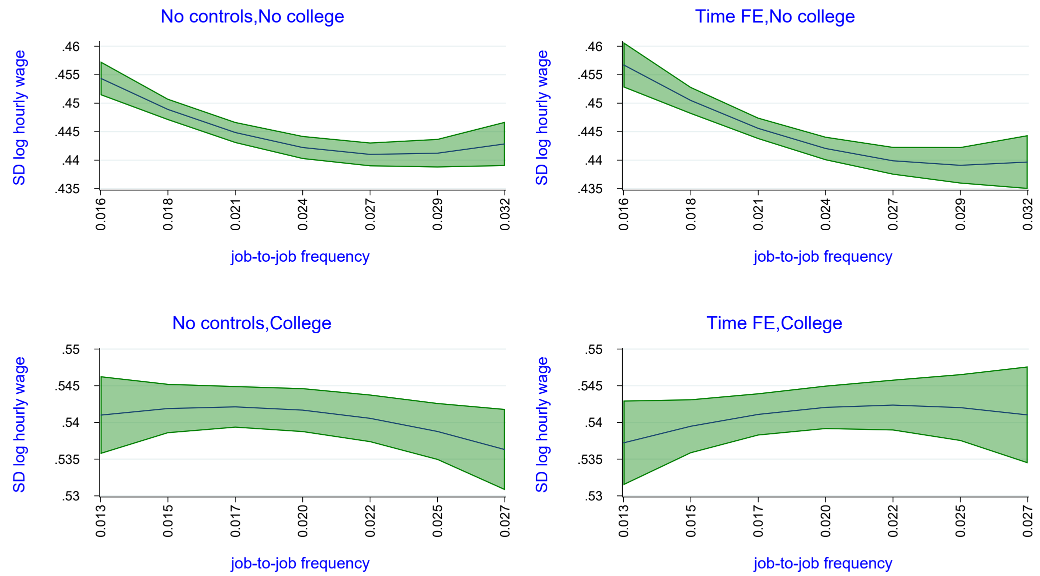
Sample 1976-2019. US real log hourly wages.

Figure 32: Standard deviation of wages vs EU frequency by State & Year



Sample 1976-2019. US real log hourly wages.

Figure 33: Standard deviation of wages vs JJ frequency by State & Year



Sample 1994-2019. Region real log hourly wages.

Appendix B Deriving Free-Entry Condition (12)

$$-\kappa + \beta \left\{ \sum_{k=1}^{\infty} \frac{e^{-\phi q}(\phi q)^k}{k!} \left[-\xi q + \int_0^{\infty} J(v) k (\phi G_A(v) + (1-\phi))^{k-1} g_A(v) dv \right] + e^{-\phi q} V \right\} = V$$

$$\kappa + \beta(1-e^{-\phi q})\xi \phi q + \chi(1-\beta e^{-\phi q}) = \beta \left(\sum_{k=1}^{\infty} \frac{e^{-\phi q}(\phi q)^k}{k!} \int_0^{\infty} J(v) k (\phi G_A(v) + (1-\phi))^{k-1} g_A(v) dv \right)$$

$$\kappa + \beta(1-e^{-\phi q})\xi \phi q + \chi(1-\beta e^{-\phi q}) = \beta \left(\sum_{k=1}^{\infty} \frac{e^{-\phi q}(\phi q)^{k-1}}{(k-1)!} \int_0^{\infty} J(v) \phi q (\phi G_A(v) + (1-\phi))^{k-1} g_A(v) dv \right)$$

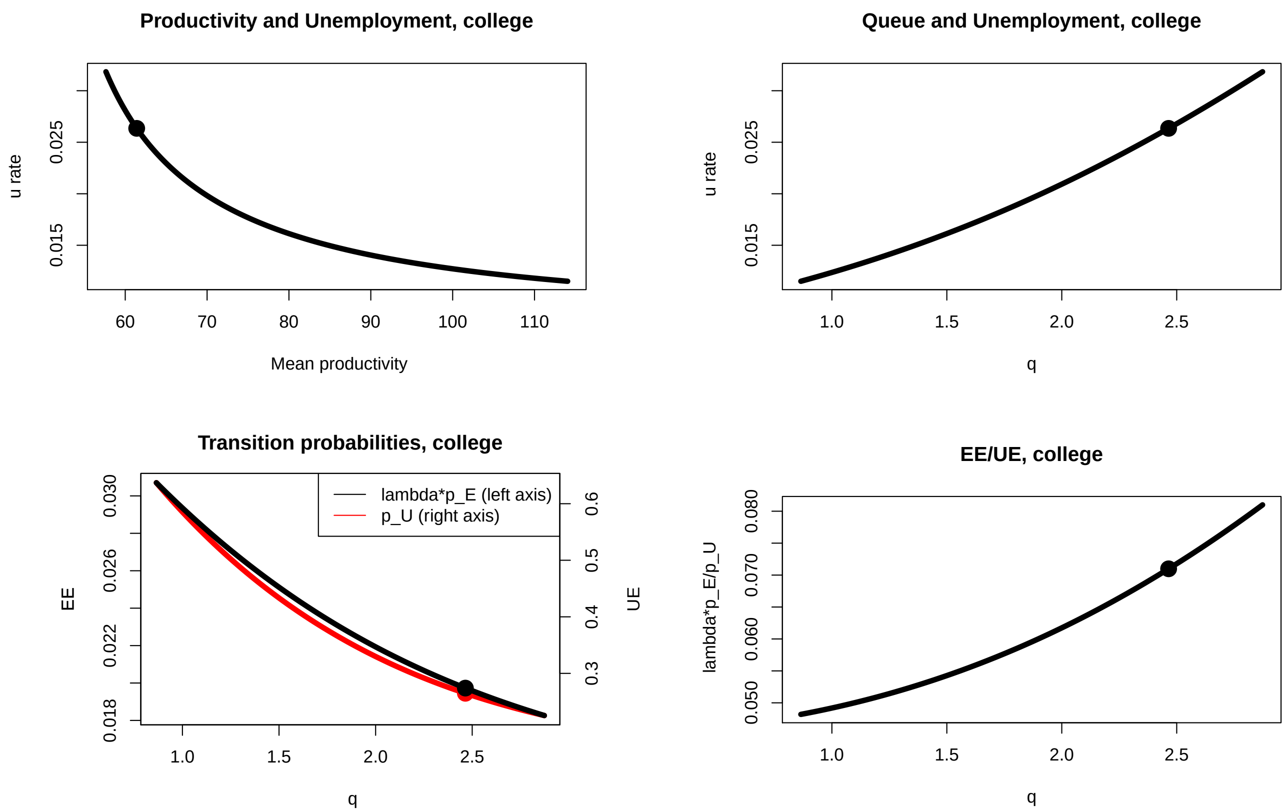
$$\kappa + \beta(1-e^{-\phi q})\xi \phi q + \chi(1-\beta e^{-\phi q}) = \beta \left(\int_0^{\infty} J(v) \phi q e^{-\phi q(1-G_A(v))} dG_A(v) \right)$$

$$\kappa + \beta(1-e^{-\phi q})\xi \phi q + \chi(1-\beta e^{-\phi q}) = \beta \phi q \left(\int_0^1 J(G^{-1}(x;q)) e^{-\phi q(1-x)} dx \right)$$

Appendix C Counterfactual experiments

A The effects of an increase in mean productivity for college

Figure 34: The effects of an increase in average productivity on unemployment and transition probabilities (college)

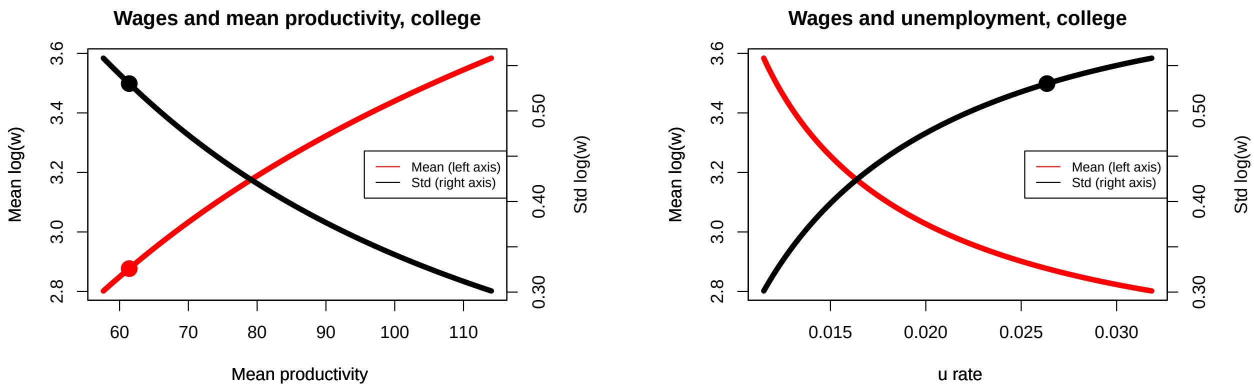


The dots represent the values for the benchmark calibration and estimation exercise.

B The effects of an increase in poaching probability λ

If the probability of applying to a job while working exogenously increases, there is more competition for jobs, since more employed agents enter into the applicants pool. This stronger competition increases the average length of the queue, as well as the unemployment rate, as we can see in the top left panel of Figure 36. When the unemployment rate is higher firms can screen more: the increase in selectivity means that the distribution of applicants differs more from the distribution of the whole population when λ, and therefore the unemployment rate, is higher, as it can be seen in the top right panel of Figure 36. The job finding probability of

Figure 35: The effects of an increase in average productivity on wages for college



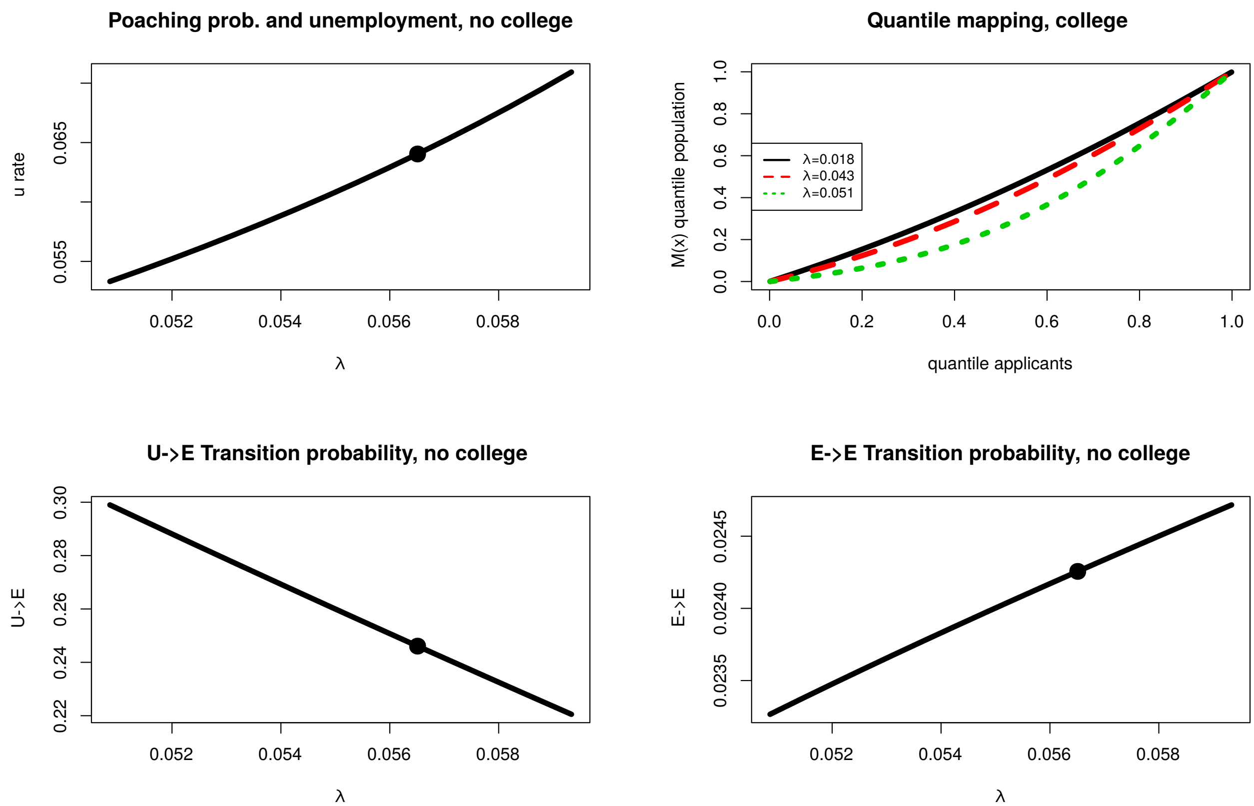
The dots represent the values for the benchmark calibration and estimation exercise.

the unemployed (in our notation, the probability $\bar{p}_U$) decreases, while the job-to-job transition probability, which is given by $\lambda\bar{p}_E$ increases, as it can be seen in the bottom panels of Figure 36.

In terms of the wage distribution, this additional competition and therefore selectivity implies that there are fewer employed (a higher unemployment rate), but those who have a job earn more, because they are the most productive, and are more similar: the mean of log-wages increases and the standard deviation of log-wages decreases as the parameter λ increases, as we can see in Figure 37.

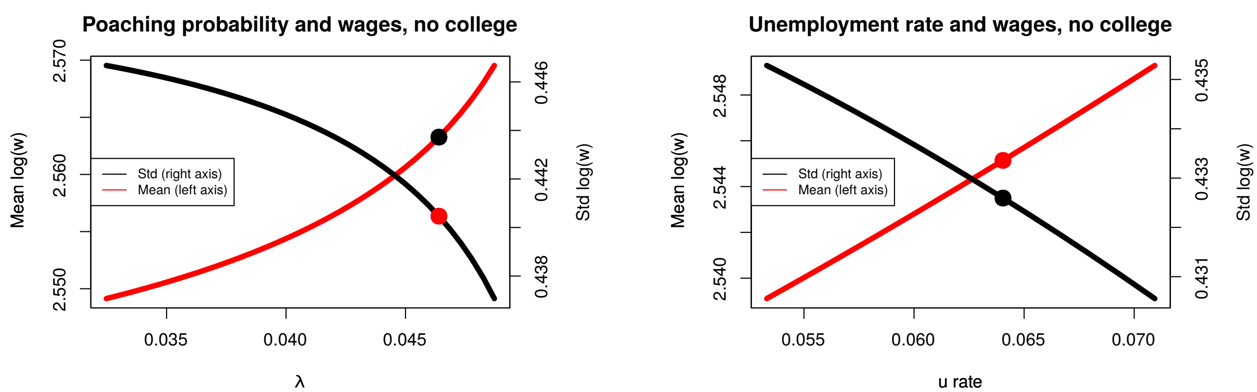
Even without calculating the implied correlation between the change in the standard deviation of log wages and the change in the unemployment rate, it appears clear that imputing the variability of the endogenous variables to changes in the poaching probability would imply a *negative* correlation between our measure of inequality and the unemployment rate, at odds with the empirical facts we described in Section 2.

Figure 36: The effects of poaching probability on the labor market



The dots represent the values for the benchmark calibration and estimation exercise.

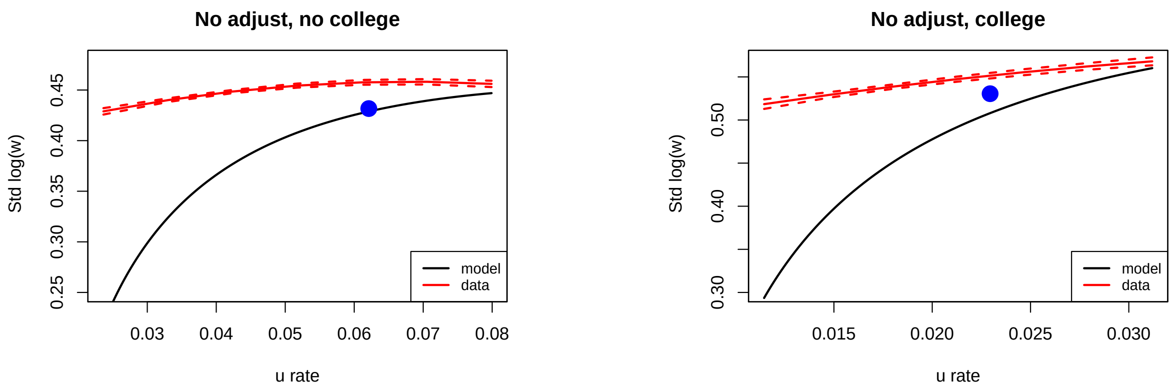
Figure 37: The effects of poaching probability on wages



The dots represent the values for the benchmark calibration and estimation exercise.

C Robustness checks with no adjusted data

Figure 38: The effects of an increase in average productivity: wage inequality and unemployment rate



The dots represent the values for the benchmark calibration and estimation exercise.