On the Right Jump Tail Inferred from the VIX Market

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Abstract

This paper addresses the role of the right jump tail under the risk-neutral measure, as a proxy for fear-of-fear, in the return predictability implicit in the VIX market. A simulation establishes that the right jump tail dominates the left jump tail in explaining various risk measures and their associated term structures. Using VIX futures and options from 2006 until 2020, the superior predictive power afforded by the variance-of-variance risk premium (VVRP) and the VVRP term structure, is shown to arise predominantly from the right jump tail risk.

Keywords: Jump tail risk; return predictability, variance risk premium, VIX derivatives JEL Classification: C15; C32; G15

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1 Introduction

An extensive literature indicates a key role for volatility in determining asset valuation and economic activity. The properties and economic drivers of volatility are key issues, see Bansal and Yaron (2004), Drechsler and Yaron (2010) and Bollerslev, Li, and Zhao (2017) among others. Investor sentiment and asset market volatility are often captured by the VIX index which is published by the Chicago Board Options Exchange (CBOE). Derived by the cross section of SPX options, the VIX nonparametrically approximates the expected future index volatility over the next 30 days. Since the VIX is not a directly tradeable instrument, futures and options, subsequently introduced by the CBOE in 2004 and 2006, provide investors with tradeable exposure to volatility. VIX futures and options are currently considered to be among the most successful products launched by the CBOE. They are widely exploited in risk management strategy.

Although the VIX and VIX futures, VIX options and SPX options are all closely associated with the S&P 500 index, Bardgett, Gourier, and Leippold (2019) argue that these data sets contain different information on the dynamics and distributions of the SPX returns. Furthermore, adding VIX options to the estimation not only improves the pricing of the VIX and SPX options, it also improves the representation of the variance term structure. A similar observation, based upon different information, is indicated in Huang et al. (2019). Using VIX options instead of SPX options, Huang et al. (2019) apply an analogous method to the VIX and construct the volatility-of-volatility index (VVIX), as used to measure the risk-neutral expectations of stock market volatility of volatility. In short, they show that the VVIX exhibits different dynamics to the VIX, where the former serves as a significant risk factor beyond market volatility risks in affecting VIX option returns.

The extensive study of volatility-of-volatility risk in recent years is attributed to its crucial role in asset pricing. In providing evidence for time-varying volatility in the VIX, Mencía and Sentana (2013) argue that this volatility has greater significance for VIX options than VIX futures. Treating the VVIX as a tail risk indicator, Park (2015) shows that the VVIX carries great predictive power for returns on tail risk hedging options such as the SPX puts and VIX calls. At stock level, Baltussen, Van Bekkum, and Van Der Grient (2018) show that volatility calculated from implied volatilities predicts future individual stock returns. Theoretical justification for the predictions afforded by the volatility-of-volatility may be traced to Bollerslev, Tauchen, and Zhou (2009). Exploiting a version of the Bansal and Yaron (2004) long-run risks (LRR) model (that characterizes time-varying volatility and volatility-of-volatility in the process of aggregate consumption), Bollerslev, Tauchen, and Zhou (2009) show that the volatility-of-volatility is the source of a genuine variance risk premium (VRP) that drives the predictive power of the VRP for market returns.

As indicated by Park (2015) and Huang et al. (2019), investors dislike volatility-of-volatility risk and are willing to pay a premium for downside protection. This indicates that the VVIX contains information not only on a physical expectation of future volatility-of-volatility risk but also on its associated risk premium. The latter is defined as the difference between the physical and risk-neutral variances of the VIX index, the so-called variance-of-variance risk premium (VVRP). Kaeck (2018) demonstrates that strategies based on the VVRP contracts may generate attractive returns for VIX option investors. In investigating the forecasting power of the VVIX for VIX option returns, Park (2015) finds the contribution of the VVRP to be less significant than that of the actual expectation of the volatility-of-volatility risk. Empirical work dedicated to the predictability inherent in the VVRP is rather limited. This contrasts sharply with mounting evidence on the usefulness of the VRP as a predictor for aggregate stock market returns, see, Bollerslev, Tauchen, and Zhou (2009), Drechsler and Yaron (2010), Bollerslev et al. (2014), and Li, Izzeldin, and Yao (2020), among others. Given that investors' aggregate risk aversion could vary differently with the time horizon, the VRP term structure receives increasing attention. Li and Zinna (2018) show a higher degree of short-run return predictability induced by the inclusion of the VRP term structure. Consistent with Li and Zinna (2018), there is a suggestion from Bardgett, Gourier,

and Leippold (2019) that the slope of the VRP term structure contains additional information on future returns. They also indicate that, although the VRP has a downward-sloping term structure during low volatility times, it displays a non-monotonic term structure during market turmoil.

Despite widespread interest in the variation of the volatility-of-volatility and its risk premia, little progress is apparent in understanding the tail risk manifest in VIX options. Given that the VIX is often referred to as the "investor fear gauge", the VVIX based on the VIX options might therefore be considered the "fear-of-fear". However, we argue that only a small fraction of the VVIX is attributable to the genuine fear-of-fear component; and that fear per se accounts for much of the predictive power underlying the VVRP. Disentangling the part of the VVIX associated with normal sized price variations, from that associated with extreme tail events, may provide a better guide to investment decisions. That idea is consistent with recent studies showing that the predictability implied by the VRP arises largely from the left jump tail which hinges on the SPX deep out-of-the-money (OTM) put options (see Bollerslev, Todorov, and Xu (2015), Andersen, Fusari, and Todorov (2015), Andersen, Fusari, and Todorov (2019) and Andersen, Todorov, and Ubukata (2021)). Among the few studies of jump dynamics for the VIX derivatives, Park (2016) proposes a pioneer dynamic model that accommodates both asymmetric volatility and jumps, to account for the positive skewness observed in VIX options. This study delivers strong evidence for the importance of upward jumps in VIX derivatives pricing. In the empirical results of Park (2015) and Huang et al. (2019), jumps are considered control variables in predictive regressions for VIX option returns. Neither formal treatment of the jump tail risk underlying the VIX market, nor a thorough analysis of jump tails in the VIX return predictability, are to be found in the literature.

The current paper seeks to fill this void by examining the impact of jump tails upon the dynamic properties of the VVIX, VVRP and their predictive power for future returns. The objectives are threefold. First, in following Bollerslev, Todorov, and Xu (2015), we treat the difference between left and right tail jump risk premia as a proxy for fear-of-fear since it is virtually exempt from any compensation for temporal variation in jump tail risk. This fear component can be further

approximated by the risk-neutral expectation of the right jump tail. The right jump tail depends solely on deep short-lived OTM call options that are worthless in the absence of any substantial increase in the VIX before the options expire. We view this as compensation for exposure to sudden downside movements in the market.

Second, we identify the different impact of upward and downward jump risk premia on a variety of relevant risk measures. For this, we undertake a Monte Carlo simulation based upon an extended model of VIX dynamics, as considered in Park (2016). Using simulated VIX options, we construct both left and right jump tail variations and evaluate their contributions to different risk measures and their associated term structures. With a wide range of strikes and equal numbers of the OTM calls and puts, our simulation study is less prone to the problem of measurement errors, as encountered by the empirical work where missing VIX deep OTM puts often biases the estimation of the left jump tail. Third, we assess return predictability for the VIX market, as implied by the VVIX, VVRP and their term structures. Our main focus is on the returns of VIX futures and VIX OTM calls, which help investors hedge against sharp increases in market volatility. To identify the source of return predictability, we isolate the right jump tail risk from the VIX options, then seek to ascertain its role in return predictions over different investment horizons.

Simulation evidence indicates that the upward jump risk premium has greater impact than its downward counterpart, on the properties of the VVIX, its upside and downside components, and the VVRP. Moreover, that impact increases as the jump risk premium increases; and it declines with longer investment horizons. As the upward jump risk premium increases, it delivers a steeper slope of term structures for the VVIX, its components, VVRP and the implied skewness. However, changes in the downward jump risk premium leave the shape of the term structure virtually unaffected. The dominant role played by the upward jump risk premium is indicative of the superiority of the right jump tail under the risk-neutral measure. The latter approximates the difference between truncated downward and upward jump risk premia. Indeed, we find that, regardless of the magnitude of the jump risk premium, the right jump tail subsumes all information contained in the left jump tail in explaining the VVRP and the VVRP term structure. Compared to the left jump tail, a larger fraction of the VVIX, implied skewness and their associated term structures may be attributed to the right jump tail or the fear-of-fear component.

In the empirical study, we first explore the predictive power of the VVIX, implied skewness and the VVRP for tail risk hedging returns, i.e. VIX OTM calls and VIX futures. We establish that the VVRP performs best in almost all the forecasting exercises, especially over medium or long horizons. This finding is in consistency with the superiority of the VRP in predicting aggregate stock market returns, as observed in many empirical studies. Our results contrast those reported by Park (2015) who, in splitting the VVIX into the VVRP and a statistical expectation of the volatility-of-volatility, argues that the latter component is a more significant contributor to the forecasting power of the VVIX. We suggest below a number of reasons that may explain those conflicting results. First, the data are different. Those used by Park (2015) are derived from overlapping samples containing daily volatility metrics from 2007 to 2013. On each day, the physical expectation of the volatility-of-volatility is based upon return variations over the past 22 trading days. The use of overlapping data may result in residual correlation and therefore erroneous standard errors. By contrast, with all our regression specifications, monthly risk measures recorded from 2006 to 2020 involve non-overlapping time intervals. In particular, data are purposefully selected to avoid the problem of maturity mismatch, in that VIX options and futures used to construct risk measures have similar length of remaining life. Second, the two studies differ in the way that the physical expectation of the future realized variance is estimated. Where Park (2015)adopts the random walk, we rely on the Heterogeneous Autoregressive (HAR) model of Corsi (2009) to obtain the direct forecasts for realized variance. Third, although Park (2015) employs the level of the VVIX in the predictive regressions for returns, such regressions are criticized by Bollerslev et al. (2013) as being unbalanced and less informative; i.e., they simultaneously consider a while noise variable (the return) on the LHS and a highly persistent variable (the VVIX) on

the RHS. Instead, we apply first differences to the VVIX to alleviate the issue of unbalanced regressions.

Adding the slope of the VVRP term structure (that is the difference between the long-term VVRP and the short-term VVRP) delivers greater return predictability for both VIX options and futures. At a general level, the R^2 exhibits a hump-shaped pattern and peaks around 2-3 month horizon. To disentangle the true source of the return predictability and to characterize the role of fear-of-fear, both the VVRP and its term structure are first deprived of the right jump tail component. This substantially reduces the R^2 relative to the regression based on the VVRP and the slope. We then separately consider the diffusive and right jump tail risk components of the VVRP and its term structure. This results in a significant increase, both in the degree of return predictability and for the impact of the right jump tail. For robustness, we check that our predictability results remain intact when using high-frequency VIX futures data and the raw VVIX index published by the CBOE. We therefore conclude that the fear-of-fear component proxied by the right jump tail variation is the primary source of the forecasting power inherent in the VVRP.

The rest of the paper proceeds as follows. We present our construction of the VVIX, implied skewness, VVRP and the jump tails in section 2. A simulation study on the role of jump tail risk is demonstrated in section 3. Section 4 details the data used in our study and section 5 discusses the main empirical results. Section 6 concludes.

2 Construction of Risk Measures

In what follows, we start by deriving the risk-neutral expectation of the quadratic variation for the VIX index and decomposing return variance into the components due to positive and negative returns, respectively. We then construct the realized variance for the VIX and obtain the variance-of-variance risk premium as the wedge between the conditional expectations of quadratic variation under the risk-neutral and objective measures. Finally, we extract the investors fear-of-fear component as proxied by the special compensation for jump tail risk.

2.1 Implied Variance Measures

The VIX index offers a model-free and market-determined estimate of one-month stock market volatility implied by index option prices. Britten-Jones and Neuberger (2000) and Jiang and Tian (2005) indicate that the VIX can be derived from the prices of S&P 500 call and put options covering a range of strikes as follows

$$VIX_t^2 = \frac{2e^{r_f\tau}}{\tau} \left[\int_0^{S_t^*} \frac{1}{K^2} P_t(\tau, K) dK + \int_{S_t^*}^\infty \frac{1}{K^2} C_t(\tau, K) dK \right]$$
(1)

where K denotes the strike price, τ is time-to-maturity measured in annual units, $P_t(\tau, K)$ $(C_t(\tau, K))$ stands for the time-t market price of an OTM put (call) option with strike K and maturity τ , S_t^* is the forward price of the S&P 500 index and r_f is the risk-free rate. In practice, the published VIX adopts a few approximations due to the availability of options data. Jiang and Tian (2005) show that such approximation errors can be largely alleviated by implementing their correction mechanism.

In the present paper, we calculate the implied volatility of volatility by applying the same method as the VIX in (1) to a cross-section of the VIX options. Since 2012, the CBOE published the index as the VVIX, which captures the model-free implied volatility of VIX futures over the next 30 days. The squared VVIX that represents the risk-neutral expectations of the volatility of volatility in the financial markets reads

$$VVIX_t^2 = \frac{2e^{r_f\tau}}{\tau} \left[\int_0^{F_t} \frac{1}{K^2} P_t^*(\tau, K) dK + \int_{F_t}^\infty \frac{1}{K^2} C_t^*(\tau, K) dK \right]$$
(2)

where F_t is the VIX futures price and $P_t^*(\tau, K)$ $(C_t^*(\tau, K))$ denotes the price of OTM put (call) options on the VIX. To approximate the integral on the right-hand side of equation (2), we follow a procedure that has been accepted as common practice in the related literature: a) interpolate between listed strikes employing a simple cubic spline; b) extrapolate the observed implied volatilities by assuming a flat implied volatility function beyond the available strike prices. To reduce measurement errors induced by the limited availability of strike prices, we consider strikes covering a range of three times the standard deviation around the futures price.

Following Huang and Li (2019), we decompose the squared VVIX into upside and downside semivariances by

$$VVIX_t^{2(+)} = \frac{2e^{r_f\tau}}{\tau} \int_{F_t}^{\infty} \frac{1}{K^2} C_t^*(\tau, K) dK$$
(3)

$$VVIX_t^{2(-)} = \frac{2e^{r_f\tau}}{\tau} \int_0^{F_t} \frac{1}{K^2} P_t^*(\tau, K) dK$$
(4)

Intuitively, $VVIX_t^{2(+)}$ ($VVIX_t^{2(-)}$) serves as a risk-neutral expectation of the upside (downside) semivariance of the 30-day forward VIX index. The option-implied skewness is then defined as the difference between the upside and downside risk-neutral semivariances as follows

$$SKEW_t = VVIX_t^{2(+)} - VVIX_t^{2(-)}$$

$$\tag{5}$$

2.2 Variance-of-Variance Risk Premium

Next, we characterize the variance-of-variance risk premium (VVRP) in the form of a gap between the objective and risk-neturalized expectations of the total quadratic variation for the VIX index over a fixed maturity. This premium represents compensation demanded by investor for the risk associated with fluctuations in the return variation of the volatility index.

Following Barndorff-Nielsen and Shephard (2002) and Kaeck (2018), we obtain the realized variance over the interval from t to $t + \tau$ below

$$RVVIX_{t} = \frac{252}{n} \sum_{i=1}^{n} \left(\log \left(F_{t_{i},t+\tau} \right) - \log \left(F_{t_{i-1},t+\tau} \right) \right)^{2}$$
(6)

where $F_{t,t+\tau}$ denotes the futures contract on day t with fixed maturity $t+\tau$. For each time horizon τ , the daily return is calculated between two points in the partition $[t, t+\tau]$, where $t+\tau$ is the expiry date of VIX options in the following month and t is the trading day after the expiry

date of the present month. Since VIX futures maturities are consistent with the expiry dates of the options, this approach achieves exact matching of information in the measurement of the two expectations of the future return variation of the VIX. To capture the premium that investors require to hold variance-sensitive assets, we construct the VVRP as follows

$$VVRP_t \equiv E_t^P (QV_{t,t+\tau}) - E_t^Q (QV_{t,t+\tau})$$

$$\approx RVVIX_t - VVIX_t^2$$
(7)

where $QV_{t,t+\tau}$ is the quadratic variation measuring the return variation of the log-price process over t and $t+\tau$, $E_t^P(QV_{t,t+\tau})$ and $E_t^Q(QV_{t,t+\tau})$ respectively correspond to the objective and risk-neutral expectations of $QV_{t,t+\tau}$.

2.3 Jump Tail Risk

As indicated in Bollerslev, Todorov, and Xu (2015) and Andersen, Todorov, and Ubukata (2021), the VVRP in equation (7) can be decomposed as

$$VVRP_{t} = \left[E_{t}^{P}(CV_{t,t+\tau}) - E_{t}^{Q}(CV_{t,t+\tau})\right] + \left[E_{t}^{P}(JV_{t,t+\tau}) - E_{t}^{Q}(JV_{t,t+\tau})\right]$$

$$= \left[E_{t}^{P}(CV_{t,t+\tau}) - E_{t}^{Q}(CV_{t,t+\tau})\right] + \left[E_{t}^{P}(JV_{t,t+\tau}^{P}) - E_{t}^{Q}(JV_{t,t+\tau}^{P})\right]$$

$$+ \left[E_{t}^{Q}(JV_{t,t+\tau}^{P}) - E_{t}^{Q}(JV_{t,t+\tau}^{Q})\right]$$
(8)

They conjecture that the very last term in equation (8) characterizes the compensation for time-varying jump intensity risk. Define the left and right jump variation under the risk-neutral measure as LJV_t^Q and RJV_t^Q , their counterparts under the physical measure are therefore LJV_t^P and RJV_t^P . In analogy to the definition and decomposition of the VVRP in equation (8), the left and right jump tail risk premia can by given by

$$LJP_{t} = E_{t}^{P} \left(LJV_{t}^{P} \right) - E_{t}^{Q} \left(LJV_{t}^{Q} \right)$$

$$= \left[E_{t}^{P} \left(LJV_{t}^{P} \right) - E_{t}^{Q} \left(LJV_{t}^{P} \right) \right]$$

$$+ \left[E_{t}^{Q} \left(LJV_{t}^{P} \right) - E_{t}^{Q} \left(LJV_{t}^{Q} \right) \right]$$

$$(9)$$

and

$$RJP_{t} = E_{t}^{P} \left(RJV_{t}^{P} \right) - E_{t}^{Q} \left(RJV_{t}^{Q} \right)$$

$$= \left[E_{t}^{P} \left(RJV_{t}^{P} \right) - E_{t}^{Q} \left(RJV_{t}^{P} \right) \right]$$

$$+ \left[E_{t}^{Q} \left(RJV_{t}^{P} \right) - E_{t}^{Q} \left(RJV_{t}^{Q} \right) \right]$$

$$(10)$$

In consistency with the work of Bollerslev, Todorov, and Xu (2015), we fail to reject the null hypothesis $LJV_t^P = RJV_t^P$ using truncated realized negative and positive return variation measures computed from both the daily and high-frequency VIX futures prices¹. As such, the difference between the two jump tail premia becomes

$$LJP_t - RJP_t \approx E_t^Q \left(RJV_t^Q \right) - E_t^Q \left(LJV_t^Q \right)$$
(11)

which will be nearly exempt from the compensation for temporal variation in jump intensity risk. The measure $LJP_t - RJP_t$ mimics the component of investor fears proposed in Bollerslev and Todorov (2011), which is implicit in the gap between the estimated objective and risk-neutral jump tail variations implied by the S&P 500 index. Since the VIX is called the "investor fear gauge", $(LJP_t - RJP_t)$ based on the VIX can therefore be interpreted as the "fear of fear" in the present study. In addition, OTM put options on the S&P 500, which constitute the left jump tail variation of the aggregate market portfolio, are more heavily traded in the market and provide investors with protection against large downward movements. Similar to OTM S&P 500 puts,

¹The *F*-statistics for the Wald test with the null hypothesis $LJV_t^P = RJV_t^P$ is 1.223 for the realized variance based on daily returns and 2.005 for that based on high-frequency futures returns.

OTM VIX calls are often considered another form of tail risk hedges. This can be explained by the leverage effect that negative variations in returns are closely associated with rises in volatility, in which case OTM VIX calls can hedge. As a result, for the VIX market the risk-neutral right jump tail embedded in the OTM call options dominates the left jump tail based on the OTM put options in magnitude. Given this, equation (11) can be approximated as

$$LJP_t - RJP_t \approx E_t^Q \left(RJV_t^Q \right) \tag{12}$$

To estimate RJV_t^Q , we focus exclusively on the short-dated options and employ only deep OTM VIX call options with log-moneyness greater than 1.5 times the normalized at-the-money (ATM) Black-Scholes implied volatility (BSIV). The use of deep OTM options may more effectively isolate jump tail risk since they are worthless unless jumps occur in the underlying asset, see, e.g. Bollerslev and Todorov (2011). Based on the point estimates of the tail shape parameter (α_t^+) and the level shift parameter (ϕ_t^+) detailed in Bollerslev, Todorov, and Xu (2015), we obtain the proxy for the expected right jump tail variation under the risk-neutral measure,

$$E_t^Q \left(RJV_t^Q \right) \approx \tau \phi_t^+ e^{-\alpha_t^+ |k_t|} \left(\alpha_t^+ k_t \left(\alpha_t^+ k_t + 2 \right) + 2 \right) / \left(\alpha_t^+ \right)^3$$
(13)

where the cutoff k_t is set as 7 times the normalized ATM BSIV at time t. We allow α_t^+ and ϕ_t^+ to vary on a daily basis and the monthly jump variation is constructed by averaging the daily measures within the month.

3 Simulation Study

This section presents a simulation study to examine the role of jump risk premium in affecting the time series properties of the risk measures implied by the VIX. We then respectively extract the left and right jump tail variations to highlight the key contribution of the latter as conjectured in section 2.

3.1 Design

We first extend the jump-diffusion model for the pricing of VIX derivatives in Park (2016) by allowing for risk premia in both upward and downward jumps. The dynamics under the risk-neutral measure takes the following form

$$dv_t = \kappa_v (u_t - v_t) dt + \sqrt{w_t} dB_{1t}^Q + J_1^Q dN_{1t}^Q + J_2^Q dN_{2t}^Q$$
(14)

$$-\lambda_+ \delta_+ dt - \lambda_- \delta_- dt$$

$$du_t = \kappa_u (\overline{\mu} - u_t) dt + \sigma_u dB_{2t}^Q$$

$$dw_t = \kappa_w (\overline{w} - w_t) dt + \sigma_w \sqrt{w_t} dB_{3t}^Q$$

where $v_t = \log (VIX_t)$, u_t denotes the long-run mean of the VIX and w_t captures the variation in the volatility of the VIX. The processes B_{1t}^Q , B_{2t}^Q and B_{3t}^Q are standard Brownian motions, among which B_{1t}^Q and B_{3t}^Q are correlated with the coefficient ρ . In the VIX dynamics, we accommodate both upward and downward jumps driven by independent compound Poisson processes. They are characterized by N_{1t}^Q (N_{2t}^Q) that represents a risk-neutral Poisson process generating upward (downward) jumps with intensity λ_+ (λ_-). The size of upward (downward) jumps is denoted by J_1^Q (J_2^Q), following an independent exponential distribution with a positive (negative) mean, i.e. $\delta_+ > 0$ ($\delta_- < 0$).

The corresponding system under the physical measure becomes

$$dv_{t} = \kappa_{v} \left(u_{t} - v_{t}\right) dt + \sqrt{w_{t}} dB_{1t}^{P} + J_{1}^{P} dN_{1t}^{P} + J_{2}^{P} dN_{2t}^{P}$$

$$-\lambda_{+}^{*} \delta_{+}^{*} dt - \lambda_{-}^{*} \delta_{-}^{*} dt$$

$$du_{t} = \kappa_{u} \left(\overline{\mu} - u_{t}\right) dt + \eta_{u} u_{t} dt + \sigma_{u} dB_{2t}^{P}$$

$$dw_{t} = \kappa_{w} \left(\overline{w} - w_{t}\right) dt + \eta_{w} w_{t} dt + \sigma_{w} \sqrt{w_{t}} dB_{3t}^{P}$$

$$(15)$$

where B_{1t}^P , B_{2t}^P and B_{3t}^P are standard Brownian motions, $\eta_u u_t$ and $\eta_w w_t$ drive the risk premia for the u_t and w_t processes. To introduce jump risk premia, we allow upward and downward jumps under the physical measure to have their own jump intensity and jump-size distributions specified by the parameters λ_{+}^{*} , δ_{+}^{*} , λ_{-}^{*} and δ_{-}^{*} . Similar to the simulation study conducted in Duan and Yeh (2010), we assume the means of jump sizes are the same under P and Q with $\delta_{+}^{*} = \delta_{+}$ and $\delta_{-}^{*} = \delta_{-}$, and allow for different jump intensities under the change of measure. As such, we define the upward and downward jump risk premia by $\phi_{+} = \lambda_{+} - \lambda_{+}^{*}$ and $\phi_{-} = \lambda_{-} - \lambda_{-}^{*}$, respectively. The specification in equation (15) preserves the affine structure of the framework under different measures.

The simulation of VIX is generated using an Euler discretized version of (15) based on 78 intervals² for each of the $T = \tau \times 200$ trading day in the sample. A daily series is extracted by sampling once every 78 data points. The parameter values used are consistent with those reported in the last column of Table 3 in Park (2016). The processes v_t , u_t and w_t are respectively initialized at 2, 2 and 0.2, which are given by the unconditional means of the corresponding series in our empirical study. We assume one year has 252 trading days.

We then compute the option prices of VIX corresponding to different strikes and maturities (τ) using the jump diffusion model under the risk-neutral probability measure in (14). To improve simulation accuracy, we rely on the empirical martingale simulation procedure introduced by Duan and Simonato (1998) and set the simulation path for option pricing to 10,000. Based on the simulated options on each trading day, we construct the implied variance measures with various maturities as in section 2 and compute the realized variance comprising the price information in the next τ days. Finally, we select both the implied and realized variances on the trading day that follows the previous maturity date so that we obtain non-overlapping samples with size equal to 200.

²We assume 1 day consists of 6.5 hours of open trading and consider a sparse sampling at a frequency of once every 5 minutes. This results in 78 intraday intervals in a day, i.e. $\frac{6.5 \times 3600}{300} = 78$.

3.2 Results

Table 1 reports the mean values of the risk measures considered in the present study with 5 different maturities. By setting ϕ_+ and ϕ_- to zero, we assume there exist no jump risk premia in Table 1 and therefore the results can be used as benchmark. To pin down the impact of upward (downward) jump risk premium on the properties of the risk measures, we vary the magnitude of the upward (downward) jump risk premium ϕ_+ from 2 to 6 while restricting the downward (upward) jump risk premium ϕ_- to zero. The risk premium is classified into low (ϕ_+ or $\phi_- = 2$), medium (ϕ_+ or $\phi_- = 4$) and high (ϕ_+ or $\phi_- = 6$).

Values of the risk measures with different sizes of jump risk premia are presented in Table 2 where the percentage changes relative to their corresponding values in the benchmark are reported in the parenthesis. We show that the upward jump risk premium generates a more substantial impact on the $VVIX_t^2$, its components and $VVRP_t$ when compared to the downward jump risk premium. Such evidence is particularly strong for short maturities when $\tau = 30$, 60 and 90. As maturity grows, our risk measures are generally less sensitive to the presence of jump risk premia. Given this, for longer maturities such as $\tau = 180$ and 360, the greater effects of the upward jumps are only observed for medium or high jump risk premium. It is worth noting that, unlike other risk measures, the implied skewness appears more sensitive to downward jump risk premium for short maturities. It could be due to the fact that the effects of upward jumps on the upside and downside components of the $VVIX_t^2$ are similar in magnitude; however, downward jumps impact $VVIX_t^2$ mainly through the downside component. Defined as the difference between the upside and downside semivariances of $VVIX_t^2$, the implied skewness is therefore more evidently affected by the downward jumps.

Figure 1 plots the time series of the risk measures for 30-, 60-, 90-, 180- and 360-day horizons. The upper panel displays the impact of the upward jump risk premium on the shape of the term structures whereas the lower panel concentrates on that of the downward jump risk premium. The recent literature provides considerable evidence suggesting that the term structure of the implied volatility and variance risk premium have great reliance on the economic conditions. In particular, Bardgett, Gourier, and Leippold (2019) find a downward-sloping term structure for the variance risk premium during normal times whereas its slope switches sign in periods of high volatility. Johnson (2017) indicates an upward-sloping term structure for the VIX in times of market calm but this is no longer true in market turmoil. In addition, a hump-shaped VIX term structure during early 2009 is documented in the work of Zhang, Shu, and Brenner (2010). Our results in Figure 1 are generally in consistency with the empirical findings in the existing literature and complement Christoffersen, Jacobs, and Ornthanalai (2012) for the important role of jump risk premium on the implied volatility term structure. In a sharp contrast, the term structure of the VVIX, its upside component, the VVRP and the implied skewness are highly responsive to upward jump risk premium, exhibiting a greater slope in magnitude as the jump premium grows. However, the downward jump risk premium seems to deliver only trivial effects on the shape of the term structure with the slope virtually unaffected by the variation in the risk premium.

In the simulation considered above, we only allow for the presence of one type of jump risk premium, i.e. upward or downward, to ascertain their different roles in affecting the dynamics of the risk measures. This is clearly in contradiction with the real-life observations where the upward and downward jump premia often coexist. However, such exercise clearly reveals the dominant role played by the upward jump risk premium and supports our earlier hypothesis that the fear-of-fear component, as approximated by the difference between the two jump tail risk premium, may constitute the primary source of variation in the risk measures considered. To further confirm our conjecture, we then simultaneously incorporate the two different jumps in the VIX dynamics and construct the right and left jump tails using the method discussed in section 2. Unlike the empirical study in which the VIX OTM puts are much less traded, our simulation study ensures that there are equal numbers of the OTM puts and calls, which alleviates the issue of measurement errors in the comparison of right and left jump tails. To assess the contribution of the two tails to different risk measures, we run the following regression with a focus on the monthly horizon only

$$y_t = C + \alpha_1 L J T_t + \alpha_2 R J T_t + \varepsilon_t \tag{16}$$

where y_t denotes the risk measures such as $VVRP_t$, $SKEW_t$ and $VVIX_t^2$. The results with alternative sets of regressors are reported in Table 3. To account for the issue of serial correlation, we derive the statistical significance using Newey and West (1987) robust *t*-statistics with an optimal lag. Table 3 shows that the right jump tail clearly dominates the left jump tail in explaining the dynamics underlying $VVRP_t$, $SKEW_t$ and $VVIX_t^2$, as evidenced by the higher adjusted R^2 observed for the univariate regressions based on the right jump tail. For multivariate regressions in which the regressors consist of the left and right jump tails, we show that the right tail subsumes all information contained in the left tail in capturing the variation in $VVRP_t$ whereas this observation does not hold for the other two measures. This finding is supportive of our interpretation of the right jump tail as the key constitute of the VVRP.

Finally, we investigate the role of different tails in determining the shape of the term structure for $VVRP_t$, $SKEW_t$ and $VVIX_t^2$. Taking $VVRP_t$ as an example, we construct the slope defined as the difference between the long-term and short-term $VVRP_t$. Specifically, the VVRP slope is computed as $VVRPS_t = VVRP_t^{(360\text{-day})} - VVRP_t^{(30\text{-day})}$, and the slopes for $SKEW_t$ and $VVIX_t^2$ are constructed in a similar fashion. We then estimate the regression (16) again by treating the slope as our dependent variable and present the output in Table 4. Analogous to the results in Table 3, a dominant role of the right jump tail is also found for forming the shape of the term structure of the three risk measures. Recent studies have established growing evidence for the importance of the use of term structure in return predictions. For example, Vasquez (2017) indicates that the slope of the implied volatility term structure is positively related to future option returns. Li and Zinna (2017) find that the slope of the variance risk premium enhances the short-run predictability of equity returns. Since the right jump tail is a key contributor to the level and slope of our risk measures, it may also perform as the primary component providing return predictive power for these measures, which we verify below in our empirical study.

4 Data

VIX futures data are collected from the CBOE website and span from March 26, 2004 through December 31, 2020. On each trading day during the sample period, three to six different maturities are traded. We rely on the daily settlement prices to obtain the realized variance of VIX. In addition, the raw VIX options data origin from OptionMetrics covering the period of February 24, 2006 to December 31, 2020. As a result, our sample is restricted to the shorter period when examining the joint information content from the data of futures and options. For robustness, we also consider an alternative measure of statistical volatility-of-volatility based on the 5-minute VIX futures returns. The data is sourced from Tick Data Inc. and starts in July 2012.

We apply standard filters to the raw options data to eliminate inaccurate or illiquid options. First, we delete the VIX options for which the price, defined as the midpoint of the option bid and ask quotes, is less than 0.2 or the trading volume is zero. Second, options with BSIV lower than 10% or greater than 150% are excluded from the sample. Third, we focus on options with 8 to 180 days to expiration. In the end, this leaves us with more than a million VIX option quotes, with a daily average of 102.7 VIX OTM calls and 41.3 puts over the full sample. The number of VIX OTM options on a given date increases with time, with around 25.9 calls (11.2 puts) at the beginning of the data set and around 136.5 calls (87.9 puts) at the end.

As of today, OTM VIX calls become a popular form of tail risk hedges as they provide investors with assurance against a large downturn of the market. To investigate its return predictability on different moneyness bins, we classify the VIX OTM calls into the slight OTM (1.0 < k < 1.2), the medium OTM (1.2 < k < 1.4), and the deep OTM (1.4 < k < 1.6), where the moneyness is defined as $k = K/F_t(\tau)$. To assess whether the return predictability previously ascribed to the popular risk measures is effectively arising from the right jump tail, we follow Bollerslev, Todorov, and Xu (2015) in constructing the jump tails using OTM options with maturities between 8 and 49 calendar days. To isolate from the diffusive risk, we only consider calls with log-moneyness in excess of the 1.5 times the maturity-normalized ATM BSIV. It is worth noting that all of our risk measures are non-overlapping. Taking the monthly horizon as an example, the implied variance measures are given by the values at the end of the month and the realized variance is derived over the following month and annualized.

5 Empirical Results

5.1 Preliminary data analysis

As option returns are heavily influenced by shocks in the underlying asset price and volatility, we follow Bakshi and Kapadia (2003) and Park (2015) to employ the delta-neutral option returns that are unaffected by the underlying asset's price risk. The delta-neutral returns below are computed for a portfolio of a long position in a VIX option, hedged by a short position in the underlying asset.

$$R_{t+1}^C(k,\tau) = \frac{C_{t+1}(k,\tau) - C_t(k,\tau) - \Delta_t(k,\tau) \left(F_{t+1}(\tau) - F_t(\tau)\right)}{C_t(k,\tau) - \Delta_t(k,\tau)F_t(\tau)} - \frac{r_t^f \tau}{365}$$
(17)

where $C_t(k, \tau)$ denotes the VIX call option price with maturity τ and moneyness k; $F_t(\tau)$ represents the futures price and $\Delta_t(k, \tau)$ stands for the option delta that is available from OptionMetrics. We then take a simple average of all the option returns that fall within the *i*th moneyness bin as follows

$$R_{i,t}^{C} = \frac{1}{N_{i,t}} \sum_{j=1}^{N_{i,t}} R_{t}^{C}(k_{j}, \tau_{j})$$
(18)

where $N_{i,t}$ is the number of options that belong to the *i*th moneyness bin in month *t*. To obtain the returns on VIX futures, we make use of the front contracts and roll over to the next maturity contract in the case where the shortest contract has less than 5 days to maturity, see also in Taylor (2019).

Panel A of Table 5 reports the summary statistics of the delta-neutral option returns of VIX calls and puts across different moneyness bins. Overall, negative returns on both call and put options occur more frequently than positive returns. We show that on average, OTM delta-hedged VIX calls have significantly negative returns and call options lose more money as they go progressively out of the money. Notably, the average returns for OTM puts are significantly positive. To investigate whether the positive gains are induced by the illiquidity of OTM put options, we further consider the delta-hedged gains for in-the-money (ITM) calls, which are equivalent to OTM puts. As reported in Panel A of Table 5, the ITM calls are more actively traded compared to OTM puts. However, the average returns of ITM calls are also significantly positive except for the mild moneyness bin.

Our results can be supported by Coval and Shumway (2001) who argue that call options on market indices are expected to generate positive returns greater than those of their underlying assets whereas put options that are used to hedge against systematic risks have returns lower than the risk-free rate. The general assumption in Coval and Shumway (2001) is that the price of the given security is negatively correlated with the stochastic discount factor (SDF) which is high in bad states of the world and low in good states. In the present paper, the VIX futures serving as the underlying asset of the VIX options is positively correlated with the SDF, which explains the different signs of average gains on call and put options relative to those indicated in Coval and Shumway (2001).

Not surprisingly, both call and put options exhibit a substantial degree of positive skewness. Combining with the evidence for the negative average gains on VIX OTM calls, our results suggest that volatility-of-volatility risks are negatively priced in the tail risk hedging options. In other words, market participants dislike volatility-of-volatility risk and are willing to pay a premium to hedge against innovations in the volatility of volatility. Barberis and Huang (2008) attribute the association between the positively skewed security and its negative average excess return to the cumulative prospect theory. In particular, investors tend to overweight the right tail of VIX options and find it highly attractive. As a result, they are willing to pay a high price for VIX OTM calls and to accept a negative excess return on the security. Since OTM calls are overpriced, some systematic risk factors can be important drivers of the delta-neutral returns on OTM calls. On the other hand, the VIX OTM puts are often regarded as tail risk taking assets that generate positive payoffs during booms. The positive average return on VIX OTM puts together with its positive skewness indicate that these options may be fairly valued. In line with Park (2015), we find that neither VVIX nor VVRP is priced in the OTM puts and their predictive power for returns on VIX OTM puts is trivial³. In addition, option returns exhibit mild negative serial correlation, which will be dealt with in the subsequent predictive regressions. Panel B of Table 5 presents the results of VIX futures returns. Similar to returns of S&P 500, VIX returns are approximately serially uncorrelated, with a mean indistinguishable from zero.

Moving to the construction of the various risk measures, we consider 30, 60, 90 and 120 days to maturity when comparing their time series properties in Table 6. In line with the work of Kaeck (2018), we provide evidence for the downward-sloping term structure for the VVIX and its upside and downside components, which is attributable to the fact that the shorter-dated VIX futures contracts are associated with higher volatility (Alexander and Korovilas (2013)). In addition, the VVRP is negative for all the maturities considered and exhibits a upward-sloping term structure with standard deviation monotonically decreasing for growing maturities. Unlike Park (2016) who document a nearly flat term structure for the implied skewness, we find a clearly downward-sloping term structure. This indicates that informed traders decrease demand for option contracts with increasing maturity. Before downward jumps occur in the underlying asset (VIX futures), hedging demand causes informed traders to buy VIX OTM calls, driving up the volatility of OTM calls. The observed downward-sloping term structure is consistent with informed trader preference for speculating by trading short-maturity options, see more details in Xing, Zhang, and Zhao (2010)

³For brevity, results for the returns on VIX OTM puts are not reported but are available upon request.

and Stilger, Kostakis, and Poon (2017).

In addition to above, we also report the time series plots of the main predictor variables in Figure 2. The first panel depicts the monthly series of $VVIX_t$ and the second panel reports the estimated RJT_t that averages the within-month values. Missing values in the series of RJT_t at the beginning of the sample comes from the fact that VIX options only started in 2006 and the deep OTM options were not often traded in early stage. Generally, we document a coherence between the series of $VVIX_t$ and RJT_t although a few differences are noted. For example, $VVIX_t$ attains its maximum during 2009-2010 whereas the fear-of-fear component proxied by RJT_t reaches the peak around 2012, coinciding with the European debt crisis. The last panel compares the $VVRP_t$ and its component stripped of the right jump tail variation, $VVRP_t - RJT_t$. The latter is interpreted as the part associated with the normal sized price movements.

5.2 Return predictability: evidence from options

To ascertain the role of right jump tail variation in pricing the tail risk hedging options, we follow French, Schwert, and Stambaugh (1987) and Amihud (2002) in exploiting the h-period-ahead predictive regressions of option returns onto each of the risk measures for every moneyness bin considered

$$\frac{1}{h}\sum_{n=1}^{h}R_{i,t+n}^{C} = \beta_{i,0} + \beta_{i,1}\Delta VIX_t + \beta_{i,2}\Delta VIX_t^2 + \beta_{i,3}x_t + \varepsilon_{i,t+n}$$
(19)

where x_t denotes various risk measures including RJT_t . We rely on a monthly observation frequency for all the return regressions considered. It is worth noting that the introduction of ΔVIX_t and ΔVIX_t^2 is to remove the effect on option returns induced by changes in the underlying asset price and volatility. In constructing $VVRP_t$, defined as the difference between $E_t^P(QV_{t,t+\tau})$ and $E_t^Q(QV_{t,t+\tau})$, for forecasting purpose, we rely on the HAR model of Corsi (2009) to obtain direct forecast for $RVVIX_t$ that can be approximated as $E_t^P(QV_{t,t+\tau})$.

Figures 3-5 report the values of the adjusted R^2 for VIX option returns on each moneyness bin. The left panel depicts the return predictability over different horizons implied by regression (19) where x_t is proxied by $VVRP_t$, $\Delta VVIX_t^2$, $SKEW_t$ and RJT_t , respectively⁴. We show that $VVRP_t$ and $VVIX_t^2$ exhibit similar predictive power for future returns over short horizons whereas the superiority of the former becomes more evident after 4 months. Similar to $VVIX_t^2$, $SKEW_t$ hardly predicts returns over medium and long horizons. In addition, the left panel in Figures 3-5 also demonstrate the non-trivial predictive power underlying RJT_t for slight and deep OTM calls, where RJT_t even dominates $VVRP_t$ over several horizons.

In the subsequent analysis, we concentrate on $VVRP_t$ as a key risk factor given its top performance in option return predictions. Inspired by Li and Zinna (2018) who point out the significance of the VRP term structure for return predictability, we also consider $VVRP_t$ and its slope $VVRPS_t$, with the later defined as $VVRPS_t = VVRP_t^{(90\text{-day})} - VVRP_t^{(30\text{-day})}$. To assess the contribution of the right jump tail risk to the forecast power of $VVRP_t$, we construct the variance-of-variance risk premium and its slope stripped of the right jump tail variation, i.e. $VVRP_t^n = VVRP_t - RJT_t$ and $VVRPS_t^n = VVRP_t^{(90-\text{day})} - \left(VVRP_t^{(30-\text{day})} - RJT_t\right)$. As manifest in our simulation results, the impact of upward and downward jump risk premia on the VVRP diminishes as maturity grows. As a result, the difference between the two premia, approximated as the right jump tail variation, may only play a negligible role in the dynamics of the long-term VVRP. Given this, we do not account for the jump tail risk in $VVRP_t^{(90\text{-day})}$ for simplicity. The measures $VVRP_t^n$ and $VVRPS_t^n$ correspond to the components ascribed to normal-sized price fluctuations in the VIX market. The right panel of Figures 3-5 report values of the adjusted R^2 for regressions in which the regressors consist of $VVRP_t$, $(VVRP_t \text{ and } VVRPS_t)$, $(VVRP_t^n \text{ and } VVRPS_t)$ $VVRPS_t^n$) and $(VVRP_t^n, VVRPS_t^n \text{ and } RJT_t)$. Regardless of moneyness levels, we find that the VVRP term structure helps enhance the return predictions over various horizons and the removal of the RJT_t results in a lower degree of predictability. By separately accounting for RJT_t and the diffusive component of $VVRP_t$ and $VVRPS_t$, we document even greater increases in the

⁴We make use of $\Delta VVIX_t$ to render the series stationary, and in turn, balance the regression where an essentially white noise variable (the VIX return) is on the left-hand side.

magnitude of return predictability.

Going one step further, we select one-, three- and six-month horizons as examples to report the estimation results in Tables 7 and 8. To account for the potential issue of serial correlation, we rely on the Newey and West (1987) heteroskedasticity consistent covariance matrix estimator with an optimal lag. For ease of interpretation, we divide each explanatory variable by its standard deviation and therefore the estimated coefficient represents the effect of a one standard deviation change in that variable. In what follows, we always preprocess the data in such a manner when conducting estimations unless otherwise stated. Table 7 presents the results for regressions where $VVRP_t$, or $VVRP_t$ together with $VVRPS_t$, serve as regressor(s). The measure $VVRP_t$ and its slope are found significant for returns of VIX OTM calls, with an expected sign. We then isolate the right jump tail risk from the two risk factors and report the estimation output in Table 8. The removal of RJT_t results in merely no predictive power for $VVRP_t$ or $VVRPS_t$ as evidenced by less significant t-statistics for coefficient estimates and lower values of R^2 . Finally, we re-introduce RJT_t to regressions based on $VVRP_t^n$ and $VVRPS_t^n$. For different degrees of moneyness, RJT_t plays a significant role in predicting future option returns. Over long horizons, the inclusion of RJT_t generates even higher predictability than that afforded by $VVRP_t$ and $VVRPS_t$ in Table 7.

5.3 Return predictability: evidence from futures

We next assess whether the right jump tail component can predict the VIX futures returns by running the predictive regression given by

$$\frac{1}{h}\sum_{n=1}^{h}r_{t+n} = \beta_0 + \beta_1 x_t + \varepsilon_{t,t+n}$$

$$\tag{20}$$

where r_t is the monthly VIX futures return. The left panel in Figure 6 reports the values of adjusted R^2 for regression (20) where $VVRP_t$, $\Delta VVIX_t^2$, $SKEW_t$ or RJT_t serves as the single explanatory variable. The risk factor $VVRP_t$ continues to outperform $VVIX_t^2$ and $SKEW_t$ in predicting futures returns and its R^2 displays a tent-shaped pattern: it achieves the maximum at the 9-month horizon and decreases thereafter. Relative to the case of option returns, the right jump tail does a much better job for futures returns by dominating $VVRP_t$ in most of the forecasting horizons. In the right panel of Figure 6, we show that the predictability jointly afforded by $VVRP_t$ and its slope is much higher in magnitude and peaks at a shorter horizon as compared with that by $VVRP_t$ alone. By subtracting RJT_t from $VVRP_t$ and its slope, values of R^2 reduce and plateau at the 9-month horizon, which display a similar pattern to that from the simple regression based on $VVRP_t$ only. In line with our findings for VIX option returns, the top performance is observed for the case where the right jump tail risk and the diffusive component of $VVRP_t$ together with its slope are separately accommodated in the predictive regression.

Table 9 reports the estimation results of regression (20) for one-, three- and six-month horizons. In the upper panel of Table 9, we find that higher levels of $VVRP_t$ or $VVRPS_t$ are associated with higher subsequent returns on the VIX futures, indicating that VIX prices fall on news of negative volatility-of-volatility shocks. The results are statistically significant at a 95% confidence level with a single exception of $VVRP_t$ over three-month horizon.

Relative to the regression based on $VVRP_t$ or $VVRPS_t$, removing RJT_t results in less significant t-statistics for $VVRP_t^n$ or $VVRPS_t^n$ and lowers values of R^2 to nearly zero. When reintroducing RJT_t to the multiple regression based on $VVRP_t^n$ or $VVRPS_t^n$, we show an overwhelming increase in the degree of predictability, which is even substantially higher than that implied by $VVRP_t$ or $VVRPS_t$. In addition, RJT_t is significantly positive over different horizons. This suggests that higher compensations for jump tail risk predicts higher future VIX market returns. Altogether, our results suggest that much of the return predictability previously attributable to $VVRP_t$ or $VVRPS_t$ is effectively arising from the component of the right jump tail risk.

5.4 Robustness

In this subsection, we investigate the robustness of our predictability results using alternatives measures of volatility-of-volatility. For the risk-neural expectation of the volatility-of-volatility, we make use of the VVIX index from the CBOE. The index is published since 2012 and back-filled until 2006. For the physical expectation, we rely on the 5-minute front-month VIX futures returns to compute $RVVIX_t$. In line with the previous analysis, all the variation measures are at a monthly frequency. Table 10 outlines the summary statistics for the alternative measures. The CBOE VVIX is on average much higher than the volatility index, which suggests that VIX futures returns are more volatile than aggregate stock market returns. Aside from this, the VVIX itself is more volatile but less persistent than the volatility index. To facilitate the construction of the long-term VVRP, we obtain the VVIX with maturity of 90 days from the CBOE directly. In line with the previous finding, the VVIX is lower in magnitude with longer maturity. The high-frequency realized variance of the VIX is much lower than the VVIX, indicating that the risk-neutral expectation of the volatility of return variation systematically exceeds the statistical expectation.

Figure 7 depicts the time series plots of the CBOE VVIX index and our measure of the VVIX calculated using OTM options. Overall, the calculated VVIX qualitatively match the evolution of the reported VVIX index with the correlation coefficient of 70%. The fact that our measure of VVIX is on average lower and displays more rapid spikes than the CBOE VVIX may be attributable to the following points. First, we obtain a broader strike range by including most of the option quotes that meet the conditions specified in section 4. To approximate the integral on the RHS of equation (2), we consider interpolation and extrapolation procedure as commonly adopted in the recent literature. However, the CBOE adopts a particular cutoff rule which may induce distortions, see Jiang and Tian (2005) and Andersen, Bondarenko, and Gonzalez-Perez (2015) for details. Second, our measure of the VVIX is obtained using the options from the Optionmetrics database. It includes the last daily bid-ask quotes only, which might not perfectly match the data published by the CBOE for their final end-of-day computation.

Next, we estimate the predictive regressions using new measures of the volatility-of-volatility risk and report the values of adjusted R^2 in Figure 8. Consistent with the earlier empirical results,

the inclusion of the VVRP term structure brings substantially better return predictions relative to the regressions based on the VVRP alone. Further to this, removing RJT_t from the VVRP and its slope lowers the R^2 and the greatest magnitude of return predictability is again observed for the case where RJT_t and the diffusive components are separately accommodated.

6 Conclusion

Within the properties of the variance-of-variance risk premium (VVRP) and volatility-of-volatility (VVIX) together with their predictive power for returns in the VIX market, the paper examines the role of fear-of-fear. With plausible assumptions, the fear component embedded in the VIX market can be proxied by the right jump tail variation under the risk-neutral measure. In a Monte Carlo simulation, the upward jump risk premium clearly outperforms its downward counterpart in determining the time-series dynamics and the term structures of the risk factors. Since the difference between the two jump risk premia may be interpreted as special compensation for bearing jump tail risk or fear-of-fear, we conjecture that the right jump tail variation may account for a nontrivial fraction of the risk measures considered. This conclusion is supported by the observation that the right jump tail component dominates the left tail as a key driver of the VVIX and implied skewness, and subsumes all the information contained in its counterpart in explaining the VVRP.

Given the simulation evidence for the dominant role of right jump tail and the lack of VIX OTM put options, we focus on the right jump tail only in the empirical study using the VIX options and futures from 2006 to 2020. We present new evidence for the superior performance of VVRP in the return predictions of VIX futures and options. The addition of the VVRP term structure further enhances return predictability. We also show that the predictive power underlying the VVRP and its term structure primarily arises from that part of the premium linked to right jump tail. In separately considering the diffusive component of the risk factors and the right jump tail risk, we show a substantial increase in the degree of predictability over different horizons. Our results hold for returns of VIX OTM calls across different levels of moneyness and returns on VIX futures, where the impact of the right jump tail is stronger for the latter.

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Benchmark Results for Simulation Study. This table reports the mean values of various measures generated in the simulation, i.e. RV_t , $VVIX_t^2$, $VVRP_t$, VC_t , VP_t and $SKEW_t$. Values of parameters in the simulation are set following the results in Table 4 of Park (2016). $VVRP_t$ is measured as the difference between RV_t and $VVIX_t^2$, where $VVIX_t^2$ is the implied variance and RV_t the realized variance over a certain period; VC_t is defined as the risk-neutral upside semi-variance derived by out-of-the-money call options and VP_t the risk-neutral downside semi-variance derived by out-of-the-money put options; $SKEW_t$ is the difference between VC_t and VP_t . All the measures are expressed in annualized terms.

Məturity	BV_{i}	$VVIX^2$	VC	VP.	VVRP	SKEW
Maturity	$I v_t$	VVIAt	V Ot	VI t	v v ItI t	DREWt
30	0.851	1.291	0.679	0.589	-0.440	0.089
60	0.799	1.416	0.677	0.722	-0.618	-0.045
90	0.843	1.560	0.704	0.841	-0.717	-0.137
180	0.868	1.672	0.627	1.036	-0.804	-0.405
360	0.958	1.468	0.458	1.004	-0.509	-0.541

Simulation Results of Varying Jump Risk Premia. This table reports the mean values of various risk measures with low, medium and high jump risk premia over different horizons. Numbers in parenthesis are the percentage changes of the values with respect to those in the benchmark results, i.e. Table 1. For the purpose of comparison, upward (downward) jump risk premium is set as zero when the downward (upward) jump risk premium is varying.

	1	Upward j	ump risk	premium]	Downward	jump risl	c premiur	n
Panel A: maturity=30										
	$VVIX_t^2$	VC_t	VP_t	$VVRP_t$	$SKEW_t$	VVIX	VC_t	VP_t	$VVRP_t$	$SKEW_t$
low	1.548	0.827	0.696	-0.697	0.130	1.360	0.678	0.658	-0.509	0.020
	(0.199)	(0.217)	(0.182)	(0.583)	(0.467)	(0.053)	(-0.002)	(0.118)	(0.156)	(-0.780)
medium	1.861	0.997	0.836	-1.010	0.156	1.493	0.710	0.759	-0.641	-0.046
	(0.441)	(0.468)	(0.420)	(1.295)	(0.753)	(0.156)	(0.045)	(0.289)	(0.458)	(-1.520)
high	2.154	1.153	0.970	-1.303	0.181	1.649	0.755	0.868	-0.797	-0.112
	(0.668)	(0.698)	(0.648)	(1.961)	(1.045)	(0.277)	(0.111)	(0.474)	(0.812)	(-2.263)
Panel B: maturity=60										
low	1.671	0.806	0.846	-0.872	-0.041	1.564	0.732	0.815	-0.766	-0.082
	(0.179)	(0.190)	(0.172)	(0.411)	(-0.090)	(0.105)	(0.081)	(0.128)	(0.240)	(0.837)
medium	1.890	0.891	0.979	-1.091	-0.086	1.643	0.740	0.885	-0.845	-0.144
	(0.334)	(0.316)	(0.355)	(0.766)	(0.929)	(0.160)	(0.093)	(0.225)	(0.367)	(2.218)
high	2.066	0.953	1.093	-1.268	-0.135	1.732	0.759	0.955	-0.934	-0.195
_	(0.459)	(0.407)	(0.513)	(1.052)	(2.016)	(0.223)	(0.120)	(0.322)	(0.511)	(3.373)
Panel C: maturity=90										
low	1.742	0.767	0.959	-0.899	-0.189	1.645	0.722	0.907	-0.801	-0.184
	(0.117)	(0.090)	(0.140)	(0.254)	(0.377)	(0.054)	(0.026)	(0.079)	(0.118)	(0.341)
medium	1.904	0.818	1.069	-1.061	-0.247	1.717	0.731	0.970	-0.874	-0.238
	(0.220)	(0.163)	(0.271)	(0.479)	(0.797)	(0.101)	(0.039)	(0.154)	(0.219)	(0.731)
high	2.071	0.867	1.186	-1.228	-0.313	1.815	0.757	1.042	-0.972	-0.281
-	(0.328)	(0.233)	(0.410)	(0.713)	(1.274)	(0.164)	(0.076)	(0.238)	(0.356)	(1.047)
Panel D: maturity=180										
low	1.699	0.635	1.054	-0.831	-0.417	1.642	0.613	1.020	-0.774	-0.399
	(0.016)	(0.013)	(0.018)	(0.034)	(0.030)	(-0.018	(-0.022)	(-0.015)	-(0.037)	(-0.015)
medium	1.806	0.656	1.140	-0.937	-0.481	1.722	0.635	1.078	-0.853	-0.438
	(0.080)	(0.047)	(0.100)	(0.166)	(0.188)	(0.030)	(0.013)	(0.040)	(0.062)	(0.081)
high	1.969	0.701	1.259	-1.101	-0.555	1.779	0.644	1.126	-0.911	-0.473
-	(0.178)	(0.118)	(0.215)	(0.370)	(0.370)	(0.064)	(0.027)	(0.087)	(0.134)	(0.167)
Panel E: maturity=360										
low	1.505	0.469	1.031	-0.547	-0.554	1.412	0.441	0.966	-0.453	-0.514
	(0.026)	(0.023)	(0.027)	(0.074)	(0.024)	(-0.038	(-0.038)	(-0.038)	(-0.110)	(-0.048)
medium	1.453	0.452	0.996	-0.494	-0.531	1.514	0.468	1.041	-0.556	-0.567
	(-0.010)	(-0.014)	(-0.008)	(-0.029)	(-0.019)	(0.032)	(0.022)	(0.036)	(0.091)	(0.049)
high	1.596	0.482	1.108	-0.638	-0.619	1.514	0.465	1.043	-0.555	-0.576
	(0.088)	(0.053)	(0.104)	(0.253)	(0.146)	(0.031)	(0.015)	(0.039)	(0.091)	(0.064)

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Explanatory Power of Jump Tails for Various Risk Measures. This table reports the regression results of $VVRP_t$, $SKEW_t$ and $VVIX_t^2$. Upward and downward jump risk premia exist simultaneously in each of the regression. RJT_t and LJT_t denote the right and left risk-neutral jump variation put forth by Bollerslev et al. (2015). Newey and West (1987) robust t-statistics with an optimal lag are shown in parenthesis. *, ** and *** indicate statistical significance at 90%, 95% and 99% confidence -

levels, respectively.									
Dependent Variable: $VVRP_t$									
Jump Risk Premium		low			medium			high	
C	-0.646***	-0.439***	-0.375^{**}	-0.744***	-0.597***	-0.367	-1.173^{***}	-0.907***	-0.832^{***}
	(-5.109)	(-3.172)	(-2.178)	(-3.093)	(-4.246)	(-1.491)	(-7.617)	(-7.907)	(-5.577)
LJT_t	-0.705		-0.373	-1.279^{**}		-0.919	-0.788***		-0.227
	(-1.620)		(-0.930)	(-2.168)		(-1.523)	(-2.719)		(-0.701)
RJT_t		-1.982***	-1.863^{***}		-2.407^{***}	-1.897***		-2.105^{***}	-1.922^{***}
		(-3.028)	(-2.978)		(-4.354)	(-3.481)		(-5.353)	(-4.381)
Adj. R^2	0.009	0.058	0.057	0.081	0.099	0.134	0.047	0.129	0.127
Dependent Variable: $SKEW_t$									
Jump Risk Premium		low			medium			high	
Ŭ	0.117^{***}	-0.112^{***}	-0.033***	0.076	-0.204^{***}	-0.124^{***}	0.061^{**}	-0.243^{***}	-0.106^{***}
	(7.131)	(-9.685)	(-3.306)	(2.175)	(-10.835)	(-4.099)	(2.125)	(-11.305)	(-6.216)
LJT_t	-0.285^{***}		-0.468***	-0.129		-0.320^{***}	-0.138^{***}		-0.414^{***}
	(-3.870)		(-16.072)	(-1.250)		(-3.320)	(-3.082)		(-8.115)
RJT_t		0.881^{***}	1.031^{***}		0.830^{***}	1.007^{***}		0.609^{***}	0.943^{***}
		(14.840)	(20.778)		(10.582)	(18.977)		(9.255)	(16.522)
Adj. R^2	0.101	0.563	0.833	0.023	0.392	0.545	0.050	0.381	0.757
Dependent Variable: $VVIX_t^2$									
Jump Risk Premium		low			medium			high	
C	1.422^{***}	1.206^{***}	1.089^{***}	1.498^{***}	1.341^{***}	1.062^{***}	1.615^{***}	1.487^{***}	1.223^{***}
	(40.177)	(56.125)	(55.791)	(19.847)	(29.141)	(15.980)	(25.892)	(31.160)	(29.739)
LJT_t	1.101^{***}		0.693^{***}	1.532^{***}		1.117^{***}	1.449^{***}		0.802^{***}
	(8.824)		(9.346)	(7.031)		(5.670)	(14.691)		(7.316)
RJT_t		2.510^{***}	2.289^{***}		2.813^{***}	2.193^{***}		2.867^{***}	2.219^{***}
c 		(29.895)	(28.658)		(13.203)	(22.199)		(23.121)	(17.622)
Adj. R^2	0.245	0.720	0.813	0.482	0.556	0.786	0.537	0.762	0.888

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and West (1987) robust t-statistics with an optimal lag are shown in parenthesis. *, ** and *** indicate statistical significance Explanatory Power of Jump Tails for the Term Structures of Risk Measures. This table reports the regression results of the term structures of $VVRP_t$, $SKEW_t$ and $VVIX_t^2$. Upward and downward jump risk premia exist simultaneously in each of the regression. RJT_t and LJT_t denote the right and left risk-neutral jump variation put forth by Bollerslev et al. (2015). Newey at 90%, 95% and 99% confidence levels, respectively.

Dependent Variable: $VVRPS_t$									
Jump Risk Premium		low			medium			high	
C	-0.002	-0.092	-0.227	0.097	-0.077	-0.292	0.688^{***}	0.321^{**}	0.348^{**}
	(-0.015)	(-0.572)	(-1.229)	(0.366)	(-0.451)	(-1.066)	(4.476)	(1.978)	(2.279)
LJT_t	1.072^{**}		0.797	1.233^{*}		0.862	0.478^{*}		-0.083
	(2.193)		(1.653)	(1.935)		(1.321)	(1.653)		(-0.225)
RJT_t		1.799^{***}	1.544^{**}		2.436^{***}	1.957^{***}		1.855^{***}	1.922^{***}
		(2.871)	(2.367)		(3.927)	(3.250)		(4.349)	(3.469)
Adj. R2	0.019	0.032	0.039	0.055	0.075	0.097	0.010	0.076	0.071
Dependent Variable: $SKEWS_t$									
Jump Risk Premium		low			medium			high	
C	-0.728***	-0.456^{***}	-0.563^{***}	-0.691^{***}	-0.432	-0.493^{***}	-0.698***	-0.397***	-0.526^{***}
	(-19.228)	(-14.490)	(-15.589)	(-17.834)	(-13.275)	(-11.820)	(-20.135)	(-9.866)	(-14.783)
LJT_t	0.432^{***}		0.633^{***}	0.055		0.243^{**}	0.105^{**}		0.389^{***}
	(3.003)		(4.484)	(0.530)		(2.350)	(1.979)		(6.386)
RJT_t		-0.930***	-1.133^{***}		-0.859***	-0.994***		-0.661^{***}	-0.976***
		(-7.553)	(-9.565)		(-6.798)	(-8.675		(-6.624)	(-9.797)
Adj. R2	0.061	0.166	0.297	-0.003	0.175	0.209	0.008	0.178	0.308
Dependent Variable: $VVIXS_t^2$									
Jump Risk Premium		low			medium			high	
C	0.160^{**}	0.284^{***}	0.461^{***}	0.096	0.267^{***}	0.529^{***}	-0.003	0.134	0.388^{***}
	(2.123)	(4.560)	(6.408)	(0.968)	(3.248)	(4.711)	(-0.034)	(1.587)	(5.113)
LJT_t	-1.416^{***}		-1.048^{***}	-1.460^{***}		-1.048^{***}	-1.413^{***}		-0.770***
	(-4.718)		(-3.610)	(-5.156)		(-3.986)	(-10.632)		(-4.766)
RJT_t		-2.404***	-2.070^{***}		-2.758***	-2.176^{***}		-2.827***	-2.205^{***}
		(-10.492)	(-8.783)		(-8.273)	(-9.019)		(-14.831)	(-9.914)
Adj. R2	0.167	0.271	0.356	0.253	0.309	0.426	0.354	0.514	0.593

Summary Statistics for VIX Returns. The table reports the summary statistics for delta-neutral option returns and VIX futures returns. The time span is Mar 2006 through Dec 2020 on a daily basis. The moneyness is defined as $k = K/F_t(\tau)$, where K is the strike price and $F_t(\tau)$ denotes the futures price. The *t*-statistics are testing the null hypothesis that returns are equal to zero. The % < 0 column shows the proportion of data that is less than zero.

	Mean (%)	No. of Contracts	<i>t</i> -Stat.	%< 0	Min.	Max.	Std.	Skew.	Kurt.	AR(1)
Panel A: VI	X Options									
OTM Put										
0.4 < k < 0.6	0.589	6,322	6.459	53%	-0.359	0.693	0.073	1.765	15.549	-0.100
0.6 < k < 0.8	0.198	51,661	7.584	59%	-0.369	2.047	0.059	3.832	61.615	-0.088
0.8 < k < 1.0	0.238	83,613	10.043	60%	-0.320	1.251	0.069	2.443	20.447	-0.091
ITM Call										
0.4 < k < 0.6	0.638	15,082	6.914	53%	-0.439	1.054	0.113	1.084	8.771	-0.284
0.6 < k < 0.8	0.137	70,177	4.855	54%	-0.430	0.880	0.075	1.020	10.299	-0.333
0.8 < k < 1.0	-0.017	85,646	-6.444	53%	-0.380	1.119	0.054	0.910	12.787	-0.257
OTM Call										
1.0 < k < 1.2	-0.080	71,251	-4.452	53%	-0.413	1.058	0.048	0.794	15.741	-0.261
1.2 < k < 1.4	-0.123	60,797	-6.754	53%	-0.395	0.819	0.045	0.569	13.264	-0.281
1.4 < k < 1.6	-0.169	51,239	-8.844	53%	-0.581	0.658	0.043	0.134	13.066	-0.277
Panel B: VIX	K Futures									
	0.001	37,693	0.001	53%	-0.329	0.753	0.054	1.417	16.581	-0.080

Summary Statistics of Risk Measures. This table reports the summary statistics for monthly $VVRP_t$, $VVIX_t^2$, VC_t , VP_t and $SKEW_t$ over the sample period from Mar 2006 until Dec 2020. $VVRP_t$ is measured as the difference between monthly RV_t and $VVIX_t^2$, where $VVIX_t^2$ is the implied variance and RV_t the realized variance over a monthly period; VC_t is defined as the risk-neutral upside semi-variance derived by out-of-the-money call options and VP_t the risk-neutral downside semi-variance derived by out-of-the-money put options; $SKEW_t$ is the difference between VC_t and VP_t .

Maturity (days)	Mean	Std	Skew	Kurtosis
Panel A: $VVRP_t$				
30	-0.219	0.682	1.970	42.809
60	-0.130	0.387	4.393	29.146
90	-0.097	0.313	3.583	18.689
120	-0.075	0.217	2.301	9.965
Panel B: $VVIX_t^2$				
30	0.677	0.415	9.496	114.223
60	0.495	0.136	0.189	3.088
90	0.397	0.107	0.119	2.984
120	0.326	0.080	0.454	4.413
Panel C: VC_t				
30	0.332	0.093	-0.486	3.789
60	0.273	0.082	-0.104	2.542
90	0.216	0.063	-0.267	3.126
120	0.179	0.050	0.176	3.344
Panel D: VP_t				
30	0.219	0.263	11.115	140.403
60	0.165	0.060	1.024	4.027
90	0.141	0.049	0.807	3.268
120	0.121	0.035	1.008	4.644
Panel E: $SKEW_t$				
30	0.113	0.286	-11.504	146.631
60	0.108	0.058	-0.082	2.513
90	0.075	0.049	-0.397	2.423
120	0.058	0.035	-0.127	2.706

Options Return Predictability: right jump tails included. This table reports the forecasting regression results for delta-neutral option returns over three horizons. VIX_t denotes the CBOE VIX index; ΔVIX_t and ΔVIX_t^2 are included in each regression as the control variables. $VVRP_t$ denotes the variance-of-variance risk premium; $VVRPS_t$ is defined as the difference between the long-term (90-day) and short-term (30-day) VVRPs. Each explanatory variable is divided by its standard deviation so that each coefficient can be considered as the effect of a one standard deviation change in that variable. Newey and West (1987) robust t-statistics with an optimal lag are shown in parenthesis. *, ** and *** indicate statistical significance at 90%, 95% and 99% confidence levels, respectively.

			one-m	onth		
Moneyness	[1.0	1.2]	[1.2	1.4]	[1.4	1.6]
C	-0.002	-0.004*	-0.003	-0.004^{**}	-0.003	-0.005**
	(-0.957)	(-1.720)	(-1.531)	(-2.418)	(-1.455)	(-1.982)
$VVRP_t$	-0.001^{*}	-0.011^{***}	-0.002^{***}	-0.010^{***}	-0.003^{**}	-0.011^{***}
	(-1.690)	(-3.827)	(-2.881)	(-5.369)	(-2.450)	(-2.336)
$VVRPS_t$		-0.011^{***}		-0.009***		-0.009**
		(-4.056)		(-5.242)		(-2.336)
Adj. R^2	0.063	0.106	0.096	0.127	0.152	0.171
No. obs.	161	161	161	161	161	161
			three- r	nonth		
Moneyness	1.	0 1.2	[1.	2 1.4]	[1.4	1.6]
C	-0.002	-0.004	-0.002	-0.004*	-0.002	-0.004^{*}
	(-0.806)	(-1.646)	(-0.999)	(-1.780)	(-0.968)	(-1.829)
$VVRP_t$	-0.002^{***}	-0.012^{***}	-0.001	-0.011^{***}	-0.002	-0.012^{***}
	(-1.794)	(-4.981)	(-1.384)	(-5.599)	(-1.325)	(-3.559)
$VVRPS_t$		-0.012^{***}		-0.012^{***}		-0.012^{***}
		(-4.711)		(-5.902)		(-3.473)
Adj. R^2	0.035	0.147	0.035	0.179	0.055	0.145
No. obs.	159	159	159	159	159	159
			six-m	onth		
Moneyness	[1.0	1.2]	[].	2 1.4]	[1.4	1.6]
C	-0.002	-0.003	-0.002	-0.003	-0.002	-0.003
	(-0.535)	(-0.759)	(-0.810)	(-1.353)	(-0.800)	(-1.096)
$VVRP_t$	-0.002^{**}	-0.006***	-0.002^{**}	-0.006***	-0.002^{**}	-0.005***
	(-2.082)	(-3.158)	(-2.132)	(-3.770)	(-2.254)	(-3.112)
$VVRPS_t$		-0.005**		-0.005***		-0.004***
		(-2.475)		(-3.420)		(-2.632)
Adj. R^2	0.017	0.041	0.032	0.073	0.035	0.054
No. obs.	156	156	156	156	156	156

difference between the long-term (90-day) and short-term (30-day) VVRPs; $VVRP_t^n$ and $VVRP_t^n$ represents the components Options Return Predictability: right jump tails considered separately. This table reports the forecasting regression results for delta-neutral option returns over three horizons. VIX_t denotes the CBOE VIX index; ΔVIX_t and ΔVIX_t^2 are included in each regression as the control variables. $VVRP_t$ denotes the variance-of-variance risk premium; $VVRPS_t$ is defined as the of $VVRP_t$ and $VVRPS_t$ that are attributable to normal sized price fluctuations. RJT_t denotes the right risk-neutral jump variation put forth by Bollerslev et al. (2015). Each explanatory variable is divided by its standard deviation so that each coefficient can be considered as the effect of a one standard deviation change in that variable. Newey and West (1987) robust t-statistics with an optimal lag are shown in parenthesis. *, ** and *** indicate statistical significance at 90%, 95% and 99% confidence levels, respectively.

					one-mont	ļ			
		[1 0 1 9]			[1 9 1 4]			[1 4 1 6]	
C	0.001	-0.002	0.002	-0.002	-0.004	-0.001	0.000	-0.001	0.005
	(0.278)	(-0.479)	(0.635)	(-0.523)	(-1.056)	(-0.141)	(0.035)	(-0.130)	(0.884)
$VVRP_t^n$	0.001	-0.000	-0.002	-0.000	-0.001	-0.003	0.000	0.000	-0.002
	(0.704)	(-0.00)	(-0.971)	(-0.163)	(-0.670)	(-1.587)	(0.209)	(0.044)	(-1.021)
$VVRPS_t^n$		-0.004^{*}	-0.007***		-0.003**	-0.005***		-0.001	-0.005***
<i>Е</i> 1 С		(-1.734)	(-3.565)		(-2.070)	(-4.308)		(-0.728)	(-2.850)
TUL			-0.003			-0.004			-0.000-
Adj. R^2	-0.003	0.015	(110.2-)	0.002	0.010	(0.027)	0.025	0.020	0.049
No. obs.	139	139	139	139	139	139	139	139	139
					three-mon	$^{\mathrm{th}}$			
		$[1.0\ 1.2]$			$[1.2 \ 1.4]$			[1.4 1.6]	
C	-0.002	-0.005	0.002	-0.005	-0.007**	-0.001	-0.005	-0.007	0.001
	(-0.371)	(-0.755)	(0.260)	(-0.917)	(-1.989)	(-0.213)	(-0.843)	(-0.875)	(0.121)
$VVRP_t^n$	-0.000	-0.001	-0.004	-0.001	-0.002	-0.005*	-0.001	-0.002	-0.005
	(-0.161)	(-0.554)	(-1.285)	(-0.887)	(-1.084)	(-1.870)	(-0.839)	(-0.834)	(-1.386)
$VVRPS_t^n$		-0.004	-0.008**		-0.004	-0.007***		-0.002**	-0.007**
		(-1.158)	(-2.471)		(-1.501)	(-3.528)		(-0.748)	(-2.328)
RJT_t			-0.008***			-0.007***			-0.009***
			(-4.272)			(-4.982)			(-4.156)
Adj. R^2	-0.001	0.037	0.174	-0.001	0.049	0.190	0.014	0.018	0.176
No. obs.	137	137	137	137	137	137	137	137	137
					six-mont]	T			
		$[1.0\ 1.2]$			$[1.2 \ 1.4]$			[1.4 1.6]	
C	-0.002	-0.003	0.000	-0.005	-0.006	-0.003	-0.004	-0.005	-0.002
	(-0.337)	(-0.853)	(0.001)	(-0.771)	(-0.916)	(-0.977)	(-0.668)	(-0.833)	(-0.500)
$VVRP_t^n$	-0.000	-0.001	-0.002^{*}	-0.001	-0.002	-0.003**	-0.001	-0.001	-0.003**
	(-0.174)	(-0.363)	(-1.698)	(-0.756)	(-0.873)	(-2.382)	(-0.617)	(-0.654)	(-2.139)
$VVRPS_t^n$		-0.001	-0.003**		-0.001	-0.003**		-0.001	-0.002***
		(-0.717)	(-2.128)		(-0.883)	(-2.516)		(-0.389)	(-3.542)
RJT_t			-0.004***			-0.003**			-0.004***
c			(-2.783)			(-3.234)			(-4.861)
Adj. R^{2}	0.001	0.003	0.066	0.019	0.029	0.093	0.032	0.028	0.080
No. obs.	134	134	134	134	134	134	134	134	134

the long-term (90-day) and short-term (30-day) VVRPs; $VVRP_t^n$ and $VVRP_t^n$ represents the components of $VVRP_t$ and $VVRP_t$ that are attributable to normal sized price fluctuations. RJT_t denotes the right risk-neutral jump variation put varying horizons. $VVRP_t$ denotes the variance-of-variance risk premium; $VVRPS_t$ is defined as the difference between forth by Bollerslev et al. (2015). Each explanatory variable is divided by its standard deviation so that each coefficient can be considered as the effect of a one standard deviation change in that variable. Newey and West (1987) robust t-statistics with Predictability of VIX Futures Returns. This table reports the predictive regression results for returns of VIX futures across an optimal lag are shown in parenthesis. *, ** and *** indicate statistical significance at 90%, 95% and 99% confidence levels, respectively.

		one-month	I		three-mon	$_{ m th}$		six-month	T
C	0.001	0.001^{*}		0.000	0.001^{**}		0.000	0.000	
	(0.848)	(1.885)		(0.718)	(2.248)		(0.434)	(0.799)	
$VVRP_t$	0.001^{**}	0.006^{***}		0.001	0.004^{***}		0.001^{*}	0.002^{***}	
	(2.548)	(2.821)		(1.307)	(5.865)		(1.829)	(3.226)	
$VVRPS_t$		0.005^{**}			0.004^{***}			0.001^{**}	
		(2.437)			(5.376)			(2.321)	
Adj. R^2	0.004	0.078		0.017	0.176		0.035	0.064	
No. obs.	161	161		159	159		156	156	
C	0.000	0.000	-0.003**	0.002	0.002	-0.001	0.001	0.001	-0.001
	(0.090)	(0.000)	(-2.426)	(1.113)	(1.114)	(0.635)	(0.694)	(0.310)	(-0.174)
$VVRP_t^n$	0.000	0.000	0.002^{**}	0.001	0.001	0.002^{***}	0.000	0.000	0.001
	(0.264)	(0.209)	(2.354)	(1.458)	(1.443)	(2.921)	(0.996)	(0.469)	(1.045)
$VVRPS_t^n$		-0.000	0.002^{**}		0.000	0.002^{***}		-0.000	0.001^{**}
		(-0.269)	(2.325)		(0.278)	(4.218)		(-0.308)	(2.187)
RJT_t			0.004^{***}			0.004^{***}			0.001^{***}
			(4.556)			(7.831)			(4.946)
Adj. R^2	-0.007	-0.013	0.122	0.020	0.014	0.393	0.019	0.007	0.179
No. obs.	139	139	139	137	137	137	134	134	134

 RV_t stands for the realized variance that is annualized and computed using 5-minute VIX futures returns. The data are Summary Statistics for the Implied and Realized Variances of the VIX. VIX_t^2 corresponds to $(VIX/100)^2$ and $VVIX_t^2$ denotes the volatility-of-volatility index $((VVIX/100)^2)$. $VVIX_t^{2*}$ represents the VVIX based on options with maturity of 90 days. monthly from July 2012 to December 2020.

Variable	Mean	Std	AR(1)	Corr. VIX_t^2	Corr. $VVIX_t^2$
VIX_t^2	0.034	0.038	0.573	1.000	0.809
$VVIX_t^2$	0.902	0.334	0.518	0.809	1.000
$VVIX_t^{2*}$	0.507	0.121	0.741	0.680	0.861
$RVVIX_t$	0.166	0.403	0.263	0.335	0.355

Figure 1 Term Structure of Risk Measures. The figure shows the term structure of various risk measures generated in the simulation study.





2020. The top panel shows the $VVIX_t^2$ derived using the VIX options; the second panel depicts the estimated right jump tail Predictor Variables. All the measures are plotted at a monthly frequency and cover the period from February 2006 to December variation, i.e. RJT; the third panel plots the variance-of-variance risk premium (VVRP) and the VVRP minus RJT. All the monthly measures are reported in annualized squared form.



Return Predictability for VIX Call Options with 1.0 < k < 1.2. The left panel depicts the adjusted R^2 from regressions of the VIX call option returns with moneyness of [1.0, 1.2] over a 1-12 month horizon based on various risk measures for the full sample. The right panel provides results from regressions based on $VVRP_t$, $(VVRP_t$ and $VVRP_t$), $(VVRP_t^n$ and $VVRP_t^n)$ and $(VVRP_t^n, VVRPS_t^n \text{ and } RJT_t)$.



Predictability of VIX Options Return (1.0<k<1.2)

Return Predictability for VIX Call Options with 1.2 < k < 1.4. The left panel depicts the adjusted R^2 from regressions of the VIX call option returns with moneyness of [1.2, 1.4] over a 1-12 month horizon based on various risk measures for the full sample. The right panel provides results from regressions based on $VVRP_t$, $(VVRP_t \text{ and } VVRPS_t)$, $(VVRP_t^n \text{ and } VVRP_t^n)$ and $(VVRP_t^n, VVRPS_t^n \text{ and } RJT_t)$.



Predictability of VIX Options Return (1.2<k<1.4)

Return Predictability for VIX Call Options with 1.4 < k < 1.6. The left panel depicts the adjusted R^2 from regressions of the VIX call option returns with moneyness of [1.4, 1.6] over a 1-12 month horizon based on various risk measures for the full sample. The right panel provides results from regressions based on $VVRP_t$, $(VVRP_t$ and $VVRP_t$), $(VVRP_t^n)$ and $VVRP_t^n$) and $(VVRP_t^n, VVRPS_t^n \text{ and } RJT_t)$.



Predictability of VIX Options Return (1.4<k<1.6)

Return Predictability for VIX Futures. The left panel depicts the adjusted R^2 from regressions of the VIX futures returns over a 1-12 month horizon based on various risk measures for the full sample. The right panel provides results from regressions based on $VVRP_t$, $(VVRP_t$ and $VVRP_t$), $(VVRP_t^n$ and $VVRPS_t^n$) and $(VVRP_t^n, VVRPS_t^n$ and RJT_t).





Volatility-of-Volatility Measures. The figure shows the time series of volatility-of-volatility measures. The blue line represents the measure calculated using VIX options and the black line represents the official VVIX index published by the CBOE in 2012. The official index is back-filled until 2006.



Return Predictability using the VVIX Index and High-Frequency Realized Variance. The plots depict the adjusted R^2 from regressions for the returns on VIX futures and options based on $VVRP_t$, $(VVRP_t$ and $VVRPS_t)$, $(VVRP_t^n$ and $VVRP_t^n)$ and $(VVRP_t^n, VVRPS_t^n \text{ and } RJT_t)$.



High-Frequency VIX Futures and the VVIX Index