Job Market Cheap Talk*

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Abstract

In recruitment processes, due to the inherent conflict of interests between the candidates and the recruiter, recruiters typically use pre-employment tests to evaluate the candidates' competence levels. Alas, the candidates' performances in these tests significantly depend on their test-taking skills, a feature that impairs these tests' validity. We show that despite its adverse effect, tests' dependency on test-taking skills can induce candidates to reveal reliable information about their values by reporting their ex-ante prospects of succeeding in the test before taking it. Thus recruiters can benefit from including a reporting stage before the test in the recruitment process.

KEYWORDS: cheap talk, information design, strategic learning, strategic information transmission.

JEL Codes: D82, D83, C72

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*We want to thank Elchanan Ben-Porath, Alex Gershkov, Ilan Kremer, Benny Moldovanu, Motty Perry, Assaf Romm, Roland Strausz, and participants of various seminars for their valuable comments. Boaz Zik gratefully acknowledges funding by the German Research Foundation (DFG) through CRC TR 224 (Project B01).

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1 Introduction

Recruitment and placement processes, where an organization aims to hire candidates for vacant positions, are of great economic importance.[1] The main goal of the recruitment process is to supply the hiring manager with as much information as possible about the candidates to make better employment decisions.[2] Clearly, the task of extracting information from a candidate is not trivial as the objectives of the hiring manager and the candidate are not aligned. Specifically, the candidate aims at maximizing his chances of being hired and getting a better placement, while the hiring manager wishes to make the best decision for the organization. In such an environment, it is well established that if the only way in which the manager tries to obtain information from the candidate is via unverifiable, non-costly messages, i.e., cheap talk, then the manager cannot extract credible information from the candidate.[3]

As part of their effort to gain information about candidates, managers tend to conduct a pre-employment test in the recruitment process.[4] Typically, the candidate's prospect of succeeding in the test depends not only on skills related to the position but also on test-taking skills, i.e., skills that allow an examinee to undertake any test-taking situation appropriately, e.g., the level of test anxiety.[5,6] The inherent dependency

[1] According to *Statista,* the staffing and recruiting industry market size in the United States in 2019 was 151.8 billion dollars.

[2] According to a survey by *Harris Poll*, in 2017, companies lost on making a bad hire an average of 14,900 dollars and on losing a good hire 29,600 dollars. Additionally, 74 percent of employers reported they hired the wrong person for a position.

[3] Assume by contradiction that there is an influential equilibrium, i.e., an equilibrium in which the manager develops different expected beliefs about the candidate's value for different messages of the candidate. Since the candidate's payoff is increasing in the manager's expected belief, he will always send the message that corresponds to the highest expected belief of the manager irrespective of the true value. Therefore, the manager's beliefs are inconsistent with the candidate's strategy, in contradiction to the equilibrium requirement.

[4] According to surveys by the *American Management Association* (AMA), 70 percent of employers use pre-employment tests as part of their recruitment process.

[5] The *American Test Anxiety Association* reported that about 35 percent of students suffer from high or moderately-high test anxiety.

[6] Test-taking skills also refer to the ability to effectively implement test-taking strategies, e.g., to manage time efficiently, to survey all questions before responding, to solve easy questions first, to check and review answers.

of the test on the candidate's test-taking skills tends to impair the test's validity, i.e., the extent to which the test's scores reflect the candidate's competence for the specific position (henceforth the *candidate's value*).[7]

In this paper, we show that despite its abovementioned adverse effect, tests' inherent dependency on test-taking skills may also contribute to the manager's ability to obtain information in the recruitment process by facilitating a channel for meaningful (cheap talk) communication between the candidate and the manager. Specifically, we show that if the test is sufficiently dependent on the test-taking skill, then the candidate may report credible information about his prospects of succeeding in the test before he approaches it, even though the candidate is completely biased. These reports enable a better interpretation of the candidate's test results. They also convey direct information about the candidate's value.

The paper's findings have two implications for the design of recruitment processes. The first is that the recruitment process should include a stage, which occurs before the test, in which the candidate reports information about his chances of succeeding in the test. The second is that if the recruitment process includes such a reporting stage, then the manager may prefer to conduct a test whose dependency on test-taking skills is significant even though such a test is intrinsically less informative because under this test the candidate would reveal credible information in the reporting stage.

From a theoretical perspective, our paper contributes to existing literature by identifying a novel channel for cheap talk communication in an important economic setting where the common belief seemed to be that the conflict of interests is too severe to allow for non-costly non-verifiable communication.

We consider the following model of a recruitment process. A candidate, who is either high or low on each of two attributes – value and test-taking skill, and a manager, whose optimal action depends only on the candidate's value, engage in the following

[7]There is a vast literature in education and psychology that deals with test-taking skills and tests validity, see, e.g., Naylor (1997), Sternberg (1998), Cohen (2006), Dodeen (2008), Wu and Stone (2016), and Stenlund et al. (2018). This literature provides evidence that examinees' test-taking skills significantly affect their performance in tests and thus adversely affect the tests' validity. For example, Hembree (1988) shows that highly anxious students score about 12 percentile points below their low-anxiety peers.

recruitment process. In the first stage, the manager chooses a binary test – a mapping from the candidate's types to a distribution over the grades success or failure, whose success probability increases in each of the candidate's attributes, from a set of feasible tests. In the second stage, the candidate reports his cheap talk message. In the third stage, the test's result is realized, and the manager chooses an action that is a best reply given her information.[8] In this environment, any test that the manager chooses induces a subgame in which the candidate communicates with the manager via cheap talk messages.

At the beginning of our analysis, we show that a necessary condition for an influential equilibrium to exist in the subgame that a test induces is that the test depends sufficiently on the candidate's test-taking skill.[9] We then present an additional condition that requires that the test is more informative about the candidate's value when the candidate's test-taking skill is high. This condition seems to hold in many economic environments. For example, it seems natural that when the candidate does not suffer from test anxiety, the result of the test would be more indicative of his value.[10] We show that the fulfillment of both conditions is sufficient for the test to induce an influential equilibrium.

Next, we study how inserting the reporting stage to the recruitment process affects the manager's preference relation over tests. Specifically, we analyze the tension that arises when the test's dependency on the test-taking skill increases, between the test's intrinsic informativeness level and its ability to induce meaningful reporting from the candidate.[11] When the test becomes more dependent on the test-taking skill its intrinsic

[8]In this environment, the manager cannot incentivize the candidate to report truthful information by conditioning the properties of the test on the candidate's messages as in Egorov and Carroll (2019). Additionally, since the manager cannot commit to her action she cannot punish the candidate if his message is inconsistent with the test's result, as in the costly verification literature, see, e.g., Ben-Porath, Dekel, and Lipman (2014).

[9]An equilibrium is *influential* if there exist two different messages of the candidate and a grade of the test which induce different manager's beliefs (see Definition 3).

[10]An example of the opposite case is a candidate with a very good short-term memory. Such a candidate would tend to succeed in most tests regardless of how knowledgeable he is in the relevant material.

[11]In Section 2, where we present the setup, we give a precise definition of what it means for a test to be more informative than another test in our framework.

informativeness level decreases. However, the dependency of the test on the candidate's test-taking skill may have a positive effect on the manager, as it can promote meaningful information transmission in the reporting stage. We show that the latter effect may lead the manager to prefer a test that is more dependent on the candidate's test-taking skill even though it is intrinsically less informative.

Intuitively, the novel mechanism that enables the sorting of the candidate's types works in the following way. In an influential equilibrium, types with low (high) prospects of succeeding in the test pool in a message that associates them with a lower (higher) expected value and a lower (higher) expected test-taking skill. The types that report the low message benefit from a lenient interpretation of the test results, as the manager knows that their low test-taking skill impairs their performance in the test. This lenient interpretation compensates them for the initial low belief associated with their message. The types that report the high message face a more strict interpretation of their test results, as the manager realizes that their high test-taking skill enhances their performance in the test. However, the higher belief associated with their message makes up for the strict inference of their test's results.

In the main model, we assume that the candidate's value and his test-taking skill are independent. In this case, the lower expected belief associated with the low message is obtained endogenously in equilibrium as the low message is sent by both types whose test-taking skill is low and also, with some probability, by the type which has a low value and a high test-taking skill. Alternatively, if there is a positive correlation between the test-taking skill and the value, this connection can form exogenously. Specifically, when there is a positive correlation, truthfully revealing a low (high) test-taking skill exogenously associates the candidate with a lower (higher) expected value. We consider the case of correlation in Section 6 and show that a *pure equilibrium*, in which the candidate truthfully reveals the level of his test-taking skill may arise.

In the main text, we consider a stylized model of a recruitment process. We believe, however, that the economic messages that the paper's results convey are of general interest. In the last part of the paper, we show, by considering several extensions, how the paper's results extend to more general environments.

The rest of the paper is as follows. In the rest of this section, we present the related literature. Section 2 presents the model. Section 3 includes an example. In Section 4, we analyze the existence of influential equilibria. Section 5 discusses the manager's preference relation over tests. Section 6 discusses extensions of the model to allow correlation and non-linear candidate's payoff. Section 7 deals with the robustness of the paper's results. Section 8 concludes. Proofs appear in the Appendix.

Related Literature

Several other papers in the theoretical information economics literature deal with recruitment processes. Carroll and Egorov (2019) consider the following environment. The candidate is initially fully informed about his quality, which depends on multiple dimensions. The manager can verify only one dimension and can commit to a verification policy, a probability distribution over which dimension to verify. Unlike in our model, the cheap talk phase occurs before the manager chooses her verification policy, and thus the manager can condition the verification policy on the candidate's messages. They characterize conditions that guarantee the existence of a verification policy that enables full learning by the manager. Moran and Morgan (2003) consider a different environment than ours in the sense that the candidates incur a cost for misrepresenting their true quality and show that there exists a unique symmetric equilibrium in which the most qualified candidate is hired. In this equilibrium, each candidate misrepresents his quality, a candidate's strategy increases in his type, and the manager hires the candidate who reports the highest quality.

Our paper joins other papers that deal with information design problems in sender-receiver cheap talk environments. Krähmer (2021) considers the following sender-receiver environment: a state-independent sender and a receiver are initially uninformed about the state; the receiver can commit to a lottery over the possible tests; the receiver observes the result of the lottery, i.e., the realized test, but not its realized grade; the sender observes the test's realized grade but not the realized test. Krähmer characterizes a condition on the sender's payoff function that is necessary and sufficient for a receiver to design a lottery over tests under which the receiver obtains full information in equilibrium.[12] Jain (2018) studies a Bayesian persuasion problem of a state-dependent sender where the sender's signal and its realization are publicly observed before the cheap talk phase. She shows that some beliefs facilitate effective cheap talk communication as they induce alignment between the sender's and the receiver's preferences and analyze the implications of cheap talk on the optimal signal of the sender.

Other papers study the possibility of influential cheap talk equilibria and their effect on the ex-ante expected payoff of the sender in environments where the sender's payoff function is state-independent and where, as opposed to our model, the receiver is uninformed. Chakraborty and Harbaugh (2010) consider environments with a multidimensional state. Unlike in our model, the receiver's action is also multidimensional and equals the vector of means of the various dimensions given her belief. They show that influential cheap talk equilibria exist. In these equilibria, the type of communication is comparative, i.e., the sender admits to being low in some dimensions and higher in others. The different messages correspond to beliefs whose vectors of means are not monotonically ordered, and the sender is indifferent between these messages. The sender benefits from influential cheap talk equilibria if his payoff function is quasi-convex in the receiver's action. Lipnowski and Ravid (2020) use an abstract belief-based approach to study the above questions and provide characterizations of when influential equilibria exist and when the sender benefits from these equilibria. In the environment we consider, where the receiver's action is unidimensional and increasing in her belief's mean about a unidimensional statistic and where the sender's payoff is increasing in the receiver's action, influential cheap talk equilibria do not exist if the receiver is uninformed about the state.[13] Moreover, in the influential cheap talk equilibria that we identify when the receiver is partially informed, the type of communication is not comparative but vertical, e.g., low sender's types admit to having low expected values.

[12]In this case, the sender observes the state in an encrypted form but does not observe the encryption code. The receiver observes the encryption code but not the encrypted state. Watson (1994) considers a similar environment where the receiver has no control over the design of the encryption code.

[13]See footnote 3 for an explanation of why influential equilibria do not exist in our model if the receiver is uninformed.

That is, the equilibrium messages are strictly monotonically ordered in terms of the means of the beliefs they correspond to.

Our paper also connects to papers that consider a receiver's learning problem in strategic communication environments different than cheap talk and show that the receiver may prefer to coarse her information to induce a more informative sender's equilibrium strategy. Weksler and Zik (2021) study the receiver's preference relation over tests in a signaling environment where the sender's signaling costs are state-independent and show, among other results, that it does not comply with Blackwell's (1951) partial order. Ball (2021) and Whitmeyer (2021) study the problem of choosing a scoring rule in a signaling environment à la Frankel and Kartik (2019) and find, among other findings, that a less informative scoring rule may induce a more informative equilibrium. Rosar (2017) and Harbaugh and Rasmusen (2018) both study, in different environments, a receiver's optimal test choice where the sender can decide whether to participate in the test and find that the manager's optimal test uses coarse grading to increase participation.

2 Model and Preliminary Analysis

2.1 The Environment

There is a manager (she) and a candidate (he). The candidate has two independent attributes T and V, each can be either low or high, where low (high) is associated with the number 0 (1). The first attribute, $T \in \{T_0, T_1\}$, corresponds to the candidate's test-taking skill, where T_0 corresponds to a low test-taking skill and T_1 corresponds to a high test-taking skill. The second attribute, $V \in \{V_0, V_1\}$, corresponds to the candidate's value, where V_0 corresponds to a low value of 0 and V_1 corresponds to a high value of 1. We denote the state space by $\Omega = \{T_0, T_1\} \times \{V_0, V_1\}$ with a generic element ω. The recruitment process is described by the following sequential game, which has four periods. At period 0, nature draws the state according to the prior distribution $\mu^0 \in \Delta\Omega$. The candidate observes the state while the manager does not.

At period 1, the manager chooses a binary test $\pi : \Omega \to \triangle G$ where $G = \{s, f\}$, which correspond to *success* and *failure*, from a set of feasible test $\Pi^F \subseteq \Pi$ and we denote $p_\pi^\omega := \pi(s|\omega) \in (0,1)$. We consider Π to be the set of all binary tests whose probability of success weakly increases in each attribute. At period 2, the candidate observes the manager's test choice and sends a costless message $m \in \mathcal{M}$, where $|\mathcal{M}| \geq |\Omega|$, to the manager. The manager observes the candidate's message and develops an interim belief $\mu^\pi(m) \in \Delta\Omega$ about the state. At period 3, a test's grade $g \in G$ is realized according to the test π and the state ω, and is observed by the manager. The manager forms a posterior belief $\mu^\pi(g, m)$ about the state and takes an optimal action, e.g., whether to hire the candidate, what position to place the candidate in, or which salary to assign to the candidate.

2.2 Payoffs

Given a belief $\mu \in \Delta\Omega$ we denote the marginal belief of $K \in \{T, V\}$ by $\mu_K \in \Delta\{K_0, K_1\}$. Since $K = \{K_0, K_1\}$ for every attribute $K \in \{T, V\}$, we slightly abuse notation and also denote the probability of the event $K = K_1$ given the belief μ by $\mu_K \in [0, 1]$ which is also the mean of attribute K given μ.

Since we want our model to encompass managers who take different types of decisions, e.g., hiring, placement, or assigning salaries, we do not explicitly model the action that the manager is taking. Rather, we use a reduced-form approach by assuming that the payoffs of both the candidate and the manager are directly linked to the posterior belief of the manager. The manager's expected payoff given her optimal action conditional on a belief $\mu \in \Delta\Omega$ is denoted by $W^M(\mu)$. We assume that the manager's payoff from her action depends only on the candidate's value, V, i.e., W^M depends only on μ_V. We assume that $W^M(\mu_V)$ is a strictly convex function of μ_V. The last assumption captures the property that the manager strictly benefits from learning about the candidate's value, as the manager's expected payoff strictly increases from any additional information about V if and only if W^M is strictly convex. The candidate wants the manager's beliefs about his value to be as high as possible independently of the state, i.e., the candidate's payoff function from a manager's belief about his value μ_V, denoted by $W^C(\mu_V)$, is strictly increasing in μ_V. In the main model, we assume that W^C is linear, i.e. $W^C(\mu_V) \coloneqq \mu_V$. In Subsection 6.2, we discuss the case where W^C is non-linear. We refer to μ_V as the *expected value* given μ.

Two explicit models that our reduced-form model encompass are the following. The first model is where the manager wants to choose the action that matches the candidate's value, and the sender's payoff is equal to the manager's action. Specifically, the manager's payoff is the quadratic loss, i.e., her payoff from action a and a state (T, V) is $-(V-a)^2$ and the candidate's payoff is a. In this case, the manager's optimal action is equal to the expected value of her belief, i.e., to μ_V. Therefore, we have that $W^M(\mu_V) = -\mu_V(1-\mu_V)$ and[14] $W^C(\mu_V) = \mu_V$. The second model is where the manager's decision is whether or not to hire the candidate. If the manager hires the candidate, her payoff equals the candidate's value. If the manager does not hire the candidate, she will obtain her payoff from the realization of an outside option, e.g., the expected productivity of another candidate, whose value for the manager is distributed in $[0, 1]$. The candidate's payoff if he is hired is 1 and 0 otherwise. Hence, his expected payoff is the probability of being hired, which is equal to the probability that the manager's belief about his value will be greater than the realized value of the manager's outside option. Specifically, our main model corresponds to the case where manager's value from the outside option is uniformly distributed in $[0, 1]$ in which case $W^M(\mu_V) = \frac{1+\mu_Y^2}{2}$ and[15] $W^C(\mu_V) = \mu_V$. The analysis in Subsection 6.2 corresponds to other distributions of the manager's outside option.

[14] The manager's expected payoff given her belief is $-\left(\mu_V(1-\mu_V)^2 + (1-\mu_V)(0-\mu_V)^2\right) = -\mu_V(1-\mu_V)$.

[15] The manager's expected payoff given her belief about the candidate's productivity level is equal to the probability that the outside option is smaller than the belief's mean multiplied by the value of the belief's mean, plus the probability of the event that the outside option is greater than belief's mean multiplied by the mean of the outside option conditional on this event, i.e., if the outside option is distributed uniformly then the expected payoff is equal to $\mu_Y \cdot \mu_Y + (1-\mu_Y) \cdot \frac{(1+\mu_Y)}{2} = \frac{1+\mu_Y^2}{2}$.

2.3 Equilibrium

In our model, every test $\pi \in \Pi^F$ induces a cheap talk subgame. Our solution concept for this subgame is perfect Bayesian equilibrium which consists of a strategy for the candidate, a profile interim beliefs of the manager, and a profile of posterior beliefs of the manager.

Definition 1. A *strategy for the candidate* is a mapping $\sigma_\pi : \Omega \to \Delta\mathcal{M}$ that assigns to each candidate's type $\omega \in \Omega$ a probability distribution over the possible messages.

Given a strategy σ_π. We denote by $\sigma_\pi(m|\omega)$ the probability that type ω sends the message m according to strategy $\sigma_\pi(\cdot)$, by $\sigma_\pi^{-1}(m)$ the set of types for which $\sigma_\pi(m|\omega) > 0$, and by $\operatorname{supp}(\sigma(\pi))$ the set of all messages $m \in \mathcal{M}$ for which there is $\omega \in \Omega$ such that $\sigma_\pi(m|\omega) > 0$.

Definition 2. We say that a strategy $\sigma_\pi(\cdot)$, a profile of interim beliefs $(\mu^\pi(m))_{m\in\mathcal{M}}$, and a profile of posterior beliefs $(\mu^\pi(g,m))_{g\in G,\, m\in\mathcal{M}}$ form an *equilibrium* if and only if the following conditions hold:

1. If $m \in \operatorname{supp}(\sigma(\pi))$, then the manager's interim belief $\mu^\pi(m)$ is obtained from her prior belief μ^0 using Bayes' rule.

2. Given $g \in G$, if $m \in \operatorname{supp}(\sigma(\pi))$, then the manager's posterior belief $\mu^\pi(g,m)$ is obtained from her interim belief $\mu^\pi(m)$ using Bayes' rule.

3. For every $\omega \in \Omega$, if $\sigma_\pi(m|\omega) > 0$, then

$$m \in \arg\max_{m'\in M} p_\pi^\omega \cdot \mu_V^\pi(s,m') + (1-p_\pi^\omega) \cdot \mu_V^\pi(f,m')$$

Definition 3. We say that an equilibrium $\left\{\sigma_\pi(m|\omega), (\mu^\pi(m))_{m\in\mathcal{M}}, (\mu^\pi(g,m))_{g\in G,\, m\in\mathcal{M}}\right\}$ is *influential* if there are at least two messages m and m' in $\operatorname{supp}(\sigma(\pi))$ each of them sent with a strictly positive probability, and some $g \in G$ such that $\mu_V^\pi(g,m) \neq \mu_V^\pi(g,m')$. We say that a test $\pi \in \Pi$ *induces an influential equilibrium* if there exists an influential equilibrium in the subgame that the test π induces.

2.4 Partial Order of Informativeness

We now define a notion of informativeness of a test with respect to the candidate's value. Given a belief μ and a test $\pi \in \Pi$ we denote by $q^{V_i}(\pi, \mu_T)$ the marginal probability of success conditional on $V = V_i$ where $i \in \{0, 1\}$.[16] We denote by $\hat{\pi}(\pi, \mu_T) : \{V_0, V_1\} \rightarrow \Delta\{s, f\}$ the binary test under which the probability of success conditional on the event $V = V_i$ is equal to $q^{V_i}(\pi, \mu_T)$.

Definition 4. Consider a prior belief $\mu^0 \in \Delta\Omega$. We say that a test π is *more informative about the candidate's value* than a test π' if and only if the test $\hat{\pi}(\pi, \mu_T^0)$ Blackwell dominates the test $\hat{\pi}(\pi', \mu_T^0)$.

Recall that a test Blackwell dominates another test if and only if any decision-maker whose optimal action depends on the state prefers the former test to the latter. Therefore, since our model considers a given prior belief, a test π is more informative about the candidate's value than a test π' if and only if any manager that our model considers prefers π to π' in the absence of a cheap talk phase. The following lemma presents a simple condition that implies the ranking of two tests according to their informativeness about the candidate's value.

Lemma 1. *Let π and π' be two tests in Π. If $q^{V_0}(\pi, \mu_T^0) \leq q^{V_0}(\pi', \mu_T^0)$ and $q^{V_1}(\pi, \mu_T^0) \geq q^{V_1}(\pi', \mu_T^0)$ and one of these inequalities holds strictly, then π is more informative about the candidate's value than π'.*

The proof of Lemma 1 is as follows. Under the test π the probability of success conditional on the event $V = V_1$ ($V = V_0$) is greater (smaller) than under the test π'. Therefore, the expected value of the posterior belief that follows grade s (f) is larger (smaller) under the test π than under the test π'. Since the expected values of the posterior beliefs average back to the prior belief's expected value and because $W^M(\mu_V)$ is strictly convex, the expected payoff of the manager is higher under the test π than under the test π'.

[16] That is, $q^{V_0}(\pi, \mu) = (1 - \mu_T) \cdot p_\pi^{(T_0,V_0)} + \mu_T \cdot p_\pi^{(T_1,V_0)}$ and $q^{V_1}(\pi, \mu) = (1 - \mu_T) \cdot p_\pi^{(T_0,V_1)} + \mu_T \cdot p_\pi^{(T_1,V_1)}$.

3 Example

In this section, we provide an illustration of the paper's results by considering a particular setting of our model. The manager's prior belief is $\mu^0(\omega) = \frac{1}{4}$ for each $\omega \in \Omega$, her payoff function[17] is $-\mu_V(1-\mu_V)$, and her set of feasible test, Π^F, includes two tests, π and π', where,

$$p_\pi^{(T_0,V_0)} = 0.01;\ \ p_\pi^{(T_1,V_0)} = 0.1;\ \ p_\pi^{(T_0,V_1)} = 0.2;\ \ p_\pi^{(T_1,V_1)} = 0.99;$$

and

$$p_{\pi'}^{(T_0,V_0)} = 0.01;\ \ p_{\pi'}^{(T_1,V_0)} = 0.2;\ \ p_{\pi'}^{(T_0,V_1)} = 0.2;\ \ p_{\pi'}^{(T_1,V_1)} = 0.99.$$

By Lemma 1, the test π is more informative about the candidate's value than π'. Hence, when the manager does not include a reporting stage in the recruitment process, she strictly prefers π to π'. As we show below, our results imply that the manager can strictly improve her payoff by including a reporting stage before the test. Moreover, our results show that including a reporting stage would lead the manager to prefer π' over π, even though π is more informative than π'. The reason for this result is that π' induces an influential equilibrium in the reporting stage, while π does not. The information that the test π' produces by facilitating information transmission in the reporting stage is large enough to compensate for its intrinsic informational inferiority.

The influential equilibrium induced by π'

We begin by presenting the influential equilibrium that is induced by π'. Consider the following candidate's strategy and the manager's beliefs that correspond to it. There are two messages, L and H, where $\sigma_{\pi'}^{-1}(L) = \{(T_0,V_0),(T_0,V_1),(T_1,V_0)\}$ and $\sigma_{\pi'}^{-1}(H) = \{(T_1,V_0),(T_1,V_1)\}$ and $\sigma_{\pi'}(L|\,(T_1,V_0)) = b$; i.e., type (T_1,V_0) sends L with probability b and H with probability $1-b$. Consider the difference in type (T_1,V_0)'s payoff from sending L and sending H as a function of its level of mixing b.

[17]This payoff function is derived from the case where the manager's payoff is the quadratic loss, i.e., her payoff from action a and a state (T,V) is $-(V-a)^2$

Assume that $b = 0$. In this case, under both messages, the expected value unconditional on the test's result is equal to $\frac{1}{2}$. Among the types in $\sigma_{\pi'}^{-1}(L)$, type (T_0, V_1) has the highest probability of success, thus, its expected payoff under L is greater than $\frac{1}{2}$. Since type (T_1, V_0)'s success probability is weakly greater than that of type (T_0, V_1), type (T_1, V_0) obtains an expected payoff greater than $\frac{1}{2}$ under L. Among the types in $\sigma_{\pi'}^{-1}(H)$, type (T_1, V_0) has the lowest probability of success, thus, its expected payoff under H is smaller than $\frac{1}{2}$.

Assume that $b = 1$. In this case, sending H identifies the candidate with the type (T_1, V_1), which has the highest value possible. Therefore, each type gets a higher expected payoff under H than under L, in particular, type (T_1, V_0).

We conclude that when $b = 0$ the difference in type (T_1, V_0)'s payoff from sending L and sending H is greater than 0, and when $b = 1$, it is smaller than 0. Since the difference is a continuous function of b, there exists a level of mixing b^* such that this difference is equal to 0, at this point, type (T_1, V_0) obtains the same expected payoff from both messages.

We now argue that the candidate's strategy that corresponds to b^* is an equilibrium. This result relies on the single crossing property which corresponds to the expected value given success (failure) is greater under H (L) than under L (H). This property implies that if type (T_1, V_0) is indifferent between L and H, then every type ω with a lower probability of success strictly prefers L to H and every type ω with a higher probability of success strictly prefers H to L. This completes the argument.

The mechanism that enables the sorting in equilibrium relies on a tradeoff between a more positive interpretation of the test's results, associated with the low message, and a higher ex-ante expected value, associated with the high message. In the above example, when the test-taking skill does not affect the candidate's value, this tradeoff arises endogenously as type (T_1, V_0) strictly mixes between the low and the high message, thus reducing the ex-ante expected value of the low message. In Section 6, we consider the case when the candidate's value is positively correlated with the test-taking skill. In such a case, admitting to having a low test-taking skill associates the candidate with a low ex-ante expected value. Hence, the above tradeoff arises even if type (T_1, V_0) sends purely the high message. This property facilitates the existence of a a *pure equilibrium*, in which the candidate credibly reveals his level of test-taking skill.

No influential equilibrium under π

We proceed by presenting the argument that shows that π does not induce an influential equilibrium. The argument starts with deriving the property that in an influential equilibrium for any two messages, m and m', either $\max\{p_\pi^\omega |\, \omega \in \sigma^{-1}(m)\} \leq \min\{p_\pi^\omega |\, \omega \in \sigma^{-1}(m')\}$ or vice versa. That is, the candidate's types are sorted into the equilibrium messages according to their probabilities of success. This property follows from the feature that in an influential equilibrium the difference in the candidate's expected payoff between sending m and m', which is equal to

$$p_\pi^\omega \cdot [\mu_V^\pi(s,m) - \mu_V^\pi(s,m') - \mu_V^\pi(f,m) + \mu_V^\pi(f,m')] + \mu_V^\pi(f,m) - \mu_V^\pi(f,m'),$$

is monotonic in the success probability of his types. Therefore, if some type ω prefers m to m', then so does any type ω' with $p_\pi^{\omega'} > p_\pi^\omega$. Now, under π types with higher values face strictly higher probabilities of success. Therefore, the above property implies that in an influential equilibrium there would necessarily be two messages, m and m', such that each of the types that send m would have a higher value than each of the types that send m'. Therefore, the expected value of each grade's posterior belief would be higher under m than under m'. Hence, the message m' cannot be sent in equilibrium.

4 Information Transmission in the Reporting Stage

In this section, we show that including a reporting stage that takes place before the test in the recruitment process can be strictly beneficial for the manager, as the candidate can transmit credible information by reporting his chances of succeeding in the test. We characterize conditions on the properties of a test that would allow obtaining meaningful information in the reporting stage. We start with the following definition:

Definition 5. We say that a test $\pi \in \Pi$ is *sensitive enough to the test-taking skill* if and only if $p_{\pi}^{(T_0,V_1)} \leq p_{\pi}^{(T_1,V_0)}$.

As we explained in Section 3, given a binary test, types are sorted into messages according to their success probabilities. Hence, a necessary condition for a test to induce an influential equilibrium is that the candidate's probability of success would not be strictly increasing in his types' values and so we obtain the following lemma.

Lemma 2. *A test $\pi \in \Pi$ induces an influential equilibrium only if it is sensitive enough to the test-taking skill.*

Remark 1. The essential requirement of the condition that the test is sensitive enough to the test-taking skill is that type (T_0, V_1)'s grades distribution would not dominate type (T_1, V_0)'s grades distribution. In our model, since tests are binary, this requirement also implies that type (T_1, V_0)'s grades distribution weakly dominates type (T_0, V_1)'s grades distribution. However, generally, the condition that type (T_1, V_0)'s grades distribution weakly dominates type (T_0, V_1)'s grades distribution is not necessary for the test to induce an influential equilibrium. Indeed, as we show in Subsection 7.1, a test with more than two grades can induce an influential equilibrium, even if type (T_1, V_0)'s grades distribution does not first-order stochastically dominate type (T_0, V_1)'s grades distribution.

We now move to characterize a sufficient condition for a test to induce an influential equilibrium.

Definition 6. Consider a test $\pi \in \Pi$. We say that the *informativeness of test π about the candidate's value is increasing in the test-taking skill* if and only if the test $\hat{\pi}(\pi, 1)$ Blackwell dominates the test $\hat{\pi}(\pi, 0)$.

In many environments, it is natural to assume that the above condition holds, i.e., that having a high test-taking skill enables better identification of the candidate's value. For example, when the test-taking skill is test anxiety, a test provides better information about a candidate's value when the candidate does not suffer from test anxiety. The following proposition presents a sufficient condition for a test to induce an influential equilibrium.

Proposition 1. *If a test* $\pi \in \Pi$ *is sensitive enough to the test-taking skill and its informativeness about the candidate's value is increasing in the test-taking skill, then it induces an influential equilibrium.*

The argument of the proposition is as follows. Given that the test is sensitive enough to the test-taking skill, we can apply the same construction that we presented in Section 3 to find a strategy for the candidate with two messages, L and H, where types (T_0, V_0) and (T_0, V_1) send the message L, type (T_1, V_1) sends the message H and type (T_1, V_0) is strictly mixing between the messages, such that when the manager's beliefs are derived from this strategy via Bayes' rule, type (T_1, V_0) is indifferent between these two messages.

For this strategy and beliefs to form an equilibrium, type (T_1, V_1) needs to prefer the message H and types (T_0, V_0) and (T_0, V_1) need to prefer the message L, i.e., the difference between the candidate's expected payoffs under the messages H and L should be increasing in his probability of success. A necessary and sufficient condition for this to occur is that the difference between the expected values of the posterior beliefs that follow the test's grades would be greater under the message H than under the message L. In the proof, we show that the property that the test's informativeness about the candidate's value increases in the test-taking skill guarantees this condition.[18]

[18]In our model, we abstract from moral hazard considerations of the candidate, e.g., the candidate may choose to fail the test on purpose if it is strategically appropriate. Accounting for moral hazard may rule out equilibria where, under some message, the expected value of the posterior belief given failure is higher than the expected value of the posterior belief given success. To rule out these equilibria, we can add another condition to the conditions of Proposition 1. For example, that $p_{\pi}^{(T_0,V_1)} \geq q^0(\pi, \mu_T)$. All the results of section 5 hold even in a model that allows for moral hazard, as they rely on the existence of an influential equilibrium in which the expected value of the posterior belief given success is greater than the expected value of the posterior belief given failure for each of the equilibrium messages.

5 Manager's Preference Relation over Tests

In the previous section, we showed that including a reporting stage in the recruitment process can be strictly beneficial for the manager. In this section, we study how including a reporting stage alters the manager's preferences over tests. Since the manager designs the recruitment process, we assume that she can select the equilibrium in the subgame that the test induces; i.e., we assume that if there are several equilibria in the subgame that a test induces, then the chosen equilibrium is the one that maximizes the manager's payoff.[19] Given this selection criterion, we identify each test $\pi \in \Pi$ with its corresponding *effective signal*, i.e., the signal that incorporates the information that arises from both the cheap talk channel and the test. We also denote by $U^M(\pi)$ the manager's expected payoff given the effective signal of π.

To study the effect of the cheap talk phase on the manager's preference relation over tests, we partition the set Π such that each cell of the partition includes tests that have the same marginal probability of success when $V = V_1$ and $V = V_0$. Specifically, each cell is identified with two parameters q^1 and q^0 that correspond to the marginal success probability of types whose $V = V_1$ and $V = V_0$, respectively. We denote each cell of the partition by

$$\Pi\left(q^0, q^1\right) \coloneqq \left\{\pi \in \Pi | q^{V_0}\left(\pi, \mu_T^0\right) = q^0 \text{ and } q^{V_1}\left(\pi, \mu_T^0\right) = q^1\right\}$$

Each such cell includes tests that are informationally equivalent about the candidate's value as, in the absence of a cheap talk phase, each test in $\Pi\left(q^0, q^1\right)$ results in the same distribution of the posterior beliefs. Therefore, in the absence of a cheap talk phase, the manager is indifferent between tests that belong to the same cell.

In the presence of a cheap talk phase, when the cell $\Pi\left(q^0, q^1\right)$ includes a test that induces an influential equilibrium, the manager is no longer indifferent between the tests in $\Pi\left(q^0, q^1\right)$, but rather strictly prefers a test that is dependent on the test-taking

[19]This assumption is standard in the mechanism design literature, where the concept of implementation basically assumes that, given a mechanism, the equilibrium that is played is selected by the designer.

skill. The following definition formalizes such preferences.

Definition 7. Assume that $\Pi^F = \Pi\left(q^0, q^1\right)$; we say that the manager *strictly prefers a test that depends on the test-taking skill* if and only if for every $\pi' \in \text{argmax}_{\pi \in \Pi(q^0,q^1)} U^M\left(\pi\right)$ we have that $\hat{\pi}\left(\pi', 1\right) \neq \hat{\pi}\left(\pi', 0\right)$.

We now present a proposition that shows an equivalence between the inclusion of a test that is sensitive enough to the test-taking skill in a cell $\Pi\left(q^0, q^1\right)$ and the inclusion of a test that induces an influential equilibrium in this cell.

Proposition 2. *Let $q^0, q^1 \in (0, 1)$. The following are equivalent:*

1. *The cell $\Pi\left(q^0, q^1\right)$ includes a test that is sensitive enough to the test-taking skill.*
2. *The cell $\Pi\left(q^0, q^1\right)$ includes a test that induces an influential equilibrium.*
3. *If $\Pi^F = \Pi(q^0, q^1)$, then the manager strictly prefers a test that depends on the test-taking skill.*

Note that the property that a test is sensitive enough to the test-taking skill does not imply that the test induces an influential equilibrium. Rather, the result that condition 1 implies condition 2 means that if $\Pi\left(q^0, q^1\right)$ includes a test that is sensitive enough to the test-taking skill, then it is possible to find a test in $\Pi\left(q^0, q^1\right)$ that is sensitive enough to the test-taking skill which induces an influential equilibrium.

Next, we present a lemma that identifies a necessary and sufficient condition for a cell $\Pi\left(q^0, q^1\right)$ to include a test that induces an influential equilibrium.

Lemma 3. *The cell $\Pi\left(q^0, q^1\right)$ includes a test that induces an influential equilibrium if and only if $q^1 - q^0 < \delta\left(q^0\right)$.*[20]

The necessity result follows from the property that if the difference between q^1 and q^0 is too large, then it is impossible to find a test sensitive enough to the test-taking skill which preserves the marginal probabilities q^1 and q^0 and, therefore, to find a test in

[20]Where $\delta\left(q^0\right) = \frac{1 - 2\mu_T q^0 + \mu_T^2}{\mu_T}$

$\Pi(q^0, q^1)$ that induces an influential equilibrium. Sufficiency follows from the property that when the condition holds, it is possible to find a test that is sensitive enough to the test-taking skill in $\Pi(q^0, q^1)$ and, therefore, by Proposition 2, to find a test in $\Pi(q^0, q^1)$ that induces an influential equilibrium.

5.1 Incompliance with Partial Order of Informativeness

We now move to present our main results about the manager's preference relation over tests. Specifically, we show two results that illustrate how the ability to induce an influential equilibrium modifies the manager's preference relation over tests so that it does not comply with the partial order of informativeness about the candidate's value.

The first result deals with the manager's preferences between two tests. To set the ground for the result we present two orders over the tests in Π.

Definition 8. Let $\pi, \pi' \in \Pi$. We denote $\pi \succ_i \pi'$ if and only if the test π is more informative about the candidate's value than the test π'.

Definition 9. Let $\pi, \pi' \in \Pi$. We denote $\pi \succ_m \pi'$ if and only if the manager strictly prefers the test π to the test π'.

The result is the following.

Corollary 1. *Assume that $\Pi(q^0, q^1)$ satisfies $q^1 - q^0 < \delta(q^0)$. There exist a test $\pi \in \Pi(q^0, q^1)$ and a test $\pi' \in \Pi$ such that $\pi' \succ_i \pi$ and $\pi \succ_m \pi'$.*

The argument for the result is the following. Any test in any cell $\Pi(q'^0, q'^1)$ with $q'^0 < q^0$ and $q'^1 > q^1$ is more informative about the candidate's value than any test in $\Pi(q^0, q^1)$, and each cell includes a test that is not sensitive enough to the test-taking skill and thus does not induce an influential equilibrium. Hence, the corollary is an immediate conclusion of Proposition 2 and Lemma 3.

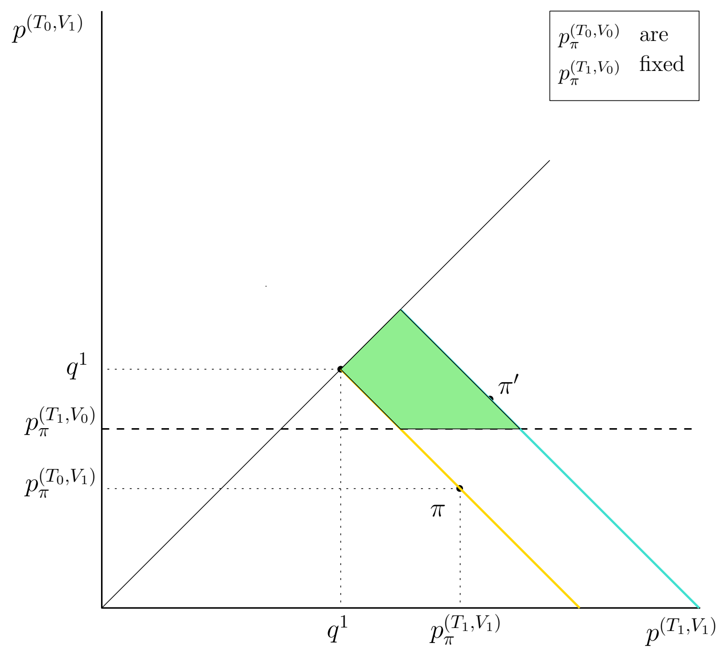

Figure 1

Figure 1 further illustrates the manager's preferences over the tests in Π. Assume that $\Pi(q^0,q^1)$ satisfies $q^1-q^0<\delta(q^0)$ and consider a test $\pi\in\Pi(q^0,q^1)$ that induces an influential equilibrium (by Lemma 3 such a test exists). Consider the set of the tests in Π that assign the success probability of π to the types that have a low value, $\left\{\dot{\pi}\in\Pi|\ p_{\dot{\pi}}^{(T_0,V_0)}=p_\pi^{(T_0,V_0)} \text{ and } p_{\dot{\pi}}^{(T_1,V_0)}=p_\pi^{(T_1,V_0)}\right\}$. This set is depicted in Figure 1 as points on the plain whose horizontal axis corresponds to type (T_1,V_1)'s success probability, $p^{(T_1,V_1)}$, and whose vertical axis corresponds to type (T_0,V_1)'s success probability, $p^{(T_0,V_1)}$. Since the success probability of the tests in Π increases in the attribute T, all these tests lie below the 45 degrees line. Consider all the tests that are informationally equivalent to the test π. These tests are located on the straight line that crosses the points (q^1,q^1) and $\left(p_\pi^{(T_1,V_1)},p_\pi^{(T_0,V_1)}\right)$, which is colored yellow. Now, since the test π induces an influential equilibrium it is sensitive enough to the test-taking skill, i.e., $p_\pi^{(T_0,V_1)}\leq p_\pi^{(T_1,V_0)}$, where $p_\pi^{(T_1,V_0)}$ is depicted by the dashed horizontal line. Now, the test π induces an influential equilibrium and, therefore, its effective signal is strictly more informative than it. Therefore, there exists a test π', that is more informative than the test π and that does not induce an influential equilibrium, such that the manager is indifferent between the effective signals of π and π'. In the figure, the test π' lies above the yellow line, and thus it is more informative than π. Additionally, π' lies above the dashed line, i.e., it is not sensitive enough to the test-taking skill, and therefore does not induce an influential equilibrium. The tests that are informationally equivalent to the test π' are located on the straight line that crosses the point π', which is colored in light blue. Now consider the tests that are located in the green trapezoid that lies below the 45 degrees and the blue lines and above the dashed and yellow lines. The test π' is more informative than the tests in the green trapezoid as they lie below the blue line. Moreover, all the tests in the green trapezoid do not induce influential equilibria as they lie above the dashed line. Therefore, the test π' is preferred by the manager over all the tests in the green trapezoid. Hence, also the test π is preferred by the manager over all the tests in the green trapezoid. Moreover, all the tests in the green trapezoid are more informative than the test π as they lie above the yellow line.

To demonstrate the economic implications of the above result, consider the following scenario. Assume that the manager can conduct a fixed exam and can decide on the time limit of the exam. Setting a stricter time limit makes the performance in the exam more dependent on the candidate's ability to perform under pressure and thus impairs the exam's accuracy level. The above result imply that even if the candidate's competency for the job does not depend on his ability to perform under pressure and although setting a strict time limit would impair the exam's accuracy, it may still be worthwhile for the manager to set a strict time limit for the exam because it would induce the candidate to reveal credible information in the reporting stage.

We proceed to show another result that deals with the manager's preferences between two cells in the partition $\{\Pi\left(q^0, q^1\right) | 0 < q^0 < q^1 < 1\}$. To set the ground for the result we present two orders over subsets of Π.

Definition 10. Let $A, B \subset \Pi$. We denote $A \succ_I B$ if and only if any test is A is more informative about the candidate's value than any test in B.

Definition 11. Let $A, B \subset \Pi$. We denote $A \succ_M B$ if and only if $\max\left\{U^M(\pi) \,|\pi \in A\right\} > \max\left\{U^M(\pi) \,|\pi \in B\right\}$

The order $\succ_M$ represents the manager's preference relation over the subsets of Π if she was asked to pick a subset from which she could choose her test. The following proposition shows that there exist two cells one more informative about the candidate's value than the other such that if the manager could choose her test from one of these cells she would prefer to choose it from the less informative cell.

Proposition 3. *There exist two cells* $\Pi(q^0, q^1)$ *and* $\Pi(q'^0, q'^1)$ *such that*

$$\Pi\left(q^0, q^1\right) \succ_I \Pi\left(q'^0, q'^1\right) \text{ and } \Pi\left(q'^0, q'^1\right) \succ_M \Pi\left(q^0, q^1\right).$$

A generic case of Proposition 3 is the following. Consider a cell $\Pi(q'^0, q'^1)$, which satisfies the inequality of Lemma 3, and thus includes a test π^* that induces an influential equilibrium. Moreover, the difference between its conditional probabilities, $q'^1 - q'^0$ is sufficiently close to $\delta(q'_0)$ to find a cell $\Pi(q^0, q^1) \succ_I \Pi(q'^0, q'^1)$ which satisfies $q^1 > q'^1$ and $q^0 < q'^0$ such that $\Pi(q^0, q^1)$ does not satisfy the inequality of Lemma 3 and thus does not include a test that induces an influential equilibrium. Additionally, $q^1 - q'^1$ and $q'^0 - q^0$ are sufficiently small to satisfy the property that the difference between the manager's expected payoffs from an arbitrary test in $\Pi(q^0, q^1)$ and her expected payoff if she only observes test π^*'s result is smaller than the difference between the manager's expected payoff from the effective signal of the test π^* and her expected payoff if she only observe test π^*'s result, so we get that $\Pi(q'^0, q'^1) \succ_M \Pi(q^0, q^1)$.

6 Extensions

6.1 Correlation

So far, we have assumed that the candidate's value and his test-taking skill are independent. We showed that even in this case, an influential equilibrium can arise. In this subsection, we consider the perhaps more plausible scenario where the candidate's value and his test-taking skill are positively correlated. We perform our analysis by considering the case where this correlation corresponds to a direct effect of the test-taking skill on the candidate's value. We then discuss the equivalent case where the test-taking skill is correlated stochastically with the candidate's value.

6.1.1 Multidimensional Valuation

We start by considering the case where the candidate's test-taking skill directly affects his competency for the job. For example, for some jobs, the ability to perform under pressure is a desired feature of the candidate. We consider the model of Section 2 with the exception that the candidate's value, denoted by U^C, is a convex combination of the attributes T and V, i.e., $U^C(T,V) = (1-\alpha)\cdot T + \alpha \cdot V$ with $\alpha \geq 1/2$. Note, that the value function in the main model is a special case of this formulation as it corresponds to the case where $\alpha = 1$. The following results show that the conditions for the test to induce an influential equilibrium, which appears in Section 4, extend to this more general case.

Lemma 4. *A test π induces an influential equilibrium only if it is sensitive enough to the test-taking skill.*

We now show that when the candidate's value also depends on his test-taking skill, then the sufficient condition that appears in Proposition 1 is also sufficient for the existence of an influential equilibrium as long as the effect of attribute V on the candidate's value is large enough relative to the effect of the test-taking skill, i.e., for a sufficiently large α. For some values of α new type of influential equilibrium emerges with two messages L and H such that types (T_0, V_0) and (T_0, V_1) send the message L and types (T_1, V_0) and (T_1, V_1) send the message H. That is, in this equilibrium the sender truthfully reveals the level of his test-taking skill. We call such an equilibrium a *pure equilibrium.* We first present the following definition.

Definition 12. Consider a test $\pi \in \Pi$. We say that the *informativeness of test π about attribute V is increasing in the test-taking skill* if and only if the test $\hat{\pi}(\pi, 1)$ Blackwell dominates the test $\hat{\pi}(\pi, 0)$.[21]

Proposition 4. *If a test $\pi \in \Pi$ is sensitive enough to the test-taking skill and its informativeness about attribute V is increasing in the test-taking skill, then there exists a cutoff $1/2 < \underline{\alpha}_\pi < 1$ such that an influential equilibrium exists for every $\underline{\alpha}_\pi \leq \alpha$. Moreover, there exists an interval $[\underline{\alpha}_\pi, \overline{\alpha}_\pi]$ such that a pure equilibrium exists if and only if $\alpha \in [\underline{\alpha}_\pi, \overline{\alpha}_\pi]$.*

The equilibria we construct to show that an influential equilibrium exists if the above condition holds consist of two messages, L and H, where types (T_0, V_0) and (T_0, V_1) send the message L, type (T_1, V_1) sends the message H, and type (T_1, V_0) is mixing between L and H. As we explain in Section 3, the reason for the existence of such equilibria is a tradeoff between the more positive overall inference of the test's result that accompanies the message L and a higher ex-ante expected value that accompanies the message H. When $\underline{\alpha}_\pi \leq \alpha \leq \overline{\alpha}_\pi$ types (T_0, V_0) and (T_0, V_1) prefer the overall more positive inference of the test that accompanies message L while types (T_1, V_0) and (T_1, V_1) prefer the higher expected value that accompanies the message H. Therefore, a pure equilibrium exists. When $\underline{\alpha}_\pi < \alpha$, for an influential equilibrium to exist, type (T_1, V_0) must send the message L with a positive probability to decrease the ex-ante expected value of message L.

6.1.2 Stochastic Correlation

We now discuss the case where the candidate's test-taking skills are stochastically positively correlated with skills that affect his value. For example, the ability to implement test-taking strategies, such as managing time efficiently in the test, is related to the ability to work in an organized fashion. The case where such a correlation exists is essentially equivalent to the case of multidimensional valuation in the sense that admitting to having a low test-taking skill associates the candidate with an ex-ante lower expected value. As in the multidimensional valuation case, this feature enables the

[21] Note that the condition of the definition is the condition that appears in Definition 6 . However, since in the model we consider in this subsection both attributes are payoff relevant we changed the label of the condition to fit the model.

existence of a pure equilibrium in which the candidate truthfully reveals the level of his test-taking skill. Specifically, it is possible to show that the conditions of Proposition 1 ensure that a pure equilibrium exists for a sufficiently moderate level of correlation.

6.2 Non-Linear Candidate's Payoff Function

In the main model, the candidate's payoff function $W^C(\mu_V)$ is assumed to be linear. In this subsection, we discuss the case where $W^C(\mu_V)$ is not linear. Specifically, we discuss the cases where $W^C(\mu_V)$ is concave and where it is convex. The cases of concavity and convexity are motivated by the following scenario. Assume that there are other n candidates who are competing with our candidate over k open positions, where $n \geq k$. Moreover, assume that the manager will obtain information about these candidates' values, such that the distribution of the manager's posterior means about each candidate's value is distributed in $[0, 1]$ according to some probability distribution F with density $f > 0$. Assume that the candidate's payoff is equal to the probability that he would be hired, i.e., to the probability that μ_V is among the k highest posterior means of the candidates. For a given k, as the number of candidates, n, increases, the marginal value of this probability increases in μ_V and the candidate's payoff becomes more convex. That is, the case where the candidate's payoff function is concave corresponds to the scenario where the competition for the position is relatively mild, while the case where the candidate's payoff function is convex corresponds to the scenario where the competition for the position is relatively fierce.

We start by considering the model of Section 2 with the exception that the candidate's payoff $W^C(\mu_V)$ is concave. As the following proposition shows, the conditions of Proposition 1, that guarantee the existence of an influential equilibrium when the candidate's payoff is linear, ensure that an influential equilibrium exists in the more general case where the candidate's payoff function can be strictly concave.

Proposition 1*. *If a test $\pi \in \Pi$ is sensitive enough to the test-taking skill and its informativeness about the candidate's value is increasing in the test-taking skill, then it induces an influential equilibrium.*

The argument in the proof of Proposition 1 can be generalized to prove Proposition 1^* in the following way. The concavity of $W^C(\mu_V)$ enables us to apply the same construction of an equilibrium strategy that we carried out in Section 3 and in the proof of Proposition 1. To see this, consider the strategy presented in Section 3 as a function of type (T_1, V_0)'s level of mixing b. Assume that $b = 0$. Since the test's informativeness about the candidate's value increases in the test-taking skill, the lottery that type (T_1, V_0) faces under the message L second-order stochastically dominates the lottery it faces under the message H. Therefore, type (T_1, V_0) strictly prefer the message L over the message H. When $b = 1$, the message H identifies the candidate with the highest type. Therefore, type (T_1, V_0) strictly prefers the message H to the message L. These features ensure a level of mixing b^* under which type (T_1, V_0) is indifferent between the two messages exists. The property that the informativeness about the candidate's value is increasing in the test-taking skill ensures that under the strategy that corresponds to b^* the single crossing property holds. Therefore, the strategy that corresponds to b^* is an equilibrium.

When the candidate's payoff function $W^C(\mu_V)$ is convex the conditions of Proposition 1 do not guarantee the existence of an influential equilibrium. To see this, consider again the strategy presented in Section 3 as a function of type (T_1, V_0)'s level of mixing b. Assume that $b = 0$. Since the test's informativeness about the candidate's value increases in the test-taking skill, the expected value of the posterior belief that follows success and the message H, $\mu_V^\pi(s, H)$, is strictly greater than the expected value of the posterior belief that follows success and the message L, $\mu_V^\pi(s, L)$. When $W^c(\mu_V)$ is convex enough, then although type (T_0, V_1)'s mean value is greater under L than under H, its expected payoff is greater under H than under L. Therefore, for some level of convexity, type (T_0, V_1)'s would prefer H to L under the strategy that corresponds to $b = 0$. This implies that type (T_0, V_1)'s would prefer H to L under all the strategies that correspond to $b \geq 0$, which implies that none of these strategies are an equilibrium. Moreover, this property imply that no separating strategy can be an equilibrium.

From an economic perspective, the above results show that the manager's possibility to extract information via cheap talk in a job recruitment process depends on the level of competition for the position. The manager can obtain information via cheap talk when the competition is sufficiently mild. When the competition becomes fierce, then at some point, we get that under any separating strategy, each of the candidate's types would want to send the message that includes the highest posterior belief. This property terminates the possibility of an influential cheap talk equilibrium in which information is transmitted from the candidate to the manager.

7 Robustness

In this paper, we argue that pre-employment tests' inherent dependency on test-taking skills can facilitate a channel for meaningful information transmission from the candidate if a reporting stage, where the candidate reports his prospects of succeeding in the test, takes place before the test. We thus recommend that recruiters would consider including such a reporting stage as part of their evaluation process. This channel arises when a test is sufficiently dependent on the test-taking skill and when the test becomes more informative about the candidate's value as the test-taking skill increases. To illustrate this economic result, we used a somewhat stylized model, where each of the state's attributes is binary, and the manager's test is binary. This model facilitates clean and simple characterizations. We believe, however, that the paper's insights are relevant for more general environments. Specifically, in this section, we present two examples of settings that satisfy parallel requirements to those that appear in Section 4, one where the manager's test is not binary and the other where the state's attributes are not binary, in which an influential equilibrium exists.

7.1 Non-Binary Tests

We now show that an influential equilibrium can arise when the manager's test is not binary. For example, a pure equilibrium arises in the following setting: The prior belief is $\mu^0(T_0, V_0) = \mu^0(T_1, V_1) = 0.3$ and $\mu^0(T_0, V_1) = \mu^0(T_1, V_0) = 0.2$. The test has three grades $\{f, m, s\}$ and its grades distribution as a function of the state is summarized by the following matrix:

V_1	$(0.4, 0.2, 0.4)$	$(0, 0.15, 0.85)$
V_0	$(0.4, 0.3, 0.3)$	$(0.15, 0.5, 0.35)$
	T_0	T_1

This test satisfies monotonicity in each attribute in the sense of first-order stochastic dominance. This property is parallel to the property that we imposed in the main model that the test's success probability would be increasing in each attribute. The property that the test is sensitive enough to the test-taking skill is manifested by the property that type (T_0, V_1)'s grades distribution does not first-order stochastically dominate type (T_1, V_0)'s grades distribution. As mentioned in Remark 1, the above example also illustrates that the condition that type (T_1, V_0)'s grades distribution weakly dominates type (T_0, V_1)'s grades distribution is not necessary for the test to induce an influential equilibrium as type (T_1, V_0)'s grades distribution does not first-order stochastically dominate type (T_0, V_1)'s distribution.

7.2 Non-Binary Candidate's Value

We now illustrate that our results extend to environments where the candidate's value and test-taking skill take more than two possible levels. Consider an environment where $T \times V = \{0, 1, 2\}^2$, where the states are uniformly distributed, and where the candidate's payoff is equal to the manager's expected value of attribute V given her belief. Consider the following binary test whose probability of success as a function of the state is summarized by the following matrix:

V_2	a	1	1
V_1	a	0.5	1
V_0	a	a	0.5
	T_0	T_1	T_2

Consider the strategy where types (T_0, V_0), (T_0, V_1), (T_0, V_2) and (T_1, V_0) send the message L and the rest of the types send the message H. The manager's posterior belief under the message L is equal to $\frac{3}{4}$ for any grade. The manager's posterior belief under the message h and a grade success is equal to $\frac{11}{8}$ and her posterior belief under the message H and a grade failure is equal to $\frac{1}{2}$. Assume that $a \leq \frac{2}{7}$. The test is monotone in the state. Additionally, when the manager's beliefs are derived from the candidate's strategy, the candidate's strategy maximizes the expected payoff of each type. That is, the above strategy and its corresponding beliefs consist an equilibrium.

8 Conclusion

Many irrelevant factors, such as test anxiety and implementing test-taking strategies, affect examinees' performance levels in tests in a non-proportionate way relative to their effect on the value that a test comes to examine. Hence, these factors adversely affect tests' validity. In this paper, we showed that despite the direct negative effect of these factors on tests' validity, they could open a channel for information transmission via cheap talk. Evaluators can exploit this effect to obtain more information via cheap talk in the evaluation process by adding a reporting stage before the test takes place.

Appendix

Proof of Lemma 2 and Lemma 4

Since our main model is a special case of the model we consider in Subsection 7.1 Lemma 4 implies Lemma 2 we thus prove Lemma 4.

Proof of Lemma 4

Given $\pi \in \Pi$, we denote the mean of the candidate's value, U^C, given the posterior belief $\mu^\pi(g, m)$, by $u^\pi(g, m)$. We start with presenting the following lemma:

Lemma 5. *Consider a test $\pi \in \Pi$. For every two messages m and m' that are sent in an influential equilibrium we have either*

$$\max\left\{p_\pi^\omega \middle| \, \omega \in \sigma^{-1}(m)\right\} \leq \min\left\{p_\pi^\omega \middle| \, \omega \in \sigma^{-1}(m')\right\}$$

or

$$\max\left\{p_\pi^\omega \middle| \, \omega \in \sigma^{-1}(m')\right\} \leq \min\left\{p_\pi^\omega \middle| \, \omega \in \sigma^{-1}(m)\right\}$$

Proof. In an influential equilibrium, for any two messages, $m \neq m'$, we have that either $u^\pi(s, m) - u^\pi(s, m') - u^\pi(f, m) + u^\pi(f, m') \neq 0$ or $u^\pi(f, m) - u^\pi(f, m') \neq 0$ or both. Otherwise, we get that $u^\pi(s, m) = u^\pi(s, m')$ and $u^\pi(f, m) = u^\pi(f, m')$ in contradiction to the definition of an influential equilibrium. Assume that, in equilibrium, there exist ω and ω' that send m and $\tilde{\omega}$ an $\tilde{\omega}'$ that send m' such that $p_\pi^{\tilde{\omega}} < p_\pi^\omega$ and $p_\pi^{\tilde{\omega}'} > p_\pi^{\omega'}$; then incentive compatibility implies

$$p_\pi^\omega \cdot [u^\pi(s, m) - u^\pi(s, m') - u^\pi(f, m) + u^\pi(f, m')] + u^\pi(f, m) - u^\pi(f, m') \geq 0$$

$$p_\pi^{\tilde{\omega}} \cdot [u^\pi(s, m) - u^\pi(s, m') - u^\pi(f, m) + u^\pi(f, m')] + u^\pi(f, m) - u^\pi(f, m') \leq 0$$

which implies that $u^\pi(s, m) - u^\pi(s, m') - u^\pi(f, m) + u^\pi(f, m') > 0$. Incentive compatibility also implies

$$p_\pi^{\omega'} \cdot [u^\pi(s, m) - u^\pi(s, m') - u^\pi(f, m) + u^\pi(f, m')] + u^\pi(f, m) - u^\pi(f, m') \geq 0$$

$$p_{\pi}^{\tilde{\omega}'} \cdot \left[u^{\pi}\left(s,m\right) - u^{\pi}\left(s,m'\right) - u^{\pi}\left(f,m\right) + u^{\pi}\left(f,m'\right)\right] + u^{\pi}\left(f,m\right) - u^{\pi}\left(f,m'\right) \leq 0$$

which implies that $u^{\pi}\left(s,m\right) - u^{\pi}\left(s,m'\right) - u^{\pi}\left(f,m\right) + u^{\pi}\left(f,m'\right) < 0$. A contradiction. $\square$

We now show that any test $\pi \in \Pi$ whose success probability is increasing in the candidate's value cannot induce an influential equilibrium.

Assume that $\pi \in \Pi$ is increasing in the candidate's value. Lemma 5 implies that in an influential equilibrium there are two messages m and m' for which $\max\left\{U^{C}\left(\omega\right) \mid \omega \in \sigma^{-1}(m)\right\} \leq \min\left\{U^{C}\left(\omega\right) \mid \omega \in \sigma^{-1}(m')\right\}$. For these equilibrium messages we get that $u^{\pi}\left(g,m\right) \geq u^{\pi}\left(g,m'\right)$ for every $g \in \{s,f\}$ and for at least one $g \in \{s,f\}$ we have that $u^{\pi}\left(g,m\right) > u^{\pi}\left(g,m'\right)$. Therefore, each of the candidate's type prefers to send the message m over m' in contradiction to m' being a message in the support of the candidate's equilibrium strategy.

Proof of Proposition 1 and Proposition 4

Since our main model is a special case of the model we consider in Subsection 7.1 Proposition 4 implies Proposition 1 we thus prove Proposition 4.

Proof of Proposition 4

Assume the manager's pure equilibrium belief, i.e., (T_1, V_0) and (T_1, V_1) send a message h and (T_0, V_0) and (T_0, V_1) send a message l. Since the test π's informativeness about attribute V is increasing in attribute T we have $\mu_V^{\pi}(s,h) > \mu_V^{\pi}(s,l)$ and $\mu_V^{\pi}(f,l) > \mu_V^{\pi}(f,h)$.

Consider the means of the candidate's value of the posterior beliefs $\mu^{\pi}\left(g,m\right)$ for $m \in \{h,l\}$ and $g \in \{s,f\}$ as a function of α:

$$u^{\pi}(g,h) = \alpha \cdot \mu_V^{\pi}(g,h) + 1 - \alpha$$

and

$$u^{\pi}(g,l) = \alpha \cdot \mu_V^{\pi}(g,l)$$

Assume $\alpha = 1/2$. For every p_π^ω we have

$$p_\pi^\omega \cdot [\mu_V^\pi(s,h) - \mu_V^\pi(s,l)] + (1 - p_\pi^\omega) \cdot [\mu_V^\pi(f,h) - \mu_V^\pi(f,l)] + 1 > 0$$

this is because

$$\mu_V^\pi(s,h) - \mu_V^\pi(s,l) > 0$$

and since

$$\mu_V^\pi(f,l) - \mu_V^\pi(f,h) < 1$$

that is, when $\alpha = 1/2$, all types prefer h to l.

Assume that $\alpha = 1$. We show that (T_0, V_1) and (T_1, V_0) prefer l to h. Since

$$\mu_V^\pi(s,h) > \mu_V^\pi(s,l)$$

and

$$\mu_V^\pi(f,l) > \mu_V^\pi(f,h)$$

we get that if type (T_0, V_1) prefers l to h then so does type (T_1, V_0). Therefore, it is sufficient to show that (T_1, V_0) prefers l to h. To see this, note that the mean of attribute V given both messages, l and h, is μ_V^0. Since (T_1, V_0) is the type with the lowest probability of success in the set $\sigma_\pi^{-1}(h)$, its expected payoff has to be lower than the unconditional mean, i.e.,

$$p_\pi^{(T_1,V_0)} \cdot \mu_V^\pi(s,h) + \left(1 - p_\pi^{(T_1,V_0)}\right) \cdot \mu_V^\pi(f,h) < \mu_V^0$$

and since (T_0, V_1) is the type with the highest probability of success in the set $\sigma_\pi^{-1}(l)$, its expected payoff has to be greater that the unconditional mean, i.e.,

$$p_\pi^{(T_0,V_1)} \cdot \mu_V^\pi(s,l) + \left(1 - p_\pi^{(T_0,V_1)}\right) \cdot \mu_V^\pi(f,l) > \mu_V^0$$

since $p_\pi^{(T_1,V_0)} \geq p_\pi^{(T_0,V_1)}$ we get

$$p_\pi^{(T_1,V_0)} \cdot \mu_V^\pi(s,l) + \left(1 - p_\pi^{(T_1,V_0)}\right) \cdot \mu_V^\pi(f,l) > \mu_V^0$$

so (T_1, V_0) prefers l to h.

We now look at the difference function of type ω utility from sending h and l as a function of α:

$$\alpha \cdot [p_\pi^\omega \cdot (\mu_V^\pi(s,h) - \mu_V^\pi(s,l) - \mu_V^\pi(f,h) + \mu_V^\pi(f,l)) + \mu_V^\pi(f,h) - \mu_V^\pi(f,l) - 1] + 1$$

This is a linear decreasing function in α which for type (T_0, V_1) ((T_1, V_0)) is positive at $\alpha = 1/2$ and negative at $\alpha = 1$. Therefore, there exists one point $\underline{\alpha}_\pi$ ($\overline{\alpha}_\pi$) where the difference function of type (T_0, V_1) ((T_1, V_0)) equals 0. So at this point, type (T_0, V_1) ((T_1, V_0)) is indifferent between l and h. Note that for any α, we have $u^\pi(s,h) > u^\pi(s,l)$ which implies that whenever type (T_0, V_1) ((T_1, V_0)) difference function equals 0 we also have $u^\pi(f,h) < u^\pi(f,l)$. These inequalities implies single crossing, i.e., that if some type prefers l (h) to h (l) than so does any other type with a lower (higher) probability of success. These inequalities also imply that, $\underline{\alpha}_\pi < \overline{\alpha}_\pi$. We therefore get that for any $\alpha \in (\underline{\alpha}_\pi, \overline{\alpha}_\pi)$, type (T_1, V_0) strictly prefers h to l and (T_0, V_1) strictly prefers l to h and single-crossing holds. We conclude that for every $\alpha \in [\underline{\alpha}_\pi, \overline{\alpha}_\pi]$ there exists a pure equilibrium. For every $\alpha < \underline{\alpha}_\pi$ ($\alpha > \overline{\alpha}_\pi$), types (T_0, V_1) and (T_1, V_0) prefer h (l) to l (h) so a pure equilibrium doesn't exist.

For every $\alpha > \overline{\alpha}_\pi$ there exists an influential equilibrium in which types (T_0, V_0) and (T_0, V_1) sends l, type (T_1, V_1) sends h, type (T_1, V_0) is mixing between h and l. We introduce a parameter $0 \leq b \leq 1$ that corresponds to the proportion at which (T_1, V_0) sends the message h. Let $\alpha > \overline{\alpha}_\pi$. When $b = 0$, type (T_1, V_0) strictly prefers l to h. When $b = 1$, the message h identifies type (T_1, V_1) with certainty. Thus, we get that given that $b = 1$, $u^\pi(s,h) = u^\pi(f,h) = 1$, i.e., each posterior mean coincides with the highest possible value. Thus, type (T_1, V_0) prefers h to l. The difference in type (T_1, V_0)'s expected payoff between sending h and l is continuous in b. Additionally, as we showed, when $b=0$ it is negative and when $b=1$ it is positive. Therefore, we get that there exists a point $b(\alpha)$ at which type (T_1,V_0) is indifferent between h and l. To prove that this level of mixing corresponds to an equilibrium it is left to show that single-crossing holds. For every $b\in[0,1]$ we have that the posterior means of the beliefs given success for the different messages are $u^{\pi}(s,h)=\alpha\cdot\mu_V^{\pi}(s,h)+1-\alpha$ and $u^{\pi}(s,l)=\alpha\cdot\mu_V^{\pi}(s,l)+(1-\alpha)\cdot g(b)$ for $g(b)<1$. Since $\mu_V^{\pi}(s,h)>\mu_V^{\pi}(s,l)$ when the belief corresponds to $b=0$ and since $\mu_V^{\pi}(s,h)$ is increasing in b and $\mu_V^{\pi}(s,l)$ is decreasing in b, we get that $u^{\pi}(s,h)>u^{\pi}(s,l)$ for every b. Since at the point $b(\alpha)$ type (T_1,V_0) is indifferent between the messages we get that for this level of mixing, $u^{\pi}(s,h)>u^{\pi}(s,l)$ implies $u^{\pi}(f,l)>u^{\pi}(f,h)$. That is, we get that the single crossing holds.

Proof of Proposition 2

We first prove that (1) if and only if (2). The first direction follows from Lemma 2 which implies that if there does not exist a test in $\Pi\left(q^0,q^1\right)$ that is sensitive enough to the test-taking skill, then there does not exist a test in $\Pi\left(q^0,q^1\right)$ that induces an influential equilibrium.

We now show that if there exists a test in $\Pi\left(q^0,q^1\right)$ that is sensitive enough to the test-taking skill then there exists a test in $\Pi\left(q^0,q^1\right)$ that induces an influential equilibrium. We start by showing the property that a cell $\Pi\left(q^0,q^1\right)$ does not include a test that is sensitive enough to the test-taking skill if $q^0\le\frac{\mu_T\cdot q^1-(\mu_T)^2}{(1-\mu_T)}$. We denote by $\tilde{\pi}\in\Pi\left(q^0,q^1\right)$ that test in $\Pi\left(q^0,q^1\right)$ for which $p_{\tilde{\pi}}^{(T_0,V_0)}=\varepsilon$ and $p_{\tilde{\pi}}^{(T_1,V_1)}=1-\varepsilon$. Under this test we get that $p_{\tilde{\pi}}^{(T_0,V_1)}=\frac{q_1-\mu_T(1-\varepsilon)}{1-\mu_T}$ and $p_{\tilde{\pi}}^{(T_1,V_0)}=\frac{q^0-(1-\mu_T)\cdot\varepsilon}{\mu_T}$. The fact that $q^0\le\frac{\mu_T\cdot q^1-(\mu_T)^2}{(1-\mu_T)}$ implies that even when $\varepsilon\to0$ we get that $p_{\tilde{\pi}}^{(T_0,V_1)}>p_{\tilde{\pi}}^{(T_1,V_0)}$. Therefore we get that for every test $\pi\in\Pi\left(q^0,q^1\right)$ we have that $p_{\pi}^{(T_0,V_1)}>p_{\pi}^{(T_1,V_0)}$, i.e., there does not exist a test in $\Pi\left(q^0,q^1\right)$ that is sensitive enough to the test-taking skill

Assume that the cell $\Pi\left(q^0,q^1\right)$ satisfies, $q^0>\frac{\mu_T\cdot q^1-(\mu_T)^2}{(1-\mu_T)}$. We will construct a test in $\Pi\left(q^0,q^1\right)$ that induces an influential cheap talk. Assume that $\frac{\mu_T\cdot q^1-(\mu_T)^2}{(1-\mu_T)}<q^0\le\frac{q^1-\mu_T}{1-\mu_T}$

and consider the following test $\tilde{\pi} \in \Pi\left(q^0, q^1\right)$

$$\left\{p_{\tilde{\pi}}^{(T_1,V_1)} = 1-\varepsilon;\ p_{\tilde{\pi}}^{(T_0,V_1)} = \frac{q^1-\mu_T\left(1-\varepsilon\right)}{1-\mu_T};\ p_{\tilde{\pi}}^{(T_1,V_0)} = \frac{q^1-\mu_T}{1-\mu_T};\ p_{\tilde{\pi}}^{(T_0,V_0)} = \frac{q^0-\mu_T\left(\frac{q^1-\mu_T}{1-\mu_T}\right)}{1-\mu_T}\right\}$$

We now show that for a small enough ε, the test that corresponds to ε induces an influential equilibrium. For a small enough ε this test is sensitive enough to the test-taking skill. Therefore, by the same construction that appears in Section 3 and in the proof of Proposition 1 (and Proposition 4) we can find a strategy with two messages, l and h, where $\sigma_{\tilde{\pi}}^{-1}\left(l\right) = \left\{\left(T_0,V_0\right),\left(T_0,V_1\right),\left(T_1,V_0\right)\right\}$ and $\sigma_{\tilde{\pi}'}^{-1}\left(h\right) = \left\{\left(T_1,V_0\right),\left(T_1,V_1\right)\right\}$ and $\sigma_{\tilde{\pi}}\left(l|\left(T_1,V_0\right)\right) = b^*\left(\tilde{\pi}\right)$, such that given the manager's beliefs that correspond to this strategy, type $(1,0)$ is indifferent between the messages. Moreover, given the interim beliefs $\left(\mu_{\tilde{\pi}}(m)\right)_{m\in\{l,h\}}$ that are implied by the level of mixing $b^*\left(\tilde{\pi}\right)$ we get that $\mu_V^{\tilde{\pi}}(f,l) > \mu_V^{\tilde{\pi}}(f,h)$, where the inequality follows from the fact that $\mu_V^{\tilde{\pi}}(f,h) \underset{\varepsilon\to 0}{\longrightarrow} 0$ as type (T_1,V_0)'s probability of failure is fixed and $p_{\tilde{\pi}}^{(T_1,V_1)} \underset{\varepsilon\to 0}{\longrightarrow} 1$, while $\mu_V^{\tilde{\pi}}(f,l)$ is fixed and strictly greater than 0. Since $\mu_V^{\tilde{\pi}}(f,l) > \mu_V^{\tilde{\pi}}(f,h)$ and since type (T_1,V_0) is indifferent between the messages we get that $\mu_V^{\tilde{\pi}}(s,l) < \mu_V^{\tilde{\pi}}(s,h)$, i.e., the single crossing property, which ensures that if some type prefers l (h) to h (l) than so does any other type with a lower (higher) probability of success, holds. Hence, the above strategy is an equilibrium.

Assume that $\frac{q^1-\mu_T}{1-\mu_T} < q^0 < q^1$ and consider the following test $\tilde{\pi} \in \Pi\left(q^0, q^1\right)$

$$\left\{p_{\tilde{\pi}}^{(T_1,V_1)} = \frac{q^1-\left(1-\mu_T\right)q^0}{\mu_T};\ p_{\tilde{\pi}}^{(T_0,V_1)} = q^0;\ p_{\tilde{\pi}}^{(T_1,V_0)} = q^0;\ p_{\tilde{\pi}}^{(T_0,V_0)} = q^0\right\}$$

this test is sensitive enough to the test-taking skill. Therefore, by the same construction that appears in Section 3 and in the proof of Proposition 1 (and Proposition 4) we can find a strategy with two messages, l and h, where $\sigma_{\tilde{\pi}}^{-1}\left(l\right) = \left\{\left(T_0,V_0\right),\left(T_0,V_1\right),\left(T_1,V_0\right)\right\}$ and $\sigma_{\tilde{\pi}'}^{-1}\left(h\right) = \left\{\left(T_1,V_0\right),\left(T_1,V_1\right)\right\}$ and $\sigma_{\tilde{\pi}}\left(l|\left(T_1,V_0\right)\right) = b^*\left(\tilde{\pi}\right)$, such that given the manager's belief given this strategy type (T_1,V_0) is indifferent between the messages. To show that single crossing holds note that under the message l the test is not informative, i.e., $\mu_V^{\tilde{\pi}}(s,l) = \mu_V^{\tilde{\pi}}(f,l)$ while under the message h the test is informative, i.e., $\mu_V^{\tilde{\pi}}(s,h) >$

$\mu_V^{\tilde{\pi}}(f,h)$. The fact that type (T_1, V_0) is indifferent between the two messages implies that $\mu_V^{\tilde{\pi}}(f,l) > \mu_V^{\tilde{\pi}}(f,h)$ and that $\mu_V^{\tilde{\pi}}(s,l) < \mu_V^{\tilde{\pi}}(s,h)$. Hence, the above strategy is an equilibrium.

We now prove that (2) if and only if (3). If $\Pi(q^0, q^1)$ does not include a test that induces an influential equilibrium, then, the manager is indifferent between all the tests in $\Pi(q^0, q^1)$. This result follows from the fact that all the tests in $\Pi(q^0, q^1)$ have the same marginal probabilities of success conditional on $V = V_0$ and $V = V_1$, so they provide the same expected payoff for the manager. Since the manager is indifferent between all the tests in $\Pi(q^0, q^1)$ she does not strictly prefer to learn about the irrelevant attribute.

Assume that $\Pi(q^0, q^1)$ includes a test π that induces an influential equilibrium. The influential equilibrium provides the manager with additional information about attribute Y in the form of the signal the corresponds to the candidate's equilibrium strategy. Therefore, since the manager benefits from any additional information about V as captured by the property that $W^M(\mu_V)$ is strictly convex, we obtain that the manager's expected payoff from the effective signal of the test π is strictly greater than her expected payoff if she only observes the test π's result. Since in the absence of cheap talk, the manager is indifferent between all the tests in $\Pi(q^0, q^1)$ we get that $\text{argmax}_{\pi \in \Pi(q^0,q^1)} U^M(\pi)$ include only tests that induce influential equilibria and any test π' that induces an influential equilibrium satisfies that $\tilde{\pi}(\pi', 1) \neq \tilde{\pi}(\pi', 0)$.

Proof of Lemma 3

In the proof of Proposition 2 we showed that a cell $\Pi(q^0, q^1)$ includes a test that is sensitive enough to the test-taking skill and thus a test that induces an influential equilibrium if and only if $q^0 > \frac{\mu_T \cdot q^1 - (\mu_T)^2}{(1-\mu_T)}$ which is equivalent to $q^1 - q^0 < \frac{1-2\mu_T q^0 + \mu_T^2}{\mu_T} \equiv \delta(q^0)$.

Proof of Proposition 3

To prove the proposition we abstract for a moment from our assumption that $p_{\pi}^{\omega} \in (0,1)$ for every $\omega \in \Omega$ and allow tests to have $p_{\pi}^{\omega} \in \{0,1\}$. For each cell $\Pi(q^0, q^1)$ which satisfies $\frac{\mu_T \cdot q^1 - (\mu_T)^2}{(1-\mu_T)} \leq q^0 \leq \frac{q^1 - \mu_T}{1-\mu_T}$ we consider the following test $\tilde{\pi} \in \Pi(q^0, q^1)$

$$\left\{ p_{\tilde{\pi}}^{(T_1,V_1)} = 1;\ p_{\tilde{\pi}}^{(T_0,V_1)} = \frac{q_1 - \mu_T}{1-\mu_T};\ p_{\tilde{\pi}}^{(T_1,V_0)} = \frac{q^1 - \mu_T}{1-\mu_T};\ p_{\tilde{\pi}}^{(T_0,V_0)} = \frac{q^0 - \mu_T\left(\frac{q^1-\mu_T}{1-\mu_T}\right)}{1-\mu_T} \right\}$$

For the same argument that we presented in the proof of Proposition 2 (where we presented a similar test only with a parameter ε), each such test induces an influential equilibrium. Therefore, its effective signal provides the manager with a strictly greater expected payoff than in the case where the manager only observes the test's result. We now define a function $\tilde{V}^M(q^0, q^1)$ as follows: whenever $\frac{\mu_T \cdot q^1 - (\mu_T)^2}{(1-\mu_T)} \leq q^0 \leq \frac{q^1 - \mu_T}{1-\mu_T}$ the function $\tilde{V}^M(q^0, q^1)$ value is the expected payoff of the manager from the effective signal of the test $\tilde{\pi} \in \Pi(q^0, q^1)$; for $0 < q^0 < \frac{\mu_T \cdot q^1 - (\mu_T)^2}{(1-\mu_T)}$ the function value is the expected payoff of the manager from the signal of some test $\pi' \in \Pi(q^0, q^1)$. Now, fix $q^1 = \tilde{q}^1$ and look at the function $\tilde{V}^M(q^0, \tilde{q}^1)$. For $\frac{\mu_T \cdot \tilde{q}^1 - (\mu_T)^2}{(1-\mu_T)} < q^0 \leq \frac{\tilde{q}^1 - \mu_T}{1-\mu_T}$ it is continuous, at the point $\frac{\mu_T \cdot \tilde{q}^1 - (\mu_T)^2}{(1-\mu_T)} = \tilde{q}^0$ it is not continuous as $q^0 \to \tilde{q}^0$ from below, and for $q^0 < \frac{\mu_T \cdot q^1 - (\mu_T)^2}{(1-\mu_T)}$ it is continuous. These properties imply that there exist $\delta > 0$ and $q'^0 < \frac{\mu_T \cdot \tilde{q}^1 - (\mu_T)^2}{(1-\mu_T)}$ and $q''^0 > \frac{\mu_T \cdot \tilde{q}^1 - (\mu_T)^2}{(1-\mu_T)}$ such that $\tilde{V}^M(q''^0, \tilde{q}^1) - \tilde{V}^M(q'^0, \tilde{q}^1) > \delta$. Now in the cell $\Pi(q''^0, \tilde{q}^1)$ we can find a test with $\varepsilon > 0$ sufficiently small such that the test $\tilde{\pi}_\varepsilon \in \Pi(q''^0, \tilde{q}^1)$, which satisfies $p_{\tilde{\pi}_\varepsilon}^{\omega} \in (0,1)$ for every $\omega \in \Omega$, and that is defined to be

$$\left\{ p_{\tilde{\pi}_\varepsilon}^{(T_1,V_1)} = 1-\varepsilon;\ p_{\tilde{\pi}_\varepsilon}^{(T_0,V_1)} = \frac{\tilde{q}^1 - \mu_T(1-\varepsilon)}{1-\mu_T};\ p_{\tilde{\pi}_\varepsilon}^{(T_1,V_0)} = \frac{\tilde{q}^1 - \mu_T}{1-\mu_T};\ p_{\tilde{\pi}_\varepsilon}^{(T_0,V_0)} = \frac{q''^0 - \mu_T\left(\frac{\tilde{q}^1-\mu_T}{1-\mu_T}\right)}{1-\mu_T} \right\},$$

induces an influential equilibrium, and the expected payoff its effective signal provides the manager is greater than $\tilde{V}^M(q''^0, \tilde{q}^1) - \delta$. Therefore, we get that $\Pi(q''^0, \tilde{q}^1) \succ_M \Pi(q'^0, \tilde{q}^1)$ and $\Pi(q'^0, \tilde{q}^1) \succ_I \Pi(q''^0, \tilde{q}^1)$.

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