Optimal Regulation of Credit Lines*

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Job Market Paper
This version: October 17, 2021

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Abstract

During the Global Financial Crisis, concerns related to the ability of banks to honor committed loans led to a spike in credit line drawdowns (credit line runs), as reported by Ivashina and Scharfstein (2010). In response to large liquidity risk exposure during distress periods, the Basel Committee on Banking Supervision introduced a framework for liquidity risk regulation as part of the post-crisis regulatory reforms. This paper presents a model of credit lines in which runs can emerge. In the model, firms face shocks that require funding to avert liquidation. Due to a pecuniary externality on their liquidation value, atomistic banks hold inadequate levels of pre-arranged liquidity compared to a constrained efficient allocation chosen by a social planner. The paper shows that a regulator can implement the social planner's solution by means of a liquidity ratio. Though credit lines become more expensive with such regulation, social welfare increases due to (i) more lending is channeled to firms in need of funds during distress periods and (ii) a decrease in the probability of a credit line run.

Keywords: credit lines, credit line runs, liquidity risk, liquidity regulation.

JEL Codes: G01, G21, G28, G32.

*I am grateful to my advisor Rafael Repullo for his support and supervision on this project, as well as to Javier Suarez and David Martinez-Miera. I thank participants at the CEMFI Banking and Finance Reading Group, the CEMFI Ph.D. Workshop, and the 2020 European Winter Meeting of the Econometric Society for insightful feedback. Funding from Spain's Ministerio de Economía, Industria y Competitividad (BES−2017-082176), María de Maeztu Programme for Units of Excellence in R&D (MDM−2016-0684), and from the Santander Research Chair at CEMFI is gratefully acknowledged.

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1. Introduction

At the onset of a financial crisis, banks may experience difficulties to meet their loan commitments, such as drawdowns of credit lines, due to liquidity pressure on both sides of their balance sheet. On the one hand, credit line usage increases following adverse macroeconomic shocks (Jiménez et al., 2009; Mian and Santos, 2011; Greenwald et al., 2020; Kapan and Minoiu, 2020).[1] On the other hand, investors may flight to quality withdrawing funds from banks (Ashcraft et al., 2010; Acharya and Mora, 2015). Thus, stable sources of funding are important to maintain bank lending during crises (Ivashina and Scharfstein, 2010; Cornett et al., 2010; Acharya and Mora, 2015). In response to the asset-side liquidity pressure that arises from credit lines, banks can reduce their exposure to them by waiving fewer covenants violations or not renewing existing credit lines (Acharya et al., 2014; Huang, 2010; Campello et al., 2011).[2] Losing access to external funds during a crisis has negative effects on firms, leading to investment spending cuts or cancellations (Campello et al., 2010; Almeida et al., 2012). Consequently, due to concerns about future credit restrictions, firms may run on their credit lines during a crisis (Ivashina and Scharfstein, 2010; Ippolito et al., 2016). Overall, liquidity risk that arises from unused credit lines can have important real effects during episodes of financial turmoil.

To mitigate liquidity risk during distress periods, the Basel Committee on Banking Supervision (BCBS) introduced a framework for liquidity risk regulation (BCBS, 2013, 2014). In such framework, two liquidity standards were proposed: the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR). The LCR requires banks to hold a minimum of liquid assets to meet cash outflows over a 30 calendar day liquidity stress scenario. For the case of credit lines, banks are required to hold liquid assets between 5% and 30%

[1] Recent papers show that a healthy banking sector was able to accommodate the sudden and sizable increase in drawdowns that followed the COVID-19 outbreak. However, banks that were more exposed to this sudden increase in drawdowns tightened their lending standards afterwards (Greenwald et al., 2020; Kapan and Minoiu, 2020).

[2] Credit lines incorporate financial covenants that, following a violation, allow lenders to restrict further access to funds.

of the undrawn credit lines.[3] The NSFR requires banks to fund their long-term assets with more stable funding (e.g., long-term debt or equity). Such regulation requires banks to fund the undrawn credit lines with at least 5% of stable funding.

Despite the large share of bank lending that comes from credit lines, their specificities have not been analyzed in the bank regulation literature.[4] This paper's novel contribution is to provide a rationale for the regulation of credit lines in a contract-theoretic model of contingent bank lending.

The paper proposes a simple model of credit lines in which their regulation can be justified from a normative perspective. In the model, firms sign credit line contracts with banks to finance their contingent liquidity needs. In case a liquidity need is not covered, firms are liquidated at fire sale prices and these liquidations contribute to depress the liquidation value of other firms. Credit lines require firms to pay the bank a fee if funds are not used and an interest rate if funds are used. Banks finance drawdowns with pre-arranged and ex-post funding. Pre-arranged funding (e.g., long-term debt or equity) is junior to ex-post funding and it helps to sustain lending in high liquidity need states. Credit line payments and pre-arranged funding decisions are determined by competition among atomistic banks.

Demand for drawdowns may not be accommodated by banks in high liquidity need states, leading to costly liquidations. Due to the insurance nature of credit lines, banks are exposed to losses when credit line usage spikes making it harder for banks to raise additional funding.[5] In such situation, claims on pre-arranged funding can be diluted to raise new funding.[6] If that remains insufficient to meet drawdowns, loan requests are served in random order until banks cannot finance any additional loans. As a consequence, some firms in need of cash do

[3]This requirement varies depending on the type of the commitment (credit vs. liquidity facility) and client (non financial vs. financial firm).

[4]For instance, in the U.S., credit lines account for more than half of all bank credit, and undrawn credit lines are more than 40% larger than the total used credit lines and term loans combined (Greenwald et al., 2020).

[5]Note that, to have the right to drawdown, payments are made even when no drawdown happens. Thus, bank competition implies that a credit line loan has a negative net return for the lender.

[6]If dilution were not feasible, fewer funds would be raised from new investors, reducing bank's borrowing capacity and increasing liquidations.

not receive a loan leading them to liquidation.

Credit line runs can arise in very high liquidity need states. If liquidation risk is high due to a shortage of bank liquidity, firms may find it optimal to request funds earlier (run), that is, before cash is actually needed. In such way, firms try to reduce the possibility of not receiving a loan and, consequently, shutting down their operations. However, early drawdowns are harder to meet by banks, because borrowing for longer periods is more expensive. Hence, when drawdowns are concentrated at an early stage, fewer funds will be granted by banks compared to a situation where everyone waits until cash is actually needed. Thus, more liquidations are expected after a run.

Regulation of credit lines is justified due to a pecuniary externality on firms' liquidation value. In high liquidity need states, firms in need of cash that do not receive a loan will be liquidated at a value that falls with aggregate liquidations. In such case, pre-arranged funding helps to sustain lending and mitigate the impact on liquidation values. However, when choosing pre-arranged funding, competitive banks do not internalize the effect that their decisions have on aggregate liquidations and, consequently, on firms' liquidation values. As a consequence, banks hold insufficient levels of pre-arranged funding compared to a constrained efficient allocation chosen by a social planner that considers such positive effect of pre-arranged funding. The social planner's solution can be implemented with a minimum requirement on pre-arranged funding. Such requirement resembles Basel III liquidity ratios. Hence, a liquidity regulation of credit lines increases social welfare by abating the negative impact of aggregate liquidations in high liquidity need states.

Although such liquidity requirement improves social welfare, credit lines become more expensive. Providers of pre-arranged funding are compensated with high returns in low liquidity need states. Such returns are paid from larger fee income in those states. Hence, an increase of pre-arranged funding will require commitment fees to increase. For this reason, a liquidity regulation will make credit lines costlier. Nonetheless, social welfare is improved due to a reduction of liquidation risk in high liquidity need states.

The paper is related with models of credit lines that are based on insurance motives (Campbell, 1978; Boot et al., 1987; Holmström and Tirole, 1998). However, these models do not incorporate the fact that access to funds is contingent on both firm's and lender's financial health, which is an important feature of credit lines. The model in this paper is built on a modified version of Holmström and Tirole (1998), in which an observable but unverifiable aggregate state determines firms' demand for liquidity. Thus, in high liquidity need states, the mechanism in which firms in need of cash are financed by those without a cash need breaks down. In such situation, pre-arranged funding becomes important to sustain lending. The model assumes that firms' liquidity needs are not verifiable and available funds can be diverted into private consumption, as opposed to the reference model. This assumption yields a realistic payment scheme, in which using the credit line is costlier than not using it. Also, it has implications on the maximum fee that can be charged to firms, which can limit the amount of pre-arranged funding that a bank can raise. The assumption also opens the possibility of credit line runs, because funds can be requested by firms even though a cash need has not actually arisen.

The paper is also closely related with the literature that studies bank runs. This literature mainly focuses on runs that occur on the liability-side of banks' balance sheet (Diamond and Dybvig, 1983; Allen and Gale, 1998; Rochet and Vives, 2004). However, recent empirical papers show that banks can be also exposed to credit line runs (Ivashina and Scharfstein, 2010; Ippolito et al., 2016). In the model, runs are motivated by fear that banks will not be able to fully meet demand for drawdowns and, consequently, some firms in need of cash will be liquidated. Though panic-based runs can arise, the model focuses on fundamental runs, that is, runs that arise in high liquidity need states when banks find it harder to sustain lending to firms. Similar to traditional models of runs, credit line runs are also costly, because banks have to finance early drawdowns by early borrowing, which is assumed to be more expensive.

A model of credit line runs is proposed by Huang (2018). In his model, credit line clients run as a response to future liquidity tightening by banks during downturns. Though the reason behind a credit line run is similar, he takes credit line contractual terms (i.e., fees and interest rates) as exogeneous. In this paper, due to competition among atomistic banks, credit line payments and holdings of pre-arranged funds are determined such that the expected payoff of the representative firm is maximized. As a consequence, credit lines do not provide full insurance, because it would require banks to maintain large and costly pre-arranged liquidity holdings in order to meet firms' demands in high, but unlikely, liquidity need states. Although double runs (by depositors and borrowers) can exist (Ippolito et al., 2016), the model in this paper exclusively focuses on credit line runs. However, in high liquidity need states, banks find it harder to raise new funding to meet drawdowns. Though this situation is not strictly a run of depositors, banks cannot entirely rely on ex-post funding to finance drawdowns; hence, the importance of pre-arranged funding.

The paper also contributes to the literature on bank regulation. In several recent papers, the combination of non-pecuniary and pecuniary externalities with financial frictions provide a rationale for bank regulation (Perotti and Suarez, 2011; Stein, 2012; Gersbach and Rochet, 2012; Segura and Suarez, 2017; Kara and Ozsoy, 2010). To the best of my knowledge, this is the first paper that analyzes the regulation of credit lines. In the model, due to a pecuniary externality on firms' liquidation value, competitive atomistic banks hold insufficient levels of pre-arranged funding compared to the constrained efficient allocation chosen by a social planner. This finding motivates the regulation of credit lines. Additionally, a liquidity requirement similar to Basel III liquidity ratios can implement the social planner's solution.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the optimal credit line in the laissez-faire equilibrium. Section 4 characterizes the social planner's solution and discusses its implementation by means of a liquidity regulation. Finally, section 5 concludes. The proofs of the results are in the Appendix.

2. The model

Consider an economy with four dates ($t = 0, 1, 2, 3$) and a large number of three types of risk-neutral agents: *firms*, *investors*, and *banks*. Firms have ongoing investment projects that mature at $t = 3$ and might require a cash injection at $t = 1$ or 2 to prevent their liquidation. The fraction of firms in need of liquidity depends on an aggregate state, which is known at $t = 1$. Banks provide contingent funds to firms by means of credit lines, which are signed at $t = 0$. Banks, in turn, finance their lending with *pre-arranged* and *ex-post funding* from investors. Pre-arranged funding is raised at $t = 0$, in which case investors demand an expected return R_0 at $t = 3$. If necessary, additional funds can be raised from investors at $t = 1$ and $t = 2$ in exchange for returns R_1 and R_2, respectively. It is assumed that borrowing for a longer period is more expensive, that is,

$$R_0 > R_1 > R_2 = 1,$$

where R_2 has been normalized to one.

2.1. Firms

Each firm has an ongoing investment project that matures at $t = 3$. Moreover, this project faces liquidity risk; specifically, a cash injection $\ell = 1$ could be needed at $t = 1$ or 2 to avert liquidation. Let $\tau \in \{1, 2\}$ be the arrival date of the liquidity shock ℓ, which is privately revealed to firms at $t = 1$ after the aggregate state is realized. Conditional on the aggregate state, τ is independent and identically distributed as follows

$$\tau = \begin{cases} 1, & \text{with probability } \alpha_1, \\ 2, & \text{with probability } \alpha_2, \end{cases}$$

where $\alpha \equiv \alpha_1 + \alpha_2 \leq 1$; hence, no cash will be needed with probability $1 - \alpha$.

An aggregate state, that determines the fraction α of firms in need of liquidity, is realized at $t = 1$. While $\alpha_1 < 1$ is known at $t = 0$, α_2 is a random variable that is distributed according to a probability density function $f(\cdot)$ with support over the range $[0, 1 - \alpha_1]$ and whose realization is publicly revealed at $t = 1$. Moreover, $f(\cdot)$ is decreasing in α_2, that is, high liquidity risk is less likely to happen. Though α_2 is observable, it is not verifiable; hence, contracts cannot be contingent on the aggregate state.

When the cash injection is not met, the firm is liquidated, in which case a liquidation value Q is produced at $t = 3$. This liquidation value depends on the number of firms short of liquidity, which is denoted by z. Moreover, $Q(z)$ is decreasing and concave, that is, liquidation value decreases when many firms do not obtain liquidity to meet the required cash injection ℓ.

When there is no liquidity shock or the cash injection is met, the firm produces a verifiable cash flow X at $t = 3$. Thus, the only source of uncertainty in the model comes from liquidity risk. The cash flow satisfies the following assumption.

Assumption 1. $X - R_1 > Q(0)$.

Assumption 1 states that, if a cash need surges at $t = 1$, the net return of meeting ℓ is larger than the liquidation return. Similarly, it is efficient to meet ℓ if the cash need surges at $t = 2$, because $R_2 < R_1$.

Moreover, firms can pledge at most a portion Y of the cash flow X to outsiders.[7] This *pledgeable income* satisfies the next assumption.

Assumption 2. $Y < 1$.

Assumption 2 implies that firms cannot raise funds from investors at $t = 1$ or 2 to finance ℓ, because pledgeable income is insufficient to pay investors their required return. Therefore, in spite that financing ℓ is efficient, Assumption 2 prevents investors to directly finance firms'

[7] The pledgeable income Y can be obtained from an unobservable effort choice made by a firm's manager at $t = 3$ as in Holmström and Tirole (1998).

liquidity needs. As in Holmström and Tirole (1998), the existence of credit lines is justified by Assumption 2. Specifically, a fee is paid to the bank when liquidity is not needed, in exchange for the right to draw down funds when a cash need arises.

Additionally, firms have access at $t = 2$ to an investment opportunity that only generates private benefits. A unit invested in this alternative investment produces a private return $\rho < 1$ at $t = 3$. It should be noted that this investment opportunity is inefficient. Moreover, ρ satisfies the next assumption.

Assumption 3. $\rho < X - Y$ and $\rho < Y$.

Assumption 3 implies that, even if the entire pledgeable income Y is promised, a firm with access to funds prefers meeting ℓ over investing funds at the rate ρ when liquidity is needed. Nonetheless, conditional on having access to funds, a firm without a cash need could invest the excess liquidity at this rate. As it will be seen, the credit line payment scheme will prevent this to happen by requiring a payment sufficiently high when the credit line is used.

2.2. Banks and credit lines

At $t = 0$ a representative bank from a competitive banking industry offers a credit line contract with sequential service constraint to firms. This contract specifies the amount of committed funds, a payment scheme, and the amount of pre-arranged funding per committed funds held by the bank.

In such contract, firms have access to funds at will up to an amount equal to 1. In exchange, a payment that depends on the date of usage s is made by firms at $t = 3$. This payment scheme, which is denoted by B, is agreed at $t = 0$ and is given by

$$B_s = \begin{cases} B_1, & \text{if the drawdown happens at } s = 1, \\ B_2, & \text{if the drawdown happens at } s = 2, \\ B_3, & \text{if the credit line is not used.} \end{cases}$$

Payments B_1 and B_2 account for the principal plus interest; whereas, payment B_3 is a fee that is paid for having the right to draw down funds. Note that, by making the payment contingent on usage, firms' decision to draw down funds can be affected. For instance, if it is costlier to use the credit line relative to not use it, a firm without a cash need is less likely to draw down funds to invest them at the rate ρ. Also, it is important to remark that the payment scheme satisfies the pledgeable income constraint, that is, $B_s \leq Y < 1$ for $s = 1, 2, 3$. Thus, by Assumption 2, when a loan is granted, the bank will suffer a loss. However, by pooling liquidity risks, firms with a cash need are financed by those that do not suffer the liquidity shock.

The bank finances drawdowns from credit lines with pre-arranged and ex-post funding. Pre-arranged funding (e.g., long-term debt or equity), which is denoted by E, is raised from *initial investors* at $t = 0$ and stored as cash to meet future drawdowns. If E is not completely used after all drawdowns are served, excess funds can be invested at the rate R_2. If additional funds are needed, new funding can be raised at $t = 1$ and 2 from *new investors*. Furthermore, pre-arranged funding is junior to funding raised at $t = 1$ and 2, helping the bank to meet higher realizations of α. Investors are compensated from revenues generated at $t = 3$. Thus, the maximum amount of ex-post funding that the bank can raise, its *borrowing capacity*, will be determined by revenues at $t = 3$.[8]

The bank sequentially serves, in random order, loan requests from its credit lines clients until its borrowing capacity is exhausted. Recall that lending to firms is possible due to the income generated from commitment fees. Therefore, in low liquidity need states, large fee income allows the bank to obtain funds from investors to meet firms' liquidity needs. However, as α_2 increases, fee income shrinks and bank's borrowing capacity might not be sufficient to meet firms' demand for liquidity. Thus, if α_2 is high, some projects may not obtain the cash needed to avert their liquidation. In such case, firms who request and do not receive a loan are exempted from any payment.

[8] If dilution were not feasible, the bank's revenue would not be entirely pledgeable to new investors, reducing the bank's borrowing capacity and its ability to meet drawdowns.

Pre-arranged funding helps the bank to better meet demand for drawdowns, especially in high liquidity need states. Specifically, claims on E can be diluted to obtain additional funding from new investors at $t = 1$ and 2. Hence, high pre-arranged funding helps to better sustain lending to firms, which increases the insurance of the credit line contract. Nonetheless, excessive holdings of pre-arranged funding are not desirable due to their high cost (recall that $R_0 > R_1$). In return for their funds, providers of E are compensated with higher payments in low liquidity need states, that is, when many firms pay fees and do not request funds from the bank.

If large liquidations are expected at $t = 2$, firms with a cash need at $t = 2$ might find it optimal to draw down (run) at $t = 1$ to secure funding and avoid liquidation. That is to say, a *credit line run* could be described as a situation where all firms hit by ℓ request a loan at $t = 1$, independently of when the cash is needed, due to a large liquidation risk at $t = 2$. It is worth mentioning that, if the demand for drawdowns is concentrated at $t = 1$ rather than at $t = 2$, fewer loans will be granted by the bank, because financing such drawdowns are more expensive (recall that $R_1 > R_2$). As a consequence, liquidations spike after a run, so credit line runs are inefficient. Moreover, following a run, the bank suspends its credit line services, because no additional loan can be extended after $t = 1$. In such situation, firms are exempted from any payment to the bank, in which case fee income will be also lost.

Figure 1 summarizes the timing of events in the model. At $t = 0$, credit lines are signed. At the beginning of $t = 1$, aggregate and individual uncertainty are revealed. Then, firms decide whether to draw down funds at $t = 1$ or wait until $t = 2$ to decide on credit line usage. If demand for drawdowns is higher than pre-arranged funding, banks can raise new funding at $t = 1$. At $t = 2$, those who did not draw down at $t = 1$ decide on credit line usage. If additional funds are needed and borrowing capacity is not exhausted, banks can raise new funding at $t = 2$. Finally, production happens and payments are made at $t = 3$.

To simplify the exposition, the core of the analysis focuses on the case in which $\alpha_1 \approx 0$. That is to say, the fraction of firms that requires a cash injection at $t = 1$ is negligible. It

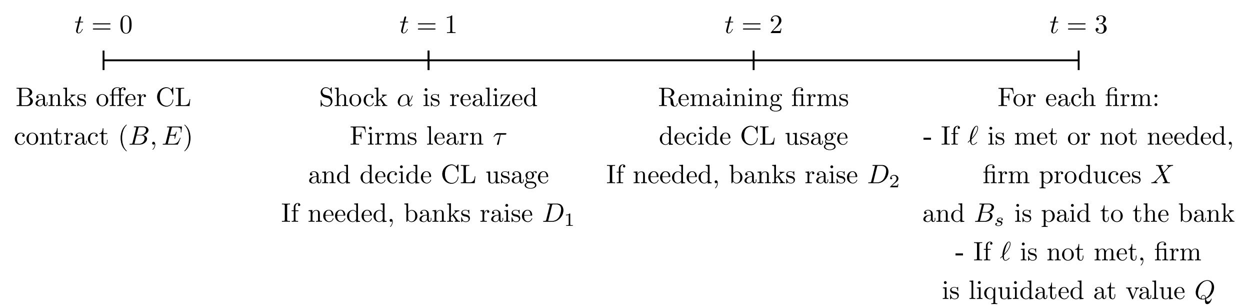

Figure 1: The sequence of events

is noteworthy that such simplification does not have major implications for the main results of the analysis. Therefore, if $\alpha_1 \approx 0$, demand for funds is essentially driven by firms that require cash at $t = 2$; hence, $\alpha \approx \alpha_2$.

3. Equilibrium analysis

3.1. The laissez-faire credit line contract

To attract firms, atomistic banks compete at $t = 0$ by offering credit lines with payment scheme B and pre-arranged funding E. However, when designing their credit lines, banks do not internalize the effect of their decisions on total liquidations z and, consequently, the liquidation value $Q(z)$.

Payment scheme B and pre-arranged funding E affect firms' drawdown decisions. On the one hand, B can be designed such that funds are never diverted into the inefficient investment. On the other hand, E helps to finance demand for liquidity for a wider range of α's; hence, reducing the occurrence of runs (recall that firms in need of cash are likely to run on their credit lines if loan requests are likely to be rejected).

To solve for the equilibrium contract, a backward induction approach is followed. First, a set of relevant incentive compatibility constraints that prevent firms to divert funds into the alternative investment at $t = 1, 2$ is obtained. Second, given a credit line contract (B, E) that satisfies these constraints, firms' drawdown decisions are derived for every α. Finally, due to competition, the equilibrium contract is obtained by maximizing the expected payoff of the representative firm at $t = 0$ subject to the incentive compatibility constraints and the participation constraint of investors who provide funding.

Along the analysis, I consider the design of the contract for a representative bank. This bank takes total liquidations z as given, which is represented by a non-decreasing function $z(\alpha)$, that is, more firms in need of cash do not obtain funding as liquidity risk increases.

3.1.a. Events at date 2

Consider firms that did not use the credit line at $t = 1$. Firms with a cash need at $t = 2$ will always request funding to meet ℓ. First, a firm in need of cash prefers meeting ℓ over liquidation, because

$$X - B_2 \geq X - Y \geq X - R_2 \geq Q(0),$$

by Assumptions 1 and 2. Second, conditional on obtaining the loan, meeting ℓ is preferred over investing funds at the rate ρ, because

$$X - B_2 \geq X - Y \geq \rho,$$

by Assumption 3.

Firms that do not need liquidity could invest funds in the alternative investment. Since such investment is inefficient, the payment scheme B is designed such that this investment is deterred. Let w_2 be the probability of obtaining a loan at $t = 2$ when it is requested. Firms that do not suffer the liquidity shock will not request funding if

$$w_2(X - B_2 + \rho) + (1 - w_2)(X - B_3) \leq X - B_3 \longleftrightarrow B_3 \leq B_2 - \rho. \tag{IC$_2$}$$

That is, if the payoff of not requesting funds is higher than the expected payoff of requesting them. Note that, if a loan is not disbursed, only the fee B_3 is paid by the firm; whereas, if a loan is granted, a payment B_2 has to be made and funds can only be invested at the rate

ρ. Condition (IC$_2$) requires that using the credit line is costlier than not using it for firms that do not have a cash need.

The bank is obliged by contract to meet loan requests. Thus, if feasible, the bank will raise additional funds from investors to fully meet the demand for drawdowns. Let L_1 be the amount of loans disbursed at $t = 1$. Therefore, the amount of pre-arranged funding that is left at $t = 2$ after serving drawdowns at $t = 1$ is equal to $M_1 = \max\{E - L_1, 0\}$; whereas, funds raised at $t = 1$ to meet such drawdowns are equal to $D_1 = \max\{L_1 - E, 0\}$. If B satisfies condition (IC$_2$), the demand for drawdowns at $t = 2$ is given by $\alpha - L_1$, that is, the total amount of funds needed by firms hit by ℓ that have not previously drawn down funds. Therefore, to meet their loan commitments, the bank requires funds $D_2 = \alpha - L_1 - M_1$ from investors. Raising this amount will be feasible if

$$L_1 B_1 + (\alpha - L_1) B_2 + (1 - \alpha) B_3 \geq \underbrace{\alpha - L_1 - M_1}_{D_2} + R_1 D_1.$$

That is, if revenue at $t = 3$ is enough to pay investors their required rates of returns for funds raised at $t = 1$ and 2 (recall that $R_2 = 1$). In such case, the amount of loans disbursed at $t = 2$ will coincide with the demand for drawdowns at $t = 2$, that is, $L_2 = \alpha - L_1$. It should be noted that the former condition is difficult to be met when either L_1 or α is high, because $B_s \leq Y < R_s$ for $s = 1$ and 2, due to Assumption 2.

In case revenue is not sufficient to raise funding to meet drawdowns, loan requests are served sequentially, in random order, until the bank exhausts its borrowing capacity. That is to say, the bank will grant loans until no more funds can be raised, that is,

$$L_1 B_1 + L_2 B_2 + (1 - \alpha) B_3 = \underbrace{L_2 - M_1}_{D_2} + R_1 D_1.$$

Note that demand for drawdowns at $t = 2$ is not fully met in such cases; hence, $L_2 < \alpha - L_1$. As a consequence, some firms in need of cash will be liquidated. It is worth mentioning that, because the bank is obliged by contract to meet drawdowns, the entire revenue is promised to new investors in these situations such that the amount of loan requests that are denied is minimized. Hence, claims on E are fully diluted in such scenario.

Proposition 1 summarizes the main results of the events occurring at $t = 2$.

Proposition 1. *If the payment scheme satisfies* (IC_2)*, only firms hit by ℓ that have not previously drawn down funds request a loan at $t = 2$. Moreover, depending on L_1 and α, demand for drawdowns at $t = 2$ is not always satisfied. The amount of loans granted at $t = 2$ is given by*

$$L_2 = \begin{cases} \alpha - L_1, & \textit{if borrowing capacity is not exhausted,} \\ \dfrac{L_1 B_1 + (1-\alpha)B_3 + M_1 - R_1 D_1}{1 - B_2}, & \textit{if borrowing capacity is exhausted.} \end{cases}$$

3.1.b. Events at date 1

Along the following analysis, it is assumed that the payment scheme satisfies condition (IC_2), that is, firms that do not use their credit lines at $t = 1$ will draw down at $t = 2$ only if funding to meet ℓ is needed.

After aggregate and individual uncertainty are revealed, firms decide on the usage of their credit lines at $t = 1$. They can either draw down (run) at $t = 1$ or wait until $t = 2$ to take a drawdown decision. Because $\alpha_1 \approx 0$, $L_1 = 0$ will indicate that firms do not run on their credit lines and drawdowns occur at $t = 2$ when firms are in need of cash. Next, I will discuss under which conditions every firm choosing not to run constitutes an equilibrium.

Consider that no firm runs (i.e., $L_1 = 0$). This will constitute an equilibrium if, given that every other firm postpones its drawdown decision, no firm finds it optimal to run. First, consider a firm without a liquidity need. This firm will not request funds at $t = 1$ if the payoff of requesting them is lower than the payoff of not requesting them, that is, if

$$X - B_1 + \rho \leq X - B_3 \longleftrightarrow B_3 \leq B_1 - \rho. \tag{IC_1'}$$

Note that, because no firm demands credit at $t = 1$, the bank could always grant a loan request at $t = 1$. It is also noteworthy that, by requesting funds at $t = 1$, whose cost is B_1, a firm without a cash need can only invest these funds in the inefficient investment; whereas, by postponing its decision to draw down, the firm gets $X - B_3$, because funds will not be requested at $t = 2$ due to condition (IC_2). As before, if using the credit line is sufficiently costly, firms that do not need liquidity will not request funding.

Next, consider a firm that needs cash at $t = 2$. Concerns about the bank's ability to meet drawdowns at $t = 2$ can push this type of firms to run. Recall that, if funding to meet ℓ is not obtained, these firms are liquidated at value Q, which depends on the aggregate amount of firms that are liquidated in state α, represented by a function $z(\alpha)$. Such function will be characterized when the laissez-faire contract is obtained (see subsection 3.1.c).[9] Nonetheless, $z(\alpha)$ is exogeneous and known when decisions are made at this stage. Thus, a firm in need of cash will not find it optimal to run if the payoff of drawing down early is lower than the expected payoff of waiting until $t = 2$ to request funds, that is, if

$$X - B_1 \leq \frac{L_2}{\alpha}(X - B_2) + \left(1 - \frac{L_2}{\alpha}\right) Q\big(z(\alpha)\big), \tag{NR}$$

where L_2 is defined as in Proposition 1 for $L_1 = 0$. On the one hand, by running, funding will be secured and liquidation will be avoided, but at the cost of a potentially higher payment. Note that, because $L_1 = 0$, a loan is always obtained when the firm chooses to run. On the other hand, if the firm decides to wait, it faces the risk of not obtaining a loan. Specifically, with probability $\frac{L_2}{\alpha}$, the loan is granted and the firm gets $X - B_2$; whereas, with probability $1 - \frac{L_2}{\alpha}$, the firm is liquidated and gets Q. Therefore, if liquidation risk remains low, which is measured by the term $1 - \frac{L_2}{\alpha}$, an equilibrium in which every firm does not run exists.

It should be noted that condition (NR) requires payments to satisfy $B_2 \leq B_1$. If $B_2 > B_1$, firms hit by ℓ will always draw down at $t = 1$. That is, even if no loan is rejected at $t = 2$,

[9] For instance, in a symmetric equilibrium, when firms in need of cash do not run and draw down at $t = 2$, aggregate liquidations are computed as $z = \alpha - L_2$, that is, the demand for liquidity that is not met by banks in state α.

drawing down at $t = 1$ is cheaper. Such scenario cannot be cost efficient, because the bank will have to finance such drawdowns with a more expensive borrowing (recall that $R_1 > R_2$). Thus, the contract must additionally set

$$B_2 \leq B_1. \tag{IC$_1$}$$

It should be noted as well that condition (IC_1') will be automatically satisfied if conditions (IC_1) and (IC_2) hold, that is, if using the credit line is costly, but using it earlier is costlier.

It is important to highlight that the decision to run of firms in need of cash depends on the drawing down decision of the rest of firms and the aggregate state. Recall that financing drawdowns with borrowing at $t = 1$ is more expensive. Thus, a loan granted at $t = 1$ exhausts faster the bank's borrowing capacity, making it more difficult to meet demand for drawdowns at $t = 2$. As a consequence, if sizable early drawdowns are expected, firms in need of cash will run to avoid high liquidation risk at $t = 2$.[10] Though these self-fulfilling panics can arise, the focus will be on fundamental runs; hence, if two equilibriums coexist, it is assumed that firms will behave according to the equilibrium without a run, which requires condition (NR) to hold.

The equilibrium without a run does not survive in high liquidity need states. Instead, a credit line run will constitute an equilibrium. A high realization of α shrinks fee income, which makes it harder for the bank to fully meet drawdowns at $t = 2$, increasing liquidation risk. Thus, the value of waiting for a firm in need of cash decreases with aggregate state α; hence, condition (NR) will not hold for high realizations of α. That is to say, even if no firm runs ($L_1 = 0$), a firm in need of cash prefers to run as a way to secure funding and avoid liquidation, deviating from the no run equilibrium. Instead, every firm in need of cash choosing to run will constitute an equilibrium. Note that fewer loans will be granted to firms in need of cash when drawdowns are concentrated at $t = 1$ due to more expensive borrowing.

[10] Note that for low α's, the bank's borrowing capacity may not be exhausted even if all firms in need of cash demand credit at $t = 1$ due to large revenues. In such case, liquidation risk should not be a concern and the run equilibrium does not exist.

Therefore, following a run, liquidations spike and no additional loan can be extended by the bank after $t = 1$, leading to the suspension of credit lines. In such scenario, a firm in need of cash will prefer to run, because, by running, funds can be obtained with some probability; whereas, by waiting, no loan will be received.[11] In sum, if condition (NR) does not hold, a credit line run will constitute an equilibrium.

Following a run, the bank faces a demand for drawdowns equal to α at $t = 1$ and concedes loans until its borrowing capacity is exhausted, that is,

$$L_1 B_1 = R_1 \underbrace{(L_1 - E)}_{D_1} \longleftrightarrow L_1 = \frac{R_1 E}{R_1 - B_1} < \alpha$$

where D_1 represents funds raised at $t = 1$. Recall that a fundamental run emerges when, given $L_1 = 0$, liquidation risk at $t = 2$ is high. Because $R_1 > R_2$, even fewer loans will be granted when drawdowns are concentrated at $t = 1$ rather than at $t = 2$; hence, bank's borrowing capacity will be exhausted at $t = 1$ and credit lines will be suspended after $t = 1$.

In Figure 2, curve NR represents the combinations of α and L_2 that make condition (NR) to hold with equality. Pairs above such curve satisfy condition (NR); whereas, pairs below do not. Also, it should be noted that the convexity of curve NR is inherited from the properties of function $Q(z)$. If $Q(z)$ were a constant function, (NR) would be linear. However, Q decreases with α as the total number of firms short of cash in the economy increases. Hence, as α increases, a higher fraction of firms in need of cash has to be financed for condition (NR) to hold.

As it can be appreciated, the range of values for α is split into three regions. In the first region, nobody runs, $L_1 = 0$; hence, borrowing capacity and pre-arranged funding are intact until $t = 2$. Due to a large fee income in this region, demand for liquidity can be met by the bank, that is, $L_2 = \alpha$. Also, because no risk of liquidation is faced and drawing down early is costly, firms do not find it profitable to draw down at $t = 1$, which is consistent with

[11] It should be noted that, in this situation, firms without a cash need will not request a loan, because the net return of investing funds at ρ is negative; i.e., $\rho - B_1 < 0$.

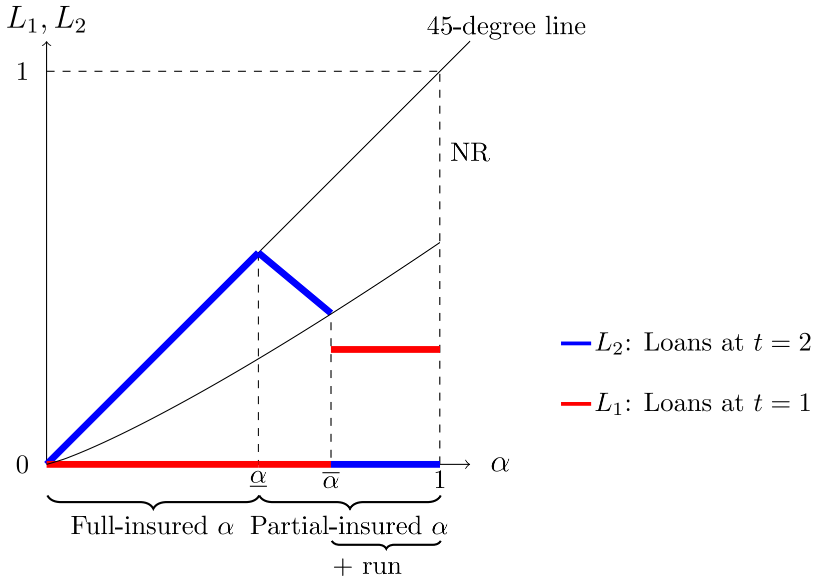

Figure 2: Loan disbursements at $t = 1$ and 2

$L_1 = 0$. However, fee income decays with α, which makes it harder to meet drawdowns at $t = 2$. As a consequence, demand for drawdowns will be fully met until $\alpha = \underline{\alpha}$. At such point, borrowing capacity is exhausted. Moreover, in such situation, the entire revenue is used to pay fund raising at $t = 2$, that is, claims on E are fully diluted.

In the second region, the bank cannot accommodate enough funds to finance firms in need of cash. Thus, with probability $1 - \frac{L_2}{\alpha} > 0$, some firms will be liquidated. Moreover, as α increases, the probability of not obtaining funds goes up. Hence, condition (NR) will eventually not hold. This happens when L_2 crosses curve NR in Figure 2. In such case, even if nobody runs, any firm finds it optimal to run due to high liquidation risk at $t = 2$. Thus, the equilibrium without a run does not survive.

Runs happen in the third region when condition (NR) does not longer hold. Thus, all firms in need of cash request funds at $t = 1$. Such demand for drawdowns is financed with fund raising at $t = 1$, which is costlier. As a consequence, fewer loans are granted at $t = 1$, which explains the drop in lending at $\alpha = \overline{\alpha}$. Moreover, borrowing capacity will be exhausted at $t = 1$; hence, no loans will be granted after $t = 1$; i.e., $L_2 = 0$.

Pre-arranged funding E can improve the insurance of the credit line. Because claims on E

can be diluted to obtain additional funding at $t = 1, 2$, lending to firms can be sustained for a wider range of $\alpha's$. Specifically, the first region in Figure 2 becomes wider as pre-arranged funding E increases. Moreover, if demand for drawdowns is not fully met, a higher amount of pre-arranged funding helps to increase lending, which reduces costly liquidations of firms.

The results in this section can be summarized in Proposition 2.

Proposition 2. *If $E < 1 - B_2$ and payment scheme B satisfies* (IC_1) *and* (IC_2)*, then there exist $\underline{\alpha} \leq \overline{\alpha}$ such that*

I. *If $\alpha \leq \underline{\alpha}$, there is a subgame perfect equilibrium in which only firms hit by ℓ request funding at $t = 2$ and all loan requests are met;*

II. *If $\underline{\alpha} < \alpha \leq \overline{\alpha}$, there is a subgame perfect equilibrium in which only firms hit by ℓ request funding at $t = 2$, but not all loan requests are met;*

III. *If $\alpha > \overline{\alpha}$, there is a subgame perfect equilibrium in which only firms hit by ℓ run on their credit lines and not all loan requests are met.*

Moreover, lending at $t = 1$ and 2 are defined in each region as

$$L_1 = \begin{cases} 0, & \textit{in region I}, \\ 0, & \textit{in region II}, \\ \dfrac{R_1 E}{R_1 - B_1}, & \textit{in region III}, \end{cases} \quad \textit{and} \quad L_2 = \begin{cases} \alpha, & \textit{in region I}, \\ \dfrac{(1-\alpha)B_3 + E}{1 - B_2}, & \textit{in region II}, \\ 0 & \textit{in region III}. \end{cases}$$

Note that, if $E < 1 - B_2$, the range of values for α can be split into three regions. Otherwise, any α can be fully met, that is, only Region I will exist. Specifically, even when $\alpha = 1$, the bank's revenue will be enough to pay back new investors for additional funds; i.e., $B_2 \geq 1 - E$. However, as it is discuss later when solving for the optimal contract, high levels of E are not efficient given that high realizations of α are very unlikely, making credit lines only costlier.

3.1.c. Solving for the laissez-faire contract

At $t = 0$ the competitive representative bank offers a credit line contract (B, E) to firms. Such contract sets the amount of pre-arranged funding E per committed funds and a payment scheme B that satisfies incentive compatibility constraints (IC_1) and (IC_2).

Pre-arranged funding E is raised from investors at $t = 0$. In return, initial investors demand a minimum expected return R_0 for their funds at $t = 3$. This pre-arranged funding is junior to funds raised at $t = 1, 2$. As a consequence, to comply with drawdowns, claims on E are fully diluted in regions II and III in Proposition 2, that is when $\alpha > \underline{\alpha}$. Therefore, in return for their funds, initial investors are compensated with payments that come from low liquidity need states $\alpha \leq \underline{\alpha}$; i.e, region I. Thus, pre-arranged funding E can be raised if [12]

$$R_0 E = \int_0^{\underline{\alpha}} \big(\alpha B_2 + (1-\alpha)B_3 - (\alpha - E)\big) f(\alpha) \mathrm{d}\alpha. \tag{$\mathrm{PC_I}$}$$

In particular, providers of E are entitled to the bank's revenue net of interest payments to new investors. Note that, if $\alpha > E$, the bank raises $D_2 = \alpha - E$ from new investors at $t = 2$ in Region I. Due to competition, such investors receive a payment equal to $\alpha - E$.[13] Recall as well that, if $\alpha \leq E$, excess of liquidity is invested at rate R_2, which is equal to 1.

The representative firm obtains an expected payoff V equal to

$$V(B,E) = \int_0^{\underline{\alpha}} V_{\mathtt{I}}(B,E;\alpha) f(\alpha)\mathrm{d}\alpha + \int_{\underline{\alpha}}^{\overline{\alpha}} V_{\mathtt{II}}(B,E;\alpha,z(\alpha)) f(\alpha)\mathrm{d}\alpha + \int_{\overline{\alpha}}^{1} V_{\mathtt{III}}(B,E;\alpha,z(\alpha)) f(\alpha)\mathrm{d}\alpha$$

from a credit line contract (B, E) that satisfies incentive compatibility constraints (IC_1) and (IC_2), where $V_{\mathtt{I}}$, $V_{\mathtt{II}}$, and $V_{\mathtt{III}}$ are the payoffs for a realization of α in regions I, II and III,

[12] Due to competition, this constraint holds with equality.

[13] Let P be the payment made by the bank to new investors for their funds $D_2 = \alpha - E > 0$. To raise such funds, P must satisfy

$$D_2 \leq P.$$

Hence, due to competition, new investors receive $P = D_2 = \alpha - E$ from the bank at $t = 3$.

respectively. These payoffs are defined as

$$
\begin{aligned}
V_{\mathtt{I}}(B,E;\alpha) &= (1-\alpha)(X-B_3)+\alpha(X-B_2),\\
V_{\mathtt{II}}(B,E;\alpha,z(\alpha)) &= (1-\alpha)(X-B_3)+\alpha\Big(\frac{L_2}{\alpha}(X-B_2)+\big(1-\frac{L_2}{\alpha}\big)Q(z(\alpha))\Big),\\
V_{\mathtt{III}}(B,E;\alpha,z(\alpha)) &= (1-\alpha)X+\alpha\Big(\frac{L_1}{\alpha}(X-B_1)+\big(1-\frac{L_1}{\alpha}\big)Q(z(\alpha))\Big),
\end{aligned}
$$

where L_1 and L_2 depend on B, E and α as defined in Proposition 2 and $z(\alpha)$ denotes aggregate liquidations. It should be noted that firms hit by ℓ will obtain a loan with probability 1, $\frac{L_2}{\alpha}$ and $\frac{L_1}{\alpha}$ in regions I, II and III, respectively. That is, firms face liquidation risk only in regions II and III.

Due to competition, the representative bank chooses (B,E) to maximize the expected payoff V of the representative firm subject to initial investors' participation constraint ($\mathrm{PC_I}$) and a payment scheme B that satisfies conditions (IC_1), (IC_2), and $B_1 \leq Y$. However, when choosing B and E, the bank does not internalize the effect on aggregate liquidations $z(\alpha)$ and, consequently, liquidation value $Q(z(\alpha))$. Hence, from the point of view of the representative bank, aggregate liquidations $z(\alpha)$ are taken as given when the contract is designed. Therefore, for equilibrium, aggregate liquidations should be consistent with the choice (B,E) of the representative bank. Definition 1 formally states the conditions that are needed for a symmetric equilibrium.

Definition 1. A *symmetric laissez-faire equilibrium* consists of a choice (E^U,B^U) for the representative bank and aggregate liquidations $z^U(\alpha)$ such that

1. Given aggregate liquidations $z^U(\alpha)$, (E^U,B^U) solves the bank's optimization problem, that is,

$$\max_{E,\{B_t\}_{t=1,2,3}} V(B,E)$$

subject to investors' participation constraint ($\mathrm{PC_I}$) and a payment scheme B that satisfies conditions (IC_1), (IC_2), and $B_1 \leq Y$.

2. Given (E^U, B^U), aggregate liquidations are computed as $z^U(\alpha) = \alpha - L_1 - L_2$ for all α, where L_1 and L_2 are defined in Proposition 2.

It is noteworthy that payment scheme B is required to satisfy conditions (IC_1), (IC_2), and $B_1 \leq Y$. First, if condition (IC_2) does not hold, firms that do not need cash will always request funds to divert them into the inefficient investment, in which case risk sharing will break down and implementing the credit line will not be feasible. Second, because fund raising at $t = 2$ is cheaper (recall $R_2 < R_1$), lending to firms can be sustained for more realizations of α if drawdowns occur at $t = 2$ rather than at $t = 1$. Thus, condition (IC_1), $B_2 \leq B_1$, will deter drawdowns at $t = 1$ when liquidation risk is low, which increases the insurance of the credit line contract. Finally, payments have to satisfy the pledgeable income constraint; i.e., $B_t \leq Y$ for $t = 1, 2$ and 3. It should be noted that conditions (IC_1) and (IC_2) imply $B_3 < B_2 \leq B_1$; hence, it will be enough to require $B_1 \leq Y$.

The next proposition characterizes the credit line contract in the laissez-faire equilibrium.

Proposition 3. *The credit line contract in the laissez-faire regime sets $B_1^U = B_2^U = Y$. Also, depending on parameters, the solution is characterized by one of the following cases:*

1. ***Interior solution:*** *$B_3^U < Y - \rho$ and E^U are chosen to equalize marginal benefit to marginal cost of E, that is,*

$$\underbrace{\frac{\partial V}{\partial E}}_{\textit{Marginal benefit of } E} = \underbrace{-\frac{\partial V}{\partial B_3}\frac{dB_3}{dE}\bigg|_{\text{(PC)}}}_{\textit{Marginal cost of } E} \tag{UR}$$

2. ***Corner solution:*** *$B_3^U = Y - \rho$ and E^U is pinned down by constraint (PC_I).*

First, payment B_1 will exhaust pledgeable income Y. It is noteworthy that B_1 is paid in region III by firms with a cash need when they obtain funds from the bank. A higher B_1 allows the bank to finance more firms in need of cash in region III; hence, reducing the number of liquidations. Therefore, despite a higher payment, firms will benefit from more lending, because the value of not liquidating the project, $X - Y > Q(0)$, is positive due to Assumption 1 and 2. Moreover, increasing B_1 relaxes constraint (IC_1).

Second, B_2 will be set as high as possible, that is, $B_2^U = B_1^U$. Recall that fee income compensates the bank for the losses that are incurred when credit lines are used. Therefore, increasing B_2, which is mainly paid in high liquidity need states, allows to slightly reduce payment B_3 in (PC_I). However, for high realizations of α, this increase in B_2 more than offsets the reduction in B_3, which allows the bank to increase lending in region II. Moreover, such increment in B_2 increases $\underline{\alpha}$ and $\overline{\alpha}$, that is, region I expands and region III shrinks. Overall, firms' welfare is improved due to more insurance. Additionally, because using the credit line costs the same at $t = 2$ as at $t = 1$, firms with a cash need will run if there exists liquidation risk ($L_2 < \alpha$); see condition (NR). Thus, runs occur when demand for drawdowns at $t = 2$ cannot be fully met by the bank, which happens whenever $\alpha > \underline{\alpha}$. Hence, region II will vanish and the range of values of α will be divided into two regions, namely, region I and III.

Finally, pre-arranged funding E helps to sustain lending in high liquidity need states, which improves the expected payoff V of the representative firm. However, increasing E comes at a cost. To compensate initial investors for their funds, commitment fee B_3 must increase; see constraint (PC_I). Such increment in the commitment fee decreases the expected payoff V of the representative firm. Hence, the marginal cost of increasing E is represented by the right-hand side of condition (UR). Thus, if feasible, the representative bank chooses B_3 and E to equalize marginal benefit to marginal cost of E, case 1 in Proposition 3. The comparative statics results on the laissez-faire equilibrium in an interior solution are analyzed and derived in Appendix B.

Nonetheless, constraint (IC_2), $B_3 \leq B_2 - \rho$, limits the maximum amount of pre-arranged funding that can be raised. As a consequence, if this constraint is binding, E cannot be increased until its marginal benefit and marginal cost equalize. In such case, contractual terms E^U and B_3^U will be pinned down by constraints (PC_I) and (IC_2), case 2 in Proposition 3.

It should be noted that commitment fee and pre-arranged funding would not be needed if loans were granted without incurring in any loss, that is, if $B_2 = 1$. In such case, costly liquidations would not occur and pre-arranged funding E would not be needed. However, Assumption 2, $Y < 1$, rules out this situation and justifies the existence of credit lines.

4. Social welfare analysis

In this section, a social planner problem is set up to determine if the previous laissez-faire equilibrium is constrained efficient. Later in this section, it is discussed how the constrained efficient credit line contract can be implemented by means of a regulatory requirement.

4.1. Constrained social planner problem

Consider a social planner that maximizes the expected payoff V of the representative firm. This social planner chooses pre-arranged funding E and payment scheme B at $t = 0$ subject to the same constraints as the representative bank in the laissez-faire regime, that is, constraints (PC_I), (IC_1), (IC_2), and $B_1 \leq Y$. However, when choosing (E, B), the planner internalizes the effect on aggregate liquidations $z(\alpha)$ and, consequently, on liquidation value $Q(z(\alpha))$. It is noteworthy that new investors will provide funding at $t = 1$ and 2 only if they are paid their required rate of return; hence, such investors break even. Additionally, providers of E also break even; see constraint (PC_I). As a consequence, firms are the only relevant class of agents with a nontrivial stake in social welfare.

In sum, the social planner's problem consists in maximizing V, that is,

$$\max_{E, \{B_t\}_{t=1,2,3}} V\big(B, E\big),$$

where function V has been defined in the preceding section, subject to the investors' participation constraint (PC_I), a payment scheme B that satisfies conditions (IC_1), (IC_2), and

$B_1 \leq Y$, and aggregate liquidations z that are computed as

$$z = \alpha - L_1(B, E) - L_2(B, E) \text{ for all } \alpha,$$

where L_1 and L_2 are defined in Proposition 2.

Let B^* and E^* be the payment scheme and the amount of pre-arranged funding that solve the problem of the social planner. The next proposition characterizes the *constrained efficient credit line contract* (B^*, E^*).

Proposition 4. *The constrained efficient credit line contract sets $B_1^* = B_2^* = Y$. Moreover, depending on parameters, the solution is characterized by one of the two following cases:*

1. ***Interior solution:*** *$B_3^* < Y - \rho$ and E^* are chosen to equalize social marginal benefit to social marginal cost of E, that is,*

$$\underbrace{\frac{\partial V}{\partial E} + \underbrace{\frac{\partial V}{\partial z}\frac{\partial z}{\partial E}}_{\textit{Pecuniary externality}}}_{\textit{Social marginal benefit of } E} = \underbrace{-\frac{\partial V}{\partial B_3}\frac{dB_3}{dE}\bigg|_{\text{(PC)}}}_{\textit{Social marginal cost}} \tag{SP}$$

2. ***Corner solution:*** *$B_3^* = Y - \rho$ and E^* is pinned down by constraint (PC_I).*

As in the laissez-faire regime, payment B_1 and B_2 exhaust pledgeable income Y. Recall that, by increasing B_1, lending to firms in need of cash can be increased in region III and costly liquidations can be reduced. Additionally, payment B_2 is set as high as possible, that is, constraint (IC_1) holds with equality; i.e., $B_2^* = B_1^*$. Note that, if $B_2 < B_1^*$, B_2 can be increased and B_3 reduced such that expected payment in region I is unaltered. However, as in the previous section, such variation improves welfare by increasing lending and reducing liquidations in region II, expanding region I, and shrinking region III.

Consequently, the social planner's problem is reduced to optimally choose pre-arranged funding E and commitment fee B_3 subject to one equality and one inequality constraint, namely, constraints (PC_I) and (IC_2), respectively. It is important to recall that E is financed with large fee revenues that are obtained in low liquidity need states; see constraint ($\mathrm{PC_I}$). Thus, constraint ($\mathrm{IC_2}$) limits the amount of E that can be raised, because, in order to implement the contract, fee B_3 cannot be set excessively high, or otherwise, firms without a cash need will always divert funds from the credit line into the inefficient investment. Because $B_2^* = Y$, the maximum fee that can be charged by the social planner is equal to $Y - \rho$, which coincides with the maximum fee that the representative bank can charge in the laissez-faire regime.

Therefore, if constraint ($\mathrm{IC_2}$) is not binding, pre-arranged funding E will be increased until its social marginal benefit equalizes its social marginal cost, as in case 1 of Proposition 4. It is noteworthy that the social marginal benefit of E adds to its private marginal benefit the effect of E on aggregate liquidations, which is represented by the term $\frac{\partial V}{\partial z}\frac{\partial z}{\partial E}$ in (SP). However, constraint ($\mathrm{IC_2}$) might prevent the equalization of the social marginal benefit and social marginal cost of E. In such scenario, increasing E will require to increase B_3 above $Y - \rho$, which is not feasible due to ($\mathrm{IC_2}$). Thus, in spite of being welfare improving, E cannot be increased. In such situation, B_3 and E are pinned down from both constraints ($\mathrm{PC_I}$) and ($\mathrm{IC_2}$), as in case 2 of Proposition 4.

By comparing condition (UR) in Proposition 3 with condition (SP) in Proposition 4, it can be appreciated that the sole difference between the social and private marginal benefit of E is given by the term $\frac{\partial V}{\partial z}\frac{\partial z}{\partial E}$ in condition (SP). This difference arises because the representative bank in the laissez-faire regime does not internalize the effect of its decision on aggregate liquidations and, consequently, liquidation values $Q(z(\alpha))$, as opposed to the social planner. This effect represents a pecuniary externality. As it was mentioned, pre-arranged funding E helps to sustain lending to firms, specially in high liquidity need states; hence, reducing aggregate liquidations and increasing liquidation values $Q(z(\alpha))$, which improves welfare. Thus, the sign of this term is positive.

As a consequence, due to this positive pecuniary externality, it is socially desirable to increase pre-arranged funding E relative to the choice of the representative bank in the

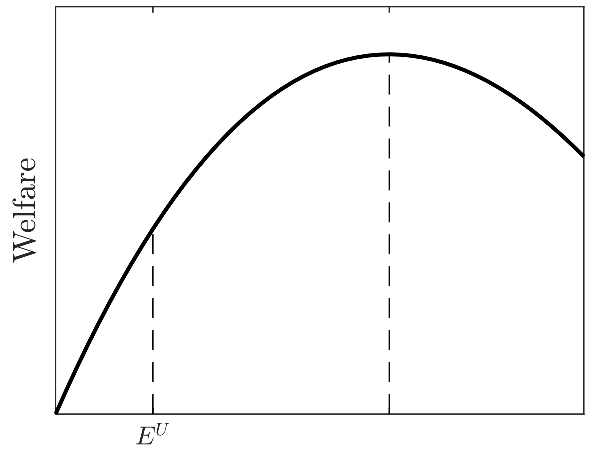

Figure 3: Welfare as a function of pre-arranged funding E

laissez-faire equilibrium. This is shown in Figure 3 for a particular parametrization of the model.[14] However, whether the social planner can select higher levels of pre-arranged funding E relative to E^U will depend on the incentive compatibility constraint (IC_2). Specifically, increasing E above E^U will require to increase commitment fee B_3, which may not be feasible due to (IC_2).

Define the *welfare gain* of implementing the solution of the social planner as

$$\Delta = V(B^*, E^*) - V(B^U, E^U),$$

that is, the difference between the social welfare achieved in the constrained planner's solution and the social welfare achieved in the laissez-faire equilibrium. The next proposition states under which circumstances such gains exist.

Proposition 5. *Depending on parameters, one of the two following cases can arise.*

1. *If the laissez-faire equilibrium is characterized by an interior solution, then welfare gains will exist; i.e., $\Delta > 0$. Moreover, low levels of pre-arranged funding are chosen in the laissez-faire equilibrium compared to the social planner's choice; i.e., $E^U < E^*$.*

[14] For the construction of Figure 3 the following parameters are chosen $X = 1.90$, $Y = 0.95$, $\rho = 0.93$, $Q_0 = 0.86$, $\gamma_0 = 2.50$, $\gamma_1 = 2.25$, $b = 4$, $R_0 = 1.03$, and $R_1 = 1.02$. These values are selected for the sole purpose of illustrating the solution of the model.

2. *If the laissez-faire equilibrium is characterized by a corner solution, the social planner's solution will coincide with the laissez-faire equilibrium; i.e.,* $\Delta = 0$.

It is important to recall that the credit line contract in the laissez-faire equilibrium and the one chosen by the planner set payments B_1 and B_2 equal to pledgeable income Y. Yet, they may differ in the their choice of pre-arranged funding and the fee required to pay for those pre-arranged funds. In case 1 of Proposition 5, the planner can increase E compared to the laissez-faire regime, because increasing the fee is still feasible; i.e., $B_3^U < Y - \rho$. This situation can be appreciated in Figure 3. If feasible, the planner will choose E such that the peak of the welfare function is reached (case 1 in Proposition 4). Otherwise, E will be increased until increasing E is not longer feasible; i.e., $B_3^* = Y - \rho$ (case 2 in Proposition 4). It is important to remark that a higher fee is paid in the planner's solution because larger values of E are chosen. Contrary, in case 2 of Proposition 5, if the fee in the laissez-faire equilibrium is such that $B_3^U = Y - \rho$, the planner will not be able to increase E without violating constraint (IC_2), even though the social marginal benefit of E is higher than its social marginal cost. Thus, no welfare gain can be achieved by the planner in this situation. Moreover, in this case, contractual terms of the credit line contract chosen by the social planner are pinned down by constraints (PC_I) and (IC_2) as in the laissez-faire equilibrium; hence, $B_3^U = B_3^*$ and $E^U = E^*$.

4.2. Implementation

This section will focus on case 1 of Proposition 5 in which a regulation of credit lines can be justified, since the laissez-faire equilibrium is not constrained efficient.

Consider a bank regulator that sets a minimum requirement $\underline{E}$ on pre-arranged funding, that is,

$$\underline{E} \leq E. \tag{LR}$$

Therefore, in order to grant credit lines to firms, banks are required to comply with this requirement. As it was highlighted before, banks in the laissez-faire equilibrium choose insufficient levels of pre-arranged funding. Hence, a regulation that requires banks to maintain a minimum amount of pre-arranged funds, such as requirement (LR), may directly deal with the main source of the externality.

With the introduction of regulatory requirement (LR), the representative bank chooses (E, B), taking aggregate liquidations as given, to maximize the expected payoff of the representative firm subject to the same constraints as in the laissez-faire case, but with the addition of constraint (LR).

The next definition enumerates the conditions that are needed for the construction of a symmetric equilibrium when a regulation that requires banks to hold minimum levels of pre-arranged funding is incorporated.

Definition 2. A *symmetric regulated equilibrium* consists of a choice (E^R, B^R) for the representative bank and aggregate liquidations $z^R(\alpha)$ such that

1. Given aggregate liquidations $z^R(\alpha)$, (E^R, B^R) solves the bank's optimization problem, that is,

$$\max_{E,\{B_t\}_{t=1,2,3}} V(B, E)$$

   subject to investors' participation constraint ($\mathrm{PC_I}$), a payment scheme B that satisfies conditions (IC_1), (IC_2), and $B_1 \leq Y$, and pre-arranged funding E that satisfies regulatory requirement (LR).

2. Given (E^R, B^R), aggregate liquidations are computed as $z^R(\alpha) = \alpha - L_1 - L_2$ for all α, where L_1 and L_2 are defined as in Proposition 2.

The *optimal regulation* of credit lines is designed by choosing regulatory requirement $\underline{E}$ such that the solution of the social planner can be implemented. The next proposition describes the optimal regulatory requirement $\underline{E}$ chosen by the bank regulator.

Proposition 6. *If $\underline{E} = E^*$, then the regulated equilibrium is constrained efficient.*

Proposition 6 states that a regulation that requires banks to maintain a minimum of E^* units of pre-arranged funds can implement the social's planner solution. In such case, the credit line in the regulated equilibrium coincides with the constrained efficient credit line contract. As in the social planner's solution, the credit line in the regulated case sets payments B_1 and B_2 equal to pledgeable income Y. Hence, if $E = E^*$ is chosen, then commitment fee must be equal to B_3^* due to ($\mathrm{PC_I}$). Therefore, if the representative bank optimally decides to raise pre-arranged funds equal to E^* when a regulatory requirement $\underline{E} = E^*$ is introduced, then the equilibrium contract will coincide with the one chosen by the social planner, and the regulated equilibrium will be constrained efficient. Note that, given $\underline{E} = E^*$, the representative bank chooses E^* if the representative firm's expected payoff V cannot be improved by increasing pre-arranged funding. First, consider the case where the solution of the social planner's problem is characterized by a corner solution, that is, E cannot be increased without violating constraint ($\mathrm{IC_2}$) even though the social marginal benefit of E is higher than its social marginal cost. In this scenario, only one choice of E will be feasible for the representative bank. Specifically, a choice of E lower than E^* is ruled out by regulation; whereas, any choice of E above E^* will require to set a commitment fee higher than B_3^*, which will violate constraint ($\mathrm{IC_2}$). Second, consider the case where the solution of the social planner is characterized by an interior solution. Hence, from Proposition 4, the social planner will choose E such that its social marginal benefit and cost equalize, that is,

$$\frac{\partial V}{\partial E} + \frac{\partial V}{\partial z}\frac{\partial z}{\partial E} = -\frac{\partial V}{\partial B_3}\frac{\mathrm{d}B_3}{\mathrm{d}E}\bigg|_{(\mathrm{PC})}.$$

However, because the sign of the externality is positive, the private marginal benefit of E at E^* is lower than its private marginal cost, that is,

$$\frac{\partial V}{\partial E} \leq -\frac{\partial V}{\partial B_3}\frac{\mathrm{d}B_3}{\mathrm{d}E}\bigg|_{(\mathrm{PC})}.$$

Thus, the representative bank does not find it optimal to increase pre-arranged funding above E^*. Note that decreasing E below E^* is not feasible due to regulatory requirement $\underline{E} = E^*$. Therefore, a regulatory requirement $\underline{E} = E^*$ can implement the social planner's solution.

As it was mentioned, the pecuniary externality on firms' liquidation value provides a rationale for credit line regulation. This externality arises from the negative effect of aggregate liquidations on firms' liquidation value, which is not internalized by atomistic competitive banks. The next result states when a regulation of credit lines will not be needed.

Corollary 1. *A regulation of credit lines will not be needed if aggregate liquidations z do not contribute to depress firms' liquidation value $Q(z)$; i.e., $Q(z) = Q_0$ for all z.*

Corollary 1 states that, if firms' liquidation value does not decay with the size of liquidations in the economy, then the channel whereby a reduction of aggregate liquidations z improve the expected payoff V of the representative firm by increasing firms' liquidation values in high liquidity need states will not operate. Thus, increasing pre-arranged funding E will not indirectly improve V by reducing liquidations. In such situation, the private marginal benefit and social marginal benefit of E will coincide; see conditions (UR) and (SP). Thus, even in an interior solution, the laissez-faire equilibrium will be constrained efficient and a regulation of credit lines will not be needed.

For a better understanding of the role of the pecuniary externality, the following liquidation value function is assumed

$$Q(z) = Q_0(1 - \gamma_0 z^{\gamma_1}).$$

Specifically, γ_0 measures the effect that a liquidation has on other firms' liquidation value. For instance, if γ_0 is zero, aggregate liquidations z do not affect firms' liquidation values; whereas, a liquidation will have a larger impact on liquidation value when γ_0 increases.

Figure 4 depicts how welfare V and pre-arranged funding E vary with respect γ_0 for each

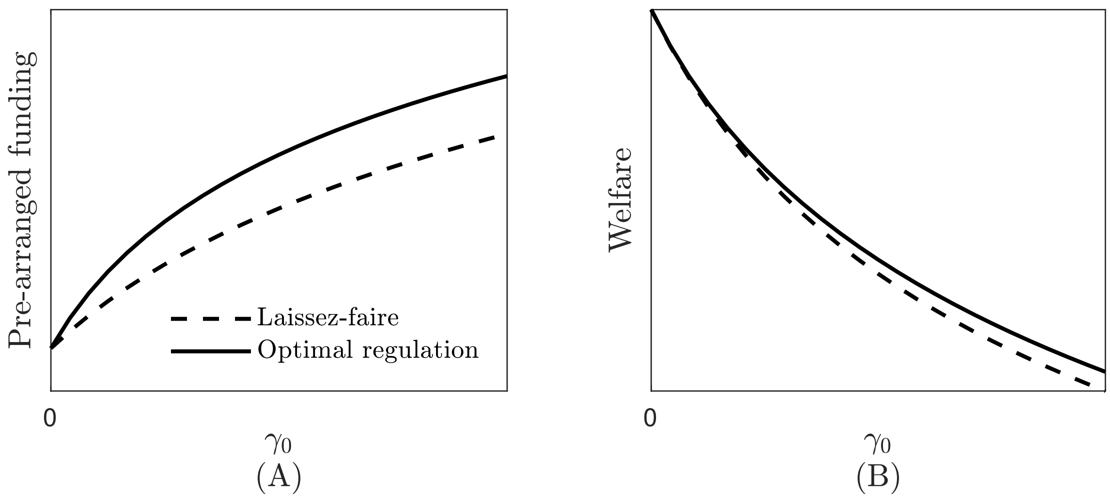

Figure 4: Effect of γ_0 on welfare and pre-arranged funding for each regime

regime.[15] As it can be appreciated, when $\gamma_0 = 0$, the representative bank's choice of pre-arranged funding E in the laissez-faire equilibrium coincides with the minimum requirement of pre-arranged funding optimally chosen by a regulator; see panel (A). As a consequence, welfare under both regimes coincide, which is the result stated in Corollary 1; see panel (B). However, as γ_0 increases, which measures the importance of the pecuniary externality, the representative bank in the laissez-faire equilibrium chooses sub-optimal levels of pre-arranged funding compared to the social planner's choice; see panel (A). Therefore, the regulator optimally sets a minimum requirement of pre-arranged funding that later becomes binding in the regulated equilibrium. These higher levels of pre-arranged funding in a regulated equilibrium with an optimal regulation contributes to increase social welfare relative to the laissez-faire regime; see panel (B). It is important to remark that pre-arranged funding in both regimes increases with γ_0, because increasing E helps to abate the more deleterious effect that liquidations have on firms' liquidation values.

4.3. Discussion

The regulatory requirement previously discussed can be matched to Basel III liquidity ratios. For instance, the regulatory requirement in the model can be interpreted as a liquidity

[15] To elaborate Figure 4, the same parameters that are used in the construction of Figure 3 are chosen.

coverage ratio. Recall that in the model banks raise funds E at date 0 from investors to store them as cash. Later, these funds are used to partially, or fully depending on α, meet demand for drawdowns at date 1 and 2. Therefore, regulatory requirement $\underline{E}$ requires banks to hold minimum levels of cash to meet loan obligations that arise from committed funds under credit lines. Moreover, these cash holdings are particularly important for sustaining lending to firms, specially in high liquidity need states.

Similarly, the regulatory requirement in the model can be interpreted as a net stable funding ratio. In the model, banks partly finance drawdowns with pre-arranged funding that is raised at date 0. This source of funding in turn is complemented with additional funds that are raised at date 1 and 2. Moreover, pre-arranged funding is junior to fund raising at date 1 and 2. Hence, banks can dilute claims on pre-arranged funding in order to obtain additional funds from investors at $t = 1, 2$ and meet their loan commitments.[16] Thus, in high liquidity need states, losses that arise from an increase in credit line usage are borne by providers of pre-arranged funding. Therefore, pre-arranged funding in the model can be seen as equity, which is considered a stable source of funding. Hence, the regulatory requirement in the model demands banks to maintain a minimum *required stable funding* (RSF) to finance their loan commitments. Specifically, committed funds under credit lines will be assigned a RSF factor of $\underline{E}$, that is, a fraction $\underline{E}$ of such bank obligations has to be financed with stable funding (e.g., equity or long-term debt). Currently, Basel III suggest a RSF factor of 5% for the undrawn portion of credit lines.

Finally, a capital requirement for the undrawn portion of credit lines can also approximate the regulatory requirement $\underline{E}$. Note that providers of pre-arranged funding in the model have a residual claim on bank revenue; hence, these investors can be equivalently seen as bank equity holders. Consequently, in low liquidity need states, these investors make large profits that come from fee income; whereas, in high liquidity need states, they bear the losses from an increase in credit line usage. Consider an economy with a capital requirement κ. For the

[16] Note that, if debt dilution were not feasible, meeting demand for drawdowns would be harder because a lower portion of bank revenue would be available for raising ex-post funding.

computation of capital requirements, undrawn credit lines are converted into an on-balance-sheet equivalent by using a *credit conversion factor*. Therefore, in the model, banks will require to hold a minimum capital $\underline{E}$ for undrawn credit lines if the credit conversion factor (CCF) satisfies

$$\underline{E} = \kappa \times CCF \rightarrow CCF = \frac{\underline{E}}{\kappa}.$$

5. Concluding remarks

This paper presents a simple model of credit lines in which a rationale for the regulation of credit lines is provided. In the model, pre-arranged junior funding (e.g., long-term debt or equity) helps banks to sustain lending to firms in the form of credit line drawdowns, specially when firms' liquidity needs are highly correlated. However, banks hold insufficient levels of pre-arranged funding in the laissez-faire equilibrium. Specifically, banks do not internalize the effect that their funding decisions have on the liquidation value of firms that do not receive funding from banks. As a consequence, in high liquidity need states, large liquidations are expected, which depress the liquidation value of firms. A liquidity requirement that links pre-funded cash reserves to committed loans can implement the constrained efficient allocation. Therefore, this pecuniary externality justifies the regulation of credit lines.

Furthermore, the liquidity requirement that is proposed in the model resembles the liquidity ratios that are proposed in Basel III. In particular, banks should partly finance credit line drawdowns with pre-arranged funds, which are stored as cash. Moreover, such pre-arranged funds should be financed with a stable source of funding, such that banks have the flexibility to raise additional funds in high liquidity need states by diluting claims on stable funding.

Though the liquidity requirement improves welfare, credit lines become costlier. In particular, commitment fees have to increase to pay for the higher levels of pre-arranged funding. In the model, investors who provide pre-arranged funding bear the losses from an increase in credit line usage in high liquidity need states and, as a consequence, they are compensated with high returns in low liquidity need states, which comes from large fee incomes. Therefore, firms will pay higher fees, but they will be better off due to an increase in lending in high liquidity need states.

To the best of my knowledge, this is the first paper that provides a rationale for the regulation of credit lines. Nevertheless, further research has to be done in this area. For instance, one could analyze the interaction of credit line and deposit runs and how liquidity regulation can help to prevent their occurrence. Similarly, other type of bank lending activities can be added to the analysis. In such case, fire sales of banks' illiquid assets to meet loan commitments can make credit line runs even costlier.

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Appendix

A. Proofs

Proof of Proposition 1

Let w_2 be the probability of obtaining a loan at $t = 2$ when it is requested. Firms that do not suffer the liquidity shock will not request funding at $t = 2$ if

$$w_2(X - B_2 + \rho) + (1 - w_2)(X - B_3) \leq X - B_3 \longleftrightarrow B_3 \leq B_2 - \rho.$$

Consider a firm who did not use the credit line at $t = 1$, but cash will be needed at $t = 2$. Such firm will always request funds to pursue continuation, because the payoff of pursing its project is higher than its liquidation value or the return on the inefficient investment, that is,

$$\max\{Q(z), \rho\} \leq X - B_2,$$

due to Assumption 1 and 3.

Thus, the bank will face a demand for drawdowns at $t = 2$ equal to $\alpha - L_1$. Such demand will be satisfied if

$$L_1 B_1 + (\alpha - L_1)B_2 + (1 - \alpha)B_3 \geq \alpha - L_1 - M_1 + R_1 D_1,$$

where $M_1 = \{E - L_1, 0\}$ denotes the available pre-arranged funding at $t = 2$ and $D_1 = \{L_1 - E, 0\}$ is the amount of fund raising at $t = 1$. If previous condition is not met, drawdowns will be granted until the whole revenue is promised to new investors, that is,

$$L_1 B_1 + L_2 B_2 + (1 - \alpha)B_3 = \alpha - L_1 - M_2 + R_1 D_1 \longleftrightarrow L_2 = \frac{L_1 B_1 + (1 - \alpha)B_3 + M_1 - R_1 D_1}{1 - B_2},$$

where $L_2 < \alpha - L_1$; hence, some liquidations will happen at $t = 2$ in such situation.

Proof of Proposition 2

Consider a payment scheme that satisfies conditions (IC_1) and (IC_2); i.e., $B_3 \leq B_2 - \rho$ and $B_2 \leq B_1 \leq Y$.

Suppose that no firm runs at $t = 1$, that is, $L_1 = 0$. This will constitute an equilibrium if, given $L_1 = 0$, every firm prefers to wait until $t = 2$ to decide on the usage of the credit line. From Proposition 1, only firms with projects that require cash will demand a loan at $t = 2$ if B satisfies (IC_2).

Consider a firm whose project will not need cash. Such firm will not request funds at

$t = 1$ if

$$X - B_1 + \rho \leq X - B_3 \longleftrightarrow B_3 \leq B_1 - \rho,$$

that is, if the payoff of requesting the loan at $t = 1$ is lower than the payoff of waiting until $t = 2$ to decide on the usage of the credit line, in which case funds will not be requested. Given the design of the payment scheme, the aforementioned condition will be satisfied.

Consider a firm whose project does not need cash at $t = 1$, but it will require funding at $t = 2$. Such firm will decide to wait if

$$X - B_1 \leq \frac{L_2}{\alpha}(X - B_2) + \Big(1 - \frac{L_2}{\alpha}\Big) Q\big(z(\alpha)\big),$$

that is, if the expected payoff of waiting and requesting the loan at $t = 2$ is higher than the payoff of drawing down earlier, in which case funding is secured. It should be noted that, because $B_2 \leq B_1$, every firm will prefer to draw down at $t = 2$ if no liquidation is expected at $t = 2$ (i.e., $L_2 = \alpha$).

Let $\alpha = \underline{\alpha}$ be the highest demand for drawdowns that the bank can fully meet at $t = 2$ when drawdowns occur according to the arrival of ℓ. That is to say, $\underline{\alpha}$ must satisfy

$$\underline{\alpha} B_2 + (1 - \underline{\alpha}) B_3 = \underline{\alpha} - E \longleftrightarrow \underline{\alpha} = \frac{B_3 + E}{1 - B_2 + B_3}.$$

It is noteworthy that, if $E < 1 - B_2$, then $\underline{\alpha} < 1$. Thus, if firms draw down according to the arrival of ℓ, the bank can fully meet demand for drawdowns for values of $\alpha \leq \underline{\alpha}$. Moreover, given that no liquidation is expected in such situation, every firm will find it optimal to wait when everyone else also decides to wait. Moreover, only firms with projects in need of cash will demand a loan at $t = 2$.

For values of $\alpha > \underline{\alpha}$, those firms that require cash at $t = 2$ may not receive funding. Yet, if the probability of not receiving the loan remains low, waiting when everyone else also decides to wait may still be preferred to drawing down early and paying B_1. Let $\alpha = \overline{\alpha}$ be the highest demand for drawdowns that does not trigger a run, that is, $\overline{\alpha}$ must satisfy

$$X - B_1 = \frac{L_2(\overline{\alpha})}{\overline{\alpha}}(X - B_2) + \Big(1 - \frac{L_2(\overline{\alpha})}{\overline{\alpha}}\Big) Q\big(z(\overline{\alpha})\big),$$

where $L_2(\overline{\alpha}) = \frac{(1-\overline{\alpha})B_3 + E}{1 - B_2}$ is defined in Proposition 1 assuming $L_1 = 0$. Because $\frac{L_2(\alpha)}{\alpha}$ is decreasing with α, such firms will not run for values of $\alpha \in (\underline{\alpha}, \overline{\alpha}]$, even though liquidation risk is not zero. Thus, if nobody runs, the bank will partially meet demand for drawdowns at $t = 2$ when $\alpha \in (\underline{\alpha}, \overline{\alpha}]$. Moreover, because liquidation risk remains low, firms will prefer to wait until $t = 2$ to decide on the usage of the credit line, in which case only those firms that need cash will demand a loan.

Finally, for values of $\alpha > \overline{\alpha}$, requesting funds at $t = 1$ is a strictly dominant strategy for any firm with a cash need at $t = 2$. That is to say, even if $L_1 = 0$, any firm with a future cash need will prefer to draw down at $t = 1$ to secure funds. Thus, such firms request funds at $t = 1$ when $\alpha > \overline{\alpha}$. In such case, the bank will grant loans until the entire revenue is promised to new investors at $t = 1$, that is,

$$B_1 L_1 = R_1(L_1 - E) \longleftrightarrow L_1 = \frac{R_1}{R_1 - B_1} E.$$

Because borrowing at $t = 1$ is more expensive, fewer funds will be allocated to firms in need of cash; hence, $L_1 < \alpha$. Therefore, no loan request will be granted after $t = 1$; hence, fee income will be lost. Moreover, any firm with a project that does not need cash will not request a loan at $t = 1$, because

$$\frac{L_1}{\alpha}(X - B_1 + \rho) + \Big(1 - \frac{L_1}{\alpha}\Big) X < X.$$

Therefore, for values of $\alpha > \overline{\alpha}$, only firms with a future cash need will request a loan at $t = 1$ and the bank will concede loans until its borrowing capacity is exhausted.

Proof of Proposition 3

(1) $\boldsymbol{B_1^U = Y}$. Note that payment B_1 affects only payoff V_{III}, because drawdowns at $t = 1$ occur only in the third region. As it was mentioned, the bank will serve loan requests until its borrowing capacity is exhausted; hence, lending in the third region satisfies

$$L_1 B_1 = R_1(L_1 - E) \longrightarrow L_1 = \frac{R_1 E}{R_1 - B_1}.$$

Thus, payoff V_{III} can be rewritten as

$$V_{\text{III}} = X - \Big(\alpha\big(X - Q(z)\big) - L_1\big(X - Q(z) - R_1\big) - R_1 E\Big),$$

which is increasing in B_1 because $\frac{\partial L_1}{\partial B_1} > 0$ and $X - Q(z) - R_1 > 0$ due to Assumption 1. Furthermore, B_1 increases threshold $\overline{\alpha}$ by making costlier to draw down at $t = 1$.

Therefore, the marginal benefit of increasing B_1 is positive, that is,

$$\frac{\partial V}{\partial B_1} = \int_{\overline{\alpha}}^{1} \frac{\partial L_1}{\partial B_1}\big(X - Q(z) - R_1\big) f(\alpha) \mathrm{d}\alpha + \big(V_{\text{II}}(\overline{\alpha}) - V_{\text{III}}(\overline{\alpha})\big) f(\overline{\alpha}) \frac{\partial \overline{\alpha}}{\partial B_1} > 0,$$

where $V_{\text{II}}(\overline{\alpha}) - V_{\text{III}}(\overline{\alpha}) > 0$ due to the spike in liquidations at $\alpha = \overline{\alpha}$, which is consequence of the drop in lending when a run happens. Thus, it is optimally to set $B_1^U = Y$.

(2) $\boldsymbol{B_2^U = B_1^U}$. Suppose $B_2 < B_1^U$. Thus, B_2 can be increased and B_3 can be appropriately reduced such that constraint (PC_I) is still satisfied. To do that, we apply the implicit function theorem to condition (PC_I) to obtain

$$\frac{\mathrm{d}B_3}{\mathrm{d}B_2} = -\frac{\mathbb{E}\big[\alpha|\alpha \in [0, \underline{\alpha}]\big]}{1 - \mathbb{E}\big[\alpha|\alpha \in [0, \underline{\alpha}]\big]} \equiv -\Delta.$$

Therefore, to prove $B_2^U = B_1^U$, we will show that the next expression is positive

$$\begin{aligned}\frac{\partial V}{\partial B_2} - \Delta\frac{\partial V}{\partial B_3} = & \int_0^{\underline{\alpha}} \left(\frac{\partial V_\text{I}}{\partial B_2} - \Delta\frac{\partial V_\text{I}}{\partial B_3}\right) f(\alpha)\mathrm{d}\alpha + \big(V_\text{I}(\underline{\alpha}) - V_\text{II}(\underline{\alpha})\big) f(\underline{\alpha})\left(\frac{\partial \underline{\alpha}}{\partial B_2} - \Delta\frac{\partial \underline{\alpha}}{\partial B_3}\right) + \\ & \int_{\underline{\alpha}}^{\overline{\alpha}} \left(\frac{\partial V_\text{II}}{\partial B_2} - \Delta\frac{\partial V_\text{II}}{\partial B_3}\right) f(\alpha)\mathrm{d}\alpha + \big(V_\text{II}(\overline{\alpha}) - V_\text{III}(\overline{\alpha})\big) f(\overline{\alpha})\left(\frac{\partial \overline{\alpha}}{\partial B_2} - \Delta\frac{\partial \overline{\alpha}}{\partial B_3}\right),\end{aligned} \tag{A.1}$$

that is, the contract can be improved by increasing B_2 and appropriately reducing B_3.

First, it should be noted that B_2 and B_3 are changed such that the expected payment to the bank in the first region is not altered; hence, the first term in the previous expression is zero. Furthermore, due to the continuity of the payoff function at $\alpha = \underline{\alpha}$, $V_\text{I}(\underline{\alpha}) = V_\text{II}(\underline{\alpha})$, the second term is also zero.

Next, because the whole bank revenue is used to raise funds at $t = 2$, V_II can be rewritten as

$$V_\text{II} = X - \Big(\alpha\big(X - Q(z)\big) - L_2\big(X - Q(z) - 1\big) - E\Big).$$

Hence, by using the expression for L_2 in Proposition 2, the third term in (A.1) is equal to

$$\begin{aligned}\int_{\underline{\alpha}}^{\overline{\alpha}} \left(\frac{\partial V_\text{II}}{\partial B_2} - \Delta\frac{\partial V_\text{II}}{\partial B_3}\right) f(\alpha)\mathrm{d}\alpha &= \int_{\underline{\alpha}}^{\overline{\alpha}} \left(\frac{\partial L_2}{\partial B_2} - \Delta\frac{\partial L_2}{\partial B_3}\right)\big(X - Q(z) - 1\big) f(\alpha)\mathrm{d}\alpha \\ &= \int_{\underline{\alpha}}^{\overline{\alpha}} \Big((1-\alpha)\big(B_3 - \Delta(1 - B_2)\big) + E\Big)\frac{X - Q(z) - 1}{(1 - B_2)^2} f(\alpha)\mathrm{d}\alpha.\end{aligned}$$

Moreover, the previous expression is positive, because

$$B_3 - \Delta(1 - B_2) = \frac{1}{1 - \int_0^{\underline{\alpha}} f(\alpha)\mathrm{d}\alpha} \int_0^{\underline{\alpha}} \big(\alpha B_2 + (1-\alpha)B_3 - \alpha\big) f(\alpha)\mathrm{d}\alpha$$

is positive due to constraint (PC_I).

Finally, the last term in expression (A.1) will be positive if $\frac{\partial \overline{\alpha}}{\partial B_2} - \Delta\frac{\partial \overline{\alpha}}{\partial B_3} \geq 0$. By applying the implicit function theorem to condition (NR),

$$X - B_1^U = \frac{L_2(B_2, B_3, \overline{\alpha})}{\overline{\alpha}}(X - B_2) + \Big(1 - \frac{L_2(B_2, B_3, \overline{\alpha})}{\overline{\alpha}}\Big)Q\big(z(\overline{\alpha})\big),$$

partial derivatives $\frac{\partial \overline{\alpha}}{\partial B_2}$ and $\frac{\partial \overline{\alpha}}{\partial B_3}$ can be obtained; hence,

$$\frac{\partial \overline{\alpha}}{\partial B_2} - \Delta \frac{\partial \overline{\alpha}}{\partial B_3} = \frac{\big(X - B_2 - Q\big)\big((1-\overline{\alpha})(B_3 - \Delta(1 - B_2)) + E\big)}{(1 - B_2)^2\big(X - B_1^U - Q + \frac{B_3}{1-B_2}(X - B_2 - Q) - (\overline{\alpha} - L_2(\overline{\alpha}))Q_z \frac{\partial z}{\partial \alpha}\big)},$$

where Q is the liquidation value evaluated at $z(\overline{\alpha})$ and Q_z is the first derivative of Q respect z, which is negative. It can be easily verified that the former expression is positive. Hence, it is optimally to increase B_2 until (IC$_1$) holds with equality; i.e., $B_2^U = B_1^U$.

(3) **Optimal choice of $\boldsymbol{E}$ and $\boldsymbol{B_3}$**. First, it should be noted that, because $B_2^U = B_1^U$, a credit line run will arise whenever

$$X - B_1^U > \frac{L_2}{\alpha}(X - B_2^U) + \Big(1 - \frac{L_2}{\alpha}\Big)Q\big(z(\alpha)\big) \longleftrightarrow L_2 < \alpha,$$

that is, whenever the demand for drawdowns cannot be fully met at $t = 2$ by the bank. Consequently, the range of values of α will be divided into two region: region I, $\alpha \leq \underline{\alpha}$, where demand for drawdowns is totally met and region III, $\alpha > \underline{\alpha}$, where runs occur. Thus, $\underline{\alpha}$ can be computed as the highest realization of α that the bank can accommodate, that is,

$$\underline{\alpha} B_2^U + (1 - \underline{\alpha})B_3 = \underline{\alpha} - E \longrightarrow \underline{\alpha} = \frac{B_3 + E}{1 - B_2^U + B_3}.$$

Therefore, the optimization problem for the representative bank consists of maximizing

$$\max_{E, B_3} \int_0^{\underline{\alpha}} \Big(X - (1-\alpha)B_3 - \alpha B_2^U\Big) f(\alpha)\mathrm{d}\alpha + \int_{\underline{\alpha}}^1 \Big(X - (\alpha - L_1)\big(X - Q(z)\big) - R_1(L_1 - E)\Big) f(\alpha)\mathrm{d}\alpha$$

subject to the following constraints

$$R_0 E = \int_0^{\underline{\alpha}} \big(\alpha B_2^U + (1-\alpha)B_3 - (\alpha - E)\big) f(\alpha)\mathrm{d}\alpha, \tag{PC$_{\text{LT}}$}$$

$$B_3 \leq B_2^U - \rho, \tag{IC$_2$}$$

where $L_1 = \frac{R_1}{R_1 - B_1^U}E$ and $\underline{\alpha} = \frac{B_3 + E}{1 - B_2^U + B_3}$, and taking aggregate liquidations z as given.

Define the lagrangian L for the constrained optimization problem as

$$L(E,B_3,\theta_1,\theta_2)=V(E,B_3)+\theta_1\Big(\int_0^{\underline{\alpha}}\big(\alpha B_2^U+(1-\alpha)B_3-(\alpha-E)\big)f(\alpha)\mathrm{d}\alpha-R_0E\Big)+\theta_2(B_2^U-\rho-B_3)$$

and whose first order conditions are

$$\begin{aligned}
\{B_3\}:&\frac{\partial V}{\partial B_3}+\theta_1\int_0^{\underline{\alpha}}(1-\alpha)f(\alpha)\mathrm{d}\alpha-\theta_2=0,\\
\{E\}:&\frac{\partial V}{\partial E}-\theta_1\big(R_0-F(\underline{\alpha})\big)=0,\\
\{\theta_1\}:&\int_0^{\underline{\alpha}}\big(\alpha B_2^U+(1-\alpha)B_3-(\alpha-E)\big)f(\alpha)\mathrm{d}\alpha-R_0E=0,\\
\{\theta_2\}:&(B_2^U-\rho-B_3)\theta_2=0,\text{where }\theta_2\geq 0\text{ and }B_2^U-\rho-B_3\geq 0.
\end{aligned}$$

where θ_1 and θ_2 are the lagrange multipliers of constraints ($\mathrm{PC_{LT}}$) and($\mathrm{IC_2}$), respectively.

By combining the first order conditions of E and B_3, we obtain

$$\frac{\partial V}{\partial E}=-\frac{\partial V}{\partial B_3}\frac{\mathrm{d}B_3}{\mathrm{d}E}\bigg|_{(\mathrm{PC})}+\lambda^U \tag{A.2}$$

where $\frac{\partial V}{\partial B_3}$, $\frac{\partial V}{\partial E}$ and $\frac{\mathrm{d}B_3}{\mathrm{d}E}$ are equal to

$$\begin{aligned}
\frac{\partial V}{\partial B_3}&=-\int_0^{\underline{\alpha}}(1-\alpha)f(\alpha)\mathrm{d}\alpha+\big(V_{\mathrm{I}}(\underline{\alpha})-V_{\mathrm{III}}(\underline{\alpha})\big)f(\underline{\alpha})\frac{\partial\underline{\alpha}}{\partial B_3},\\
\frac{\partial V}{\partial E}&=\big(V_{\mathrm{I}}(\underline{\alpha})-V_{\mathrm{III}}(\underline{\alpha})\big)f(\underline{\alpha})\frac{\partial\underline{\alpha}}{\partial E}+R_1\int_{\underline{\alpha}}^1\frac{X-Q(z)-B_1^U}{R_1-B_1^U}f(\alpha)\mathrm{d}\alpha,\\
\frac{\mathrm{d}B_3}{\mathrm{d}E}&=\frac{R_0\big/F(\underline{\alpha})-1}{\mathbb{E}\big[1-\alpha|\alpha\leq\underline{\alpha}\big]}
\end{aligned}$$

respectively, and $\lambda^U=\theta_2\frac{R_0/F(\underline{\alpha})-1}{\mathbb{E}[1-\alpha|\alpha\in[0,\underline{\alpha}]]}$.

Moreover, due to the definition of the symmetric laissez-faire equilibrium, aggregate liquidations are computed as

$$z(\alpha)=\begin{cases}0, & \alpha\leq\underline{\alpha}\\ \alpha-\frac{R_1E}{R_1-B_1^U}, & \alpha>\underline{\alpha}.\end{cases}$$

It should be noted that, when $\alpha\leq\underline{\alpha}$, demand for liquidity is fully met in the economy; hence, no liquidations occur in this region. Contrary, if $\alpha>\underline{\alpha}$, total liquidations will be equal to $\alpha-L_1$, where $L_1=\frac{R_1E}{R_1-B_1^U}$.

Thus, given the equilibrium aggregate liquidation function $z(\alpha)$, equation (A.2) and the first order conditions of θ_1 and θ_2 pin down the equilibrium contractual terms B_3 and E. Furthermore, it is noteworthy that, if constraint (IC_2) is not binding (i.e., $\lambda^U = 0$), the solution is characterized by equations (A.2) and ($\mathrm{PC_I}$). However, if the commitment fee B_3 required to obtain such pre-arranged funding E violates constraint (IC_2), a corner solution is obtained, that is, constraints (IC_2) and ($\mathrm{PC_I}$) pin down B_3 and E.

Proof of Proposition 4

To prove this proposition, the same steps as in Proposition 4 will be followed. First, it should be noted that aggregate liquidations are equal to

$$z = \begin{cases} 0, & \text{if } \alpha \leq \underline{\alpha}, \\ \alpha - L_2, & \text{if } \alpha \in (\underline{\alpha}, \overline{\alpha}], \\ \alpha - L_1, & \text{if } \alpha > \overline{\alpha}, \end{cases}$$

where $L_2 = \frac{(1-\alpha)B_3+E}{1-B_2}$ and $L_1 = \frac{R_1}{R_1-B_1}E$. As opposed to the representative bank in the laissez-faire equilibrium, the social planner will take into account the effect of B and E on z when designing the constrained efficient credit line contract.

(1) $\boldsymbol{B_1^* = Y}$. As in Proposition 1, increasing B_1 has a positive effect on V due to an increase in lending in region III. Additionally, increasing B_1 reduces aggregate liquidations in region III, which has a positive effect on Q. Overall, the effect of B_1 on V is positive, that is,

$$\frac{\partial V}{\partial B_1} = \int_{\overline{\alpha}}^{1} \left(\frac{\partial L_1}{\partial B_1}\big(X - Q(z) - R_1\big) + (\alpha - L_1) Q_z \frac{\partial z}{\partial B_1} \right) f(\alpha) \mathrm{d}\alpha + \big(V_{\mathrm{II}}(\overline{\alpha}) - V_{\mathrm{III}}(\overline{\alpha})\big) f(\overline{\alpha}) \frac{\partial \overline{\alpha}}{\partial B_1} > 0.$$

(2) $\boldsymbol{B_2^* = B_1^*}$. Suppose $B_2 < B_1^*$. As in Proposition 3, B_2 can be increased and B_3 can be appropriately reduced such that ($\mathrm{PC_I}$) is satisfied, that is,

$$\frac{\mathrm{d}B_3}{\mathrm{d}B_2} = -\frac{\mathbb{E}\big[\alpha | \alpha \in [0, \underline{\alpha}]\big]}{1 - \mathbb{E}\big[\alpha | \alpha \in [0, \underline{\alpha}]\big]} \equiv -\Delta.$$

As before, it can be shown that such change in the payment scheme does not alter the firm's expected payoff in region I, because the expected payment to the bank in such region remains the same. However, such change helps to expand lending in region II. Consequently, it helps to decrease liquidations. Therefore, such change increases welfare, that is,

$$\frac{\partial V}{\partial B_2} - \Delta\frac{\partial V}{\partial B_3} = \int_{\underline{\alpha}}^{\overline{\alpha}} \left(\left(\frac{\partial L_2}{\partial B_2} - \Delta\frac{\partial L_2}{\partial B_3} \right) (X - Q(z) - 1) + (\alpha - L_2) Q_z \left(\frac{\partial z}{\partial B_2} - \Delta\frac{\partial z}{\partial B_3} \right) \right) f(\alpha) \mathrm{d}\alpha + \big(V_{\mathrm{II}}(\overline{\alpha}) - V_{\mathrm{III}}(\overline{\alpha})\big) f(\overline{\alpha}) \left(\frac{\partial \overline{\alpha}}{\partial B_2} - \Delta\frac{\partial \overline{\alpha}}{\partial B_3} \right) > 0.$$

Thus, the contract can always be improved by increasing B_2 and appropriately reducing B_3.

(3) **Optimal choice of $\boldsymbol{E}$ and $\boldsymbol{B_3}$**. As in Proposition 3, because $B_2^* = B_1^*$, the range of values of α will be divided into two regions: region I, $\alpha \leq \underline{\alpha}$, and region III, $\alpha > \underline{\alpha}$, where $\underline{\alpha}$ is the highest demand of drawdowns that the bank can accommodate, that is,

$$\underline{\alpha} B_2^* + (1 - \underline{\alpha}) B_3 = \underline{\alpha} - E \longrightarrow \underline{\alpha} = \frac{B_3 + E}{1 - B_2^* + B_3}.$$

Hence, the optimization problem for the social planner consists of maximizing V

$$\max_{E, B_3} \int_0^{\underline{\alpha}} \left(X - (1 - \alpha) B_3 - \alpha B_2^* \right) f(\alpha) \mathrm{d}\alpha + \int_{\underline{\alpha}}^1 \left(X - (\alpha - L_1)\big(X - Q(z)\big) - R_1(L_1 - E) \right) f(\alpha) \mathrm{d}\alpha$$

subject to the following constraints

$$R_0 E = \int_0^{\underline{\alpha}} \left(\alpha B_2^* + (1 - \alpha) B_3 - (\alpha - E) \right) f(\alpha) \mathrm{d}\alpha, \qquad (\mathrm{PC}_{\mathrm{LT}})$$

$$B_3 \leq B_2^* - \rho, \qquad (\mathrm{IC}_2)$$

and aggregate liquidations, which are computed as

$$z = \begin{cases} 0, & \text{if } \alpha \leq \underline{\alpha}, \\ \alpha - L_1, & \text{if } \alpha > \underline{\alpha}, \end{cases}$$

where $L_1 = \frac{R_1}{R_1 - B_1^*} E$ and $\underline{\alpha} = \frac{B_3 + E}{1 - B_2^* + B_3}$.

Define the lagrangian L for the constrained optimization problem as

$$L(E, B_3, \vartheta_1, \vartheta_2) = V(E, B_3) + \vartheta_1 \left(\int_0^{\underline{\alpha}} \left(\alpha B_2^* + (1 - \alpha) B_3 - (\alpha - E) \right) f(\alpha) \mathrm{d}\alpha - R_0 E \right) + \vartheta_2 (B_2^* - \rho - B_3)$$

and whose first order conditions are

$$\begin{aligned}
\{B_3\} :& \frac{\partial V}{\partial B_3} + \vartheta_1 \int_0^{\underline{\alpha}} (1-\alpha) f(\alpha)\mathrm{d}\alpha - \vartheta_2 = 0, \\
\{E\} :& \frac{\partial V}{\partial E} + \frac{\partial V}{\partial z}\frac{\partial z}{\partial E} - \vartheta_1\big(R_0 - F(\underline{\alpha})\big) = 0, \\
\{\vartheta_1\} :& \int_0^{\underline{\alpha}} \big(\alpha B_2^* + (1-\alpha)B_3 - (\alpha - E)\big) f(\alpha)\mathrm{d}\alpha - R_0 E = 0, \\
\{\vartheta_2\} :& (B_2^* - \rho - B_3)\vartheta_2 = 0, \text{where } \vartheta_2 \geq 0 \text{ and } B_2^* - \rho - B_3 \geq 0.
\end{aligned}$$

where ϑ_1 and ϑ_2 are the lagrange multipliers of constraints ($\mathrm{PC_{LT}}$) and($\mathrm{IC_2}$), respectively.

By combining the first order conditions of E and B_3, we obtain

$$\frac{\partial V}{\partial E} + \frac{\partial V}{\partial z}\frac{\partial z}{\partial E} = -\frac{\partial V}{\partial B_3}\frac{\mathrm{d}B_3}{\mathrm{d}E}\bigg|_{(\mathrm{PC})} + \lambda^* \tag{A.3}$$

where $\frac{\partial V}{\partial B_3}$, $\frac{\partial V}{\partial E}$, $\frac{\partial V}{\partial z}\frac{\partial z}{\partial E}$, and $\frac{\mathrm{d}B_3}{\mathrm{d}E}$ are equal to

$$\begin{aligned}
\frac{\partial V}{\partial B_3} &= -\int_0^{\underline{\alpha}} (1-\alpha) f(\alpha)\mathrm{d}\alpha + \big(V_{\mathrm{I}}(\underline{\alpha}) - V_{\mathrm{III}}(\underline{\alpha})\big) f(\underline{\alpha})\frac{\partial \underline{\alpha}}{\partial B_3}, \\
\frac{\partial V}{\partial E} &= \big(V_{\mathrm{I}}(\underline{\alpha}) - V_{\mathrm{III}}(\underline{\alpha})\big) f(\underline{\alpha})\frac{\partial \underline{\alpha}}{\partial E} + R_1 \int_{\underline{\alpha}}^1 \frac{X - Q(z) - B_1^*}{R_1 - B_1^*} f(\alpha)\mathrm{d}\alpha, \\
\frac{\partial V}{\partial z}\frac{\partial z}{\partial E} &= \int_{\underline{\alpha}}^1 (\alpha - L_1) Q_z \frac{\partial z}{\partial E} f(\alpha)\mathrm{d}\alpha, \\
\frac{\mathrm{d}B_3}{\mathrm{d}E} &= \frac{R_0/F(\underline{\alpha}) - 1}{\mathbb{E}\big[1-\alpha|\alpha \leq \underline{\alpha}\big]},
\end{aligned}$$

respectively, and $\lambda^* = \vartheta_2 \frac{R_0/F(\underline{\alpha})-1}{\mathbb{E}[1-\alpha|\alpha\in[0,\underline{\alpha}]]}$. Thus, if constraint ($\mathrm{IC_2}$) is not binding (i.e., $\lambda^* = 0$), contractual terms B_3 and E are pinned down from equations (A.3) and ($\mathrm{PC_I}$). Otherwise, B_3 and E are pinned down from constraints ($\mathrm{IC_2}$) and ($\mathrm{PC_I}$).

Proof of Proposition 5

(1) Assume that $\lambda^U = 0$ in (A.2). In such case, equations (A.2) and ($\mathrm{PC_I}$) pin down B_3^U and E^U. Because $\frac{\partial V}{\partial z}\frac{\partial z}{\partial E}$ is positive, the social marginal benefit of E is higher than its private marginal benefit. Moreover, the social and private marginal cost coincide. Thus, the social planner can improve welfare by increasing E above E^U, that is,

$$\underbrace{\frac{\partial V}{\partial E}(B_3^U, E^U)}_{\text{Private marginal benefit of } E} = \underbrace{-\frac{\partial V}{\partial B_3}(B_3^U, E^U)\frac{\mathrm{d}B_3}{\mathrm{d}E}\bigg|_{(\mathrm{PC})}}_{\text{Social \& private marginal cost of } E} < \underbrace{\frac{\partial V}{\partial E}(B_3^U, E^U) + \frac{\partial V}{\partial z}\frac{\partial z}{\partial E}(B_3^U, E^U)}_{\text{Social marginal benefit of } E}.$$

Also, commitment fee B_3 has to be increased to finance the higher holdings of E.

(2) Suppose that the solution in the laissez-faire equilibrium is given by a corner solution. Hence, B_3 and E are pinned down from constraints (IC_2) and (PC_I). Also, because $\lambda^U > 0$, the private marginal benefit of E is higher than its private marginal cost, that is,

$$\frac{\partial V}{\partial E}(B_3^U, E^U) > -\frac{\partial V}{\partial B_3}(B_3^U, E^U)\frac{\mathrm{d}B_3}{\mathrm{d}E}\bigg|_{(\text{PC})}.$$

Due to the positive externality, the social marginal benefit of E is higher than its private marginal benefit. However, the social planner cannot increase E above E^U. Note that any $E > E^U$ will require to increase B_3 above B_3^U, which is unfeasible due to constraint (IC_2). Consequently, the social planner cannot improve welfare in this situation.

Proof of Proposition 6

We will show that, given $\underline{E} = E^*$, the equilibrium credit line contract in the regulated equilibrium will coincide with the constrained efficient credit line contract. Therefore, such regulation implements the social planner's solution.

Following exactly the same steps as in Proposition 3, it can be proved that $B_1^R = B_2^R = Y$. Therefore, taking aggregate liquidations as given, the representative bank chooses E and B_3 such that the representative firm's expected payoff is maximized,

$$\max_{E,B_3} \int_0^{\underline{\alpha}} \Big(X - (1-\alpha)B_3 - \alpha B_2^R\Big) f(\alpha)\mathrm{d}\alpha + \int_{\underline{\alpha}}^1 \Big(X - (\alpha - L_1)\big(X - Q(z)\big) - R_1(L_1 - E)\Big) f(\alpha)\mathrm{d}\alpha,$$

subject to the following constraints

$$R_0 E = \int_0^{\underline{\alpha}} \big(\alpha B_2^R + (1-\alpha)B_3 - (\alpha - E)\big) f(\alpha)\mathrm{d}\alpha, \tag{PC_LT}$$

$$B_3 \le B_2^R - \rho, \tag{IC_2}$$

$$E^* \le E, \tag{LR}$$

where $L_1 = \frac{R_1}{R_1 - B_1^R}E$ and $\underline{\alpha} = \frac{B_3 + E}{1 - B_2^R + B_3}$.

Therefore, for $E^R = E^*$ (hence, $B_3^R = B_3^*$) to be a solution, it must be the case that the representative bank cannot increase the representative firm's expected payoff by increasing E above E^*. First, assume that the social planner's solution is characterized by an interior solution, that is,

$$\frac{\partial V}{\partial E}(B_3^*, E^*) + \frac{\partial V}{\partial z}\frac{\partial z}{\partial E}(B_3^*, E^*) = -\frac{\partial V}{\partial B_3}(B_3^*, E^*)\frac{\mathrm{d}B_3}{\mathrm{d}E}\bigg|_{(\text{PC})}.$$

Therefore, the private marginal benefit at (B_3^*, E^*) must satisfy

$$\frac{\partial V}{\partial E}(B_3^*, E^*; z(B_3^*, E^*)) < -\frac{\partial V}{\partial B_3}(B_3^*, E^*; z(B_3^*, E^*))\frac{\mathrm{d}B_3}{\mathrm{d}E}\bigg|_{\text{(PC)}},$$

that is, increasing E will decrease the representative firm's expected payoff when aggregate liquidations are taken as given. Thus, the representative bank will find it optimal to increase E above E^*; hence, choosing (B_3^*, E^*) will constitute an equilibrium. On the other hand, if the social planner's solution is characterized by a corner solution, no choice of $E \geq E^*$ is feasible without violating constraint (IC_2). Thus, in such situation, the only feasible choice of E is E^*.

B. Comparative statics of the laissez faire equilibrium

In this section, the properties of the equilibrium credit line contract are discussed. To that purpose, it is assumed that α is beta-distributed with parameters $a = 1$ and $b \geq 1$, that is,

$$\alpha \sim \text{Beta}(1, b),\ b \geq 1.$$

Such probability density function has support over the range $[0, 1]$ and is decreasing in α. Moreover, as b increases, high realizations of α are less likely to occur. That is to say, extremely high liquidity need states are rare events. Note that the uniform case, in which any realization of α is equally likely, can be obtained as a special case when $b = 1$.

Furthermore, it is assumed that the liquidation value function $Q(z)$ satisfies

$$Q(z) = Q_0(1 - \gamma_0 z^{\gamma_1}),$$

where $Q_0 > 0$, $\gamma_0 > 0$ and $\gamma_1 \geq 1$, that is, $Q(z)$ is decreasing and concave. It is important to remark that γ_0 measures the effect that a liquidation has on other firms' liquidation value. For instance, if $\gamma_0 = 0$, no liquidation would contribute to depress firms' liquidation value; i.e., $Q(z) = Q_0$ for all z.

Table 1 summarizes the comparative statics of the equilibrium contract chosen by a representative bank in an interior solution. These results are derived at the end of the section. The table shows the signs of the derivatives $dE^U/d\theta$ and $dB_3^U/d\theta$ with respect to a parameter denoted generically by θ.

As it can be observed, other things equal, an increase in the shape parameter b of the beta distribution function decreases E, because high realizations of α become rare; hence, having

	θ								
	b	R_0	R_1	γ_0	γ_1	Q_0	Y	X	ρ
$\frac{dE^U}{d\theta}$	$-$	$-$	$+$	$+$	$-$	$-$	$-$	$+$	0
$\frac{dB_3^U}{d\theta}$	$-$	?	$+$	$+$	$-$	$-$	$-$	$+$	0

Table 1: Comparative statics of contractual terms E and B_3^U (in an interior equilibrium)

high levels of E to insure firms against high liquidity need events becomes less attractive. The cost of facing liquidations is represented by parameters X, Q_0, γ_0, γ_1. For instance, more value is lost after a liquidation if continuation cash flow X is higher, the intercept of the liquidation value function Q_0 is lower, or the slope of the liquidation value function is steeper; i.e., higher γ_0 or lower γ_1. In such situations, it is optimal to increase E to reduce the negative effect of liquidations. Similarly, borrowing cost R_1 impacts positively on E, because fewer loans can be granted after a run if borrowing at $t = 1$ becomes more expensive, which makes runs costlier. Moreover, a higher pledgeable income Y reduces the need of pre-arranged funding E. Additionally, if the cost of raising pre-arranged funding R_0 increases, a lower E is chosen. Finally, as long as the equilibrium fee B_3^U does not hit constraint (IC_2), the return on the inefficient investment ρ does not affect equilibrium contractual terms.

The comparative statics of equilibrium fee B_3^U share the same signs as the comparative statics of equilibrium pre-arranged funding E^U, except for parameter R_0. A higher borrowing cost at $t = 0$ will require to increase the commitment fee to compensate investors who provide pre-arranged funding. However, at the same time, pre-arranged funding is reduced when R_0 is higher, which pushes down the commitment fee. As a result, the sign of the total effect of R_0 on equilibrium fee B_3^U cannot be determined.

Furthermore, recall that the commitment fee B_3 cannot be set excessively high; otherwise, the incentive compatibility constraint (IC_2) may not be satisfied. In such case, the equilibrium credit line will be characterized by a corner solution (see Proposition 3). Consequently, Table 1 provides also information about when the equilibrium contract will be characterized by a corner solution. For instance, if high realizations of α are likely to occur (a low b), large values of E are optimally chosen. Simultaneously, the commitment fee B_3 must also increase to finance them, thereby making less likely that the incentive compatibility constraint (IC_2) holds with slack. Similarly, a high R_1, γ_0, and X and a low Q_0, γ_1, and Y make it more likely that the equilibrium contract is characterized by a corner solution; see the second row in Table 1. Although the return on the inefficient investment ρ does not affect contractual terms in an interior equilibrium, it reduces the maximum fee that a firm can pay, which increases the possibility that the equilibrium contract is characterized by a corner solution.

Proof:

Assume that contractual terms in the laissez-faire regime are characterized by an interior solution. Hence, according to Proposition 3, E_3^U and B_3^U satisfy the following system of equations

$$\Psi_1(B_3^U, E^U; \theta) \equiv \frac{\partial V}{\partial E} + \frac{\partial V}{\partial B_3} \frac{\mathrm{d}B_3}{\mathrm{d}E}\bigg|_{(\mathrm{PC})} = 0, \tag{A.4}$$
$$\Psi_2(B_3^U, E^U; \theta) \equiv \int_0^{\alpha} \big(\alpha Y + (1-\alpha)B_3^U - (\alpha - E^U)\big) f(\alpha)\mathrm{d}\alpha - R_0 E^U = 0,$$

where θ is a vector of parameters.

From equation Ψ_2, B_3^U can be expressed as a function of E^U, that is, $B_3^U = g(E^U;\theta)$. Therefore, Ψ_1 can be expressed as a function of E^U; i.e., $\Psi_1(g(E^U;\theta), E^U;\theta)$.

The sign of the derivative of E^U respect to a generic parameter θ_k is obtained by total differentiation of equation Ψ_1:

$$\frac{\mathrm{d}E^U}{\mathrm{d}\theta_k} = -\frac{1}{\partial\Psi_1/\partial E}\Big(\frac{\partial \Psi_1}{\partial \theta_k} - \frac{\partial \Psi_1}{\partial B_3}\frac{\partial\Psi_2/\partial\theta_k}{\partial\Psi_2/\partial B_3}\Big),$$

where $\frac{\partial\Psi_1}{\partial E} < 0$ by the second order condition, which implies

$$\mathrm{sign}\Big(\frac{\mathrm{d}E^U}{\mathrm{d}\theta_k}\Big) = \mathrm{sign}\Big(\frac{\partial \Psi_1}{\partial \theta_k} - \frac{\partial \Psi_1}{\partial B_3}\frac{\partial\Psi_2/\partial\theta_k}{\partial\Psi_2/\partial B_3}\Big). \tag{A.5}$$

Similarly, by total differentiating Ψ_2, the sign of the derivative of B_3^U respect to θ_k can be obtained as

$$\mathrm{sign}\Big(\frac{\mathrm{d}B_3^U}{\mathrm{d}\theta_k}\Big) = \mathrm{sign}\Big(-\frac{\partial\Psi_2/\partial\theta_k}{\partial\Psi_2/\partial B_3} - \frac{\partial\Psi_2/\partial E}{\partial\Psi_2/\partial B_3}\frac{\mathrm{d}E^U}{\mathrm{d}\theta_k}\Big), \tag{A.6}$$

where the first and second term are the direct and indirect (through movements in E) effect of θ_k on B_3^U, respectively.

It can be proved that $\frac{\partial\Psi_1}{\partial E}$, $\frac{\partial\Psi_1}{\partial B_3}$, and $\frac{\partial\Psi_2}{\partial E}$ have negative signs, whereas $\frac{\partial\Psi_2}{\partial B_3}$ has a positive sign. Moreover, it can be easily shown that $\frac{\partial\Psi_2}{\partial\theta_k} = 0$ for $\theta_k = R_1, \gamma_0, \gamma_1, Q_0, X$. Hence, for such parameters

$$\mathrm{sign}\Big(\frac{\mathrm{d}E^U}{\mathrm{d}\theta_k}\Big) = \mathrm{sign}\Big(\frac{\mathrm{d}B_3^U}{\mathrm{d}\theta_k}\Big) = \mathrm{sign}\Big(\frac{\partial\Psi_1}{\partial\theta_k}\Big).$$

Lastly, it can be shown $\frac{\partial \Psi_1}{\partial \theta_k} > 0$ for $\theta_k = \gamma_0, R_1, X$ and $\frac{\partial \Psi_1}{\partial \theta_k} < 0$ for $\theta_k = \gamma_1, Q_0$, which deliver the results in Table 1.

For the remaining three parameters, it can be demonstrated that $\frac{\partial \Psi_1}{\partial R_0} < 0$, $\frac{\partial \Psi_2}{\partial R_0} < 0$, $\frac{\partial \Psi_1}{\partial b} < 0$, $\frac{\partial \Psi_2}{\partial b} > 0$, $\frac{\partial \Psi_1}{\partial Y} < 0$, and $\frac{\partial \Psi_2}{\partial Y} > 0$. By using the expressions in (A.5) and (A.6), we can obtain the results in Table 1 for the remaining parameters.