Information Transmission in Voluntary Disclosure Games*

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Abstract

Does a better-informed sender transmit more accurate information in equilibrium? We show that, in a general class of voluntary disclosure games, unlike other strategic communication environments, the answer is positive. If the sender's evidence is more Blackwell-informative then the receiver's equilibrium utility increases. We apply our main result to show that an uninformed sender who chooses a test from a Blackwell-ordered set does so efficiently.

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1 Introduction

Voluntary disclosure plays a central role in many markets with information asymmetries. Even when agents' interests are not aligned, if an informed agent holds hard evidence, she can disclose pieces of it to promote her interests. The commonness of voluntary disclosure in communication environments, e.g., of annual reports by public companies, verifiable curricula vitae by job candidates, etc., has inspired a vast body of literature in economics, finance, and accounting.

A question that has not been answered yet by this literature is whether a sender with better information transmits more of it in equilibrium. In cheap-talk models, for example, the answer is trivially negative.[1] Recent literature addressing other communication models, such as costly disclosure and signaling,[2] also points to a non-monotone link between the sender's access to information and equilibrium communication. Nevertheless, we show that in voluntary disclosure environments, better-informed agents transmit more accurate information *in equilibrium*.

We study a generalized Shin (1994) model of voluntary disclosure: in each state of the world, nature performs two conditionally independent lotteries. The first determines the realizations of a given set of signals (Blackwell experiments), and the second determines which subset of those signals is in the sender's possession. The sender decides which subset of the signals in her possession she should disclose, and then the receiver chooses an action (a real number). The two players' interests regarding this action are not aligned. Whereas the receiver's goal is to coordinate the action with the state, the sender aims to maximize the action. Our main result states that whenever the signals are more Blackwell-informative the receiver's expected utility increases.

To provide the reader with some intuition for the reason a more informative sender communicates more information, we begin with a simple example (Section 2). We study a model with an evidence structure à la Dye (1985), where the set of signals is a singleton. As

[1] In simple examples of Crawford and Sobel's (1982) model, limiting the sender's information can ease her incentive compatibility constraints and allow for a more refined equilibrium partition.

[2] For examples of each, see Harbaugh and Rasmusen (2018) and Ball (2020), respectively.

is well known, the equilibrium in Dye's (1985) model is characterized by a threshold; i.e., an informed sender discloses her evidence if and only if it has an implied posterior mean above some value. This threshold characterization allows us to decompose the effects of an increase in the signal's informativeness to its direct effect and its strategic effect, and to show that both benefit the receiver.

However, such a decomposition method cannot accommodate the general model since the equilibrium does not have a simple characterization and usually involves mixing. Therefore, we apply a recent result by Hart et al. (2017) to show that the equilibrium question can be reduced to a mechanism design question. In this way, we can prove that the receiver's utility is increasing in the signals' informativeness without providing a characterization of the equilibrium.

Hart et al. (2017) show that commitment power does not help the receiver obtain higher utility. That is, the receiver's utility under the optimal (deterministic) mechanism is the same as his utility in (his most preferred) equilibrium. To prove our general claim, we construct a direct (potentially) random mechanism that generates the exact same distribution of the state of the world and the receiver's action as the optimal mechanism of any given less informative evidence structure. Subsequently, we construct a deterministic incentive compatible mechanism that gives the receiver a higher utility than the mimicking mechanism. Therefore, the mimicking mechanism gives the receiver a lower utility than the optimal deterministic mechanism, and the reduction implies that the receiver's *equilibrium* utility is increasing in the sender's informativeness.

We apply our main result to DeMarzo et al.'s (2019) model of an endogenous Dye (1985) evidence structure, where an uninformed sender chooses a test in private and then decides whether she should disclose its result. They show that, in equilibrium, the sender's choice minimizes the disclosure threshold. Therefore, our main result implies that when the sender faces a Blackwell-ordered set her choice is efficient. Since the disclosure threshold is decreasing in the test's informativeness (Jung and Kwon, 1988), the sender chooses the most informative test which, according to our result, maximizes the receiver's expected utility.

This paper contributes to the literature on strategic disclosure. Starting with Grossman (1981) and Milgrom (1981), the economic literature discusses environments in which communication is verifiable. Though the early models indicate information unraveling, Dye (1985) and Jung and Kwon (1988) establish an equilibrium with partial disclosure by allowing for uncertainty regarding the sender's informativeness. Verrecchia (1983) shows that such an equilibrium can be obtained if disclosure is costly.

Our model generalizes the evidence structure of Shin (1994), who studies a model of binary signals where each one signifies whether the state is above or below some value. Shin's (1994) structural assumptions guarantee the existence of an equilibrium in which the sender discloses all, and only, her "good" evidence. By contrast, our general model does not admit such assumptions, and therefore the equilibrium does not have a simple characterization.

Our proof applies findings in the disclosure literature on the value of commitment power. Glazer and Rubinstein (2004, 2006) study verifiable communication models in which the receiver's action is binary, and they show that the receiver does not gain from commitment power. This result is extended to multi-action environments by Sher (2011) and is further generalized by Hart et al. (2017). Ben-Porath et al. (2019) prove that the "no value for commitment" result can be extended to a multi-sender environment, in which some senders wish to maximize the receiver's action and some wish to minimize it. A similar result is shown to hold in a special case by Bhattacharya and Mukherjee (2013). To the best of our knowledge, our paper is the first to show that these equivalency results can constitute a tool for answering natural questions in the disclosure literature.

The closest paper to ours in the disclosure literature is Rappoport (2020) who shows that a more informed sender encounters more pessimism on the receiver's part. Unlike our model, Rappoport's (2020) does not distinguish between the distribution of the underlying state and the sender's ability to provide evidence. He studies a reduced-form evidence model in the spirit of Hart et al. (2017) where the sender's type is defined as a pair: the first dimension corresponds to the receiver's utility, and the second dimension specifies the set of types the sender can mimic.[3] In this model, Rappoport (2020) defines the evidence structure as more

[3] A somewhat similar reduced-form approach to evidence modeling is taken by Lipman and Seppi (1995).

informative if, for every pair of types of the sender, such that one can mimic the other, the relative probability of the former is higher. He shows that if the evidence structure is more informative according to this definition then the receiver's equilibrium action is lower for each report of the sender.

Our model differs from the reduced-form model in a way that allows us to compare the receiver's utility across different evidence structures. In principle, one can interpret the reduced-form model as if the sender's type corresponds to an actual state of the world or a distribution over a state space. However, under such interpretation, Rappoport's (2020) definition of a more informed sender would typically imply a change in the prior distribution of the state and thus does not allow for a comparison of the receiver's utility in the two evidence structures. Moreover, in the reduced-form model, an alternative definition of an increase in the informativeness of an evidence structure, using the Blackwell order (similar to the one in our model), is also not applicable for such a comparison. Since the sender's type also determines which types she can mimic, a garbling of the types' distribution changes not only the quality of the evidence but also the strategic environment. Therefore, to study the effect of an informativeness change in the evidence structure on the receiver's utility, one has to define an explicit model of evidence that disentangles the quality of the sender's information from her available strategies.

Our paper also connects to the literature that studies the relation between the sender's ability to transmit information and equilibrium performance. Harbaugh and Rasmusen (2018) study a model of voluntary certification with a disclosure cost in the spirit of Verrecchia (1983), and show that grade-coarsening is optimal. That is, the receiver may observe more information in equilibrium if the quality of the sender's information worsens. Ball (2020) studies a communication model with costly distortion, in which an intermediary observes the sender's message and aggregates it into a score. He shows that a partly informative score can be optimal for the receiver. In a related framework, Whitmeyer (2019) shows how a receiver can profit from garbling the sender's message.[4]

[4] See also Frankel and Kartik (2019) who show that, in a costly distortion environment, commitment can help the receiver achieve better results.

The rest of the paper is organized as follows. In Section 2 we present an example. In Section 3 we discuss our model. In Section 4 we study the effect of a change in the informativeness of the evidence structure. In Section 5 we present an application. Section 6 concludes.

2 Example

Before we present the general model, we start with an analysis of a simple evidence structure, the Dye (1985) model. With probability q, the sender (she) obtains a verifiable realization of a signal X containing information about the state of the world ω, and is otherwise uninformed. An informed sender can either disclose her evidence truthfully or pretend to be uninformed. Then, the receiver (he) observes the sender's disclosure (or lack thereof) and chooses an action $a \in \mathbb{R}$. In contrast to the sender who wishes to maximize a regardless of the state, the receiver wishes to coordinate a with ω. Specifically, assume that the receiver's utility takes the quadratic form,[5] i.e., $u_R(a, \omega) = -(a-\omega)^2$.

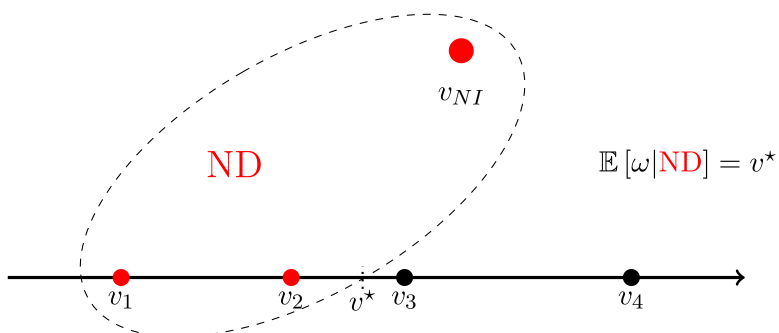

Figure 1: Equilibrium

The "no-disclosure" set: informed types with implied posterior mean $v_i \leq v^\star$ and the uninformed type do not disclose and thereby induce action $v^\star$.

Let v_i denote the posterior mean implied by evidence $x_i \in X$, and let H denote the distribution (CDF) of v. Dye (1985) and Jung and Kwon (1988) show that the equilibrium in this game is defined by a threshold $v^\star$, depicted in Figure 1. Informed types disclose

[5] Recall that quadratic preferences imply that the receiver's optimal action is the posterior-mean given his belief.

their evidence if and only if its implied posterior mean is above the no-disclosure action $v^\star$, which is defined by the (unique; see Acharya et al., 2011) solution of

$$v^\star = \frac{qH(v^\star)\,\mathbb{E}[\omega|v \le v^\star] + (1-q)\,\mathbb{E}[\omega]}{qH(v^\star) + (1-q)}. \tag{1}$$

A Direct Approach To see why the receiver is better off if the sender's signal is more informative, consider the following example, depicted in Figure 2. Assume that evidence x_3 is replaced with two more accurate pieces of evidence, $\underline{x_3}$ and $\overline{x_3}$, where $\underline{v_3} < v_3 < \overline{v_3}$, and further assume that $\underline{v_3} < v^\star$. We can analyze the effect of this mean-preserving spread in two steps. First, suppose that the threshold does not change, and upon no disclosure, the receiver chooses action $v^\star$. In such a case, it is clear that the receiver is better off. Following the disclosure of $\overline{x_3}$ the receiver's action is $\overline{v_3}$, and, since $\overline{v_3}$ is the optimal action given this event, it is a more appropriate action than v_3. Also, even though $\underline{x_3}$ does not disclose her evidence, the threshold action $v^\star$ is a more appropriate action in this event than v_3 as $\underline{v_3} < v^\star < v_3$. Therefore, when holding both players' strategies fixed, the informational effect of splitting x_3 is beneficial for the receiver.

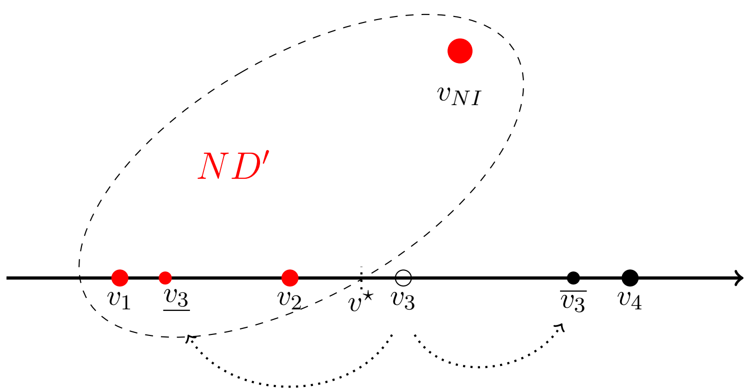

Figure 2: Direct Effect

Fixing the disclosure threshold at $v^\star$, i.e., $ND' = ND \cup \{\underline{x_3}\}$, the direct effect of additional information is positive.

Second, we argue that the strategic effect also works in the receiver's favor. As was shown by Jung and Kwon (1988), a mean-preserving spread of the evidence distribution induces a decrease in the disclosure threshold. By adding $\underline{x_3}$ to the no-disclosure set, we decrease the expectation of ω conditional on $v \le v^\star$, and thereby obtain the equilibrium at a lower disclosure threshold $\widetilde{v}^\star$. Next, we split ND' (the union of the original no-disclosure set and $\underline{x_3}$) into two: the "new" no-disclosure set $\widetilde{ND}$, and the informed types with posterior mean between the two thresholds (v_2 in Figure 3). For the set $\widetilde{ND}$, action $\widetilde{v}^\star$ is the optimal action and hence more appropriate than action $v^\star$. In addition, informed types between the two thresholds disclose in the new equilibrium and induce their optimal action. That is, the receiver's utility, relative to the first step, necessarily increases.

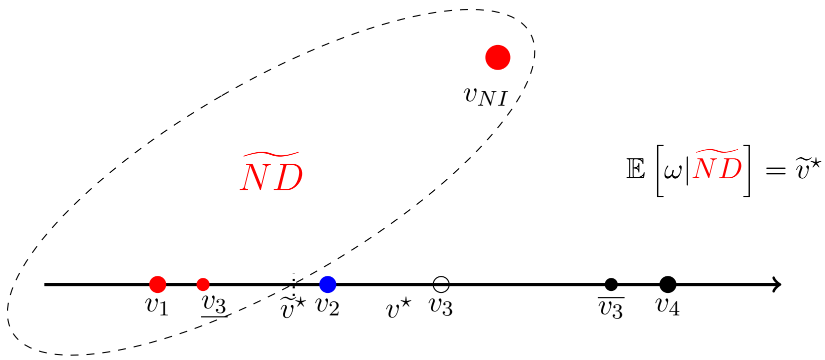

Figure 3: Strategic Effect

The receiver is better off when the no-disclosure action is $\widetilde{v}^\star$.

Similar arguments apply to other mean-preserving spreads, and one can deduce that the receiver is better off when the sender's signal is more informative. However, as mentioned in the introduction, this intuitive reasoning cannot be generalized. The equilibrium of the general model is quite involved, and we cannot decompose the effect of additional information into such two simple channels. Therefore, we take an indirect approach.

An Indirect Approach There is another way to see why the receiver is better off if the sender's signal is more informative. Consider a receiver who, for some reason, can commit to the action he would take after the disclosure of each piece of evidence. In the more informative evidence structure, such a receiver can commit to the following direct mechanism.

$$\psi\left(x\right)=\begin{cases} v^{\star}, & x\in\left\{ x_{1},x_{2}\right\} \cup\emptyset\\ v_{3}, & x\in\left\{ \underline{x_{3}},\overline{x_{3}}\right\} \\ v_{4}, & x=x_{4} \end{cases} \tag{2}$$

Mechanism ψ mimics the equilibrium payoffs of the less informative structure. Evidence $\underline{x_3}$ and $\overline{x_3}$ are treated as their corresponding evidence in the less informative signal, x_3. If the sender does not disclose her evidence ($x = \emptyset$), or if she discloses evidence with posterior mean below $v^{\star}$ (other than $\underline{x_3}$), the receiver chooses the corresponding no-disclosure action $v^{\star}$. And, if the sender discloses evidence with posterior-mean above $v^{\star}$ (other than $\overline{x_3}$), the receiver chooses the optimal action.

Since each informed type obtains a weakly higher payoff when disclosing truthfully, mechanism ψ is incentive compatible. In addition, mechanism ψ mimics the equilibrium of the less informative signal state by state, and therefore it guarantees the receiver the same utility. Now, we can apply Hart et al.'s (2017) result on the equivalency between the optimal mechanism and equilibrium. They show that, in voluntary disclosure games, commitment power does not help the receiver obtain a higher payoff. The optimal (deterministic) mechanism in the game with the more informative signal, which is better than ψ by definition, provides the receiver the same utility that he achieves in equilibrium. Therefore, we can deduce that the receiver's *equilibrium* utility is higher in the game with the more informative signal.

Unlike the direct approach, this method can apply to a quite general class of voluntary disclosure games. We show that, even without a characterization of the equilibrium, we can construct a mechanism that mimics the joint distribution of the receiver's action and the state induced by any less informative evidence structure. The general "mimicking" mechanism is a bit more complex and involves randomization. Therefore, in addition to the argument above, we still need to show that the incentive-compatibility constraints hold and that we can apply Hart et al.'s (2017) result.

3 Model and Preliminary Analysis

Next, we study a general model in which we allow for a rich structure of evidence and a more general functional form of the receiver's utility.

3.1 Model

A voluntary disclosure game is a communication game between an informed player (the sender, or she) and a decision-maker (the receiver, or he). In the first stage of the game, the sender decides which evidence to disclose. Then, the receiver chooses an action $a \in \mathbb{R}$.

State of the World and Preferences The state of the world is $\omega \in \Omega$, where Ω is a finite set and $f \in \Delta\Omega$ is its prior distribution. We assume that the sender's preferences do not depend on the state of the world and she wishes to maximize the receiver's action; i.e., $u_S(a)$ is a strictly increasing function. The receiver's utility, however, depends also on the state. Specifically, we assume that, for every state ω, $u_R(a, \omega)$ is differentiable, single-peaked,[6] and concave.

Evidence There is a set of n conditionally independent signals $\Sigma = \{\sigma_1, \sigma_2, \dots, \sigma_n\}$. For each signal $\sigma_i \in \Sigma$, we denote by S_i its finite set of possible realizations, and by s_i a generic realization. A signal σ_i is a mapping $\sigma_i : \Omega \longrightarrow \Delta S_i$, where $\sigma_i(s_i|\omega)$ denotes the probability of a realization $s_i \in S_i$ given a state $\omega \in \Omega$. In addition, there is a mapping $Q : \Omega \to \Delta 2^N$, where $N = \{1, 2, \dots, n\}$, that determines which signals the sender obtains given a state ω. That is, for every $A \subseteq N$, $Q(A|\omega)$ denotes the conditional probability that the sender observes the realizations of the subset of signals $\{\sigma_i \in \Sigma | i \in A\}$.

The set of signals Σ and the mapping Q induce a conditional probability distribution $G(\cdot|\omega)$ over the set of possible profiles of evidence $\mathcal{E} := \times_{i=1}^{n}(S_i \cup \{\emptyset\})$, with a generic element E. The interpretation is as follows: the sender can obtain at most one realization of each signal, and if she does not obtain any realization of signal σ_i then the ith coordinate

[6] For every ω, there exists a' such that, $\frac{\partial}{\partial a} u_R(a', \omega) = 0$, $\frac{\partial}{\partial a} u_R(a, \omega) > 0$ for $a < a'$, and $\frac{\partial}{\partial a} u_R(a, \omega) < 0$ for $a > a'$.

of the profile $E \in \mathcal{E}$ is $\emptyset$, i.e., $E_i = \emptyset$. We denote the set of signals for which $E_i \neq \emptyset$ by $A_E \subseteq N$. Consider a profile of evidence $E \in \mathcal{E}$ such that for every $i \in A_E$ we have $E_i = s_i \in S_i$. The probability that the sender obtains evidence profile E in state $\omega \in \Omega$ is given by

$$G(E \mid \omega) := Q(A_E|\omega) \cdot \Pi_{i \in A_E} \sigma_i (s_i|\omega) . \tag{3}$$

As can be seen from (3), the distribution of the profile of evidence conditional on the state, $G(\cdot|\cdot)$, is pinned down by the set of signals Σ and the mapping Q. We call this distribution an *evidence structure* and denote it by $G(\Sigma, Q)$.

Strategies The set of strategies available to the sender depends on her type, that is, the profile of evidence in her possession. Intuitively, we assume that the sender must disclose the truth but not necessarily the whole truth. Formally, the set of pure strategies of a sender who possesses evidence profile $E \in \mathcal{E}$, denoted by $\Gamma_E \subseteq \mathcal{E}$, is defined as follows.

Definition 1. *$E' \in \mathcal{E}$ is a pure strategy that is available for sender of type $E \in \mathcal{E}$ if and only if the following conditions hold:*

- $A_{E'} \subseteq A_E$.
- $\forall i \in A_{E'}$, $E'_i = E_i$.

Thus type E's set of available strategies is $\Delta\Gamma_E$, where γ_E denotes the generic element of this set, and $\gamma_E(E')$ denotes the probability that the strategy assigns to message $E' \in \Gamma_E$. The set of available strategies for the sender is $\Gamma := \times_{E \in \mathcal{E}} \Delta\Gamma_E$, with a generic element γ.

The set of available strategies for the receiver, Π, is the set of all mappings[7] $\pi : \mathcal{E} \to \mathbb{R}$. Following the disclosure decision of the sender, the receiver forms a belief about the state, $\mu : \mathcal{E} \to \Delta\Omega$. We denote by $\mu_E(\omega)$ the probability that the belief μ assigns to state $\omega \in \Omega$ after the sender has disclosed the profile $E \in \mathcal{E}$.

[7]The concavity of u_R implies that we can assume without loss of generality that the receiver can play only pure strategies.

3.2 Preliminary Analysis

Equilibria An equilibrium is defined as a pair, $(\gamma^\star, \pi^\star)$, together with a belief $\mu^\star$.

The strategy of the sender, $\gamma^\star$, satisfies

$$\gamma_E^\star\left(E'\right) > 0 \implies E' \in \underset{E'' \in \Gamma_E}{\arg\max}\ u_S\left(\pi^\star\left(E''\right)\right). \tag{4}$$

The receiver's strategy, $\pi^\star$, satisfies

$$\pi^\star(E) = \underset{a \in \mathbb{R}}{\arg\max}\ \mathbb{E}_{\mu_E^\star}\left[u_R(a,\omega)\right], \tag{5}$$

where $\mu^\star$ is consistent with $\gamma^\star$ along the equilibrium path. That is, for every $E \in \mathcal{E}$ that is played with positive probability,

$$\mu_E^\star(\omega) = \frac{f(\omega) \sum\limits_{E' \in \{E'' \mid E \in \Gamma_{E''}\}} G\left(E' \mid \omega\right) \gamma_{E'}^\star(E)}{\sum\limits_{\omega' \in \Omega} f(\omega') \sum\limits_{E' \in \{E'' \mid E \in \Gamma_{E''}\}} G\left(E' \mid \omega'\right) \gamma_{E'}^\star(E)}. \tag{6}$$

Equilibrium Selection As is well known, communication games admit a plethora of equilibria. However, because we are interested in studying the effect of the sender's informativeness on the receiver's utility, we limit our discussion to the receiver's most preferred equilibrium. The receiver-optimal equilibrium is also the focus of recent literature on disclosure games; see, for example, Jiang (2019) and Rappoport (2020). In addition, the receiver-optimal equilibrium is the unique equilibrium that survives the truth-leaning refinement (Hart et al., 2017),[8] and is the unique equilibrium in which there are no self-signaling sets (Bertomeu and Cianciaruso, 2018).[9] We denote the utility of the receiver in this equilibrium by $\widetilde{U}_R(G)$.

[8] A truth-leaning equilibrium is defined as an equilibrium in which the sender discloses truthfully unless she can strictly gain from hiding some evidence, and her off-path messages are taken at face value.

[9] This refinement is in the spirit of Grossman and Perry (1986), where off-path beliefs must satisfy the following consistency requirement. Consider an equilibrium and a message E that is not played in this equilibrium. If there exists a set of sender's types $D \subseteq \mathcal{E}$ and a belief $\mu \in \Delta D$ with full support such that, given belief μ, all the sender's types in D and only those types find it profitable to deviate to E, then μ must be the off-path belief following such a deviation. That is, an equilibrium does not survive the "no self-signaling sets" refinement if there exists a deviation with the above property.

4 Informativeness

We now turn to study the effect of changes in the evidence structure. We show that an increase in the Blackwell-informativity of the sender's signals implies an increase in the receiver's utility.

Proposition 1. *Let $Q : \Omega \to \Delta 2^N$ and let $\Sigma = \{\sigma_1, ..., \sigma_n\}$ and $\widehat{\Sigma} = \{\widehat{\sigma}_1, ..., \widehat{\sigma}_n\}$ be two sets of signals. If, for every i, σ_i is more Blackwell-informative than $\widehat{\sigma}_i$, then $\widetilde{U}_R\left(G(\Sigma, Q)\right) \geq \widetilde{U}_R\left(G(\widehat{\Sigma}, Q)\right)$.*

We defer the proof of Proposition 1 to the appendix. Here, We provide a sketch of it. First, due to a standard transitivity argument, it is sufficient to prove the claim for the case where there exists $i \in N$ such that σ_i is more Blackwell-informative than $\widehat{\sigma}_i$ and, for every $j \neq i$, we have that $\sigma_j = \widehat{\sigma}_j$. We prove Proposition 1 for this case using Hart et al.'s (2017) equivalence between the optimal *deterministic* mechanism and the receiver-optimal equilibrium.[10] That is, instead of comparing the receiver's equilibrium payoff under both evidence structures, we compare his payoffs in the optimal *deterministic* mechanisms, where he can commit in advance to an action given any report of the sender.

Let $\psi^{\star}(G)$ denote the optimal deterministic mechanism for evidence structure G. Since σ_i is more Blackwell-informative than $\widehat{\sigma}_i$, evidence structure $G(\widehat{\Sigma}, Q)$ can be obtained by a garbling of $G(\Sigma, Q)$. We construct a direct mechanism for evidence structure $G(\Sigma, Q)$ that mimics this garbling. For every report of the sender that includes an element of S_i, the mechanism performs a lottery that "garbles" S_i into $\widehat{S}_i$, and, for each realization, the mechanism commits the action that corresponds to the resulting profile of evidence under $\psi^{\star}\left(G\left(\widehat{\Sigma}, Q\right)\right)$. Otherwise, if the sender's report does not include an element of S_i, i.e. $E_i = \emptyset$, the mechanism chooses the same action as in $\psi^{\star}\left(G\left(\widehat{\Sigma}, Q\right)\right)$. This construction shows that, as long as the garbling mechanism is incentive compatible, a receiver with commitment power can obtain, under evidence structure $G(\Sigma, Q)$, the same payoff as in $\psi^{\star}\left(G\left(\widehat{\Sigma}, Q\right)\right)$.

[10] In the proof we show that our framework is a special case of the framework considered at Hart et al. (2017) and thus we can apply their result.

However, the mimicking mechanism is potentially random. Therefore, we can not apply Hart et al.'s (2017) equivalence result directly, since, in general, the receiver's utility from a random mechanism might be strictly higher than in equilibrium.[11] To get around this problem, we construct a *deterministic* mechanism that improves upon the mimicking mechanism. For each report of the sender, the deterministic mechanism executes an action equal to the expectation of the actions induced by the corresponding lottery under the mimicking mechanism. Assuming it is incentive compatible, the deterministic mechanism improves upon the mimicking mechanism since the receiver's preferences are concave. It follows that the receiver's payoff in $\psi^\star\left(G\left(\widehat{\Sigma}, Q\right)\right)$ is (weakly) lower than his payoff in the constructed deterministic mechanism, which is (weakly) lower than the receiver's payoff in $\psi^\star\left(G\left(\Sigma, Q\right)\right)$ (and in the equilibrium of $G\left(\Sigma, Q\right)$.) Therefore, all we are left to show is that both mechanisms are indeed incentive compatible.[12]

In the proof, we go over all possible sender types and show that the incentive constraints hold. Here, we concentrate on the most challenging case in which a type $E \in \mathcal{E}$ with $E_i \neq \emptyset$ is contemplating whether she should report $E' \in \Gamma_E$ where $E'_i \neq \emptyset$. Since the garbling that generates $\widehat{\sigma}_i$ from σ_i is independent of the state, it is also independent of coordinates different than i. Therefore, reporting E' induces the same lottery as E but with lower action for each lottery realization. Fix such a realization; for both E and E', the mimicking mechanism "replaces" the i'th coordinate in the sender's report with the same piece of evidence, and commits the action that corresponds to the resulting profile under $\psi^\star\left(G\left(\widehat{\Sigma}, Q\right)\right)$. Meaning, reporting E induces an action that corresponds to a truthful disclosure of some type under $\psi^\star\left(G\left(\widehat{\Sigma}, Q\right)\right)$ while reporting E' induces an action that corresponds to a possible deviation of the same type. Since $\psi^\star\left(G\left(\widehat{\Sigma}, Q\right)\right)$ is incentive compatible, we know that, for each realization of the lottery, reporting E induces a (weakly) higher action than reporting E', and thus this deviation is not profitable. Using similar arguments, we show that no

[11] In different environments, Hart et al. (2016) and Sher (2011) prove an equivalence between the receiver-optimal equilibrium and the optimal random mechanism. However, they impose additional assumptions that do not hold in our model. In particular, we require that the sender's preferences be monotone in the receiver's action and that the receiver's preferences be concave without any additional restriction, while they require a "relative" concavity condition between the receiver's and the sender's preferences.

[12] In fact, it is enough to show that the constructed deterministic mechanism is IC, but our way to show it goes through the incentive compatibility of the (potentially) random mechanism.

type of sender has a profitable deviation from truthful disclosure. Moreover, the action distribution induced by truthful disclosure dominates the action distribution induced by a deviation. Therefore, our *deterministic* mechanism, which replaces the lotteries with their expectation, is also incentive compatible. Applying Hart et al.'s (2017) equivalence result, we show that Proposition 1 follows.

5 Application: Evidence Gathering

In this section, we apply our main result to discuss test choice efficiency. We show that, in DeMarzo et al.'s (2019) endogenous evidence model, the sender's choice is efficient. A sender who chooses a test from a Blackwell-ordered set chooses the test that maximizes the receiver's utility.[13]

We study the implications of our result for DeMarzo et al.'s (2019) model of evidence gathering. A sender chooses a test in private from a given set $\mathcal{T} = \{T_1, T_2, \ldots, T_m\}$, where each test induces a Dye (1985) disclosure game in which the sender obtains evidence with probability q. As discussed in Section 2, if it is known that the sender's choice was T_i, then the equilibrium of the induced sub-game is defined by the unique solution to[14]

$$v_i^\star = \frac{qH_i\left(v_i^\star\right)\mathbb{E}\left[\omega|v_i\left(s_i\right) \leq v_i^\star\right] + (1-q)\,\mathbb{E}\left[\omega\right]}{qH_i\left(v_i^\star\right) + 1 - q}, \tag{7}$$

where $v_i\left(s_i\right)$ denotes the posterior mean of ω implied by realization s_i in test T_i, and H_i denotes its CDF.

For a sender whose preferences are linear, e.g., $u\left(a\right) = a$, her equilibrium choice can be characterized by what DeMarzo et al. (2019) call the "minimum principle." That is, the sender chooses a test $\widetilde{T}$ that minimizes the equilibrium action for no-disclosure, i.e., $\widetilde{T} \in \arg\min_i v_i^\star$. This characterization allows us to show the following result concerning their framework. Denote by $\widetilde{U}_R\left(T_i\right)$ the expected utility of the receiver in the sub-game induced by an observed choice of T_i, and denote by $\widehat{U}_R$ his expected utility in the equilibrium

[13] We adopt here DeMarzo et al.'s (2019) terminology of "tests," but these are equivalent to the signals discussed above.

[14] As in Section 2, we assume that the receiver's utility is quadratic; hence, his optimal action is the posterior-mean given his beliefs.

of the evidence gathering game. Proposition 1 implies that the sender's choice maximizes the receiver's expected utility.

Proposition 2. *Assume that the set of available tests, $\mathcal{T}$, is Blackwell-ordered. In the equilibrium of the evidence gathering game, $\widehat{U}_R = \arg\max_{T_i \in \mathcal{T}} \widetilde{U}_R(T_i)$; i.e., the sender chooses the most informative test and maximizes the receiver's utility.*

Proof. By Jung and Kwon's (1988) result discussed in Section 2, a more informative test implies a lower disclosure threshold. Thus, the sender's choice is the most informative test, in which the fixed point $v_i^\star$ is minimized. By Proposition 1, we deduce that the sender's choice maximizes the receiver's utility. ∎

Note that DeMarzo et al. (2019) already characterize choice efficiency in the sense that the sender chooses the most informative test. However, without our Proposition 1, this result does not imply that the receiver's utility is maximized.[15]

6 Conclusion

In this paper, we have asked whether a better-informed sender communicates more information in voluntary disclosure games. Applying recent results in the disclosure literature, we have shown that this question can be reduced to a mechanism design problem and proved that a better-informed sender communicates information more effectively. If the sender's evidence is more Blackwell-informative then the receiver's expected utility in equilibrium increases. We have also applied our findings to discuss choice efficiency in a model with an endogenous evidence structure.

[15] See also Ben-Porath et al. (2018) who study a project choice in a disclosure environment and, among other results, characterize the sender's choice between projects that have the same expectation and are ranked according to second-order stochastic dominance. This choice is closely related to a choice between information structures that are Blackwell-ordered.

References

Acharya, V. V., P. DeMarzo, and I. Kremer (2011): "Endogenous information flows and the clustering of announcements," *American Economic Review*, 101, 2955–2979.

Ball, I. (2020): "Scoring strategic agents," *arXiv preprint arXiv:1909.01888*.

Ben-Porath, E., E. Dekel, and B. L. Lipman (2018): "Disclosure and choice," *The Review of Economic Studies*, 85, 1471–1501.

——— (2019): "Mechanisms with evidence: Commitment and robustness," *Econometrica*, 87, 529–566.

Bertomeu, J. and D. Cianciaruso (2018): "Verifiable disclosure," *Economic Theory*, 65, 1011–1044.

Bhattacharya, S. and A. Mukherjee (2013): "Strategic information revelation when experts compete to influence," *The RAND Journal of Economics*, 44, 522–544.

Crawford, V. P. and J. Sobel (1982): "Strategic information transmission," *Econometrica*, 50, 1431–1451.

DeMarzo, P. M., I. Kremer, and A. Skrzypacz (2019): "Test design and minimum standards," *American Economic Review*, 109, 2173–2207.

Dye, R. A. (1985): "Disclosure of nonproprietary information," *Journal of Accounting Research*, 23, 1431–1451.

Frankel, A. and N. Kartik (2019): "Improving information from manipulable data," *arXiv preprint arXiv:1908.10330*.

Glazer, J. and A. Rubinstein (2004): "On optimal rules of persuasion," *Econometrica*, 72, 1715–1736.

——— (2006): "A game theoretic approach to the pragmatics of debate: An expository note," in *Game Theory and Pragmatics*, Springer, 248–262.

GROSSMAN, S. J. (1981): "The informational role of warranties and private disclosure about product quality," *Journal of Law and Economics*, 24, 461–483.

GROSSMAN, S. J. AND M. PERRY (1986): "Perfect sequential equilibrium," *Journal of Economic Theory*, 39, 97–119.

HARBAUGH, R. AND E. RASMUSEN (2018): "Coarse grades: Informing the public by withholding information," *American Economic Journal: Microeconomics*, 10, 210–235.

HART, S., I. KREMER, AND M. PERRY (2016): "Evidence games with randomized rewards," *Working Paper*.

——— (2017): "Evidence games: Truth and commitment," *American Economic Review*, 107, 690–713.

JIANG, S. (2019): "Disclosure games with large evidence spaces," *arXiv preprint arXiv:1910.13633*.

JUNG, W.-O. AND Y. K. KWON (1988): "Disclosure when the market is unsure of information endowment of managers," *Journal of Accounting Research*, 26, 146–153.

LIPMAN, B. L. AND D. J. SEPPI (1995): "Robust inference in communication games with partial provability," *Journal of Economic Theory*, 66, 370–405.

MILGROM, P. R. (1981): "Good news and bad news: Representation theorems and applications," *Bell Journal of Economics*, 12, 380–391.

RAPPOPORT, D. (2020): "Evidence and skepticism in verifiable disclosure games," *Available at SSRN 2978288*.

SHER, I. (2011): "Credibility and determinism in a game of persuasion," *Games and Economic Behavior*, 71, 409–419.

SHIN, H. S. (1994): "News management and the value of firms," *The RAND Journal of Economics*, 25, 58–71.

Verrecchia, R. E. (1983): "Discretionary disclosure," *Journal of Accounting and Economics*, 5, 179–194.

Whitmeyer, M. (2019): "Bayesian elicitation," *arXiv preprint arXiv:1902.00976*.

Appendix

Proof of Proposition 1

As mentioned in the main text, it is enough to prove the proposition for Σ and $\widehat{\Sigma}$, which differ only in the first coordinate; i.e., σ_1 is more Blackwell-informative than $\widehat{\sigma}_1$. We prove Proposition 1 in four steps: we reduce the equilibrium question to a mechanism design question, construct a (potentially) random mimicking mechanism, construct a deterministic mechanism that improves upon the mimicking mechanism, and show that both mechanisms are incentive compatible.

Reduction To confirm that our model is a special case of Hart et al.'s (2017) reduced-form model, one only needs to check the transitivity property of the disclosure order that Hart et al. (2017) impose in their model. Consider $E, E', E'' \in \mathcal{E}$ such that $E \in \Gamma_{E'}$ and $E' \in \Gamma_{E''}$. We need to show that $E \in \Gamma_{E''}$. If $E \in \Gamma_{E'}$ then $A_E \subseteq A_{E'}$, and if $E' \in \Gamma_{E''}$ then $A_{E'} \subseteq A_{E''}$. It follows that $A_E \subseteq A_{E''}$. Additionally, for every $i \in A_E$ it holds that $E_i = E'_i$ and for every $i \in A_{E'}$ it holds that $E'_i = E''_i$. Because $A_E \subseteq A_{E'}$ it follows that for every $i \in A_E$ $E_i = E''_i$. We showed that $A_E \subseteq A_{E''}$ and that for every $i \in A_E$ we have that $E_i = E''_i$, it follows that $E \in \Gamma_{E''}$. Therefore, we can use Hart et al.'s (2017) result. That is, consider a receiver with commitment power, who states in advance which action would follow each disclosure of the sender. A deterministic mechanism is defined by a function $\psi : \mathcal{E} \to \mathbb{R}$, where $\psi(E)$ is the action the receiver takes in the case where the sender discloses E. The optimal mechanism is given by

$$\psi^{\star} := \underset{\psi:\mathcal{E}\to\mathbb{R}}{\arg\max} \mathbb{E}\left[u_R\left(\psi\left(E\right), \omega\right)\right], \tag{8}$$

$$s.t.$$

$$(IC): \ \forall E \in \mathcal{E}, \forall E' \in \Gamma_E, \psi\left(E\right) \geq \psi\left(E'\right). \tag{9}$$

Hart et al. (2017) show that in the optimal deterministic mechanism $\psi^\star$, the receiver chooses the same actions as in the unique (in payoff terms) receiver-optimal equilibrium, and thus the expected payoff of the receiver is the same in both circumstances.

The Mimicking Mechanism Next, we construct a (potentially) random mechanism for evidence structure $G(\Sigma, Q)$ that mimics the joint distribution of the state and actions that is induced by the optimal deterministic mechanism of $G(\widehat{\Sigma}, Q)$. The signal σ_1 is more Blackwell-informative than $\widehat{\sigma}_1$. That is, there exists a $|S_{\sigma_1}|$ by $|S_{\widehat{\sigma}_1}|$ "garbling" matrix L where $l^{k,j}$ denotes the probability that a realization $s^1_k \in S_{\sigma_1}$ is "garbled" to a realization $\widehat{s}^1_j \in S_{\widehat{\sigma}_1}$. Specifically, for every $\widehat{s}^1_j \in S_{\widehat{\sigma}_1}$ and a state of the world $\omega \in \Omega$ it holds that:

$$\widehat{\sigma}_1(\widehat{s}^1_j \mid \omega) = \sum_{s^1_k \in S_{\sigma_1}} \sigma_1(s^1_k \mid \omega) \cdot l^{k,j}. \tag{10}$$

Let $\psi^\star(G(\widehat{\Sigma}, Q)) : \widehat{\mathcal{E}} \to \mathbb{R}$ be the optimal deterministic mechanism of the less informative evidence structure $G(\widehat{\Sigma}, Q)$, and define the (potentially) random mimicking mechanism $\widetilde{\psi} : \mathcal{E} \to \Delta\mathbb{R}$ for the more informative evidence structure $G(\Sigma, Q)$ in the following way: let $E = (E_1, E_2, ..., E_n) \in \mathcal{E}$. If $E \in \widehat{\mathcal{E}}$, i.e., $E_1 = \phi$, then $\widetilde{\psi}(E) = \psi^\star(G(\widehat{\Sigma}, Q))(E)$. If this is not the case, that is, if there exists $s^1_k \in S_{\sigma_1}$ such that $E_1 = s^1_k$, then the mechanism runs a lottery and with probability $l^{k,j}$ the receiver's action is $\psi^\star(G(\widehat{\Sigma}, Q))(\widehat{s}^1_j, E_2, ..., E_n)$.

A Deterministic Mechanism Hart et al.'s (2017) result establishes an equivalence between the optimal *deterministic* mechanism and the receiver-optimal equilibrium. That is, to use this result, we need to find a deterministic mechanism that is incentive compatible and improves upon the mimicking mechanism. Such a mechanism, by definition, gives the receiver a (weakly) lower expected payoff than the optimal deterministic mechanism. Thus, we can conclude that the expected payoff of the receiver is (weakly) lower under the (potentially) random mimicking mechanism than under the optimal *deterministic* mechanism. We define this deterministic mechanism in the following way. For every $E \in \mathcal{E}$, the action of the receiver given the report E is the expectation of the (potentially degenerate) lottery

$\widetilde{\psi}(E)$. If this mechanism is incentive compatible, it (weakly) improves the expected payoff of the receiver relative to the mimicking mechanism since his preferences are concave.

Incentive Compatibility It is left to show that both mechanisms we have constructed are incentive compatible. First consider $E = (E_1, E_2, ..., E_n) \in \mathcal{E}$ where $1 \notin A_E$, i.e., $E \in \widehat{\mathcal{E}}$. For such a profile of evidence and for every available strategy to a sender of type E, the mechanisms $\widetilde{\psi}$ and $\psi^\star(G(\widehat{\Sigma}, Q))$ coincide. It follows that such a type of sender would find it optimal to report truthfully because the mechanism $\psi^\star(G(\widehat{\Sigma}, Q))$ is incentive compatible. Note also that given a report of such an evidence profile the action of the receiver is deterministic also in $\widetilde{\psi}$ as it coincides with $\psi^\star(G(\widehat{\Sigma}, Q))$. Thus, given such a report the deterministic mechanism we have constructed coincides with $\widetilde{\psi}$, and thus such a type of sender would find it optimal to report truthfully also in the deterministic mechanism. Now consider a sender of type $E = (E_1, E_2, ..., E_n) \in \mathcal{E}$ where $1 \in A_E$. First, it is not profitable to omit the realizations of any $B \subset A_E$ with $1 \in B$. By the definition of mechanism $\widetilde{\psi}$, if the sender chooses to omit the realizations of such $B \subset A_E$ and to report some $E'' \in \Gamma_E$, where $E''_1 = \emptyset$, her payoff is $\psi^\star(G(\widehat{\Sigma}, Q))(E'')$ while if she reports truthfully she gets a lottery. Each realization of this lottery yields a payment that a type who can report E'' receives under the mechanism $\psi^\star(G(\widehat{\Sigma}, Q))$. Since this is a lottery over payments that are weakly larger than $\psi^\star(G(\widehat{\Sigma}, Q))\,(E'')$, it follows that omitting the realizations of such $B \subset A_E$ is not profitable both in $\widetilde{\psi}$ and the constructed deterministic mechanism. It is left to show that it is not profitable to omit any $B \subset A_E$ where $1 \notin B$. If the sender reports such $E' \in \Gamma_E$ and if she reports E then, by the definition of the mechanism $\widetilde{\psi}$, she gets the same lottery in terms of the probability of each result, but the action given each result is different. If the sender reports E' she gets the lottery[16]

$$\bigoplus_{\widehat{s}^1_j \in S_{\widehat{\sigma}_1}} (\psi^\star(G(\widehat{\Sigma}, Q))(\widehat{s}^1_j, E'_2, ..., E'_N), l^{k,j}), \tag{11}$$

[16] We denote by $\bigoplus_{i \in N}(x_i, p_i)$ the lottery in which for every $i \in N$ the probability to get the prize x_i is p_i.

and if the sender reports truthfully she gets the lottery

$$\bigoplus_{\widehat{s}_j^1 \in S_{\widehat{\sigma}_1}} (\psi^\star(G(\widehat{\Sigma}, Q))(\widehat{s}_j^1, E_2, ..., E_N), l^{k,j}). \tag{12}$$

Again, because $(\widehat{s}_j^1, E_2', ..., E_N') \in \Gamma_{(\widehat{s}_j^1, E_2, ..., E_N)}$ and because the mechanism $\psi^\star(G(\widehat{\Sigma}, Q))$ is incentive compatible we have that, for every j,

$$\psi^\star(G(\widehat{\Sigma}, Q))(\widehat{s}_j^1, E_2, ..., E_N) \geq \psi^\star(G(\widehat{\Sigma}, Q))(\widehat{s}_j^1, E_2', ..., E_N'). \tag{13}$$

It follows that the lottery that the sender gets if she reports truthfully dominates the lottery she gets if she reports E', and thus such a deviation from truthful disclosure is not profitable both under $\widetilde{\psi}$ and under the deterministic mechanism. Since we have covered every possible deviation from truthful disclosure, we can conclude that both mechanisms are indeed incentive compatible. It follows that $\psi^\star(G(\Sigma, Q))$ is at least as good for the receiver as $\psi^\star(G(\widehat{\Sigma}, Q))$. This concludes the proof.