Easy Money: the Inefficient Supply of Inside Liquidity*

Alessio Galluzzi†

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Abstract

In modern market economies, money supply depends more on liquid debt securities, such as deposits and commercial paper, created by financial intermediaries. However, the recent financial crisis has exposed the fragility of this source of liquidity. This paper outlines a model in which currency, safe liabilities, and risky liabilities all provide liquidity services. During normal times, intermediaries can fully satisfy the demand for liquidity, while during a crisis there is a large drop in the liquidity supply because of defaults from risky securities. Optimal policy aims to reduce these fluctuations in the supply of liquid assets by reducing the supply of risky securities. When studied individually, lump-sum taxes, liquidity, or equity requirements all restore efficiency. However, the optimal policy rates are sensitive to the model parameterization and, in the case of capital requirements, do not rule out the inefficient equilibrium.

Keywords: Currency; Inside money; Liquidity requirements; Capital requirements
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†The University of Sydney Business School. Email: alessio.galluzzi@sydney.edu.au

1 Introduction

Lucas and Nicolini (2015) show that liquid liabilities issued by intermediaries should be included into the money aggregates. There is an agreement in the literature that these liabilities provide liquidity for financial transactions, as documented in Herrenbrueck and Geromichalos (2017) and Piazzesi and Schneider (2018).[1] As such, they are collectively known as inside money or inside liquidity.[2] However, as the financial system conditions deteriorate, these liabilities quickly lose both their value and their ability to provide liquidity services.[3] As argued in Brunnermeier (2009), it was the loss in liquidity that transformed the initial housing price shock into the Great Recession.

This paper aims to study the welfare implications of such liquidity provision and what the optimal policy response is. Therefore, I set up a model in the spirit of Benigno and Robatto (2019) where financial intermediaries issue liabilities that are traded for liquidity purposes. A distinguishing feature of the model is that there are two sources of liquidity: public liquidity in the form of fiat money issued by the central bank, and private liquidity in the form of safe and risky liabilities issued by banks. For example, deposits are a safe liability, while repos are a risky one. Money is more liquid than safe liabilities, and safe liabilities are more liquid than risky ones. This is because when a risky liability is defaulted upon, it ceases to provide liquidity services. The composition of liquidity is endogenously determined, leaving competition among intermediaries to govern the quantities of money, safe, and risky securities in equilibrium.

The first result of the model is that the central bank sets the overall quantity of liquidity available in the economy by targeting inflation. This is because risk-averse households demand liquidity to finance consumption. Since households can imperfectly substitute among the liquid securities, the realized liquidity premium is proportional to the marginal bene-

[1]Other studies on the topic include Lucas (1990), Geromichalos, Licari, and Suárez-Lledó (2007), Bigio (2015), and Lagos and Zhang (2018).

[2]See Gurley and Shaw (1960) and Holmstrom and Tirole (2011).

[3]Gorton and Metrick (2012) and Gorton, Laarits, and Metrick (2018) document this phenomenon in the repo market at the onset of the Great Recession.

fit of aggregate liquidity (i.e. the sum of money, safe and risky securities). Moreover, the money demand Euler equation determines what the expected value of such marginal benefit is. That is, money demand determines the average quantity of aggregate liquidity in the model. This result is crucial for the first set of new implications of the model. In this economy, the supply of liquid assets must be either constant in all states of the world and set to some intermediate amount, or it must be abundant in some states and scarce in others. Additionally, the key determinant of money demand and aggregate liquidity is the growth path for money supply set by the central bank. In equilibrium, this growth rate is equal to the inflation rate. Therefore, by targeting inflation, the central bank sets the amount of aggregate liquidity. Importantly, the nominal quantity of money is irrelevant, since its real value will adjust to satisfy the condition imposed by the money demand. Consequently, the central bank cannot provide more liquidity in the market by simply printing more money.

Because the aggregate amount of liquidity is set, but intermediaries compete among each other, a previously unexplored externality arises. The profit-seeking intermediary that issues an additional risky security does not internalize the real cost imposed on the household that buys that security but it is unable to use it in the event of a default. As an intermediary issues more of one security, the supply of the other liquid securities must drop to keep the total amount of liquidity consistent with equilibrium. That is, an increased issuance of risky securities decreases the available amount of safe securities. Furthermore, the composition of liquidity across the different securities has real effects, since agents are unable to buy consumption goods with an illiquid security. Specifically, households make their portfolio decision before the state of the world is revealed. Then, when risky securities become illiquid because of default, households cannot use them to finance current consumption. Any recovery value can only be used for consumption at a later date, which causes the welfare loss.

In equilibrium, there is an abundant amount of liquidity in good times and "easy money" to finance high levels of consumption, while in bad times liquidity dries up and consumption collapses. However, this equilibrium is inefficient. To show this, I study how a central planner would allocate liquidity securities and consumption under the demand constraints given by the household problem. The planner puts no weight on the intermediaries and only maximizes the households' expected utility. Optimal policy prescribes consumption to be as constant as possible between good and bad times. To achieve this, liquidity must be less volatile across states. Therefore, the planner reduces the amount of risky securities in favor of a larger supply of safe securities. In other words, in the competitive equilibrium there is an excessive issuance of risky securities which is indeed imposing an externality on the economy.

To correct the externality, I first show that lump-sum taxes implement the planner's solution. Then I study whether bank regulations in the form of liquidity and capital requirements can also implement the planner's solution. I focus on these two sets of regulations since they are some of the key instruments in the Basel III regulatory framework. Each regulatory policy is studied individually, and I show that they can also generally address the inefficiency that would otherwise emerge in equilibrium. However, capital requirements on their own may not be sufficient to achieve efficiency. This is because, under some model parameterizations, the baseline competitive equilibrium remains an equilibrium even after capital requirements are introduced. Therefore, such capital requirements would not change the ultimate allocation of liquidity.

Related Literature

My work is related to the new monetarist literature pioneered by Lagos and Wright (2005), in which households demand liquid assets for transaction services. While this literature focuses on the micro-foundations to determine when fiat currency is valued, I take a reduced form approach similar to Lucas and Stokey (1987) to focus on the role of intermediaries. Within this class of models, Lagos and Rocheteau (2008) study how money and capital can be a competing medium of exchange. Their analysis is further refined in Gu, Mattesini, Monnet, and Wright (2013) and Gu, Mattesini, and Wright (2016), whose work focuses on the role of banking and credit in expanding the set of feasible allocations. In particular, Gu et al. (2016) compare money with credit to show how real balances adjust to keep the amount of liquidity constant.[4] My model provides a similar result, with the additional feature that intermediaries can default on their liabilities.[5] In this context, the key equilibrium condition is that the expected marginal value of liquidity is constant, implying that liquidity can fluctuate across states and create a welfare loss. Geromichalos and Herrenbrueck (2016) consider an economy where an illiquid asset can be exchanged for a liquid one in a frictional search model. Similarly, my model includes securities that can provide liquidity and therefore demand a liquidity premium that increases with inflation. Finally, Andolfatto, Berentsen, and Waller (2016) study monetary policy where money is backed by an illiquid capital, which is exactly the type of asset-backed security that intermediaries issue in my model.

This paper also relates to a long literature on the creation and demand for liquidity, starting from the work of Gorton and Pennacchi (1990). Holmström and Tirole (1998) address whether governments should create or regulate liquidity to stimulate efficient investment. Eisfeldt (2004), Bigio (2015) and Kurlat (2017) identify adverse selection as the key element for asset illiquidity. Bianchi and Bigio (2017) look at how intermediaries manage their liquidity risk and how monetary policy affects the issuance of credit. I synthesize these works to develop a model where securities can lose their liquidity properties. Finally, Benigno and Robatto (2019) consider the efficient supply of liquidity when safe government bonds[6] are available together with safe and risky securities from intermediaries.[7] They then study tax-based policies to correct potential inefficiencies. This paper follows their base framework while using fiat currency as the asset of choice for the public supply of liquidity. This allows me to address bank level regulations, like liquidity and capital requirement policies, that

[4]Lacker and Schreft (1996) look at a similar problem, but where money and credit have different user cost.

[5]These liabilities are claims backed by a risky asset, as in Lagos (2011).

[6]Krishnamurthy and Vissing-Jorgensen (2012) document how the treasury bond market is driven by the demand for safe and liquid assets.

[7]Magill, Quinzii, and Rochet (2016) consider the case where only private debt provides a liquidity service and analyze the consequences for monetary policy.

have not been studied from a consumption-based point of view.

The safe assets literature (such as Caballero (2006); Caballero and Farhi (2017); Diamond (2016); Farhi and Maggiori (2017); Li (2017); Magill et al. (2016); Stein (2012); and Woodford (2001)) has modeled how these assets provide liquidity services. These papers all stress the importance of fiscal capacity to implement corrective policies, while I focus my attention on the outcomes from regulation that do not require direct government intervention. Gorton (2017) provides historical evidence about the liquidity provision of safe assets. I then allow risky assets to provide liquidity services, as long as the economy is in a good state. Gorton and Metrick (2012) show how risky assets quickly lost their liquidity value in the repo market during the 2008 financial crisis. Finally, my results also highlight the issues with an excessive amount of credit, as in Lorenzoni (2008) and Moreira and Savov (2017).

The remainder of the paper is organized as follows. Section 2 describes the model environment, with Section 3 highlighting the main economic forces. Then Section 4 derives the competitive equilibrium and Section 5 shows how the equilibrium is inefficient. Section 6 discusses how capital and liquidity requirements can implement the planner solution, and Section 7 concludes.

2 Model

2.1 Environment

The model is an extension of Benigno and Robatto (2019), with discrete time over an infinite horizon. As in the new monetarist models pioneered in Lagos and Wright (2005), each period is divided into two sub-periods, morning and evening. There is a single consumption good, which is produced in the morning of every period and can be freely stored until the evening, after which it fully depreciates. Production transforms a fixed and non-depreciating supply of capital $\bar{K}$ into the consumption good through a linear technology $Y_t = A_t\bar{K}$, where A_t is an aggregate shock on capital productivity and the only source of uncertainty in the model.

The shocks are independent and identically distributed according to

$$A_t = \begin{cases} A_h & \text{with probability } 1-\pi \\ A_\ell & \text{with probability } \pi \end{cases},$$

with $A_h > A_\ell$. Define $\bar{A} \equiv (1-\pi)A_h + \pi A_\ell$ as the average productivity of capital. In what follows, I refer to a realization of A_h as the good or high state and a realization of A_ℓ as the bad or low state.

The economy is populated by an infinitely lived representative household, a continuum of two-period lived, overlapping generations of intermediaries, and a central bank. The central bank controls the supply of fiat currency, M_t, through lump-sum transfers to households. Intermediaries are the only agent that can manage capital,[8] which they finance through debt securities and equity. The household then invests in the intermediaries' liabilities to transfer resources intertemporally. For clarity of exposition, I will call the intermediary issuing of a safe debt security "commercial bank" and the issuer of a risky debt security "shadow bank". The corresponding securities are denoted as b^c and b^s respectively. To create safe debt securities, a commercial bank has to issue equity n^c as well. The rules that govern the intermediaries' behavior are detailed in Section 2.3.

Table 1 summarizes the timing of the model. At the beginning of the morning, the aggregate shock realizes, resolving all uncertainty for the period. Then, production occurs and all prices are determined so that the household can make its morning consumption choice. In the evening, a new generation of banks is born and the central bank makes the lump-sum monetary transfer. Subsequently, the household makes its portfolio choice, so that the new generation of banks can acquire capital from the old generation of banks.

[8]That is, the household has an infinitely high management cost for capital. A similar setup can be found in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011).

Table 1: Model Timing

Morning	Evening
- Aggregate shock is publicly observed - Production - Prices & default determined - Household's morning consumption	- New generation of intermediaries enters - Lump-sum monetary transfer - Evening consumption and portfolio choice - Old generation of intermediaries dies

2.2 Household's Problem

Household's preferences are characterized by a quasi-linear per period utility over morning and evening consumption[9]

$$U_t = \log c_t^{am} + c_t^{pm}. \tag{1}$$

Morning consumption c_t^{am} is subject to a liquidity constraint, and cannot exceed the sum of currency m and debt securities b. These two assets are not perfect substitutes when it comes to their liquidity value. First, only a fraction $0 < \theta < 1$ of a security face value can be used to finance morning consumption. That is, securities provide fewer liquidity services than fiat currency. This is a common assumption in the Kiyotaki and Moore (1997) literature and in the new monetarist models with multiple sources of liquidity, like Lagos and Rocheteau (2008). Moreover, Barnett (1982) argues that different securities provide different amounts of liquidity services, as measured by their user cost, and this should be taken into account when measuring aggregate liquidity. Second, a security may be defaulted upon in the bad state, in which case it loses all of its liquidity value and cannot be used in morning transactions. Although this is a stark assumption, it is a useful simplification to highlight the main forces of the model. Geromichalos, Herrenbrueck, and Lee (2018) show that assuming a one-to-one link between safety and liquidity of an asset is generally justified while highlighting important exceptions that lead a defaulted asset to be liquid as well. Furthermore, it is the case that an asset liquidity deteriorates quickly as its rating declines,

[9]The log utility is a useful device to recover closed form solutions, but the results of this paper can be extended to a more general quasi-linear utility $U = u(c^{am}) + c^{pm}$, where $u(\cdot)$ is an increasing and concave function that satisfies standard Inada conditions.

as documented in Benmelech and Bergman (2018). After default, the household partially recovers its loan but the proceeds can only be used to finance evening consumption.

Since commercial bank securities are safe, while shadow bank securities are risky, the morning liquidity constraint is

$$c_t^{am} \leq \varphi_t M_{t-1} + \theta \left(1 + r_{t-1}^c\right) b_{t-1}^c + \theta \left(1 - \mathbb{I}_t\right) \left(1 + r_{t-1}^s\right) b_{t-1}^s, \tag{2}$$

where $\mathbb{I}_t$ is an indicator function such that $\mathbb{I}_t = 1$ when the shadow bank defaults on its securities. Taking the morning consumption as the numeraire good, the real value of a unit of money is denoted by φ_t.

The asset timing follows the cash-in-advance and new monetarist tradition. The asset allocations are made in the previous period before the aggregate capital productivity realizes. The household lends b_{t-1}^c to the commercial bank and its first-order condition pins down the equilibrium interest rate $1 + r_{t-1}^c$. Another way to interpret these amounts is to normalize the price of the liability to one at issuance. Then $1 + r_{t-1}^c$ represents the face value of an equivalent zero-coupon bond issued by the commercial bank and b_{t-1}^c is the quantity issued. The same holds for a shadow bank-issued security b_{t-1}^s.

Once the period moves forward to the evening, the household makes its evening consumption and asset allocation decisions subject to a budget constraint[10]

$$c_t^{pm} + \varphi_t M_t + b_t^s + b_t^c + n_t^c \leq W_t + T_t, \tag{3}$$

where W_t is the household's wealth at the beginning of the evening

$$W_t = \varphi_t M_{t-1} + \left(1 + r_{t-1}^c\right) b_{t-1}^c + \left(1 - \chi_t\right) \left(1 + r_{t-1}^s\right) b_{t-1}^s + (1 + r_t^n) n_{t-1}^c - c_t^{am}. \tag{4}$$

That is, the household's wealth is defined by the value of the assets carried over from the

[10]Since the consumption good is freely storable between morning and evening, consumption prices must equalize as long as consumption is nonzero

previous period minus the amount used for morning consumption. The transfer T_t includes the real value of the monetary transfer from the central bank and any taxation levied on the intermediaries and then rebated to the household. The monetary component of the lump-sum transfer implements the desired monetary policy, either as a helicopter drop if the central bank is expanding the available currency in circulation or as a tax if the amount of currency is reduced.[11] The realized return on commercial bank equity is denoted as r_t^n. This is a random variable and is determined as a residual from the intermediaries' profits, as I will detail in the next section. Finally, $1 - \chi_t$ represents the recovery rate of the risky security after default. If no default happens, then the full face value of the risky security is paid and $\chi_t = 0$. If instead the intermediary is in a state of default, then $\chi_t > 0$ and the recovery rate is determined endogenously from the value of the assets in the defaulted intermediary's balance sheet.

The household chooses a plan for state-contingent consumption and asset holdings to maximize its expected utility over the infinite time horizon

$$\max E\left[\sum_{t=0}^{\infty} \beta^t U_t\right] = E\left[\sum_{t=0}^{\infty} \beta^t \left(\log c_t^{am} + c_t^{pm}\right)\right]$$

subject to the morning liquidity constraint (2) and the evening budget constraint (3), where $0 < \beta < 1$ represents the discount factor. Define η_t as the Lagrange multiplier for the liquidity constraint (2) and λ_t as the multiplier for the budget constraint (3). Taking the

[11] Clearly, it is necessary to assume that the central bank is credible when announcing its policy and can then enforce it with the household. This assumption is not crucial for the results of this paper. The model can be adjusted to account for imperfect enforcement, but it would be an additional mechanism that masks some of the economic forces.

first-order conditions gives the following Euler equations:

$$\begin{aligned} c_t^{am}: \quad \tfrac{\beta^t}{c_t^{am}} &= \lambda_t + \eta_t && (5) \\ c_t^{pm}: \quad \beta^t &= \lambda_t && (6) \\ n_t^c: \quad 1 &\geq E\left\{\beta\left(1+r_{t+1}^n\right)\right\} && (7) \\ M_t: \quad 1 &\geq E\left\{\frac{\varphi_{t+1}}{\varphi_t}\left(\frac{\eta_{t+1}}{\beta^t}+\beta\right)\right\} && (8) \\ b_t^c: \quad 1 &\geq E\left\{\left[\theta\frac{\eta_{t+1}}{\beta^t}+\beta\right](1+r_t^c)\right\} && (9) \\ b_t^s: \quad 1 &\geq E\left\{\left[\theta\left(1-\mathbb{I}_{t+1}\right)\frac{\eta_{t+1}}{\beta^t}+\left(1-\chi_{t+1}\right)\beta\right](1+r_t^s)\right\} && (10) \end{aligned}$$

Equity return r^n is pinned down by a simple stochastic discount factor, as in traditional asset price models. Then currency and securities returns have a liquidity premium component identified by the terms multiplied by the Lagrange multiplier η_{t+1} from the morning liquidity constraint. The liquidity premium is the largest for currency, the most liquid asset in the economy. Securities follow, with risky securities always having a smaller liquidity premium than safe ones, as implied by the presence of the indicator function. Furthermore, risky securities returns include a default premium component, measured by the default loss rate χ_{t+1}.

After combining the consumption first order conditions, morning consumption c_t^{am} is

$$c_t^{am} = \frac{\beta^t}{\lambda_t + \eta_t} = \frac{1}{1+\frac{\eta_t}{\beta^t}}. \tag{11}$$

The expression is decreasing in the Lagrange multiplier $\eta_t \geq 0$. Therefore, the maximum level of morning consumption is $c_t^{am} = 1$ and it can be achieved if and only if $\eta_t = 0$ (i.e. when the morning liquidity constraint is not binding). If so, marginal utilities of consumption are equalized between morning and evening.

This result is straightforward since under a standard utility maximization problem with equal prices the optimum is reached by equalizing marginal utilities. Any additional binding constraint makes the chosen allocation deviate from this benchmark. Therefore, the household would always like to increase its holdings of assets that can relax the morning constraint, even though it results in higher demanded returns for those assets. Whether or not the supply of liquid assets is positive in equilibrium is an outcome of the interaction between monetary policy and intermediaries.

2.3 Intermediaries' Problem

The main role of financial intermediaries is to manage capital and to provide liquidity in the economy. They buy capital from the households that cannot manage it and transform it into liquid debt securities that relax the household's morning liquidity constraint. In the context of this paper, liquidity is defined by the degree to which it facilitates transactions, rather than the classic definition of liquidity as the ability to quickly transform an asset into currency with little or no losses on its face value. Examples of assets with such properties include demand deposits, certificates of deposit, and commercial paper.

The intermediary then structures its balance sheet to offer a menu of securities to the household. Some have a low but safe return, while others have a high but risky return. I assume that intermediaries honor their obligations as long as they have sufficient assets on their balance sheet and that they have limited liability.[12] Consequently, the riskiness of a security is entirely determined by the intermediary's balance sheet structure. Here I separate the intermediaries into two types: a commercial bank that issues safe securities, and a shadow bank that issues risky ones. All securities are implicitly backed by the return on assets, which is risky. Therefore, a commercial bank must raise equity to issue safe securities. This set of assumptions greatly simplifies the optimal security design problem that an intermediary faces, as studied in Allen and Gale (1988). In Appendix B, I show how this design is indeed optimal, by allowing for tranching of the debt securities. Intermediaries live for two periods, with overlapping generations. At a given time period t there is a unit measure of competitive

[12] Intermediaries' securities can be interpreted as collateralized loans, as in Gorton and Ordonez (2014). Thus, in the event of a default, the borrower can still obtain the value of the collateral.

intermediaries that issue securities and raise equity to invest in capital. At time $t+1$, the intermediary observes the return on capital, repays its creditors, and liquidates its equity with the residual returns, if any.

More formally, a newborn intermediary has a choice between two contracts. If the intermediary chooses to operate as a commercial bank, it raises safe securities and equity to make its securities default-free. Furthermore, a commercial bank is subject to some balance sheet costs τ proportional to the size of its assets, to be paid in units of the consumption good. This is a social cost that goes to waste. The balance sheet costs reflect the regulatory cost of banking activity. In the United States, the major source of regulatory costs for institutions that offer demand deposits and other safe instruments is FDIC insurance.[13] If instead the intermediary chooses to operate as a shadow bank, it issues risky securities that default in the bad state, and it is not subject to any balance sheet cost. In both cases debt is implicitly collateralized by capital, and the modeling choice can be thought of as a simplified version of the collateral equilibrium framework described in Geanakoplos and Zame (2002), Geanakoplos (2003), and Geanakoplos and Zame (2014).

In this environment, an intermediary that chooses to operate as a commercial bank is subject to the balance sheet constraint

$$(1+\tau)\, q_t^k K_t^c = b_t^c + n_t^c, \tag{12}$$

where τ represents the balance sheet costs. The price of capital q_t^k is an ex-dividend price since the capital production for the current period is enjoyed by the older generation of intermediaries. The safe securities b_t^c offer a real return $1+r_t^c$ promised at the time of issuance, before the aggregate shock realization A_{t+1} . The return on capital $1+r_{t+1}^k$ and the return on equity $1+r_{t+1}^n$ are instead stochastic and depend on state realization. Consequently,

[13]See Afonso, Armenter, and Lester (2018) and Banegas and Tase (2017).

the expected profits of a newborn commercial bank are given by

$$E\left[\Pi_{t+1}^c\right] = E\left[1 + r_{t+1}^k\right] q_t^k K_t^c - \left(1 + r_t^c\right) b_t^c - E\left[1 + r_{t+1}^n\right] n_t^c, \tag{13}$$

where the different returns are to be determined in equilibrium. The return on securities $1 + r_t^c$ is not subject to an expectation term, since the promised return on securities is fully paid in every state of the world. Therefore, the lowest possible return on capital is sufficient to repay the promised return on safe securities, or

$$\left(1 + r_\ell^k\right) q_t^k K_t^c \geq \left(1 + r_t^c\right) b_t^c, \tag{14}$$

where $1 + r_\ell^k = {\left(A_\ell + q_{t+1}^k\right)}/{q_t^k}$ is the return on capital if the low state materializes in period $t+1$. The problem for the commercial bank is to choose capital K^c, safe securities b^c, and equity n^c to maximize equation (13) subject to balance sheet constraint (12) and liability constraint (14), taking as given prices and returns.

Now look at the problem of an intermediary that chooses to operate as a shadow bank. The balance sheet constraint at time t is

$$q_t^k K_t^s = b_t^s, \tag{15}$$

and the expected profits in the following period are

$$E\left[\Pi_{t+1}^s\right] = E\left[1 + r_{t+1}^k\right] q_t^k K_t^s - E\left[\left(1 - \chi_{t+1}\right)\left(1 + r_t^s\right)\right] b_t^s, \tag{16}$$

where $1 + r_t^s$ is the promised return on the shadow bank securities and $0 < 1 - \chi_{t+1} < 1$ is the recovery rate on the promised return. The contract is designed such that the shadow bank securities do not default in the good state. Thus $\chi_h = 0$. However, when there is low productivity, the shadow bank partially defaults on its liabilities and limited liability determines the recovery rate $1-\chi_\ell$

$$\left(1+r_l^k\right) q_t^k K_t^s = \left(1-\chi_\ell\right)\left(1+r_t^s\right) b_t^s. \tag{17}$$

The objective of a shadow bank is to choose securities b^s and capital K^s to maximize its profits (16), given the balance sheet (15), taking as given the recovery rate as in (17), prices and returns.

Finally, a newborn intermediary maximizes its expected profits when choosing between operating as a commercial or as shadow banking

$$E\left[\Pi_{t+1}\right] = \max\left\{E\left[\mathit{\Pi}_{t+1}^c\right], E\left[\mathit{\Pi}_{t+1}^s\right]\right\}. \tag{18}$$

2.4 Market Clearing and Equilibrium Definition

To complete the model, the central bank sets a constant growth rate for fiat currency

$$M_{t+1} = (1+\mu) M_t, \tag{19}$$

where μ is positive when the central bank is expanding the quantity of nominal currency in circulation and negative when reducing it. As this is fiat currency, the central bank can potentially implement any policy, as long as the implied financial returns do not violate household transversality conditions. Since the objective of this paper is to study ex-ante prudential regulation, I will simply take the central bank policy as a given and solve for the resulting equilibrium.

Now that the description of all the economic actors is complete, the definition of an equilibrium is:

Definition 1. An equilibrium is a set of

- Sequences of state contingent prices $\varphi_t,\ q_t^k$and returns $r_t^s,\ r_t^c,\ r_t^n$

- Default rates χ_{t+1}
- Household's choices of $M_t,\ b_t^s,\ b_t^c,\ n_t^c,\ c_t^{am}, c_t^{pm}$
- Intermediaries' choices of $K_t^c,\ b_t^c,\ n_t^c,\ K_t^s,\ b_t^s$

Such that:

- Households maximize their utility, given prices, returns and default rates
- Intermediaries maximizes profits, given prices, returns and default rates
- The free entry condition holds
- The government budget is satisfied

$$T_t = \mu M_{t-1}$$

- Markets clear, including

$$\begin{aligned} \bar{K} &= K_t^c + K_t^s \\ Y_t = A_t \bar{K} &= c_t^{am} + c_t^{pm} + \tau q^k K^c \end{aligned}$$

While this definition allows for a variety of equilibrium paths, I will focus on stationary equilibria, where real quantities are constant over time, while nominal quantities grow at a constant rate. Since the quantity and price of currency are not constant, in a stationary equilibrium only the real value of money $m_t = \varphi_t M_t$ is constant over time.

3 Main Economic Forces

In this section, I consider simplified versions of the model to illustrate the basic economic forces at work and derive some basic results that are helpful to understand the complete model. First, I look at the case where only a commercial bank operates. Second, I focus on the case where only a shadow bank operates. The aim here is to highlight the degree of complementarity and substitutability between currency and debt securities, and outline the consequences on consumption outcomes. As I show later in the paper, both cases are actual corner solutions of the general model, so the following discussion is also beneficial to fully understand the results.

Before moving to the specific cases of the model, it is useful to derive some general results that apply regardless of the simplifications that I make in the following sections. First consider any monetary policy that satisfies constant growth of nominal currency from (19), where $\mu \geq \beta - 1$ is the growth rate of money. The lower bound is defined by the Friedman rule, where the nominal interest rates hit zero. To achieve a stationary equilibrium, where the real value of money is constant, it must be the case that the price of currency φ_t is moving in the equal and opposite direction, or

$$\frac{\varphi_{t+1}}{\varphi_t} = \frac{1}{1+\mu}. \tag{20}$$

Note that this growth rate is exactly the inverse of the inflation rate, thus in this model issuance of currency generates an equal amount of inflation.

Having established a path for money prices, I revisit the money holding Euler equation (10). Assuming that the household wants to hold a strictly positive quantity of currency, the equation simplifies to

$$1 + \mu = E\left[\frac{\eta_{t+1}}{\beta^t} + \beta\right]. \tag{21}$$

For the remainder of the paper, I redefine the morning constraint multiplier η_{t+1} to its current value multiplier form, such that $\eta_{t+1} = \beta^t \kappa_{t+1}$. $\kappa_t \geq 0$ then represents the value of relaxing the morning constraint, that is, the marginal value of liquidity for the household. Thus equation (10) further simplifies to

$$\bar{\kappa} = E\left[\kappa\right] = 1 + \mu - \beta, \tag{22}$$

where I will refer to $\bar{\kappa}$ as the expected or average liquidity premium in the economy. This equation captures a crucial aspect of this model: in any equilibrium, the expected marginal utility of liquidity must be constant. Therefore, the real balances of all the liquid assets will adjust in all states of the world to satisfy this condition.[14]

Then, morning consumption from equation (11) can be rewritten as a function of the wedge created by the realized liquidity premium ${}^{\kappa_t}/_{\beta}$:

$$c_t^{am} = \frac{1}{1 + {}^{\kappa_t}/_{\beta}}. \tag{23}$$

This highlights how the realized liquidity premium acts a wedge that reduces morning consumption and is therefore crucial in determining the welfare outcomes.

3.1 A Model with Only Commercial Banks

Consider a model where the only choice for an intermediary is to operate as a commercial bank. To further simplify the analysis, also assume that there are no balance sheet costs ($\tau = 0$). Using equation (9) from the household problem, in a stationary equilibrium the commercial bank security needs to offer a return equal to

$$1 + r^c = \frac{1}{\theta\bar{\kappa} + \beta}. \tag{24}$$

Given that equity has an expected return $E\left[1 + r^n\right] = {}^1/_{\beta}$, the term $\theta\bar{\kappa}$ is a measure for the liquidity premium of safe securities. In this context, it is even more transparent how the parameter θ can be interpreted as the relative liquidity value between currency and

[14] This result is in line with Gu et al. (2016)

commercial bank securities, since the implied return of currency is $1/(\bar{\kappa}+\beta)=1/(1+\mu)$.

Now, look at the profit maximization problem of the commercial bank. Because the expected profits of commercial banks are linear in the asset allocation, and the return on securities is lower than the expected return of equity, the bank would issue only safe securities if there were no further constraints. However, the commercial banking contract requires the bank to provide safe securities that never default, as implied by constraint (14). Thus, the constraint must be binding, and the amount of safe securities as a fraction of the total assets is

$$\frac{b^c}{q^k \bar{K}} = \frac{1+r_\ell^k}{1+r^c}, \tag{25}$$

where I replace $K^c = \bar{K}$ to account for the market clearing condition. Along the equilibrium path, this fraction must be less than one. This equation also identifies the maximum leverage a commercial bank can carry while ensuring the return of the safe security.

Because of free entry, expected profits need to be zero in equilibrium, thus

$$E\left[1+r^k\right] q^k \bar{K} = (1+r^c)\, b^c + E\left[1+r^n\right] n^c.$$

Divide this equation by the total value of capital $q^k K$ to write the expected return on capital as a weighted average between the safe return on securities and the expected return on equity

$$E\left[1+r^k\right] = (1+r^c)\frac{b^c}{q^k \bar{K}} + E\left[1+r^n\right]\left(1-\frac{b^c}{q^k \bar{K}}\right). \tag{26}$$

The first relevant feature of the model is that the return on capital is pinned down by the demand coming from the financial sector. In an economy without banks in which the households can hold capital but cannot provide liquidity transformation, the expected return on capital would be equal to the expected return of equity in the model. Thus, in the presence of a liquidity premium, the return on capital must be below the return implied by the household's discount factor. That implies that there is some charter value in the banking activity that would be destroyed without the financial system. General equilibrium forces connect this charter value directly to the demand for liquidity from the representative household.

In equilibrium, the expected return on capital is

$$E\left[1+r^k\right] = \frac{1}{\beta} - \frac{1-\beta}{\beta}\frac{\bar{A}-\theta(\bar{A}-A_\ell)\bar{\kappa}}{\bar{A}\beta + A_\ell\theta\bar{\kappa}}. \tag{27}$$

First, note how the expected liquidity premium $\bar{\kappa}$ appears in this formula. With no liquidity premium, the formula collapses to the standard return $^1/_\beta$. Otherwise, capital returns are a decreasing function of the average liquidity premium. Indeed, this is what the general equilibrium forces would suggest, as the increased but unsatisfied demand for securities constricts their returns and in turn puts downward pressure on the return for capital. Second, the low state productivity A_ℓ appears explicitly in the formula. This is because the limited liability constraint pins down the structure of the balance sheets, thus the portfolio weights in equation (26). From the return on capital, one derives the solution for the price of capital, and the real amount of securities and equity issued.

Since safe securities never default, the household holds the same amount of liquidity regardless of the state. There is no variance in the liquidity premium term, and $\bar{\kappa} = \kappa_l = \kappa_h = 1+\mu-\beta$. Therefore, equation (23) implies that morning consumption is equalized between the two states and the expected one-period utility in the stationary equilibrium is

$$E\left[U\right] = \log\left(\frac{\beta}{\beta+\bar{\kappa}}\right) + \bar{A}\bar{K} - \frac{\beta}{\beta+\bar{\kappa}}, \tag{28}$$

where the linear term is expected evening consumption. This illustrates how the liquidity premium in the economy is the determinant factor for welfare. Furthermore, it also highlights how monetary policy has welfare implications since the expected utility decreases according to the inflation rate μ. The effects of μ on welfare arise mechanically in this environment from the presence of the liquidity constraint that governs morning consumption. However, it still highlights the role of an inflation tax in the economy. Namely, higher levels of inflation in this model push consumption into the future (i.e., the evening), as the return of holding currency becomes smaller and the representative household does not want to hold as much of it. A second consequence is that the highest expected utility is achieved when the growth rate of currency μ is the smallest, at $\underline{\mu} = \beta - 1$. This is not a surprising result, since it is a standard outcome in monetary models. As the nominal interest rate hits zero, the return of currency is equalized to the expected return of a Lucas tree, thus the rate of return dominance disappears. Then also the wedge from the morning liquidity constraint must disappear. Otherwise, the household would demand more money to satisfy its liquidity needs. If the morning constraint is no longer binding, then the morning and evening marginal utilities are equalized, which is the general condition for optimality.

After discussing both the banks' and household's problems, I can complete the model by looking at the real value of money m. The discussion above assumed that the household is willing to hold a strictly positive amount of real currency. However, it is possible that the supply of liquidity from a commercial bank is so large that money is worthless.[15] Using the optimal level of consumption in equation (23) and the morning liquidity constraint (2), it is possible to solve for the real value of money and derive the following result:

Proposition 1. *Given a monetary policy μ and parameters of the model, there exists a threshold for the security liquidity $\bar{\theta}^c$ such that*

- *If $\theta \geq \bar{\theta}^c$, no monetary equilibrium exists*
- *If $\theta < \bar{\theta}^c$ there is a monetary equilibrium and the value of money is decreasing in θ*

For the proof and the closed-form solution for the real value of money m, see Appendix A.1. Intuitively, if securities can provide abundant liquidity services, then there is no need for fiat currency. If instead securities provide few liquidity services, then the household demands

[15] A non-monetary equilibrium always coexists with the monetary equilibrium I am describing, as is the case for models with fiat currency.

more aggregate liquidity than the commercial bank can supply. Thus, fiat currency and securities coexist.

3.2 A Model with Only Shadow Banks

Now consider an environment where only shadow banks operate. Then the only assets that can relax the household morning liquidity constraint are fiat currency and securities that default in the bad state. Unlike the previous case, consumption cannot be equalized between the two states, since one asset loses its liquidity value in the event of a negative shock. Consequently, holding currency becomes an insurance instrument against negative shocks. However, the expected return of currency is still pinned down by equation (22), so the insurance value of currency is constrained by the expected return that currency needs to have in a monetary equilibrium.

From equation (10), the promised return on risky securities is

$$1 + r^s = \frac{1}{\theta\,(1-\pi)\,\kappa_h + (1-\pi\chi_\ell)\,\beta}. \tag{29}$$

In the high state, risky securities do not default, thus they enjoy a liquidity premium as measured by $(1-\pi)\,\kappa_h$ and no default premium component. On the other hand, in the low state these securities lose their liquidity value and gain a default premium component $-\pi\chi_\ell\beta$ that measures the amount of return loss in a bankruptcy. The shadow bank is only able to capture a fraction of the liquidity premium and needs to pay an additional amount to compensate for the risk of default. Since the value lost in the bankruptcy χ_ℓ is determined in equilibrium, the overall return of the risky security may exceed the discount factor.

The return on securities directly pins down the return on capital. From equation (15), securities are the only source of financing for the shadow bank, and the free entry condition implies zero expected profits. Then combining (15), (16), (29), and the zero-profit condition solves for the expected return of capital

$$E\left[1+r^{k}\right]=\frac{1-\pi\chi_{\ell}}{\theta\left(1-\pi\right)\kappa_{h}+\left(1-\pi\chi_{\ell}\right)\beta}. \tag{30}$$

Note that if the liquidity needs of a household are completely satisfied in the high state ($\kappa_h = 0$), then the expected return to pay out for a bank is exactly equal to $1/\beta$, the return of an asset when there are no liquidity concerns and the expected return consumer demands on equity.

While equation (29) pins down the promised return to the household, the shadow bank is only paying it in the high state. In the low state, a fraction $1-\chi_\ell$ of the promised return is paid out as the result of default. Combine (15) with (17) to get

$$1+r_{\ell}^{k}=\frac{1-\chi_{\ell}}{\theta\left(1-\pi\right)\kappa_{h}+\left(1-\pi\chi_{\ell}\right)\beta}. \tag{31}$$

This equation, together with (30), summarizes the interaction of the different general equilibrium forces in the financial assets market. Taking the price of capital as given, the liquidity premium in the high state κ_h and the default loss rate χ_ℓ adjust to jointly guarantee that no more securities are issued and that the limited liability constraint is binding. If equation (30) fails, then the supply of securities, and thus the overall supply of liquidity, must change to bring profits to zero. If (31) fails, then default loss rate χ_ℓ adjusts so that no value is destroyed in the bankruptcy process.

Because of the risky securities default, it must be that the household is more liquidity constrained in the bad state, or $\kappa_\ell > \kappa_h$. This reflects the differences in the amount of aggregate liquidity available in each state. Furthermore, money fully derives its value from the household's morning consumption in the low state. Thus, the real value of money is tied to the liquidity premium in the low state κ_ℓ. Also, since the liquidity premium κ is a factor of the Lagrange multiplier η, it must be the case that $\kappa \geq 0$ in all states. The details of how to solve for the high state liquidity premium κ_h and the low state loss rate after default χ_ℓ

are in Appendix A.2. Studying these conditions leads to the following proposition:

Proposition 2. *Given a monetary policy μ and parameters of the model, there exists a threshold for the security liquidity $\bar{\theta}^s$ such that*

- *If $\theta > \bar{\theta}^s$, no monetary equilibrium exists*
- if $\theta = \bar{\theta}^s$ *there is a monetary equilibrium with $\kappa_h = 0$ and $\kappa_\ell = {}^{(1+\mu-\beta)}/_{\pi}$*
- *If $\theta < \bar{\theta}^s$ there is a monetary equilibrium with $\kappa_\ell > \kappa_h > 0$*

This result is similar to the one derived under commercial banking, since when securities have very high liquidity value θ they can fully replace currency. However, the real value of money is no longer necessarily decreasing in security liquidity. When θ is small enough, the value of money is increasing in security liquidity. At those initial levels for θ, the liquidity value of securities is so small that the insurance motive for holding money dominates, driving up demand and therefore its price. Low security liquidity also makes them more expensive to issue, as the shadow bank can only capture a small fraction of the liquidity premium. This reduces the supply of securities and drives the household towards currency.

A second difference concerns morning consumption and welfare. In a monetary equilibrium, welfare depends on the amount of liquidity a shadow bank can issue, as parameterized by θ. Moreover, welfare can either increase or decrease as the amount of shadow bank securities, as measured by θ, increases. Starting from the case where $\theta = \bar{\theta}^s$, the liquidity needs of the household are fully satisfied in the high state, since $\kappa_h = 0$. Thus, morning consumption is also maximized in the same state. The downside of maximum morning consumption in the good state is variance since morning consumption is minimized in the low state. Consequently, the liquidity premium in the low state is maximized.. Now look at the case where $\theta < \bar{\theta}^s$. While morning consumption in the low state is still smaller than the one in the high state, the difference between the two goes down. While reducing the variance in marginal utilities is always welfare improving, level effects may prevail. I discuss the issue of equilibrium ranking and efficiency in the following sections.

4 Competitive Equilibrium

In the previous section, I established the role of each type of security in providing liquidity services. Safe securities and fiat currency are close to perfect substitutes, therefore there is a role for both only if the general equilibrium forces constrain the commercial bank to a limited issuance. Risky securities provide the household with additional consumption only in one state of the world, thus complementing fiat currency but never being a substitute for it.

In light of these facts, I now consider the case where an intermediary has the choice of operating either as a commercial bank or as a shadow bank. The economy that emerges has a positive supply of all of the asset types, with consumption outcomes similar to those observed in Proposition 2. However, different equilibria are possible, with only one type of bank as described previously. In fact, under some parameterizations, there may even be multiple equilibria.

4.1 No Balance Sheet Costs ($\tau = 0$)

First, consider the case where there are no additional balance sheet costs on the commercial bank operations. The main reason to look at this special case is to highlight the equilibrium that emerges purely from the different funding structures of each intermediary.

Three conditions need to hold to have a monetary equilibrium with both commercial and shadow banks. First, banks' profits must equalize. Second, free entry pins the profits at zero, thus setting the expected return of capital. Third, the total amount of safe and risky securities must be small enough to require a positive value for fiat currency. These three conditions also provide the framework to solve for an equilibrium. The general solution method follows a guess and verify approach, where I postulate the structure of the liquidity premia and which intermediaries operate in the stationary equilibrium.

I illustrate this procedure in the following example, where I show how it can also be used to rule out candidate equilibria. Suppose there exists an equilibrium where safe and risky securities are issued and the liquidity premium in the high state is zero ($\kappa_h = 0$). Monetary policy is away from the Friedman rule ($\mu > \beta - 1$). Equation (30) needs to hold, as the shadow bank must make zero profits. Since $\kappa_h = 0$, Equation (30) implies that the expected cost of issuing risky securities is equal to the inverse of the discount rate β. The shadow bank zero profit condition requires the expected return on capital to be equal to the inverse of the discount rate as well. From (7), the return on equity is also equal to the inverse of the discount rate. Yet, there is still a positive liquidity premium that the commercial bank can capture from the low state, thus the return on commercial bank securities is lower than the return on equity. Consequently, the portfolio weighted cost of funding for a commercial bank is always lower than the expected return on capital, which means that the commercial bank's expected profits are strictly positive. As a result, the shadow bank should operate as a commercial one, which contradicts the assumed equilibrium outcome.

The example rules out any equilibrium where the liquidity demand from the household is fully satisfied in one state by any combination of liquid securities. Thus, in equilibrium, the household morning constraint (2) is always binding and the associated liquidity premium must be strictly positive. More generally, it is possible to construct the following equilibria involving a commercial bank:

Proposition 3. *If an equilibrium with positive issuance of safe securities exists, then it takes one of the following forms*

- *If* $\pi \geq \bar{\pi}$ *and* $\theta < \bar{\theta}^c$, *then only safe securities are issued with* $\kappa_\ell = \kappa_h = 1 + \mu - \beta$
- *If* $\pi < \bar{\pi}$ *and* $\underline{\theta}^{cs} < \theta < \bar{\theta}^{cs}$, *then both safe and risky securities operate with* $\kappa_\ell > \kappa_h > 0$

The details of the derivation are in Appendix A.3. The two equilibria are mutually exclusive since the threshold for the probability of a low state π is the same for the two possible equilibria. However, the liquidity threshold is different, since, when both banks operate, the increased availability of privately issued liquid instruments reduces the role of currency.

The first equilibrium is the same as the one described in section 3.1, with the additional constraint that an entrant shadow bank must make negative expected profits. This will happen if the shadow bank cannot capture enough of the existing liquidity premium so that the expected return paid on risky securities is greater than the average cost of funding for a commercial bank. As risky securities only have access to the liquidity premium in the high state, the less is the high state likely, the less premium they can capture. A highly unlikely good state also increases the risk premium, further increasing the cost of issuing a risky security. Therefore, an equilibrium with a commercial bank exists only if the probability of a low state is large enough.

If instead the probability of a low state is small, then the issuer of risky securities wants to enter the market. The result is an equalization of the cost of funding, as long as one type of bank is not incentivized to expand its balance sheet beyond feasibility. This is the mechanism that drives the existence of the lower bound for the liquidity of securities $\underline{\theta}^{cs}$. As the liquidity value of securities θ decreases, the shadow banking sector controls a larger share of the capital in the economy. This is driven by an increase in the difference between the expected return on capital and the realized return in the low state. This forces the commercial bank to issue more equity to ensure the return of its safe securities. The increased operational cost reduces the size of commercial banking to the point that shadow banks would want to control more than the available capital. On the contrary, the role of the upper bound $\bar{\theta}^{cs}$ is the same as the one seen in the previous sections, where if securities bring too much liquidity value, then there is no place for fiat currency.

In terms of consumption, both types of securities circulate in the second equilibrium, thus morning consumption is differentiated between the two states. As in the equilibrium described in Proposition 2, the household is able to consume more in the high state mornings, but not as much as to completely fulfill its liquidity needs. However, the mechanism is different from the one in Proposition 2. There fiat currency was fully responsible for the consumption in the low state, so the value of currency was more sensitive to the changes in the values of parameters like the liquidity value of securities θ. In this case, the value of currency is less elastic to such changes, since part of the change is absorbed by the commercial bank's securities. In other words, the real value of money is more stable with respect to changes in the environment when other similarly safe sources of liquidity are available.

4.2 Positive Balance Sheet Costs ($\tau > 0$)

The previous sections serve as a baseline to understand the interaction between different types of liquid debt securities. However, the issuance of different types of securities is often connected to management and regulatory costs that go beyond the simple difference in returns. For instance, a financial intermediary may implement stronger monitoring practices when investing in capital backed by high-grade debt, as in Benigno and Robatto (2019). In terms of regulatory costs, the biggest for bank holding companies is the FDIC insurance fee, as documented by Afonso et al. (2018) and Banegas and Tase (2017).

To model these differences in the cost of funding, I assume that the commercial bank needs to pay an additional cost measured as a fraction $\tau > 0$ of the capital it acquires. This is effectively a capital tax that is paid in units of the consumption good, which is assumed to be wasted.[16] All things being equal, the additional cost increases the return on capital required for a commercial bank to break even, thus it creates the space for new types of equilibria that were impossible in the previous case. This is crucial, as the model can express its full richness of results only when the balance sheet costs are positive. To be more precise, it can be shown that a competitive equilibrium would exist over a broader region of the parameter space when the balance sheet costs are positive and small. Furthermore, the existence of this cost is going to introduce a new competitive equilibrium that features a stark allocation, which allows for a clear prediction regarding the inefficiency of such allocation. As different equilibria have different welfare implications, this introduces a role for government policies.

[16]The results would not be different if the capital tax is rebated as a lump sum to the household.

Finally, the balance sheet costs open the possibility for multiple equilibria over the same parameter space, which is going to impact the effectiveness of government policies.

First, I describe the main equilibrium that emerges under this market structure.

Proposition 4. *Given a monetary policy $\mu > \beta - 1$ and parameters of the model, there exists $\bar{\tau}^{sc}$, $\underline{\theta}^{sc}_{\tau}$, and $\bar{\theta}^{sc}_{\tau}$ such that if $0 < \tau < \bar{\tau}^{sc}$ and $\theta \geq \underline{\theta}^{sc}_{\tau}$, or $\tau \geq \bar{\tau}^{sc}$ and $\underline{\theta}^{sc}_{\tau} \leq \theta \leq \bar{\theta}^{sc}_{\tau}$, there exists an equilibrium where both banks operate and the liquidity premia are given by*

$$\kappa_h = 0 \text{ and } \kappa_\ell = \frac{1 + \mu - \beta}{\pi}.$$

I prove the proposition in Appendix A.4.The intuition for the boundary on the liquidity of securities is the same as Proposition 3. If the liquidity services are low, then the shadow bank has an incentive to expand its balance sheet beyond what is feasible in the economy. The same holds true if the balance sheet costs τ are large. If that is the case, then the financial sector as a whole may also issue too many securities, rendering money useless. Thus, an upper bound for security liquidity exists in this parameter region.

In this equilibrium, morning consumption reaches its unconstrained optimal level $c^{am} = 1$ in the high state. That is, the liquidity demand is fully satiated because shadow banks acquire enough capital to provide ample liquidity. However, this comes at the cost of the morning consumption in the low state, which is at the minimum possible level. As risky securities are defaulted upon, a large fraction of the aggregate liquidity evaporates. Such a variance in consumption is due to household money demand pinning down the average liquidity premium. If there is to be fiat currency or another form of outside money, then the provision of inside liquidity, in the form of safe and risky securities, must follow the constraint imposed by Equation (22). Then banks make their decision on the intensive and extensive margin accordingly.

Other equilibria exist in regions of the parameter space not covered by Proposition 4. One such case is an equilibrium where the only intermediaries are shadow banks. As detailed in section 3.2, this equilibrium is characterized by the circulation of risky securities only, thus currency is necessary to achieve positive morning consumption in the low state. Furthermore, the equilibrium liquidity will generally not lead to the zero liquidity premium in the high state of Proposition 4. The existence of this equilibrium is limited by the commercial bank's incentives to entry, which are decreasing in the balance sheet cost τ.

Furthermore, equilibria with commercial banks only or with both banks but positive liquidity premium in both states detailed in Proposition 3 are also possible with positive balance sheet costs. They are still mutually exclusive, but they can each exist in regions where the equilibrium of Proposition 4 exists as well. That is, there is a region of the parameter space where multiple equilibria exist. The balance sheet costs τ are the main responsible for this since they introduce a wedge in an otherwise linear problem.

To better understand the mechanism that drives the multiplicity, start from an economy where the household is not liquidity constrained in the good state. Then reduce the amount of liquidity available in the good state, for instance by reducing the quantity of money. Now there exists a positive liquidity premium in the good state that makes it cheaper for the shadow bank to operate. The commercial bank funding cost also goes down, since the return on safe securities decreases. Everything else equal, the shadow bank or the commercial bank can expand and profit. As they expand, the liquidity premium decreases and the cost of funding increases. If the commercial bank expands, it can't drive the liquidity premium down too much, or profits become negative because of the balance sheet costs. If instead the shadow bank expands, then it will expand as much as possible and drive the premium to zero. General equilibrium forces do not rule out either behavior, which leads to multiplicity.

This equilibrium multiplicity can only be shown numerically since parameter restrictions on the different equilibria cannot always be solved in closed form. Appendix C.1 provides an illustration of how different equilibria exist in parameter space. Here I provide a numerical example to highlight the differences in capital allocation and consumption between equilibria. Consider a parameterization where multiple equilibria are possible, by setting the liquidity

Variable	Description	Worse Equilibrium	Better Equilibrium
K^c	Commercial Capital	18%	64.22%
K^s	Shadow Capital	82%	35.78%
m	Real Currency	0.3206	0.1571
c_h^{am}	High State am Consumption	1	0.9474
c_ℓ^{am}	Low State am Consumption	0.3519	0.6627
	Per Period Expected Utility	0.0802	0.0916

Table 2: Multiple Equilibria Illustration

value of securities to $\theta = 0.035$ and the probability of a crisis to $\pi = 0.04$.[17] Two equilibria exist. In the first one, labeled "Worse Equilibrium", liquidity is organized as in Proposition 4 and consumption is maximized in the high state. In the second one, labeled "Better Equilibrium", both banks still operate, but morning consumption is not maximized in the low state since shadow banks are constrained. First, the composition of the financial sector is dramatically different between the two states. In the better equilibrium, the commercial banks hold a much larger share of capital, which means that shadow banks are smaller and provide a small fraction of the available aggregate liquidity. This is a welfare improvement for the household, since consumption is less volatile between the two states. Table 2 summarizes these results.

Which equilibrium is picked can be the outcome of a sunspot, or the result of government policies that constrain asset issuance. This is the goal of the final section of this paper, where I discuss the welfare implications of the market equilibrium and address its inefficiencies.

5 Inefficient Supply of Liquidity

In the previous sections I have detailed the equilibrium structure of the financial sector and what is the liquidity provision in each equilibrium. The numerical illustration at the end

[17]The remaining parameters are as in the Appendix C.1 illustration. Furthermore, Appendix C.2 shows a simple calibration that justifies the parameter choice.

of Section 4.2 shows how there is a welfare rank among equilibria. Therefore, I now study the optimal provision of liquidity in two steps. First, I consider a simple planner's problem where the objective is to maximize the household's utility to provide a benchmark result. Then I study the planner's problem to allocate the resources in the economy, accounting for the costs and constraints of running the banking sector.

5.1 Simple Planner's Problem and Inefficient Liquidity

To begin with, consider a planner whose objective is to maximize the household's utility by allocating the overall level of liquidity. That is, the planner chooses the state-contingent liquidity premium κ_t, without considering how the allocation is achieved among money, safe, and risky securities. The household's optimal morning consumption is given by Equation (23). After replacing and removing the constant terms, the per period expected utility in a stationary equilibrium given a monetary policy μ, and the liquidity premia κ_h and κ_ℓ is

$$W = (1-\pi)\left[\log\left(\frac{\beta}{\beta+\kappa_h}\right) - \frac{\beta}{\beta+\kappa_h}\right] + \pi\left[\log\left(\frac{\beta}{\beta+\kappa_\ell}\right) - \frac{\beta}{\beta+\kappa_\ell}\right], \tag{32}$$

where $\kappa_h \in [0, 1+\mu-\beta]$. The bounds on the liquidity premium arise from the characterization of the liquidity premia from the household's problem, which requires Equation (22) and $\kappa_\ell \geq \kappa_h$ to hold in a monetary equilibrium.

First, let me discuss which monetary policy achieves the highest levels of welfare, as in the monetary policy chosen by a welfare-maximizing central bank. As shown in section 2.2, the highest level of welfare in a given state is achieved only if the liquidity premium is zero. Thus, if there exists a monetary policy such that the liquidity premium is zero in both states, that would immediately be a candidate for the first best monetary policy. That policy is the Friedman rule, or setting the money growth rate to $\mu = \beta - 1$. Any other policy, with $\mu > \beta - 1$, requires a positive average liquidity premium, thus morning consumption must be less than optimal in at least one of the productivity states.

However, no bank as defined in the model would find it profitable to enter under the Friedman rule. To have a meaningful welfare comparison between the competitive equilibrium and the planner's choice, I set $\mu > \beta - 1$ and look at the second-best supply of liquid at the chosen monetary policy. First, Equation (23), combined with Equation (22), implies that the expected marginal utility with respect to morning consumption is constant in any equilibrium and equal to

$$E\left[U'\right] = \frac{1+\mu-\beta}{\beta} = \frac{\bar{\kappa}}{\beta}. \tag{33}$$

There are two channels that operate in selecting the welfare maximizing liquidity premium. On one hand, different liquidity premia in the two states increase the variance in the marginal utility realizations, which negatively impacts welfare. On the other hand, there is an inflation cost from holding liquidity. The welfare maximization problem, shown in detail in Appendix A.5, reflects these two forces and leads to the following result:

Proposition 5. *If*

$$\mu \leq \begin{cases} \left(1+2\sqrt{(1-\pi)\,\pi}\right)\beta - 1 & \text{if } \pi < \frac{1}{2} \\ 2\beta - 1 & \text{if } \pi \geq \frac{1}{2} \end{cases},$$

then $\kappa_h = 1+\mu-\beta$ is the unique welfare maximizer. If not, then welfare is maximized for some interior liquidity premium $\kappa_h \in (0, 1+\mu-\beta]$.

The proposition states that if the growth rate of money is small enough, then the best outcome is achieved by equalizing the liquidity premium, and therefore consumption, across all states.[18] However, as inflation and the required average liquidity premium increase, there is an inflation cost from holding liquidity in every state of the world. Thus, it becomes beneficial to cluster the losses in the low state and consume more in the good state. Also. when inflation μ is finite, it is never welfare-maximizing to set the liquidity premium to zero in the good state. Therefore:

[18] Berentsen and Waller (2011) also find that the optimal policy is to choose monetary injections that smooth consumption

Corollary 1. *The competitive equilibrium in Proposition 4 is inefficient, in the sense that there exists a different liquidity allocation that improves the household's welfare.*

The source of the inefficiency is the excessive entry from the shadow banking sector.[19] Shadow banks only consider the additional unit of profit from issuing risky securities, without internalizing the welfare loss from the additional volatility in morning consumption. Furthermore, the shadow bank entry lowers the liquidity premium, which makes the return of risky securities higher and consequently keeps the return on capital high, and its price low. Commercial banks do not find it profitable to issue more safe securities, because with high returns on capital they must keep the leverage low. This is not dissimilar from the pecuniary externality in Park (2020), where banks securities issuance only supports consumption in one state, keeping asset prices low and consumption volatility high. The main difference is that in this model the externality propagates through capital, the asset that acts as implicit collateral for the entire banking sector.

5.2 Planner's Problem with Banking

Now that I have a benchmark result for the planner's problem, I consider how the planner's solution is affected once the banking sector is taken into account by the planner. The planner's objective must now include the wasted resources due to the commercial bank balance sheet costs τ, which reduce the available evening consumption. The new objective function is

$$W = (1-\pi)\left[\log\left(\frac{\beta}{\beta+\kappa_h}\right) - \frac{\beta}{\beta+\kappa_h}\right] + \pi\left[\log\left(\frac{\beta}{\beta+\kappa_\ell}\right) - \frac{\beta}{\beta+\kappa_\ell}\right] - \tau q^k K^c, \qquad (34)$$

As in the previous section, the planner is constrained by the household's demand. Therefore, Equation (22) must still apply to guarantee positive money holdings, along with the morning consumption constraint in Equation (2). Furthermore, Equations (24) and (29) pin down the

[19] As in the excessive entry in the Berentsen and Waller (2015) search framework

demand for the safe and risky securities respectively. Then the planner must also account for the banking sector problem. The allocation of capital must satisfy the balance sheet constraints in Equations (12) and (15). Furthermore, the allocation must also ensure that safe securities are guaranteed in every state of the world with the implicit leverage constraint

$$\frac{b_t^c}{q_t^k K_t^c} \leq \frac{1 + r_\ell^k}{1 + r_t^c}. \tag{35}$$

Finally, the last set of constraints ensure the feasibility of the solution. To summarize, the planner chooses an allocation of liquidity $\kappa_h, \ \kappa_\ell$, capital $K^c, \ K^s$, securities b^c, n^c, b^s, and real currency m to maximize (34) under the previously illustrated set of constraints.

To study the properties of the problem, consider a solution where all the feasibility constraints are satisfied and both banks exist. Call λ_1, λ_2 the two multipliers on Equations (35) and (2) respectively. After some manipulations, it can be shown that the first order conditions for the commercial bank safe securities and equity are

$$b^c : \quad \tfrac{\tau}{1+\tau} \quad = \lambda_1 \frac{(1+\tau)\, n^c}{(b^c + n^c)^2} + \lambda_2 \frac{\theta R^s(\kappa_h)}{1+\tau} \tag{36}$$

$$n^c : \quad \tfrac{\tau}{1+\tau} \quad = -\lambda_1 \frac{(1+\tau)\, b^c}{(b^c + n^c)^2} + \lambda_2 \frac{\theta R^s(\kappa_h)}{1+\tau}. \tag{37}$$

$R^s(\kappa_h)$ represents the gross rate of return of risky securities as a function of the high state liquidity κ_h. The left-hand side is the marginal cost of having a commercial bank system, which is proportional to the balance sheet size and constant. The right-hand side is the marginal benefit of increasing the amount of each security. Both securities make the shadow banking sector and the returns it can offer smaller, with a benefit captured by the λ_2 term. However, only safe securities increase the morning consumption, which leads to a benefit from increasing the amount of safe securities and a cost from increasing equity. Since both conditions hold at the same time, the conclusion is that in the planner solution $\lambda_1 = 0$, which implies that the leverage constraint in (35) is not binding. The planner forces the commercial bank to be overcollateralized and ultimately offer less safe securities.

Consequently, the planner tolerates some variance in morning consumption to reduce the cost of having a large commercial bank. A shadow bank sector exists to supply the economy with liquid securities to buy additional morning consumption in the good state of the economy. The overall liquidity allocation is summarized in the following proposition:

Proposition 6. *A planner that considers the banking sector allocation chooses $\kappa_\ell > \kappa_h > 0$ as the unique welfare maximizer.*

Because of the number of constraints involved in the problem, the above solution can only be recovered numerically. Nevertheless, this does not change the conclusion that the competitive equilibrium in Proposition 4 is inefficient since the main externality is the excessive entry by shadow banks, and the consequent lack of safe securities.

5.3 Implementing the Planner's Solution

To implement the planner's solution, I introduce lump-sum taxes and transfer to the banking sector. The budget constraints then become

$$\begin{aligned} (1+\tau)\, q^k K^c + T^c &= b^c + n^c && (38) \\ q^k K^s + T^s &= b^s && (39) \end{aligned}$$

for the commercial and shadow bank respectively. The lump-sum is in units of the consumption good, and it represents a subsidy if $T^i < 0$ or a tax if $T^i > 0$, for $i \in \{c, s\}$. Taxes and transfers are financed from a lump-sum tax on the household T^h, such that the government budget is balanced

$$T^c + T^s = T^h \tag{40}$$

In order to solve for the taxation that implements the planner's solution, I ignore the tax on the household and verify that the solution found does not violate the household's Euler equations[20] and market clearing. Then the following holds:

Proposition 7. *There generally exists a unique* T^{c*} and T^{s*} that implement the planner's solution

The values for T^{c*} and T^{s*} are recovered numerically, so while they generically exist, there are counterexamples in extreme regions of the parameter space where the social optimal cannot be implemented. The values for T^{c*} and T^{s*} are generally positive, which means that the planner is taxing the intermediaries and then rebating the tax to the household. A tax on shadow banks is intuitive since the planner desires to reduce the level of entry from the shadow banking sector. The tax on commercial banks is less immediate, but recall that issuing safe securities has a social cost. Therefore, the planner needs to also limit commercial banks expansion after the shadow banks reduce their presence in the economy.

6 Government Policies

In the previous section, I have determined that the competitive equilibrium in Proposition 4 is inefficient and that the planner's solution can be implemented via a lump-sum tax. I now turn my attention to interventions that policymakers have implemented to strengthen the financial system to see if they can implement the social optimum. As a starting point, I choose to focus on the competitive equilibrium described in Proposition (4), since it provides a better description of what takes place in reality. There is abundant liquidity during normal times, but during a crisis privately created liquidity dries up, without completely disappearing. Because the welfare analysis from the previous section is ex-ante, I study if and how prudential policies can achieve the second-best welfare of Proposition 6. In particular, I focus on liquidity requirements and capital requirements. Liquidity requirements reduce the volatility of the total value of assets in the banks' balance sheet and therefore reduce losses

[20] In particular by ensuring that $c^{pm} > 0$ in every state

in the event of a default.[21] Similarly, capital requirements protect security holders by issuing a junior asset that is the first to absorb the losses.

6.1 Liquidity Requirements

In this section, I focus on liquidity requirements as a policy to address the inefficiencies in the financial markets. As currently implemented, the main objective of liquidity requirements is to avoid self-fulfilling prophecies that would lead to a bank run.[22] In fact, an intermediary can be solvent, with assets valued more than liabilities, but not able to cover unexpected cash flows.

As such, the Basel III accords introduce a liquidity coverage ratio (LCR) requirement, where intermediaries need to hold liquid assets for a value greater or equal to their net cash flow over a 30-day stress period.[23] To implement this regulation in the model, I consider cash as the only asset that counts toward the liquidity requirement and require intermediaries to hold a fraction of their liabilities in the liquid asset. Thus, every intermediary must hold an amount of fiat currency greater or equal to a fraction $0 < \delta < 1$ of the issued securities, or

$$m^c \geq \delta^c b^c \text{ and } m^s \geq \delta^s b^s. \tag{41}$$

This allows for potentially different regulations to be imposed on the two banking sectors. The constraint is always going to be binding for both banking sectors since in equilibrium the return on currency is always lower than the expected return on capital. Thus, liquidity

[21] Of course, liquidity requirements may also be useful to prevent other causes of a financial collapse, such as bank runs.

[22] As intended in the literature stemming from the Diamond and Dybvig (1983) model.

[23] Similar to my model, different assets have different likelihood of losing their liquidity value and are consequently classified differently. Level 1 assets are the safest and most liquid, thus they fully count towards the liquidity requirements. Examples include cash, central bank reserves, and high-quality government securities. Level 2 assets carry some risk of losing their liquidity value, thus only a fraction of their value counts towards the liquidity coverage ratio, with haircuts up to 50%. Starting from the most liquid instruments, examples include securities issued or guaranteed by specific multilateral development banks or sovereign entities, securities issued by U.S. government-sponsored enterprises, publicly traded common stocks, and investment-grade corporate debt securities issued by non-financial sector corporations.

requirements can be interpreted as an additional cost that is imposed on intermediaries to push the intermediaries to issue more or only safe liabilities.

After the new regulation is imposed, I let the economy adjust to a new stationary competitive equilibrium. Then, the following proposition holds:

Proposition 8. *Suppose that the planner wants to achieve the allocation described in Proposition 6. There generally exists a unique* $\delta^{c*} > 0$ and a $\delta^{s*} > 0$ *that implement the welfare maximizing liquidity allocation as a competitive equilibrium.*

The values for δ^{c*} and δ^{s*} are recovered numerically, so while they generically exist, there are counterexamples in extreme regions of the parameter space where the social optimal cannot be implemented. The optimal policy generally imposes stricter regulation on shadow banks, or $\delta^{s*} > \delta^{c*}$. Intuitively, the planner wants to make it more expensive for a shadow bank to operate, so that the commercial bank can expand. However, there are parameter regions where the opposite is true, or $\delta^{s*} < \delta^{c*}$. This happens because the planner also wants to limit the size of a commercial bank since there is a social cost associated with the amount of capital that a commercial bank holds.

The approach outlined here to restore efficiency has strong limitations in practical applications. Since different liquidity requirements need to be imposed on each type of bank, the supervising authority must be able to identify which securities should be identified as safe and which as risky. The cost of such activity may outweigh the benefit of a more efficient allocation of liquidity.[24] Furthermore, this model does not take into account the beneficial effect credit expansion has on investment. Consequently, a regulator must be mindful of the negative spillovers on credit availability that liquidity requirements may introduce.

6.2 Capital Requirements

After establishing that liquidity requirements can restore efficiency but may be difficult to implement, I now consider capital requirements. These have been long used to ensure the

[24]As previously documented, the current regulatory framework combines securities into broad categories

stability of the financial sector, and they have been subject to numerous revisions. The underlying principle is to make sure financial institutions have enough skin in the game to avoid excessive risk-taking and hold enough resources to withstand a negative shock.

Under the Basel III agreement, financial institutions must hold a minimum amount of capital relative to their risk-weighted assets.[25] This type of regulation can be almost directly implemented in the model, by mandating intermediaries to issue equity for at least a fraction $0 < \gamma < 1$ of their assets, or

$$n^c \geq \gamma^c q^k K^c \text{ and } n^s \geq \gamma^s q^k K^s. \tag{42}$$

As for the previous policy, I allow for differential regulation between the two banking sectors. Additionally, this constraint will always be binding for the shadow banks, but not necessarily for the commercial ones. Issuers of safe liabilities are still subject to the leverage constraint defined in Equation (14), so market forces may push them to issue more equity than what the policy requires.

After the new regulation is imposed, I let the economy adjust to a new stationary competitive equilibrium. Then, the following proposition holds:

Proposition 9. *Suppose that the planner wants to achieve the allocation described in Proposition 6. There generally exists a* $\gamma^{c*} > 0$ and a $\gamma^{s*} > 0$ *that implement the welfare maximizing liquidity allocation as a competitive equilibrium. However, an inefficient competitive equilibrium may coexist under the same policy choice.*

The values for γ^{c*} and γ^{s*} are recovered numerically, so while they generically exist, there are counterexamples in extreme regions of the parameter space where the social optimal cannot be implemented. Furthermore, the policy is often not unique, as the capital requirement

[25] Financial institutions must have a ratio of common equity tier 1 over risk-weighted assets greater than 4.5%. That means, an intermediary must hold common stocks and earnings for a value greater than 4.5% of the value of its assets, weighted by the risk. A broader requirement also mandates a Tier 1 capital (which includes equity-like securities such as non-redeemable non-cumulative preferred stocks) ratio over the risk-weighted assets over 6%.

is not binding for commercial banks. That is, any $\gamma^{c*} \in [0, \bar{\gamma}^c(\gamma^{s*})]$ implements the social optimum, where $\bar{\gamma}^c(\gamma^{s*})$ is the equilibrium commercial bank fraction of capital under the policy γ^{s*}.

Importantly, capital requirements may not eliminate inefficient equilibria. It can be shown that, if the capital requirement on shadow banks is not too large, the inefficient equilibrium of Proposition 4 still exists. Therefore, if the economy starts from that equilibrium, capital requirements may not be sufficient to induce a more efficient allocation of liquidity. The mechanism that drives equilibrium multiplicity under the capital requirement policy is similar to what drives multiplicity in the baseline model. Since capital requirements impose a cost that is proportional to the value of capital, they introduce the same wedge that the balance sheet costs τ adds. The difference is that the wedge now operates on the shadow banks, while it is mostly irrelevant for commercial banks. Because of equilibrium multiplicity, capital requirements are a potentially ineffective policy tool.[26]

7 Conclusion

This paper outlines a model where financial intermediaries issue liabilities that are traded for liquidity purposes, along with fiat currency. Some of these liabilities take the form of safe and always liquid securities, such as deposits. Other liabilities are risky, such as repo and commercial paper, and cease to provide liquidity services when defaulted upon. Intermediaries compete to determine the endogenous composition of liquidity, as demanded by the household. Moreover, the household money demand Euler pins down the average quantity of aggregate liquidity in the economy. Consequently, the amount of liquid assets must be either constant in all states of the world and set to some intermediate amount, or it is abundant in some states and scarce in others.

Because the aggregate amount of liquidity is set, but intermediaries compete among each

[26]Kashyap, Tsomocos, and Vardoulakis (2017) find a similar result, where capital requirements are insufficient and the inefficiency can only be corrected by the joint introduction of capital and liquidity requirements

other, an externality arises. As an intermediary issues more of a given security, the supply of the other liquid securities must drop to keep the total amount of liquidity consistent with equilibrium. Furthermore, the composition of liquidity across the different securities has real effects, since agents are unable to buy consumption goods with an illiquid security. In equilibrium, there is ample issuance of risky securities and relatively few safe securities. Thus, in good times households can finance high levels of consumption, while in bad times liquidity dries up and consumption collapses.

However, the competitive equilibrium is inefficient, since a planner that maximizes the household's expected utility chooses consumption to be as constant as possible. To achieve this result, liquidity needs to be less volatile across states, and the planner reduces the amount of risky securities in favor of a larger amount of safe securities. Consequently, government regulation can be used to address the inefficiency. Lump-sum taxes implement the planner's solution, but they are not likely to be adopted by a policymaker. Thus I look at liquidity and capital requirements. The former can restore efficiency when appropriately designed, but requires separate regulation for each type of security. This is likely to generate moral hazard if the regulator is unable to verify intermediaries' balance sheets. The latter is instead insufficient on its own since the economy would remain in the inefficient equilibrium, even if a dominating one exists.

References

Afonso, G., R. Armenter, and B. R. Lester (2018). A model of the federal funds market: yesterday, today, and tomorrow.

Allen, F. and D. Gale (1988). Optimal security design. *The Review of Financial Studies 1*(3), 229–263.

Andolfatto, D., A. Berentsen, and C. Waller (2016). Monetary policy with asset-backed money. *Journal of Economic Theory 164*, 166–186.

Banegas, A. and M. Tase (2017). Reserve balances, the federal funds market and arbitrage in the new regulatory framework.

Barnett, W. A. (1982). The optimal level of monetary aggregation. *Journal of Money, Credit and Banking 14*(4), 687–710.

Benigno, P. and R. Robatto (2019). Private money creation, liquidity crises, and government interventions. *Journal of Monetary Economics 106*, 42–58.

Benmelech, E. and N. Bergman (2018). Debt, information, and illiquidity. Technical report, National Bureau of Economic Research.

Berentsen, A. and C. Waller (2011). Price-level targeting and stabilization policy. *Journal of Money, Credit and Banking 43*, 559–580.

Berentsen, A. and C. Waller (2015). Optimal stabilization policy with search externalities. *Macroeconomic Dynamics 19*(3), 669–700.

Bianchi, J. and S. Bigio (2017). Banks, liquidity management, and monetary policy.

Bigio, S. (2015). Endogenous liquidity and the business cycle. *American Economic Review 105*(6), 1883–1927.

Board of Governors of the Federal Reserve System (2019a). Velocity of m2 money stock. Retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/M2V, May 26, 2019.

Board of Governors of the Federal Reserve System (2019b). Velocity of mzm money stock. Retrieved from FRED, Federal Reserve Bank of St. Louis;

https://fred.stlouisfed.org/series/MZMV, May 26, 2019.

Brunnermeier, M. K. (2009). Deciphering the liquidity and credit crunch 2007-2008. *Journal of Economic perspectives 23*(1), 77–100.

Caballero, R. J. (2006). On the macroeconomics of asset shortages. Technical report, National Bureau of Economic Research.

Caballero, R. J. and E. Farhi (2017). The safety trap. *The Review of Economic Studies 85*(1), 223–274.

Diamond, D. W. and P. H. Dybvig (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy 91*(3), 401–419.

Diamond, W. (2016). Safety transformation and the structure of the financial system.

Eisfeldt, A. L. (2004). Endogenous liquidity in asset markets. *The Journal of Finance 59*(1), 1–30.

Farhi, E. and M. Maggiori (2017). A model of the international monetary system. *The Quarterly Journal of Economics 133*(1), 295–355.

Geanakoplos, J. (2003). Liquidity, default, and crashes endogenous contracts in general. In *Advances in economics and econometrics: theory and applications: Eighth World Congress*, Volume 170.

Geanakoplos, J. and W. Zame (2002). Collateral and the enforcement of intertemporal contracts. Technical report, Yale University working paper.

Geanakoplos, J. and W. R. Zame (2014). Collateral equilibrium, i: A basic framework. *Economic Theory 56*(3), 443–492.

Geromichalos, A. and L. Herrenbrueck (2016). Monetary policy, asset prices, and liquidity in over-the-counter markets. *Journal of Money, Credit and Banking 48*(1), 35–79.

Geromichalos, A., L. Herrenbrueck, and S. Lee (2018). Asset safety versus asset liquidity. Technical report, Working paper.

Geromichalos, A., J. M. Licari, and J. Suárez-Lledó (2007). Monetary policy and asset prices. *Review of Economic Dynamics 10*(4), 761–779.

Gertler, M. and P. Karadi (2011). A model of unconventional monetary policy. *Journal of Monetary Economics 58*, 17–34.

Gertler, M. and N. Kiyotaki (2010). Financial intermediation and credit policy in business cycle analysis. In *Handbook of monetary economics*, Volume 3, pp. 547–599. Elsevier.

Gorton, G. (2017). The history and economics of safe assets. *Annual Review of Economics 9*, 547–586.

Gorton, G., T. Laarits, and A. Metrick (2018). The run on repo and the fed's response. Technical report, National Bureau of Economic Research.

Gorton, G. and A. Metrick (2012). Securitized banking and the run on repo. *Journal of Financial Economics 104*(3), 425–451.

Gorton, G. and G. Ordonez (2014). Collateral crises. *American Economic Review 104*(2), 343–78.

Gorton, G. and G. Pennacchi (1990). Financial intermediaries and liquidity creation. *The Journal of Finance 45*(1), 49–71.

Gu, C., F. Mattesini, C. Monnet, and R. Wright (2013). Banking: A new monetarist approach. *Review of Economic Studies 80*(2), 636–662.

Gu, C., F. Mattesini, and R. Wright (2016). Money and credit redux. *Econometrica 84*(1), 1–32.

Gurley, J. and E. Shaw (1960). Money in a theory of finance. *Brookings Institution*.

Herrenbrueck, L. (2019). Interest rates, moneyness, and the fisher equation. *Simon Fraser University Discussion Papers*.

Herrenbrueck, L. and A. Geromichalos (2017). A tractable model of indirect asset liquidity. *Journal of Economic Theory 168*, 252–260.

Holmström, B. and J. Tirole (1998). Private and public supply of liquidity. *Journal of Political Economy 106*(1), 1–40.

Holmstrom, B. and J. Tirole (2011). *Inside and outside liquidity.* MIT press.

Jordà, Ò., M. Schularick, and A. M. Taylor (2011). Financial crises, credit booms, and external imbalances: 140 years of lessons. *IMF Economic Review 59*(2), 340–378.

Kashyap, A. K., D. P. Tsomocos, and A. Vardoulakis (2017). Optimal bank regulation in the presence of credit and run risk.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*(2), 211–248.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012). The aggregate demand for treasury debt. *Journal of Political Economy 120*(2), 233–267.

Kurlat, P. (2017). Liquidity as social expertise. *The Journal of Finance 73*(2), 619–656.

Lacker, J. M. and S. L. Schreft (1996, August). Money and credit as means of payment. *Journal of Monetary Economics 38*(1), 3–23.

Lagos, R. (2011). Asset prices, liquidity, and monetary policy in an exchange economy. *Journal of Money, Credit and Banking 43*, 521–552.

Lagos, R. and G. Rocheteau (2008). Money and capital as competing media of exchange. *Journal of Economic Theory 142*(1), 247–258.

Lagos, R. and R. Wright (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy 113*(3), 463–484.

Lagos, R. and S. Zhang (2018). On money as a medium of exchange in near-cashless credit economies.

Li, Y. (2017). Procyclical finance: The money view.

Lorenzoni, G. (2008). Inefficient credit booms. *The Review of Economic Studies 75*(3), 809–833.

Lucas, R. E. (1990). Liquidity and interest rates. *Journal of Economic Theory 50*(2), 237–264.

Lucas, R. E. and J. P. Nicolini (2015). On the stability of money demand. *Journal of Monetary Economics 73*, 48–65.

Lucas, R. E. and N. L. Stokey (1987). Money and interest in a cash-in-advance economy. *Econometrica: Journal of the Econometric Society*, 491–513.

Magill, M. J., M. Quinzii, and J.-C. Rochet (2016). Unconventional monetary policy and the safety of the banking system.

Moreira, A. and A. Savov (2017). The macroeconomics of shadow banking. *The Journal of Finance 72*(6), 2381–2432.

Park, J. (2020). Inside money, business cycle, and bank capital requirements. *Review of Economic Dynamics 36*, 103–121.

Piazzesi, M. and M. Schneider (2018). Payments, credit and asset prices.

Queralto, A. (2019). A model of slow recoveries from financial crises. *Journal of Monetary Economics*.

Stein, J. C. (2012). Monetary policy as financial stability regulation. *The Quarterly Journal of Economics 127*(1), 57–95.

Woodford, M. (2001). Monetary policy in the information economy. Technical report, National Bureau of Economic Research.

A Proofs

A.1 Proof of Proposition 1

Solve for b^c using Equations (24), (25), and (27). Then combine the result with Equation (2) and $\kappa_\ell = \kappa_h = 1 + \mu - \beta$ to get

$$\frac{\beta}{1+\mu} = \varphi_t M_{t-1} + \theta \frac{b^c}{\theta (1+\mu-\beta) + \beta}.$$

Multiply and divide the money term M_{t-1} by φ_{t-1}, noting that ${}^{\varphi_{t-1}}/_{\varphi_t} = 1 + \mu$ and $m = \varphi_{t-1} M_{t-1}$, to get

$$\frac{\beta}{1+\mu} = \frac{m}{1+\mu} + \theta \frac{b^c}{\theta (1+\mu-\beta) + \beta}.$$

Replace the value of b^c and simplify to get

$$m = \frac{\beta - \beta^2 (1-\theta) - \beta\theta (1+\mu) \left(1 + \bar{A}\bar{K}\right) - A_\ell \bar{K} \theta (1+\mu)(1-\beta)}{1 - \beta - \theta (1+\mu-\beta)},$$

which, given the general assumptions about the parameters, is positive if and only if

$$\theta < \frac{(1-\beta)\beta}{A_\ell \bar{K} (1+\mu)(1-\beta) + \beta \left[1 + \mu - \beta + \bar{A}\bar{K} (1+\mu)\right]} = \bar{\theta}^c.$$

Which proves the first part of the proposition. Then differentiate m with respect to θ to get

$$\frac{\partial m}{\partial \theta} = -\frac{(1-\beta)(1+\mu)\left[\bar{A}\beta + A_\ell (1-\beta)\right]}{\left[\beta (1-\theta) + \theta (1+\mu) - 1\right]^2} \bar{K}.$$

Numerator and denominator are both positive, thus the real value of money is decreasing in security liquidity θ.

A.2 Solution Steps for the Equilibrium in Proposition 2

Note that

$$E\left[1+r^{k}\right]=\frac{\bar{A}+q^{k}}{q^{k}} \text{ and } 1+r_{\ell}^{k}=\frac{A_{\ell}+q^{k}}{q^{k}}.$$

Then Equations (30) and (31) both solve for the price of capital q^k as a function of the liquidity premium in the good state κ_h and the loss rate of default χ_ℓ:

$$\begin{aligned}
q^{k} &= \frac{\bar{A}\left[\beta\left(1-\pi\chi_{\ell}\right)+\theta\kappa_{h}\left(1-\pi\right)\right]}{1-\pi\chi_{\ell}-\beta(1-\pi\chi_{\ell})-\theta\kappa_{h}\left(1-\pi\right)} \\
q^{k} &= \frac{A_{\ell}\left[\beta\left(1-\pi\chi_{\ell}\right)+\theta\kappa_{h}\left(1-\pi\right)\right]}{1-\chi_{\ell}-\beta(1-\pi\chi_{\ell})-\theta\kappa_{h}\left(1-\pi\right)}
\end{aligned}$$

Equating the two solves for the liquidity premium κ_h as a function of the default loss rate χ_ℓ

$$\kappa_{h}=\frac{\bar{A}\left[1-\beta\left(1-\pi\chi_{\ell}\right)-\chi_{\ell}\right]-A_{\ell}\left(1-\beta\right)\left(1-\pi\chi_{\ell}\right)}{\theta\left(1-\pi\pi\right)\left(A-A_{\ell}\right)}.$$

Thus $\kappa_\ell$is obtained by solving $\bar{\kappa}=1+\mu-\beta=(1-\pi)\,\kappa_h+\pi\kappa_\ell$. Since in an equilibrium it must be that $\kappa_h\geq 0$ and $\kappa_\ell>\kappa_h$, then it must be that $0<\chi_\ell<1$ and

$$\frac{\left(\bar{A}-A_{\ell}\right)\left[1-\beta+\beta\theta\left(1-\pi\right)-\theta\left(1+\mu\right)\left(1-\pi\pi\right)\right]}{\bar{A}-\pi A_{\ell}-\left(\bar{A}-A_{\ell}\right)\beta\pi}<\chi_{\ell}\leq\frac{\left(\bar{A}-A_{\ell}\right)\left(1-\beta\right)}{\bar{A}-\pi A_{\ell}-\left(\bar{A}-A_{\ell}\right)\beta\pi}.$$

The only unknown left to solve for is the default loss rate χ_ℓ. To do so, I can solve for the value of securities issued by the shadow banks in two ways. First, using the shadow bank balance sheet constraint (15), it must be that $b^s=q^k\bar{K}$. Second, using the household morning liquidity constraint (2) realization in the low state

$$\frac{\beta}{\beta+\kappa_{\ell}}=\frac{m}{1+\mu}$$

with the same constraint in the high state and the return on securities (29), shadow banks securities must satisfy

$$b^s = \beta \frac{\kappa_\ell - \kappa_h}{(\beta + \kappa_\ell)(\beta + \kappa_h)} \frac{\theta (1 - \pi) \kappa_h + (1 - \pi \chi_\ell) \beta}{\theta}. \tag{43}$$

Equating the two expressions for securities b^s returns a cubic equation in the object of interest, the default loss rate χ_ℓ. While this equation is unwieldy to even report in this paper, it can be decomposed into a linear and a quadratic factor. The solution to the linear term is never acceptable in equilibrium, which leaves the two solutions from the quadratic equation. One of these solutions either is negative or implies a negative value for liquidity premium in the low state κ_ℓ, which numerically verifies that the solution is unique when it exists.

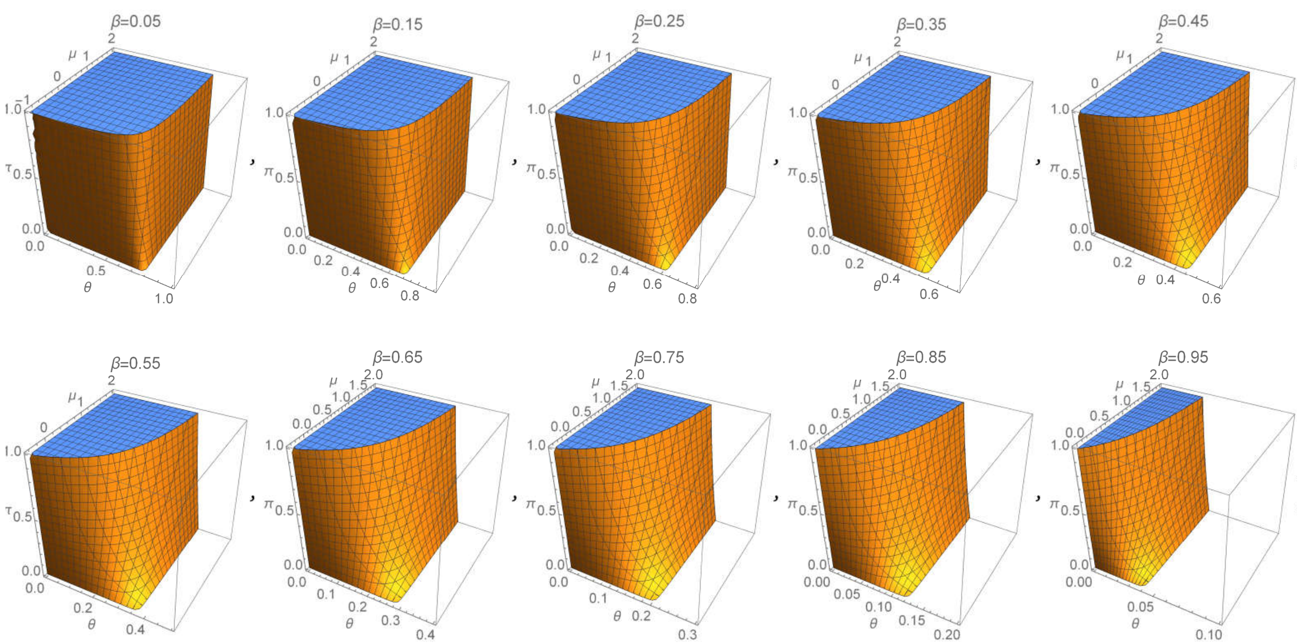
To determine the security liquidity threshold $\bar{\theta}$, suppose one constructs an equilibrium with $\kappa_h = 0$ and $\kappa_\ell = {}^{(1+\mu-\beta)}/_{\pi}$. This hugely simplifies the previous analysis, since many elements of the solutions can be solved for directly. In particular, the return on equity is now ${}^{1}/_{\beta}$, thus the price of capital is

$$q^k = \frac{\beta}{1 - \beta} \bar{A}$$

securities are defined as in Equation (43), which are then used to solve for the default loss rate χ_l from the bank balance sheet constraint (15) as a function of the parameters. However, the limited liability constraint (31) can also be used to obtain another closed form solution for χ_l as a function of the parameters. Since the two expressions for χ_l are not the same algebraically, there must be a parameter value that makes them equal in an equilibrium. The parameter of choice is of course arbitrary, but focusing on the liquidity of the securities returns

$$\bar{\theta}^s = \frac{(1 - \beta)(1 - \pi)(1 + \mu - \beta)}{\bar{K} \left[1 + \mu - \beta (1 - \pi)\right] \left[\bar{A} - \pi A_l - (A - A_l) \beta \pi\right]}$$

Finally, I take a numerical approach to verify that an equilibrium exists only if $\theta \leq \bar{\theta}^s$. While a closed-form solution exists, it is as impractical as the equation generating it to study how it evolves over the entire parameter space. Thus, I test the hypothesis with the assistance of Mathematica to span a reasonable set of the parameter space. The threshold, and therefore the region, is much more sensitive to the discount factor β, the probability of a low state π, and the money growth rate μ, rather than capital $\bar{K}$ and productivity levels A_h and A_l. Consequently, I focus my analysis on the first group of parameters. Here I graphically report the results when setting $\bar{K} = 1$, $A_\ell = 1$, and $A_h = 1.2$. The shaded region is where the equilibrium is satisfied, that is where $\kappa_h \geq 0$, $\kappa_\ell > \kappa_h$, and $0 < \chi_\ell < 1$. The visible boundary is the threshold value $\bar{\theta}^s$.



The scale of the graphs over θ changes to keep the boundary visible, while the scale for μ increases to account for the shifting Friedman Rule. Indeed, the shaded region is to the left of the boundary, or where the value of the security liquidity is below the threshold.

A.3 Deriving the Equilibrium in Proposition 3

As a first step to prove Proposition 3, consider first the commercial bank only equilibrium from Proposition 1 with the details provided in Appendix A.1. The only step missing is verifying that there is no incentive to operate as a shadow bank. First compute what the recovery rate after default $1-\chi_\ell$ is using Equation (31) to get

$$1-\chi_\ell = \frac{(1-\pi)\left[A_\ell(1-\beta)+\bar{A}\beta\right]\left[\beta(1-\theta)+\theta(1+\mu)\right]}{\bar{A}\beta(1-\beta\pi)+A_\ell\theta(1+\mu-\beta)-A_\ell(1-\beta)\beta\pi}.$$

Then compute the expected profits as in (30)

$$E\left[\Pi^s\right] = E\left[1+r^k\right] - \frac{1-\pi\chi_\ell}{\theta(1-\pi)\kappa_h+(1-\pi\chi_\ell)\beta},$$

with the expected return on capital defined by Equation (27). To see where a deviation exists, set $E\left[\Pi^s\right]>0$ and solve the inequality for the probability of the low state π to get

$$\pi < \frac{\left(\bar{A}-A_\ell\right)\left[1-\beta(1-\theta)-\theta(1+\mu)\right]}{\bar{A}\beta+A_\ell(1-\beta)} = \bar{\pi}.$$

The shadow bank profits are positive if the probability of a low state π is below the threshold $\bar{\pi}$, thus an equilibrium with commercial banks exists only if $\pi \geq \bar{\pi}$.

Now move to the second equilibrium in the proposition, where both type of banks operate. Equations (30) and (31) pin down the expected return on capital and the return on capital in the low state respectively as a function of the default loss rate χ_ℓ and the liquidity premium in the good state κ_h. Then Equation (25) pins the fraction of the commercial bank assets financed with safe securities. The complement fraction then identifies the equity issuance as a fraction of the commercial bank assets. Use Equation (26) to solve for the liquidity premium in the high state κ_h as a function of the default loss rate χ_ℓ

$$\kappa_h = \frac{(1+\mu-\beta)(1-\chi_\ell)}{1-\pi}.$$

Since $\bar{\kappa} = 1+\mu-\beta$, the liquidity premium in the low state is

$$\kappa_\ell = \frac{(1+\mu-\beta)\chi_\ell}{\pi}.$$

Plug the liquidity premia in Equation (43) to get an expression for the securities issued by the shadow bank b^s. Also, the liquidity premia can be used back in Equations (30) and (31) to obtain two expressions for the price of capital q^k as in Appendix A.2. Equating the two expressions solves for the default loss rate

$$\chi_l = \frac{\left(\bar{A}-A_\ell\right)[1-\beta(1-\theta)+\theta(1+\mu)]}{\bar{A}[1-\theta(1+\mu-\beta)-\beta\pi]+A_\ell[\theta(1+\mu-\beta)-(1-\beta)\pi]}.$$

Given the solution for χ_l, check for the necessary but not sufficient condition for equilibrium $\kappa_\ell > \kappa_h > 0$ and $0 < \chi_\ell < 1$. The inequalities are verified if either

$$\theta < \frac{\bar{A}(1-2\beta)-2A_\ell(1-\beta)}{\left(\bar{A}-A_\ell\right)(1+\mu-\beta)}$$

or

$$\begin{aligned} \pi \quad &< \quad \frac{\left(\bar{A}-A_\ell\right)[1-\beta(1-\theta)-\theta(1+\mu)]}{\bar{A}\beta+A_\ell(1-\beta)} = \bar{\pi} \\ \frac{\bar{A}(1-2\beta)-2A_\ell(1-\beta)}{\left(\bar{A}-A_\ell\right)(1+\mu-\beta)} \quad &< \theta < \quad \frac{1-\beta}{1+\mu-\beta}, \end{aligned}$$

where these last two inequalities must hold jointly. The first inequality is relevant only if the capital productivity in the high state A_h is at least twice the productivity in the low state, and for low values of the discount factor β, thus I focus on the second set of conditions. These define the upper bound for π from the proposition and conditions on θ that end up being irrelevant for the equilibrium.

To find the relevant conditions on the security liquidity θ it is necessary to solve for the last unknowns in the model. Equation 15 solves for the amount of capital held by the shadow bank K^s, given the solution for the shadow bank securities b^s and the price of capital q^k. Market clearing then returns the capital held by the commercial bank K^c. The equilibrium condition $0 < K^c < \bar{K}$ implicitly determines the equilibrium lower bound for the security liquidity $\underline{\theta}^{cs}$.

After determining the asset side of a commercial bank's balance sheet, Equation (25) pins down the amount of safe securities b^c issued and thus Equation (12) solves for the amount of equity n^c issued. Finally, the household's morning liquidity constraint (2) at the low state solves for the real value of money

$$m = \left(\frac{\beta}{\beta + \kappa_\ell} - \theta \frac{b^c}{\theta \left(1 + \mu - \beta\right) + \beta} \right) \left(1 + \mu\right),$$

where the condition $m > 0$ implicitly determines the upper bound value for the security liquidity $\bar{\theta}^{cs}$.

A.4 Proof of Proposition 4

The proof follows similar steps as detailed in Appendix section A.3 to prove Proposition 3. However, I start not only from guessing that both banks operate, but also that the liquidity premia in each state are given by

$$\kappa_h = 0 \text{ and } \kappa_\ell = \frac{1 + \mu - \beta}{\pi}.$$

Then the zero profit condition on shadow banks (30) immediately implies

$$E\left[1+r^{k}\right]=\frac{1}{\beta}\Rightarrow q^{k}=\frac{\beta}{1-\beta}\bar{A}.$$

Now the price of capital is simply the discounted value of the future expected revenues. The return on capital in the low state pins down the value of commercial bank securities relative to commercial bank capital. Then use the zero profits condition for the commercial bank to recover the default loss rate

$$\chi_{\ell}=\frac{\beta\tau-\theta\left(1+\mu-\beta\right)}{\beta\pi\tau-\theta\left(1+\mu-\beta\right)}.$$

in equilibrium $0<\chi_{\ell}<1$, which gives a first condition on the lower bound for the security liquidity θ

$$\theta>\frac{\beta\tau}{1+\mu-\beta} \tag{44}$$

The second part of the lower bound is derived from the solution for the capital acquired by the shadow bank. The solution for the default loss rate χ combined with the household's liquidity constraint (2) in the high and low state, which solves for the shadow bank securities b^{s}. Then the shadow bank balance sheet constraint (15) solves for the capital owned by the shadow bank K^{s}. In equilibrium this solution must be feasible, or $0<K^{s}<\bar{K}$, which returns

$$\theta>\frac{\left(1-\beta\right)\left(1-\pi\right)\left(1+\mu-\beta\right)}{\bar{K}\left[1+\mu-\beta\left(1-\pi\right)\right]\left[\bar{A}\left(1-\beta\pi\right)-A_{\ell}\left(1-\beta\right)\pi\right]}. \tag{45}$$

The combination of (44) and (45) defines the equilibrium lower bound $\underline{\theta}_{\tau}^{sc}$. The remaining part of the model is solved as in Appendix section A.3. Here I will only detail the conditions such that the real value of money is positive, or $m>0$, which holds true when either

$$\tau\leq\frac{\pi\left(1-\beta\right)\left(1+\mu-\beta\right)}{\bar{A}\bar{K}\left[1+\mu-\beta\left(1-\pi\right)\right]}=\bar{\tau}^{sc},$$

or

$$\theta < \frac{\bar{A}\bar{K}\left[1+\mu-\beta\left(1-\pi\right)\right]\beta\pi\tau-\left(1-\beta\right)\left(1+\mu-\beta\right)\left[\beta-\beta\left(1-\pi\right)\pi-\left(1+\mu\right)\left(1-\pi\pi\right)\right]}{\left(\beta-\mu-1\right)\left[\left(1-\beta\right)\left(1+\mu-\beta\right)\pi-\bar{A}\bar{K}\left(1+\mu-\beta\left(1-\pi\right)\right)\tau\right]}\tau = \bar{\theta}_{\tau}^{sc}.$$

A.5 Proof of Proposition 5

Take the per period expected utility (32) and take the standard first order conditions to recover the following candidate maxima for the liquidity premium in the high state

$$\begin{aligned}
\kappa_{h,1} &= 1+\mu-\beta \\
\kappa_{h,2} &= \frac{1+\mu-\beta+\sqrt{\left(1+\mu-\beta\right)^2-4\beta^2\left(1-\pi\right)\pi}}{2\left(1-\pi\right)} \\
\kappa_{h,3} &= \frac{1+\mu-\beta-\sqrt{\left(1+\mu-\beta\right)^2-4\beta^2\left(1-\pi\right)\pi}}{2\left(1-\pi\right)}
\end{aligned}$$

Looking at existence and feasibility of the solution, $\kappa_{h,2}$ satisfies the constraint (that is $0 \leq \kappa_{h,2} \leq 1+\mu-\beta$) if

$$\sqrt{\frac{\left(1+\mu\right)\left(2\beta-\mu-1\right)}{\beta^2}} \leq 1-2\pi$$

While $\kappa_{h,3}$ is acceptable if

$$\sqrt{\frac{\left(1+\mu\right)\left(2\beta-\mu-1\right)}{\beta^2}} \leq 1-2\pi \text{ or } 1+\mu \geq 2\beta$$

Since the second derivative evaluated at $\kappa_{h,1}$ is positive if $1+\mu < 2\beta$, $\kappa_{h,1}$is the unique interior maximizer if $\sqrt{\frac{(1+\mu)(2\beta-\mu-1)}{\beta^2}} > 1-2\pi$. If the latter condition fails, but $1+\mu < 2\beta$, $\kappa_{h,1}$ and $\kappa_{h,3}$ are both local maxima and $\kappa_{h,2}$ is a local minimum. Finally, if $1+\mu > 2\beta$, $\kappa_{h,3}$ is a local maximum and $\kappa_{h,1}$ is a local minimum. An illustration of the possible optimum is given in the picture below. The left panel shows the case where $\kappa_{h,1}$ is the welfare maximizing liquidity premium in the high state (which requires a relatively small value for μ), while the right panel shows the case of an interior maximizer at $\kappa_{h,3}$ (which exists at higher levels of inflation μ).

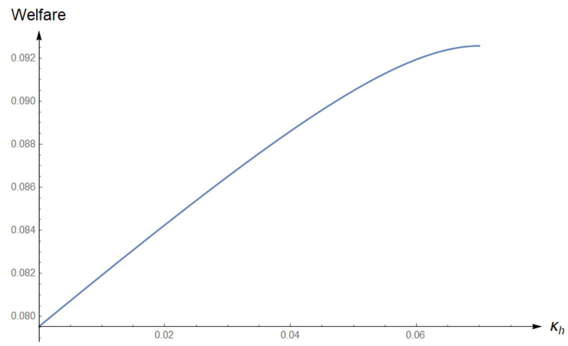


Figure 1: Low μ

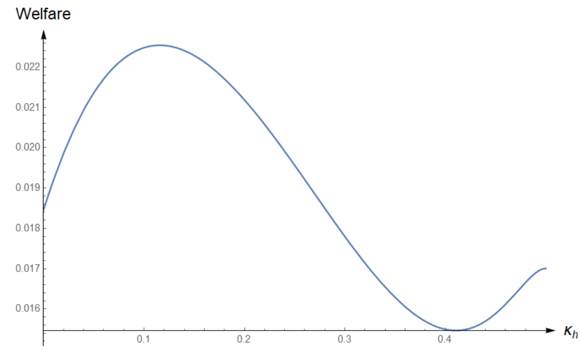


Figure 2: High μ

B Model with Tranching

Consider a commercial bank that can design its balance sheet in the spirit of Allen and Gale (1988). That is, this bank can finance itself with a combination of safe and risky securities, and equity. Following the tranching nomenclature and the potential rating of each tranche, I will denote the safe securities as the senior tranche b^A and the risky securities as the junior tranche b^B.The balance sheet constraint becomes

$$q^k K^c + \tau \left(b^A + n\right) = b^A + b^B + n. \tag{46}$$

Note how the balance sheet cost only applies to the fraction of capital financed with the senior tranche and equity, to keep the model as close as possible to the baseline. As in the main model, the same rules of security design apply. The senior tranche pays out its promised return in every state of the world, therefore

$$\left(1 + r_\ell^k\right) q^k K^c \geq \left(1 + r^A\right) b^A. \tag{47}$$

Then the default rate of the junior trance in the low productivity state is

$$\left(1+r_{\ell}^{k}\right) q^{k} K^{c}-\left(1+r^{A}\right) b^{A}=\left(1-\chi_{\ell}^{B}\right)\left(1+r^{B}\right) b^{B}. \tag{48}$$

A similar constraint potentially applies if the junior tranche is defaulted upon in the high productivity state as well. However, a junior tranche that defaults in every state of the world is clearly equivalent to equity, and therefore I will rule it out[27]. The bank chooses how much of the senior and junior tranche to issue, along with equity, to maximize its expected profits.

The household demanded returns for the senior and junior tranches are defined in (24) and (29) respectively, while the expected return on equity is simply $E\left[1+r^{n}\right]={}^{1}\!/\!_{\beta}$. Finally, the expected return paid by the bank on the junior tranche is defined in (30). Then some algebra manipulations, I can show the following result:

Proposition 10. *For a bank issuing securities in a senior and junior tranche, together with equity, the following holds:*

- *From the bank's perspective, the cheapest security to issue is the senior tranche, followed by the junior tranche and equity.*

- *The return of the junior tranche is decreasing in the default rate, thus the bank will fully default* $(\chi_{\ell}=1)$ *on it.*

The combination of these two results implies that in equilibrium this bank will issue as much as possible of the senior tranche, while the junior tranche takes the role of the residual claimant. In equilibrium, this bank is almost identical to the commercial bank discussed in the main paper, except for a reduction in the wasted resources induced by equity. All of the results then follow, after minor adjustments to account for this difference.

[27]It can be easily shown that a security that defaults in both states of the world has the same expected cost as equity in this model. Thus the bank would be at best indifferent between the two alternatives.

C Numerical Illustrations

C.1 Equilibrium Multiplicity with Positive Balance Sheet Costs

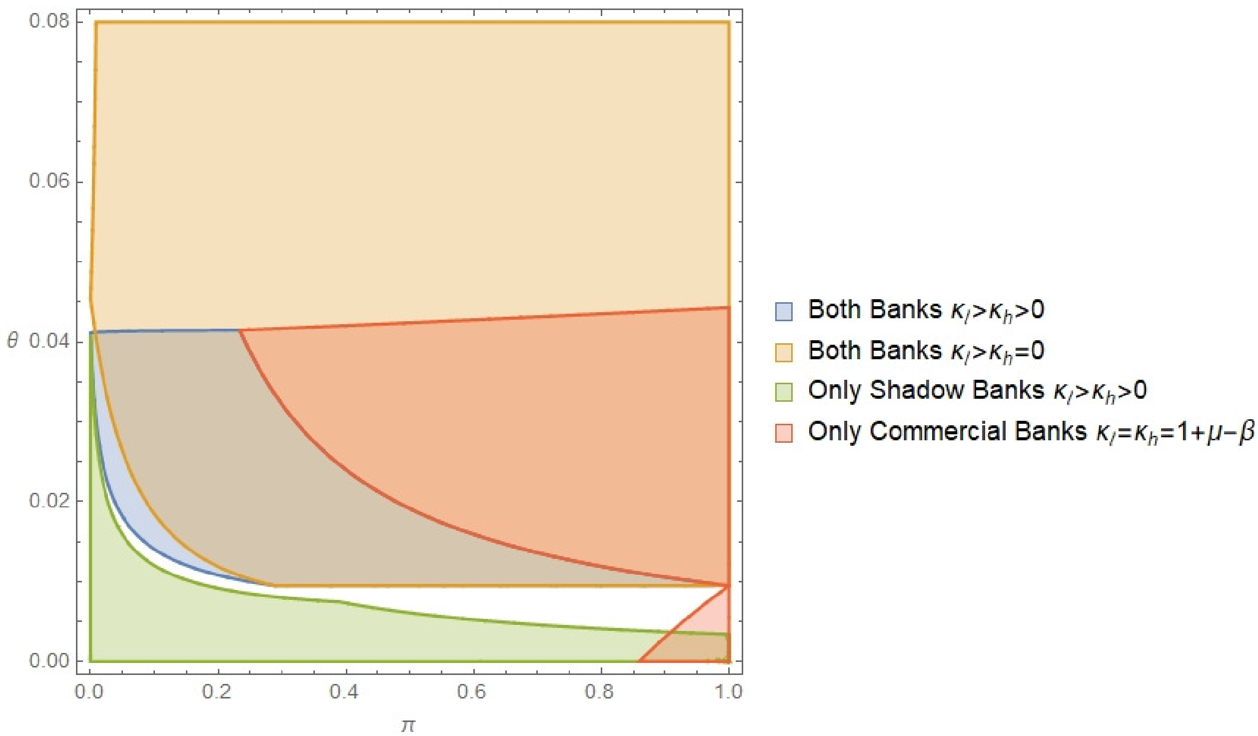
The following figure illustrates the regions where different equilibria exist in the probability of the low state π and security liquidity θ plane. The other parameters are chosen as follows

Parameter	Value
A_ℓ	1.0007
A_h	1.1
$\bar{K}$	1
τ	0.0007
β	0.95
μ	0.02

Note how the region in yellow, which represents the equilibrium where both commercial and shadow banks exist and liquidity is fully satisfied in the good state, partially overlaps with the blue region, where the two banks still operate, but the liquidity premium is always positive, and the red region, where commercial banking is the only profitable type of intermediary.

C.2 Numerical Example

The model is calibrated at a yearly frequency. I choose the balance sheet cost τ to match the cost of FDIC insurance in the United States. Banegas and Tase (2017) estimate this cost at 7 basis points over the entire asset composition of the average balance sheet of an insured intermediary. Then, I pick the productivity in the low state A_ℓ such that consumption must be positive in both sub-periods in any equilibrium. To have a significant difference between a boom and a crisis, I make the high state productivity A_h approximately 10% larger than that of the low state. A numerical exploration of the model suggests that the



outcomes are not strongly dependent on the size of the shock.[28] Furthermore, my choice of productivity parameters is consistent with Queralto (2019), who estimates a drop of 9% in the total factor productivity after banking crises in a panel of advanced and emerging economies. The probability of a low state is chosen so that the average time between two crises is 6 years and one quarter. Given that the realization of a low state is a Bernoulli random variable with independent draws, the expected time between crises (in quarters) is given by $1/\pi$. Jordà, Schularick, and Taylor (2011) look at a panel data of financial crises to unveil an average duration between crises of 28 years, which reduces to 15 years once the no financial crises period from 1940 to 1973 is removed from the sample. Equilibrium conditions imply that the probability of a low state cannot be calibrated to these values. Nevertheless, under the calibration choice, the time between recessions is longer than the approximately 4 years and 3 quarters observed in the U.S. economy after the Great Depression.[29] Finally,

[28]The model is robust for values of A_h up to 30% larger than A_ℓ.

[29]See the NBER U.S. Business Cycle Expansions and Contractions

Parameter	Value	Source
A_ℓ	1.0007	Model condition
A_h	1.1	Queralto (2019)
$\bar{K}$	1	Normalization
τ	0.0007	Banegas and Tase (2017)
β	0.9709	3% Discount Rate
μ	0.02	Federal Reserve 2% Inflation Target
π	0.16	6 Years and 1 Quarter Between Crises

Table 3: Numerical Example: Chosen Parameters

I pick a standard discount factor to target a 3% discount rate[30] and set the growth rate of currency to the Federal Reserve inflation target of 2%. The set of parameters is summarized in Table 3.

With this choice of parameters, I then use the banking data from the Federal Reserve FR Y-9C Consolidated Report of Condition and Income form to recover appropriate values for the liquidity of safe and risky securities θ . Specifically, I extract the consolidated balance sheet from Schedule HC in the third quarter of 2017 and construct the composition of liabilities in the financial sector as a whole to target the ratio between the total amount of safe and risky securities and equity.[31] In the model there is no direct match to such ratio, given that only a subset of financial intermediaries issues equity. However, since the collected data represents a snapshot of the financial industry as a whole, it is appropriate to consider the equity issued by commercial banks in the model as the total equity in the economy. The target value and the resulting parameter is reported in Table 4.

The targeted parameter results may seem surprising, since it is quantitatively small. However, recall how the securities in the model also represent a much wider class of bank liabilities. The parameter θ then combines the liquidity of all of these securities under one

[30]Herrenbrueck (2019) discusses how this choice matters when targeting the Fisher rate. The choice of parameters implies a Fisher interest rate of 5%, not far from the Herrenbrueck (2019) estimation for the same period that includes the effect of consumption growth

[31]Further details on the data are outlined in Appendix C

Parameter	Value	Target
θ	0.0169	$\frac{b^c+b^s}{n^c} = 7.8$

Table 4: Numerical Example: Targeted Parameters

estimate. Consequently, the targeting procedure will likely underestimate the true liquidity of demand deposits and the other liquid securities. Looking at the data, in the third quarter of 2017 the timed and money market deposits were approximately 2.5 times larger than demand deposits.[32] Similarly, long-term risky liabilities are about twice the size of short term liabilities.[33]

Furthermore, the size of the liquidity parameter is a result of the characteristics of the model. In order to guarantee that afternoon consumption is always greater than zero, the productivity of capital needs to be relatively large. Therefore, also the balance sheet of the banking sector is much larger than consumption per time period. Consequently, a monetary equilibrium requires the liquidity parameter to be very small in order for currency and the other assets to circulate at the same time.

Under the calibrated parameterization, the competitive equilibrium is characterized as expected by the presence of both types of financial intermediaries. Specifically, shadow banks flood the market with liquidity in good times, such that the demand for liquidity is completely satisfied and morning consumption reaches its maximum value $c_h^{am} = 1$. However, these securities are unable to provide liquidity services in a crisis, thus morning consumption collapses after a negative aggregate shock.

The equilibrium outcome is summarized in Table 5. Approximately 63% of capital is held by the commercial banking sector, with the remaining part in the shadow banking sector. While the shadow banking sector is roughly half the size of the commercial banking sector, its default causes the morning consumption in the low state to be about one quarter smaller

[32]This measure includes interest and non-interest bearing deposits, NOW accounts and other transaction accounts.

[33]Short term risky liabilities include Federal Funds, reverse repo, and trading liabilities. Long term risky liabilities are made of other borrowings, subordinated notes and other liabilities.

Variable	Description	Equilibrium Value
K^c	Commercial Capital	63.09%
K^s	Shadow Capital	36.91%
$1 - \chi_\ell$	Recovery Rate	79.26%
$\bar{A}\bar{K}/(m+\theta b^c)$	Velocity of Money	1.4277
c_h^{am}	High State am Consumption	1
c_ℓ^{am}	Low State am Consumption	0.7598

Table 5: Calibration: Equilibrium Outcome

than consumption in the good state. The impact on morning consumption is so large because the shadow banks issue slightly less than half of the aggregate amount of privately issued liquid securities. Of course, the total of morning and evening consumption in the low state only falls by 10%, which is equal to the drop in production. This is reflected in a substantial recovery rate on defaulted securities, as the household is able to recover approximately 79% of the promised payment. Finally, as an external validation of the results, I compute the velocity of "M2", intended as currency and traded safe securities. The velocity of 1.43 is in line with what is observed in the data,[34] matching the decline in velocity that has been observed after the global financial crisis.

D Data Sources and Aggregation

I recovered the Fr Y-9C data from the Wharton Research Data Services (WRDS), with a focus on the Schedule HC Consolidated Balance Sheet, as measured at the end day of the filing quarter, using data from Q1 2006 to Q3 2017. Q4 2017 was only partially available for the sample of Bank Holding Companies and therefore it was dropped. While the schedules about balance sheet details (such as a detailed decomposition of loans, securities held and deposit liabilities) have changed multiple times across the relevant time frame, the consolidated

[34]See the M2 and MZM velocity data from the Board of Governors of the Federal Reserve System in the third quarter of 2017

Variable	Construction and Reference Codes
Total Assets	BHCK2170
Total Equity	BHDMG105
Deposits	BHDM6631 + BHDM6636 + BHFN6631 + BHFN6636
Risky Liabilities	BHDMB993 + BHCKB995 + BHCK3548 + BHCK3190 + BHCK4062 + BHCK2750 + BHCKC699

Table 6: Variables Construction with Reference Codes

balance sheet schedule has not, and therefore data is fully comparable across the entire time series. Total equity capital required some reconstruction, as older reports only include its two components, the total holding company equity capital and the non-controlling interests in consolidated subsidiaries.

With the fully uniform data, I compute new aggregated variables to look at trends as informed by the model. Specifically, liabilities are separated into deposits and risky liabilities. The deposits aggregate is given by the interest and non-interest bearing deposits in domestic and foreign offices. The risky liabilities aggregate is composed of purchased Federal Funds, reverse repo, trading liabilities, other borrowed money, and subordinated notes. Table 6 summarizes the full list of relevant variables with the FR Y-9C codes. Then, the relevant ratios are computed to obtain the calibration targets.