Incentives for Contract Designers and Contractual Design*

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Abstract

This paper examines the optimal provision of incentives for contract designers and the implications for contractual design. A buyer hires an agent to draft a contract for the seller that is incomplete because the ex-ante specified design might not be appropriate ex-post. The degree of contract incompleteness is endogenously determined by the effort exerted by the agent, who can manipulate the buyer's beliefs because his effort is not observable (moral hazard), and he is better informed at the outset (adverse selection). We discuss how the asymmetric information generated during the contract drafting stage explains some empirical observations and contracting phenomena.

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1 Introduction

Contracts are critical business instruments, yet those in charge of drafting them are rarely the people responsible for delivering the anticipated outcome. Contract designers are typically not the residual claimants, and their motivation to draft optimal contracts depends on the design of their incentive packages. Moreover, relationships between contract designers and those who employ them typically involve several contractual frictions such as moral hazard, adverse selection, and limited liability.

Consider a buyer who hires an agent to design a contract for the seller who will implement the project. A buyer-seller contract is incomplete because the ex-ante specified project design might not be appropriate ex-post. The contract designer has better information about whether there will be a need for a new design and covertly exerts effort to learn it. For example, in the construction industry, the "Design-Bid-Build" is a project delivery method in which the buyer contracts with separate entities for the design and implementation of a project.[1] The initial project design might be changed due to newly discovered site conditions that were not initially specified. Although the unusual site conditions are foreseeable, the initial contract and price are not necessarily indexed appropriately.[2] The engineers working on the project design may choose to save the cost of the seismic surveys and announce that the site conditions are standard. Moreover, due to their experience from similar projects, the engineers may interpret the publicly available information differently and, as a result, may start the contract-drafting process with superior information.

What is the most efficient way to incentivize the contract designers to exert effort and specify the appropriate design at the outset? Should their compensation be contingent on whether the initially specified design is appropriate ex-post or not? How is the buyer-seller contract affected by the contract designer's incentives, and what are the trade-offs involved?

The novelty of our paper is to endogenize the costs of writing a contract by presenting the drafting of contracts as a contracting problem itself. This allows us to study how the asymmetric information generated during the contract drafting stage impacts the buyer-seller contractual relationship and how the value of the buyer-seller relationship affects the contract designer's incentives. Our key contribution is to study how the asymmetric information at the contract-drafting stage impacts the price and the degree of contract incompleteness.

Our model introduces a contract drafting stage in an incomplete contract environment

[1] The delegation of contract drafting to an agent inside a firm is common as well. For example, contract managers handle pre-contractual matters including the review and drafting of contracts.

[2] An illustrative example is the building of the Getty Center Art Museum in Los Angeles in the 1990s. The project design had to be changed due to site conditions that were not initially described. In addition, the project design also was modified due to the change in the regulatory environment.

of Tirole (2009). There are three players: a buyer, a contract designer (agent), and a seller. A buyer-seller contract specifies the desired project design to be delivered by the seller and the corresponding price. At the outset, the parties are aware of a default design that is appropriate with some probability only. A default design could be a previously used contract or an "industry standard." Something might go wrong, and a new, initially indescribable design might turn out to be more appropriate, in which case the parties engage in costly renegotiation. A contract is, therefore, incomplete because the ex-ante specified design might not be appropriate ex-post. We will say that the contract is more incomplete the higher the probability that the initially specified design is renegotiated ex-post. Furthermore, in our model, the price and the degree of contract incompleteness are inversely related: anticipating that renegotiation is likely the seller accepts a smaller initial price.

The buyer must address two incentive problems. First, a higher effort is required to be more likely to discover the appropriate design, and the agent's effort is not observable (moral hazard). Second, the agent is better informed regarding the probability of discovering the new design at the outset (adverse selection), and we assume that the probability might be high (high-type projects) or low (low-type projects).

We summarize our main results next. First, we show that despite the simultaneous presence of moral hazard and adverse selection, the buyer offers the agent a simple incentive scheme. The agent is rewarded for getting the design right at the outset, and the value of the reward is the same regardless of the project's type. Intuitively, paying the agent unless he discovers the design right only exacerbates the moral hazard problem. The reason the value of the reward does not depend on the project's type is that in our setting, the probability of discovering the appropriate design directly enters the buyer's objective function. As a result, we have what is called a common values problem in contract theory.[3] It is known that in this case, there is a strong conflict between the buyer's preference for efficiency and the screening role of contracts. This results in pooling contracts, i.e., the buyer offers the same contract to agents working on both project types.

Second, we examine how the interaction of moral hazard and adverse selection at the contract drafting stage distorts the degree of buyer-seller contract incompleteness and the corresponding price. The agent devotes less effort to learning the appropriate design relative to the first-best scenario due to moral hazard. Furthermore, adverse selection polarizes the effort choices across the two project types. The agent's reward in the pooling contract is lower than the reward of the agent working on a low-type project in the environment without adverse selection. Then, the agent working on the low-type project exerts less effort. The

[3]See, e.g., Laffont and Martimort (2002).

opposite is true for the agent working on the high-type project as he exerts more effort than in the environment without adverse selection. Therefore, the adverse selection problem results in a contract being either closer to the first-best benchmark (for the high-type project) or more incomplete (for the low-type project). For example, if the high type project is relatively likely, this polarization results in a less incomplete contract and a higher price than in the environment without adverse selection.

Third, we study how the degree of contract incompleteness is affected by the buyer's bargaining power, i.e., the portion of surplus she collects in the case of renegotiation. If the buyer is learning the appropriate design herself, there is interdependence between the effort devoted to learning and the degree of contract incompleteness. Intuitively, a buyer who obtains a smaller portion of the surplus in case of renegotiation has more incentives to learn the appropriate design and, as a result, offers the seller a less incomplete contract and pays a higher price. We find that this intuitive monotonicity of the first-best environment might no longer hold due to the adverse selection problem. If the buyer's bargaining power decreases, she benefits from a higher agent's effort, since it mitigates the chances of renegotiation. However, a higher effort also increases the agent's rent. Therefore, the buyer trades off the efficiency in learning the appropriate design and the agent's rent. When the adverse selection problem is severe enough and the value of the buyer-seller relationship is relatively small, the latter effect becomes pivotal, and the agent's effort decreases. Then, a buyer-seller contract becomes more incomplete. Interestingly, when this is the case, a buyer with a smaller bargaining power might end up paying the seller a smaller price.

Empirical implications. Presenting the drafting of contracts as a contracting problem itself leads to three main empirical implications: (1) the buyer does not screen the contract designer by offering a menu of contracts, and the latter is rewarded for getting the design right at the outset; (2) the contract might be less incomplete and the price higher due to adverse selection; (3) if the adverse selection problem is severe enough, a buyer with a smaller bargaining power might end up paying a smaller price to the seller.

The first implication is consistent with the contract management literature for both public and private project management, which emphasizes the critical role contract managers play in preparing contracts and the importance of providing the appropriate incentives.[4] In particular, it is acknowledged that, in smaller companies, procurement employees (buyers) are "generalists" who might lack relevant expertise so that the contract designer who provides specifications has more knowledge regarding the subject matter.[5]

[4]See Turner (2003), Cohen and Eimicke (2008), and Webb (2015).

[5]For example, when buying software, a procurer might have little knowledge of relevant IT specifications.

The literature emphasizes that the top driver for improvements in the management of contracts is the pressure to mitigate the delivery risk, i.e., the chances of renegotiation of the initial design. Contract managers responsible for the pre-project planning are rewarded for mitigating the likelihood of renegotiation.[6] This is consistent with our economic intuition that the contract designers are rewarded for getting the design right at the outset.

The second implication is consistent with the empirical industrial organization literature on procurement. Alternation of the project design and renegotiation are ubiquitous in both public and private procurement contracts.[7] It is documented that the anticipation of renegotiation by the seller might increase the costs for the procuring agency, with the adaptation costs being one of the main channels. For example, Bajari et al. (2014) provide a quantitative analysis of the price markups due to the expectation of renegotiation using highway paving contracts. Miller (2014) estimates the effect of ex-post contract revisions on the costs for work items on bridge projects procured in California. Jung et al. (2019) study the effect of anticipated contract renegotiations on strategic bidding in construction projects in Vermont.[8] Our analysis suggests that higher prices emerge in an environment where there is an adverse selection problem in the buyer-contract designer relationship. Consistent with Bajari et al. (2014), our analysis suggests that the higher prices due to adverse selection are associated with a lower probability of ex-post renegotiation (see Corollary 3).

The third implication suggests that a buyer with a smaller bargaining power might end up paying a smaller price. The existing literature has explained this phenomenon using the countervailing buyer power hypothesis (see Galbraith (1952)). The intuition provided in the literature is that large retailers are beneficial to consumers since they offer smaller prices to the buyers because their market power allows them to pay smaller prices in the wholesale market. For example, von Ungern-Sternberg (1996) develops a model where a decrease in the number of retailers leads to a decrease in equilibrium consumer prices (see also de Fontenay and Gans (2004), Raskovich (2007), Iozzi and Valletti (2014), and Gaudin (2017)).[9] We highlight a novel channel that might prevent prices from increasing after the consolidation in the retailing sector. In particular, if the adverse selection problem in the buyer-contract designer relationship is severe enough, the price the buyer pays might become lower despite

[6]See Turner (2003) and Dalcher (2008) for a review of contracting specifically for contract management, and Shtub and Rosenwein (2016) for a description of the modern project management practices.

[7]See Bajari and Tadelis (2001), Bajari et al. (2008), and Bajari et al. (2014).

[8]See also Decarolis and Palumbo (2015) and Baltrunaite et al. (2020).

[9]Chipty and Snyder (1999) showed that large buyers do not necessarily benefit from positive bargaining effects in the cable television industry. Dobson and Waterson (2003) argue that final prices fall following a reduction in the number of retailers only if the retailer services are very close substitutes. Erutku (2005) illustrates that buying power at the retail level can lead to a rise in wholesale price.

the increase in the seller's bargaining power (see Corollary 4).

Related Literature. Our paper is related to three strands of the literature. First, it is linked to the literature on incomplete contracts. Three main causes of contract incompleteness are typically considered: unforeseen contingencies, costs of writing contracts, and costs of enforcing them (see Tirole (1999) for an early paper, while Hart (2017) contains recent citations). We contribute to a set of papers studying the costs of writing contracts.[10] Battigalli and Maggi (2002) explicitly model the language used to describe contracts, i.e., primitive sentences and logical connectives, and Battigalli and Maggi (2008) study costly contracting in a dynamic setting.[11] We endogenize the cost of writing contracts by presenting contract drafting as an agency problem between the buyer and the contract designer.[12] Importantly, in our model, a contract might be too incomplete (complete) if the buyer's bargaining power is smaller (higher).

Second, our paper is related to the literature on procurement which studies the effect of the transaction costs due to renegotiation when either the buyer or the seller can affect the chances of renegotiation. Bajari and Tadelis (2001) develop a model where the buyer incurs a cost of providing a comprehensive design and is faced with a trade-off between providing incentives for the seller and reducing ex-post transaction costs due to costly renegotiation. De Chiara (2018) studies how various courts' approaches motivate sellers to make relationship-specific investments to reduce the probability that the design of the goods they procure is defective.[13] Herweg and Schmidt (2020) develop a model where a potential seller might privately discover flaws in the design proposed by the buyer before agreeing to produce. We contribute to the literature by connecting the probability of renegotiation (the default design not being appropriate ex-post) to the asymmetric information generated at the contract-drafting stage.

Third, our paper is related to the literature on principal-agent contracts with endogenous information gathering. Early papers on this topic are Cremer and Khalil (1992) and Cremer et al. (1998).[14] Gromb and Martimort (2007) study the optimal design of incentive

[10] Early papers are Dye (1985), Spier (1992), and Anderlini and Felli (1994).

[11] Heller and Spiegler (2008) argue that contradictory instructions might be viewed as a form of contract incompleteness. Bolton and Faure-Grimaud (2010) study contracting between two "boundedly rational" agents who face time costs of deliberating transactions.

[12] To the best of our knowledge, ours is the first paper that explicitly models the agency conflict between the buyer and the contract designers in settings with moral hazard and adverse selection. Relatedly, in a model with a three-level hierarchy, Khalil et al. (2013) study contracts offered by a bureaucrat to her agent.

[13] Relatedly, Ganglmair (2017) studies an environment in which sellers can reduce the probability of defective delivery through cooperative investment. De Chiara (2020) considers a game in which a buyer must decide whether to procure goods whose design may prove defective through auctions or negotiations. Herweg and Schwarz (2018) study procurement auctions with renegotiations.

[14] See also Terstiege (2012) and Terstiege (2016).

contracts for experts in different collusion environments. Gerardi and Maestri (2012) study how an agent can be incentivized to acquire and truthfully report an unverifiable signal in an environment with moral hazard and adverse selection.[15] In a very general framework, Gottlieb and Moreira (2017) derive conditions for pooling contracts to be optimal in environments with adverse selection and moral hazard when agents have limited liability.[16] Khalil et al. (2020) study how to incentivize an agent to learn the project profitability when he has private information about the efficiency of learning. Chade and Swinkels (2021) study a principal-agent problem with both moral hazard and adverse selection when the agent is risk-averse. None of these papers study how endogenous information gathering affects the degree of contract incompleteness and, therefore, the economic focus is different.

The rest of the paper is organized as follows. In Section 2, we present the base model and the first-best benchmark. In Section 3, we describe the optimal buyer-agent and buyer-seller contracts and our main results. Section 4 concludes. All the proofs are relegated to the Appendix.

2 Model

The buyer hires the seller to implement a project. Initially, all parties are aware of a default design D that delivers value $V > 0$ to the buyer with probability $0 < 1 - \beta_0 < 1$. The default design could be a previously used contract or a so-called "industry standard." With probability β_0, however, some different, initially indescribable, design N delivers value V to the buyer, whereas design D delivers only δV, where $0 \leqslant \delta < 1$. The probability β_0, therefore, reflects how likely a standard form contract might be modified. Before the buyer approaches the seller, an agent is hired to learn the appropriate design. We call this the *contract drafting stage*, which is described next.

Contract Drafting Stage. The agent might learn the appropriate design by exerting effort $e \in (0, 1)$, and we denote by $c(e)$ his cost-of-effort. If N is the appropriate design (with probability β_0), the agent discovers it with probability e. If D is the appropriate design (with probability $1 - \beta_0$), the agent never discovers design N no matter the effort he exerts. We assume that learning design N generates "hard" information that can be presented to

[15] Unlike in Gerardi and Maestri (2012), the effort choice is continuous and the agent's report is verifiable, i.e., information is "hard" in our model.

[16] The optimality of pooling contracts caused by a mix of moral hazard and adverse selection is also observed in Ollier and Thomas (2013), Escobar and Pulgar (2017), and Castro-Pires and Moreira (2021). Foarta and Sugaya (2020) show that this pooling result might no longer hold if the principal has additional screening instruments. Relatedly, Terstiege (2014), Bhaskar and Mailath (2019), and Rodivilov (2021b) develop models where the agent gets a rent because his and the principal's beliefs diverge.

the buyer, but it cannot be fabricated.

The cost function $c(e)$ is three-times continuously differentiable with respect to $e \in (0,1)$:

$$c'(e) > 0, c''(e) > 0, c'''(e) \geqslant 0, \tag{1}$$

with $c'(0) = 0$ and $c(e) \to \infty$ as $e \to 1$.[17]

The buyer does not directly observe the agent's chosen effort level e, and, therefore, a moral hazard problem emerges. The buyer must also address an adverse selection problem. We assume that the agent is privately informed about the probability that design N is the appropriate one, represented by the parameter β_0. The probability parameter β_0 determines the project's type, and we will refer to a project with high or low probability as a high or low-type project. With probability $\nu \in (0,1)$, the project is a high type, $\theta = H$. With probability $(1-\nu)$, it is a low type, $\theta = L$. Thus, we define the probability parameter with the type superscript:

$$\beta_0^\theta = Pr(\text{design } N \text{ is appropriate}|\text{type } \theta \text{ project}).$$

Renegotiation Stage. If the buyer specifies the delivery of design N when contracting with the seller, the good is delivered without any renegotiation. However, if the contract with the seller specifies design D to be delivered but design N turns out to be the appropriate one, the buyer and the seller will renegotiate the delivery. Converting design D to design N requires the seller to incur additional adjustment cost γ. For example, in the construction industry, the adjustment costs include the time and resources devoted to changing the project design and the additional new materials. We assume that there are ex-post renegotiation gains:

Assumption (A1). $$(1-\delta)V - \gamma > 0.$$

That is, the increase in the buyer's value, $V - \delta V$, minus the adjustment cost is positive.

A hold-up problem emerges as a result. We apply the generalized Nash bargaining solution and assume that the buyer and the seller have bargaining powers α and $1-\alpha$, respectively. That is, if renegotiation occurs, the buyer collects $\alpha[(1-\delta)V-\gamma]$, and the seller collects $(1-\alpha)[(1-\delta)V-\gamma]$. We assume that the bargaining powers are the same ex-ante and ex-post.

2.1 Contracts and Payoffs

The Buyer-Agent (B-A) Contract. We first describe the contract the buyer offers to the agent. Without loss of generality, we use a direct truthful mechanism, where the agent

[17] These conditions guarantee the effort level is strictly positive and strictly less than one in equilibrium.

is asked to announce the project's type, denoted by $\hat{\theta}$. A contract specifies, for each type, the transfer as a function of whether or not the agent discovers design N, and whether D turns out to be the appropriate design. A contract is defined formally by

$$\omega^{\hat{\theta}} = \{w_N^{\hat{\theta}}, w_{DD}^{\hat{\theta}}, w_{DN}^{\hat{\theta}}\}, \tag{2}$$

where $w_N^{\hat{\theta}}$ is the agent's wage in case he specifies N as the appropriate design; $w_{DD}^{\hat{\theta}}$ is the agent's wage if he specifies design D and it is appropriate ex-post; and $w_{DN}^{\hat{\theta}}$ is the agent's wage if he specifies design D but design N is appropriate ex-post.

An agent who observes type θ project, announcing type $\hat{\theta}$, receives the expected utility $U^\theta(\omega^{\hat{\theta}}, e)$ from a contract $\omega^{\hat{\theta}}$ given the effort level e:

$$U^\theta(\omega^{\hat{\theta}}, e) := \underbrace{(1-\beta_0^\theta)}_{D \text{ appropriate}} \underbrace{w_{DD}^{\hat{\theta}}}_{D \text{ specified}} + \underbrace{\beta_0^\theta}_{N \text{ appropriate}} \Big[\underbrace{e w_N^{\hat{\theta}}}_{N \text{ specified}} + \underbrace{(1-e) w_{DN}^{\hat{\theta}}}_{D \text{ specified}} \Big] - c(e). \tag{3}$$

If the appropriate design is indeed D (with probability $1-\beta_0^\theta$), then the agent specifies it at the outset and collects $w_{DD}^{\hat{\theta}}$. If, however, the appropriate design is N (with probability β_0^θ), the agent either specifies it (with probability e) and collects $w_N^{\hat{\theta}}$, or fails to specify it (with probability $1-e$) and collects $w_{DN}^{\hat{\theta}}$.

The Buyer-Seller (B-S) Contract. Production costs $k > 0$ for the seller (regardless of the initially specified design), and we assume that the production cost k is high enough so that the seller does not trade without a contract, which is described next.[18]

Degree of Contract Incompleteness. If the agent exerts effort $e^{\hat{\theta}}$ given type θ project and does not discover the new design N, the posterior probability of renegotiation is given by

$$\beta^\theta(e^{\hat{\theta}}) = \frac{\beta_0^\theta(1-e^{\hat{\theta}})}{1-\beta_0^\theta e^{\hat{\theta}}}, \tag{4}$$

which is monotonically decreasing in the agent's effort level:

$$\frac{d\beta^\theta(e^{\hat{\theta}})}{de^{\hat{\theta}}} < 0. \tag{5}$$

Since, in equilibrium, all parties correctly anticipate the effort level chosen for each project's type, the expected probability of renegotiation becomes

$$\beta := \beta(e^L, e^H) = \mathbf{E}_\theta \beta^\theta(e^\theta) = \nu\beta^H(e^H) + (1-\nu)\beta^L(e^L). \tag{6}$$

The probability of renegotiation reflects the degree of contract incompleteness, and a buyer-seller contract is more incomplete the higher the probability of renegotiation.

Prices. Suppose the agent does not discover design N and, therefore, the seller is asked to deliver design D. The default price p_D accounts for the hold-up problem, i.e., guarantees

[18]That is, the seller would not invest k without a contract to avoid a hold-up problem on the seller's side.

the seller collects a fraction $(1-\alpha)$ of the ex-ante total surplus:

$$(1-\alpha)\Bigg[V - k - \underbrace{\beta(e^L, e^H)\gamma}_{\text{expected adjustment cost}}\Bigg] = p_D - \Bigg[\underbrace{k - \beta(e^L, e^H)(1-\alpha)[(1-\delta)V - \gamma]}_{\text{seller's opportunity cost}}\Bigg], \quad (7)$$

where the left hand side is the seller's expected share of the total surplus and the right hand side is the seller's expected profit. Given that with probability β the seller holds the buyer up for an amount $(1-\alpha)[(1-\delta)V-\gamma]$, the seller's opportunity cost is $k-\beta(1-\alpha)[(1-\delta)V-\gamma]$.

The price p_D is formally defined as:

$$p_D := p_D(\beta) = k + (1-\alpha)\Bigg[V - k - \beta(1-\delta)V\Bigg], \quad (8)$$

and it is decreasing in the degree of contract incompleteness:

$$\frac{dp_D}{d\beta} < 0. \quad (9)$$

Therefore, the price p_D and the degree of contract incompleteness β are inversely related: anticipating that renegotiation is less likely the seller demands a higher price.

Suppose next the agent discovers design N at the outset. The price p_N then reflects the production cost as well as the seller's share of the total surplus:

$$p_N := k + (1-\alpha)\big(V - k\big). \quad (10)$$

Two aspects of pricing are worth noting. First, the price p_N does not depend on the degree of contract incompleteness since by asking the seller to deliver design N the buyer fully reveals that it is the appropriate design at the outset. Second, since the price p_D accounts for the possibility of the hold-up, it is smaller than the price p_N:

$$p_N - p_D = \beta(1-\alpha)(1-\delta)V > 0, \quad (11)$$

and the price differences is monotonically increasing in the degree of contract incompleteness.

To sum up, the buyer either offers the seller a contract $\{D, p_D\}$ that specifies design D to be delivered, which is renegotiated with probability β; or a contract $\{N, p_N\}$ that specifies design N to be delivered. A representative time-line is plotted in Figure 1.

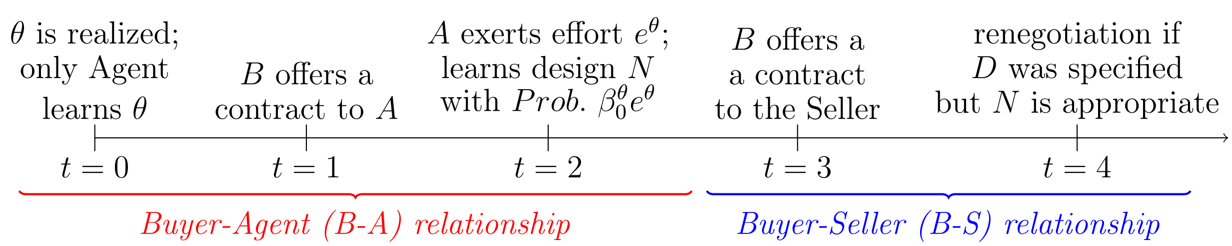

Figure 1. The time-line of the game.

2.2 The First-Best Benchmark

Suppose the project's type θ and the agent's effort choice are directly observed by the buyer. The seller, however, only knows the distribution of the project's type. Given that the seller correctly anticipates the equilibrium effort level $e = e^{\theta}_{FB}$ for each project's type θ and denoting the first-best probability of renegotiation as $\beta_{FB} = \beta(e^L_{FB}, e^H_{FB})$, the buyer is choosing e to maximize the following expected profit for each θ:

$$(1-\beta^{\theta}_0)\big(V-p_D\big)+\beta^{\theta}_0\bigg(e(V-p_N)+(1-e)\big(V-p_D-\gamma-(1-\alpha)[(1-\delta)V-\gamma]\big)\bigg)-c(e)$$

subject to $p_D = k + (1-\alpha)\big[V - k - \beta_{FB}(1-\delta)V\big]$ and $p_N = k + (1-\alpha)\big(V-k\big)$.

If the appropriate design is D (with probability $1-\beta^{\theta}_0$), then the agent specifies it at the outset, and the buyer collects $V - p_D$. If the appropriate design is N (with probability β^{θ}_0), and the agent specifies it at the outset (with probability e), the buyer collects $V - p_N$. If the appropriate design is N but the agent specifies design D then the buyer collects $V - p_D$ and bears the adjustment cost γ and the seller's share of the surplus $(1-\alpha)[(1-\delta)V-\gamma]$.

The optimal e^L_{FB} and e^H_{FB} are determined by the following conditions for $\theta = L, H$:[19]

$$\beta^{\theta}_0\Bigg(\underbrace{\gamma}_{\text{social benefit}}+\underbrace{(1-\alpha)[(1-\delta)V-\gamma]}_{\text{the seller's share}}-\underbrace{\beta_{FB}(1-\alpha)(1-\delta)V}_{p_N-p_D}\Bigg)=c'(e^{\theta}_{FB}). \tag{12}$$

The first-best effort level equalizes the marginal cost of discovering design N, the right-hand side of (12), and its marginal benefit, the left-hand side of (12), that includes three components: (i) the social benefit, (ii) the seller's share of the renegotiation gains, and (iii) the price difference. The first component reflects that if the buyer has all the bargaining power ($\alpha = 1$), i.e., without the hold-up problem, the benefit of discovering design N is only in avoiding the adjustment cost γ. The second component reflects that in the case of renegotiation, the seller holds the buyer up for an amount of $(1-\alpha)[(1-\delta)V-\gamma]$. The third component reflects that if design N is discovered, the buyer pays p_N instead of p_D, where the price difference $p_N - p_D$ is given by (11). Note that the price difference component appears with a negative sign on the left-hand side of (12) and, therefore, this component discourages the buyer from learning design N. Intuitively, discovering design N also brings "bad news" since the buyer pays a higher price that does not account for the possibility of renegotiation.

The following assumption guarantees the uniqueness of the first-best effort:[20]

[19] Given that $c'(0) = 0$, $c'(1) = +\infty$, and the marginal cost is increasing ($c''(e) > 0$), the first-best effort level is well defined and is such that $0 < e^{\theta}_{FB} < 1$ for $\theta = L, H$.

[20] The assumption $(A2)$ guarantees that the expected profit is concave, and is equivalent to $\frac{d\left(\beta^{\theta}_0(\gamma+(1-\alpha)[(1-\delta)V-\gamma]-\beta(1-\alpha)(1-\delta)V)-c'(e^{\theta})\right)}{de^{\theta}} < 0$ for $\forall e^{\theta} \in (0,1)$ and $\theta \in \{L, H\}$.

Assumption (A2). $\frac{(\beta_0^\theta)^2(1-\beta_0^\theta)(1-\alpha)(1-\delta)Pr(\theta)V}{(1-\beta_0^\theta e)^2} < c''(e)$ for $\forall\ e \in (0,1)$ and $\theta \in \{L, H\}$.

Condition (12) illustrates the intuitive monotonicity of the first-best effort level, which is decreasing in the buyer's bargaining power α:[21]

$$\frac{de^\theta_{FB}}{d\alpha} < 0. \tag{13}$$

That is, if the buyer obtains a higher portion of the surplus in case of renegotiation, her incentives to exert effort to avoid renegotiation go down. Since the probability of renegotiation is decreasing in the agent's effort level (see (5)), this immediately implies that a buyer with a lower bargaining power offers a less incomplete contract:

$$\frac{d\beta_{FB}}{d\alpha} > 0. \tag{14}$$

Finally, since price p_D is decreasing in the degree of contract incompleteness (see (9)), a buyer with a lower bargaining power also offers a higher price:

$$\frac{dp_D(\beta_{FB})}{d\alpha} < 0. \tag{15}$$

We summarize the results in Lemma 1 below.

Lemma 1 (The First-Best Scenario). If the buyer's bargaining power is higher, the *B-S* contract is more incomplete, and the default price is smaller.

3 The Second-Best Inefficiency

We now turn to the main model with both moral hazard and adverse selection and illustrate the nature of second-best inefficiency arising in this environment.

Denote

$$e^{\theta\hat{\theta}} := argmax_e U^\theta(\omega^{\hat{\theta}}, e) \tag{16}$$

as the optimal effort level for the agent working on type-θ project under contract $\omega^{\hat{\theta}}$.

For a given contract ω^θ, the moral hazard constraint determining the optimal effort choice can be presented as:

$$(MH^\theta) \qquad e^\theta \in e^{\theta\theta}.$$

The optimal contract will have to satisfy the following incentive compatibility constraints for all θ and $\hat{\theta}$:

$$(IC^{\theta,\hat{\theta}}) \qquad U^\theta(\omega^\theta, e^\theta) \geqslant U^\theta(\omega^{\hat{\theta}}, e^{\theta\hat{\theta}}).$$

[21] Similar monotonicity holds with respect to the adjustment cost γ and the buyer's retained value δ.

We also assume the agent's ex ante participation constraint that guarantees the agent accepts the contract in equilibrium must be satisfied:

$$(IR^{\theta}) \qquad U^{\theta}(\omega^{\theta}, e^{\theta}) \geqslant 0;$$

and the agent's payments are non-negative:[22]

$$(LL^{\theta}) \qquad w^{\theta}_{N}, w^{\theta}_{DD}, w^{\theta}_{DN} \geqslant 0.$$

The buyer maximizes the following objective function

$$\mathbf{E}_{\theta}\Bigg[(1-\beta_0^{\theta})\big(V-p_D(\beta)\big)+\beta_0^{\theta}e^{\theta}(V-p_N)+\beta_0^{\theta}(1-e^{\theta})\big(V-p_D(\beta)-\gamma-(1-\alpha)[(1-\delta)V-\gamma]\big) \\ -\big[\underbrace{(1-\beta_0^{\theta})w^{\theta}_{DD}+\beta_0^{\theta}\big(e^{\theta}w^{\theta}_{N}+(1-e^{\theta})w^{\theta}_{DN}\big)}_{\text{the agent's expected wage}}\big]\Bigg]$$

subject to, for all $\theta, \hat{\theta} \in \{H, L\}$, the (MH^{θ}), $(IC^{\theta,\hat{\theta}})$, (IR^{θ}), and (LL^{θ}) constraints.

The following two benchmarks lay the ground to our characterization of the main model.

3.1 Moral Hazard (No Adverse Selection)

To highlight the role of moral hazard, we first briefly outline a benchmark case without adverse selection. If the buyer directly observes the project's type, she motivates the agent to exert effort by paying a higher reward for discovering design N at the outset and a lower one for failure to do so. In particular, the (MH^{θ}) constraint can be replaced with the following First Order Condition for $\theta = L, H$:[23]

$$w^{\theta}_{N} - w^{\theta}_{DN} = \frac{c'(e^{\theta}_{MH})}{\beta^{\theta}_{0}}. \tag{17}$$

where e^{θ}_{MH} denotes the agent's effort in a benchmark with moral hazard only.

It is optimal to pay the agent as little as possible if he fails to discover design N:

$$w^{\theta}_{DN} = w^{\theta}_{DD} = 0. \tag{18}$$

Intuitively, the cheapest way to motivate the agent to work is to reward him only when the buyer is certain the agent has worked. Indeed, the positive value of w^{θ}_{DN} makes it more difficult to satisfy (MH^{θ}), while w^{θ}_{DD} does not affect the agent's incentives to work.[24]

[22] Without the limited liability constraints, the buyer can receive first-best profit since learning design N is a random event correlated with the agent's type (see Cremer and McLean (1985)).

[23] The First Order Condition is also sufficient given that the cost function $c(e)$ is strictly convex.

[24] In Gerardi and Maestri (2012), the agent is rewarded if he fails to obtains a signal that provides definitive evidence in favor of one state but if his report matches the true state ex-post, which is analogous to w^{θ}_{DD} being strictly positive in our model. The reason is that information is "soft" in Gerardi and Maestri (2012) (the agent's report is not verifiable), whereas information is "hard" in our model.

Denoting $\beta_{MH} = \beta(e^L_{MH}, e^H_{MH})$, the agent's effort level is determined by

$$\underbrace{\beta_0^\theta\left(\gamma + (1-\alpha)[(1-\delta)V - \gamma] - \beta_{MH}(1-\alpha)(1-\delta)V\right) = c'(e^\theta_{MH})}_{FB \text{ effort level}} + \underbrace{e^\theta_{MH}c''(e^\theta_{MH})}_{MH \text{ distortion}}. \quad (19)$$

The role of the moral hazard problem can be illustrated by comparing the first-best effort level in (12) with the one defined in (19) above, where the additional term $e^\theta_{MH}c''(e^\theta_{MH})$ on the right-hand side determines the downward distortion in e^θ_{MH}.

Given that the agent working on the high type project is more likely to discover new design ($\beta_0^H > \beta_0^L$), a smaller payment is required to incentivize him to exert effort:

$$w^H_N < w^L_N. \quad (20)$$

We summarize the main results in Proposition 1 below.

Proposition 1. ***The optimal contract with moral hazard (no adverse selection).*** The agent is rewarded for getting the design right at the outset and gets nothing otherwise:

$$w^\theta_N(e^\theta_{MH}) = \frac{c'(e^\theta_{MH})}{\beta_0^\theta} > w^\theta_{DD} = w^\theta_{DN} = 0 \text{ for } \theta = L, H.$$

The *B-S* contract is more incomplete and the default price is lower due to moral hazard:

$$\beta_{MH} > \beta_{FB} \text{ and } p_D(\beta_{MH}) < p_D(\beta_{FB}).$$

Proof: See Appendix A.

Condition (19) also implies that the intuitive monotonicity of the first-best scenario remains intact without adverse selection:

Corollary 1. Moral Hazard (no Adverse Selection). If the buyer's bargaining power is higher, the *B-S* contract is more incomplete, and the default price is smaller.

3.2 Adverse Selection (No Moral Hazard)

Consider next the case with adverse selection but no moral hazard: the agent's cost-of-effort is still determined by the cost function $c(e)$ but is observable and contractible.[25] The first-best effort efficiency is restored in this case, since the buyer could use the fact that the agent working on the high type project is relatively more likely to discover design N (conditional on it being the appropriate one) to screen the agent without distorting the effort level. In other words, since success in discovering design N is a random event that is correlated with

[25]For example, the internal monitoring practices are in place, and the buyer hires a monitor who collects a signal on the agent's effort. See Rodivilov (2021a) for a recent monitoring literature review.

the project's type, we can apply well-known ideas from mechanisms a la Cremer and McLean (1985) that says the buyer can still receive the first-best profit.

To implement the first-best effort level, the buyer has to counter the incentive of the agent working on the high type project to claim he is working on a low type. Relative to the first-best payments, the buyer can change the payments in the following way. She can increase the payment for discovering design N to the agent working on the high type project w_N^H and, simultaneously, increase the payment to the agent working on the low type project for failing to discover design N when D is indeed the appropriate one, w_{DD}^L.

To summarize, moral hazard is essential to generate distortions in the degree of contract incompleteness:

Corollary 2. ***Without moral hazard, the buyer implements the first-best.***

3.3 General Case: Adverse Selection and Moral Hazard

We now return to the main model with both moral hazard and adverse selection. Note that adverse selection is reflected in the $(IC^{\theta,\hat{\theta}})$ constraints, just as in a standard model with asymmetric information only. Moral hazard, however, is explicitly reflected in the (MH^{θ}) constraints, and implicitly in the $(IC^{\theta,\hat{\theta}})$ constraints. We illustrate this in more detail next.

Consider the $(IC^{\theta,\hat{\theta}})$ constraint:

$$(IC^{\theta,\hat{\theta}}) \qquad (1-\beta_0^{\theta})w_{DD}^{\theta} + \beta_0^{\theta}\big(e^{\theta}w_N^{\theta} + (1-e^{\theta})w_{DN}^{\theta}\big) - c(e^{\theta}) \geqslant$$
$$(1-\beta_0^{\theta})w_{DD}^{\hat{\theta}} + \beta_0^{\theta}\big(e^{\theta\hat{\theta}}w_N^{\hat{\theta}} + (1-e^{\theta\hat{\theta}})w_{DN}^{\hat{\theta}}\big) - c(e^{\theta\hat{\theta}}),$$

where $e^{\theta\hat{\theta}}$ is the effort the agent working on the type θ project chooses off-equilibrium:

$$e^{\theta\hat{\theta}} = argmax_e\big\{(1-\beta_0^{\theta})w_{DD}^{\hat{\theta}} + \beta_0^{\theta}\big(ew_N^{\hat{\theta}} + (1-e)w_{DN}^{\hat{\theta}}\big) - c(e)\big\}. \qquad (21)$$

Moral hazard is, therefore, also reflected on the right-hand side of $(IC^{\theta,\hat{\theta}})$ via the optimal off-equilibrium effort choice $e^{\theta,\hat{\theta}}$.

We start solving the main model by explaining why both the (IC) constraints are binding. In doing so, we illustrate that the agent is paid only if he discovers design N at the outset and gets nothing otherwise. Next, we characterize the degree of contract incompleteness and prove that the buyer-seller contract might be less incomplete in an environment with both moral hazard and adverse selection than in the presence of moral hazard only. Finally, we prove that a buyer with a lower bargaining power might offer a more incomplete contract if the adverse selection problem is severe enough. The key results are presented in Propositions 2 and 3 following the discussion.

3.3.1 Both (IC) Constraints are Binding: Implications for the Agent's Rewards

We now explain why both (IC) constraints are binding and discuss implications for the buyer-agent contract. The $(IC^{H,L})$ constraint is binding for the standard reason in adverse selection models. The agent working on the high type project has an incentive to lie in order to collect the higher transfer given to the agent working on the low type project. To illustrate, suppose the buyer offers the menu of contracts described in Proposition 1 optimal when the project's type is public. The $(IC^{H,L})$ constraint then can be rewritten as

$$\beta_0^H e^H w_N^H - c(e^H) \geqslant \beta_0^H e^{HL} w_N^L - c(e^{HL}), \tag{22}$$

which is equivalent to[26]

$$e^H \geqslant e^{HL}. \tag{23}$$

Thus, if offered the menu of contracts optimal when the project's type is public, the agent working on the high type project obtains a higher payoff if he lies and exerts a higher effort off-equilibrium, $e^{HL} > e^H$. To counter these incentives, the buyer must then set w_N^L less than w_N^H to make the agent working on the high type project telling the truth:[27]

$$w_N^L \leqslant w_N^H. \tag{24}$$

Conditions (20) and (24) together illustrate a strong conflict between the buyer's desire for the agent working on the high type project to exert a higher effort level for the efficiency reasons and the monotonicity condition imposed by the presence of asymmetric information.[28] In particular, condition (20) states that the agent working on the low type project must be paid more if the project's type is observable, $w_N^H < w_N^L$, since he is less likely to discover design N, $\beta_0^L < \beta_0^H$. However, condition (24) states the opposite must hold, and a higher payment is required for the agent working on the high type project to guarantee truth-telling. Given the conflicting conditions (20) and (24), the agent's rewards for learning

[26] First, given that $w_N^H = \frac{c'(e^H)}{\beta_0^H}$ and $w_N^L = \frac{c'(e^{HL})}{\beta_0^H}$), the $(IC^{H,L})$ constraint simplifies to

$$e^H c'(e^H) - c(e^H) \geqslant e^{HL} c'(e^{HL}) - c(e^{HL}).$$

Second, the $(IC^{H,L})$ constraint can be rewritten as $f(e^H) \geqslant f(e^{HL})$, where $f(e) = ec'(e) - c(e)$. Third, since the function $f(e)$ is strictly increasing in e, $f'(e) = ec''(e) > 0$, the $(IC^{H,L})$ is equivalent to $e^H \geqslant e^{HL}$.

[27] Given that $w_N^H = \frac{c'(e^H)}{\beta_0^H}$ and $w_N^L = \frac{c'(e^{HL})}{\beta_0^H}$, condition $e^H \geqslant e^{HL}$ is equivalent to $w_N^H \geqslant w_N^L$ since the function $c'(e)$ is increasing in e.

[28] The reason for the conflict is that a common value problem emerges since the agent's type β_0^θ directly enters the buyer's objective function. In this common-value setting, there can be a conflict if, for instance, rent minimization necessitates the low type to be paid less than the high type, but the efficiency requires the opposite, leading to both (IC) constraints binding (see, e.g., Laffont and Martimort (2002), page 53).

design N become identical for both types,

$$w_N^H = w_N^L, \tag{25}$$

and the high type's $(IC^{H,L})$ constraint becomes binding.

To illustrate why the $(IC^{L,H})$ constraint is binding, we first argue that, if the project's type is privately observed by the agent, it remains optimal to reward him for discovering design N at the outset and pay nothing otherwise:[29]

$$w_{DD}^H = w_{DD}^L = w_{DN}^H = w_{DN}^L = 0. \tag{26}$$

Intuitively, paying the agent unless he discovers design N only increases the adverse selection rent and, in addition, exacerbates the moral hazard problem. Recall that if the agent's effort is observable, the buyer optimally implements the first-best. Thus, the payments are chosen primarily to mitigate the moral hazard rent, and the agent is paid only if the buyer is certain the agent worked.

Given that the agent is rewarded only for discovering design N, the $(IC^{L,H})$ becomes

$$\beta_0^L e^L w_N^L - c(e^L) \geqslant \beta_0^L e^{LH} w_N^H - c(e^{LH}), \tag{27}$$

which, following similar steps as for the $(IC^{H,L})$ constraint, can be rewritten as

$$w_N^L \geqslant w_N^H. \tag{28}$$

Therefore, the $(IC^{L,H})$ constraint is binding as well since $w_N^L = w_N^H$ in equilibrium.

We summarize the main results in Proposition 2 below.

Proposition 2. ***The B-A contract with moral hazard and adverse selection.*** The agent is paid for getting the design right at the outset regardless of the project's type:

$$w_N^H(e^H) = \frac{c'(e^H)}{\beta_0^H} = \frac{c'(e^L)}{\beta_0^L} = w_N^L(e^L) > 0 = w_{DD}^H = w_{DN}^H = w_{DD}^L = w_{DN}^L.$$

Proof: See Appendix B.

Note that, because of the pooling contract, the reward for discovering design N is higher (lower) than the reward promised to the agent working on a high (low) type project in the benchmark with moral hazard only:

$$w_N^H(e_{MH}^H) < w_N^H(e^H) = w_N^L(e^L) < w_N^L(e_{MH}^L). \tag{29}$$

This property plays an important role as it affects the degree of the buyer-seller contract incompleteness, as we illustrate next.

[29] See Section 6.1.4 in Appendix B for a formal proof.

3.3.2 The Degree of Contract Incompleteness

We now describe how the incentive conflict between the buyer and the agent affects the degree of the buyer-seller contract incompleteness. In particular, we illustrate how the adverse selection polarizes the degree of contract incompleteness across the two project types.

The effort of the agent working on the low-type project is distorted downward due to moral hazard and is distorted downward further due to adverse selection (see the upper part of Figure 2). Therefore, adverse selection exacerbates the moral hazard problem for the low-type project. However, the interaction of adverse selection and moral hazard results in a different distortion in the case of the high type project. While the effort of the agent working on the high type project is distorted downward due to moral hazard, it is distorted *upward* due to adverse selection (see the lower part of Figure 2).

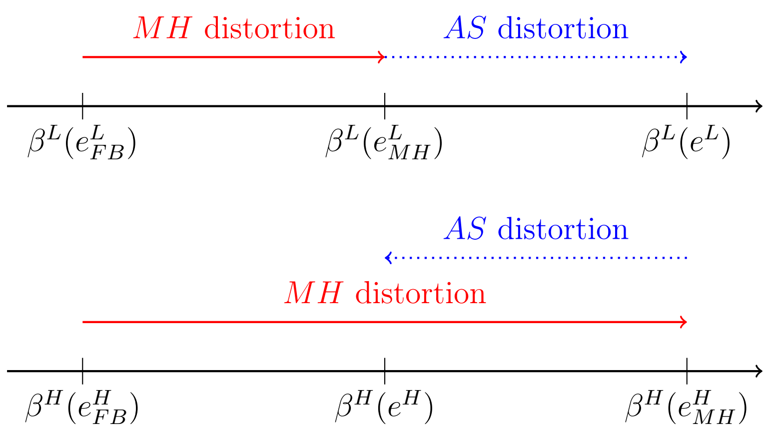

Figure 2. Distortions in $\beta^H(e^H)$ and $\beta^H(e^L)$ due to moral hazard (MH) and adverse selection (AS).

The reason this polarization emerges is the pooling contract offered to the agent. Recall that the agent's reward is higher (lower) than the reward promised to the agent working on a high (low) type project in the benchmark with moral hazard only (see (29)). Since the agent working on the low type project is promised a smaller reward, $w^L_N(e^L) < w^L_N(e^L_{MH})$, he exerts less effort than in the benchmark with moral hazard only. The agent working on the high type project, however, is promised a higher reward than in the benchmark with moral hazard only, $w^H_N(e^H) > w^H_N(e^H_{MH})$, and, as a result, he chooses a higher effort level.

Therefore, adverse selection mitigates the downward distortion in the effort emerging due to moral hazard in the case of the high type project. Consequently, in an environment with both moral hazard and adverse selection, the degree of contract incompleteness is polarized: contracts are either closer to the first-best scenario (high-type project) or more incomplete (low-type project) than in the benchmark with moral hazard only.

We summarize the results in Proposition 3 below.

Proposition 3. ***The Equilibrium Degree of Contract Incompleteness.*** The contract drafted by the agent working on the high (low) type project is less (more) incomplete than in the benchmark with moral hazard only:

$$\beta^H(e^H) < \beta^H(e^H_{MH}) \text{ and } \beta^L(e^L) > \beta^L(e^L_{MH}).$$

Proof: See Appendix B.

Interestingly, this polarization of effort might result in contracts being less incomplete in an environment with both moral hazard and adverse selection than in the presence of moral hazard only. In particular, this is the case when the high type project is relatively likely (ν is high enough). Then, the adverse selection results in a less incomplete contract coupled with a higher default price. We summarize the results in Corollary 3 below.

Corollary 3. ***If the portion of the high type project is high enough, the contract is less incomplete and the default price is higher due to adverse selection:***

$$\exists\ 0 < \underline{\nu} < 1 \text{ such that } \beta < \beta_{MH} \text{ and } p_D(\beta) > p_D(\beta_{MH}) \text{ if } \nu > \underline{\nu}.$$

Proof: See Appendix B.

3.3.3 The Role of Adverse Selection: More Incomplete Contract if the Buyer's Bargaining Power is Lower.

Recall that, in the first-best scenario, the contract is less incomplete if the buyer's bargaining power is lower (see Lemma 1). If the agent is drafting the contract, that may not hold, and a contract might become more incomplete if the buyer's bargaining power becomes lower. We explain this next in detail.

To distinguish the effects of moral hazard and adverse selection on the degree of contract incompleteness, we rewrite the equilibrium effort level e^θ for $\theta = L, H$ as:

$$e^\theta = \underbrace{e^\theta_{MH}}_{MH} + (\underbrace{e^\theta - e^\theta_{MH}}_{AS}). \tag{30}$$

Then, if α becomes smaller, two effects emerge.

First, recall that in the benchmark without adverse selection, the contract is less incomplete if the buyer's bargaining power is smaller (see Corollary 1):

$$\frac{de^\theta_{MH}}{d\alpha} < 0. \tag{31}$$

Thus, if moral hazard is the only incentive problem, the contract is less incomplete if the buyer's bargaining power is smaller.

Second, the distortion in the agent's effort due to adverse selection, $e^\theta - e^\theta_{MH}$, also changes. If $e^\theta - e^\theta_{MH}$ is increasing in α, it becomes costlier to incentivize a higher effort when the buyer's bargaining power is smaller due to a higher agent's rent.

Thus, whether the contract is more or less incomplete if the buyer's bargaining power is smaller depends on the interaction of the two effects caused by moral hazard and adverse selection, respectively. We find that the latter effect becomes pivotal if the adverse selection problem is severe enough (β_0^L is significantly smaller than β_0^H). Intuitively, with a higher agent's effort, the buyer mitigates the chances of renegotiation but pays a higher rent to the agent. This rent is high if the probabilities of discovering design N differ significantly across high- and low-type projects. If the value of the project V is relatively high, it remains optimal to incentivize the agent to work more if the buyer's bargaining power becomes smaller. However, it is optimal to discover the appropriate design with a smaller probability in order to lower the agent's rent if the value of the project V is not too high.[30]

The following Proposition 4 provides formal sufficient conditions.

Proposition 4.
If the adverse selection problem is severe enough and value V is not too high, the contract is more incomplete if the buyer's bargaining power is smaller:
For any $\beta_0^H \in (0,1)$ there exist $0 < \overline{\beta}_0^L(\beta_0^H) < \beta_0^H$, $\overline{V} > 0$, $0 < \underline{\gamma}(\beta_0^H) < (1-\delta)V$ such that

$$\text{if } \beta_0^L < \overline{\beta}_0^L,\ V < \overline{V}, \text{ and } \underline{\gamma} < \gamma < (1-\delta)V \text{ then } \frac{d\beta}{d\alpha} < 0.$$

Proof: See Appendix C.

Thus, a severe adverse selection problem might result in a more incomplete contract if the buyer's bargaining power is lower. This result illustrates how the extent of contract incompleteness depends on the firms' internal organizational structure when, for example, the contract designer is an employee rather than an external contractor hired by the buyer. For instance, if the internal monitoring practices are in place and the agent cannot shirk, then a contract is less incomplete if the buyer's bargaining power is lower. However, if the agent chooses the effort covertly and the adverse selection problem is severe enough, the opposite might be true. Proposition 4, therefore, highlights the pivotal role of asymmetric information in contract drafting.

Having established that the contract might be more incomplete if the buyer's bargaining power is lower, we now discuss implications for the optimal price. In particular, we prove that the price might become smaller if the buyer's bargaining power is lower:

[30] There is also a condition on the parameter γ which ensures the adverse selection rent is monotonic in α.

Corollary 4. ***If conditions of Proposition 4 hold and either k is not too low or α is not too high, the price p_D is smaller the lower the buyer's bargaining power is: $\frac{dp_D}{d\alpha} > 0$.***
Proof: See Appendix C.

To see why a buyer with a lower bargaining power might pay a lower price, recall definition (8):

$$p_D = k + \underbrace{(1-\alpha)}_{\text{decreasing in } \alpha} \left[\underbrace{V - k - \beta(e^L, e^H)(1-\delta)V}_{\text{increasing in } \alpha \text{ (see Proposition 4)}} \right]. \tag{32}$$

Notice that if the buyer's bargaining power becomes smaller, two effects emerge. First, given that the term $(1-\alpha)$ increases, the price should increase since the seller now captures a larger portion of the surplus. Second, given that the term $V - k - \beta(1-\delta)V$ decreases, the price should decrease since the contact becomes more incomplete. Therefore, the price decreases when the buyer's bargaining power becomes smaller if the second effect dominates. Intuitively, this is the case if the price is more sensitive to the degree of contract incompleteness. Then, a buyer with a lower bargaining power offers a more incomplete contract and pays a smaller price. Any of the two conditions on the primitives is sufficient for this to be the case: (1) k not too low, or (2) α not too high.

4 Conclusion

In this paper, we have studied the interaction between the provision of incentives for contract designers and the optimal buyer-seller contract. While there has been much attention on studying optimal contracts, details of contract drafting are typically suppressed. Contract drafting and the buyer-seller relationship are intertwined, and our paper is a step towards studying this interaction.

When contract drafting is delegated to the third party, the buyer must address two incentive problems: moral hazard and adverse selection. We find that the contract designer is rewarded for getting the design right at the outset, and the value of the reward is the same regardless of the project's type. This pooling contract might result in a less incomplete buyer-seller contract and a higher price than in the environment without adverse selection.

If the buyer is drafting the contract herself, the buyer-seller contract is less incomplete if the buyer's bargaining power is lower. This might no longer hold if contract drafting is delegated to the third party. If the adverse selection problem is severe enough and the value of the buyer-seller relationship is not too high, the opposite is true.

5 Appendix A. Proof of Proposition 1.

Given that the seller correctly anticipates the equilibrium effort level $e = e^\theta_{MH}$, the buyer maximizes the following objective function subject to, for $\theta \in \{H, L\}$, the (MH^θ), (IR^θ), and (LL^θ) constraints given below:

$$(1-\beta^\theta_0)\big(V - p_D(\beta_{MH})\big) + \beta^\theta_0 e(V-p_N) + \beta^\theta_0(1-e)\big(V - p_D(\beta_{MH}) - \gamma - (1-\alpha)[(1-\delta)V-\gamma]\big)$$
$$-(1-\beta^\theta_0)w^\theta_{DD} - \beta^\theta_0\big(ew^\theta_N + (1-e)w^\theta_{DN}\big)$$

(MH^θ) $\qquad e \in argmax_{\tilde{e}} \left\{(1-\beta^\theta_0)w^\theta_{DD} + \beta^\theta_0\big((1-\tilde{e})w^\theta_{DN} + \tilde{e}w^\theta_N\big) - c(\tilde{e})\right\};$

(IR^θ) $\qquad (1-\beta^\theta_0)w^\theta_{DD} + \beta^\theta_0\big((1-e)w^\theta_{DN} + ew^\theta_N\big) - c(e) \geqslant 0;$

(LL^θ) $\qquad w^\theta_N, w^\theta_{DD}, w^\theta_{DN} \geqslant 0.$

We first simplify the buyer's optimization problem by replacing the (MH^θ) constraint with the following First Order Condition:

$$\beta^\theta_0\big(w^\theta_N - w^\theta_{DN}\big) = c'(e^\theta_{MH}), \tag{33}$$

which is also sufficient given that the cost function $c(e^\theta)$ is convex.[31]
In addition, assumptions on the the cost function $c(e^\theta)$ guarantee that $e^\theta > 0$ in equilibrium.

Labeling by λ_{IR}, λ_{MH}, λ_N, λ_{DD}, λ_{DN} Lagrange multipliers of the constraints associated with the (IR^θ), (MH^θ), and the corresponding (LL^θ) constraints, respectively, the Lagrangian for the buyer's optimization problem becomes

$$\begin{aligned}\mathcal{L} = (1-\beta^\theta_0)\big(V - p_D(\beta_{MH})\big) + \beta^\theta_0 e(V-p_N) + \beta^\theta_0(1-e)\big(V - p_D(\beta_{MH}) - \gamma - (1-\alpha)[(1-\delta)V-\gamma]\big)\\ -(1-\beta^\theta_0)w^\theta_{DD} - \beta^\theta_0\big(ew^\theta_N + (1-e)w^\theta_{DN}\big)\\ +\lambda_{IR}\big[(1-\beta^\theta_0)w^\theta_{DD} + \beta^\theta_0\big((1-e)w^\theta_{DN} + ew^\theta_N\big) - c(e)\big]\\ +\lambda_{MH}\big[\beta^\theta_0\big(w^\theta_N - w^\theta_{DN}\big) - c'(e)\big] + \lambda_N w^\theta_N + \lambda_{DD}w^\theta_{DD} + \lambda_{DN}w^\theta_{DN}.\end{aligned}$$

Differentiating the Lagrangian (together with the equilibrium condition $e = e^\theta_{MH}$), we obtain the following Kuhn-Tucker conditions for the optimization problem:

$[w^\theta_N] : (\lambda_{IR} - 1)\beta^\theta_0 e^\theta_{MH} + \lambda_{MH}\beta^\theta_0 + \lambda_N = 0;$

$[w^\theta_{DD}] : (\lambda_{IR} - 1)(1-\beta^\theta_0) + \lambda_{DD} = 0;$

$[w^\theta_{DN}] : (\lambda_{IR} - 1)\beta^\theta_0(1-e^\theta_{MH}) - \lambda_{MH}\beta^\theta_0 + \lambda_{DN} = 0;$

[31] The sufficiency of the First Order Condition follows from the second order derivative being negative

$$\frac{d\left[\beta^\theta_0\left(w^\theta_N - w^\theta_{DN}\right) - c'(e^\theta_{MH})\right]}{de^\theta_{MH}} = -c''(e^\theta_{MH}) < 0.$$

$[e^{\theta}_{MH}] : \beta^{\theta}_{0}\big(\gamma+(1-\alpha)[(1-\delta)V-\gamma]-\beta_{MH}(1-\alpha)(1-\delta)V\big)-\beta^{\theta}_{0}w^{\theta}_{N}+\beta^{\theta}_{0}w^{\theta}_{DN} = \lambda_{MH}c''(e^{\theta}_{MH})$; complemented by the constraints of the problem and the corresponding complementary slackness conditions.

We solve the buyer's optimization problem in two steps. First, in Section (5.1), we derive the optimal payments w^{θ}_{N}, w^{θ}_{DD}, and w^{θ}_{DN} for an arbitrary effort level e^{θ}_{MH}. Second, in Section (5.2), we characterize the optimal effort level e^{θ}_{MH}.

5.1 Payments, w^{θ}_{N}, w^{θ}_{DD}, and w^{θ}_{DN}.

Note that from equation (33) above it follows that $w^{\theta}_{N} = w^{\theta}_{DN} + \frac{c'(e^{\theta}_{MH})}{\beta^{\theta}_{0}}$ and, therefore

$$w^{\theta}_{N} > 0. \tag{34}$$

As a result, the (LL) constraint associated with w^{θ}_{N} must be slack in equilibrium:

$$\lambda_{N} = 0. \tag{35}$$

Thus, condition $[w_{N}]$ can be rewritten as $(\lambda_{IR}-1)\beta^{\theta}_{0}e^{\theta}_{MH} = -\lambda_{MH}\beta_{0}$, which implies that

$$(\lambda_{IR}-1)\beta^{\theta}_{0}e^{\theta}_{MH} < 0, \tag{36}$$

since $\lambda_{MH} > 0$.

Condition $[w^{\theta}_{DD}]$ then immediately implies that

$$\lambda_{DD} = -(\lambda_{IR}-1)(1-\beta^{\theta}_{0}) > 0, \tag{37}$$

and, as a result, the (LL) constraint associated with w^{θ}_{DD} must be binding:

$$w^{\theta}_{DD} = 0. \tag{38}$$

Given (36) above, condition $[w^{\theta}_{DN}]$ implies that

$$\lambda_{DN} = \lambda_{MH}\beta_{0} - (\lambda_{IR}-1)\beta_{0}(1-e^{\theta}_{MH}) > 0, \tag{39}$$

and, as a result, the (LL) constraint associated with w^{θ}_{DN} must be binding:

$$w^{\theta}_{DN} = 0, \tag{40}$$

which together with (33) implies that

$$w^{\theta}_{N} = \frac{c'(e^{\theta}_{MH})}{\beta^{\theta}_{0}}. \tag{41}$$

5.2 The Optimal Effort Level e^{θ}_{MH}.

To characterize the optimal effort level e^{θ}_{MH}, we first prove by contradiction that the (IR) constraint is slack in equilibrium. Suppose the (IR) constraint is binding. Then, $\lambda_{IR} > 0$, and the equilibrium effort level e^{θ}_{MH} and w^{θ}_{N} are jointly determined by

$$\beta_0^\theta w_N^\theta = c'(e_{MH}^\theta) \text{ and } \beta_0^\theta e^\theta w_N^\theta = c(e_{MH}^\theta).$$

Therefore, the equilibrium effort level e^θ is a solution to the following equation

$$e_{MH}^\theta c'(e_{MH}^\theta) = c(e_{MH}^\theta). \tag{42}$$

We next prove by contradiction that there is no strictly convex function $c(e)$ such that (42) has an interior solution. Differentiating (42) with respect to e_{MH}^θ we obtain:

$$c'(e_{MH}^\theta) + e_{MH}^\theta c''(e_{MH}^\theta) = c'(e_{MH}^\theta) \implies e_{MH}^\theta c''(e_{MH}^\theta) = 0, \tag{43}$$

which leads to a contradiction since $c''(e_{MH}^\theta) > 0$.

Thus, $\lambda_{IR} = 0$, and the equilibrium effort level e_{MH}^θ and w_N^θ are jointly determined by

$$\beta_0^\theta w_N^\theta = c'(e_{MH}^\theta),\ \lambda_{MH} = e_{MH}^\theta, \text{ and}$$
$$\beta_0^\theta\big(\gamma + (1-\alpha)[(1-\delta)V - \gamma] - \beta_{MH}(1-\alpha)(1-\delta)V\big) - \beta_0^\theta w_N^\theta = e_{MH}^\theta c''(e_{MH}^\theta).$$

Therefore, the equilibrium effort level e^θ is a solution to the following equation

$$\beta_0^\theta\big(\gamma + (1-\alpha)[(1-\delta)V - \gamma] - \beta_{MH}(1-\alpha)(1-\delta)V\big) = \beta_0^\theta w_N^\theta + e_{MH}^\theta c''(e_{MH}^\theta), \tag{44}$$

which can be rewritten as

$$\beta_0^\theta\big(\gamma + (1-\alpha)[(1-\delta)V - \gamma] - \beta_{MH}(1-\alpha)(1-\delta)V\big) = c'(e_{MH}^\theta) + e_{MH}^\theta c''(e_{MH}^\theta). \tag{45}$$

This completes the proof of Proposition 1.

6 Appendix B. Proof of Propositions 2, 3, and Corollary 3.

6.1 Proof of Proposition 2.

Given that the seller correctly anticipates the equilibrium effort levels $e = e^\theta$, the buyer maximizes the following objective function

$$\mathbf{E}_\theta\Big[(1-\beta_0^\theta)\big(V - p_D(\beta)\big) + \beta_0^\theta e^\theta (V - p_N) + \beta_0^\theta (1-e^\theta)\big(V - p_D(\beta) - \gamma - (1-\alpha)[(1-\delta)V - \gamma]\big)$$
$$- \big[(1-\beta_0^\theta) w_{DD}^\theta + \beta_0^\theta\big(e^\theta w_N^\theta + (1-e^\theta) w_{DN}^\theta\big)\big]\Big]$$

subject to, for all $\theta, \hat\theta \in \{H, L\}$, the (MH^θ), $(IC^{\theta,\hat\theta})$, (IR^θ), and (LL^θ) constraints.

Outline of the proof. First, we simplify the buyer's optimization problem by replacing the moral hazard constraints for each project's type with the necessary and sufficient First Order Conditions. Second, we prove in Claim 1 that the $(IC^{H,L})$ is violated and the $(IC^{L,H})$

is automatically satisfied with the contract described in Proposition 1 (when the project's type is public). This implies that either $(IC^{H,L})$ or both the (IC) constraints are binding in equilibrium. Third, in Claim 2, we derive the optimal contract assuming only the $(IC^{H,L})$ is binding. Fourth, in Section 6.3, we prove that the $(IC^{L,H})$ is satisfied with equality in the contract defined in Claim 2. This implies that Claim 2 characterizes the optimal payment scheme, and that both the (IC) constraints are binding in equilibrium.

We begin to solve the the buyer's optimization problem by replacing the (MH^{θ}) constraints for $\theta = L, H$ with the following First Order Conditions:

$$\beta_0^{\theta}\big(w_N^{\theta} - w_{DN}^{\theta}\big) = c'(e^{\theta}), \tag{46}$$

that is also sufficient given that is convex.

Assumptions on the the cost function guarantee that $e^{\theta} > 0$ in equilibrium. In addition, the participation constraints (IR^{θ}) are automatically satisfied since the agent gets a positive rent even in absence of private information (see Appendix A for the details).

We denote by e^{HL} the off-equilibrium effort chosen by the agent working on the high type project:

$$e^{HL} = argmax_e\big\{(1-\beta_0^H)w_{DD}^L + \beta_0^H\big((1-e)w_{DN}^L + ew_N^L\big) - c(e)\big\}, \tag{47}$$

which is characterized by the following First Order Condition:

$$\beta_0^H\big(w_N^L - w_{DN}^L\big) = c'(e^{HL}); \tag{48}$$

and by e^{LH} the off-equilibrium effort level chosen by the agent working on the low type project:

$$e^{LH} = argmax_e\big\{(1-\beta_0^L)w_{DD}^H + \beta_0^L\big((1-e)w_{DN}^H + ew_N^H\big) - c(e)\big\}, \tag{49}$$

which is characterized by the following First Order Condition:

$$\beta_0^L\big(w_N^H - w_{DN}^H\big) = c'(e^{LH}). \tag{50}$$

We next prove in Claim 1 below that the $(IC^{H,L})$ is violated and the $(IC^{L,H})$ is automatically satisfied with the contract described in Proposition 1 (when the project's type is public). This implies that the $(IC^{H,L})$ constraint must be binding in equilibrium.

6.1.1 The $(IC^{H,L})$ Constraint is Binding.

Claim 1. Given the contract described in Proposition 1,
the $(IC^{H,L})$ is violated and the $(IC^{L,H})$ is automatically satisfied.
Proof: We first prove by contradiction that the $(IC^{H,L})$ constraint is violated. Suppose to the contrary that the $(IC^{H,L})$ constraint is automatically satisfied in the contract defined in

Proposition 1 (see Appendix A). Given that $w_{DD}^H = w_{DD}^L = w_{DN}^H = w_{DN}^L = 0$, the $(IC^{H,L})$ constraint simplifies to

$$\beta_0^H e^H w_N^H - c(e^H) \geqslant \beta_0^H e^{HL} w_N^L - c(e^{HL}). \tag{51}$$

Using $w_N^H = \frac{c'(e^H)}{\beta_0^H}$ and $w_N^L = \frac{c'(e^{HL})}{\beta_0^H}$, the (51) above can be rewritten as

$$e^H c'(e^H) - c(e^H) \geqslant e^{HL} c'(e^{HL}) - c(e^{HL}). \tag{52}$$

Since the function $f(e) = ec'(e) - c(e)$ is increasing in e:

$$f'(e) = (ec'(e) - c(e))' = ec''(e) + c'(e) - c'(e) = ec''(e) > 0,$$

the $(IC^{H,L})$ constraint finally simplifies to

$$e^H \geqslant e^{HL}. \tag{53}$$

We next prove that the equilibrium values of e^H and e^{HL} violate (53). In particular, we prove that $e^{HL} > e^H$ with the optimal contract defined in Proposition 1. Given that $w_N^H = \frac{c'(e^H)}{\beta_0^H}$, $w_N^L = \frac{c'(e^{HL})}{\beta_0^H}$, and $c''(e) > 0$,

$$e^H \geqslant e^{HL} \text{ if and only if } w_N^H \geqslant w_N^L.$$

We prove next that $w_N^H < w_N^L$, which will lead to a contradiction. Consider the ratio $\frac{w_N^L}{w_N^H}$. Using (45) for $\theta = L, H$, $\frac{w_N^L}{w_N^H}$ can be rewritten as

$$\frac{w_N^L}{w_N^H} = \frac{c'(e^L)\beta_0^H}{c'(e^H)\beta_0^L} = \frac{c'(e^L)}{c'(e^H)} \frac{\frac{c'(e^H)+e^H c''(e^H)}{\left(\gamma+(1-\alpha)[(1-\delta)V-\gamma]-\beta(1-\alpha)(1-\delta)V\right)}}{\frac{c'(e^L)+e^L c''(e^L)}{\left(\gamma+(1-\alpha)[(1-\delta)V-\gamma]-\beta(1-\alpha)(1-\delta)V\right)}} = \frac{c'(e^L)}{c'(e^H)} \left(\frac{c'(e^H)+e^H c''(e^H)}{c'(e^L)+e^L c''(e^L)} \right).$$

Therefore, $w_N^L > w_N^H$ if and only if

$$\frac{c'(e^L)}{c'(e^H)} \left(\frac{c'(e^H)+e^H c''(e^H)}{c'(e^L)+e^L c''(e^L)} \right) > 1,$$

$$\frac{e^H c''(e^H)}{c'(e^H)} > \frac{e^L c''(e^L)}{c'(e^L)}.$$

Given that the function $\phi(e) = \frac{ec''(e)}{c'(e)}$ is strictly increasing in e:

$$\phi'(e) = \frac{(ec'''(e)+c''(e))c'(e)-ec''(e)c''(e)}{[c'(e)]^2} = \frac{ec'(e)c'''(e)+c''(e)(c'(e)-ec''(e))}{[c'(e)]^2} > 0 \text{ since } c'''(e) \geqslant 0,$$

it must be that $\frac{e^H c''(e^H)}{c'(e^H)} > \frac{e^L c''(e^L)}{c'(e^L)}$ if and only if $e^H > e^L$. Therefore, $w_N^L > w_N^H$ if and only if $e^H > e^L$, which is the case in the optimal contract defined in Proposition 1.

Thus, we proved that

$$w_N^L > w_N^H \tag{54}$$

in the optimal contract defined in Proposition 1, which in turn implies that $e^H < e^{HL}$, and we have a contradiction with (53).

We now prove that the $(IC^{L,H})$ is automatically satisfied in the contract defined in Proposition 1. Given that $w^H_{DD} = w^L_{DD} = w^H_{DN} = w^L_{DN} = 0$, the $(IC^{L,H})$ constraint simplifies to

$$\beta^L_0 e^L w^H_L - c(e^L) \geqslant \beta^L_0 e^{LH} w^{H_N - c(e^{LH})}.(55)$$

Using $w^L_N = \frac{c'(e^L)}{\beta^L_0}$ and $w^H_N = \frac{c'(e^{LH})}{\beta^L_0}$, the (55) above can be rewritten as

$$e^L c'(e^L) - c(e^L) \geqslant e^{LH} c'(e^{LH}) - c(e^{LH}), \tag{56}$$

which simplifies to

$$e^L \geqslant e^{LH}. \tag{57}$$

Given that (54) implies $e^L > e^{LH}$, the $(IC^{L,H})$ is automatically satisfied in the contract defined in Proposition 1.

Q.E.D.

6.1.2 The Optimal Contract with Binding (IC^{HL}) Constraint.

Claim 2. The optimal contract with binding (IC^{HL}):

$$w^\theta_N = \frac{c'(e^\theta)}{\beta^\theta_0} > w^L_{DD} = w^H_{DD} = w^L_{DN} = w^H_{DN} = 0.$$

Proof: Given that the $(IC^{H,L})$ constraint is binding, from (53) it follows that

$$e^{HL} = e^H, \tag{58}$$

and, therefore, the $(IC^{H,L})$ constraint can be rewritten as

$(IC^{H,L})$:

$$(1 - \beta^H_0)(w^H_{DD} - w^L_{DD}) + \beta^H_0 \big((1 - e^H) w^H_{DN} + e^H w^H_N\big) - \beta^H_0 \big((1 - e^H) w^L_{DN} + e^H w^L_N\big) = 0. \tag{59}$$

Using the binding (MH^θ) constraints for $\theta = L, H$:

$$w^\theta_{DN} = w^\theta_N - \frac{c'(e^\theta)}{\beta^\theta_0}, \tag{60}$$

we express w^H_{DD} from the $(IC^{H,L})$ constraint as

$$w^H_{DD} = w^L_{DD} + \frac{\beta^H_0}{(1-\beta^H_0)} \left((1 - e^H)[w^L_N - \frac{c'(e^L)}{\beta^L_0}] + e^H w^L_N \right)$$
$$- \frac{\beta^H_0}{(1 - \beta^H_0)} \left((1 - e^H)[w^H_N - \frac{c'(e^H)}{\beta^H_0}] + e^H w^H_N \right). \tag{61}$$

Labeling λ^θ_{DD}, λ^θ_{DN}, λ^θ_N as the Lagrange multipliers of the constraints associated with (LL) constraints, the Lagrangian is:

$$
\begin{aligned}
\mathcal{L} = \nu\Big\{&(1-\beta_0^H)\big(V-p_D\big)+\beta_0^H e^H(V-p_N-w_N^H)\\
&+\beta_0^H(1-e^H)\big(V-p_D-\gamma-(1-\alpha)[(1-\delta)V-\gamma]-[w_N^L-\tfrac{c'(e^H)}{\beta_0^H}]\big)\Big\}\\
+\nu\Big\{&\tfrac{\beta_0^H}{(1-\beta_0^H)}\big((1-e^H)[w_N^H-\tfrac{c'(e^H)}{\beta_0^H}]+e^H w_N^H\big)-w_{DD}^L-\tfrac{\beta_0^H}{(1-\beta_0^H)}\big((1-e^H)[w_N^L-\tfrac{c'(e^L)}{\beta_0^L}]+e^H w_N^L\big)\Big\}\\
+(1-\nu)\Big\{&(1-\beta_0^L)\big(V-p_D-w_{DD}^L\big)+\beta_0^L e^L(V-p_N-w_N^L)\\
&+\beta_0^L(1-e^L)\big(V-p_D-\gamma-(1-\alpha)[(1-\delta)V-\gamma]-[w_N^L-\tfrac{c'(e^L)}{\beta_0^L}]\big)\Big\}\\
+\lambda_{DD}^H\Big[&w_{DD}^L+\tfrac{\beta_0^H}{(1-\beta_0^H)}\big((1-e^H)[w_N^L-\tfrac{c'(e^L)}{\beta_0^L}]+e^H w_N^L\big)-\tfrac{\beta_0^H}{(1-\beta_0^H)}\big((1-e^H)[w_N^H-\tfrac{c'(e^H)}{\beta_0^H}]+e^H w_N^H\big)\Big]\\
&+\lambda_{DN}^H[w_N^H-\tfrac{c'(e^H)}{\beta_0^H}]+w_N^H\lambda_N^H+\lambda_{DD}^L w_{DD}^L+\lambda_{DN}^L[w_N^L-\tfrac{c'(e^L)}{\beta_0^L}]+w_N^L\lambda_N^L.
\end{aligned}
$$

The Kuhn-Tucker conditions for the optimization problem are:

$[w_{DD}^L] : -\nu-(1-\nu)(1-\beta_0^L)+\lambda_{DD}^H+\lambda_{DD}^L=0;$

$[w_N^L] : -\nu\frac{\beta_0^H}{(1-\beta_0^H)}-(1-\nu)\beta_0^L+\lambda_{DD}^H\frac{\beta_0^H}{(1-\beta_0^H)}+\lambda_N^L+\lambda_{DN}^L=0;$

$[w_N^H] : -\nu\beta_0^H+\nu\frac{\beta_0^H}{(1-\beta_0^H)}-\lambda_{DD}^H\frac{\beta_0^H}{(1-\beta_0^H)}+\lambda_{DN}^H+\lambda_N^H=0,$

complemented by the constraints of the problem and the corresponding complementary slackness conditions.

We now characterise the optimal payment structure for both types. Note that from (46) it follows that $w_N^\theta = w_{DN}^\theta + \frac{c'(e^\theta)}{\beta_0^\theta}$ for type each type $\theta \in \{H, L\}$ and, therefore,

$$w_N^\theta > 0. \tag{62}$$

As a result, the (LL^θ) constraints associated with w_N^θ must be slack in equilibrium:

$$\lambda_N^\theta = 0. \tag{63}$$

We next prove that Lagrange multipliers determined by $[w_{DD}^L]$, $[w_N^L]$, and $[w_N^H]$ are positive. We express λ_{DD}^L, λ_{DN}^L, and λ_{DN}^H as a function of λ_{DD}^H only from $[w_{DD}^L]$, $[w_N^L]$, and $[w_N^H]$ as follows:

$$[w_{DD}^L] : \lambda_{DD}^L = (1-\beta_0^L(1-\nu)) - \lambda_{DD}^H, \tag{64}$$

$$[w_N^L] : \lambda_{DN}^L = (1-\nu)\beta_0^L + \nu\frac{\beta_0^H}{(1-\beta_0^H)} + \lambda_{DD}^H\frac{\beta_0^H}{(1-\beta_0^H)}, \tag{65}$$

$$[w_N^H] : \lambda_{DN}^H = -\frac{\nu(\beta_0^H)^2}{(1-\beta_0^H)} + \lambda_{DD}^H\frac{\beta_0^H}{(1-\beta_0^H)}. \tag{66}$$

Note that from $[w_N^H]$ it follows directly that

$$\lambda_{DD}^H > 0, \tag{67}$$

since otherwise $\lambda_{DN}^H < 0$.

Next, for any $\nu\beta_0^H < \lambda_{DD}^H < 1 - \beta_0^L(1-\nu)$, all Lagrange multipliers are strictly positive and, therefore, all payments w_{DD}^θ and w_{DN}^θ are zero for $\theta = H, L$:

$$w_{DD}^L = w_{DD}^H = w_{DN}^L = w_{DN}^H = 0. \tag{68}$$

Finally, from (68) and (46) for $\theta \in \{H, L\}$ it follows that

$$w_N^\theta = \frac{c'(e^\theta)}{\beta_0^\theta}. \tag{69}$$

Q.E.D.

6.1.3 The (IC^{LH}) is satisfied with equality in the contract defined in Claim 2.

Given that $w_{DD}^H = w_{DD}^L = w_{DN}^H = w_{DN}^L = 0$, the $(IC^{L,H})$ constraint simplifies to (see proof of Claim 1)

$$e^L \geqslant e^{LH}. \tag{70}$$

We next prove that $e^L = e^{LH}$ in the contract defined in Claim 2, which will imply that the $(IC^{L,H})$ constraint is satisfied with equality.

First, given that $w_{DD}^H = w_{DD}^L = w_{DN}^H = w_{DN}^L = 0$, the $(IC^{H,L})$ constraint simplifies to:

$$w_N^H = w_N^L, \tag{71}$$

which implies

$$w_N^L = \frac{c'(e^L)}{\beta_0^L} = \frac{c'(e^H)}{\beta_0^H} = w_N^H. \tag{72}$$

Second, from the the First Order Condition characterising e^{LH} we have (50):

$$\beta_0^L w_N^H = c'(e^{LH}). \tag{73}$$

Combining (72) and (73), we obtain

$$e^{LH} = e^L.$$

Thus, the $(IC^{L,H})$ constraint is satisfied with equality in the contract defined in Claim 2.

To summarize, we established that Claim 2 characterizes the optimal payment scheme, and that both the (IC) constraints are binding in equilibrium.

This completes the proof of Proposition 2.

6.2 Proof of Proposition 3.

We start by combining the moral hazard constraints $\beta_0^H w_N^H = c'(e^H)$ and $\beta_0^L w_N^L = c'(e^L)$ with the equilibrium wage pooling result $w_N^L = w_N^H$ to derive the following condition:

$$\beta_0^H c'(e^L) - \beta_0^L c'(e^H) = 0. \tag{74}$$

Labeling by μ as the Lagrange multiplier associated with condition (74), the Lagrangian for the buyer's optimization problem is:

$$\mathcal{L} = \nu\Big\{(1-\beta_0^H)\big(V - p_D\big) + \beta_0^H e^H (V - p_N - \tfrac{c'(e^H)}{\beta_0^H})$$
$$+\beta_0^H(1-e^H)\big(V - p_D - \gamma - (1-\alpha)[(1-\delta)V-\gamma]\big)\Big\}$$
$$(1-\nu)\Big\{(1-\beta_0^L)\big(V - p_D\big) + \beta_0^L e^L (V - p_N - \tfrac{c'(e^L)}{\beta_0^L})$$
$$+\beta_0^L(1-e^L)\big(V - p_D - \gamma - (1-\alpha)[(1-\delta)V-\gamma]\big)\Big\}$$
$$+\mu\big[\beta_0^H c'(e^L) - \beta_0^L c'(e^H)\big].$$

Differentiating the Lagrangian (together with the equilibrium condition the the seller correctly anticipates the effort levels), we obtain the following Kuhn-Tucker conditions for the optimization problem:

$$[e^L] : \beta_0^L\big(\gamma + (1-\alpha)[(1-\delta)V-\gamma] - \beta(1-\alpha)(1-\delta)V\big) = c'(e^L) + e^L c''(e^L) + \tfrac{\mu\beta_0^H}{(1-\nu)} c''(e^L);$$

$$[e^H] : \beta_0^H\big(\gamma + (1-\alpha)[(1-\delta)V-\gamma] - \beta(1-\alpha)(1-\delta)V\big) = c'(e^H) + e^H c''(e^H) - \tfrac{\mu\beta_0^L}{\nu} c''(e^H),$$

complemented by the constraints of the problem and the corresponding complementary slackness conditions.

We now characterise the optimal effort level for both types.

L-type. Comparing $[e^L]$ below

$$\beta_0^L\big(\gamma + (1-\alpha)[(1-\delta)V-\gamma] - \beta(1-\alpha)(1-\delta)V\big) = c'(e^L) + e^L c''(e^L) + \frac{\mu\beta_0^H}{(1-\nu)} c''(e^L) \tag{75}$$

with the optimal effort e_{MH}^L when the project's type is known (see Proposition 1)

$$\beta_0^L\big(\gamma + (1-\alpha)[(1-\delta)V-\gamma] - \beta(1-\alpha)(1-\delta)V\big) = c'(e_{MH}^L) + e^L c''(e_{MH}^L), \tag{76}$$

and given that $\mu > 0$, it follows that

$$e^L < e_{MH}^L.$$

H-type. Comparing $[e^H]$ below

$$\beta_0^H\big(\gamma + (1-\alpha)[(1-\delta)V-\gamma] - \beta(1-\alpha)(1-\delta)V\big) = c'(e^H) + e^H c''(e^H) - \frac{\mu\beta_0^L}{\nu} c''(e^H), \tag{77}$$

with the optimal effort e^H_{MH} when the project's type is known (see Proposition 1)

$$\beta_0^H\big(\gamma + (1-\alpha)[(1-\delta)V - \gamma] - \beta(1-\alpha)(1-\delta)V\big) = c'(e^H_{MH}) + e^H c''(e^H_{MH}), \tag{78}$$

and given that $\mu > 0$, it follows that

$$e^H > e^H_{MH}.$$

This completes the proof of Proposition 3.

6.3 Proof of Corollary 3.

We now derive sufficient condition for the contract to be less incomplete due to adverse selection. First, given that $e^L < e^L_{MH}$, it must be that

$$\beta^L(e^L_{MH}) < \beta^L(e^L).$$

Second, given that $e^H > e^H_{MH}$, it must be that

$$\beta^H(e^H_{MH}) > \beta^H(e^H).$$

The contract is less incomplete due to adverse selection if:

$$\nu\beta^H(e^H_{MH}) + (1-\nu)\beta^L(e^L_{MH}) > \nu\beta^H(e^H) + (1-\nu)\beta^L(e^L)$$

We define a value of ν, called $\underline{\nu}$, such that

$$\underline{\nu}: \;\; \underline{\nu}\beta^H(e^H_{MH}) + (1-\underline{\nu})\beta^L(e^L_{MH}) = \underline{\nu}\beta^H(e^H) + (1-\underline{\nu})\beta^L(e^L).$$

Note that $\underline{\nu} < 1$ because $\beta^H(e^H_{MH}) > \beta^H(e^H)$, and $\underline{\nu} > 0$ because $\beta^L(e^L_{MH}) < \beta^L(e^L)$.

Therefore, if $\nu > \underline{\nu}$, the contract is less incomplete due to adverse selection:

$$\beta < \beta_{MH} \text{ if } \nu > \underline{\nu}.$$

Finally, since the price $p_D(\beta)$ is decreasing in β, we have

$$p_D(\beta) > p_D(\beta_{MH}) \text{ if } \nu > \underline{\nu}.$$

7 Appendix C. Proof of Proposition 4 and Corollary 4.

7.1 Proof of Proposition 4

We prove Proposition 4 in three steps. In Step 1, we prove that e^H and e^L move in the same direction if α changes. In Step 2, we prove that that for the high type $\frac{de^H}{d\alpha} > 0$ if β_0^L is small, γ is not too high, and V is not too high. In Step 3, we conclude.

Step 1: $sgn(\frac{de^H}{d\alpha}) = sgn(\frac{de^L}{d\alpha})$**.** Directly applying the Implicit Function Theorem to (74), we obtain

$$\frac{de^L}{de^H} = \frac{\beta_0^L c''(e^H)}{\beta_0^H c''(e^L)} > 0. \tag{79}$$

Therefore, e^H and e^L move in the same direction if α changes.

Step 2. We next prove that for any $\beta_0^H \in (0,1)$ there exist $0 < \overline{\beta}_0^L(\beta_0^H) < \beta_0^H$, $\overline{V} > 0$, and $\underline{\gamma} > 0$ such that if $\beta_0^L < \overline{\beta}_0^L$, $V < \overline{V}$, and $\underline{\gamma} < \gamma < (1-\delta)V$ then $\frac{de^H}{d\alpha} > 0$.

To determine the sign of $\frac{de^H}{d\alpha}$ and $\frac{de^H}{d\delta}$, we rewrite the buyer's optimization problem with respect to the high type's effort level e^H. First, given the condition (74), we can express e^L as a function of e^H as follows:

$$\hat{e}^L := \hat{e}^L(e^H) = c'^{-1}\left(\frac{\beta_0^L}{\beta_0^H} c'(e^H)\right). \tag{80}$$

Then, the buyer is choosing e^H to maximize

$$\nu\Big\{(1-\beta_0^H)\big(V-p_D\big) + \beta_0^H e^H (V - p_N - \tfrac{c'(e^H)}{\beta_0^H})$$

$$+\beta_0^H(1-e^H)\big(V - p_D - \gamma - (1-\alpha)[(1-\delta)V-\gamma]\big)\Big\}$$

$$(1-\nu)\Big\{(1-\beta_0^L)\big(V-p_D\big) + \beta_0^L \hat{e}^L (V - p_N - \tfrac{c'(\hat{e}^L)}{\beta_0^L})$$

$$+\beta_0^L(1-\hat{e}^L)\big(V - p_D - \gamma - (1-\alpha)[(1-\delta)V-\gamma]\big)\Big\}.$$

Therefore, the First Order Condition that determines the optimal value of e^H becomes:

$$\nu\beta_0^H\Big[\big(\gamma + (1-\alpha)[(1-\delta)V-\gamma] - \beta(1-\alpha)(1-\delta)V\big) - \tfrac{c'(e^H)}{\beta_0^H} - e^H\tfrac{c''(e^H)}{\beta_0^H}\Big]$$

$$+(1-\nu)\beta_0^L\frac{d\hat{e}^L}{de^H}\Big[\big(\gamma+(1-\alpha)[(1-\delta)V-\gamma]-\beta(1-\alpha)(1-\delta)V\big)-\frac{c'(\hat{e}^L)}{\beta_0^L}-\hat{e}^L\frac{c''(\hat{e}^L)}{\beta_0^L}\Big] = 0. \tag{81}$$

Next, using the Implicit Function Theorem, we obtain

$$\frac{de^H}{d\alpha} = -\frac{\frac{\partial\Phi}{\partial\alpha}}{\frac{\partial\Phi}{\partial e^H}}, \tag{82}$$

where $\Phi =$

$$\nu\beta_0^H\left[\nu\beta_0^H\left[\Big(\gamma+(1-\alpha)[(1-\delta)V-\gamma]-\beta(1-\alpha)(1-\delta)V\Big)-\tfrac{c'(e^H)}{\beta_0^H}-e^H\tfrac{c''(e^H)}{\beta_0^H}\right]-\tfrac{c'(e^H)}{\beta_0^H}-e^H\tfrac{c''(e^H)}{\beta_0^H}\right]$$
$$+(1-\nu)\beta_0^L\tfrac{d\hat{e}^L}{de^H}\times$$
$$\times\left[\nu\beta_0^H\left[\Big(\gamma+(1-\alpha)[(1-\delta)V-\gamma]-\beta(1-\alpha)(1-\delta)V\Big)-\frac{c'(e^H)}{\beta_0^H}-e^H\frac{c''(e^H)}{\beta_0^H}\right]-\frac{c'(\hat{e}^L)}{\beta_0^L}-\hat{e}^L\frac{c''(\hat{e}^L)}{\beta_0^L}\right].\tag{83}$$

We next determine the signs of $\frac{\partial\Phi}{\partial\alpha}$, and $\frac{\partial\Phi}{\partial e^H}$.

$\boldsymbol{\frac{\partial\Phi}{\partial\alpha}>0}$ ***if*** $\boldsymbol{\underline{\gamma}<\gamma<(1-\delta)V}$. We first prove that $\frac{\partial\Phi}{\partial\alpha}>0$ if γ is not too low:

$$\tfrac{\partial\Phi}{\partial\alpha}=\nu\beta_0^H[\gamma-(1-\beta)(1-\delta)V]+(1-\nu)\beta_0^L\tfrac{d\hat{e}^L}{de^H}[\gamma-(1-\beta)(1-\delta)V].$$

Since $\frac{d\hat{e}^L}{de^H}=\frac{\beta_0^L c''(e^H)}{\beta_0^H c''(\hat{e}^L)}>0$, for $\frac{\partial\Phi}{\partial\alpha}>0$ it is sufficient that $\gamma-(1-\beta)(1-\delta)V>0$. We define a value of γ, called $\underline{\gamma}$, such that

$$\underline{\gamma}:\quad \underline{\gamma}=(1-\beta)(1-\delta)V>0.$$

In addition, assumption ($A1$) requires $\gamma<(1-\delta)V$.
Therefore, $\frac{\partial\Phi}{\partial\alpha}>0$ if γ is not too low:

$$\tfrac{\partial\Phi}{\partial\alpha}>0 \text{ if } \underline{\gamma}<\gamma<(1-\delta)V.$$

$\boldsymbol{\frac{\partial\Phi}{\partial e^H}}$: We now determine the sign of $\frac{\partial\Phi}{\partial e^H}$.

$$\begin{aligned}\tfrac{\partial\Phi}{\partial e^H}&=\nu\beta_0^H\left[-\tfrac{d\beta}{de^H}(1-\alpha)(1-\delta)V-\tfrac{c''(e^H)}{\beta_0^H}-\tfrac{c''(e^H)}{\beta_0^H}-e^H\tfrac{c'''(e^H)}{\beta_0^H}\right]\\
&+(1-\nu)\beta_0^L\frac{d\left(\frac{d\hat{e}^L}{de^H}\right)}{de^H}\left[\Big(\gamma+(1-\alpha)[(1-\delta)V-\gamma]-\beta(1-\alpha)(1-\delta)V\Big)-\tfrac{c'(\hat{e}^L)}{\beta_0^L}-\hat{e}^L\tfrac{c''(\hat{e}^L)}{\beta_0^L}\right]\\
&+(1-\nu)\beta_0^L\tfrac{d\hat{e}^L}{de^H}\left[-\tfrac{d\beta}{d\hat{e}^L}(1-\alpha)(1-\delta)V\tfrac{d\hat{e}^L}{de^H}-\tfrac{c''(\hat{e}^L)}{\beta_0^L}\tfrac{d\hat{e}^L}{de^H}-\Big(\tfrac{c''(\hat{e}^L)}{\beta_0^L}+\hat{e}^L\tfrac{c'''(\hat{e}^L)}{\beta_0^L}\Big)\tfrac{d\hat{e}^L}{de^H}\right]\\
&=-\nu\big[\tfrac{d\beta}{de^H}\beta_0^H(1-\alpha)(1-\delta)V+2c''(e^H)+e^Hc'''(e^H)\big]\\
&+(1-\nu)\beta_0^L\frac{d\left(\frac{d\hat{e}^L}{de^H}\right)}{de^H}\left[\Big(\gamma+(1-\alpha)[(1-\delta)V-\gamma]-\beta(1-\alpha)(1-\delta)V\Big)-\tfrac{c'(\hat{e}^L)}{\beta_0^L}-\hat{e}^L\tfrac{c''(\hat{e}^L)}{\beta_0^L}\right]\\
&-(1-\nu)(\tfrac{d\hat{e}^L}{de^H})^2\big[\tfrac{d\beta}{d\hat{e}^L}(1-\alpha)(1-\delta)\beta_0^LV+2c''(\hat{e}^L)+\hat{e}^Lc'''(\hat{e}^L)\big].\end{aligned}$$

Given that

$$\frac{d\left(\frac{d\hat{e}^L}{de^H}\right)}{de^H}=\frac{d\left(\frac{\beta_0^Lc''(e^H)}{\beta_0^Hc''(\hat{e}^L)}\right)}{de^H}=\frac{\beta_0^L}{\beta_0^H}\frac{c'''(e^H)c''(\hat{e}^L)-c''(e^H)c'''(\hat{e}^L)\frac{d\hat{e}^L}{de^H}}{\big(c''(\hat{e}^L)\big)^2}$$

$$=\tfrac{\beta_0^L}{\beta_0^H}\frac{c'''(e^H)c''(\hat{e}^L)-c''(e^H)c'''(\hat{e}^L)\frac{\beta_0^Lc''(e^H)}{\beta_0^Hc''(\hat{e}^L)}}{\big(c''(\hat{e}^L)\big)^2},$$

the expression for $\frac{\partial \Phi}{\partial e^H}$ simplifies to

$$\frac{\partial \Phi}{\partial e^H} = -\nu\big[\frac{d\beta^H(e^H)}{de^H}\beta_0^H(1-\alpha)(1-\delta)V + 2c''(e^H) + e^H c'''(e^H)\big]$$

$$-(1-\nu)(\frac{d\hat{e}^L}{de^H})^2\big[\frac{d\beta^L(\hat{e}^L)}{d\hat{e}^L}(1-\alpha)(1-\delta)\beta_0^L V + 2c''(\hat{e}^L) + \hat{e}^L c'''(\hat{e}^L)\big]$$

$$+(1-\nu)\beta_0^L\Bigg[\big(\gamma + (1-\alpha)[(1-\delta)V-\gamma] - \beta(1-\alpha)(1-\delta)V\big) - \frac{c'(\hat{e}^L)}{\beta_0^L} - \hat{e}^L\frac{c''(\hat{e}^L)}{\beta_0^L}\Bigg]\times$$

$$\times\frac{\beta_0^L}{\beta_0^H}\left(\frac{c'''(e^H)c''(\hat{e}^L) - c''(e^H)c'''(\hat{e}^L)\frac{\beta_0^L c''(e^H)}{\beta_0^H c''(\hat{e}^L)}}{\big(c''(\hat{e}^L)\big)^2}\right)$$

$$= -\nu\big[\frac{d\beta}{de^H}\beta_0^H(1-\alpha)(1-\delta)V + 2c''(e^H) + e^H c'''(e^H)\big]$$

$$-(1-\nu)\left(\frac{\beta_0^L c''(e^H)}{\beta_0^H c''(\hat{e}^L)}\right)^2\big[\frac{d\beta}{d\hat{e}^L}(1-\alpha)(1-\delta)\beta_0^L V + 2c''(\hat{e}^L) + \hat{e}^L c'''(\hat{e}^L)\big]$$

$$+(1-\nu)\beta_0^L\Bigg[\big(\gamma + (1-\alpha)[(1-\delta)V-\gamma] - \beta(1-\alpha)(1-\delta)V\big) - \frac{c'(\hat{e}^L)}{\beta_0^L} - \hat{e}^L\frac{c''(\hat{e}^L)}{\beta_0^L}\Bigg]\times$$

$$\times\frac{\beta_0^L}{\beta_0^H}\left(\frac{c'''(e^H)c''(\hat{e}^L) - c''(e^H)c'''(\hat{e}^L)\frac{\beta_0^L c''(e^H)}{\beta_0^H c''(\hat{e}^L)}}{\big(c''(\hat{e}^L)\big)^2}\right).$$

$$= -\nu\big[\frac{d\beta}{de^H}\beta_0^H(1-\alpha)(1-\delta)V + 2c''(e^H) + e^H c'''(e^H)\big]$$

$$+(1-\nu)\frac{(\beta_0^L)^2}{(\beta_0^H)^2(c''(\hat{e}^L))^2}\times$$

$$\times\Bigg(\big(\beta_0^H c'''(e^H)c''(\hat{e}^L) - \beta_0^L c''(e^H)c'''(\hat{e}^L)\frac{c''(e^H)}{c''(\hat{e}^L)}\big)\times$$

$$\times\big[\big(\gamma + (1-\alpha)[(1-\delta)V-\gamma] - \beta(1-\alpha)(1-\delta)V\big) - \frac{c'(\hat{e}^L)}{\beta_0^L} - \hat{e}^L\frac{c''(\hat{e}^L)}{\beta_0^L}\big]$$

$$-(c''(e^H))^2\big[\frac{d\beta}{d\hat{e}^L}(1-\alpha)(1-\delta)\beta_0^L V + 2c''(\hat{e}^L) + \hat{e}^L c'''(\hat{e}^L)\big]\Bigg).$$

We prove next that if β_0^L and V are not too high, then $\frac{\partial \Phi}{\partial e^H} < 0$.

First, $\beta_0^H c'''(e^H)c''(\hat{e}^L) - \beta_0^L c''(e^H)c'''(\hat{e}^L)\frac{c''(e^H)}{c''(\hat{e}^L)} > 0$ if β_0^L is small enough. We define a value of β_0^L, called $\overline{\beta}_0^L(\beta_0^H)$, such that

$$\overline{\beta}_0^L(\beta_0^H):\ \ \beta_0^H c'''(e^H)c''(\hat{e}^L) - \overline{\beta}_0^L(\beta_0^H)c''(e^H)c'''(\hat{e}^L)\frac{c''(e^H)}{c''(\hat{e}^L)} = 0 \text{ or, equivalently}$$

$$\overline{\beta}_0^L(\beta_0^H) \equiv \beta_0^H\frac{c'''(e^H)(c''(\hat{e}^L))^2}{c'''(\hat{e}^L)(c''(e^H))^2} > 0.$$

Since $e^H > \hat{e}^L$, we have $\frac{c'''(e^H)(c''(\hat{e}^L))^2}{c'''(\hat{e}^L)(c''(e^H))^2} < 1$ and, as a result, for any $\beta_0^H \in (0,1)$:

$$\overline{\beta}_0^L(\beta_0^H) < \beta_0^H.$$

Thus, $\beta_0^H c'''(e^H)c''(\hat{e}^L) - \beta_0^L c''(e^H)c'''(\hat{e}^L)\frac{c''(e^H)}{c''(\hat{e}^L)} > 0$ if $\beta_0^L < \overline{\beta}_0^L$.

Second, since $\frac{d\beta}{de^H} < 0$, there exists a small enough value of V such that:

$$\tfrac{d\beta}{de^H}\beta_0^H(1-\alpha)(1-\delta)V + 2c''(e^H) + e^H c'''(e^H) > 0.$$

We define a value of V, called V_1, such that

$$V_1\colon\ \ \tfrac{d\beta}{de^H}\beta_0^H(1-\alpha)(1-\delta)V_1 + 2c''(e^H) + e^H c'''(e^H) = 0.$$

Thus, $\frac{d\beta}{de^H}\beta_0^H(1-\alpha)(1-\delta)V + 2c''(e^H) + e^H c'''(e^H) > 0$ if $V < V_1$.

Third, since $\gamma + (1-\alpha)[(1-\delta)V - \gamma] - \beta(1-\alpha)(1-\delta)V = \alpha\gamma + (1-\beta)(1-\alpha)(1-\delta)V$, there exists a small enough value of V such that:

$$\big(\gamma + (1-\alpha)[(1-\delta)V - \gamma] - \beta(1-\alpha)(1-\delta)V\big) - \tfrac{c'(\hat{e}^L)}{\beta_0^L} - \hat{e}^L\tfrac{c''(\hat{e}^L)}{\beta_0^L} < 0.$$

We define a value of V, called V_2, such that

$$V_2\colon\ \ \big(\gamma + (1-\alpha)[(1-\delta)V - \gamma] - \beta(1-\alpha)(1-\delta)V\big) - \tfrac{c'(\hat{e}^L)}{\beta_0^L} - \hat{e}^L\tfrac{c''(\hat{e}^L)}{\beta_0^L} = 0.$$

Thus, $\big(\gamma + (1-\alpha)[(1-\delta)V - \gamma] - \beta(1-\alpha)(1-\delta)V\big) - \frac{c'(\hat{e}^L)}{\beta_0^L} - \hat{e}^L\frac{c''(\hat{e}^L)}{\beta_0^L} < 0$ if $V < V_2$.

Fourth, since $\frac{d\beta}{d\hat{e}^L} < 0$, there exists a small enough value of V such that:

$$\tfrac{d\beta}{d\hat{e}^L}(1-\alpha)(1-\delta)\beta_0^L V + 2c''(\hat{e}^L) + \hat{e}^L c'''(\hat{e}^L) > 0.$$

We define a value of V, called V_3, such that

$$V_3\colon\ \ \tfrac{d\beta}{d\hat{e}^L}(1-\alpha)(1-\delta)\beta_0^L V_3 + 2c''(\hat{e}^L) + \hat{e}^L c'''(\hat{e}^L) = 0.$$

Thus, $\frac{d\beta}{d\hat{e}^L}(1-\alpha)(1-\delta)\beta_0^L V + 2c''(\hat{e}^L) + \hat{e}^L c'''(\hat{e}^L) > 0$ if $V < V_3$.
Therefore, $\frac{\partial\Phi}{\partial e^H} < 0$ if $\beta_0^L < \overline{\beta}_0^L$, $V < V_1$, $V < V_2$, and $V < V_3$.

We define the smallest value of V as

$$\overline{V} = min\{V_1, V_2, V_3\}.$$

Consequently, $\frac{\partial\Phi}{\partial e^H} < 0$ if $\beta_0^L < \overline{\beta}_0^L$ and $V < \overline{V}$.

To summarize, we proved that for any $\beta_0^H \in (0,1)$ there exist $0 < \overline{\beta}_0^L(\beta_0^H) < \beta_0^H$, $\overline{V} > 0$, and $\underline{\gamma} > 0$ such that if $\beta_0^L < \overline{\beta}_0^L$, $V < \overline{V}$, and $\underline{\gamma} < \gamma < (1-\delta)V$ then $\frac{de^H}{d\alpha} > 0$.

Step 3. In Step 1, we proved that e^H and e^L move in the same direction if either α changes. In Step 2, we proved that for any $\beta_0^H \in (0,1)$ there exist $0 < \overline{\beta}_0^L(\beta_0^H) < \beta_0^H$, $\overline{V} > 0$, and $\underline{\gamma} > 0$ such that if $\beta_0^L < \overline{\beta}_0^L$, $V < \overline{V}$, and $\underline{\gamma} < \gamma < (1-\delta)V$ then $\frac{de^H}{d\alpha} > 0$.

Therefore, if $\beta_0^L < \overline{\beta}_0^L$, $V < \overline{V}$, and $\underline{\gamma} < \gamma < (1-\delta)V$ then

$$\frac{de^\theta}{d\alpha} > 0 \text{ for } \theta = L, H.$$

Finally, since the probability of renegotiation is decreasing in the effort level, $\frac{d\beta}{de^H} < 0$ and $\frac{d\beta}{de^L} < 0$, it must be that if $\beta_0^L < \overline{\beta}_0^L$, $V < \overline{V}$, and $\underline{\gamma} < \gamma < (1-\delta)V$ then

$$\frac{d\beta}{d\alpha} < 0 \text{ for } \theta = L, H.$$

This completes the proof of Proposition 4.

7.2 Proof of Corollary 4.

We now derive sufficient condition for the price p_D to be increasing in α. Differentiating both sides of (8) with respect to α, we obtain

$$\begin{aligned}\frac{dp_D}{d\alpha} &= \frac{d\left((1-\alpha)\left[V-k-\beta(e^L,e^H)(1-\delta)V\right]\right)}{d\alpha} \\ &= (1-\alpha)\left[V-k-\frac{d\beta}{d\alpha}(1-\delta)V\right] - \left[V-k-\beta(1-\delta)V\right] \\ &= (1-\delta)V\left[\beta-(1-\alpha)\frac{d\beta}{d\alpha}\right] - \alpha(V-k). \end{aligned} \tag{84}$$

Therefore, $\frac{dp_D}{d\alpha} > 0$ if and only if

$$(1-\delta)V\left[\beta-(1-\alpha)\tfrac{d\beta}{d\alpha}\right] - \alpha(V-k) > 0,$$

$$\beta-(1-\alpha)\frac{d\beta}{d\alpha} > \frac{\alpha}{(1-\delta)}\frac{(V-k)}{V}. \tag{85}$$

Any of the following conditions on the primitives is sufficient for (85) to hold: (1) k high enough, or (2) α small enough,

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