On the Heterogeneous Interest Rate Sensitivity of Firms' Investments*

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September 1, 2021

Abstract

Investment rates of young or small firms are more sensitive to monetary policy shocks. Conventional perspective views these findings as supporting the financial accelerator mechanism, based on the narrative that these firms are financially constrained and monetary policy affects financial conditions. In this paper, we present two mechanisms which make firms typically classified as financially constrained more sensitive to monetary policy even in the absence of a financial accelerator. First, with decreasing returns to scale, firms that operate below their optimal size have a higher marginal return to investing. This makes them more sensitive to monetary policy-induced changes in the discount rate. Second, fixed capital adjustment costs, as necessary to replicate lumpy investment behavior, create additional heterogeneous effects via the extensive margin of investment. We quantify both mechanisms in a calibrated heterogeneous firm New Keynesian model and provide supporting empirical evidence using firm-level investment data.

Keywords: Adjustment Costs, Lumpy Investment, Heterogeneous Sensitivity, Observational Equivalence, Monetary Policy

JEL Classification: E52, E22, D21, D22

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*We thank Christian Bayer, Yongsung Chang, Russell Cooper, Joachim Jungherr, Keith Kuester, Emi Nakamura, Haozhou Tang, Petr Sedláček as well as seminar/workshop participants at the Peking University for insightful discussions. We thank Lixing Wang for his research assistance in constructing the Section 3 of the paper.

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1 Introduction

The empirical literature has documented that young firms' investment is more sensitive to monetary policy than old firms', see e.g., Cloyne et al. (2020). Gertler and Gilchrist (1994) show a similar result for small and large firms.[1] Conventional wisdom views these findings as supporting the financial accelerator mechanism, based on the narrative that young firms are financially constrained[2] and monetary policy affects financial conditions.

In this paper, we present two mechanisms which make firms typically classified as financially constrained more sensitive to monetary policy *even in the absence* of a financial accelerator. Thus, there is an issue of observational equivalence: The observed heterogeneous sensitivity can arise not only due to a financial accelerator mechanism but also due to two non-financial mechanisms described next.

First, if there are decreasing returns to scale and new firms enter the economy below optimal size, young firms have a higher marginal return to productive capital. Due to this, young firms not only grow faster unconditionally (as in the data), but are also more sensitive to changes in the stochastic discount factor triggered by monetary policy. Technically speaking, the heterogeneous sensitivity emerges because young firms' *marginal return* curves shift more than old firms' in response to interest rate changes. The classical financial accelerator instead predicts a heterogeneous sensitivity due to differential effects on the *marginal cost* curve.[3]

Second, the presence of an *extensive margin* investment decision creates heterogeneous effects on *average* investment rates among young and old firms. The group-specific average investment rate is now the fraction of investing firms (hazard rate) times the investment rate conditional on investing. On the one hand, if a monetary shock causes one additional young and one additional old firm to make an investment, we estimate a higher *average* sensitivity of young firms. That is because conditional on adjusting, young firms have a higher investment rate. On the other hand, the mon-

[1]Clearly, these findings are connected, as age and size are strongly correlated in the data. In this paper, we focus on age but emphasize and show that our results are similar when comparing small and large firms.

[2]Rauh (2006), Fee et al. (2009), Hadlock and Pierce (2010), and more recently Cloyne et al. (2020) argue that young firms are more likely financially constrained than old firms. Gertler and Gilchrist (1994) rely on the narrative that "*...the costs of external finance apply mainly to younger firms, firms with a high degree of idiosyncratic risk, and firms that are not well collateralized. These are, on average, smaller firms...*" to motivate the use of firm size as a proxy for financial frictions.

[3]Financial constraints make additional investments increasingly expensive for firms. Technically speaking, they generate an upward-sloping marginal cost curve for investment. According to the financial accelerator, monetary policy shifts or flattens out this marginal cost curve, which makes financially constrained firms more sensitive than unconstrained ones. For more details, see Ottonello and Winberry (2020).

etary shock may affect hazard rates across groups differently, generating additional heterogeneous sensitivity.

We illustrate both mechanisms in a simple two-period partial equilibrium model. Moreover, we quantify them in a general equilibrium heterogeneous firm model calibrated to match moments of the cross-sectional investment rate distribution and firm life-cycle patterns. According to the model, young firms are almost twice as sensitive to a monetary policy shock as old firms, explaining a large chunk of the observed heterogeneity in the data. Decomposing this total effect, we find that the extensive margin effect is quantitatively more important than the intensive margin effect.

Understanding the source of the heterogeneous sensitivity in firm-level investments is relevant for policymakers. To the extent that financial frictions cause the observed heterogeneity, an effective stimulus policy is to provide credit to constrained firms (i.e., young firms). However, the issue of observational equivalence we put forward suggests a biased estimate regarding the quantitative importance of the financial accelerator. As a result, ignoring the mechanisms that we highlight in this paper will overstate the effectiveness of stimulus policies that solely address credit constraints.

Empirically, we confirm that young firms are more sensitive to monetary policy shocks than old firms (Stylized Fact 1), building on previous empirical work by Cloyne et al. (2020). The same finding emerges for small firms as opposed to large firms. It is noteworthy that the heterogeneous sensitivity is documented in a sample of public firms, which are relatively unconstrained compared to private firms. (Caglio et al., 2021) This supports the idea that next to the financial accelerator there are additional mechanisms at work. To further support the model we develop, we document two important features of firm investment behavior in the same data set. First, young firms grow faster than old firms, i.e., young firms have unconditionally higher investment rates (Stylized Fact 2). Second, even in our sample of relatively large and public firms, investment is lumpy (Stylized Fact 3).

To the extent that age is correlated with popular proxies of financial frictions, as is documented by Cloyne et al. (2020), the issue of observational equivalence extends beyond the comparison of firms by age or size. However, one should not interpret our results as rejecting the financial accelerator mechanism. It is likely that both financial frictions and the non-financial mechanisms that we emphasize in this paper are responsible for the observed heterogeneity in the data. Our findings highlight that the challenges in identifying the financial accelerator mechanism remain an open issue.

Literature Review. Our paper contributes to the literature that aims to document the financial accelerator mechanism in the data. The first generation of the empirical literature on the relevance of financial constraints relied on reduced-form regressions. For example, Fazzari et al. (1988) interpret the correlation between firm-level investment rates and cash flows as evidence supporting the financial accelerator. However, this kind of reduced-form evidence is problematic for two reasons. First, there is a simultaneity problem (Sargent 1980, Shapiro et al. 1986, Garber and King 1983) as both the investment rate and the proxy for financial frictions are correlated with unobserved productivity shocks. Therefore, the reduced-form correlations may be driven by unobserved shocks. Second, there is an issue arising from model misspecification, as pointed out by Gomes (2001). Firms' investment decisions are solutions to their optimization problems, which depend on many states in a non-linear fashion. In consequence, linear reduced-form regressions might be subject to a model misspecification issue. Gomes (2001) shows that financial frictions are neither necessary nor sufficient to estimate a cash-flow effect from reduced-form regressions due to model misspecification. The new generation of this literature has moved from unconditional firm behavior to firm behavior conditional on macroeconomic shocks. Several recent papers compare the investment behavior of groups of constrained and unconstrained firms after monetary policy shocks. Ottonello and Winberry (2020) use leverage and distance to default to group firms, Jeenas (2018) uses liquidity, and Cloyne et al. (2020) use age and dividend-paying-status. Excess sensitivity among constrained firms is often taken as evidence supporting the financial accelerator. To the extent that the macroeconomic shocks are uncorrelated with the proxies of financial frictions, the simultaneity issue discussed above is resolved. We raise the concern about the observational equivalence issue for the new generation of the literature, which uses evidence conditional on macroeconomic shocks. More specifically, we argue that the heterogeneous sensitivity might be driven by non-financial factors.[4] In this sense, our paper relates to Crouzet and Mehrotra (2020), who argue that large firms are less cyclical than small firms because they are better diversified across industries, but not due to financial frictions.

Moreover, our paper contributes to the literature that emphasizes the extensive margin of firms' investment behavior or the relevance of non-convex adjustment costs. A long debate has focused on whether lumpy firm-level investment behavior mat-

[4]A long literature uses firm age as a proxy for financial constraints. Rauh (2006), Fee et al. (2009), Hadlock and Pierce (2010), and more recently Cloyne et al. (2020) argue that young firms are more likely financially constrained than old firms. There are appealing reasons to use firm age to proxy for financial constraints. Most importantly, firm age is reasonably exogenous. However, there are also critical voices. Farre-Mensa and Ljungqvist (2016) argue that supposedly financially constrained firms do not behave as if they were constrained. This includes the Hadlock and Pierce (2010) index of financial constraints, which uses firm age as an input. Moreover, Dinlersoz et al. (2018) argue that only private firms, but not public ones (covered in Compustat) appear financially constrained.

ters for the behavior of aggregate investment and its responsiveness to shocks over the business cycle. Important contributions include Caballero et al. (1995), Caballero and Engel (1999), Thomas (2002), Khan and Thomas (2003), Khan and Thomas (2008), Bachmann et al. (2013), House (2014), Koby and Wolf (2020), Winberry (2021). Monetary policy shocks in models with non-convex adjustment costs have also been analyzed in Reiter et al. (2013), Reiter et al. (2020), and Fang (2020). We contribute to this literature by building a heterogeneous firm model that combines non-convex adjustment costs, firm life-cycle dynamics, and a New Keynesian sticky-price setup. Moreover, we consider the heterogeneous interest rate sensitivity of firms' investment along the life cycle. The existing literature finds that heterogeneous sensitivity to macroeconomic shocks across *aggregate states* of the economy is due to a more sensitive hazard rate in booms than in recessions. In contrast, we find that the heterogeneous sensitivity *across firms* arises at the extensive margin even if holding the responses of the hazard rate homogenous.

The remainder of this paper is organized as follows. Section 2 presents our stylized empirical facts. Section 3 outlines the simple model and explains its key mechanisms. Section 4 presents the full New Keynesian heterogeneous firm model. Section 5 calibrates the model and analyzes the effects of a monetary policy shock. Section 6 concludes.

2 Empirical Motivation

We document three stylized facts, which motivate our subsequent model-based analyses. First, we show that young (small) firms are more sensitive to monetary policy shocks than their old (large) counterparts (Stylized Fact 1). In the context of the financial accelerator literature, these are rather established findings. Evidence that small firms are more sensitive dates back to Gertler and Gilchrist (1994). The finding that young firms are more sensitive is emphasized in Cloyne et al. (2020). As is well-known, age and size are strongly correlated. In this section, we focus on age and provide results by size in Appendix D.1.

While the higher sensitivity of young (or small) firms is typically related to financial frictions, we argue that young firms differ from old firms along other dimensions which affect their interest rate sensitivity. Therefore, we show secondly that there are important differences in the investment behavior of young and old firms. On the one hand, young firms unconditionally have higher investment rates (Stylized Fact 2). On the other hand, investment rates among young firms display more lumpiness. Nevertheless, there is clear evidence of lumpy investment among firms of all ages (Stylized

Fact 3).

2.1 Data Description

We use quarterly firm-level data from Compustat. Our sample begins with 1986Q1 and ends with 2018Q4. We exclude firms with incomplete or questionable information (e.g. negative reported sales) and those not suitable for our analysis (e.g. financial firms) from the sample. Details on the sample selection are relegated to Appendix C.1. Since information on firm age in Compustat is scarce, we merge age information from WorldScope and Jay Ritter's database, as explained in Appendix C.2.

Capital stocks reported in Compustat are *accounting* capital stocks and do not perfectly reflect *economic* capital stocks. On the one hand, accounting depreciation is driven by tax incentives and usually exceeds economic depreciation. On the other hand, accounting capital stocks are reported at historical prices, not current prices. With positive inflation, both issues make the economic capital stock exceed the accounting capital stock. To not understate the capital stock (and overstate investment rates), we use a Perpetual Inventory Method (PIM) to compute real economic capital stocks, building on Bachmann and Bayer (2014). Details of the procedure can be found in Appendix C.3. Our baseline measure of the investment rate is $i_{jt} = \frac{CAPX-SPPE}{INVDEF*L.k_{jt}}$, thus, real capital expenditures (CAPX) net of real sales of capital (SPPE) divided by the lagged real economic capital stock. We measure size as the log of total assets (AT). More details are given in Appendix C.4.

For parts of the subsequent analysis, we aggregate the firm-level micro data to quarterly group-specific cross-sectional investment rate distributions and moments thereof. Moments sensitive to outliers, such as the mean, are winsorized.[5] We use the monetary policy shocks implied by the Proxy SVAR in Gertler and Karadi (2015). We first update the time series used in the VAR and the high-frequency instruments. Then, we run the SVAR and compute the implied monetary policy shocks. Details are relegated to Appendix C.5.

2.2 Monetary Policy Shocks

To estimate the effects of monetary policy shocks, we run the following simple local projection (LP):

$$y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + QuarterDummy + e_{j,t+h} \tag{1}$$

[5] Quite importantly, winsorizing is done by group and quarter. This ensures that the process does not systematically bias our sample.

where ϵ_t^{MP} is the monetary policy shock. The shocks are scaled to reduce the 1-year Treasury rate by 25 basis points. Throughout, we use Newey-West standard errors to account for heteroskedasticity and autocorrelation. We first report results using aggregate data before turning to firm-level data.

Aggregate Effects. Using time series data from FRED, we document the aggregate effects of the monetary policy shocks we utilize. Qualitatively, these are quite similar to Gertler and Karadi (2015). Panel (a) of Figure 1 shows that a monetary policy shock decreases the 1-year Treasury rate (FRED: GS1) for roughly 4 quarters. Thereafter, it overshoots, as observed in Gertler and Karadi (2015). Panels (b) and (c) show that (real) investment (FRED: PNFI) and the relative price of capital goods (FRED: PIRIC) increase strongly. The peak effect on investment is roughly 1.4%. As we will show in the model, the endogenous response of the relative price of capital generates a heterogeneous effect on young and old firms. Panel (d) shows that real GDP (FRED: GDPC1) also increases following an expansionary shock. The peak effect is about 0.35%.

Firm-Level Effects. To investigate the heterogeneous effects of monetary policy shocks, we work with the quarterly cross-sectional distribution of investment rates. In contrast to working with the firm-level data directly, this evades difficulties stemming from lumpy (i.e. nonlinear) firm-level investment behavior. We focus on young vs. old firms and relegate results for small vs. large firms to Appendix D.1.

Figure 2a shows that the average investment rate among all firms increases significantly following an expansionary monetary policy shock. Reassuringly, the trajectory looks very similar to the trajectory of aggregate investment. Figure 2b shows that young firms on average increase their investment rates by much more than old firms. The increase in average investment rates is significant for young firms at most horizons, while it is insignificant for old firms at most horizons. This finding was first documented by Cloyne et al. (2020) and is throughout this paper referred to as Stylized Fact 1. Quantitatively, the monetary shock increases the average investment rate at the peak by 0.35 percentage points. As a point of reference, note that the average (quarterly) investment rate is about 6.23%.

The bottom panels of Figure 2 show that these differences are to a sizeable extent driven by the extensive margin. Panel (2c) shows that there is a relatively larger increase in investment spikes among young firms. Panel (2d) shows that in addition, there is a relatively larger decrease in the inaction rate.

(a) 1-Year Treasury Rate **(b)** Relative Price of Investment Goods

(c) Investment **(d)** GDP

Figure 1: Aggregate Effects of a Monetary Policy Shock

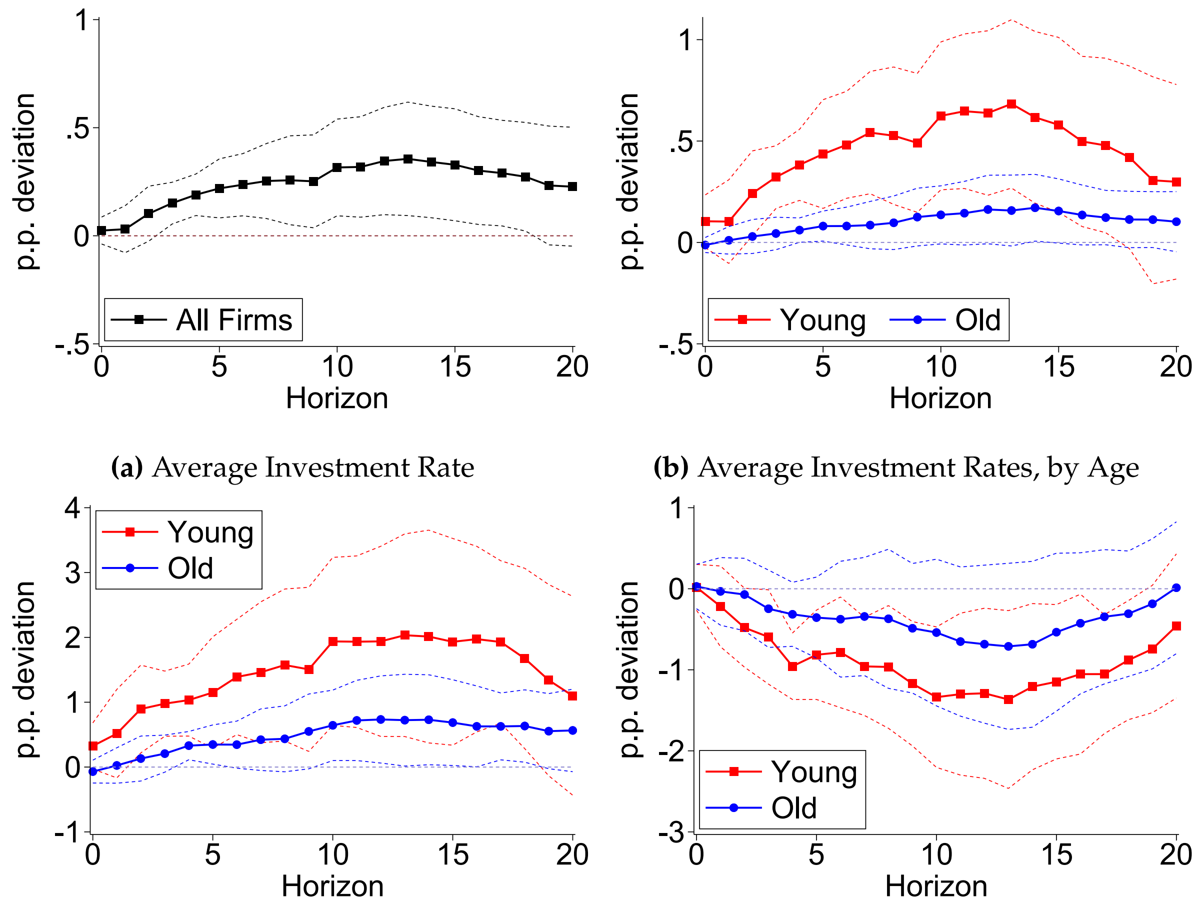
Notes: Dashed lines indicate 90 % confidence intervals. All variables except for the 1-year Treasury rate are in logs.

2.3 Life-Cycle Investment Behavior

We examine how investment behavior changes over a firm's life-cycle. Figure 3a displays the mean and four quantiles of the investment rate distribution by age in our sample. We emphasize two key features that emerge.

First, the *average* investment rate is higher among young firms (Stylized Fact 2). More precisely, the average investment rate is highest among new (age 0) firms, falls almost monotonically in age, and levels off roughly after age 20 at around 5%. Comparing young (less than 15 years old) and old (larger equal 15 years old) firms, we see that young firms' average investment rate is almost twice as high (8.7% vs. 4.9%).

Second, investment behavior among firms of all ages is *lumpy* (Stylized Fact 3). That is, there is a sizeable amount of large investment rates (investment spikes), but also many near zero investment rates (inaction periods). Figure 3b shows that this is



(a) Average Investment Rate

(b) Average Investment Rates, by Age

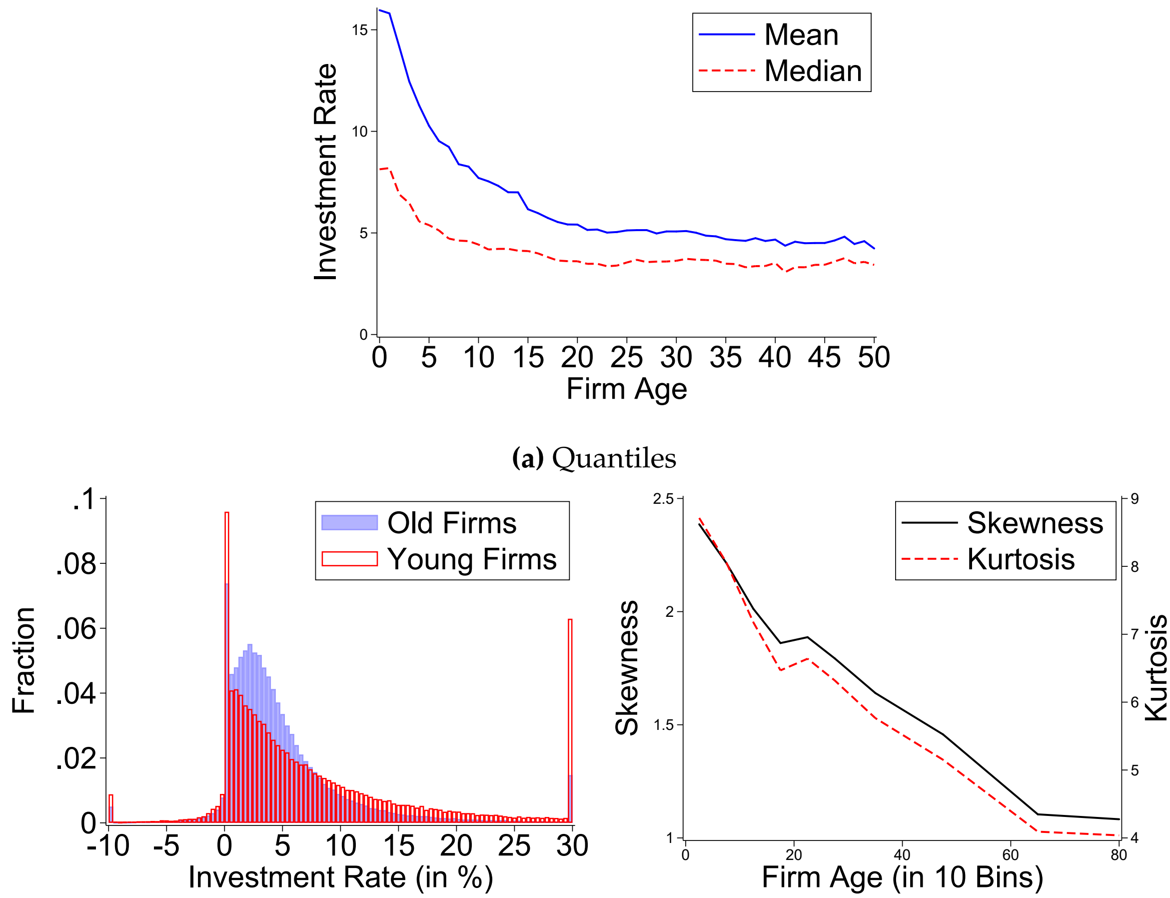
(c) Spike Rate, by Age

(d) Inaction Rate, by Age

Figure 2: Effect of a Monetary Policy Shock on Group-Specific Average Investment Rates, Spike, and Inaction Rates

Notes: Young (old) firms are firms less (more) than 15 years old. Dashed lines indicate 90 % confidence intervals.

the case among young and among old firms. Nevertheless, investment appears to be more lumpy among young firms. Figure 3c displays the skewness and kurtosis of the investment rate distribution, two statistics which are associated with lumpy investment, by age. It confirms that investment is lumpy, even among old and established firms, as there is positive skewness and excess kurtosis throughout.



(a) Quantiles

(b) Distribution of Investment Rate

(c) Skewness & Kurtosis

Figure 3: Investment Rate Distribution and Moments thereof by Age

3 A Simple Model

In Section (4), we build a heterogeneous firms life-cycle model with nominal rigidity and capital adjustment costs. The purpose is to explain the observed interest rate sensitivity of investment differences between young and old (or small and big) firms *without* introducing a financial accelerator mechanism. In the current section, we illustrate the mechanisms at work through the lens of a simple two-period model. In the simple model we focus on size. Since age and size are strongly correlated both in the data and in the full model, all the intuitions we provide in the simple model hold true when comparing young and old firms in the full model. Section (3.1) illustrates the heterogeneity through the intensive margin. Section (3.2) introduces fixed adjustment cost, which brings about the extensive margin.

3.1 The Intensive Margin

The model consists of two periods. In period one, firms are endowed with k_0 units of capital and choose next period's capital k_1. The price of one unit of capital relative to the price of the consumption goods is q. Moreover, there is a convex capital adjustment cost (CAC) $\frac{\phi}{2}\frac{(k_1-k_0)^2}{k_0}$.[6] In period two, firms transform capital into consumption goods (y) using a decreasing returns to scale production technology $y = k_1^\theta$ with $\theta < 1$. Future sales are discounted at the real interest rate r. The resulting firms' optimization problem is

$$\max_{k_1} \frac{1}{1+r}k_1^\theta - q(k_1 - k_0) - \frac{\phi}{2}\frac{(k_1-k_0)^2}{k_0}. \tag{2}$$

The first order condition for k_1 reads:

$$q + \phi\left(\frac{k_1}{k_0} - 1\right) = \frac{1}{1+r}\theta k_1^{\theta-1}. \tag{3}$$

This optimality condition can be reformulated in terms of the investment rate $i = \frac{k_1}{k_0}$:

$$\underbrace{q + \phi(i-1)}_{Marginal\ Cost\ (MC)} = \underbrace{\frac{1}{1+r}\theta(ik_0)^{\theta-1}}_{Marginal\ Benefit\ (MB)}. \tag{4}$$

which implicitly defines the optimal investment rate as a function of the size of a firm $i(k_0)$.

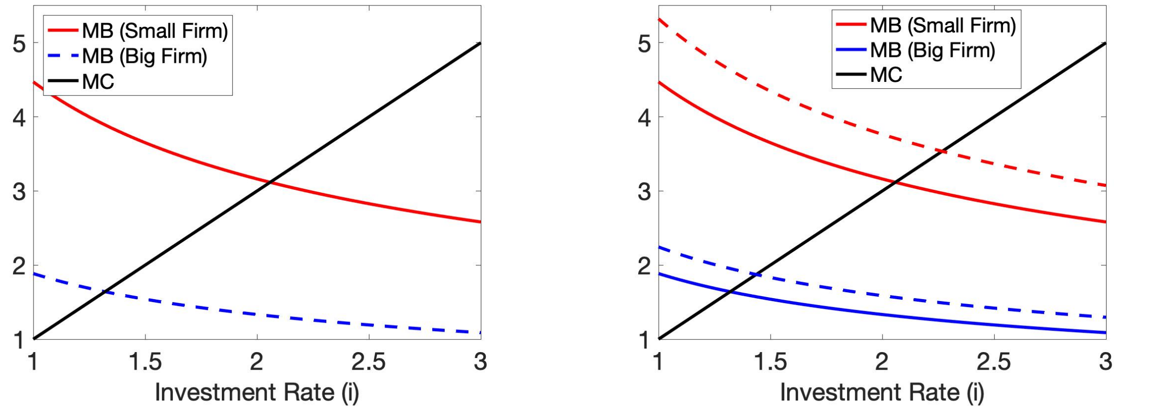
Proposition 1. *Consider a simple two-period partial equilibrium model populated by firms whose* $i \equiv \frac{k_1^*}{k_0} > 1$. *With the decreasing returns to scale* ($\theta < 1$)*, the following properties about interest rate sensitivity of investment hold:* $i'(r) < 0$ *and* $\frac{\partial i'(r)}{\partial k_0} > 0$.

Proof. See Appendix B □

Figure (4) illustrates Proposition (1) visually. Figure (4a) shows that the marginal benefit of investment is higher for small firms due to the decreasing returns to scale. As a result, small firms choose a higher investment rate. This result rationalizes why small/young firms have higher investment rates (stylized Fact 2). Figure (4b) shows that a change in r shifts the MB curves proportional to k_0, making small firms increase investment rates more than big firms: consistent with stylized Fact 1 in the data.

The mechanism that generates heterogeneous r-sensitivity differs from the classical financial accelerator. The latter relies on the heterogeneous effects of an interest

[6] This convex capital adjustment cost makes the marginal cost curve upward-sloping. Cooper and Haltiwanger (2006) show that such an adjustment cost is necessary to match the micro-data. The results that we drive in the section holds true in the absence of CAC ($\phi = 0$).



(a) The Baseline **(b) Interest Rate Sensitivity of Investment**

Figure 4: Panel (a) plots the MC curve $q + \phi(i-1)$ together the MB curve $\frac{1}{1+r}\theta(ik_0)^{\theta-1}$ for two firms with different sizes (low k_0 and high k_0). Panel (b) considers a shock to the interest rate.

rate change on the marginal cost curve; see e.g., Ottonello and Winberry (2020) for a graphical illustration.

3.2 Introducing the Fixed Adjustment Cost

We now introduce the following features to the model discussed above. First, there is a continuous mass of firms within each size category k_0. Second, the production function is $y(j) = z(j)k(j)^\theta$, where the firm-level productivity $z(i)$ is drawn from a random distribution with mean equals to one. Third, capital adjustments are subject to a fixed adjustment cost ξ. Fourth, the manager of a firm drafts an investment proposal before the realization of the idiosyncratic productivity $z(i)$. The firm's CEO decides whether to implement the investment project after the realization of $z(i)$. At the investment proposal drafting stage, the manager decides the optimal amount of capital to acquire (if the proposal is approved) based on the unconditional expected value of $z(i)$ that is equal to one. The last assumption ensures that the investment decision conditional on adjusting is the same as the one we solved above. We will relax this simplifying assumption in the quantitative general equilibrium model.

The manager's problem is the same as the problem described in (2):

$$\max_{k_1(j)} \frac{1}{1+r}k_1(j)^\theta - q(k_1 - k_0(j)) - \frac{\phi}{2}\frac{(k_1(j) - k_0(j))^2}{k_0(j)} - \xi, \tag{5}$$

resulting the same optimality condition (3).

Let VA denote the value added of adjusting capital in the absence of fixed adjustment cost:

$$VA = \frac{1}{1+r} z(j) k_1(j)^{*\theta} - q(k_1^* - k_0(j)) - \frac{\phi}{2} \frac{(k_1^*(j) - k_0(j))^2}{k_0(j)} - \frac{1}{1+r} z(j) k_0(j)^{\theta}, \tag{6}$$

where k_1^* is the optimal amount of capital that the manager wants to achieve, which satisfies the condition (3). Let $V_n(z(j), k_0(j)) = \frac{1}{1+r} z(j) k_0(j)^{\theta}$ denote the value of not adjusting capital. The optimization problem of the CEO is:

$$\max\{VA, \xi\}. \tag{7}$$

Solving this maximization problem leads a cutoff value $\bar{z}$, such that for a firm j, the CEO will decide to adjust capital if and only if $z(j) > \bar{z}$. This cutoff value depends on k_0, r, q, ξ among other parameters of the model.

The average investment rate of the category of firms ($\bar{i}$) with size k_0 is:

$$\bar{i}(k_0) = \lambda(k_0) \times i(k_0) \tag{8}$$

where $\lambda \in [0,1]$ denotes the fraction of firms chooses to invest — the hazard rate. Conditional on investing the firm chooses the level of investment rate i that satisfies condition (4).

The group-specific interest rate sensitivity of investment is:

$$\bar{i}'(r) = \underbrace{\lambda'(r) i}_{\text{Extensive Margin}} + \underbrace{\lambda i'(r)}_{\text{Intensive Margin}}. \tag{9}$$

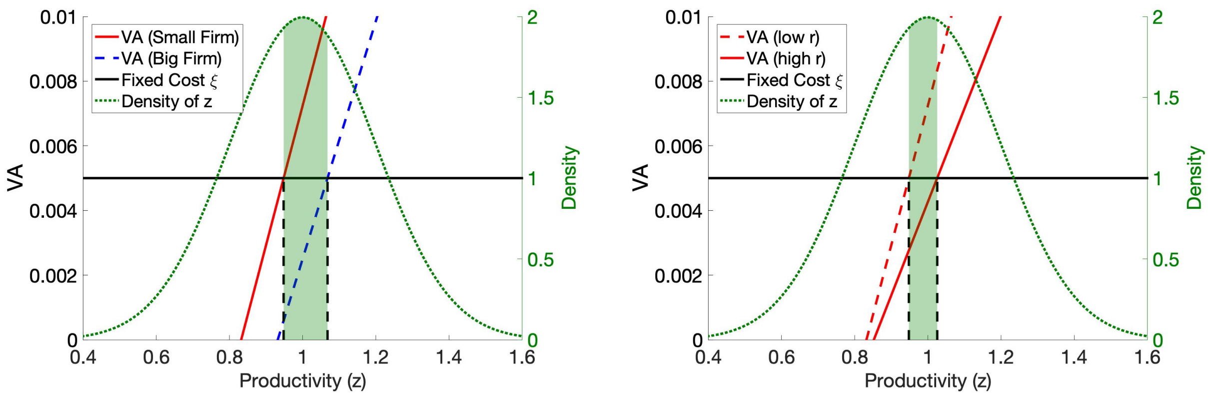
We are ultimately interested in the heterogeneous sensitivities across different groups of firms:

$$\frac{\partial \bar{i}'(r)}{\partial k_0} = \frac{\partial \lambda'(r)}{\partial k_0} i + \lambda'(r) i'(k_0) + \lambda'(k_0) i'(r) + \lambda \frac{\partial i'(r)}{\partial k_0} \tag{10}$$

In the previous subsection, we have discussed $i'(k_0), i'(r)$, and $\frac{\partial i'(r)}{\partial k_0}$. We now discuss how the hazard rate depends on r, k_0 and q. Similar to before, we restrict the analysis to the parameter region where firms always decide to invest in the absence of fixed adjustment.

Proposition 2. *In the absence of the convex adjustment cost ($\phi = 0$), consider an economy populated by firms whose $i \equiv \frac{k_1^*}{k_0} > 1$, i.e., $k_0 < \left(\frac{\theta}{(1+r)q}\right)^{\frac{1}{1-\theta}}$, then the following properties about the sensitivity of the hazard rate holds: $\lambda'(k_0) < 0$ and $\lambda'(r) < 0$.*

Proof. See Appendix B □



(a) Small v.s. Big Firms' Adj. Hazard **(b) r-Sensitivity of Adj. Hazard**

Figure 5: This figure plots the VA of a firm against its productivity z. The black horizontal line indicates the fixed adjustment cost ξ. The intercept of the two curves pins down the threshold value of z. The green dotted line plots the density function of z (normal distribution). The area under the density function to the right of the threshold value of z is the adjustment hazard. The shaded area in Panel (a) plots the difference in adjustment hazard between a small and a big firm. The shaded area in Panel (b) plots the difference in adjustment hazard after an interest rate shock.

Figure (5) illustrates Proposition (2) graphically. In the Figure, we plot the VA of a firm against its productivity z. The black horizontal line indicates the fixed adjustment cost ξ.The intercept of the VA and the fixed adjustment cost curves pins down the threshold value of z. The green dotted line plots the density function of z (normal distribution). The area under the density function to the right of the threshold value of z is the adjustment hazard. The shaded area in Figure (5a) plots the difference in adjustment hazard between a small and a big firm. Small firms have higher hazard rates ($\lambda'(k_0) < 0$): other things equal, a small firm's value-added of adjusting capital (VA) is higher than a big firm's VA. Figure (5b) shows that a lower interest rate increases the hazard rate ($\lambda'(r) < 0$): the VA increases after a reduction in the interest rate, i.e., the VA curve shifts to the upper left corner.

We are now ready to summarize the source of the heterogeneous interest rate sensitivity of firms' investments. The following Corollary summarizes the results that we derived analytically.

***Corollary* 1.** *The following properties about the interest rate sensitivity of investment hold:* $i'(r) < 0$, $\lambda'(r)i'(k_0) > 0$, $\lambda'(k_0)i'(r) > 0$, *and* $\lambda\frac{\partial i'(r)}{\partial k_0} > 0$.

Proof. It follows directly from Proposition (1) and Proposition (2). □

The following expression, which indicates the difference in r-sensitivity between small and big firms ($\bar{i}'(r)_S - \bar{i}'(r)_L$) implied by equation (10), is useful to understand Corollary (1).

$$\bar{i}'(r)_S - \bar{i}'(r)_L = \underbrace{(\lambda_S - \lambda_L) i'(r) + \lambda \left(i'(r)_S - i'(r)_L \right)}_{\text{Intensive Margin}} + \underbrace{\lambda'(r)(i_S - i_L) + (\lambda'(r)_S - \lambda'(r)_L) i}_{\text{Extensive Margin}} \tag{11}$$

Consider the interest rate sensitivity of investment, and consider a monetary easing ($\triangle r < 0$) that has an expansionary effect on firms' investments ($i'(r)\triangle r > 0$). This effect is heterogeneous across firms of different sizes. In particular, small firms' average r-sensitivity can be higher for three reasons. First, a bigger fraction of small firms ($\lambda_S - \lambda_L > 0$) is affected/treated by monetary shocks—the *level effect at the intensive margin*. Second, among the adjusters, small firms are more affected ($|i'(r)_S - i'(r)_L| > 0$) — the *investment rate increase channel*.

Third, among the *new* adjusters, small firms' investment rates are higher ($i_S - i_L > 0$)—the *level effect at the extensive margin*. Note that the third effect arises due to fixed adjustment costs. In the absence of the non-convex adjustment costs, the hazard rate is constant and equals to one. In contrast, the hazard rate is endogenous in the presence of fixed adjustment costs, which gives rise to the additional mechanisms leading to the heterogeneous effects of interest rate changes.

The sign of the last expression of the equation (11), which we label as the *hazard rate increase channel*, cannot be shown analytically. We solve this term numerically, and the results are reported in Figure (6a). In response to an interest rate cut, the change in the hazard rate among small firms is smaller than that of big firms because the same interest rate cut shifts the big firm's VA curve more.

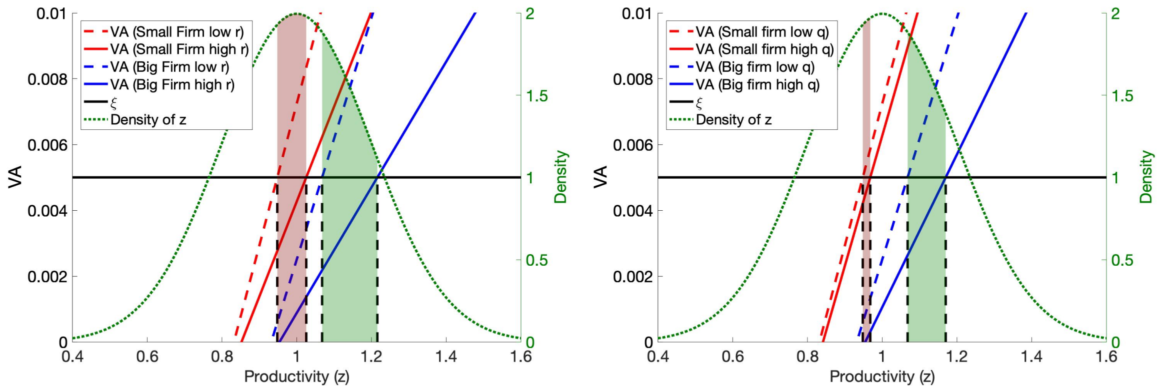
Now consider the heterogeneity in the capital price (q) sensitivity of investment:

$$\frac{\partial \bar{i}'(q)}{\partial k_0} = \frac{\partial \lambda'(q)}{\partial k_0} i + \lambda'(q) i'(k_0) + \lambda'(k_0) i'(q) + \lambda \frac{\partial i'(q)}{\partial k_0} \tag{12}$$

Similar to the discussion conditional on interest rate shocks, the average investment among small firms might be bigger due the last three terms in equation (12): on average, small firms have a higher investment rate ($i'(k_0) < 0$), a bigger hazard rate ($\lambda'(k_0) < 0$), and small firms more sensitive to changes in capital price ($\frac{\partial i'(q)}{\partial k_0} > 0$) under the assumption that $\phi = 0$[7]. Moreover, the first term is quantitatively small for the same reason we discussed above conditional on interest rate shocks.

In Appendix A, we consider the case when the fixed adjustment cost is drawn

[7]With $\phi > 0$, $\frac{\partial i'(q)}{\partial k_0}$ might be smaller than 0 depending on q and ϕ that we discussed in proposition (1).



(a) r-Sensitivity of Adjustment Hazard **(b) q-Sensitivity of Adjustment Hazard**

Figure 6: This figure plots the VA of a firm against productivity z. The black horizontal line indicates the fixed adjustment cost ξ.The intercept of the VA and the fixed adjustment cost curves pins down the threshold value of z.. The green dotted line plots the density function of z (normal distribution). The area under the density function to the right of the threshold value of z is the adjustment hazard. The shaded area in Panel (a) plots the difference in r-sensitivity of adjustment hazard between a small and a big firm. The shaded area in Panel (b) plots the difference in q-sensitivity of adjustment hazard between a small and a big firm.

from a distribution. This assumption is to mimic the fact that investments are subject to different levels of fixed adjustment cost in the real world. The results we derived in Proposition (2) hold true.

In sum, we show analytically that three channels might give rise to the heterogeneous interest rate sensitivity we observe in the data (Stylized Fact 1). Whether our model is consistent with the data is a quantitative question that we will address in Section (4) through the lens of a general equilibrium model.

4 Model

We build a New Keynesian model with heterogeneous firms subject to capital adjustment costs and endogenous entry and exit. These features have been studied separately; see, e.g., Khan and Thomas (2008), Clementi and Palazzo (2016), and Ottonello and Winberry (2020). The novelty of our model is to combine all these ingredients that are relevant for the understanding of the heterogeneous sensitivity of firms' investments.

4.1 Investment Block

Production Firms There exists a continuum of production firms[8] in the economy. Each firm j produces a quantity y_{jt} of the intermediate good using the production function

$$y_{jt} = z_{jt} k_{jt}^{\theta} n_{jt}^{\nu} \qquad \text{with } \theta, \nu > 0 \text{ and } \theta + \nu < 1 \tag{13}$$

where z_{jt} is total factor productivity (TFP), k_{jt} is the capital stock, and n_{jt} is labor input. Productivity z_{jt} is subject to idiosyncratic shocks and follows an AR(1) process in logs

$$\log z_{jt} = \rho_z \log z_{jt-1} + \sigma_z \epsilon_{jt}^{z} \qquad \text{with } \epsilon_{jt}^{z} \sim \mathcal{N}(0,1) \tag{14}$$

Labor n_{jt} can be adjusted frictionlessly in every period. Capital k_{jt} is accumulated according to

$$k_{jt+1} = (1-\delta) k_{jt} + i_{jt} \tag{15}$$

where i_{jt} is investment and δ the depreciation rate. The relative price of capital is q_t. There are capital adjustment costs, which need to be paid if $i_{jt} \neq 0$.[9] Total adjustment costs consist of a random fixed adjustment cost $w_t \xi_{jt}$, where ξ_{jt} is distributed uniformly between 0 and $\bar{\xi}$, and a convex adjustment cost $\frac{\phi}{2} \frac{i_{jt}^2}{k_{jt}}$:

$$AC(k_{jt}, k_{jt+1}, \xi_{jt}) = w_t \xi_{jt} \mathbb{1}\{k_{jt+1} \neq (1-\delta) k_{jt}\} + \frac{\phi}{2} \frac{(k_{jt+1} - (1-\delta) k_{jt})^2}{k_{jt}} \tag{16}$$

where w_t is the real wage.

Entry & Exit There are voluntary (endogenous) and involuntary (exogenous) firms exits. Firms face i.i.d. exit shocks ϵ_{jt}^{exit} and are forced to exit the economy at the end of the period with probability π^{exit}. Firms which are allowed to continue into next period may nevertheless choose to exit voluntarily. Thereby, they avoid paying the fixed operating cost c_f. The voluntary exit decision is discussed in more detail in 4.1.

Each period, a fixed mass of newborn firms enters the economy. These entrants are endowed with k_0 units of capital and draw their initial productivity level from the distribution $\mu^{ent} \sim \mathcal{N}(-m \frac{\sigma_z}{\sqrt{1-\rho_z^2}}, \frac{\sigma_z^2}{1-\rho_z^2})$. This distribution is the ergodic distribution of (14), shifted to the left by m standard deviations. This ensures that firms are born with

[8] We normalize the mass of firms to 1 in steady state. Following aggregate shocks, the mass of firms can deviate from 1 due to endogenous exit. While our model also features retailers, a final good producer, and a capital good producer, we only refer to intermediate good producers as firms.

[9] Matching the empirical distribution of investment rates requires a rich adjustment cost specification, as discussed in Cooper and Haltiwanger (2006).

low productivity, which grows over time to the long-run average.[10]

Timing Within any period, the timing is as follows. At stage one, idiosyncratic TFP shocks to incumbent firms realize. At stage two, a fixed mass firms enters the economy. Entrants draw their initial productivity from μ^{ent} and are endowed with k_0 units of capital from the household. Henceforth, they are indistinguishable from incumbent firms. At stage three, firms hire labour and production takes place. At stage four, exit shocks realize and random fixed adjustment costs are drawn. Firms which are allowed to continue decide whether to exit voluntarily or pay the operating cost. At the last stage, all exiting firms sell their capital stock and leave the economy. Continuing firms decide whether to adjust their capital stock or remain inactive.

Value Functions The beginning-of-period firm value function is

$$\begin{aligned} V_0(z,k;\cdot) = \max_n \; pzk^\theta n^\nu - wn + \pi^{exit} E_\xi \left[CV^{Exit}(z,k,\xi;\cdot) \right] \\ + (1-\pi^{exit}) E_\xi \left[CV(z,k,\xi;\cdot) \right] \end{aligned} \tag{17}$$

where

$$CV^{Exit}(z,k,\xi;\cdot) = (1-\delta)qk - w\xi \tag{18}$$

$$CV(z,k,\xi;\cdot) = \max \left\{ CV^{Exit}(z,k,\xi;\cdot), CV_a(z,k,\xi;\cdot), CV_n(z,k;\cdot) \right\} \tag{19}$$

and

$$CV_n(z,k;\cdot) = E_{z'} \left[\Lambda V_0(z',(1-\delta)k;\cdot) \right] - c_f \tag{20}$$

$$CV_a(z,k,\xi;\cdot) = E_{z'} \left[\Lambda V_0(z',k';\cdot) \right] - q\left(k' - (1-\delta)k\right) - AC(k,k',\xi) - c_f \tag{21}$$

Time and firm subscripts are dropped for readability and primes denote next period's values. Note that exiting firms do need to pay the random fixed, but not the convex capital adjustment cost.[11]

[10] Foster et al. (2016) document that young firms have low levels of measured productivity and discuss potential explanations.

[11] We do not make exiting firms pay convex adjustment costs, because this would make exiting extremely expensive in our current calibration. In contrast, we do make exiting firms pay random fixed adjustment costs. This adds randomness to the exit decision, similar to the random operating cost in Clementi and Palazzo (2016).

Policy Functions The labor decision in (17) is static and independent of the exit and capital decision

$$n^*(z,k;\cdot) = \left(\frac{p\nu z k^\theta}{w}\right)^{\frac{1}{1-\nu}} \tag{22}$$

Thus, earnings net of labor costs are

$$\pi(z,k;\cdot) \equiv pzk^\theta (n^*)^\nu - wn^* \tag{23}$$

The optimal exit and capital decisions are computed as follows. First of all, the solution to (21) is the policy function $k_a^*(z,k;\cdot)$, which is independent of ξ. Thus, we can compute $CV_a(z,k,\xi;\cdot)$. Now note that both $CV_a(z,k,\xi;\cdot)$ and $CV^{Exit}(z,k,\xi;\cdot)$ depend on ξ linearly $(-w\xi)$. It follows that a firm with productivity z and capital k will either under no ξ draw adjust $(CV_a(z,k,\xi;\cdot) < CV^{Exit}(z,k,\xi;\cdot)\ \forall\ \xi)$ or under no ξ draw exit voluntarily $(CV_a(z,k,\xi;\cdot) > CV^{Exit}(z,k,\xi;\cdot)\ \forall\ \xi)$. This splits the firm state space in two parts. When $\pi_{vol}^{exit} = 1$, firms potentially exit voluntarily, depending on ξ. When $\pi_{vol}^{exit} = 0$, firms do not exit voluntarily, irrespective of ξ.

$$\pi_{vol}^{exit}(z,k;\cdot) = \begin{cases} 1 & \text{if } CV_a(z,k;\cdot) < CV^{Exit}(z,k;\cdot) \\ 0 & \text{if } CV_a(z,k;\cdot) > CV^{Exit}(z,k;\cdot) \end{cases} \tag{24}$$

In both cases, the actual solution to (19) still depends on the fixed cost ξ and we can formulate a cutoff rule. Firms with $\pi_{vol}^{exit} = 0$ adjust capital if and only if their fixed adjustment cost draw ξ is smaller or equal $\xi^T(z,k;\cdot)$:

$$k^*(z,k,\xi;\cdot) = \begin{cases} k_a^*(z,k,\xi;\cdot) & \text{if } \xi \leq \xi^T(z,k;\cdot) \\ (1-\delta)k & \text{if } \xi > \xi^T(z,k;\cdot) \end{cases} \tag{25}$$

where

$$\xi^T(z,k;\cdot) = \frac{CV_a(z,k,\xi=0;\cdot) - CV_n(z,k;\cdot)}{w} \tag{26}$$

Firms with $\pi_{vol}^{exit} = 1$ exit if and only if their fixed adjustment cost draw ξ is smaller or equal $\xi_{Exit}^T(z,k;\cdot)$, so

$$k^*(z,k,\xi;\cdot) = \begin{cases} 0 & \text{if } \xi \leq \xi_{Exit}^T(z,k;\cdot) \\ (1-\delta)k & \text{if } \xi > \xi_{Exit}^T(z,k;\cdot) \end{cases} \tag{27}$$

where

$$\xi_{Exit}^T(z,k;\cdot) = \frac{CV^{Exit}(z,k,\xi=0;\cdot) - CV_n(z,k;\cdot)}{w} \tag{28}$$

For future reference, we define the adjustment hazard rate $h(z,k)$ and the total exit rate $\pi^{exit}_{tot}(z,k;\cdot)$ as:

$$h(z,k;\cdot) = (1-\pi^{exit}_{vol}(z,k;\cdot))\frac{\xi^T(z,k;\cdot)}{\bar{\xi}} \tag{29}$$

$$\pi^{exit}_{tot}(z,k;\cdot) = \min\Big\{\pi^{exit} + \pi^{exit}_{vol}(z,k;\cdot)\frac{\xi^T_{Exit}(z,k;\cdot)}{\bar{\xi}},1\Big\} \tag{30}$$

4.2 New Keynesian Block

We separate nominal rigidities from the investment block of the model. A fixed mass of retailers $i \in [0,1]$ produces differentiated varieties $\widetilde{y}_{it}$ from the undifferentiated intermediate goods produced by the production firms. There is a one-to-one production technology $\widetilde{y}_{it} = y_{it}$, where y_{it} is the amount of the intermediate good retailer i purchases. Retailers face Rotemberg quadratic price adjustment costs $\frac{\varphi}{2}\left(\frac{\widetilde{p}_{it}}{\widetilde{p}_{it-1}}-1\right)^2 Y_t$, where $\widetilde{p}_{it}$ is the relative price of variety i.

A representative final good producer aggregates the differentiated varieties optimally into the final good according to

$$Y_t = \left(\int \widetilde{y}_{it}^{\frac{\gamma-1}{\gamma}}\, di\right)^{\frac{\gamma}{\gamma-1}} \tag{31}$$

The resulting demand function for retail good $\widetilde{y}_{it}$ is:

$$\widetilde{y}_{it} = \left(\frac{\widetilde{p}_{it}}{P_t}\right)^{-\gamma} Y_t, \tag{32}$$

where $P_t = \left(\int_0^1 \widetilde{p}_{it}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$ is the price of the final good.

The optimization problem of a monopolistically competitive retailer i is:

$$\max_{\{\widetilde{p}_{it}\}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\Lambda_t\left\{(\widetilde{p}_{it}-p_t)\widetilde{y}_{it} - \frac{\varphi}{2}\left(\frac{\widetilde{p}_{it}}{\widetilde{p}_{it-1}}-1\right)^2 Y_t\right\}\right] \tag{33}$$

subject to the demand curve (32). We log-linearize the optimality condition of the retailer's problem to obtain the familiar New Keynesian Phillips Curve (NKPC):

$$\log(1+\pi_t) = \frac{\gamma-1}{\varphi}\log\frac{p_t}{p^*} + \beta\,\mathbb{E}_t \log(1+\pi_{t+1}) \tag{34}$$

where $\pi_t \equiv P_t/P_{t-1} - 1$ is the inflation rate, $p^* = \frac{\gamma-1}{\gamma}$ is the relative price (in terms of the final good) of the intermediate good in steady state.

4.3 Capital Good Producer

There is a representative capital good producer operating in a perfectly competitive market. It transforms units of the final good (I_t^Q) into new capital (I_t) subject to investment adjustment costs:

$$I_t = I_t^Q \left[1 - \frac{\kappa}{2} \left(\frac{I_t^Q}{I_{t-1}^Q} - 1 \right)^2 \right], \tag{35}$$

where I_t^Q represents the amount of the final good used and I_t the amount of new capital produced. The capital good producer's optimization problem is:

$$\max_{\left\{ I_t^Q \right\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \Lambda_t \left\{ q_t I_t - I_t^Q \right\} \right] \quad s.t. \ (35), \tag{36}$$

where Λ_t is stochastic discount factor to be specified below in the household optimality condition. (We assume that the capital good producer is owned by the household.) New capital is sold to production firms at the relative price q_t. The presence of investment adjustment costs generates time-variation in q_t. The solution of the capital good producer's optimization problem implicitly defines q_t:

$$\begin{aligned} \Lambda_t = \Lambda_t q_t & \left[1 - \frac{\kappa}{2} \left(\frac{I_t^Q}{I_{t-1}^Q} - 1 \right)^2 - \kappa \left(\frac{I_t^Q}{I_{t-1}^Q} - 1 \right) \frac{I_t^Q}{I_{t-1}^Q} \right] \\ & + \mathbb{E}_t \left[\Lambda_{t+1} q_{t+1} \kappa \left(\frac{I_{t+1}^Q}{I_t^Q} - 1 \right) \left(\frac{I_{t+1}^Q}{I_t^Q} \right)^2 \right]. \end{aligned} \tag{37}$$

4.4 The Central Bank

The central bank sets the nominal interest rate r_t^n according to a Taylor rule

$$\log(1 + r_t^n) = \rho_r \log(1 + r_{t-1}^n) + (1 - \rho_r) \left[\log \frac{1}{\beta} + \varphi_\pi \log(1 + \pi_t) \right] + \epsilon_t^m \tag{38}$$

where ϵ_t^m is a monetary policy shock, ρ_r is the interest rate smoothing parameter, and φ_π is the reaction coefficient to inflation.

4.5 Household

There is a representative household, which consumes C_t^h, supplies labor N_t^h, and saves or borrows in one-period non-contingent bonds B_t^h. Its objective is to maximize expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log(C_t^h) - \psi N_t^h \right) \tag{39}$$

subject to the flow budget constraint:

$$P_t C_t^h + Q_t^B B_t^h \leq B_{t-1}^h + w_t N_t^h + \Pi_t \tag{40}$$

where Q_t^B is the nominal one-period risk-free bond price (one unit of B_t pays one unit of currency at $t+1$), w_t is the nominal wage, and Π_t subsumes additional transfers to and from the household.[12] Optimization gives two Euler equations

$$\Lambda_t \equiv \beta \mathbb{E}_t \left[\frac{C_t^h}{C_{t+1}^h} \right] \tag{41}$$

$$w_t = \psi C_t^h \tag{42}$$

where Λ_t is the household's stochastic discount factor between period t and $t+1$, and w_t is the real wage.

4.6 Equilibrium Definition

A recursive competitive equilibrium in this model is a set of value functions $\{V_0(z,k;\cdot)$, $CV^{Exit}(z,k,\xi;\cdot)$, $CV_a(z,k,\xi;\cdot)$, $CV_n(z,k;\cdot)\}$, policy functions $\{n^*(n,z;\cdot)$, $k^*(z,k,\xi;\cdot)$, $\xi^T(z,k;\cdot)$, $\xi^T_{Exit}(z,k;\cdot)\}$, quantities $\{C, Y, I\}$, prices $\{p, w, \pi, \Lambda, q\}$, and a distribution μ such that all agents in the economy behave optimally, the distribution of firms is consistent with decision rules, and all markets clear. A more precise equilibrium definition is relegated to Appendix E.1.

[12] Π_t includes dividends from intermediate good producers, retailers, and the final good producer, as well as the initial capital endowment k_0, which entering firms receive from the household. We follow Winberry (2021) and do not rebate back adjustment costs to the household in a lump-sum manner. Therefore, convex adjustment costs do exhaust the aggregate resource constraint.

5 Model Analysis

5.1 Calibration

We fix a subset of parameters to conventional values. These parameters are listed in Table 1. Given these fixed parameters, we fit the remaining parameters to match the moments listed in Table 3. The fitted parameters are listed in Table 2.

Fixed Parameters Since a model period corresponds to a quarter, the discount factor β is set to 0.99. The labor disutility parameter ψ is set to 0.75.[13] Capital and labor coefficients are set to standard values, that is, $\theta = 0.21$ and $\nu = 0.64$. (Ottonello and Winberry (2020)) The depreciation rate δ generates an annual aggregate investment rate of 7.7% as reported in Zwick and Mahon (2017). We target the standard deviation of TFP shocks σ_z, but fix their persistence ρ_z due to the identification problem discussed in Clementi and Palazzo (2015). We set ρ_z to 0.95. (Khan and Thomas (2008); Bloom et al. (2018)) The exogenous exit probability π^{exit} is set to 0.011 to match the annual exit probability of the oldest firms of 4.4% in the Business Dynamics Statistics (BDS).

We choose standard values for the parameters of the New Keynesian block, i.e. $\varphi = 90$ and $\gamma = 10$. (Ottonello and Winberry (2020)) The coefficient on inflation in the Taylor rule φ_π is set to 1.69, the interest rate smoothing parameter ρ_r is set to 0.86. External investment adjustment costs κ are set to XXX to roughly match the peak response of investment relative to the peak response of output documented in Section 2.

Calibrated Parameters The parameters listed in Table 2 are chosen to match the targeted moments listed in Table 3. The fitted parameters align with those typically estimated in the literature. Only the convex adjustment cost parameter, $\phi = 8.978$, is quite high compared to 2.34 in Winberry (2021) or 2.40 in Fang (2020). These models do not feature a firm life-cycle and thus only look at established firms. Due to decreasing returns to scale, young firms have a strong incentive to grow immediately and would choose extremely large investment rates in the absence of convex adjustment costs. To bring young firms' investment rates down and average investment rates in line with the data, we therefore need large convex adjustment costs.

Table 3 compares a number of targeted and untargeted empirical moments with simulated moments from our model. Overall, the model matches the data very well,

[13] This value follows from normalizing the steady state real wage w to 1.

Table 1: Fixed Parameters

Parameter	Description	Value
Household		
β	Discount factor	0.99
ψ	Labor Disutility	0.75
Investment Block		
θ	Capital Coefficient	0.21
ν	Labor Coefficient	0.64
δ	Depreciation Rate	0.0192
ρ_z	Persistence of TFP Shock	0.95
π^{exit}	Exogenous Exit Probability	0.011
New Keynesian Block		
φ	Price Adjustment Cost	90
γ	Elasticity of Substitution over Intermediate Goods	10
φ_π	Taylor Rule Coefficient on Inflation	1.69
ρ_r	Interest Rate Smoothing	0.86
κ	Aggregate Investment Adjustment Costs	4

even though we target eight moments with six parameters. We slightly overestimate the mean, standard deviation, skewness, and autocorrelation of investment rates. Moreover, we slightly underestimate the share of age 0, age 1, and young firms and slightly overestimate the share of employment in age 0 firms.

Table A.1 in the Appendix illustrates in more detail which parameter ensures the fit of which moment.

Table 2: Fitted Parameters

Parameter	Description	Value
σ_z	Volatility of TFP Shock	0.092
k_0	Initial Capital of Entrants	1.458
$\bar{\xi}$	Upper Bound on Fixed Adjustment Cost	1.407
ϕ	Convex Adjustment Cost	8.978
$\bar{c_f}$	Upper Bound on Operating Cost	0.251
m	TFP Mean Shift of Entrants	0.450

5.2 Firm Life-Cycle Profiles

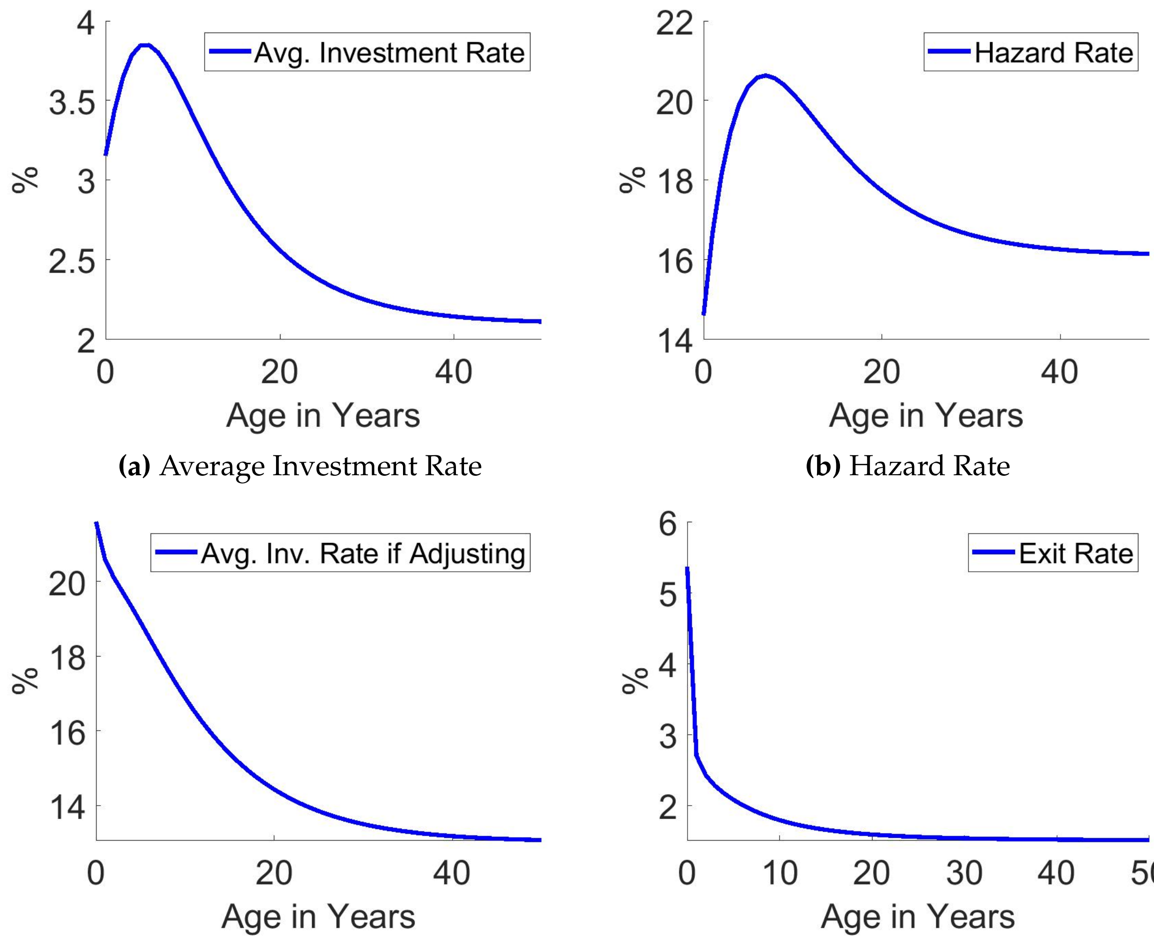
Figure 7 shows that our model is able to replicate several untargeted empirical patterns. Panel (a) shows that the average investment rate is on average higher for young firms and after the first few years falls in age. Panels (b) and (c) decompose this average investment rate into the average investment rate conditional on adjusting capital

Table 3: Empirical & Simulated Moments

Moment	**Data**	**Model**
Targeted		
Share of employment in age 0	0.023	0.026
Share of firms in age 0	0.092	0.078
Share of firms in age 1	0.071	0.067
Share of firms in ages 0-15	0.683	0.663
Average Investment Rate (%)	0.119	0.124
Standard Deviation of Investment Rates	0.200	0.218
Skewness of Investment Rates	3.230	3.335
Autocorrelation of Investment Rates	0.380	0.402
Untargeted		
Average Investment Rate (%)	0.104	0.117
Standard Deviation of Investment Rates	0.160	0.206
Spike Rate (%)	0.144	0.192
Positive Rate (%)	0.856	0.808
Inaction Rate (%)	0.237	0.538
Skewness of Investment Rates	3.600	3.338
Autocorrelation of Investment Rates	0.400	0.406
Share of employment in age 1-15	0.306	0.531
Share of employment in age 16+	0.671	0.469
Share of employment in age 0-15	0.329	0.557
Share of employment in age 0-5	0.135	0.193
Share of employment in age 6-15	0.194	0.338
Share of firms in age 16+	0.317	0.337
Share of firms in ages 0-5	0.377	0.355
Spike Rate (%)	0.174	0.204
Positive Rate (%)	0.826	0.796
Inaction Rate (%)	0.302	0.538

Notes: Data moments related to investment rates are taken from Zwick and Mahon (2017) (Appendix, Table B.1, Unbalanced Sample). Corresponding model moments are computed from a simulation of a large panel of firms. All other moments are computed from Business Dynamics Statistics (BDS). Corresponding model moments are computed from the steady state distribution.

and the average (ex ante) probability to adjust capital. Evidently, the observation that young firms have on average higher investment rates is driven in part by a higher hazard rate and in part by a higher investment rate conditional on investing. The hazard rate profile is hump-shaped because very young firms have high exit rates which discourages large investments. Panel (d) shows that young firms have a high, old firms a low probability to exit the economy. Since the exogenous exit shock is independent and identically distributed, this heterogeneity is driven by voluntary exits.



(a) Average Investment Rate

(b) Hazard Rate

(c) Avg. Inv. Rate (Conditional on Adjusting)

(d) Exit Rate

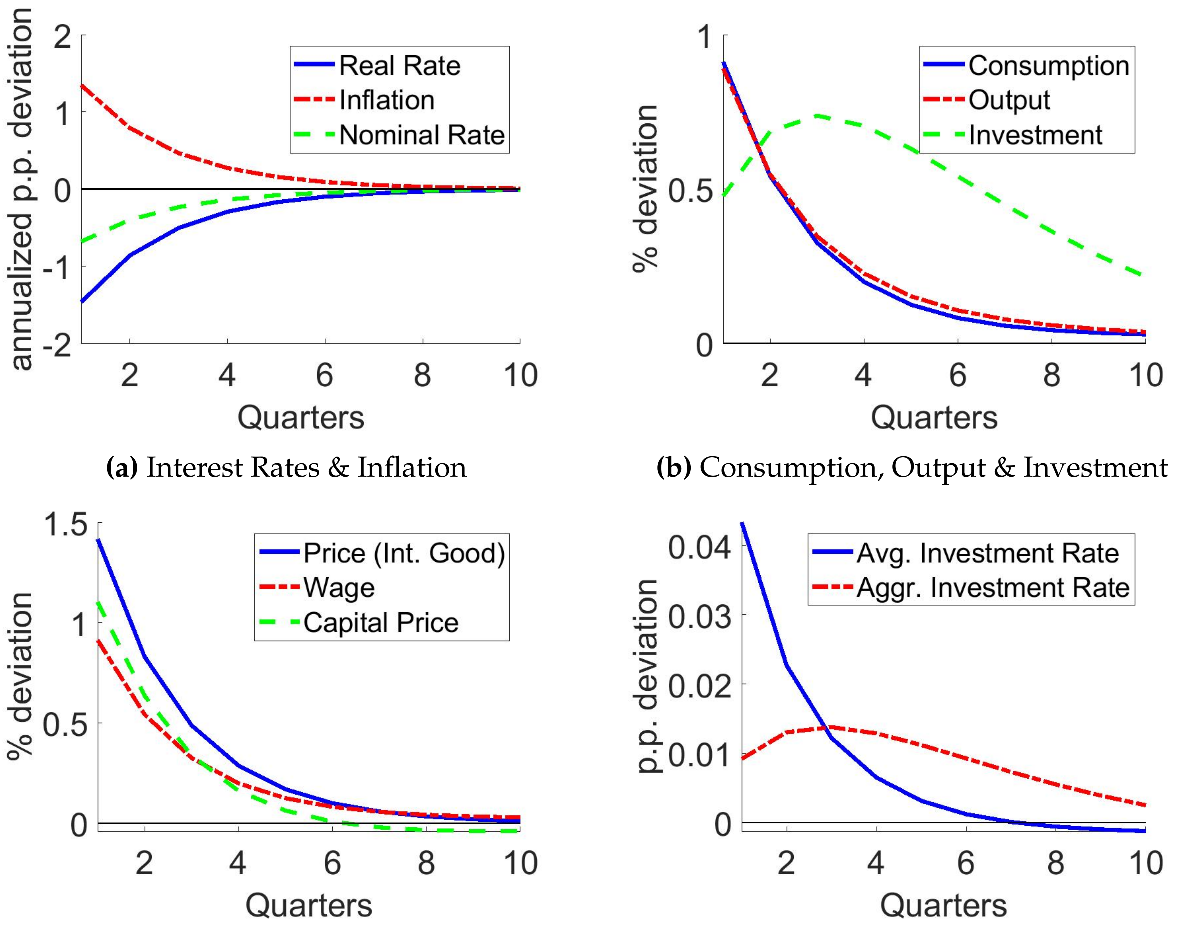
Figure 7: Life-Cycle Profiles

Notes: Investment rates, the hazard rate, and the exit rate refer to a quarter and are computed from the steady state distribution.

5.3 The Aggregate Effects of Monetary Policy Shocks

We study the effects of an unexpected expansionary monetary policy shock $\epsilon_t^m = -0.0025$, followed by a perfect foresight transition back to steady state. Figure 8 plots the impulse response functions of aggregates, prices, and the average and aggregate investment rate[14]. These confirm that our model produces the typical New Keynesian effects to a monetary policy shock. Aggregate capital adjustment costs generate a hump-shaped response of investment. Impulse response functions of output and consumption are not hump-shaped as in Christiano et al. (2005), because our model does not feature habit formation. The aggregate (quarterly) investment rate (Panel (d)) rises

[14]In line with our empirical analysis, we use *gross* investment rates, i.e. $i_{jt} = \frac{k_{j,t+1}-(1-\delta)k_{jt}}{k_{jt}}$. The average investment rate is $\bar{i}_t = \int i_t(k,z) \times \frac{\xi_t^T(k,z)}{\bar{\xi}} \times d\mu_t(k,z)$.



(a) Interest Rates & Inflation

(b) Consumption, Output & Investment

(c) Prices

(d) Average & Aggregate Investment Rate

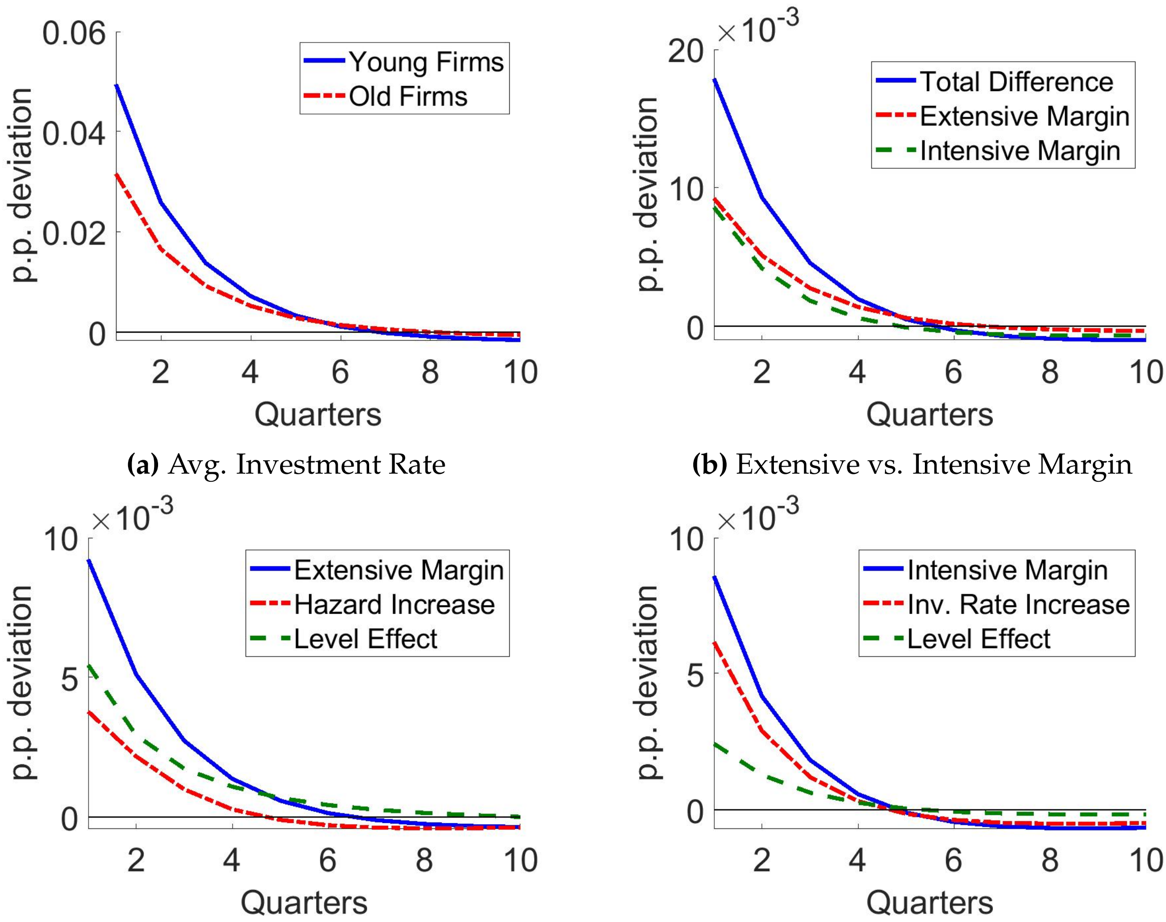
Figure 8: Aggregate Effects of an Expansionary Monetary Policy Shock

from 1.925% to about 1.938%. The *average* quarterly investment rate rises a lot more, because young firms, which are typically small, increase their investment rates more strongly. This finding is discussed in detail in the following subsection.

5.4 Heterogeneous Sensitivity

Figure 9 plots the effect of the expansionary monetary policy shock on firm-level investment rates by age group. Panel (a) shows that young firms on average increase their investment rates more strongly than old firms. Panels (b), (c), and (d) decompose this total effect into the four components identified in equation (10). Panel (b) shows that both the intensive margin and the extensive margin contribute, although the extensive margin effect is quantitatively slightly more important. This decomposition is computed by holding either the investment rate conditional on investing, or the hazard rate at steady state. Panel (c) decomposes the extensive margin effect into the effect due to a different hazard rate and the level effect. Evidently, the level effect is slightly larger. Panel (d) decomposes the intensive margin effect into the effect due to a higher investment rate conditional on investing and the level effect. Both point towards young firms being more sensitive over the first few quarters. This means that young firms increase their investment rates (conditional on investing) relatively more strongly and that this increase affects more firms, because young firms invest more frequently (level effect).

These findings confirm that the results from the simple model in Section 3 hold in a full general equilibrium heterogeneous firm model. Young firms are more sensitive to monetary policy shocks even in the absence of a financial accelerator mechanism. There is an extensive and an intensive margin effect, while the former is quantitatively more relevant.



(a) Avg. Investment Rate

(b) Extensive vs. Intensive Margin

(c) Extensive Margin

(d) Intensive Margin

Figure 9: Heterogeneous Effect (by Age Group) of an Expansionary Monetary Policy Shock

6 Conclusion

In this paper, we put forward the issue of observational equivalence: The observed heterogeneous sensitivity of young and old firms in response to monetary policy shocks do not only arise due to a financial accelerator mechanism but also due to non-financial mechanisms. We formulate our key mechanisms in a New Keynesian heterogeneous firm model with realistic firm life-cycle profiles and non-convex capital adjustment costs. The heterogeneous sensitivity emerges at the intensive margin because young (small) firms' marginal benefit curves shift more than old (large) firms' in response to interest rate changes. Moreover, non-convex capital adjustment costs give rise to heterogeneous sensitivity due to an extensive margin mechanism. We show in a quantitative model that the extensive margin effect is quantitatively more important. Moreover, we provide supporting empirical evidence using firm-level investment data from Compustat.

One should not interpret our results as rejecting the financial accelerator mechanism. In fact, there exist empirical evidence that suggests that the financial accelerator is highly plausible. We believe that both financial frictions and the mechanisms that we emphasize in this paper are responsible for the observed heterogeneity in the data. Decomposing the quantitative relevance of these alternative channels is an interesting question that we reserve for future research. Such an analysis needs to be done based on a model with financial frictions and non-convex adjustment costs as in Bayer (2006) and Whited (2006). Our findings highlight that the challenges in identifying the financial accelerator mechanism remain an open issue in the new generation of empirical literature. We cast doubt on the interpretation that the excess sensitivity of young firms' investment is entirely due to the financial frictions. Moreover, our results are informative about the design of optimal stimulus policy.

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Appendices

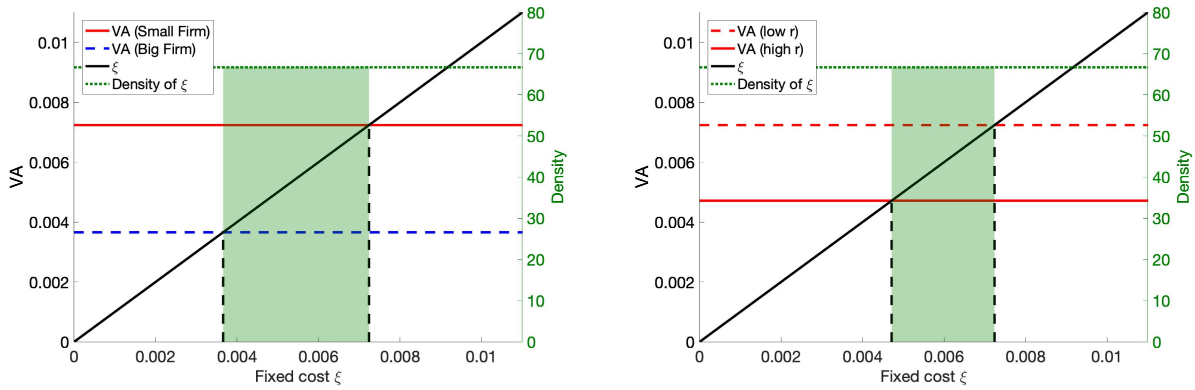
A A Simple Model with Random Fixed Adjustment Costs

We now consider the case when the fixed adjustment cost is draw from a distribution. This assumption is to mimic the fact that investments are subject to different levels of fixed adjustment cost in the real world. For simplicity, we fix the productivity $z(j) = 1$ $\forall j$. The next proposition shows that the results we derived in Proposition (2) can be carried over to the case with random fixed adjustment costs.

Proposition 3. *In the absence of the convex adjustment cost ($\phi = 0$), consider an economy populated by firms whose $i \equiv \frac{k_1^*}{k_0} > 1$, i.e., $k_0 < \left(\frac{\theta}{(1+r)q}\right)^{\frac{1}{1-\theta}}$. In addition, firms face random fixed adjustment cost ξ. Then, the following properties about the sensitivity of the hazard rate holds: $\lambda'(k_0) < 0$ and $\lambda'(r) < 0$*

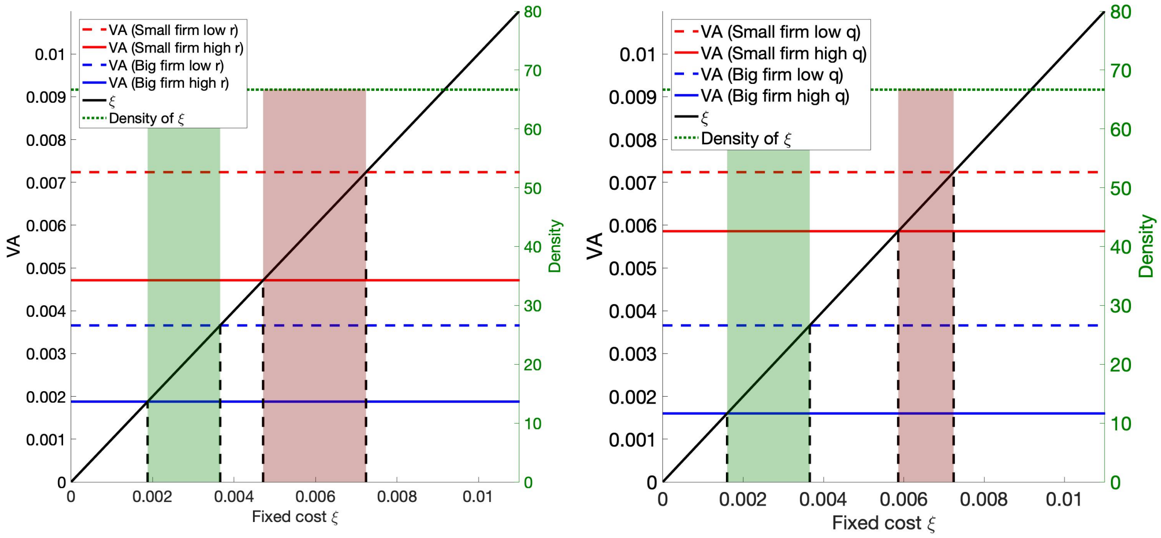
Proof. See Appendix B □

Figure (A.1) illustrates the Proposition 3 graphically. Figure A.2 illustrates the heterogeneous r-sensitivity and q-sensitivity of adjustment hazard rates.



(a) Small v.s. Big Firms' Adj. Hazard **(b) r-Sensitivity of Adj. Hazard**

Figure A.1: This figure plots the VA of a firm against random fixed cost ξ. The black upward slopping line is the 45° line indicating the points where VA equals to ξ. The intercept of the two curves pins down the threshold value of ξ. The green dotted line plots the density function of ξ (uniform distribution). The area under the density function to the left of the threshold value of ξ is the adjustment hazard. The shaded area in Panel (a) plots the difference in adjustment hazard between a small and a big firm. The shaded area in Panel (b) plots the difference in adjustment hazard after an interest rate shock.



(a) r-Sensitivity of Adjustment Hazard: **(b) q-Sensitivity of Adjustment Hazard**

Figure A.2: This figure plots the VA of a firm against the fixed adjustment cost ξ. The black line is the 45 degree line. The intercept of the two curves pins down the threshold value of ξ. The green dotted line plots the density function of ξ (uniform distribution). The area under the density function to the left of the threshold value of ξ is the adjustment hazard. The shaded area in Panel (a) plots the r-sensitivity of adjustment hazard for a small and a big firm. The shaded area in Panel (b) plots the q-sensitivity of adjustment hazard for a small and a big firm.

B Proofs

Proposition 1 *Consider a simple two-period partial equilibrium model populated by firms whose $i \equiv \frac{k_1^*}{k_0} > 1$. With the decreasing returns to scale ($\theta < 1$), the following properties about interest rate sensitivity of investment hold: $i'(r) < 0$ and $\frac{\partial i'(r)}{\partial k_0} > 0$.*

Proof. Let $G(i,r,q) \equiv q + \phi(i-1) - \frac{1}{1+r}\theta(ik_0)^{\theta-1} = 0$, the implicit function that defines i as a function of exogenous variables r and q. Then,

$$\frac{\partial G}{\partial i} = \phi + (1-\theta)(1+r_0)^{-1}\theta(ik_0)^{\theta-1}i^{-1}$$

$$\frac{\partial G}{\partial k_0} = (1-\theta)(1+r_0)^{-1}\theta(ik_0)^{\theta-1}k_0^{-1}$$

$$\frac{\partial G}{\partial r_0} = (1+r_0)^{-2}\theta(i_0k_0)^{\theta-1}.$$

The proposition can be proved by applying the Implicit Function Theory.

Step 1, show $\frac{\partial i_0}{\partial k_0} < 0$.

$$\begin{aligned}
\frac{\partial i}{\partial k_0} &= -\frac{\frac{\partial G}{\partial k_0}}{\frac{\partial G}{\partial i}} \\
&= -\frac{(1-\theta)(1+r_0)^{-1}\theta(ik_0)^{\theta-1}k_0^{-1}}{\phi+(1-\theta)(1+r_0)^{-1}\theta(ik_0)^{\theta-1}i_0^{-1}} \\
&= -\frac{(1-\theta)\big(q+\phi(i-1)\big)k_0^{-1}}{\phi+(1-\theta)\big(q+\phi(i-1)\big)i^{-1}} < 0
\end{aligned}$$

The last inequality holds since $i > 1$ and $\theta < 1$.

Step 2, show $i'(r) = \frac{\partial i}{\partial r_0} < 0$.

$$\begin{aligned}
i'(r) &= -\frac{\frac{\partial G}{\partial r_0}}{\frac{\partial G}{\partial i}} \\
&= -\frac{(1+r_0)^{-2}\theta(ik_0)^{\theta-1}}{\phi+(1-\theta)(1+r_0)^{-1}\theta(ik_0)^{\theta-1}i^{-1}} \\
&= -\frac{(1+r_0)^{-1}\big(q+\phi(i-1)\big)}{\phi+(1-\theta)\big(q+\phi(i-1)\big)i^{-1}} < 0
\end{aligned}$$

Step 3, show $\frac{\partial i'(r)}{\partial k_0} > 0$. $i'(r) = -\frac{(1+r_0)^{-1}\big(q+\phi(i-1)\big)i}{\phi i+(1-\theta)\big(q+\phi(i-1)\big)}$. Denote $A \equiv \big(q+\phi(i-1)\big)i$ and $B \equiv \phi i + (1-\theta)\big(q+\phi(i-1)\big)$. The partial derivatives with respect to k_0 are:

$$\begin{aligned}
\frac{\partial A}{\partial k_0} &= \frac{\partial i}{\partial k_0}\big(q+\phi(i-1)\big) + i\phi\frac{\partial i}{\partial k_0} \\
&= \frac{\partial i}{\partial k_0}\big(q+\phi(i-1)+i\phi\big) \\
\frac{\partial B}{\partial k_0} &= \frac{\partial i}{\partial k_0}(2-\theta)\phi.
\end{aligned}$$

Then

$$\begin{aligned}
\frac{\partial A}{\partial k_0}B - \frac{\partial B}{\partial k_0}A &= \frac{\partial i}{\partial k_0}\big(q+\phi(i-1)+i\phi\big)\big(\phi i+(1-\theta)\big(q+\phi(i-1)\big)\big) - \frac{\partial i}{\partial k_0}(2-\theta)\phi\big(q+\phi(i-1)\big)i \\
&= \frac{\partial i}{\partial k_0}\Big[(q+2\phi i-\phi)\big((2-\theta)\phi i+(1-\theta)(q-\phi)\big)-(2-\theta)\phi i\big(q+\phi i-\phi\big)\Big] \\
&= \frac{\partial i}{\partial k_0}\Big\{\phi i\Big[\phi i+(1-\theta)(q+\phi(i-1))\Big] \\
&+ \Big[q+\phi(i-1)\Big]\Big[\phi i+(1-\theta)\big(q+\phi(i-1)\big)-(2-\theta)\phi i\Big] \\
&= \frac{\partial i}{\partial k_0}\Big\{(\phi i)^2+\Big[q+\phi(i-1)\Big]\Big[\phi i(1-\theta)+\phi i+(1-\theta)(q+\phi(i-1))-(2-\theta)\phi i\Big]\Big\} \\
&= \frac{\partial i}{\partial k_0}\Big\{(\phi i)^2+\Big[q+\phi(i-1)\Big]\Big[(1-\theta)(q+\phi(i-1))\Big]\Big\} > 0
\end{aligned}$$

The last inequality holds because $i > 1$ and $\frac{\partial i}{\partial r_0} < 0$. Therefore,

$$\frac{\partial i'(r)}{\partial k_0} > 0$$

$\square$

Proposition 2 *In the absence of the convex adjustment cost ($\phi = 0$), consider an economy populated by firms whose $i \equiv \frac{k_1^*}{k_0} > 1$, i.e., $k_0 < \left(\frac{\theta}{(1+r)q}\right)^{\frac{1}{1-\theta}}$, then the following properties about the sensitivity of the hazard rate holds: $\lambda'(k_0) < 0$ and $\lambda'(r) < 0$*

Proof. The threshold level of productivity $\bar{z}$ is the one that sets $VA = \xi$. The solution for $\bar{z}$ reads as: $\bar{z} = (1+r)\frac{q(k_1^*-k_0)+\xi}{k_1^{*\theta}-k_0^\theta}$, where $k_1^* = \left(\frac{\theta}{(1+r)q}\right)^{\frac{1}{1-\theta}}$. The hazard rate is

$$\lambda = 1 - F(z),$$

where the function F is the cdf of z. Since F is monotonically increasing in z, $\lambda'(k_0) < 0$, $\lambda'(r) < 0$, and $\lambda'(q) < 0$ if and only if $\bar{z}'(k_0) > 0$, $\bar{z}'(r) > 0$, and $\bar{z}'(q) > 0$.

Step 1: show $\bar{z}'(r) > 0$. $\bar{z}'(r) = \frac{q(k_1^*-k_0)+\xi}{k_1^{*\theta}-k_0^\theta} > 0$

Step 2: show $\bar{z}'(k_0) > 0$. It can be shown that the sufficient condition for $\bar{z}'(k_0) > 0$ is $\xi\theta k_0^{\theta-1} > q(k_1^{*\theta} - k_0^\theta) - q\theta k_0^{\theta-1}(k_1^* - k_0)$. In what follows, we will show $q(k_1^{*\theta} - k_0^\theta) -$

$q\theta k_0^{\theta-1}(k_1^* - k_0) < 0$, hence the sufficient conditional holds.

$$\begin{aligned}
& q(k_1^{*\theta} - k_0^\theta) - q\theta k_0^{\theta-1}(k_1^* - k_0) < 0 \\
& \Leftrightarrow (k_1^{*\theta} - k_0^\theta) < \theta k_0^{\theta-1}(k_1^* - k_0) \\
& \Leftrightarrow (k_1^{*\theta} - k_0^\theta) < \theta k_0^\theta \frac{k_1^*}{k_0} - \theta k_0^\theta \\
& \Leftrightarrow \left(\frac{k_1^*}{k_0}\right)^\theta - 1 < \theta \frac{k_1^*}{k_0} - \theta \\
& \Leftrightarrow \theta i - i^\theta > \theta - 1,
\end{aligned}$$

we have used $i \equiv \frac{k_1^*}{k_0}$ in the last line. Since $\frac{\partial \theta i - i^\theta}{\partial i} > 0$ for $i > 1$, it follows that $\theta i - i^\theta > \theta - 1$ holds.

□

Proposition 3 *In the absence of the convex adjustment cost ($\phi = 0$), consider an economy populated by firms whose $i \equiv \frac{k_1^*}{k_0} > 1$, i.e., $k_0 < \left(\frac{\theta}{(1+r)q}\right)^{\frac{1}{1-\theta}}$. In addition, firms face random fixed adjustment cost ξ. Then, the following properties about the sensitivity of the hazard rate holds: $\lambda'(k_0) < 0$ and $\lambda'(r) < 0$.*

Proof. Consider the investment decision problem of firm j

$$\max\{(1+r)^{-1}z(j)k_1^{*\theta}(j) - q(k_1^*(j) - k_0(j)) - \frac{\phi}{2}\frac{(k_1^*(j) - k_0(j))^2}{k_0(j)} - \xi(j), (1+r)^{-1}z(j)k_0^\theta(j)\}$$

The CEO will decide to invest if and only if $\xi(j) < \bar{\xi}(j)$, where

$$\begin{aligned}
\bar{\xi}(j) &= (1+r)^{-1}z(j)\left(k_1^{*\theta}(j) - k_0^\theta(j)\right) - q(k_1^*(j) - k_0(j)) - \frac{\phi}{2}\frac{(k_1^*(j) - k_0(j))^2}{k_0(j)} \\
&= (1+r)^{-1}z(j)\left(i^{*\theta}(j) - 1\right)k_0^\theta(j) - q(i^*(j)k_0(j) - k_0(j)) - \frac{\phi}{2}\left(i^*(j) - 1\right)^2 k_0(j)
\end{aligned}$$

For simplicity, we consider the baseline case in which $z = 1$ and we omit the firm index (j) in the following. Recall that by definition

$$\lambda = F_\xi(\bar{\xi}).$$

It follows that $\frac{\partial \lambda}{\partial r} < 0, \frac{\partial \lambda}{\partial k_0} < 0$ if and only if $\frac{\partial \bar{\xi}}{\partial r} < 0, \frac{\partial \bar{\xi}}{\partial k_0} < 0$

Step 1: show $\frac{\partial \bar{\xi}}{\partial r} < 0$.

$$\begin{aligned}\frac{\partial \bar{\xi}}{\partial r} &= -(1+r)^{-2}(i^{*\theta}-1)k_0^{\theta} + \underbrace{\left[(1+r)^{-1}\theta i^{*\theta-1}k_0^{\theta} - qk_0 - \phi(i^*-1)k_0\right]}_{=0}\frac{\partial i^*}{\partial r} \\ &= -(1+r)^{-2}(i^{*\theta}-1)k_0^{\theta} < 0\end{aligned}$$

Step 2: show $\frac{\partial \bar{\xi}}{\partial k_0} < 0$.

$$\begin{aligned}\frac{\partial \bar{\xi}}{\partial k_0} &= (1+r)^{-1}\left(i^{*\theta}-1\right)\theta k_0^{\theta-1} - q(i^*-1) - \frac{\phi}{2}(i^*-1)^2 \\ &\quad + \left[(1+r)^{-1}\theta i^{*\theta-1}k_0^{\theta} - qk_0 - \phi(i^*-1)k_0\right]\frac{\partial i^*}{\partial k_0} \\ &= (1+r)^{-1}\left(i^{*\theta}-1\right)\theta k_0^{\theta-1} - q(i^*-1) - \frac{\phi}{2}(i^*-1)^2 \\ &= \left(i^{*\theta}-1\right)\left[q+\phi(i-1)\right]i^{*1-\theta} - q(i^*-1) - \frac{\phi}{2}(i^*-1)^2 \\ &= \left(i^{*\theta}-1\right)\left[q+\phi(i-1)\right]i^{*1-\theta} - \left[q+\frac{\phi}{2}(i^*-1)\right](i^*-1) \\ &= \left(i^{*}-i^{*1-\theta}\right)\left[q+\phi(i-1)\right] - \left[q+\frac{\phi}{2}(i^*-1)\right](i^*-1) \\ &< 0 \text{ for } \phi = 0\end{aligned}$$

From the second to the third equality, we have used the fact that $(1+r)^{-1}\theta k_0^{\theta-1} = [q+\phi(i-1)]\, i^{*1-\theta}$ implied from the manager's optimization problem.

□

C Data

We use the Compustat North America Fundamentals Quarterly database. Observations are uniquely identified by GVKEY & DATADATE.

C.1 Sample Selection

In line with the literature, we exclude observations which fall under the following criteria

1. not incorporated in the United States (based on FIC)

2. native currency not U.S. Dollar (based on CURNCDQ)

3. fiscal quarter does not match calendar quarter (based on FYR)
4. specific sectors
    - Utilities (SIC 4900-4999)
    - Financial Industry (SIC 6000-6999)
    - Non-operating Establishments (SIC 9995)
    - Industrial Conglomerates (SIC 9997)
    - Non-classifiable (NAICS > 999900)
5. missing industry information (SIC or NAICS code)
6. missing capital expenditures (based on CAPX)
7. missing or non-positive total assets (AT) or net capital (PPENT)
8. negative sales (SALEQ)
9. acquisitions (based on AQCY) exceed 5% of total assets (in absolute terms)
10. missing or implausible age information (see Appendix C.2)
11. outlier in the Perpetual Inventory Method (see Appendix C.3)

Our sample begins with 1986Q1 and ends with 2018Q4. In a final step, we exclude firm which we observe for less than 20 quarters, unless they are still in the sample in the final period. This ensures that we do not mechanically exclude all firms incorporated in the last five years of our sample.

C.2 Firm Age

We use data on firm age from WorldScope and Jay Ritter's database[15]. WorldScope provides the date of incorporation (Variable: INCORPDATE), while Jay Ritter's database provides the founding date. Both are merged with Compustat based on CUSIP. We define as the firm entry quarter the minimum of both dates if both are available. We do not use information on the initial public offering (IPO) of a firm to determine its age, since the time between incorporation and IPO can vary substantially. However, we use the IPO date to detect implausible age information. We exclude firms for which the IPO date reported in Compustat (IPODATE) precedes the firm entry quarter by more than four quarters. In similar fashion, we exclude firms which appear in Compustat

[15] https://site.warrington.ufl.edu/ritter/

more than four quarters before the firm entry quarter.[16] Finally, we merge information on the beginning of trading from CRSP (Variable: BEGDAT) based on CUSIP and likewise exclude firms with trading more than four quarters before the firm entry quarter.

C.3 Perpetual Inventory Method

Accounting capital stocks $k^a_{j,t}$ as reported in Compustat deviate from *economic* capital stocks for at least two reasons. First, accounting depreciation is driven by tax incentives and usually exceeds economic depreciation. Second, accounting capital stocks are reported at historical prices, not current prices. With positive inflation, both issues make the economic capital stock exceed the accounting capital stock. Therefore, we use a Perpetual Inventory Method (PIM) to compute real economic capital stocks, building on Bachmann and Bayer (2014).

Investment. In principle, there are two options to measure net nominal quarterly investment. First, investment can be measured directly ($I^{dir}_{j,t}$) from the Statement of Cash Flows as capital expenditures (CAPX) less the sale of PPE (SPPE)[17]. Second, investment can be backed out ($I^{indir}_{j,t}$) from the change in PPE (D.PPENT) plus depreciation (DPQ), using Balance Sheet and Income Statement information. Either measure needs to be deflated to obtain real investment. We use INVDEF from FRED, which has the advantage of being quality-adjusted. We prefer the direct investment measure, since the indirect measure basically captures any change to PPE, including changes due to acquisitions. Nevertheless, we want to exclude observations where both investment measures differ strongly. To this end, we compute investment rates using lagged net accounting capital (L.PPENT), compute the absolute difference between both and discard the top 1% of that distribution.

Depreciation Rates. We obtain economic depreciation rates from the Bureau of Economic Analysis' (BEA) Fixed Asset Accounts. Specifically, we retrieve current-cost net stock and depreciation of private fixed assets by year and industry.[18] We calculate annual depreciation rates by industry and assume a constant depreciation rate within the calendar year to calculate quarterly depreciation rates.

Real Economic Capital Stocks. We initialize a firm's capital stock with the net

[16] We do not construct firm age from the first appearance in Compustat. An inspection of the data reveals that this would result in wrongly classifying a number of old and established firms as young. Cloyne et al. (2020) do exactly this. However, they show in an earlier working paper version that results are unchanged if only age information from WorldScope is used.

[17] We follow Belo et al. (2014) and set missing values of SPPE to zero.

[18] The Fixed Asset Accounts also provide depreciation rates by asset type (Equipment, Structures, Intellectual Property Products), which we do not use since the firm-level data does not include information on capital stocks or capital expenditure by asset type.

(real) accounting capital stock $k^a_{j,1}$ (PPENT / INVDEF) whenever this variable is first observed. We iterate forward using deflated investment and the economic depreciation rate.

$$k^{(1)}_{j,1} = k^a_{j,1} \tag{43}$$

$$k^{(1)}_{j,t+1} = (1-\delta^e_t)k^{(1)}_{j,t} + \frac{p^I_t}{p_{2009,t}} I^{dir}_{j,t} \tag{44}$$

Comparing $k^{(1)}_{j,t}$ and $k^a_{j,t}$ shows non-negligible discrepancies. On average, the economic capital stock is larger, confirming the hypothesis that accounting capital stocks are understated. This makes it problematic to use the accounting capital stock as a starting value in the PIM. As a remedy, we again follow Bachmann and Bayer (2014) and use an iterative procedure to re-scale the starting value. We compute a time-invariant scaling factor ϕ at the sector-level and use it to re-scale the starting value as follows. We iterate until ϕ converges. The procedure is initialized with $k^{(0)}_{j,t} = k^a_{j,t}$ and $\phi^{(0)} = 1$.

$$\phi^{(n)} = \frac{1}{NT} \sum_{j,t} \frac{k^{(n)}_{j,t}}{k^{(n-1)}_{j,t}} \quad \text{[and not in top or bottom 1\%]} \tag{45}$$

$$k^{(n+1)}_{j,1} = \phi^{(n)} k^{(n)}_{j,1} \tag{46}$$

Outliers. We exclude firms for which the economic capital stock becomes negative at any point in time. This can arise if there is a sale of capital, which exceeds current economic capital. Further, we compute the deviation between (real) book and economic capital stocks and discard the top 1% of that distribution. Finally, we discard firms for which we have less than 20 observations, unless they are still in the sample in the final quarter.

Evaluation. Our estimated real economic capital stock is still highly correlated with the real accounting capital stock. A simple regression has an R^2 of above 0.96 and shows that the economic capital stock is on average slightly higher (by about 4%), as expected. The investment rate (net real investment over lagged real economic capital) is highly correlated ($\rho > 0.98$) with the accounting investment rate used in Cloyne et al. (2020). A simple regression shows that on average, the economic investment rate is lower (by about 13%) than the accounting investment rate, also as expected due to the underreporting of accounting capital stocks.

C.4 Variable Construction

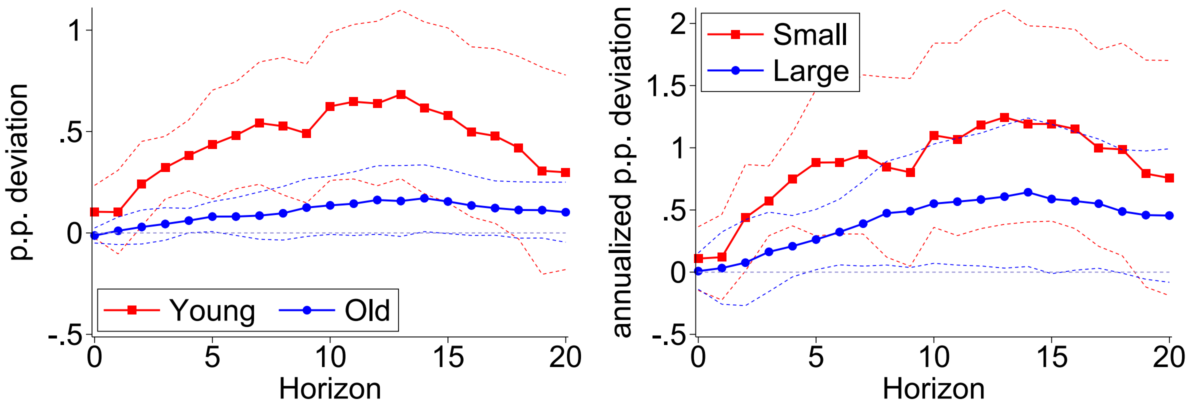
Most of our variables follow the definitions in the literature. For our baseline investment rate, we use $i_{jt} = \frac{CAPX - SPPE}{INVDEF * L.k_{jt}}$, thus, real net investment over the lagged real economic capital stock, computed as described previously. As a comparison, we use the accounting capital stock investment rate $i^a_{jt} = \frac{CAPX - SPPE}{L.PPENT}$. In addition, we use the following investment rate, following Bachmann and Bayer (2014): $i_{jt} = \frac{2(CAPX - SPPE)}{INVDEF * (k_{jt} + L.k_{jt})}$

C.5 Identification of Monetary Policy Shocks

We use the monetary policy shocks implied by the proxy SVAR used in Gertler & Karadi (2015). We calculate them according to the following procedure.

First, we update the data used in the Gertler & Karadi (2015) baseline SVAR. They use monthly data from 1979M7 to 2012M6. We update all time series to 2019M12. The SVAR includes (the log of) industrial production (FRED: INDPRO), (the log of) the consumer price index (FRED: CPIAUCSL), the one-year government bond rate (FRED: GS1), and the excess bond premium (Source: https://www.federalreserve.gov/econresdata/notes/f notes/2016/files/ebp_csv.csv, retrieved in February 2020). Moreover, we update the instrument (cumulative high-frequency FF4 surprises) to 2015M10.

Then, we run the SVAR and compute the implied structural monetary policy shocks. See the appendix of Mertens & Ravn (2013) for details. Importantly, even though the instrument is only available until 2015M10, we can compute the structural monetary policy shock until 2019M12.



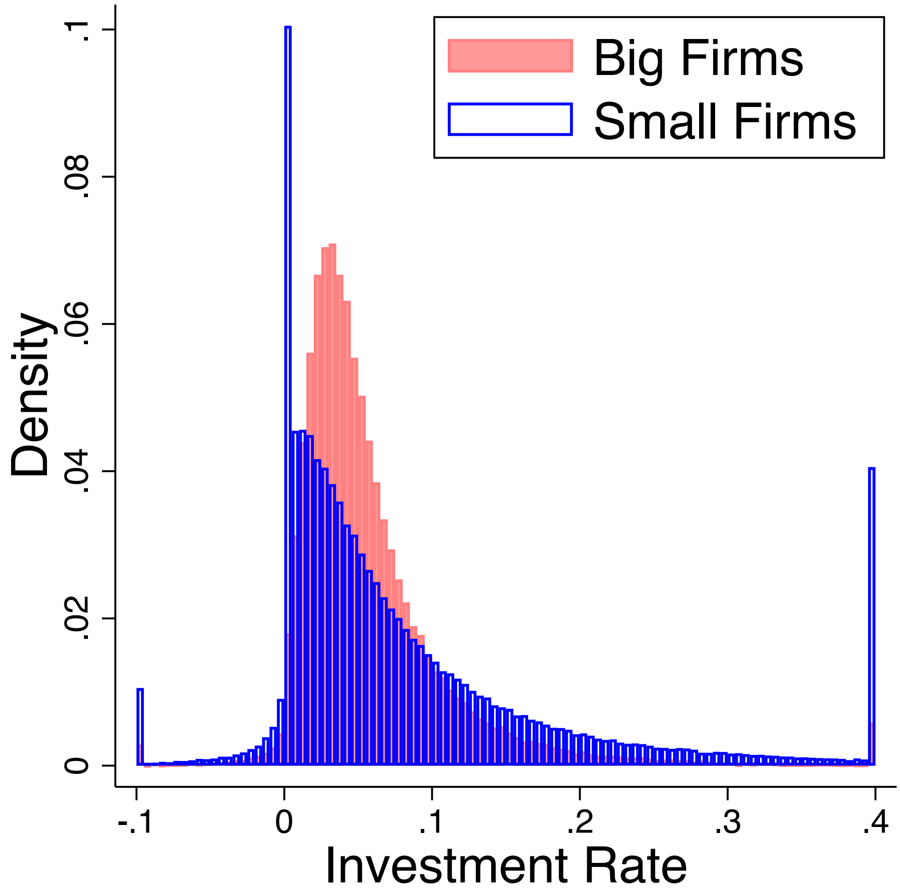
(a) Young vs. Old Firms **(b)** Small vs. Large Firms

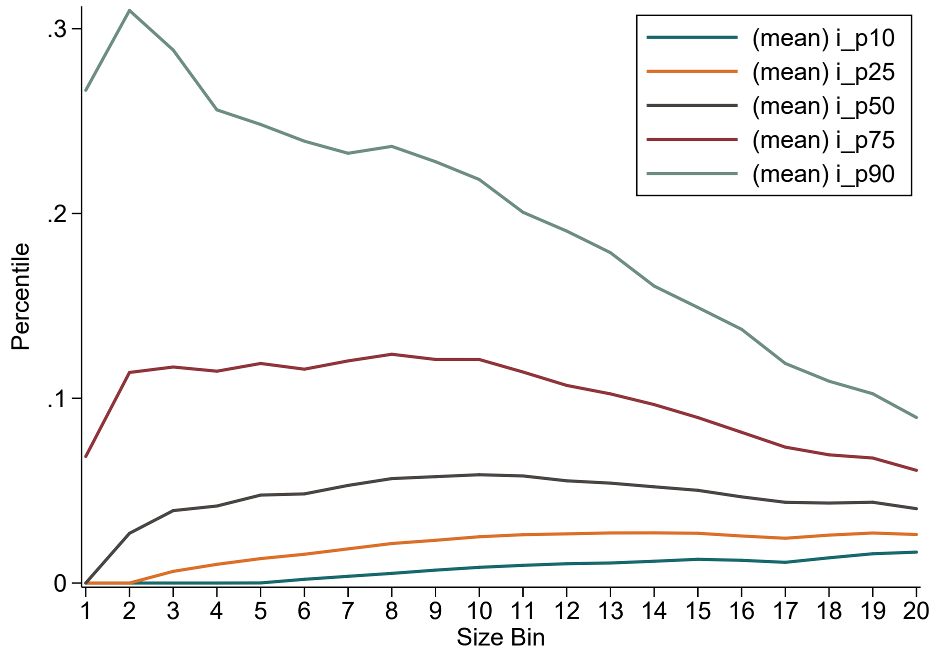
Figure A.3: Average Effect of a Monetary Policy Shock on Group-Specific Average Investment Rates

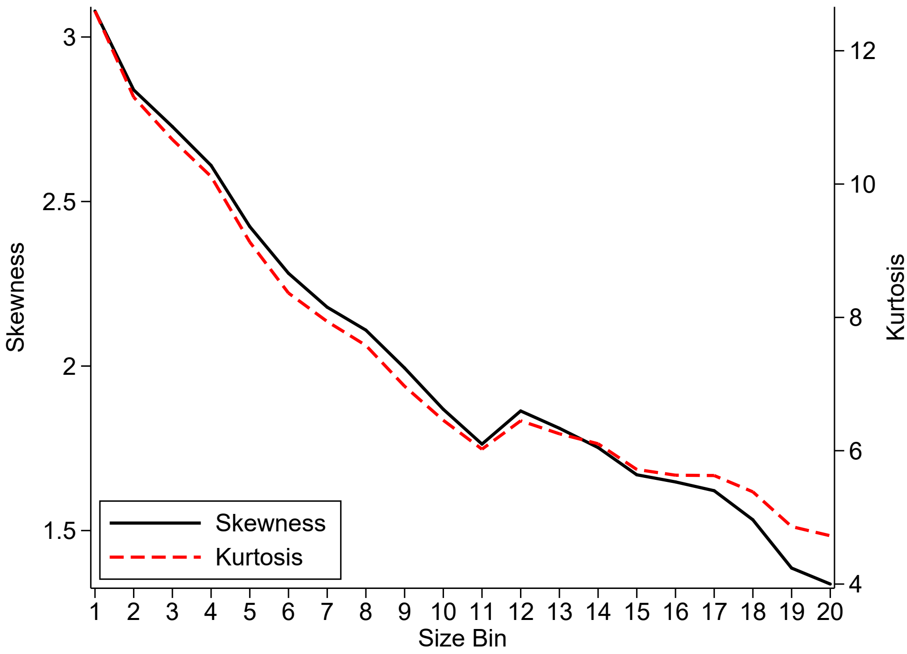
Notes: Young (old) firms are firms less (more) than 15 years old. Small (large) firms are firms smaller (larger) than the median. Dashed lines indicate 90 % confidence intervals.

D Additional Empirical Evidence

D.1 Size as a Proxy for Lumpiness of Investment







E Analysis of the Calibrated Model

E.1 Equilibrium Definition

A recursive competitive equilibrium in this model is a set of value functions $\{V_0(z,k;\cdot)$, $CV^{Exit}(z,k,\xi;\cdot), CV_a(z,k,\xi;\cdot), CV_n(z,k;\cdot)\}$, policy functions $\{n^*(n,z;\cdot), k^*(z,k,\xi;\cdot), \xi^T(z,k;\cdot)$, $\xi^T_{Exit}(z,k;\cdot)\}$, quantities $\{C, Y, I\}$, prices $\{p, w, \pi, \Lambda\}$, and a distribution μ such that all agents in the economy behave optimally, the distribution of firms is consistent with decision rules, and all markets clear.

1. Investment Block: Taking all prices as given, $V_0(z,k;\cdot)$ solves the Bellman equation (14) with associated decision rules $n^*(n,z;\cdot), k^*_t(z,k,\xi;\cdot), \xi^T_t(z,k;\cdot), \xi^T_{t\ Exit}(z,k;\cdot)$
2. Household Block: C satisfies the household's optimality conditions (41) and (42).
3. New Keynesian Block: The NKPC holds. The Taylor rule holds.
4. All markets (for the final good and labour) clear.
5. The distribution of firms evolves as implied by XXX.

E.2 Identification of the Fitted Parameters

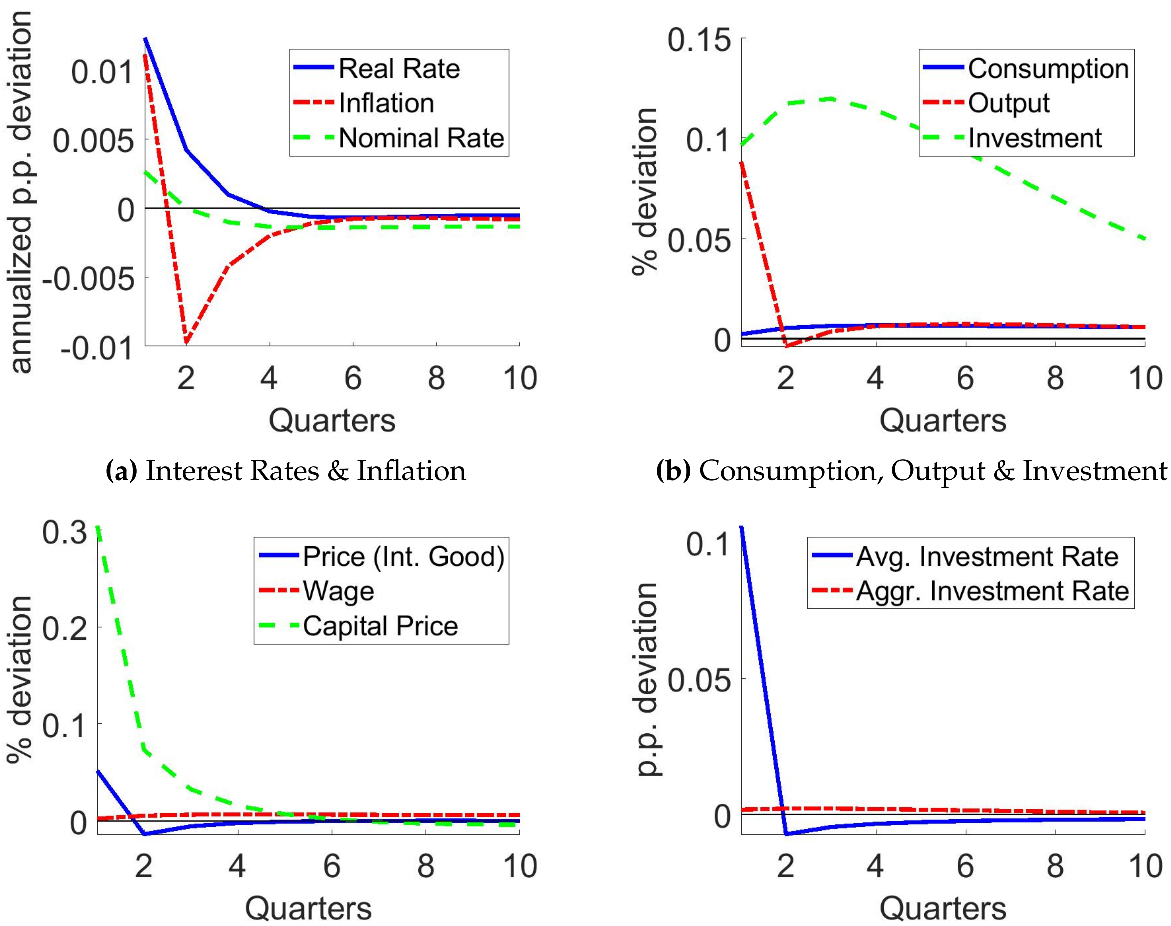
Table A.1: Identification of the Fitted Parameters

Target	σ_z	k_0	$\bar{\xi}$	ϕ	c_f	m
Targeted						
Average Investment Rate (%)	0.569	-0.192	-0.036	-0.841	-0.228	-0.175
Standard Deviation of Investment Rates	0.402	-0.318	0.132	-0.966	0.286	-0.253
Skewness of Investment Rates	-1.068	-0.336	0.090	-0.082	1.008	-0.096
Autocorrelation of Investment Rates	-0.073	-0.254	-0.398	0.422	1.011	0.217
Share of employment in age 0-15	-1.178	-0.174	0.258	0.450	2.446	0.008
Share of firms in ages 0-15	-0.305	-0.068	0.115	0.185	0.752	0.078
Untargeted						
Spike Rate (%)	1.084	-0.081	-0.044	-0.861	-0.595	-0.079
Positive Rate (%)	-0.182	0.014	0.007	0.145	0.100	0.013
Inaction Rate (%)	-0.252	0.001	0.709	-0.070	0.147	0.056
Share of employment in age 0	-3.947	-0.692	0.958	1.596	8.393	0.672
Share of employment in age 1-15	-0.970	-0.135	0.205	0.364	1.999	-0.042
Share of employment in age 16+	0.442	0.061	-0.093	-0.166	-0.912	0.019
Share of employment in age 0-5	-1.766	-0.220	0.370	0.647	3.307	-0.012
Share of employment in age 6-15	-0.301	-0.060	0.065	0.124	0.852	-0.058
Share of firms in age 0	-3.274	-1.024	0.826	1.330	8.664	0.984
Share of firms in age 1	-0.430	-0.038	0.248	0.324	-0.006	0.066
Share of firms in ages 16+	0.657	0.147	-0.249	-0.398	-1.620	-0.169

Notes: This matrix shows the local elasticities of the simulated moments w.r.t. the fitted parameters. Thus, an element in this matrix informs about the amount (and direction) of the change of the simulated moment (column), if the parameter (row) changes by 1%. (TABLE NEEDS TO BE UPDATED.)

F Stimulus Policy

We consider conditional lump-sum transfers as a potential stimulus policy. Firms which adjust their stock of capital receive a government transfer u_t, which reduces their total fixed adjustment cost. Thus, instead of paying $w_t \xi_{jt}$, firms pay $w_t(\xi_{jt} - u_t)$. We consider u_t to follow an autoregressive process with persistence $\rho_u = 0$, i.e. $u_t = \rho_u u_{t-1} + \epsilon_t^u$. In the following, we simulate the perfect foresight transition path after $\epsilon_t^u = 0.01\bar{\xi}$. The government runs a balanced budget and finances the subsidy with a lump-sum tax on the household. Therefore, the subsidy does not show up in the resource constraint of the economy.



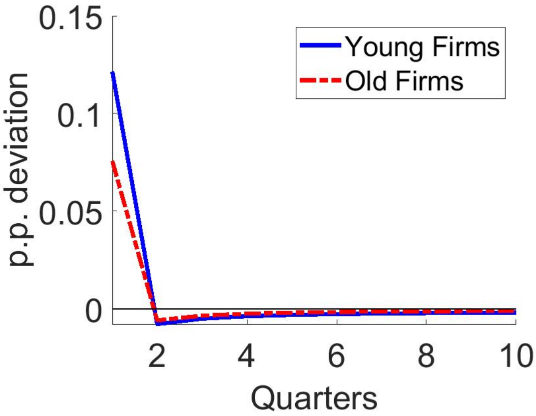
(a) Interest Rates & Inflation

(b) Consumption, Output & Investment

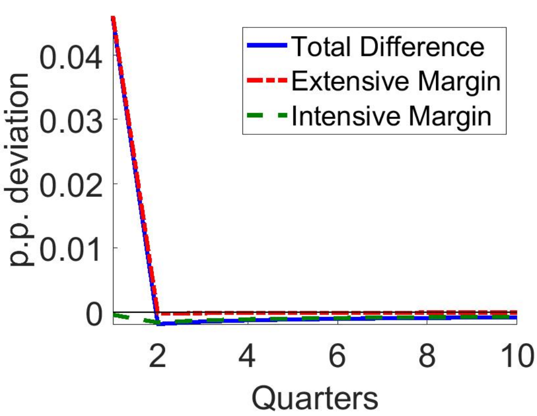
(c) Prices

(d) Average & Aggregate Investment Rate

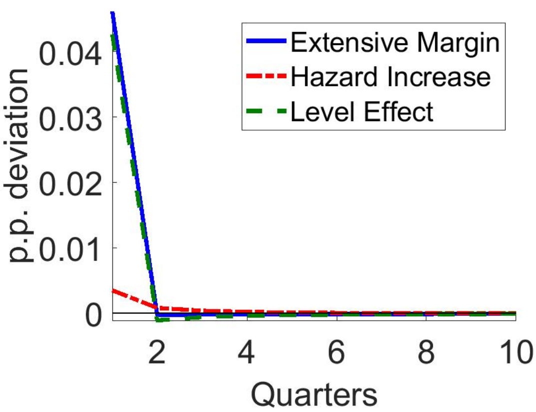
Figure A.4: Aggregate Effects of Stimulus Policy



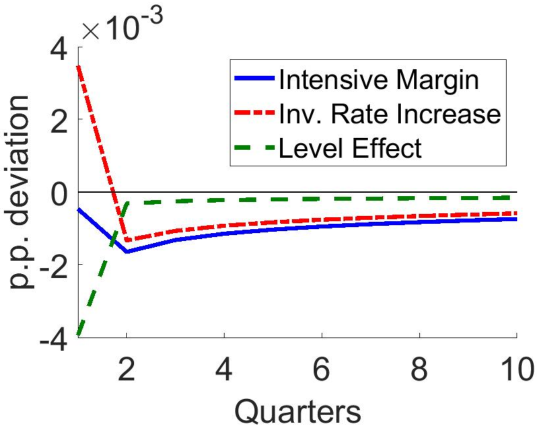
(a) Avg. Investment Rate



(b) Extensive vs. Intensive Margin



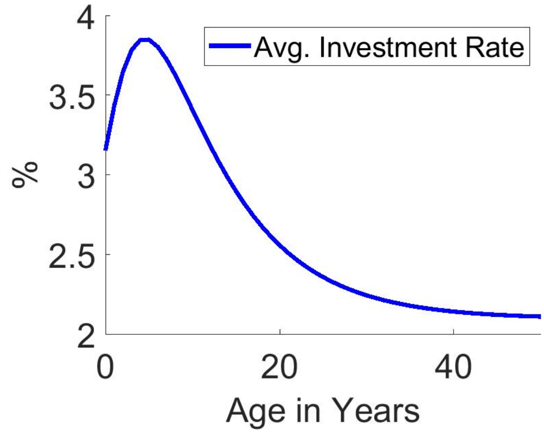
(c) Extensive Margin



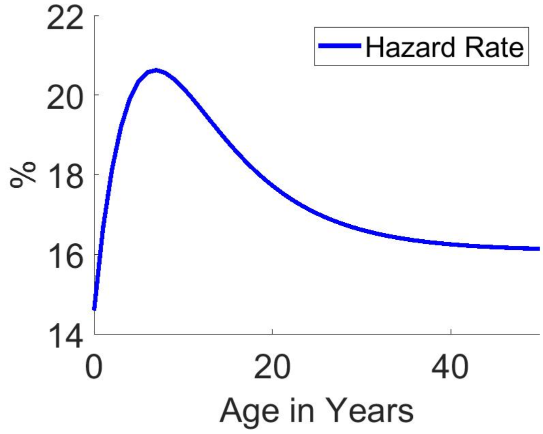
(d) Intensive Margin

Figure A.5: Heterogeneous Effect (by Age Group) of Stimulus Policy

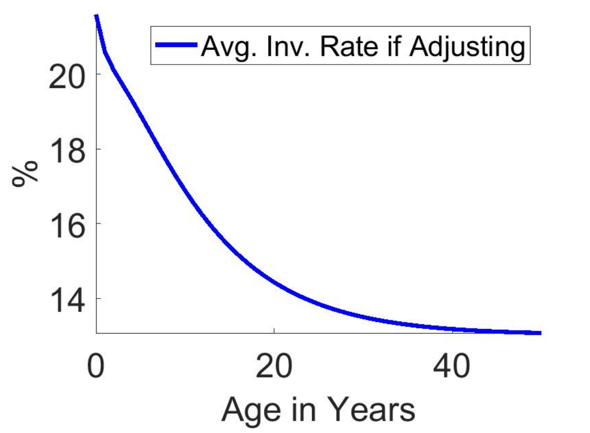
G No Aggregate Adjustment Costs (i.e. q=1)



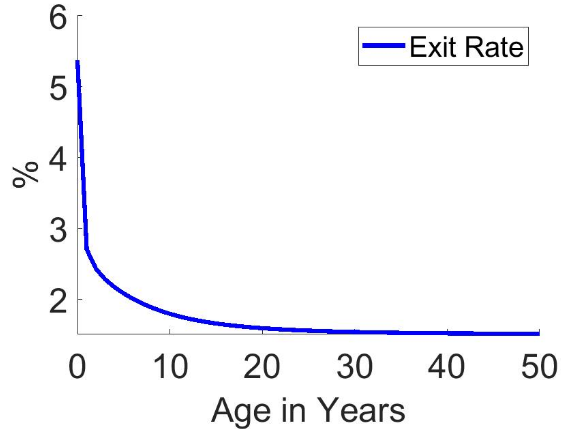
(a) Average Investment Rate



(b) Hazard Rate



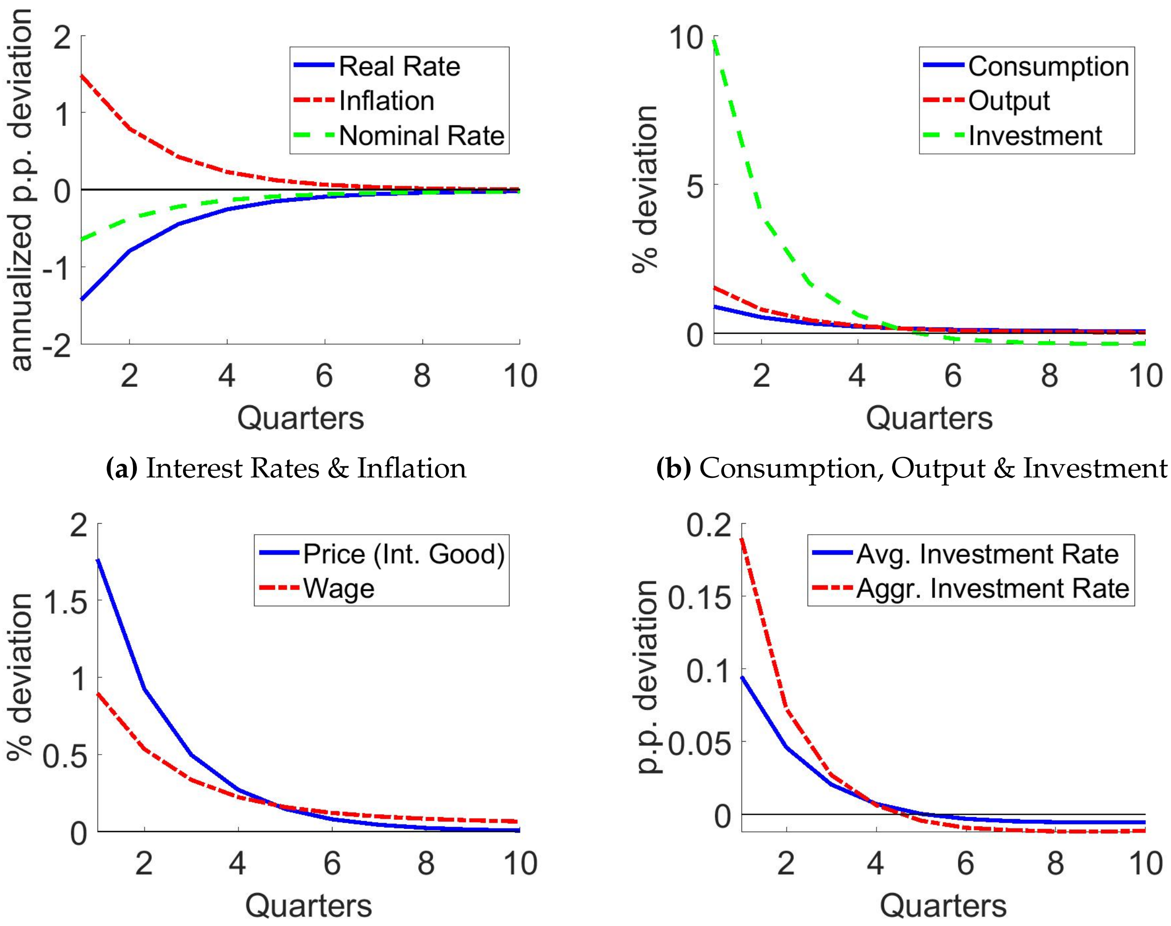
(c) Avg. Inv. Rate (Conditional on Adjusting)



(d) Exit Rate

Figure A.6: Life-Cycle Profiles

Notes: Investment rates, the hazard rate, and the exit rate refer to a quarter and are computed from the steady state distribution.



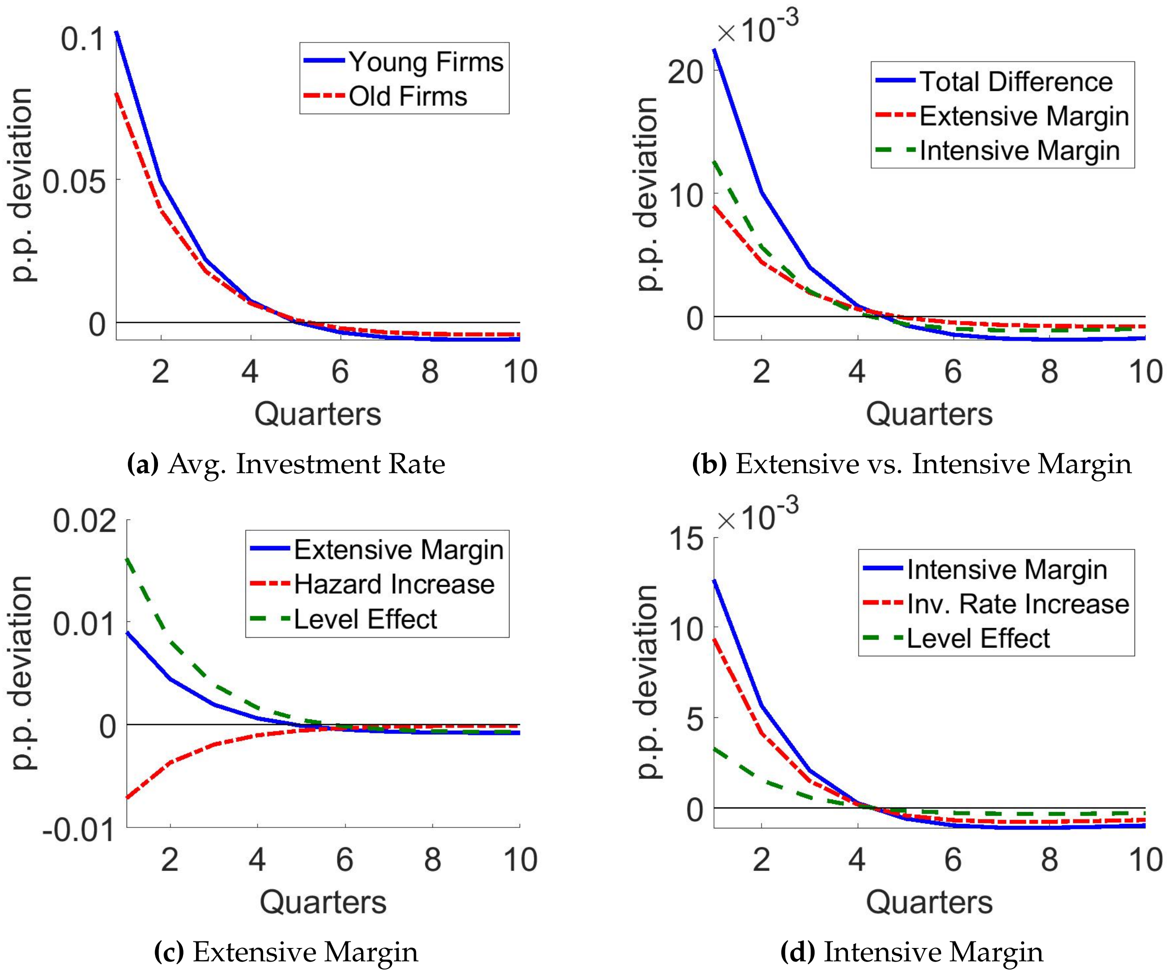
(a) Interest Rates & Inflation

(b) Consumption, Output & Investment

(c) Prices

(d) Average & Aggregate Investment Rate

Figure A.7: Aggregate Effects of an Expansionary Monetary Policy Shock



(a) Avg. Investment Rate

(b) Extensive vs. Intensive Margin

(c) Extensive Margin

(d) Intensive Margin

Figure A.8: Heterogeneous Effect (by Age Group) of an Expansionary Monetary Policy Shock