Multi-Product Establishments and Product Dynamics*

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February 5, 2022

Abstract

The current paper builds a general equilibrium model based on heterogeneous productivities of establishments and heterogeneous tastes at the product level. Establishments choose endogenously their product mix over the business cycle given different income elasticities across products in consumer preferences. We calibrate and estimate the model's shock processes with Japanese data and find that (de)regulation policy at entry, incumbent firms or establishments and each product level provide substantially different outcomes, thereby providing a caveat for policy debate.

Keywords: Firm heterogeneity, multiple-product, business cycle, Nonhomothetic CES preferences

JEL: D24, E23, E32, L11, L60.

*We would like to thank very much all seminar participants and comments at the 4th Sophia Workshop, the research seminar at Sophia University and the AFES meeting in 2021. We also thank Kongphop Wongkaew, Gao Fei and Yuki Murakami for their excellent research assistance. The present project was supported by JSPS KAKENHI Grant Numbers 21K01394 and a grant-in-aid from TCER. Of course all remaining errors are our own.

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1 Introduction

Multiple product aspects of firms, establishments and plants are a salient feature of modern economies. The importance of simultaneously promoting the entry and exit of firms, and thus having a dynamic turnover, is often considered as a good indicator of "creative destruction" (Schumpeter (1942)). Little is known, however, about how firms, establishments or plants adjust their product portfolio over the business cycle. The current paper provides a novel theoretical model to explore this issue. Importantly, we distinguish products with respect to their income elasticities. Our model not only embeds the entry and exit of production units, but also allows the adjustment at product level within them. As we observe in data, firms, establishments or plants add and/or drop products and, hence, change their product mix over the business cycle. Our research questions are the following. What is the consequence of a policy that aims to enhance the entry of production units? Instead of regulating entry, what happens if regulation is made to maintain incumbent production units? What is the consequence of these policies on the product mix in the economy? How do firms or establishments in the economy react to a policy with which a particular product is targeted? The paper tackles these revived questions in a stylized DSGE model.

To start, using the underexplored Japanese data, the Current Survey of Production (*Seisan Dotai Tokei* in Japanese), we document extensively a multi-product aspect of establishments and firms and heterogeneous dynamics across products over the business cycle. Next, we provide a novel theoretical model that captures the multi product aspect of establishments and the asymmetric product dynamics based on different income elasticities across products. Along the business cycle, establishments or firms change their product mix by adding and/or dropping products depending on product specific profitability. The theoretical model embeds a number of exogenous processes. Specifically, we focus on the regulation shock at entry, incumbent establishment and for production of each product. The theoretical model is calibrated based on the parameter values used in the literature while the shock processes are estimated relying on the Bayesian methods.

Our findings and contributions are the following. First, we document the multiple product aspect of establishments and firms and how their product mix changes over the business cycle. The Japanese product dynamics measured at establishment level is isomorphic to those found in Broda and Weinstein (2010) measured at household level. Using their definition, we find that "creation" is procyclical and "destruction" is acyclical. Also, we demonstrate a substantial heterogeneity of business cycles across products, that depend on different income elasticities at the product level. Second, we provide a novel theoretical model that captures heterogeneous business cycles across products with establishments producing multiple products. We estimate the shock processes and entry adjustment costs for Japanese economy. Through simulation, we confirm that heterogeneous income elasticities are the source of heterogeneous fluctuations of products over the business cycle. Third, our paper sheds light on the policy outcome that aims to regulate the market. Specifically we consider (de)regulation policies at the entry, incumbent establishment and product level. We show that 1) the regulation policy that aims to increase the number of entrants results in wiping out the products with lower income elasticity. As a result, the product portfolio in the economy ends up putting a higher weight on the products with higher income elasticities. We also show that 2) the policy that aims to reduce operating costs for incumbent establishment increases the number of producing establishments. On the other hand, the operating establishments reduce their production lines and, hence, decrease the number of product varieties, in particular, that have a higher income elasticities. Another finding is that 3) product specific regulation induces a reallocation of resources from the regulated product toward unregulated products. The extent of reallocation is greater toward products with a lower income elasticity with which the product is protected.

Our paper is related to the literature that explores firm, establishment or plant entry and exit over the business cycles such as Bilbiie et al. (2012), Lee and Mukoyama (2015), Clementi and Palazzo (2016) and Hamano and Zanetti (2017). Specifically, the theoretical model in the paper is based on heterogeneous establishments and embeds endogenous product creation and destruction as Hamano and Zanetti (2017) as well as multi-product establishments *à* la Bernard et al. (2010). Further, following Matsuyama (2015) and

Comin et al. (2021), we introduce non-homothetic preferences with different income elasticities across products to capture heterogeneous product dynamics. Minniti and Turino (2013) provides a DSGE model that captures the dynamics of multiproduct firms from a different modeling strategy and research interest. From an empirical standpoint, our paper is related to DEKLE et al. (2015), Bernard and Okubo (2016) and Hamano and Okubo (2021) that document the business cycles of products based on a complementary plant level Japanese data source. We find a similar product dynamics as found in Broda and Weinstein (2010) and Ueda et al. (2019) that investigate product turnover using the data corrected at the household level in U.S. and Japan, respectively.

The remainder of the paper is organized as follows. Section 2 describes the CSP data. Section 3 documents multiproduct aspects of firms and establishments and heterogeneous business cycles across products. The theoretical model is provided in Section 4. Section 5 shows the calibration. Section 6 documents impulse response functions of the theoretical model induced with various types of shocks. Section 7 provides the result of Bayesian estimation and quantitative assessments of the simulated model. Section 8 concludes.

2 Data

The Current Survey of Production (henceforth, CSP, *Anise Dotai Tokei* in Japanese) surveys the production status of materials and manufacturing products produced within manufacturing establishments on a monthly basis. The origin of the CSP can be found in the Monthly Textile Production Survey (*Orimono Tsukibetsu Sangaku Chosa* in Japanese) starting in 1927. Subsequently, the Monthly Survey of Important Production (*Jyuyou Anise Tsukibetsu Chosa* in Japanese) started in June 1930, surveying the production status of important products on a monthly basis. After WWII, the CSP started in January 1943 to grasp the production status and adjust the supply and demand of products and materials at the request of the Supreme Commander for the Allied Powers.

The CSP asks information about production value and quantity, sales value and quantity, inventory, machinery, equipment, production capacity, and the number of workers at the end of the month for each product produced by each establishment. The CSP covers approximately 12,000 manufacturing establishments and 1,600 categories of products defined at the seven digit level. Establishments are required to complete the questionnaire by law under the Japanese Statistics Act. Ministry of Economy, Trade and Industry, conducted the survey. The response rate of the CSP is approximately 94 percent. Establishments to be surveyed vary depending on both the industry and the number of employees. For example, establishments producing midget passenger cars, which are classified as the motor vehicle industry, with more than 50 workers are required to answer the CSP questionnaires. In the case of music instruments, establishments with more than 20 workers are required to respond. The details of the scope of surveyed establishments by industry are shown in Table 7 in the Appendix.

In principle, the CSP covers establishments producing products whose total production or sales values are large in the Census of Manufacturers (hereafter, CMF) which is conducted annually and covers manufacturing establishments with more than 4 workers. Note that the CMF is considered as one of the most comprehensive survey of manufacturers in the Japanese economy.[1] The total sales value in the CSP is approximately one-quarter of the value in the CMF.[2] This is not attributable only to the simple coverage rate of manufacturing establishments, but does not cover the food manufacturing, pharmaceuticals, or medical equipment manufacturing industries.[3] The products to be surveyed are revised every year. The basic criterion for the revision is that products

[1] The CMF started as a census for collecting information about manufacturing plants with 5 or more employees. The CMF from 1939 to 1980 was conducted on all manufacturing plants. The CMF from 1981 to 2009 was conducted on all manufacturing plants in years ending with 0, 3, 5, and 8. Since 2010, the CMF has been conducted on manufacturing plants with 4 or more employees. The Economic Census for Business Activity, which collects information about all establishments in all industries in Japan, is conducted every five years.

[2] The total sales value in the 2017 CSP is 74,132,212 million yen while the value in the 2017 CMF is 283,062,628 million yen.

[3] The current survey of production for the food manufacturing products is conducted by the Ministry of Agriculture, Forestry and Fisheries. The current survey of production for the pharmaceuticals and medical equipment manufacturing products is conducted by the Ministry of Health, Labour and Welfare.

whose annual shipment values in the CMF are less than 10 billion yen are to be excluded. Meanwhile, the product items to be added are the ones whose annual shipment values are more than 100 billion yen. Therefore, the products covered in the CSP tend to be the core lines of manufacturing products in the Japanese economy.

Establishments are required to fill out one or several types of questionnaires, depending on their products. Each questionnaire is prepared for each of the 117 manufacturing industries in the 2017 CSP. For example, an establishment producing only midget passenger cars has to answer to the questionnaire for the motor vehicle industry. In the case of another establishment producing both midget trucks and excavators, it must answer to the questionnaires for both the motor vehicle industry and the construction equipment industry. Table 8 in the Appendix reports the number of products covered in the CSP by the 2-digit level industrial classifications of the Japan Standard Industrial Classification (hereafter JSIC).[4] Following the definition of Bernard et al. (2010), we refer to the 2-digit level JSIC classifications as "sectors" and 4-digit level JSIC classification as "industries". We also add information on the number of products covered in the CMF for the purpose of comparison. As mentioned above, the food sector and the beverage, tobacco, and feed sectors are excluded from the coverage of CSP. The number of products whose data on production quantity is the largest among the categories of data. Meanwhile, using the information on sales values is a good choice for both maximizing the number of products and obtaining product values.

Each questionnaire for the 117 manufacturing industries has its list of products. Table 9 in the Appendix gives an example of product classifications of the CSP in a form that is comparable with those of the CMF. As mentioned above, the industrial classification of the CMF is consistent with the JSIC. The first and second columns of the tables are the sector-industry-product classification codes of the CMF and their classification category names. The transportation equipment sector (CMF 31) contains the motor vehicles, including the motorcycle industry (CMF 3111) which contains 9 products larger than 125 ml., including

[4]The JSIC is consistent with the International Standard Industrial Classification of All Economic Activities (ISIC).

those with side cars and motor scooters (CMF 311118). Meanwhile, the motor vehicle industry, including motorcycle industry of the CSP, contains 15 product categories, such as midget, small, and large passenger cars. As shown in Table 8, the number of CSP product categories belonging to core Japanese manufacturing industries, such as general-purpose machinery, production machinery, and electrical machinery, equipment and suppliers, tends to be larger than the number of the CMF product categories.

3 Stylized Facts

We construct a dataset from the CSP which is able to track when new products enter the market, when old products exit the markets, and how establishments or firms change their product portfolios from 2000Q1 to 2017Q4.[5] We aggregate the original monthly data into quarterly data. The numbers of sector, industry, and product categories of our dataset are 18, 73, and 905, respectively. The number of product categories in our dataset is smaller than the approximately 1,600 original product categories since we 1) use the products registered with their sales values and 2) remove from the dataset both of the product categories that were newly-surveyed and dropped during the periods from 2000Q1 to 2017Q4 and, hence, use only the continuously observed product categories to avoid noise created by newly-surveyed and dropped products.

Fact 1: Firms or establishments may produce multiple products in different sectors/industries.

Table 1 reports descriptive statistics of our dataset by single and multiple product establishments and firms in 2017. The share of multiproduct (MP) establishments is 50 percent while multisector (MS) and multi-industry (MI) establishments account for 9 and 6 percent, respectively. We call an establishment-product combination a "product variety". The last column in the table reports the number of product varieties depending on establishment characteristics. In 2017, the total number of product varieties was 16,366 of which 13,525 number of product varieties are produced by MP establishments. The table reports the characteristics of not only establishments but also firms in 2017. We

[5] The establishment- and firm-level data of the CSP is available only from January 2000 to the present. We apply the secondary use of official statistics to the Ministry of Economy, Trade, and Industry.

observe a similar pattern for firms: almost the half of firms produce more than one product category.

Table 1: Descriptive Statistics by Single- and Multiple-Product Establishments and Firms in 2017

	Units		Sales		Employees		Products	
	Number	%	Value (mil. JPY)	%	Number	%	Number	%
(1) Establishments								
Single product	3,214	50	12,672,987	24	605,297	38	3,214	20
Multiple product	3,153	50	39,448,775	76	1,002,972	62	13,152	80
Multiple industry	460	7	13,989,518	27	306,627	19	2,292	14
Multiple sector	248	4	8,086,380	16	188,940	12	1,039	6
Total	6,367	100	52,121,762	100	1,608,269	100	16,366	100
(2) Firms								
Single product	2,306	47	5,815,738	11	366,235	23	2,306	14
Multiple product	2,595	53	46,306,024	89	1,242,035	77	14,060	86
Multiple industry	453	9	27,405,368	53	663,260	41	4,179	26
Multiple sector	308	6	21,204,771	41	543,202	34	2,838	17
Total	4,901	100	52,121,762	100	1,608,269	100	16,366	100

Notes: A single product plant (firm) is a plant (firm) that produces only one type of product at the 6-digit JSIC level. A multiple industry plant (firm) is a plant (firm) that is active in multiple industries (4-digit JSIC level). A multiple sector plant (firm) is a plant (firm) that is active in multiple sectors (2-digit JSIC level).

Table 1 also reports the sales and employment shares of establishments and firms. The share of Multi Product (MP) establishments is 50 percent, while the sales share of MP establishments is 76 percent. MI and MS establishments are only 7 and 4 percent whereas they account for 27 percent and 16 percent of sales, respectively. The percentages of MP, MI, and MS firms are almost the same as of MP, MI, and MS establishments. Meanwhile, the sales shares of MP, MI, and MS establishments are smaller than those of MP, MI, and MS firms. Needless to say, this is because an establishment is a production unit of a firm and a firm can have several establishments. A firm can increase its production scale not only by increasing the scale of its incumbent establishments but by operating a new establishment. Similar patterns are observed for both the number of employees and products. Further, Table 2 presents the mean, median, and several percentiles of the number of products per establishment or firm in 2017. The mean of all establishments is 2.6 products while the top 1 percent of establishments have more than 21 products. The mean of all firms is 3.3, which is slightly larger than that of all establishments, while the top 1 percent firms has more than 33, witch is much larger than of establishments. Regarding MI or MS firms, the mean is 9.2 and the top 1 percent MI or MS firms have more than 68 number of products.[6]

[6] Our dataset shows similar results shown by the dataset in Bernard et al. (2010). According to their U.S. dataset, the share of MP firms in 1997 is 39 percent, indicating that MP firms are not majority. Meanwhile, the sales share of MP firms is 87 percent, which implies that MP firms dominate the manufacturing market.

Table 2: Number of Products per **Establishment**/Firm in 2017

	Mean	Median	p75	p90	p95	p99
(1) Establishments						
All	2.6	1	3	5	7	21
Multiple product	4.2	3	4	7	12	32
Multiple industry	5.0	4	6	9	13	22
Multiple sector	4.2	3	5	8	9	18
(2-1) Firms						
All	3.3	2	3	6	11	33
Multiple product	5.4	3	5	11	18	34
Multiple industry	9.2	4	9	22	32	68
Multiple sector	9.2	5	10	22	32	68
(2-2) Firms -- Number of plants						
All	1.3	1	1	2	3	6
Multiple product	1.6	1	2	3	4	8
Multiple industry	2.6	2	3	5	8	14
Multiple sector	2.8	2	3	6	8	15

Notes: A multiple product plant (firm) is a plant (firm) that produces more than one type of product at the 6-digit JSIC level. A multiple industry plant (firm) is a plant (firm) that is active in multiple industries (4-digit JSIC level). A multiple sector plant (firm) is a plant (firm) is active in multiple sectors (2-digit JSIC level).

Fact 2: Product creation is procyclical while product destruction is acyclical.

Figures 1–1c show the cyclicality of product creation and destruction at macro level. Product creation (destruction) is defined as the real value of new (dropped) products in period t relative to period $t-4$ divided by the total value of products in period $t-4$, following the definition of Broda and Weinstein (2010). Creation, destruction and total sales growth thus have the following relationship:

$$\underbrace{\frac{\mathcal{Y}_t - \mathcal{Y}_{t-4}}{\mathcal{Y}_{t-4}}}_{\text{Total sales growth}} = \underbrace{\frac{NEW_t}{\mathcal{Y}_{t-4}}}_{\text{Creation}} - \underbrace{\frac{DROP_t}{\mathcal{Y}_{t-4}}}_{\text{Destruction}} + \underbrace{\frac{COM_t - COM_{t-4}}{\mathcal{Y}_{t-4}}}_{\text{Common products growth}},$$

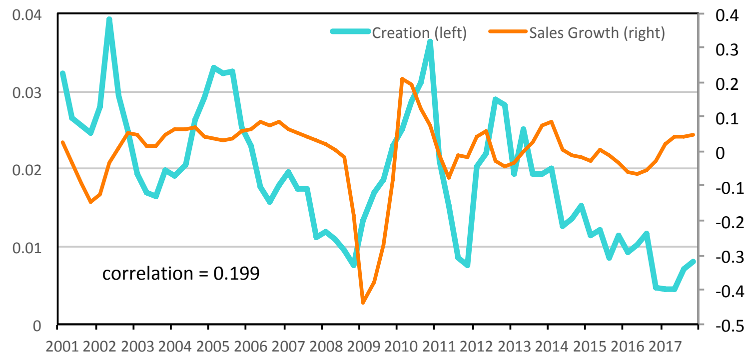
where $\mathcal{Y}_t$ is the value of total sales of all the products in period t, NEW_t is the value of new products in period t relative to period $t-4$, $DROP_t$ is the value of dropped products in period t relative to period $t-4$, and COM_t (COM_{t-4}) is the value of products in period t ($t-4$) produced in both periods (common products). At the macro level, the cyclicality of creation has a relatively positive correlation with that of total sales growth given that their correlation coefficient is 0.199. Meanwhile, the correlation coefficient between destruction and total sales growth is 0.001, and it seems that destruction is not correlated with total sales growth rates. Since creation is procyclical and destruction is acyclical, net creation, or creation minus destruction, has a positive correlation with total sales growth.

Further, Figure 2 shows the number of new establishments H, the number of producing establishments S, the number of inoperative establishments D, and the number of product varieties M, together with the growth rate of the total sales and total employment from a year ago implied by the CSP. There are 855 products that have complete registration among 905 products over the sample period of 2000Q1 to 2017Q4. Throughout the sample period, on average, we have 7,986 number of establishments, 7,383 producing establishments and 18,560 product-varieties every quarter. While the previous measure of "product creation" shows procyclical pattern with total sales growth rate, the contemporaneous correlation of the growth rate of the number of producing establishments S and that of establishment entry H with total sales growth are countercyclical showing the correlation of -0.30 and -0.30, respectively. The growth rate of establishment exit D is countercyclical with a correlated total sales growth of -0.62. Further, the number of products produced per establishment M/S is 2.43 on average and the number of product varieties M is procyclical showing its correlation with total sales growth of 0.25. Compared to substantially volatile entry H and exit D, the number of product varieties M is less volatile. The standard deviation of entry H and exit D are 62.2 and 23.6, respectively while that of the number of product varieties is 0.99. However, this does not necessarily mean that establishments keep the same product mix over the business cycles.[7]

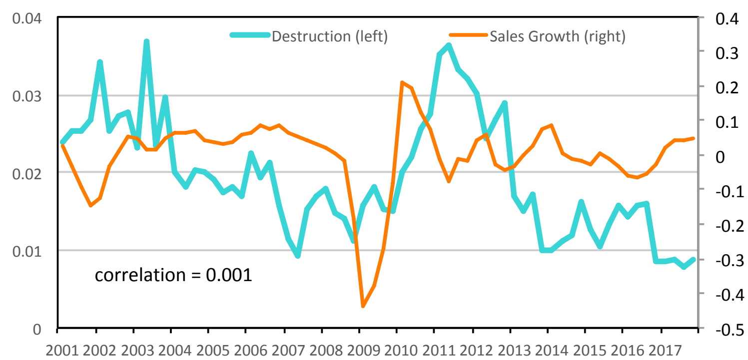
[7]The Japanese product dynamics measured at establishment level is isomorphic to those found in Broda and Weinstein (2010) measured at household level. Using their definition, "creation" is procyclical

Figure 1: Product Creation and Destruction

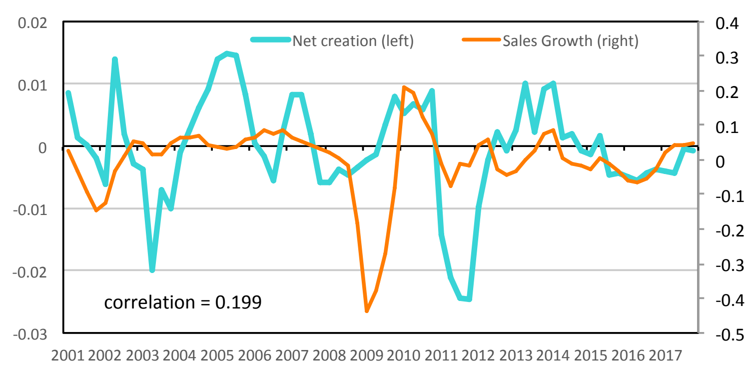
(a) Product Creation



(b) Product Destruction

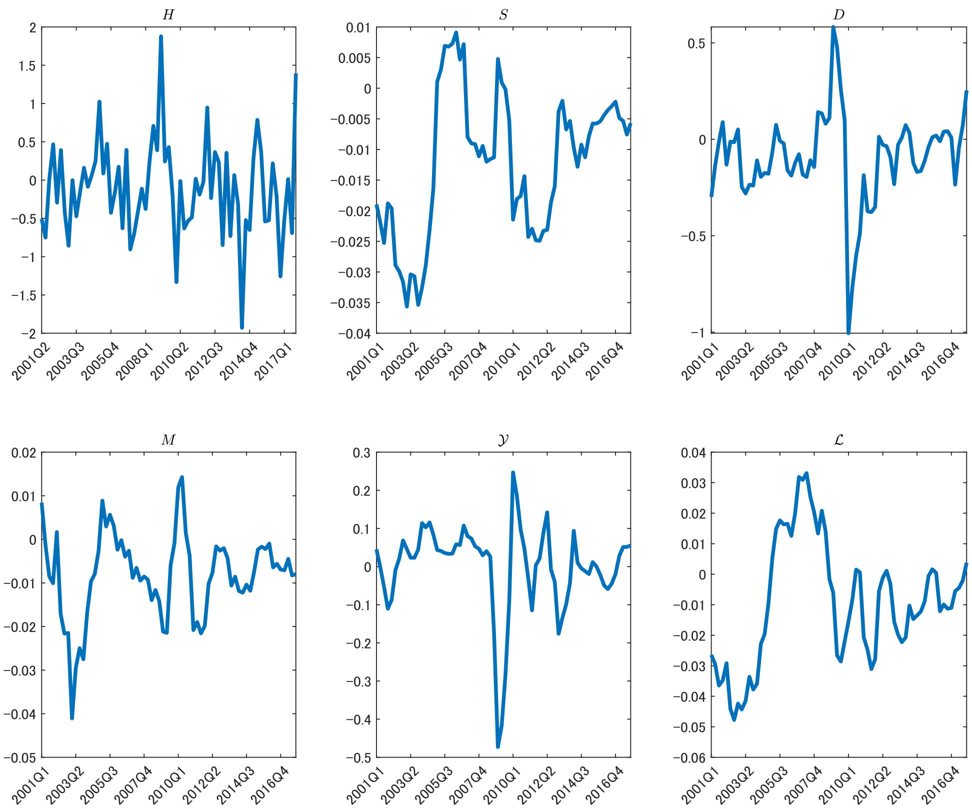


(c) Net Creation



The figure shows cyclicality of product creation, destruction and net creation implied by the Current Survey of Production. Product creation (destruction) is defined as the growth rate of real value of new (dropped) products from a year ago.

Figure 2: Establishment and Product Dynamics in Japan



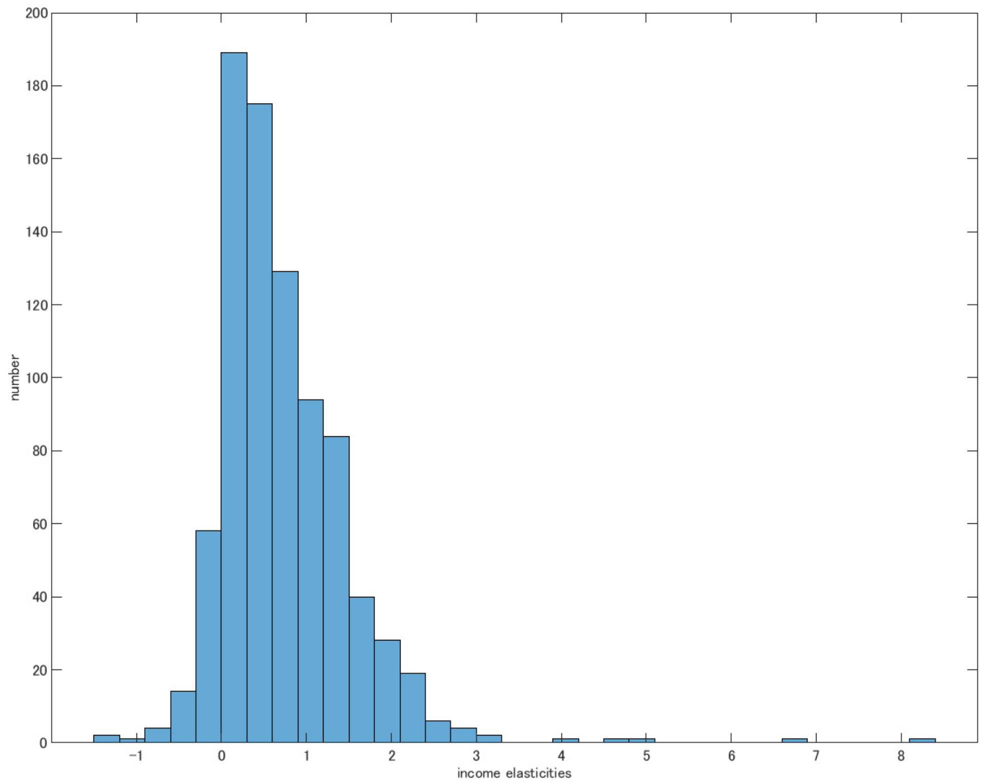
The figure shows the number of new establishments H, the number of producing establishments S, the number of inoperative establishments D, and the number of product-varieties M, together with the growth rate of total sales $\mathcal{Y}$ and total employment $\mathcal{L}$ from a year ago implied by the Current Survey of Production.

Fact 3: There is a substantial heterogeneity over the business cycles across products.

Behind the above mentioned aggregate patterns, indeed, there exists substantial heterogeneity across products. Figure 3 documents the histogram of the elasticities of real sales growth of products with respect to real total sales growth. These elasticities are estimated separately for each product category. They range from 38.88 to -1.347, with a medium of 0.5633. We categorize products into three groups upon the ranking of the income elasticities from the highest to the lowest. The medium income elasticity of the first group is 1.347 while that of the second and the third group are 0.563 and 0.1346, respectively. The number of product varieties M_1 in the first group is 4,473 while that of

and "destruction" is countercyclical. See Figure A1 in the Appendix.

Figure 3: Estimated income elasticities of products

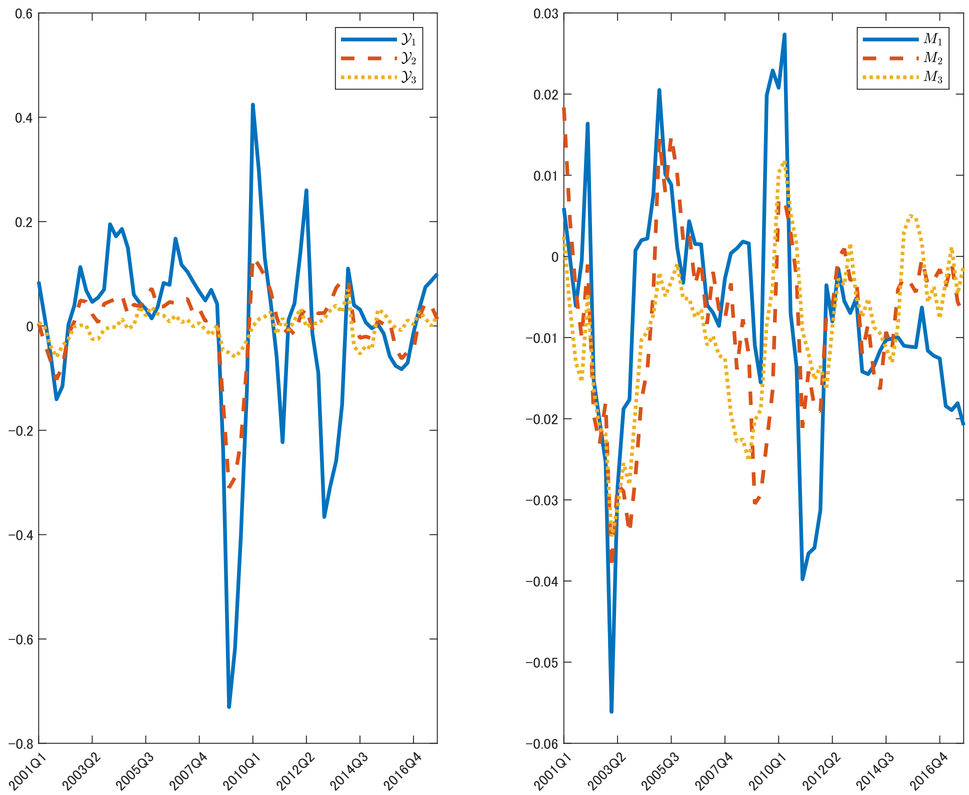


The figure shows the histogram of the estimated income elasticities of 905 products registered in the CSP data.

the second M_2 and the third group M_3 are 5,770 and 7,675, respectively every quarter on average. Thus the number of products per establishment for each product group (M_1/S, M_2/S and M_3/S) amounts to 0.61, 0.78 and 1.04, respectively for each quarter on average.

Figure 4 documents the cyclical pattern of product groups. The real sales of the first group $\mathcal{Y}_1$ is the most volatile with its standard deviation of 18.40 and show a strong comovement with real total sales with its correlation of 0.99. The standard deviation of the real sales of the second group $\mathcal{Y}_2$ and its correlation with total sales are lower, at 7.78 and 0.87, respectively while those for the third group $\mathcal{Y}_3$ are the lowest, at 0.68 and 0.49, respectively. Compared to the sales growth, the number of product varieties in each group shows a lower volatility. The standard deviation of the number of product varieties in the first group M_1 is the highest with 1.49, in the second group M_2 it is 1.18 and the

Figure 4: Heterogeneity across product groups



The figure shows the growth rate of the total sales ($\mathcal{Y}_1$,$\mathcal{Y}_1$ and $\mathcal{Y}_3$)and the number of product-varieties of each product group (M_1, M_2 and M_3) from a year ago implied by the Current Survey of Production.

third group M_3 is the lowest with 0.95. Furthermore, these numbers of product varieties uniformly show a procyclical pattern: the correlation of the first group product varieties with the sales growth is 0.10, that of the second group is 0.33 and that of the third group is 0.20.

What kind of products are categorized in each income elasticity group? Figure 5 ranks 18 sector-level categories of products according to the three product groups' relative concentration. For instance, the high-income elasticity group has 5.0% of products categorized into the transportation equipment sector, while the whole economy has 2.9% of products classified in the same sector. In this case, the high-income elasticity product group's relative concentration is 1.7, obtained by dividing 5.0% with 2.9%, meaning the transportation equipment products are 1.7 times more concentrated in the high-income elasticity group than the whole economy on average. As seen in Figure 5, investment and durable goods and their intermediates are more concentrated in the high-income elasticity group than in the whole economy. The most concentrated product category in the high-income elasticity group is electronic parts/devices, widely used in investment and durable products. The relative concentration coefficient is 2.4, which means that electronic parts/devices are 2.4 times more concentrated in the high-income elasticity group than in the economy as a whole, on average. Non-ferrous metals/products are the second most concentrated product category in the high-income elasticity product group. In fact, aluminum products are heavily used in transport equipment, including automobiles. The third, fourth, and fifth most concentrated product categories are production machinery, transportation equipments, and general-purpose machinery, respectively. These product categories are also typical investment or durable goods. In contrast, the most concentrated product category in the low-income elasticity group is textile products, which is the least concentrated in the high-income elasticity group.

In what follow, we built a theoretical model that captures the multiple product aspects of establishments and heterogeneous business cycles across products, as documented.

4 The Model

We employ the terms, "firms", "establishments" and "plants" interchangeably. The model embeds endogenous product creation and destruction as Hamano and Zanetti (2017) as well as multi-product establishments *à* la Bernard et al. (2010)). There is an exogenously determined J number of product categories. Within each product category i, there is $M_{i,t}$ number of product varieties (defined as a combination of firm and product) which is endogenously determined. In each period, there is H_t mass of new entrant firms. Upon entry, firms draw an idiosyncratic productivity φ and a specific taste λ_i for each product i. Among total N_t number of firms only a subset of S_t number of firms are operating, incurring fixed headquarter cost. Firms may produce multiple product varieties depending

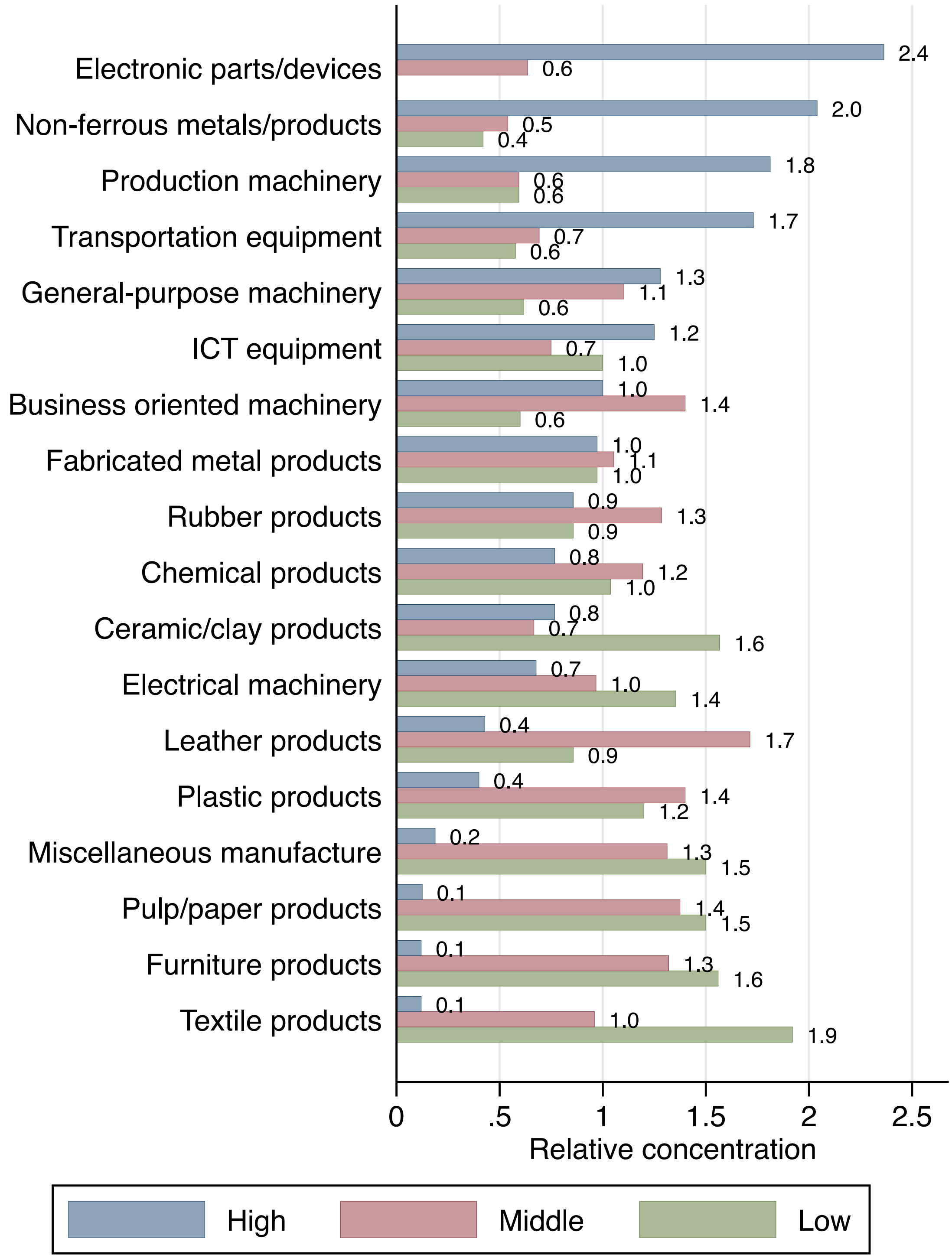


Figure 5: Product Groups' Relative Concentration of Products by Sector-Level Product Categories

on the profitability of each product. Further, to capture the different cyclical pattern across products, we introduce non-homothetic preferences following Matsuyama (2015) and Comin et al. (2021). Because of different income elasticities, some products are more in demand than others over the business cycles. Accordingly to different profitabilities across products, establishments adjust their product mix.

4.1 Households

The representative household maximizes expected utility, $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t(j)$, where $0 < \beta < 1$ is the exogenous discount factor. The utility of each individual household j at time t depends on consumption $C_t(j)$ and the supply of labor $L_t(j)$, as follows:

$$U_t(j) = A_t \ln C_t(j) - \chi_t \frac{L_t(j)^{1+\varsigma}}{1+\varsigma},$$

where A_t is an exogenous demand shifter at time t. $\chi_t > 0$ represents the disutility of supplying labor, and $\varsigma > 0$ is the inverse of the Frisch elasticity of labor supply.

Following Matsuyama (2015) and Comin et al. (2021), the consumption is defined with *implicitly, additively separable with constant elasticity of substitution (CES)*, introduced by Hanoch (1975) as

$$\left(\sum^{J} C_t(j)^{\frac{\epsilon_i - \nu}{\nu}} \alpha_i^{\frac{1}{\nu}} C_{i,t}(j)^{1-\frac{1}{\nu}} di \right)^{\frac{1}{1-\frac{1}{\nu}}} \equiv 1,$$

α_i stands for the preference weight on each product and ν represents the elasticity of substitution across products. ϵ_i stands for the demand elasticity of product i with respect to the aggregate consumption basket $C_t(j)$, i.e., "income elasticity". It is assumed that $\epsilon_i > \nu$ when $0 < \nu < 1$ or $0 < \epsilon_i < \nu$ for $\nu > 1$ which implies $(\epsilon_i - \nu) / (1 - \nu) > 0$. These parameters restrictions ensure that $C_t(j)$ is globally monotone increasing and globally quasi-concave in $C_{i,t}(j)$.

Each product variety is defined over a continuum of product varieties, Ω, and during each period t, only a subset of product varieties, $\Omega_t \subset \Omega$, is available. Each product variety is indexed by $\omega \in \Omega_t$. The consumption aggregator is

$$C_{i,t}(j)=\left(\int_{\omega\in\Omega_t}\left(\lambda_i\left(\omega\right)c_{i,t}\left(j,\omega\right)\right)^{1-\frac{1}{\sigma}}d\omega\right)^{\frac{1}{1-\frac{1}{\sigma}}},$$

where $c_t\left(\omega\right)$ is individual demand for each product variety ω. $\lambda_i\left(\omega\right)$ is taste or "quality" of each product variety. In particular, $\sigma>1$ is the elasticity of substitution among varieties. We assume that $\sigma>\nu>0$.

The demand for each product variety, ω, is

$$\lambda_i\left(\omega\right)c_{i,t}\left(j,\omega\right)=\left(\frac{p_{i,t}\left(\omega\right)/\lambda_i\left(\omega\right)}{P_{i,t}}\right)^{-\sigma}C_{i,t}(j), \tag{1}$$

where $p_t\left(\omega\right)$ denotes the price of variety ω. The price index of each product is

$$P_{i,t}=\left(\int_{\omega\in\Omega_t}\left(\frac{p_{i,t}\left(\omega\right)}{\lambda_i\left(\omega\right)}\right)^{1-\sigma}d\omega\right)^{\frac{1}{1-\sigma}}, \tag{2}$$

The relative expenditure in each product basket is found to be

$$C_{i,t}(j)=\left(\frac{P_{i,t}}{P_t}\right)^{-\nu}\alpha_iC_t(j)^{\epsilon_i} \tag{3}$$

where price index of aggregate basket $C_t(j)$ is also implicitly defined as[8]

$$P_tC_t=\left(\sum^{J}P_{i,t}^{1-\nu}\alpha_iC_t(j)^{\epsilon_i-\nu}di\right)^{\frac{1}{1-\nu}} \tag{4}$$

Equation (2) is consistent with a welfare-basis index and shows that for a given variety ω, the price index rises (decreases) when the number of available varieties decreases (rises). We choose P_t as numeraire.

[8]Note that by setting "income elasticity" $\epsilon_i=1$, the expression collapses to the standard demand function and price index.

The demand of each product basket is

$$C_{i,t}(j)=\left(\frac{P_{i,t}}{P_t}\right)^{-\nu}\alpha_iC_t(j).$$

The price index that minimizes the consumption expenditure is

$$P_t=\left(\int_0^1\alpha_iP_{i,t}^{1-\nu}di\right)^{\frac{1}{1-\nu}}.$$

4.2 Production, Pricing and Producing Decision

Each establishment can produce more than one product. The number of products within a establishment is endogenously determined. In each period, a number of new establishments, H_t, enters in the market. Prior to entry, these new establishments are identical and face a sunk entry cost of $f_{E,t}$ in effective labor units. The entry cost is therefore equal to $w_t f_{E,t}$ units of consumption goods where w_t stands for real wage. Upon entry, each establishment draws a productivity level, φ, from a cumulative distribution, $G(\varphi)$, with support on $[\varphi_{\min}, \infty)$ and consumer taste level for each product, λ_i, from a cumulative distribution, $Z_i(\lambda_i)$, with support on $[\lambda_{i\min}, \infty)$.

The production of product i requires a fixed operational cost of $f_{i,t}/Z_t$ in effective labor units for each product in every period. Furthermore, establishments are required to pay a headquarter fixed operational cost also defined in terms of effective labor units as $f_{h,t}/Z_t$. Total labor demand for a establishment with productivity level φ is thus given by

$$l_t(\varphi) = \sum^{J} I_i \left[\frac{y_{i,t}(\varphi, \lambda_i)}{Z_t \varphi} + \frac{f_{i,t}}{Z_t} \right] di + \frac{f_{h,t}}{Z_t}. \tag{5}$$

I_i is indicator that takes 1 if the establishment produces product i otherwise 0.

4.2.1 Product Production

Demand of each establishment specific product variety is characterized by equation (1). Profit maximization yields the following optimal price:

$$\rho_{i,t}(\varphi, \lambda_i) = \frac{\sigma}{\sigma - 1} \frac{w_t}{Z_t \varphi}, \tag{6}$$

where $\rho_{i,t}(\varphi, \lambda_i)$ stands for the real price of product i produced by establishment with productivity φ which has drawn a consumer taste λ_i for this product. Depending on the level of product-specific productivity, φ, and consumer taste λ_i, a product may or may not be produced. Thus, using equation (5), (6) and (3), if production materializes, the following real operational establishment-product specific profits are generated:

$$d_{i,t}(\varphi, \lambda_i) = \frac{1}{\sigma} \left(\frac{\rho_{i,t}(\varphi, \lambda_i)}{\lambda_i} \right)^{1-\sigma} \rho_{i,t}^{\sigma-\nu} \alpha_i \int_0^1 C_t(j)^{\epsilon_i} dj - w_t \frac{f_{i,t}}{Z_t}.$$

where $\rho_{i,t} \equiv \frac{P_{i,t}}{P_t}$, which is the real price of the basket of product i. Since the elasticity of substitution among varieties is assumed to be more than unitary ($\sigma > 1$), a lower taste-adjusted real price implies higher profits.

Total operational profits of producing establishment with productivity φ is thus given by

$$d_{s,t}(\varphi) = \sum^{J} I_i d_{i,t}(\varphi, \lambda_i)\, di - w_t \frac{f_{h,t}}{Z_t}.$$

4.3 Product Entry and Exit

We assume that products entered at time t only start producing at time $t+1$. These products are discounted by the stream of their expected profits $\left\{\widetilde{d}_{s,k}\right\}_{k=t+1}^{\infty}$, using the stochastic discount factor of households adjusted by exogenous exit inducing shock δ. Thus, their expected post entry value is

$$v_t = E_t \sum_{k=t+1}^{\infty} \left[\beta (1-\delta)\right]^{k-t} \left(\frac{\Lambda_t}{\Lambda_k}\right) \widetilde{d}_k, \tag{7}$$

which represents the share price of equities and mutual funds across different products. Λ_k is a discount factor of the representative household which is defined below. Product entry occurs until the expected product value (7) is equal to the entry cost, leading to the free entry condition,

$$v_t = w_t \frac{f_{E,t}}{Z_t} \left(\frac{H_t}{H_{t-1}}\right)^{\theta_E}. \tag{8}$$

The timing of entry and of production implies that the number of products evolves according to the law of motion:

$$N_t = (1-\delta)\left(N_{t-1} + H_{t-1}\right). \tag{9}$$

Among N_t number of potential producers, only a subset number of S_t firms produce. Each product variety i may or may not be produced by firm with productivity φ. Only products with $d_{i,t}(\varphi) > 0$ are produced. For firm with productivity φ, there exists a zero profit consumer taste cutoff $\lambda^*_{i,t}(\varphi)$ for product i as

$$d_{i,t}\left(\varphi, \lambda_{i,t}^{*}\left(\varphi\right)\right) = \frac{1}{\sigma}\left(\frac{\rho_{i,t}\left(\varphi, \lambda_i\right)}{\lambda_{i,t}^{*}\left(\varphi\right)}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\nu}\alpha_i \int_0^1 C_t(j)^{\epsilon_i} dj - w_t\frac{f_{i,t}}{Z_t} = 0. \tag{10}$$

Also firm with productivity φ produces only when $d_{s,t}\left(\varphi\right) > 0$ from which we can determine zero profit productivity cutoff φ_t^* as

$$d_{s,t}\left(\varphi_t^*\right) = \sum^{J} \int_{\lambda_{i,t}^*(\varphi_t^*)}^{\infty} d_{i,t}\left(\varphi_t^*, \lambda_i\right) dZ_i(\lambda_i) di - w_t\frac{f_{h,t}}{Z_t} = 0.$$

4.4 Product Average

A specific average of productivities weighted by consumer tastes of all producers for product i is defined following Bernard et al. (2010), as in their technical Appendix:

$$\widetilde{\varphi}_{i,t} \equiv \left[\int_{\varphi_t^*}^{\infty} \widetilde{\lambda}_{i,t}(\varphi)\frac{dG(\varphi)}{1-G(\varphi_t^*)}\right]^{\frac{1}{\sigma-1}}, \text{where} \quad \widetilde{\lambda}_{i,t}(\varphi) \equiv \int_{\lambda_{i,t}^*(\varphi)}^{\infty} (\lambda_i\varphi)^{\sigma-1}\frac{dZ_i(\lambda_i)}{1-Z_i(\lambda_{i,t}^*(\varphi))}.$$

In the above expression $\widetilde{\lambda}_{i,t}(\varphi)$ stands for the average productivity-weighted taste of product i for establishment with productivity φ. It summarizes the space of the taste that is capable for production of product i by firm with φ. The term, $\widetilde{\varphi}_{i,t}$ thus contains all the information about the distribution of productivities and consumer tastes. In short, it can be interpreted as the taste-weighted-average productivity of product i. Using this taste weighted average productivity, the taste-adjusted real price for product i of average producer is defined as

$$\widetilde{\rho}_{i,t} = \frac{\sigma}{\sigma-1}\frac{w_t}{Z_t\widetilde{\varphi}_{i,t}}.$$

Based on this real price, we can define average profits for each product i as

$$\widetilde{d}_{i,t} = \frac{1}{\sigma}\frac{\rho_{i,t}\int_0^1 C_{i,t}(j)dj}{M_{i,t}} - \frac{w_t f_{i,t}}{Z_t} \tag{11}$$

In the above expression of average profits, we have used the demand for the basket of product i, $C_{i,t}(j) = \rho_{i,t}^{-\nu}\alpha_i C_t(j)^{\epsilon_i}$ and the price index of each product basket i, $\rho_{i,t}^{1-\sigma} = M_{i,t}\widetilde{\rho}_{i,t}^{1-\sigma}$. Similarly, the average real profits among surviving producers are expressed as follows:

$$\widetilde{d}_{s,t} = \sum^{J} \frac{M_{i,t}}{S_t} \widetilde{d}_{i,t} di - w_t \frac{f_{h,t}}{Z_t} \tag{12}$$

Finally, we define the average operational profits among potential producers:

$$\widetilde{d}_t = (S_t/N_t)\, \widetilde{d}_{s,t} \tag{13}$$

where we have used the fact hat$M_{i,t} = \int_{\varphi_t^*}^{\infty} \left[1 - Z_i(\lambda_{i,t}^*(\varphi))\right] \frac{dG(\varphi)}{1-G(\varphi_t^*)} S_t$ and $S_t = [1 - G(\varphi_t^*)]\, N_t$.[9]

4.5 Parametrization of Productivity and Taste Draw

To solve the model, we must assume a distribution of productivity levels, φ and λ_i. We assume the following Pareto distribution for $G(\varphi)$ and $Z_i(\lambda_i)$, respectively as

$$G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^{\kappa}, \quad Z_i(\lambda_i) = 1 - \left(\frac{\lambda_{i\,\min}}{\lambda_i}\right)^{\upsilon}$$

where $\varphi_{\min}$ and $\lambda_{i\,\min}$ are the minimum productivity level and κ and υ determine the shape of the distribution. The parameter κ and υ index the dispersion of productivity across products. The dispersion decreases as these parameters increase, and the productivity or tastes are concentrated toward the lower bound $\varphi_{\min}$ and $\lambda_{i\,\min}$. We set $\varphi_{\min} = \lambda_{i\,\min} = 1$ without loss of generality. To ensure that variance of the productivity distribution are finite and that the number of products is positive, we assume that $\kappa > \upsilon > \sigma - 1$. With this parametrization, we can express the taste-weighted-average productivity, $\widetilde{\varphi}_{i,t}$, in equation as[10]

$$\widetilde{\varphi}_{i,t} = \left[\frac{\upsilon}{\upsilon - (\sigma - 1)}\right]^{\frac{1}{\sigma-1}} \varphi_t^* \lambda_{i,t}^*(\varphi_t^*),$$

[9] See Appendix A for detailed derivation.

[10] Using the zero profits consumer taste cutoff (10) for establishment with productivity φ_t^*, the consumer taste cutoff of establishment with productivity φ_t, i.e., $\lambda_{i,t}^*(\varphi)$, can be expressed as a function of cutoff productivity level φ_t^* and the consumer taste cutoff of this cutoff firm $\lambda_{i,t}^*(\varphi_t^*)$ as $\lambda_{i,t}^*(\varphi) = \frac{\varphi_t^*}{\varphi} \lambda_{i,t}^*(\varphi_t^*)$. The expression has an intuitive interpretation. The cutoff consumer taste of a firm is decreasing with respect to her own productivity because it allows to produce even with a lower end of taste. It is increasing with respect to φ_t^* and $\lambda_{i,t}^*(\varphi_t^*)$ since a higher value of each intensifies the competition. The above characteristic in turn means that the average taste-weighted productivity $\widetilde{\varphi}_{i,t}$ is expressed in terms of φ_t^* and $\lambda_{i,t}^*(\varphi_t^*)$. Specifically, with the Pareto distribution as in the paper, $\widetilde{\lambda}_{i,t}(\varphi) = \frac{\upsilon}{\upsilon-(\sigma-1)} \left[\varphi_t^* \lambda_{i,t}^*(\varphi_t^*)\right]^{\sigma-1}$ and thus $\widetilde{\varphi}_{i,t}^{\sigma-1} = \int_{\varphi_t^*}^{\infty} \widetilde{\lambda}_{i,t}(\varphi) \frac{dG(\varphi)}{1-G(\varphi_t^*)} = \frac{\upsilon}{\upsilon-(\sigma-1)} \left[\varphi_t^* \lambda_{i,t}^*(\varphi_t^*)\right]^{\sigma-1} \int_{\varphi_t^*}^{\infty} \frac{dG(\varphi)}{1-G(\varphi_t^*)} = \frac{\upsilon}{\upsilon-(\sigma-1)} \left[\varphi_t^* \lambda_{i,t}^*(\varphi_t^*)\right]^{\sigma-1}$

and the fraction of surviving products as

$$\frac{M_{i,t}}{S_t} = \frac{\kappa}{\kappa - \upsilon} \lambda^*_{i,t} (\varphi^*_t)^{-\upsilon}, \quad \frac{S_t}{N_t} = \varphi^{*-\kappa}_t. \tag{14}$$

By combining (??) and (14), we get

$$\widetilde{\varphi}_{i,t} = \left[\frac{\upsilon}{\upsilon - (\sigma - 1)} \right]^{\frac{1}{\sigma - 1}} \left(\frac{S_t}{N_t} \right)^{-\frac{1}{\kappa}} \left(\frac{M_{i,t}}{S_t} \frac{\kappa - \upsilon}{\kappa} \right)^{-\frac{1}{\upsilon}}. \tag{15}$$

As mentioned earlier, for the firm with cutoff level productivity, we can define zero profit consumer taste cutoff condition as $d_{i,t} \left(\varphi^*_t, \lambda^*_{i,t} \left(\varphi^*_t \right) \right) = 0$. This implies:

$$\widetilde{d}_{i,t} = \frac{\sigma - 1}{\upsilon - (\sigma - 1)} w_t \frac{f_{i,t}}{Z_t}. \tag{16}$$

Furthermore, the product with cutoff productivity level earns zero profits from production, such that $d_{s,t} \left(\varphi^*_t \right) = 0$. The condition implies[11]

$$\widetilde{d}_{s,t} = \frac{\upsilon}{\kappa - \upsilon} w_t \frac{f_{h,t}}{Z_t}. \tag{17}$$

4.6 Household Budget Constraints and Intertemporal Problems

The household receives income by supplying labor $L_t(j)$, at the real wage rate w_t, by acquiring average dividends income $\widetilde{d}_t$, and by selling its initial share position v_t, of shareholdings x_t, of the firm composed of existing products, N_t. The household spends its income on consumption $C_t(j)$, buying $x_{t+1}(j)$ shares of the firm composed of existing products N_t, and new products H_t, at share price v_t. The household budget constraint is thus

$$L_t(j) w_t + x_t(j) N_t \left(v_t + \widetilde{d}_t \right) = C_t(j) + x_{t+1}(j) v_t \left(N_t + H_t \right). \tag{18}$$

During each period t, the representative household chooses consumption $C_t(j)$, shareholdings $x_{t+1}(j)$, and the labor supply $L_t(j)$, to maximize the expected utility function subject to the budget constraint (18). The first-order conditions with respect to consumption and labor supply yield the standard labor supply equation

$$\chi_t L_t(j)^{\varsigma} = w_t C_t(j)^{-1}.$$

[11] We place the detailed derivation in Appendix B.

The first-order condition with respect to shareholdings once combined with the product law of motion (9) and the first-order condition for consumption yields

$$v_t = \beta\,(1-\delta)\,E_t\left[\frac{\Lambda_{t+1}(j)}{\Lambda_t(j)}\left(v_{t+1}+\widetilde{d}_{t+1}\right)\right], \qquad (19)$$

where $\Lambda_t(j) = A_t/C_t(j)$ stands for the shadow value of the budget constraint. which, once iterated forward, shows that share prices are the expected discounted sum of future dividends.

4.7 Model equilibrium and solution

In equilibrium, households are symmetric and $C_t(j) = C_t$, $C_{i,t}(j) = C_{i,t}$, $L_t(j) = L_t$ and $\Lambda_t(j) = \Lambda_t$. To derive the aggregate equilibrium, we impose labor market clearing. Aggregate labor supply, L_t, is employed in either the production of consumption goods that include both fixed operational headquarter as well as product specific cost or the creation of new firms:

$$L_t = \sum^{J} M_{i,t}\left(\frac{\widetilde{y}_{i,t}}{Z_t\widetilde{\varphi}_{i,t}}+\frac{f_{i,t}}{Z_t}\right)di + S_t\frac{f_{h,t}}{Z_t} + H_t\frac{f_{E,t}}{Z_t}\left(\frac{H_t}{H_{t-1}}\right)^{\theta_E},$$

which can be expressed as[12]

$$L_t = \sum^{J} M_{i,t}\left[(\sigma-1)\frac{\widetilde{d}_{i,t}}{w_t}+\sigma\frac{f_{i,t}}{Z_t}\right]di + S_t\frac{f_{h,t}}{Z_t} + \frac{H_t v_t}{w_t}. \qquad (20)$$

Equation (20) is equivalent to the aggregated accounting identity of GDP obtained by aggregating budget constraints among households. Further we define total real sales, real sales of each product and employment used for production as

$$\mathcal{Y}_t = \sum^{J}\mathcal{Y}_{i,t}di, \quad \mathcal{Y}_{i,t} \equiv M_{i,t}\widetilde{y}_{i,t}$$

$$\mathcal{L}_t = \sum^{J} M_{i,t}\left[(\sigma-1)\frac{\widetilde{d}_{i,t}}{w_t}+\sigma\frac{f_{i,t}}{Z_t}\right]di + S_t\frac{f_{h,t}}{Z_t}$$

[12]Note that $\widetilde{d}_{i,t} = \frac{\widetilde{\rho}_{i,t}}{\sigma}\widetilde{y}_{i.t} - \frac{w_t f_{i,t}}{Z_t}$.

Finally, we assume the following shock process

$$\begin{pmatrix} \ln(A_t) \\ \ln(Z_t) \\ \ln(\chi_t) \\ \ln(f_{E,t}) \\ \ln(f_{h,t}) \\ \ln(f_{i,t}) \end{pmatrix} = \begin{pmatrix} \rho_A & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_Z & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_\chi & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_{f_E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{f_h} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{f_i} \end{pmatrix} \begin{pmatrix} \ln(A_{t-1}) \\ \ln(Z_{t-1}) \\ \ln(\chi_{t-1}) \\ \ln(f_{E,t-1}) \\ \ln(f_{h,t-1}) \\ \ln(f_{i,t-1}) \end{pmatrix} + \begin{pmatrix} \sigma_A \varepsilon_{A,t} \\ \sigma_Z \varepsilon_{Z,t} \\ \sigma_\chi \varepsilon_{\chi,t} \\ \sigma_{f_E} \varepsilon_{f_E,t} \\ \sigma_{f_h} \varepsilon_{f_h,t} \\ \sigma_{f_i} \varepsilon_{f_i,t} \end{pmatrix}$$

where ρ_A, ρ_χ, ρ_Z, ρ_{f_E}, ρ_{f_h} and ρ_{f_i} refer to the shock persistence and $\varepsilon_{A,t}$, $\varepsilon_{\chi,t}$, $\varepsilon_{Z,t}$, $\varepsilon_{f_E,t}$, $\varepsilon_{f_h,t}$ and $\varepsilon_{f_i,t}$ are normally distributed innovations with zero mean whose variances equal to σ_A^2, σ_χ^2, σ_Z^2, $\sigma_{f_E}^2$, $\sigma_{f_h}^2$ and $\sigma_{f_i}^2$. The model consists of 13 equations and 13 endogenous variables among which the number of products, N_t, is a state variable. Table 1 summarizes the benchmark model.

Table 3: Summary of the benchmark model

Average pricing ($\times J$)	$\widetilde{\rho}_{i,t} = \frac{\sigma}{\sigma-1}\frac{w_t}{Z_t \widetilde{\varphi}_{i,t}}$
Real taste-adjusted price ($\times J$)	$\rho_{i,t}^{1-\sigma} = M_{i,t}\widetilde{\rho}_{i,t}^{1-\sigma}$
Demand for product i ($\times J$)	$C_{i,t} = \rho_{i,t}^{-\nu}\alpha_i C_t^{\epsilon_i}$
Price index	$1 = \sum^J \rho_i^{1-\nu}\alpha_i C^{\epsilon_i - 1}$
Average product profits ($\times J$)	$\widetilde{d}_{i,t} = \frac{1}{\sigma}\frac{\rho_{i,t}C_{i,t}}{M_{i,t}} - \frac{w_t f_{i,t}}{Z_t}$
Average survivor's profits	$\widetilde{d}_{s,t} = \sum^J \frac{M_{i,t}}{S_t}\widetilde{d}_{i,t} di - w_t \frac{f_{h,t}}{Z_t}$
Average profits	$\widetilde{d}_t = \frac{S_t}{N_t}\widetilde{d}_{s,t}$
Consumer taste cutoff ($\times J$)	$\widetilde{d}_{i,t} = \frac{\sigma-1}{\upsilon-(\sigma-1)} w_t \frac{f_{i,t}}{Z_t}$
Productivity cutoff	$\widetilde{d}_{s,t} = \frac{\upsilon}{\kappa-\upsilon} w_t \frac{f_{h,t}}{Z_t}$
Taste weighted productivity ($\times J$)	$\widetilde{\varphi}_{i,t} = \left[\frac{\upsilon}{\upsilon-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \left(\frac{S_t}{N_t}\right)^{-\frac{1}{\kappa}} \left(\frac{M_{i,t}}{S_t}\frac{\kappa-\upsilon}{\kappa}\right)^{-\frac{1}{\upsilon}}$
Labor market clearing	$L_t = \sum^J M_{i,t}\left[(\sigma-1)\frac{\widetilde{d}_{i,t}}{w_t} + \sigma\frac{f_{i,t}}{Z_t}\right] di + S_t \frac{f_{h,t}}{Z_t} + \frac{H_t v_t}{w_t}$
Free entry condition	$v_t = w_t \frac{f_{E,t}}{Z_t}\left(\frac{H_t}{H_{t-1}}\right)^{\theta_E}$
Motion of establishments	$N_{t+1} = (1-\delta)(N_t + H_t)$
Euler equation	$v_t = \beta(1-\delta)E_t\left[\frac{A_{t+1}}{A_t}\frac{C_t}{C_{t+1}}\left(v_{t+1} + \widetilde{d}_{t+1}\right)\right]$
Optimal labor supply	$\chi_t L_t^{\varsigma} = w_t C_t^{-1}$

5 Calibration

We calibrate the model based on the CSP. The data length we use is 2000Q1 to 2017Q4. Over the sample periods, among 905 products, 855 products that completed registration in entire sample period.

We calibrate the parameters of the theoretical model as in Table 4. The value of discount factor β takes the standard value of calibration in quarterly basis. The inverse of the Frisch elasticity of labor supply ς is taken from Sugo and Ueda (2008), which estimates the elasticity for the Japanese economy. The elasticity of substitution across product varieties σ is set according to Ghironi and Melitz (2005). The parameter υ that shapes the taste distribution is set so that it matches the within establishment sales variation in data. The parameter κ that shapes the productivity distribution is set according to Hamano and Zanetti (2017). These values satisfy the restriction on the parameter such that $\kappa > \upsilon > \sigma - 1$.[13]

In the calibration, we assume that $J = 3$, namely the total number of products is three. We calibrate the consumption weight of each product group, α_1, α_2 and α_3 to match the sales share of each product group at the steady state ($P_1C_1/PC = 0.51$, $P_2C_2/PC = 0.27$ and $P_3C_3/PC = 0.22$). Based on the sales information of each product, we estimate the income elasticities. We regroup the elasticities from the highest to the lowest into three groups and take the medium elasticities of each group estimated to calibrate the income elasticity of each product group, ϵ_1, ϵ_2 and ϵ_3. We assume that products are substitute

[13] Given the Pareto distribution in the paper, the standard deviation of the logarithm of sales for product i is given by

$$\text{std}\left(\log \rho_{i,t}\left(\varphi, \lambda_i\right) y_{i,t}\left(\varphi, \lambda_i\right)\right) = (\sigma - 1)\sqrt{\frac{1}{\kappa^2} + \frac{1}{\upsilon^2}}$$

The standard deviation of logarithm of sales for establishment φ is given by

$$\text{std}\left(\log \rho_{i,t}\left(\varphi, \lambda_i\right) y_{i,t}\left(\varphi, \lambda_i\right)\right) = \frac{\sigma - 1}{\upsilon}$$

The productivity dispersion κ implied by the above first equation fails to meet the restriction of the parameter, however.

and set $\nu = 2$ with which the restriction on the parameter's value, $\sigma > \nu > \epsilon_1$ is satisfied.

The exogenous establishment destruction rate δ is calibrated so that it replicates the average establishment creation rate in the data, $H/N = 0.0056$ which amounts to 2.24 % of the annual establishment creation rate. We set entry fixed costs at the steady state as $f_e = 1$ without loss of generality. The operational fixed costs for production of each three group of product, f_1, f_2 and f_3 are set to match the average number of product produced by producing establishments $M_1/S = 0.61$ $M_2/S = 0.78$ and $M_3/S = 1.04$, respectively. As a result, the number of products produced per establishment M/S is 2.43 at the steady state. Similarly, we set the headquarter fixed costs f_h so that it replicates the steady state surviving rate $S/N = 0.93$. Finally, the parameter that determines the disutility for the labor supply χ is set to 0.9588 with which labor supply at the steady state is unity. We provide a detailed derivation of the steady state in Appendix B. The other parameters which are related to the propagation of shocks are estimated with Bayesian methods.

Table 4: Calibration

β	Discount factor	0.99
ς	Inverse of Frisch elasticity of labor	2.15
σ	Elasticity of substitution of product varieties	3.8
κ	Productivity dispersion	11.51
υ	Taste dispersion	4.18
α_1	consumption weight of product 1	0.49194
α_2	consumption weight of product 2	0.40725
α_3	consumption weight of product 3	0.41862
ϵ_1	income elasticity of product 1	1.3470
ϵ_2	income elasticity of product 2	0.5633
ϵ_3	income elasticity of product 3	0.1346
ν	Elasticity of substitution of products	2
δ	Exogenous death shock	0.0056
f_e	fixed cost for establishment entry	1
f_h	fixed cost for establishment exit	0.0297
f_1	fixed cost for product 1	0.0265
f_2	fixed cost for product 2	0.0080
f_3	fixed cost for product 3	0.0042
χ	disutility in supplying labor	0.9588

6 Multi-Product Establishments, Product Switching and Macroeconomic Dynamics

Before moving on to the quantitative assessment of the theoretical model with estimated shock processes and adjustment costs, to shed light on the basic propagation mechanism of the model, we provide impulse response functions with a simple shock process of productivity and various types of regulation.

6.1 Aggregate Productivity Shock

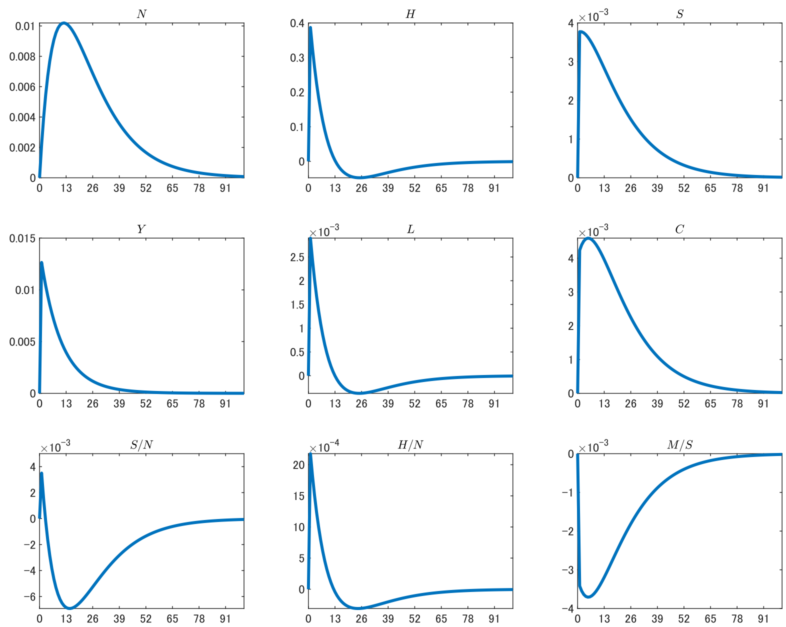
Figure 6a shows the impulse response functions of aggregate variables following one standard deviation of labor productivity shock. We set the shock persistence as $\rho_Z = 0.9$ and entry adjustment costs as $\theta_E = 0$. Following such a positive productivity shock, both the number of new entrants (H) and their share of the number of total establishments (H/N) increases substantially on impact by around 40 %. As a result of the household's consumption smoothing, the number of establishments (N) increases gradually with a hump shaped pattern, as is the case for consumption (C). Given the Schumpeterian feature of the model, however, only a subset number of establishments (S) operates. This, in turn, provides a reduction in the share of operating establishments in transitory dynamics except for a few initial periods (S/N). The number of product varieties per establishment (M/S) falls indicating the selection of products produced. A higher real income (Y) and a lower level of employed workers (L) in transitory dynamics are realized following such a positive labor productivity shock.

Behind the above dynamics of aggregate variables, the theoretical model reveals a substantial heterogeneity across products. Figure 6b provides the IRFs of average sales, the number of product varieties, the average taste-weighted productivity and the number of product per establishment for each product group. On the one hand, with the highest income elasticity, the number of establishments that produce the first product group, i.e., the number of product varieties of the first product group (M_1), increases on impact by around 0.6 %. The average sales of this product varieties ($\widetilde{y}_1$) fall for initial several quarters, however. On the other hand, the third product group which has the lowest income elasticity shows contrasting dynamics. The number of the third group of product varieties (M_3) slightly increases on impact while the number becomes lower from the steady state level in transitory dynamics while the average sales of this group ($\widetilde{y}_3$) increases gradually. Given the middle value of income elasticity, the second group of product reiterate shows the dynamics somehow in between of the first and the third group of product varieties.

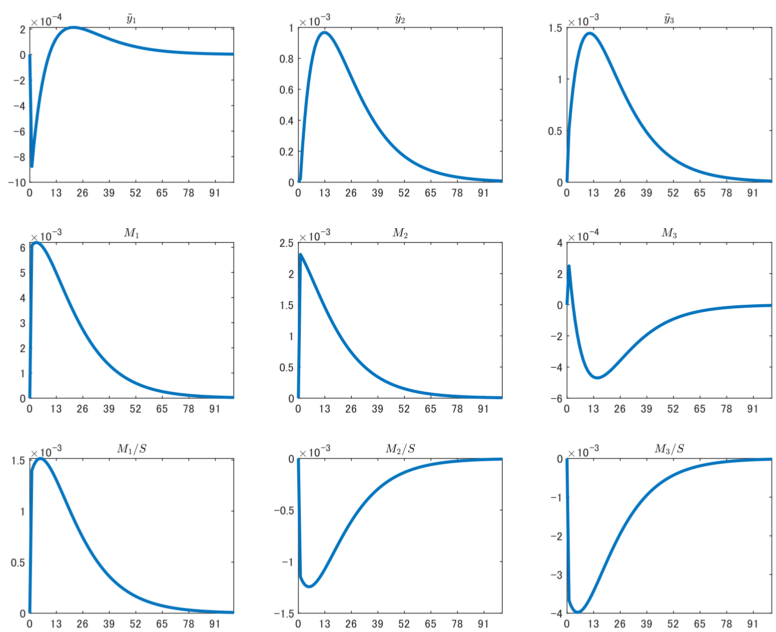
Thus following the aggregate productivity shock, we see contrasting dynamics across

Figure 6: IRFs of Productivity Shock

(a) IRFs of Aggregate Variables



(b) IRFs of Different Products



The figure shows the impulse response functions expressed as a percent deviation from the steady state level following one standard deviation of labor productivity shock Z_t with the shock persistence of $\rho_Z = 0.9$. The IRFs are obtained with the benchmark parameter values as in Table 4 and with zero entry adjustment costs such that $\theta_E = 0$.

products. As the number of product varieties per establishment increases only for the first group of product, the operating establishments switch their product portfolio from the second and the third group of products toward the first group of products.

6.2 Regulation Shocks

The theoretical model embeds various types of regulatory shocks that allow to control the production of each product group as well as the entry and exit of establishments. In this subsection we look at the impact of these regulatory shocks on each type of product as well as aggregate macroeconomic variables.

6.2.1 Entry Regulation Shock

Figure 7 provides the IRFs following a permanently 1 % higher fixed cost for entry, f_E, compared to its initial steady state level. In producing the IRFs, we set the entry adjustment costs as $\theta_E = 0$. As expected, and shown in Figure 7a, such an entry regulatory shock reduces the number of entrants (H) and the total number of establishments (N) at the new steady state. The number of producing establishments (S) increases on impact but it remains unchanged in the long run. Following the entry regulation shock, total income (Y) and consumption (C) decrease permanently. This, in turn, means a permanently lower wage cost, allowing survival of less efficient incumbent establishments at the new steady state. This is the reason why we don't see any change in the number of producers (S). Put differently, a lower number of entrants due to entry regulations is just replaced by the survival of less efficient incumbent establishments as argued in Hamano and Zanetti (2017). These producing establishments, in turn, expand their product portfolio, and the number of product varieties per establishment (M/S) increase at the new steady state.

At disaggregated product level, however, we see contrasting dynamics across products. As Figure 7b indicates, the number of the first group of product varieties (M_1) decreases while the number of the second and the third group of product varieties (M_2 and M_3) increase at the new steady state. The extent of the increase is greater for the third group of product. Note that fixed operational costs of each product ($f_1 w/Z$, $f_2 w/Z$ and $f_3 w/Z$) fall exactly in the same way following the entry regulatory shock due to the reduction in income and thus wage costs. However, producers only marginally enjoy such profit-enhancing regulatory shock because they experience simultaneously a reduction in revenue. This is the reason why the selection is more severe for the first group of product due to the high income elasticity to such an extent that we see a reduction in the number of product varieties at the new steady state. On the other hand, it allows the highest entry for the third group of products. Note also that the average sales of each product group ($\widetilde{y}_1$, $\widetilde{y}_2$ and $\widetilde{y}_3$) decreases uniformly but with a different extent. The sales reduction is the lowest for the first group of product given the substitutability between the intensive and extensive margins.

The number of product varieties per plant for the first group (M_1/S) decrease permanently.The entry regulation shock reduces the number of entrants (H) but at the same time it results in wiping out the products with the highest income elasticities. The composition of product portfolio in the economy ends up shifting toward the products with lower income elasticities.

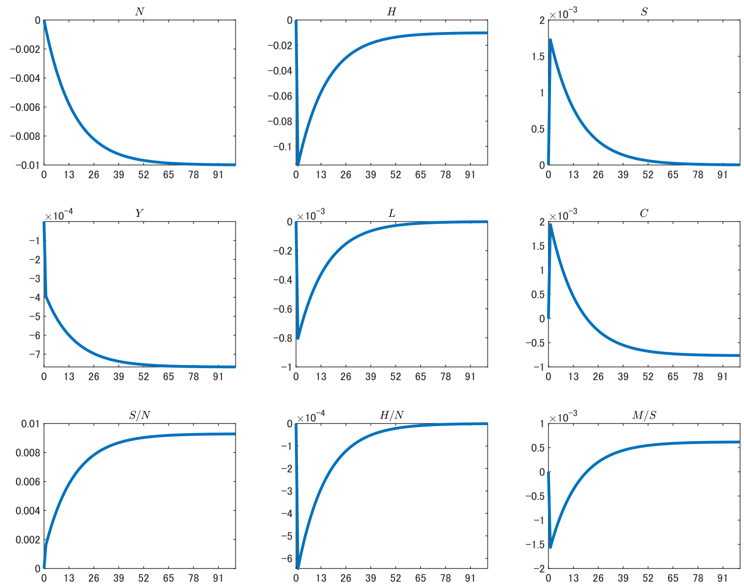
6.2.2 Establishment Regulation Shock

Figure 8 provides the IRFs following a permanent increase in establishment regulations f_h, which is higher by 1 % compared to its initial steady state level. In producing the IRFs, we again set the entry adjustment costs as $\theta_E = 0$. As Figure 8a indicates, the number of producing establishments (S) decrease permanently while leaving the number of entrants (H), and thus the total number of establishments (N), unchanged. As a result, the establishment survival rate (S/N) decreases. As explained in Hamano and Zanetti (2017), establishment specific-regulations are similar to a negative subsidy which reduces the profitability of establishments and thus leads to a higher efficiency of operating establishments on average.

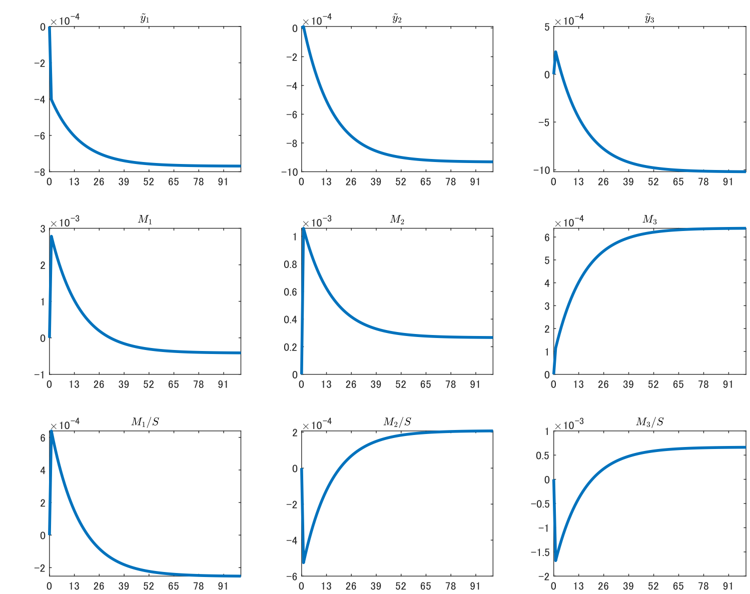
In the theoretical model, establishments endogenously add and drop each product and thus adjust their product portfolio. As Figure 8b shows, following the establishment

Figure 7: Entry Regulation Shock

(a) IRFs of Aggregate Variables



(b) IRFs of Different Products



The figure shows the impulse response functions expressed as a percent deviation from the steady state level following one standard deviation following a permanently 1 % higher fixed cost for entry, f_E. The IRFs are obtained with benchmark parameter values as in Table 4 and with zero entry adjustment costs such that $\theta_E = 0$.

specific regulatory shock, the scale of production of all products ($\widetilde{y}_1$, $\widetilde{y}_2$ and $\widetilde{y}_3$) is reduced uniformly. In contrast, only the number of product varieties in the first group (M_1) decreases while the number of the second and the third group of product varieties (M_2 andM_3) increase permanently. Again, different income elasticities across products result in these differences. As is the case for the entry regulation shock, following a negative subsidy shock, the fixed operational costs of each product ($f_1 w/Z$, $f_2 w/Z$ and $f_3 w/Z$) fall in the exact same way because of a fall in wage costs. However, the fall in revenue is the greatest for the first group of product because of the highest income elasticity. As a result, despite the same reduction in product specific fixed costs, the number of product varieties in the first group falls while this is not the case for the second and the third group of products. Observe that the rise in the number is higher for the third group of product than that of the second group of product, which is a result of a lower income elasticity of the third group of product compared to the second. Due to a relatively strong reduction in the number of producing establishments (S), not only the share of establishments that produce the second and third group of product varieties (M_2/S andM_3/S), but also the share of the first group of product varieties (M_1/S) expands.

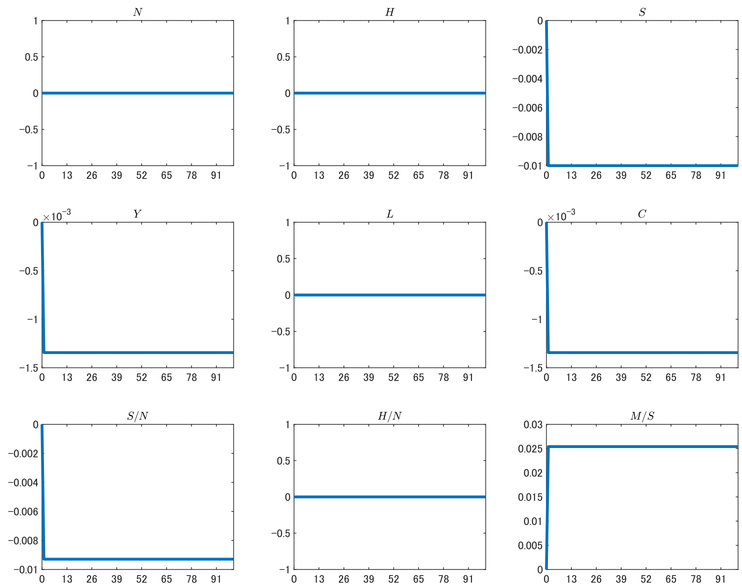
To summarize, the establishment regulation shock reduces the number of producing establishments (S) and income (Y) in the economy. Simultaneously, the surviving establishments expand their production lines and increase the number of product varieties, in particular, that have a lower income elasticities.

6.2.3 Product Specific Regulation Shock

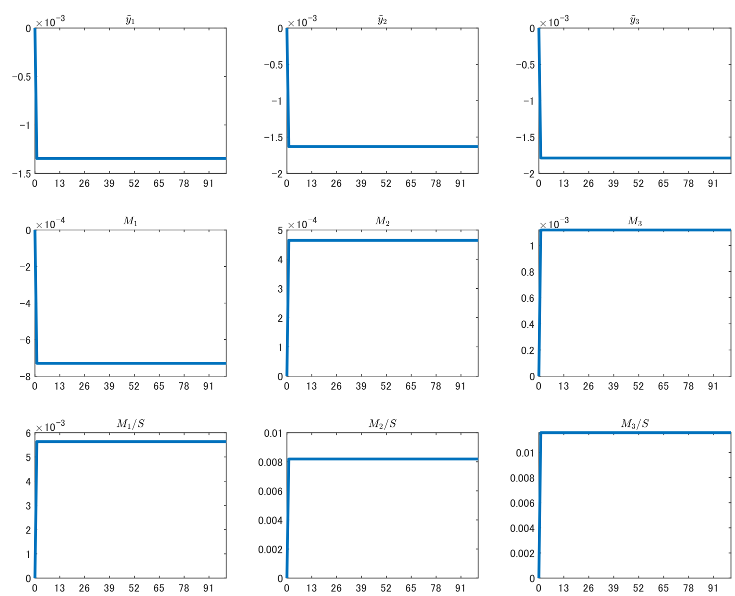
To understand the specificity of the model with entry and exit of establishments producing multiple-products, we consider a regulation shock for production of a specific product. Specifically, the product specific operational fixed cost for the production of the second group of product, $f_{2,t}$, is assumed to be permanently higher by 1 % compared to its steady state level. As Figure 9a shows, such a product specific regulation is powerless to influence the number of entry (H) and thus the total number of establishments (N) and producers (S) in the economy. However, here again we see contrasting adjustment across

Figure 8: Establishment Regulation Shock

(a) IRFs of Aggregate Variables



(b) IRFs of Different Products



The figure shows the impulse response functions expressed as a percent deviation from the steady state level following one standard deviation following a permanently 1 % higher establishment regulation cost, f_h. The IRFs are obtained with benchmark parameter values as in Table 4 and with zero entry adjustment costs such that $\theta_E = 0$.

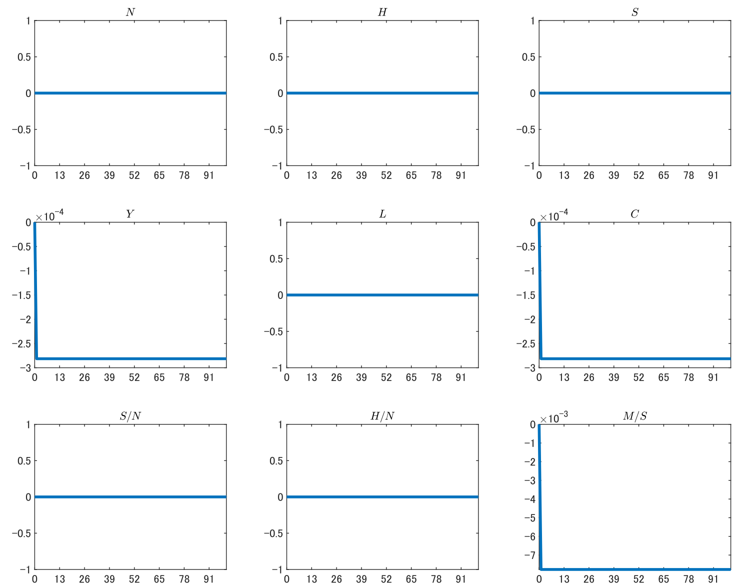
different types of products. As Figure 9b shows, following such a shock, the number of the second group of product varieties (M_2) decreases while the number of the first and third group of product varieties (M_1 and M_3) increase. Because of the substitutability between extensive and intensive margins, the average sales of the second group of product ($\widetilde{y}_2$) increases while those of the first and third group of products ($\widetilde{y}_1$ and $\widetilde{y}_3$) decrease. Observe also that although the directions of changes are the same for the first and third group of products, the extent of change is greater for the third group of product. As Figure 9a indicates, following such a recessionary shock, the aggregate income (Y) and consumption (C) fall, which leads to a lower product specific operational fixed costs for the first and third group of products ($f_1 w/Z$ and $f_3 w/Z$) that are not regulated. For the same extent of reduction in fixed costs and the same extent of fall in the aggregate consumption (C), the fall in revenue is smaller for the third group of product due to a lower income elasticity than the first group of product that has the highest income elasticity. This is the reason why we see a higher increase in the number of product varieties for the third group of product (M_3) than the first group of product (M_1).

The above exercise reveals the consequence of the regulation of a targeted product and its spillover toward other products. Establishments change their product mix in responding such a regulation. As a result of product specific regulation shock, there is a reallocation of resources from the second group of product toward the first and third group of products. The reallocation is greater toward the third group of product due to the lower income elasticity. Accordingly, the composition of product portfolio in the economy shifts from the regulated second group of product (M_2/S) toward less regulated products among which the product with a lower income elasticity gains the highest increase of the share.

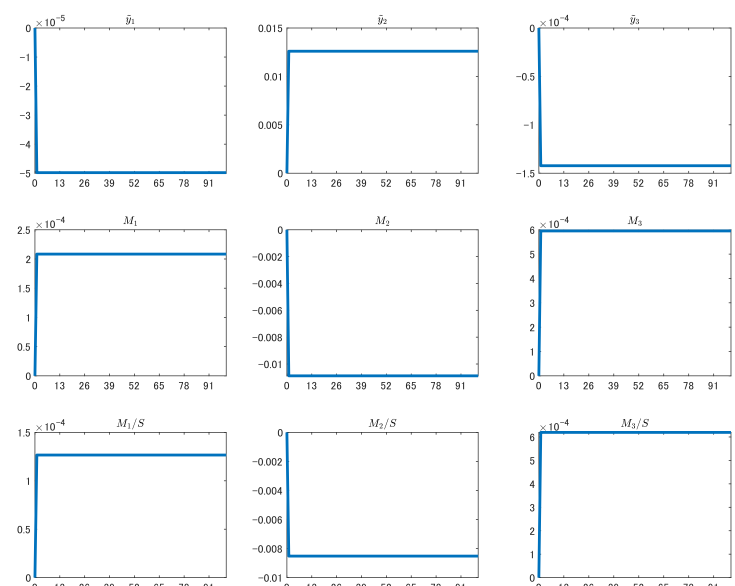
IRFs of other shocks and other variables are available upon request.

Figure 9: Product 2 Regulation Shock

(a) IRFs of Aggregate Variables



(b) IRFs of Different Products



The figure shows the impulse response functions expressed as a percent deviation from the steady state level following one standard deviation following a permanently 1 % higher product specific operational fixed cost for the production of the second group of product, f_2. The IRFs are obtained with benchmark parameter values as in Table 4 and with zero entry adjustment costs such that $\theta_E = 0$.

7 Quantitative Assessment

7.1 Estimation

The other parameters which are related to the propagation of shocks are estimated with Bayesian methods. We use real total sales growth, total employment growth, the number of entrants in total establishments and the number of producing establishment in total establishments, the share of establishments that produces product 1, the share of establishments that produces product 2 and the share of establishments that produce product 3 as observables. We also estimate the measurement errors between these observables and the theoretical counterpart in the model. Table 3 summarizes the priors and the results of posterior simulation.[14] Priors are shown with distribution and its upper and lower quantile. Posteriors report the mode and 90 % confidence intervals. Some parameters are only weakly identified with relatively wide 90 % of confidence intervals. Bergin et al. (2018) set the parameter for entry adjustment costs to 2.24 to match the second moments of entry. While the model is different, our posterior mode of entry adjustment cost is 2.76.

[14]MCMC is conducted with 1,000,000 draws of posterior simulation.

Table 5: Estimation

		Priors			Posteriors	
		Distr	Low	High	Mode	90% of CI
σ_A	std D. of preference shock	Invgamma	0.0001	2.0000	1.0710e-04	0.0001 0.0105
σ_Z	Std D. of productivity shock	Invgamma	0.0001	2.0000	0.0066	0.0003 9.5729
σ_χ	Std D. of labor disutility shock	Invgamma	0.0001	2.0000	0.0255	0.0177 0.0594
σ_{f_E}	Std D. of entry regulation shock	Invgamma	0.0001	2.0000	0.1505	0.0505 9.0482
σ_{f_h}	Std D. of establishment regulation shock	Invgamma	0.0001	2.0000	0.0070	0.0054 0.0125
σ_{f_1}	Std D. of product regulation shock 1	Invgamma	0.0001	2.0000	0.0066	0.0028 0.0200
σ_{f_2}	Std D. of product regulation shock 2	Invgamma	0.0001	2.0000	2.5030e-04	0.0001 0.0387
σ_{f_3}	Std D. of product regulation shock 3	Invgamma	0.0001	2.0000	0.0038	0.0005 0.0049
ρ_A	Persistence of demand shock	Beta	0.0256	0.7761	0.1950	0.0000 0.8757
ρ_Z	Persistence of productivity shock	Beta	0.0256	0.7761	0.9855	0.2381 0.9903
ρ_χ	Persistence of labor disutility shock	Beta	0.0256	0.7761	0.9105	0.7327 0.9783
ρ_{f_E}	Persistence of entry regulation shock	Beta	0.0256	0.7761	0.3674	0.0362 0.9362
ρ_{f_h}	Persistence of establishment regulation shock	Beta	0.0256	0.7761	0.9825	0.5212 0.9943
ρ_{f_1}	Persistence of product regulation shock 1	Beta	0.0256	0.7761	0.9987	0.5691 0.9997
ρ_{f_2}	Persistence of product regulation shock 2	Beta	0.0256	0.7761	0.1455	0.0000 0.9689
ρ_{f_3}	Persistence of product regulation shock 3	Beta	0.0256	0.7761	0.9983	0.3845 0.9996
θ_E	Adjustment cost for establishment entry	Gamma	1.0000	5.0000	2.7573	1.2449 9.4537
σ_{GYS}	Measurement errors	Invgamma	0.0001	2.0000	0.1108	0.1012 9.1814
σ_{GLS}	Measurement errors	Invgamma	0.0001	2.0000	1.3249e-04	0.0001 0.0059
σ_{M_1S}	Measurement errors	Invgamma	0.0001	2.0000	1.4134e-04	0.0001 0.0340
σ_{M_2S}	Measurement errors	Invgamma	0.0001	2.0000	1.2028e-04	0.0001 0.0052
σ_{M_3S}	Measurement errors	Invgamma	0.0001	2.0000	8.4294e-05	0.0001 0.0128
σ_{HN}	Measurement errors	Invgamma	0.0001	2.0000	0.0027	0.0029 9.7126
σ_{SN}	Measurement errors	Invgamma	0.0001	2.0000	0.0153	0.0051 0.0195

7.2 Second Moments

Table 6 reports the second moments of the data and the theoretical model. In the theoretical model, the standard deviation of total sales ($\mathcal{Y}$) employments ($\mathcal{L}$) and establishment entries (H) are lower while the number of producing establishments, their exit and the number of product varieties are higher compared to the data. This would indicate the presence of adjustment costs at establishment and product level in addition to entry. Further, in the theoretical model these variables show intuitive correlation with the growth of total sales: employment $\mathcal{L}$, entry H, the number of producing establishments S and product varieties M are correlated positively while establishment exit D is correlated negatively. However, in the data, establishment entry H and the number of producing establishment S show a puzzling negative correlation (-0.30 and -0.30, respectively).

Table 6: Second moments

		$\mathcal{Y}$	$\mathcal{L}$	H	S	D	M
St. dev. (%)	Data	11.5	2.04	62.2	1.17	23.6	0.99
	Model	4.64	1.98	38.6	5.70	120	4.66
Relative to $\mathcal{Y}$	Data	1.00	0.18	5.42	0.10	2.06	0.09
	Model	1.00	0.43	8.31	1.22	26.1	1.00
Corr($\mathcal{Y}, X_t$)	Data	1.00	0.23	-0.30	-0.30	-0.62	0.25
	Model	1.00	0.35	0.20	0.77	-0.04	0.27
		$\mathcal{Y}_1$	$\mathcal{Y}_2$	$\mathcal{Y}_3$	M_1	M_2	M_3
St. dev. (%)	Data	18.4	7.78	0.68	1.49	1.18	0.95
	Model	6.69	3.03	1.89	15.7	2.79	7.22
Relative to $\mathcal{Y}$	Data	1.60	0.68	0.23	0.13	0.10	0.08
	Model	1.44	0.65	0.41	3.37	0.60	1.56
Corr($\mathcal{Y}, X_t$)	Data	0.99	0.87	0.49	0.10	0.33	0.20
	Model	1.00	1.00	0.75	0.99	0.97	1.00

At each product group level, the theoretical model replicates accurately the pattern of the standard deviation of the sales growth in the data (Table 6). The first group $\mathcal{Y}_1$ shows the highest standard deviation with 18.4, the second group $\mathcal{Y}_2$ gives the middle value with 7.78 and the third group $\mathcal{Y}_3$ shows the lowest with 0.68 in the data. In the theoretical model, these standard deviations are 6.69, 3.30 and 1.89, respectively. The standard deviation of the number of the first product varieties M_1 is the highest with 1.49, that of the second group M_2 is 1.18 and the third group M_3 is the lowest with 0.95 in the data. In the theoretical model, the first product group gives the highest standard deviation with 15.7 while the the second product group M_2 gives the lowest with 2.79. The standard deviation of the number of product varieties tends to be higher than the sales growth in the theoretical model compared to the data. This is due to a substitutability in adjustment between intensive (production scale) and extensive margins (number of product varieties) in the theoretical model.

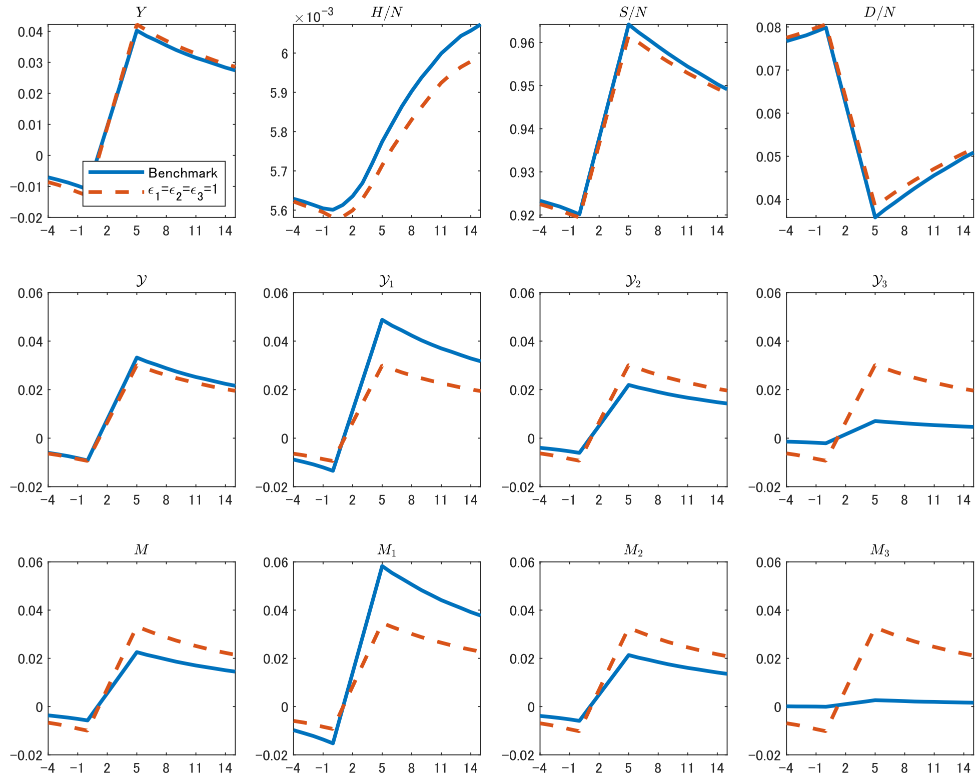
Remember that we are not targeting specific moment conditions in estimating the parameters of the model. Instead, our estimation strategy is Bayesian, relying on a posterior maximization of the likelihood. This would lead us to conclude safely that the theoretical model replicates well, at least qualitatively, the second moments of the CSP data.

7.3 Income Elasticities and Product Specific Business Cycles

In order to demonstrate the importance of different income elasticities to capture the observed heterogeneous pattern of products, we conduct a simulation analysis. Fig 10 provides the simulated pass of major economic variables following the expansionary period. We define the expansionary period when the growth rate of GDP peaks at more than 3 % in three quarter after the below trend. With the parameter values calibrated and estimated, we simulate the theoretical model for 2 million times and average out the pass of each economic variable that corresponds to the defined expansionary pass of GDP. The figure also shows the simulation result with the uniform income elasticity across products: $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1$ with dashed lines. As the first row panel in the figure shows, the benchmark simulation and the simulation with the uniform income elasticity demonstrate a very similar pattern. When the economy booms and GDP Y increases, entry rate H/N increases gradually and exit rate D/N decreases sharply and the surviving rate of establishments S/N increases in both cases. However, as the panels in the second and the third rows shows, following the expansionary periods, the adjustment is different across products in the bench mark case. The sales growth is the most emphasized for the first group of products $\mathcal{Y}_1$ with the highest income elasticity. The expansionary sales growth becomes milder for the second group of product $\mathcal{Y}_2$ and is the lowest for the third group of product $\mathcal{Y}_3$. We observe a similar pattern for the number of product varieties: the number of product varieties in the first group M_1 shows the strongest expansionary adjustment, followed by the second M_2 and the third group of product varieties M_3. Note that these simulation results at different product levels are consistent with the moments presented in Table 6. However, these heterogeneous adjustments in the growth in sales and in the number of product varieties disappear once we assume the uniform income elasticities across products, as the dashed lines in the figure indicates. Intuitively, following the heterogeneous shifts in the profitabilities across products, establishments change their product mix. Our result indicates the importance of heterogeneous income elasticities to replicate product specific dynamics.[15]

[15] We observe an opposite pattern for the recessionary period. The result of the simulation is available upon request.

Figure 10: Heterogeneous business cycles across products



The figure provides the simulated pass of GDP Y, entry rate H/N, surviving rate S/N, exit rate D/N, total sales$\mathcal{Y}$ and sales of each product group ($\mathcal{Y}_1$,$\mathcal{Y}_2$ and $\mathcal{Y}_3$) and the total number of product varieties (M) and the number of product varieties of each product group (M_1, M_2 and M_3) in the expansionary period. We define the expansionary period when the growth rate of GDP peaks at more than 3 % in three quarter from below the trend. The figure also shows the simulation result with uniform income elasticity across product: $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1$ with dashed lines.

8 Conclusion

In the paper, we document extensively a multiproduct aspect of establishments or firms and heterogeneous dynamics across products over the business cycle. With a novel theoretical model that captures the multi product aspect of firms or establishments, along the business cycle, establishments or firms change their product mix depending on product specific profitabilities due to different income elasticities across products. The theoretical model embeds a number of exogenous processes; in particular we focus on the regulation shock at entry, incumbent establishment and for production of each product. The theoretical model is calibrated based on the parameter values used in the literature while the shock processes are estimated relying on the Bayesian methods.

We find that heterogeneous income elasticities across products are crucial to shape the general equilibrium outcome of the various types of regulation. Namely, entry regulation reduces the number of entrants but at the same time results in wiping out the products with greater income elasticity. Also with establishment regulation, it reduces the number of producing establishments, on the one hand. On the other hand, the surviving establishments expand their production lines and increase the number of product varieties that have lower income elasticity. Finally, product specific regulation induces a reallocation of resources from the regulated product toward unregulated products. The reallocation is greater for the products with a lower income elasticity.

For the future research, it would be interesting to investigate the propagation mechanism of exogenous disturbances and how it is modified due to heterogeneous income elasticities. Also, we can consider a number of extensions including adding nominal rigidity, to explore the impact of monetary policy shock in a more general environment with multi-product firms, establishments or plants.

References

Bergin, P. R., L. Feng, and C. Y. Lin (2018, March). Firm Entry and Financial Shocks. *Economic Journal 128*(609), 510–540.

Bernard, A. B. and T. Okubo (2016, September). Product Switching and the Business Cycle. NBER Working Papers 22649, National Bureau of Economic Research, Inc.

Bernard, A. B., S. J. Redding, and P. K. Schott (2010, March). Multiple-product firms and product switching. *American Economic Review 100*(1), 70–97.

Bilbiie, F. O., F. Ghironi, and M. J. Melitz (2012). Endogenous Entry, Product Variety, and Business Cycles. *Journal of Political Economy 120*(2), 304 – 345.

Broda, C. and D. E. Weinstein (2010). Product creation and destruction: Evidence and price implications. *American Economic Review 100*(3), 691–723.

Clementi, G. L. and B. Palazzo (2016, July). Entry, Exit, Firm Dynamics, and Aggregate Fluctuations. *American Economic Journal: Macroeconomics 8*(3), 1–41.

Comin, D., D. Lashkari, and M. Mestieri (2021, January). Structural Change With LongâRun Income and Price Effects. *Econometrica 89*(1), 311–374.

DEKLE, R., K. Atsushi, K. Nobuhiro, and M. Tsutomu (2015, December). Product Dynamics and Aggregate Shocks: Evidence from Japanese product and firm level data. Discussion papers 15137, Research Institute of Economy, Trade and Industry (RIETI).

Ghironi, F. and M. J. Melitz (2005). International trade and macroeconomic dynamics with heterogeneous firms. *The Quarterly Journal of Economics 120*(3), 865–915.

Hamano, M. and T. Okubo (2021). In search of lost time: Firm vintage and macroeconomic dynamics. Technical report, RIETI.

Hamano, M. and F. Zanetti (2017). Endogenous Turnover and Macroeconomic Dynamics. *Review of Economic Dynamics 26*, 263–279.

Hanoch, G. (1975, May). Production and Demand Models with Direct or Indirect Implicit Additivity. *Econometrica 43*(3), 395–419.

Lee, Y. and T. Mukoyama (2015). Entry and exit of manufacturing plants over the business cycle. *European Economic Review 77*(C), 20–27.

Matsuyama, K. (2015, August). The Home Market Effect and Patterns of Trade between Rich and Poor Countries. Discussion Papers 1519, Centre for Macroeconomics (CFM).

Minniti, A. and F. Turino (2013). Multi-product firms and business cycle dynamics. *European Economic Review 57*(C), 75–97.

Schumpeter, J. A. (1994 (1942)). *Capitalism, socialism, and democracy.* New York: Routledge.

Sugo, T. and K. Ueda (2008, December). Estimating a dynamic stochastic general equilibrium model for Japan. *Journal of the Japanese and International Economies 22*(4), 476–502.

Ueda, K., K. Watanabe, and T. Watanabe (2019, April). Product Turnover and the Cost-of-Living Index: Quality versus Fashion Effects. *American Economic Journal: Macroeconomics 11*(2), 310–347.

A Data

Table 7: The Scope of the CSP by Industry

Sector (JSIC)	Industry	Establishment Size (Workers)
	Toys	10+ workers
11 Textile products		
	Chemical fibers	30+ workers *1
	Spun yarn	20+ workers *2
	Woven fabrics	10+ workers *3
	Tufted carpet, felt and non-woven fabrics	20+ workers
	Dyeing and finishing process	20+ workers
	Knit and sewn clothing products	30+ workers
	Secondary products (wadding and futon)	20+ workers
	Secondary products (nets and ropes)	20+ workers
	Secondary products (narrow fabrics, braids and lace fabrics)	10+ workers
13 Lumber and wood products, except furniture		
	Furniture	50+ workers
14 Pulp, paper, and papr products		
	Pulp	All

Continued on next page

Table 7 – continued from previous page

Sector (JSIC)	Industry	Establishment Size (Workers)
	Toys	10+ workers
	Paper	All
	Paperboard	All
	Corrugated cardboard	50+ workers
	Disposable diaper	All
15 Printing and allied industries		
	Printing	100+ workers
16 Chemical and allied products		
	Chemical fertilizers, lime and industrial sodium chemicals (lime and precipitated calcium carbonate)	15+ workers
	Chemical fertilizers, lime and industrial sodium chemicals (except for lime and precipitated calcium carbonate)	All
	Coal tar products, cyclic chemicals and synthetic dyes	All
	Industrial organic chemicals and sensitive materials for photography	All
	Petrochemical products	All
	Industrial inorganic chemicals and gunpowders	All
	Catalyst	All
	High pressure gas	All
	Plastic (materials)	All
	Oil and fat products, soap, synthetic detergents, and surface-active agents	10+ workers
	Cosmetics	30+ workers
	Paints and printing inks	10+ workers
17 Petroleum and coal products		

Continued on next page

Table 7 – continued from previous page

Sector (JSIC)	Industry	Establishment Size (Workers)
	Toys	10+ workers
	Petroleum products	All
18 Plastic products, except otherwise classified		
	Plastic products	50+ workers
19 Rubber products		
	Rubber products (automibile tires)	5+ workers
	Rubber products (except for automibile tires)	5+ workers
20 Leather tanning, leather products and fur skins		
	Leather boots and shoes	10+ workers
	Leather	10+ workers
21 Ceramic, stone and clay products		
	Flat glass, safety glass, multiple glass and glass fiber products	All
	Glass products	10+ workers
	Enameled iron products	20+ workers
	Cement	All
	Cement products	30+ workers
	Ceramic wares	10+ workers
	Fine ceramics	5+ workers
	Refractory bricks and monolithic refractories	All
	Carbon products and grinding wheels	All
	Board and panels	All
22 Iron and steel		
	Pig iron, ferro-alloys, crude steel, semi-finished steel, steel forgings and casting	All
	Ordinary hot-rolled steel	All

Continued on next page

Table 7 – continued from previous page

Sector (JSIC)	Industry	Establishment Size (Workers)
	Toys	10+ workers
	Ordinary cold-finished, metallic-coated steel sheets and shapes	All
	Special rolled steel	All
	Special pipes and tubes	All
	Cold-finished steel bars, wires, cast iron pipes and tubes, secondary steel products	30+ workers
23 Non-ferrous metals and products		
	Non-ferrous metals	All
	Aluminum	All
	Elongated copper products	All
	Aluminum mill products	All
	Electric wires and cables products	30+ workers
	Optical fiber products	All
	High-purity polycrystal silicon, silicon wafers, solders and copper ally ingots	All
	Light metal plate products	20+ workers
24 Fabricated metal products		
	Steel structures	50+ workers
	Transmission line hardware	30+ workers
	Springs	30+ workers
	Valves and pipe fittings	30+ workers
	Pneumatic tools, saw blades and knives for industry (machines)	20+ workers

Continued on next page

Table 7 – continued from previous page

Sector (JSIC)	Industry	Establishment Size (Workers)
	Toys	10+ workers
	Machinist hand tools	30+ workers
	Gas and oil equipment for cooking,	50+ workers
	Boiloing and heating, and solar water heaters	
	Sintered products (excluding cemented carbide tips)	30+ workers
	Forgings from billets and bars	20+ workers
	Iron castings	30+ workers
	Malleable iron castings and precision castings	20+ workers
	Copper, copper alloy castings	10+ workers
	Aluminum alloy castings	20+ workers
	Die castings	30+ workers
	Metal building materials	30+ workers
25 General-purpose machinery		
	Boilers and power units	50+ workers
	(except engines for motor vehicles,	
	motorcycles, railroad cars and aircraft)	
	Pumps, compressors, fans and blowers	50+ workers
	Oil-hydraulic and pneumatic equipment (except for aircraft)	50+ workers
	Conveyance machines and industrial robots	50+ workers
	Power transmission equipment	50+ workers
	Refrigerating machines, appliances and equipment	50+ workers
	Bearings, bearing metals and bushings	50+ workers
26 Production machinery		
	Construction equipment, mining machinery and crushers	50+ workers
	Chemical machinery and storage tanks	50+ workers

Continued on next page

Table 7 – continued from previous page

Sector (JSIC)	Industry	Establishment Size (Workers)
	Toys	10+ workers
	Paper making machines, plastic processing machinery	50+ workers
	Printing, plate making, bookbinding and paper converting machines	30+ workers
	Agricultural machinery and wood working machinery	30+ workers
	Metal cutting machine tools	50+ workers
	Metal forming machinery and foundry equipment	30+ workers
	Food products machinery, wrapping and packing machinery	30+ workers
	Textile machinery	50+ workers
	Sewing machines	30+ workers
	Mold and die	30+ workers
	Tools for machines	30+ workers
	Semiconductor, flat-panel display manufacturing system	50+ workers
27 Business oriented machinery		
	Office machinery	50+ workers
	Business service equipment	50+ workers
	Measuring equipment, instruments, Testing machines and surveying equipment	50+ workers
	Optical appliances and instruments, watches and clocks	50+ workers
28 Electronic parts, devices and electronic circuits		
	Parts for electronic equipment	50+ workers
	Electronic tubes, semiconductor devices and integrated circuits (electronic active devices)	50+ workers
29 Electrical machinery, equipment, and supplies		
	Rotating electrical machineries (except for aircraft)	50+ workers

Continued on next page

Table 7 – continued from previous page

Sector (JSIC)	Industry	Establishment Size (Workers)
	Toys	10+ workers
	Electrical stationary machines (except for aircraft)	50+ workers
	Switchgears and controlling equipment (except for aircraft)	50+ workers
	Consumer electric appliances	50+ workers
	Electric lamps, wiring equipment and luminaires	50+ workers
	Electric measuring instruments and associated electronic equipment	50+ workers
	Cells and batteries	50+ workers
30 Information and communication electronics equipment		
	Communication and related equipment	50+ workers
	Consumer electronic appliances	50+ workers
	Electronic computers and information terminals	50+ workers
31 Transportation equipment		
	Motor vehicles (except for combat vehicles)	50+ workers
	Parts for motor vehicles and electric equipment for internal combustion engines	50+ workers
	Bicycles	30+ workers
	Wheelchairs	10+ workers
	Industrial vehicles	50+ workers
	Aircraft	All
32 Miscellaneous manufacturing industries		
	Musical instruments	20+ workers
	Stationery	20+ workers

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Table 8: The Number of Products Covered in the CMF and CSP (2017)

Sector (JSIC)		CMF	CSP			
		Shipment	Production		Shipment	
		Value	Quantity	Value	Quantity	Value
9	Food	93	-	-	-	-
10	Beverages, Tobacco, and Feed	29	-	-	-	-
11	Textile products	174	62	-	62	30
12	Lumber and wood products, except furniture	39	-	-	-	-
13	Furniture and Fixtures	21	25	-	25	25
14	Pulp, paper, and papr products	56	66	1	59	59
15	Printing and allied industries	9	-	8	-	-
16	Chemical and allied products	200	249	-	282	244
17	Petroleum and coal products	23	20	-	-	-
18	Plastic products, except otherwise classified	43	30	-	30	30
19	Rubber products	39	34	-	28	28
20	Leather tanning, leather products and fur skins	34	7	-	10	7
21	Ceramic, stone and clay products	109	95	4	91	91
22	Iron and steel	64	87	-	87	-
23	Non-ferrous metals and products	53	58	-	58	53
24	Fabricated metal products	104	134	111	40	38
25	General-purpose machinery	74	145	118	80	64
26	Production machinery	143	242	160	132	98
27	Business oriented machinery	75	31	31	30	30
28	Electronic parts, devices and electronic circuits	56	81	82	37	38
29	Electrical machinery, equipment, and supplies	109	156	158	69	69
30	Information and communication electronics equipment	56	43	48	18	15
31	Transportation equipment	76	89	85	26	26
32	Miscellaneous manufacturing industries	106	17	-	17	17
	Total	1785	1671	806	1181	962

Table 9: An Example of Product Classification of CMF and CSP

Census of Manufactures		Current Survey of Production
31	Transportation Equipment	
3111	Motor vehicles, including motorcycles	
311111	Light and small passenger cars, less than 2000 ml cylinder capacity, including chassis	Midget passenger cars (cylinder capacity less than or equal to 660ml)
		Small passenger cars (cylinder capacity greater than 660ml and less than or equal to 2,000ml)
311112	Ordinary passenger cars, 2000 ml cylinder capacity or more, including chassis	Large passenger cars (cylinder capacity greater than 2,000ml)
311113	Buses	Small bus chassis (including complete buses)
		Large bus chassis (including complete buses)
311114	Trucks, including tractors	Midget truck chassis with gasoline engines (including complete trucks)
		Midget truck chassis with diesel engines (including complete trucks)
		Large truck chassis with gasoline engines (including complete trucks)
		Large truck chassis with diesel engines (including complete trucks)
		Tractor truck chassis (including complete tractor trucks)
311116	Bus and truck chassis	
311115	Motor vehicles for special-use	Special passenger cars
311117	Motorcycles, less than 125 ml, including motor bicycles and motor scooters	Motorcycles (cylinder capacity less than or equal to 50ml)
		Motorcycles (cylinder capacity greater than 50ml and less than or equal to 125ml)
311118	Motorcycles, more than 125 ml, including ones with side cars and motor scooters	Motorcycles (cylinder capacity greater than 125ml and less than or equal to 250ml)
		Motorcycles (cylinder capacity greater than 250ml)

Notes: The product ID number 31116 (Bus and truck chassis) of the Census of Manufactures corresponds to both bus chassis (including complete busses) and truck chassis (including complete trucks) of the Current Survey of Production.

B Average Profits

With $S_t = [1 - G(\varphi_t^*)]\, N_t$, by defining $\widetilde{d}_t(\varphi)$ as the expected profits of firm with productivity φ, the expected profit of potential producers is

$$\widetilde{d}_t = [1 - G(\varphi_t^*)] \int_{\varphi_t^*}^{\infty} \widetilde{d}_t(\varphi) \frac{dG(\varphi)}{1 - G(\varphi_t^*)} = (S_t/N_t)\, \widetilde{d}_{s,t}$$

Also, by defining $\widetilde{d}_{i,t}\left(\varphi, \lambda_{i,t}^*(\varphi)\right)$ as the average realized profits of firm with productivity φ for product i, the average realized profits of surviving producers are

$$\widetilde{d}_{s,t} = \int_{\varphi_t^*}^{\infty} \widetilde{d}_t(\varphi) \frac{dG(\varphi)}{1 - G(\varphi_t^*)}$$
$$= \int_{\varphi_t^*}^{\infty} \sum^{J} \left[1 - Z_i(\lambda_{i,t}^*(\varphi))\right] \widetilde{d}_{i,t}\left(\varphi, \lambda_{i,t}^*(\varphi)\right) di \frac{dG(\varphi)}{1 - G(\varphi_t^*)} - \frac{w_t f_{h,t}}{Z_t}$$

Note that by plugging the definition of $\widetilde{\lambda}_{i,t}(\varphi)$,

$$\widetilde{d}_{i,t}\left(\varphi, \lambda_{i,t}^*(\varphi)\right) = \int_{\lambda_{i,t}^*(\varphi)}^{\infty} \left[\frac{1}{\sigma}\left(\frac{\rho_{i,t}\left(\varphi, \lambda_i\right)}{\lambda_i}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\nu} \alpha_i C_t^{\epsilon_i} - \frac{w_t f_{i,t}}{Z_t}\right] \frac{dZ_i(\lambda_i)}{1 - Z_i(\lambda_{i,t}^*(\varphi))}$$
$$= \frac{1}{\sigma}\left(\frac{\sigma}{\sigma - 1}\frac{w_t}{Z_t}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\nu} \alpha_i C_t^{\epsilon_i} \int_{\lambda_{i,t}^*(\varphi)}^{\infty} (\lambda_i \varphi)^{\nu-1} \frac{dZ_i(\lambda_i)}{1 - Z_i(\lambda_{i,t}^*(\varphi))} - \int_{\lambda_{i,t}^*(\varphi)}^{\infty} \frac{w_t f_{i,t}}{Z_t} \frac{dZ_i(\lambda_i)}{1 - Z_i(\lambda_{i,t}(\varphi_t^*))}$$
$$= \frac{1}{\sigma}\left(\frac{\sigma}{\sigma - 1}\frac{w_t}{Z_t}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\nu} \alpha_i C_t^{\epsilon_i} \widetilde{\lambda}_{i,t}(\varphi) - \frac{w_t f_{i,t}}{Z_t}$$

Using the above expression, the average realized profits of surviving producers are

$$\widetilde{d}_{s,t} = \int_{\varphi_t^*}^{\infty} \sum^{J} \left[1 - Z_i(\lambda_{i,t}^*(\varphi))\right] \left[\frac{1}{\sigma}\left(\frac{\sigma}{\sigma - 1}\frac{w_t}{Z_t}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\nu} \alpha_i C_t^{\epsilon_i} \widetilde{\lambda}_{i,t}(\varphi) - \frac{w_t f_{i,t}}{Z_t}\right] di \frac{dG(\varphi)}{1 - G(\varphi_t^*)} - \frac{w_t f_{h,t}}{Z_t}$$
$$= \sum^{J} \frac{1}{\sigma}\left(\frac{\sigma}{\sigma - 1}\frac{w_t}{Z_t}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\nu} \alpha_i C_t^{\epsilon_i} \int_{\varphi_t^*}^{\infty} \left[1 - Z_i(\lambda_{i,t}^*(\varphi))\right] \int_{\varphi_t^*}^{\infty} \widetilde{\lambda}_{i,t}(\varphi) \frac{dG(\varphi)}{1 - G(\varphi_t^*)} di$$
$$- \sum^{J} \int_{\varphi_t^*}^{\infty} \left[1 - Z_i(\lambda_{i,t}^*(\varphi))\right] \frac{dG(\varphi)}{1 - G(\varphi_t^*)} \frac{w_t f_{i,t}}{Z_t} di - \frac{w_t f_{h,t}}{Z_t}$$
$$= \sum^{J} \int_{\varphi_t^*}^{\infty} \left[1 - Z_i(\lambda_{i,t}^*(\varphi))\right] \frac{dG(\varphi)}{1 - G(\varphi_t^*)} \left[\frac{1}{\sigma}\left(\frac{\sigma}{\sigma - 1}\frac{w_t}{Z_t \widetilde{\varphi}_{i,t}}\right)^{1-\sigma} \rho_{i,t}^{\sigma-\nu} \alpha_i C_t^{\epsilon_i} - \frac{w_t f_{i,t}}{Z_t}\right] di - \frac{w_t f_{h,t}}{Z_t}$$

$$= \sum^{J} \frac{M_{i,t}}{S_t} \left[\frac{1}{\sigma} \widetilde{\rho}_{i,t}^{1-\sigma} \rho_{i,t}^{\sigma-\nu} \alpha_i C_t^{\epsilon_i} - \frac{w_t f_{i,t}}{Z_t} \right] di - \frac{w_t f_{h,t}}{Z_t}$$

$$= \sum^{J} \frac{M_{i,t}}{S_t} \widetilde{d}_{i,t} di - \frac{w_t f_{h,t}}{Z_t}$$

where we have used that $\widetilde{\varphi}_{i,t}^{\sigma-1} = \int_{\varphi_t^*}^{\infty} \widetilde{\lambda}_{i,t}(\varphi) \frac{dG(\varphi)}{1-G(\varphi_t^*)}$

To sum up we have the following relationship,

$$\widetilde{d}_{i,t} = \frac{1}{\sigma} \widetilde{\rho}_{i,t}^{1-\sigma} \rho_{i,t}^{\sigma-\nu} \alpha_i C_t^{\epsilon_i} - \frac{w_t f_{i,t}}{Z_t}$$

which can be further rewritten with (3)and $\rho_{i,t}^{1-\sigma} = M_{i,t} \widetilde{\rho}_{i,t}^{1-\sigma}$ as

$$\widetilde{d}_{i,t} = \frac{1}{\sigma} \frac{\rho_{i,t} C_{i,t}}{M_{i,t}} - \frac{w_t f_{i,t}}{Z_t}$$

And

$$\widetilde{d}_{s,t} = \sum^{J} \frac{M_{i,t}}{S_t} \widetilde{d}_{i,t} di - \frac{w_t f_{h,t}}{Z_t}$$

with $M_{i,t} = \int_{\varphi_t^*}^{\infty} \left[1 - Z_i(\lambda_{i,t}^*(\varphi)) \right] \frac{dG(\varphi)}{1-G(\varphi_t^*)} S_t$.

C Zero Profit Consumer Taste Cutoff and Zero Profit Cutoff

ZPCT of the above firm with the cutoff productivity implies

$$d_{i,t} \left(\varphi^*, \lambda_{i,t}^* \left(\varphi^* \right) \right) = \frac{1}{\sigma} \left(\frac{\rho_{i,t} \left(\varphi^*, \lambda_i^* \right)}{\lambda_{i,t}^* \left(\varphi^* \right)} \right)^{1-\sigma} \rho_{i,t}^{\sigma-\nu} \alpha_i C_t^{\epsilon_i} - \frac{w_t f_{i,t}}{Z_t} = 0.$$

By plugging the equilibrium price

$$\frac{1}{\sigma} \left(\frac{\sigma}{\sigma - 1} \frac{w_t}{Z_t \varphi_t^* \lambda_{i,t}^* (\varphi_t^*)} \right)^{1-\sigma} \rho_{i,t}^{\sigma-\nu} \alpha_i C_t^{\epsilon_i} = \frac{w_t f_{i,t}}{Z_t}.$$

Plugging the above relation in the average realized product profits gives

$$\widetilde{d}_{i,t} = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_t}{Z_t\widetilde{\varphi}_{i,t}}\right)^{1-\sigma}\rho_{i,t}^{\sigma-\nu}\alpha_i C_t^{\epsilon_i} - \frac{w_t f_{i,t}}{Z_t}$$
$$= \left[\left(\frac{\varphi_t^*\lambda_{i,t}^*(\varphi_t^*)}{\widetilde{\varphi}_{i,t}}\right)^{1-\sigma} - 1\right]\frac{w_t f_{i,t}}{Z_t}$$
$$= \left[\frac{\sigma-1}{\upsilon-(\sigma-1)}\right]\frac{w_t f_{i,t}}{Z_t}$$

where we have used $\left(\frac{\varphi_t^*\lambda_{i,t}^*(\varphi_t^*)}{\widetilde{\varphi}_{i,t}}\right)^{1-\sigma} = \frac{\upsilon}{\upsilon-(\sigma-1)}$ implied by Pareto distribution.

Also zero profit cutoff condition implies that

$$d_{s,t}(\varphi_t^*) = \sum^J\left[1 - Z_i(\lambda_{i,t}^*(\varphi_t^*))\right]\int_{\lambda_{i,t}^*(\varphi_t^*)}^{\infty} d_{i,t}(\varphi_t^*,\lambda_i)\frac{dZ_i(\lambda_i)}{1-Z_i(\lambda_{i,t}^*(\varphi_t^*))}di - \frac{w_t f_{h,t}}{Z_t}$$
$$= \sum^J\left[1 - Z_i(\lambda_{i,t}^*(\varphi_t^*))\right]\int_{\lambda_{i,t}^*(\varphi_t^*)}^{\infty}\left[\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_t}{Z_t\varphi_t^*\lambda_i}\right)^{1-\sigma}\rho_{i,t}^{\sigma-\nu}\alpha_i C_t^{\epsilon_i} - \frac{w_t f_{i,t}}{Z_t}\right]\frac{dZ_i(\lambda_i)}{1-Z_i(\lambda_{i,t}^*(\varphi_t^*))}di - \frac{w_t f_{h,t}}{Z_t}$$
$$= \sum^J\left[\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_t}{Z_t\varphi_t^*\lambda_{i,t}^*(\varphi_t^*)}\right)^{1-\sigma}\rho_{i,t}^{\sigma-\nu}\alpha_i C_t^{\epsilon_i}\frac{\upsilon}{\upsilon-(\sigma-1)} - \frac{w_t f_{i,t}}{Z_t}\right]\lambda_{i,t}^*(\varphi_t^*)^{-\upsilon}di - \frac{w_t f_{h,t}}{Z_t}$$
$$= \sum^J\left[\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_t}{Z_t\widetilde{\varphi}_{i,t}}\right)^{1-\sigma}\rho_{i,t}^{\sigma-\nu}\alpha_i C_t^{\epsilon_i} - \frac{w_t f_{i,t}}{Z_t}\right]\frac{\kappa-\upsilon}{\kappa}\frac{M_{i,t}}{S_t}di - \frac{w_t f_{h,t}}{Z_t}$$
$$= \frac{\kappa-\upsilon}{\kappa}\sum^J\widetilde{d}_{i,t}\frac{M_{i,t}}{S_t}di - \frac{w_t f_{h,t}}{Z_t} = 0$$

From the first to the second line, we have used implied integral by Pareto distribution, $Z_i(\lambda_i) = 1-\left(\frac{\lambda_{i\,\min}}{\lambda_i}\right)^{\upsilon}$. From the second to the third line, we have used $\frac{M_{i,t}}{S_t} = \frac{\kappa}{\kappa-\upsilon}\lambda_{i,t}^*(\varphi_t^*)^{-\upsilon}$ together with $\left(\frac{\varphi_t^*\lambda_{i,t}^*(\varphi_t^*)}{\widetilde{\varphi}_{i,t}}\right)^{1-\sigma} = \frac{\upsilon}{\upsilon-(\sigma-1)}$. Finally, by plugging the expression of $\widetilde{d}_{s,t}$ found previously we obtain (17).

D Steady State

We start by deriving the steady state of the benchmark model. The Euler equation (19) provides:

$$\frac{1}{\beta} = (1-\delta)\left(1+\frac{\widetilde{d}}{\upsilon}\right). \tag{21}$$

Using the average profit equation (13), ZCP equation (17) and the free entry condition (8) at the steady state, we can express equation (21) as:

$$\frac{1}{\beta} = (1-\delta)\left(1 + \frac{S}{N}\frac{\upsilon}{\kappa - \upsilon} f_h\right),$$

which provides the steady state endogenous destruction rate, S/N, given operational fixed costs, f_h.

Also by plugging (16) and (17) into (12), we have

$$\frac{\kappa}{\kappa - \upsilon} f_h = \frac{\sigma - 1}{\upsilon - (\sigma - 1)} \sum^{J} \frac{M_i f_i}{S} di$$

which provides $\sum^{J} \frac{M_i f_i}{S} di$, given f_h.

From the law of motion of products (9), we derive the number of new products, $H = \delta N / (1-\delta)$. Using these relations and (16) in the labor market clearing condition (20), it yields:

$$\frac{L}{N} = \left[\frac{(\sigma-1)^2}{\upsilon - (\sigma-1)} + \sigma\right] \frac{S}{N} \sum^{J} \frac{M_i f_i}{S} di + \frac{S}{N} f_h + \frac{\delta}{1-\delta} f_E, \tag{22}$$

Also from (11), (16) and $C_i = \rho_i^{-\nu} \alpha_i C^{\epsilon_i}$,

$$\left[\frac{\sigma - 1}{\upsilon - (\sigma-1)} + 1\right] w f_i = \frac{1}{\sigma} \frac{\rho_i^{1-\nu} \alpha_i C^{\epsilon_i}}{M_i}$$

which can be further transformed as

$$\left[\frac{\sigma - 1}{\upsilon - (\sigma-1)} + 1\right] \frac{w}{C} S \frac{M_i f_i}{S} di = \frac{1}{\sigma} \rho_i^{1-\nu} \alpha_i C^{\epsilon_i - 1}$$

By summing up both side of the equation across products and using the definition of the price index (4) and $\chi L^{\varsigma} = w C^{-1}$, we obtain

$$\left[\frac{\sigma - 1}{\upsilon - (\sigma-1)} + 1\right] \chi L^{\varsigma} \frac{S}{N} N \sum^{J} \frac{M_i f_i}{S} di = \frac{1}{\sigma}$$

Plugging L from (22), finally we have

$$\left[\frac{\sigma-1}{\upsilon-(\sigma-1)}+1\right]\chi\left\{\left[\frac{(\sigma-1)^2}{\upsilon-(\sigma-1)}+\sigma\right]\frac{S}{N}\sum^{J}\frac{M_i f_i}{S}di+\frac{S}{N}f_h+\frac{\delta}{1-\delta}f_E\right\}^{\varsigma}N^{\varsigma+1}\frac{S}{N}\sum^{J}\frac{M_i f_i}{S}di=\frac{1}{\sigma}$$

This gives the solution for N provided f_h, f_E and χ since S/N and $\sum^{J}\frac{M_i f_i}{S}di$ are a function of f_h, respectively. Once we get N, it is easy to solve for L and S.

We setf_i to pin down M_i. In calibration, first we set $\frac{M_i}{S}$ with data. From

$$\widetilde{\varphi}_i=\left[\frac{\upsilon}{\upsilon-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{S}{N}\right)^{-\frac{1}{\kappa}}\left(\frac{M_i}{S}\frac{\kappa-\upsilon}{\kappa}\right)^{-\frac{1}{\upsilon}}$$

we can compute $\widetilde{\varphi}_i$ and M_i.

Next, we solve the steady state value of w. With price index (4), the price index of each product basket i, $\rho_{i,t}^{1-\sigma}=M_{i,t}\widetilde{\rho}_{i,t}^{1-\sigma}$, the average pricing of product i , $\widetilde{\rho}_i=\frac{\sigma}{\sigma-1}\frac{w_t}{\widetilde{\varphi}_{i,t}}$ and labor supply, $\chi L^{\varsigma}=wC^{-1}$, we have

$$1=\sum^{J}\rho_i^{1-\nu}\alpha_i C^{\epsilon_i-1}$$

$$1=\sum^{J}\left(\frac{\sigma}{\sigma-1}\frac{w}{\widetilde{\varphi}_i}M_i^{\frac{1}{1-\sigma}}\right)^{1-\nu}\alpha_i\left(\frac{w}{\chi L^{\varsigma}}\right)^{\epsilon_i-1}$$

The above is a non-linear equation with respect to w. Given, $\widetilde{\varphi}_i$, M_i and L, we can compute w. The remaining variables are relatively easy to compute. Specifically, fixed costs for each product i are computed from

$$f_i=\left[\frac{\sigma-1}{\upsilon-(\sigma-1)}+1\right]^{-1}\frac{1}{\sigma}\frac{\rho_i C_i}{wM_i}.$$