Price setting frequency and the Phillips curve*

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Abstract

We develop a New Keynesian (NK) model with endogenous price setting frequency. Whether a firm updates its price in a given period depends on an analysis of expected cost and benefits modelled by a discrete choice process. A firm decides to update the price when expected benefits outweigh expected cost and then resets the price optimally. The model predicts that prices are more flexible during expansions and less flexible during recessions. Our quantitative analysis shows that contrary to the standard NK model, the assumed price setting behaviour: (i) is consistent with micro data on price setting frequency; (ii) gives rise to a non-linear Phillips curve that is steeper during expansions and flatter during recessions; (iii) explains shifts in the Phillips curve associated with different historical episodes without relying on implausible high cost-push shocks and nominal rigidities inconsistent with micro data; (iv) improves the macroeconomic time series fit of a medium-scale NK model over the sample 1959 to 2019.

Keywords: Price setting, inflation dynamics, monetary policy, Phillips curve.
JEL Classification: E31, E32, E52.

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1 Introduction

> *'Another key development in recent decades is that price inflation appears less responsive to resource slack. That is, the short-run price Phillips curve [...] appears to have flattened, implying a change in the dynamic relationship between inflation and employment.'* (Clarida 2019, Vice Chair, Board of Governors, Federal Reserve System)

The flattening of the Phillips curve and historical shifts in this relationship between the output gap and inflation are well documented in the data. As pointed out by Clarida (2019) and others, these observations pose a challenge to frameworks for monetary policy analysis and they are now put under scrutiny. This certainly includes frameworks such as the New Keynesian (NK) model and its theory of the Phillips curve. At the heart of the NK model are assumptions about price setting behavior such as the popular Calvo (1983)-Yun (1996) pricing model that give rise to the Phillips Curve. The Calvo (1983) parameter θ governing the price stickiness, in turn, is the key determinant of the Phillips curve slope.

Standard NK models predict a Phillips curve relationship that is much steeper than in the data observed in recent decades. This has undesirable implications such as the *missing deflation puzzle* (Hall 2011), i.e., while NK models predict high deflation along with a dramatic downturn such as the *Great Recession*, one can actually observe surprisingly modest declines in inflation and a subsequent inflation-less recovery.

A well-known remedy to reconcile the NK model with the data are implausible high cost-push shocks, high price indexation and nominal rigidities that are by-and-large inconsistent with observed price setting frequency at the micro level. For instance, Del Negro, Giannoni & Schorfheide (2015) or Guerrieri & Iacoviello (2017) estimate Calvo (1983) parameters as high as $\theta = 0.87$ or 0.9. Yet, this remedy creates an unfortunate tension. On the one hand, large and highly auto-correlated cost-push shocks and high degrees of price stickiness reduce the covariance between inflation and output and improve the model's fit to inflation. On the other hand, the inflation dynamics are then mostly explained by large cost-push shocks (see, e.g., King & Watson 2012, Lindé, Smets & Wouters 2016, Fratto & Uhlig 2020). For example, Del Negro et al. (2015, p.169) argue that explaining inflation mainly with cost-push shocks is unfortunate, because these shocks lack a clear economic interpretation and fail to explain a lot of variation in other variables.

Next, explaining inflation mainly through cost-push shocks and high degrees of price rigidities and indexation also seems implausible from the viewpoint of the *Great Recession*. The latter is perceived as a demand-driven downturn that caused the observed inflation and output gap dynamics during and after the crisis. Explaining inflation via high degrees of nominal price rigidities and indexation also seems implausible in light of empirical evidence on the price setting frequency and indexation at the micro level.

Admittedly, the insight that Calvo (1983) pricing models are notoriously difficult to reconcile with observed price setting at the micro level is not new, but nevertheless important in this context.[1] A model that is consistent with macro data (e.g., flattening of the Philips curve, missing deflation puzzle) may still be subject to observational equivalence with many other models. If this very same model were also consistent with micro data (e.g., price setting frequency), it would clearly outperform these other models along an important dimension (see Christiano, Eichenbaum & Trabandt 2018). For instance, Nakamura, Steinsson, Sun & Villar (2018) use US CPI micro data from the BLS to analyze the evolution, dispersion, heterogeneity and duration of US prices. They conclude that the magnitude and frequency of price changes are heterogeneous and time-varying. Figure 1 reconstructs the frequency of price adjustment based on the Nakamura et al. (2018) data and its relation to inflation.[2]

[1] Standard menu cost models à la Rotemberg (1982) suffer from the same problem. At the macro level, estimates of the quadratic cost have increased a lot. At the micro level, these models fail to account for price dispersion.

[2] Nakamura et al. (2018) define price changes as any entry with $\ln(p_{i,t}/p_{i,t-1}) \neq 0$ within the BLS consumer goods' price tags database. They discard $\ln(p_{i,t}/p_{i,t-1}) > 1$ as inputs errors. Based on the mean frequency of price change in each CPI Entry Level Items of the remaining values, they compute the monthly expenditure-weighted medians across CPI Entry Level Items. Expenditure weights are fixed at their year 2000 value. The series is seasonally-adjusted by averaging monthly values over the

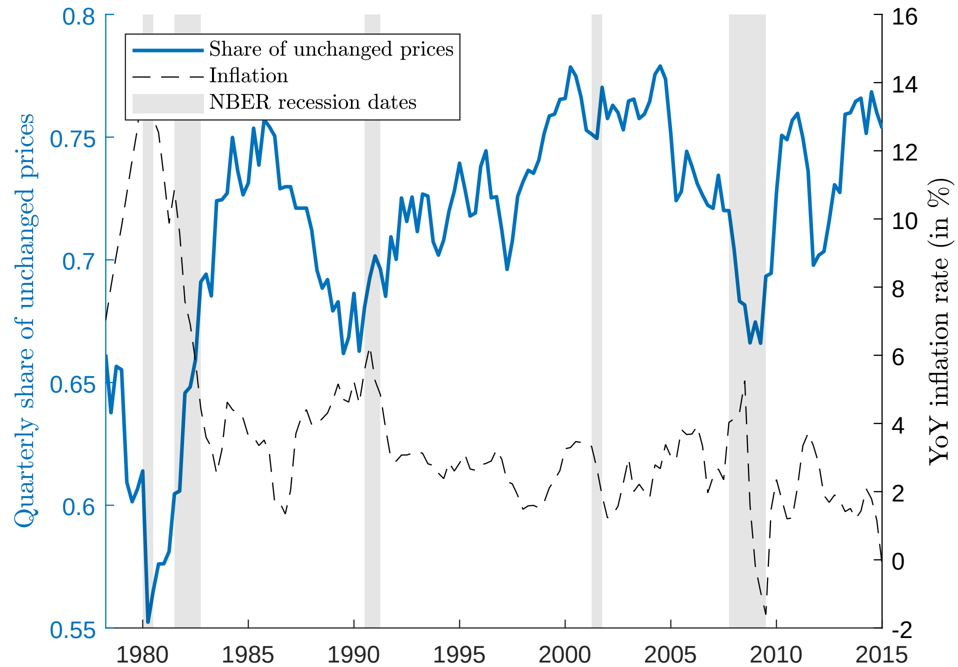

Figure 1: Quarterly share of unchanged prices and inflation, United States, 1978-2014. Inflation is the seasonally-adjusted year to year CPI growth based on CPIAUCSL obtained via FRED. The quarterly share of unchanged prices is computed by multiplying one minus the sum of the monthly seasonally-adjusted frequencies of price in- and decreases of Nakamura et al. (2018) for the respective months.

Most strikingly, the share of non-updated prices corresponding to the Calvo (1983) parameter varies from $\theta = 0.55$ to $\theta = 0.78$, which implies a very large variation in the slope of the NK Phillips curve. Clearly, this variation and the negative correlation, -0.808, between the two variables is inconsistent with the Calvo (1983) pricing model that assumes a constant θ. Moreover, based on macro-data, Fernández-Villaverde & Rubio-Ramírez (2007) show that the price setting frequency varies over time and is negatively correlated with inflation and price indexation. It is then natural to conjecture that a time-varying price setting frequency may be an alternative explanation for the observed flattening and shifts in the Phillips curve.

Against this background, we propose to augment the Calvo (1983) model of time-

previous 12 months. See Nakamura et al. (2018) (also their Figure XV) for further details.

dependent price changes by a state-dependent component. We argue that this extension can reconcile the NK model with the observed flattening of the Phillips curve and the evidence on time-varying price setting frequency at the micro level. The key novelty relative to the standard NK model is that the price setting frequency - *discussed in this paper as the Calvo share* - is endogenous and time-varying. Whether a firm updates its price in a given period depends on its assessment of expected cost and benefits modelled by a discrete choice process (Brock & Hommes 1997). We denote this the *Calvo law of motion* and interpret it as an approximation to firms' managerial decision of whether to update the price. Firms are more likely to update their prices when expected benefits outweigh expected costs and then set the price optimally. This assumption is broadly in line with random menu cost models (e.g., Costain & Nakov 2011a, Nakamura & Steinsson 2010). Yet, the model remains amenable to business cycle analysis with full information Bayesian methods.

We implement the *Calvo law of motion* in a NK model with trend inflation (Ascari & Sbordone 2014). Relative to the Calvo (1983) pricing model, our model has several advantages. First, the price setting frequency is no longer constant, but state-dependent and time-varying. Second, the *Calvo law of motion* captures the managerial decision process regarding price setting in line with micro evidence. This evidence shows that posting a new price is the result of a complex cost-benefit analysis by the firms' managers rather than a random process.[3] The *Calvo law of motion* models this idea by taking into account the expected present value of profits. We assume that there exists a trade off between updating and not updating current prices. Updating prices requires firms to spend resources (e.g., gather information, renegotiate contracts, etc.). In a sense, updating prices is an inherently costly dynamic process where firms face heterogeneous opportunity costs. We assume that firms decide to update their prices when it will

[3]For instance see Blinder, Canetti, Lebow & Rudd (1998) and Zbaracki, Ritson, Levy, Dutta & Bergen (2004) for qualitative and quantitative surveys with managers about their prices setting decisions.

increase the firm's expected present value of profits by more than maintaining price. For plausible parametrizations, profits are countercyclical due to the shape of the cost function. Around the steady state, for any given price, profits are relatively higher in a recession and relatively lower in an expansion. Thus, in present value terms, the benefit of updating the price optimally net of updating cost outweighs the cost of maintaining the price in expansions. However, the same only holds true for relatively large recessions. In consequence, the model predicts that prices are more flexible during expansions and less flexible during recessions.

Third, another appealing feature of our approach is that the aggregate equilibrium conditions of the model are isomorphic to the standard NK model with trend inflation, except for the time-varying price setting frequency following the *Calvo law of motion*. On the one side, this implies that the proposed mechanism can be easily embedded into any DSGE model with a Calvo (1983) pricing model including large-scale models used in policy making institutions. On the other side, this implies that the model can be analyzed and estimated with standard tools. We exploit this fact in our quantitative analysis and estimate the model over the micro time series in Figure 1 and standard macro time series under full information. In turn, we can assess the *Calvo share*'s role in explaining shifts in the Phillips curve.

Our main theoretical finding is the model's prediction of more flexible prices during expansions and less flexible prices during recessions, which can explain the non-linearity in the Phillips curve documented in the data. The price setting frequency is positively related with inflation. It accelerates during booms implying an accelerating inflation. In contrast, the model permits a decelerating price setting frequency during recessions and thus allows for low, but stable inflation during times of slack.

The quantitative main results of our paper are as follows. First, we find that our setup with the *Calvo law of motion* provides a good approximation of the observed price setting frequency depicted in Figure 1. The endogenous correlation between the share of unchanged prices and inflation is -0.581. Second, our model, despite its small scale, also fits the observed dynamics in inflation and output well. Third, the *Calvo law of motion* enables the model to explain observed inflation data to a large extent by discount factor and monetary policy shocks as well as the endogenous evolution of the price setting frequency. The role of cost-push shocks is very limited.[4] Finally, we show that the Calvo law of motion largely improves the macroeconomic time series fit of the medium-scale NK model developed in Fernández-Villaverde & Rubio-Ramírez (2006) over the sample 1959 to 2019.

Related literature. Our paper is related to a large literature relying on the seminal Calvo (1983)-Yun (1996) pricing model to generate a Phillips curve. We contribute to this literature by proposing a modification of the pricing model that gives rise to a time-varying price setting frequency. This modification is in part motivated by discussions over the stability of the original Calvo parameter as in Fernández-Villaverde & Rubio-Ramírez (2007), Alvarez, Lippi & Paciello (2011) or Berger & Vavra (2018) and its consistency with the paradigm of micro-founded models.[5] Therefore, also Davig (2016)'s proposal to implement shifts in the Phillips curve by modeling a quadratic price adjustment cost as a two state Markov process, is closely related to ours.

In contrast, the *Calvo law of motion*, our proposed modification to the NK model is essentially a discrete choice model inspired by Brock & Hommes (1997). Moreover, our proposal is within the realm of the Calvo (1983) pricing model and introduces an explicit cost-benefit analysis of price updating. While modelling the decision of whether to update the price as a discrete choice is a novelty within the NK model, a well-established literature has used discrete choice processes in NK models for modelling

[4]These results are consistent with the findings in Del Negro, Lenza, Primiceri & Tambalotti (2020) on the flattening of the price Phillips Curve.

[5]See Chari, Kehoe & McGrattan (2009), Plosser et al. (2012) and Lubik & Surico (2010) for discussion of sticky price models being subject to the Lucas Critique and see Caplin & Spulber (1987) and Gertler & Leahy (2008) for sticky price models explicitly aimed at addressing the Lucas Critique. Finally, see Bakhshi, Khan & Rudolf (2007) and Levin & Yun (2007) for a model with endogenous foundation of price setting frequency with respect to its relation to the trend inflation.

expectations and belief formation (see, e.g., Branch 2004, Branch & McGough 2010, Branch & Evans 2011, Hommes & Lustenhouwer 2019, Branch & Gasteiger 2019).

Although the use of the Calvo law of motion is inspired by a different strand of the literature, it is reminiscent of the random menu cost models by Costain & Nakov (2011a,b, 2015), or, more recently Costain & Nakov (2019), Costain, Nakov & Petit (2021). In these models, individual firms are subject to control cost and decide optimally about when and how they reset their price. Thereby a firm also takes the effect on its own future probability of adjustment into account. Consequently, the price setting frequency has a micro-foundation. Moreover, next to aggregate shocks, these models use idiosyncratic shocks to account for the cross sectional distribution of price adjustment observed in the data. Despite these advantages, it is not straightforward how one can solve and estimate such models with full information Bayesian methods. As the latter is an essential exercise in our paper, we deliberately approximate these important aspects of price resetting in reduced form.

Our quantitative work also relates to state or time-dependent sticky price models based on micro-econometric evidence.[6] In a series of papers, Nakamura & Steinsson (2008), Nakamura & Steinsson (2013) and Nakamura et al. (2018) develop a profound analysis of the implications of *heterogeneous* menu costs models and their fit to micro data constructed using BLS prices tag data. We apply the Nakamura et al. (2018) data to match one dimension of it: the price setting frequency. In related work, Gagnon (2009), Klenow & Kryvtsov (2008) and Alvarez & Burriel (2010) obtain similar conclusions about the inconsistency of the Calvo (1983) pricing model with pricing data at the micro level as, for instance, Nakamura et al. (2018). The models proposed in that literature fit better the cross-sectional price dynamics because of the heterogeneity in price stickiness and idiosyncratic shocks.[7] The proposed *Calvo law of motion* in this

[6]Theoretical and empirical implications of those models are extensively discussed in Alvarez, Lippi & Passadore (2017).

[7]Another related branch of the literature are the sticky information models (see, e.g., Mankiw & Reis 2002, Mankiw, Reis & Wolfers 2003). These papers introduce sticky price models based on the

paper captures this heterogeneity in reduced form.

Our paper is also related to multisector menu cost models such as Nakamura & Steinsson (2010)'s *CalvoPlus* model or more recently Gautier & Le Bihan (2020). Therein firms face a menu cost and idiosyncratic shocks on their price resetting cost. This combination of state- and time-dependent pricing matches the observed distribution of frequency and size of price changes in the cross-section. Moreover, it generates a plausible degree of money non-neutrality, mostly because of the time-dependent component. We use a combination of state- and time-dependent pricing to reconcile the NK model with the observed shifts of the Phillips curve and the evidence on time-varying price setting frequency at the micro level.

Finally, our model speaks to the rapidly expanding discussion on the explanations and implications of the empirically documented nonlinearity and flattening of the Phillips curve. For instance, Mavroeidis, Plagborg-Møller & Stock (2014) discuss inflation expectations as an explanation of the observed data. Aruoba, Bocola & Schorfheide (2017) and Forbes, Gagnon & Collins (2021) point toward the non-linearity in price and nominal wages adjustment costs. Moreover, Harding, Lindé & Trabandt (2021) resolve the missing deflation puzzle with the use of the Kimball aggregator.

The rest of the paper is organized as follows. Section 2 embeds the endogenous price setting frequency in a small-scale NK model with trend inflation. Section 3 contains the quantitative analysis of the small-scale NK model based on micro and macro data. Section 4 provides a horse-race between a standard medium-scale DSGE model with and without the Calvo law of motion. Section 5 concludes.

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frequency of forecast updating by firms. Firms have a probability to update their forecasts and thus their prices. Those models generate meaningful price dispersion, forecast behaviour, cross-sectional dynamics and stickiness. Yet, the updating property is fixed as in the Calvo-Yun model because observing the world is costly. Thus, the concerns regarding the Calvo-Yun model also apply to this branch of the literature.

2 An augmented small-scale NK model

We begin with developing a standard small-scale NK model augmented with the Calvo law of motion. We simulate the model to illustrate the key features of the proposed Calvo law of motion and to build intuition for the results derived in this paper.[8]

In the subsequent section, we use this model to examine the extent to which the Calvo law of motion helps to make the NK model consistent with both macroeconomic and microeconomic data. The novelty in the model is that the time-varying Calvo share θ_t enters in the forward looking profit maximization problem of intermediate firms. Most parts of the model are identical to Ascari & Sbordone (2014). Therefore we focus on the departures from this model, namely the firms' pricing problem, the Calvo law of motion and the resulting price dispersion.

2.1 The firm's pricing problem

First we discuss the intermediate firms' price setting problem. The novelty is that we consider θ_t as an endogenous variable and not as a parameter. These firms maximize the expected present value of profits over an infinite horizon by applying the stochastic discount factor and the current and expected future frequency of price setting in an inflationary world. The price setting frequency and therefore the optimal reset price depends on the current and expected markup generated by the pricing decision. Those assumptions generate a complex feedback loop between the pricing decision and the resetting decision. Formally the problem is

$$\max_{P_t^*} \ \mathbb{E}_t \sum_{j=0}^{\infty} \mathcal{D}_{t,t+j} \left(\prod_{k=0}^{j} \theta_{t+k} \right) \theta_t^{-1} \left[\frac{P_t^*}{P_{t+j}} - \frac{\Gamma'_{t+j}}{P_{t+j}} \right] Y_{i,t+j}$$

$$\text{s.t. } Y_{i,t+j} = \left(\frac{P_t^*}{P_{t+j}} \right)^{-\epsilon} Y_{t+j}$$

[8] In a previous version we also presented a simplified augmented model with similar predictions. We have relegated the this model to Appendix A for the sake of brevity.

where $\mathcal{D}_{t,t+j} \equiv \beta^j \frac{\lambda_{t+j}}{\lambda_t}$ is the stochastic discount factor with λ_{t+j} denoting the $t + j$ marginal utility of consumption. Γ'_t is the marginal cost, P_t is the price level, Y_t is the aggregate output level, $Y_{i,t+j}$ us demand for the good of firm i and ϵ is the price elasticity of demand. The optimal price for the resetting firm, P_t^*, has to satisfy the first-order necessary condition for an optimum

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \left(\prod_{k=0}^{j} \theta_{t+k} \right) \theta_t^{-1} \mathcal{D}_{t,t+j} (P_{t+j}^{\epsilon} Y_{t+j} \Gamma'_{t+j})}{\mathbb{E}_t \sum_{j=0}^{\infty} \left(\prod_{k=0}^{j} \theta_{t+k} \right) \theta_t^{-1} \mathcal{D}_{t,t+j} (P_{t+j}^{\epsilon-1} Y_{t+j})}. \tag{1}$$

We note that $\Gamma'_{t+j} = w_{t+j}$ holds because we assume a linear production function for intermediate goods producers. Moreover, the aggregate price level evolves according to

$$P_t = \left(\theta_t P_{t-1}^{1-\epsilon} + (1 - \theta_t) P_t^{*\,1-\epsilon} \right)^{\frac{1}{1-\epsilon}}. \tag{2}$$

We define $\Pi_{t,t+j-1}$ as the cumulative gross inflation between t and $t + j - 1$

$$\Pi_{t,t+j-1} \equiv \begin{cases} \frac{P_t}{P_{t-1}} \times \frac{P_{t+1}}{P_t} \times ... \times \frac{P_{t+j-1}}{P_{t+j-2}} & \text{for } j = 1, 2, ... \\ 1 & \text{for } j = 0. \end{cases}$$

Dividing both sides of (1) by P_t, we obtain

$$p_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \left(\prod_{k=0}^{j} \theta_{t+k} \right) \theta_t^{-1} \mathcal{D}_{t,t+j} \Pi_{t+1,t+j}^{\epsilon} Y_{t+j} w_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \left(\prod_{k=0}^{j} \theta_{t+k} \right) \theta_t^{-1} \mathcal{D}_{t,t+j} \Pi_{t+1,t+j}^{\epsilon-1} Y_{t+j}},$$

where $p_t^* \equiv P_t^*/P_t$ is the relative price level implied by the optimal price decision. By applying the definition of gross inflation, $\pi_t \equiv P_t/P_{t-1}$, and by using (2), we obtain

$$1 = (\theta_t \pi_t^{\epsilon-1} + (1 - \theta_t) p_t^{*\,1-\epsilon})^{\frac{1}{1-\epsilon}}.$$

It follows that we can rewrite (1) as

$$p_t^* = \frac{\epsilon}{\epsilon - 1}\frac{\psi_t}{\phi_t}, \qquad \text{where} \tag{3}$$

$$\psi_t = Y_t^{1-\sigma} w_t + \mathbb{E}_t \beta \theta_{t+1} \pi_{t+1}^{\epsilon} \psi_{t+1}, \tag{4}$$

$$\phi_t = Y_t^{1-\sigma} + \mathbb{E}_t \beta \theta_{t+1} \pi_{t+1}^{\epsilon-1} \phi_{t+1}. \tag{5}$$

2.2 The Calvo law of motion

This paper proposes to model firms as being run by managers that, in principle, consider to reset the price for their firm's good in each period. Managers base the strategic decision of updating or not updating the price optimally on a cost-benefit analysis.

The benefit of optimally updating the price is quantified based on the expected present value of the firm's profits when setting p_t^*, which we denote U_t^*. Computing the latter requires coordination within the firm that comes at a cost τ that has to be taken into account, say, a meeting to establish what is the optimal price in period t. More generally, τ may capture information acquisition, contract revisions, negotiations, working time, agency cost, or, simply menu costs (Rotemberg 1982). Thus, only if the expected present value of the firm's profits implied by p_t^* net of the cost outperforms the expected benefit of maintaining the price, managers will reset the price.

Yet, there is an additional subtle but essential point that has to be taken into account when computing the benefit of maintaining the price. Even in a model with a fixed parameter θ, maintaining the price has fundamentally different implications for each individual firm as long as there is non-zero trend inflation. Each firm i has a different old price and thus faces a different opportunity cost of changing the price $U_{i,t}^f$. This heterogeneity among firms increases the complexity in quantifying the benefit of maintaining the price at the cost of model tractability.

We propose to sidestep this complex issue for the sake of tractability. In particular, we quantify the benefit of maintaining the price as the expected present value of the firm's profits when choosing the average relative old price level in the economy p_t^f at no cost. We denote this benefit $U_{i,t}^f = U_t^f \forall i$. This assumption allows us to approximate the Calvo share variation θ_t in reduced form by building on Brock & Hommes (1997) and assuming the following *Calvo law of motion* for the price setting frequency

$$\theta_t = \frac{\exp\left(\omega U_t^f\right)}{\exp\left(\omega U_t^f\right) + \exp\left(\omega\left(U_t^* - \tau + \varepsilon_t^\theta\right)\right)}, \tag{6}$$

where $\theta_t \in [0,1]$ and $(1-\theta_t)$ denotes the share of updated prices. Parameter $\omega \geq 0$ is denoted the *intensity of choice* and captures the idea that every period some firms update their prices and others do not as long as $\omega < \infty$. Thus, this parameter captures the above discussed heterogeneity of firms in reduced form. ε_t^θ denotes a contract shock, which follows an $AR(1)$ stationary process. The shock captures exogenous variation in the managers' relative cost of updating their price. We set this shock to zero for now, but use it for estimation purposes in later sections. The present value of expected profits implied by the pricing decision $x \in \{*, f\}$ is

$$U_t^x = \mathbb{E}_t \sum_{k=0}^{\infty} \mathcal{D}_{t,t+k} \left(\prod_{j=0}^{k} \theta_{t+j}\right) \theta_t^{-1} \left[Y_{t+k}\left(\frac{p_t^x}{\left(\Pi_{t,t+k-1}\right)\Pi_t^{-1}}\right)^{1-\epsilon} - Y_{t+k} w_{t+k} \left(\frac{p_t^x}{\left(\Pi_{t,t+k-1}\right)\Pi_t^{-1}}\right)^{-\epsilon}\right], \tag{7}$$

i.e., consistent with the firm's pricing problem discussed above, real profits are discounted by the subjective discount factor, the time-varying resetting frequency and the aggregate price variation.[9]

[9]This specification nests the standard Calvo pricing model for $\omega \to 0$. Moreover, note that (6) implies $(1-\theta_t) = 1/\left[\exp\left(-\omega\left(U_t^* - U_t^f - \tau + \varepsilon_t^\theta\right)\right) + 1\right]$. This functional form corresponds to the one assumed or derived for the individual firm price resetting probability in random menu cost models such as Costain & Nakov (2011a,b), or, more recently Costain & Nakov (2019) and Costain et al. (2021). In particular to a random menu cost model with a logistic distribution of menu costs with standard deviation $\pi/(\omega\sqrt{3})$, where one can interpret τ as the mean, $1/\omega$ as the scale parameter. However, in our reduced form approach, we do not endogenize θ_t at the individual firm level. Therefore, in contrast to random menu cost models, in our model firms do not consider the effect of the price setting decision on future θ_{t+j}. The quantitative importance of the latter channel is an open empirical question beyond the scope of our paper.

Figure 2 illustrates the properties of (6). One can observe several worthwhile features

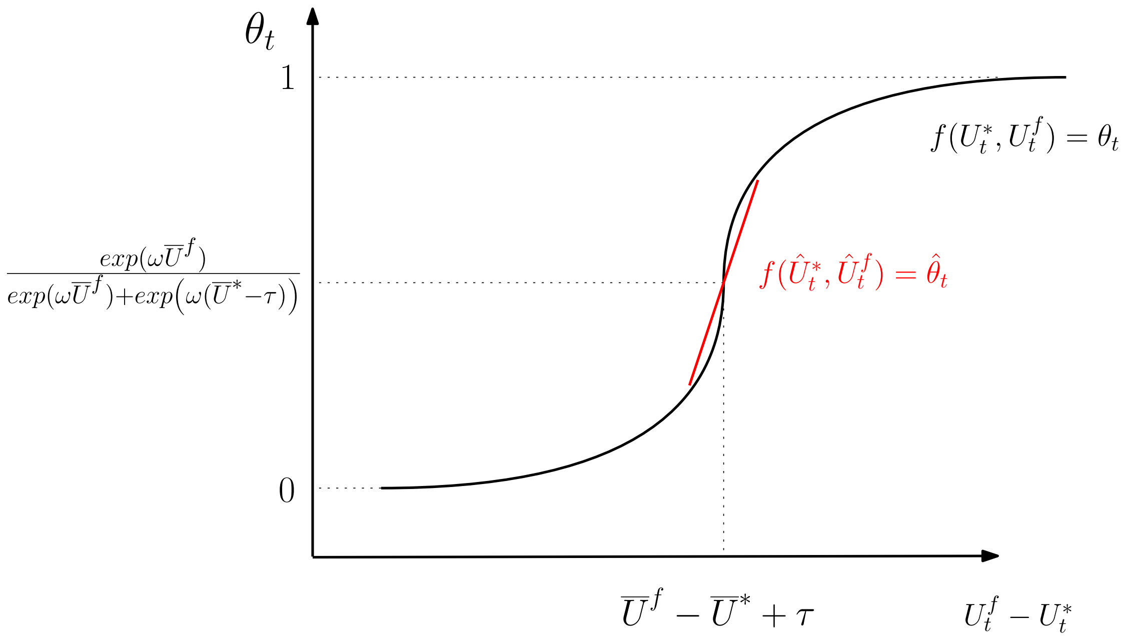


Figure 2: The Calvo law of motion (black) and its linearised form (red).

from Figure 2. The function is bounded between zero and one. In steady state, θ is determined by the intensity of choice ω, the updating cost τ and the present values of profits $\overline{U}^*$ and $\overline{U}^f$.[10] For instance, a zero inflation steady state implies $\overline{p}^* = \overline{p}^f$ and therefore $\overline{U}^* = \overline{U}^f$. With zero updating cost, $\tau = 0$, this in turn implies a share of $\theta = 1/2$. Moreover, in steady state the Calvo law of motion nests pure time-dependent pricing for $\omega \to 0$ as in the standard Calvo model.

However, out of steady state, managers' cost-benefit analysis implies state-dependent pricing. In states where the benefit of updating the price outweighs the cost, the share of firms that update their price increases. In states where the cost of updating the price outweighs the benefit, the share of firms that maintain the price increases. As we show below, for standard parametrizations, the incentive to optimally update the price is

[10]Note that trend inflation has a direct effect on the steady state price setting frequency. The higher trend inflation, $\overline{\pi} > 1$, the higher the difference between relative prices, $\overline{p}^* > \overline{p}^f$, and the larger the difference in implied steady state present values of profits, $\overline{U}^* > \overline{U}^f$. Thus, the higher trend inflation, the higher the price setting frequency. This is an interesting implication in line with the ones by Levin & Yun (2007) and Bakhshi et al. (2007), where higher trend inflation also leads to higher price resetting frequency.

stronger in expansions and weaker in recessions. Period profits are countercyclical: in the neighborhood of the steady state, for any given price, profits are relatively higher in a recession and relatively lower in an expansion. Via (7) this leads to the prediction that in recessions U_t^f exceeds $U_t^* - \tau$ and the opposite is true in expansions.

While finite ω and τ as well as modest variations of profits imply that θ_t varies between zero and one, the two polar cases $\theta_t = 0$ and $\theta_t = 1$ are feasible. Fully flexible prices, $\theta_t = 0$, emerge if either $U_t^* \to +\infty$ or $U_t^f \to -\infty$. In these extreme cases the benefit of optimally resetting the price will always outweigh the cost and the economy behaves similar to a flexible price economy.

In the case of fixed prices, $\theta_t = 1$, the optimal price is not evolving and is equal to the steady state value of the marginal cost. This becomes feasible if either $\tau \to +\infty$, $U_t^* \to -\infty$ or $U_t^f \to +\infty$. These are extreme cases, where the cost of optimally resetting the price will always outweigh the benefit.

Also ω is a crucial parameter in determining price setting behavior in our model. Above we have interpreted it as a measuring how rational and heterogeneous agents are in the strategy selection (Brock & Hommes 1997). If $\omega = 0$, then θ is constant as in Calvo (1983) and pricing is entirely time-dependent. On the other hand, when $\omega \to +\infty$, all managers consider the whole set of information and do the optimal trade off between both strategies. This leads to the extreme case where $\theta_t = \{0, 1\}$. However, while the true value of ω is an empirical question, we do not consider $\omega \to +\infty$ to be a likely case even if strategy selection is entirely rational.[11]

2.3 Price dispersion

Given the Calvo law of motion, price dispersion is a more complex process relative to the standard trend inflation NK model. The time-varying θ_t, can amplify or mute the non-monotonic behavior of price dispersion. In order to illustrate this point, consider

[11] Brock & Hommes (1997) argue that when $\omega \to +\infty$ the Calvo law of motion reaches the *neoclassical limit* where $\theta_t = \{0, 1\}$ is *rational* because it is always optimal.

the definition of relative price dispersion

$$s_t \equiv \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} di. \tag{8}$$

Under the Calvo pricing this can be expressed as

$$s_t = \frac{1}{P_t^{-\epsilon}} \left(\sum_{k=0}^{\infty} \theta_{t|t-k}(1-\theta_{t-k})(P^*_{i,t-k})^{-\epsilon} \right), \quad \text{where } \theta_{t|t-k} = \begin{cases} \Pi_{s=0}^{k-1}\theta_{t-s}, & \text{if} \quad k \geq 1, \\ 1, & \text{if} \quad k = 0, \end{cases}$$

or, recursively as

$$s_t = (1-\theta_t)(p_t^*)^{-\epsilon} + \theta_t \pi_t^{\epsilon} s_{t-1}.$$

From the above expression for s_t one can see that the time-varying Calvo share θ_t implies time-varying effects on price dispersion that can amplify or mute the non-monotonic effects of p_t^* and π_t on s_t. Suppose that a shock creates an incentive for firms to lower p_t^* and consequently leads to a decline in π_t. First, a lower p_t^* tends to raise s_t. Second, a lower π_t tends to decrease s_t. A higher θ_t implies that less firms update to the new optimal price and therefore mutes the first effect and amplifies the second. The reverse is true for a lower θ_t. A similar reasoning applies to a shock that creates an incentive for firms to increase p_t^*.

2.4 The linearised Phillips curve

In order to understand how the Calvo law of motion affects the model dynamics, we linearise the NK Phillips curve around a trend inflation steady state as in Ascari & Sbordone (2014) (see Appendix B.2).[12] Throughout the linearisation, we assume $0 < \overline{\theta} < 1$ to avoid the empirically implausible polar cases $\overline{\theta} = \{0, 1\}$.[13] Thus, the NK

[12]A hat ($\hat{\cdot}$) indicates that a variable is expressed in log-deviation from their steady state.

[13]Based on Figure 1 this seems to be a reasonable assumption.

Phillips curve can be written as

$$\hat{\pi}_t = \alpha_1 \hat{w}_t + \alpha_2 \mathbb{E}_t \hat{\pi}_{t+1} + \alpha_3 \mathbb{E}_t \hat{\phi}_{t+1} + \alpha_4 \hat{\theta}_t + \alpha_5 \mathbb{E}_t \hat{\theta}_{t+1} \tag{9}$$

with $\alpha_1, \alpha_2, \alpha_3 > 0$ and $\alpha_4, \alpha_5 < 0$ being the composite parameters displayed and discussed in Appendix C. The last two terms in (9) emerge because of the Calvo law of motion. In addition, as we discuss below, also $\mathbb{E}_t \hat{\phi}_{t+1}$ is affected by the time-varying price setting frequency.

As in a standard trend inflation model, inflation $\hat{\pi}_t$ is positively linked to expected inflation $\mathbb{E}_t \hat{\pi}_{t+1}$, marginal cost $\hat{w}_t$ and the additional term $\hat{\phi}_t$. Moreover, we can disentangle the relation between $\hat{\theta}_t$, $\mathbb{E}_t \hat{\theta}_{t+1}$ and $\hat{\pi}_t$. First of all, there is a negative relation between $\hat{\theta}_t$ and $\hat{\pi}_t$. Consistent with our discussion of the effect of θ_t on price dispersion s_t in (8), the higher $\hat{\theta}_t$, the less frequent price changes are and thus the less inflation we observe. The relation is also negative between $\mathbb{E}_t \hat{\theta}_{t+1}$ and $\hat{\pi}_t$. Thus, if the economy is expected to be less flexible in the next period, inflation will also be lower.

The Calvo law of motion and a positive trend inflation steady state together have an additional effect on inflation in (9) via

$$\hat{\phi}_t = \beta \theta \bar{\pi}^{\epsilon - 1} (\mathbb{E}_t \hat{\theta}_{t+1} + (\epsilon - 1) \mathbb{E}_t \hat{\pi}_{t+1} + \mathbb{E}_t \hat{\phi}_{t+1}).$$

Indeed, the higher expected values of $\hat{\theta}_t$ are, the higher current inflation is. This is generated by the same effect as a "fear of missing out" on price adjustment. If a firm expects less flexibility of the economy in the future in an inflationary environment, it may increase the price now.

Finally, it is important to mention that, while considering a non-zero trend inflation steady state appears generally plausible in light of the positive inflation targets proclaimed by many central banks, it is essential for our purposes. With a zero inflation steady state, there is no difference in the steady state price of a price re-setter and a non price re-setter, i.e., $\overline{p}^f = \overline{p}^*$. Thus, in a first order approximation, the effect of the time-varying price setting frequency would simply cancel.

2.5 The complete model

Our model is very similar to a standard NK model with trend inflation, (see, e.g., Ascari & Sbordone 2014) as we only add the Calvo law of motion. The complete non-linear system of model equations is as follows:

$$
\begin{aligned}
\textbf{Euler equation:} \quad & Y_t^{-\sigma} \exp(\varepsilon_t^d) = \beta \mathbb{E}_t \frac{i_t}{\pi_{t+1}} Y_{t+1}^{-\sigma} \exp(\varepsilon_{t+1}^d) \\
\textbf{Marginal cost:} \quad & w_t = \exp(\varepsilon_t^s) \chi N_t^{\varphi} Y_t^{\sigma} \\
\textbf{Labour supply:} \quad & Y_t = N_t / s_t \\
\textbf{Relative prices:} \quad & \frac{P_t^x}{P_t} = p_t^x \quad \text{for} \quad x \in \{*, f\} \\
\textbf{Calvo law of motion:} \quad & \theta_t = \frac{\exp\left(\omega U_t^f\right)}{\exp\left(\omega U_t^f\right) + \exp\left(\omega\left(U_t^* - \tau + \varepsilon_t^\theta\right)\right)}, \\
\textbf{Agg. price dynamics:} \quad & 1 = (\theta_t \pi_t^{\epsilon - 1} + (1-\theta_t) p_t^{*\,1-\epsilon})^{\frac{1}{1-\epsilon}} \\
\textbf{Opt. price setting:} \quad & p_t^* = \frac{\epsilon}{\epsilon - 1} \frac{w_t Y_t^{1-\sigma} + \mathbb{E}_t \beta \theta_{t+1} \pi_{t+1}^{\epsilon} \psi_{t+1}}{Y_t^{1-\sigma} + \mathbb{E}_t \beta \theta_{t+1} \pi_{t+1}^{\epsilon-1} \phi_{t+1}} \\
\textbf{Price law of motion:} \quad & p_t^f = \frac{1}{\pi_t} \\
\textbf{Price dispersion:} \quad & s_t = (1-\theta_t) p_t^{*\,-\epsilon} + \theta_t \pi_t^{\epsilon} s_{t-1} \\
\textbf{Monetary policy:} \quad & i_t - \bar{i} = (1-\rho)\{\phi_\pi(\pi_t - \overline{\pi}) + \phi_y(\frac{Y_t - \overline{Y}}{\overline{Y}})\} \\
& \qquad + \rho(i_{t-1} - \bar{i}) + \varepsilon_t^r \\
\textbf{Shocks:} \quad & \varepsilon_t^j = \rho_j \varepsilon_{t-1}^j + u_{\varepsilon^j,t}, \qquad \text{where} \qquad j \in \{d, r, \theta\}, \\
& \varepsilon_t^s = \rho_s \varepsilon_{t-1}^s - \mu_s u_{\varepsilon^s,t-1} + u_{\varepsilon^s,t},
\end{aligned}
$$

with $0 \leq \rho_j < 1$, $0 \leq \mu_s < 1$ and $u_{\varepsilon^j,t}, u_{\varepsilon^s,t} \sim$ iid $\mathcal{N}(0, \sigma_j^2)$. Note that we model the cost-push shock as an ARMA(1,1) process, which is in line with standard practices (see, e.g., Smets & Wouters 2007). In particular, this allows the model to better capture the high-frequency movements in inflation. In doing so, we also give more potential to the model to explain inflation simply by cost-push shocks and without relying on the Calvo law of motion.

2.6 Asymmetric dynamics in the Phillips curve

In this section, we use the Fair & Taylor (1983) method to simulate non-linear impulse responses in order to to illustrate an important feature of the small-scale NK model: asymmetric dynamics in the Phillips curve implied by the Calvo law of motion.

Calibration. We use a standard calibration, see Table 1. The results are robust to different calibrations. Here, we assume log utility. The intensity of choice, $\omega = 10$, is taken from the heuristic switching learning literature. The price elasticity of demand tunes the level of inflation and the optimal relative price. We choose τ in such a way that it implies a steady state value of $\overline{\theta} = 0.75$, which is standard in the NK literature. Most parameters are taken from Galí (2015). The parametrization of shocks is solely for illustrative purposes, but in line with findings in the literature.[14]

Results. Figure 3 displays the simulated impulse response functions to a *positive* and to a *negative* 5 percent demand shock. We start with the benchmark of time invariant θ (black dashed line). A *positive* demand shock raises output and real marginal cost, equal to w_t, on impact above their steady state level. Firms that can reset the price, raise their price to stabilize their markups and thus profits. In consequence, on impact, p_t^* and π_t increase, p_t^f must decline and price dispersion increases. Due to our calibration of the monetary policy rule, the nominal interest rate increases in response to rising

[14] Equation (7) is a complex infinite sum. In order to make the numerical and empirical analysis feasible, we reduce the horizon to 8 quarters. This is common practice in sticky information models à la Mankiw & Reis (2002), where the infinite backward sum cannot be handled analytically. The results are robust to larger and shorter horizons.

		Values	Sources
β	Discount factor	0.99	Galí (2015)
σ	Relative risk aversion	1	Galí (2015)
φ	Frisch elasticity	1	Galí (2015)
ϕ_π	Policy stance on inflation	1.5	Galí (2015)
ϕ_y	Policy stance on output	0.125	Galí (2015)
$\overline{\pi}$	Inflation target	1	No inflation
ϵ	Price elasticity of demand	9	Galí (2015)
$\overline{\theta}$	Steady state Calvo share	0.75	Galí (2015)
ω	Intensity of choice	10	illustrative purpose
ρ_d	Discount factor shock, AR(1)	0.8	illustrative purpose

Table 1: Calibrated parameters for dynamic simulations (quarterly basis).

inflation and output. The subsequent periods show a persistent monotonic convergence of endogenous variables toward their steady state levels. This is due to the persistence in the demand shock, which implies that a fixed share of firms will revise their price upward each period until marginal cost have returned to their steady state value.

Relative to the standard model, a time-varying Calvo share θ_t (blue dashed line) has novel and important implications: while the responses of output and real marginal cost are muted on impact, the responses of nominal variables are amplified on impact. The boom in demand implies that the cost-benefit analysis modelled by (6) leads more managers to the conclusion that raising the price, net of the cost τ, implies a higher benefit relative to not raising the price. This can be seen from the $U_t^* - U_t^f - \tau$ in Panel g. Therefore θ_t declines, which translates into higher inflation and price dispersion on impact. The latter rationalizes both the more aggressive response of monetary policy and the muted response of output and real marginal cost on impact. Subsequent periods are again characterized by a similar monotonic convergence.

Next, we turn to the impulse response functions to a *negative* 5 percent demand shock in Figure 3. In the benchmark case with time invariant θ (black solid line), the impulse responses and the economic intuition behind them are exactly the opposite of the positive demand shock. Firms that can update the price, do so optimally. However,

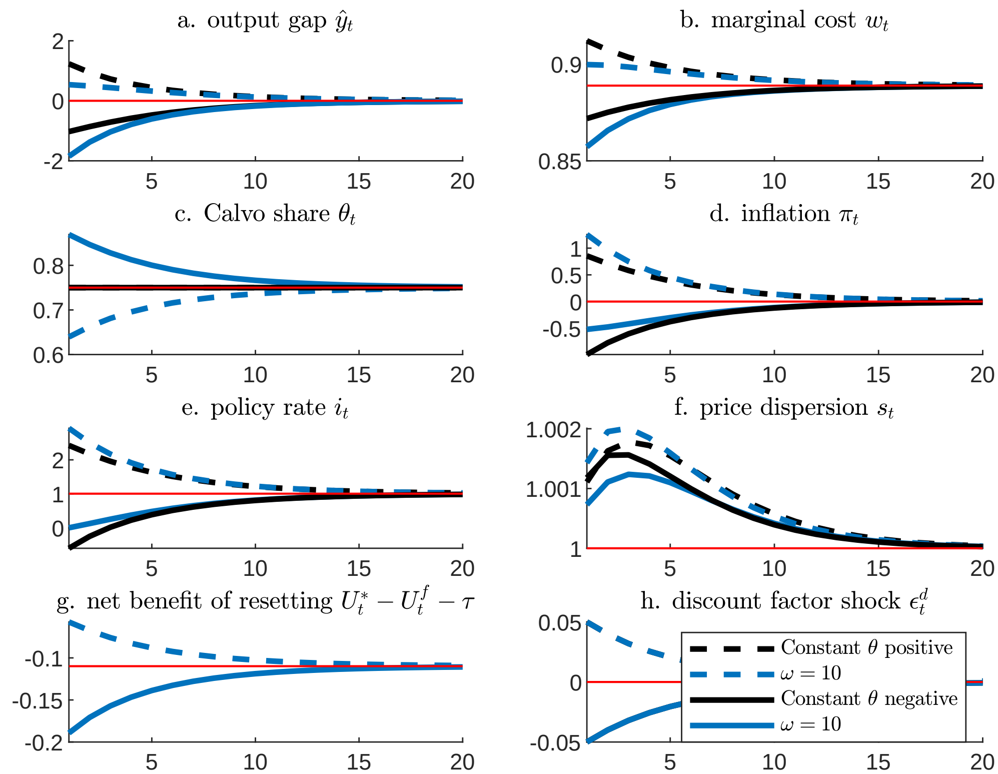


Figure 3: Asymmetric impulse responses to a positive or negative ($\pm 5\%$) discount factor shock in the small-scale NK model.

in the case of time-varying θ_t (blue solid line), the responses in a recession reveal a striking asymmetry compared to a boom.

In contrast to the effects of a positive demand shock, the decline in output and real marginal cost on impact is amplified relative to the standard model. Due to the similar mechanism as described above, the decline in inflation and prices is muted on impact.

In particular, the cost-benefit analysis of managers leads them to the conclusion that lowering the price net of the cost implies a lower benefit relative to maintaining the price, see $U_t^* - U_t^f - \tau$ in Panel g. Thus, a lower share of managers will reset the price and less firms will actually do so. Thus, θ_t increases, which translates into lower inflation. The relative advantage of not resetting the price dies out as real marginal cost monotonically increase toward their steady state. Thus, θ_t reverts back to its steady state as well.

One may wonder why there is no symmetry in the responses to the negative demand shock and why there is no increase in the price setting frequency? The reason is the shape of the profit function in the NK model. In order to clarify this point, consider the nominal profit function $\Pi_t^x = P_t^x Y_t^x - \Gamma_t Y_t^x$ for $x \in \{*, f\}$, which we can express in real terms as $P_t^{-1}\Pi_t^x = (p_t^x)^{1-\epsilon} Y_t - w_t (p_t^x)^{-\epsilon} Y_t$.

Figure 4 depicts the latter in steady state as a function of the relative price (blue). Moreover, it depicts the real profit as a function the relative price when holding all variables at their steady state, but considering a lower (black) or higher (red) output level. In steady state, $Y = 1$, it holds that $\overline{p}^* = \overline{p}^f = 1$. All else equal, in an

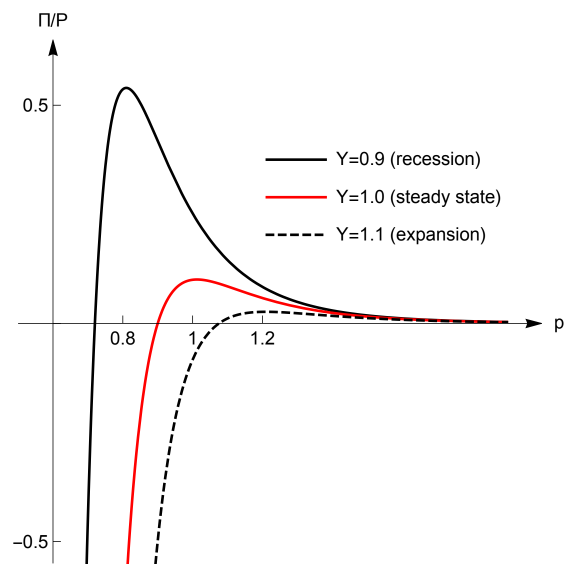


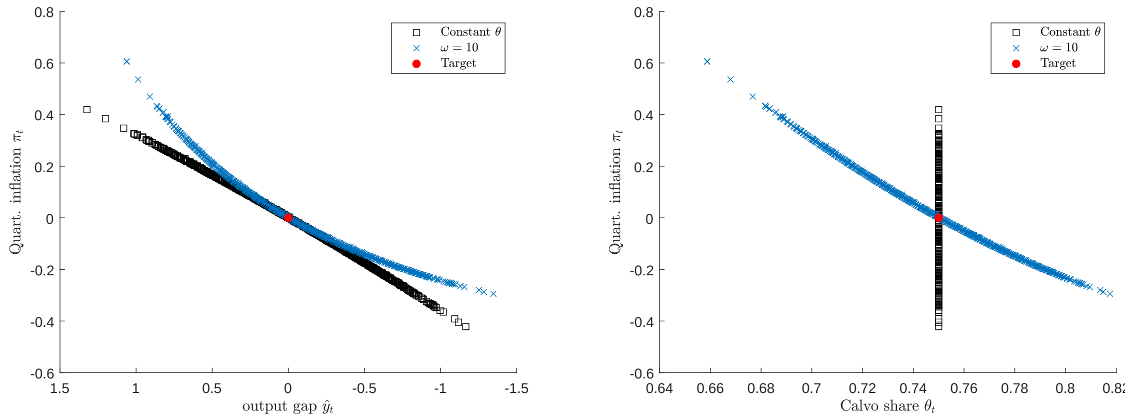
Figure 4: Comparative statics: real profit as function of relative price at different levels of output.

expansion, $Y = 1.1$, a higher relative price, say $p = 1.2$, implies a positive real profit in the expansion, steady state and recession. Contrary, in a recession, $Y = 0.9$, a lower relative price, say $p = 0.8$, implies a relatively large positive real profit. However, in steady state and expansion such a price implies a large negative real profit. Finally, maintaining the steady state price, $p = 1$ implies a positive real profit in the recession and relatively modes negative real profit in expansion. Assume equal probability of $Y \in \{0.9, 1, 1.1\}$. The figure then suggests that the expected present value of real profits (recall (7)) implied by optimally lowering the price today may still be positive even when the risk of non-resetting in the future is taken into account. However, net of adjustment costs, the expected present value may not exceed the expected present value of maintaining the relative price at the steady state. For this to be true, the recession must be relatively large or very long.

Consistent with this intuition, in small recessions as illustrated in Figure 3, lowering the price implies lower profits (net of cost) than keeping the price (see Figure Panel g.). The opposite is true in a large recession. Figure D.1 shows results for three times larger recession. In this case, lowering the price generates a higher benefit. In consequence, the price setting frequency in large recessions increases.

The above exercise makes clear that the Calvo law of motion implies an asymmetry in price setting by firms. The source of this behaviour is rooted in the countercyclical behavior of firm profits. Raising prices in booms raises firm profits relative to keeping the price unchanged. In contrast, whether lowering prices in recessions is more beneficial relative to maintaining the price depends on the size of recession. As a consequence, the model with time-varying θ_t generates larger responses of inflation relative to the benchmark case of the invariant θ in booms, but smaller responses in recessions.

This asymmetry in impulse response functions to a demand shock translates into a prediction for the Phillips curve, which is illustrated in Figure 5. To produce this figure, we simulate the model for $10,000$ periods under demand shocks only. The Phillips curve in Panel 5a is flat in recessions and steep in booms. This prediction can be rationalized by the evolution of the Calvo share, see Panel 5b. When inflation is high, the benefit implied by the past average price level is low and the of price resetting frequency is high. In contrast, when inflation is low, the benefit implied by the past average price level is high and the price resetting frequency is low. The correlation between the share



(a) Phillips curve

(b) Relation between inflation and the Calvo share

Figure 5: Global dynamics in the augmented ($\omega = 10$, blue) and standard (constant θ, black) small-scale NK model in levels in response to a discount factor shock with standard deviation of 0.1.

of unchanged prices and inflation in this exercise is -0.974.

Figure 6 is a density plot for price setting frequency in relation to inflation for the standard and the augmented model and illustrates the prediction in a distinct way. The model features an asymmetric accelerating Phillips curve where deflation is limited and inflation is self re-enforcing.[15] Inflation in the standard model is modestly negatively skewed (-0.078), whereas inflation in the augmented model is positively skewed (0.473).[16] In consequence, our model predicts only modest declines in inflation during large recessions. Therefore our modelling approach has the potential to explain important patterns in the observed Phillips curve dynamics such as low, but positive inflation during times of persistent slack as observed during the Great Recession, the subsequent recovery without high inflation, or, the flattening of the Phillips curve. Widely used models (incl. the standard NK model) fail to explain such patterns (see, e.g., Hall 2011).

[15] It is not overly surprising that the visible differences in Figure 6 are rather small. In this simulation exercise we just change one parameter modestly.

[16] Those results are significantly different at 99%.

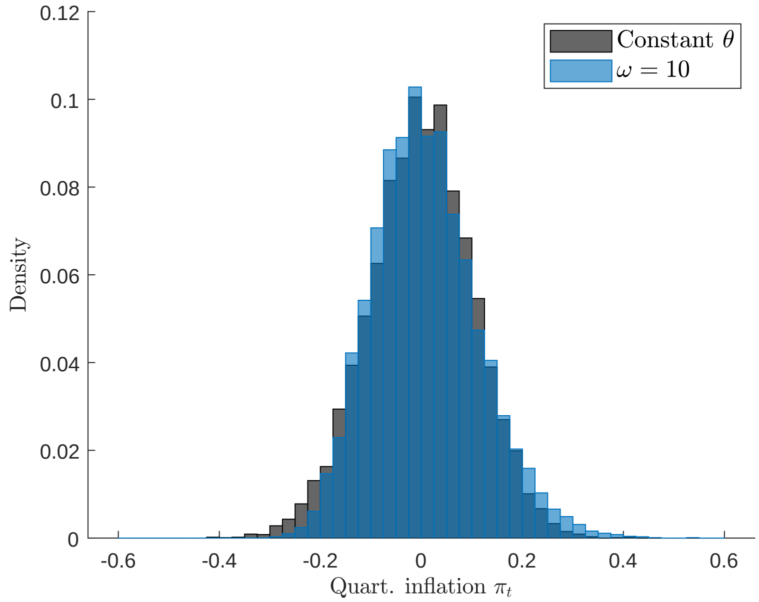


Figure 6: Density of inflation in the small-scale NK model. Augmented model ($\omega = 10$) in blue and standard model (constant θ) in gray.

Monetary non-neutrality. Finally, it is worthwhile to point out the implications of adding a state-dependent component to a time-dependent pricing model via the Calvo law of motion for monetary non-neutrality. It is well known that, due to the selection effect, it is challenging to reconcile textbook state-dependent pricing models with the evidence on the real effects of monetary policy. Contrary, textbook time-dependent pricing models such as the standard NK model feature a high degree of monetary non-neutrality. Against this background, Nakamura & Steinsson (2010) show that extending a state-dependent model with a time-dependent component (i.e., the CalvoPlus model) only generates a high degree of monetary non-neutrality for a high share of time-dependent price changes. Figure E.1 in Appendix E shows that, similar to the discount factor shock, the impulse responses to a positive or negative ($\pm 1\%$) monetary policy shock are again asymmetric due to the properties of the profit function. Relative to the standard NK model, the degree of monetary non-neutrality is higher (lower) for a contractionary (expansionary) shock.

3 Empirical analysis of the small-scale NK model

The augmented NK model predicts asymmetric responses to aggregate shocks that, even in a linearized model, can translate into quantitative differences relative to the standard NK model. Moreover, the augmented NK model has another key prediction that distinguishes it from the standard NK model: the price setting frequency is time-varying. Thus, a natural question presents itself: which model is more consistent with the data? The remainder of the paper provides an answer to this question by comparing estimated versions of the augmented NK model to the standard NK model.

3.1 Data and measurement equations

We use four quarterly time series in log-levels: the output gap, inflation, the Federal Funds rate and the share of unchanged prices depicted in Figure 1. The sample ranges from 1964Q1 to 2019Q4. The output gap (GDPC1)[17], inflation (PCE)[18] and the Federal Funds (FEDFUNDS) rate are taken from Fred.

The main innovation of our estimation is that we use the share of unchanged prices in the estimation in order to assess the consistency of our model with microeconomic next to macroeconomic data. To construct this time series, we use the data on monthly prices changes from Nakamura et al. (2018) between 1978 to 2014 (see the note in Figure 1 for methodological details). Conceptually this share of unchanged prices corresponds to the Calvo share θ_t, which accounts for the share of prices that are not updated per quarter. Note that θ_t is not available for the periods 1964 to 1977 and 2015 to 2019. Thus, for these periods we treat θ_t as a latent state variable and exclude it from the likelihood optimization problem.[19]

[17]The output gap is the log deviation of the real GDP time series from an HP filtered trend computed by the authors in order to keep a zero mean time series (GDPC1).

[18]We use the log growth of the personal consumption expenditures implicit price deflator (DPCERD3Q086SBEA).

[19]An alternative is to estimate the model solely for the sample 1978 to 2014. However, such short samples raise many general identification problems.

The observables are related to the model variables by the measurement equations

$$
\begin{aligned}
y_t^{obs} &= \hat{y}_t \\
\pi_t^{obs} &= 100 \times \ln(\overline{\pi}) + \hat{\pi}_t \\
r_t^{obs} &= 100 \times \overline{r} + \hat{i}_t \\
\theta_t^{obs} &= \theta_t,
\end{aligned}
$$

where $\overline{\pi} = 1 + \gamma_\pi/100$ and $\overline{r} = (\overline{\pi}/\beta) - 1$ is the quarterly risk free rate.

3.2 Parameter estimates

We estimate a linearised version of the model using a linear Kalman filter with Bayesian Priors and Monte-Carlo Markov chain sampling. The linearisation, optimization and sampling are handled by Dynare (Juillard et al. 1996) using the Metropolis Hastings algorithm with a diagonal covariance matrix.

Priors. For the parameters shared by the augmented and the standard NK model, we define priors according to Table 2. Our choices are broadly in line with the Smets & Wouters (2007) priors.[20] In addition, for the augmented NK model, we choose a prior for ω normally distributed around 10 with a standard deviation of 0.5. This choice is in line with empirical and experimental evidence of $\omega \in [0, 10]$ using the heuristic switching model (see, e.g., Hommes 2011, Cornea-Madeira, Hommes & Massaro 2019, Hommes 2021). Results are robust for a prior range of $5 < \omega < 15$, but the identification is fairly challenging and we need to use a relatively tight prior. Consequently, our choice is motivated by delivering the best fit in the range for ω. As is standard in the literature, we calibrate the price elasticity of demand to $\epsilon = 9$, which implies a mark-up of 12% in line with empirical estimates by Basu & Fernald (1997).

[20] Relative to Smets & Wouters (2007), we reduced some standard deviations in order to guarantee plausible parameter estimates and to avoid unit root processes in the shocks.

Price- and wage-setting		Prior Shape	Prior Mean	Prior STD	*Posterior: Dynamic Calvo* Mean	5%	95%
ω	Intensity of choice	$\mathcal{N}$	10	.5	8.5235	7.6928	9.3751
$\overline{\theta}$	Calvo share	$\mathcal{B}$	.5	.1	0.7484	0.7463	0.7501
Monetary authority							
ϕ_π	MP. stance, π_t	$\mathcal{N}$	1.5	.15	2.7212	2.5406	2.9053
ϕ_y	MP. stance, Y_t	$\mathcal{N}$	.12	.05	0.0997	0.0201	0.1726
ρ	Interest-rate smoothing	$\mathcal{B}$	.75	.1	0.2825	0.2159	0.3494
γ_π	Quarterly inflation trend	$\mathcal{G}$	.62	.1	1.0552	0.9887	1.1228
Preferences and technology							
$100((\overline{\pi}/\beta)-1)$	Natural interest rate	$\mathcal{G}$	.75	.1	1.1320	0.9581	1.3046
σ	Relative risk aversion	$\mathcal{N}$	1.5	.25	1.0406	1.0000	1.0923
φ	Inverse of Frisch elasticity	$\mathcal{N}$	2	.37	1.7479	1.2536	2.2372
Exogenous processes							
σ_d	Discount factor shock, std.	$\mathcal{IG}$	.1	2	0.0196	0.0157	0.0232
σ_s	Cost-push shock, std.	$\mathcal{IG}$	.1	2	0.0247	0.0201	0.0291
σ_r	MP shock, std.	$\mathcal{IG}$	.1	2	0.0079	0.0071	0.0087
σ_θ	Contract shock, std.	$\mathcal{IG}$	.1	2	0.0623	0.0556	0.0690
ρ_d	Discount factor shock, AR(1)	$\mathcal{B}$	.5	.1	0.8550	0.8120	0.8991
ρ_s	Cost-push shock, AR(1)	$\mathcal{B}$	.5	.1	0.9248	0.8988	0.9517
μ_s	Cost-push shock, MA(1)	$\mathcal{B}$	.5	.1	0.1707	0.0927	0.2464
ρ_r	MP shock, AR(1)	$\mathcal{B}$	.5	.1	0.5719	0.4781	0.6673
ρ_θ	Contract shock, AR(1)	$\mathcal{B}$	.5	.1	0.8660	0.8315	0.9006
Log-likelihood						-214.702230	

Table 2: Estimated parameters of the augmented small-scale NK model (US: 1964-2019). $\mathcal{B}$, $\mathcal{G}$, $\mathcal{IG}$, $\mathcal{N}$ denote beta, gamma, inverse gamma and normal distributions, respectively.

Results. Our estimated parameter values are reported in Table 2. The parameters shared with the standard NK model are all broadly in line with the existing literature. Also the parameter estimates for the Calvo law of motion are plausible. The posterior mean of the Calvo share $\overline{\theta} = 0.7484$ is fairly close to the historical average in various datasets and also in line with estimates of the corresponding parameter in random menu cost models.[21] The intensity of choice $\omega = 8.5235$ is strictly positive and in line with the evidence on dynamic predictor selection. Our estimates for the standard shock processes are also in line with existing literature, and, most important, these shocks are the main drivers of the variation in the Calvo share. Figure 7 illustrates that monetary policy and discount factor shocks play an important role in explaining the

[21]For instance, Costain et al. (2021) estimate a the rate of decision making of 0.2707, which corresponds to our posterior mean of the frequency of price change $(1-\overline{\theta}) = 0.2516$.

variation of the Calvo share over the sample. This finding demonstrates the consistency of the Calvo law of motion with the US business cycle. Another finding pointing to the empirical relevance of the model is that the endogenous correlation between the share of unchanged prices and inflation at the posterior means is -0.581, which is in line with the stylized fact in Figure 1.

Remarkably, contract shocks appear to play a key role mostly during the Volcker disinflation and the Great Recession. We rationalize these findings by the extraordinary events in both cases. During the Volcker disinflation the contract shocks push the frequency downward. Thus, they help to match the extraordinary sharp decline in the frequency of price increases during this period (see Nakamura et al. 2018, Figure XV). The Great Recession features extraordinary events on commodity markets underlying the dynamics in the price setting frequency data (see Nakamura et al. 2018, pp.1968-1969). By construction, our model is too abstract to capture these extraordinary dynamics as this is not our objective in this paper. The contract shock seems to absorb these dynamics.[22]

3.3 Consistency with the data

We next demonstrate that the Calvo law of motion improves the consistency of the NK model with macroeconomic and microeconomic data by two exercises. First, we re-estimate the model without contract shocks, but with the possibility of measurement errors in the Nakamura et al. (2018) data. We then compare the predicted path for the latent state variable θ_t to the Nakamura et al. (2018) series depicted in Figure 1. Second, we assess the Calvo Law of Motion's contribution in fitting the post-WWII US Phillips curve.

[22]Recall that we estimate a linearized model. This estimation cannot fully exploit the asymmetry in the profit function. As discussed above, for deep recessions such as the Great Recession, the non-linear model would allow for a decline in θ_t as observed in the Nakamura et al. (2018) data. Thus, the non-linear would most likely be able to explain the history of θ_t with an even less prominent role of contract shocks.

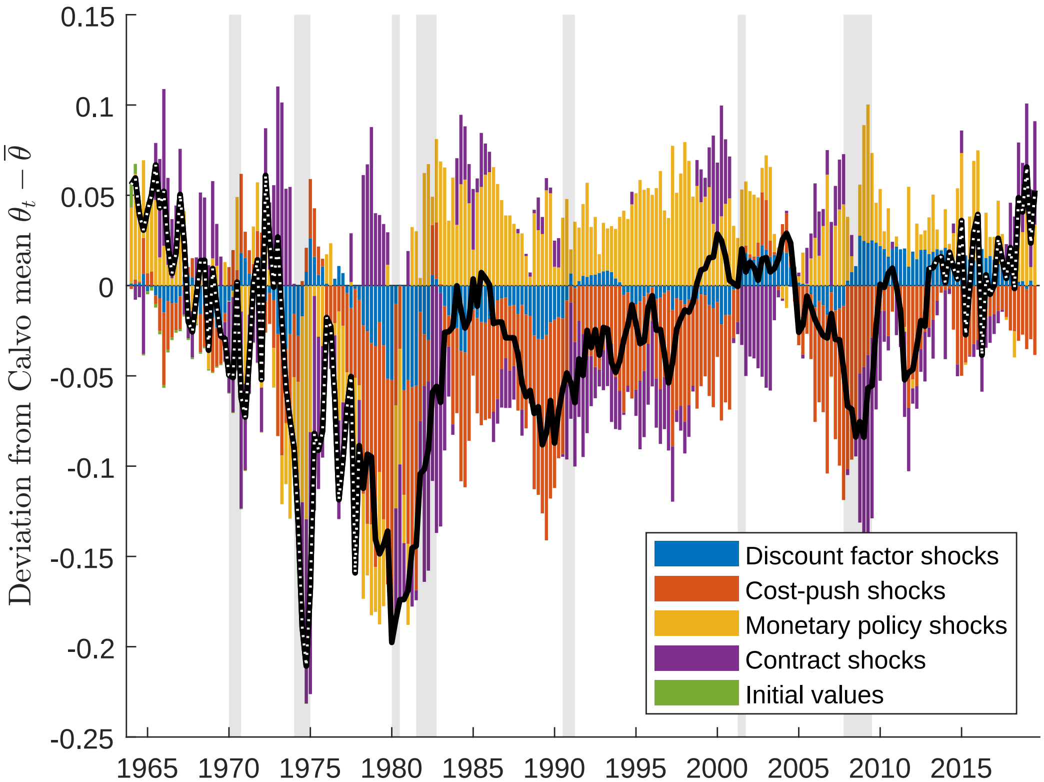


Figure 7: Historical decomposition of the Calvo share in deviation from the posterior mean, United States, 1964-2019. Dotted line indicates generated/unobserved data.

Relevance of the Calvo law of motion. The estimated model naturally raises the question of whether the augmented NK model is consistent with the Nakamura et al. (2018) data. We provide an answer by comparing the predicted path for the latent state variable θ_t from the re-estimation of the augmented NK model to the Nakamura et al. (2018) data in Figure 8, θ_t^{obs}.[23]

Overall, the predicted path and the data line up fairly good both in qualitative and quantitative terms. Taking the rather small range of the vertical axis into account, the notable deviations/measurement errors are again the Volcker disinflation and the Great

[23]The measurement equation in this exercise is $\theta_t^{obs} = \theta_t + \varepsilon_t^{\theta_{obs}}$ with $\varepsilon_t^{\theta_{obs}}$ an AR(1) process. The estimation algorithm minimizes the difference between θ_t^{obs} and θ_t. However, when the model implied dynamics are occasionally at odds with empirical data, the differences are treated as an error in the data and not as an aggregate exogenous shock.

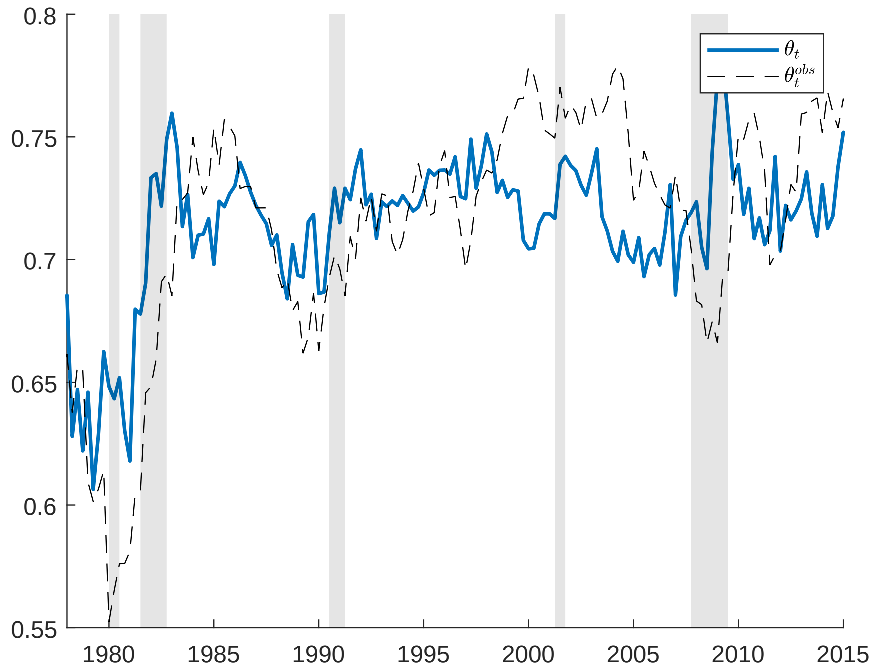
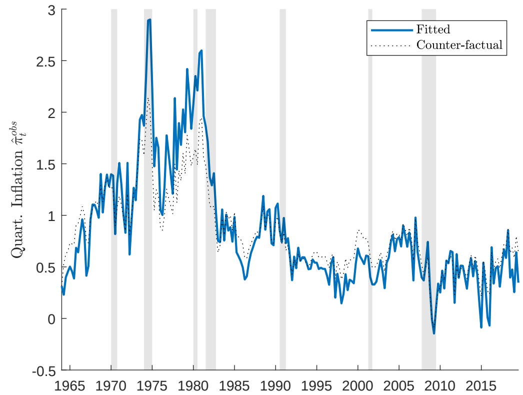


Figure 8: Observed share of unchanged prices θ_t^{obs} and the Calvo share θ_t generated as latent state variable, United States, 1978-2014.

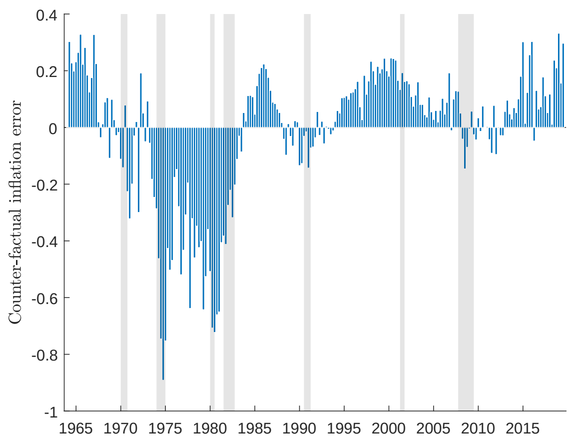
Recession. In both cases the augmented NK model generates a spike in θ_t (less price updating) that is much larger than the increase in the observed θ_t^{obs}. As above, the discrepancy during the Volcker Disinflation and the Great Recession can be rationalized by extraordinary events (see Nakamura et al. 2018). Therefore, in sum, Figure 8 suggests that the Calvo law of motion is a relevant and reasonable modelling device as it makes the NK model consistent with microeconomic data on price setting frequency.

Fitting the post-WWII US Phillips curve. We now show that the Calvo law of motion also improves the consistency of NK model with macroeconomic data in the sense that it enables the NK model to better explain post-WWII US inflation and output gap and therefore the Phillips curve over the sample.

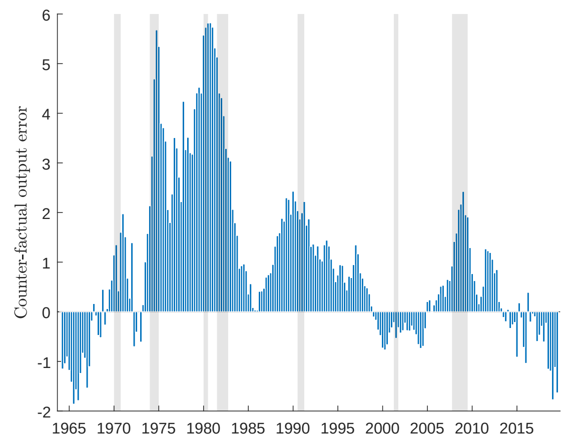
In order to do so, we compare the observed data for inflation and the output gap, π_t^{obs} and y_t^{obs}, to counter-factual predictions derived from an exercise in which the Calvo



(a) Inflation



(b) Inflation (difference)



(c) Output gap (difference)

Figure 9: Observed and counter-factual data, United States, 1964-2019. Counter-factual errors are computed as the difference between counter-factual and observed data.

parameter is equal to the posterior mean $\theta_t = \overline{\theta}$. We then apply the sequences of aggregate shocks, initial values and parameters from the estimation reported in Table 2 to generate the time paths for the counter-factual scenario.

Figure 9 contrasts the observed data and the data generated by the counter-factual exercise. Panels 9a and 9b make clear that counter-factual inflation is too low during the Great Inflation and early Volcker Disinflation, whereas it is too high during the late Volcker Disinflation and most of the Great Moderation. Through most of the Great Recession the counter-factual scenario is more deflationary, whereas in the subsequent recovery the model is more inflationary. Put differently, consistent with the data, the model with time-varying θ_t features more inflation during the Great Recession and less inflation in the recovery thereafter. Thus, the counter-factual features the missing deflation puzzle during the Great Recession and the missing inflation puzzle for the longest expansion in US history so far. In contrast, the Calvo law of motion can help to resolve these puzzles of the linearized NK model.

Likewise Panel 9c establishes that the augmented NK model is more consistent with the observed output gap. The counter-factual scenario predicts a substantially less recessionary Volcker Disinflation, a more expansionary Great Moderation, a less severe Great Recession and a smaller expansion thereafter.

In sum, these findings demonstrate that the standard NK model with a fixed Calvo share fails to predict important patterns in the Phillips curve, i.e., inflation and output gap dynamics, during the post-WWII period. The augmented NK model fits these patterns, such as the well-documented flattening of the Phillips curve, better than the standard NK model. This is no surprise as it is known that for the standard NK model with fixed θ, the only way to change the slope of the Phillips curve is through implausible high (residual) cost-push shocks. This is why standard estimates with time-invariant price setting frequency tend to exhibit Calvo parameter estimates that are inconsistent with microeconomic data on price setting frequency (to reduce the co-movement between inflation and output) and large cost-push shocks that are negatively correlated with the output gap.

In contrast, in the augmented NK model, inflation is not predominantly driven by cost-push shocks (which is in the end the unexplained inflation residual of the model), but to a large extent driven by discount factor and monetary policy shocks, see Figure 10. This suggests that during these periods, inflation is to a large extent driven by the time-varying price setting frequency, which depends on discount factor and monetary

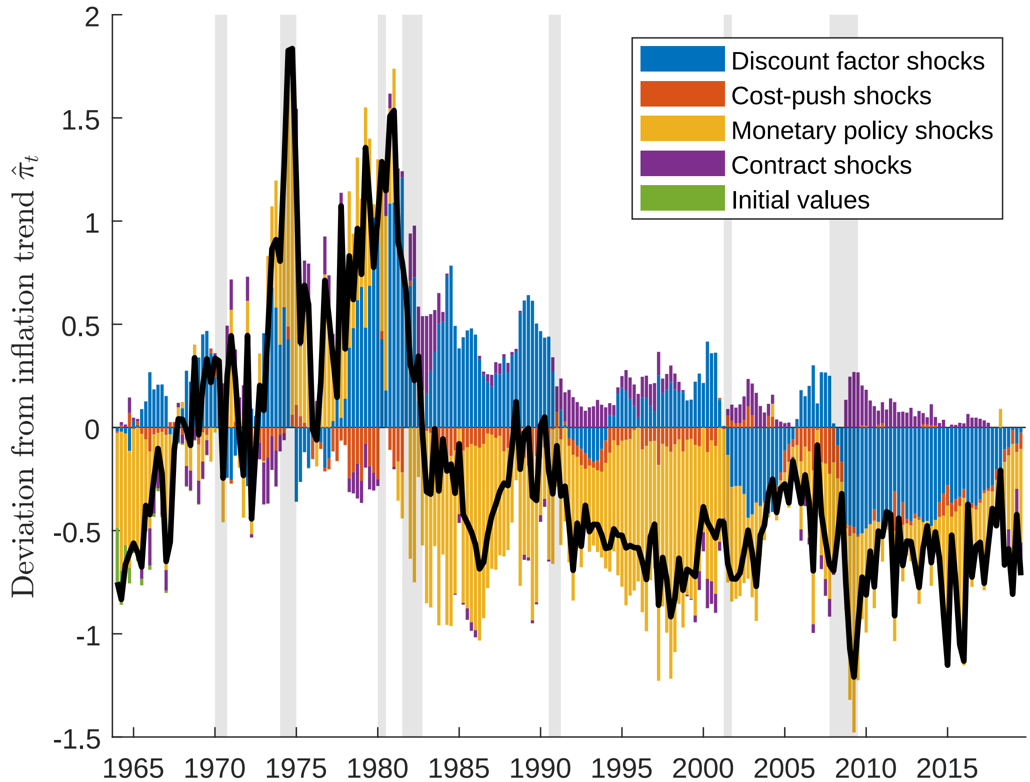


Figure 10: Historical decomposition of inflation, United States, 1964-2019. Values are in log-deviation from the posterior mean of quarterly trend inflation.

policy shocks.[24] Thus, in the augmented NK model, inflation to a large extent can be explained directly by the dynamics of the output gap driving price dispersion and the price setting frequency.

Overall, our results suggest that the Calvo law of motion offers great potential to improve the NK model's macroeconomic time series fit. This could be highly relevant for estimated medium-scale NK models. An in-depth assessment of this potential can be done by comparing the marginal likelihood for the augmented and the standard NK model. We pursue this comparison right below.

[24]This is consistent with the empirical findings in Del Negro et al. (2020). They explain the change in the relation between inflation and unemployment by a flattening of the price Phillips curve.

4 Empirical analysis of a medium-scale NK model

The analysis of the augmented small-scale NK model above suggests that the Calvo law of motion improves the NK model's fit to both macro and micro data. However, the result that cost-push shocks do not play a major role in accounting for inflation contrasts the findings of estimated medium-scale DSGE models. We explain this discrepancy in findings with the Calvo law of motion's ability to approximate the evolution in observed price setting behavior of firms. Alternatively, one may rather attribute the discrepancy to the unrealistic nature of the small-scale model than to the Calvo law of motion. In particular, the small-scale model restricts the number of exogenous shock processes relative to more realistic medium-scale DSGE models. Moreover, the small-scale model lacks many potentially important realistic features (e.g., investment, sticky wages), as well as popular mechanical sources of persistence (e.g., habit formation, price indexation) that have been shown to be useful in order to account for observed inflation dynamics.

In order to investigate this issue, we estimate a standard and an augmented version of a popular medium-scale NK model with many of the aforementioned features. The augmented version has again an endogenous price setting frequency due to the Calvo law of motion. However, we now refrain from relating this variable to the observed price setting frequency via a measurement equation. This has the additional advantage that both models can be compared based on the same number of shocks. We then compare the empirical fit of the two estimated versions based on the marginal likelihood.

As a standard medium-scale DSGE model, we take the model developed in Fernández-Villaverde & Rubio-Ramírez (2006) and estimated numerous times (see, e.g., Fernández-Villaverde & Rubio-Ramírez 2007, Fernández-Villaverde 2010, Fernández-Villaverde, Guerron-Quintana & Rubio-Ramírez 2010) off the shelf.[25] Fernández-Villaverde (2010)

[25]We choose this model because, contrary to Smets & Wouters (2007), a non-linear version is publicly available. The implementation of our dynamic Calvo share is therefore straightforward and we can avoid cumbersome linearisation. The initial code has been provided by the Macroeconomic Model

provides a detailed model description and we stick to the notation therein. We emphasize that the model assumes staggered price and wage setting due to Calvo (1983). In the augmented version, we use the Calvo law of motion to endogenize the price setting frequency for goods prices, θ_p, but not for wages in order to remain consistent with our analysis in the previous sections. This model also contains five exogenous shocks: discount factor shock, labour supply shock, investment-specific technological shock, neutral technology shock and a monetary policy shock.

We estimate the model based on five time series observed for the US over the period 1959Q1 to 2019Q4: real output growth, CPI growth, Fed fund rate, hourly real compensation growth and real investment growth since 1959 in the US.[26] We also estimate both models for the period 1959Q1 to 2007Q3 to guarantee that our conclusions are not altered by changing the sample to the pre-Great Recession period. This also facilitates comparison with the posterior estimates in Fernández-Villaverde (2010). We mostly stick to the Fernández-Villaverde (2010) protocol based on Smets & Wouters (2007) priors.[27]

Tables 3 and 4 present information on the parameter prior and posterior distributions. As far as the priors are concerned, note that we do not choose them in order to fine-tune one particular estimation. Rather the opposite, to the extent possible, we choose to set all four estimations on equal footing. Therefore, our prior distribution is a compromise in order to ensure that we achieve convergence and plausible estimates

Data Base (see Wieland, Afanasyeva, Kuete & Yoo 2016).

[26]Fernández-Villaverde & Rubio-Ramírez (2007), Fernández-Villaverde (2010) use the relative price of investment with respect to the price of consumption. Instead we follow the majority of papers in the literature and use investment.

[27]As in Fernández-Villaverde (2010), we calibrate the rate of depreciation of capital ($\delta = 0.025$), the price elasticity of demand ($\epsilon = 10$), the fixed cost of production ($\Phi = 0$) and the elasticity of substitution among different types of labor ($\eta = 10$). With the original priors, the model is prone to generate unit-root dynamics for the discount factor and labour disutility shocks in all four estimations. We solve this issue by reducing the prior for the standard deviation of the AR(1) coefficients from 0.2 to 0.1. In order to prevent $\hbar \to 1$, we reduce the prior for the standard deviation of $\hbar$ from 0.1 to 0.05. Finally, we reduce the prior for the standard deviation of γ_π from 0.125 to 0.1 to correct for implausible estimates of γ_π.

Price- and wage-setting		Prior Shape	Prior Mean	Prior STD	*Posterior: dynamic Calvo* Mean	5%	95%	*Posterior: constant Calvo* Mean	5%	95%
ω	Intensity of choice	$\mathcal{N}$	10	2.5	9.3641	5.0712	13.6465			
$\overline{\theta}_p$	Calvo share, prices	$\mathcal{B}$	.5	.1	0.7098	0.6777	0.7423	0.7241	0.6931	0.7551
χ	Indexation, prices	$\mathcal{B}$	.5	.15	0.1100	0.0434	0.1738	0.1417	0.0596	0.2204
θ_w	Calvo share, wages	$\mathcal{B}$	.5	.1	0.6838	0.6605	0.7075	0.6831	0.6619	0.7048
χ_w	Indexation, wages	$\mathcal{B}$	.5	.1	0.1488	0.0898	0.2074	0.1477	0.0902	0.2041
Monetary authority										
γ_R	Interest-rate smoothing	$\mathcal{B}$	.75	.1	0.3518	0.2615	0.4429	0.4398	0.3417	0.5398
γ_y	MP. stance, output gap	$\mathcal{N}$	.125	.05	0.2816	0.2369	0.3244	0.3126	0.2641	0.3608
γ_π	MP. stance, inflation	$\mathcal{N}$	1.5	.1	1.0128	1.0080	1.0175	1.0126	1.0072	1.0177
$100(\overline{\Pi}-1)$	Quarterly inflation trend	$\mathcal{G}$	.95	.05	1.0093	0.9247	1.0966	1.0101	0.9213	1.0964
Preferences and technology										
$100(\beta^{-1}-1)$	Time preference	$\mathcal{N}$	.25	.1	0.1430	0.0557	0.2258	0.1265	0.0489	0.2007
$\hbar$	Consumption habit	$\mathcal{B}$	.7	.05	0.7387	0.6981	0.7795	0.8561	0.8317	0.8817
ψ	Scaling for labour supply	$\mathcal{N}$	9	3	9.0990	4.3474	13.7849	8.9962	4.1123	13.8728
ϑ	Inverse of Frisch elasticity	$\mathcal{N}$	1	.1	1.1116	0.9520	1.2688	1.1828	1.0241	1.3445
κ	Capital adjustment cost	$\mathcal{N}$	4	1.5	0.2660	0.1972	0.3346	0.3093	0.2121	0.4045
α	Capital share	$\mathcal{N}$	.3	.025	0.1275	0.1098	0.1450	0.1220	0.1063	0.1376
$100\Lambda_\mu$	Investment growth trend	$\mathcal{N}$	.34	.1	0.2086	0.0808	0.3365	0.1928	0.0654	0.3144
$100\Lambda_A$	Technology growth trend	$\mathcal{N}$	.178	.075	0.3094	0.2140	0.4040	0.3451	0.2507	0.4398
Exogenous processes										
σ_d	Discount factor shock, std.	$\mathcal{IG}$	.1	2	0.0212	0.0179	0.0245	0.0346	0.0281	0.0413
σ_φ	Labour supply shock, std.	$\mathcal{IG}$	.1	2	0.2815	0.2575	0.3000	0.2839	0.2631	0.3000
σ_μ	Investment techno. shock, std.	$\mathcal{IG}$	.1	2	0.0089	0.0080	0.0097	0.0093	0.0084	0.0102
σ_A	Neutral techno. shock, std.	$\mathcal{IG}$	.1	2	0.0221	0.0187	0.0253	0.0239	0.0201	0.0275
σ_e	MP shock, std.	$\mathcal{IG}$	.1	2	0.0096	0.0086	0.0106	0.0094	0.0084	0.0104
ρ_d	Discount factor shock, AR(1)	$\mathcal{B}$	.5	.1	0.7462	0.6357	0.8593	0.3438	0.2187	0.4657
ρ_φ	Labour supply shock, AR(1)	$\mathcal{B}$	.5	.1	0.8959	0.8690	0.9234	0.8952	0.8700	0.9205
Log-likelihood					-1493.744099			-1490.114046		

Table 3: Estimated parameters of the Fernández-Villaverde (2010) model (US: 1959-2008Q3). $\mathcal{B}$, $\mathcal{G}$, $\mathcal{IG}$, $\mathcal{N}$ denote beta, gamma, inverse gamma and normal distributions, respectively.

Price- and wage-setting		Shape	*Prior* Mean	STD	*Posterior: dynamic Calvo* Mean	5%	95%	*Posterior: constant Calvo* Mean	5%	95%
ω	Intensity of choice	$\mathcal{N}$	10	2.5	9.0526	5.2165	12.9788			
$\overline{\theta}_p$	Calvo share, prices	$\mathcal{B}$	.5	.1	0.7409	0.7163	0.7658	0.5480	0.3984	0.6251
χ	Indexation, prices	$\mathcal{B}$	.5	.15	0.0611	0.0219	0.0985	0.4170	0.4000	0.4389
θ_w	Calvo share, wages	$\mathcal{B}$	.5	.1	0.7018	0.6780	0.7257	0.3114	0.2047	0.3767
χ_w	Indexation, wages	$\mathcal{B}$	.5	.1	0.1324	0.0780	0.1852	0.5884	0.5082	0.6595
Monetary authority										
γ_R	Interest-rate smoothing	$\mathcal{B}$	.75	.1	0.3400	0.2520	0.4281	0.4618	0.3365	0.6661
γ_y	MP. stance, output gap	$\mathcal{N}$	.125	.05	0.2767	0.2424	0.3109	0.2385	0.0832	0.3108
γ_π	MP. stance, inflation	$\mathcal{N}$	1.5	.1	1.0526	1.0500	1.0560	1.1007	1.0193	1.3371
$100(\overline{\Pi}-1)$	Quarterly inflation trend	$\mathcal{G}$	.95	.05	1.0735	0.9883	1.1575	0.9765	0.8840	1.0510
Preferences and technology										
$100(\beta^{-1}-1)$	Time preference	$\mathcal{N}$	.25	.1	0.1033	0.0396	0.1637	0.1585	0.0615	0.2156
$\hbar$	Consumption habit	$\mathcal{B}$	.7	.05	0.6741	0.6397	0.7079	0.7702	0.7472	0.7957
ψ	Scaling for labour supply	$\mathcal{N}$	9	3	9.6008	4.9782	14.2230	8.4015	3.8055	12.7165
ϑ	Inverse of Frisch elasticity	$\mathcal{N}$	1	.1	1.2756	1.1375	1.4141	0.8645	0.6691	1.0081
κ	Capital adjustment cost	$\mathcal{N}$	4	1.5	0.2391	0.1790	0.2987	0.3962	0.2651	0.6554
α	Capital share	$\mathcal{N}$	.3	.025	0.1285	0.1142	0.1427	0.1849	0.1362	0.2113
$100\Lambda_\mu$	Investment growth trend	$\mathcal{N}$	.34	.1	0.0913	0.0000	0.1729	0.1229	0.0215	0.2317
$100\Lambda_A$	Technology growth trend	$\mathcal{N}$	.178	.075	0.3622	0.2724	0.4535	0.1526	0.0516	0.2206
Exogenous processes										
σ_d	Discount factor shock, std.	$\mathcal{IG}$	.1	2	0.0247	0.0212	0.0282	0.0313	0.0250	0.0424
σ_φ	Labour supply shock, std.	$\mathcal{IG}$	.1	2	0.2856	0.2671	0.3000	0.0458	0.0408	0.0522
σ_μ	Investment techno. shock, std.	$\mathcal{IG}$	.1	2	0.0089	0.0082	0.0097	0.0089	0.0074	0.0120
σ_A	Neutral techno. shock, std.	$\mathcal{IG}$	.1	2	0.0293	0.0254	0.0331	0.0157	0.0117	0.0186
σ_e	MP shock, std.	$\mathcal{IG}$	.1	2	0.0293	0.0254	0.0331	0.0086	0.0078	0.0092
ρ_d	Discount factor shock, AR(1)	$\mathcal{B}$	.5	.1	0.8910	0.8746	0.9079	0.9127	0.8725	0.9834
ρ_φ	Labour supply shock, AR(1)	$\mathcal{B}$	.5	.1	0.9610	0.9523	0.9697	0.9954	0.9945	0.9972
Log-likelihood						-1919.524492			-2062.121051	

Table 4: Estimated parameters of the Fernández-Villaverde (2010) model (US: 1959-2019Q4). $\mathcal{B}$, $\mathcal{G}$, $\mathcal{IG}$, $\mathcal{N}$ denote beta, gamma, inverse gamma and normal distributions, respectively.

for all four estimations.[28]

In doing so, we find that the Calvo law of motion does not lead to an improvement for the subsample 1959Q1 to 2007Q3. The log-likelihood for the augmented and the standard model are pretty close. This finding is consistent with the view that NK models with fixed price and wage setting frequency do well in fitting the data up to the Great Recession even without relying heavily on cost-push shocks.

Contrary, there is a striking difference in log-likelihoods for the entire sample including the Great Recession and the New Normal. The Calvo law of motion largely improves the model fit to the data for the entire sample. Thus, when then sample includes periods that are characterized by relatively big shifts in the Phillips curve a model with time-varying price setting frequency improves the model fit to the data. Another striking result is that the Calvo law of motion leads to price and wage setting parameter stability across samples, whereas the version with fixed price and wage setting frequency is prone to parameter instability. We now discuss these and other parameter estimates in greater detail.

Price- and wage-setting. For the augmented model, the estimate of the posterior mean of the Calvo share $\overline{\theta}_p$ is arguably close to the average of 0.7142 found in the Nakamura et al. (2018) data plotted in Figure 1 and again in line with estimates of the rate of decision making in random menu cost models such as Costain et al. (2021).[29] Price indexation is estimated to be relatively unimportant in this model and it is remarkable that both the Calvo share $\overline{\theta}_p$ and the price indexation parameter χ remain rather unaffected by the sample length. This finding stands in sharp contrast

[28] Alternatively, one could choose differing priors and fine-tune each estimation individually. However, we think that this would not lead to a fair comparison between the four estimations, because of the additional degrees of freedom introduced for each individual estimation.

[29] We follow Fernández-Villaverde & Rubio-Ramírez (2007) who interpret the estimated θ_p as a measure of price setting frequency. However, this interpretation is imprecise as the model features price indexation and therefore all prices are changed each period. But at least for the low degrees of price indexation that we estimate for the augmented NK model, this should be a negligible issue. An alternative would be to estimate the model without price indexation.

to the standard model.

Both the Calvo share and price indexation vary dramatically with the sample length in the standard model. In particular, for the standard model to match the dynamics of inflation over the entire sample, a high price indexation parameter in combination with a rather low constant Calvo share is required. This seems implausible for at least two reasons: first, the estimated Calvo share is inconsistent with the average in the Nakamura et al. (2018) data. Second, such a high degree of price indexation seems inconsistent with previous findings (see, e.g., Smets & Wouters 2007, Del Negro et al. 2015).

This exercise also allows us to shed light on how a dynamic price setting frequency interacts with the labor market. In the augmented model, the Calvo probability for wages, θ_w, is rather constant across samples. For the standard model, it is 0.68 for the subsample and 0.31 for the entire sample. Contrary, to fit the data over the entire sample, wage indexation has to be dramatically larger compared to the subsample estimate in the standard model (0.15 vs. 0.59).

To sum up, the subsample estimate regarding wage-setting parameters for both the augmented and standard model are in line with the literature on estimated NK models. This literature assigns an important role to nominal wage rigidities. However, to fit the data over the entire sample, the standard model relies heavily on a dramatic decline in the wage-setting frequency and a dramatic increase in exogenous wage indexation. Contrary, the augmented model's estimates for wage-setting behavior, θ_w and χ_w, are virtually unaffected by the change in the sample length. Thus, the Calvo law of motion is a mechanism that contributes to parameter stability across samples.

We rationalize this result by a change in the trade off between fitting the wage and inflation data over the entire sample. The latter includes the Great Recession and the New Normal with the large shifts in inflation dynamics. For the standard model, when the posterior sampler maximizes the likelihood in light of these shifts, it seems to explore both the space of price and wage setting parameters and arrives at very different posterior estimates relative to the subsample estimate.

Contrary, relative to the standard model, the time-varying price setting frequency, all else equal, improves the model's ability to fit the shifts in inflation dynamics. Thus, in the estimation, independent of the sample length, the posterior sampler settles on similar price and wage setting parameters and maximizes the likelihood function by exploring other areas of the parameter space.

Is the parameter stability in this context particularly desirable? We believe that in light of the evidence provided in Del Negro et al. (2020) the answer is yes. For example, Del Negro et al. (2020) provide evidence from an SVAR and an estimated NK model that does not support the hypothesis that there has been structural change on the labor market in recent years. Therefore the labor market parameter estimates in the standard model should be unaffected by the sample length.

Notice that our findings are also consistent with the results in Fernández-Villaverde & Rubio-Ramírez (2007) who find that there is an inverse relationship between θ_p and χ and between θ_w and χ_w, once one allows these parameters to vary over time. Fernández-Villaverde & Rubio-Ramírez (2007) conclude that a high exogenous price and wage indexation reflect important price and wage dynamics not captured in the model. We find that the Calvo law of motion generates price- and wage setting parameter stability over different sample lengths. This suggests that the Calvo law of motion can capture these dynamics to some extent.[30]

Another important question is, whether the augmented NK model relies on implausible estimates of the intensity of choice, ω, to fit the data? The answer is clearly no. Independent of the sample length and even with a wider prior the parameter is arguably stable and in the range of existing estimates (e.g., Hommes 2011, Cornea-Madeira et al.

[30] Consequently, one possible extension of our paper could be a reassessment of our findings in a version where also θ_w is endogenized with the Calvo law of motion.

2019, Hommes 2021).[31]

Monetary authority. Does the augmented NK model yield plausible estimates for the monetary authority parameters? We estimate coefficients for the output gap and interest-rate smoothing that are frequently reported to in the literature. The coefficient on inflation is close to, but above unity. This value is arguably at the lower end of the typical range of estimates found in the literature. However, our sample covers subsamples where this coefficient is typically clearly below unity or clearly above unity (e.g., Lubik & Schorfheide 2004). Therefore, our estimates are arguably within the plausible range. Another plausible explanation is that the coefficients are affected by the zero lower bound period in the data that we do not incorporate in the model.

Preferences and technology. The preference parameters are all in a reasonable range. This holds for both model versions and independent of the sample length. The technology parameters imply estimated average annual growth rates of real GDP per quarter, $400(\Lambda_A + \alpha\Lambda_\mu)/(1-\alpha)$. For the subsample we obtain 1.54% for the augmented and 1.68% for the standard NK model. The corresponding rates for the full sample are 1.72% and 0.86%. Fernández-Villaverde (2010) estimates 1.7% over a slightly different sample (1959Q1 to 2007Q1) and also uses different data for investment. Nevertheless, although the sample length differs, our estimates based on the augmented model are of comparable size for the subsample and the entire sample. Contrary, our estimate based on standard model for the entire sample is clearly lower.

[31]Notice that, while keeping the prior mean of $\omega = 10$, we widen its standard deviation relative to the estimation of the small-scale NK model from 0.5 to 2.5 due to over-identification issues. In the former case, the estimated posterior mean $\omega = 9.97$ $[9.16; 10.81]$ overlaps with the prior mean. Nonetheless, the resulting posterior mean $\overline{\theta}_p = 0.74$ $[0.72; 0.77]$ and the Log-likelihood (-1919.21) are close to the posterior estimates reported in Table 4. In order to make sure that our results for the augmented model are robust to different priors within a plausible range, we re-estimated the augmented model with a prior mean $\omega = 5$ and $\omega = 15$. The former case yields $\omega = 4.78$ $[1.10; 8.19]$ together with $\overline{\theta}_p = 0.73$ $[0.71; 0.76]$ and a Log-likelihood of -1919.20. The latter case yields $\omega = 14.16$ $[10.17; 18.07]$ together with $\overline{\theta}_p = 0.75$ $[0.72; 0.77]$ and a Log-likelihood of -1919.22. These estimates are again close to the posterior estimates reported in Table 4. Thus, we conclude that our results are robust.

All told, we conclude that the augmented NK model improves the model fit to the data. Moreover, while not the primary objective of this exercise, the augmented NK model also makes the model less prone to parameter instability and at the same time it yields estimated parameters for the monetary authority, preferences, technology, and, also for exogenous processes that are within a plausible range.

5 Conclusion

We develop a New Keynesian model with endogenous price setting frequency by augmenting the time-dependent price setting mechanism with a state-dependent component. The augmented NK model is consistent with the data both at the macro and micro level. In this way the NK model can potentially be reconciled with phenomena such as shifts in the Phillips curve associated with different historical episodes.

In our model, the present value of firm profit and costly updating drive heterogeneity and stickiness in price setting. A firm decides to update the price when expected benefits outweigh expected cost and then resets the price optimally. We model the updating decision with a discrete choice process that we denote the Calvo law of motion. The process approximates the idiosyncratic trade offs that firms face when deciding about price updating well.

As profits are countercyclical, the model predicts that prices are more flexible during expansions and less flexible during recessions. This in turn gives rise to a non-linear Phillips curve. The price setting frequency accelerates during booms implying an accelerating inflation. In contrast, the model permits a decelerating price setting frequency during recessions and thus allows for low, but stable inflation. This mechanism remains effective in a linearised version of model that we take to the data.

We find that our setup with the Calvo law of motion provides a good approximation of the observed price setting frequency based on micro data. Second, our model, besides its small scale, also fits the observed dynamics in inflation and output well. Third, the Calvo law of motion enables the model to explain the dynamics in inflation data to a large extent by discount factor and monetary policy shocks as well as the endogenous evolution of the price setting frequency, while the contribution of cost-push shocks to the shifts in the Phillips curve is very limited. Fourth, the Calvo law of motion improves the macroeconomic time series fit of a medium-scale NK model over the sample 1959-2019.

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A A simplified model

This discusses the model in the paper in its (perhaps) simplest setting. This model allows us to illustrate the key features of the proposed Calvo law of motion and to build intuition for the results derived in the paper. Two simplifications relative to a standard DSGE model are worth mentioning. In this simple model firms are myopic. They do not take the future into account, when they set their prices. Moreover, aggregate demand is assumed to be an exogenous stationary AR(1) process.

Model outline. Aggregate demand for consumption Y_t is normalized and follows

$$\begin{aligned} Y_t &= \overline{Y} e^{\varepsilon_t} \\ \varepsilon_t &= \rho \varepsilon_{t-1} + u_t, \end{aligned}$$

where $\overline{Y} = 1$ is the steady state, ε is a preference perturbation that follows an AR(1) stationary process with $0 \leq \rho < 1$ and u_t i.d.d and normally distributed. Labor supply is determined by the following schedule[32]

$$N_t^{\varphi} Y_t^{\sigma} = \frac{W_t}{P_t},$$

where W_t denotes the nominal wage and P_t is the aggregate price level.

The production technology is linear, where labour N_t is the only input

$$Y_t = N_t.$$

This implies that the real marginal cost are $w_t \equiv W_t/P_t$.

We assume that firms operate under monopolistic competition. The aggregate price level evolves according to equation (A.1) similar to the Calvo (1983) model, where a share of θ_t firms keep their former price and $1-\theta_t$ firms update their price, i.e.,

$$\begin{aligned} P_t &= (\theta_t P_{t-1}^{1-\epsilon} + (1-\theta_t) P_t^{*\,1-\epsilon})^{\frac{1}{1-\epsilon}} \qquad (A.1) \\ \Leftrightarrow 1 &= (\theta_t \pi_t^{\epsilon-1} + (1-\theta_t) p_t^{*\,1-\epsilon})^{\frac{1}{1-\epsilon}} \\ \Leftrightarrow \pi_t &= \left(\frac{1-(1-\theta_t) p_t^{*1-\epsilon}}{\theta_t} \right)^{\frac{1}{\epsilon-1}}, \end{aligned}$$

where ϵ is the price elasticity of demand of goods and, P_t^* is the optimal re-setting price, $p_t^* \equiv P_t^*/P_t$ is the relative optimal price and $\pi_t \equiv P_t/P_{t-1}$ denotes inflation. Firms are myopic and therefore their optimal price is not set in a forward-looking way. Given the firms' market power, it is simply optimal to charge a constant markup over real marginal cost, i.e., $p_t^* = \frac{\epsilon}{\epsilon-1} w_t$.[33] Finally, note that the relative price of non price

[32]This schedule could be derived from assuming instantaneous utility $U(C_t, N_t) = \frac{C_t^{1-\sigma}-1}{(1-\sigma)} - \frac{N_t^{1+\varphi}}{(1+\varphi)}$, aggregate goods market clearing $Y_t = C_t$, and the budget constraint $w_t N_t = C_t$.

[33]This could be derived from a Dixit & Stiglitz (1977) model of monopolistic competition.

resetting firms is given by $p_t^f \equiv 1/\pi_t$ and that the relative prices $p_{i,t}^*$ and p_t^f determine the respective firms' share in aggregate demand and their respective labor demand.

The Calvo law of motion. In the simplified model firms are myopic. Thus, we assume that the performance measure is based on the firm's markups. That is, the price setting frequency is driven by the difference between relative prices with $\hat{p}_t^f$ denoting the average relative past price and $\hat{p}_t^*$ denoting the relative optimal price. Next, we assume that managers cannot observe the resetting price before updating it, but they have expectations about the relative resetting price $E_{t-1}\hat{p}_t^*$ and the average old price $E_{t-1}\hat{p}_t^f$. They observe the past form expectations according to $\mathbb{E}_{t-1}\hat{p}_{i,t}^* = \hat{p}_{i,t-1}^*$ and $\mathbb{E}_{t-1}\hat{p}_t^f = \hat{p}_{t-1}^f$.

Once we take into account that firms in the model have an identical cost structure, and that in equilibrium markets clear, (6) can be equivalently expressed as

$$\theta_t = \frac{e^{\omega \hat{p}_{t-1}^f}}{e^{\omega \hat{p}_{t-1}^f} + e^{\omega\left(\hat{p}_{t-1}^* - \tau\right)}}. \tag{A.2}$$

Steady state. The simplified model has the following steady states: $Y = 1$, $N = Y$, $w = N^{\varphi} Y^{\sigma}$, as well as

$$\begin{aligned} p^* &= \frac{\epsilon}{\epsilon - 1} w \\ \overline{\pi} &= \left(\frac{1 - (1 - \overline{\theta}) p^{*1-\epsilon}}{\overline{\theta}} \right)^{\frac{1}{\epsilon - 1}} \\ p^f &= \frac{1}{\overline{\pi}} \\ \overline{\theta} &= \frac{1}{1 + e^{-\omega\tau}}. \end{aligned}$$

Asymmetric dynamics in the Phillips curve. The model can be solved recursively after defining the size of the shock at every period.

We again use simulated impulse responses to illustrate an important feature of this simplified model (similar to the small-scale NK model above): asymmetric dynamics in the Phillips curve implied by the Calvo law of motion. As this analysis is solely for illustrative purposes, we again parametrize the model with values that are frequently used in the literature as can be seen from Table 5.

Figure A.1a displays the simulated impulse response functions to a *positive* 10 percent demand shock. The benchmark of time invariant θ (black dashed line) is similar to the effects in the small-scale NK model. However, it is important to note that because of an exogenous aggregate demand side (i.e., the absence of feedback loop between prices and demand), output, marginal cost, and the optimal price decision are the same between the benchmark and the model enriched with θ_t.

Relative to the standard model, a time-varying Calvo share θ_t (blue solid line) has novel and important implications: while the responses of output and marginal cost

		Values
ω	Intensity of choice	2
θ	Calvo share steady state	$\frac{1}{1+e^{-\omega\tau}} = 0.75$
σ	Relative risk aversion	1
φ	Frisch elasticity	1
ϵ	Price elasticity of demand	9
ρ	Demand shock, AR(1)	0.8

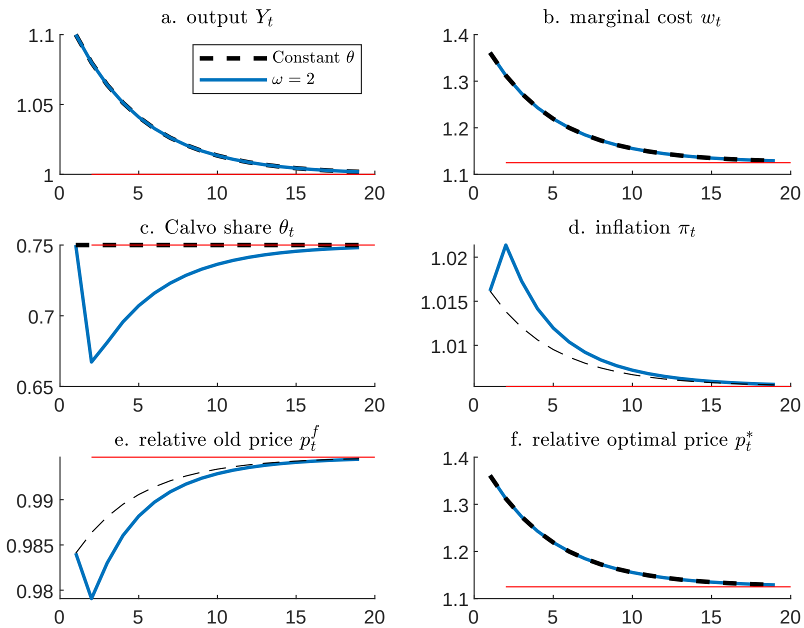
Table 5: Calibrated parameters (quarterly basis)

are identical, the responses of nominal variables are strikingly different after the initial impact of the shock in $t = 1$. The boom in demand implies that the performance review of managers modelled by (A.2) after the impact period leads managers to the conclusion that raising the price net of the cost τ implies a higher markup relative to not raising the price. This implies that managers will setup meetings to reset the price and more firms will actually do so. Therefore θ_t declines, which translates into even higher inflation relative to the impact period and an even larger share of firms that have reset the price since the shock occurred. As more and more firms have already reset their price and marginal cost monotonically decline, more managers refrain from organizing meetings as their performance review modelled by (A.2) suggests that maintaining the price is the better strategy. This implies a hump-shaped response of inflation to a positive demand shock.

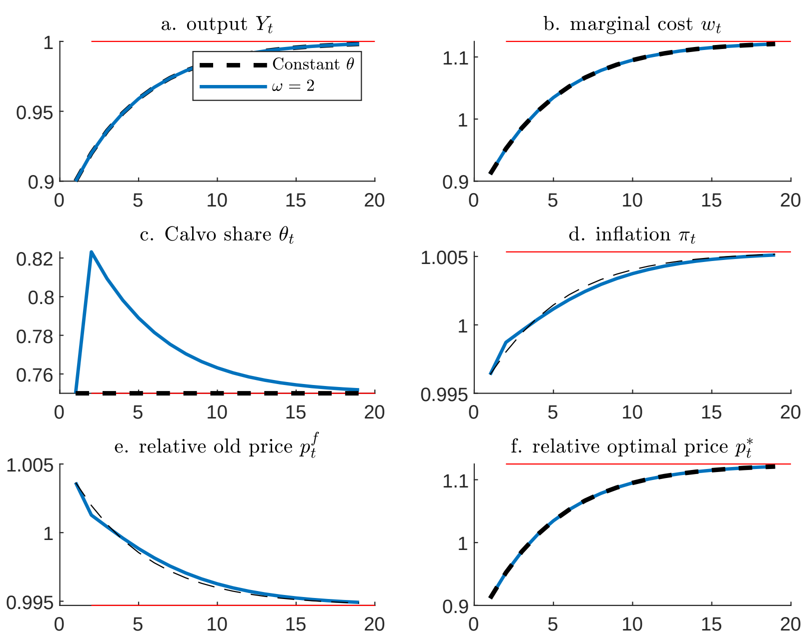
Next, we report simulated impulse response functions to a *negative* 10 percent demand shock in Figure A.1b. In the benchmark with time invariant θ (black dashed line), the impulse responses and the economic intuition behind them are exactly the opposite of the positive demand shock. However, in the case of time-varying θ_t (blue solid line) the responses in the recession are strikingly different compared to a boom, but more in line with the standard model.

The initial effects are again identical to the standard model. In subsequent periods, the performance review of managers leads them to the conclusion that lowering the price net of the cost τ implies a lower markup relative to maintaining the price. Thus, a lower share of managers will set up meetings to reset the price and less firms will actually do so. Thus, θ_t increases, which translates into lower inflation relative to the impact period and a lower share of firms that have reset the price since the shock occurred. The relative advantage of not resetting the price dies out as marginal cost monotonically increase toward their steady state. It follows that more managers organize meetings and more firms reset their price. Thus, θ_t reverts back to its steady state as well.

The above exercise makes clear that the Calvo law of motion implies an asymmetry in price setting by firms. The source of this behaviour is rooted in the countercyclical markups. Raising prices in booms raises markups (and therefore profits) relative to keeping the price unchanged. In contrast, lowering prices in recessions lowers markups relative to maintaining the price. As a consequence, the model with time-varying θ_t generates hump-shaped and larger responses of inflation relative to the benchmark case



(a) Response to a positive +10% demand shock



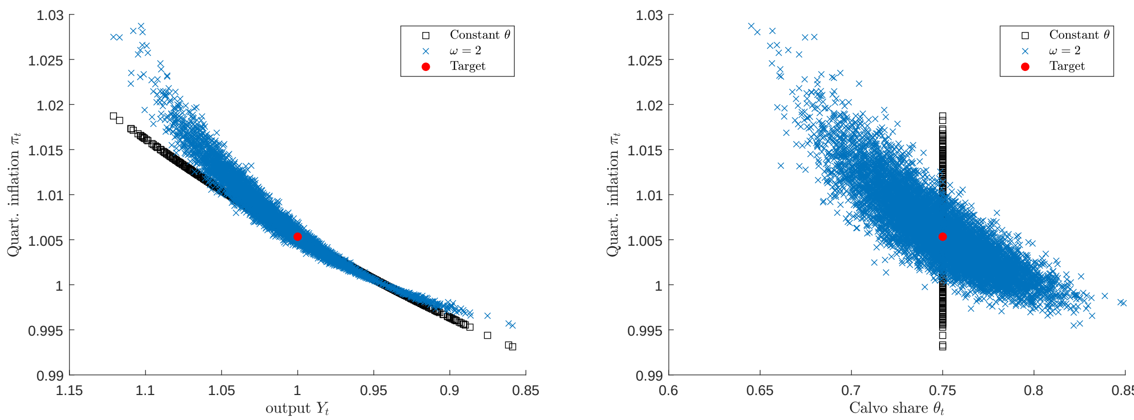
(b) Response to a negative -10% demand shock

Figure A.1: Asymmetric impulse responses to a positive or negative (±10%) discount factor shock in the simplified model. IRFs are displayed in levels. The red solid line depicts the steady state.

of the invariant θ in booms (see Figure A.1a), but responses close to the standard model in recessions (see Figure A.1b).

Similar to the small-scale NK model in the paper, this asymmetry in impulse response functions to a demand shock again translates into a prediction for the Phillips curve of this simple model, which is illustrated in Figure A.2a. The Phillips curve is flat in recessions and steep in booms, which can be rationalized by the adjustment of the Calvo share over time, see Figure A.2b. When inflation is high, the markup implied by the past average price level is low and the of price resetting frequency is high. In contrast, when inflation is low, the markup implied by the past average price level is high and the price resetting frequency is low.

It is remarkable that even without any forward-looking private sector behavior or features such as price indexation, our model displays an asymmetric accelerating Phillips curve where deflation is limited and inflation is self re-enforcing.



(a) Phillips curve

(b) Inflation and the Calvo share

Figure A.2: Global dynamics in the augmented ($\omega = 2$, blue) and standard (constant θ, black) simplified model in levels in response to a discount factor shock with standard deviation of 0.1.

B Small-scale NK model

B.1 Steady state

For $Y = 1$ and $\theta_t = \overline{\theta}$, the steady state of the model variables is determined by

$$
\begin{aligned}
p^* &= \frac{\epsilon}{\epsilon - 1}\frac{\psi}{\phi} \\
\psi &= \frac{wY^{1-\sigma}}{1 - \overline{\theta}\beta\overline{\pi}^{\epsilon}} \\
\phi &= \frac{Y^{1-\sigma}}{1 - \overline{\theta}\beta\overline{\pi}^{\epsilon-1}} \\
1 &= (\overline{\theta}\overline{\pi}^{\epsilon-1} + (1-\overline{\theta})p^{*\,1-\epsilon})^{\frac{1}{1-\epsilon}} \\
w &= (\epsilon - 1)/\epsilon \\
i &= \overline{\pi}/\beta \\
N &= Ys \\
s &= \frac{(1-\overline{\theta})p^{*\ -\epsilon}}{(1-\overline{\theta}\overline{\pi}^{\epsilon})} \\
p^f &= 1/\overline{\pi}.
\end{aligned}
$$

B.2 Linearisation

B.2.1 The Phillips curve

We linearize (3)

$$\hat{p}^*_{i,t} = \hat{\psi}_t - \hat{\phi}_t \tag{B.1}$$

then we linearize (4) :

$$\hat{\psi}_t = (1 - \theta\beta\overline{\pi}^{\epsilon})\hat{w}_t + \beta\theta\overline{\pi}^{\epsilon}(\mathbb{E}_t\hat{\theta}_{t+1} + \epsilon\mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t\hat{\psi}_{t+1})$$

then we linearize (5) :

$$\hat{\phi}_t = \beta\theta\overline{\pi}^{\epsilon-1}(\mathbb{E}_t\hat{\theta}_{t+1} + (\epsilon-1)\mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t\hat{\phi}_{t+1})$$

then we linearize (2) :

$$
\begin{aligned}
0 &= \theta(\epsilon-1)\overline{\pi}^{\epsilon-1}\hat{\pi}_t + [(1-\theta)(1-\epsilon)p^{*\,1-\epsilon}]\hat{p}^*_t + \overline{\pi}^{1-\epsilon}\theta\hat{\theta}_t - p^*\theta\hat{\theta}_t \\
0 &= \theta(\epsilon-1)\overline{\pi}^{\epsilon-1}\hat{\pi}_t + [(1-\theta)(1-\epsilon)p^{*\,1-\epsilon}]\hat{p}^*_t + (\overline{\pi}^{1-\epsilon} - p^*)\theta\hat{\theta}_t \\
0 &= \theta(\epsilon-1)\overline{\pi}^{\epsilon-1}\hat{\pi}_t + [(1-\theta)(1-\epsilon)(\frac{1-\theta\overline{\pi}^{\epsilon-1}}{1-\theta})]\hat{p}^*_t + (\overline{\pi}^{1-\epsilon} - p^*)\theta\hat{\theta}_t
\end{aligned}
$$

$$\hat{p}_t^* = \frac{\theta\overline{\pi}^{\epsilon-1}}{1-\theta\overline{\pi}^{\epsilon-1}}\hat{\pi}_t - \frac{\overline{\pi}^{1-\epsilon} - p^*}{(1-\epsilon)(1-\theta\overline{\pi}^{\epsilon-1})}\theta\hat{\theta}_t \tag{B.2}$$

then we substitute (B.2) into (B.1)

$$\hat{\psi}_t = \hat{\phi}_t + \frac{\theta\overline{\pi}^{\epsilon-1}}{1-\theta\overline{\pi}^{\epsilon-1}}\hat{\pi}_t - \frac{\overline{\pi}^{1-\epsilon} - p^*}{(1-\epsilon)(1-\theta\overline{\pi}^{\epsilon-1})}\theta\hat{\theta}_t \tag{B.3}$$

Now we plug(B.3) into (4)

$$\begin{aligned}\hat{\phi}_t + \frac{\theta\overline{\pi}^{\epsilon-1}}{1-\theta\overline{\pi}^{\epsilon-1}}\hat{\pi}_t - \frac{\overline{\pi}^{1-\epsilon} - p^*}{(1-\epsilon)(1-\theta\overline{\pi}^{\epsilon-1})}\theta\hat{\theta}_t = {} & (1-\theta\beta\overline{\pi}^{\epsilon})\hat{w}_t ... \\ & ... + \beta\theta\overline{\pi}^{\epsilon}(... \\ & ... \mathbb{E}_t\hat{\theta}_{t+1} + \epsilon\mathbb{E}_t\hat{\pi}_{t+1} + ... \\ & ... \mathbb{E}_t[\hat{\phi}_{t+1} + \frac{\theta\overline{\pi}^{\epsilon-1}}{1-\theta\overline{\pi}^{\epsilon-1}}\hat{\pi}_{t+1} - \frac{\overline{\pi}^{1-\epsilon} - p^*}{(1-\epsilon)(1-\theta\overline{\pi}^{\epsilon-1})}\theta\hat{\theta}_{t+1}] ... \\ & ...)\end{aligned}$$

$$\begin{aligned}\hat{\phi}_t = {} & (1-\theta\beta\overline{\pi}^{\epsilon})\hat{w}_t - \frac{\theta\overline{\pi}^{\epsilon-1}}{1-\theta\overline{\pi}^{\epsilon-1}}\hat{\pi}_t + \frac{\overline{\pi}^{1-\epsilon} - p^*}{(1-\epsilon)(1-\theta\overline{\pi}^{\epsilon-1})}\theta\hat{\theta}_t ... \\ & ... + \beta\theta\overline{\pi}^{\epsilon}(\mathbb{E}_t\hat{\theta}_{t+1} + \epsilon\mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t[\hat{\phi}_{t+1} + \frac{\theta\overline{\pi}^{\epsilon-1}}{1-\theta\overline{\pi}^{\epsilon-1}}\hat{\pi}_{t+1} - \frac{\overline{\pi}^{1-\epsilon} - p^*}{(1-\epsilon)(1-\theta\overline{\pi}^{\epsilon-1})}\theta\hat{\theta}_{t+1}])\end{aligned}$$

and then we substitute (5)

$$\begin{aligned}\beta\theta\overline{\pi}^{\epsilon-1}(\mathbb{E}_t\hat{\theta}_{t+1} + (\epsilon-1)\mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t\hat{\phi}_{t+1}) = {} & (1-\theta\beta\overline{\pi}^{\epsilon})\hat{w}_t - \frac{\theta\overline{\pi}^{\epsilon-1}}{1-\theta\overline{\pi}^{\epsilon-1}}\hat{\pi}_t + \frac{\overline{\pi}^{1-\epsilon} - p^*}{(1-\epsilon)(1-\theta\overline{\pi}^{\epsilon-1})}\theta\hat{\theta}_t ... \\ & ... + \beta\theta\overline{\pi}^{\epsilon}(... \\ & ... \mathbb{E}_t\hat{\theta}_{t+1} + \epsilon\mathbb{E}_t\hat{\pi}_{t+1} + ... \\ & ... \mathbb{E}_t[\hat{\phi}_{t+1} + \frac{\theta\overline{\pi}^{\epsilon-1}}{1-\theta\overline{\pi}^{\epsilon-1}}\hat{\pi}_{t+1} - \frac{\overline{\pi}^{1-\epsilon} - p^*}{(1-\epsilon)(1-\theta\overline{\pi}^{\epsilon-1})}\theta\hat{\theta}_{t+1}] ... \\ & ...)\end{aligned}$$

$$\begin{aligned}\frac{\theta\overline{\pi}^{\epsilon-1}}{1-\theta\overline{\pi}^{\epsilon-1}}\hat{\pi}_t = {} & (1-\theta\beta\overline{\pi}^{\epsilon})\hat{w}_t + \frac{\overline{\pi}^{1-\epsilon} - p^*}{(1-\epsilon)(1-\theta\overline{\pi}^{\epsilon-1})}\theta\hat{\theta}_t - \beta\theta\overline{\pi}^{\epsilon-1}(\mathbb{E}_t\hat{\theta}_{t+1} + (\epsilon-1)\mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t\hat{\phi}_{t+1}) ... \\ & ... + \beta\theta\overline{\pi}^{\epsilon}(\mathbb{E}_t\hat{\theta}_{t+1} + \epsilon\mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t[\hat{\phi}_{t+1} + \frac{\theta\overline{\pi}^{\epsilon-1}}{1-\theta\overline{\pi}^{\epsilon-1}}\hat{\pi}_{t+1} - \frac{\overline{\pi}^{1-\epsilon} - p^*}{(1-\epsilon)(1-\theta\overline{\pi}^{\epsilon-1})}\theta\hat{\theta}_{t+1}])\end{aligned}$$

$$\hat{\pi}_t = \frac{1-\theta\overline{\pi}^{\epsilon-1}}{\theta\overline{\pi}^{\epsilon-1}}\{...$$
$$(1-\theta\beta\overline{\pi}^{\epsilon})\hat{w}_t + \frac{\overline{\pi}^{1-\epsilon}-p^*}{(1-\epsilon)(1-\theta\overline{\pi}^{\epsilon-1})}\theta\hat{\theta}_t - \beta\theta\overline{\pi}^{\epsilon-1}(\mathbb{E}_t\hat{\theta}_{t+1} + (\epsilon-1)\mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t\hat{\phi}_{t+1})...$$
$$... + \beta\theta\overline{\pi}^{\epsilon}(\mathbb{E}_t\hat{\theta}_{t+1} + \epsilon\mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t[\hat{\phi}_{t+1} + \frac{\theta\overline{\pi}^{\epsilon-1}}{1-\theta\overline{\pi}^{\epsilon-1}}\hat{\pi}_{t+1} - \frac{\overline{\pi}^{1-\epsilon}-p^*}{(1-\epsilon)(1-\theta\overline{\pi}^{\epsilon-1})}\theta\hat{\theta}_{t+1}])...$$
$$...\}$$

$$\hat{\pi}_t = \frac{(1-\theta\overline{\pi}^{\epsilon-1})(1-\theta\beta\overline{\pi}^{\epsilon})}{\theta\overline{\pi}^{\epsilon-1}}\hat{w}_t - \frac{\overline{\pi}^{1-\epsilon}-p^*}{(1-\epsilon)\pi^{\epsilon-1})}\beta\theta\overline{\pi}^{\epsilon}\mathbb{E}_t\hat{\theta}_{t+1} + \frac{\overline{\pi}^{1-\epsilon}-p^*}{\overline{\pi}^{\epsilon-1}}\hat{\theta}_t...$$
$$... + \beta\overline{\pi}\mathbb{E}_t\hat{\pi}_{t+1} + \beta(\overline{\pi}-1)(1-\theta\overline{\pi}^{\epsilon-1})[(\epsilon-1)\mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t\hat{\phi}_{t+1} + \mathbb{E}_t\hat{\theta}_{t+1}]$$

simplifying :

$$\hat{\pi}_t = \kappa\hat{w}_t + \beta\overline{\pi}\mathbb{E}_t\hat{\pi}_{t+1} + \eta[(\epsilon-1)\mathbb{E}_t\hat{\pi}_{t+1} + \mathbb{E}_t\hat{\phi}_{t+1} + \mathbb{E}_t\hat{\theta}_{t+1}] - \iota\frac{\beta\theta\overline{\pi}^{\epsilon}}{1-\epsilon}\mathbb{E}_t\hat{\theta}_{t+1} + \iota\hat{\theta}_t$$

with $\kappa = \frac{(1-\theta\overline{\pi}^{\epsilon-1})(1-\theta\beta\overline{\pi}^{\epsilon})}{\theta\overline{\pi}^{\epsilon-1}}$, $\eta = \beta(\overline{\pi}-1)(1-\theta\overline{\pi}^{\epsilon-1})$ and $\iota = \frac{\overline{\pi}^{1-\epsilon}-p^*}{\overline{\pi}^{\epsilon-1}}$.

C Details for the linearised augmented NK Phillips Curve

When the trend inflation $\overline{\pi}$ increases the values of $|\alpha_4|$ and $|\alpha_5|$ increase and thus inflation reacts more to change in θ_t. Indeed, the optimal price is higher relative to the existing prices and thus by construction a change in the share of non-updater generates more change in inflation.

An increase of the steady state share of non updating firms θ generates larger $|\alpha_4|$ and $|\alpha_5|$ and thus, the response of inflation is higher. There is a proportional effect: a 1% deviation of a larger number is larger in absolute value. There is also an effect on the optimal relative price p^* that tends to be farther from the other price if the resetting probability is lower.

An increase in the value of the price elasticity of goods ϵ generates a lower steady state markup and thus increase the response from change in marginal cost deviation of the optimal pricing decision from the distribution of relative prices. This increases $|\alpha_4|$ and increases the response the of inflation to the change in the Calvo share. On the other side, it decreases the value of $|\alpha_5|$ and thus decreases the response of inflation toward expected Calvo share. This is explain by the lower markups generated by the change in ϵ and smaller expected deviations implies by the new optimal pricing decision.

Phillips curve parameters			*Value of the parameter*	*Sign*	*Relative to parameter* $\overline{\pi}$	θ	ϵ	β
α_1	-	Relation to marginal cost	$\frac{(1-\theta\overline{\pi}^{\epsilon-1})(1-\theta\beta\overline{\pi}^{\epsilon})}{\theta\overline{\pi}^{\epsilon-1}}$	$\alpha_1 > 0$	$-$	$-$	$-$	$-$
α_2	-	Relation to expected inflation	$\beta\overline{\pi} + \beta(\overline{\pi}-1)(1-\theta\overline{\pi}^{\epsilon-1})(\epsilon-1)$	$\alpha_2 > 0$	$+$	$+$	$+$	$+$
α_3	-	Relation to trend inflation variable	$\beta(\overline{\pi}-1)(1-\theta\overline{\pi}^{\epsilon-1})$	$\alpha_3 > 0$	$+$	$-$	$-$	$+$
α_4	-	Relation to value of the Calvo	$\frac{\overline{\pi}^{1-\epsilon}-p^*}{\overline{\pi}^{\epsilon-1}}$	$\alpha_4 < 0$	$-$	$-$	$-$	$=$
α_5	-	Relation to the expected value of the Calvo	$\frac{\overline{\pi}^{1-\epsilon}-p^*}{\overline{\pi}^{\epsilon-1}}\frac{\beta\theta\overline{\pi}^{\epsilon}}{1-\epsilon} + \beta(\overline{\pi}-1)(1-\theta\overline{\pi}^{\epsilon-1})$	$\alpha_5 < 0$	$-$	$-$	$+$	$-$

Table 6: NKPC parameters and their relations to other structural parameters

D A large negative shock to demand

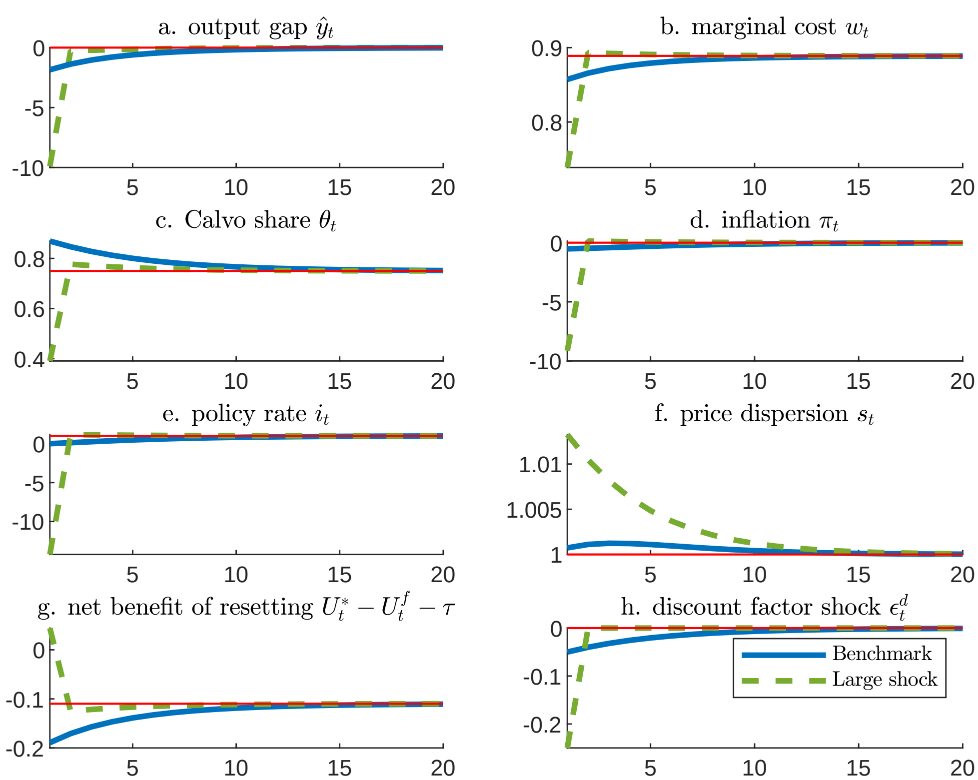


Figure D.1: Impulse responses to a *large* negative (-25%) discount factor shock in the augmented small-scale NK model in comparison to the benchmark (-5%) discount factor shock in Figure 3.

E Monetary non-neutrality

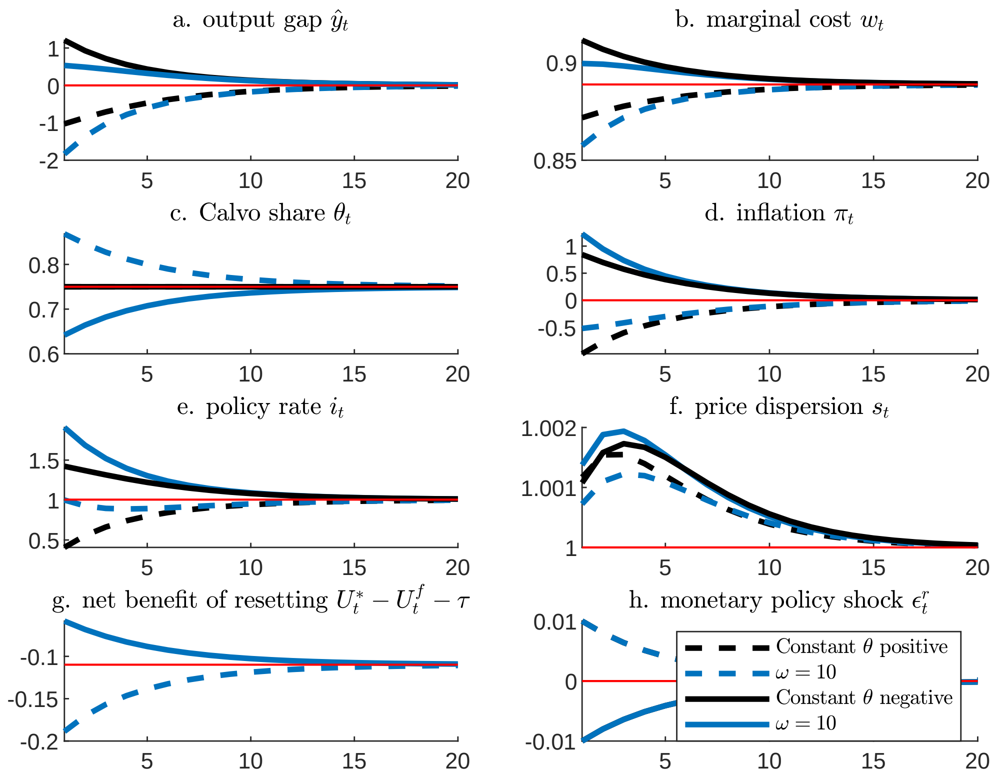


Figure E.1: Asymmetric impulse responses to a positive or negative (±1%) monetary policy shock with $\rho_r = 0.80$ in the small-scale NK model.