The Financial (In)Stability Real Interest Rate, R**

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February 11, 2022

Abstract

We introduce the concept of financial stability real interest rate using a macroeconomic banking model with an occasionally binding financing constraint as in Gertler and Kiyotaki (2010). The financial stability interest rate, r**, is the threshold interest rate that triggers the constraint being binding. We discuss r** and its dynamics, and show that persistently low real rates induce an increase in financial vulnerabilities and a consequent decline in the level of r**. We also provide a measure of r** for the US economy and discuss its evolution over the past 50 years, highlighting that during periods of financial stress that are associated with a decline in r** , the real rate tracks r** —a feature of monetary policy known as "Greenspan's put".

1 Introduction

One of the key aspects that has characterized the global economy and in particular advanced economies in the last two decades is the secular decline in real interest rates. The decline in global real interest rates has largely occurred in a context of relatively low and stable inflation suggesting that the drop in observed real interest rates reflects a fall in what researchers refer to as the "natural real interest rate," also known as r* (see, for example, Holston et al., 2017, and Del Negro et al. 2019). The concept of natural real interest rate dates to Wicksell (1898) and it is usually defined as the "real rate consistent with real GDP equals to its potential in the absence of shocks to demand. In turn potential GDP is defined to be the level of output consistent with stable price inflation absent transitory supply shocks" (see Laubach and Williams (2003)). In short, the concept of natural real interest rate is associated with the notion of macroeconomic stability.

In this paper we propose a complementary concept that we call the "financial stability real interest rate, r**." The core idea relies on determining the underlying level of real interest rate that might generate financial instability dynamics. Both conceptually and observationally r** differs from the "natural real interest rate" and from the observed real interest rate reflecting a tension in terms of macroeconomic stabilization versus financial stability objectives.

First, we discuss r** from a conceptual standpoint. To define the financial stability real interest rate one first needs to develop a concept of financial stability. To this end, we consider an environment in which some agents in the economy face a credit constraint that gives rise to debt-deflation or asset fire-sale dynamics. Importantly, the credit constraint is occasionally binding. This implies that the economy is characterized by two states: when the constraint is not binding the economy is in a normal state or tranquil period; when the constraint binds the economy is in a crisis mode and a financial instability dynamic arises. The financial stability real interest rate is the interest rate that, for a given state of the economy (for example for a given amount of private debt), would be consistent with the constraint being just binding.

Just like the natural rate of interest provides a benchmark for monetary policy in terms of macroeconomic stability, r** is meant to provide a benchmark for financial stability: if the real rate in the economy is at or above r**, the tightness of financial conditions may generate financial instability. Like the natural interest rate, the financial (in)stability real interest rate is state dependent: it evolves with the conditions of the economy, and in particular with the degree of imbalances in the financial system.

The notion of a financial (in)stability real interest rate broadly applies to any model where the economy fluctuates between normal state and a crisis state (see for example Mendoza (2010), Benigno et al. (2013), and Akinci and Chahrour (2018) in the context of the sudden stop literature). For concreteness, in this paper we use a particular model to illustrate how r** is constructed. The specific approach that we will follow in developing the concept of the financial stability real interest rate builds upon the banking framework developed by Gertler and Karadi (2011) and Gertler and Kiyotaki (2015). One of the virtues of using the Gertler-Kiyotaki framework is that it allows to relate the concept of financial stability real interest rate also to key variables for financial intermediaries such as the net worth or the asset/liability ratio.

In this framework, financial intermediaries channel funds from households to firms. The key imperfection is that banks have a limit in their ability to raise funds because of a moral hazard problem. It is assumed that after raising funds and buying assets at the beginning of the period, and then the banker decides whether to operate honestly or divert assets for personal use. This moral hazard problem gives rise to an incentive compatibility constraint that creates a link between the value of the bank and the value of the assets that can be diverted.

In their seminal work, Gertler and Karadi (2011) and Gertler and Kiyotaki (2015) always assume that the constraint is binding. The key departure that we would like to consider, as in Akinci and Queralto (forthcoming), is to allow for the constraint to be occasionally binding so that the economy can display both a tranquil and a crisis state. This departure requires using a global solution method for solving the model and takes into account the non-linearity generated by the occasionally binding constraint.

As a first pass we focus on a version of the model in which there is no need to determine nominal variables as contracts are expressed in real terms. Therefore by construction, the real rate in this economy coincides with the natural rate of interest r*, that is, the underlying interest rate consistent with macroeconomic stability. In order to further simplify the exposition, in this paper we illustrate the mechanism in a situation where the real interest rate is exogenous as, for example, in a small open economy. We leave the discussion of the rich interactions between monetary policy and the financial (in)stability real interest rate to further research.

Within our modeling approach, we characterize some key properties of our conceptualization. Given the non-linearity built in our approach, the response of the economy to a shock differs depending on the underlying state. In particular the level and the evolution of the financial stability real rate r** depends on the economy being in a high or low fragility state. For example, when the economy is in a high fragility state (high leverage), a negative shock that triggers a binding credit constraint is associated with a spike in credit spreads and a relatively low level of the financial stability real interest rate, r**. Interestingly, during the period of financial stress r** stands below the natural real interest rate. This suggests that, under these circumstances, a policy rate that tracks the natural real interest rate leads to financial instability. Moreover, prolonged period of low real interest rate leads eventually to an increase in leverage of the banking sector and a lower level of the financial stability real interest rate. Low for long (in terms of real interest rates) tends then to reduce the policy space as the gap between the natural and the financial stability real interest rate shrinks.

We then provide an empirical measure of r** for the US economy and discuss its evolution over the past 50 years. We construct our measure by building on the properties of our model economy to identify in the data episodes of financial instability. First, periods of financial stress coincides with very volatile credit spreads. Such volatility is a consequence of the non linearity of the model: when financing constraints are binding, the financial accelerator mechanism amplifies the impact of shocks to the economy and to credit spreads in particular. Second, the level of credit spreads is connected with the tightness of the financial constraint, and therefore with the gap between r** and the real rate r. Again, this relationship is non linear as it changes depending on the constraint being binding or not: during periods of financial stress, credit spreads predict the latent r**-r gap very well. The relationship becomes much looser during tranquil financial periods, where movements in spreads are much noisier proxies for the r**-r gap. Our empirical measure of r** shows that in post-1970s US data as r** falls during periods of financial stress the real rate tends to track r** —a feature of monetary policy known as "Greenspan's put". This has been a feature of all financial stress episodes in the US, with the only exception being the later part of the Great Financial Crisis, when the nominal interest was stuck at the zero lower bound. In general we note that financial stress episodes are associated with periods in which the real interest rate is above our measure of r**.

The next section describes the model, section 3 discusses our calibration strategy and section 4 presents the quantitative properties of r**. In section 5 we construct the empirical measure of r**. Section 6 concludes.

2 Model

We propose a framework that builds upon the banking model developed in Gertler and Kiyotaki (2010). In this setting, banks make risky loans to nonfinancial firms and collect deposits from domestic households. In addition, banks may also hold a perfectly safe asset, supplied by the foreign sector (or, under an equivalent formulation, by the government sector). Because of an agency problem, banks may be constrained in their access to external funds. The key aspect for the purpose of our analysis is to allow for this constraint to be occasionally binding, as in Akinci and Queralto (forthcoming). In normal, or "tranquil," times, banks' constraints do not bind: credit spreads are small and the economy's behavior is similar to a frictionless neoclassical framework. When the constraint binds the economy enters into financial stress mode: credit spreads become large and volatile, and investment and credit drop, consistent with the evidence.

As we mentioned above for simplicity at this stage we consider a real model in which there is no nominal determination.

2.1 Households

Each household is composed of a constant fraction $(1-f)$ of workers and a fraction f of bankers. Workers supply labor to the firms and return their wages to the household. Each banker manages a financial intermediary ("bank") and similarly transfers any net earnings back to the household. Within the family there is perfect consumption insurance.

Households do not hold capital directly. Rather, they deposit funds in banks. The deposits held by each household are in intermediaries other than the one owned by the household. Bank deposits are riskless one-period securities. Consumption, C_t, deposits, D_t, and labor supply, L_t, are given by maximizing the discounted expected future flow of utility

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, L_{t+i}),$$

subject to the budget constraint $C_t + D_t \leq W_t L_t + R^d_{t-1} D_{t-1} + \Pi_t \; \forall t$.

$\mathbb{E}_t$ denotes the mathematical expectation operator conditional on information available at time t, and $\beta \in (0,1)$ represents a subjective discount factor. The variable W_t is the real wage, R^d_t is the (gross) real interest rate received from holding one-period deposits, and Π_t is total profits distributed to households from their ownership of both banks and firms.

2.2 Banks

Banks are owned by the households and operated by the bankers within them. In addition to its own equity capital, a bank can obtain external funds from domestic households, d_t. In each period the bank uses its net worth n_t and deposits d_t, to purchase securities issued by nonfinancial firms, s_t, at price Q_t, as well as safe assets b_t. In turn, nonfinancial firms use the proceeds to finance their purchases of physical capital.

2.2.1 Agency friction and incentive constraint

We follow Gertler and Kiyotaki (2010) in assuming that banks are "specialists" who are efficient at evaluating and monitoring nonfinancial firms and at enforcing contractual obligations with these borrowers. For this reason firms rely solely on banks to obtain funds and there are no contracting frictions between banks and firms. However, as in Gertler and Kiyotaki (2010), we introduce an agency problem whereby the banker managing the bank may decide to default on its obligations and instead transfer a fraction of assets to the households, in which case it is forced into bankruptcy and its creditors can recover the remaining funds. In recognition of this possibility, creditors potentially limit the funds they lend to banks. In our setup, banks may or may not be credit constrained, depending on whether or not they are perceived to have incentives to disregard their contractual obligations.

More specifically, after having borrowed external funds but before repaying its creditors, the bank may decide to default on its obligations and divert fraction $\Theta(x_t)$ of risky loans. In this case, the bank is forced into bankruptcy and its creditors recover the remaining funds. To ensure that the bank does not divert funds, the incentive constraint must hold:

$$V_t \geq \Theta(x_t) Q_t s_t \tag{1}$$

where V_t stands for the continuation value of the bank. This constraint requires that the bank's continuation value be higher than the value of the diverted funds.

The variable x_t is the economywide ratio of risky to safe assets held by the banking sector: $x_t \equiv \frac{Q_t K_t}{B_t}$. We assume that the agency friction worsens as the banking sector portfolio becomes more risky: $\Theta'(x_t) > 0$. The rationale for this assumption is that risky loans are more opaque and hard to monitor relative to safe assets, which leads creditors to turn more cautious when the banking sector's portfolio becomes riskier.[1]

2.2.2 The banker's problem

The bank pays dividends only when it exits. If the exit shock realizes, the banker exits at the *beginning* of $t+1$, and simply waits for its asset holdings to mature and then pays the net proceeds to the household. The objective of the bank is to maximize expected terminal payouts to the household. Formally, the bank chooses state-contingent sequences $\{s_t, b_t, d_t\}$ to solve

$$V_t(n_t) = \max \quad (1-\sigma)\mathbb{E}_t \Lambda_{t,t+1} \left(R_{K,t+1} Q_t s_t + R_t b_t - R_t^d d_t \right) + \sigma \mathbb{E}_t \Lambda_{t,t+1} V_{t+1}(n_{t+1})$$

subject to

$$Q_t s_t + b_t + R_{t-1}^d d_{t-1} \leq R_{K,t} Q_{t-1} s_{t-1} + R_{t-1} b_{t-1} + d_t \tag{2}$$

and the incentive constraint given in equation (1). Here R_t is the return on the safe asset and $\Lambda_{t,t+1}$ is the household's stochastic discount factor, given by the marginal rate of substitution between consumption at dates $t+1$ and t. Equation (2) is the bank's budget constraint, stating that the bank's expenditures (consisting of asset purchases, $Q_t s_t + b_t$, and repayment of deposit financing, $R_{t-1}^d d_{t-1}$) cannot exceed its revenues, stemming from payments of previous-period asset holdings, $R_{K,t} Q_{t-1} s_{t-1} + R_{t-1} b_{t-1}$ and deposits d_t. The bank's problem is also subject to the balance sheet identity

$$Q_t s_t + b_t \equiv n_t + d_t \tag{3}$$

This equation is equivalent to a definition of net worth, and states that the bank's assets are funded by the sum of net worth and deposits.

In this setting, banks can perfectly arbitrage between deposits and holdings of the safe asset. As a consequence, we will have that $R_t^d = R_t$. Thus, R_t is effectively the economywide risk-free rate. Using this condition and combining (2) with the balance sheet identity we obtain the law of motion

[1] The assumption that x_t is the *aggregate* ratio, as opposed to the bank-specific one, implies that banks do not internalize the effect on Θ of their choice of x_t. We make this assumption for convenience, but expect our key results to also apply to the case in which the effect of banks' choice of x_t is internalized.

of net worth:

$$n_t = (R_{K,t} - R_{t-1})\, Q_{t-1} s_{t-1} + R_{t-1} n_{t-1} \tag{4}$$

We next guess that the value function satisfies, $V_t(n_t) = \alpha_t n_t$, where α_t is a coefficient to be determined. Let

$$\mu_t \equiv \mathbb{E}_t[\Lambda_{t,t+1}(1 - \sigma + \sigma\alpha_{t+1})(R_{K,t+1} - R_t)] \tag{5}$$

$$\nu_t \equiv \mathbb{E}_t[\Lambda_{t,t+1}(1 - \sigma + \sigma\alpha_{t+1})]R_t \tag{6}$$

Note that α_{t+1}, capturing the value to the bank of an extra unit of net worth the following period, acts by "augmenting" banks' stochastic discount factor (SDF) so that their effective SDF is given by $\Lambda_{t,t+1}\,(1 - \sigma + \sigma\alpha_{t+1})$.

With these definitions, the problem simplifies to

$$\alpha_t n_t = \max_{s_t} \quad \mu_t Q_t s_t + \nu_t n_t \tag{7}$$

subject to the incentive constraint

$$\mu_t Q_t s_t + \nu_t n_t \geq \Theta(x_t) Q_t s_t \tag{8}$$

and (4). The solution is as follows. Let the banker's (risky) leverage be $\phi_t \equiv \frac{Q_t s_t}{n_t}$. If $\mu_t = 0$, the banker is indifferent as to its leverage choice. If $\mu_t > 0$, the banker leverages up as much as allowed by the constraint. Rearranging (8), maximum leverage, denoted $\overline{\phi}_t$, is given by

$$\overline{\phi}_t = \frac{\nu_t}{\Theta(x_t) - \mu_t}. \tag{9}$$

Observe that $\overline{\phi}_t$ is decreasing in $\Theta(x_t)$, and therefore falls as the banking sector's porftolio shifts toward risky assets (i.e. as x_t rises).

Since the bankers problem is linear, we can easily aggregate across banks. For surviving banks, the evolution of net worth is given by (4). We assume entering bankers receive a small exogenous equity endowment, given by fraction ξ/f of the value of the aggregate capital stock. Thus the law of motion of aggregate net worth is the following:

$$N_t = \sigma\big[\,(R_{K,t} - R_{t-1})\underbrace{Q_{t-1}K_{t-1}}_{=Q_{t-1}S_{t-1}} + R_{t-1}N_{t-1}\big] + (1-\sigma)\xi Q_{t-1}K_{t-1} \tag{10}$$

2.2.3 Credit spreads and the financial constraint

The model highlights how the behavior of credit spreads depends crucially on whether the financial constraint binds. We define the credit spread as the (annualized) expected return on nonfinancial firms' securities, $\mathbb{E}_t(R_{K,t+1})$, minus the risk-free rate, R_t. When the constraint is not binding, banks can fully arbitrage away this excess return, and the following condition holds:

$$\mathbb{E}_t\left(\boldsymbol{\Omega}_{t+1}R_{Kt+1}\right)=\mathbb{E}_t\left(\boldsymbol{\Omega}_{t+1}\right)R_t, \tag{11}$$

where $\boldsymbol{\Omega}_{t+1}\equiv 1-\sigma+\sigma\alpha_{t+1}$ is the banker's SDF. If the economy is far a way from the constraint, the credit spread $\mathbb{E}_t(R_{K,t+1})-R_t$ will tend to be low on average, and relatively stable. The model then implies a behavior of investment similar to standard (frictionless) models, with the condition $\mathbb{E}_t(R_{K,t+1})\approx R_t$ determining the response of investment to movements of the real rate: a higher R_t, for example, raises the required expected return on investment, triggering a fall in Q_t and I_t.

By contrast, when the constraint binds, banks' lending is constrained by their net worth: from the constraint at equality, we have $\Theta(x_t)Q_tS_t=\alpha_tN_t$. Banks cannot fully arbitrage away excess returns: we have

$$\mathbb{E}_t\left(\boldsymbol{\Omega}_{t+1}R_{Kt+1}\right)>\mathbb{E}_t\left(\boldsymbol{\Omega}_{t+1}\right)R_t,$$

and the credit spread will tend to be large and volatile. In this regime, investment behavior is heavily influenced by financial accelerator / fire-sale dynamics: a lower asset price Q_t erodes net worth and tightens the constraint further, which pushes investment down, triggering another round of decline in Q_t. Along the way, credit spreads skyrocket.

2.3 Nonfinancial Firms

There are two categories of nonfinancial firms: final goods firms and capital producers. In turn, within final goods firms we also distinguish between "capital leasing" firms and final goods producers, in order to clarify the role of bank credit used to finance capital goods purchases.

2.3.1 Final Goods Firms

We assume that there are two types of final goods firms: capital leasing firms and final goods producers. The first type of firm purchases capital goods from capital good producers, stores them for one period, and then rents them to final goods firms. The second type uses physical capital (rented from capital leasing firms) and labor to produce final output. Importantly, capital leasing firms have to rely on banks to obtain funding to finance purchases of capital, as explained below. In addition, final goods producers need to rely on banks to finance working capital.

In period $t-1$, a representative capital leasing firm purchases K_{t-1} units of physical capital at price Q_{t-1}. It finances these purchases by issuing S_{t-1} securities to banks which pay a state-contingent return $R_{K,t}$ in period t. At the beginning of period t, the realization of the capital quality shock ψ_t determines the effective amount of physical capital in possession of the firm, given by $e^{\psi_t}K_{t-1}$. The firm rents out this capital to final goods firms at price Z_t, and then sells the undepreciated capital $(1-\delta)e^{\psi_t}K_{t-1}$ in the market at price Q_t. The payoff to the firm per unit of physical capital purchased is thus $e^{\psi_t}\left[Z_t+(1-\delta)Q_t\right]$. Given frictionless contracting between firms and banks, it follows that the return on the securities issued by the firm is given by the following (note that this equation implies that capital leasing firms make zero profits state-by-state): $R_{K,t} = e^{\psi_t} \frac{Z_t + (1-\delta)Q_t}{Q_{t-1}}$.

The capital quality shock $\psi_t \sim N(0, \sigma_\psi)$, which provides a source of fluctuations in returns to banks' assets, is a simple way to introduce an exogenous source of variation in the value of capital.[2] These variations are enhanced by the movements in the endogenous asset price Q_t triggered by fluctuations in ψ_t.

In the aggregate, the law of motion for capital is given by

$$K_t = \Gamma(I_t) + (1-\delta)e^{\psi_t} K_{t-1} \tag{12}$$

Final goods firms produce output Y_t using capital and labor: $Y_t = F(e^{\psi_t} K_{t-1}, L_t)$. The first order conditions for labor and for physical capital are as follows:

$$F_1(K_t, L_t) = Z_t \tag{13}$$

$$F_2(K_t, L_t) = W_t \tag{14}$$

2.3.2 Capital Goods Producers

Capital producers, owned by households, produce new investment goods using final output, and they sell those goods to firms at the price Q_t. The quantity of newly produced capital, $\Gamma(I_t)$, is an increasing and concave function of investment expenditure to capture convex adjustment costs.

The objective of the capital producer is then to choose $\{I_t\}$ to maximize profits distributed to households:

$$\max \{Q_t \Gamma(I_t) - I_t\} \tag{15}$$

The resulting first-order condition yields a positive relation between Q_t and I_t:

$$Q_t = [\Gamma'(I_t)]^{-1} \tag{16}$$

2.4 Interest rate determination

We assume that the safe rate, R_t, evolves (mostly) exogenously. The goal is to capture a first pass at fluctuations in the natural real interest rate, without taking a stance on their causes. Accordingly, R_t satisfies

$$R_t = \mathcal{R}_t + \beta^{-1} + f(B_t/\overline{B}), \tag{17}$$

where $\mathcal{R}_t$ follows the stochastic process

$$\log(\mathcal{R}_t) = \rho_R \log(\mathcal{R}_{t-1}) + \epsilon_{R,t},$$

[2] This may be thought of as capturing some form of economic obsolescence. Gertler et al. (2012) provide an explicit microfoundation of fluctuations in capital quality ψ_t based on time-varying obsolescence of intermediate goods.

with $\epsilon_{R,t} \sim N(0, \sigma_R)$. The (endogenous) term $f(B_t/\overline{B})$ is a small portfolio cost we introduce for technical reasons, as it helps ensure stationarity of safe asset holdings B_t (Schmitt-Grohe and Uribe (2003)).

2.5 Resource Constraint, Market Clearing, and Equilibrium

The resource constraint and the balance of payments equations, respectively, are given by:

$$Y_t = C_t + I_t + \mathbf{T}_t \tag{18}$$

$$T_t = B_t - R_{t-1} B_{t-1} \tag{19}$$

where $\mathbf{T}$ stands for net exports (or transfers (taxes) under an equivalent formulation where safe assets are provided by the government sector). An equilibrium is defined as stochastic sequences for the eight quantities $Y_t, C_t, I_t, \mathbf{T}_t, B_t, L_t, K_t, N_t$, four prices $R_{K,t}, Q_t, R_t, W_t$, and four banking sector coefficients $\mu_t, \nu_t, \alpha_t, \phi_t$ such that households, banks, and firms solve their optimization problems, and all markets (for short-term debt, securities, new capital goods, final goods, and labor) clear, given exogenous stochastic sequences for ψ_t, and $\mathcal{R}_t$.

3 Functional Forms and Parameter Values

In this section we describe, in turn, the functional forms and the parameter values used in the model simulations.

3.1 Functional Forms

The functional forms of preferences, production function, and investment adjustment cost are the following:

$$U(C_t, L_t) = \frac{\left(C_t - \chi \frac{L_t^{1+\epsilon}}{1+\epsilon}\right)^{1-\gamma} - 1}{1-\gamma} \tag{20}$$

$$F(K_t, H_t) = (e^{\psi_t} K_{t-1})^{\eta} L_t^{1-\eta} \tag{21}$$

$$\Gamma(I_t) = a_1 (I_t)^{1-\vartheta} + a_2 \tag{22}$$

$$\Theta(x_t) = \theta \left(\frac{x_t}{\bar{x}}\right)^{\kappa} \tag{23}$$

The utility function, equation (20), is defined as in GHH(1988), which implies non-separability between consumption and leisure. This assumption eliminates the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor independent of consumption. The parameter γ is the coefficient of relative risk aversion, and ϵ determines the wage elasticity of labor supply, given by $1/\epsilon$. The production function, equation (21), takes the Cobb-Douglas form. The coefficient η is the elasticity of output with respect to capital. Equation (22) defines the the investment technology, with the ϑ corresponding to the elasticity of the price of capital with

Table 1: Calibrated Model Parameters

Parameter	**Symbol**	**Value**	**Source/Target**
Conventional			
Discount factor	β	0.995	Interest rate 2%, ann.)
Risk aversion	γ	2	Standard RBC value
Capital share	η	0.33	Standard RBC value
Capital depreciation	δ	0.025	Standard RBC value
Debt elast. of interest rate	φ	0.01	Standard RBC value
Reference debt/output ratio	$\bar{b}$	2	Steady state B/Y of 200%
Labor disutility	χ	2.22	Steady state labor of 33%
Inverse Frisch elast.	ϵ	1/8	Gertler and Kiyotaki (2010)
Elasticity of Q w.r.t. I	ϑ	0.25	Gertler, Kiyotaki, Prestipino (2019)
Investment technology	a_1	0.925	$Q = 1$
Investment technology	a_2	−0.975%	$\Gamma(I) = I$
Financial Intermediaries			
Survival rate	σ	0.95	Exp. survival of 5 yrs
Transfer rate	ξ	0.1827	
Fraction divertable	θ	0.20	{ Frequency of crises around 3%,
Elasticity of Θ_x w.r.t. x	κ	1.5	Leverage of 5}
Shock Processes			
Persistence of interest rate	ρ_R	0.915	
SD of interest rate innov. (%)	σ_R	0.175	
SD of capital quality (%)	σ_ψ	0.75	

respect to investment. Finally equation (23) defines the positive relationship between the ratio of risky-to-safe assets, x_t, and the degree of financial frictions, Θ_x, in the economy, as described before.

We assign values to the structural parameters of the model using a combination of calibration and econometric estimation techniques. We calibrate several preference, production, and financial sector parameters to standard values when possible, and report them in Table 1.

We set the discount factor, β, to 0.995, which implies an annual real neutral rate of interest rate of 2%. The following four parameters are standard values in business cycle literature: The risk aversion parameter, γ, the capital share, η, and the depreciation of capital, δ, are set to 2, 0.33, and 0.025, respectively. The reference debt to output ratio is set to 2, which yields a ratio of external debt to GDP of 50% annually—a conservative estimate.

The values we assign to the Frisch labor supply elasticity (given by $1/\epsilon$) is at the upper end of a wide range of values used in the literature. We found that these parameters are hard to identify in the estimation procedure described below. Accordingly, the Frisch labor supply elasticity is set to 8, a value that is above the range typically found in the literature. As in Gertler and Kiyotaki (2010), this relatively high value represents an attempt to compensate for the absence of frictions such as nominal wage and price rigidities, which are typically included in quantitative DSGE models. While our framework excludes these frictions to preserve simplicity, they likely have a role in accounting for employment and output volatility in the countries we study, hence, as Gertler and Kiyotaki (2010) do, we partly compensate for their absence by setting a relatively high elasticity of labor supply.

We follow Gertler et al. (2019) in choosing the parameters governing the investment technology. More specifically, we set ϑ, which corresponds to the elasticity of the price of capital with respect to investment rate, equal to 0.25, a value within the range of estimates from panel data. We then choose a_1 and a_2 to hit two targets: first, a ratio of quarterly investment to the capital stock of 2 percent and, second, a value of the price of capital Q equal to unity in the risk-adjusted steady state.

We then assign values to the four parameters relating to financial intermediaries: the survival rate of bankers, σ, the transfer to entering bankers, ξ, the fraction of assets that bankers can divert, θ, and the parameter determining the responsiveness of the degree of financing frictions to the ratio of risky-to-safe assets, κ. We calibrate σ to 0.95 as in Gertler and Kiyotaki (2010), implying that bankers survive for about 5 years on average. This value of banks' survival rate is around the mid-point of values found in the literature.

The start-up transfer rate ξ ensures that entering bankers have some funds to start operations. We set κ to 1.5. We calibrate the remaining parameters related to financial intermediaries to hit two targets: a leverage ratio of about 5 in the risk-adjusted steady state, and a frequency of financial crises of 3 percent annually. The target leverage ratio is an estimate of the average financial sector leverage.

Finally, as we have direct observations on real country interest rates, we fix the persistence and standard deviation of innovation for the interest rate shocks, ρ_R and σ_R, to the real interest rate from the U.S. data. We then choose the standard deviation of innovation for the capital quality shock to broadly match the standard deviation of output growth in the U.S.

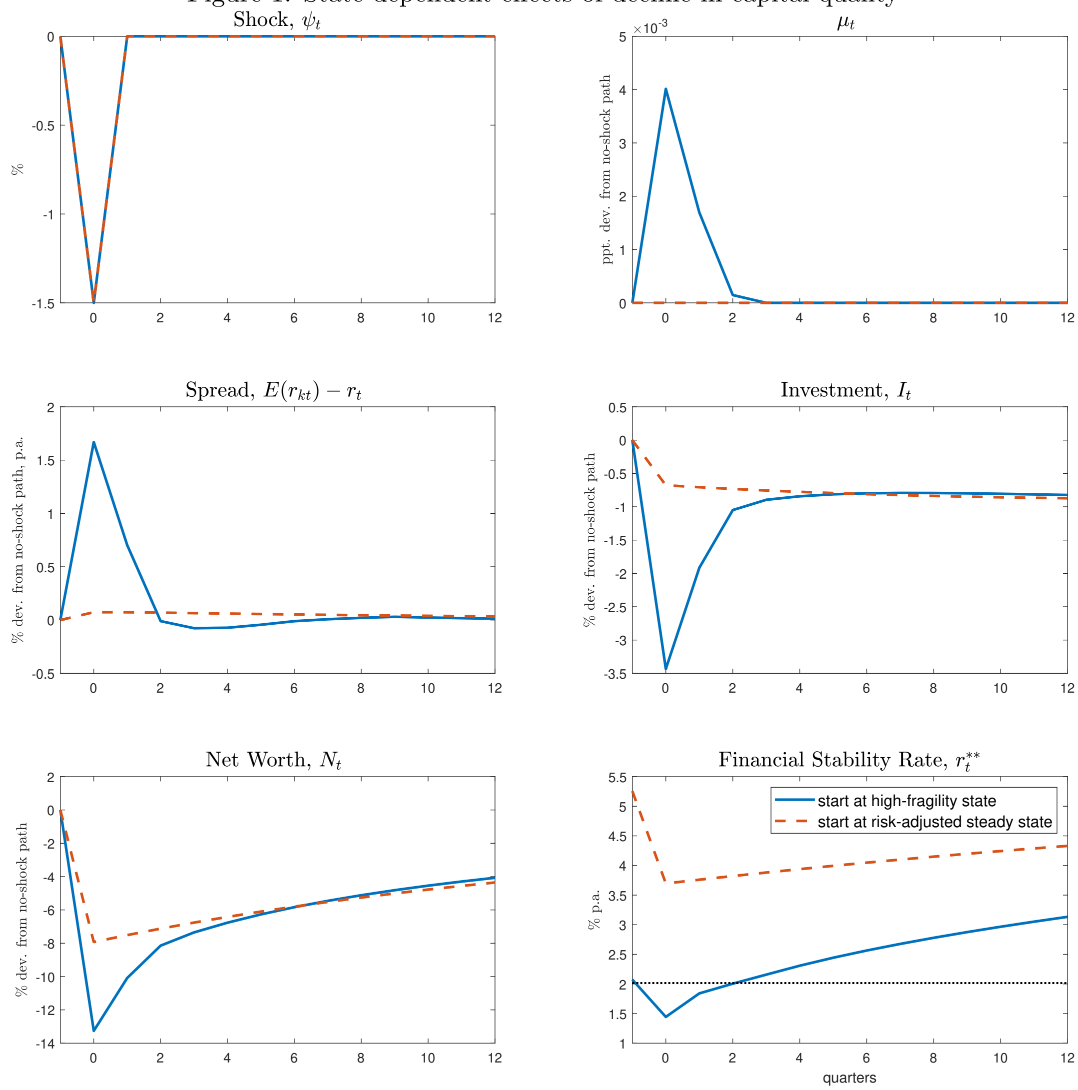
4 Model Results

4.1 The financial stability rate, r^{**}: Dynamics

In this section we define and then characterize the dynamic properties of the financial stability interest rate in a calibrated version of the simple model as described above. We calculate the implied real interest rate in the economy that makes the constraint just binding, and call it the financial stability rate, r^{**}. When the leverage constraint is slack, this implied real interest rate is a benchmark rate for financial stability: if the real rate is to increase beyond r^{**}, the tightness of financial conditions would generate financial instability. In those states in which the constraint binds, conversely, we compute the counterfactual real interest rate that makes the constraint nonbinding. Thus, we define r^{**} as the *threshold* rate above which financial instability arises.

Before showing the financial stability rate dynamics, it is useful to illustrate the nonlinearity and state-dependence induced by the financial constraint, as well as the amplification via the financial accelerator mechanism that occurs when the constraint binds. Figure 1 shows the responses to a 1.5 percent capital quality shock which hits when the economy is initially at the risk-adjusted steady state (red dashed line in Figure 1). The shock leads net worth to drop about 8 percent on impact. The decline in net worth, however, is not large enough to push the economy into the constrained region. As a consequence, the shock has only modest effects on investment and credit spreads.

Figure 1: State-dependent effects of decline in capital quality



Note: Responses to a 1.5% capital quality shock when the economy is at the risk-adjusted steady state (red dashed line) and when its initial capital stock is lower than its risk-adjusted steady state value (blue solid line). Variables indicated % dev. computed as percent deviations relative to their no-shock path. Dotted black line in the last panel shows the level of the real interest rate in the model economy.

We next perform a similar experiment, i.e. we hit the model with a 1.5 percent capital quality shock at $t = 0$, but we now assume that the economy is initially in a state of high financial fragility—with bank leverage very close to the maximum allowed by the constraint. Formally, we assume that the quality-adjusted capital stock $e^{\psi_t} K_{t-1}$ (a key state variable in the model), is about five percent below its steady-state value in period $t = -1$, right before the shock hits at $t = 0$.

The blue solid line in Figure 1 shows the dynamic effects of the capital quality shock when the economy starts from this high-fragility state (in deviation from the path that the economy would have followed absent the shock). The decline in bank net worth is now large enough to bring banks up against their constraints. As a consequence, the spread jumps by about 175 basis points annually. The decline in net worth is roughly 14 percent on impact, almost twice as much as the decline that occurs with a capital quality shock of the same size but with a less-fragile initial state. The sharp decline in net worth is explained by the financial accelerator mechanism that operates when the constraint binds: falling net worth leads investment to drop, which drives asset prices down, leading net worth to drop further. As a consequence, there is a severe drop in investment, of about 3.5 percent –several times larger than the decline of 0.7 percent that occurs when the economy is not in a fragile state.

The last panel of the figure shows the dynamic evolution of financial stability interest rate, r^{**}, when the shock hits starting at risk-adjusted state (high-fragility state) as shown by the red dashed line (blue solid line). The economy's initial states are different in these two cases, thus the model-implied financial stability rate differs markedly at these initial points (denoted by $t = -1$). As it is just illustrated, the economy is further away from the constraint at the risk-adjusted steady state, causing r^{**} to take much higher values compared to the high-fragility state (around 5 percent vs just above 2 percent). Note that the constraint is not binding in either cases at the initial points, as a result r^{**} is still higher than the real interest rate of 2 percent before the shock hits.

The shock that hits at $t = 0$ at the risk-adjusted steady state leads $r**$ to fall from 5 percent to 3.7 percent, which is still above the underlying real interest rate in the model economy. When the same shock hits at the high-fragility state, on the other hand, the constraint binds and r^{**} falls below the real interest rate of 2 percent. It is because the real interest rates consistent with financial stability has to be much smaller than the underlying real interest rate to be able to alleviate financial instability pressures generated by the binding financing constraints by boosting asset prices and bank equity valuations.

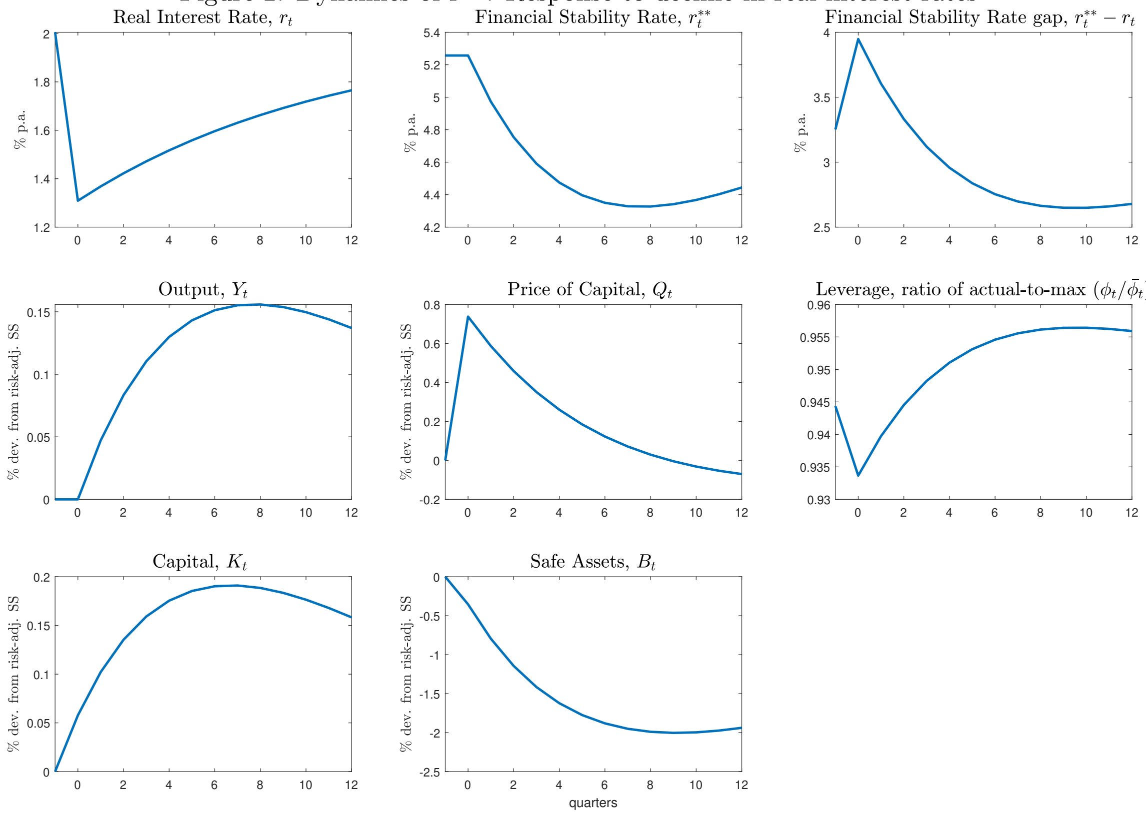
This exercise illustrates that responses of the economy to a shock can be very different depending on the underlying state – in the example, the stock of quality-adjusted capital. More importantly, this figure displays that both the level and the evolution of financial stability real rate, r**, could be different depending on the underlying state of the economy.

We next show the dynamic evolution of the endogenous variables in the model, such as output, asset prices, leverage, capital and safe assets, as well as the the financial stability interest rate, r^{**}, to an unexpected fall in the real rate of interest. These results are displayed in Figure 2. The figure also shows the dynamics of financial stability interest rate gap, defined as the difference between the financial stability rate and the underlying real rate of interest in the model economy.

Before the shock arrives at time $t = 0$, the economy is at the risk-adjusted state state, which features a real rate of interest of 2 percent and a financial stability rate of 5 percent (shown in the first two panels of the figure, respectively). The real rate then falls by 1 standard deviation at $t = 0$ and stays low for an extended period before going back to its steady-state level of 2 percent. As show in Figure 2, persistent reductions in real rates (such as those associated with accommodative monetary policy) lead to an improvement in financial conditions (price of capital, Q, rises sharply on impact) and banks' balance sheets improve (not shown).

As the economy's state variables remain unchanged at the time of the shock, financial stability interest rate remains at around 5 percent. This, along with a fall in real rate, leads to a short-run

Figure 2: Dynamics of r**: Response to decline in real interest rates

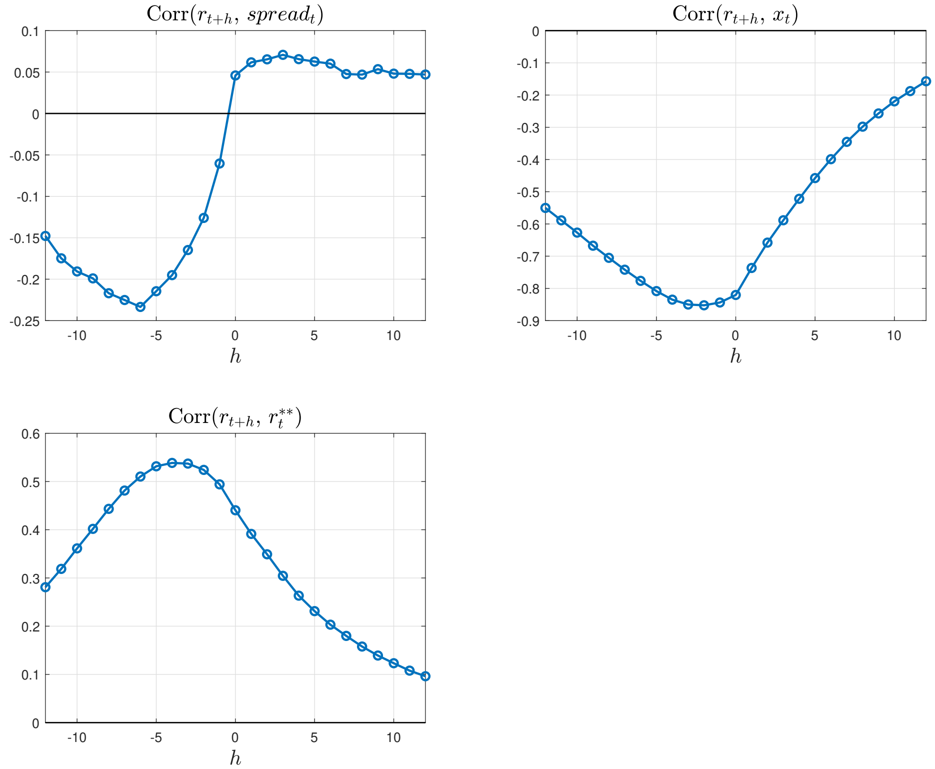


Note: Responses to a one standard deviation shock to the real interest rate, r_t, at time 0. The economy is at the risk-adjusted steady in the initial period. Variables indicated % dev. computed as percent deviations relative to their risk-adjusted steady state values.

increase in financial stability rate gap (r-gap). In the medium-to-long run, however, the financial stability real rate starts to fall. This happens in the model both because the positive impact of the shock on asset prices and bank equity values fades over time (Q goes back to steady state relatively quickly, leading N to decline gradually after its initial rise), and because banks shift their portfolios from safe assets (B falls) towards riskier capital (K increases), as real interest rates fall. This type of "reach for yield" behavior arises naturally in the model: in the face of persistently lower return on safe assets, agents respond by saving less in safe assets and more in risky assets. Thus, the ratio of risky-to-safe assets, x_t, builds up. As a result, the distance of banks' actual leverage to the maximum leverage that the bank can assume (due to the agency friction) shrinks over time, leading both the financial stability real rate and r-gap to decline markedly. As a result, the model implies that persistent declines in real interest rates today cause the real rate consistent with financial stability to be at low levels in the future. Note that while the model we are currently working with features flexible prices, this result suggests that the extension with nominal rigidities may feature an interesting tradeoff between macroeconomic and financial stability.

To further illustrate the dynamic relationship between the real interest rate and the financial

Figure 3: Real Rate, Credit Spreads and r**, Lead-Lag Correlations

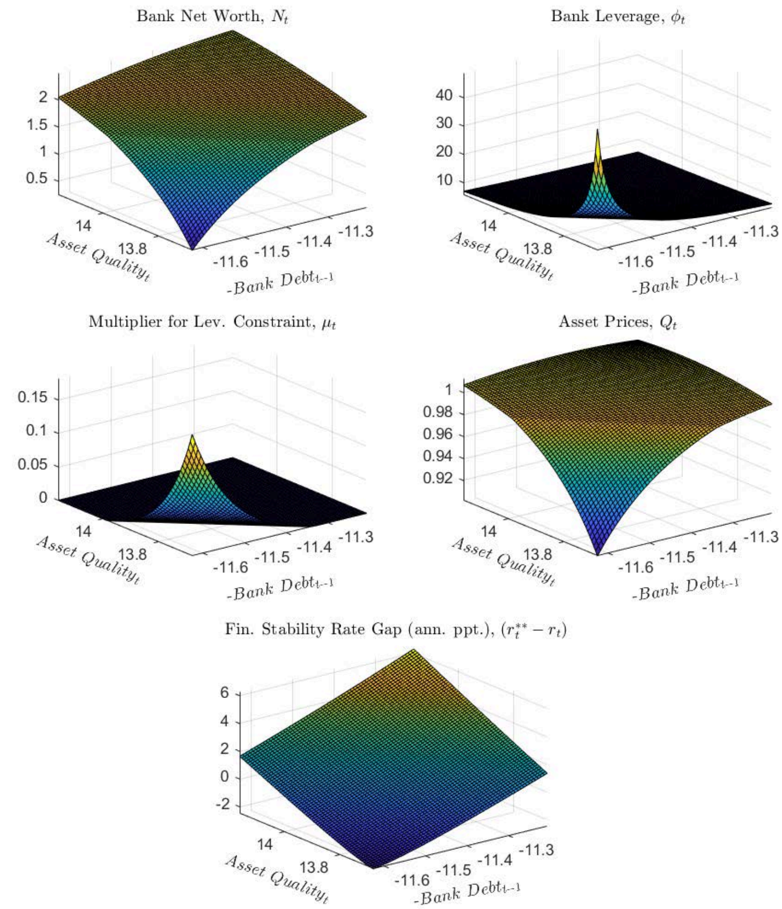


stability rate, Figure 3 shows the model-implied cross-correlogram between the real interest rate, the credit spread (a measure of financial stress), the ratio of risky-to-safe assets in banks' portfolio (x_t), and the financial stability rate (r^{**}). The first panel in the figure, for example, shows the correlation between the real interest rate r_{t+h} and the period-t credit spread, for a range of values of h. The contemporaneous correlation between these two variables is positive: higher interest rates today erode asset prices and bank net worth, leading the economy to move toward the constrained region and pushing up credit spreads. By contrast, the correlation between current credit spreads and the lagged interest rate is *negative*: *low* levels of interest rates today are associated with greater *future* financial stress. The reason is that low interest rates are associated with a higher ratio of risky-to-safe assets in the banking sector (second panel of Figure 3). This buildup of risky lending moves the economy closer to the financial stress region, as it raises the extent of financial frictions (measured by $\Theta(x_t)$). Accordingly, the model implies a positive correlation between r and r^{**}, a shown in the bottom panel, as well as a *lead* of r over r^{**}: lower current real interest rates are associated with lower future values of the financial stability rate.

4.2 The financial stability rate in a quantitative model

We next augment the model with additional features shown to improve the empirical realism of DSGE models, such as introducing adjustment cost at the level of investment, working capital

Figure 4: Equilibrium objects as a function of states

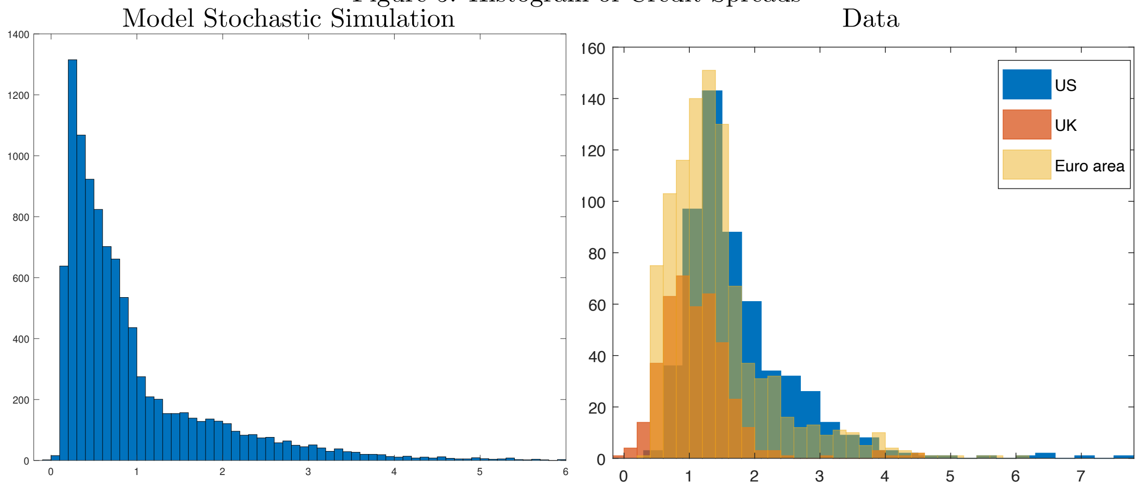


Note: Model endogenous variables as a function of quality of Bank Assets, $\overline{K}_t$, and the debt of the banking sector from the previous period, $\overline{N}_{t-1}$. All other states kept at risk-adjusted-steady-state value.

frictions for financing labor input, and adding shocks to total factor productivity. We also estimate some of the parameters of the model to match long run business cycle moments in the data. We then show while the model is still extremely simple, it is quantitatively realistic, enabling us to use it for constructing an empirical measure of the financial stability rate in the data.

We first describe banks' behavior as a function of the endogenous states in our calibrated economy. Figure 4 displays the three dimensional policy functions for a given level of the banking sector debt and the quality of bank assets. The constrained region is not only characterized by very low values of asset quality or by very high values of banking sector debt, but also by a combination of relatively low values the former and relatively high values of the latter. The threshold of banking sector debt for which the constraint becomes binding, and hence the level of r**, is a function of the level of asset quality. Note that interestingly, while all the other charts are very non linear, the r** chart looks very linear. This is because the power of changes in the real interest rate affecting the

Figure 5: Histogram of Credit Spreads



Note: Credit spreads stand for corporate bond spreads for non-financial firms. Euro area includes Germany, France, Spain, and Italy. Spreads are calculated as the average spreads between the yield of private-sector bonds in Italy, Spain, Germany and France relative to the yield on German government securities, in the UK relative to UK government securities, and in the US relative to US government securities, of matched maturities. Data sources: Gilchrist and Mojon (2014), Bank of England, Gilchrist and Zakrajsek (2012).

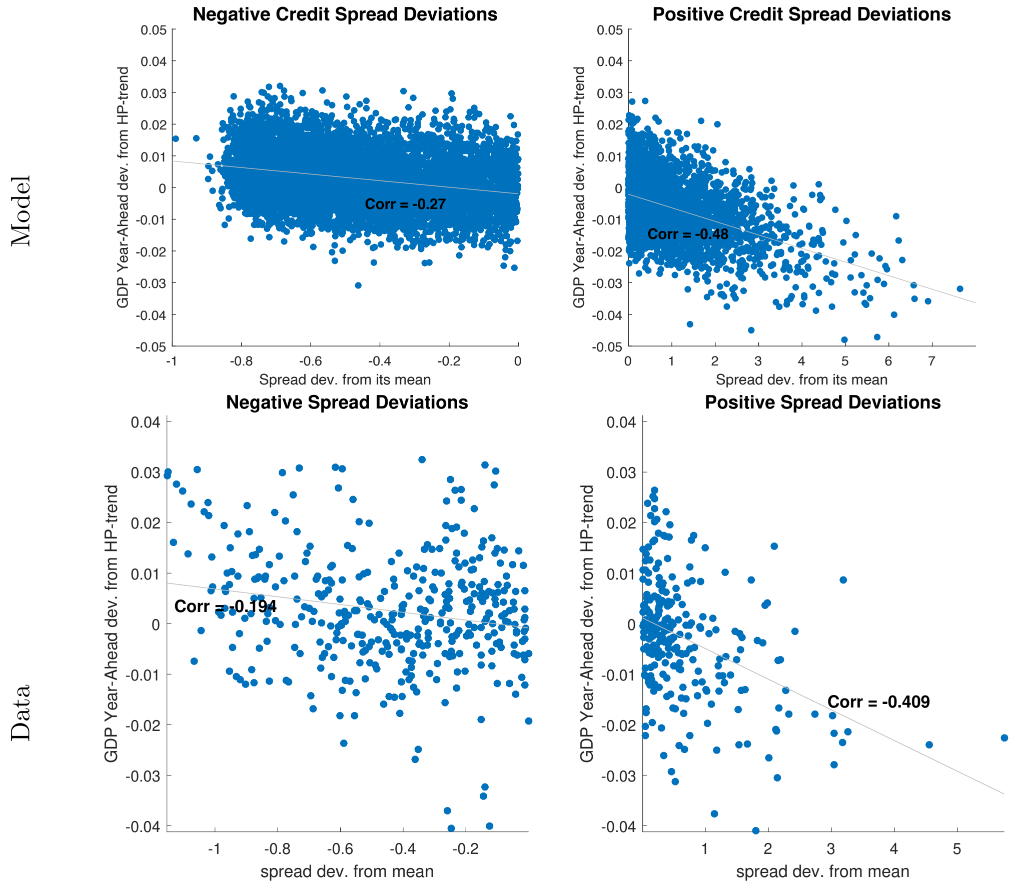
financing conditions varies with the extent to which the economy is constrained: When the economy is deep in the constrained region, the financial accelerator becomes very powerful, thus it benefits tremendously from a rate cut.

Figure 5 shows histogram of credit spreads. As shown in the panel on the right, credit spreads display occasional large spikes in the data. Spreads hover around 100 basis points a large fraction of the time, while they infrequently take values as large as 700 basis points. The panel on the left shows a histogram of credit spreads obtained from model stochastic simulations. As shown, the model delivers an asymmetric distribution of credit spreads, as in the data.

Our model economy displays strong nonlinearities consistent with the evidence from the macro-finance literature (see, for example Merton (2009), Kenny and Morgan (2011), Hubrich et al. (2013), He and Krishnamurthy (2019), or more recently Adrian et al. (2019)). Figure 6 illustrates the asymmetric relation between credit spreads and economic activity: when financial stresses are relatively elevated, they tend to be more strongly associated with real activity than when they are relatively compressed. In particular, considering positive values of spreads yields a correlation between credit spreads and real economic activity (calculated as year-ahead deviation of real investment from its HP trend) of about -0.41, compared with -0.19 obtained when we consider negative values of credit spreads, consistent with empirical results shown in the lower panel of the same figure. Key to explaining the model's ability to generate this asymmetry is the occasionally binding incentive constraint: a binding constraint tends to be associated with elevated levels of financial stresses, and at the same time leads to amplified movements in real activity (via the financial accelerator).

Finally, Figure 7 shows how our model produces occasional financial crisis episodes that are extreme manifestations of the asymmetric and nonlinear behavior in the model economy. The model

Figure 6: Credit Spreads and Output

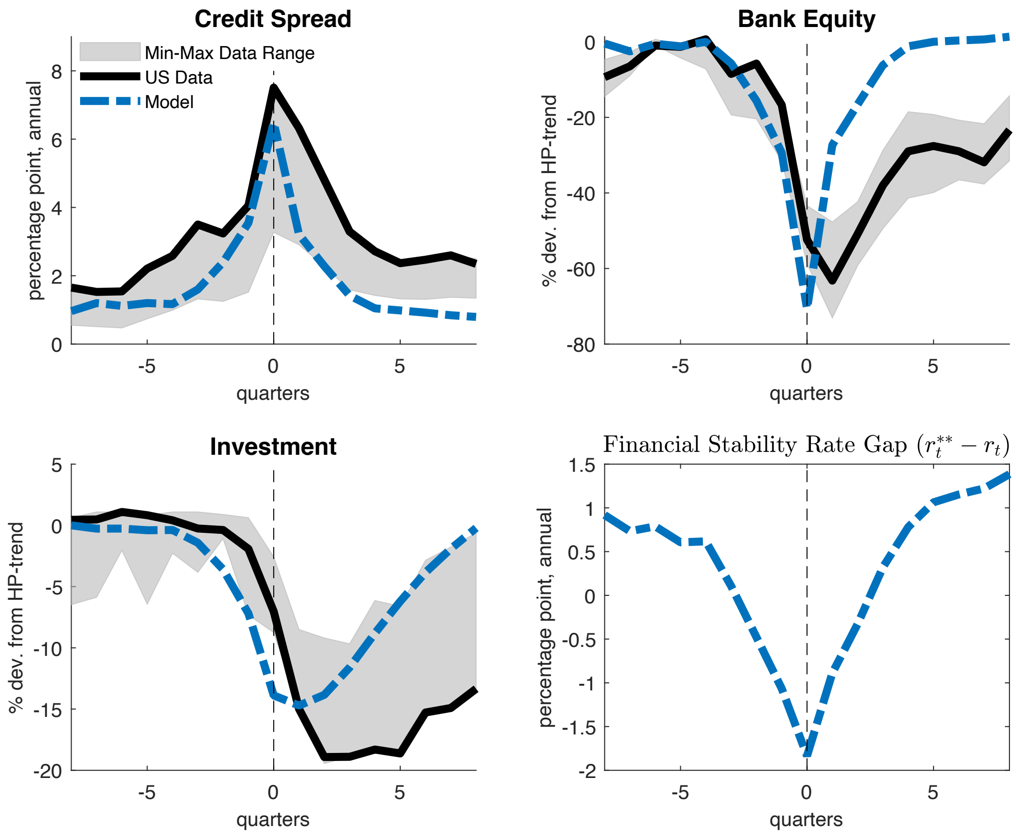


Note: The upper (lower) left (right) panel shows the relationship between year-ahead real GDP, expressed as a deviation from its HP trend, and the negative (positive) deviations of the credit spread from its mean in the model (data). The lower panel shows Data sources: Haver Analytics, Gilchrist and Mojon (2014), Bank of England, Gilchrist and Zakrajsek (2012), authors' calculations.

can broadly reproduce quantitatively realistic crisis dynamics, particularly the size of the increase in spreads. In the quarters leading up to the crisis, bank equity (first panel, right column) deteriorates sharply. These equity losses eventually put banks up against their borrowing constraints, leading credit spreads (first panel, left column) to jump significantly: the spread increases from just below 2 percentage points annually to about 6 percentage points in only two quarters. Along the way, with a binding constraint, the financial accelerator mechanism operates, with the drops in net worth, investment, and asset prices reinforcing each other. All told, investment at the trough is about 15 percent below trend in the simulation, close to the average drop in investment in the data.

The lower right panel of Figure 7 shows the behavior of the financial stability real rate gap, $r^{**} - r$. In the period preceding or following the crisis, when the constraint is not binding, r-gap is fairly constant. As soon as the crisis hits and the constraint becomes binding, r-gap drops suddenly. This behavior of the financial stability interest rate gap is qualitatively consistent with the evidence

Figure 7: Average Financial Crisis: Model versus Data



Note: A financial crisis event in the model is defined as an event in which banks' constraint binds for at least four consecutive quarters and the spike in the credit spread is at least one-and-three-quarters standard deviations above average. We simulate the economy for 10,000 periods and compute averages across identified financial crisis events. Dashed-dotted blue lines show the dynamics of macro aggregates surrounding the identified financial crisis episodes in the model.

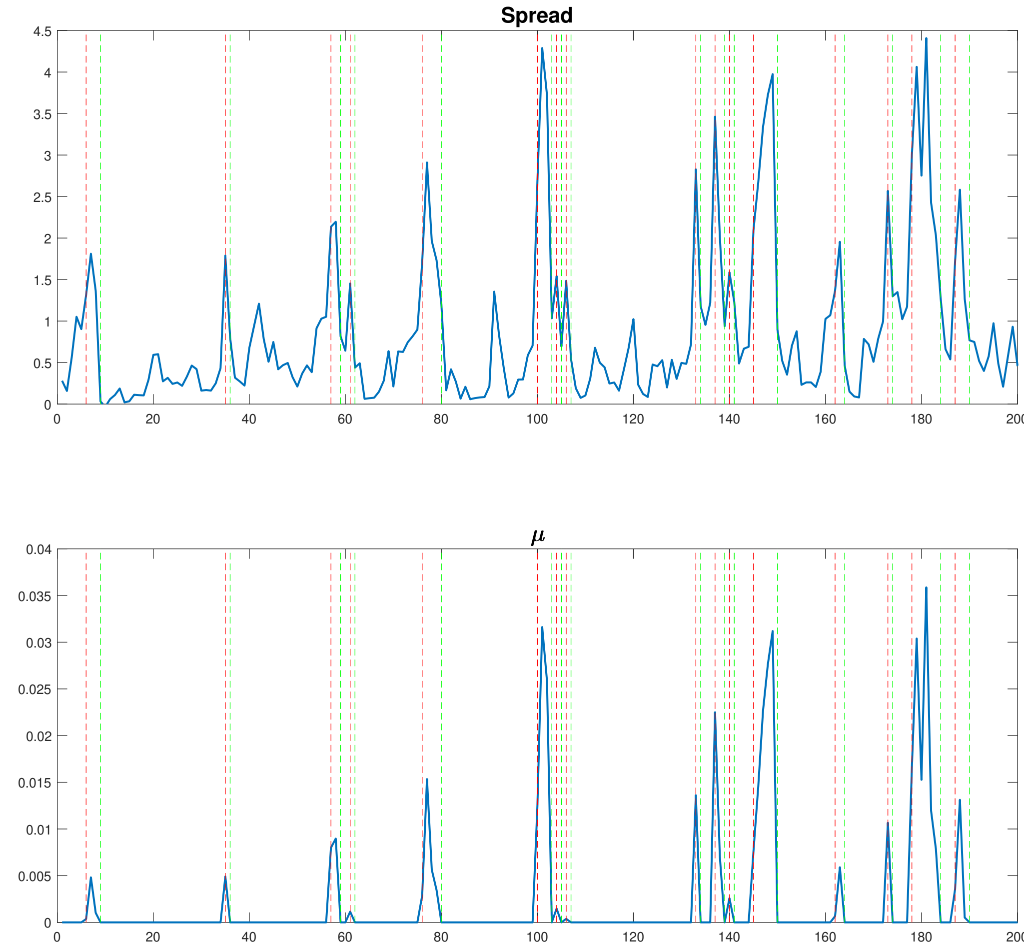
that central banks tend to cut interest rates quite rapidly at the onset of financial crises.

5 Measuring r**

The previous sections defined r** and discussed its properties. This section provides a measure r** for the US economy and discusses its evolution over the past 50 years.

Within the context of the model of course measuring r** is straightforward: there is a mapping between the model's state variables (debt and quality adjusted assets) and the financial instability real interest rate on which we elaborate in the previous sections. In principle one could measure these very same variables in the data and use the same mapping to derive r**. In practice this

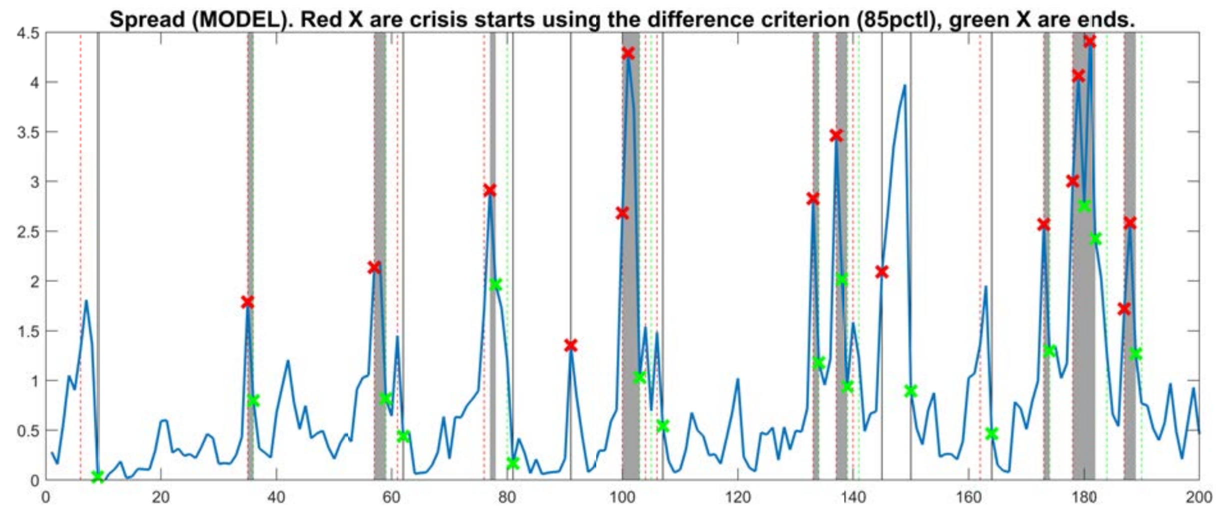
Figure 8: Spreads and financial constraints in the model



approach may not be very promising however, because it is hard to construct empirical counterparts for these state variables.

A possibly more promising avenue which we pursue here is to identify a variable that is easy to measure in the data and that is tightly associated with r**. In the reminder of this section we argue that this variable is credit spreads. Given the model's nonlinearities however, the relationship between any observable, and specifically spreads, and r** is likely to be different depending on whether the constraint is binding. For this reason it is important to first identify periods where the economy is likely to be under financial stress, in the sense that the intermediaries' leverage constraints are binding. As it turns out, credit spreads are very helpful for this task as well. In the first part of the section we will argue that the *volatilty* of spreads helps identify episodes of financial stress. In the second part of the section, we show that the *level* of spreads is tightly associated with r**, and more precisely with the gap between r** and the real rate r, especially during episodes

Figure 9: Credit Spreads and Financial Stress Episodes, Model



of financial stress. Finally, we will present estimates of r** in the data and discuss some specific historical episodes, such as the Great Recession.

5.1 Identifying Financial Stress Episodes

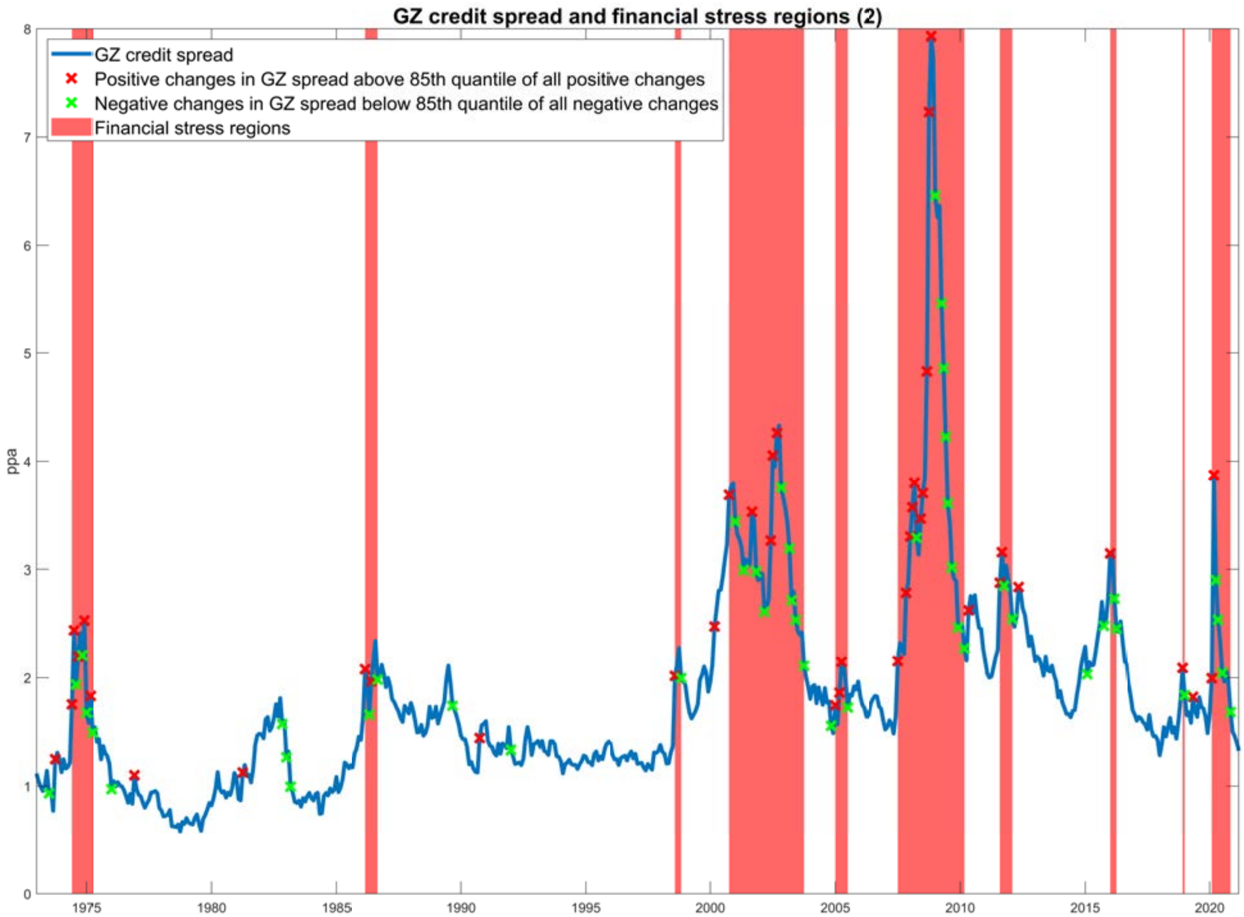
As discussed in the section 2 there is a tight relationship in this non linear model between credit spreads and financial constraints. When financial constraints are not binding, intermediaries can arbitrage between riskless and risky assets, thereby keeping spreads very tight.[3] When the constraint is binding however this arbitrage is neither possible—because of the binding constraint—nor desirable as risky assets become a very poor hedge for intermediaries, so spreads open up. Moreover, as discussed before, financial accelerator dynamic kick in this non linear model, with the result that the economy becomes very sensitive to shocks and spreads turn very volatile.

Figure 8 illustrates these dynamics using data simulated from the model. The top panel displays credit spreads and the bottom panel shows the value of the lagrange multiplier on the leverage constraint μ. The red and green dashed lines mark the beginning and the end of a financial stress episode, respectively. The figure shows that whenever μ is positive spreads are volatile, with the beginning of an episode generally characterized by a large increase, and the end by a decrease.

Of course μ is not directly observable in the data. We therefore want to construct a heuristic rule for identifying financial stress episodes that works correctly the model and that can be applied to the data. The above observations lead us to construct such a rule as follows. Call "spread jumps" changes in spread $\Delta spread_t$ that are above some quantile q of the distribution, i.e., $|\Delta spread_t| > q$.

[3]In the model we are missing features inducing liquidity or default premia that would keep spreads positive even when financing constraints are not biding.

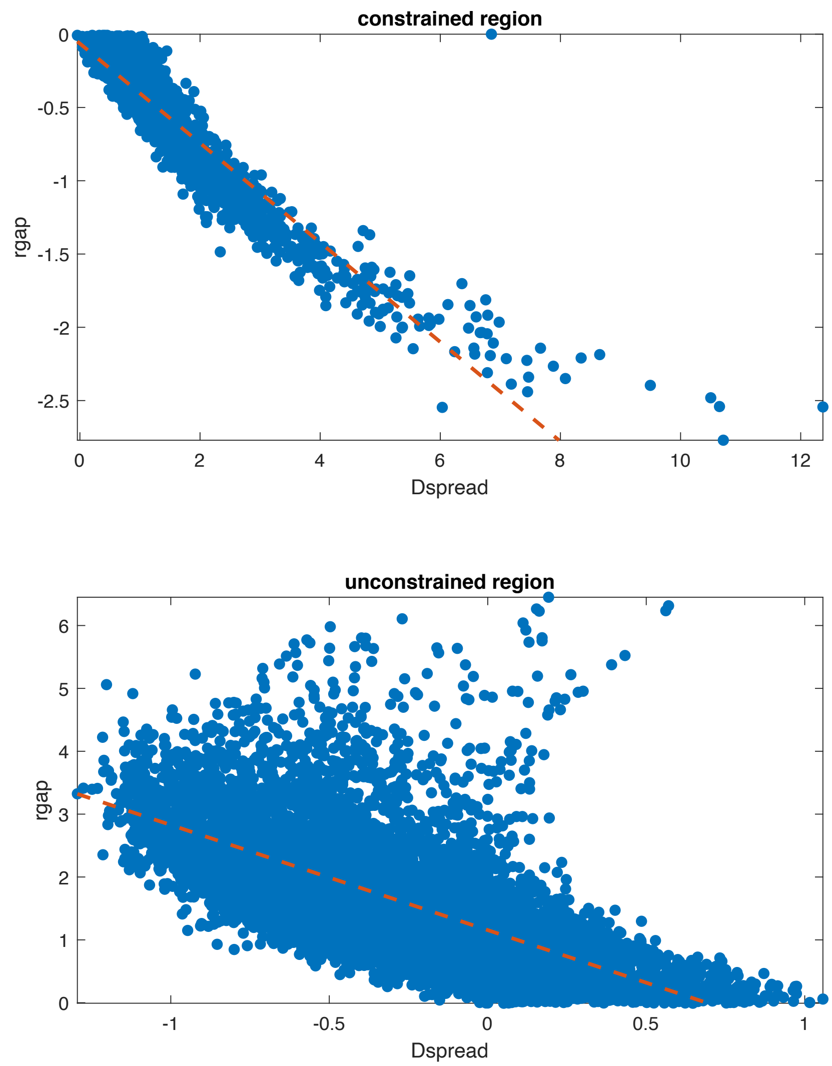
Figure 10: Credit Spreads and Financial Stress Episodes, Data



We then define a financial stress region as a sequence of jumps no more than two quarters/six months apart, beginning with an upward jump and ending with a downward jump. The requirement that jumps are no more than two quarters apart is dictated by the desire to avoid including in our definition non constrained regions in which spreads are less volatile. One can think of this heuristic approach as an alternative to estimating a regime switching model where the spread data is divided into high and low volatility regions.

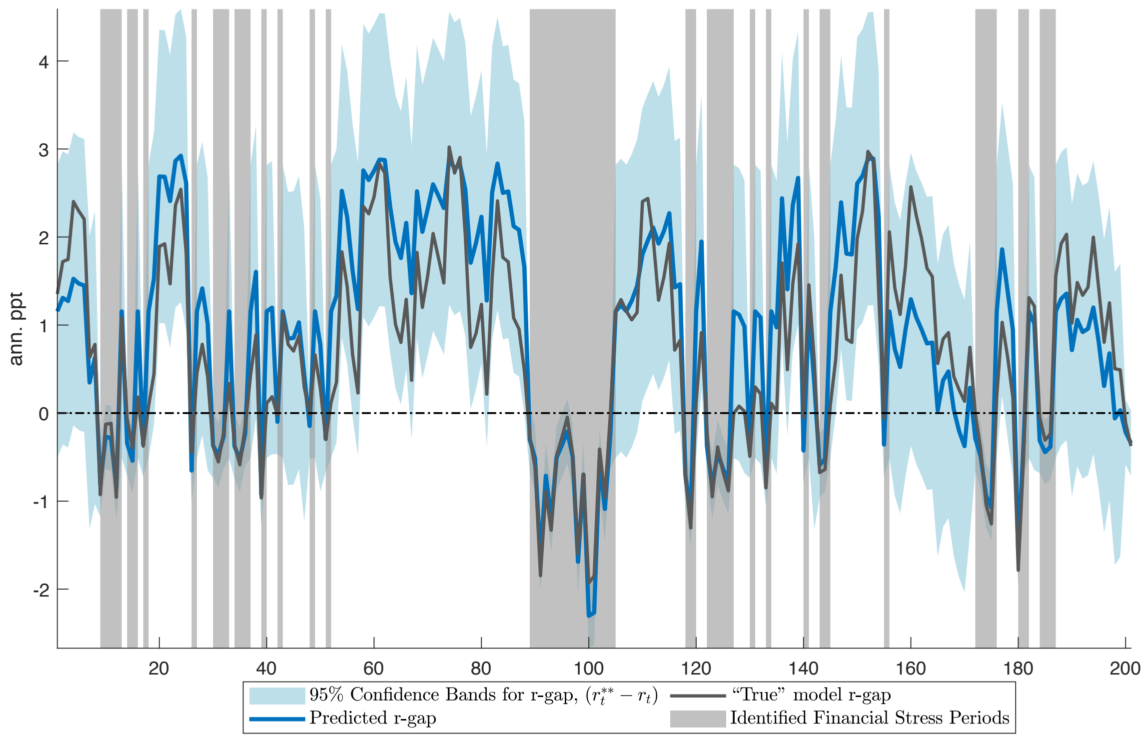
Figure 9 shows how well the rule works in the model: the shaded areas are the financial stress episodes as identified by the rule when we pick $q = q_{85}$, that is, the 85^{th} quantile of the distribution. As before, the red and green dashed lines mark the beginning and the end of true financial stress episodes. The red and green crosses mark spread jumps satisfying $|\Delta spread_t| > q_{85}$ (red are positive, green are negative). The figure shows that the episodes defined as crisis by the heuristic rule are indeed $\mu > 0$ periods, that is, the rule entails no false positives (we verified that this is the case in

Figure 11: Spreads and and r**-r in the model



a much longer simulation). For instance, the solitary jump in spread around period 90 is correctly not recognized as the start of a financial stress episode. However, the rule entails quite a few false negatives, that is, stress episodes that are not recognized as such. This occurs both because the initial rise in spread is not large enough to satisfy the $|\Delta spread_t| > q_{85}$ requirement (eg, see the episode around t=5 in Figure 9) or more often because more than two quarters pass between jumps

Figure 12: True vs Predicted r-gap, $r_t^{**} - r_t$, **Model**



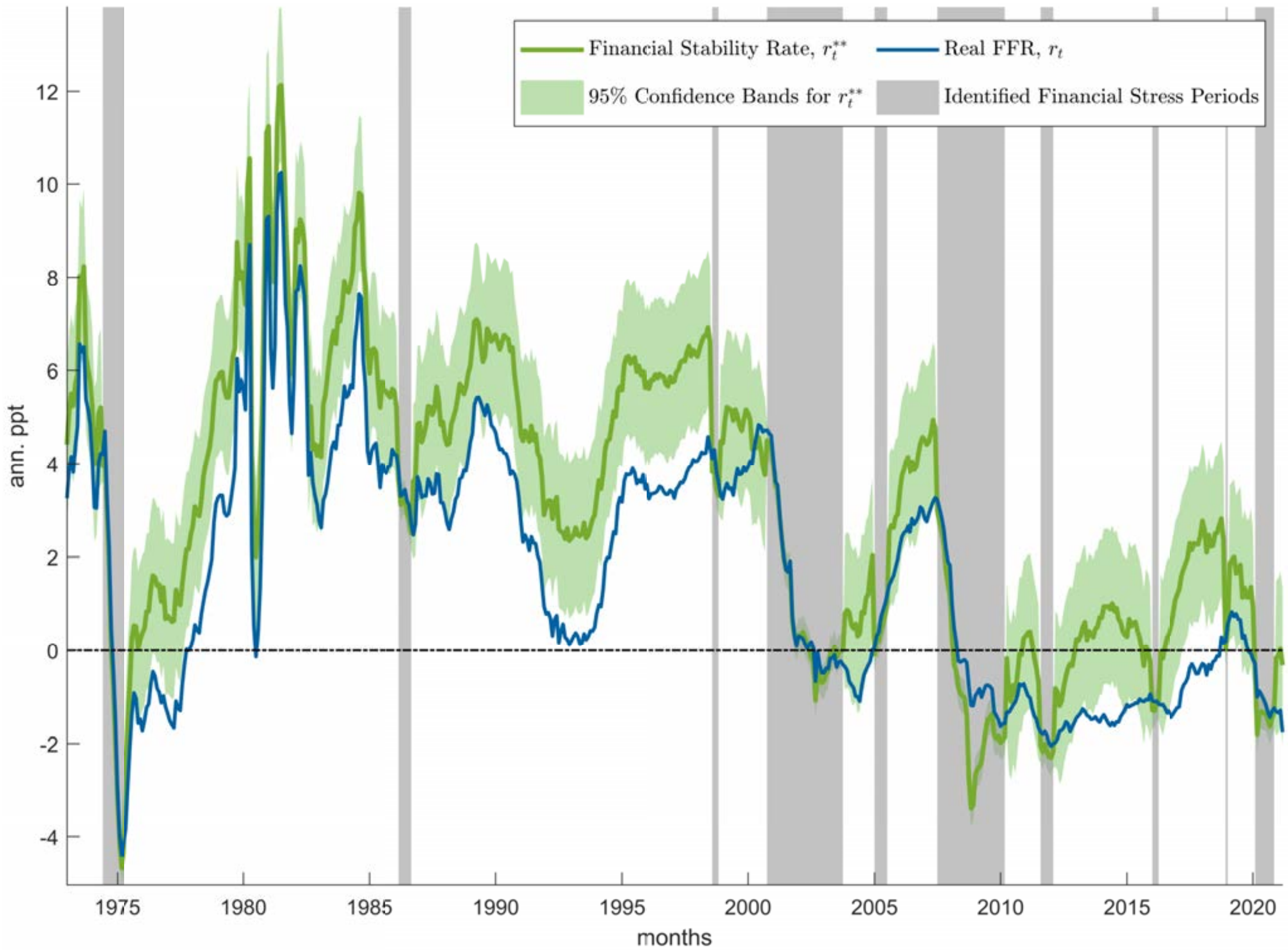
in spreads (eg, the stress episode around t=150 is not recognized as there are more than two quarters that do not satisfy the $|\Delta spread_t| > q_{85}$ requirement in between the beginning and the end of the crisis). If course, lowering the required quantile to, say, q_{80} would reduce false negatives but would also introduce some false positives. We want to avoid doing so for reasons discussed in the next section.

Figure 10 shows the result of the heuristic rule when applied to Gilchrist and Zakrajsek (2012)'s GZ spread in the period for which this spread is available. The shaded areas are all arguably periods associated with some degree of financial stress, from the LTCM crisis in the late 1990s to the period following 9/11/2001 to the Great Recession and its aftermath.

5.2 Credit Spreads and r**

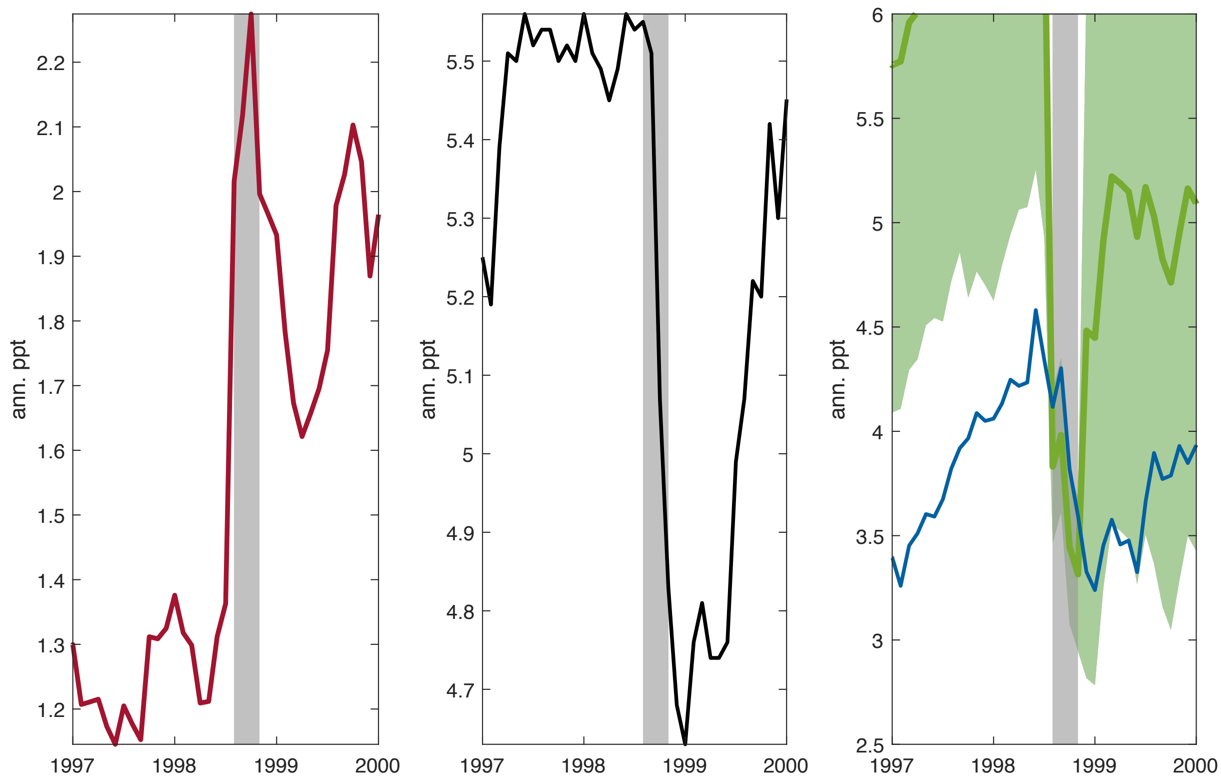
Figure 8 shows that the level of spreads is very correlated with the leverage multiplier μ *when the constraint is binding*, but of course not correlated at all (since $\mu = 0$) when the constraint does

Figure 13: Financial stability rate, r_t^{**}, vs real FFR, r_t, Data



not bind. In the constrained region, the level of the multiplier μ is likely to be correlated with the (negative) gap between r** and the current level of the real rate: how much r would need to fall

Figure 14: Episode 1: LTCM

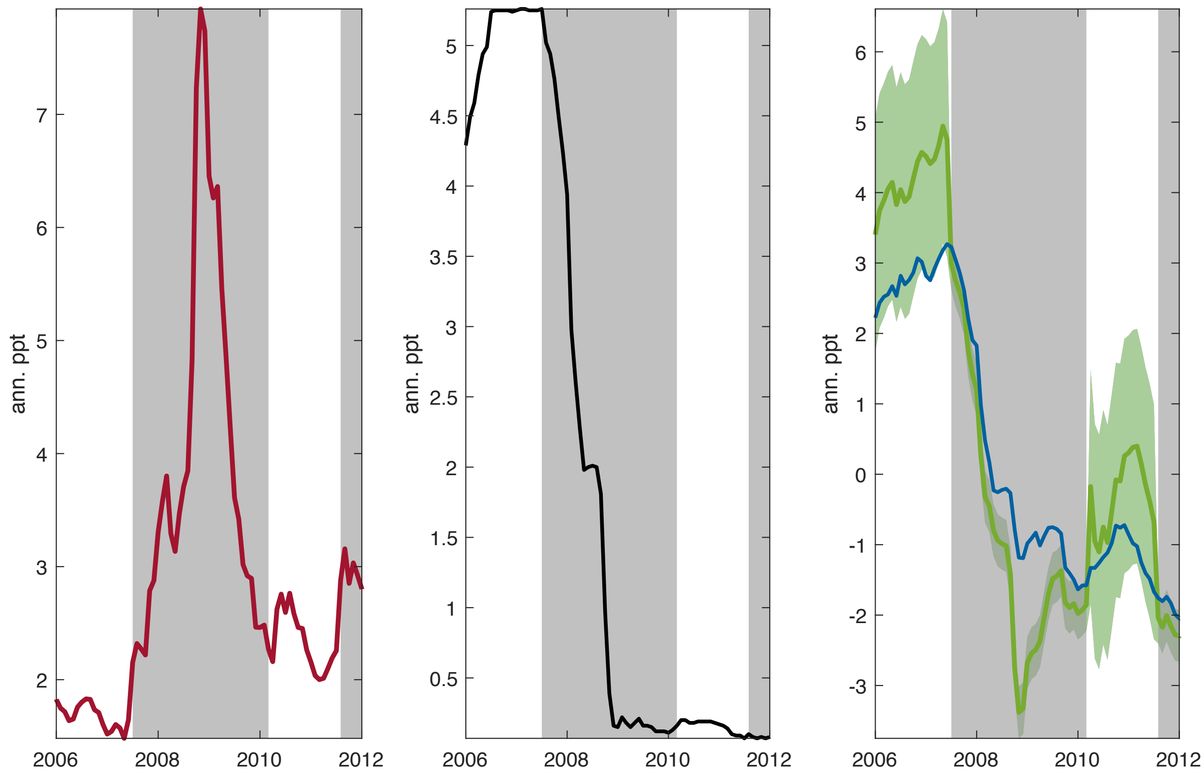


from its current level in order to improve the intermediaries' balance sheet (via its effect on asset prices) depends on how binding the constraint is. If the constraint is just binding, a small cut in the real rate may suffice. If the economy is in the throngs of a financial crisis, a larger cut in real rates may be needed to restore the health of the financial system.

These considerations lead us to run two separate regressions of the r**-r gap on the level of spreads, one for financial stress periods and one for "normal" periods. Ultimately, we want to use the estimated relationship on the data and map the observed level of spreads onto a measure for r**-r, and then use the level of the real rate to infer r** itself.

When considering US data, this approach runs into the following problem: if in the simple model we built spreads are stationary, there is ample evidence that in US data they are not (e.g., Del Negro et al., 2017). Looking at Figure 10 it is apparent that the peak of spreads during the LTCM crisis in the late 1990s amounts to a relatively low level of spreads in the 2010s, for instance. For this reason we amend the above strategy as follows: instead of using the level of spreads, we use (both in the model and in the data) the level of spreads relative to what they were in the period right before the economy first entered the current regime. To the extent that right before entering the constrained regime (or right after exiting the constrained regime) the constraint is "close to" being

Figure 15: Episode 2: Financial Crisis



binding, then in that period the gap between r** and r is close to zero, and therefore normalizing spreads using their initial level is harmless in the model, and beneficial in the data as it effectively removes the trend.

In sum, we run the separate regressions using model-generated data for the *financially constrained* regime ($\mu > 0$)

$$(r_t^{**} - r_t) = \alpha_c + \beta_c Dspread_t + \epsilon_t,\ Var(\epsilon_t) = \sigma_c^2;\quad \hat{\beta}_c = -0.34,\ \hat{\sigma}_c^2 = 0.034 \tag{24}$$

and *unconstrained* ($\mu = 0$) regime

$$(r_t^{**} - r_t) = \alpha_u + \beta_u Dspread_t + \epsilon_t,\ Var(\epsilon_t) = \sigma_u^2;\quad \hat{\beta}_u = -1.67,\ \hat{\sigma}_u^2 = 0.693 \tag{25}$$

where $Dspread_t = (spread_t - spread_{\underline{\tau}})$, with $\underline{\tau}$ being the period before/after the economy enters/exits a financial stress episode. The estimates coefficients are displayed next to each regression and the fitted regression line is shown in Figure 11 together with the simulated data.

The results of the regressions confirm the intuition outlined above. In periods of financial stress there is a very tight relationship between credit spreads and the tightness of financial constraints, implying that the r**-r gap is well predicted by the level of spreads. Conversely, in unconstrained periods the relationship is much looser, as indicated by the higher standard deviations of the regression errors. The slope is negative in both regions—when spreads are low/high, r** is well above/below r and hence the gap is high/low—but the slope is very different. It is much lower in the constrained region indicating that in this region an increase in the tightness of the constraint (and therefore a decrease in the r**-r gap) leads to large increases in credit spreads. Since such spreads are in the right hand side in the above regression, this translates into a low slope. Vice versa, in the unconstrained region the slope is more negative: when the economy is close to the constraint (that is, the r**-r gap is positive but close to zero), in this non linear model intermediaries incorporate the risk that the constraint may become binding in pricing assets, leading to an increase in spreads. But this increase is still relatively mild, so that when spreads are on the right hand side of the regression the slope is more negative.

Figure 12 shows how well the fitted regressions do in capturing the r**-r gap in the model. Specifically, the black solid lines display the true gap, the blue line is the fitted gap, and the shaded areas are the 95 percent coverage intervals implied by the estimated $\hat{\sigma}_c$ and $\hat{\sigma}_u$. Shaded gray areas identify financial stress episodes. The figure shows that at least in the model spreads work very well in essentially nailing the r**-r gap during the periods of financial stress. The distance between true and fitted values is generally small in these regions, and almost always within the relatively narrow bands. Outside of financial stress periods the fit becomes much poorer. This ignorance is reflected however in the wider bands, so that at least in those simulated data it is never the case that the true value of the gap falls outside the bands.

5.3 r** in the Data

The bottom line of the previous two sections is that the volatility of credit spreads helps to identify regions of financial distress, and that especially in these regions the level of spreads can quite accurately pin down the gap between r** and r, at least in the model. In this section we will make use of these results to provide an estimate of the time series of r** for US data over the past fifty years, and argue that this estimate is sensible. We will also show that the popular notion of a "Greenspan's put", namely that the central bank cuts rates whenever financial intermediaries become constrained, seems to be supported by the data: when financial constraints become binding and r** falls, the real rate soon follows it down so to close the gap between the two and ameliorate impact of the constraint.

The blue line in Figure 13 shows the real rate, as measured by the ex-post real federal funds rate. The green line shows the point estimate of r** implied by the regressions (24) and (25), with the green shaded areas being the 95 percent coverage intervals. Vertical shaded gray areas identify financial stress episodes as in Figure 10. By construction, r** is below r during periods of financial stress, and above it otherwise, although the uncertainty is often large enough that that the 95 intervals include r. Broadly speaking, it appears that during the first part of the Great Moderation period, in the mid to late 80s and the 90s, r** is significantly above r except for short-lived episodes of stress such as the LTCM crisis. In the 2000s and right after the Great Recession the gap between r** and r is close to zero, meaning that the constraints is close to being binding, even in periods that are not classified as financial stress episodes. In the mid to late 2010s r** is generally well above r, except again for a couple of very short-lived periods of stress, until the Covid pandemic hits the economy in March 2020. We also note that in most financial stress episode r** is rarely if ever significantly below r for extended periods of time, with the Great Recession being the only exception, when monetary policy was constrained by the zero lower bound.

The bird's eye view on r** afforded by Figure 13 makes it difficult to disentangle what happens during specific episodes. For this reasons in the reminder of the section we will zoom into two such episodes. The first, shown in Figure 14, is the LTCM financial stress period in the late 1990s. Because of the currency crisis in Russia and related turmoil in emerging markets in the summer of 1998, the hedge fund LTCM ran into liquidity and solvency problems and had to be bailed out. As LTCM had large trades with a number of important counterparties, the events of 1998 put the US financial system under considerable stress. The left panel of Figure 14 shows that credit spreads jumped by about 100 basis points within two months. The right panel shows that r** (green line) falls by about 75 bps from the beginning to the end of the financial stress episode. That is exactly by how much Greenspan cut interest rates during this period, thereby quelling the financial distress (middle panel). During the first part of the Great Recession (Figure 15) the story is quite similar. Spreads increase and, as a consequence, r** falls. Initially the real rate r follows r** downward, thereby closing the r**-r gap and limiting the effects of the financial turmoil. In mid 2008 the nominal rate hit the zero lower bound however, and as a consequence r could not fall any longer. When the Lehman crisis hit the economy, spreads increased further, r** fell and a persistent gap between r** and r opened until late 2009 and early 2010.

6 Conclusion

In this paper, we introduce the concept of financial stability real interest rate, r**. As a vehicle to illustrate our idea, we use a macroeconomic banking model based on Gertler and Kiyotaki (2010) where the banking sector faces a constraint in terms of a limit on the amount of funds that it can raise. When the constraint binds the economy experiences financial instability with increasing credit spreads, declining asset prices and contraction in economy activity.

We show that as the banking sector becomes more leveraged, the financial stability interest rate becomes lower. This has implications for monetary policy, in that even relatively low levels of the real interest rate could trigger financial instability.

Our analysis is conducted within a simple real model where the natural real interest rate is exogenously determined. In future work we plan to explore the interaction between macroeconomic stability and financial stability within a richer framework in which monetary policy is endogenously specified.

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Appendix: Model State Variables

Let $\overline{K}_t \equiv e^{\psi_t} K_{t-1}$ denote the effective amount of physical capital at the beginning of period t (after the capital quality shock is realized), and define $\overline{B}_{t-1} \equiv R_{t-1} B_{t-1}$ to be the stock of external debt plus interest. Let also $\overline{N}_{t-1}$ refer to the predetermined part of aggregate net worth (i.e., the component of net worth that does not depend on time-t variables like Q_t), given by the following:

$$\overline{N}_{t-1} = \sigma \left[x_{t-1} N_{t-1} + R_{t-1} \left(\underbrace{N_{t-1} - Q_{t-1} K_{t-1}}_{=-D_{t-1}} \right) \right] + (1-\sigma)\xi Q_{t-1} K_{t-1}$$

Note that $\overline{N}_{t-1}$ is equal to aggregate new equity issued by surviving banks ($\sigma x_{t-1} N_{t-1}$), plus startup transfers to entering banks ($(1-\sigma)\xi Q_{t-1} K_{t-1}$), minus the total stock of debt (with interest) carried over by surviving banks ($\sigma R_{t-1} D_{t-1}$). Given our calibration the latter term will always be large relative to the first two, so that $\overline{N}_{t-1} < 0$.

Given these definitions, let $\mathcal{S}_t$ denote the model's aggregate state vector, given by seven variables:

$$\mathcal{S}_t \equiv \left\{ \overline{K}_t, -\overline{N}_{t-1}, \overline{B}_{t-1}, R_t \right\}$$

We use the negative of $\overline{N}_{t-1}$ so that $\mathcal{S}_t > 0$. Our solution method relies on using parametric functions to approximate the model's one-step-ahead expectations (see Akinci and Queralto (forthcoming), Judd, Maliar and Maliar (2011) and Den Haan (2007) for details of our solution method).