# Financial innovations in a world with limited commitment

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**Abstract**

We examine an economy with a limited commitment friction where fiat money, bank deposits and short-term and long-term nominal government bonds coexist. In such economy, we examine the effects of financial development. Our focus is on the way in which trades are executed; specifically, the means of payment consists of either cash or claims against deposit accounts. The limited commitment friction means that bankers can abscond with a portion of the collateral accumulated to back deposit claims. So, we consider two types of financial developments; one permits a larger fraction of trades to use deposit claims while the other reduces the fraction of collateral with which the bank could abscond. Interestingly, we find that when a larger fraction of trades use deposit claims, the welfare and inequality results are indeterminate. The indeterminacy owes to countervailing forces that exist between the extensive margin and the intensive margin. More buyers can obtain the higher returns that occur when using deposit claims, but without an injection of collateral, the quantity consumed in such trades declines. Reducing the ability to abscond collateral results in higher expected welfare, but also means inequality increases. Lastly, we consider a model in which the fraction of sellers in connected matches is endogenized. This allows us to examine the effects of changes in the distribution of costs that are important to the choice of participating in connected matches or not.

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**Keywords**: financial innovation, inequality .

## 1 Introduction

Innovations in the financial sector are generally regarded as good things. By lowering transactions costs, easing information problem, improving enforcement of contract and improving efficiency of payment system, there are improvements in identifying good borrowers, depositors have greater access to accounts paying interest, etc. that can result in improved outcome. Two questions arise when we look more closely at such perceived welfare gains that are realized as a result of financial innovation. First, there is the distribution of gains. With heterogeneity in the access to financial markets across consumers, those with better access to such markets are likely to benefit more. Maybe Wall St. vs. Main Street is partially based on the distribution of gains from financial innovations. A second issue arises when we consider economies with limited commitment.

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In this paper, we consider two different types of financial innovation. The first innovation we consider directly impacts the nature of how we pay for things. The analysis is largely motivated by changes in the payments made in cash relative to payments made with a deferred payment that includes transaction accounts. In other words, we compare between people who have access to the banking system and those who do not. Since 2009, the Federal Deposit Insurance Corporation has conducted a biennial survey, including a question on whether anyone on the household owns a checking or savings account. If no, the households is referred to as unbanked. Figure 1 reports the percentage of United States' households that are unbanked for the period 2009 through 2019. In general, there is a downward trend in the fraction of households without checking or savings accounts over the ten-years span. Does greater access to banking services unambiguously raise expected welfare?

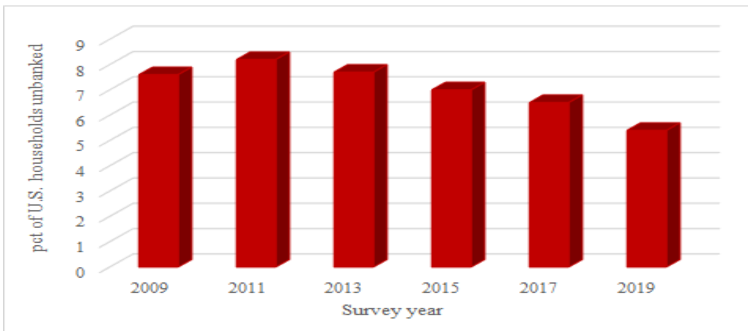


Figure 1: Welfare and Inequality for varying $\rho$

In addition, we consider an economy in which there is a financial innovation that reduces the haircuts on securities that serve as collateral. This kind of financial innovation could owe to improved information, or artificial intelligence that is better at calculating risks. Over the years, we have experienced significant decline in the cost of acquiring information and increased pledgeability of collateral as a result, especially the long-term collateral. Such innovation leads to an improved value of collateral and hence, consumers' ability to buy goods against those collateral.

Our chief contribution is that we examine how these changes in the financial innovations interact with the nature of how we pay for things. To delve a little deeper, one means of payment–bank deposits–are subject to a limited commitment problem. Banks hold collateral in order to back deposit claims. But with limited commitment, the quality of collateral matters. In a given period, there are two types of buyers, those with access to the deposit claims and those who must pay for things with cash. Those buying with cash are technically accessing the bank, but must withdraw so that when the transactions occur, they are equivalent to being unbanked. Within this framework, we study the effect of the two financial innovations on consumption in money- and deposit-backed trades, expected welfare and inequality.

In addition to the basic model economy, we extend our framework by allowing sellers to share some of the total surplus and to face a distribution of idiosyncratic cost shocks. Together, these forces allow us to characterize a seller's decision problem in which they choose to pay the realized cost shock and observe the DM buyers deposit account. In short, we examine an economy in which there is an endogenous payment-system margin. We show that there exists a threshold level of the cost shock such that realization of cost less than the threshold level results in the seller choosing the technology to observe the DM buyer's deposits.

Our results are easily summarized. We study economies in with financial development occurs through innovations that change the payment landscape. In particular, consider the case in which a larger fraction of sellers can observe deposit accounts. This corresponds to a larger measure of buyers using deposit claims to make purchases. In this financial innovation, both an intensive and extensive margin are operating. Along the extensive margin, more buyers have access to the preferred method of payment. However, the extensive margin adds pressure to a banking system that has a limited supply of collateral. Indeed, it is through the collateral shortage that we observe the intensive margin; same collateral spread across more connected buyers results in less consumption per trade. The two countervailing forces result in the overall effect being indeterminate. Our numerical results show that the intensive margin dominates the extensive margin, resulting in financial innovation that lowers expected welfare. When the intensive margin dominates, consumption inequality declines as the gap shrinks between buyers in connected matches and those in unconnected matches.

With the endogenous payment-system margin, the financial innovation is characterized as a change in the distribution of cost that places greater probability mass over low-cost shocks. For cases in which the extensive margin is small enough, the impact of this financial innovation is qualitatively the same as in the exogenous change in payment-system margin. Indeed, the key difference is that intensive-margin dominance is no longer sufficient to result in a decrease in consumption in connected matches. Rather, the intensive margin impact must be greater than the marginal impact of the change in total surplus in connected matches on the threshold level of the cost shock. The financial innovation is more likely to cause a reduction in consumption inequality when the distribution changes. In addition, an increase in pledgeability is more likely to result in an increase in consumption, but actually dampens the gain because a larger measure of sellers choose connected matches.

We additionally find that there exists a fiscal policy action that can undo the expected welfare loss. In general, an increase in lump-sum taxes improves expected welfare. Specifically, there exists a lump-sum tax increase that offsets the effects of the intensive margin that is present when there is an increase in the measure of matches with connected sellers. With an increase in lump-sum taxes, for example, the outstanding stock of government debt increases. In other words, lump-sum tax increase can add collateral to the economy so that consumption in these connected matches is held constant. Expected welfare increases when the fiscal policy is implemented simultaneously with the financial innovation. Compared with the innovation alone, inequality increases. With a greater measure of connected matches, the Gini coefficient is greater with the fiscal policy accommodation relative to financial innovation alone. The relationship is non-monotonic, however. With the measure of connected matches equal to one, inequality vanishes as each DM buyer purchases the same quantity. It is also true that inequality vanishes when the measure of connected matches goes to zero. But the measure of connected matches equal to one will Pareto dominate the case in which the measure equals zero.

In our setup, the financial innovation that allows a larger measure of sellers to access bank deposit information can be deleterious to expected welfare because of the role that collateral plays in the economy. With limited commitment, a fixed quantity of collateral must be shared between those using deposit claims to pay in those connected matches. It is not a liquidity problem, it is a collateral problem. Consequently, a fiscal policy that increases collateral will effectively accommodate the deleterious effects of the financial innovation.

The second financial development applies to the bank's ability to abscond. Put another way, suppose that long-term debt becomes more pledgeable. In this financial innovation, the results are determinate. Here, only the intensive margin is at work. Buyers able to use deposit claims are able to consume larger quantities. As a result, expected welfare increases and inequality increases.

## 1.1 Related Literature

Researchers have devoted considerable effort to understanding how financial development affects welfare. At risk of oversimplifying this rich literature, financial development reduces a friction that results in higher returns. The frictions could be information frictions like those in Boyd and Prescott (1986), Allen (1990), Ramakrishnan and Thakor (1984) among others, or traditional risk pooling as in Townsend (1983), King and Levine (1993), Acemoglu et al. (2006), Allen and Gale (1997), for example.

Greenwood and Smith (1997) study the connection between exchange, specialization and innovation. Since transactions are costly, financial innovation that reduces transaction cost related to exchange promotes specialization. The study however, does not explain the financial instruments or institutions that lowers transaction cost.

Greenwood and Jovanovic (1990) study how financial development can affect income inequality. Building on the Kuznets Curve (Kuznets (1955)), the authors endogenize organizational capital, focusing on how its buildup can reduce information costs. With the investment in the financial superstructure, entrepreneurs are better informed with respect to capital projects, idiosyncratic risk can be better diversified and consumers can better smooth consumption.

Another literature has focused on how financial innovations affect welfare and other economic outcomes in economies with incomplete markets. In **?**, the effects of financial innovations are examined in economies with incomplete markets. The main finding s that welfare generally declines in response to financial innovations when there is an incomplete set of markets in the economy. In a dynamical setting, **?** report that financial innovations can destablize an economy; specifically, adding an additional Arrow-Debreu security to economy results in greater volatility in economic activity over time. The point of this literature is to show that there are ambiguous welfare implications associated with financial innovations in economies with existing market failures.

In this paper, we focus on how the financial intermediary serves depositors. In the same spirit as Diamond and Dybvig (1983) and **?**, there exist idiosyncratic risk that a bank can diversify by offering state-contingent deposit contracts. Following Williamson (2016), we examine an economy in which financial development occurs with a limited commitment friction. Our focus is on the way in which trades are executed; specifically, the means of payment consists of either cash or claims against deposit accounts. The limited commitment friction means that bankers can abscond with a portion of the collateral accumulated to back deposit claims. So, we consider two types of financial developments; one permits a larger fraction of trades to use deposit claims while the other reduces the fraction of collateral with which the banks could abscond.

In this paper, the lack of commitment allows for hidden action. For bankers, the inefficiency owes to the quality of collateral available. With haircuts, absconding is an option. What our results indicate is that there are financial innovations that stress the collateral backing certain kinds of transactions. What is driving our results, therefore, is not a liquidity shortage in the payment system, but a collateral shortage. Innovations that create additional collateral are welfare improving because they reduce the value of the hidden action. Innovations that reduce collateral per buyer will adversely affect welfare, but can be offset by a policy action that increases the quantity of collateral.

The paper develops the model in Section 2. We derive optimal decision in Section 3. In Section 4, we characterize the equilibrium. The two types of financial innovations are examined in Sections 5 and 6. A brief summary is presented in Section 7.

## 2 Economic Environment

Our model builds on the frictional and incomplete-market framework of Williamson (2016) and analyzed in Dhital et al. (2021). [1]

There are an infinite number of discrete time periods and indexed by $t = 0, 1, 2, ....$ Each period is divided into three parts; hereafter, morning, afternoon, and night. Agents trade sequentially in various markets that are characterized by different frictions and trading protocols. In the morning, a specialized good is produced and consumed. For this specialized good, trades are executed through bilateral matches. The morning market is called the DM market. In the afternoon, there is a frictionless, competitive market where production, consumption and saving occurs. This market is called the CM market. In the evening, there is a realization of an idiosyncratic random shock that identifies the type of trade in the next period's DM.

The economy is populated by three types of infinitely-lived agents: sellers, buyers, bankers. Sellers are identified by the fact that they uniquely possess the technology to produce the DM good. There is a measure one of Sellers are permanently identified by one additional characteristic; specifically, sellers are either endowed with a technology that allows them to (costlessly) confirm a buyer's deposit holdings in the bank or not. Let $0 < \rho < 1$ be the measure of sellers not endowed with the bank-confirmation technology, hereafter referred to as unconnected sellers, and $1 - \rho$ be the measure of sellers endowed with the bank-confirmation technology, hereafter referred to as connected sellers. Because connected sellers can observe bank deposits, they are willing to accept claims against those deposits as payment in the DM. DM sellers can redeem the deposit claims for a specified quantity of goods in the CM. Without the bank-confirmation technology, sellers will not extend unsecured credit, and hence, only currency is accepted.

Buyers are so named because they uniquely derive utility from consuming the DM good. There is a measure one of buyers. In the evening sub-period, the buyer learns which type of seller they will meet in the next morning's DM. The shock is independently and identically distributed across time. Accordingly, it follows that by the law of large numbers, $\rho$ is the likelihood that a buyer will be matched with a seller who cannot confirm bank deposits and $1 - \rho$ is the probability that a buyer will be matched with a seller endowed with the bank-confirmation technology.

Bankers operate in the CM like buyers and sellers. Specifically, bankers can produce and consume the CM good. In addition, bankers can accept deposits, acquire money balances and electronically record purchases of short- and long-term government bonds. Each buyer can be matched with at most one bank. However, a

[1] The interested reader is directed to Dhital et al. (2021) for a thorough description of the economic environment.

banker can match with any number of buyers. The bankers have access to a costless record-keeping technology that allows them to register the identity of agents.[2] In addition to their activities in the afternoon sub-period, a banker passively and costlessly operates an ATM machine during the evening sub-period that allows depositors to withdraw currency. The deposit contract specifies the return given to depositors depending on when the withdrawal is made. At date $t$, deposits made in the afternoon sub-period can be withdrawn during the date-$t$ evening sub-period.

There is also a government operating with two branches, a central bank and a treasury. The government issues three fundamental assets that are liabilities of the government; currenct, short-term and long-term government bonds. The government bonds are electronic book-entries in the governments record. These records are only available in afternoon and evening sub-periods. [3]

The government collects lump-sum taxes and issues quantities of short- and long-term government bonds through its treasury. Meanwhile, the central bank sets the path for the quantity of fiat money. Together, these decisions constitute the revenues collected by the government. Revenues are used to finance lump-sum transfers. Only buyers pay the lump-sum tax and receive the lump-sum transfer. Let $M_0, B_0^s$ and $B_0^l$ represent the initial nominal quantities of aggregate fiat money, aggregate short-term bonds and aggregate long-term bonds. Throughout this analysis, short-term bonds are one-period pure discount obligation that promise to pay one unit of money in the following period. Long-term bonds are consols that pay one unit of currency in perpetuity. All principal and interest payments are made in the afternoon. Let $z_t^s(z_t^l)$ be the date-$t$ nominal price of the short-term (long-term) government bonds. The government budget constraint is then given by

$$\phi_t\Big(M_t - M_{t-1} + z_t^s B_t^s - B_{t-1}^s + z_t^l B_t^l - (z_t^l + 1)B_{t-1}^l\Big) - \tau_t = 0 \quad (1)$$

where $z_t^s$ ($z_t^l$) represents the price of short (long) term bonds at time $t$. Implicit in this formulation is the assumption that the government has no assets or liabilities at the beginning of period $0$.

The final aspect of the physical environment is limited commitment. Anyone holding assets can abscond with those assets. In this paper, the extent to which the asset is pledgeable differs. Currency is perfectly pledgeable. However, both short- and long-term government bonds are less pledgeable than currency. Here, we also assume that long-term bonds are less pledgeable than the short-term bonds. Let $\theta_l > \theta_s$ denote the fraction of long-term and short-term government bonds respectively that can be absconded. In other words, $1 - \theta_s > 1 - \theta_l$ represents the fraction of the bond that can be pledged as collateral.

**Preferences:** All private agents discount the future at a rate $\beta \in (0, 1)$. As in Lagos and Wright (2005) and Rocheteau and Wright (2005), each DM buyer derives utility from consuming the DM perishable good and obtains disutility from CM effort. An individual buyer has preferences is given by

[2]Similar technologies are found in Berentsen et al. (2007), Williamson (2012), Williamson (2016), Aït Lahcen and Gomis-Porqueras (2019) among others.

[3]The reference to electronic recording of government bonds is another way of saying that government bonds are not physical objects that can be carried back and forth. The electronic record will play an important role in the physical description of a connected match. This approach is also utilized in Berentsen and Waller (2011), Martín (2011), Domínguez and Gomis-Porqueras (2019) and Carli and Gomis-Porqueras (2021) among others

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{Q_t^{1-\sigma}}{1-\sigma} - H_t \right] \tag{2}$$

where $Q$ stands for the quantity consumed by DM buyers such that $Q_t \in \{q_t, q_t^u\}$ where $q_t$ is consumption by DM buyers in connected trades and $q_t^u$ is in unconnected trades, $H_t$ denotes effort exerted in CM, and $E_0$ is the expectation operator. Sellers, on the other hand, derive utility from consuming the CM perishable good and obtain disutility from DM effort. Throughout our analysis, we assume $0 < \sigma < 1$. Their preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[ -h_t + X_t^p \right]$$

where $h_t$ denotes DM effort and $X_t^p$ represents consumption of the CM good. We use the superscript $p$ to identify sellers.

Bankers derive payoffs from CM consumption and effort. Their preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[ X_t^b - H_t^b \right].$$

where $X_t^b$ represents consumption of the CM good by bankers and $H_t^b$ denotes their CM effort.

**Saving and Deposit Contract:** Saving decisions are implemented through an asset market, which opens for agents to acquire money balances, purchases of short-term and long-term electronic government bonds, or deposit goods into a bank in the CM. In the CM of period $t$, buyers accumulate assets to finance their purchases in the ensuing DM $(t+1)$. Since bankers face limited commitment, depositors require that deposits be collateralized. Thus, once agents deposit CM goods, the bank buys fiat money, short-term bonds and long-term government bonds from financial markets and uses them as collateral.

Since agents who have deposited goods in the bank can still can withdraw currency in the evening after the shock is realized, buyers who deposited CM goods with a bank can partly undo some of their savings decisions. In particular, the one period deposit contract offered by banks are such that, after the realization of idiosyncratic trading shocks, buyers can withdraw some or all of their deposits through ATM in the evening. The deposit contract also allows buyers to keep some or all of their deposits with the bank for one period. Deposits left in the bank until maturity (one period) earn an interest in CM. Note that this type of deposit contract is quite valuable to buyers as they face uncertainty and they are risk averse.[4] As a result, buyers will deposit goods in the bank as they have the ability to withdraw if needed. This is preferable to carrying fiat money outright.[5]

In the morning, buyers and sellers are matched in the DM market. We assume that buyers make TIOLI offers. Sellers in connected trades will accept deposits or currency while sellers in unconnected trades will accept currency.

The afternoon consists of CM production by bankers and buyers. Here, our aim is to make the set of

[4]Thus, this type of bank deposit is then an option for buyers. As in Diamond and Dybvig (1983), the deposit contract provides insurance against the buyer's shock.

[5]Note that claims to public debt are not accepted by sellers as they cannot verify their ownership. This is the case as sellers do not have access to government records in DM. As a result, claims to buyers' bond holdings cannot be used to facilitate trade in DM.

decisions and actions more concrete. For the sake of illustration, suppose the treasury is running a primary deficit at date $t$ and the central bank is injecting new currency into the economy. Asset sales and tax collections by the treasury combined with principal payments on (discounted) short-term bonds and interest payments on long-term bonds. Currency injections by the central bank. For the narrative to hold together, we can think of the banker as operating a depository institution and a dry goods shop. Net production by buyers are deposited into the bank net of goods collected by the treasury. Sellers purchase the CM goods from bankers by presenting either currency and deposit claims. Thus, deposit claim settlement is accomplished. Any currency injections, for example, represent CM goods sold to the central bank, which are part of the government's revenues. The treasury uses goods to make principal payments on discounted short-term bonds and interest payments on long-term bonds. the bank uses its own production along with principal and interest payments to purchase newly issued short-term bonds and long-term bonds.[6]

Lastly, in the evening, buyers learn with whom they matched in the next period's morning DM. Those in unconnected matches will proceed to the ATM and withdraw currency.

Through the rest of the paper, we focus on monetary equilibria where the central bank does not implement the Friedman rule. Thus, positive nominal interest rates are observed in equilibrium; i.e, $\frac{\beta\phi_{t+1}}{\phi_t} < 1$. Finally, we consider two different policy regimes and different operating procedures for monetary and fiscal policies. In particular, we consider an active monetary policy regime and an active fiscal one. In addition, we study different operating procedures for monetary and fiscal policy, which will be described later on the paper.

For completeness, we assume that each banker is endowed with deposits at date $t = 0$. In this period, the deposit endowment will be applied to date-$t = 0$ morning DM trades with connected sellers. In the afternoon-subperiod, sellers, buyers and bankers have access to the CM-good production technology, which is linear in effort. Let $n_0$ denote the deposits and $\phi_0(B_0^s + B_0^l)$ denote the real value of government bonds held by the bank with $\phi_t$ being the price of money. Note also that bankers have access to a costless record-keeping technology that allows them to register the identity of agents.[7]

# 3 The Decision Problems

In this section, we provide a formal representation of the model economy and the decision problems. With a complete description of the economic environment, it is possible to summarize the decisions that each participant will face in each subperiod for an arbitrary date $t \geq 0$. For completeness, we assume that the economy begins in the afternoon of period $0$.

## 3.1 Households' Problem

Given the sequential nature of the environment, we solve agents' optimal decisions backwards. In what follows, we first analyze the decisions by buyers and sellers. Finally, we characterize the bankers' problem.

[6]Note that if currency is being destroyed, then the central bank is giving CM goods to the bank and collecting currency. While a primary surplus means the treasury is issuing fewer short- and long-term bonds than it is paying in principal and interest. Even though the assets are denominated in dollars, $\phi_t$ denotes the rate at which CM goods are exchange for dollar denominated assets.

[7]Similar technologies are found in Berentsen et al. (2007), Williamson (2012), Williamson (2016), Aït Lahcen and Gomis-Porqueras (2019) among others.

Note that depending on the shock buyers have experienced, these agents may have traded with a connected or unconnected seller. Throughout our analysis, the superscript $u$ denotes the state of the world where the buyer has traded with an unconnected seller. In contrast, the absence of a superscripts represents the state in which the buyer has traded with a connected seller.

## 3.2 Evening problem

After buyers have learned the realization of their idiosyncratic shock, they must decide whether to withdraw funds from the bank or not. With knowledge of their match, the buyer chooses

$$\max[V(c_t, d_t), V^u(c_t, d_t)] \tag{3}$$

where $V(c_t, d_t)$ and $V^u(c_t, d_t)$ represent the value function for a DM buyer who is entering the next DM and trades with a connected or unconnected DM seller with currency $c_t$ and deposit $d_t$, respectively. It is worth highlighting that a positive nominal interest rate is sufficient to induce DM buyers to withdraw all of their deposits in the form of currency in the evening when they are going to trade with unconnected DM sellers in the next DM. This is the case as fiat money is the only payment instrument unconnected sellers will accept. In contrast, in connected matches, claims to buyers' deposits (bank checks) can be offered as payment in the ensuing DM. This is the case as connected sellers can verify the buyer's deposit holdings with banks. Consequently, in a connected match, the seller receives the proceeds of the buyer's deposit contract in the following CM. Throughout the rest of the paper, we characterize economies where the return on bank deposits, $R_t$, dominates the return on fiat money. In such scenario there is no need for DM buyers to simultaneously hold both currency and deposits; i.e., $c_t = 0$ and $d_t > 0$. The buyer either holds deposits, denoted $d_t$, or withdraws early with currency, denoted $c_t^w$, depending on the realization of the shock.

### 3.2.1 Afternoon problem

In period $t$, depending on the shock that they have experienced, buyers enter CM with some combination of cash holdings ($c_{t-1}$), deposits net of claims ($d_{t-1} - n_{t-1}$) and claims to their bank deposits ($n_{t-1}$). These differ depending on the shock that DM buyers received in $t - 1$.

In this competitive market, buyers choose labor effort ($H_t$), currency holdings ($c_t$), public debt holdings of different maturities ($\mathcal{B}_t^s, \mathcal{B}_t^l$) and deposits ($d_t$). Buyers also pay lump sum taxes. To fund $t+1$ DM consumption, buyers must save. Fiat money, public debt of different maturities and deposits are the only savings instruments available to these private agents. Since the government bonds are book entries in the government's record, claims to public debt are not accepted by either unconnected or connected sellers. This is the case as sellers cannot verify the ownership of a buyer's bond holdings in DM. These government records are only available in CM. As a result, buyers do not directly hold any public debt, $\mathcal{B}_t^s = \mathcal{B}_t^l = 0$.

Given the timing of CM savings decisions and asset market, buyers must make the deposit decision (and hence, asset-holding decision) before the end of afternoon period (before the idiosyncratic shock is realized). To deal with the idiosyncratic risk, buyers can deposit goods into the bank. The state-contingent deposit contract offers the buyer the option of withdrawing "early" (in the evening of date-$t$) or "late" in the following date-$t + 1$ CM. For buyers withdraw early only if they are in unconnected matches. In connected matches, the check (claim to deposit) is offered as payment and the seller receives the proceeds of the deposit contract in the following CM subperiod.

In this paper, we study economies in which the state-contingent return (if held until maturity) on banks deposits, $R_t$, dominates fiat money. As a result, risk-averse buyers prefer the deposit contract offered by competitive banks, partly insuring them against the shocks they experience.

Let $V(c_t, d_t)$ and $V^u(c_t, d_t)$ again represent the value function for a buyer who is entering the next DM and trading with a connected or unconnected DM seller, respectively. The buyer either holds deposits, denoted $d_t$, or withdraw currency, denoted $c_t^w$, depending on the realized match state. We can then write the resulting date-$t$ CM value function (before a buyer realizes the trade shock) as

$$W(c_{t-1}, d_{t-1} - n_{t-1}, n_{t-1}) = -H_t + \beta \, [\, (1-\rho)\, V(0, d_t) + \, \rho \, V^u(c_t^w, 0)] \tag{4}$$

where $H_t$ is CM effort by DM buyers where $H_t = d_t - R_{t-1}(d_{t-1} - n_{t-1}) - \tau_t$, $\tau_t$ denotes CM taxes and $c_t^w$ is the currency withdrawn at the end of the period after the trade shock has been revealed and $d_t$ is the deposit held until next period. The buyer, thus, expends effort equal to the quantity of deposits carried over to the next CM less the return on net deposits; that is, the stock of date-$t-1$ deposits less the date-$t-1$ stock of claims against those deposits. Because currency withdrawn cannot exceed the quantity deposited, we have that $c_t^w \leq d_t$. It is important to emphasize that the decision on how much of the new deposits $d_t$ to withdraw is made after shocks are revealed. Given that banks face free entry and trade in a competitive market, the optimal deposit is found in the bank's problem.

Sellers neither face any uncertainty nor hold assets across periods. To pay for CM consumption, they can simply use the proceeds from their previous DM production. Thus, these agents do not carry fiat money across periods, nor will sellers deposit with private banks. As a result, we have that $c_t^p = d_t^p = 0$. Thus, the resulting date-$t$ CM value function of a connected seller is given by

$$W^p(c_{t-1}, d_{t-1} - n_{t-1}, n_{t-1}) = X_t^p + \beta \, V^p(0, 0) \tag{5}$$

where CM consumption is given by $X_t^p = R_{t-1} n_{t-1}$. Similarly, the date-$t$ CM value function of an unconnected seller is given by

$$W^p(c_{t-1}, d_{t-1} - n_{t-1}, n_{t-1}) = X_t^p + \beta \, V_p(0, 0) \tag{6}$$

where CM consumption is given by $X_t^p = \phi_t c_{t-1}$, where $c_{t-1}$ denotes the cash payment he receives for producing DM goods in the previous sub-period.[8]

### 3.2.2 Morning problem

In an unconnected match, the buyer makes a take it or leave it offer to the seller. The terms of trade specify the date-$t$ quantity of DM goods to be exchanged and the corresponding payment. Because the state of trade shock is known, the buyer withdraws all his deposits in the form of currency. This is the case as fiat money is costly to carry across periods and it is the only medium of exchange that will be accepted when trading with an

[8] We assume throughout that sellers do not pay lump-sum taxes.

unconnected seller. Recall that $c^w_{t-1}$ is the quantity of currency withdrawn from the bank by a buyer during the date $t-1$ CM for trades at date $t$ DM. Let $q^u_t$ denote the quantity of DM goods produced by the seller and $l^u_t$ denote the unit of currency paid by buyer in exchange for the DM goods. The optimal terms of trade at date $t$ DM when buyer is matched with an unconnected seller then solves the following problem.

$$V^u(c^w_{t-1},0)=\max_{q^u_t,l^u_t}\left\{\frac{(q^u)^{1-\sigma}}{1-\sigma}+W(c^w_{t-1}-l^u_t,0,0)\right\}\quad \text{s. t.} \tag{7}$$

$$-q^u_t+W^p(l^u_t,0,0)\geq W^p(0,0,0)$$

$$l^u_t\leq c^w_{t-1}$$

Note that the first constraint represents the seller's incentive compatibility constraint, which is required to induce DM production. The second one highlights the fact that the buyer cannot hand in more fiat money than what he has brought into the match. Finally, the third constraint takes into account the optimal withdraw decision of the buyer in the previous CM at $t-1$.

It is easy to show that the optimal terms of trade imply the following DM consumption schedule

$$q^u_t(m_{t-1})=\begin{cases} q^* & \text{if } \phi_t c^w_{t-1}\geq q^* \\ \frac{\phi_t}{\phi_{t-1}}m_{t-1} & \text{if } \phi_t m_{t-1}<q^* \end{cases} \tag{8}$$

where $m_{t-1}=\phi_{t-1}c^w_{t-1}$ is the real money balance in term of $t-1$ CM goods, $q^*$ is the efficient DM allocation, which is implicitly defined by $u'(q^*)=1$.

Next, consider the case in which the buyer is assigned to a connected match. In a connected match, the buyer can use his deposits to fund DM consumption. In particular, the buyer does not need to transfer his deposits to the seller. Instead, he can offer claims to them. Let $n_{t-1}$ denote the quantity of claims to the buyer's deposits, which the seller can obtain from the bank in the next CM market. Under a take it or leave offer made by the buyer, the optimal terms of trade at the beginning of period $t$ solves the following problem.

$$V(0,d_{t-1})=\max_{q_t,n_{t-1}}\left\{\frac{q_t^{1-\sigma}}{1-\sigma}+W(0,d_{t-1}-n_{t-1},n_{t-1})\right\} \tag{9}$$

$$\text{s.t} -q_t+W^p(0,0,n_{t-1})\geq W^p(0,0,0)$$

$$n_{t-1}\leq d_{t-1}$$

where the first constraint represents the seller's incentive compatibility constraint that induces production. The second one reflects the fact that the buyer may not offer more claims than his deposits.[9]

It is easy to show that the optimal terms of trade imply the following DM consumption schedule

$$q_t(n_{t-1})=\begin{cases} q^* & \text{if } R_{t-1}n_{t-1}\geq q^* \\ R_{t-1}n_{t-1} & \text{if } R_{t-1}n_{t-1}<q^*. \end{cases} \tag{10}$$

[9]Here, the notation suffers from the timing issue, but we will adopt it. Deposits are made in the date-$t-1$ CM subperiod. However, claims are issued to sellers in the date-$t$ DM subperiod. Yet, we adopt the notation with $n_{t-1}$. Our thinking is that claims are made against accumulated deposits. Consequently, we identify the claims as "originating" in date-$t-1$ rather than their implementation date. We use this notation through the analysis.

### 3.3 Bankers

Bankers operate in a free-entry and competitive environment and only trade in CM. Without loss of generality, given perfect competition among bankers, we consider the banking industry operating as a single bank.[10] While DM buyers can trade at most with one banker, a banker can contract with all DM buyers and DM sellers. As in Williamson (2012), banks maximize the expected utility of depositors, offering state-contingent deposit contracts subject to their balance sheet constraints. Since bankers face limited commitment, the incentive compatibility constraint has to be considered by depositors. As in Diamond and Dybvig (1983), a deposit contract provides insurance against DM buyer shocks.

Because of free entry and prefect competition, the deposit contract maximizes the expected utility of a representative DM buyer. The deposit contract specifies a state-contingent return, $R$, per unit of CM good deposited if held until maturity. Claims to deposit are redeemed by the bank in CM for $R$ units of CM goods per unit claim. As stated above, note that the deposit contract allows currency withdrawals before the end of the CM period. Thus, if a buyer deposits with a bank, after he receives his shock in the evening period, he can either withdraw currency or leave his deposit in the bank until the next CM, depending on the shock he has received. In particular, if matched with a connected seller in the ensuing DM, the buyer can use claims to his deposits to fund DM consumption. In contrast to Diamond and Dybvig (1983), bankers face limited commitment. As a result, when offering deposit contracts, buyers require that bank deposits (loans) be collateralized, as in Kiyotaki and Moore (1997), among others. Long-term government bonds are less pledgeable forms of collateral than short-term public debt; i.e, $(1-\theta_s) > (1-\theta_l)$.[11] Using the law of large numbers, the banker's objective function in period $t$ is given by

$$\mathcal{U}_t = -d_t + \beta \left[ \rho \, u\Big( q^u_{t+1}(m_t) \Big) + (1-\rho) \, u\big( q_{t+1}(n_t) \big) \right] \tag{11}$$

where $d_t$ represents the quantity of goods deposited by the representative depositor, which is funded by exerting effort in CM. $q^u_{t+1}(m_t)$ and $q_{t+1}(n_t)$ are the DM consumption in the two different states of the world, which are given by optimal terms of trade; i.e, (50) and (52), respectively.

Here, the bank accepts deposits and allocates the resources in order to maximize buyers expected utility. More specifically, the banker uses the buyers' deposits to acquire fiat money and government debt. At date $t+1$, the banker receives proceeds from all the holdings of public debt to fund the payments to his depositors. Recall that buyers matched in the unconnected meetings withdraw all their deposits. Since return on bonds generally dominates return on money, a bank that wishes to maximize buyer's utility would only acquire minimum amount of currency necessary to honor the withdrawal from buyers in unconnected meetings, $\rho m_t$, and would not carry any money balance across periods. Incorporating these facts, the banker's balance sheet and incentive compatibility constraints are given by

$$d_t - \rho m_t - z^s_t b^s_t - z^l_t b^l_t - (1-\rho) R_t n_t + \frac{\phi_{t+1}}{\phi_t} b^s_t + \frac{\phi_{t+1}}{\phi_t} b^l_t (1 + z^l_{t+1}) = 0 \tag{12}$$

[10] A similar insurance scheme is found in the random-relocation models of Bencivenga and Smith (1991); Schreft and Smith (1998) and Gomis-Porqueras (2000), among others.

[11] $\theta_s$ and $\theta_l$ represent haircuts on short and long-term bonds respectively.

$$-(1-\rho)R_t n_t + \frac{\phi_{t+1}}{\phi_t} b_t^s (1-\theta_s) + \frac{\phi_{t+1}}{\phi_t} b_t^l (1+z_{t+1}^l)(1-\theta_l) \geq 0 \tag{13}$$

where $m_t = \phi_t c_t$ denotes real balances expressed in terms of CM goods that the banker holds, and $b_t^s = \phi_t B_t^s$ ($b_t^l = \phi_t B_t^l$) represents real short- (long-)term public debt that the banker holds in his portfolio.

The balance sheet constraint can be characterized in the usual way. The proceeds of the deposits accepted by the banker are used to acquire fundamental assets–money sufficient to honor early withdrawal, short-term bonds and long-term bonds. In the next period CM, the payoffs from the assets are used to redeem the claims to the deposits. The incentive-compatibility constraint represents the limited commitment problem, establishing that the net payoff for the bank from paying off all its liabilities in the CM of period $t+1$ is at least as large as payoff from absconding. In other words, the bank is never worse off by honoring its liabilities in the CM of period $t+1$ than absconding with what it could.

Here, we assume that the banker could abscond with assets after currency withdrawals. The way the incentive-compatibility constraint is written, it is the claims against deposits for those in connected trades that the banker is considering. By absconding, the banker can take the unpledgeable value of the short- and long-term government bonds.[12] So this "run-away" value is not greater than the net payoff earned by redeeming claims on deposits with the proceeds from the bank's assets.

We can write the banker's problem as follows

$$\max_{d_t, c_t^w, n_t, m_t, b_t^s, b_t^l} \mathcal{U}_t \quad \text{s.t.} \quad (11) \text{ and } (12)\ .$$

It is easy to show that balance-sheet constraint will always be binding. After substituting deposits from the balance-sheet condition into the objective function, the banker's first-order conditions are given by

$$n_t: \quad -R_t + \beta\, q_{t+1}^{-\sigma}(n_t)\big)\, \frac{\partial q_{t+1}(n_t)}{\partial n_t} - \Lambda_t R_t \leq 0 \tag{14}$$

$$m_t: \quad -1 + \beta \left(\frac{\phi_{t+1} m_t}{\phi_t}\right)^{-\sigma} \frac{\phi_{t+1}}{\phi_t} \leq 0 \tag{15}$$

$$b_t^s: \quad -z_t^s + \frac{\phi_{t+1}}{\phi_t} - \Lambda_t \frac{\phi_{t+1}}{\phi_t}(1-\theta_s) \leq 0 \tag{16}$$

$$b_t^l: \quad -z_t^l + \frac{\phi_{t+1}}{\phi_t}(1+z_{t+1}^l) + \Lambda_t \frac{\phi_{t+1}}{\phi_t}(1-\theta_l)(1+z_{t+1}^l) \leq 0 \tag{17}$$

where $\Lambda_t$ is the Lagrange multiplier associated with the bank's incentive constraint represented in by equation (13). Finally, the DM consumption in connected trade is given by

$$q_{t+1}(n_t) = \begin{cases} q^* & \text{if } R_t n_t \geq q^* \\ R_t n_t & \text{if } R_t n_t < q^*. \end{cases}$$

Given that the balance-sheet constraint binds, the discounted payoff to the banker is zero. With limited commitment, the deposits received by the bank are less than the pledgeable value of assets acquired. The

[12]Or alternatively, currency is completely pledgeable.

difference is bank capital, which the banker can acquire at a constant marginal cost (disutility from effort). Here, the banker's incentive constraint will bind in equilibrium because of the scarcity of collateral, not because the bank capital is scarce.

The solution to the banker's problem yields the deposit contract, $d_t, n_t, m_t$ and the quantity of each fundamental asset acquired by the banker. The solution provides us with the means of characterizing whether the consumption of buyers in connected trades is first best or not. In this paper, we focus on the case where buyers in connected matches trade for an inefficient quantity, $q < q^*$. [13]

# 4 Monetary Equilibrium

Having solved for the optimal decisions of agents, we can now characterize the resulting monetary equilibrium. We begin by representing the optimal decision rules for DM buyers, bankers and market clearing conditions in which participants take government policies as given. After adjusting for time periods, Equations (50) and (52) provide the optimal DM quantities, taking as given the government policies, which are given by

$$q_{t+1}(n_t) = \begin{cases} q^* & \text{if } R_t n_t \geq q^* \\ R_t n_t & \text{if } R_t n_t < q^* \end{cases} \tag{18}$$

$$q^u_{t+1}(m_t) = \frac{\phi_{t+1} m_t}{\phi_t}. \tag{19}$$

Note that the efficient quantity is not supported in an unconnected meeting in the equilibrium we consider as we are away from the Friedman rule. From Equation (15), we have the following

$$\frac{\phi_t}{\phi_{t+1}} = \beta u' \left( \frac{\phi_{t+1} m_t}{\phi_t} \right). \tag{20}$$

From Equation (14), the banker's optimal asset allocation and optimal terms of trade satisfies the following condition

$$(1-\rho) R_t n_t \Big( \beta u'(R_t n_t)(1-\theta_s) + \theta_s \Big) + \frac{z^l_t b^l_t (\theta_l - \theta_s)}{\beta u' \left(R_t n_t\right)(1-\theta_l) + \theta_l} = (z^s_t b^s_t + z^l_t b^l_t)(1-\theta_s). \tag{21}$$

The equation indicates a binding incentive compatibility constraint. When $\Lambda_t > 0$, the return on deposits carried over to the next period will dominate the return on fiat money. Moreover, public debt of different maturities, which act as collateral, is scarce. As a result, the gross nominal bond yields for the short-term and long-term

[13] There is a case in which the DM buyer can afford the efficient quantity because the marginal value of additional collateral is zero. We refer to this special case as $\Lambda_t = 0$. Though it cannot be excluded as a possible case in this model economy, it corresponds to an economy marked by relaxing the incentive-compatibility constraint that is not valued by the banker. In other words, the banker is on the cusp of having plentiful collateral. It represents a tipping point in the sense that collateral is scarce because of the limited commitment/pledgeability issue and simultaneously additional collateral is not valued by the bank. We refer the readers to Dhital et al. (2021) for the detailed analysis of the case.

government bonds exhibit a premium. In particular we have that

$$\frac{1}{z_t^s} \equiv 1 + r_t^s = \frac{\phi_t}{\phi_{t+1}\Big(\beta\ u'\big(R_t n_t\big)(1-\theta_s) + \theta_s\Big)} \tag{22}$$

$$\frac{1 + z_{t+1}^l}{z_t^l} \equiv 1 + r_t^l = \frac{\phi_t}{\phi_{t+1}\Big(\beta\ u'\big(R_t n_t\big)(1-\theta_l) + \theta_l\Big)} \tag{23}$$

where $r^s(r^l)$ are the net interest for short-term (long-term) government bonds.

Given these conditions, the price of short-term and long-term government bonds can be found from Equations (22) and (23) respectively, which solve

$$z_t^s = \frac{\beta u'\left(R_t n_t\right)\left(1-\theta_s\right) + \theta_s}{\beta u'\left(\frac{\phi_{t+1} m_t}{\phi_t}\right)} \tag{24}$$

$$\frac{1 + z_{t+1}^l}{z_t^l} = \frac{\beta u'\left(\frac{\phi_{t+1} m_t}{\phi_t}\right)}{\beta\ u'\big(R_t n_t\big)(1-\theta_l) + \theta_l}. \tag{25}$$

Market clearing conditions for money, short-term bonds and long-term bonds are given by

$$\phi_t M_t = \rho m_t \tag{26}$$

$$\phi_t B_t^s = b_t^s \tag{27}$$

$$\phi_t B_t^l = b_t^l. \tag{28}$$

To complete the characterization of the monetary equilibria, we must specify the policy rules of the government. To simplify exposition, we assume that one branch (central bank or treasury) actively makes choices, while the other branch passively responds in order to ensure the period-by-period government budget constraint is satisfied. We further assume that all private agents understand the various policy regimes and that the government is committed to implementing such policies. In what follows, we examine an economy in which the central bank chooses the money growth rate. The treasury chooses taxes and the path for short-term and long-term government debt. Regardless of which policy regime we consider, throughout our analysis, we assume that the government takes the money-to-total-value-of-nominal-government-liabilities ratio (M-L ratio) as given and implements policies that are consistent with this ratio. Let $\delta$ denote the M-L ratio. For a money growth rate $\mu$, M-L ratio implies the following

$$M_{t+1} = \mu M_t = \mu\delta(M_t + z_t^s B_t^s + z_t^l B_t^l). \tag{29}$$

For a given money growth rate, $\mu$, and a money to total value of nominal government liabilities (M-L ratio), $\delta$, there is one additional piece of information needed to solve for the stationary equilibrium. Specifically, we must assume that either short-term (Policy $\mathcal{S}$) or long-term (Policy $\mathcal{L}$) government debt is fixed. The implication is that an increase in any of the government securities, for example, implies an increase in one other type of securities as well, while taxes (or transfers) adjusts to satisfy the government budget constraint.[14] In this paper, we focus on equilibrium in which the *value* of long-term government debt is constant over time. It follows that short-term debt adjusts in order to ensure that the government budget constraint is satisfied.[15]

Throughout the rest of the paper, we focus on stationary equilibria. We define a stationary equilibrium in the following way.

**Definition 1.** *Given monetary and fiscal policies, a stationary monetary equilibrium is a set of CM and DM consumption bundles* $\{x, q^u, q\}$*, real money balances and real bond holdings* $\{m, b^s, b^l\}$*, deposits, claims to deposits, return on deposit and prices* $\{d, n, R, z^s, z^l\}$ *that are constant over time and satisfy the agents' optimization problems satisfying Equations* (18) - (25)*, market clearing conditions* (26) - (28) *and the government budget constraints* (1).

Consider an equilibrium in the economy in which there is an increase in government bonds. With the additional collateral, consumption opportunities of connected trades are greater. Because public debt is not priced fundamentally, Ricardian equivalence does not hold in this environment. Households are willing to pay a premium for public debt because of its collateral value in the face of limited commitment. Thus, the revenues generated by issuing nominal debt, when compared to taxing, are quite different from a situation when government bonds are priced fundamentally. As pointed out by Bassetto and Sargent (2020), the lines between monetary and fiscal policy are no longer blurred. The underlying monetary and fiscal interactions may deliver quite different equilibrium properties depending on which policy regime is followed.

Once we solve for the equilibrium allocations, total deposit can be derived from the bank's balance sheet constraint, Equation (12), and is given by

$$d_t = \rho m_t + z_t^s b_t^s + z_t^l b_t^l + (1-\rho) R_t n_t - \frac{\phi_{t+1}}{\phi_t} b_t^s - \frac{\phi_{t+1}}{\phi_t} b_t^l (1 + z_{t+1}^l) \tag{30}$$

After substituting the equilibrium objects in a stationary equilibrium, we get

$$d = q^u (q^u)^{-\sigma} + z^s b^s + z^l b^l + (1-\rho) q - \frac{1}{\mu} b_t^s - \frac{1}{\mu} b_t^l (1 + z_{t+1}^l) \tag{31}$$

Using the expression for M-L ratio and prices of government bonds, equilibrium total deposit becomes,

$$d = q^u (q^u)^{-\sigma} + (1-\rho) q (\beta (q)^{-\sigma} (1-\theta_s) + \theta_s) + \frac{z^l b^l}{\beta (q)^{-\sigma} (1-\theta_l) + \theta_l} \tag{32}$$

We focus on cases in which the central bank commits to a fixed money growth rate and the treasury implements Policy $\mathcal{L}$; that is the treasury set the sets the value of long-term government debt to be a constant. Formally, let $\phi_t z_t^l B_t^l = A \;\; \forall t$. After imposing stationarity on Equations (18) to (28) and after repeated substi-

[14] We refer you to Dhital et al. (2021) for justifications of the assumptions and robustness checks.

[15] In our previous work, we considered two alternatives: Policy $\mathcal{L}$ sets the value of long-term debt while Policy $\mathcal{S}$ sets the value of short-term debt. For this paper, the focus is on financial innovation. It is worth noting that the two different policy settings imply different paths for future after-tax deficits and surpluses because the interest expenses are different for each policy. Fiscal actions affect the quantity of collateral available to banks through Policy $\mathcal{L}$ and $\mathcal{S}$. Hence, different bond supply processes for long and short-term public debt matter for real allocations. This is the case as Ricardian equivalence does not hold in this environment. We keep the two types of government debt in order to consider financial innovations in the form of changing relative haircuts for the two types.

tution, the stationary equilibria can be summarized by DM consumption, $q$ and $q^u$. In particular, the stationary equilibria satisfies the following equations

$$(1-\rho)\ q\ (\beta u'(q)(1-\theta_s)+\theta_s)+\frac{A(\theta_l-\theta_s)}{\beta u'(q)(1-\theta_l)+\theta_l}=\frac{\rho(1-\delta)q^u\ u'(q^u)(1-\theta_s)}{\delta} \tag{33}$$

$$\beta u'(q^u)=\mu \tag{34}$$

where equation (34) pins down the buyer's consumption in unconnected meetings, while equation (33) then determines consumption in connected trades. It is through the binding banker's incentive compatibility constraint, equation (33), that the DM consumption in the two states of the world are linked to each other.

In general, there are two opposing forces that link these two different consumption allocations. To provide more intuition, suppose there is an increase in $q^u$. The money growth rate needed to satisfy the government budget constraint declines, reducing the future price level. As a result, the real value of government debt obligations increases. This in turn loosens the bank's incentive compatibility constraint. As a result, buyers in connected trades can increase their consumption. The size of such an increase depends on how risk averse agents are. This is the case as claims to deposit contracts are effectively claims to a financial instrument that acts like insurance. To illustrate the other opposing effect, the collateral channel, suppose there is a decrease in the money growth rate that causes a decrease in the supply of short-term debt. Because the quantity of collateral is reduced, the incentive compatibility constraint tightens and buyers in connected trades will consume less.

# 5 Innovation in the measure of sellers

Here, we consider a case in which the measure of sellers capable of observing deposit claims increases. Consider a financial innovation that reduces the cost associated with verifying the buyer's deposit accounts. We examine the cost reduction as manifesting itself into a larger measure of buyers using deposit claims to pay for DM consumption. Formally, consider a decline in $\rho$. Such innovation can be thought of capturing the financial inclusion and improvement in payment system. We summarize the effects of a change in $\rho$ in the following lemma.

**Lemma 1.** *The quantity consumed by buyers in connected trades is positively related to $\rho$.*

With this type of financial development, the decrease in $\rho$ affects the quantity consumed in the connected DM matches. Specifically, with a decline in $\rho$, we find that along the intensive margin, a decline in the quantity consumed in connected matches results.[16] With more sellers in connected trades, our results show that greater financial development results in less consumption by buyers in the connected matches. The result seems counter-intuitive. However, with limited commitment, DM consumption is backed by a fixed value of collateral. Hence, with a larger measure of DM buyers in connected matches, there is less collateral per depositor. Consequently, for DM buyers in connected matches, consumption declines. We refer to this impact as the intensive margin. With a fixed quantity of inputs–that is, pledgeable collateral–each connected match is supported by less collateral, which results in less consumption supported.

[16]The intensive margin is a straightforward application of stochastic dominance. With an increase in the likelihood that a DM buyer will be in a connected match, the distribution facing DM buyers will dominate the distribution facing DM buyers with a lower value of $\rho$. So, a decline in $\rho$ corresponds with an increase in consumption.

The welfare effects take into account the intensive margin and the extensive margin. To see this, consider welfare for the typical DM buyer. Let the expected, per-period welfare be represented by $\Xi = (1-\rho)\frac{q(\rho)^{1-\sigma}}{1-\sigma} + \rho\frac{(q^u)^{1-\sigma}}{1-\sigma}$. Effect on welfare is summarized by the proposition below.

**Proposition 1.** *A positive financial innovation in the form of a smaller measure of DM buyers in unconnected trades has an indeterminate effect on expected welfare.*

The impact on expected welfare owes to the countervailing forces present in the intensive and extensive margins. The extensive margin effect is negative and captured within the second term in the derivative of $\Xi$ with respect to $\rho$ in the proof of the proposition. A decrease in the fraction of buyers in unconnected trades results in a reduced quantity consumed by each connected buyer. The intensive margin is captured by the first and third terms; because consumption in connected trades is greater than consumption in unconnected trades–that is, $q > q^u$–there are expected welfare gains from the switch from unconnected matches to connected matches. The opposing forces render the total expected welfare indeterminate.

In order to understand the effect on the consumption inequality, We can represent the area under the Lorenz curve in the model economy by

$$L = 0.5\left[\frac{2\rho q^u}{q^u + q(\rho)} + 1 - \rho - \frac{q^u}{q^u + q(\rho)}\right] + (1-\rho)\left[\frac{q^u}{q^u + q(\rho)}\right] \tag{35}$$

It follows that the Gini coefficient is $(1/2 - L) \div 1/2$. The following proposition illustrates the effect of changes in $\rho$ on the consumption inequality.

**Proposition 2.** *A positive financial innovation in the form of a smaller measure of DM buyers in unconnected trades has an indeterminate effect on consumption inequality.*

In the equation for the area under the Lorenz curve, note that $L$ depends on changes along the intensive and extensive margin. This is why the relationship between the measure of buyers in unconnected matches and the Gini coefficient is indeterminate.

With such indeterminacy, we turn to numerical analysis in order to gain some insight into the quantitative effects that financial innovations in the form of a larger share of DM sellers capable of verifying deposit accounts will have on welfare and on inequality.

Table 1 reports the results of a numerical experiment in the financial innovation is an increase in the measure of sellers capable of observing the deposit accounts of DM buyers in matches. For these experiments, we choose values of $\beta = 0.99$, $\sigma = 0.4$, $\theta_s = 0.01$, $\theta_l = 0.05$, $\delta = 0.3$, and $A = 0.1$. Over the period of 1970 to present, the average ratio of M1 money stock to the sum of M1 stock and Federal Debt held by the public in the U.S. is about 0.3. Hence, the M-L ratio ($\delta$) is set at 0.3 in this exercise. Different pledgeability owes to the fact that the central bank generally applies different haircuts to debt with different maturities. In the U.S., the Federal Reserve has historically applied one percent haircuts on government bonds of maturity less than five years and four percent on longer term bonds. We consider the initial value $\rho_0 = 0.15$ and change it to $\rho_1 = 0.10$. The measure of the unbanked households in the U.S. is about 6.5% and that of the underbanked is up to 17% in 2017.[17] Hence, the range of $\rho$ considered here captures the US data well.

[17] See https://www.fdic.gov/householdsurvey/

Table 1: Results for an increase in the measure of sellers in connected DM matches.

| | | | |
|---|---|---|---|
| Case I: $\rho = 0.15$ | | | |
| $q^u = 0.9752$ | $q = 1.6574$ | $expW = 2.1646$ | $Gini = 0.3123$ |
| | | | |
| Case II: $\rho = 0.10$ | | | |
| $q^u = 0.9752$ | $q = 1.2645$ | $expW = 1.8912$ | $Gini = 0.1917$ |

Based on the numerical results, an increase in the measure of sellers in connected matches (a decrease in $\rho$), results in a decrease in the expected welfare of buyers. We see that consumption in connected matches declines by 13 percent for a given five percentage decline in $\rho$. Table 1 further indicates that consumption inequality decreases as the Gini coefficient declines from 0.3123 to 0.1917 with a five-percentage point decline in $\rho$.

Figure 2 shows the results over a wider range of values of $\rho$. Here, we plot combinations of $\rho$ and consumption in connected matches for two different money growth rates. With $\mu = 1$ and $\mu = 1.05$, we see that consumption in connected matches is monotonically, positively related to values of $\rho \in [0.1, 0.11, 0.12, ..., 0.29, 0.3]$. Figure 3 reports the combinations of the Gini coefficient and $\rho$. As Figure 3 shows, there is a positive, monotonic relationship between inequality and $\rho$. Thus, the intensive margin appears to dominate the extensive margin for the measure of unconnected matches–that is, money trades–that account for the measure of matches between 0 percent and 30 percent.

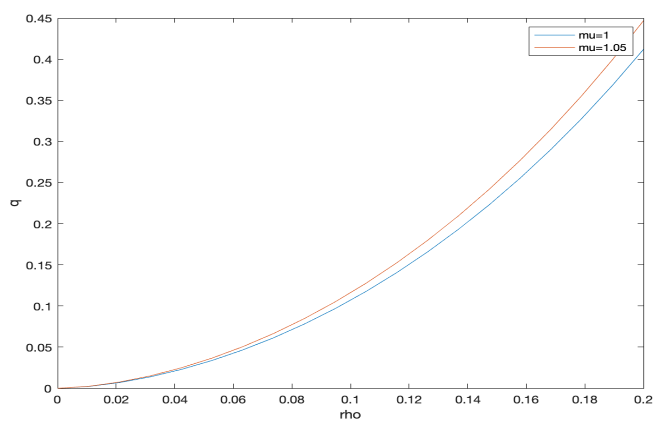


Figure 2: Consumption in connected matches for varying $\rho$

The findings above shows that a financial innovation does not always lead to a welfare improvement. When parametrized to match the US data, the findings show that welfare is lower when the economy experiences an improved financial inclusion. It is worth noting that the findings are for the case where the value of collateral are fixed. It is plausible to assume that if more collateral are available to accommodate the financial innovation, we could see an improvement in expected welfare. Since the collateral are supplied by the treasury, it is possible that they can supply more collateral and help loosen the bank's incentive compatibility constraint, and hence, consumption in connected trades. Next, we study the influence fiscal government can have on the expected welfare. The Lemma below shows that an increase in taxes results in an increase in expected welfare.

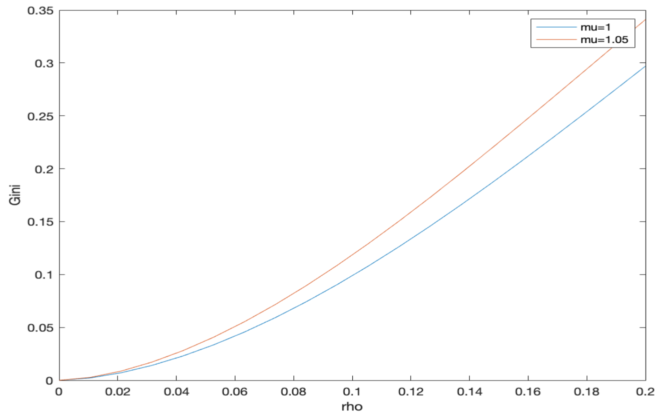


Figure 3: Gini coefficient for varying $\rho$

**Lemma 2.** *Consider an increase in lump-sum taxes, the corresponding increase in collateral results in greater consumption in connected matches and greater expected welfare.*

The intuition is straightforward in this policy experiment. With higher taxes, the quantity of collateral supported will also increase. With more collateral, those in connected matches will consume greater quantities.

Armed with this lemma, the following proposition holds.

**Proposition 3.** *In an experiment in which there is a financial development resulting in a larger share of connected matches in the DM market, there exists an increase in lump-sum taxes, denoted $\tau^*$, that leaves the quantity consumed by buyers in that DM market unchanged.*

As shown earlier, a financial development resulting in a larger share of connected matches in the DM market results in decline in consumption in those trades ($\frac{dq}{d\rho} < 0$). With $\frac{dq}{d\tau} > 0$, it follows that there exists an increase in lump-sum taxes such that the quantity remains unchanged. With an increase in lump-sum taxes, for example, the amount of collateral increases. Consequently, for connected matches in the DM, the increase in the measure of buyers is "matched" by an increase in collateral. The lump-sum tax increase is engineered so that the intensive-margin effects are mitigated. Thus, the same quantity of goods can be purchased by buyers using deposit claims as before.

The accommodation policy effectively shuts down the intensive margin. Expected welfare unambiguously increases when only the extensive margin is operational. In comparison to the experiments with both margins operating, the inequality increases as a larger fraction of matches are with connected sellers and quantities are held constant. The Gini coefficient here is non-monotonically changing with $\rho$. A potential decline in inequality due to intensive margin previously is attributed to decline in the gap between the two quantities. When the intensive margin is shut down, the declining effect is also shut down. For the measure of matches considered, inequality is larger when the government accommodates the financial innovation. However, the non-monotonicity is evident when we consider the case in which $\rho = 0$–that is, when everyone is in connected DM matches– and when $\rho = 1$, when everyone is in unconnected DM matches. In both cases, consumption inequality vanishes because quantities are equal for all DM buyers. The case with $\rho = 0$ Pareto dominates the case with $\rho = 1$.

## 6 Fraction absconding

In this section, we consider the impact that a decrease in $\theta_l$ would have on consumption, welfare and the inequality. Here, the financial innovation takes the form of reducing the risk associated with absconding. Through a marginal reduction in the limited commitment problem, the haircut on the long-term government bond is reduced. This type of financial innovation captures instances where long-term debt is more valuable as a collateral asset.

We summarize the effect on stationary equilibrium in the following proposition.

**Proposition 4.** *For a change in the fraction of long-term government debt that can be absconded (a change in $\theta_l$), the results are as follows:*

- *consumption in connected trades $q$ is inversely related to $\theta_l$;*
- *welfare is inversely related to $\theta_l$;*
- *inequality is inversely related to $\theta_l$.*

The intuition is straightforward. By reducing the haircut on long-term bonds (or, alternatively, decreasing the bank's ability to abscond), the value of pledgeable collateral increases. With this increased backing, the support for consumption for those in connected matches increases.

In this experiment, only the intensive margin is operating. With the increase in consumption in connected matches, it follows that expected welfare increases with a decrease in the ability to abscond as the consumption in unconnected matches are unaffected. Similarly, inequality increases as a given fraction of buyers in connected matches will consume larger quantities, and hence, the wedge between two types of consumption increases.

## 7 Endogenous connected matches

Consider a model in which the fraction of connected matches is endogenized. To do this, we build on the work by **?** where they consider a costly record-keeping technology adoption that allows trade credit to exist. the intent to demonstrate that changes in the distribution of the costs of record-keeping can be a source of the type of financial innovations considered in the case with a change in the fraction of buyers using deposits as the means of payment.

We modify the environment presented above in two ways. First, we relax the take-it-or-leave-it assumption in the DM market and divide the total surplus by Kalai bargaining. Second, we assume that each DM seller decides whether to adopt a record-keeping technology by paying a fixed cost. By paying the cost, the DM seller is allowed to observe the buyers deposit account. Formally, at the end of the CM, sellers can invest in a technology that allows them to be in a connected match. We assume that the cost is measured as a loss of flow utility, represented by $\kappa \geq 0$. Each seller receives an idiosyncratic draw from the distribution of costs, where the date-$t$ draw is taken from a time-invariant distribution; let $F(\kappa) : \mathbb{R} \to [0, 1]$. The costs are defined over non-negative support represented by $\kappa \in [0, \overline{\kappa}]$. We assume that at the beginning of each period, sellers decide whether or not to invest in the record-keeping technology, taking the buyers' choices as given.

The seller chooses whether to adopt the record-keeping technology by solving the following problem:

$$\max\{-\kappa + (1-\omega)S(d), (1-\omega)S(z)\} \tag{36}$$

where $0 \le \omega \le 1$ is the bargaining power of buyer and $S(d)$ ($S(z)$) is the DM surplus when trade is settled using bank deposits (cash). Think of the seller's problem as choosing between two options: there is the (net) utility from paying the cost and obtaining the portion of total surplus received in a connected match and there is the utility of surplus received in an unconnected match.

There exists a cost threshold, $\hat{\kappa}$, such that sellers invest in the record-keeping technology. Let $\hat{\kappa}$ denote the threshold level. The solution for $\hat{\kappa}$ is

$$\hat{\kappa} = (1-\omega)[S(d) - S(z)]. \tag{37}$$

In this problem, the seller treats the total surplus as given. With a CRRA utility function, the DM surplus is represented as $S(d) = \frac{(Q_t)^{1-\sigma}}{1-\sigma} - Q_t$ for connected matches and $S(z) = \frac{(Q_t^u)^{1-\sigma}}{1-\sigma} - Q_t^u$. for unconnected matches. (We use the notation of upper case $Q_t$ and $Q_t^u$ to signify that these are DM quantities that the seller takes as given.)

The seller's decision to invest in the record-keeping technology is represented by $\lambda(\kappa) \in [0,1]$. In other words, for a given value of $\kappa$, the seller's decision problem is given by

$$\lambda(\kappa) = \begin{cases} 1 & \text{if } \kappa < \hat{\kappa}, \\ [0,1] & \text{if } \kappa = \hat{\kappa}, \\ 0 & \text{if } \kappa > \hat{\kappa}. \end{cases} \tag{38}$$

By the law of large numbers, the measure of sellers that invest in the costly technology is given by the probability that a seller draws $\kappa \le \hat{\kappa}$. It follows that the aggregate fraction of sellers choosing connected matches is given by

$$1 - \rho = \int_0^\infty \lambda(\kappa) dF(\kappa) = F(\hat{\kappa}). \tag{39}$$

In general, the fraction of sellers adopting the record-keeping technology will depend on taxes, money growth rates, pledgeability of collateral, the composition of government debt, and the probability distribution function. With an endogenous fraction of sellers in connected matches, recall that the quantity consumed in unconnected matches is invariant to changes in $\rho$ (see equation (34)). With endogenous fraction of connected matches, equation (33) and equation (39) are solved simultaneously. Put simply, $\rho$ affects the quantity consumed in connected matches and $q$ affects the size of the consumer surplus, which in turn affects $\hat{\kappa}$.

To illustrate the interaction between measure of connected matches and the quantity consumed in those matches, consider the case in which $q$ is positively related to $\rho$. Recall that this means the intensive margin quantitatively dominates the extensive margin. Now, suppose there is a change in the distribution of record-keeping costs. For example, consider two distributions of cost of record keeping, denoted $F_0(\kappa)$ and $F_1(\kappa)$. A financial innovation corresponds to a change characterized by first-order stochastic dominance; formally, suppose at date $t$, there is an unexpected and permanent change in the distribution of costs from $F_0(\kappa)$ to $F_1(\kappa)$ with $F_0(\kappa)\, FSD\, F_1(\kappa)$. Thus, the first experiment is a treatment so that $F_1(\kappa) \ge F_0(\kappa)$ for all $\kappa$. Accordingly,

$F_1$ has a larger probability mass over the lower values of $\kappa$. It follows that $\Lambda$ will increase.

The following proposition gives the complete picture of the effect caused by an exogenous change in the distribution of costs.

**Proposition 5.** *If the intensive margin dominates the extensive margin for the DM buyers, a stochastic dominant change in the distribution of costs in the first-degree sense results in a decrease in the measure of sellers in unconnected trades, a decrease in the quantity consumed by DM buyers in connected trades, a decrease in consumption inequality and a decrease in expected welfare.*

In compact form, we write the two equations as

$$1-\rho = F\{(1-\omega)[S(q)-S(q^u)]\} = F(q;,q^u,\Delta,\omega) \tag{40}$$

We define $\Delta$ so that $F_\Delta$ is the marginal impact of the change in the distribution function from $F_0$ to $F_1$.

and (33) as

$$q = H(\rho; q^u, \theta_s, \theta_l, Z) \tag{41}$$

where $Z = A, \beta, \delta, \tau$ is the set of parameters included in the solution for the for $q$. The findings are qualitatively similar to the case where $\rho$ is exogenously set when intensive margin dominates.

If the extensive margin dominates, $H_\rho < 0$, and $\frac{d\rho}{d\Delta} > 0$ if $H_\rho + F_q$ is positive. In other words, financial innovation could result in a larger fraction of DM buyers in unconnected matches if the extensive margin dominates and the impact on total surplus for buyers in connected matches is marginally small. The counter-intuitive result is more likely as $q \to q^*$ since the gain in the surplus would be approaching is maximum. With $H_\rho + F_q$ is positive, $\frac{dq}{d\Delta}$ will be positive.

Next, we consider a decrease in the haircut on long-term government bonds. Quantities consumed in the unconnected matches are not affected. As a first-order effect, collateral increases and consumption in connected matches increases. The general-equilibrium effects owe to the following chain: with $q \leq q^*$, total surplus increases with the increase in consumption in connected matches. With the increase in surplus, $\hat{\kappa}$ increases, resulting in $F(\hat{\kappa})$ to increase. In other words, the model predicts an increase in the measure of sellers choosing the record-keeping technology. As such, the given increase in collateral is spread over a larger measure of DM buyers, mitigating the first-order impact on consumption in connected matches.

In the next experiment, we consider the impact that a change in haircut on long-term government bonds would have on measure of connected matches and on the quantity of consumption in those matches.

**Proposition 6.** *Given an increase in the pledgeability of long-term government bonds, the equilibrium measure of sellers in connected matches increases, expected welfare increases and consumption inequality increases. Correspondingly, the expected welfare gains and the increase in the Gini coefficient are smaller compared with the case in which the pledgeability increase occurs in an economy in which the measure of connected matches is exogenously set.*

With $H_\rho > 0, F_q \geq 0, and F_{\theta_l} > 0$, we find that $\frac{d\rho}{d\theta_l} > 0$ and $\frac{dq}{d\theta_l} < 0$ . Thus, the findings reported in 6 are qualitatively similar to those of the baseline case where measure of sellers are set exogenously. What is new is that a decrease in $\theta_l$ here results in an increase in measure of connected sellers $(1-\rho)$. With this additional channel, the measure of connected sellers has a dampening effect on the increase in consumption in connected matches with endogenous payment-margin compared with the case in which the payment margin was exogenously set. Consequently, the increase in consumption inequality and expected welfare are also dampened in the endogenous case.

In the final experiment, we consider the effect that fiscal policy has on the economy with an endogenous payment margin. Given that the government policy affects the quantities consumed in connected matches, the measure of sellers choosing to adopt the communication technology is likely to change. Indeed, as shown in the Lemma 2, an increase in lump-sum taxes results in an increase in consumption in connected trades. As such, the DM surplus increases, resulting in an increase in $\hat{\kappa}$ and hence, the measure of connected sellers. The proposition below summarizes the result.

**Proposition 7.** *An increase in lump-sum taxes results in an increase in the measure of connected sellers.*

Hence, an expansionary fiscal policy improves financial inclusion. Contrary to the case where the measure of connected sellers are exogenously set, there's again a dampening effect at work here. The resulting increase in the measure of connected sellers dampens the increase in $q$ due to the fiscal policy.

# 8 Conclusion

In the case of financial innovations, the starting premise is that lower transaction costs will expand the set of feasible trades in which people can engage. There are open questions regarding the distribution of those gains. In this paper, we examine two types of financial innovation under an environment with limited commitment. We consider a change in the measure of sellers capable of accessing information on deposit claims. This first innovation mirrors a kind of expansion in the types of payments offered to sellers. The second type of innovation occurs in the haircuts on financial instruments. In this model economy, a smaller haircut corresponds to reducing the friction that is present because of limited commitment. In short, the security serves as a better form of collateral because of the financial innovation.

At the heart of problem is limited commitment. Banks must provide collateral because of their ability to abscond with deposits. Our setup creates ex post heterogeneity between those matched with sellers capable of accessing financial data and those without. We then allow for a larger measure of sellers that are capable of observing deposit claims and therefore, willing to accept these claims as payment.

An interesting result is that a financial innovation that results in a more sophisticated payment system – that is, one with a larger fraction of matches using bank deposits–can result in lower expected welfare. These financially sophisticated buyers are more tied to financial institutions, but without an increase in collateral, their consumption declines and the distribution of output is more equally distributed. Those who now can access the financial technology are better off along the extensive margin. But along the intensive margin, the gains are smaller. Indeed, as more buyers utilize deposit claims, the banking system is stressed as the deposits are backed by limited collateral. Given the level of collateral, intensive margin implies that the consumption in such deposit backed trades now decline. Inequality declines as the wedge between deposit-backed and money-backed consumption declines.

We show that there exists a fiscal policy accommodation that increases collateral by imposing a larger lump-sum tax on buyers. They are willing to pay the larger tax to access more collateral. Indeed, the increase in collateral means greater backing for deposit claims, thus supporting more consumption compared with the case in which the financial innovation occurs and everything else is held constant. Such accommodation policy shuts down the intensive margin and results in improved welfare but higher inequality.

In the case of a smaller haircut, the financial innovation results in more consumption by buyers in connected matches. Accordingly, with less quantity to abscond, those in connected matches are backed by better assets. Inequality increases with this type of financial innovation and expected welfare is higher.

We extend the model economy to consider how financial innovation affects consumption and consumption inequality when sellers face an idiosyncratic shock and surplus sharing. The modification permits sellers to choose the type of payment they are willing to accept. In other words, with a payment-system margin, we see how financial innovations affect consumption and consumption inequality. The slope of the distribution function becomes an important factor that characterizes the nature of the financial innovation. We derive conditions in which the financial innovation–in terms of greater probability over low-cost shocks–results in an increase in a decrease in expected welfare and inequality. It is important to note that the results depend qualitatively on the relationship between the intensive and extensive margins. If, for example, the extensive margin dominates the marginal increase in total surplus through changes in threshold level of costs, financial innovations can have the opposite effects on consumption in connected matches and therefore, consumption inequality. We also derive conditions in which innovations in the form of greater collateral pledgeability result in dampening the effect on consumption inequality and expected welfare. Because greater pledgeability induces an increase in seller surplus, the measure of connected sellers increases. Along this payment-system margin, the given increase in collateral is now spread across a larger measure of depositors.

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# 9 Appendix

## 9.1 Proof of Lemma 1

To prove this lemma, totally differentiate (33) and simplify, yielding

$$\frac{dq}{d\rho} = \frac{\mathcal{A}}{\mathcal{B}} \tag{42}$$

where $\mathcal{A} = \frac{(1-\delta)(q^u)^{(1-\sigma)}(1-\theta_s)}{\delta} + \beta(1-\theta_s)(q)^{(1-\sigma)} + \theta_s q > 0$ and $\mathcal{B} = (1-\rho)\beta(1-\theta_s)(1-\sigma)(q)^{-\sigma} + (1-\rho)\theta_s + \frac{A(\theta_l-\theta_s)\sigma\beta(1-\theta_l)(q)^{-\sigma-1}}{(\beta(1-\theta_l)(q)^{-\sigma}+\theta_l)^2} > 0$

**Proof of Proposition 1**

We differentiate expected per period DM welfare with respect to $\rho$, yielding

$$d\Xi = -\frac{q(\rho)^{1-\sigma}}{1-\sigma} + (1-\rho)q(\rho)^{-\sigma}\frac{\partial q}{\partial \rho} + \frac{(q^u)^{1-\sigma}}{1-\sigma} \tag{43}$$

In general, the sign of this expression is indeterminate.

**Proof of Proposition 2**

Immediately follows from Lemma 1 and Proposition 1.

**Proof of Lemma 2** Without loss of generality, we consider the tax increase experiment with policy $\mathcal{L}$. Note that with a fixed money growth rate, Equation (34) indicates that $q^u$ is unaffected by the tax increase.

The effect on $q$ can then be analyzed by considering Equation (33). From the government budget constraint (1), $z_t^s b_t^s + z_t^l b_t^l$, we can write,

$$\mathcal{B} = z_t^s b_t^s + z_t^l b_t^l = \tau + b_{t-1}^s + b_{t-1}^l + \rho(\mu-1)m_{t-1} \tag{44}$$

Equation (33) can then be written as,

$$(1-\rho)\ q\ (\beta u'(q)(1-\theta_s)+\theta_s) + \frac{A(\theta_l-\theta_s)}{\beta(q)^{-\sigma}(1-\theta_l)+\theta_l} - (1-\theta_s)\mathcal{B} = 0 \tag{45}$$

Taking the derivative with respect to $\tau$, we get

$$\frac{\partial q}{\partial \tau} = \frac{(1-\theta_s)\frac{\partial \mathcal{B}}{\partial \tau}}{(1-\rho)\Big((\beta(q)^{-\sigma}(1-\theta_s)+\theta_s)+\beta-\sigma(q)^{-\sigma}(1-\theta_s)\Big) - \frac{\beta A(\theta_l-\theta_s)-\sigma(q)^{-\sigma-1}(1-\theta_l)}{\left(\beta(q)^{-\sigma}(1-\theta_l)+\theta_l\right)^2}}. \tag{46}$$

The numerator is positive. With $0 < \sigma < 1$, the denominator is positive and hence the derivative is positive. Thus, an increase in lump-sum taxes raises $q$. The effect on welfare follows immediately as $q^u$ is unaffected while $q$ increases with an increase in $\tau$.

### 9.2 Proof of Proposition 3

Given that $\frac{\partial q}{\partial \rho} < 0$ and $\frac{\partial q}{\partial \tau} > 0$, by the continuity argument, there exists a level of increase in tax ($\tau^*$), such that the decline in $q$ due to decline in $\rho$ is exactly offset by an increase in $q$ due to the increase in taxes.

### 9.3 Proof of Proposition 4

From (34), it is clear that DM consumption in unconnected trades is invariant to changes in $\theta_l$. Totally differentiating (33) yields

$$\Delta 1 dq + \Delta_2 d\theta_l = 0 \tag{47}$$

where $\Delta_1 = (1-\rho)[\beta q^{\sigma}(1-\theta_s)+\theta_s] - (1-\rho)(\sigma)[\beta q^{-\sigma}(1-\theta_s)] + A(\theta_l-\theta_s)\beta\sigma q^{-\sigma-1}(1-\theta_l)$ and $\Delta_2 = \frac{(\beta q^{-\sigma}-1)[A(\theta_l-\theta_s)+A[\beta q^{-\sigma}(1-\theta_l)+\theta_l]}{[\beta q^{-\sigma}(1-\theta_l)+\theta_l]^2}$. With $0<\sigma<1$, it follows that $\Delta_1 > 0$. We can simplify $\Delta_2$ as follows

$$\Delta_2 = \beta q^{-\sigma} A(1-\theta_s) + A\theta_s > 0 \tag{48}$$

With the positive denominator, we get $\frac{dq}{d\theta_l} = -\frac{\Delta_2}{\Delta_1} < 0$. Thus, for a given decrease in the haircut on long-term government bonds, consumption in connected matches increases.

### 9.4 Solution for consumption quantities with endogenous $\rho$

In an economy in which the DM surplus is divided by proportional bargaining between buyers and sellers, we need the seller to get some surplus from DM trade in order for there to be any incentive to pay the cost of the record-keeping technology.

The terms of trade when only cash is accepted–that is, unconnected matches–is given by

$$V^u(c^w_{t-1}, 0) = \max_{q^u_t, l^u_t} \left\{ \frac{(q^u)^{1-\sigma}}{1-\sigma} + W(c^w_{t-1} - l^u_t, 0, 0) \right\} \quad \text{s. t.} \tag{49}$$

$$-q^u_t + \phi_t l^u_t = \frac{1-\omega}{\omega}\left[\frac{(q^u)^{1-\sigma}}{1-\sigma} - \phi_t l^u_t\right],$$

$$l^u_t \le c^w_{t-1}.$$

It is easy to show that the optimal terms of trade imply the following DM consumption schedule

$$q^u_t(m_{t-1}) = \begin{cases} q^* & \text{if payment constraint does not bind,} \\ Q^u_t & \text{if payment constraint binds} \end{cases} \tag{50}$$

where $q^*$ is the efficient DM allocation, which is implicitly defined by $u'(q^*) = 1$, and $m_{t-1} = \phi_{t-1} c^w_{t-1}$ is the real money balance in term of $t-1$ CM goods. Finally, $Q_t^u$ solves

$$\frac{\phi_t}{\phi_{t-1}} m_{t-1} = (1-\omega)\frac{(Q_t^u)^{1-\sigma}}{1-\sigma} + \omega\, Q_t^u.$$

The terms of trade when bank deposits are accepted is given by

$$V(0,d_{t-1}) = \max_{q_t, n_{t-1}} \left\{ \frac{q_t^{1-\sigma}}{1-\sigma} + W(0, d_{t-1}-n_{t-1}, n_{t-1}) \right\} \quad \text{s. t.} \tag{51}$$

$$-q_t + n_{t-1} = \frac{1-\omega}{\omega}\left[\frac{q_t^{1-\sigma}}{1-\sigma} - n_{t-1}\right],$$

$$n_{t-1} \leq d_{t-1}.$$

It is easy to show that the optimal terms of trade imply the following DM consumption schedule

$$q_t(n_{t-1}) = \begin{cases} q^* & \text{if payment constraint does not bind,} \\ Q_t & \text{if payment constraint binds.} \end{cases} \tag{52}$$

where $n_{t-1} = d_{t-1}$ and $Q_t$ solves the following condition

$$R_{t-1} n_{t-1} = (1-\omega)\frac{(Q_t)^{1-\sigma}}{1-\sigma} + \omega\, Q_t.$$

### 9.5 Proof of Proposition 5

In compact form, we write the Equations (33) and (39) as

$$1-\rho = F\{(1-\omega)[S(q) - S(q^u)]\} = F(q;, q^u, \Delta, \omega) \tag{53}$$

We define $\Delta$ so that $F_\Delta$ is the marginal impact of the change in the distribution function from $F_0$ to $F_1$. and (33) as

$$q = H(\rho; q^u, \theta_s, \theta_l, Z) \tag{54}$$

where $Z = A, \beta, \delta, \tau$ is the set of parameters included in the solution for the for $q$. The solution for $\frac{d\rho}{d\Delta}$ and $\frac{dq}{d\Delta}$ are obtained by applying Cramer's rule. With $H_\rho > 0, F_q \geq 0$ and $F_\Delta > 0$, the results in Proposition 5 follows immediately. With intensive margin dominating, the consumption inequality and expected welfare decline.

### 9.6 Proof of Proposition 6

Consider Equations (33), (37) and (39). Plug equation (37) into (39) to get,

$$(1-\rho) = F\left((1-\omega)\left(S(d) - S(z)\right)\right) 0 \tag{55}$$

Let locus $\mathbf{F}(\rho)$ define Equation (33) and $\mathbf{G}(\rho)$ define (**??**). Differentiating the loci with respect to $q$ and $\rho$, we get,

$$\frac{\partial q}{\partial \rho} = \frac{\partial \mathbf{F}(\rho)}{\partial \rho} = -\delta \frac{(1-\rho)\left[\beta(1-\theta_s)(u'(q)+qu''(q))+\theta_s\right] - \frac{A(\theta_l-\theta_s)}{\beta u''(q)(1-\theta_l)}}{-\beta u'(q)(1-\theta_s)+\theta_s-(1-\delta)(1-\theta_s)u'(q^u)}$$

The numerator is positive for CRRA function. Hence, the derivative is positive. This implies that locus $\mathbf{F}(\rho)$ is upward sloping.

$$\frac{\partial q}{\partial \rho} = \frac{\partial \mathbf{G}(\rho)}{\partial \rho} = -\frac{\frac{dF(.)}{dS(d)}\frac{dS(d)}{dq}}{-1}$$

The derivative is negative, and hence, locus **G** is downward sloping.

Now consider a decrease in $\theta_l$. Locus $\mathbf{G}(\rho)$ is invariant to changes in $\theta_l$. Locus $\mathbf{F}(\rho)$ shifts up due to a decrease in $\theta_l$, resulting in higher $q$ and lower $\rho$.

Note that $q^u$ is invariant to changes in $\theta_l$. It follows then that consumption inequality and expected welfare both increase.

Now let's compare the case of endogenous versus exogenous measure of connected sellers. Contrary to the exogenous case, a decrease in $\theta_l$ in endogenous case results in an increase in measure of connected sellers $(1-\rho)$ in addition to an increase in $q$. From the proof of Proposition 5, such increase in the measure of connected sellers lowers $q$. Hence, the increase in $q$ in this case is dampened by resulting decrease in $q$ due to an in increase in the measure of connected sellers. No such dampening effect is present in the exogenous case. Consequently, the increase in consumption inequality and expected welfare are also dampened in the endogenous case.

## 9.7 Proof of Proposition 7

Consider an increase in $\tau$. From Lemma 2, $\mathbf{F}(\rho)$ shifts up. $\mathbf{G}(\rho)$ is invariant to change in $\tau$. As a result, $q$ increases and $\rho$ decreases.