Fear of Hiking?
Monetary Policy and Sovereign Risk

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Abstract

What are the implications of a rise in interest rates by the central bank of a monetary union for sovereign borrowing decisions and sovereign default risk in a union member? We study this question in a quantitative sovereign default model and obtain two results. First, the sovereign's incentives to borrow following a monetary tightening are shaped by two competing effects, a positive income and a negative substitution effect. Second, a critical threshold for debt to GDP exists above which the income effect is dominant, implying that a monetary tightening increases debt levels and the risk of a sovereign default. We quantify this "Fear of Hiking" zone and study its business cycle properties in an application of the model to the euro area.

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"*As former central bankers and as European citizens, we are witnessing the ECB's ongoing crisis mode with growing concern. [...] In contrast, the suspicion that behind this measure lies an intent to protect heavily indebted governments from a rise in interest rates is becoming increasingly well founded.*" Open letter to the ECB in October 2019—see footnote 1.

1 Introduction

The aftermath of the Great Recession has been characterized by high levels of public debt and volatile spreads in several euro members. The ongoing Covid crisis accompanied by unprecedented public support measures has reinforced these trends. In this environment, if the ECB raises key interest rates, what will be the implications for debt accumulation and sovereign risk (the risk of a sovereign default)? As exemplified by the above quote, this question is at the forefront of a heated debate.[1]

In this paper, we put forward a framework to study systematically the implications of a monetary tightening in a currency union for sovereign borrowing decisions and default incentives in a small union member. The core of our framework is the standard sovereign default model originally due to Eaton and Gersovitz (1981). We first show that, when the union-wide central bank raises interest rates, whether public debt declines or rises in the small union member is ambiguous, as substitution and income effects are pushing the incentives to borrow in opposite directions. We then show that a threshold for (external) public debt to GDP exists above which the income effect is dominant, implying that a rate rise increases debt accumulation and sovereign risk.[2] We call the region of the state space where the income effect is dominant the "Fear of Hiking" zone. We then apply our insights to the euro area. We first study whether countries are currently in the Fear of Hiking zone, by contrasting their current debt levels with the model-implied thresholds. We then study various policy scenarios with the help of our framework.

We construct a model of a small open economy within a currency union. The domestic government borrows without commitment, and its debt is held by risk-neutral investors inside the union but outside the domestic country. When the central bank changes interest rates, the outside option of investors changes. Through this channel, a monetary tightening has implications for government finances in the small union member in our framework. As in Chatterjee and Eyigungor (2012), the government issues long-term debt. Moreover, as in recent work by Bianchi and Mondragon (2021), Bianchi et al. (2019) and others, we consider a sticky-wage environment, with two implications. First, wage stickiness implies also price stickiness, hence a monetary tightening changes the price of debt also in real terms. Second, changes in aggregate demand affect output of goods which are

[1]In October 2019, several former senior European central bankers formulated an open letter to the ECB, criticizing then-President Mario Draghi for the ECB's ultra-loose monetary stance. Only two weeks later, former-ECB president Jean-Claude Trichet formulated a response letter, criticizing in turn the open letter. The open letter can be found here: https://www.ft.com/content/71f90f42-e68f-11e9-b112-9624ec9edc59; and Trichet's response letter can be found here: http://prod-upp-image-read.ft.com/c8b98512-ec31-11e9-85f4-d00e5018f061.

[2]As is standard in the Eaton and Gersovitz (1981) framework, all public debt in our model is external, that is, held by foreign investors.

produced domestically and therefore incomes in the domestic country.

We start by showing that the effects on sovereign borrowing following *temporary*[3] monetary tightenings can be framed in terms of two familiar forces which push the incentives to take on debt in opposite directions: a substitution and an income effect. The substitution effect is that borrowing becomes more expensive, implying that borrowing declines. The income effect is that the domestic country becomes poorer, implying that borrowing rises.[4] We then characterize the conditions under which one or the other effect is dominant. In particular, we show that a threshold for debt to GDP exists above (below) which the income (substitution) effect is dominant. Hence a monetary tightening raises debt accumulation, and by implication sovereign risk, whenever debt to GDP is high enough.[5]

The threshold has an intuitive interpretation and is governed by three parameters i) the elasticity of intertemporal substitution (EIS) ii) the fraction of debt which needs to be refinanced (coupon payment) in a given period and iii) the fraction of domestic demand that falls on domestically-produced goods (home bias). Intuitively, the EIS governs the strength of the substitution effect, hence a higher EIS implies that the threshold rises (i.e., the region where the substitution effect dominates becomes larger). In turn, a larger coupon payment means that a larger share of debt becomes exposed to the rise in interest rates, which strengthens the income effect and thus reduces the threshold. Last, home bias determines to what extent changes in domestic demand feed into domestic incomes. If this feedback is large, it becomes more costly for the domestic government to reduce borrowing following a rise in interest rates, as this also reduces domestic incomes. Hence a larger home bias acts like a strengthening of the income effect, reducing the threshold. In addition to parameters, the threshold also depends on the state of the business cycle. In particular, we show that the threshold inherits the business cycle properties of the trade balance to GDP ratio. Everything else equal, a more negative trade balance reduces the threshold, making it more likely that a monetary tightening raises borrowing and sovereign risk. Intuitively, this happens because a negative trade balance is financed with net capital inflows. Hence a larger part of the debt becomes exposed to the rise in interest rates, strengthening the income effect.

We next generalize our results to the case of *persistent* monetary tightenings. We show that adding persistence tends to increase the threshold characterizing the behavior of sovereign borrowing, as it implies a strengthening of the substitution effect. Hence a monetary tightening is more likely to reduce sovereign borrowing when the rate rise is persistent. However, we also show that adding persistence decouples borrowing decisions from sovereign risk; following a monetary tightening, sovereign risk may actually *rise* even though sovereign borrowing *declines*. When the rate hike is persistent, therefore, two separate thresholds for debt to GDP exist. The first threshold

[3]When the monetary tightening is temporary, then debt levels and the risk of a sovereign default always move in the same direction. When the monetary tightening is persistent, this may no longer be the case, as we discuss below.

[4]The income and substitution effects of changes in interest rates are familiar from the two-period consumption model that is typically taught to undergraduates. We explain how our results are linked to the two-period consumption model in the paper.

[5]Of course, a monetary tightening may also trigger an *immediate* default, a case which we also discuss in the paper. However, the main focus of our paper is on how monetary policy influences borrowing decisions and the risk of a sovereign default going forward, i.e. conditional on repayment in the current period.

defines the level of debt to GDP above which sovereign borrowing rises following a rate hike. The second threshold - necessarily below the first one - defines the level of debt to GDP above which sovereign risk rises following a rate hike.

To quantify these effects we turn to an application of the model to countries in the euro area. We first conduct a back-of-the envelope calculation, contrasting model-implied thresholds for various countries. For example, we find that thresholds are currently between 51% and 65% for Italy - for rate hikes with expected persistence between 1 and 3 years.[6] We then solve the model numerically by calibrating it to Italy, matching some key moments such as mean external debt and the counter-cyclical trade balance. In the calibrated model, both debt to GDP and the threshold are equilibrium objects, depending on the economy's state variables. We identify the set of states which imply that debt to GDP lies above the threshold in the equilibrium of the model, and call this the "Fear of Hiking" zone. We then describe the properties of the Fear of Hiking zone over the business cycle. We find that the economy is in the Fear of Hiking zone about 71% of the time. We also show that visiting the Fear of Hiking zone correlates negatively with the business cycle. As a result, in the calibrated model, a monetary tightening is more likely to raise sovereign borrowing and sovereign risk when the economy is in a recession.

Finally, we use the calibrated model to study three policy scenarios. In line with our theoretical insights, we show that a monetary tightening in a *run-up* phase of the cycle - when debt levels are still below the threshold but are quickly expanding - acts as a break on debt accumulation, reducing debt to GDP and by implication sovereign risk. We next study the effects of monetary policy in an "acute" phase, when debt levels are currently high and the risk of default is elevated (i.e. the economy is in the Fear of Hiking zone). We show that in this case, it is through a *cut* in the interest that debt to GDP and the risk of a sovereign default can be reduced significantly. Third, we study the implications of a decline in long-term interest rates. We find that the economy becomes more likely to be in the Fear of Hiking zone, thus making it harder for the central bank to raise interest rates.

Our paper adds to the literature at the intersection of monetary policy and sovereign default. Within this literature, closest to us are contributions with an explicit focus on monetary unions, as well as papers which construct small open economy models and apply them to the euro zone context. For instance, de Ferra and Romei (2020) build a model of a monetary union and study the interaction between monetary policy and sovereign default in some member countries. While in our framework monetary policy is effectively exogenous, they also study optimal monetary policy. Among other results, they find that the possibility of default tends to make monetary policy more expansionary. In their framework, the central bank affects the economy through the relative price of non-tradables, while it has no effect on the safe interest rate which enters the price of debt. Our analysis is therefore complementary, because in our model the central bank affects the economy exclusively through the price of debt.[7] Among the group of papers studying small

[6]Recall that this number refers to *external* debt to GDP - not total debt to GDP. At the time of writing (spring 2022), external private-held debt to GDP is slightly above 50% in Italy, while total debt to GDP is above 150%.

[7]Another paper studying the link between monetary policy and sovereign default in a monetary union is Aguiar

open economies, Na et al. (2018) show that a sovereign default has negative effects on employment when a country cannot devalue the exchange rate due to lack of an independent monetary policy (such as membership in a monetary union); Bianchi and Mondragon (2021) focus on rollover crises, showing that joining a monetary union leaves countries more vulnerable to such crises; Kriwoluzky et al. (2019) show that a sovereign debt crisis is reinforced when there is a break-up risk of the monetary union.[8] To the best of our knowledge, we are the first paper in this literature to study systematically the comparative statics of sovereign borrowing and default risk with respect to the safe interest rate. In our framework, understanding this link is important because the safe rate is the policy instrument set by the central bank. More generally, our findings may also inform other frameworks where an important driver of the business cycle is exogenous variation in the safe interest rate.[9]

In terms of results, we also add to a second literature which highlights the presence of non-linearities when public debt levels are high enough. For instance, the literature on "fiscal limits" highlights that when debt levels are high enough, countries are no longer able to stabilize their stock of debt through higher taxes which may trigger expectations of inflation (Davig et al., 2011). In models with self-fulfilling default, the "Crisis zone" is typically reached when debt levels are high enough, whereas self-fulfilling default may not occur for low debt level (Cole and Kehoe, 2000; Bianchi and Mondragon, 2021). Relatedly, a "gambling for redemption" motive arises when debt levels are high enough (Conesa and Kehoe, 2017). In contrast to these papers, in our analysis high debt levels change how monetary policy influences borrowing decisions and thereby sovereign risk, because a high debt stock implies that the income effect of bond price changes becomes dominant. When this happens, a monetary tightening leads to even higher borrowing by the sovereign and a higher risk of a sovereign default.

The rest of the paper is structured as follows. Section 2 introduces the model. Section 3 presents our main analytical insights, notably the existence of a threshold for debt to GDP above which the effects of monetary policy flip. In Section 4 we quantify the threshold and introduce the "Fear of Hiking" zone. Section 5 discusses some implications of our results for monetary policy and makes concluding remarks. An accompanying Appendix collects the proofs of all propositions as well as additional materials.

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et al. (2015). However, their focus is non non-fundamental equilibria (rollover crises). In our model, we focus our attention on default due to bad fundamentals.

[8] Other papers at the intersection of monetary policy and sovereign default are Corsetti and Dedola (2016), Bacchetta et al. (2018) and Arellano et al. (2020), among others. Corsetti and Dedola (2016) focus on the effects of unconventional monetary policy, showing that purchases of risky debt by the central bank can rule out self-fulfilling default. Bacchetta et al. (2018) show that by generating inflation, a central bank may rule out slow-moving debt crises (in the sense of Lorenzoni and Werning 2019). Arellano et al. (2020) develop a New Keynesian model with sovereign default risk ("NK-Default"). They show that in presence of default risk, inflation volatility increases.

[9] For instance, Johri et al. (2020) study the dependence of emerging market spreads on fluctuations in the U.S. treasury bill rate. They find the safe rate to be an important driver of emerging market borrowing decisions and spreads.

2 The model

In this section, we develop a model of a small open economy inside a monetary union. The economy is small, because domestic developments have no repercussions on the rest of the monetary union. The economy issues defaultable debt which is held by investors in the rest of the monetary union. A union-wide central bank sets short-term nominal interest rates. We assume the existence of nominal rigidities which implies that monetary policy has real effects. Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$.

2.1 Households

The domestic economy is populated by a large number of identical households. Each household supplies one unit of labor in-elastically and there is no dis-utility from working. However, due to the presence of nominal wage rigidities to be described below, households may be able to sell less than their endowment on the labor market. For future reference, we say that the economy operates below potential / in a slack labor market whenever equilibrium employment is below households' unit-endowment. Households' preferences over consumption are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}, \qquad 0 < \beta < 1, \quad \sigma > 0, \tag{1}$$

where C_t is a composite of tradable goods produced domestically $C_{h,t}$ and tradable goods produced in the rest of the monetary union $C_{f,t}$

$$C_t = \zeta C_{h,t}^{1-\gamma} C_{f,t}^{\gamma}, \qquad 0 < \gamma < 1, \tag{2}$$

where $\zeta \equiv (1-\gamma)^{-(1-\gamma)}\gamma^{-\gamma}$. The parameter β captures impatience, the parameter σ measures risk aversion (equivalently, the inverse of the elasticity of intertemporal substitution), and the parameter $1-\gamma$ captures home bias in consumption. Note that we assume the elasticity of substitution between domestically produced and foreign tradable goods (the trade elasticity) is equal to one. This is without loss of generality, because the relative price between the two goods is going to be constant in equilibrium.

Each households' budget constraint is given by

$$P_{h,t}C_{h,t} + P_{f,t}C_{f,t} = W_t L_t + P_{h,t}T_t, \tag{3}$$

where $W_t L_t$ is labor income and where T_t is a lump-sum transfer payment from the government (tax if negative). In the budget constraint, the prices of domestic and foreign consumption goods are denoted $P_{h,t}$ and $P_{f,t}$, respectively. Cost minimization then leads to the demand functions for domestically produced and foreign goods

$$C_{h,t} = (1-\gamma)\frac{P_t}{P_{h,t}}C_t \tag{4}$$

$$C_{f,t} = \gamma\frac{P_t}{P_{f,t}}C_t, \tag{5}$$

with the cost-minimizing price index (the consumer price index) $P_t = P_{h,t}^{1-\gamma}P_{f,t}^{\gamma}$.

As is standard in the sovereign default literature, we assume households have no direct access to international financial markets. Rather, the decision to borrow or save is taken by the government, and households receive the proceeds from these transactions in the form of transfer payments T_t.

2.2 Firms

The economy is populated by a large number of identical firms which are owned by the households. Firms produce the domestic consumption good by using the technology $Y_t = L_t$, where L_t is labor demand. Firms' profits are $P_{h,t}Y_t - W_tL_t$. The first order condition is

$$P_{h,t} = W_t. \tag{6}$$

Firms make zero profits in equilibrium.

2.3 Nominal rigidities

We assume the presence of nominal wage rigidities in the domestic economy.[10] The presence of nominal wage rigidities plays two roles in the model. First, it creates the possibility of involuntary unemployment. Second, it implies that monetary policy has real effects. Indeed, as we will see, prices inherit the wage stickiness, so that the central bank can affect the real interest rate of the domestic country through movements in the nominal interest rate.

As in recent sovereign default frameworks by Na et al. (2018), Bianchi and Mondragon (2021) and Bianchi et al. (2019), we assume the presence of downward nominal wage rigidity

$$W_t \geq \bar{W}. \tag{7}$$

By combining (7) with labor demand (6), this implies that $P_{h,t} \geq \bar{W}$, that is, the price of domestically produced goods is downward sticky as well. As it is standard, we close the labor market with the following complementary slackness condition

$$(W_t - \bar{W})(L_t - 1) = 0, \tag{8}$$

[10] A growing body of evidence emphasizes how nominal wage rigidities represent an important transmission channel through which monetary policy affects the real economy. For instance, this conclusion is reached by Olivei and Tenreyro (2007), who show that monetary policy shocks in the US have a bigger impact on output in the aftermath of the season in which wages are adjusted. Micro-level evidence on the importance of nominal wage rigidities is provided by Fehr and Goette (2005), Gottschalk (2005), Barattieri et al. (2014) and Fabiani et al. (2010).

implying that when the wage rigidity is slack, households must supply their labor endowment; and conversely, that when households supply less than their endowment the wage rigidity must bind.

For the rest of the union, in turn, we assume the price level is constant at all times, and we normalize the price level to one, $P_{f,t} = 1$. Because we consider aggregate shocks (changes in the central bank's policy rate), this amounts to assuming some form of nominal rigidity in the rest of the union as well. For simplicity, we are not making the underlying nominal rigidity in the rest of the union explicit, but restrict our attention to modeling in detail the domestic country.

Last, we normalize $\bar{W} = 1$. This implies that, in states when the wage rigidity binds, all prices in the monetary union are equal to one.

2.4 Domestic government

The government collects taxes $-P_{h,t}T_t$ from households and issues nominal, defaultable long-term debt B_t which is held by foreign investors. To model long-term debt, we follow Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) and assume that debt pays geometrically declining coupons. In particular, a bond issued in period t promises to pay $\tilde{\mu}(1-\mu)^{j-1}$ in period $t+j$ for all $j \geq 1$, where $\tilde{\mu} \equiv \mu + \iota$ and $\iota > 0$ is the nominal interest rate in steady state.[11] At the start of each period, the government can choose to default on its debt. Conditional on repayment, the budget constraint of the government is

$$\tilde{\mu}B_{t-1} + P_{h,t}T_t = q_t(B_t - (1-\mu)B_{t-1}), \qquad 0 < \mu < 1, \tag{9}$$

where we denote q_t the price of debt. The left-hand side captures the government's expenditure, consisting of coupon payments plus transfer payments. In turn, the right-hand side captures the government's income, given by the net issuance of new debt.

In case the government defaults, the domestic country is excluded from credit markets for a random number of periods. Moreover in this case, a utility loss $\kappa(Y_t, \xi_t)$ materializes, which depends non-negatively on output Y_t and is subject to exogenous shifts through the variable ξ_t. The utility loss captures various un-modeled costs of default, such as a reputation loss or turmoil in domestic financial markets. As is well known, without the utility loss from default, plausible levels of debt can not be generated in equilibrium.[12] The dependence of the utility loss on Y_t implies that the government' incentives to default are higher in a recession. Finally, the fact that κ can fluctuate through the variable ξ_t implies that the default decision becomes (somewhat) detached from the business cycle. The fact that default cannot be perfectly explained by fundamentals is a key feature in emerging market data (Aguiar et al., 2016) and it helps us to calibrate the model to observed spread levels in the quantitative application of the model in Section 4.

[11] We normalize the debt service payment of the bond to $\mu + \iota$, so that the default-free bond price in steady state equals 1. This allows us to interpret B_{t-1}/Y_t as debt to GDP, as we do in the rest of the paper.

[12] In the original work of Eaton and Gersovitz (1981) and Arellano (2008), rather than assuming a utility loss, the authors assumed an output loss that materializes in the event of default. Both versions of the model have been extensively used and produce very similar results. We follow Bianchi and Mondragon (2021) and assume that the loss is in terms of utility.

2.5 Foreign lenders

Debt issued by the domestic government is held by foreign investors. Foreign investors understand the governments' incentives to default, inducing them the charge an ex-ante lower price (a spread over the risk free rate). Assuming risk neutral foreign lenders, the price of debt is given by

$$q_t = \frac{\mathbb{E}_t(1-\delta_{t+1})(\tilde{\mu}+(1-\mu)q_{t+1})}{1+i_t}, \tag{10}$$

where δ_{t+1} is an indicator variable that takes the value of 1 if the government defaults in period $t+1$.

The opportunity cost of funds for foreign investors is the nominal interest rate i_t. It is the nominal interest rate set by the union-wide monetary authority. The fact that investors' opportunity cost is i_t reflects that investors are also members of the monetary union. Indeed, we assume that all debt issued by the domestic economy is held inside the monetary union, and that the monetary union as a whole is a closed economy.[13]

2.6 Market clearing

Domestic goods market clearing is given by

$$Y_t = (1-\gamma)P_{h,t}^{-\gamma}C_t + P_{h,t}^{-\zeta}X_t, \qquad \zeta > 0. \tag{11}$$

It implies that domestic output is equal to domestic plus foreign demand for domestically produced goods. Domestic demand is governed by equation (4), and is decreasing in the price of domestic goods. In turn, foreign demand is given by X_t, and is treated as an exogenous variable. In addition, we assume that also foreign demand for domestic goods is decreasing in the domestic price level, with elasticity ζ.

When the wage rigidity is slack, households supply their unit endowment implying that $Y_t = 1$. In this case, equation (11) can be seen as the equation determining $P_{h,t}$. Prices increase crowding out aggregate demand, to the point where demand is compatible with households' unit endowment. In contrast, when the wage rigidity binds this implies $P_{h,t} = 1$, and equation (11) can be seen as the equation determining Y_t: output becomes endogenous and is driven by (domestic and foreign) aggregate demand. Firms satisfy this demand by hiring the appropriate number of hours, which households supply as long as labor demand is below their unit endowment.

In the quantitative application of the model in Section 4, X_t will be the main driver of the domestic business cycle. In what follows, we assume that X_t takes values in the interval $[X_{\min}, X_{\max}]$. Moreover, we assume that $X_{\max} \leq \gamma$, implying that the domestic economy suffers from a negative

[13] Without this assumption, investors from outside the monetary union would rush to buy assets inside the monetary union in case their opportunity cost of funds lies below the rate set by the union-wide monetary authority. As stressed by Fornaro (2021), euro area countries receive the majority of their capital inflows from within the union. A model of Eurozone debt dynamics and default, where capital flows are possible both inside the monetary union and between the monetary union and investors outside the monetary union, is provided by de Ferra (2020).

output gap when the trade balance is zero. This assumption implies that the domestic government has incentives to run trade balance deficits, as we explain in detail in Section 2.8.

The second equilibrium condition is the domestic resource constraint. Combining equations (3) and (9), using that $W_t L_t = P_{h,t} Y_t$, and using equations (4)-(5), we obtain

$$P_{h,t}^{1-\gamma} C_t = P_{h,t} Y_t - \tilde{\mu} B_{t-1} + q_t (B_t - (1-\mu) B_{t-1}), \tag{12}$$

which holds in periods when the government chooses to repay. For future reference, we note that the resource constraint can also be written in terms of the trade balance. By using equations (5) and (11), we see that $P_{h,t}^{1-\gamma} C_t = P_{h,t} Y_t + C_{f,t} - P_{h,t}^{1-\zeta} X_t$. Inserting we obtain

$$P_{h,t}^{1-\gamma} X_t - C_{f,t} = \tilde{\mu} B_{t-1} - q_t (B_t - (1-\mu) B_{t-1}). \tag{13}$$

The left-hand side is the trade balance, the difference between exports and imports. The right-hand side captures how the trade balance is financed. For instance, a trade balance deficit is financed by issuing new debt in excess of the coupon payment.

Finally, conditional on default, the resource constraint is simply $P_{h,t}^{1-\gamma} C_t = P_{h,t} Y_t$ and the trade balance is equal to zero: $P_{h,t}^{1-\gamma} X_t = C_{f,t}$.

2.7 Recursive government problem and Markov equilibrium

We study the optimal policy of a benevolent government that chooses sequentially without commitment. In each period, the government decides whether to default on its debt. The equilibrium concept is therefore the one of Markov perfect equilibrium. Intuitively, in each period the government takes as given its own action in the following period, while internalizing that its current action influences its action in the future through endogenous state variables. In a rational expectations equilibrium, the government's current and expected future actions, conditional on state variables, are compatible.

As is standard in the literature, we state the government's problem using recursive notation, that is, we henceforth omit time subscript t. Denote $s = (X, \xi)$ the exogenous state. The other state variables are B, the current level of debt, and i, the nominal interest rate set by the central bank. When the government has access to financial markets, at the start of the period, it compares the value of repayment and the value of default, denoted by respectively $V^r(B, i, s)$ and $V^\delta(i, s)$

$$V(B, i, s) = \max_{\delta \in \{0,1\}} \left\{ (1-\delta) V^r(B, i, s) + \delta V^\delta(i, s) \right\}. \tag{14}$$

Let $U(C)$ denote the utility flow. The value of repayment is

$$V^r(B, i, s) = \max_{B', C} \left\{ U(C) + \beta \mathbb{E} V(B', i', s') \right\} \tag{15}$$

subject to the set of constraints

$$\begin{aligned}
&i) && P_h^{1-\gamma}C = P_hY - \tilde{\mu}B + q(B', i, s)(B' - (1-\mu)B)\\
&ii) && Y = (1-\gamma)P_h^{-\gamma}C + P_h^{-\zeta}X\\
&iii) && P_h \geq 1\\
&iv) && Y \leq 1\\
&v) && (P_h - 1)(Y - 1) = 0.
\end{aligned}$$

In the set of constraints, the price of debt q is taken as given by the government, but the government understands that q is a function of its borrowing decision B'. Constraints iii) and iv) capture that prices cannot fall below one due to downward nominal wage rigidity and that labor demand cannot be larger than households' unit endowment. Finally, constraint v) is the complementary slackness condition (8).

The value of default is

$$V^{\delta}(i, s) = U\left(\frac{X}{\gamma}\right) - \kappa\left(\frac{X}{\gamma}, \xi\right) + \beta\mathbb{E}\left(pV(0, i', s') + (1-p)V^{\delta}(i', s')\right), \tag{16}$$

where we denote p the probability of re-entering credit markets in the next period. Here we have inserted $Y = C = X/\gamma$, by using goods market clearing (11) and the fact that $P_h = 1$ under financial autarky, that is, the wage rigidity necessarily binds (see Section 2.8 for an explanation).

Definition 1 (Markov-perfect equilibrium) *For a given law of motion governing i and s, a Markov-perfect equilibrium is a set of value functions $\{V(B, i, s), V^r(B, i, s), V^{\delta}(i, s)\}$, a set of policy functions $\{\delta(B, i, s), B'(B, i, s), C(B, i, s)\}$ and a pricing function $q(B', i, s)$ such that*

1. *Given the bond price schedule, the value and policy functions solve equations (14)-(16)*
2. *The bond price schedule satisfies*

$$q(B', i, s) = \frac{\mathbb{E}(1 - \delta(B', i', s'))(\tilde{\mu} + (1-\mu)q(B'', i', s'))}{1+i},$$

where $B'' = B'(B', i', s')$.

2.8 Discussion of economic environment

In the analysis that follows, we inspect how changes in the interest rate set by the union-wide monetary authority affect debt accumulation and default incentives in the small union member. We study this issue in light of recent events in the euro area. Our main period of interest is the time after the 2008 Global Financial Crisis, when interest rates have been low, debt levels have been elevated and spreads have been volatile in several euro countries (notably in Spain, Italy, Portugal and Greece).

To be able to speak to developments in these countries, we have made two assumptions to try to capture these countries' experiences, which we add to an otherwise standard model of sovereign default.

The first assumption is that inflation is low and stable due to nominal rigidities. While these countries have experienced high inflation rates before 2008, inflation rates have been low and stable after 2008, in spite of high unemployment. This finding has triggered interests in the macro effects of downward nominal wage rigidity during the European crisis (see for instance Schmitt-Grohé and Uribe (2016), Na et al. (2018), Bianchi et al. (2019) and Wolf (2020)). We follow this literature by assuming that nominal wages are rigid downwards. At the same time, in our model surges of inflation are possible in "run-up" phases of the business cycle, when debt levels are currently low but are quickly expanding (see below).

The second assumption is that the economy has a tendency to run external imbalances. Before 2008, the countries of interest have experienced large trade balance deficits, while the deficits have subsided or turned into surpluses after 2008. To see how we capture this in the model, assume that the wage rigidity binds implying that $P_h = 1$. Combine equations (11)-(13), to obtain

$$Y_t = -\frac{1-\gamma}{\gamma}(X_t - C_{f,t}) + \frac{X_t}{\gamma}. \tag{17}$$

As this equation shows, in periods when the trade balance is zero ($X_t - C_{f,t} = 0$), output is given by X_t/γ. Recalling that we assumed $X_{\max} \leq \gamma$, this shows that $Y_t < 1$ under a zero trade balance for typical realizations of X_t. The labor market is slack, confirming that the wage rigidity binds when the trade balance is zero.[14] Intuitively, the economy suffers a structural demand shortage, in the sense that demand is not strong enough to maintain the economy at full employment when the trade balance is zero. How can the government respond to the demand shortage? As equation (17) shows, the government can fight the demand shortage and raise domestic incomes by running a trade balance deficit ($X_t - C_{f,t} < 0$). This feature of the model therefore implies a tendency by the government to take on debt and run external imbalances. Note that, when the trade balance deficit is large enough, this may even trigger inflation. As shown in equation (17), a large enough deficit implies that $Y_t > 1$. Hence the wage rigidity ceases to bind, and the domestic economy experiences a surge of inflation.

A similar "demand channel" operates in the model by Bianchi et al. (2019), who focus explicitly on the experience of Spain. In their model, the government faces a trade-off between stabilizing domestic incomes through higher government spending and incurring higher spreads. The trade-off arises, because government spending triggers a trade balance deficit, that is, it is financed with foreign debt. In our model, the government can raise consumption demand and domestic incomes by running a trade balance deficit, but it also faces the trade-off that this implies more borrowing and thus potentially higher spreads.[15]

[14] It is for this reason that the wage rigidity binds under financial autarky, which we used in equation (16).

[15] We note that the demand channel can be shut off in our model once $\gamma = 1$. In this case, households' consumption falls exclusively on foreign goods implying that domestic incomes become independent of domestic aggregate demand

In sum, when debt levels are currently low (for instance in the periods after a sovereign default), the economy has a tendency to run (large) trade balance deficits, which may even be inflationary. As debt levels climb, so do spreads which stabilizes debt levels and implies a well-defined stationary distribution. In this stationary distribution - because trade balances remain close to zero or are even positive - the wage rigidity mostly binds, implying zero inflation and a slack labor market. We believe that this setup captures well the environment in several euro members. We thus believe that our model provides a good laboratory to ask about the effects of changes in the safe interest rate on borrowing decisions and on sovereign risk.

3 Monetary policy, sovereign borrowing and sovereign risk

In this section, we study analytically how the government's incentives to borrow and default are shaped by the union-wide monetary policy. We show that a monetary tightening can either raise or reduce borrowing and sovereign risk. Our main insight is that a threshold for debt to GDP exists above which the effects of monetary policy flip. When debt to GDP is below the threshold, a monetary tightening reduces borrowing and sovereign risk, whereas the opposite happens when debt to GDP is above the threshold.

3.1 Income and substitution effect

To illustrate the forces shaping debt accumulation, we focus on the government's optimality condition in a period when it has access to financial markets. Moreover, we consider a state where the wage rigidity binds.[16] By combining constraints $i)$ and $ii)$ and inserting them in the value function (15), we then obtain

$$V^r(B,i,s) = \max_{B'} \left\{ U\left(\frac{1}{\gamma}(X - \tilde{\mu}B + q(B',i,s)(B' - (1-\mu)B))\right) + \beta \mathbb{E} V(B',i',s') \right\}$$

The first order condition is[17]

$$\frac{U'(C)}{\gamma}\left(q(B',i,s) + \frac{\partial q(B',i,s)}{\partial B'}(B' - (1-\mu)B)\right) + \beta \frac{\partial}{\partial B'}\mathbb{E}V(B',i',s') = 0, \tag{18}$$

which relates the marginal utility of consumption $U'(C)$ to the bond price schedule $q(B',i,s)$ and the continuation value $V(B',i',s')$. As we can see, the government borrows according to a standard Euler equation, but subject to an endogenous borrowing limit as the price of debt is

(see equation (17)). Hence our model reduces to a model in which output is exogenous, as in for instance Chatterjee and Eyigungor (2012).

[16] As explained in Section 2.8, the wage rigidity binds in our model except in run-up phases when debt levels are low. Because our main period of interest is the time after the Global Financial crisis (when debt levels have been high), we derive our analytical insights assuming that the wage rigidity is currently binding.

[17] To derive the first order condition, we implicitly assume that all functions in the government's problem are differentiable. We do not make these assumptions in Section 4 where we solve the model numerically, as we solve the model using value function iteration. This being said, we verify in the numerical solution of the model that all conditions derived in this section describe well the equilibrium of the model; that is, we find that the Euler equation does in fact describe the choices made by the sovereign.

endogenous. Specifically, a large (negative) derivative $\partial q(B', i, s)/\partial B'$ indicates a steep price curve which, everything else equal, discourages further borrowing.

In what follows, we study the effects of a temporary monetary tightening, that is, of a rise in i which leaves agents' expectations about the future path of monetary policy unchanged. Stated differently, the future interest rate i' becomes independent of the current interest rate i. In this case, a change in i has no *direct* effect on future variables, but only *indirect* ones through its effect on borrowing B' (the endogenous state variable). This makes the problem tractable. We study persistent interventions below, in Section 3.6.

The following proposition establishes that a rise in i can either raise or reduce B', due to an income and a substitution effect which push the incentives to borrow in opposite directions. Moreover, we characterize the conditions under which one or the other effect is dominant. Because it is instructive, we provide a sketch of the proof in the main text. A full proof can be found in Appendix A.1.

Proposition 1 *Assume that the government takes borrowing decisions according to the Euler equation* (18)*. Consider a temporary monetary policy change. Then*

$$\frac{\partial B'}{\partial i} > 0 \iff \frac{\gamma}{\sigma} < \frac{q(B', i, s)(B' - (1-\mu)B)}{C}. \tag{19}$$

Proof (sketch). Start by extracting the interest rate $1+i$ in equation (18)

$$\frac{1}{\gamma}U'\left(\frac{1}{\gamma}\left(X - \tilde{\mu}B + \frac{\tilde{q}(B', s)}{1+i}(B' - (1-\mu)B)\right)\right)\left(\tilde{q}(B', s) + \frac{\partial \tilde{q}(B', s)}{\partial B'}(B' - (1-\mu)B)\right) \\ + (1+i)\beta\frac{\partial}{\partial B'}\mathbb{E}V(B', i', s') = 0,$$

where we define the bond price exclusive of the interest rate $\tilde{q}(B', s) = q(B', i, s)(1+i)$. Because the previous equation must hold in all states (B, i, s), it can be viewed as defining borrowing B' as a function of i implicitly. It reveals that changes in i affect B' through two competing channels. First, a rise in i reduces the amount of resources that can be raised for a given amount of new debt $B' - (1-\mu)B$, the term inside the marginal utility of consumption. This corresponds to an *income effect* of higher interest rates. Second, the interest rate i multiplies the time preference rate β, thus changing the relative price of consumption across periods. This corresponds to a *substitution effect* of higher interest rates. There is also an interaction with the fact that output is endogenous in our model, which we called the demand channel in Section 2. Indeed, borrowing B' raises consumption by a factor $1/\gamma$, reflecting that domestic incomes increase when domestic consumption increases (recall equation (11)).

By differentiating with respect to i, the income effect appears as

$$\mathcal{I} \equiv \frac{U''(C)}{\gamma^2}\left(-\frac{\tilde{q}(B', s)}{(1+i)^2}(B' - (1-\mu)B)\right)\left(\tilde{q}(B', s) + \frac{\partial \tilde{q}(B', s)}{\partial B'}(B' - (1-\mu)B)\right). \tag{20}$$

In turn, the substitution effect is simply $\beta(\partial/\partial B')\mathbb{E}V(B', i', s')$. By using the Euler equation, the substitution effect can also be written as

$$\mathcal{S} \equiv -\frac{1}{1+i}\frac{U'(C)}{\gamma}\left(\tilde{q}(B',s) + \frac{\partial \tilde{q}(B',s)}{\partial B'}(B' - (1-\mu)B)\right). \tag{21}$$

When the government is a net borrower in the current period ($B' - (1-\mu)B > 0$), the income effect is strictly positive ($\mathcal{I} > 0$).[18,19] In contrast, the substitution effect is strictly negative ($\mathcal{S} < 0$). Which effect dominates? Using that $\sigma = -U''(C)(C/U'(C))$ and factorizing terms, the income effect dominates ($\mathcal{I} + \mathcal{S} > 0$) if and only if

$$\frac{U'(C)}{\gamma}\left(q(B',i,s) + \frac{\partial q(B',i,s)}{\partial B'}(B' - (1-\mu)B)\right)\left(\frac{\sigma}{\gamma}\frac{q(B',i,s)(B' - (1-\mu)B)}{C} - 1\right) > 0.$$

Because the first two factors are positive, we can write

$$\frac{\partial B'}{\partial i} > 0 \iff \frac{\gamma}{\sigma} < \frac{q(B',i,s)(B' - (1-\mu)B)}{C},$$

which is the final expression.[20] ■

3.2 A threshold for debt to GDP

The interesting aspect of equation (19) is that it implies a threshold for debt to GDP above which the effects of monetary policy flip. Specifically, the substitution effect is dominant on one side of the threshold, whereas the income effect is dominant on the other side. In this section we explore this result in depth.

Corollary 1 *Assume that the domestic country runs a zero trade balance in the current period, $X - C_f = 0$. Then the threshold for debt to GDP above which the effects of monetary policy flip is*

$$\mathcal{T} = \frac{\gamma}{\tilde{\mu}\sigma}. \tag{22}$$

Whenever $B/Y < \mathcal{T}$, a monetary tightening reduces B' (the substitution effect is dominant). Whenever $B/Y > \mathcal{T}$, a monetary tightening increases B' (the income effect is dominant).

We start with a special case which is particularly instructive. Assume that the trade balance in the current period is equal to zero, $X - C_f = 0$. From the resource constraint (12), this is the

[18] Being a net borrower means that the government does not repurchase debt. In our calibrated model, this is always the case.

[19] To see this result, note that $U''(C) < 0$. Note moreover that

$$\tilde{q}(B',s) + \frac{\partial \tilde{q}(B',s)}{\partial B'}(B' - (1-\mu)B) > 0.$$

This follows because in equation (18), the (expected) continuation value is declining in B'.

[20] The careful reader will have noticed that in this sketch of proof, we have omitted indirect derivatives due to the fact that B' is a function of i in equilibrium. As it turns out, due to an Envelope-type of condition, these indirect derivatives are irrelevant for the argument. Note moreover that on two occasions, we have used that i' is independent of i, without mentioning it explicitly. See the full proof in Appendix A.1 for all these details.

case whenever new debt issuance by the government $q(B' - (1-\mu)B)$ exactly covers the coupon payment $\tilde{\mu}B$. Intuitively, this corresponds to a tranquil state of the world in which the government intends net debt issuance to be equal to zero. By inserting $q(B' - (1-\mu)B) = \tilde{\mu}B$ in equation (19), and using that $C = Y$ when the trade balance is zero, we obtain the condition $\gamma/\sigma < \tilde{\mu}B/Y$. This yields the threshold (22) from Corollary 1.

The threshold (22) is governed by three parameters. We explain the influence of each of these parameters in turn.

The elasticity of intertemporal substitution. The first parameter is the EIS, given by $1/\sigma$. The influence of the EIS is easy to understand. The higher the EIS, the more the government is willing to shift consumption across periods. This parameter therefore determines the strength of the substitution effect. The higher the EIS, the stronger the substitution effect, making it more likely that the substitution effect dominates the income effect. This implies that the threshold $\mathcal{T}$ increases in the EIS.

Coupon payment. The parameter $\tilde{\mu}$ determines the coupon payment, that is, the fraction of debt which is re-financed in a period where the trade balance is equal to zero. When a small part of the debt is re-financed, then a rise in interest rates only affects this small part of debt, making the income effect weaker. It follows that a low $\tilde{\mu}$ increases the threshold $\mathcal{T}$ above which the income effect is dominant. As pointed out by Chatterjee and Eyigungor (2012), the fact that debt becomes insulated from changes in the bond price q is what enables models with long-term debt to explain large equilibrium amounts of debt. For the threshold that we derived, what matters is that long-term debt insulates the government from changes in the safe interest rate i.

Demand channel. The last parameter is the home bias coefficient $1-\gamma$, which governs the degree to which changes in domestic consumption change domestic income. Recall from equation (11) that the lower γ, the more consumption matters for domestic income due to a Keynesian-cross type of effect. This feedback of consumption into output enters the threshold $\mathcal{T}$ as strengthening the income effect (hence reducing the threshold). Intuitively, the demand channel makes it more costly for the domestic government to reduce borrowing following a rise in interest rates, as this also reduces domestic incomes. This makes it more attractive for the government to keep borrowing high following a rise in interest rates, which is *as if* the income effect of the interest rate change became stronger.

Corollary 2 *If the trade balance $X - C_f$ is not equal to zero, the threshold for debt to GDP above which the effects of monetary policy flip is*

$$\mathcal{T} = \frac{\gamma}{\tilde{\mu}\sigma}\left(1 + \frac{X - C_f}{Y}\left(\frac{\sigma}{\gamma} - 1\right)\right) \tag{23}$$

Whenever $B/Y < \mathcal{T}$, a monetary tightening reduces B' (the substitution effect is dominant). Whenever $B/Y > \mathcal{T}$, a monetary tightening increases B' (the income effect is dominant).

We next generalize the threshold to the case where the trade balance is non-zero in the current period. Again using the resource constraints (12) and (13), we can see that condition (19) can be written as $\gamma/\sigma < (\tilde{\mu}B - (X - C_f))/(Y - (X - C_f))$. Rearranging terms yields the threshold in equation (23).

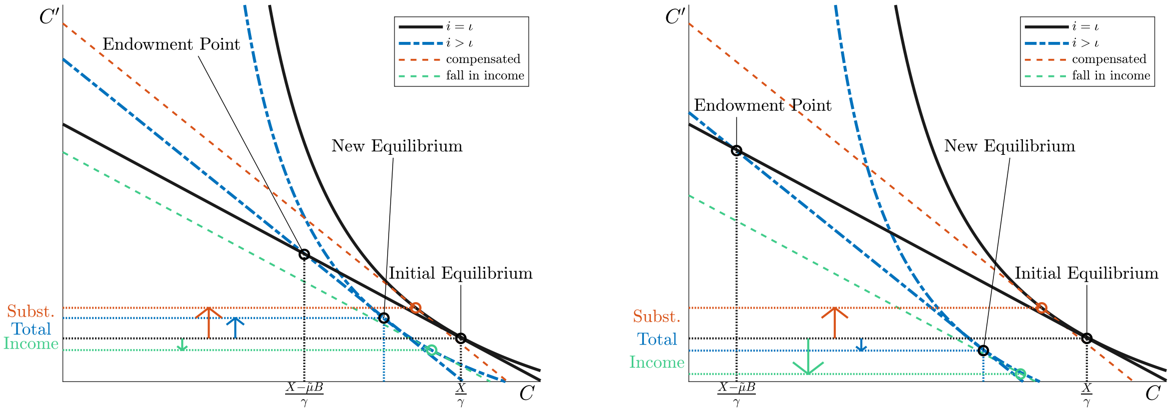
Equation (23) shows that in general, the threshold is a function of the state of the economy. In particular, the threshold depends on the trade balance to GDP ratio (positively so in the plausible case that $\sigma \geq 1$, that is, assuming an EIS smaller than 1). The intuition is the following. When the trade balance is negative, the domestic country experiences capital inflows in the form of fresh government debt (in excess of those flows which re-finance the coupon payment). Hence the government is exposed more strongly to changes in interest rates in such a period, making the income effect stronger. This explains that the threshold becomes *smaller* in a period of capital inflows (negative trade balance).

Because the trade balance is endogenous in the equilibrium of the model, it is hard to characterize the properties of the threshold further from equation (23). However, what becomes clear is that it inherits the business cycle properties of the trade balance to GDP ratio. For instance, if the trade balance is counter-cyclical the threshold becomes larger in a recession. We explore the implications of this dependence on the cycle in our quantitative application in Section 4.

3.3 Link to the two-period consumption model

The fact that interest rate changes have contrasting effects on borrowing should not come as a surprise. Indeed this result squares with undergraduate teaching of the effects of interest rate changes in the two-period consumption model. In this model, the substitution and income effects are typically explained in terms of consumption. The substitution effect of higher interest rates means that first-period consumption becomes more expensive relative to second-period consumption. Hence first-period consumption falls, whereas second-period consumption rises. In turn, for a *borrowing* agent, the income effect implies that the agent becomes poorer, which reduces consumption in both periods. The overall effect on *second-period* consumption is therefore ambiguous. Because borrowing and second-period consumption are directly related (second-period consumption equals second-period income net of debt repayment), the ambiguous response of second-period consumption maps into an ambiguous response of debt repayment. In addition, the income effect scales with the level of debt: the higher the debt stock, the stronger the income effect. In contrast, the substitution effect is inherently a marginal effect, and operates even when the debt stock is zero. This makes it clear that the income effect may start to dominate the substitution effect when the stock of debt is sufficiently high.

Perhaps the best way to make this link to the two-period consumption model is by using a diagram, which we do in Figure 1. To make the analogy as clear as possible, in the figure we make assumptions to reduce our model effectively to a two-period setup. We do so by assuming that all stochastic variables are constant (implying in particular no default risk), and by assuming that the interest rate equals its steady state value $\iota = 1/\beta - 1$ from next period onward. By implication, next-period consumption is $C' = Y' - \iota B' = (X' - \iota B')/\gamma$, where we have used equation (11) to replace Y'. In the current period, instead, consumption is given by $C = (X - \tilde{\mu}B + q(B' - (1-\mu)B))/\gamma$, where



(a) Low B: Substitution effect dominates. **(b) High B: Income effect dominates.**

Figure 1: Illustration of the effects of interest rate changes. Shown are the effects on current and future consumption of a rise in current-period interest rates i. We distinguish a case of low incoming debt (left panel), and high incoming debt (right panel). The red and green dashed lines furthermore decompose the overall effect into a compensated effect and the effect due to a lower present value of income. A full Slutsky decomposition of the effects of changes in i on C' can be found in Appendix A.3.

$q = (1+\iota)/(1+i)$. Combining both, we obtain the usual budget line in (C, C') space, connecting different levels of B'. Last, by using an indifference curve, we also plot the consumption bundle in the initial equilibrium. For simplicity, in the figure we focus on the case of zero trade balance (see Corollary 1), in which case current consumption in the initial equilibrium is $C = X/\gamma$.

Now turn to the effects of a rise in current-period interest rates i. As it is standard, the budget line tilts around the endowment point. In our model, however, the endowment point is a function of the model's endogenous state variable B. Precisely, it is given by $C = (X - \tilde{\mu}B)/\gamma$ (as this implies that $q(B' - (1-\mu)B) = 0$). The endowment point is necessarily to the left of the initial equilibrium, and the more so, the higher the level of B. We thus distinguish between a low level of B (left panel) and a high level of B (right panel). When interest rates rise, from the perspective of the initial equilibrium, the budget line not only tilts but it also *shifts*. Moreover, the shift is larger, the larger is the level of B. This implies that, depending on the level of B, the new equilibrium can be characterized either by a higher level of C' (the tilt dominates, left panel), or by a lower level of C' (the shift dominates, right panel). This is just the graphical representation of the substitution and income effects discussed before.[21] Last, recalling that $C' = (X' - \iota B')/\gamma$, the ambiguous response of C' translates into an ambiguous response of B' in the new equilibrium. The figure thus makes it clear that a rise in interest rates *reduces* B' when current debt B is low, whereas it *raises* B' when current debt B is high.

In sum, there is a clear link between our findings and the conventional two-period consumption model. However, there is also a key difference. In our model, the endowment point around which

[21] In the figure, we also decompose the overall response of the equilibrium consumption point into the compensated (Hicksian) response plus the response triggered by the fall in (the present value of) income. We refer the interested reader to Appendix A.3, where we provide the full Slutsky decomposition of the rise in interest rates on future consumption C'.

the budget line tilts is *endogenous*, as it depends on the endogenous state variable B. This implies that - depending on the current debt stock - either the substitution effect or the income effect of a change in interest rates may be dominant.[22,23]

3.4 Implications for the risk of default and spreads

We have shown that monetary policy has ambiguous effects on borrowing B'. Moreover, we saw that the current stock of debt to GDP disciplines this ambiguity. What are the implications of this finding for the risk of a sovereign default?

A conventional measure of sovereign default risk is the spread, defined as the yield-to-maturity of the bond with price q relative to a bond of equivalent maturity that does not carry a risk of default (see for instance Hatchondo and Martinez, 2009). As it can be easily shown, the spread in our model solves the expression[24]

$$spread = (\mu + \iota)\left(\frac{1}{q} - \frac{1}{q^f}\right),$$

where q^f is the price of a risk-free bond which is the solution of $q^f = (\mu + \iota + (1-\mu)q^{f\prime})/(1+i)$. However, the spread is not a good measure to assess the effects of monetary policy on default risk in the context of our analysis. To understand this point, consider the special case of the model where debt maturity is one period ($\mu = 1$). In this case, the spread becomes

$$spread = (1+\iota)\left(\frac{1+i}{\mathbb{E}(1-\delta')(1+\iota)} - \frac{1+i}{1+\iota}\right) = (1+i)\left(\frac{1}{\mathbb{E}(1-\delta')} - 1\right).$$

As this equation shows, the spread is increasing in $1+i$. This implies that a monetary tightening increases the spread, even when expected default $\mathbb{E}\delta'$ is completely unaffected by monetary policy. This effect is well known and is explained in for instance Arora and Cerisola (2001). In intuitive terms, when there is a risk of default, investors receive the safe rate only in states of the world in which default does not actually occur. To compensate, when the safe rate rises, the rate on the risky bond must increase more than one-by-one, which shows up as a rise in the spread.

To assess the impact of monetary policy on the risk of default, we therefore use an alternative

[22] The influence of the three parameters $1/\sigma$, $\tilde{\mu}$ and γ discussed in the last section can also be understood by looking at Figure 1. A larger EIS $1/\sigma$ implies less-curved indifference curves, hence when the budget line tilts there is a larger effect on C'. A larger coupon $\tilde{\mu}$ increases the distance between the initial equilibrium and the endowment point, implying a larger shift when the budget line tilts. Finally, a larger γ reduces the distance between the initial equilibrium and the endowment point, implying a smaller shift when the budget line tilts.

[23] What about the case of a non-zero trade balance (see Corollary 2)? Going back to Figure 1, a negative trade balance implies that $C > X/\gamma$ in the initial equilibrium. Hence the distance between the initial equilibrium and the endowment point becomes larger, implying a larger shift when the budget line tilts. This makes it clear that the income effect becomes stronger (the threshold falls) in a period where the trade balance is in deficit.

[24] The yield to maturity is defined as the interest rate y_t which solves:

$$q_t = \sum_{s=1}^{\infty}(1+y_t)^{-s}(1+\mu)^{s-1}(\mu+\iota).$$

Inserting q_t and q_t^f, where q_t is defined in equation (10) and where q_t^f is defined in the text, we obtain the spread by using the equation $spread_t = y_t - y_t^f$.

measure than the spread. This measure is given by

$$S \equiv (\mu + \iota)\left(\frac{1}{\tilde{q}} - 1\right) \tag{24}$$

where $\tilde{q}$ is given by

$$\tilde{q} = \frac{\mathbb{E}(1-\delta')(\mu + \iota + (1-\mu)\tilde{q}')}{1+\iota}. \tag{25}$$

The difference to before is that the numerator in the bond price is $1 + \iota$ (the steady state interest rate), rather than the actual interest rate $1 + i$. Intuitively, this modification isolates the change in the spread following a monetary tightening that is due to a change in default risk.[25] In the following, we say that a monetary tightening increases sovereign risk if it leads to a rise in S.

We next argue that the ambiguous response of borrowing B' documented before maps directly into an ambiguous response of S when monetary interventions are temporary. To understand this point, consider the probability of repayment next period

$$\mathbb{E}(1 - \delta(B', i', s')) = \textit{probability}\ (V^\delta(i', s') < V^r(B', i', s')).$$

The two values V^δ and V^r depend on i', but they do not depend directly on i. Because we study a case where i' is independent of i, a change in i impacts the repayment probability only through changes in the endogenous state variable B'. In turn, the probability of repayment falls in B', because the value of repayment V^r falls in B'. It follows that a monetary tightening reduces the probability of a default in the next period whenever it induces a decline in B', whereas it raises the default probability whenever it induces a rise in B'. The same argument applies to the probability of default two periods ahead, three periods ahead, and so forth in all periods after.[26]

Corollary 3 *Continue to assume temporary monetary interventions. Then the threshold in equation (23) applies not just to borrowing B', but equivalently to sovereign risk S. Whenever $B/Y < \mathcal{T}$, a monetary tightening reduces S. Whenever $B/Y > \mathcal{T}$, a monetary tightening increases S.*

In sum, when monetary interventions are temporary, the threshold in equation (23) describes not just the behavior of borrowing, but equivalently the reaction of sovereign risk.

3.5 Immediate default

To this point, we have taken it for granted that the government chooses to repay in the current period, such that it maintains access to the credit market from this period to the next. However, of course the government may also choose to default in the current period, and this decision will be influenced by monetary policy as well. We discuss this issue briefly in this section.

[25] Note that $q^f = 1$ by using this modification, hence no q^f appears in the spread as defined by equation (24)

[26] Formally, the following corollary assumes that $\tilde{q}'$ is downward sloping in B'. That this property is satisfied in models with long-term debt such as the one studied here has been shown in Chatterjee and Eyigungor (2012).

The government chooses to default in the current period whenever the value of default is higher than the value of repayment, that is if

$$V^{\delta}(s) < V^{r}(B, i, s). \tag{26}$$

Here we have focused on temporary monetary interventions, such that the value of default is not directly a function of the interest rate i (see equation (16)). As long as the government is a net borrower ($B' - (1-\mu)B > 0$), a rise in interest rates i reduces the value of repayment V^r.[27] Hence a sufficiently large monetary tightening may imply that the value of repayment falls below the value of default. Conversely, a cut in the interest rate by the central bank may help prevent an imminent default.

In what follows, we focus on scenarios in which monetary policy does not alter the current default decision of the government. Even in this case, the fact that monetary policy may directly affect the government's incentives to default will be relevant for understanding the implications of persistent monetary interventions, which we discuss in the next section.

3.6 The effects of persistence

We now relax the assumption that the monetary intervention is temporary. Instead, we assume that the intervention is persistent with persistence parameter $0 \leq \rho \leq 1$. Because a persistent intervention can be understood as the sum of a temporary intervention (analyzed before) plus an anticipated future intervention, we frame our discussion around the effects of anticipated future interventions on B' and on S.

The effects of anticipated future interventions can be understood as the sum of two channels. First, a future monetary tightening implies that borrowing decisions in the future change, which because the government is forward looking, changes borrowing decisions already today. This effect would even be present in the absence of uncertainty, that is, if X and ξ were not random variables implying that the risk of default would be zero. Second, in the case of persistent interventions, monetary policy has a *direct* effect on future default risk that is independent of the response of borrowing. This follows from the arguments made in Section 3.5. A future monetary tightening raises the government's future incentives to default, which raises sovereign risk S today even when debt levels are unaffected.[28] Of course, this feeds back into the response of borrowing as well, because the government's incentives to borrow change following a change in the risk of a future default.

Because of the interaction of these effects, the problem becomes analytically intractable. However, we can obtain a partial characterization by isolating the implications of the first channel, the fact that a future monetary tightening changes current borrowing even in the absence of uncer-

[27] This follows because a rise in i strictly reduces the choice set by the government in equation (15), which must be welfare reducing. In the calibrated model in Section 4, we find that the government is always a net borrower in equilibrium.

[28] Moreover, a future monetary tightening may also trigger an immediate default. We proceed under the assumption that this is not the case (see Section 3.5).

tainty. This allows us to generalize the threshold (23) to the case where monetary interventions are persistent.

Proposition 2 *Assume that X and ξ are constant. Assume that $\iota = 1/\beta - 1$, implying that the model without uncertainty is stationary. Consider a persistent monetary tightening, with persistence parameter $0 \leq \rho \leq 1$. Define the auxiliary parameter $\tilde{\rho}$ such that*

$$\tilde{\rho} \equiv \frac{1 + \iota - (1-\mu)\rho}{1 + \iota - \rho} \geq 1.$$

Then the threshold in equation (23) can be generalized to

$$\mathcal{T} = \frac{\gamma\tilde{\rho}}{(\tilde{\mu} + \iota(1-\tilde{\rho}))\sigma}\left(1 + \frac{X - C_f}{Y}\left(\frac{\sigma}{\gamma\tilde{\rho}} - 1\right)\right). \tag{27}$$

Whenever $B/Y < \mathcal{T}$, a monetary tightening reduces B'. Whenever $B/Y > \mathcal{T}$, a monetary tightening increases B'.

Proof. In the Appendix A.2. ■

The threshold in equation (27) is a generalization of the threshold in equation (23), in the sense that the latter is nested when persistence becomes equal to zero $\rho = 0$.[29,30]

As before, the threshold in equation (27) equals a constant plus a term that depends on the trade balance. By inspecting the former, we note that the effect of persistence is to *raise* the threshold $\mathcal{T}$. This is because $\tilde{\rho}$ is increasing in the persistence parameter ρ. Hence, adding persistence to the monetary intervention acts like a strengthening of the substitution effect, making it more likely that a monetary tightening reduces borrowing. What is the intuition for this effect? A monetary tightening in the future reduces future consumption unambiguously.[31] The government reacts by reducing its borrowing today, in order to smooth consumption. Hence persistence makes it more likely that the government reduces its borrowing today following a monetary tightening, which appears as a strengthening of the substitution effect.

However, as we noted above, the threshold in equation (27) represents only a partial characterization of the effects of persistent interventions on borrowing, as it abstracts from the direct effect of monetary policy on future default risk that may matter for current borrowing decisions as well. Taking into account this effect may shift the threshold in equation (27), implying that it becomes only an approximation.[32]

[29] To see this, note that $\tilde{\rho} = 1$ when $\rho = 0$.

[30] This shows that the threshold in equation (23) would hold also in a model without uncertainty, implying that default *per se* does not matter for the threshold. Default only matters for the threshold insofar, as it changes the business cycle properties of the trade balance. See Section 4 for a discussion.

[31] Going back to the two-period consumption model, we noted that the effect of a rise in interest rates for *current* consumption are unambiguous for a borrowing agent. Hence a monetary tightening in period $t + k$ unambiguously reduces consumption in this period.

[32] This being said, in our numerical analysis of the model, we find that the threshold (27) predicts perfectly the behavior of borrowing even once we allow for uncertainty. Without proving it, we therefore suspect that the response of default risk only affects the magnitude of the change in debt, but not its sign (whether borrowing rises or declines).

What about the effect of persistent monetary interventions on sovereign risk S? As we noted above, when monetary interventions are persistent, a monetary tightening can raise S even when borrowing is unchanged. Unlike in the case of temporary interventions, the threshold in equation (27) may thus not be used to predict the behavior of sovereign risk. We take up this issue numerically in the calibrated model in Section 4. To anticipate, we find that a threshold for debt to GDP describing the behavior of sovereign risk still exists. Hence when debt to GDP is low enough, a monetary tightening reduces sovereign risk even when the monetary tightening is persistent. We also find that this threshold does not overlap with the threshold that describes the behavior of borrowing B', but instead is strictly *lower*. This result squares with the intuition developed in this section, as it implies the existence of states (intermediate levels of debt to GDP) where a monetary tightening *raises* sovereign risk S, but it *reduces* borrowing B'.

4 Quantitative analysis

We have shown that the effects of a monetary tightening on sovereign borrowing and sovereign risk in the small union member are ambiguous. We have also shown that a threshold for debt to GDP exists above which the effects of monetary policy flip. This begs the question how *large* is the threshold in a reasonable calibration, and what the existence of such a threshold implies for the conduct of monetary policy. We take up this issue in this section.

4.1 Back-of-the-envelope calculation

Before solving the model numerically, we first conduct a back-of-the-envelope calculation of the threshold implied by our model. This can be done because the threshold depends only on parameters and on a variable which is observable (the trade balance to GDP ratio) - see equation (27). The parameters are the home bias coefficient $1-\gamma$, the coupon payment $\tilde{\mu}$ and the EIS $1/\sigma$. We discuss next how we obtain the three parameters.

Using an annual calibration, we assume that the nominal interest rate is 2% in steady state, implying $\iota = 0.02$.[33] The average maturity of debt $1/\mu$ is observable in Eurozone countries. For instance, it is 7 years in Italy, implying that $\mu = 0.14$. This implies $\tilde{\mu} = \mu + \iota = 0.16$ for the coupon payment. Next, equation (17) establishes a link between γ and gross exports to GDP and the trade balance to GDP ratio in our economy. We can measure the evolution of both in the post-2008 period, and take an average in order to obtain a plausible value for γ.[34] This yields $\gamma = 0.27$ (again for Italy), implying that about three quarters of the consumption basket are domestic goods. The

[33] Recall that we abstract from trend inflation. In turn, a two-percent yearly real rate is a standard target in many studies.

[34] Solving equation (17) for γ, we obtain

$$\gamma = \left(1 - \frac{X_t - C_{f,t}}{Y_t}\right)^{-1} \left(\frac{X_t}{Y_t} - \frac{X_t - C_{f,t}}{Y_t}\right).$$

In states where the trade balance is zero ($X_t - C_{f,t} = 0$), the parameter γ therefore measures gross exports to GDP.

	Italy	Spain	Greece	Portugal
Debt to GDP (non-residents)	55%	55%	88%	59%
Debt to GDP (non-residents + ECB)	86%	78%	101%	88%
Threshold $\mathcal{T}$ (1 year)	51%	55%	0%	11%
Threshold $\mathcal{T}$ (2 years)	58%	62%	0%	18%
Threshold $\mathcal{T}$ (3 years)	64%	68%	6%	24%

Table 1: Public debt and thresholds in 2020. Shown are external public debt to GDP, total and according to holdings, in Italy, Spain, Greece and Portugal. The table also shows the model-implied threshold $\mathcal{T}$, defined in equation (27), for three values of ρ (0, 0.5 and 0.66). Data sources are in Appendix B.

last parameter is the EIS. The evidence suggests that this parameter is quite close to zero.[35] For instance, Best et al. (2019) recently estimated the EIS to be 0.1. More broadly, Havránek (2015) conducts a meta analysis and finds an average estimate for the EIS of 0.3 to 0.4. We proceed by placing ourselves in the middle of these estimates, by using $1/\sigma = 0.25$.

We combine these parameters with the trade balance to GDP ratio in the year 2020 (the latest available) in order to compute the threshold for this year. We proceed in this way not just for Italy, but also for Spain, Greece and Portugal, all countries in which debt levels are currently quite high. We then contrast the threshold with actual debt levels. The result is in Table 1.

When inspecting the numbers, two things are important to bear in mind. First, in our model all debt is held by foreign creditors. The relevant statistic to consider is thus *external* debt - debt held by non-residents, plus potentially debt held by foreign public creditors such as the ECB.[36] Second, our model predicts that the threshold is *endogenous* to the monetary stance. In particular, equation (27) implies that the threshold is higher when the monetary intervention is more persistent. We thus report three different thresholds in the table, the first following a purely transitory intervention (1 year, $\rho = 0$), as well as two longer-term interventions (expected duration of 2 years, $\rho = 0.5$, and 3 years, $\rho = 0.66$).

For Italy and Spain we find thresholds in the order of 50% - 70% external debt to GDP, the same ballpark as the current debt levels in these countries (excluding holdings by the ECB). When including ECB holdings, debt levels exceed the threshold even for a 3-year intervention, implying that a rate hike is likely to increase debt levels further in these countries. For Greece we find that the threshold in 2020 was almost zero, reflecting that the trade balance was very negative (minus 8% to GDP in this year). As explained above, in a period of net capital inflows the income effect becomes stronger, implying that the threshold declines. The same was true for Portugal, where we also find thresholds in 2020 to be very low.

Of course, this exercise is only suggestive and the numbers should not be taken too literally. In

[35] See Hall (1988) for early evidence that the EIS is not much different from zero.

[36] Bocola et al. (2019) develop a model of the European debt crisis in which the total stock of debt, rather than external debt, is the relevant state variable for the government's incentives to default. We suspect that a threshold for debt to GDP separating the effects of monetary policy exists also in their model. Hence the relevant statistic may also be the total stock of debt, which is considerably higher than external debt in all economies considered.

particular, one caveat is that both debt to GDP and the trade balance to GDP ratio which shapes the threshold are endogenous, and respond to the monetary intervention as well. To obtain further insights, we therefore proceed by solving the model numerically.

4.2 Calibration

We proceed with the case of Italy. Following Section 4.1, we set the parameter values $\iota = 0.02$, $\mu = 0.14$, $\gamma = 0.27$ as well as $1/\sigma = 0.25$. Furthermore, following Bianchi et al. (2019), we set the annual probability of reentry after default $p = 0.18$. For lack of a better number, we assume the elasticity of foreign and domestic demand for domestically produced goods is identical, implying that $\zeta = 0.27$ (see equation (11)).[37]

All remaining parameters are set to match key moments in the data through simulation. Details on the construction of moments are provided in Appendix B.3. We calibrate the discount factor β to match the average level of external debt to GDP (non-residents) between 2000 and 2019, given by 49.4%. The logarithm of foreign demand follows an AR(1) process

$$\log(X_t/\mu_X) = \rho_X \log(X_{t-1}/\mu_X) + \sigma_X \varepsilon_t$$

with $\varepsilon_t \sim_{i.i.d.} \mathcal{N}(0,1)$. The parameters μ_X, ρ_X and σ_X are set to match mean unemployment (our measure for the output gap) as well as the standard deviation and autocorrelation of detrended GDP. For the utility loss in default, we follow Bianchi et al. (2019) and use

$$\kappa(Y_t, \xi_t) = \max(0, L_0 + L_1 \log(Y_t)) + \xi_t.$$

As explained in Section 2, we assume that the cost of default is subject to shocks through the variable ξ_t. This shock plays a dual role in our analysis. First, it partly delinks the default decision from the economy's current state, reducing the sensitivity of the spread to economic fundamentals. This reduces the disciplining effect of spreads on borrowing which means higher spread levels are reached in equilibrium.[38] Second, as is well known, this shock improves the convergence properties of the solution algorithm in models with long-term debt.[39] Following this literature, we assume ξ_t is i.i.d. and follows a logistic distribution with location parameter 0 and scale parameter s_x.

We set s_x to match the Italian spread over German bonds during 2006-2021. We calibrate the parameters L_0 and L_1 to match the standard deviation of the spread as well as the correlation of the trade balance with GDP. To understand the last point, note that the emergence of spreads tends to make the trade balance counter-cyclical, a well-known feature of sovereign default models. Hence the loss function κ which matters for spreads can be used to discipline the cyclicality of the

[37] This elasticity is not important for our results, as we calibrate the model such that the wage rigidity binds most of the time in the equilibrium of the model.

[38] As discussed in Aguiar et al. (2016), the spread tends to react very steeply to borrowing in the absence of this shock. As a result, optimal debt levels tend to lie just below this steep part, at a low spread level.

[39] See Chatterjee and Eyigungor (2012) for an explanation. See Gordon (2019), Dvorkin et al. (2021) and Arellano et al. (2020) for recent applications of this method to sovereign default models.

Parameter	Value	Target	Data	Model
ι	0.02	Risk free rate	-	-
σ	4	EIS	-	-
γ	0.27	Home bias	-	-
μ	0.14	Debt maturity	-	-
p	0.18	Exclusion period	-	-
ζ	0.27	Elasticity of domestic demand	-	-
β	0.945	mean(B_t/Y_t)	0.499	0.529
μ_X	0.257	mean$(1-L_t)$	0.094	0.088
σ_X	0.022	std(Y_t)	0.023	0.021
ρ_X	0.65	corr(Y_t, Y_{t-1})	0.640	0.610
s_ξ	0.66	mean$(spread_t)$	0.014	0.014
L_0	2.262	corr$(X_t - C_{f,t}, Y_t)$	-0.170	-0.140
L_1	20	std$(spread_t)$	0.011	0.006

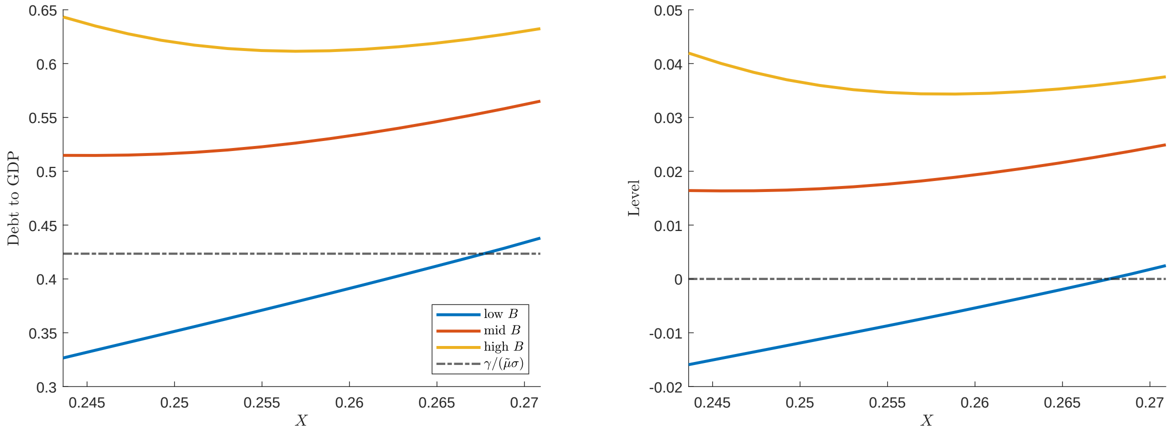
Table 2: Parameters. Details on the calibration can be found in Appendix B.

trade balance. Matching this cyclicality is important for us, because the cyclicality of the trade balance maps directly into the cyclicality of the threshold, from equation (23). The calibrated parameters and the fit of the model are summarized in Table 2. This calibration implies - in line with our earlier explanations - that the wage rigidity always binds in the equilibrium of the model except in the periods succeeding a default (see Appendix B.3).

To solve the model numerically, we discretize the state space for B and X. We approximate the process of X by using the Tauchen (1986) algorithm. In our calibration, the highest level of X is smaller than γ, in line with our earlier assumptions. We do not need a grid for ξ because this shock does not matter for the solution of the model conditional on repayment in the current period. In turn, we solve the model only at the steady state for the interest rate, $i = \iota$. To study monetary interventions, we use "MIT shocks", shocks that are zero-probability events. To study such shocks, it is not necessary to solve for policy functions outside of the steady state value for i. This implies that in all policy functions that follow, we omit the two arguments $i = \iota$ and ξ for better readability. More details on the numerical algorithm can be found in Appendix B.5.

4.3 The Fear of Hiking zone

In the equilibrium of the model, the threshold is a function of the state variables and hence there exists a policy function $\mathcal{T}(B, X)$. To obtain this policy function, we evaluate equation (23) at the policy function of the trade balance to GDP ratio. The result is in Figure 2. In the left panel, we plot the equilibrium threshold against the foreign demand variable X, by contrasting three different levels of current indebtedness B. In this figure, we also plot the baseline threshold $\mathcal{T} = \gamma/(\tilde{\mu}\sigma)$ (see equation (22)), which applies in states of the world where the trade balance is equal to zero



(a) Threshold. **(b) Trade balance to GDP.**

Figure 2: Policy functions for threshold and trade balance. Shown are policy functions $\mathcal{T}(B, X)$ (by using equation (23)) and $(X - C_f(B, X))/Y(B, X)$, both drawn against X for three different values of B.

(the dashed-dotted line). In turn, in the right panel we show the trade balance to GDP ratio in the equilibrium of the model.[40]

The first thing the figure shows is that the threshold may depart substantially from its baseline value, reflecting that the trade balance may be substantially different from zero. Turning to the comparative statics, for a given level of foreign demand X, a higher level of debt B implies a higher trade balance and thereby a higher threshold. The trade balance response reflects that the country starts to repay when its current debt stock is high. But capital outflows imply that the income effect becomes weaker, hence that the threshold rises (recall Section 3.2). The comparative statics with respect to X (keeping fixed B) are more involved as they depend on the level of B itself. Specifically, when current debt levels are low, then a higher X raises the trade balance and therefore the threshold. However, when current debt levels are high, then a higher X *reduces* the trade balance and therefore the threshold. This behavior of the trade balance (and by implication of the threshold) highlights the role of sovereign default in our model. When debt levels are low, spreads are low and the country borrows in a recession for consumption-smoothing purposes, which explains that both variables are pro-cyclical.[41] In contrast, when debt levels are high then spreads start to climb in a recession which creates incentives for the government to repay. Hence the trade balance becomes larger in a recession, making both the trade balance and the threshold counter-cyclical.

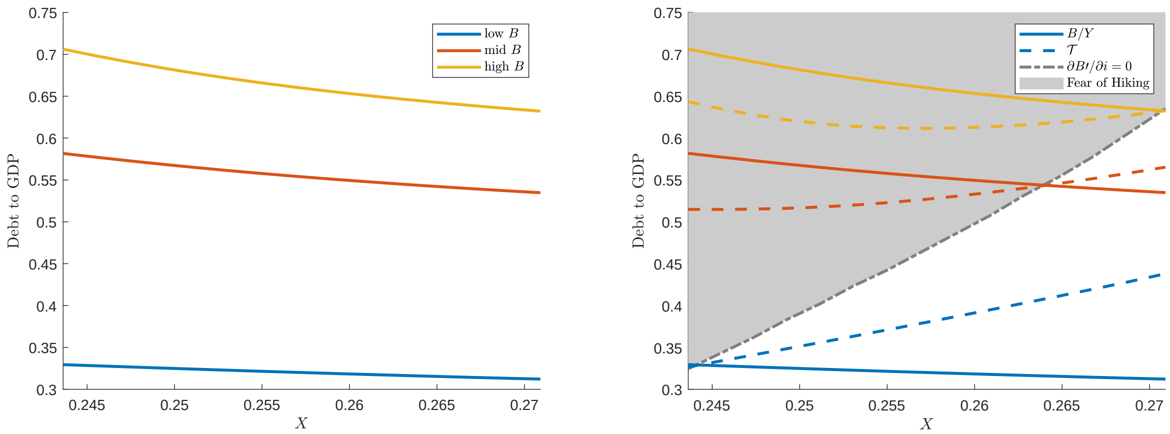
We also study unconditional statistics, in Table 3. The first row shows the mean of the stationary distribution of $\mathcal{T}$. The mean threshold is 0.51 and is thus 9 percentage points higher than the baseline threshold (22), which is 0.42. This reflects that on average, the country is running trade

[40]We verify numerically that in all points shown in the figure, the government chooses to repay in the current period (no immediate default). More precisely, the government chooses to repay unless the utility loss shock ξ_t is very negative. The same holds true for all figures that follow in this section.

[41]We are not showing it, but the policy function for output Y shows the expected pattern: it is increasing in foreign demand X, as it is intuitive from equation (11). Hence the country is in a recession when X is low, and we may infer pro-cyclicality and counter-cyclicality from looking at Figure 2.

Statistic	Value
mean($\mathcal{T}$)	0.5116
corr($\mathcal{T}, Y$)	-0.2122
mean($\mathbb{1}_{\mathcal{I}}$)	0.7056
corr($\mathbb{1}_{\mathcal{I}}, Y$)	-0.6781

Table 3: Unconditional statistics. Shown are unconditional statistics of the threshold (23) and the indicator variable (28). The indicator variable equals 1 in states where the income effect is dominant, i.e. where the economy is in the Fear of Hiking zone.



(a) Debt to GDP.

(b) Debt to GDP and threshold.

Figure 3: Contrasting debt to GDP and the threshold. Shown are policy functions $B/Y(B, X)$ in the left panel, and $B/Y(B, X)$ as well as $\mathcal{T}(B, X)$ in the right panel, all drawn against X on the horizontal axis for three different values of B. In the right panel, the intersections of debt to GDP and the threshold indicate the set of points where $\partial B'/\partial i = 0$. Moreover, the Fear of Hiking zone indicates the set of points where the income effect is dominant, hence where $\partial B'/\partial i > 0$ as well as $\partial S/\partial i > 0$.

balance surpluses. The second row shows the unconditional correlation between the threshold and GDP. The correlation is negative, hence the threshold co-moves negatively with the cycle. Looking at equation (23), it is clear that the threshold inherits this property directly from the trade balance to GDP ratio (where a negative correlation with output was a calibration target). In turn, the last two rows in Table 3 are explained below.

From Figure 2 alone, one cannot determine the states of the world in which the country is on either side of the threshold. To do so, one must compare the threshold with actual levels of debt to GDP, which are also determined in the equilibrium of the model. The policy function for debt to GDP is shown in the left panel of Figure 3. The figure is constructed in the same way as Figure 2. We see that a higher B implies that debt to GDP rises. Moreover, we see that for a given B, a lower X implies that debt to GDP rises. This happens because a lower X implies that also output Y is lower, hence debt to GDP must increase mechanically.

The interesting aspect is to compare the equilibrium for debt to GDP with the equilibrium for the threshold, which is done in the right panel of Figure 3. To create this figure, we simply draw the left panels of Figures 2-3 on top of each other. Let us again go through the comparative statics. For a given level of B (movement along the lines), debt to GDP tends to lie above the threshold for low levels of X, but below the threshold for high levels of X. At the intersections of the curves, debt to GDP and the threshold coincide. In the figure, we connect these intersections through a gray dashed-dotted line. Hence, one can find the region where the income effect dominates (implying that a rate rise raises borrowing and sovereign risk) to the left of the gray dashed-dotted line. We call this part of the state space the "Fear of Hiking" zone, and make it gray in the figure for better visibility. Because output Y is low when X is low, we see that the economy is more likely to be in the Fear of Hiking zone when the economy is in a recession. In turn, a higher level of B also makes it more likely that the economy is in the Fear of Hiking zone.

We can also make the point that the economy is more likely to visit the Fear of Hiking zone in a recession by looking at unconditional statistics. To do so, we define an indicator variable $\mathbb{1}_{\mathcal{I}}$ which takes the value of 1 in states of the world where the income effect is dominant (the economy is in the Fear of Hiking zone)

$$\mathbb{1}_{\mathcal{I}}(B,X) \equiv \mathbb{1}\left\{\frac{B}{Y(B,X)} > \mathcal{T}(B,X)\right\}. \tag{28}$$

We study the properties of $\mathbb{1}_{\mathcal{I}}$ in the last two rows in Table 3. The mean value of $\mathbb{1}_{\mathcal{I}}$ in the stationary distribution of the model is 0.7059, implying that 71 percent of the time, the economy finds itself in the Fear of Hiking zone. The last row shows the correlation between $\mathbb{1}_{\mathcal{I}}$ and output. The correlation is -0.68, confirming that the substitution effect tends to dominate in good times, whereas the income effect tends to dominate in a recession.[42]

We conclude this discussion with a remark on the construction of the gray dashed-dotted line in the right panel of Figure 3. As we explained above, we obtain this line by using the policy function for $\mathcal{T}$, from equation (23). An alternative - and more direct - way of obtaining this line is by using numerical differentiation. Indeed, one may compute the derivative $\partial B'/\partial i$ in the solution of the model and check if the derivative is positive, zero, or negative. We verified that the set of points where the derivative is zero equals exactly the gray dashed-dotted line in the figure. This testifies to the accuracy of our numerical algorithm, and it also confirms that equation (23) provides a correct description of the equilibrium of the model. A subtle issue is that by using the numerical approach, one can obtain *only* the gray dashed-dotted line, but not the rest of the policy function of the threshold. This is because this method can only find the threshold points which are attained by the model in equilibrium. Still, knowing this line is enough to be able to find the Fear of Hiking zone in the equilibrium of the model. We exploit this fact below, in Section 4.4.

[42]It might be confusing to the reader that we found $\mathcal{T}$ to be counter-cyclical (second row in Table 3), but $\mathbb{1}_{\mathcal{I}}$ to be counter-cyclical as well (fourth row). After all, a counter-cyclical $\mathcal{T}$ implies that the threshold is higher in a recession, hence that the income effect should be less likely to be dominant in a recession (implying a pro-cyclical $\mathbb{1}_{\mathcal{I}}$). What this argument ignores is that not only $\mathcal{T}$ is counter-cyclical, but also debt to GDP. Indeed as shown in Figure 3 right panel, debt to GDP is even *more* counter-cyclical than $\mathcal{T}$. Thus, while $\mathcal{T}$ tends to rise in a recession, debt to GDP tends to rise by *even more*, implying that debt to GDP tends to exceed the threshold in a recession. This explains why we find $\mathbb{1}_{\mathcal{I}}$ to be counter-cyclical.

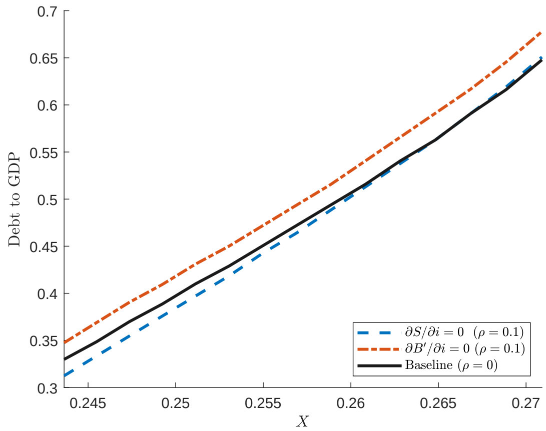


Figure 4: Persistent interventions. The figures shows the set of points where $\partial B'/\partial i = 0$ as well as $\partial S/\partial i = 0$, and contrasts both with those obtained under temporary interventions ($\rho = 0$, see Figure 3, right panel).

4.4 Persistent interventions

We next extend our quantitative results to the case of persistent monetary interventions. As we discussed in Section 3.6, adding persistence to the monetary intervention has two effects. First, it shifts the threshold for debt to GDP which characterizes the behavior of B'. Second, it decouples the response of borrowing decisions B' and the response of sovereign risk S. We illustrate these results by setting the autocorrelation of the monetary shock to $\rho = 0.1$. Because we do not have closed-form expressions for the thresholds for the case of persistent interventions, we proceed by using numerical differentiation. Specifically, as explained at the end of the last section, we search for the set of points in the state space where $\partial B'/\partial i = 0$ as well as $\partial S/\partial i = 0$ in the equilibrium of the model.

The result is in Figure 4. For transparency, we also show again the case of temporary monetary interventions (the gray dashed-dotted line from Figure 3, the right panel). In line with our analytical insights (see Section 3.6), the line characterizing the response of B' has shifted upward relative to the case of temporary interventions. Adding persistence therefore *shrinks* the Fear of Hiking zone for B'. At the same time, the line characterizing the response of S no longer overlaps with the one describing the behavior of B', but is instead strictly lower. As we discussed in Section 3.6, this implies the existence of states where following a monetary tightening, sovereign borrowing declines but sovereign risk still rises. Another result is that the line characterizing the response of S may even shift downward relative to the case of temporary interventions, implying that a monetary tightening is now more likely to increase sovereign risk. Interestingly, we obtain this result despite the fact that B' is more likely to decline when the rate hike is persistent, which by itself tends to reduce sovereign risk. In sum, adding persistence may even *enlarge* the Fear of Hiking zone for S relative to the case of temporary monetary interventions.

4.5 Policy scenarios

What does the existence of the threshold and hence of a Fear of Hiking zone imply for the conduct of monetary policy? Here we take a first stab at this issue by going through three policy experiments. To be clear, the objective of this section is not to provide a careful quantitative evaluation of actual policy proposals or to replicate any particular historical event. Moreover, we keep discussing our findings from a purely positive standpoint, without touching on normative questions or optimal monetary policy. In fact, such tasks are beyond the scope of this paper. Rather, the objective of this section is to show what certain types of monetary interventions imply for the dynamics of debt and sovereign risk in this economy.

4.5.1 Rate hike and run-up dynamics

We first study the effects of monetary policy in a "run-up" phase - when debt levels are initially low but are quickly expanding. We illustrate that a rate hike can put a break on debt accumulation and thereby reduce the risk of a sovereign default.

We fix an initial level of debt implying that the economy is outside of the Fear of Hiking zone. We then compute an impulse response function, expressing variables in percent deviation from the paths they would have followed in the absence of intervention (i.e., keeping i at its steady state level ι). More precisely, we compute a generalized impulse response, simulating paths for X after the start of transition and computing an average response. To obtain quantitatively meaningful results, we study a rate hike which is both large (2 percentage points on impact) and persistent ($\rho = 0.8$). The result is in Figure 5. We display the dynamic response of four variables. The first three variables are the nominal interest rate i, debt to GDP B/Y and sovereign risk S. We show the paths of these variables conditional on default not occurring in sample, that is, we assume the default cost variable ξ_t realizes low enough such that the government always finds it desirable to repay. The fourth variable, in turn, reports the number of defaults. To construct this variable, we assume that ξ_t follows its law of motion implying that defaults occur in sample. We then count the number of paths along which a default occurred and divide by the total number of simulated paths.

As the figure illustrates, the rate hike triggers a decline in public debt and in sovereign risk, in line with our earlier insights. Both variables revert as the rate hike dies out. At their troughs, debt to GDP declines by 1 percentage point and sovereign risk by 0.025 percentage points, respectively. Our measure counting the number of defaults is initially unaffected but then declines steadily to minus 0.42 percentage points. What this shows is that the monetary intervention has prevented 0.42 percentage points of default events until 30 years after the start of the intervention.[43] While these numbers may appear not overly large, recall that we consider an initially low level of debt in this experiment, which implies that sovereign default risk is low to begin with.

[43] In levels, 13.2% of all simulated paths involved a default in the 30 years of the experiment, but only 12.78% under the monetary intervention.

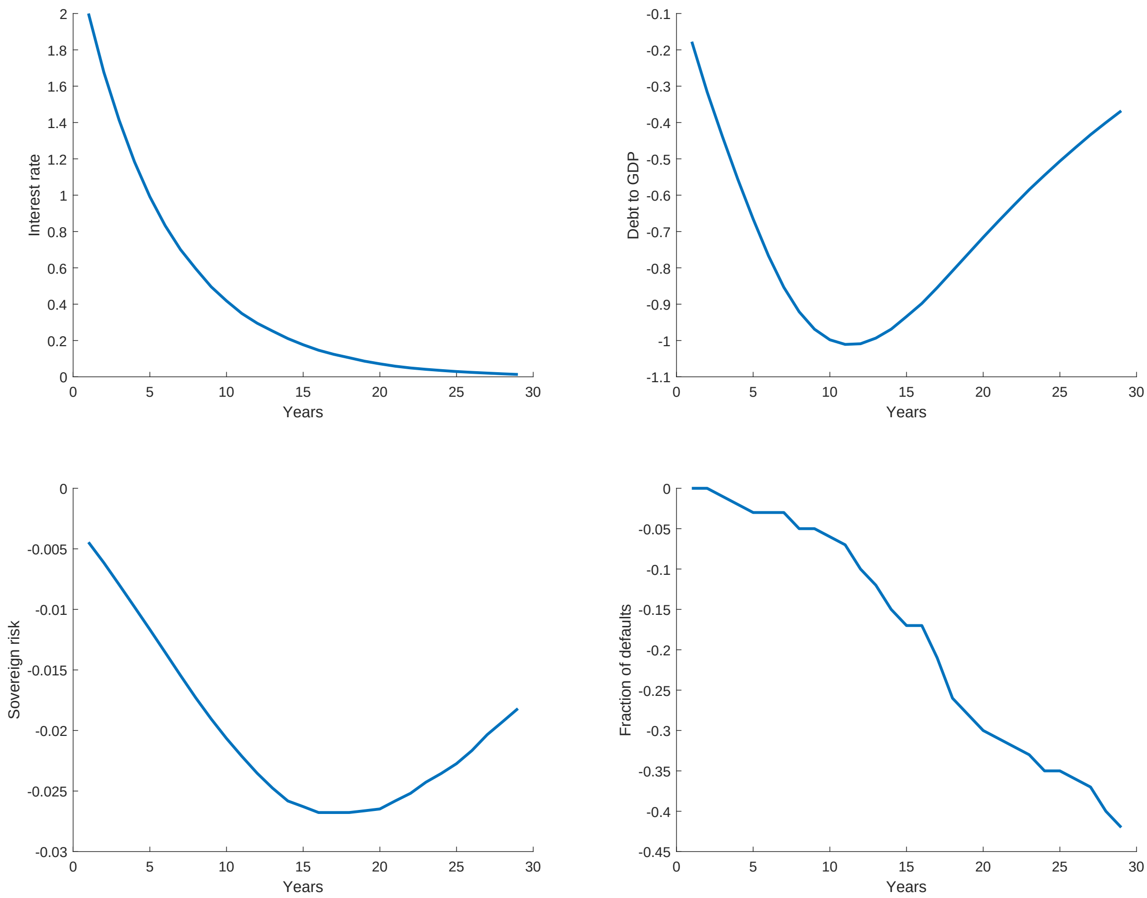


Figure 5: Rate hike outside the Fear of Hiking zone. Shown are the effects of a rate hike on debt to GDP, sovereign risk and the fraction of defaults when debt levels are initially low. All variables are expressed in percentage point deviation from the trend they would have followed in the absence of intervention.

4.5.2 Rate cut in the Fear of Hiking zone

We next study the effects of monetary policy in an “acute” phase, when public debt is high and the risk of default is elevated (i.e. the economy is in the Fear of Hiking zone). We illustrate that in this case, it is through a rate *cut* that debt levels and the risk of a sovereign default are reduced - the opposite as in the previous subsection.

We conduct the same type of analysis as in the previous subsection, except that we now assume that debt levels are initially high. The result is in Figure 6. We find that the rate cut reduces debt to GDP substantially, by more then 2 percentage points on impact. Similarly, sovereign risk drops on impact by 1.2 percentage points. Again, both variables revert as the monetary easing subsides. The fraction of defaults declines by 4.5 percentage points on impact. This implies that the rate cut has prevented 4.5 percentage points of *imminent* defaults, i.e. defaults that would otherwise have occurred in the period of intervention (recall Section 3.5). In the periods after, the fraction of defaults rises. What this reflects is that some of the defaults prevented by monetary policy in the first period now occur in later periods. However, 30 periods after the start of the intervention the fraction of defaults is still negative, at about -2.5 percentage points. Again, this reflects that the

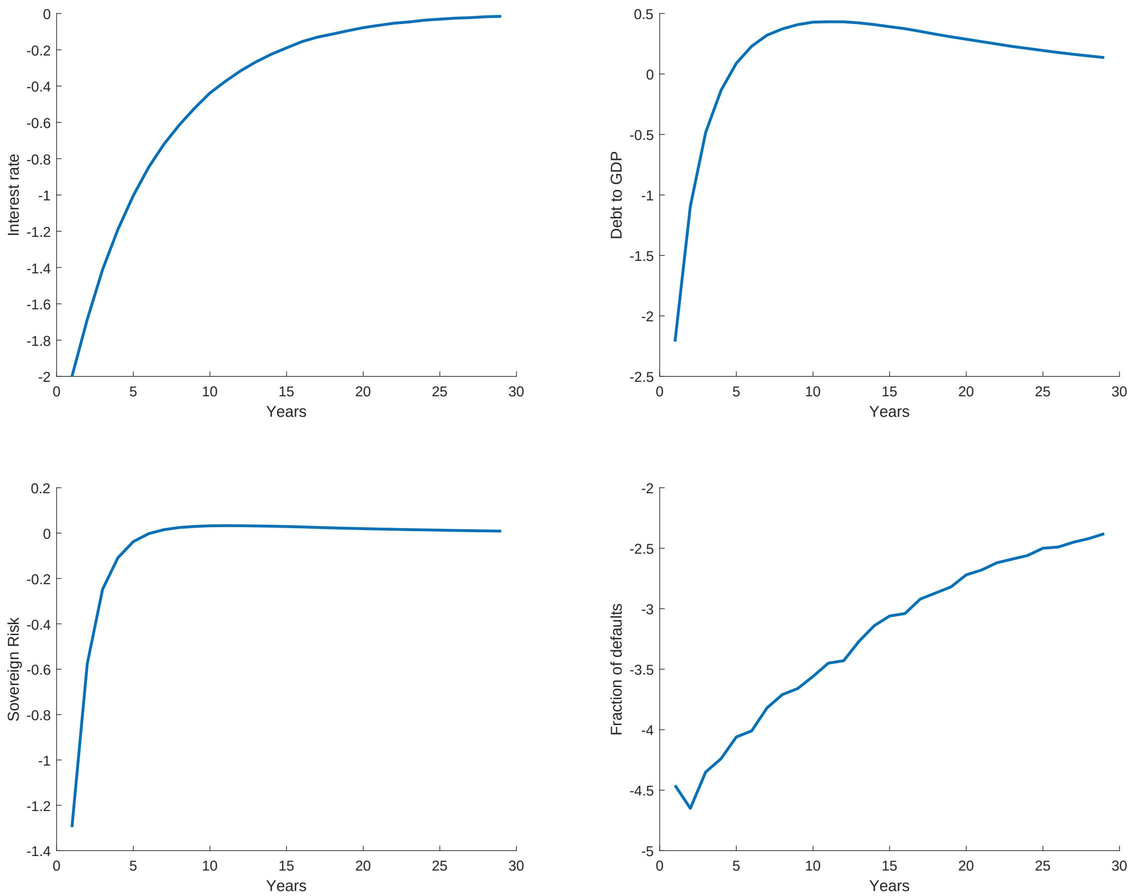


Figure 6: Rate cut in the Fear of Hiking zone. Shown are the effects of a rate hike on debt to GDP, sovereign risk and the fraction of defaults when debt levels are initially low. All variables are expressed in percentage point deviation from the trend they would have followed in the absence of intervention.

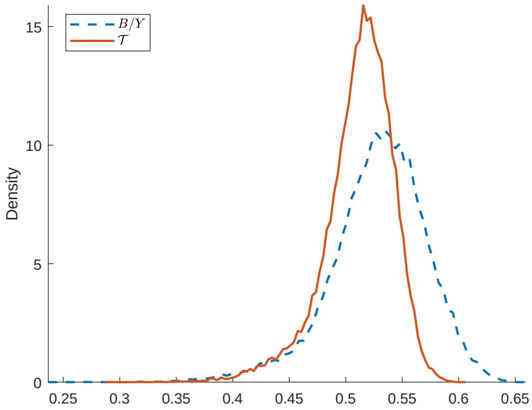
monetary intervention has prevented 2.5 percentage points of default events until 30 years after the start of the intervention.

Taken together, the results in this and the previous subsection reiterate the main point of our analysis: that monetary policy may have diametrically opposed effects on sovereign borrowing and on sovereign risk depending on the initial level of debt.

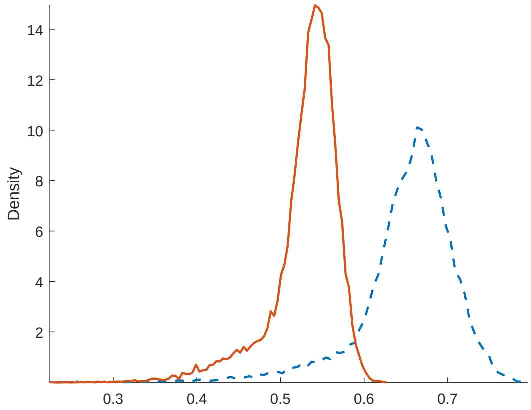
4.5.3 Decline in long-term interest rates

We last study how the economy responds to a decline in long-term interest rates. We do so in light of a recent "too-low-for-too-long" narrative (Gerke and Röttger, 2021). According to this narrative, a prolonged low-interest environment, rather than reduce, may actually increase sovereign default risk in high-debt euro countries due to lower incentives to reduce the stock of sovereign debt.

Long-term interest rates have declined steadily across major industrial economies in recent decades. The literature on secular stagnation has described a host of factors which may have led to this decline, for instance, a decline in potential output growth (Benigno and Fornaro, 2018) or rising inequality and population aging (Mian et al., 2021; Eggertsson et al., 2019). In the context



(a) Baseline long-term interest rates.



(b) Lower long-term interest rates.

Figure 7: Stationary distributions for debt to GDP and threshold following a decline in long-term interest rates. Shown are stationary distributions for B/Y and $\mathcal{T}$, baseline calibration in the left panel ($\iota = 0.02$), low long-run interest rate in the right panel ($\iota = 0$).

of our model, we capture a decline in long rates by assuming that the steady state interest rate ι has declined to a lower level, while keeping all other parameters at their calibrated values.

We illustrate our results with the help of Figure 7. The left panel captures the baseline case $\iota = 0.02$, while the right panel considers a significant drop in the steady state interest rate, to $\iota = 0$. The figure shows that a decline in long rates shifts the stationary distribution of debt to GDP to the right (the blue-dashed lines). We are not showing it, but also the stationary distribution of sovereign risk experiences a significant rightward shift (despite the fact that interest rates are lower, which by itself tends to reduce sovereign risk). Hence default risk is indeed higher in the long term, which is brought about by higher average levels of debt.

What is interesting in the context of our analysis is to compare the shift in the stationary distribution of debt to GDP with the shift in the stationary distribution of the threshold $\mathcal{T}$ (red solid lines).[44] We find that the stationary distribution of $\mathcal{T}$ also shifts to the right, but by much less than the one of debt to GDP.[45] What this implies is that, following the decline in long rates, the economy is now much more likely to be in the Fear of Hiking zone. Lower long rates may therefore limit the room to maneuver of monetary policy, to the extent that rate hikes are now more likely to have adverse effects on sovereign borrowing and the possibility of default.

5 Concluding remarks

When the central bank of a currency union raises its policy rate, what are the implications for sovereign borrowing decisions and the risk of a sovereign default in a union member? Motivated

[44] Here we show the threshold $\mathcal{T}$ for temporary monetary interventions by using equation (23); however, results would be similar by looking at more persistent interventions.

[45] From equation (23), the rightward shift of the stationary distribution for $\mathcal{T}$ reflects that the trade balance to GDP ratio is on average larger. This follows because debt levels are on average higher, requiring more net exports to maintain the stock of debt bounded.

by this question, in this paper we have constructed a quantitative sovereign default model of a small member of a monetary union. We have found that a rise in interest rates by the union-wide monetary authority may either raise or reduce sovereign borrowing and the risk of a sovereign default and that this depends on the current stock of debt to GDP in the small union member. Specifically, a rate hike may reduce default risk going forward, but this will only work if debt to GDP is low enough to begin with, that is as long as the economy is not in the "Fear of Hiking" zone. Conversely, what is needed to make a default less likely when debt levels are already high - or even to prevent an imminent default - is a cut in the interest rate.[46]

The paper opens up several avenues for future research. For example, we have studied how a given (exogenous) change in monetary policy impacts a union member in the presence of default risk, but we have not asked normative questions, particularly how default risk in a union member alters the optimal conduct of monetary policy.[47] Several aspects need to be taken into account when thinking about optimal monetary policy. First of all, one point that comes out in our analysis is that the Fear of Hiking zone is endogenous to agents' expectations about the future monetary stance. When raising interest rates, the right combination of a current rate hike and forward guidance about the future path of interest rates may thus be required in order to minimize the risk of a sovereign default. Second, such announcements of the central bank must also be credible. Ex post, the central bank may have incentives to cut the interest rate, in order to prevent an imminent default. Anticipating this, agents may not believe the central bank's announcements, i.e. its path of forward guidance. We hope that our framework will turn out useful and flexible enough in order to address such questions in the future.

Appendix (for online publication)

A Appendix analytical derivations

A.1 Proof of Proposition 1

Denote $\tilde{q}(B', i, s) \equiv q(B', i, s)(1+i)$ the bond price exclusive of the interest rate. By equation (10), it is given by

$$\tilde{q}(B', i, s) = \mathbb{E}(1 - \delta(B', i', s'))(\tilde{\mu} + (1-\mu)q(B'(B', i', s'), i', s')).$$

[46] What if member states are heterogeneous in terms of their current levels of debt to GDP? In this case, a one-size-fits-all monetary policy may not be successful at mitigating the risk of sovereign default in all countries simultaneously.

[47] Of course, a central bank's mandate is to preserve price stability, not per se financial stability (such as preventing a sovereign default). However, its imprint on sovereign default risk should still be of interest for a central bank. Moreover, a sovereign default has also feedback effects on price stability. As argued by de Ferra and Romei (2020), a sovereign default tends to be deflationary, and as shown by Na et al. (2018), a sovereign default in a country with a fixed nominal exchange rate and nominal rigidities tends to entail a negative output gap.

Because i' is by assumption independent of i, this equation makes it clear that $\tilde{q}$ is not a function of i. Hence we can write $\tilde{q}(B', s)$. Furthermore we have

$$\frac{\partial q(B', i, s)}{\partial B'} = \frac{\partial}{\partial B'} \frac{\tilde{q}(B', s)}{1+i} = \frac{1}{1+i} \frac{\partial \tilde{q}(B', s)}{\partial B'}.$$

Using both in the Euler equation (18), we can write

$$\frac{1}{\gamma} U' \left(\frac{1}{\gamma} \left(X - \tilde{\mu} B + \frac{\tilde{q}(B', s)}{1+i} (B' - (1-\mu)B) \right) \right) \left(\tilde{q}(B', s) + \frac{\partial \tilde{q}(B', s)}{\partial B'} (B' - (1-\mu)B) \right) \\ + (1+i)\beta \frac{\partial}{\partial B'} \mathbb{E} V(B', i', s') = 0.$$

We can write this as $\Lambda(B', B, i, i', s, s') = 0$, which must hold, in particular, for all levels of i. Hence, this equation defines a mapping of i into B', which is implicitly defined $\Lambda(B'(i), B, i, i', s, s') \equiv 0$. We differentiate Λ with respect to i

$$\mathcal{I} + \mathcal{S} + \frac{\partial \Lambda(B', B, i, i', s, s')}{\partial B'} \frac{\partial B'(i)}{\partial i} = 0,$$

where the *direct* derivatives $\mathcal{I}$ and $\mathcal{S}$ have been defined in equations (20) and (21) and where the last summand captures the indirect derivatives (the fact that B' depends on i in equilibrium). Notice that, in the computation of $\mathcal{S}$, we have also used that $\mathbb{E}V(B', i', s')$ is not dependent on i, because we assumed that i' is independent of i.

The last step is to pin down the indirect derivatives. As it turns out, we do not need to compute these derivatives in order to pin down their sign. This is because, by assumption, we study the Euler equation at a local maximum, hence the second order condition at the equilibrium point is negative:

$$\frac{\partial}{\partial B'} \Lambda(B', B, i, i', s, s') < 0.$$

Hence $\partial B'/\partial i > 0$ if and only if $\mathcal{I} + \mathcal{S} > 0$, which yields the statement in the proposition.

A.2 Proof of Proposition 2

In what follows we use sequence (rather than recursive) notation as this makes the proof of the proposition easier. As explained in the main text, we consider an economy where all uncertainty is resolved in period t and hence $X_{t+s} = X$ is constant for all $s \geq 0$. To rule out explosive paths, we also assume $\lim_{s\to\infty} \beta(1 + i_{t+s}) = \beta(1 + \iota) = 1$.

The proof proceeds in two steps. We start by characterizing the effects of a one-time shock in period $t + s$ that is perfectly anticipated in period t. Thereafter, we take the sum over the effects of perfectly anticipated shocks in all future periods $t + s$, for $s \geq 0$, by weighting them with the persistence term ρ^s. Hence we study the case of AR(1) shocks as a perfectly foreseen path of future temporary shocks.

Future one-time shock. The strategy to determine the effect of an expected future interest change on borrowing is to solve for the derivatives of the bond price and consumption with respect to the interest rate analytically. These can be combined to find the derivative of borrowing with respect to the interest rate change.

Iterating forward the bond pricing equation (10), the price of the sovereign bond as a function of future interest rates is

$$q_t = \sum_{s=1}^{\infty} \left(\Pi_{r=0}^{s-1}(1+i_{t+r})\right)^{-1} (\tilde{\mu})(1-\mu)^{s-1}.$$

Notice that no expectation and no default premium enter here, as we have assumed that there is no more uncertainty after period t. We can directly take the derivative with respect to a future i_{t+s}

$$\frac{\partial q_t}{\partial i_{t+s}} = -\frac{\tilde{\mu}}{1+\iota}\sum_{p=s}^{\infty}(1+\iota)^{-p-1}(1-\mu)^p = -\frac{\tilde{\mu}}{(1+\iota)^2}\left(\frac{1-\mu}{1+\iota}\right)^s \frac{1}{1-\frac{1-\mu}{1+\iota}}.$$

where the derivative (as all other derivatives in this section) is evaluated at the steady state interest rate $i_{t+s} = \iota$ for all $s \geq 0$. For readability, we do not make this explicit. The expression above simplifies to

$$\frac{\partial q_t}{\partial i_{t+s}} = -\frac{(1-\mu)^s}{(1+\iota)^{s+1}}. \tag{A.1}$$

Next, we solve for consumption as a function of future interest rates. The Euler equation (18) can be iterated in absence of uncertainty to yield

$$C_{t+s} = \left(\Pi_{r=0}^{s-1}(1+i_{t+r})\beta^s\right)^{\frac{1}{\sigma}} C_t \tag{A.2}$$

for all $s \geq 1$. Similarly, iterating forward the resource constraint (12) yields the inter-temporal resource constraint[48]

$$X - \gamma C_t + \sum_{s=1}^{\infty}\left(\Pi_{r=0}^{s-1}(1+i_{t+r})\right)^{-1}(X-\gamma C_{t+s}) = ((1-\mu)q_t + \tilde{\mu})B_{t-1}.$$

Using equation (A.2), the intertemporal resource constraint can also be written as

$$\gamma C_t + \sum_{s=1}^{\infty}\left(\Pi_{r=0}^{s-1}(1+i_{t+r})\right)^{\frac{1-\sigma}{\sigma}}\beta^{\frac{s}{\sigma}}\gamma C_t$$
$$= X + \sum_{s=1}^{\infty}\left(\Pi_{r=0}^{s-1}(1+i_{t+r})\right)^{-1}X - ((1-\mu)q_t+\tilde{\mu})B_{t-1}. \tag{A.3}$$

Notice that, evaluated in steady state $i_{t+s} = \iota$ for all $s \geq 0$, this implies the simple expression for current consumption

$$C_t = \frac{1}{\gamma}\left(X - \iota B_{t-1}\right). \tag{A.4}$$

[48]This involves solving the period $t+1$ budget constraint for $q_t B_t$ and then using the fact that $\frac{q_t}{q_{t+1}(1-\mu)+\tilde{\mu}} = \frac{1}{1+i_t}$.

The derivative of consumption with respect to a future interest change can be calculated from equation (A.3) and is given by

$$\frac{\partial C_t}{\partial i_{t+s}} = \frac{\iota}{\gamma}\left(-\frac{1}{\sigma\iota(1+\iota)^{s+2}}X + \frac{(1-\mu)^{s+1}+\frac{1-\sigma}{\sigma}}{(1+\iota)^{s+2}}B_{t-1}\right). \tag{A.5}$$

We now have all preliminaries to calculate the derivative of B_t with respect to i_{t+s}. First, we write the choice of debt as a function of other variables

$$B_t = \frac{1}{q_t}(\gamma C_t - X + \tilde{\mu}B_{t-1}) + (1-\mu)B_{t-1}.$$

Taking the derivative gives

$$\frac{\partial B_t}{\partial i_{t+s}} = -\frac{1}{q_t^2}\frac{\partial q_t}{\partial i_{t+1}}(\gamma C_t - X + \tilde{\mu}B_{t-1}) + \frac{\gamma}{q_t}\frac{\partial C_t}{\partial i_{t+s}}.$$

Plugging in the results (A.1), (A.4) and (A.5) and rewriting, we get the following final expression for a shock at a general time $t+s$ on current borrowing B_t

$$\frac{\partial B_t}{\partial i_{t+s}} = \frac{1}{(1+\iota)^{s+2}}\left((1-\mu)^s\tilde{\mu}B_{t-1} - \frac{1}{\sigma}X + \frac{1-\sigma}{\sigma}\iota B_{t-1}\right). \tag{A.6}$$

Persistent shock. To compute the derivative of B_t with respect to a shock at time t which follows an AR(1) process with persistence parameter ρ, we take a weighted sum over the effects on B_t of future expected temporary shocks. By using equation (A.6), we can write

$$\begin{aligned}
\frac{\partial B_t}{\partial i_t^{\mathrm{AR}(1)}} &= \sum_{s=0}^{\infty}\rho^s\frac{\partial B_t}{\partial i_{t+s}} = \sum_{s=0}^{\infty}\frac{\rho^s}{(1+\iota)^{s+2}}\left((1-\mu)^s\tilde{\mu}B_{t-1} - \frac{1}{\sigma}X + \frac{1-\sigma}{\sigma}\iota B_{t-1}\right)\\
&= \frac{1}{(1+\iota)}\left(\frac{1}{1+\iota-(1-\mu)\rho}\tilde{\mu}B_{t-1} - \frac{1}{1+\iota-\rho}\frac{1}{\sigma}\left(X + (\sigma-1)\iota B_{t-1}\right)\right)\\
&= \frac{1}{(1+\iota)}\left(\frac{1}{1+\iota-(1-\mu)\rho}(\tilde{\mu}B_{t-1} + X - C_{f,t} + \iota B_{t-1}) - \frac{1}{1+\iota-\rho}\left(\frac{\gamma}{\sigma}(Y-(X-C_{f,t})) + \iota B_{t-1}\right)\right),
\end{aligned}$$

where we have used $\iota B_{t-1} = X - C_{f,t}$ and $X - \iota B_{t-1} = \gamma(Y-(X-C_{f,t}))$ in the last equality. This implies that

$$\frac{\partial B_t}{\partial i_t^{\mathrm{AR}(1)}} > 0 \iff (\tilde{\mu} + \iota(1-\tilde{\rho}))B_{t-1} \leq \tilde{\rho}\left(\frac{\gamma}{\sigma}(Y-(X-C_t^f))\right) + (X - C_t^f),$$

where we define the auxiliary parameter $\tilde{\rho} = \frac{1+\iota-(1-\mu)\rho}{1+\iota-\rho}$. Dividing both sides by Y and rearranging yields the threshold stated in the main text.

A.3 Slutsky decomposition in Figure 1

We consider the same economy as in the proof of Proposition 2 in Appendix A.2, that is, a perfect foresight economy in which interest rates equal $\iota = 1/\beta - 1$ in the long run. In fact, in what follows we assume that the interest rate is equal to the steady state value ι in all future periods after the current period.

The objective is to decompose the effect of changes in the current interest rate i on future consumption C' into an income and a substitution effect via a Slutsky decomposition. To economize on notation, we write all variables as functions of the current interest rate only. For example, we write $C'(i)$.

Since C is constant for all periods after the current period t, lifetime utility is

$$U(i) = \frac{C(i)^{1-\sigma}}{1-\sigma} + \sum_{t=1}^{\infty} \beta^t \frac{C'(i)^{1-\sigma}}{1-\sigma} = \frac{C(i)^{1-\sigma}}{1-\sigma} + \frac{1}{\iota}\frac{C'(i)^{1-\sigma}}{1-\sigma} \tag{A.7}$$

Using $\beta = \frac{1}{1+\iota}$ the Euler equation becomes

$$C(i) = C'(i)\left(\frac{1+\iota}{1+i}\right)^{\frac{1}{\sigma}}.$$

Substituting for $C(i)$ in equation (A.7) and solving for $C'(i)$, we can write

$$C'(i, U(i)) = \left(\frac{U(i)(1-\sigma)}{\frac{1}{\iota} + \left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}}\right)^{\frac{1}{1-\sigma}}. \tag{A.8}$$

This expression allows us to decompose the effects of an interest rate change into an income effect (green) and a substitution effect (red)

$$\frac{dC'(i,U)}{di} = \frac{\partial C'(q,U)}{\partial U}\frac{\partial U}{\partial i} + \frac{\partial C'(i,U)}{\partial i} \tag{A.9}$$

The second term is the *compensated* effect of the interest rate on future consumption, i.e. how future consumption changes with the interest rate holding the level of lifetime utility (the present value of income) constant. Graphically, the income effect corresponds to the shifting of the green line and the substitution effect corresponds to the tilting of the red line in Figure 1. In the following, we determine the values of the derivatives in Equation (A.9).

We start with the compensated (substitution) effect which is given by

$$\frac{\partial C'(i,U)}{\partial i} = \frac{1}{\sigma} C'(i)^{\sigma} \frac{U(i)(1-\sigma)}{\left(\frac{1}{\iota} + \left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}\right)^2} \left(\frac{1+i}{1+\iota}\right)^{-\frac{1}{\sigma}} \frac{1}{1+\iota}$$

Using $\frac{U(i)(1-\sigma)}{\frac{1}{\iota}+\left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}} = C'(i)^{1-\sigma}$ this is

$$\frac{\partial C'(i,U)}{\partial i} = \frac{1}{\sigma}\frac{C'(i)}{\left(\frac{1}{\iota}+\left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}\right)}\left(\frac{1+i}{1+\iota}\right)^{-\frac{1}{\sigma}}\frac{1}{1+\iota} \tag{A.10}$$

Next, we derive the partial derivative of C' with respect to U

$$\frac{\partial C'(i,U)}{\partial U} = C'(i)^{\sigma}\left(\frac{1}{\frac{1}{\iota}+\left(\frac{1+\iota}{1+i}\right)^{\frac{\sigma-1}{\sigma}}}\right) \tag{A.11}$$

It remains to determine the derivative of U with respect to i. As in the proof of Proposition 2, consumption in all future periods is given by permanent income

$$C'(i) = \frac{X-\iota B'(i)}{\gamma}. \tag{A.12}$$

Notice that $q(i) = \frac{1+\iota}{1+i}$ holds and we can write the budget constraint as

$$B'(i) = (1-\mu B) + \frac{1+i}{1+\iota}\left(C(i)+\frac{\tilde{\mu}B-X}{\gamma}\right).$$

This yields future consumption in terms of current consumption and the interest rate

$$C'(i) = \frac{X-\iota(1-\mu)B}{\gamma} - \iota\frac{1+i}{1+\iota}\left(C(i)+\frac{\tilde{\mu}B-X}{\gamma}\right).$$

Lifetime utility then is

$$U(i) = \frac{C(i)^{1-\sigma}}{1-\sigma} + \frac{1}{\iota}\frac{\left(\frac{X-\iota(1-\mu)B}{\gamma} - \iota\frac{1+i}{1+\iota}\left(C(i)+\frac{\tilde{\mu}B-X}{\gamma}\right)\right)^{1-\sigma}}{1-\sigma}.$$

Importantly, $U(i)$ is evaluated at the optimal choice $C(i)$. Therefore the envelope theorem applies and we only have to take the direct derivative with respect to i:

$$\frac{\partial U}{\partial i} = -\frac{1}{1+\iota}C'(i)^{-\sigma}\left(C(i)+\frac{\tilde{\mu}B-X}{\gamma}\right) \tag{A.13}$$

We substitute (A.10), (A.11) and (A.13) into (A.9) to obtain

$$\frac{dC'(i,U)}{di} = -\left(\frac{1}{\frac{1}{\iota}+\left(\frac{1+\iota}{1+i}\right)^{\frac{\sigma-1}{\sigma}}}\right)\frac{1}{1+\iota}\left(C(i)+\frac{\tilde{\mu}B-X}{\gamma}\right) + \frac{1}{\sigma}\frac{C'(i)}{\left(\frac{1}{\iota}+\left(\frac{1+i}{1+\iota}\right)^{\frac{\sigma-1}{\sigma}}\right)}\left(\frac{1+i}{1+\iota}\right)^{-\frac{1}{\sigma}}\frac{1}{1+\iota}$$

Using $C(i) = C'(i)\left(\frac{1+\iota}{1+i}\right)^{\frac{1}{\sigma}}$ and evaluating at the steady state $i = \iota$

$$\frac{dC'(i,U)}{di} = -\frac{1}{\left(\frac{1}{\iota}+1\right)}\frac{1}{1+\iota}\left(C(i) + \frac{\tilde{\mu}B - X}{\gamma}\right) + \frac{1}{\sigma}C(i)\frac{1}{\left(\frac{1}{\iota}+1\right)}\frac{1}{1+\iota}$$

Collecting terms, we find

$$\frac{dC'(i,U)}{di} = \frac{\iota}{(1+\iota)^2}\left(-\left(C(i) + \frac{\tilde{\mu}B - X}{\gamma}\right) + \frac{1}{\sigma}C(i)\right) \tag{A.14}$$

Here we see that the income effect of an interest rate increase on future consumption is negative, while the substitution effect is positive. Equation (A.12) established a negative linear relationship between C' and B'. This implies that these terms have to be multiplied by $-\frac{\gamma}{\iota}$ to translate them into effects on B'.

Finally note that it is straightforward to see that equation (A.14) gives again the threshold $\mathcal{T}$ that we have found in the main text.

B Appendix quantitative analysis

B.1 Data sources

Our main source is the Eurostat database from which we get data on gross trade flows and public debt relative to GDP, as well as unemployment.[49] We complement this with data on debt maturity from the OECD (2021)[50], data on bond yields from the Bundesbank and Banca d'Italia and data on the sectoral decomposition of government debt holdings from the updated Bruegel database of sovereign bond holdings developed in Merler and Pisani-Ferry (2012).[51] In Table 4 we summarize the details on the data used along with the respective source.

B.2 Construction of Table 1

To compute debt to GDP according to holdings in Table1, we multiply total debt to GDP in Q4 2020 (obtained from Eurostat) with the shares of debt held by non-residents (and the central bank) in Q3 2019 by using the Bruegel database. In order to compute the threshold, we use the trade balance relative GDP obtained from Eurostat for 2020. The way we compute the threshold is explained in the main text.

B.3 Calibration details

For the standard deviation and autocorrelation of GDP, as well as the correlation of the trade balance to GDP ratio with GDP, we use the values provided by Uribe and Schmitt-Grohé (2017).

[49] See ec.europa.eu/eurostat/en/web/main/data/database.
[50] See www.oecd.org/finance/oecdsovereignborrowingoutlook.htm.
[51] See www.bruegel.org/publications/datasets/sovereign-bond-holdings/.

They compute these moment on annual Italian data from 1965 to 2010 after removing a log-quadratic trend. We compute the remaining moments by using our own data sources above. For the level of government debt to GDP and government debt holdings by sector we use quarterly observations from Q1 2000 to Q3 2019, the latest observations available. We compute holdings by foreigners as the product of total debt to GDP and the share of government debt held by non-residents.[52] We then compute the mean over the quarterly series. For unemployment we take the mean over the annual observations from 2000-2020. To compute the spread, we use data on 5 year bond yields (which is the closest to average maturity available) for Italy and Germany. We average over monthly observations to aggregate the data to yearly frequency. We then take the difference between the two series to obtain the spread. All data moments can be found in the second to last column of Table 2.

To compute model analogues of these moments, we simulate the model for 100 000 periods. We then exclude the first 1000 periods as well as any periods the economy spends in autarky and the first 25 periods after reentry.[53] We apply log-quadratic detrending to model generated GDP separately for each sub-sample. We report the weighted mean over the sub-samples for the standard deviation and autocorrelation. All model moments can be found in the last column of Table 2.

[52] We do not include central bank holdings in the calibration.

[53] Since no defaults occur in the data, there is no need to apply a similar procedure.

Name	Unit	Dates	Frequency	Source
Gross domestic product	Chain linked volumes (2010), million euro	2000-2020	annual	Eurostat
Exports of goods and services	Chain linked volumes (2010), million euro	2000-2020	annual	Eurostat
Imports of goods and services	Chain linked volumes (2010), million euro	2000-2020	annual	Eurostat
Unemployment	Percentage of population in the labour force	2000-2020	annual	Eurostat
Government consolidated gross debt	Percentage of gross domestic product	Q1 2000 - Q2 2021	quarterly	Eurostat
Share of government debt held by non residents	Percentage of total government debt	Q1 2000 - Q3 2019	quarterly	Bruegel database
Share of government debt held by central bank	Percentage of total government debt	Q1 2000 - Q3 2019	quarterly	Bruegel database
Average term-to-maturity of outstanding marketable debt	Years	2020	-	OECD
German Bund 5-year notes yield	ppt	9.2006-11.2021	monthly	German Bundesbank
Italy yield of 5 year benchmark BTP	ppt	9.2006-11.2021	monthly	Bank of Italy

Table 4: Data sources

B.4 Output and inflation

Figure **??** shows...

B.5 Numerical algorithm and accuracy

Our numerical solution relies on discrete choice value function iteration augmented with extreme value taste shocks. As discussed for example in Gordon (2019) and Dvorkin et al. (2021) these shocks smooth out kinks in policy functions and bond price schedules which otherwise lead to convergence problems. Our solution algorithm closely follows Arellano et al. (2020) where a full description can be found.

For borrowing B we choose a discrete grid over the interval [-0.05, 0.8] with 800 linearly spaced points. We denote the set of possible borrowing levels with $\mathcal{B}$. We discretize the autoregressive process for X using the Tauchen (1986) algorithm. We use 15 gridpoints and set the gridwidth to 1.88 standard deviations to ensure that the highest gridpoint $X_{\max}$ is smaller than γ.

The algorithm involves extending the exogenous state with a taste shock specific to each possible borrowing choice, so $s = (X, \xi, \{\psi_{B'}\}_{B' \in \mathcal{B}})$. The shocks $\psi_{B'}$ follow a Gumble (Extreme Value Type 1) distribution with standard deviation σ_ψ. The value function in repayment is then given by

$$V^r(B, i, s) = \max_{B' \in \mathcal{B}, C} \left\{ U(C) + \beta \mathbb{E} V(B', i', s') + \psi_{B'} \right\}. \tag{B.1}$$

Before taste shocks realize, the policy functions for borrowing B' and default δ are probability distributions over the possible choices at each grid point (B, X).[54] In all figures we report the mean over the borrowing policy function. We set the standard deviation of the borrowing taste shock to a very small value $\sigma_\psi = 10^{-4}$ which ensures that these shocks do not affect our quantitative results while the algorithm still convergences. Figure 8 shows the borrowing policy function at the gridpoint $(0.544, 0.255)$. It can be seen that nearly all mass lies on the three gridpoints in the narrow interval $[0.539, 0.541]$.

We iterate backward in time on the value function and bond price schedule until updating errors are smaller than 10^{-6}.

Interest rate changes are modeled as MIT shocks. To do so, we additionally add gridpoints with different interest rates. However, the probability of transitioning from the steady state to any point on the grid outside the steady state is set to zero when computing optimal choices. The grid and transition matrix we use for the interest rate depends on the experiment. For Figure 4, we use a grid of $i \in \{\iota, \iota + 10bps\}$ to compute the numerical derivative of borrowing with respect to the interest rate. Since the shock has no persistence, we set the probability of transitioning to the steady state interest rate in the following period to one. For Figure 5, we use a grid of $i \in \{\iota, \iota + 1ppt\}$, again with a probability of returning to steady state equal to one. For Figure 6, we again use the grid $i \in \{\iota, \iota + 10bps\}$. However, here the probability of remaining in the elevated interest state is equal to $\rho = 0.1$.

[54] See Arellano et al. (2020) for details on how to compute the choice probabilities.

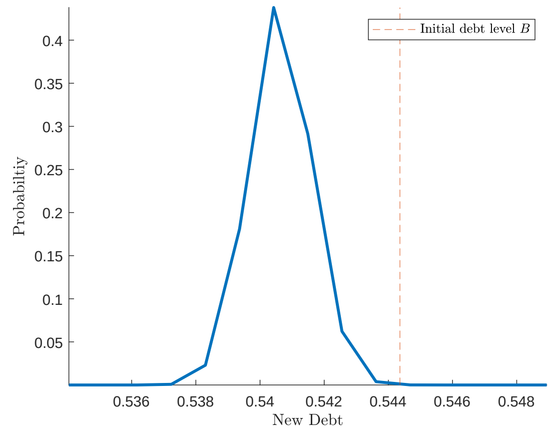


Figure 8: Borrowing policy function. The figure shows the choice probabilities for different values of B' before the realization of the taste shock at the gridpoint $(B, X) = (0.544, 0.255)$.

Our analytical analysis provides a natural test for the accuracy of our numerical solution. As pointed out in the main text, the points in the state space where $\partial B'/\partial i = 0$ can be computed in two ways. First, we can use the analytical threshold in equation (23). Second, we can numerically compute the derivative of the policy function and find the points where it is zero. In our numerical results, we find that these two approaches yield virtually identical results (see Figure **??**). This gives us confidence in our results in two ways. First, it shows that the Euler equation does in fact describe well the choices made by the sovereign.[55] Second, it shows that our numerical solution is accurate in the sense that it matches the theoretical predictions of the Euler equation.

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[55] Due to the complexity of the model and possible non-convexities, this cannot be shown analytically.

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