# Reckoning Climate Change Damages along an Envelope

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**Abstract**

We derive necessary and sufficient conditions for weather fluctuations to systematically identify the marginal long-run effect of climate on an optimized outcome. Empirical estimates of local marginal effects flexibly trace out a common long-run response function that can be used for non-marginal climate counterfactual analysis. Our empirical application considers the effect of temperature and precipitation on county-level agricultural GDP in the United States. Overall, agricultural GDP is predicted to decrease by 6–10% under a 2°C warming scenario.

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Since the seminal work of Mendelsohn et al. (1994), economists have sought ways to predict the impacts of climate change on economic outcomes using observational data. Econometric damage estimates can be informative by

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themselves, but they are also needed to calibrate larger models of the economy (Auffhammer, 2018). One of the main empirical challenges in exploiting historical patterns of climate or weather and economic outcomes is attribution. In an ideal but unrealistic situation, one could observe two otherwise identical economies subject to different climates, and track relevant outcomes.

By necessity, empirical work has departed from this ideal comparison in two main directions. Keeping with the core idea that the relevant counterfactual comparison should involve different climates, Mendelsohn et al. (1994) and the ensuing "Ricardian" literature have compared outcomes across locations exposed to different climates, controlling to the extent possible for potential cross-sectional confounders (Schlenker et al., 2005; Ortiz-Bobea, 2020). The belief that no amount of care would allay concerns about omitted variable bias has led another branch of literature to favor panel approaches with fixed effects, which exploit assumedly random weather fluctuations as a source of identifying variation (Deschênes and Greenstone, 2007).[1] A weakness of that approach is that the effect of weather is conceptually different from the effect of climate due to plausible adaptation (Hsiang, 2016; Auffhammer, 2018).[2]

The present paper proposes a novel theoretical and empirical framework intended to make the best out of both worlds. Our main empirical objects are a collection of local marginal effects identified purely from random weather fluctuations using historical data. We use these objects to trace out a common long-run climatic response function without any parametric restriction beyond that inherent in the choice of relevant climatic variables, making use of the Envelope Theorem and the Gradient Theorem, as initially suggested by Hsiang (2016). The fact that we rely on these theorems for identification of climatic effects does put restrictions on the applicability of our framework, however.

We thus begin by characterizing formally instances wherein the Envelope Theorem may be legitimately invoked to argue that the long-run, adaptation-inclusive marginal effects of permanent changes in environmental conditions

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[1]A third approach, explored in Burke and Emerick (2016), exploits county-level climatic trends, residual of state-level trends, between distant periods.

[2]Nonlinear panel approaches exploit both weather variation and climatic differences, and can be expected to deliver a response that is a mix of short-run and long-run effects (Kolstad and Moore, 2020; Mérel and Gammans, 2021).

(say, climate as measured over several decades) on an outcome of interest are identical to the marginal effects of transient, adaptation-exclusive changes in these conditions (say, yearly weather). In doing so, we are led to introduce a new nomenclature meant to clarify the discourse about long-run effects in climate impact assessment. This nomenclature defines and relates a series of outcome response functions, notably that inferable from high-frequency observational data and those that may be used to build relevant climate counterfactuals. We show that in addition to the usual smoothness assumption and the fact that the outcome of interest must be an optimized value, exploiting the envelope property requires certain functional restrictions on the relationship between weather, climatic adaptation, and the outcome of interest. These restrictions relate directly to Theil (1954)'s sufficient conditions regarding the use of expectations in social welfare maximization problems.[3] A notable contribution of our paper is to prove that these conditions are not only sufficient, as argued by Theil (1954), but also necessary.

We then demonstrate how the envelope property can be leveraged empirically to trace out a global long-run response to climate while addressing concerns about omitted variables. Intuitively, we exploit assumedly random fluctuations in weather at each given climate value to estimate a local slope, which reflects both short-run and long-run effects under the tangency property implied by the Envelope Theorem. Riemann integration of local slope estimates across adjacent climates then delivers the global long-run response function without imposing any further parametric assumption.

Our empirical implementation of this envelope-gradient-theorem approach uses agricultural GDP observed over a 19-year period across more than a thousand US counties. In spite of the flexibility of the method and the demands put on our data to estimate climate-specific marginal responses, the long-run responses to climatic variables that we obtain are surprisingly smooth and precisely estimated. In addition, they are coherent with stylized expectations as well as basic restrictions imposed in prior parametric work on temperature

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[3]Theil (1954) is concerned with the use of expectations of random variables in static, nonlinear welfare maximization programs in place of full distributions of these variables. Simon (1956) and Theil (1957) address a similar issue in the context of dynamic programming.

and agricultural outcomes such as yields or farmland values.

Previous literature has already discussed important restrictions to the use of the Envelope Theorem in climate impact assessment; these pertain to whether the outcome is optimized and whether adaptations can be considered to have a continuous, rather than discrete, effect on the outcome (Guo and Costello, 2013; Nordhaus, 2010; Hsiang, 2016; Blanc and Schlenker, 2017; Deryugina and Hsiang, 2017). Here, we show that even for optimized outcomes that depend smoothly on adaptation actions, the Envelope Theorem does not necessarily imply that random weather variation can be used to identify the slope of the long-run outcome function with respect to climate. The reason is that tangency between long-run and short-run responses at the mean weather (i.e., the first moment of climate) is guaranteed only when long-run actions are taken to optimize outcomes under such mean weather. More realistically however, long-run actions are taken to optimize *expected* outcomes under the entire weather *distribution*, as recognized in several studies (Kelly et al., 2005; Schlenker, 2017; Shrader, 2021; Maue and Kolstad, 2020; Carleton et al., 2020; Mérel and Gammans, 2021). As a result, even under risk neutrality the slopes (and the values) of the short-run and long-run responses may not coincide at the mean weather.[4] Thus, without a proper tangency condition, random weather fluctuations are of little use if analysts care about long-run responses that include adaptation, which is almost always the case (Auffhammer, 2018).

Having made that point formally, our paper goes on to derive structural restrictions on the outcome function that ensure that even under expected outcome maximization, an envelope result systematically holds at mean weather, allowing the identification of long-run effects of environmental change from random high-frequency fluctuations. These conditions are shown to be both necessary and sufficient within a broad class of smooth and convex optimization problems assumed to underlie long-run adaptation choices. Like previous authors, we assume that observed outcomes $y(x, a)$ depend structurally on a realization of weather $x$ and unobserved actions $a$ taken in response to climate, and that conditional on the weather realization, climate influences

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[4]If not, they coincide at some other weather value, but without further functional assumptions it is impossible to know where, obviating the practical benefit of the tangency property.

outcomes only through these actions. This structural relationship gives rise to a reduced-form relationship between weather, climate, and the outcome. The key structural restriction implied by the tangency property is that the actions $a$ are allowed to interact only with a *linear* function of weather, that is, $y(x, a) = \Gamma(x) + \Psi(a)x + \Phi(a)$, for some smooth but otherwise unrestricted functions $\Gamma$, $\Psi$, and $\Phi$. Said differently, tangency between the short-run response to weather and the long-run response to climate (at mean weather) requires the marginal effect of farmers' actions to depend on weather in a linear fashion.[5] This condition may seem restrictive, but it is needed if one is to exploit the tangency property implied by the Envelope Theorem to recover marginal climate impacts from the observation of weather effects. In fact, a comparable trade-off is described by Theil (1954) in the context of the social welfare maximization problem under uncertainty.[6]

Armed with a deeper understanding of what the envelope property, in its most usable form, implicitly assumes, we proceed with its empirical implications. The main consequence of local tangency is that, for the purpose of delineating marginal climate effects, weather fluctuations can be considered as good as climatic changes. Because the assumption of exogeneity is much more likely to hold for weather than for climate, this property opens the door to consistent estimation of marginal climate effects, which can then be integrated across adjacent climates to obtain a common long-run response function. In our application, we regress annual measures of US county-level agricultural GDP on average temperature (or degree days, a measure of heat accumulation rooted in agronomy) and cumulative precipitation. For each climate variable,

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[5]We further show that the marginal response to weather also identifies the marginal *expected* response to climate if and only if the function $\Gamma$ is quadratic.

[6]Specifically, Theil (1954) writes the following:

> The main conclusion is that there is a central class of cases for which the short-cut is permissible [and] is bound to lead to the same policy decisions as those taken in the case of full information. The important thing is that this result does not require any additional specifications of the policy maker's preferences. In other cases the short-cut will usually lead to choice bias. It is in principle possible to avoid this by replacing the stochastic vector [...] by a non stochastic vector different from the expectation, but such a short-cut requires specifications of the welfare function that go much farther than [our] assumption.

we estimate marginal effects for 100 climatic intervals. We then use these slopes to construct the long-run response functions of agricultural GDP to temperature and precipitation, free of functional form restrictions.

We explore more demanding models to address plausible threats to identification. Indeed, different regions may exhibit distinct responses to weather based on geographic variables such as soils. To the extent that these variables are correlated with climate, the climate-specific marginal effects we identify may be biased as they would also reflect the influence of these factors. Consequently, we allow marginal weather effects to vary systematically with soil quality. We further exploit the fact that climates may have changed during the period of observation, meaning that counties may have moved across climatic intervals over time. Namely, we expand the climatic variation present in the data by defining a county's climate as a moving average of past weather. As a result, differences in weather slopes across climatic intervals are identified from comparing a county to itself under different climates, and from comparing counties with different climates but similar soils. We find the results to be extremely close to those from our baseline model.

Our paper relates to recent and ongoing work by several authors. The idea of integrating marginal effects along a climate gradient originates in Hsiang (2016). Our empirical approach differs from the follow-up work of Deryugina and Hsiang (2017) however, as these authors directly estimate a global flexible (polynomial) panel model with fixed effects. Interpreting their global relationship as the underlying long-run response would require more than the Envelope Theorem (Mérel and Gammans, 2021), and although the counterfactual predictions may be close in some applications, they do not need to.[7] The quadratic model proposed by Mérel and Gammans (2021) is a special case of the framework proposed here whereby the tangency property holds by design. Their approach does not require integration of marginal effects, and it delivers short-run response curves in addition to the common long-run response, but at the cost of functional form assumptions regarding the data generating process. Schlenker (2017) even relaxes the tangency condition, again within the confines of a parameterized representation of the structural outcome function.

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[7]The difference between the two approaches is explained formally in Section 2.1.

Finally, Lemoine (2021) analyzes adaptation to climate and weather forecast information in the context of a dynamic optimization model. In contrast, our behavioral model is static and ignores adaptation to weather forecasts. Yet, focusing on the process of long-run climatic adjustment affords a more sophisticated treatment of it. For instance, Lemoine (2021) posits that long-run actions are taken to maximize the long-run value of payoffs under mean weather, an assumption we relinquish in favor of the more common assumption of expected outcome maximization. Another difference is that our framework allows for possibly nonlinear long-run climatic responses, whereas he recovers a single adaptation-inclusive slope coefficient for each weather variable.

The rest of the paper proceeds as follows. Section 1 investigates the theoretical underpinnings of the envelope property. Section 2 discusses its consequences in terms of empirical identification of climate effects. Section 3 presents our empirical application, and Section 4 concludes.

# 1   The envelope argument: what are we assuming?

Previous literature has already acknowledged two central assumptions underlying the use of the Envelope Theorem, namely differentiability (Guo and Costello, 2013) and the fact that outcomes must be optimized (Hsiang, 2016). Here, we argue that in addition to these caveats, functional restrictions are in fact needed to usefully invoke the envelope property as a basis for the empirical identification of a marginal long-run response to climate from random weather fluctuations. To our knowledge, we are the first to make these restrictions explicit.

For clarity's sake, we consider unidimensional weather, denoted by $x \in \mathbb{R}$. We relax this assumption in our empirical application. We denote by $F$ the cumulative distribution function of $x$ and call it the *climate*. The expectation of weather is denoted by $\mathbb{E}[x] = \mu$. In many empirical studies it is the only aspect of climate taken into consideration.[8]

The outcome is a structural function $y(x, a)$ of weather and an action $a \in$

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[8]Schlenker (2017) is a notable exception.

$U \subset \mathbb{R}$ taken prior to (or without knowledge of) the realization of weather, where $U$ denotes an open, convex subset of $\mathbb{R}$. Importantly, conditional on the weather realization $x$, the value of the outcome only depends on climate (or its moments) through the action $a$. If multiple actions can be taken, then $a$ is interpretable as an index (mapping) that varies according to this set of actions. Hence, the assumption of a univariate action does not unduly restrict the analysis. We call $y(x, a)$ the *structural outcome function*.

## 1.1 Regularity assumptions

We restrict the function $y$ to be continuously differentiable with respect to $x$ and $a$, to be concave with respect to $a$, and to have a unique maximizer in the $a$ dimension, for given $x$. As a result, for all values of $x$, this unique maximizer, to be denoted $\hat{a}(x)$, is characterized by the first-order condition

$$\frac{\partial y}{\partial a}(x, a) = 0.$$

The solution to the previous equation, $\hat{a}(x)$, thus answers the question: "what action would be optimal to maximize the outcome under weather $x$?" Note that the concavity assumption is made here without loss of generality once we require that there be a unique critical point characterizing the global maximum, as one can always redefine the index $a$ through a well-chosen change of variable such that the resulting function be concave. (For a formal proof of this claim, see Appendix A.)

We further assume that the function $\hat{a}(x)$ is surjective, though not necessarily bijective, on $U$, that is, each possible action $a \in U$ is optimal for some value(s) of weather. This implies that the equation $\frac{\partial y}{\partial a}(x, a) = 0$ has at least one solution $x \in \mathbb{R}$ for any given value $a \in U$. Solving this equation for $x$ rather than $a$ answers the question: "for what value(s) of weather would the action $a$ be optimal?"

We also make the following assumption:

**Assumption 1** *The function $\frac{\partial y}{\partial x}(x, a)$ is injective with respect to $a$, that is, $\frac{\partial y}{\partial x}(x, a) = \frac{\partial y}{\partial x}(x, a') \Rightarrow a = a'$.*

Assumption 1 requires that the marginal effect of weather on the outcome depend on the action taken, in the sense that different actions imply different marginal effects for given weather. Note that this assumption does not preclude the same action $a$ from being optimal for different values of weather, say $x$ and $x'$, as the corresponding conditions would be $\frac{\partial y}{\partial a}(x, a) = \frac{\partial y}{\partial a}(x', a) = 0$.

Finally, note that the fact that the action $a$ is taken without knowledge of the weather realization does not mean that there is no possibility of adaptation to weather once realized; if there is, the effects of such adaptation are simply accounted for in the structural outcome function $y(x, a)$. That is, $y(x, a)$ can be thought of as the value function of an optimization problem where adaptation to weather is assumed to maximize the outcome, conditional on weather $x$ and the long-run action $a$.

## 1.2   Statement of the problem

We define the *long-run outcome response function* to climate $\mu$ as the value of the following problem:[9]

$$\left\{ \max_{a \in U} y(\mu, a) \right\} = y\left(\mu, \hat{a}(\mu)\right) \equiv Y(\mu). \tag{1}$$

This definition conforms with the concept of "long run" as typically used in microeconomics textbooks, e.g., Perloff (2016) or Nicholson and Snyder (2016). For instance, although firms cannot vary capital in each production period, the long-run cost function is one that minimizes the cost of producing any given quantity when allowing all factors, including capital, to vary with output. This long-run cost function tells us how production costs would change if output quantity were to change from say, durably low levels to durably high levels. However, it is not very informative if we are interested in the effects of transient production shocks, because it assumes a degree of flexibility that the firm typically does not have. Similarly, the function $Y(\cdot)$ indicates how the outcome *would* respond to weather, if the action $a$ *could* be taken in anticipation

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[9]The squiggly brackets around the maximization problem, although perhaps redundant, are meant to emphasize that the element on the right-hand side of the equality sign is equal to the value, rather than the objective, function. The equivalency sign ($\equiv$) indicates a definition.

of weather. It is informative about how the outcome would respond if climate changed from say, a durably cool climate to a durably warm climate.

The Envelope Theorem implies that $Y'(\mu) = \frac{\partial y}{\partial x}\left(\mu, \hat{a}(\mu)\right)$. This theorem has been invoked in the literature as a way to justify the use of weather fluctuations to identify the marginal effects of changes in climate. The reason is that it is easier to observe weather shocks than climatic shocks in empirical work. Weather changes at a yearly frequency, sometimes with large swings, providing arguably exogenous variation in many settings. In contrast, climate may be stationary, or may only change at a slow pace over time. These changes may only be detectable over very long periods of time during which other relevant factors may change, raising concerns about omitted variable bias.

Unfortunately, the function observed in the data may not be $y\left(x, \hat{a}(\mu)\right)$, as implicitly assumed by the use of the Envelope Theorem. For $\hat{a}(\mu)$ is the long-run adaptation that *maximizes the outcome under mean weather*. Instead, if economic agents are taking long-run actions in response to climatic signals, a more tenable assumption is that they choose $a$ to maximize *expected outcomes*, that is, the expected realization of $y$ given the climate $F$. The same point has been made, for instance, in contributions by Kelly et al. (2005), Schlenker (2017), Shrader (2021), or Carleton et al. (2020).[10] Conceptually, agents' choice of action, denoted $\tilde{a}(F)$, would thus solve the following problem:

$$\left\{ \max_{a \in U} \int y(x, a) dF(x) \right\} = \int y\left(x, \tilde{a}(F)\right) dF(x). \tag{2}$$

There is a conceptual difference between maximizing the outcome under mean weather, as in Problem (1), and maximizing the expectation of the outcome under given climate, as in Problem (2). Notably, maximization of expected outcomes implies that higher-order moments of the climate, for instance its variance, may influence choices, whereas only the mean matters in Problem (1).

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[10]Lemoine (2021) proposes a hybrid approach in the context of a dynamic optimization model, whereby some actions affecting future payoffs are taken so as to maximize the expected discounted payoff given weather forecast information, while others are assumed, for convenience, to be taken so as to maximize the long-run value of the payoff evaluated at mean weather.

**Table 1**  Definitions of functions

| Function | Name |
| --- | --- |
| $y(x, a)$ | *structural outcome function* |
| $\hat{y}(x, \mu) \equiv y\left(x, \hat{a}(\mu)\right)$ | *conditional reduced-form outcome function* |
| $Y(\mu) \equiv \hat{y}(\mu, \mu)$ | *long-run outcome response function* |
| $\tilde{y}(x, F) \equiv y\left(x, \tilde{a}(F)\right)$ | *short-run reduced-form outcome function* |
| $\mathcal{Y}(F) \equiv \int \tilde{y}\left(x, F\right) dF(x)$ | *long-run expected outcome response function* |

We will denote $\hat{y}(x, \mu) \equiv y\left(x, \hat{a}(\mu)\right)$ the reduced-form outcome function that would obtain if $a$ were chosen to solve Problem (1). The long-run outcome response function is related to this reduced-form outcome function because $Y(\mu) = y\left(\mu, \hat{a}(\mu)\right) = \hat{y}(\mu, \mu)$, and $Y'(\mu) = \frac{\partial \hat{y}}{\partial x}(\mu, \mu)$ since $\frac{\partial \hat{y}}{\partial \mu}(\mu, \mu) = \frac{\partial y}{\partial a}\left(\mu, \hat{a}(\mu)\right) \hat{a}'(\mu) = 0$ from the first-order condition to Problem (1). In contrast, we will denote $\tilde{y}(x, F) \equiv y\left(x, \tilde{a}(F)\right)$ the reduced-form outcome function that obtains when $a$ is chosen to solve Problem (2). It is this latter function, not $\hat{y}(x, \mu)$, that we expect to observe in the data. We will thus refer to $\tilde{y}(x, F)$ as the *short-run reduced-form outcome function*, and to $\hat{y}(x, \mu)$ as the *conditional reduced-form outcome function*.

Unfortunately, the relationship between the functions $\tilde{y}(x, F)$ and $Y(\mu)$ is not as straightforward as that between $\hat{y}(x, \mu)$ and $Y(\mu)$. Due to Assumption 1, the slopes $\frac{\partial y}{\partial x}\left(\mu, \hat{a}(\mu)\right)$ and $\frac{\partial y}{\partial x}\left(\mu, \tilde{a}(F)\right)$ differ unless $\hat{a}(\mu) = \tilde{a}(F)$. If $\hat{a}(\mu) \neq \tilde{a}(F)$ and agents solve Problem (2) rather than (1), then the Envelope Theorem, which is based on Problem (1), cannot be legitimately invoked in empirical work. That is, a local slope obtained using random weather fluctuations (i.e., $\frac{\partial \tilde{y}}{\partial x}\left(\mu, F\right)$) does not identify the underlying long-run slope $Y'(\mu) = \frac{\partial \hat{y}}{\partial x}(\mu, \mu)$.

In addition to the long-run outcome response function $Y(\mu)$, one may be interested in what we may call the *long-run expected outcome response function*, defined as

$$\mathcal{Y}(F) \equiv \int y\left(x, \tilde{a}(F)\right) dF(x) = \int \tilde{y}\left(x, F\right) dF(x).$$

This function recognizes that economic agents decide on actions $a$ based on expectations, but that once actions are set optimally outcomes are still random due to the randomness of weather. Said differently, $\mathcal{Y}(F)$ is the expectation of

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the short-run reduced-form relationship $\tilde{y}(x, F)$ that we expect to observe in the data, and it is the value function of Program (2). Can the Envelope Theorem be used here to argue that the marginal effect of climate on the expected outcome, $\frac{\partial \mathcal{Y}}{\partial \mu}$, can be identified from the empirically identified marginal change in weather, namely $\frac{\partial \tilde{y}}{\partial x}(\mu, F)$? Unfortunately the answer is still no, because taking the partial derivative of the objective function in Program (2) with respect to the mean weather $\mu$ (holding actions $a$ constant at $\tilde{a}(F)$) would entail taking the derivative of the probability density function, rather than that of the function $y(x, a)$ itself. Intuitively, the objective function of Program (2) is an average, while the function we observe in the data is a particular realization.

Table 1 summarizes the definitions of the various functions discussed above. In what follows, we first derive a necessary and sufficient condition on the structural outcome function $y(x, a)$ under which $\tilde{a}(F) = \hat{a}(\mu)$ for all $F$, where $\mu$ denotes the expected weather under climate $F$.[11] This is, in essence, the same problem as that analyzed by Theil (1954) in the context of the social welfare maximization problem under uncertainty, although his specification of the structural outcome function is slightly different from ours and he does not attempt to prove necessity. While Theil (1954)'s interest lies chiefly in identifying restrictions under which a policymaker may use expected values in lieu of distribution functions in the expected welfare maximization program, in our context the property $\tilde{a}(F) = \hat{a}(\mu)$ further implies that the marginal effect of weather, evaluated at the mean weather, identifies the long-run marginal effect of climate, that is, $\frac{\partial \tilde{y}}{\partial x} = Y'$. In addition to this key property, we derive a more restrictive necessary and sufficient condition under which the marginal effect of weather also identifies the marginal effect of climate on the long-run expected outcome, that is, $\frac{\partial \tilde{y}}{\partial x} = \frac{\partial \mathcal{Y}}{\partial \mu}$.

## 1.3 Necessary condition for $\tilde{a}(F) = \hat{a}(\mu)$

Suppose that, despite the fact that Problem (1) and Problem (2) are conceptually different, it is nonetheless the case that $\tilde{a}(F) = \hat{a}(\mu)$ for all $F$, where $\mu$ denotes

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[11]Alternatively, one could imagine placing restrictions on the set of distributions $F$ for which the tangency property holds. Such an approach lies beyond the scope of the present paper.

the expectation of $F$. We now show the following key lemma:

**Lemma 1** *The function $\frac{\partial y}{\partial a}(x, a)$ must be __affine in $x$__ for all $a \in U$.*

Proof: Consider some $a \in U$. By assumption, there exists $\mu \in \mathbb{R}$ such that $\frac{\partial y}{\partial a}(\mu, a) = 0$, that is, $a = \hat{a}(\mu)$. Then for all distributions $F$ with mean $\mu$, it must be the case that $\int \frac{\partial y}{\partial a}(x, a)dF(x) = 0$ from the first-order condition of Problem (2). Define the function $\phi_a(x) = \frac{\partial y}{\partial a}(x + \mu, a)$. Then $\phi_a(0) = 0$. It is also the case that for all distributions $G$ with mean zero, $\int \phi_a(x)dG(x) = 0$. To see why, rewrite

$$\int \phi_a(x)dG(x) = \int \frac{\partial y}{\partial a}(x + \mu, a)g(x)dx = \int \frac{\partial y}{\partial a}(x, a)g(x - \mu)dx = 0$$

where $g$ is the p.d.f. associated with $G$ and the last equality obtains because the density $f(x) \equiv g(x - \mu)$ has mean $\mu$. (To see why, note that $\int xf(x)dx = \int xg(x - \mu)dx = \int (x + \mu)g(x)dx = \mu$ since $G$ has mean zero.)

Using the fact that $\int \phi_a(x)dG(x) = 0$ for all $G$ with mean zero, we will now show that $\phi_a$ is a linear function. First, for all $x \in \mathbb{R}$, we must have that $\frac{\phi_a(x)}{2} + \frac{\phi_a(-x)}{2} = 0$ since $\frac{x}{2} + \frac{-x}{2} = 0$. That is, $\phi_a$ is an odd function. Second, for all $x > 0$, consider the distribution $G_x = \left(-1, x; \frac{x}{x+1}, \frac{1}{x+1}\right)$ that takes on the value -1 with probability $\frac{x}{x+1}$ and the value $x$ with probability $\frac{1}{x+1}$. $G_x$ is a zero-mean distribution, therefore we must have

$$\frac{x}{x+1}\phi_a(-1) + \frac{1}{x+1}\phi_a(x) = 0$$

that is, $\phi_a(x) = -x\phi_a(-1) = x\phi_a(1)$ where we have used the fact that $\phi_a$ is odd. Finally, for $x < 0$, we have $\phi_a(x) = -\phi_a(-x) = -(-x)\phi_a(1) = x\phi_a(1)$. Therefore, for all $x \in \mathbb{R}$, we have that $\phi_a(x) = \phi_a(1)x$, that is, $\phi_a$ is a linear function.

Since $\phi_a(x) = \frac{\partial y}{\partial a}(x + \mu, a)$, we must have $\frac{\partial y}{\partial a}(x, a) = \phi_a(x - \mu) = \phi_a(1)(x - \mu) = \phi_a(1)x - \phi_a(1)\mu$, which shows that the function $\frac{\partial y}{\partial a}$ is affine in $x$. Q.E.D.

## 1.4 Implied structural outcome function

Given that $\frac{\partial y}{\partial a}$ is affine in $x$ for all $a \in U$, it must be that

$$y(x, a) = \Gamma(x) + \Psi(a)x + \Phi(a) \tag{3}$$

for some $C^1$ functions $\Gamma$, $\Psi$, and $\Phi$. Note that the structure in Equation (3) only imposes that actions interact with a *linear* function of weather, but the functions $\Gamma$, $\Psi$, and $\Phi$ may themselves be nonlinear.

It is easy to check that for the class of functions $y(x, a)$ defined by Equation (3), $\int \frac{\partial y}{\partial a}(x, a)dF(x) = \frac{\partial y}{\partial a}(\mu, a)$, where $\mu$ is the mean of the distribution $F$. Therefore, if $a$ maximizes $\int y(x, a)dF(x)$, it also maximizes $y(\mu, a)$, hence $\hat{a}(\mu) = \tilde{a}(F)$ and the structure in (3) is also sufficient for tangency. These results are summarized in the following proposition.

**Proposition 1** *A necessary and sufficient condition for $\frac{\partial \tilde{y}}{\partial x} = Y'$ is that the structural outcome function follow the structure in Equation* (3).

This proposition directly relates to the so-called "certainty equivalence" principle in dynamic programming, which finds its origin in the static work of Theil (1954) and was developed by Simon (1956) and Theil (1957). This principle states that under certain functional restrictions, the optimal policy strategy under uncertainty is equivalent to that which would be optimal if all uncertain variables were replaced by their expected value. In Appendix B, we show that the structure in Equation (3) is equivalent to the functional restrictions identified by Theil (1954).

## 1.5 Long-run expected outcome response function

Here we are interested in the behavior of the function $\mathcal{Y}(F)$, particularly how it changes when the mean of climate changes, that is, when there is a shift in the weather distribution, say towards higher temperatures, without a change in the shape of that distribution. We derive a necessary and sufficient condition under which $\frac{\partial \mathcal{Y}}{\partial \mu} = \frac{\partial \tilde{y}}{\partial x}$, which provides the needed envelope result (equality of slopes).

For expositional convenience, we consider distribution functions $F$ that can be parsimoniously described by two parameters $\mu$ and $\sigma^2$ reflecting the mean and variance of weather, respectively. Our derivations would still hold if we allowed higher moments to describe the set of possible distributions. We denote $f(x, \mu, \sigma^2)$ the p.d.f. of $F$. We can then view $\mathcal{Y}(F)$ as a function of the two distributional parameters, say $\mathcal{Y}(\mu, \sigma^2)$. We are interested in the change in the expected outcome with respect to $\mu$, holding $\sigma^2$ constant, that is, $\frac{\partial \mathcal{Y}}{\partial \mu}$, and therefore it may be useful to derive conditions under which the empirically identified effect $\frac{\partial \tilde{y}}{\partial x}(\mu, F)$ represents the effect $\frac{\partial \mathcal{Y}}{\partial \mu}$, in addition to representing the effect $Y'(\mu)$ discussed previously. We thus consider the set of structural outcome functions $y(x, a)$ satisfying the structure in Equation (3), and ask what additional restrictions, if any, are needed for the empirically identified marginal effect to also reflect the local behavior of the function $\mathcal{Y}(\mu, \sigma^2)$ with respect to $\mu$.

Given Equation (3), we have that $\tilde{y}(x, \mu, \sigma^2) = \Gamma(x) + \Psi\left(\tilde{a}(\mu, \sigma^2)\right) x + \Phi\left(\tilde{a}(\mu, \sigma^2)\right)$ and therefore $\frac{\partial \tilde{y}}{\partial x}(\mu, \mu, \sigma^2) = \Gamma'(\mu) + \Psi\left(\tilde{a}(\mu, \sigma^2)\right)$. We also have that $\mathcal{Y}(\mu, \sigma^2) = \int \Gamma(x) dF(x) + \Psi\left(\tilde{a}(\mu, \sigma^2)\right) \mu + \Phi\left(\tilde{a}(\mu, \sigma^2)\right)$ and therefore

$$
\begin{aligned}
\frac{\partial \mathcal{Y}}{\partial \mu}(\mu, \sigma^2) &= \int \Gamma(x) \frac{\partial f}{\partial \mu}(x, \mu, \sigma^2) dx + \Psi\left(\tilde{a}(\mu, \sigma^2)\right) \\
&\quad + \left[\Psi'\left(\tilde{a}(\mu, \sigma^2)\right) \mu + \Phi'\left(\tilde{a}(\mu, \sigma^2)\right)\right] \frac{\partial \tilde{a}}{\partial \mu}(\mu, \sigma^2) \\
&= \int \Gamma(x) \frac{\partial f}{\partial \mu}(x, \mu, \sigma^2) dx + \Psi\left(\tilde{a}(\mu, \sigma^2)\right)
\end{aligned}
$$

where we have used the fact that $\tilde{a}(\mu, \sigma^2) = \hat{a}(\mu)$ together with the first-order condition for the maximization of $y(\mu, a)$.

Therefore, it is apparent that the essential condition for $\frac{\partial \mathcal{Y}}{\partial \mu} = \frac{\partial \tilde{y}}{\partial x}$, under the structural model implied by (3), is that, for all distributions $F$, $\Gamma'(\mu) = \int \Gamma(x) \frac{\partial f}{\partial \mu}(x, \mu, \sigma^2) dx$, that is:

$$
\Gamma'(\mu) = \frac{\partial}{\partial \mu}\left(\int \Gamma(x) dF(x)\right). \tag{4}
$$

Condition (4) is trivially satisfied if $\Gamma(x)$ is affine. It is also satisfied if $\Gamma(x)$ is quadratic. To see why, write $\Gamma(x) = ax^2 + bx + c$. We then have:

$$\int \Gamma(x)dF(x) = a \int x^2 f(x, \mu, \sigma^2)dx + b \int xf(x, \mu, \sigma^2)dx + c \int f(x, \mu, \sigma^2)dx$$
$$= a(\mu^2 + \sigma^2) + b\mu + c$$

and therefore $\frac{\partial}{\partial \mu} \left( \int \Gamma(x)dF(x) \right) = 2a\mu + b = \Gamma'(\mu)$.

However, the property does not carry forward if $\Gamma(x)$ is cubic.[12] As expressed in the following proposition, it turns out that the fact that $\Gamma(x)$ be quadratic is necessary for condition (4) to hold for all $F$.

**Proposition 2** *A necessary and sufficient condition for $\frac{\partial \mathcal{Y}}{\partial \mu} = \frac{\partial \tilde{y}}{\partial x}$ for all $F = (\mu, \sigma^2)$ is that*

$$y(x, a) = \gamma x^2 + \Psi(a)x + \Phi(a) \tag{5}$$

*for some $\gamma \in \mathbb{R}$ and some $C^1$ functions $\Psi$ and $\Phi$. In that case, $\mathcal{Y}(\mu, \sigma^2) = Y(\mu) + \gamma\sigma^2$, that is, the long-run expected outcome function is a vertical translation of the long-run outcome response function.*

Proof: We have already argued sufficiency. To show necessity, consider the class of probability distributions $G_{x,x_0,p} = (x, x_0 + x; 1 - p, p)$ that take on the value $x$ with probability $1 - p$ and the value $x_0 + x$ with probability $p$, and $0 \le p \le 1$. The expectation is $\mu = px_0 + x$, while the variance is $\sigma^2 = x_0^2 p(1 - p)$. Therefore, if one holds $x_0$ and $p$ constant, then a change in the mean $\mu$, keeping the variance constant, is equivalent to a change in the value of $x$. Applying

---

[12] Consider the function $\Gamma(x) = a'x^3 + b'x^2 + c'x + d'$. Writing $x^3 = (x - \mu + \mu)^3 = (x - \mu)^3 + 3\mu(x - \mu)^2 + 3\mu^2(x - \mu) + \mu^3$ and defining $M_3 \equiv \int (x - \mu)^3 dF(x)$, we obtain:

$$\int \Gamma(x)f(x, \mu, \sigma^2)dx = a'(M_3 + \mu^3 + 3\mu\sigma^2) + b'(\mu^2 + \sigma^2) + c'\mu + d'$$

and therefore $\int \Gamma(x)\frac{\partial f}{\partial \mu}(x, \mu, \sigma^2, M_3)dx = 3a'(\mu^2 + \sigma^2) + 2b'\mu + c'$ which is different from $\Gamma'(\mu) = 3a'\mu^2 + 2b'\mu + c'$.

condition (4) to this class of distributions, we get

$$
\begin{aligned}
\Gamma'(px_0 + x) &= \left.\frac{\partial}{\partial x}\right|_{p,x_0} \left[(1-p)\Gamma(x) + p\Gamma(x_0 + x)\right] \\
&= (1-p)\Gamma'(x) + p\Gamma'(x_0 + x).
\end{aligned}
$$

If condition (4) is to hold for all distributions, it must hold for all values of the distribution parameters $x_0$ and $p$ within the $G_{x,x_0,p}$ class. That is, we must have

$$
\forall x_0,\ \forall x,\ \forall p \in [0,1] \quad \Gamma'(px_0 + x) = (1-p)\Gamma'(x) + p\Gamma'(x_0 + x),
$$

that is,

$$
\forall x_0,\ \forall x,\ \forall p \in [0,1] \quad \Gamma'\left((1-p)x + p(x_0 + x)\right) = (1-p)\Gamma'(x) + p\Gamma'(x_0 + x),
$$

that is

$$
\forall x,\ \forall y,\ \forall p \in [0,1] \quad \Gamma'\left((1-p)x + py\right) = (1-p)\Gamma'(x) + p\Gamma'(y).
$$

This last property implies that $\Gamma'$ must be affine (as a function that is both concave and convex), and thus $\Gamma$ itself must be quadratic. Given that the structure in (3) already allows for an interaction between $x$ and an unspecified function of $a$ (which may include a constant), the restriction of $\Gamma(x)$ to a single quadratic term does not constrain the model specification beyond what has been shown to be necessary. The last part of the proposition is obtained by specializing the definitions of $Y$ and $\mathcal{Y}$ to the assumed structure for $y(x, a)$. Q.E.D.

Proposition 2 provides an interesting result in light of the common use of the quadratic functional form to represent the relationship between weather and outcomes in empirical work. However, it should not be construed as implying that estimating a quadratic panel regression of the outcome on weather, even with fixed effects, will trace out the long-run expected outcome response function, or even the long-run outcome response function. Mérel and Gammans (2021) demonstrate how a quadratic weather panel leads to biased

counterfactual estimates when the structural outcome function is of the form $y(x, a) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3(x - a)^2$, with $\beta_3 < 0$. This is a special case of Equation (5) with $\gamma = \beta_2 + \beta_3$, $\Psi(a) = \beta_1 - 2\beta_3 a$, and $\Phi(a) = \beta_0 + \beta_3 a^2$. The authors show that estimates of the parameters $(\beta_1, \beta_2)$ obtained from a simple quadratic panel with fixed effects are biased, affecting identification of both the long-run outcome response function $Y(\mu)$ and the long-run expected outcome response function $\mathcal{Y}(F)$. The intuition behind this result is that although fixed effects account for the function $\Phi(a)$, structurally the slope term $\Psi(a)x$ still depends on actions and thus climate, whereas the simple quadratic specification imposes common coefficients on $x$ across panels.

An econometric panel implementation of the structural relationship in Equation (5) that remained totally agnostic about the functions $\Psi$ and $\Phi$ would be:

$$y_{it} = \alpha_i + \beta_{1i} x_{it} + \beta_2 x_{it}^2 + \epsilon_{it}$$

where $i$ indexes panels and $t$ indexes time. Assuming stationary climates, this regression would identify the function $\Gamma$ through the $\beta_2$ coefficient, but it would not identify $y(x, a)$ itself because the actions $a$ are unobserved and instead left implicit in the coefficients $\alpha_i$ and $\beta_{1i}$. Thus, the estimated relationship would be useless to recover the long-run outcome function (or the long-run expected outcome function), and so counterfactual analysis of climate impacts would be precluded.

Finally, note that although Proposition 2 establishes a result about equality of slopes, it does not ensure tangency, as there is no guarantee that $\mathcal{Y}(\mu, \sigma^2) = \tilde{y}(\mu, \mu, \sigma^2)$. Of course, equality in levels is not required for counterfactual analysis of changes in climate, so one should not view this caveat as a weakness. For the sake of completeness, the proposition below establishes that tangency requires $\Gamma(x)$ to be affine, that is, $\gamma = 0$ in Equation (5).

**Proposition 3** *A necessary and sufficient condition for $\mathcal{Y}(\mu, \sigma^2)$ to be tangent to $\tilde{y}(x, \mu, \sigma^2)$ at $x = \mu$ for all $(\mu, \sigma^2)$ is that*

$$y(x, a) = \Psi(a)x + \Phi(a) \tag{6}$$

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*for some $C^1$ functions $\Psi$ and $\Phi$.*

<u>Proof</u>: Under the structure of Proposition 2, $Y(\mu)$ is tangent to $\tilde{y}(x, \mu, \sigma^2)$ at $x = \mu$ and $\frac{\partial \mathcal{Y}}{\partial \mu}(\mu, \sigma^2) = \frac{\partial \tilde{y}}{\partial x}(\mu, \mu, \sigma^2)$. Since $\mathcal{Y}(\mu, \sigma^2) = Y(\mu) + \gamma \sigma^2$, a necessary and sufficient condition for tangency between $\mathcal{Y}$ and $\tilde{y}$ is that $\gamma = 0$.

## 1.6   How restrictive are these conditions?

Admittedly, the results of the preceding sections may be viewed as "bad news" for empiricists seeking to identify climatic impacts from random weather fluctuations. While they certainly call for some caution when invoking envelope arguments to interpret panel estimates, in our view they certainly do not close the door to empirical work on the subject. First, note that the conditions of Proposition 3 are not needed for counterfactual analysis. Second, the restriction in Proposition 2 is only needed for inference on the long-run expected outcome response function, not the long-run outcome response function itself. In addition, relative to Proposition 1 that proposition only requires the additional assumption that the outcome depend quadratically on weather, an assumption many empiricists have already been willing to make in parametric models.[13] Thus, the discussion boils down to Equation (3), which imposes that actions interact with a linear function of weather in the structural outcome function.

Another way to express this key restriction is to say that the marginal effect of weather on the marginal effect of actions is constant with respect to weather. Importantly, the restriction does not preclude actions from attenuating marginal weather effects, since $\frac{\partial y}{\partial x} = \Gamma'(x) + \Psi(a)$. In particular, it does not prevent adaptation from turning a negative marginal weather effect into a positive one; but it precludes turning a locally concave relationship into a locally convex (or less concave) one.[14] Also note that if Equation (3) is violated

---

[13]See the discussion of Section 1.5, however.

[14]Note that while it is possible for actions to affect the slopes of the weather-outcome relationship without affecting its convexity, it is not possible for actions to affect the convexity of that relationship without also affecting the slopes. Even if at some weather value the slope is invariant to actions, if convexity varies across actions then slopes must be different in a neighborhood of that weather value.

because actions interact with a nonlinear function of weather, it may still be possible to redefine weather through a change of variable so that the restriction is met. For instance, assume a structural outcome function of the form:

$$y(x, a) = \Gamma(x) + \Psi(a)\Lambda(x) + \Phi(a)$$

where $\Lambda(x)$ is a bijective function. Then, the restriction is met for the transformation of weather $z = \Lambda(x)$, that is, for the structural outcome function

$$v(z, a) = \Gamma \circ \Lambda^{-1}(z) + \Psi(a)z + \Phi(a).$$

Thus, as long as actions interact with a monotonic function of weather, there exists a transformation of weather for which the restriction holds, although this particular transformation may remain unknown to the analyst. This could call for implementing multiple model specifications involving different transformations of weather variables. One instance where the monotonicity condition would fail is if the marginal effects of the action on the outcome were identical at distant values of climate, say extremely cold and extremely hot temperatures, without being constant between those climate values. In practice, such situation would call for restricting the analysis to a climate support over which the marginal effect of the action can be assumed to be monotonic with respect to weather.

One may further assess the restrictiveness of the identifying assumptions by considering graphical depictions of outcome response functions that meet them. Figure 1 depicts a short-run reduced-form outcome function, the long-run outcome response function, and the long-run expected outcome response function in the special case where the structural outcome function has the form $y(x, a) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 (x - a)^2$, with $\beta_3 < 0$ and $\beta_2 < 0$. This structure satisfies Assumption 1 because $\frac{\partial y}{\partial x} = \beta_1 - 2\beta_3 a + 2(\beta_2 + \beta_3)x$ and $\beta_3 \neq 0$. It is a special case of the structure in Proposition 2 with $\gamma = \beta_2 + \beta_3 < 0$, $\Psi(a) = \beta_1 - 2\beta_3 a$, and $\Phi(a) = \beta_0 + \beta_3 a^2$. Because the restriction of Proposition 1 is satisfied, there is tangency between the short-run reduced-form outcome function corresponding to climate $\mu_0$ and the long-run outcome

**Figure 1**  Key outcome functions and their relative positions
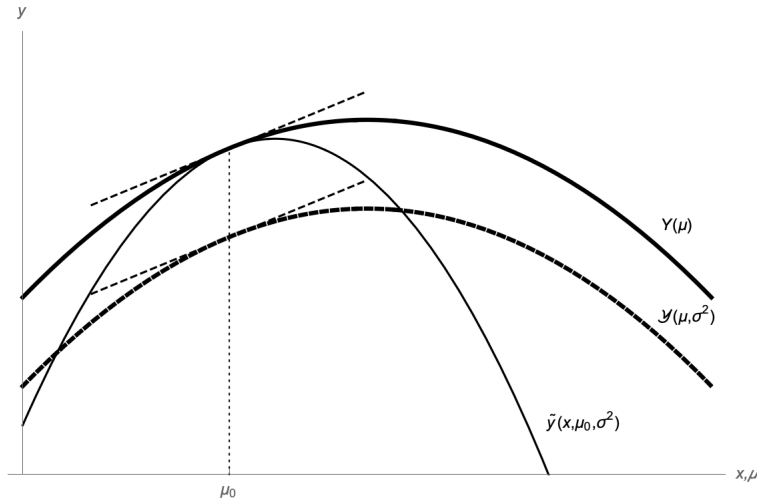


response function at $\mu_0$. Because the restriction of Proposition 2 is satisfied, there is also equality of slopes between the short-run reduced-form outcome function and the long-run expected outcome function at $\mu_0$. But because the restriction of Proposition 3 does not hold, these last two functions are not tangent to each other at $\mu_0$.

One important restriction in the previous example is that the second-order derivative of the long-run outcome response function is constant across climates. Clearly, this need not be the case under the confines of Proposition 1, as structural outcome functions of the form $y(x, a) = \Gamma(x) + \beta_3 (x - a)^2$, with $\beta_3$ a constant independent of $a$ and $\Gamma$ unrestricted, would also meet the minimum restriction for tangency between the long-run outcome response function and the short-run reduced-form outcome function. Such an example is represented in Figure 2. The figure shows that Proposition 1 still allows for substantial heterogeneity in the shape of the short-run response functions. The key restriction here is that the second-order derivative must be identical across short-run responses at any given weather value $x$ (even if this common derivative is allowed to vary with $x$). In practice however, the restriction would only apply to the set of short-run responses at locations whose weather distribution support actually includes the weather value $x$.

To conclude this section, we note that one may still be able to exploit weather

**Figure 2** Long-run outcome response function and three short-run reduced-form outcome functions
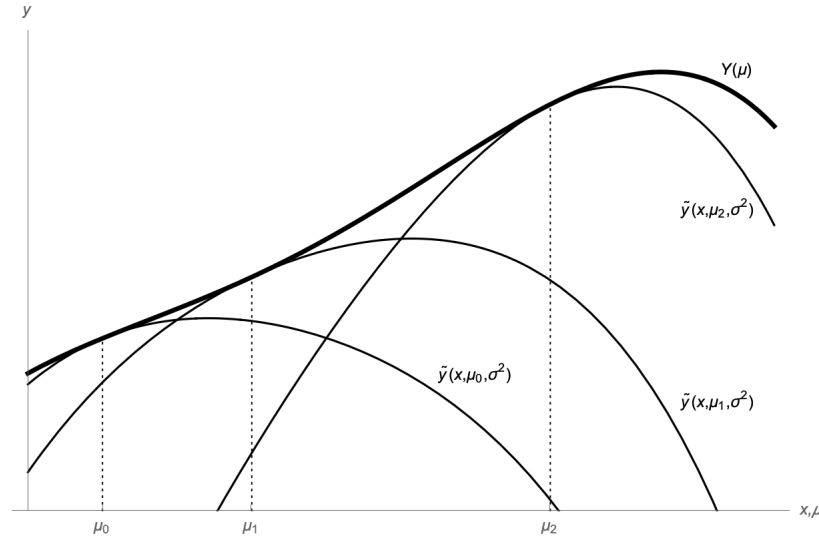


fluctuations to identify long-run responses of optimized outcomes in panel settings while relinquishing the tangency property, at the cost of an explicit parameterization of the adaptation process, as in Schlenker (2017). That study proposes a functional parameterization of $y(x, a)$ that accommodates an interaction between actions (which, like here, are not observed in the data) and a nonlinear (quadratic) function of weather. Interestingly perhaps, the results of the study indicate that relaxing the tangency restriction does not necessarily lead to large differences in the estimated long-run outcome response function.

# 2 Consequences for empirical work on optimized outcomes

A large literature has sought to exploit random weather fluctuations to identify the response of optimized or quasi-optimized outcomes to climate (Dell et al., 2014; Blanc and Schlenker, 2017; Auffhammer, 2018). While some of this literature, notably Deschênes and Greenstone (2007), acknowledges at the outset that the identified effect may not reflect long-run adaptation to climate, several papers have invoked the Envelope Theorem to argue that this effect is still

relevant for climate change analysis, at least at the margin. A recent working paper goes further by using the Envelope Theorem as a justification for non-marginal analysis, arguing that a global nonlinear relationship between an optimized outcome (GDP) and weather identified in a panel with fixed effects represents a long-run relationship inclusive of climatic adaptation (Deryugina and Hsiang, 2017).

Such a strong result requires more than the Envelope Theorem to hold (Mérel and Gammans, 2021). Nonetheless, it remains true that, to the extent that an Envelope result holds locally, as in the instances highlighted in Section 1, a global long-run relationship may be recovered by stepwise integration, as initially suggested by Hsiang (2016). This is the approach taken in the empirical part of this paper. While we cannot claim as our own the idea of integrating marginal effects estimated using local time-series variation, to the extent of our knowledge we are the first to implement it empirically. After laying out the method, we discuss the testability of the structural restriction embedded in Equation (3).

## 2.1 Recovering a long-run response by integrating marginal effects across climates

By assuming that the long-run and short-run responses of the outcome are tangent to each other at the mean weather (which, as shown above, is tantamount to assuming the structure in Equation (3) for some unspecified functions $\Gamma$, $\Phi$, and $\Psi$), we can use local time-series variation in weather to identify local marginal responses in a first step, and in a second step integrate stepwise across climates to recover the global relationship between climate and the outcome.

More specifically, assume that we have panel data, where each panel represents a location, e.g., a county, and the outcome is observed at a yearly frequency. We may estimate marginal responses for each county, or more parsimoniously we may group counties by climates by defining climatic intervals, and estimate local marginal responses for each climate interval. The first step

**Figure 3**  Stepwise integration between two climates
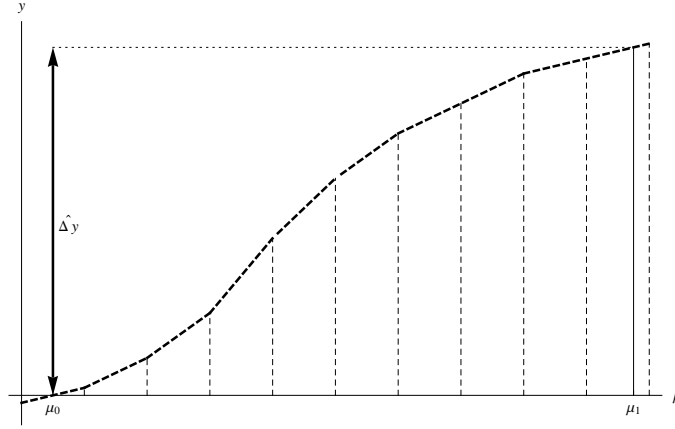


Note: Vertical dashed lines delimit climatic intervals. $\hat{\Delta}y$ represents our estimate of the change in outcome resulting from a movement from climate $\mu_0$ to climate $\mu_1$ along the estimated long-run response function.

of our analysis thus consists of estimating the following regression:

$$y_{it} = \beta_{\mu(i)}x_{it} + f_{\text{state}(i)}(t) + \alpha_i + \alpha_t + \epsilon_{it} \tag{7}$$

where $i$ denotes a county, $t$ denotes a year, $y_{it}$ denotes the outcome (in our application, agricultural GDP), $x_{it}$ denotes weather, state($i$) is the state of county $i$ and $f_{\text{state}(i)}(t)$ is a state-specific time trend, and $\alpha_i$ and $\alpha_t$ are respectively county and year fixed effects. The parameters of interest are the set of marginal weather effects $\beta_\mu$, identified from the marginal weather responses of counties whose climate $\mu(i)$ falls within the interval $\left[\mu - \frac{\Delta\mu}{2}, \mu + \frac{\Delta\mu}{2}\right]$, where $\Delta\mu$ is the width of the climatic interval. Although one could imagine identifying these marginal effects by estimating non-linear functions specific to each climatic interval $\left[\mu - \frac{\Delta\mu}{2}, \mu + \frac{\Delta\mu}{2}\right]$ and evaluating them at climate $\mu$, the specification in (7) is the most parsimonious and delivers first-order approximations to the underlying short-run response functions, which is sufficient for the purpose of computing local marginal effects. In Appendix E, we report results from models that instead use locally quadratic approximations to short-run response functions. They turn out to be extremely close to those from the locally linear

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model.[15]

In the second step, stepwise integration is performed by multiplying local marginal effects by the climatic width of each interval and summing up across intervals located between the initial and final climate values. This allows us to recover the long-run difference in outcomes between an initial climate $\mu_0$ and a final climate $\mu_1$. This procedure is represented graphically on Figure 3 for the case of unidimensional climate.

Contrary to previous work, the approach described above is essentially model-free, or non-parametric, once the choice of relevant weather variables has been made. The global response to climate is identified flexibly by the simple arrangement of short-run slopes across a large number of climates (in our case, 100 intervals for each climatic variable). The only assumptions required for the resulting relationship to identify the underlying long-run response are thus (i) the existence of a common (or global) long-run response across climates, and (ii) the local tangency result. Note that climatic variation is not in any way absent from recovery of the long-run response: although only weather fluctuations are used to identify local marginal effects, the arrangement of these slopes into a global response requires knowledge of mean weather, i.e., climates.

Relatedly, it may be useful to make explicit how this envelope-gradient-theorem approach differs, at least from a conceptual standpoint, from previous approaches that have attempted to recover a long-run response function from global flexible panels with fixed effects, notably Deryugina and Hsiang (2017). The key difference is illustrated in Figure 4 based on a data set with two climates (or counties), denoted $\mu_1$ and $\mu_2$. There are two (equiprobable) weather realizations for each climate, say $x_{i1}$ and $x_{i2}$ for climate $\mu_i$ ($i = 1, 2$), thus the sample size is four. To make the two approaches directly comparable,

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[15]While we estimate local weather slopes within a parametric framework, the global long-run response function is still determined by the flexible arrangement of these slopes across climatic intervals. This is different from estimating a global parametric panel, as much of the literature has done. Here, within each climatic bin, we essentially resort to local polynomial approximations of the true DGP to estimate the marginal effects of weather. The fact that local polynomials of order 1 and 2 yield similar results in our application lends credence to this approximation. As for nonparametric regressions, the asymptotic convergence of our estimator would rely on increasing the degree of the local polynomials adequately with the number of observations. Eliciting the conditions of convergence in our setting is beyond the scope of the present work, however.

**Figure 4** Difference between the envelope-gradient approach and the flexible panel approach
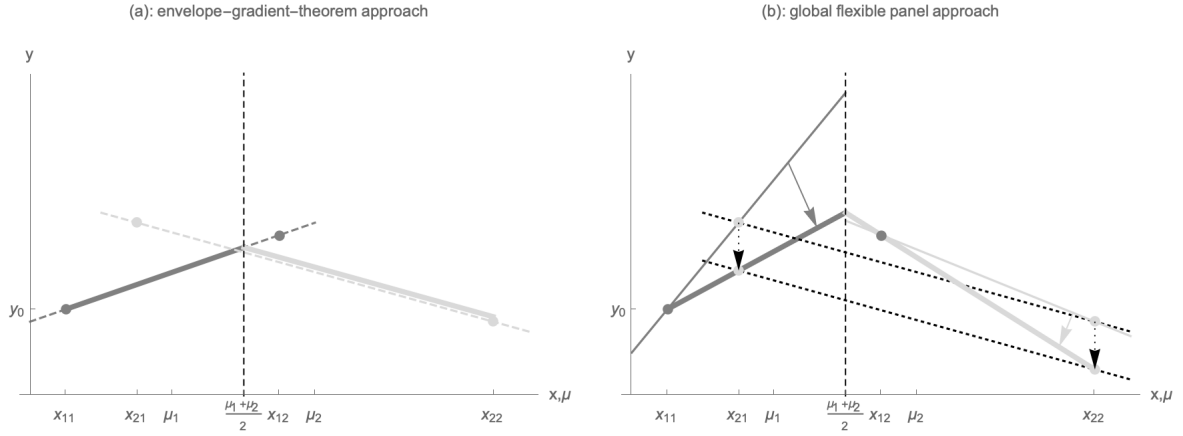


Note: Climatic response functions are represented with bold segments over the range $[x_{11}, x_{22}]$ and are normalized to take the value $y_0$ at the climatic value equal to $x_{11}$. In panel (b), the dotted arrows represent the fixed effect for climate $\mu_2$ (relative to that for climate $\mu_1$). The linear segments obtained by interpolating the raw data without imposing continuity are shown with thin solid lines; their positions after the fixed effects have been set to ensure continuity are shown with bold lines.

we consider a flexible panel implemented using a piecewise linear function. We assume that the degrees of freedom are the same in both approaches. Since the envelope-gradient-theorem approach estimates, at a minimum, two intercepts and two slopes (see Equation (7)), for the global flexible panel we consider a piecewise linear interpolation with one knot located precisely at the climatic value that separates the two climatic intervals, taken here to be the midpoint between $\mu_1$ and $\mu_2$. Because there are as many model parameters as observations, both approaches provide a perfect fit to the data. But the resulting response functions, despite being both piecewise linear, are different.

Panel (a) depicts the interpolation afforded by the envelope-gradient-theorem approach. Under this approach, climates are considered separately from each other and each linear segment simply joins the two climate-specific data points. The linear segments (the slopes) are then arranged so as to provide a continuous response at the knot, as illustrated in Figure 3 for the case of many

climatic intervals. Panel (b) of Figure 4 depicts the interpolation afforded by the piecewise-linear panel with fixed effects, for the exact same data set. The midpoint between climates is such that it does not "separate" the climate-specific data: one observation for climate $\mu_2$ lies within the $\mu_1$ interval, and vice-versa. This feature is arguably common to many panel datasets, including the one explored in the empirical part of this paper, as the supports of locational weather outcomes tend to overlap across locations. Because of this overlap, the model would fit the first segment through the data points corresponding to the weather observations $x_{11}$ and $x_{21}$, and the second segment through the data points corresponding to the weather observations $x_{12}$ and $x_{22}$, while using the intercepts to ensure continuity at the knot. Using the intercept of the first county as the reference, this implies that the intercept of the second county should be shifted downwards, as indicated by the arrows on the graph. The resulting piecewise linear function is clearly different from that arising from the arrangement of slopes in panel (a).

What is the intuition behind this difference? The flexible panel approach consists of fitting a function through the entire set of data points, irrespective of whether a given point belongs to a particular county/climate (except for the county-specific intercepts which vertically shift all observations from the same county by a constant). This approach implies that degrees of freedom may be used to arbitrage errors across data points that belong to different climates. In contrast, the gradient-theorem approach fully accounts for differences across climates: degrees of freedom are only used to arbitrage errors across data points within the same climate. As illustrated in Figure 4, the resulting response functions can, at least conceptually, be quite different. And increasing the flexibility of the parametric panel model does not necessarily help: here, the model fit is already perfect, yet the responses are distinct.

## 2.2  Addressing omitted variable bias in local slope estimates

While the integration of local slopes along a climate gradient yields a long-run response function under the functional assumptions discussed in Section 1, there remains a possibility that these slopes, which reflect a marginal response

at any given climatic value, may be biased by the presence of omitted variables. Specifically, if there exist time-invariant unobserved factors common to counties in a given climatic interval that contribute to the marginal weather response, i.e., that interact with weather, they will confound the estimated weather slope (to the extent that they are not caused by climate, of course). That is, the weather slope estimated at a given climate may reflect the effect of these factors, in addition to that of climate. This is a problem for counterfactual climate analysis because as counties' climates change under warming, making them cross climatic intervals, these other factors do not change.

A "brute-force" solution to the problem could be to exploit the fact that climates may have changed during the period of observation and completely remove all cross-sectional comparisons from identification, in the spirit of Burke and Emerick (2016). Specifically, one could define climate as a rolling average over, say, thirty years and include county-level weather slopes as additional covariates, which would non-parametrically control for all time-invariant, county-specific factors that may interact with weather, such as soils. That is, in the first step one could estimate the following augmented regression:

$$y_{it} = \beta_{\mu(i,t)} x_{it} + \gamma_i x_{it} + f_{\text{state}(i)}(t) + \alpha_i + \alpha_t + \epsilon_{it}$$

where $\mu(i, t)$ now indicates the climatic interval where county $i$ is present in year $t$. Unfortunately, in our application to US agricultural GDP, climatic trends during the observation period are too weak to allow for a precise estimation of the climate-contingent weather slopes. Even when we replace county-specific slopes with state-specific slopes, which essentially confines identification to comparisons across counties in the same state and comparisons of counties with themselves under varying climates, estimates remain extremely noisy. This outcome should not come as a surprise; Fisher et al. (2012) make a comparable remark in the context of the parametric panel approach regarding the use of state-by-year fixed effects in regressions of county agricultural profits on weather, as these fixed effects absorb a large amount of useful weather variation, leading to statistically insignificant estimates.

Thus, by necessity, the strategy employed in our application is more modest.

We choose to control for interactions between weather and observables (here, soils) explicitly while making a selection-on-observables assumption. That is, we estimate the following augmented regression:

$$y_{it} = \beta_{\mu(i)} x_{it} + \gamma' \mathbf{z}_i x_{it} + f_{\text{state}(i)}(t) + \alpha_i + \alpha_t + \epsilon_{it} \tag{8}$$

where $\mathbf{z}_i$ is a vector of time-invariant covariates interacting with weather and $\gamma$ is the associated parameter vector. We also estimate a specification that allows locational climates to vary over time, that is,

$$y_{it} = \beta_{\mu(i,t)} x_{it} + \gamma' \mathbf{z}_i x_{it} + f_{\text{state}(i)}(t) + \alpha_i + \alpha_t + \epsilon_{it} \tag{9}$$

where the assignment of an observation $(i, t)$ to a climate interval, $\mu(i, t)$, is based on a rolling climate $\mu_{it}$ calculated using the weather average over the previous thirty years, that is, $\mu_{it} = \frac{\sum_{s=t-30}^{t-1} x_{is}}{30}$.[16] In Equation (9), differences in weather slopes across climates are identified from comparisons across counties with similar covariates $\mathbf{z}_i$, as well as comparisons of counties with themselves under varying climates.

## 2.3 Empirical testability of the structural restriction

One of the key challenges in testing the restriction in Equation (3) is that we expect to observe the short-run reduced-form outcome function $\tilde{y}(x, F)$ in the data, but not the underlying structural outcome function $y(x, a)$ that is subject to the restriction. In the absence of comprehensive data on adaptation actions, it may be tempting to test the restriction by estimating a model of the form

$$y_{it} = \alpha_i + \beta_i x_{it} + \phi(x_{it}, \boldsymbol{\delta}_i) + \text{controls} + \epsilon_{it} \tag{10}$$

where $\phi$ is a nonlinear function parameterized by $\boldsymbol{\delta}_i$, and testing for the joint equality of the $\boldsymbol{\delta}_i$ coefficients across panels. The problem is that heterogeneity across panels in the non-linear portion of the response function may be due

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[16]Note that $\mu_{it}$ denotes the climate of county $i$ in year $t$, while $\mu(i, t)$ denotes the climate interval in which this climate value falls. The parameters of interest are indexed by $\mu$, not by $(i, t)$.

not only to different adaptation choices driven by climate, but also to unobserved time-invariant factors.[17] Thus, a rejection of the null hypothesis of equal $\delta_i$s could be entirely due to unobserved heterogeneity that is independent of adaptation, which would not in itself violate the key restriction of Equation (3). That is, the test could fail even if the underlying condition is met. Conversely, failure to reject the null could be due to time-invariant factors obfuscating a nonlinear interaction between weather and climate-driven adaptation actions.

# 3 Empirical implementation

Using the method described in Section 2, we derive the long-run response of US county-level agricultural GDP to temperature and precipitation. Because agricultural GDP represents the value added of the agricultural sector (value of agricultural outputs minus expenditures on intermediary inputs), we deem it interpretable as an optimized quantity. Although long time-series data on other agricultural outcomes such as crop yields are available, these outcomes are arguably not optimized, therefore we refrain from applying our envelope-gradient-theorem approach on them.

## 3.1 Data

US county-level data on agricultural GDP over the period 2001–2019 come from the Bureau of Economic Analysis of the US Department of Commerce. These data are available for a wide range of sectors and are estimates of the gross margin output, the value of sales and inventories less the costs associated with production. For the agricultural sector, these estimates are developed from USDA farm sales, farmland, and labor data (Fleck et al., 2014). As in Schlenker et al. (2006) and Schlenker and Roberts (2009), our analysis focuses on counties east of the 100th meridian, where agriculture is largely non-irrigated. We also remove counties with more than 10% missing values, which leaves us

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[17]Although not an issue with respect to the restriction of Equation (3), heterogeneity in marginal effects may cause bias in estimating climate-contingent weather slopes for the purpose of recovering the long-run outcome response function. See Section 2.2. In Equation (10), such heterogeneity is controlled through the use of county-specific slopes $\beta_i$.

with an almost balanced panel of 1,250 counties representing 69.4% of total US agricultural GDP over the period of investigation. We use historical weather data derived from the PRISM monitoring network (PRISM Climate Group, 2020) and made available by Wolfram Schlenker.[18] The raw data is available at a 4 km resolution and includes daily information on maximum temperature, minimum temperature, and precipitation. We aggregate weather data to the county level using farmland areas as weights. We focus on weather and climate during the growing season defined as April to October, and consider two variables: average temperature and total precipitation. In Appendix C, we also provide results for models that use growing degree days above 8°C in lieu of average temperature.

For each county, we calculate the 30-year rolling average of past weather in each year of our sample and use the average of this time series as the central climate value. For each of our two climatic variables, we divide the spectrum of observed central climates into 100 climatic intervals (or bins), so that roughly 12 counties are present in each bin. This choice implies that bins have varying widths; typically those located at the endpoints of the climatic spectrum are wider due to the smaller number of counties with extreme climates. US agriculture spans a wide swath of climates. Growing-season average temperature ranges from 10°C to 29°C. For growing-season precipitation, most of the mass of the distribution is centered around 700 mm, although our sample includes both dramatically drier (<400 mm) and wetter (>1,000 mm) climates. This cross-sectional variation is useful in constructing a long-run response curve whose domain includes most of the future climates likely to be seen as a result of climate change. Figure 5 depicts the geographical distribution of the central climate intervals for counties included in our sample.
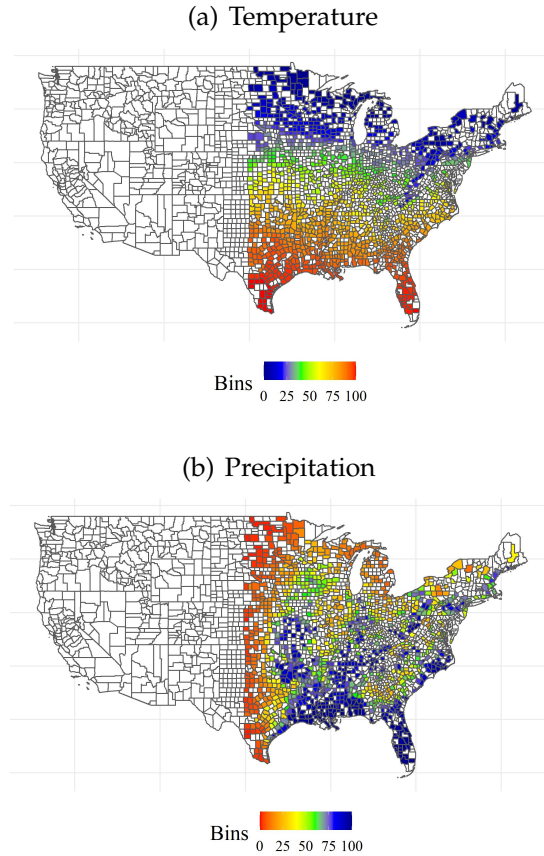
## 3.2   Models

We implement the envelope-gradient-theorem approach on several variants and compare the results to those obtained from a traditional parametric panel model. Our benchmark panel model is quadratic in temperature and precip-

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[18]See http://www.columbia.edu/~ws2162/links.html.

**Figure 5**  Geographical distribution of central climates

(a) Temperature



Bins
0  25  50  75  100

(b) Precipitation



Bins
0  25  50  75  100

itation. As argued in Mérel and Gammans (2021), the quadratic model has been used extensively in the prior climate impact assessment literature. Based on the results of our envelope-gradient-theorem approach, we also present results from a parametric cubic model. All of our estimated models, including the parametric models, include county and year fixed effects as well as state-specific linear time trends to control for variations in the unobserved determinants of agricultural GDP.

Our first envelope-gradient-theorem approach assumes that climates are stationary in each county. County climates are computed using the simple average of climate values over the 19 observation years as $\bar{\mu}_i = \frac{\sum_{t=2001}^{2019} \mu_{it}}{19}$. As a consequence, each county remains in the same climate bin across the entire

period of observation. Marginal weather effects are computed by estimating Equation (7). Climate-specific marginal effects are given by the set of estimated coefficients $\hat{\beta}_\mu$.

One shortcoming of the previous approach is that it assumes away climatic trends during the observation period. To the extent that counties have experienced trends in climate, such variation could be exploited to identify the climate-outcome relationship. In fact, variation arising from climatic trends could be viewed as more legitimate than pure cross-sectional climatic variation because it is presumably less prone to confounding factors (Burke and Emerick, 2016). In a second variant, we thus replace the time-invariant central county climates $\bar{\mu}_i$ by their value at any given point in time, $\mu_{it}$. Therefore, counties are allowed to cross climatic intervals based on variation in their rolling climatic average. The estimating equation becomes:

$$y_{it} = \beta_{\mu(i,t)} x_{it} + f_{\text{state}(i)}(t) + \alpha_i + \alpha_t + \epsilon_{it}. \tag{11}$$

This equation is nearly identical to to Equation (7), with the key difference that $\beta_{\mu(i)}$ is replaced with $\beta_{\mu(i,t)}$.

A possible improvement over the approaches taken in Equations (7) and (11) is to better control for time-invariant factors, most notably soils, that may affect the response of the outcome to weather variables. This possibility is explored in additional models that rely on estimates of the marginal weather effects obtained from Equations (8) and (9).

In the most demanding specification (Equation (9)), the marginal effect of weather at given climate $\beta_\mu$ is identified from within-county comparisons across climates visited over time and, because marginal weather effects are allowed to vary systematically across soil types, from comparisons across counties with similar soils. As in Equation (11), this strategy partially relies on temporal variation in climate to identify the marginal effect of climate, and more so since part of the cross-sectional variation in marginal effects has been removed from identification.

Importantly, climatic trends tend to be quite small relative to the large cross-sectional climatic variation in our sample. For example, the standard

deviation of the stationary climates $\bar{\mu}_i$ is 3.4°C for temperature and 119 mm for precipitation, while the average of the within-county standard deviations of $\mu_{it}$ is 0.09°C for temperature and 11.5 mm for precipitation. Despite their small size relative to the cross-sectional variation, these climate trends are statistically significant in many locations, as also evidenced in prior work by Burke and Emerick (2016) and Cui (2020). Nonetheless, when implementing this approach we verify that the variation induced by counties crossing climatic intervals is driven at least in part by climate trends, rather than movements in the rolling average driven by random weather shocks. To that effect, we compare the number of interval crossings by a county to the number of intervals "visited." For temperature (resp. precipitation) the median county experiences 4 (resp. 14) climate intervals crossings and visits 3 (resp. 9) climate intervals. On average, 60% (resp. 60%) of crossings are to a previously unvisited climate, suggesting that a meaningful share of these crossings are driven by climate trends, rather than weather fluctuations. Also note that climate trends may be non-monotonic, with periods of cooling followed by periods of warming for example, which implies that re-visits of a climatic interval by a county may be due to "genuine" climatic variation.

For statistical inference, we rely on a Conley variance-covariance matrix that allows for spatial correlation across adjacent counties, as in Schlenker and Roberts (2009) or Schlenker (2017).[19] For comparison purposes we also show state-clustered standard errors. We use our estimate of the variance-covariance matrix of the $\hat{\beta}_\mu$ coefficients to construct confidence intervals around the long-run response functions of agricultural GDP to each climatic variable. These confidence intervals demonstrate the uncertainty pertaining to a counterfactual change from the median of the county-specific climates $\bar{\mu}_i$ to any climatic value, lower or higher. They thus answer the question "how confident are we that a change away from the median climate would lead to a change in agricultural GDP?" Since predicted changes in the outcome are a linear combination of the estimated $\hat{\beta}_\mu$ parameters, the confidence intervals can be easily constructed

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[19]We allow for spatial correlation across neighboring counties using a neighboring lag structure, rather than geographical distance. Correlation is allowed up to the third-degree neighbor.

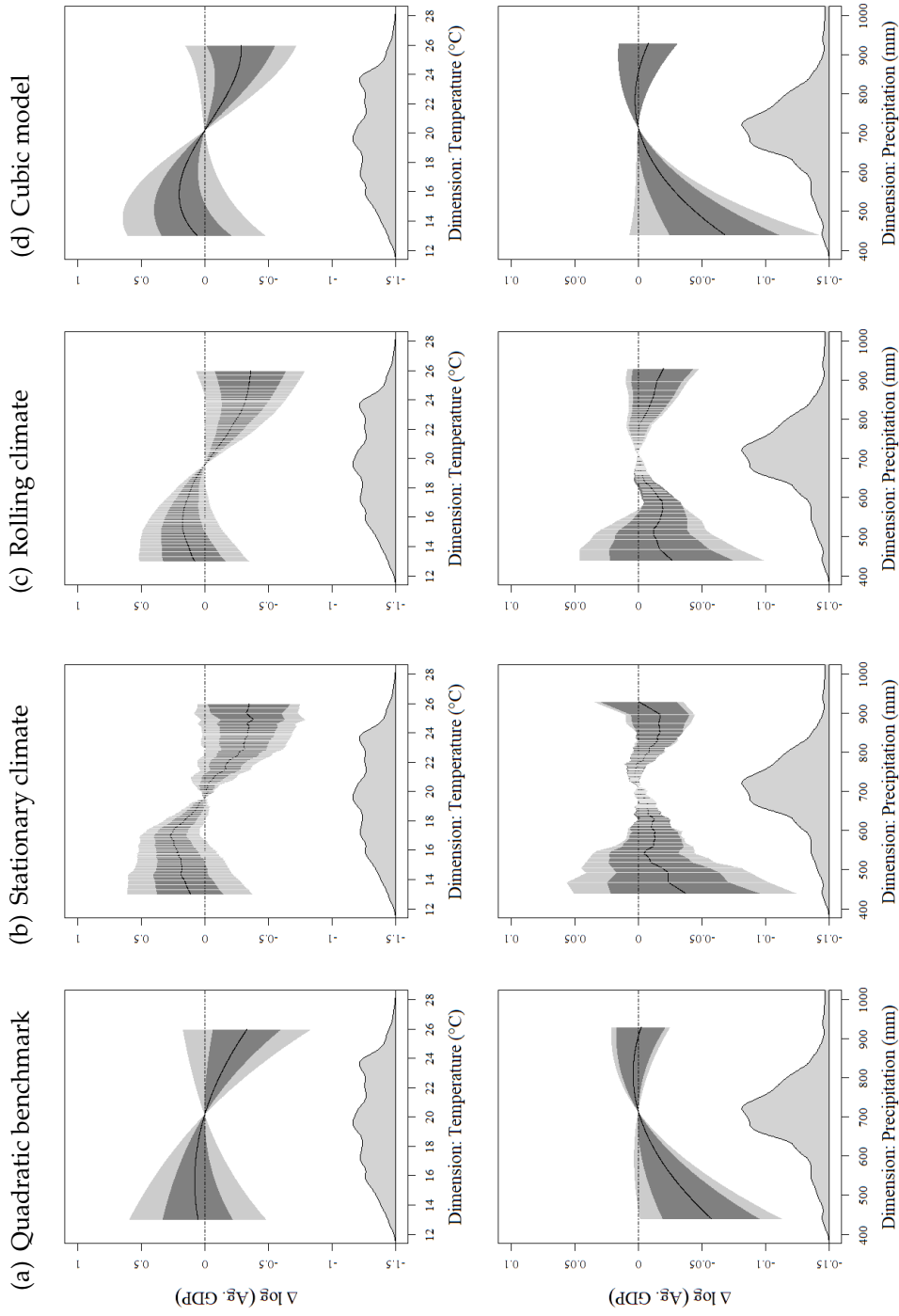using the variance-covariance matrix of coefficient estimates.

## 3.3 Long-run responses to climate

Figure 6 depicts the long-run response curves for temperature and precipitation. We present results for the benchmark quadratic panel model, for two envelope-gradient-theorem models, and for a cubic panel model. In columns (b) and (c), gray vertical lines delineate the 100 bins used in the estimation. The distribution of climates is shown at the bottom of each panel and does not depend on the model. Confidence intervals based on Conley standard errors and more conservative state-clustered standard errors are shown with dark and light shading, respectively.

Column (a) of Figure 6 shows long-run responses for the benchmark quadratic panel model. This model predicts significantly negative effects of increasing climate temperature above the median value when using Conley standard errors. The estimated precipitation response is an inverted U-shape, with the response peaking around 800 mm of cumulative precipitation. Thus, for most climates, agricultural GDP is predicted to increase with additional precipitation according to this model. Note, however, that this precipitation response is one order of magnitude smaller than the response to temperature. This finding is congruent with previous evidence on US agricultural yields, for instance Schlenker and Roberts (2009).

The patterns depicted in columns (b) and (c) of Figure 6 are reassuring about the ability of the envelope-gradient-theorem approach to deliver meaningful and useful response functions. First, despite the demands put on our data (marginal effects are only identified from the few counties in each climatic interval), the effects we derive are relatively precisely estimated, with confidence intervals comparable in size to those obtained in the benchmark quadratic model. Second, these effects largely conform with expectations grounded in previous parametric work on agricultural yields. Notably, the marginal effect of warming is significantly negative across a large swath of the temperature distribution. Interestingly, we find a more negative marginal impact in the middle of the distribution of temperatures than at the upper end. One expla-

**Figure 6** Long-run responses to climate



(a) Quadratic benchmark

(b) Stationary climate

(c) Rolling climate

(d) Cubic model

Legend: 🟦 95% CI with state-clustered SEs   ⬛ 95% CI with Conley SEs

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**Figure 7** Long-run responses to climate, controlling for weather-soil interactions



(a) Quadratic benchmark     (b) Stationary climate     (c) Rolling climate     (d) Cubic model

Legend:    ◼ 95% CI with state-clustered SEs     ◼ 95% CI with Conley SEs

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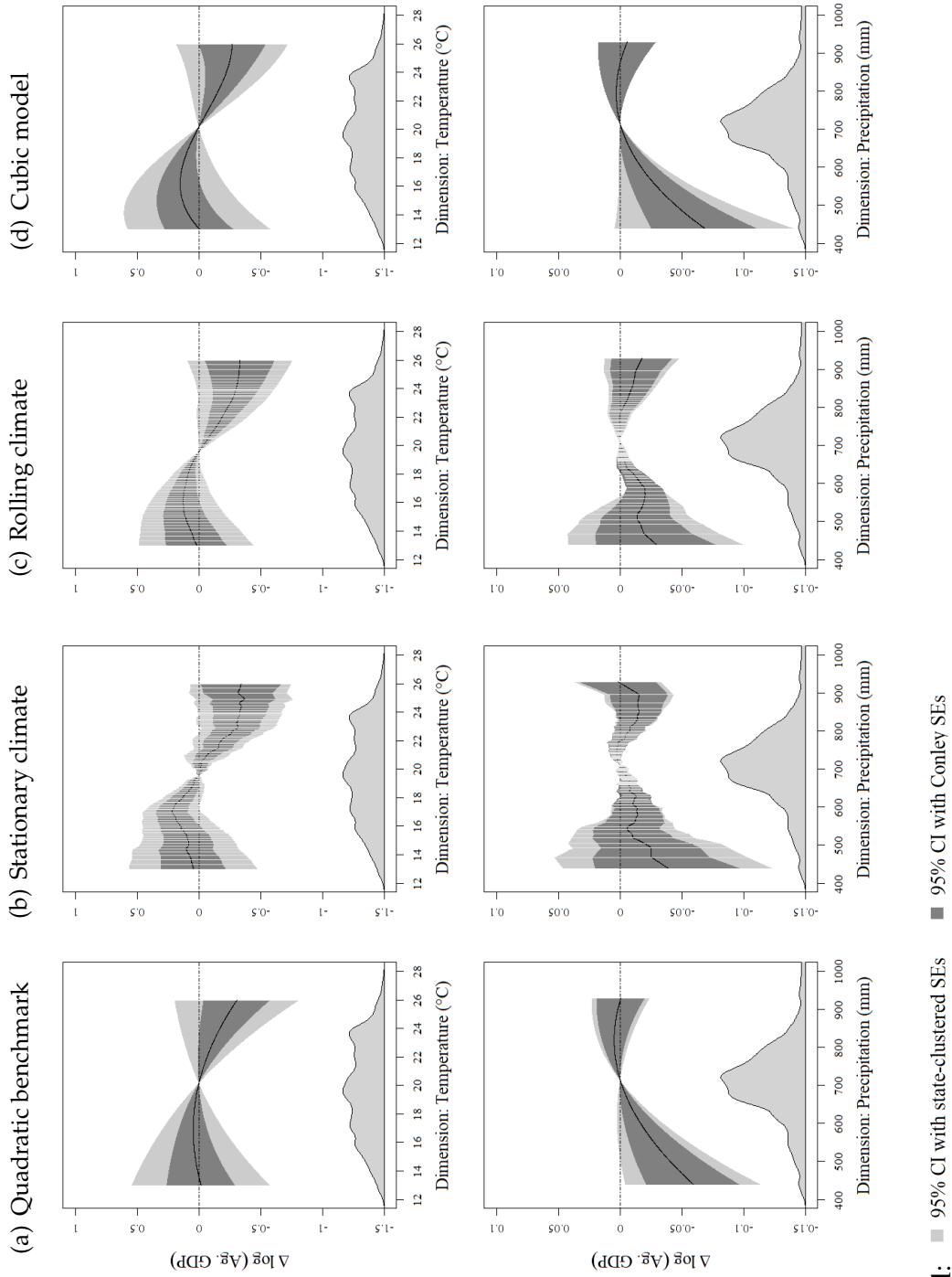
nation could be that once crop yields have suffered from heat, the marginal effect of additional heat diminishes. Intuitively, in the limit where crop yields reach zero and agricultural production ceases, there must be a tapering of the marginal effect. Since nothing in our two-step procedure forces this change in curvature to arise, these results suggest that empirical analyses of temperature and agricultural outcomes ought to allow for varying convexity in the temperature response. Based on this insight, we depict the long-run responses to climatic variables in a parametric cubic model in Column (d) of Figure 6. The climate response obtained with the cubic model is very close to that of the flexible envelope-gradient-theorem approach. In the case of temperature, we again find that warming significantly reduces GDP and that the negative marginal effect of warming is largest in the middle of the temperature distribution. For precipitation, the results generated by the cubic model are quite similar to those of the quadratic benchmark. In both models, increases in precipitation drive higher agricultural GDP in dry climates. This finding is not supported by the results of the envelope-gradient-theorem models, which imply a less steep and not statistically significant effect of increased precipitation in dry climates. Taking a broader view, given that the appropriate parametric form cannot be known with certainty, adopting the envelope-gradient-theorem approach seems preferable to estimating parametric high-order polynomials. This is particularly true as the approach seems to deliver results that are similarly precise.

Overall, results from the first two variants of the envelope-gradient-theorem approach appear comparable, even if the rolling-climate approach produces smoother curves. This finding could in fact be expected, as unlike column (b), column (c) exploits the movement of counties across neighboring climate intervals over time for identification. As a result, time-invariant, county-level idiosyncratic factors that may affect marginal responses to weather tend to be captured in adjacent intervals, as opposed to a single interval, resulting in a smoothing of effects.

Figure 7 provides the long-run curves obtained when adding a soil-weather interaction term, for each of the four specifications shown in Figure 6. For soils information, we rely on the dataset constructed by Ramcharan et al. (2018)

which provides detailed information on soil characteristics at a spatial resolution of 100 m for the conterminous United States. To capture soil quality, we follow Ortiz-Bobea (2020) and use the proportion of soils within a county classified in the top three of an eight-class index reflecting capability to produce commonly cultivated crops and pasture plants. Overall, these models produce long-run response curves and warming impacts that are comparable to the ones reported for the model without weather interactions. A model that uses alternative soil controls based on soil type, rather than soil quality, is presented in Appendix D.

In Section 1, we outlined the conditions required for weather fluctuations to identify marginal climate effects. Without observing the true data-generating process, it is not possible to know the extent to which these conditions hold in applications. However, as outlined in that section, there may exist some transformation of weather variables such that the conditions do hold. Motivated by this insight, we also estimate a growing degree-day (GDD) specification for temperature effects. We follow Schlenker et al. (2006), an analysis of climate and farmland values, and select 8°C as the lower threshold above which we measure heat accumulation. In order to allow the GDD variable to capture the negative effects of severe warming, and to avoid picking an upper threshold among the different values that have been deemed relevant for various crops, we do not include an upper threshold. Our GDD results, which are shown in Appendix C, closely match those of the average temperature specifications.

## 3.4   Climate counterfactuals

We use our calculated long-run response functions to derive the county-level impacts of a 2°C uniform warming on agricultural GDP. These impacts compare predicted values under the 2001–2019 climate to a counterfactual where the climate is 2°C warmer for all counties. At the county level, the impact on the logarithm of agricultural GDP is given by the linear combination $\sum_{b \in B} \hat{\beta}_b \Delta \text{Temp}_{b,i}$ where $B$ is the set of temperature bins, $\hat{\beta}_b$ is the estimated slope of bin $b$, and $\Delta \text{Temp}_{b,i}$ is the length of the temperature interval that county $i$ crosses in bin $b$

under a 2°C warming scenario.[20] The county-level impact on agricultural GDP is thus $\widehat{\Delta}_i = \exp\left(\sum_{b \in B} \hat{\beta}_b \Delta \text{Temp}_{b,i}\right) - 1$, expressed in percentage terms. We then aggregate these impacts at the country level by taking the weighted average $\sum_{i \in C} w_i \widehat{\Delta}_i$, where $C$ denotes the set of counties included in the simulation and $w_i$ is the share of the agricultural GDP of county $i$ in the total agricultural GDP of the included counties.[21]

In this simulation, we do not include the warmest counties for which counterfactual temperatures are projected to lie too far away in the warmest bin.[22] The rationale is that each estimated slope is only valid locally. We cannot evaluate the damage for the warmest climates without assuming a constant slope after a certain threshold, an assumption we do not wish to make. However, the convexity of the long-run response functions estimated using our approach suggests that the damage would be lower than average for these warmest counties.

Figure 8 shows county-level warming impacts for the benchmark panel, the envelope-gradient-theorem models, and the cubic model. The top panels show the results corresponding to the long-run curves depicted in Figure 6; the bottom panels account for soil quality-weather interactions and relate to the long-run curves depicted in Figure 7. Impacts for the models with an alternative characterization of soil-weather interactions are shown in Appendix D. The x-axis represents the county's reference period climate temperature value and the y-axis represents the change in the logarithm of agricultural GDP associated with a 2°C warming. For each county-level impact, we show a 95% confidence interval based on the Conley standard errors. In each panel, we also report cumulative US impacts.

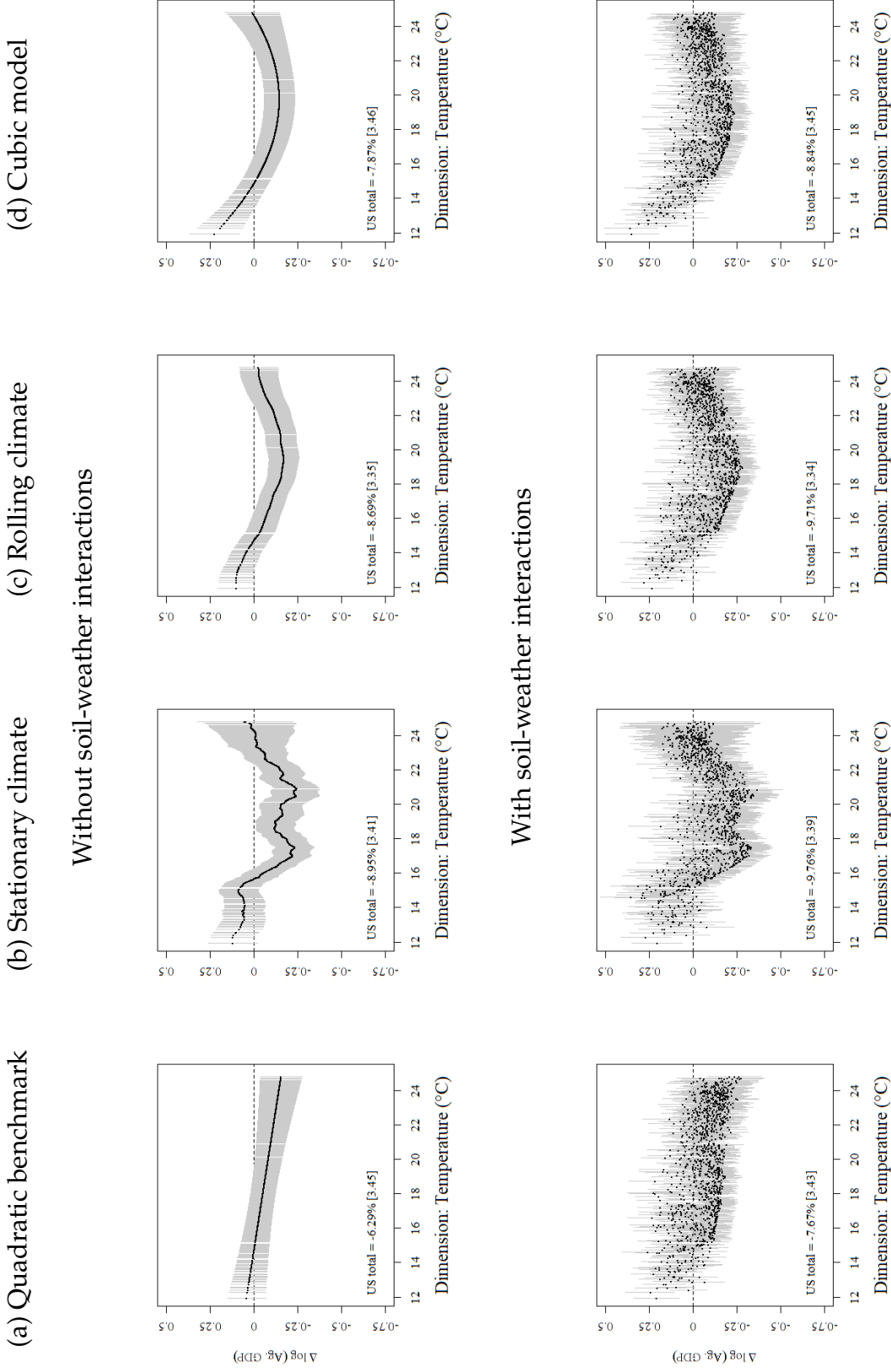Panel (a) of Figure 8 shows impacts for the quadratic panel model. This

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[20]If county $i$ does not cross bin $b$, the length is zero; if it fully crosses it the length is that of the bin itself. The only bins partially crossed are those located at the endpoints of the county's climatic trajectory. See Figure 3.

[21]To obtain $w_i$, we first compute the county-specific average of the agricultural GDP over the 19 years in our sample, say agGDP$_i$, and then construct the ratio $w_i = \frac{\text{agGDP}_i}{\sum_{j \in C} \text{agGDP}_j}$.

[22]These counties have climates that are warmer than the lower bound of the last bin (26.25°C) plus the maximum width of the middle bins (0.55°C) minus 2°C. There are 74 such counties out of the 1,250 used in the estimation (5.9%). They produced 6.3% of the US total agricultural GDP on average over 2001–2019.

**Figure 8** Simulated impact of a +2°C scenario on county agricultural GDP



(a) Quadratic benchmark     (b) Stationary climate     (c) Rolling climate     (d) Cubic model

Without soil-weather interactions

With soil-weather interactions

Note: Each dot corresponds to a county. The gray segments represent 95% confidence intervals using Conley standard errors. In each panel, the estimate of the total effect on the US national average agricultural GDP is given together with its Conley standard error in brackets.

model predicts that currently warm counties are harmed by increases in temperature, and significantly so. Predicted impacts range from near zero for counties near the median climate to losses of over 10% in the warmest counties. Overall, we find a statistically and economically significant US-wide agricultural GDP loss of 6.29%.

Panels (b) and (c) of Figure 8 show impacts for the envelope-gradient-theorem models. Panel (b) refers to the variant that uses a stationary definition of climate. This model implies much more detrimental effects than the benchmark panel model in moderate climates, but less severe declines in warmer climates. This discrepancy is due to the convex shape of the estimated long-run response to temperature that is not captured by the quadratic model. Aggregating across counties, we find that a 2°C warming is associated with a 8.95% agricultural GDP loss, a decline that is both economically and statistically significant, and slightly greater in magnitude than the aggregate loss implied by the benchmark panel model. The results from the variant that uses a rolling climate definition are shown in panel (c). Overall, county-level and aggregate results are comparable to those from the stationary climate variant. In line with our estimated long-run response functions, the distribution of impacts across climates appears smoother. In this variant, we find a statistically significant aggregate agricultural GDP loss of 8.69%.

Panel (d) shows results for the parametric cubic model. The pattern is similar to the one obtained using the flexible approach in panel (c). The aggregate damage is statistically significant, albeit slightly smaller (7.87%). In the bottom panels, the damages obtained using specifications that include soil quality-weather interactions vary across counties with similar climate due to soils effects. However, the overall patterns are similar, and the aggregate damages are all statistically significant, and slightly larger in magnitude. Appendix D reports comparable results for models that control differently for soil effects.

# 4   Conclusion

A full accounting of the consequences of future climate change on outcomes of economic interest remains an important, yet elusive, research goal. There

has been a longstanding debate on the ability of methodologies that rely on random weather fluctuations to provide information on the long-run effects of climate change, accounting for adaptation by economic agents (Kolstad and Moore, 2020; Lemoine, 2021; Mérel and Gammans, 2021). In this paper, we provide a set of necessary and sufficient conditions for the short-run response to weather and the long-run response to climate to be tangent to each other at the expected weather irrespective of the weather distribution. Delineating these conditions is important because the tangency property legitimizes the use of weather fluctuations as a way to identify long-run marginal effects. We introduce an empirical methodology to estimate marginal effects that are specific to a given climatic interval and demonstrate how, under the tangency hypothesis, these marginal effects can be integrated to construct a common long-run response to climatic variables, as initially proposed by Hsiang (2016). Because the long-run response curve results from the simple arrangement of slopes across the climate gradient, it is by design more flexible than curves obtained from parametric approaches.

We apply this envelope-gradient-theorem approach to a panel of weather and agricultural GDP data from the US. We explore models that differ along various dimensions: the choice of stationary vs. rolling climate, the inclusion of soil-weather controls, or the use of GDD instead of average temperature. We find qualitatively similar long-run responses to climate across model specifications. Our results indicate that, despite the considerable demands put on our data, the approach can deliver long-run response functions that are coherent with agronomic expectations and prior parametric work, with some important nuances. For instance, we provide empirical support for changes in convexity in the effects of temperature on agricultural GDP that would not be captured in parametric quadratic panels.

We then consider the effect of a 2°C uniform warming on US agricultural GDP. Using the long-run climate responses derived from the envelope-gradient-theorem approach, we calculate county-level impacts. We find these impacts to be comparable across model specifications, at least in the aggregate. Importantly, our approach delivers county-level impact estimates that are markedly different from those of a benchmark quadratic panel. Our specifica-

tion that allows for a rolling climate definition predicts an aggregate damage to agricultural GDP after adaptation of 8.7% under the +2°C scenario. This effect is slightly larger in magnitude than the aggregate damage estimate from the benchmark quadratic panel (6.3%). Using a cubic rather than quadratic specification delivers a damage estimate that is closer to the non-parametric estimate (7.9%). Given the inevitable uncertainty surrounding the correct parametric choice in a particular setting, our envelope-gradient-theorem approach provides empiricists with a valuable new tool to quantify climate impacts.

# Appendices

## A Concavity of $y(x, a)$ with respect to $a$

We assume that for all $x$, $y(x, a) \equiv y_x(a)$ has a unique maximizer characterized by $y'_x(a) = 0$ (that is, there is only one critical point at which the derivative function is zero). Now consider the upper concave envelope of $y_x(\cdot)$, defined as

$$\tilde{y}_x \equiv \inf \left\{ u \, | \, u \text{ is convex and } u(a) \geq y_x(a) \; \forall a \in U \right\}.$$

A representation of such an envelope is depicted in Figure A.1.

**Figure A.1**   The upper concave envelope
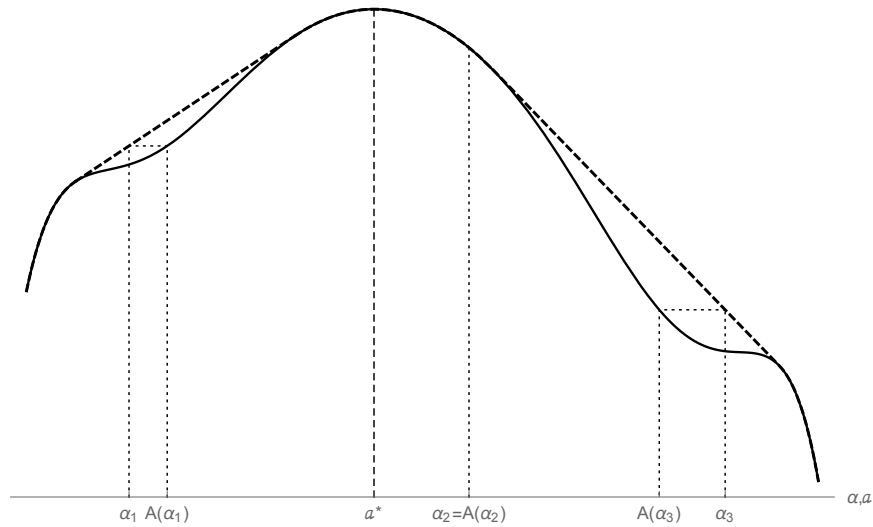


Since $y_x$ has only one critical point, say $a^*$, at which it is maximized, we must have that $y'_x(a) > 0 \; \forall a < a^*$ and $y'_x(a) < 0 \; \forall a > a^*$. Further denote by $y_-$ the restriction of $y_x$ to the range $a \leq a^*$ and $y_+$ its restriction to the range $a > a^*$. (Note that $x$ plays no particular role here.) Both $y_-$ and $y_+$ are monotone and therefore bijective. Now consider the following change of variable:

$$a = A(\alpha) = \begin{cases} y_-^{-1} \left( \tilde{y}_x(\alpha) \right) & \text{if } \alpha < a^* \\ y_+^{-1} \left( \tilde{y}_x(\alpha) \right) & \text{if } \alpha > a^* \end{cases}.$$

The change of variable is illustrated in Figure A.1 for various values of $\alpha$. It is clear that it is well defined because it is both injective (for each $\alpha$, there is a unique $A(\alpha)$) and surjective (for each $a$, there is a unique $\alpha$ such that $a = A(\alpha)$). The composite function $\alpha \mapsto y_x(A(\alpha))$ is by construction equal to the function $\tilde{y}_x(\alpha)$, therefore it is concave.

## B    Relationship with Theil (1954)'s model

Slightly adapting his notation to make it more congruent with ours, the structural outcome function considered in Theil (1954) has the following form:

$$W(\mathbf{a}, \mathbf{x}) = A(\mathbf{a}) + \sum_{h=1}^{n} A_h(\mathbf{a})y_h(\mathbf{a}, x_h) + \sum_{h=1}^{n}\sum_{k=1}^{n} A_{hk}y_h(\mathbf{a}, x_h)y_k(\mathbf{a}, x_k) \qquad \text{(B-1)}$$

where $\mathbf{x}$ and $\mathbf{y} \equiv (y_1, \ldots, y_n)$ are random vectors of the same dimension $n$ and

$$y_h(\mathbf{a}, x_h) = f_h(\mathbf{a}) + x_h, \quad h = 1, \ldots, n \qquad \text{(B-2)}$$

for some unspecified functions $f_h$. The functions $A$ and $A_h$ are also left unspecified, while the $A_{hk}$ are taken as parametric, that is, they do not depend on $\mathbf{a}$. The vector $\mathbf{a}$ is interpreted as actions (policies) while the vector $\mathbf{x}$ represents random environmental variables (weather in our case) and the vector $\mathbf{y}$ represents "indirect variables" that enter the welfare function and depend both on actions and environmental variables. In an application to farm profits, the indirect variables could be revenue (which depends on weather through yields and on long-run actions such as crop choice) and costs (which may depend on weather through short-run adaptations such as variation in irrigation intensity and on long-run actions such as crop choice). Within this structure, Theil (1954) shows that a certainty equivalence principle holds.

   We will now show that the restrictions embedded in Equations (B-1) and (B-2) are, in fact, equivalent to those in Equation (3). Plugging the identities in

Equation (B-2) into Equation (B-1), we obtain

$$
\begin{aligned}
W(\mathbf{a}, \mathbf{x}) \;=\;\; & A(\mathbf{a}) + \sum_{h=1}^{n} A_h(\mathbf{a}) f_h(\mathbf{a}) + \sum_{h=1}^{n}\sum_{k=1}^{n} A_{hk} f_h(\mathbf{a}) f_k(\mathbf{a}) + \sum_{h=1}^{n} A_h(\mathbf{a}) x_h \\
& + \sum_{h=1}^{n} x_h \sum_{k=1}^{n}(A_{hk} + A_{kh}) f_k(\mathbf{a}) + \sum_{h=1}^{n}\sum_{k=1}^{n} A_{hk} x_h x_k
\end{aligned}
$$

which can be rewritten as

$$
W(\mathbf{a}, \mathbf{x}) = B(\mathbf{a}) + \sum_{h=1}^{n} B_h(\mathbf{a}) x_h + \sum_{h=1}^{n}\sum_{k=1}^{n} A_{hk} x_h x_k \tag{B-3}
$$

where $B(\mathbf{a}) \equiv A(\mathbf{a}) + \sum_{h=1}^{n} A_h(\mathbf{a}) f_h(\mathbf{a}) + \sum_{h=1}^{n}\sum_{k=1}^{n} A_{hk} f_h(\mathbf{a}) f_k(\mathbf{a})$ and $B_h(\mathbf{a}) \equiv A_h(\mathbf{a}) + \sum_{k=1}^{n}(A_{hk} + A_{kh}) f_k(\mathbf{a})$.

Apart from the fact that the actions $\mathbf{a}$ and the random variables $\mathbf{x}$ are multivariate, Equation (B-3) has the same structure as Equation (3), where actions are restricted to interact only with a linear function of $\mathbf{x}$, but $\mathbf{a}$ and $\mathbf{x}$ are allowed to affect the outcome in an otherwise unrestricted fashion. (It could appear, in addition, that $\mathbf{x}$ is restricted to enter quadratically into the welfare function. This is not an essential restriction, as one could easily add a function $\Gamma(\mathbf{x})$ to Equation (B-1) without changing the analysis, as there would be no interaction between this new function and the actions $\mathbf{a}$.)

The necessity proof in Section 1.3 extends to the case where $\mathbf{x}$ and $\mathbf{a}$ are multivariate in a straightforward fashion. Because the criterion is to ensure that $\tilde{\mathbf{a}}(F) = \hat{\mathbf{a}}\left(\mu(F)\right)$ for *all* (joint) distributions $F$ of the random vector $\mathbf{x}$, one can successively consider degenerate distributions where only one of the random variables in $\mathbf{x}$ is truly random and all others are taken as certain. Each iteration $h$ will lead to the conclusion that the gradient of $W$ with respect to $\mathbf{a}$ must be an affine function of $x_h$.

# C   Models with growing degree-days

We obtain similar long-run curves using uncapped growing degree-days (above 8°C) instead of average temperature. Figure C.1 shows long-run responses without soil controls and Figure C.2 shows responses with soil controls. The shape of all long-run curves is comparable to that of their average-temperature counterpart. Warming impacts using the GDD specifications are shown in Figure C.3. These impacts are similar to those obtained using average temperature.

# D   Models with alternative soil controls

We check that our results are comparable when controlling for soil-weather interactions using an alternative variable indicating the type of soil. To capture soil type, we use data on the clay, sand, and silt makeup of soil at 5 cm depth and construct a categorical variable with 9 categories, making sure that there is a sufficient number of counties falling in each category.[23]

   Overall, the alternative models yield long-run response curves and warming impacts that are comparable to the ones reported for the model with soil quality-weather interactions. Figures D.1 and D.2 contrast the results obtained with the two alternative soil controls using a rolling climate definition to assign observations to climatic intervals.

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[23]The shares of clay, sand, and silt add up to one for each county. We consider the 40% and 60% quantiles of the shares of clay and sand to build 3 categories for each variable, and interact these to obtain 9 soil types. The average number of counties per category is 139. The largest category is the type with the most clay and the least sand (399 counties) and the smallest category is the one with the least clay and the least sand (57).

**Figure C.1** Long-run responses to climate using GDD instead of average temperature



(a) Quadratic benchmark    (b) Stationary climate    (c) Rolling climate    (d) Cubic model
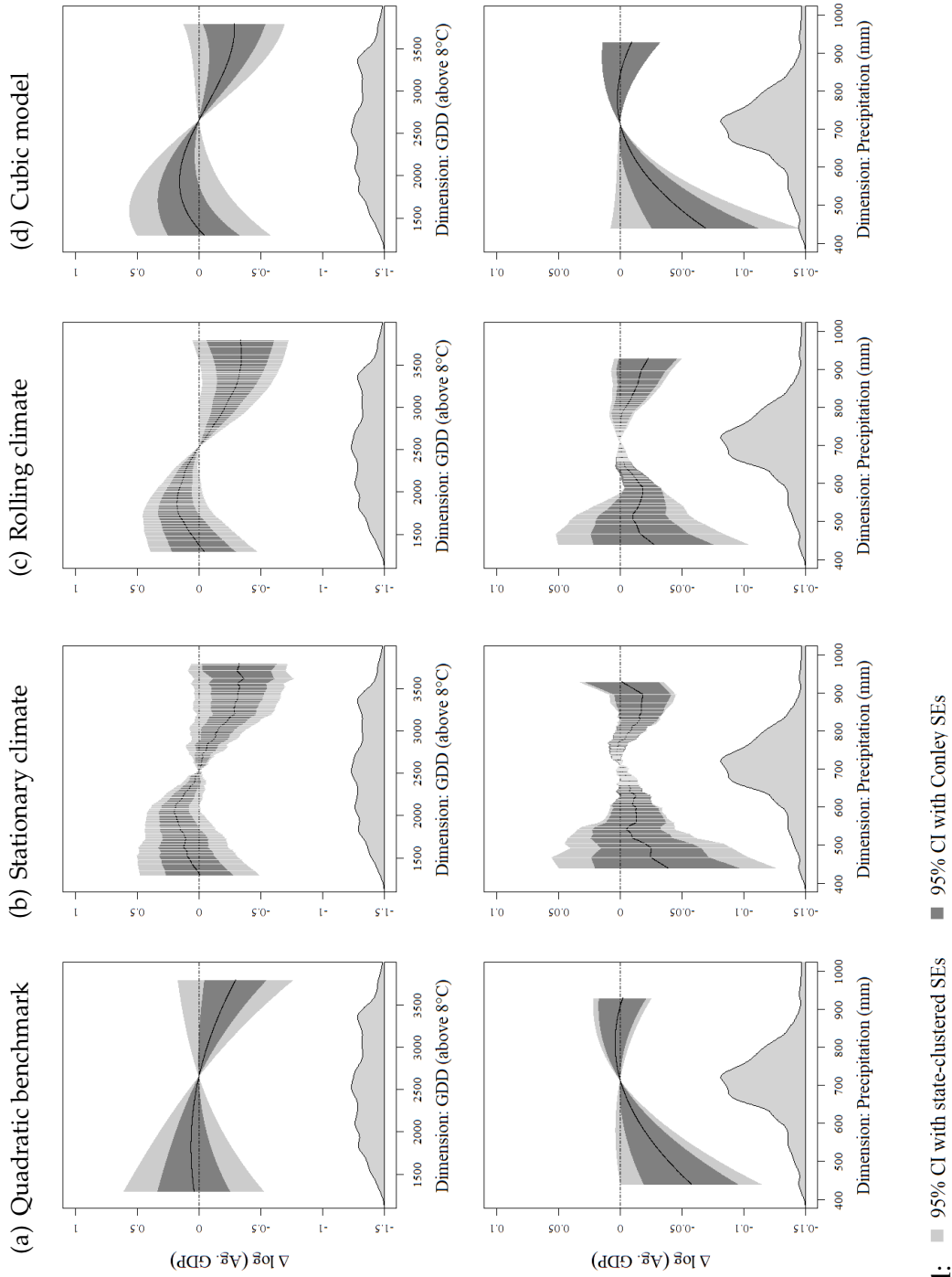
**Figure C.2** Long-run responses to climate in GDD, controlling for weather-soil interactions



(a) Quadratic benchmark     (b) Stationary climate     (c) Rolling climate     (d) Cubic model
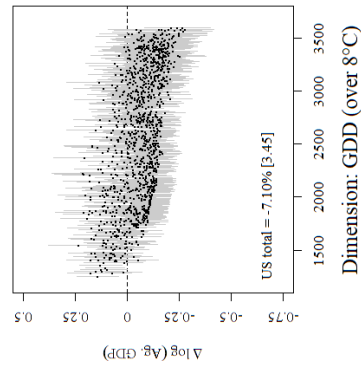
Legend: ■ 95% CI with state-clustered SEs     ■ 95% CI with Conley SEs
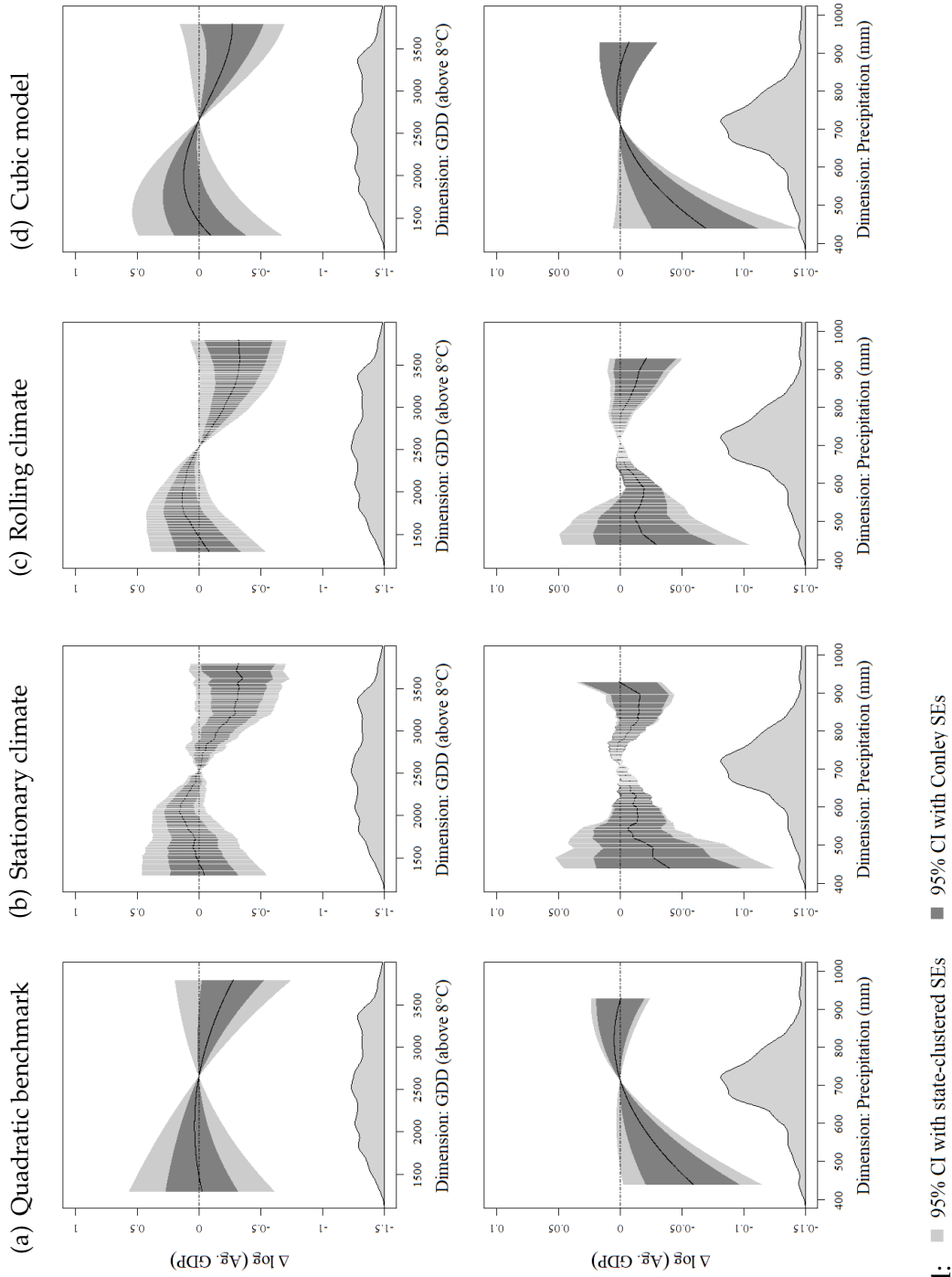
**Figure C.3** Simulated impact of a +2°C scenario on county agricultural GDP

(a) Quadratic benchmark     (b) Stationary climate     (c) Rolling climate     (d) Cubic model

Without soil-weather interactions
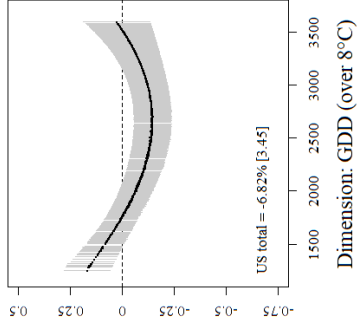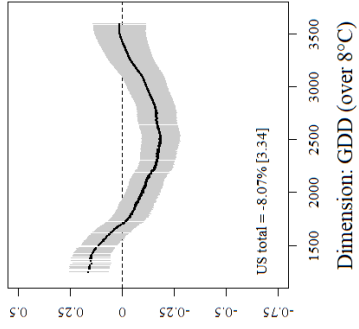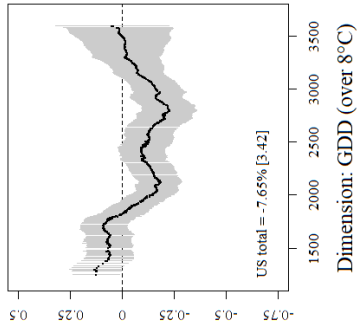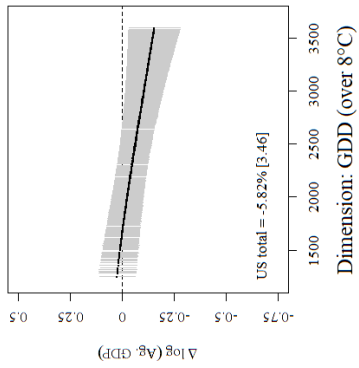
With soil-weather interactions
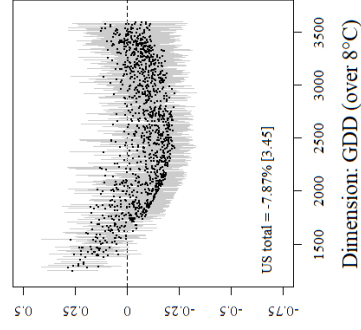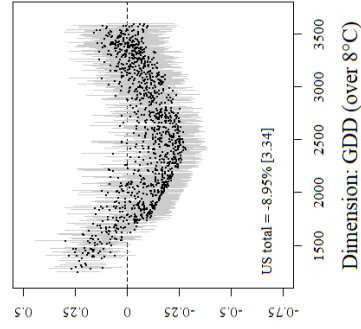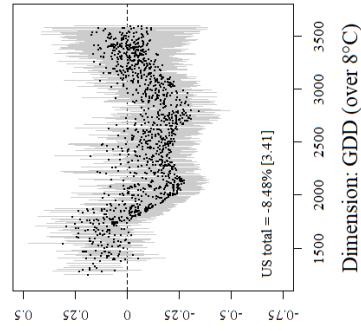


Note: Each dot corresponds to a county. The gray segments represent 95% confidence intervals using Conley standard errors. In each panel, the estimate of the total effect on the US national average agricultural GDP is given together with its Conley standard error in brackets.

**Figure D.1** Long-run response curves

(a) Soil quality-weather interactions    (b) Soil type-weather interactions
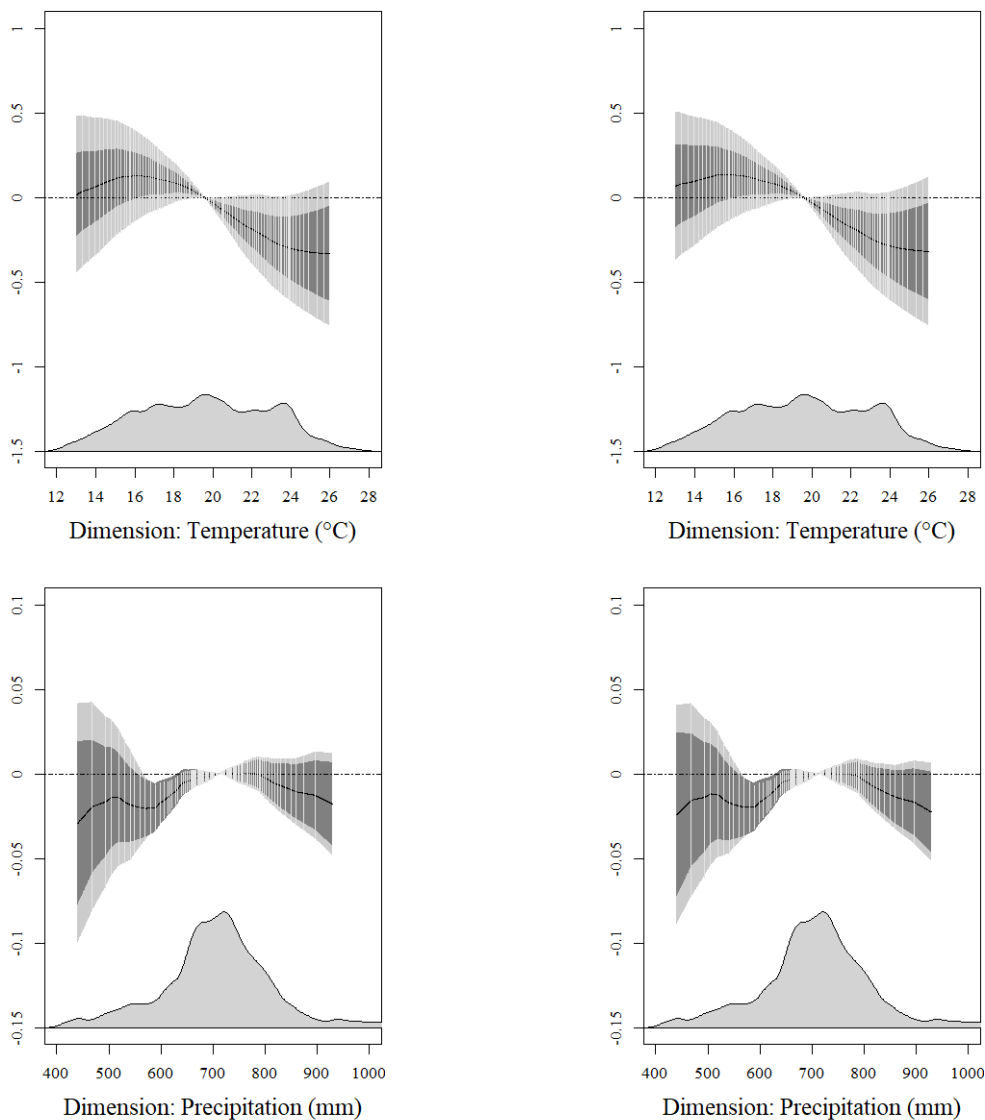


Legend:  ▢ 95% CI with state-clustered SEs    ▪ 95% CI with Conley SEs

Note: For the specification using soil quality, the figure represents the response at mean quality. For the specification using soil types, the figure represents an average response across types, where soil type-specific slopes are weighted by the number of counties falling in each category.

**Figure D.2**   Simulated impact of a +2°C scenario on county agricultural GDP

(a) Soil quality-weather interactions
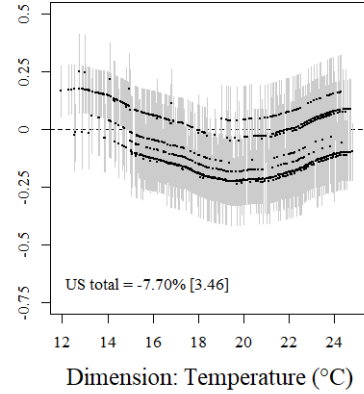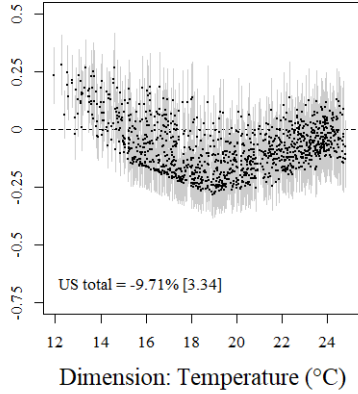
(b) Soil type-weather interactions



Note: Each dot corresponds to a county. The gray segments represent 95% confidence intervals using Conley standard errors. In each panel, the estimate of the total effect on the US national average agricultural GDP is given together with its Conley standard error in brackets.

# E   Models estimated from locally quadratic short-run responses

As an alternative to Equation (7), we derive local weather slopes from locally quadratic approximations to the short-run responses. That is, in the first step we estimate the following regression:

$$y_{it} = \beta^1_{\mu(i)} x_{it} + \beta^2_{\mu(i)} (x_{it})^2 + f_{\text{state}(i)}(t) + \alpha_i + \alpha_t + \epsilon_{it}$$

for the specification with stationary climate; for the specification with rolling climate, assignment of observations to climatic intervals is based on the rolling climate value. The local slope at climate $\mu$ is then computed as $\beta^1_\mu + 2\beta^2_\mu \mu$.

The resulting long-run response curves are shown in Figure 1, and the corresponding warming impacts are shown in Figure 2. They are very close to

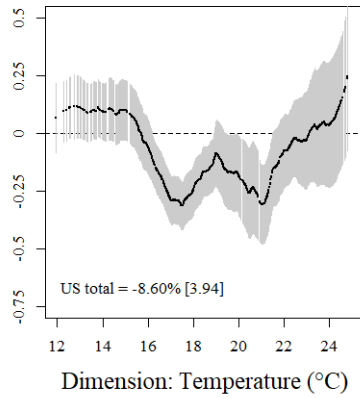those obtained using the locally linear approximation.
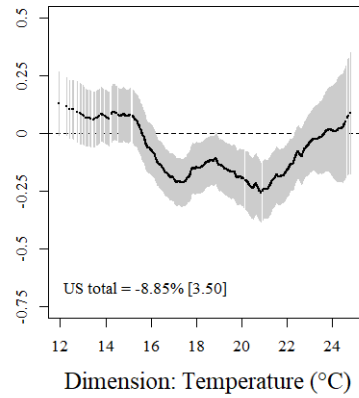
**Figure 1**    Long-run response curves

(a)  Stationary climate          (b)  Rolling climate
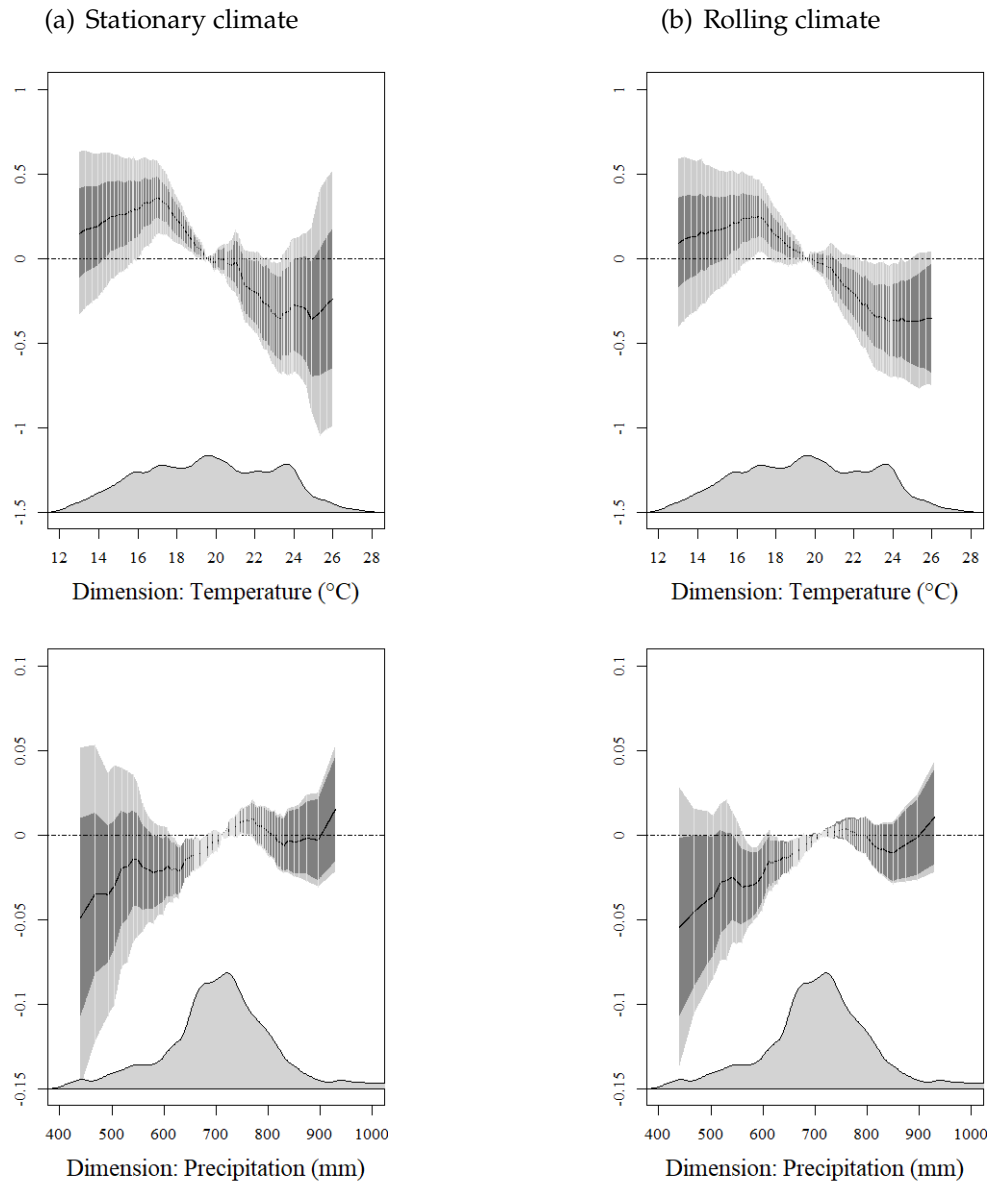


Legend:   ▫ 95% CI with state-clustered SEs    ▪ 95% CI with Conley SEs

**Figure 2**   Simulated impact of a +2°C scenario on county agricultural GDP

(a) Stationary climate          (b) Rolling climate



Note: Each dot corresponds to a county. The gray segments represent 95% confidence intervals using Conley standard errors. In each panel, the estimate of the total effect on the US national average agricultural GDP is given together with its Conley standard error in brackets.

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